Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.3-General-trinomial/124-1.2.3.2-a

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**255**]. This is test number [124].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (255)	0.00 (0)
Mathematica	100.00 (255)	0.00 (0)
Fricas	78.82 (201)	21.18 (54)
Maple	62.75 (160)	37.25 (95)
Giac	61.18 (156)	38.82 (99)
Maxima	52.94 (135)	47.06 (120)
Reduce	51.76 (132)	48.24 (123)
Mupad	38.82 (99)	61.18 (156)
Sympy	19.22 (49)	80.78 (206)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

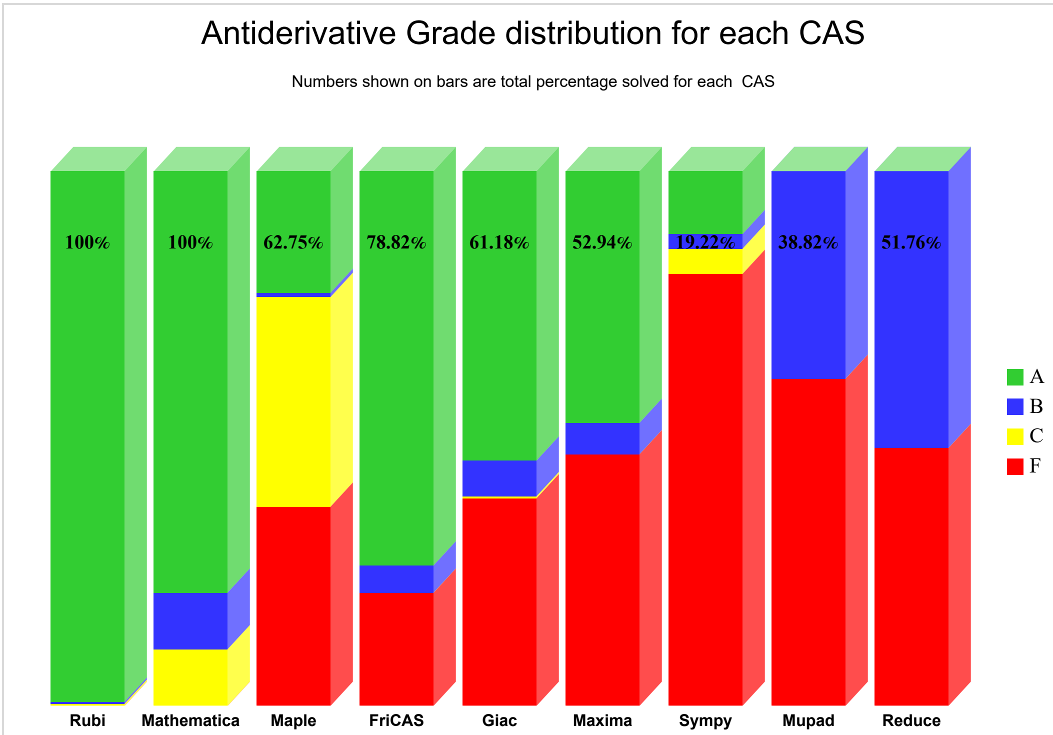
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

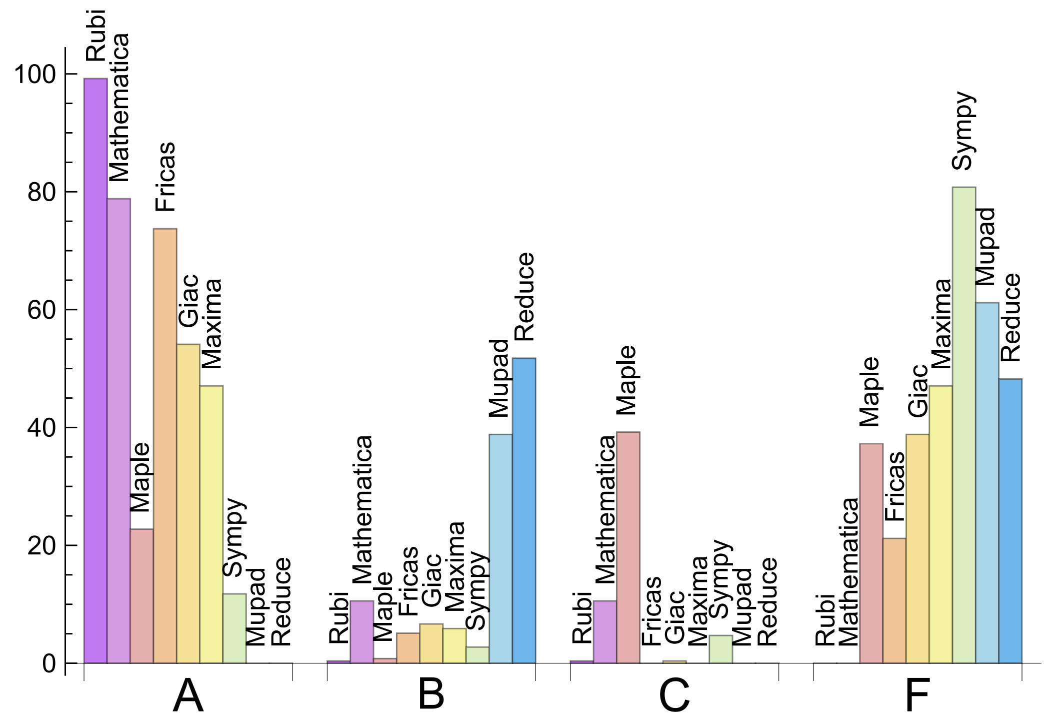
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.216	0.392	0.392	0.000
Mathematica	78.824	10.588	10.588	0.000
Fricas	73.725	5.098	0.000	21.176
Giac	54.118	6.667	0.392	38.824
Maxima	47.059	5.882	0.000	47.059
Maple	22.745	0.784	39.216	37.255
Sympy	11.765	2.745	4.706	80.784
Mupad	0.000	38.824	0.000	61.176
Reduce	0.000	51.765	0.000	48.235

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	54	100.00	0.00	0.00
Maple	95	100.00	0.00	0.00
Giac	99	97.98	0.00	2.02
Maxima	120	60.00	0.00	40.00
Reduce	123	100.00	0.00	0.00
Mupad	156	0.00	100.00	0.00
Sympy	206	84.95	15.05	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Fricas	0.09
Giac	0.13
Reduce	0.17
Rubi	0.31
Maple	0.77
Mathematica	1.44
Sympy	3.28
Mupad	18.29

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	62.97	0.48	58.00	0.38
Maxima	99.98	0.77	88.00	0.62
Reduce	110.56	0.74	59.00	0.55
Mathematica	128.40	1.01	101.00	0.88
Giac	132.10	0.82	86.00	0.61
Rubi	137.76	0.88	119.00	1.00
Sympy	138.29	1.36	122.00	0.95
Fricas	282.44	1.33	103.00	0.80
Mupad	418.55	2.46	109.00	0.93

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

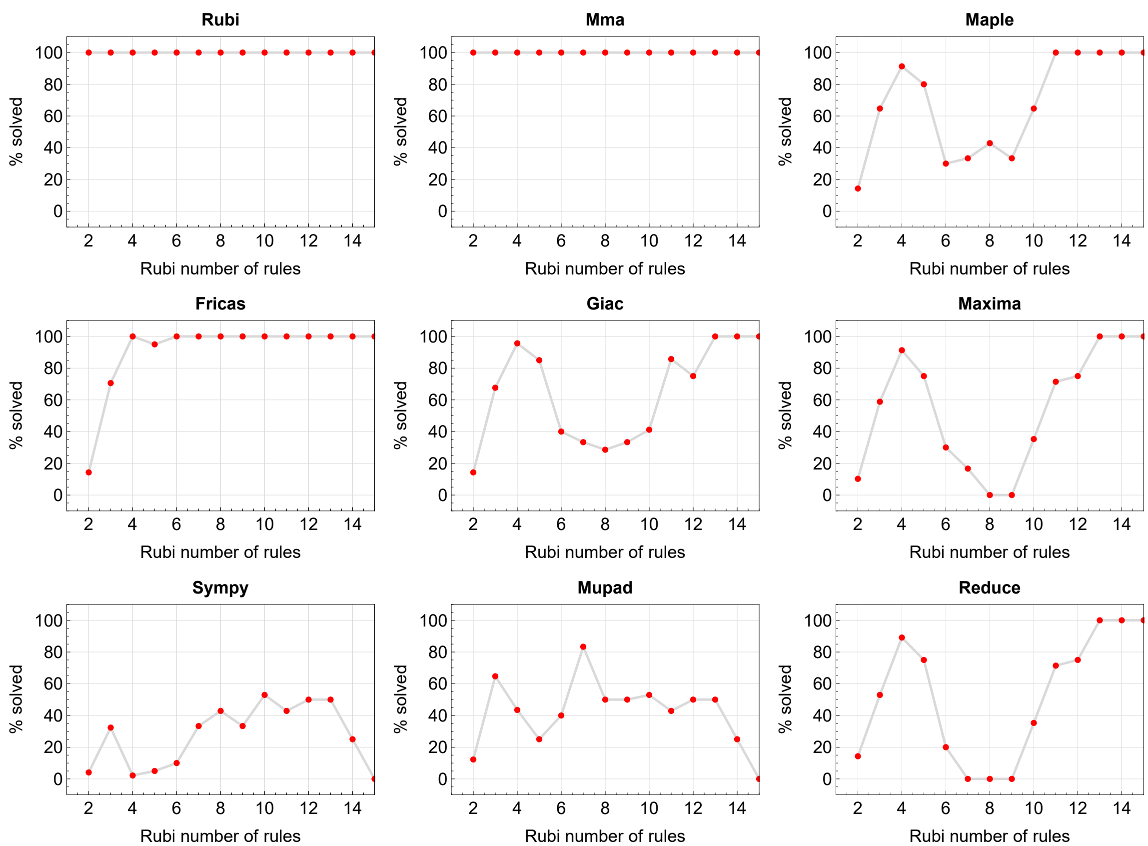


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

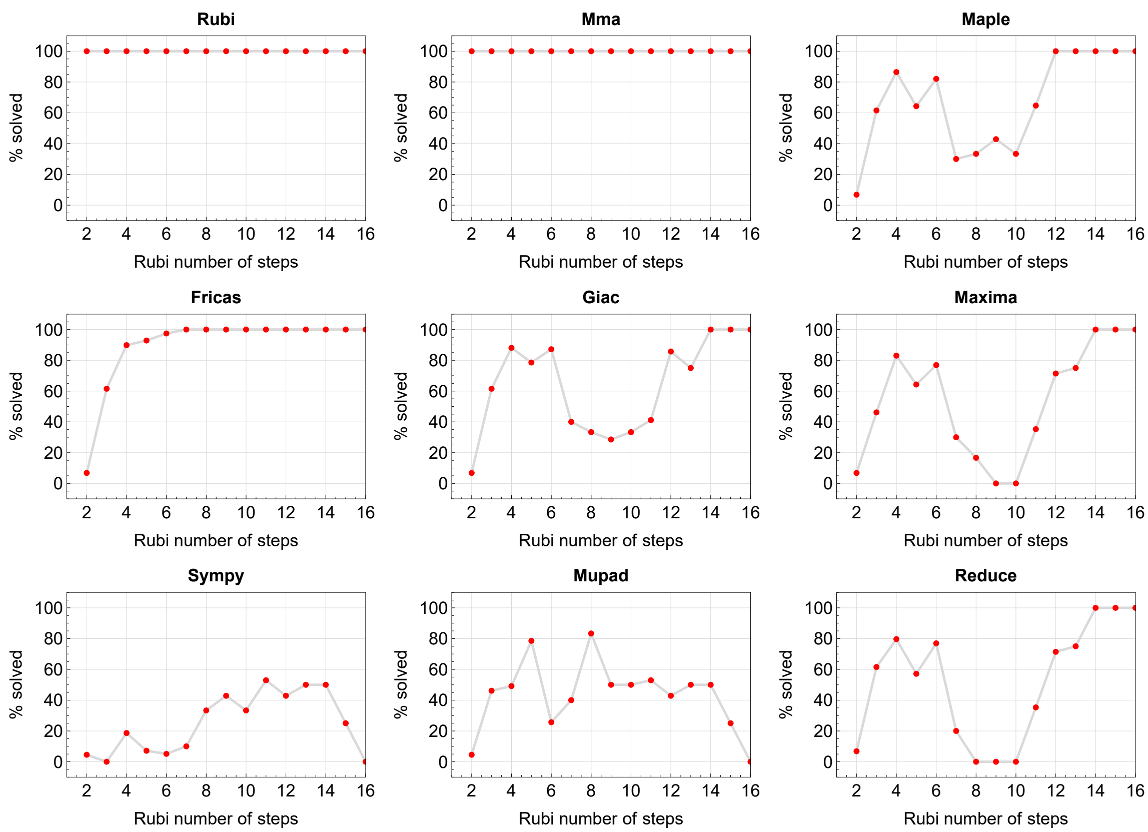


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

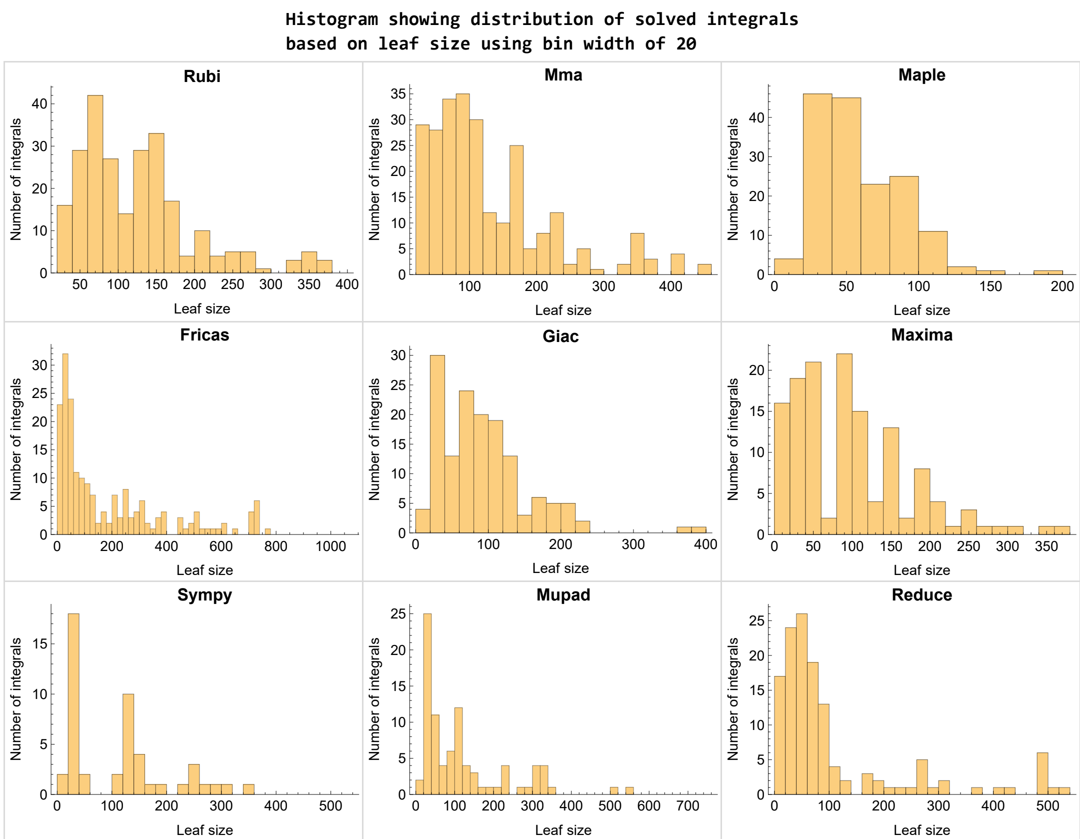


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

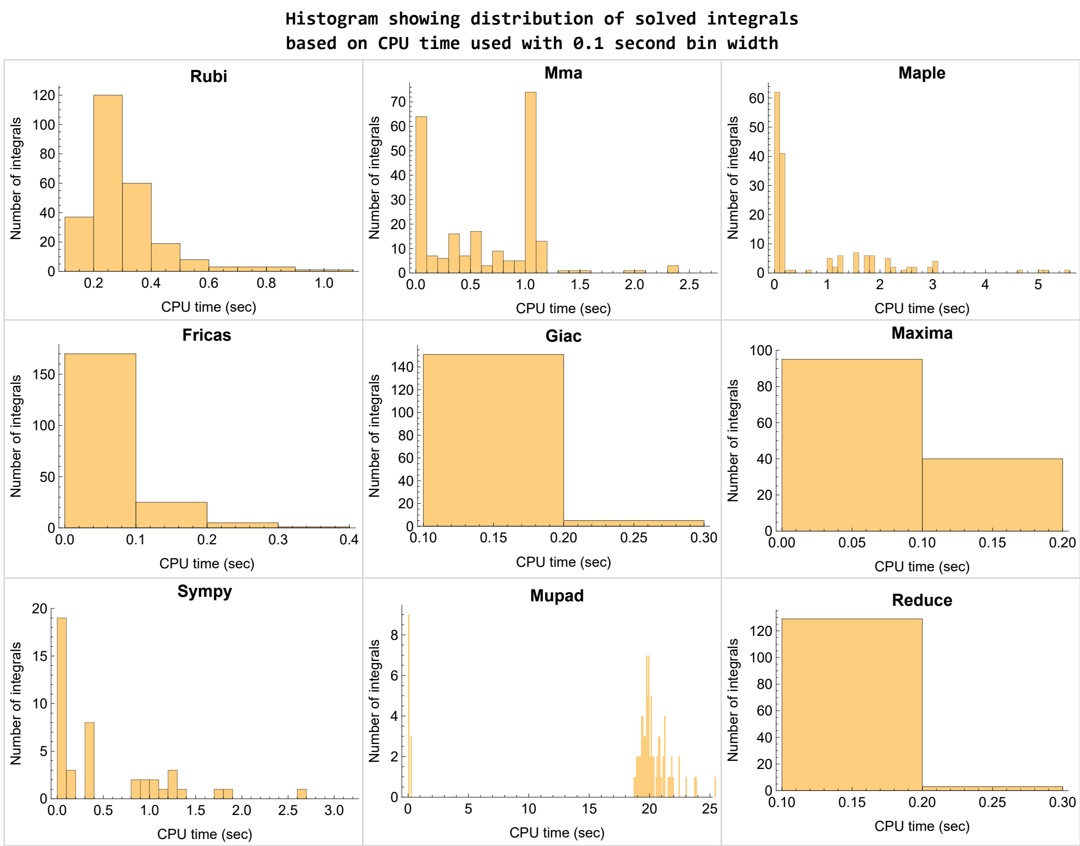


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

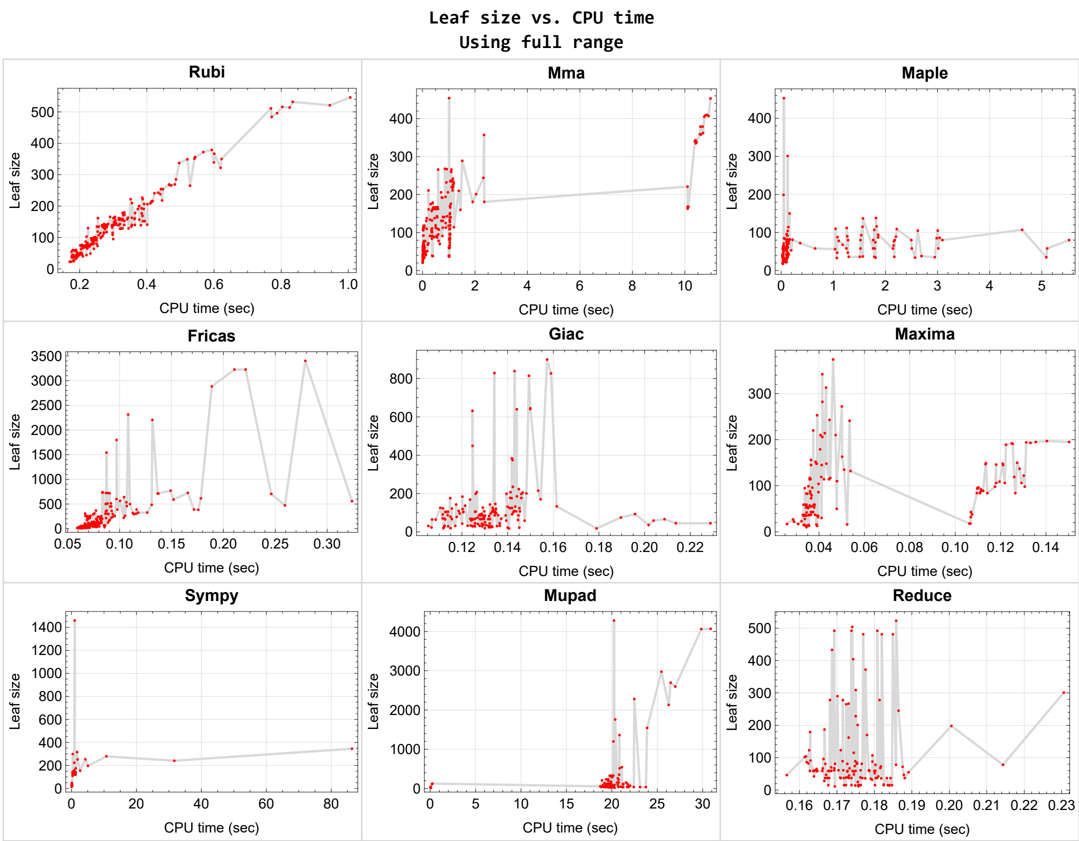


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {117, 181, 182, 183, 184, 185, 199, 200, 201, 202, 203, 217, 218, 228, 229, 230, 231, 232, 235, 237, 238, 239, 240, 241}

Maple {2, 3, 4, 5, 6, 7, 8, 18, 19, 20, 21, 23, 24, 25, 26, 27, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

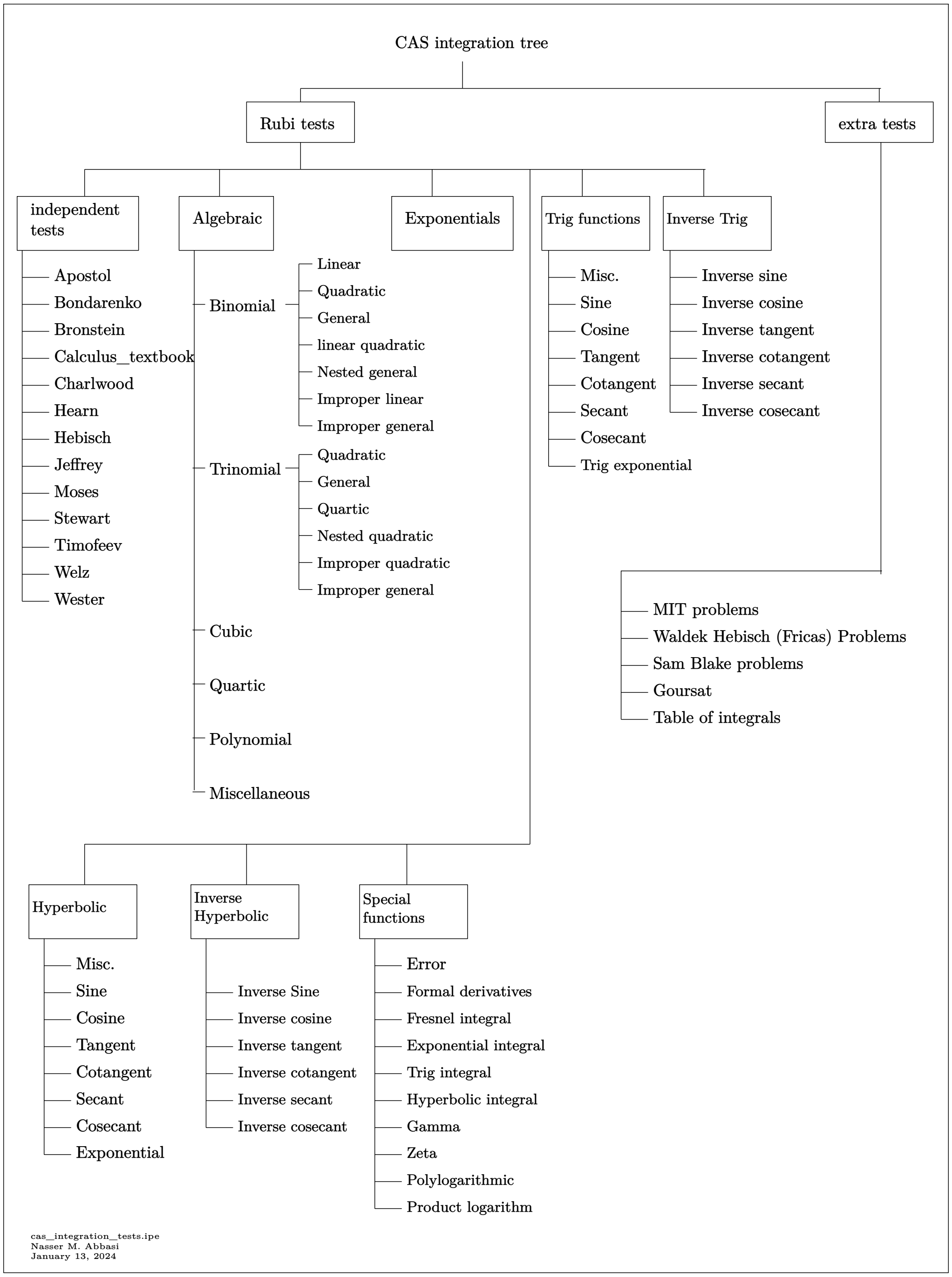
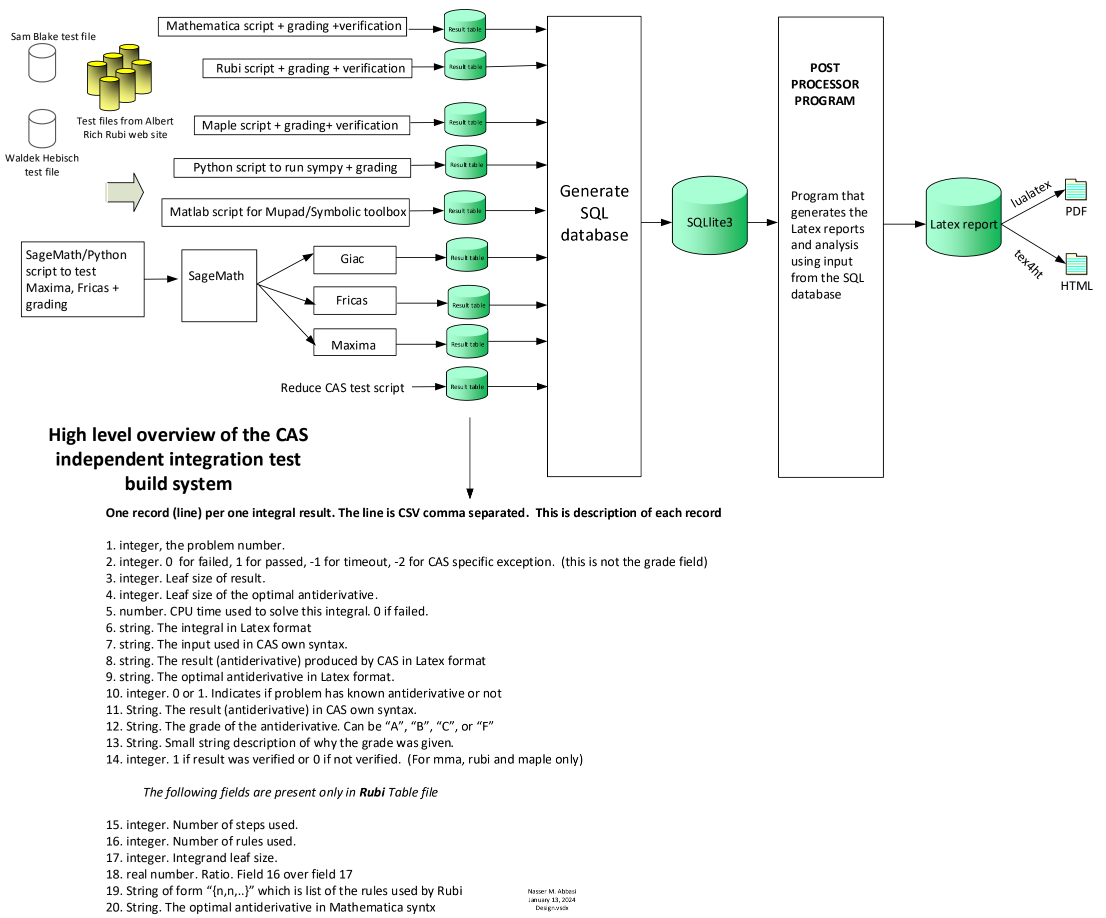


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

B grade { 139 }

C grade { 161 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 159, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 225, 226, 227, 233, 234, 238, 239, 240, 241, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

B grade { 3, 4, 5, 40, 78, 79, 91, 92, 139, 181, 182, 183, 184, 185, 199, 200, 201, 202, 203, 217, 218, 228, 229, 230, 231, 232, 237 }

C grade { 117, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 155, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 169, 235, 236, 242, 243 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 28, 29, 30, 31, 32, 33, 34, 35, 36, 43, 44, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 103, 104, 105, 110, 111, 112, 115, 123, 124, 125, 126, 127, 136, 137, 138, 140, 141, 142, 156, 159, 162, 165, 168, 222, 223, 234 }

B grade { 139, 233 }

C grade { 2, 3, 4, 5, 6, 7, 8, 18, 19, 20, 21, 23, 24, 25, 26, 27, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 128, 129, 130, 131, 132, 133, 134, 135, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 160, 161, 163, 164, 166, 167, 169 }

F normal fail { 106, 107, 108, 109, 113, 114, 116, 117, 118, 119, 120, 121, 122, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 110, 111, 112, 115, 123, 124, 125, 126, 127, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 225, 226, 227, 234 }

B grade { 40, 47, 128, 129, 130, 131, 132, 133, 134, 135, 139, 224, 233 }

C grade { }

F normal fail { 106, 107, 108, 109, 113, 114, 116, 117, 118, 119, 120, 121, 122, 181, 182, 183, 184, 185, 199, 200, 201, 202, 203, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 2, 4, 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 110, 111, 112, 115, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 159, 162, 165, 168, 233, 234 }

B grade { 1, 3, 6, 7, 8, 20, 24, 25, 26, 40, 46, 47, 48, 49, 139 }

C grade { }

F normal fail { 106, 107, 108, 109, 113, 114, 116, 117, 118, 119, 120, 121, 122, 128, 129, 130, 131, 132, 133, 134, 135, 155, 157, 158, 160, 161, 163, 164, 166, 167, 169, 181, 182, 183, 184, 185, 199, 200, 201, 202, 203, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

F(-1) timedout fail { }

F(-2) exception fail { 123, 124, 125, 126, 127, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 111, 112, 115, 123, 124, 125, 126, 127, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 159, 162, 165, 168, 173, 174, 189, 190, 207, 222, 223 }

B grade { 25, 40, 47, 103, 104, 110, 139, 155, 157, 158, 160, 163, 164, 166, 208, 233, 234 }

C grade { 161 }

F normal fail { 106, 107, 108, 109, 113, 114, 116, 117, 118, 119, 120, 121, 122, 128, 129, 130, 131, 132, 133, 134, 135, 170, 171, 172, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

F(-1) timedout fail { }

F(-2) exception fail { 167, 169 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 15, 16, 17, 19, 20, 25, 26, 27, 40, 47, 48, 49, 61, 62, 63, 64, 78, 79, 90, 91, 92, 105, 110, 111, 112, 115, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 189, 190, 207, 208, 209, 210, 221, 222, 223, 233, 234 }

C grade { }

F normal fail { }

F(-1) timedout fail { 9, 10, 11, 12, 13, 14, 18, 21, 22, 23, 24, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 113, 114, 116, 117, 118, 119, 120, 121, 122, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

F(-2) exception fail { }

Sympy

A grade { 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169 }

B grade { 123, 124, 125, 126, 127, 233, 234 }

C grade { 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

F normal fail { 3, 4, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 237, 238, 239, 240 }

F(-1) timedout fail { 1, 2, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 66, 105, 235, 236, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 110, 111, 112, 115, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 222, 223, 233, 234 }

C grade { }

F normal fail { 106, 107, 108, 109, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	91	36	114	13	0	29	15	71
N.S.	1	0.62	1.15	0.46	1.44	0.16	0.00	0.37	0.19	0.90
time (sec)	N/A	0.193	0.537	0.128	0.036	0.065	0.000	0.125	0.180	21.998

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	39	31	83	13	0	23	15	59
N.S.	1	1.06	0.58	0.46	1.24	0.19	0.00	0.34	0.22	0.88
time (sec)	N/A	0.212	1.012	0.102	0.034	0.059	0.000	0.107	0.184	21.879

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	88	23	52	13	0	22	14	33
N.S.	1	1.00	2.44	0.64	1.44	0.36	0.00	0.61	0.39	0.92
time (sec)	N/A	0.179	0.451	0.102	0.035	0.063	0.000	0.140	0.175	21.253

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	45	454	26	96	11	0	28	11	109
N.S.	1	0.60	6.05	0.35	1.28	0.15	0.00	0.37	0.15	1.45
time (sec)	N/A	0.196	1.013	0.083	0.037	0.062	0.000	0.133	0.169	21.254

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	45	178	28	99	17	0	43	17	112
N.S.	1	0.60	2.37	0.37	1.32	0.23	0.00	0.57	0.23	1.49
time (sec)	N/A	0.207	0.386	0.086	0.035	0.061	0.000	0.129	0.182	21.739

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	22	86	13	0	30	15	33
N.S.	1	1.00	0.95	0.56	2.21	0.33	0.00	0.77	0.38	0.85
time (sec)	N/A	0.188	1.016	0.086	0.040	0.062	0.000	0.126	0.176	23.097

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	24	117	15	0	31	15	35
N.S.	1	0.62	0.49	0.30	1.48	0.19	0.00	0.39	0.19	0.44
time (sec)	N/A	0.201	1.018	0.089	0.045	0.061	0.000	0.112	0.176	23.745

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	24	148	15	0	31	15	35
N.S.	1	0.62	0.49	0.30	1.87	0.19	0.00	0.39	0.19	0.44
time (sec)	N/A	0.202	1.013	0.088	0.039	0.061	0.000	0.132	0.182	21.851

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	36	13	13	0	29	15	0
N.S.	1	0.62	0.49	0.46	0.16	0.16	0.00	0.37	0.19	0.00
time (sec)	N/A	0.200	1.012	2.171	0.034	0.063	0.000	0.126	0.174	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	36	13	13	0	29	15	0
N.S.	1	0.62	0.49	0.46	0.16	0.16	0.00	0.37	0.19	0.00
time (sec)	N/A	0.200	1.012	1.787	0.034	0.064	0.000	0.129	0.184	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	36	13	13	0	29	15	0
N.S.	1	0.62	0.49	0.46	0.16	0.16	0.00	0.37	0.19	0.00
time (sec)	N/A	0.199	1.011	1.297	0.039	0.064	0.000	0.133	0.168	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	50	36	33	10	10	0	20	12	0
N.S.	1	0.68	0.49	0.45	0.14	0.14	0.00	0.27	0.16	0.00
time (sec)	N/A	0.193	0.010	1.067	0.034	0.061	0.000	0.112	0.176	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	47	38	36	14	14	0	29	14	0
N.S.	1	0.61	0.49	0.47	0.18	0.18	0.00	0.38	0.18	0.00
time (sec)	N/A	0.200	1.014	1.509	0.034	0.062	0.000	0.125	0.184	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	44	37	34	15	15	0	26	15	0
N.S.	1	0.59	0.50	0.46	0.20	0.20	0.00	0.35	0.20	0.00
time (sec)	N/A	0.198	1.011	1.812	0.033	0.063	0.000	0.133	0.172	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	47	37	34	13	13	0	30	15	33
N.S.	1	0.61	0.48	0.44	0.17	0.17	0.00	0.39	0.19	0.43
time (sec)	N/A	0.200	1.011	2.569	0.036	0.064	0.000	0.135	0.185	21.224

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	35	15	15	0	31	15	35
N.S.	1	0.62	0.49	0.44	0.19	0.19	0.00	0.39	0.19	0.44
time (sec)	N/A	0.206	1.012	2.948	0.033	0.062	0.000	0.105	0.176	21.130

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	35	15	15	0	31	15	35
N.S.	1	0.62	0.49	0.44	0.19	0.19	0.00	0.39	0.19	0.44
time (sec)	N/A	0.203	1.013	5.086	0.035	0.065	0.000	0.117	0.172	21.674

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	119	78	113	42	114	35	0	67	37	0
N.S.	1	0.66	0.95	0.35	0.96	0.29	0.00	0.56	0.31	0.00
time (sec)	N/A	0.234	0.890	0.121	0.036	0.067	0.000	0.116	0.177	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	113	31	83	35	0	45	37	46
N.S.	1	1.06	1.64	0.45	1.20	0.51	0.00	0.65	0.54	0.67
time (sec)	N/A	0.215	0.773	0.105	0.036	0.064	0.000	0.126	0.174	20.963

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	60	23	52	35	0	44	36	36
N.S.	1	1.00	1.58	0.61	1.37	0.92	0.00	1.16	0.95	0.95
time (sec)	N/A	0.181	1.027	0.103	0.035	0.065	0.000	0.132	0.182	20.727

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	160	74	60	54	152	32	0	65	32	0
N.S.	1	0.46	0.38	0.34	0.95	0.20	0.00	0.41	0.20	0.00
time (sec)	N/A	0.222	1.029	0.100	0.038	0.064	0.000	0.107	0.179	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	73	62	59	156	38	0	85	38	0
N.S.	1	0.45	0.39	0.37	0.97	0.24	0.00	0.53	0.24	0.00
time (sec)	N/A	0.230	1.029	0.116	0.036	0.062	0.000	0.129	0.171	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	162	72	61	59	220	39	0	86	39	0
N.S.	1	0.44	0.38	0.36	1.36	0.24	0.00	0.53	0.24	0.00
time (sec)	N/A	0.230	1.026	0.100	0.037	0.063	0.000	0.128	0.176	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	161	74	266	52	253	39	0	85	39	0
N.S.	1	0.46	1.65	0.32	1.57	0.24	0.00	0.53	0.24	0.00
time (sec)	N/A	0.230	0.596	0.104	0.039	0.062	0.000	0.130	0.172	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	59	41	148	35	0	68	37	151
N.S.	1	1.00	1.44	1.00	3.61	0.85	0.00	1.66	0.90	3.68
time (sec)	N/A	0.181	1.020	0.104	0.039	0.063	0.000	0.124	0.184	20.577

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	75	61	44	179	37	0	69	37	151
N.S.	1	0.89	0.73	0.52	2.13	0.44	0.00	0.82	0.44	1.80
time (sec)	N/A	0.203	1.022	0.108	0.040	0.064	0.000	0.112	0.168	20.760

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	167	78	61	44	210	37	0	69	37	151
N.S.	1	0.47	0.37	0.26	1.26	0.22	0.00	0.41	0.22	0.90
time (sec)	N/A	0.231	1.022	0.111	0.041	0.063	0.000	0.126	0.172	21.563

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	75	61	58	35	35	0	67	37	0
N.S.	1	0.45	0.37	0.35	0.21	0.21	0.00	0.40	0.22	0.00
time (sec)	N/A	0.224	1.023	2.145	0.033	0.078	0.000	0.128	0.180	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	75	61	58	35	35	0	67	37	0
N.S.	1	0.45	0.37	0.35	0.21	0.21	0.00	0.40	0.22	0.00
time (sec)	N/A	0.227	1.025	1.793	0.034	0.067	0.000	0.129	0.171	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	75	61	58	35	35	0	67	37	0
N.S.	1	0.45	0.37	0.35	0.21	0.21	0.00	0.40	0.22	0.00
time (sec)	N/A	0.218	1.021	1.253	0.033	0.067	0.000	0.132	0.183	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	78	59	56	32	32	0	64	35	0
N.S.	1	0.48	0.36	0.35	0.20	0.20	0.00	0.40	0.22	0.00
time (sec)	N/A	0.222	1.019	1.034	0.040	0.062	0.000	0.109	0.179	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	73	61	58	37	37	0	67	37	0
N.S.	1	0.44	0.37	0.35	0.22	0.22	0.00	0.41	0.22	0.00
time (sec)	N/A	0.224	1.021	1.522	0.038	0.065	0.000	0.130	0.167	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	71	61	58	37	37	0	65	37	0
N.S.	1	0.44	0.37	0.36	0.23	0.23	0.00	0.40	0.23	0.00
time (sec)	N/A	0.225	1.022	1.764	0.033	0.063	0.000	0.113	0.181	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	73	61	58	37	37	0	69	37	0
N.S.	1	0.44	0.37	0.35	0.22	0.22	0.00	0.42	0.22	0.00
time (sec)	N/A	0.227	1.020	2.506	0.036	0.065	0.000	0.137	0.167	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	71	61	58	37	37	0	68	37	0
N.S.	1	0.44	0.37	0.36	0.23	0.23	0.00	0.42	0.23	0.00
time (sec)	N/A	0.224	1.023	3.015	0.034	0.061	0.000	0.124	0.188	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	73	61	58	37	37	0	70	37	0
N.S.	1	0.44	0.37	0.35	0.22	0.22	0.00	0.42	0.22	0.00
time (sec)	N/A	0.224	1.017	5.102	0.036	0.063	0.000	0.112	0.173	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	160	107	83	53	145	57	0	105	59	0
N.S.	1	0.67	0.52	0.33	0.91	0.36	0.00	0.66	0.37	0.00
time (sec)	N/A	0.257	1.036	0.195	0.036	0.063	0.000	0.134	0.164	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	119	88	83	42	114	57	0	105	59	0
N.S.	1	0.74	0.70	0.35	0.96	0.48	0.00	0.88	0.50	0.00
time (sec)	N/A	0.240	1.037	0.148	0.042	0.067	0.000	0.114	0.179	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	83	31	83	57	0	67	59	0
N.S.	1	1.06	1.20	0.45	1.20	0.83	0.00	0.97	0.86	0.00
time (sec)	N/A	0.213	1.034	0.115	0.035	0.064	0.000	0.132	0.163	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	B	F	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	82	23	52	57	0	66	58	36
N.S.	1	1.00	2.16	0.61	1.37	1.50	0.00	1.74	1.53	0.95
time (sec)	N/A	0.179	1.034	0.109	0.035	0.069	0.000	0.128	0.165	22.417

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	251	98	82	75	206	55	0	104	55	0
N.S.	1	0.39	0.33	0.30	0.82	0.22	0.00	0.41	0.22	0.00
time (sec)	N/A	0.244	1.039	0.102	0.041	0.068	0.000	0.113	0.189	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	252	97	85	81	214	61	0	124	61	0
N.S.	1	0.38	0.34	0.32	0.85	0.24	0.00	0.49	0.24	0.00
time (sec)	N/A	0.245	1.036	0.107	0.043	0.073	0.000	0.141	0.164	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	99	85	82	282	61	0	126	61	0
N.S.	1	0.39	0.34	0.33	1.12	0.24	0.00	0.50	0.24	0.00
time (sec)	N/A	0.248	1.047	0.119	0.041	0.069	0.000	0.111	0.176	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	99	85	82	313	61	0	127	61	0
N.S.	1	0.39	0.34	0.33	1.24	0.24	0.00	0.50	0.24	0.00
time (sec)	N/A	0.249	1.038	0.165	0.043	0.068	0.000	0.129	0.170	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	252	96	85	81	342	61	0	125	61	0
N.S.	1	0.38	0.34	0.32	1.36	0.24	0.00	0.50	0.24	0.00
time (sec)	N/A	0.246	1.030	0.219	0.041	0.068	0.000	0.142	0.169	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	251	98	268	72	374	61	0	123	61	0
N.S.	1	0.39	1.07	0.29	1.49	0.24	0.00	0.49	0.24	0.00
time (sec)	N/A	0.250	0.856	0.366	0.046	0.071	0.000	0.112	0.179	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	81	58	210	57	0	106	59	231
N.S.	1	1.00	1.98	1.41	5.12	1.39	0.00	2.59	1.44	5.63
time (sec)	N/A	0.183	1.030	0.651	0.047	0.067	0.000	0.126	0.164	19.372

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	75	83	68	241	59	0	107	59	231
N.S.	1	0.89	0.99	0.81	2.87	0.70	0.00	1.27	0.70	2.75
time (sec)	N/A	0.202	1.033	1.181	0.053	0.070	0.000	0.133	0.165	19.510

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	128	103	83	66	272	59	0	107	59	231
N.S.	1	0.80	0.65	0.52	2.12	0.46	0.00	0.84	0.46	1.80
time (sec)	N/A	0.221	1.029	2.158	0.050	0.070	0.000	0.145	0.174	19.603

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	101	83	80	57	57	0	105	59	0
N.S.	1	0.40	0.33	0.31	0.22	0.22	0.00	0.41	0.23	0.00
time (sec)	N/A	0.247	1.030	2.179	0.036	0.072	0.000	0.126	0.168	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	98	83	80	56	56	0	104	59	0
N.S.	1	0.39	0.33	0.32	0.22	0.22	0.00	0.41	0.23	0.00
time (sec)	N/A	0.243	1.025	1.769	0.038	0.064	0.000	0.113	0.180	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	98	83	80	56	56	0	104	59	0
N.S.	1	0.39	0.33	0.32	0.22	0.22	0.00	0.41	0.23	0.00
time (sec)	N/A	0.234	1.026	1.283	0.037	0.069	0.000	0.121	0.173	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	101	81	78	53	53	0	101	57	0
N.S.	1	0.41	0.33	0.32	0.21	0.21	0.00	0.41	0.23	0.00
time (sec)	N/A	0.243	1.027	1.106	0.034	0.066	0.000	0.142	0.173	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	97	83	80	59	59	0	105	59	0
N.S.	1	0.39	0.33	0.32	0.24	0.24	0.00	0.42	0.24	0.00
time (sec)	N/A	0.243	1.028	1.522	0.038	0.073	0.000	0.114	0.180	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	97	83	80	59	59	0	103	59	0
N.S.	1	0.39	0.33	0.32	0.24	0.24	0.00	0.41	0.24	0.00
time (sec)	N/A	0.238	1.029	1.766	0.034	0.071	0.000	0.121	0.168	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	95	83	80	59	59	0	107	59	0
N.S.	1	0.38	0.33	0.32	0.24	0.24	0.00	0.43	0.24	0.00
time (sec)	N/A	0.239	1.031	2.497	0.034	0.071	0.000	0.144	0.175	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	97	83	80	59	59	0	106	59	0
N.S.	1	0.39	0.33	0.32	0.24	0.24	0.00	0.42	0.24	0.00
time (sec)	N/A	0.241	1.031	3.093	0.038	0.077	0.000	0.135	0.166	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	94	83	80	59	59	0	107	59	0
N.S.	1	0.38	0.33	0.32	0.24	0.24	0.00	0.43	0.24	0.00
time (sec)	N/A	0.242	1.026	5.523	0.033	0.067	0.000	0.130	0.169	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	81	67	64	95	45	0	79	73	0
N.S.	1	0.48	0.40	0.38	0.56	0.27	0.00	0.47	0.43	0.00
time (sec)	N/A	0.237	1.036	0.122	0.034	0.068	0.000	0.115	0.182	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	127	68	167	47	36	33	0	59	61	0
N.S.	1	0.54	1.31	0.37	0.28	0.26	0.00	0.46	0.48	0.00
time (sec)	N/A	0.222	0.575	0.128	0.035	0.067	0.000	0.124	0.172	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	74	113	34	42	22	0	33	48	64
N.S.	1	0.99	1.51	0.45	0.56	0.29	0.00	0.44	0.64	0.85
time (sec)	N/A	0.215	0.499	0.102	0.035	0.067	0.000	0.124	0.175	19.846

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	22	15	13	0	22	37	33
N.S.	1	1.00	1.16	0.50	0.34	0.30	0.00	0.50	0.84	0.75
time (sec)	N/A	0.178	0.186	0.098	0.034	0.067	0.000	0.145	0.171	19.948

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	80	55	134	31	43	18	0	32	45	48
N.S.	1	0.69	1.68	0.39	0.54	0.22	0.00	0.40	0.56	0.60
time (sec)	N/A	0.191	0.285	0.105	0.036	0.071	0.000	0.112	0.173	20.192

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	125	65	175	44	73	33	0	50	61	75
N.S.	1	0.52	1.40	0.35	0.58	0.26	0.00	0.40	0.49	0.60
time (sec)	N/A	0.223	0.307	0.108	0.037	0.085	0.000	0.130	0.178	20.168

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	165	79	162	58	104	45	0	65	78	0
N.S.	1	0.48	0.98	0.35	0.63	0.27	0.00	0.39	0.47	0.00
time (sec)	N/A	0.239	0.589	0.119	0.040	0.080	0.000	0.125	0.180	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	208	93	192	71	135	58	0	78	89	0
N.S.	1	0.45	0.92	0.34	0.65	0.28	0.00	0.38	0.43	0.00
time (sec)	N/A	0.248	0.738	0.144	0.051	0.069	0.000	0.118	0.172	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	277	162	140	86	122	108	0	169	88	0
N.S.	1	0.58	0.51	0.31	0.44	0.39	0.00	0.61	0.32	0.00
time (sec)	N/A	0.317	1.079	3.038	0.130	0.075	0.000	0.124	0.179	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	240	163	131	77	109	123	0	146	81	0
N.S.	1	0.68	0.55	0.32	0.45	0.51	0.00	0.61	0.34	0.00
time (sec)	N/A	0.344	1.043	2.164	0.120	0.089	0.000	0.135	0.167	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	235	153	128	74	106	106	0	143	78	0
N.S.	1	0.65	0.54	0.31	0.45	0.45	0.00	0.61	0.33	0.00
time (sec)	N/A	0.333	1.047	1.808	0.129	0.080	0.000	0.132	0.186	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	202	146	109	47	98	304	0	124	65	0
N.S.	1	0.72	0.54	0.23	0.49	1.50	0.00	0.61	0.32	0.00
time (sec)	N/A	0.309	1.031	1.277	0.131	0.116	0.000	0.145	0.165	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	202	143	109	47	98	299	0	122	67	0
N.S.	1	0.71	0.54	0.23	0.49	1.48	0.00	0.60	0.33	0.00
time (sec)	N/A	0.320	1.029	1.076	0.118	0.093	0.000	0.116	0.166	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	238	161	133	93	106	103	0	131	84	0
N.S.	1	0.68	0.56	0.39	0.45	0.43	0.00	0.55	0.35	0.00
time (sec)	N/A	0.364	1.037	1.538	0.122	0.076	0.000	0.140	0.175	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	243	158	140	94	106	143	0	125	96	0
N.S.	1	0.65	0.58	0.39	0.44	0.59	0.00	0.51	0.40	0.00
time (sec)	N/A	0.341	1.042	1.862	0.118	0.075	0.000	0.133	0.166	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	278	178	151	105	119	129	0	137	107	0
N.S.	1	0.64	0.54	0.38	0.43	0.46	0.00	0.49	0.38	0.00
time (sec)	N/A	0.365	1.049	2.621	0.126	0.077	0.000	0.121	0.179	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	196	105	93	77	145	103	0	116	201	0
N.S.	1	0.54	0.47	0.39	0.74	0.53	0.00	0.59	1.03	0.00
time (sec)	N/A	0.268	1.044	0.148	0.041	0.066	0.000	0.134	0.176	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	154	91	81	65	114	91	0	92	187	0
N.S.	1	0.59	0.53	0.42	0.74	0.59	0.00	0.60	1.21	0.00
time (sec)	N/A	0.250	1.042	0.106	0.044	0.072	0.000	0.129	0.167	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	116	80	192	54	55	69	0	62	170	0
N.S.	1	0.69	1.66	0.47	0.47	0.59	0.00	0.53	1.47	0.00
time (sec)	N/A	0.241	0.756	0.100	0.036	0.072	0.000	0.145	0.178	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	41	71	119	31	43	36	0	32	28	42
N.S.	1	1.73	2.90	0.76	1.05	0.88	0.00	0.78	0.68	1.02
time (sec)	N/A	0.212	0.502	0.090	0.037	0.070	0.000	0.122	0.167	20.120

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	143	23	16	26	0	24	25	34
N.S.	1	1.00	3.76	0.61	0.42	0.68	0.00	0.63	0.66	0.89
time (sec)	N/A	0.181	0.555	0.084	0.039	0.066	0.000	0.132	0.176	20.079

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	147	84	74	70	88	90	0	87	198	0
N.S.	1	0.57	0.50	0.48	0.60	0.61	0.00	0.59	1.35	0.00
time (sec)	N/A	0.242	1.042	0.112	0.036	0.071	0.000	0.136	0.201	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	188	98	97	92	117	119	0	120	228	0
N.S.	1	0.52	0.52	0.49	0.62	0.63	0.00	0.64	1.21	0.00
time (sec)	N/A	0.263	1.038	0.115	0.036	0.070	0.000	0.135	0.175	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	231	117	111	104	148	134	0	109	245	0
N.S.	1	0.51	0.48	0.45	0.64	0.58	0.00	0.47	1.06	0.00
time (sec)	N/A	0.280	1.046	0.126	0.043	0.073	0.000	0.119	0.186	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	275	193	235	85	137	503	0	177	278	0
N.S.	1	0.70	0.85	0.31	0.50	1.83	0.00	0.64	1.01	0.00
time (sec)	N/A	0.384	1.125	2.983	0.128	0.087	0.000	0.141	0.168	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	280	200	235	89	149	512	0	185	265	0
N.S.	1	0.71	0.84	0.32	0.53	1.83	0.00	0.66	0.95	0.00
time (sec)	N/A	0.377	1.103	2.203	0.114	0.088	0.000	0.144	0.172	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	276	191	235	87	146	503	0	176	278	0
N.S.	1	0.69	0.85	0.32	0.53	1.82	0.00	0.64	1.01	0.00
time (sec)	N/A	0.373	1.070	1.861	0.118	0.087	0.000	0.142	0.172	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	277	200	237	90	147	514	0	176	267	0
N.S.	1	0.72	0.86	0.32	0.53	1.86	0.00	0.64	0.96	0.00
time (sec)	N/A	0.378	1.079	1.268	0.113	0.092	0.000	0.115	0.173	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	273	207	235	88	145	499	0	177	278	0
N.S.	1	0.76	0.86	0.32	0.53	1.83	0.00	0.65	1.02	0.00
time (sec)	N/A	0.392	1.074	1.080	0.121	0.110	0.000	0.146	0.181	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	316	215	260	115	148	201	0	201	290	0
N.S.	1	0.68	0.82	0.36	0.47	0.64	0.00	0.64	0.92	0.00
time (sec)	N/A	0.414	1.091	1.561	0.121	0.081	0.000	0.148	0.170	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	314	212	266	116	150	242	0	184	309	0
N.S.	1	0.68	0.85	0.37	0.48	0.77	0.00	0.59	0.98	0.00
time (sec)	N/A	0.413	1.102	1.832	0.127	0.079	0.000	0.120	0.175	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	119	86	206	42	53	69	0	43	68	53
N.S.	1	0.72	1.73	0.35	0.45	0.58	0.00	0.36	0.57	0.45
time (sec)	N/A	0.239	0.970	0.099	0.038	0.067	0.000	0.120	0.182	18.720

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	232	31	43	58	0	32	57	42
N.S.	1	1.06	3.36	0.45	0.62	0.84	0.00	0.46	0.83	0.61
time (sec)	N/A	0.213	0.911	0.100	0.037	0.068	0.000	0.128	0.174	19.115

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	267	23	16	48	0	24	47	34
N.S.	1	1.00	7.03	0.61	0.42	1.26	0.00	0.63	1.24	0.89
time (sec)	N/A	0.184	0.924	0.092	0.052	0.068	0.000	0.112	0.188	18.842

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	223	116	96	92	132	178	0	109	372	0
N.S.	1	0.52	0.43	0.41	0.59	0.80	0.00	0.49	1.67	0.00
time (sec)	N/A	0.272	1.057	0.122	0.054	0.069	0.000	0.143	0.178	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	269	132	119	114	163	207	0	143	404	0
N.S.	1	0.49	0.44	0.42	0.61	0.77	0.00	0.53	1.50	0.00
time (sec)	N/A	0.297	1.059	0.138	0.050	0.070	0.000	0.117	0.174	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	365	254	226	107	197	734	0	207	481	0
N.S.	1	0.70	0.62	0.29	0.54	2.01	0.00	0.57	1.32	0.00
time (sec)	N/A	0.448	1.162	4.622	0.140	0.083	0.000	0.141	0.182	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	359	243	218	105	195	723	0	205	492	0
N.S.	1	0.68	0.61	0.29	0.54	2.01	0.00	0.57	1.37	0.00
time (sec)	N/A	0.434	1.154	3.004	0.136	0.089	0.000	0.147	0.169	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	368	254	229	109	195	734	0	207	481	0
N.S.	1	0.69	0.62	0.30	0.53	1.99	0.00	0.56	1.31	0.00
time (sec)	N/A	0.443	1.191	2.218	0.150	0.084	0.000	0.127	0.185	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	360	241	221	107	193	723	0	199	492	0
N.S.	1	0.67	0.61	0.30	0.54	2.01	0.00	0.55	1.37	0.00
time (sec)	N/A	0.423	1.162	1.787	0.133	0.087	0.000	0.140	0.174	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	359	254	219	112	191	734	0	198	481	0
N.S.	1	0.71	0.61	0.31	0.53	2.04	0.00	0.55	1.34	0.00
time (sec)	N/A	0.442	1.139	1.257	0.125	0.083	0.000	0.146	0.177	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	351	269	211	110	189	719	0	199	492	0
N.S.	1	0.77	0.60	0.31	0.54	2.05	0.00	0.57	1.40	0.00
time (sec)	N/A	0.466	1.148	1.043	0.122	0.090	0.000	0.126	0.181	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	400	269	242	137	192	311	0	223	504	0
N.S.	1	0.67	0.60	0.34	0.48	0.78	0.00	0.56	1.26	0.00
time (sec)	N/A	0.483	1.160	1.570	0.125	0.076	0.000	0.145	0.174	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	394	266	234	138	194	352	0	215	523	0
N.S.	1	0.68	0.59	0.35	0.49	0.89	0.00	0.55	1.33	0.00
time (sec)	N/A	0.473	1.180	1.820	0.131	0.087	0.000	0.153	0.186	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	159	111	453	243	369	0	900	433	0
N.S.	1	0.51	0.35	1.45	0.78	1.18	0.00	2.88	1.38	0.00
time (sec)	N/A	0.328	0.361	0.054	0.045	0.077	0.000	0.157	0.169	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	113	131	199	119	159	0	384	179	0
N.S.	1	0.55	0.64	0.97	0.58	0.78	0.00	1.87	0.87	0.00
time (sec)	N/A	0.272	0.147	0.047	0.041	0.082	0.000	0.142	0.163	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	67	53	56	35	35	0	83	36	55
N.S.	1	0.69	0.55	0.58	0.36	0.36	0.00	0.86	0.37	0.57
time (sec)	N/A	0.221	0.063	0.035	0.039	0.077	0.000	0.132	0.178	19.001

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	0	0	0	0	0	19	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.206	0.065	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	41	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.206	0.072	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	63	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.204	0.080	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	0	0	0	0	0	218	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	2.83	0.00
time (sec)	N/A	0.207	0.061	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	163	110	150	115	163	0	375	162	207
N.S.	1	0.95	0.64	0.87	0.67	0.95	0.00	2.18	0.94	1.20
time (sec)	N/A	0.304	0.159	0.162	0.036	0.079	0.000	0.142	0.173	18.918

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	128	77	96	79	108	0	235	104	137
N.S.	1	0.98	0.59	0.74	0.61	0.83	0.00	1.81	0.80	1.05
time (sec)	N/A	0.276	0.104	0.106	0.037	0.077	0.000	0.142	0.162	19.182

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	83	51	58	54	70	0	132	67	85
N.S.	1	1.08	0.66	0.75	0.70	0.91	0.00	1.71	0.87	1.10
time (sec)	N/A	0.223	0.077	0.082	0.035	0.076	0.000	0.143	0.173	19.009

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	304	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	5.07	0.00
time (sec)	N/A	0.189	0.095	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	300	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	5.00	0.00
time (sec)	N/A	0.188	0.090	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	45	32	31	30	37	0	58	39	46
N.S.	1	1.10	0.78	0.76	0.73	0.90	0.00	1.41	0.95	1.12
time (sec)	N/A	0.180	0.051	0.050	0.041	0.074	0.000	0.147	0.173	18.927

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	60	51	0	0	0	0	0	131	0
N.S.	1	1.03	0.88	0.00	0.00	0.00	0.00	0.00	2.26	0.00
time (sec)	N/A	0.181	0.081	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	55	211	0	0	0	0	0	127	0
N.S.	1	1.04	3.98	0.00	0.00	0.00	0.00	0.00	2.40	0.00
time (sec)	N/A	0.176	0.227	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	0	0	0	0	0	64	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.204	0.059	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	0	0	0	0	0	148	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	2.55	0.00
time (sec)	N/A	0.185	0.100	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	153	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	2.55	0.00
time (sec)	N/A	0.188	0.097	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	67	55	0	0	0	0	0	69	0
N.S.	1	1.05	0.86	0.00	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	0.211	0.055	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	153	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	2.55	0.00
time (sec)	N/A	0.180	0.100	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	80	78	83	0	254	316	75	77	1758
N.S.	1	0.99	0.96	1.02	0.00	3.14	3.90	0.93	0.95	21.70
time (sec)	N/A	0.250	0.061	0.069	0.000	0.082	1.701	0.190	0.171	20.374

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	64	62	60	0	197	223	59	42	1199
N.S.	1	1.02	0.98	0.95	0.00	3.13	3.54	0.94	0.67	19.03
time (sec)	N/A	0.242	0.027	0.048	0.000	0.078	0.846	0.204	0.171	20.175

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	129	131	36	20	174
N.S.	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	0.53	4.58
time (sec)	N/A	0.188	0.013	0.038	0.000	0.079	0.394	0.202	0.178	19.919

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	66	65	0	223	253	66	18	1362
N.S.	1	1.06	0.96	0.94	0.00	3.23	3.67	0.96	0.26	19.74
time (sec)	N/A	0.253	0.029	0.044	0.000	0.092	4.247	0.209	0.157	20.837

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	95	92	85	0	293	345	93	20	4281
N.S.	1	1.07	1.03	0.96	0.00	3.29	3.88	1.04	0.22	48.10
time (sec)	N/A	0.300	0.036	0.057	0.000	0.114	86.389	0.196	0.161	20.240

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	636	546	70	61	0	3402	279	0	53	4069
N.S.	1	0.86	0.11	0.10	0.00	5.35	0.44	0.00	0.08	6.40
time (sec)	N/A	1.006	0.035	0.128	0.000	0.279	10.746	0.000	0.166	30.874

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	521	70	59	0	2882	196	0	48	2280
N.S.	1	0.83	0.11	0.09	0.00	4.57	0.31	0.00	0.08	3.61
time (sec)	N/A	0.945	0.034	0.033	0.000	0.189	5.056	0.000	0.155	22.473

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	516	44	43	0	2314	175	0	20	2695
N.S.	1	0.92	0.08	0.08	0.00	4.15	0.31	0.00	0.04	4.83
time (sec)	N/A	0.803	0.021	0.034	0.000	0.108	1.312	0.000	0.156	26.474

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	496	42	43	0	1542	122	0	20	2129
N.S.	1	0.89	0.08	0.08	0.00	2.76	0.22	0.00	0.04	3.82
time (sec)	N/A	0.788	0.019	0.034	0.000	0.087	0.984	0.000	0.168	26.254

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	511	43	41	0	1798	158	0	18	1543
N.S.	1	0.92	0.08	0.07	0.00	3.22	0.28	0.00	0.03	2.77
time (sec)	N/A	0.770	0.019	0.033	0.000	0.097	0.832	0.000	0.159	23.886

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	484	45	40	0	2206	155	0	16	2597
N.S.	1	0.87	0.08	0.07	0.00	3.95	0.28	0.00	0.03	4.65
time (sec)	N/A	0.772	0.021	0.029	0.000	0.132	2.657	0.000	0.165	26.980

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	532	71	61	0	3225	252	0	55	2978
N.S.	1	0.87	0.12	0.10	0.00	5.29	0.41	0.00	0.09	4.88
time (sec)	N/A	0.834	0.035	0.049	0.000	0.211	1.871	0.000	0.163	25.457

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	612	514	75	62	0	3225	241	0	58	4063
N.S.	1	0.84	0.12	0.10	0.00	5.27	0.39	0.00	0.09	6.64
time (sec)	N/A	0.825	0.036	0.052	0.000	0.221	31.650	0.000	0.162	29.848

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	37	35	28	27	27	29	29	52	27
N.S.	1	1.06	1.00	0.80	0.77	0.77	0.83	0.83	1.49	0.77
time (sec)	N/A	0.192	0.009	0.046	0.029	0.060	0.058	0.114	0.180	0.055

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	22	22	24	47	22
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.79	0.86	1.68	0.79
time (sec)	N/A	0.183	0.008	0.034	0.029	0.061	0.057	0.132	0.167	0.049

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	25	21	18	17	17	15	19	42	17
N.S.	1	1.19	1.00	0.86	0.81	0.81	0.71	0.90	2.00	0.81
time (sec)	N/A	0.183	0.007	0.033	0.026	0.061	0.048	0.115	0.164	0.058

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	25	21	18	17	17	15	19	42	16
N.S.	1	2.50	2.10	1.80	1.70	1.70	1.50	1.90	4.20	1.60
time (sec)	N/A	0.180	0.006	0.030	0.031	0.069	0.050	0.130	0.181	20.301

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	33	27	22	23	21	20	24	46	21
N.S.	1	1.22	1.00	0.81	0.85	0.78	0.74	0.89	1.70	0.78
time (sec)	N/A	0.190	0.009	0.036	0.032	0.063	0.064	0.133	0.157	0.087

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	40	34	27	28	35	29	36	67	26
N.S.	1	1.18	1.00	0.79	0.82	1.03	0.85	1.06	1.97	0.76
time (sec)	N/A	0.195	0.008	0.044	0.034	0.071	0.079	0.129	0.169	19.576

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	47	41	33	35	40	34	41	70	32
N.S.	1	1.15	1.00	0.80	0.85	0.98	0.83	1.00	1.71	0.78
time (sec)	N/A	0.202	0.007	0.048	0.040	0.070	0.096	0.127	0.168	0.044

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	141	118	64	94	104	144	96	91	124
N.S.	1	1.14	0.95	0.52	0.76	0.84	1.16	0.77	0.73	1.00
time (sec)	N/A	0.401	0.068	0.049	0.110	0.077	0.322	0.133	0.163	0.263

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	139	114	58	92	90	129	94	89	119
N.S.	1	1.14	0.93	0.48	0.75	0.74	1.06	0.77	0.73	0.98
time (sec)	N/A	0.382	0.032	0.041	0.111	0.071	0.338	0.129	0.175	19.577

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	135	111	59	89	99	134	91	86	118
N.S.	1	1.13	0.93	0.50	0.75	0.83	1.13	0.76	0.72	0.99
time (sec)	N/A	0.354	0.031	0.040	0.112	0.071	0.348	0.112	0.162	19.708

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	129	111	57	85	83	126	87	82	104
N.S.	1	1.14	0.98	0.50	0.75	0.73	1.12	0.77	0.73	0.92
time (sec)	N/A	0.346	0.030	0.040	0.111	0.074	0.338	0.114	0.162	19.428

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	129	107	60	84	84	134	86	81	114
N.S.	1	1.15	0.96	0.54	0.75	0.75	1.20	0.77	0.72	1.02
time (sec)	N/A	0.328	0.028	0.037	0.110	0.071	0.313	0.139	0.177	0.206

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	129	106	54	84	99	110	86	81	113
N.S.	1	1.15	0.95	0.48	0.75	0.88	0.98	0.77	0.72	1.01
time (sec)	N/A	0.323	0.026	0.038	0.111	0.076	0.318	0.145	0.173	19.453

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	128	108	60	84	84	119	86	81	113
N.S.	1	1.14	0.96	0.54	0.75	0.75	1.06	0.77	0.72	1.01
time (sec)	N/A	0.320	0.027	0.039	0.114	0.068	1.228	0.127	0.170	19.458

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	128	107	58	84	99	124	86	81	110
N.S.	1	1.14	0.96	0.52	0.75	0.88	1.11	0.77	0.72	0.98
time (sec)	N/A	0.323	0.026	0.037	0.127	0.077	1.190	0.134	0.175	19.656

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	140	118	65	89	95	139	91	102	119
N.S.	1	1.18	0.99	0.55	0.75	0.80	1.17	0.76	0.86	1.00
time (sec)	N/A	0.355	0.045	0.046	0.112	0.081	1.267	0.142	0.161	0.222

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	140	113	59	89	123	128	91	115	118
N.S.	1	1.18	0.95	0.50	0.75	1.03	1.08	0.76	0.97	0.99
time (sec)	N/A	0.364	0.057	0.047	0.112	0.073	1.077	0.128	0.174	19.769

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	152	118	71	96	112	141	98	123	124
N.S.	1	1.21	0.94	0.56	0.76	0.89	1.12	0.78	0.98	0.98
time (sec)	N/A	0.388	0.050	0.049	0.110	0.073	1.281	0.135	0.163	19.930

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	152	118	65	96	128	136	98	123	121
N.S.	1	1.21	0.94	0.52	0.76	1.02	1.08	0.78	0.98	0.96
time (sec)	N/A	0.394	0.070	0.049	0.110	0.074	1.068	0.135	0.171	19.776

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	350	59	44	0	254	26	641	36	320
N.S.	1	0.85	0.14	0.11	0.00	0.62	0.06	1.56	0.09	0.78
time (sec)	N/A	0.622	0.018	0.032	0.000	0.069	0.091	0.150	0.166	19.857

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	40	39	33	32	32	37	32	34	34
N.S.	1	1.03	1.00	0.85	0.82	0.82	0.95	0.82	0.87	0.87
time (sec)	N/A	0.210	0.014	0.033	0.107	0.068	0.061	0.137	0.156	0.054

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	367	41	40	0	261	26	827	18	304
N.S.	1	0.89	0.10	0.10	0.00	0.64	0.06	2.01	0.04	0.74
time (sec)	N/A	0.601	0.012	0.028	0.000	0.067	0.096	0.159	0.161	19.990

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	349	39	40	0	217	24	640	18	327
N.S.	1	0.85	0.09	0.10	0.00	0.53	0.06	1.56	0.04	0.80
time (sec)	N/A	0.521	0.011	0.027	0.000	0.069	0.101	0.144	0.163	20.154

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	18	20
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.78	0.87
time (sec)	N/A	0.170	0.007	0.029	0.107	0.068	0.057	0.141	0.155	19.872

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	355	40	38	0	261	26	815	16	304
N.S.	1	0.95	0.11	0.10	0.00	0.70	0.07	2.17	0.04	0.81
time (sec)	N/A	0.544	0.012	0.030	0.000	0.072	0.084	0.149	0.173	19.948

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	337	42	37	0	253	20	632	14	327
N.S.	1	1.81	0.23	0.20	0.00	1.36	0.11	3.40	0.08	1.76
time (sec)	N/A	0.496	0.011	0.024	0.000	0.069	0.091	0.125	0.161	19.987

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	44	55	33	38	34	41	35	14	36
N.S.	1	1.07	1.34	0.80	0.93	0.83	1.00	0.85	0.34	0.88
time (sec)	N/A	0.220	0.015	0.035	0.107	0.062	0.076	0.131	0.156	0.056

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	372	61	35	0	217	24	829	45	286
N.S.	1	0.89	0.15	0.08	0.00	0.52	0.06	1.99	0.11	0.69
time (sec)	N/A	0.568	0.016	0.036	0.000	0.069	0.100	0.134	0.167	20.040

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	356	65	38	0	291	31	645	49	324
N.S.	1	0.85	0.16	0.09	0.00	0.70	0.07	1.54	0.12	0.78
time (sec)	N/A	0.543	0.014	0.038	0.000	0.074	0.098	0.150	0.155	19.760

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	49	51	38	43	51	48	45	16	41
N.S.	1	1.02	1.06	0.79	0.90	1.06	1.00	0.94	0.33	0.85
time (sec)	N/A	0.232	0.017	0.037	0.107	0.069	0.086	0.136	0.162	0.068

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	379	54	46	0	306	39	839	35	318
N.S.	1	0.90	0.13	0.11	0.00	0.72	0.09	1.98	0.08	0.75
time (sec)	N/A	0.593	0.017	0.039	0.000	0.067	0.111	0.143	0.177	19.848

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	339	38	33	0	253	24	0	12	513
N.S.	1	0.89	0.10	0.09	0.00	0.66	0.06	0.00	0.03	1.35
time (sec)	N/A	0.600	0.012	0.029	0.000	0.070	0.067	0.000	0.153	20.857

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	16	20
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.70	0.87
time (sec)	N/A	0.174	0.012	0.032	0.106	0.068	0.049	0.179	0.155	0.057

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	351	37	36	0	217	24	0	16	351
N.S.	1	0.88	0.09	0.09	0.00	0.54	0.06	0.00	0.04	0.88
time (sec)	N/A	0.542	0.011	0.028	0.000	0.071	0.067	0.000	0.162	20.804

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	238	206	0	0	451	0	0	1104	543
N.S.	1	1.03	0.89	0.00	0.00	1.95	0.00	0.00	4.78	2.35
time (sec)	N/A	0.440	0.710	0.000	0.000	0.092	0.000	0.000	2.957	21.103

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	180	166	0	0	367	0	0	804	315
N.S.	1	1.05	0.97	0.00	0.00	2.15	0.00	0.00	4.70	1.84
time (sec)	N/A	0.333	0.493	0.000	0.000	0.088	0.000	0.000	2.132	19.844

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	163	132	0	0	303	0	0	764	193
N.S.	1	1.07	0.86	0.00	0.00	1.98	0.00	0.00	4.99	1.26
time (sec)	N/A	0.302	0.362	0.000	0.000	0.086	0.000	0.000	1.962	19.626

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	119	101	0	0	237	0	96	494	87
N.S.	1	1.10	0.94	0.00	0.00	2.19	0.00	0.89	4.57	0.81
time (sec)	N/A	0.253	0.368	0.000	0.000	0.082	0.000	0.119	1.476	19.394

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	87	87	0	0	197	0	74	475	72
N.S.	1	1.05	1.05	0.00	0.00	2.37	0.00	0.89	5.72	0.87
time (sec)	N/A	0.218	0.560	0.000	0.000	0.078	0.000	0.130	1.207	19.277

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	105	0	0	566	0	0	0	88
N.S.	1	1.00	0.96	0.00	0.00	5.19	0.00	0.00	0.00	0.81
time (sec)	N/A	0.298	0.347	0.000	0.000	0.100	0.000	0.000	5.141	19.799

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	115	107	0	0	601	0	0	939	91
N.S.	1	1.03	0.96	0.00	0.00	5.37	0.00	0.00	8.38	0.81
time (sec)	N/A	0.298	0.344	0.000	0.000	0.097	0.000	0.000	7.636	20.296

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	92	91	0	0	215	0	0	0	0
N.S.	1	1.05	1.03	0.00	0.00	2.44	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	0.383	0.000	0.000	0.087	0.000	0.000	2.104	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	127	108	0	0	259	0	0	0	0
N.S.	1	1.09	0.93	0.00	0.00	2.23	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	0.620	0.000	0.000	0.095	0.000	0.000	5.146	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	171	141	0	0	325	0	0	0	0
N.S.	1	1.06	0.88	0.00	0.00	2.02	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	0.829	0.000	0.000	0.118	0.000	0.000	17.711	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	216	176	0	0	389	0	0	0	0
N.S.	1	1.09	0.88	0.00	0.00	1.95	0.00	0.00	0.00	0.00
time (sec)	N/A	0.417	1.084	0.000	0.000	0.172	0.000	0.000	89.306	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	358	0	0	0	0	0	151	0
N.S.	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	0.333	10.614	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	337	0	0	0	0	0	89	0
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.267	10.447	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	335	0	0	0	0	0	86	0
N.S.	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.257	10.421	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	340	0	0	0	0	0	95	0
N.S.	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.295	10.408	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	340	0	0	0	0	0	99	0
N.S.	1	1.00	2.43	0.00	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.298	10.383	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	285	289	0	0	641	0	0	0	0
N.S.	1	0.97	0.99	0.00	0.00	2.19	0.00	0.00	0.00	0.00
time (sec)	N/A	0.487	1.516	0.000	0.000	0.104	0.000	0.000	9.118	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	227	220	0	0	535	0	0	1168	0
N.S.	1	1.02	0.99	0.00	0.00	2.40	0.00	0.00	5.24	0.00
time (sec)	N/A	0.387	1.109	0.000	0.000	0.107	0.000	0.000	4.957	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	204	210	194	0	0	451	0	0	1104	0
N.S.	1	1.03	0.95	0.00	0.00	2.21	0.00	0.00	5.41	0.00
time (sec)	N/A	0.356	0.878	0.000	0.000	0.091	0.000	0.000	3.537	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	166	140	0	0	361	0	170	742	223
N.S.	1	1.11	0.93	0.00	0.00	2.41	0.00	1.13	4.95	1.49
time (sec)	N/A	0.291	0.562	0.000	0.000	0.092	0.000	0.154	2.477	20.740

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	134	114	0	0	297	0	133	702	115
N.S.	1	1.08	0.92	0.00	0.00	2.40	0.00	1.07	5.66	0.93
time (sec)	N/A	0.254	1.197	0.000	0.000	0.089	0.000	0.161	2.063	21.227

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	169	143	0	0	727	0	0	0	0
N.S.	1	1.09	0.92	0.00	0.00	4.69	0.00	0.00	0.00	0.00
time (sec)	N/A	0.387	0.938	0.000	0.000	0.166	0.000	0.000	15.623	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	157	131	0	0	713	0	0	1212	0
N.S.	1	1.05	0.87	0.00	0.00	4.75	0.00	0.00	8.08	0.00
time (sec)	N/A	0.378	0.732	0.000	0.000	0.137	0.000	0.000	13.758	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	159	131	0	0	713	0	0	0	0
N.S.	1	1.05	0.87	0.00	0.00	4.72	0.00	0.00	0.00	0.00
time (sec)	N/A	0.363	0.780	0.000	0.000	0.137	0.000	0.000	51.547	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	177	148	0	0	771	0	0	20	0
N.S.	1	1.09	0.91	0.00	0.00	4.73	0.00	0.00	0.12	0.00
time (sec)	N/A	0.385	1.038	0.000	0.000	0.149	0.000	0.000	200.031	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	143	118	0	0	319	0	0	0	0
N.S.	1	1.08	0.89	0.00	0.00	2.40	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	0.993	0.000	0.000	0.113	0.000	0.000	18.346	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	178	160	0	0	383	0	0	0	0
N.S.	1	1.10	0.99	0.00	0.00	2.36	0.00	0.00	0.00	0.00
time (sec)	N/A	0.328	1.449	0.000	0.000	0.176	0.000	0.000	87.697	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	222	201	0	0	473	0	0	20	0
N.S.	1	1.03	0.93	0.00	0.00	2.19	0.00	0.00	0.09	0.00
time (sec)	N/A	0.388	2.045	0.000	0.000	0.259	0.000	0.000	200.028	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	267	244	0	0	557	0	0	20	0
N.S.	1	1.05	0.96	0.00	0.00	2.18	0.00	0.00	0.08	0.00
time (sec)	N/A	0.467	2.320	0.000	0.000	0.324	0.000	0.000	200.034	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	453	0	0	0	0	0	323	0
N.S.	1	1.00	3.21	0.00	0.00	0.00	0.00	0.00	2.29	0.00
time (sec)	N/A	0.302	10.978	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	410	0	0	0	0	0	238	0
N.S.	1	1.00	2.91	0.00	0.00	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	0.280	10.843	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	408	0	0	0	0	0	232	0
N.S.	1	1.00	3.00	0.00	0.00	0.00	0.00	0.00	1.71	0.00
time (sec)	N/A	0.281	10.772	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	379	0	0	0	0	0	177	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.315	10.594	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	379	0	0	0	0	0	183	0
N.S.	1	1.00	2.69	0.00	0.00	0.00	0.00	0.00	1.30	0.00
time (sec)	N/A	0.307	10.678	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	191	138	0	0	303	0	0	764	0
N.S.	1	1.12	0.81	0.00	0.00	1.77	0.00	0.00	4.47	0.00
time (sec)	N/A	0.391	0.460	0.000	0.000	0.089	0.000	0.000	1.716	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	133	101	0	0	241	0	0	494	0
N.S.	1	1.10	0.83	0.00	0.00	1.99	0.00	0.00	4.08	0.00
time (sec)	N/A	0.281	0.350	0.000	0.000	0.093	0.000	0.000	1.262	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	114	91	0	0	203	0	0	476	0
N.S.	1	1.10	0.88	0.00	0.00	1.95	0.00	0.00	4.58	0.00
time (sec)	N/A	0.262	0.284	0.000	0.000	0.083	0.000	0.000	1.227	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	69	68	0	0	161	0	59	245	55
N.S.	1	1.01	1.00	0.00	0.00	2.37	0.00	0.87	3.60	0.81
time (sec)	N/A	0.213	0.204	0.000	0.000	0.090	0.000	0.139	0.840	19.461

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	0	0	118	0	74	111	34
N.S.	1	1.00	0.95	0.00	0.00	2.74	0.00	1.72	2.58	0.79
time (sec)	N/A	0.187	0.129	0.000	0.000	0.083	0.000	0.127	0.546	19.306

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	124	0	0	105	36
N.S.	1	1.00	1.00	0.00	0.00	2.82	0.00	0.00	2.39	0.82
time (sec)	N/A	0.191	0.121	0.000	0.000	0.081	0.000	0.000	0.363	19.249

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	74	72	0	0	179	0	0	227	56
N.S.	1	1.03	1.00	0.00	0.00	2.49	0.00	0.00	3.15	0.78
time (sec)	N/A	0.220	0.226	0.000	0.000	0.091	0.000	0.000	0.660	19.351

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	118	91	0	0	221	0	0	0	0
N.S.	1	1.09	0.84	0.00	0.00	2.05	0.00	0.00	0.00	0.00
time (sec)	N/A	0.280	0.362	0.000	0.000	0.104	0.000	0.000	1.414	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	163	110	0	0	263	0	0	0	0
N.S.	1	1.12	0.76	0.00	0.00	1.81	0.00	0.00	0.00	0.00
time (sec)	N/A	0.355	0.573	0.000	0.000	0.105	0.000	0.000	3.445	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	218	141	0	0	327	0	0	0	0
N.S.	1	1.14	0.73	0.00	0.00	1.70	0.00	0.00	0.00	0.00
time (sec)	N/A	0.445	0.782	0.000	0.000	0.127	0.000	0.000	11.300	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	168	0	0	0	0	0	33	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.317	10.136	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	168	0	0	0	0	0	31	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.274	10.106	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	163	0	0	0	0	0	30	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.251	10.107	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	343	0	0	0	0	0	34	0
N.S.	1	1.00	2.49	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.299	10.415	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	342	0	0	0	0	0	34	0
N.S.	1	1.00	2.44	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.299	10.391	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	206	170	0	0	591	0	0	0	0
N.S.	1	1.06	0.87	0.00	0.00	3.03	0.00	0.00	0.00	0.00
time (sec)	N/A	0.402	1.048	0.000	0.000	0.152	0.000	0.000	2.704	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	150	131	0	0	459	0	0	1171	0
N.S.	1	1.09	0.96	0.00	0.00	3.35	0.00	0.00	8.55	0.00
time (sec)	N/A	0.302	0.718	0.000	0.000	0.107	0.000	0.000	1.603	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	130	95	0	0	387	0	0	1125	84
N.S.	1	1.08	0.79	0.00	0.00	3.22	0.00	0.00	9.38	0.70
time (sec)	N/A	0.275	0.600	0.000	0.000	0.098	0.000	0.000	0.929	19.979

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	0	68	0	45	74	38
N.S.	1	1.00	1.00	0.97	0.00	1.74	0.00	1.15	1.90	0.97
time (sec)	N/A	0.184	0.366	2.693	0.000	0.086	0.000	0.229	0.175	19.714

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	37	0	67	0	45	72	37
N.S.	1	1.00	1.00	0.97	0.00	1.76	0.00	1.18	1.89	0.97
time (sec)	N/A	0.176	0.395	1.566	0.000	0.085	0.000	0.214	0.188	19.752

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	91	0	0	389	0	0	1086	0
N.S.	1	1.00	0.99	0.00	0.00	4.23	0.00	0.00	11.80	0.00
time (sec)	N/A	0.246	0.758	0.000	0.000	0.116	0.000	0.000	0.761	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	155	125	0	0	485	0	0	1153	0
N.S.	1	1.09	0.88	0.00	0.00	3.42	0.00	0.00	8.12	0.00
time (sec)	N/A	0.327	0.880	0.000	0.000	0.131	0.000	0.000	1.358	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	198	209	166	0	0	615	0	0	0	0
N.S.	1	1.06	0.84	0.00	0.00	3.11	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	1.060	0.000	0.000	0.178	0.000	0.000	3.453	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	265	210	0	0	705	0	0	0	0
N.S.	1	1.04	0.82	0.00	0.00	2.75	0.00	0.00	0.00	0.00
time (sec)	N/A	0.529	1.384	0.000	0.000	0.246	0.000	0.000	19.161	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	340	0	0	0	0	0	60	0
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.304	10.404	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	362	0	0	0	0	0	58	0
N.S.	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.273	10.694	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	359	0	0	0	0	0	57	0
N.S.	1	1.00	2.60	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.266	10.573	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	407	0	0	0	0	0	432	0
N.S.	1	1.00	2.89	0.00	0.00	0.00	0.00	0.00	3.06	0.00
time (sec)	N/A	0.306	10.905	0.000	0.000	0.000	0.000	0.000	0.261	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	405	0	0	0	0	0	433	0
N.S.	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	3.03	0.00
time (sec)	N/A	0.303	10.743	0.000	0.000	0.000	0.000	0.000	0.275	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	70	301	110	241	1459	449	301	260
N.S.	1	1.00	0.69	2.98	1.09	2.39	14.45	4.45	2.98	2.57
time (sec)	N/A	0.264	0.661	0.125	0.048	0.080	0.971	0.125	0.231	20.660

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	51	50	71	299	119	78	89
N.S.	1	1.00	0.67	0.98	0.96	1.37	5.75	2.29	1.50	1.71
time (sec)	N/A	0.203	0.068	0.051	0.048	0.079	0.370	0.120	0.214	20.638

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	84	0	0	0	0	0	24	0
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.333	0.358	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	315	322	78	0	0	0	0	0	51	0
N.S.	1	1.02	0.25	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.620	0.613	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	357	0	0	0	0	0	0	0
N.S.	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	2.348	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	259	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	1.65	0.00
time (sec)	N/A	0.317	1.914	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	37	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.322	2.356	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	221	0	0	0	0	0	64	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.322	10.103	0.000	0.000	0.000	0.000	0.000	0.297	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	179	0	0	0	0	0	473	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	3.05	0.00
time (sec)	N/A	0.307	0.390	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	222	162	0	0	0	0	0	1573	0
N.S.	1	0.99	0.72	0.00	0.00	0.00	0.00	0.00	7.02	0.00
time (sec)	N/A	0.352	0.515	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	162	162	0	0	0	0	0	448	0
N.S.	1	1.01	1.01	0.00	0.00	0.00	0.00	0.00	2.78	0.00
time (sec)	N/A	0.254	0.384	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	138	0	0	0	0	0	283	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	2.18	0.00
time (sec)	N/A	0.225	0.162	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	1029	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	7.46	0.00
time (sec)	N/A	0.280	0.419	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	1023	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	7.41	0.00
time (sec)	N/A	0.276	0.435	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	251	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	1.82	0.00
time (sec)	N/A	0.268	0.518	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0	247	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	1.86	0.00
time (sec)	N/A	0.269	0.234	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	157	0	0	0	0	0	95	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.316	0.350	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	164	0	0	0	0	0	280	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	2.06	0.00
time (sec)	N/A	0.279	0.511	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	288	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	2.09	0.00
time (sec)	N/A	0.279	0.531	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	162	162	0	0	0	0	0	568	0
N.S.	1	0.99	0.99	0.00	0.00	0.00	0.00	0.00	3.46	0.00
time (sec)	N/A	0.300	0.443	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	280	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	2.03	0.00
time (sec)	N/A	0.279	0.538	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	278	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	2.01	0.00
time (sec)	N/A	0.275	0.525	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	166	164	0	0	0	0	0	270	0
N.S.	1	0.99	0.98	0.00	0.00	0.00	0.00	0.00	1.61	0.00
time (sec)	N/A	0.305	0.510	0.000	0.000	0.000	0.000	0.000	0.192	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [143] had the largest ratio of [.8750000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	0.62	26	0.154
2	A	5	4	1.06	26	0.154
3	A	4	3	1.00	26	0.115
4	A	4	4	0.60	26	0.154
5	A	4	4	0.60	26	0.154
6	A	5	4	1.00	26	0.154
7	A	4	4	0.62	26	0.154
8	A	4	4	0.62	26	0.154
9	A	4	4	0.62	26	0.154
10	A	4	4	0.62	26	0.154
11	A	4	4	0.62	24	0.167
12	A	2	2	0.68	22	0.091
13	A	4	4	0.61	26	0.154
14	A	4	4	0.59	26	0.154
15	A	4	4	0.61	26	0.154
16	A	4	4	0.62	26	0.154
17	A	4	4	0.62	26	0.154
18	A	6	5	0.66	26	0.192
19	A	5	4	1.06	26	0.154
20	A	4	3	1.00	26	0.115
21	A	6	5	0.46	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	6	5	0.45	26	0.192
23	A	6	5	0.44	26	0.192
24	A	6	5	0.46	26	0.192
25	A	3	3	1.00	26	0.115
26	A	6	5	0.89	26	0.192
27	A	6	5	0.47	26	0.192
28	A	4	4	0.45	26	0.154
29	A	4	4	0.45	26	0.154
30	A	4	4	0.45	24	0.167
31	A	3	3	0.48	22	0.136
32	A	4	4	0.44	26	0.154
33	A	4	4	0.44	26	0.154
34	A	4	4	0.44	26	0.154
35	A	4	4	0.44	26	0.154
36	A	4	4	0.44	26	0.154
37	A	6	5	0.67	26	0.192
38	A	6	5	0.74	26	0.192
39	A	5	4	1.06	26	0.154
40	A	4	3	1.00	26	0.115
41	A	6	5	0.39	26	0.192
42	A	6	5	0.38	26	0.192
43	A	6	5	0.39	26	0.192
44	A	6	5	0.39	26	0.192
45	A	6	5	0.38	26	0.192
46	A	6	5	0.39	26	0.192
47	A	3	3	1.00	26	0.115
48	A	6	5	0.89	26	0.192
49	A	7	6	0.80	26	0.231
50	A	4	4	0.40	26	0.154
51	A	4	4	0.39	26	0.154
52	A	4	4	0.39	24	0.167
53	A	3	3	0.41	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	4	4	0.39	26	0.154
55	A	4	4	0.39	26	0.154
56	A	4	4	0.38	26	0.154
57	A	4	4	0.39	26	0.154
58	A	4	4	0.38	26	0.154
59	A	6	5	0.48	26	0.192
60	A	6	5	0.54	26	0.192
61	A	5	4	0.99	26	0.154
62	A	4	3	1.00	26	0.115
63	A	7	6	0.69	26	0.231
64	A	6	5	0.52	26	0.192
65	A	6	5	0.48	26	0.192
66	A	6	5	0.45	26	0.192
67	A	4	4	0.58	26	0.154
68	A	12	11	0.68	26	0.423
69	A	12	11	0.65	26	0.423
70	A	11	10	0.72	24	0.417
71	A	11	10	0.71	22	0.455
72	A	12	11	0.68	26	0.423
73	A	12	11	0.65	26	0.423
74	A	13	12	0.64	26	0.462
75	A	6	5	0.54	26	0.192
76	A	6	5	0.59	26	0.192
77	A	6	5	0.69	26	0.192
78	A	4	3	1.73	26	0.115
79	A	3	2	1.00	26	0.077
80	A	6	5	0.57	26	0.192
81	A	6	5	0.52	26	0.192
82	A	6	5	0.51	26	0.192
83	A	13	12	0.70	26	0.462
84	A	13	12	0.71	26	0.462
85	A	13	12	0.69	26	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
86	A	13	12	0.72	24	0.500
87	A	13	12	0.76	22	0.545
88	A	14	13	0.68	26	0.500
89	A	14	13	0.68	26	0.500
90	A	6	5	0.72	26	0.192
91	A	4	3	1.06	26	0.115
92	A	3	2	1.00	26	0.077
93	A	6	5	0.52	26	0.192
94	A	6	5	0.49	26	0.192
95	A	15	14	0.70	26	0.538
96	A	15	14	0.68	26	0.538
97	A	15	14	0.69	26	0.538
98	A	15	14	0.67	26	0.538
99	A	15	14	0.71	24	0.583
100	A	15	14	0.77	22	0.636
101	A	16	15	0.67	26	0.577
102	A	16	15	0.68	26	0.577
103	A	4	4	0.51	28	0.143
104	A	4	4	0.55	28	0.143
105	A	4	4	0.69	28	0.143
106	A	3	3	1.00	28	0.107
107	A	3	3	1.00	28	0.107
108	A	3	3	1.00	28	0.107
109	A	2	2	1.00	26	0.077
110	A	5	4	0.95	24	0.167
111	A	5	4	0.98	24	0.167
112	A	5	4	1.08	24	0.167
113	A	2	2	1.00	24	0.083
114	A	2	2	1.00	24	0.083
115	A	4	3	1.10	24	0.125
116	A	2	2	1.03	22	0.091
117	A	2	2	1.04	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
118	A	4	3	1.00	24	0.125
119	A	2	2	1.00	24	0.083
120	A	2	2	1.00	24	0.083
121	A	4	3	1.05	24	0.125
122	A	2	2	1.00	24	0.083
123	A	4	3	0.99	18	0.167
124	A	6	5	1.02	18	0.278
125	A	4	3	1.00	18	0.167
126	A	8	7	1.06	18	0.389
127	A	6	5	1.07	18	0.278
128	A	13	12	0.86	18	0.667
129	A	12	11	0.83	18	0.611
130	A	11	10	0.92	18	0.556
131	A	11	10	0.89	18	0.556
132	A	11	10	0.92	16	0.625
133	A	11	10	0.87	14	0.714
134	A	13	12	0.87	18	0.667
135	A	13	12	0.84	18	0.667
136	A	4	3	1.06	16	0.188
137	A	4	3	1.00	16	0.188
138	A	4	3	1.19	16	0.188
139	B	4	3	2.50	16	0.188
140	A	4	3	1.22	16	0.188
141	A	4	3	1.18	16	0.188
142	A	4	3	1.15	16	0.188
143	A	15	14	1.14	16	0.875
144	A	14	13	1.14	16	0.812
145	A	13	12	1.13	16	0.750
146	A	12	11	1.14	16	0.688
147	A	11	10	1.15	16	0.625
148	A	11	10	1.15	16	0.625
149	A	11	10	1.14	14	0.714

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	11	10	1.14	12	0.833
151	A	13	12	1.18	16	0.750
152	A	13	12	1.18	16	0.750
153	A	14	13	1.21	16	0.812
154	A	15	14	1.21	16	0.875
155	A	10	9	0.85	16	0.562
156	A	7	6	1.03	16	0.375
157	A	9	8	0.89	16	0.500
158	A	9	8	0.85	16	0.500
159	A	4	3	1.00	16	0.188
160	A	9	8	0.95	14	0.571
161	C	9	8	1.81	12	0.667
162	A	8	7	1.07	16	0.438
163	A	10	9	0.89	16	0.562
164	A	11	10	0.85	16	0.625
165	A	5	4	1.02	16	0.250
166	A	12	11	0.90	16	0.688
167	A	9	8	0.89	10	0.800
168	A	4	3	1.00	14	0.214
169	A	9	8	0.88	14	0.571
170	A	10	9	1.03	20	0.450
171	A	8	7	1.05	20	0.350
172	A	8	7	1.07	20	0.350
173	A	6	5	1.10	20	0.250
174	A	5	4	1.05	20	0.200
175	A	9	8	1.00	20	0.400
176	A	8	7	1.03	20	0.350
177	A	5	4	1.05	20	0.200
178	A	6	5	1.09	20	0.250
179	A	8	7	1.06	20	0.350
180	A	10	9	1.09	20	0.450
181	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	2	2	1.00	18	0.111
183	A	2	2	1.00	16	0.125
184	A	2	2	1.00	20	0.100
185	A	2	2	1.00	20	0.100
186	A	11	10	0.97	20	0.500
187	A	9	8	1.02	20	0.400
188	A	9	8	1.03	20	0.400
189	A	7	6	1.11	20	0.300
190	A	6	5	1.08	20	0.250
191	A	11	10	1.09	20	0.500
192	A	11	10	1.05	20	0.500
193	A	10	9	1.05	20	0.450
194	A	10	9	1.09	20	0.450
195	A	6	5	1.08	20	0.250
196	A	7	6	1.10	20	0.300
197	A	9	8	1.03	20	0.400
198	A	11	10	1.05	20	0.500
199	A	2	2	1.00	20	0.100
200	A	2	2	1.00	18	0.111
201	A	2	2	1.00	16	0.125
202	A	2	2	1.00	20	0.100
203	A	2	2	1.00	20	0.100
204	A	9	8	1.12	20	0.400
205	A	7	6	1.10	20	0.300
206	A	7	6	1.10	20	0.300
207	A	5	4	1.01	20	0.200
208	A	4	3	1.00	20	0.150
209	A	4	3	1.00	20	0.150
210	A	5	4	1.03	20	0.200
211	A	7	6	1.09	20	0.300
212	A	9	8	1.12	20	0.400
213	A	11	10	1.14	20	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	2	2	1.00	20	0.100
215	A	2	2	1.00	18	0.111
216	A	2	2	1.00	16	0.125
217	A	2	2	1.00	20	0.100
218	A	2	2	1.00	20	0.100
219	A	9	8	1.06	20	0.400
220	A	7	6	1.09	20	0.300
221	A	6	5	1.08	20	0.250
222	A	3	2	1.00	20	0.100
223	A	3	2	1.00	20	0.100
224	A	6	5	1.00	20	0.250
225	A	7	6	1.09	20	0.300
226	A	9	8	1.06	20	0.400
227	A	11	10	1.04	20	0.500
228	A	2	2	1.00	20	0.100
229	A	2	2	1.00	18	0.111
230	A	2	2	1.00	16	0.125
231	A	2	2	1.00	20	0.100
232	A	2	2	1.00	20	0.100
233	A	2	2	1.00	20	0.100
234	A	2	2	1.00	18	0.111
235	A	3	3	1.00	20	0.150
236	A	5	5	1.02	20	0.250
237	A	2	2	1.00	22	0.091
238	A	2	2	1.00	22	0.091
239	A	2	2	1.00	22	0.091
240	A	2	2	1.00	22	0.091
241	A	2	2	1.00	20	0.100
242	A	6	5	0.99	18	0.278
243	A	4	3	1.01	18	0.167
244	A	3	2	1.00	18	0.111
245	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	2	2	1.00	18	0.111
247	A	2	2	1.00	16	0.125
248	A	2	2	1.00	14	0.143
249	A	4	3	1.00	18	0.167
250	A	2	2	1.00	18	0.111
251	A	2	2	1.00	18	0.111
252	A	4	3	0.99	18	0.167
253	A	2	2	1.00	18	0.111
254	A	2	2	1.00	18	0.111
255	A	4	3	0.99	18	0.167

CHAPTER 3

Listing of integrals

3.1 $\int x^8\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^8\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}+\frac{bx^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12(a+bx^3)}$$

output
```
a*x^9*((b*x^3+a)^2)^(1/2)/(9*b*x^3+9*a)+b*x^12*((b*x^3+a)^2)^(1/2)/(12*b*x
^3+12*a)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

$$\int x^8\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{x^9(4a+3bx^3)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+bx^3)^2}\right)\right)}{36\left(-a^2-abx^3+\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}$$

input
```
Integrate[x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(x^9*(4*a + 3*b*x^3)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2]
)))/(36*(-a^2 - a*b*x^3 + Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \sqrt{a^2+2abx^3+b^2x^6}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int bx^8\left(bx^3+a\right)dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^8\left(bx^3+a\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(bx^{11}+ax^8\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{ax^9}{9}+\frac{bx^{12}}{12}\right)}{a+bx^3}$$

input
```
Int[x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a*x^9)/9 + (b*x^12)/12))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x^{9} \left(3 b \,x^{3}+4 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{36 b \,x^{3}+36 a}$	36
default	$\frac{x^{9} \left(3 b \,x^{3}+4 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{36 b \,x^{3}+36 a}$	36
orering	$\frac{x^{9} \left(3 b \,x^{3}+4 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{36 b \,x^{3}+36 a}$	36
pseudoelliptic	$\frac{\operatorname{csgn}\left(b \,x^{3}+a \right) \left(b \,x^{3}+a \right)^{2} \left(3 b^{2} x^{6}-2 a \,x^{3} b +a^{2}\right)}{36 b^{3}}$	42
risch	$\frac{a \,x^{9} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{9 b \,x^{3}+9 a}+\frac{b \,x^{12} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{12 b \,x^{3}+12 a}$	54

input
```
int(x^8*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/36*x^9*(3*b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^8\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{12}\,bx^{12}+\frac{1}{9}\,ax^9$$

input
```
integrate(x^8*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*b*x^12 + 1/9*a*x^9
```

Sympy [F(-1)]

Timed out.

$$\int x^8\sqrt{a^2+2abx^3+b^2x^6}\,dx=\text{Timed out}$$

input
```
integrate(x**8*((b*x**3+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(53) = 106.

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.44

$$\int x^8\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{b^2x^6+2\,abx^3+a^2}a^2x^3}{6\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}x^3}{12\,b^2}+\frac{\sqrt{b^2x^6+2\,abx^3+a^2}a^3}{6\,b^3}-\frac{5\,\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a}{36\,b^3}$$

input
```
integrate(x^8*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*x^3/b^2 + 1/12*(b^2*x^6 + 2*a*b*x^
3 + a^2)^(3/2)*x^3/b^2 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3/b^3 - 5/3
6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x^8\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{12}\,bx^{12}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{9}\,ax^9\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^8*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/12*b*x^12*sgn(b*x^3 + a) + 1/9*a*x^9*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 22.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int x^8\sqrt{a^2+2abx^3+b^2x^6}\,dx$$
$$=\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}\,\left(a^3-4\,a^2\,b\,x^3-5\,a\,b^2\,x^6+3\,b\,x^3\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)\right)}{36\,b^3}$$

input
```
int(x^8*((a + b*x^3)^2)^(1/2),x)
```

output
```
((a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)*(a^3 - 4*a^2*b*x^3 - 5*a*b^2*x^6 + 3*b*
x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)))/(36*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int x^8\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{x^9(3b\,x^3+4a)}{36}$$

input
```
int(x^8*((b*x^3+a)^2)^(1/2),x)
```

output
```
(x**9*(4*a + 3*b*x**3))/36
```

3.2 $\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=-\frac{a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b^2}+\frac{(a^2+2abx^3+b^2x^6)^{3/2}}{9b^2}$$

output
```
-1/6*a*(b*x^3+a)*((b*x^3+a)^2)^(1/2)/b^2+1/9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)
/b^2
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{(a+bx^3)^2}(3ax^6+2bx^9)}{18(a+bx^3)}$$

input
```
Integrate[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(3*a*x^6 + 2*b*x^9))/(18*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^3\sqrt{b^2x^6+2abx^3+a^2}dx^3$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{3b^2}-\frac{a\int\sqrt{b^2x^6+2abx^3+a^2}dx^3}{b}\right)$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{3b^2}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\int\left(b^2x^3+ab\right)dx^3}{b^2\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{3b^2}-\frac{a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{2b^2}\right)$$

input
```
Int[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(-1/2*(a*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/b^2 + (a^2 + 2*a*b*x
^3 + b^2*x^6)^(3/2)/(3*b^2))/3
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{2}\left(-2b\,x^{3}+a\right)}{18b^{2}}$	31
gosper	$\frac{x^{6}\left(2b\,x^{3}+3a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{18b\,x^{3}+18a}$	36
default	$\frac{x^{6}\left(2b\,x^{3}+3a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{18b\,x^{3}+18a}$	36
orering	$\frac{x^{6}\left(2b\,x^{3}+3a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{18b\,x^{3}+18a}$	36
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\,x^{9}}{9b\,x^{3}+9a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a\,x^{6}}{6b\,x^{3}+6a}$	54

input
```
int(x^5*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*csgn(b*x^3+a)*(b*x^3+a)^2*(-2*b*x^3+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.19

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{9}\,bx^9+\frac{1}{6}\,ax^6$$

input
```
integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/9*b*x^9 + 1/6*a*x^6
```

Sympy [F(-1)]

Timed out.

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\text{Timed out}$$

input
```
integrate(x**5*((b*x**3+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx = -\frac{\sqrt{b^2x^6+2\,abx^3+a^2}ax^3}{6\,b} - \frac{\sqrt{b^2x^6+2\,abx^3+a^2}a^2}{6\,b^2} + \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}}{9\,b^2}$$

input
```
integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*x^3/b - 1/6*sqrt(b^2*x^6 + 2*a*b*x^
3 + a^2)*a^2/b^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.34

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{1}{18}\left(2\,bx^9+3\,ax^6\right)\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/18*(2*b*x^9 + 3*a*x^6)*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 21.88 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}\,\left(8\,b^2\,\left(a^2+b^2\,x^6\right)-12\,a^2\,b^2+4\,a\,b^3\,x^3\right)}{72\,b^4}$$

input
```
int(x^5*((a + b*x^3)^2)^(1/2),x)
```

output
```
((a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)*(8*b^2*(a^2 + b^2*x^6) - 12*a^2*b^2 + 4
*a*b^3*x^3))/(72*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.22

$$\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{x^6(2b\,x^3+3a)}{18}$$

input
```
int(x^5*((b*x^3+a)^2)^(1/2),x)
```

output
```
(x**6*(3*a + 2*b*x**3))/18
```

3.3 $\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 36

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

output
```
1/6*(b*x^3+a)*((b*x^3+a)^2)^(1/2)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 88 vs. 2(36) = 72.

Time = 0.45 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.44

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{x^3(2a+bx^3)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+bx^3)^2}\right)\right)}{-6a^2-6abx^3+6\sqrt{a^2}\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(x^3*(2*a + b*x^3)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2]))
)/(-6*a^2 - 6*a*b*x^3 + 6*Sqrt[a^2]*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1690, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a^2+2abx^3+b^2x^6}\,dx$$

$$\Big\downarrow \ 1690$$

$$\frac{1}{3}\int \sqrt{b^2x^6+2abx^3+a^2}dx^3$$

$$\Big\downarrow \ 1079$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)dx^3}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \ 17$$

$$\frac{\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

input
```
Int[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{\left(b\,x^{3}+a\right)^{2} \operatorname{csgn}\left(b\,x^{3}+a\right)}{6 b}$	23
default	$\frac{\left(b\,x^{3}+a\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{6 b}$	24
risch	$\frac{\left(b\,x^{3}+a\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{6 b}$	24
gosper	$\frac{x^{3} \left(b\,x^{3}+2 a\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{6 b\,x^{3}+6 a}$	35
orering	$\frac{x^{3} \left(b\,x^{3}+2 a\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{6 b\,x^{3}+6 a}$	35

input
```
int(x^2*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(b*x^3+a)^2*csgn(b*x^3+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.36

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{6}\,bx^6+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*b*x^6 + 1/3*a*x^3
```

Sympy [F]

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int x^2\sqrt{(a+bx^3)^2}\,dx$$

input
```
integrate(x**2*((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt((a + b*x**3)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(23) = 46$.

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.44

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{6}\sqrt{b^2x^6+2\,abx^3+a^2}x^3+\frac{\sqrt{b^2x^6+2\,abx^3+a^2}a}{6\,b}$$

input
```
integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*x^3 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a
^2)*a/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{6}\left(bx^6+2\,ax^3\right)\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*(b*x^6 + 2*a*x^3)*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 21.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx = \left(\frac{a}{6\,b}+\frac{x^3}{6}\right)\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

input
```
int(x^2*((a + b*x^3)^2)^(1/2),x)
```

output
```
(a/(6*b) + x^3/6)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.39

$$\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{x^3(b\,x^3+2a)}{6}$$

input
```
int(x^2*((b*x^3+a)^2)^(1/2),x)
```

output
```
(x**3*(2*a + b*x**3))/6
```

3.4 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx=\frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
b*x^3*((b*x^3+a)^2)^(1/2)/(3*b*x^3+3*a)+a*((b*x^3+a)^2)^(1/2)*ln(x)/(b*x^3
+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 454 vs. $2(75) = 150$.

Time = 1.01 (sec) , antiderivative size = 454, normalized size of antiderivative = 6.05

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx$$
$$=\frac{-2a\sqrt{a^2}bx^3-2\sqrt{a^2}b^2x^6+2abx^3\sqrt{\left(a+bx^3\right)^2}-2a\left(a^2+abx^3-\sqrt{a^2}\sqrt{\left(a+bx^3\right)^2}\right)\operatorname{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+\cdots}}\right.}{}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

output
```
(-2*a*Sqrt[a^2]*b*x^3 - 2*Sqrt[a^2]*b^2*x^6 + 2*a*b*x^3*Sqrt[(a + b*x^3)^2
] - 2*a*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2])*ArcTanh[(b*x^3)/(S
qrt[a^2] - Sqrt[(a + b*x^3)^2])] - 2*((a^2)^(3/2) + a*Sqrt[a^2]*b*x^3 - a^
2*Sqrt[(a + b*x^3)^2])*Log[x^3] + (a^2)^(3/2)*Log[Sqrt[a^2] - b*x^3 - Sqrt
[(a + b*x^3)^2]] + a*Sqrt[a^2]*b*x^3*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x
^3)^2]] - a^2*Sqrt[(a + b*x^3)^2]*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)
^2]] + (a^2)^(3/2)*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] + a*Sqrt[a
^2]*b*x^3*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] - a^2*Sqrt[(a + b*x
^3)^2]*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]])/(6*(a^2 + a*b*x^3 - S
qrt[a^2]*Sqrt[(a + b*x^3)^2]))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x} \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \frac{b(bx^3+a)}{x}dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \frac{bx^3+a}{x}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(bx^2+\frac{a}{x}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a\log(x)+\frac{bx^3}{3}\right)}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((b*x^3)/3 + a*Log[x]))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a+a\ln\left(b\,x^{3}\right)\right)}{3}$	26
default	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(b\,x^{3}+3a\ln\left(x\right)\right)}{3b\,x^{3}+3a}$	34
risch	$\frac{b\,x^{3}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{3b\,x^{3}+3a}+\frac{a\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(x\right)}{b\,x^{3}+a}$	52

input
```
int(((b*x^3+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*csgn(b*x^3+a)*(b*x^3+a+a*ln(b*x^3))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \frac{1}{3}\,bx^3 + a\log\left(x\right)$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/3*b*x^3 + a*log(x)
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \int \frac{\sqrt{\left(a+bx^3\right)^2}}{x}\,dx$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x,x)
```

output
```
Integral(sqrt((a + b*x**3)**2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}a\log\left(2\,b^2x^3+2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}a\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{1}{3}\,\sqrt{b^2x^6+2\,abx^3+a^2}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/3*(-1)^(2*b^2*x^3 + 2*a*b)*a*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^
3 + 2*a^2)*a*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/3*sqrt(b^2*x^6 +
 2*a*b*x^3 + a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \frac{1}{3}\,bx^3\mathrm{sgn}\left(bx^3+a\right) + a\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/3*b*x^3*sgn(b*x^3 + a) + a*log(abs(x))*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 21.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3} - \frac{\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)\sqrt{a^2}}{3} + \frac{a\,b\,\ln\left(a\,b+\sqrt{\left(b\,x^3+a\right)^2}\,\sqrt{b^2}+b^2\,x^3\right)}{3\,\sqrt{b^2}}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x,x)
```

output
```
(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/3 - (log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a
^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)*(a^2)^(1/2))/3 + (a*b*log(a*b + ((a
+ b*x^3)^2)^(1/2)*(b^2)^(1/2) + b^2*x^3))/(3*(b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \log(x)\,a+\frac{b\,x^3}{3}$$

input
```
int(((b*x^3+a)^2)^(1/2)/x,x)
```

output
```
(3*log(x)*a + b*x**3)/3
```

3.5 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\,(a+bx^3)}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/3*a*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+b*((b*x^3+a)^2)^(1/2)*ln(x)/(b*x^
3+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 178 vs. 2(75) = 150.

Time = 0.39 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.37

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx$$
$$= \frac{a\sqrt{a^2}-a\sqrt{(a+bx^3)^2}-2abx^3\text{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+bx^3)^2}}\right)-2\sqrt{a^2}bx^3\log\left(x^3\right)+\sqrt{a^2}bx^3\log\left(a\left(\sqrt{a^2}-bx\right.\right.}{6ax^3}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

output
```
(a*Sqrt[a^2] - a*Sqrt[(a + b*x^3)^2] - 2*a*b*x^3*ArcTanh[(b*x^3)/(Sqrt[a^2
] - Sqrt[(a + b*x^3)^2])] - 2*Sqrt[a^2]*b*x^3*Log[x^3] + Sqrt[a^2]*b*x^3*L
og[a*(Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2])] + Sqrt[a^2]*b*x^3*Log[a*(S
qrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2])])/(6*a*x^3)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4} dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \frac{b(bx^3+a)}{x^4}dx}{b(a+bx^3)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \frac{bx^3+a}{x^4}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(\frac{a}{x^4}+\frac{b}{x}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(b\log(x)-\frac{a}{3x^3}\right)}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/3*a/x^3 + b*Log[x]))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.37

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-\ln\left(b\,x^{3}\right) b\,x^{3}+a\right)}{3 x^{3}}$	28
default	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(3 b \ln\left(x\right) x^{3}-a\right)}{3 x^{3} \left(b\,x^{3}+a\right)}$	38
risch	$-\frac{a \sqrt{\left(b\,x^{3}+a\right)^{2}}}{3 x^{3} \left(b\,x^{3}+a\right)}+\frac{b \sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(x\right)}{b\,x^{3}+a}$	52

input
```
int(((b*x^3+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*csgn(b*x^3+a)*(-ln(b*x^3)*b*x^3+a)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx=\frac{3\,bx^3\log\left(x\right)-a}{3\,x^3}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*b*x^3*log(x) - a)/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx=\text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx=\frac{1}{3}\left(-1\right)^{2\,b^2x^3+2\,ab}b\log\left(2\,b^2x^3+2\,ab\right)-\frac{1}{3}\left(-1\right)^{2\,abx^3+2\,a^2}b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}}{3\,x^3}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/3*(-1)^(2*b^2*x^3 + 2*a*b)*b*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^
3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) - 1/3*sqrt(b^2*x^6 +
 2*a*b*x^3 + a^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = b\log\left(|x|\right)\operatorname{sgn}\left(bx^3+a\right) - \frac{bx^3\operatorname{sgn}(bx^3+a)+a\operatorname{sgn}(bx^3+a)}{3\,x^3}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
b*log(abs(x))*sgn(b*x^3 + a) - 1/3*(b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a
))/x^3
```

Mupad [B] (verification not implemented)

Time = 21.74 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = \frac{\ln\left(a\,b+\sqrt{\left(b\,x^3+a\right)^2}\,\sqrt{b^2}+b^2\,x^3\right)\,\sqrt{b^2}}{3} - \frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3} - \frac{a\,b\,\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)}{3\,\sqrt{a^2}}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^4,x)
```

output
```
(log(a*b + ((a + b*x^3)^2)^(1/2)*(b^2)^(1/2) + b^2*x^3)*(b^2)^(1/2))/3 - (
a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*x^3) - (a*b*log(a*b + a^2/x^3 + ((a^2)
^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3))/(3*(a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^4}\,dx = \frac{3\log(x)\,b\,x^3 - a}{3x^3}$$

input
```
int(((b*x^3+a)^2)^(1/2)/x^4,x)
```

output
```
(3*log(x)*b*x**3 - a)/(3*x**3)
```

3.6 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7} dx$

Optimal result

Integrand size = 26, antiderivative size = 39

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx=-\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6ax^6}$$

output
```
-1/6*(b*x^3+a)*((b*x^3+a)^2)^(1/2)/a/x^6
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx=-\frac{\sqrt{(a+bx^3)^2}(a+2bx^3)}{6x^6(a+bx^3)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

output
```
-1/6*(Sqrt[(a + b*x^3)^2]*(a + 2*b*x^3))/(x^6*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1693, 1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{\sqrt{b^2x^6+2abx^3+a^2}}{x^9}dx^3$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^9}dx^3}{3b(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^9}dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \text{48}$$

$$-\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6ax^6}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

output
```
-1/6*((a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right) \left(2 b\,x^{3}+a\right)}{6 x^{6}}$	22
gosper	$-\frac{\left(2 b\,x^{3}+a\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{6 x^{6} \left(b\,x^{3}+a\right)}$	34
default	$-\frac{\left(2 b\,x^{3}+a\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{6 x^{6} \left(b\,x^{3}+a\right)}$	34
orering	$-\frac{\left(2 b\,x^{3}+a\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{6 x^{6} \left(b\,x^{3}+a\right)}$	34
risch	$\frac{\left(-\frac{b\,x^{3}}{3}-\frac{a}{6}\right) \sqrt{\left(b\,x^{3}+a\right)^{2}}}{x^{6} \left(b\,x^{3}+a\right)}$	35

input
```
int(((b*x^3+a)^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*csgn(b*x^3+a)*(2*b*x^3+a)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx=-\frac{2\,bx^3+a}{6\,x^6}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
-1/6*(2*b*x^3 + a)/x^6
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx=\text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**7,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(26) = 52$.

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^7}\,dx = \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^2}{6\,a^2} + \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}b}{6\,ax^3} - \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}}{6\,a^2x^6}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2/a^2 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
 + a^2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/(a^2*x^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^7}\,dx = -\frac{2\,bx^3\mathrm{sgn}(bx^3 + a) + a\mathrm{sgn}(bx^3 + a)}{6\,x^6}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="giac")
```

output
```
-1/6*(2*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^6
```

Mupad [B] (verification not implemented)

Time = 23.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^7}\,dx = -\frac{\left(2\,b\,x^3 + a\right)\,\sqrt{{\left(b\,x^3 + a\right)}^2}}{6\,x^6\,\left(b\,x^3 + a\right)}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^7,x)
```

output
```
-((a + 2*b*x^3)*((a + b*x^3)^2)^(1/2))/(6*x^6*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^7}\,dx = \frac{-2b\,x^3 - a}{6x^6}$$

input
```
int(((b*x^3+a)^2)^(1/2)/x^7,x)
```

output
```
( - a - 2*b*x**3)/(6*x**6)
```

3.7 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

output
```
-1/9*a*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-1/6*b*((b*x^3+a)^2)^(1/2)/x^6/(b*
x^3+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx=-\frac{\sqrt{\left(a+bx^3\right)^2}\left(2a+3bx^3\right)}{18x^9\left(a+bx^3\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

output
```
-1/18*(Sqrt[(a + b*x^3)^2]*(2*a + 3*b*x^3))/(x^9*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^{10}}dx}{b(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^{10}}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^{10}}+\frac{b}{x^7}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{9x^9}-\frac{b}{6x^6}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

output
```
((-1/9*a/x^9 - b/(6*x^6))*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(3 b\,x^{3}+2 a\right)}{18 x^{9}}$	24
risch	$\frac{\left(-\frac{b\,x^{3}}{6}-\frac{a}{9}\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{x^{9}\left(b\,x^{3}+a\right)}$	35
gosper	$-\frac{\left(3 b\,x^{3}+2 a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{18 x^{9}\left(b\,x^{3}+a\right)}$	36
default	$-\frac{\left(3 b\,x^{3}+2 a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{18 x^{9}\left(b\,x^{3}+a\right)}$	36
orering	$-\frac{\left(3 b\,x^{3}+2 a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{18 x^{9}\left(b\,x^{3}+a\right)}$	36

input
```
int(((b*x^3+a)^2)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/18*csgn(b*x^3+a)*(3*b*x^3+2*a)/x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10}}\,dx = -\frac{3\,bx^3 + 2\,a}{18\,x^9}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="fricas")
```

output
```
-1/18*(3*b*x^3 + 2*a)/x^9
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10}}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**10,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(53) = 106.

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10}}\,dx = -\frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^3}{6\,a^3} - \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^2}{6\,a^2x^3} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b}{6\,a^3x^6} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}}{9\,a^2x^9}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="maxima")
```

output
```
-1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3/a^3 - 1/6*sqrt(b^2*x^6 + 2*a*b*x^
3 + a^2)*b^2/(a^2*x^3) + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b/(a^3*x^6)
 - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/(a^2*x^9)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx=-\frac{3\,bx^3\mathrm{sgn}(bx^3+a)+2\,a\mathrm{sgn}(bx^3+a)}{18\,x^9}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="giac")
```

output
```
-1/18*(3*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^9
```

Mupad [B] (verification not implemented)

Time = 23.75 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx=-\frac{\left(3\,b\,x^3+2\,a\right)\,\sqrt{{\left(b\,x^3+a\right)}^2}}{18\,x^9\,\left(b\,x^3+a\right)}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^10,x)
```

output
```
-((2*a + 3*b*x^3)*((a + b*x^3)^2)^(1/2))/(18*x^9*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10}}\,dx = \frac{-3b\,x^3 - 2a}{18x^9}$$

input
```
int(((b*x^3+a)^2)^(1/2)/x^10,x)
```

output
```
( - 2*a - 3*b*x**3)/(18*x**9)
```

3.8 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{13}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{13}}\,dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\,(a+bx^3)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\,(a+bx^3)}$$

output
```
-1/12*a*((b*x^3+a)^2)^(1/2)/x^12/(b*x^3+a)-1/9*b*((b*x^3+a)^2)^(1/2)/x^9/(
b*x^3+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{13}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(3a+4bx^3)}{36x^{12}\,(a+bx^3)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^13,x]
```

output
```
-1/36*(Sqrt[(a + b*x^3)^2]*(3*a + 4*b*x^3))/(x^12*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{13}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^{13}}dx}{b(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^{13}}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^{13}}+\frac{b}{x^{10}}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{12x^{12}}-\frac{b}{9x^9}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^13,x]
```

output
```
((-1/12*a/x^12 - b/(9*x^9))*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(4b\,x^{3}+3a\right)}{36x^{12}}$	24
risch	$\frac{\left(-\frac{b\,x^{3}}{9}-\frac{a}{12}\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{x^{12}\left(b\,x^{3}+a\right)}$	35
gosper	$-\frac{\left(4b\,x^{3}+3a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{36x^{12}\left(b\,x^{3}+a\right)}$	36
default	$-\frac{\left(4b\,x^{3}+3a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{36x^{12}\left(b\,x^{3}+a\right)}$	36
orering	$-\frac{\left(4b\,x^{3}+3a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{36x^{12}\left(b\,x^{3}+a\right)}$	36

input
```
int(((b*x^3+a)^2)^(1/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/36*csgn(b*x^3+a)*(4*b*x^3+3*a)/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{13}}\,dx = -\frac{4\,bx^3 + 3\,a}{36\,x^{12}}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^13,x, algorithm="fricas")
```

output
```
-1/36*(4*b*x^3 + 3*a)/x^12
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{13}}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**13,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(53) = 106.

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{13}}\,dx = \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^4}{6\,a^4} + \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^3}{6\,a^3x^3} - \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}b^2}{6\,a^4x^6} + \frac{5\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}b}{36\,a^3x^9} - \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}}{12\,a^2x^{12}}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^13,x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4/a^4 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
 + a^2)*b^3/(a^3*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2/(a^4*x^6
) + 5/36*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b/(a^3*x^9) - 1/12*(b^2*x^6 + 2
*a*b*x^3 + a^2)^(3/2)/(a^2*x^12)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{13}}\,dx = -\frac{4\,bx^3\mathrm{sgn}(bx^3+a)+3\,a\mathrm{sgn}(bx^3+a)}{36\,x^{12}}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^13,x, algorithm="giac")
```

output
```
-1/36*(4*b*x^3*sgn(b*x^3 + a) + 3*a*sgn(b*x^3 + a))/x^12
```

Mupad [B] (verification not implemented)

Time = 21.85 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{13}}\,dx = -\frac{(4\,b\,x^3+3\,a)\,\sqrt{(b\,x^3+a)^2}}{36\,x^{12}\,(b\,x^3+a)}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^13,x)
```

output
```
-((3*a + 4*b*x^3)*((a + b*x^3)^2)^(1/2))/(36*x^12*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{13}}\, dx = \frac{-4b\, x^3 - 3a}{36x^{12}}$$

input
```
int(((b*x^3+a)^2)^(1/2)/x^13,x)
```

output
```
( - 3*a - 4*b*x**3)/(36*x**12)
```

3.9 $\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

output
```
a*x^5*((b*x^3+a)^2)^(1/2)/(5*b*x^3+5*a)+b*x^8*((b*x^3+a)^2)^(1/2)/(8*b*x^3
+8*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{\left(a+bx^3\right)^2}\left(8ax^5+5bx^8\right)}{40\left(a+bx^3\right)}$$

input
```
Integrate[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(8*a*x^5 + 5*b*x^8))/(40*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int bx^4\left(bx^3+a\right)dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4\left(bx^3+a\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(bx^7+ax^4\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{ax^5}{5}+\frac{bx^8}{8}\right)}{a+bx^3}$$

input
```
Int[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a*x^5)/5 + (b*x^8)/8))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x^{5}\left(5 b\,x^{3}+8 a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{40 b\,x^{3}+40 a}$	36
default	$\frac{x^{5}\left(5 b\,x^{3}+8 a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{40 b\,x^{3}+40 a}$	36
orering	$\frac{x^{5}\left(5 b\,x^{3}+8 a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{40 b\,x^{3}+40 a}$	36
risch	$\frac{a\,x^{5}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{5 b\,x^{3}+5 a}+\frac{b\,x^{8}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{8 b\,x^{3}+8 a}$	54

input
```
int(x^4*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*x^5*(5*b*x^3+8*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

input
```
integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*b*x^8 + 1/5*a*x^5
```

Sympy [F(-1)]

Timed out.

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\text{Timed out}$$

input
```
integrate(x**4*((b*x**3+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

input
```
integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/8*b*x^8 + 1/5*a*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{8}\,bx^8\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,ax^5\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*b*x^8*sgn(b*x^3 + a) + 1/5*a*x^5*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int x^4\,\sqrt{{\left(b\,x^3+a\right)}^2}\,dx$$

input
```
int(x^4*((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x^4*((a + b*x^3)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{x^5\left(5b\,x^3+8a\right)}{40}$$

input
```
int(x^4*((b*x^3+a)^2)^(1/2),x)
```

output
```
(x**5*(8*a + 5*b*x**3))/40
```

3.10 $\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}$$

output
```
a*x^4*((b*x^3+a)^2)^(1/2)/(4*b*x^3+4*a)+b*x^7*((b*x^3+a)^2)^(1/2)/(7*b*x^3
+7*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{(a+bx^3)^2}(7ax^4+4bx^7)}{28(a+bx^3)}$$

input
```
Integrate[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(7*a*x^4 + 4*b*x^7))/(28*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int bx^3\left(bx^3+a\right)dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(bx^3+a\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(bx^6+ax^3\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{ax^4}{4}+\frac{bx^7}{7}\right)}{a+bx^3}$$

input
```
Int[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a*x^4)/4 + (b*x^7)/7))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x^{4}\left(4b\,x^{3}+7a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{28b\,x^{3}+28a}$	36
default	$\frac{x^{4}\left(4b\,x^{3}+7a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{28b\,x^{3}+28a}$	36
orering	$\frac{x^{4}\left(4b\,x^{3}+7a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{28b\,x^{3}+28a}$	36
risch	$\frac{a\,x^{4}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{4b\,x^{3}+4a}+\frac{b\,x^{7}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{7b\,x^{3}+7a}$	54

input
```
int(x^3*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/28*x^4*(4*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{7}\,bx^7+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/7*b*x^7 + 1/4*a*x^4
```

Sympy [F(-1)]

Timed out.

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\text{Timed out}$$

input
```
integrate(x**3*((b*x**3+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{7}\,bx^7+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/7*b*x^7 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{1}{7}\,bx^7\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,ax^4\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/7*b*x^7*sgn(b*x^3 + a) + 1/4*a*x^4*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx = \int x^3\,\sqrt{\left(b\,x^3+a\right)^2}\,dx$$

input
```
int(x^3*((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x^3*((a + b*x^3)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{x^4(4b\,x^3+7a)}{28}$$

input
```
int(x^3*((b*x^3+a)^2)^(1/2),x)
```

output
```
(x**4*(7*a + 4*b*x**3))/28
```

3.11 $\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

output
```
a*x^2*((b*x^3+a)^2)^(1/2)/(2*b*x^3+2*a)+b*x^5*((b*x^3+a)^2)^(1/2)/(5*b*x^3
+5*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{\left(a+bx^3\right)^2}\left(5ax^2+2bx^5\right)}{10\left(a+bx^3\right)}$$

input
```
Integrate[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(5*a*x^2 + 2*b*x^5))/(10*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int bx\left(bx^3+a\right)dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(bx^3+a\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(bx^4+ax\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{ax^2}{2}+\frac{bx^5}{5}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(((a*x^2)/2 + (b*x^5)/5)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x^{2} \left(2 b \,x^{3}+5 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{10 b \,x^{3}+10 a}$	36
default	$\frac{x^{2} \left(2 b \,x^{3}+5 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{10 b \,x^{3}+10 a}$	36
orering	$\frac{x^{2} \left(2 b \,x^{3}+5 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{10 b \,x^{3}+10 a}$	36
risch	$\frac{a \,x^{2} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 b \,x^{3}+2 a}+\frac{b \,x^{5} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{5 b \,x^{3}+5 a}$	54

input
```
int(x*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*x^2*(2*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

input
```
integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/5*b*x^5 + 1/2*a*x^2
```

Sympy [F]

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\int x\sqrt{(a+bx^3)^2}\,dx$$

input
```
integrate(x*((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x*sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

input
```
integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/5*b*x^5 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{1}{5}\,bx^5\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,ax^2\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/5*b*x^5*sgn(b*x^3 + a) + 1/2*a*x^2*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx = \int x\,\sqrt{{\left(b\,x^3+a\right)}^2}\,dx$$

input
```
int(x*((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x*((a + b*x^3)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{x^2(2b\,x^3+5a)}{10}$$

input
```
int(x*((b*x^3+a)^2)^(1/2),x)
```

output
```
(x**2*(5*a + 2*b*x**3))/10
```

3.12 $\int \sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

output
```
a*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+b*x^4*((b*x^3+a)^2)^(1/2)/(4*b*x^3+4*a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{\sqrt{\left(a+bx^3\right)^2}\left(4ax+bx^4\right)}{4\left(a+bx^3\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(4*a*x + b*x^4))/(4*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(abx+\frac{b^2x^4}{4}\right)\sqrt{a^2+2abx^3+b^2x^6}}{b\left(a+bx^3\right)}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a*b*x + (b^2*x^4)/4)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(b*(a + b*x^3))
```

Defintions of rubi rules used

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.45

method	result	size
gosper	$\frac{x(b\,x^3+4a)\sqrt{(b\,x^3+a)^2}}{4b\,x^3+4a}$	33
default	$\frac{x(b\,x^3+4a)\sqrt{(b\,x^3+a)^2}}{4b\,x^3+4a}$	33
orering	$\frac{x(b\,x^3+4a)\sqrt{(b\,x^3+a)^2}}{4b\,x^3+4a}$	33
risch	$\frac{ax\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}+\frac{b\,x^4\sqrt{(b\,x^3+a)^2}}{4b\,x^3+4a}$	51

input
```
int(((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.14

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{1}{4}\,bx^4+ax$$

input
```
integrate(((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*b*x^4 + a*x
```

Sympy [F]

$$\int \sqrt{a^2 + 2abx^3 + b^2x^6}\, dx = \int \sqrt{(a + bx^3)^2}\, dx$$

input
```
integrate(((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.14

$$\int \sqrt{a^2 + 2abx^3 + b^2x^6}\, dx = \frac{1}{4}\, bx^4 + ax$$

input
```
integrate(((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*b*x^4 + a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.27

$$\int \sqrt{a^2 + 2abx^3 + b^2x^6}\, dx = \frac{1}{4}\left(bx^4 + 4\, ax\right) \mathrm{sgn}\left(bx^3 + a\right)$$

input
```
integrate(((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*(b*x^4 + 4*a*x)*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx=\int \sqrt{(b\,x^3+a)^2}\,dx$$

input
```
int(((a + b*x^3)^2)^(1/2),x)
```

output
```
int(((a + b*x^3)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.16

$$\int \sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{x(b\,x^3+4a)}{4}$$

input
```
int(((b*x^3+a)^2)^(1/2),x)
```

output
```
(x*(4*a + b*x**3))/4
```

3.13 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 77

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2} \, dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}+\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

output
```
-a*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+b*x^2*((b*x^3+a)^2)^(1/2)/(2*b*x^3+2*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2} \, dx = \frac{(-2a+bx^3)\sqrt{(a+bx^3)^2}}{2x(a+bx^3)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

output
```
((-2*a + b*x^3)*Sqrt[(a + b*x^3)^2])/(2*x*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^2}dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^2}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^2}+bx\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{bx^2}{2}-\frac{a}{x}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

output
```
((-(a/x) + (b*x^2)/2)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{\left(-b\,x^{3}+2a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2x\left(b\,x^{3}+a\right)}$	36
default	$-\frac{\left(-b\,x^{3}+2a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2x\left(b\,x^{3}+a\right)}$	36
orering	$-\frac{\left(-b\,x^{3}+2a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2x\left(b\,x^{3}+a\right)}$	36
risch	$-\frac{a\sqrt{\left(b\,x^{3}+a\right)^{2}}}{x\left(b\,x^{3}+a\right)}+\frac{b\,x^{2}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2b\,x^{3}+2a}$	54

input
```
int(((b*x^3+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2}\,dx = \frac{bx^3 - 2\,a}{2\,x}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
1/2*(b*x^3 - 2*a)/x
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2}\,dx = \int \frac{\sqrt{(a + bx^3)^2}}{x^2}\,dx$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a + b*x**3)**2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2}\,dx = \frac{bx^3 - 2\,a}{2\,x}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*(b*x^3 - 2*a)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2} \, dx = \frac{1}{2} bx^2 \text{sgn}\left(bx^3 + a\right) - \frac{a\text{sgn}(bx^3 + a)}{x}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
1/2*b*x^2*sgn(b*x^3 + a) - a*sgn(b*x^3 + a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2} \, dx = \int \frac{\sqrt{\left(b\,x^3 + a\right)^2}}{x^2} \, dx$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^2,x)
```

output
```
int(((a + b*x^3)^2)^(1/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2} \, dx = \frac{b\,x^3 - 2a}{2x}$$

input
```
int(((b*x^3+a)^2)^(1/2)/x^2,x)
```

output
```
( - 2*a + b*x**3)/(2*x)
```

3.14 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 74

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3} dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} + \frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

output
```
-1/2*a*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+b*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3} dx = -\frac{(a-2bx^3)\sqrt{(a+bx^3)^2}}{2x^2(a+bx^3)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
```

output
```
-1/2*((a - 2*b*x^3)*Sqrt[(a + b*x^3)^2])/(x^2*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^3}dx}{b(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^3}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^3}+b\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(bx-\frac{a}{2x^2}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
```

output
```
((-1/2*a/x^2 + b*x)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{\left(-2 b \,x^{3}+a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 x^{2} \left(b \,x^{3}+a \right)}$	34
default	$-\frac{\left(-2 b \,x^{3}+a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 x^{2} \left(b \,x^{3}+a \right)}$	34
orering	$-\frac{\left(-2 b \,x^{3}+a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 x^{2} \left(b \,x^{3}+a \right)}$	34
risch	$-\frac{a \sqrt{\left(b \,x^{3}+a \right)^{2}}}{2 x^{2} \left(b \,x^{3}+a \right)}+\frac{b x \sqrt{\left(b \,x^{3}+a \right)^{2}}}{b \,x^{3}+a}$	51

input
```
int(((b*x^3+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx=\frac{2\,bx^3-a}{2\,x^2}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*b*x^3 - a)/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx=\text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx=\frac{2\,bx^3-a}{2\,x^2}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
1/2*(2*b*x^3 - a)/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx = bx\text{sgn}\left(bx^3+a\right) - \frac{a\text{sgn}(bx^3+a)}{2\,x^2}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
b*x*sgn(b*x^3 + a) - 1/2*a*sgn(b*x^3 + a)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx = \int \frac{\sqrt{(b\,x^3+a)^2}}{x^3}\,dx$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^3,x)
```

output
```
int(((a + b*x^3)^2)^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3}\,dx = \frac{2b\,x^3-a}{2x^2}$$

input
```
int(((b*x^3+a)^2)^(1/2)/x^3,x)
```

output
```
( - a + 2*b*x**3)/(2*x**2)
```

3.15 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 77

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx = -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

output
```
-1/4*a*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-b*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx = -\frac{\sqrt{\left(a+bx^3\right)^2}\left(a+4bx^3\right)}{4x^4\left(a+bx^3\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
```

output
```
-1/4*(Sqrt[(a + b*x^3)^2]*(a + 4*b*x^3))/(x^4*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^5}dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^5}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^5}+\frac{b}{x^2}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{4x^4}-\frac{b}{x}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
```

output
```
((-1/4*a/x^4 - b/x)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{(4b\,x^3+a)\sqrt{(b\,x^3+a)^2}}{4x^4(b\,x^3+a)}$	34
default	$-\frac{(4b\,x^3+a)\sqrt{(b\,x^3+a)^2}}{4x^4(b\,x^3+a)}$	34
orering	$-\frac{(4b\,x^3+a)\sqrt{(b\,x^3+a)^2}}{4x^4(b\,x^3+a)}$	34
risch	$\frac{(-b\,x^3-\frac{a}{4})\sqrt{(b\,x^3+a)^2}}{x^4(b\,x^3+a)}$	35

input
```
int(((b*x^3+a)^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(4*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5}\,dx = -\frac{4\,bx^3 + a}{4\,x^4}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/4*(4*b*x^3 + a)/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5}\,dx = -\frac{4\,bx^3 + a}{4\,x^4}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/4*(4*b*x^3 + a)/x^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx = -\frac{4\,bx^3\mathrm{sgn}(bx^3+a)+a\mathrm{sgn}(bx^3+a)}{4\,x^4}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/4*(4*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^4
```

Mupad [B] (verification not implemented)

Time = 21.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx = -\frac{(4\,b\,x^3+a)\,\sqrt{(b\,x^3+a)^2}}{4\,x^4\,(b\,x^3+a)}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^5,x)
```

output
```
-((a + 4*b*x^3)*((a + b*x^3)^2)^(1/2))/(4*x^4*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx = \frac{-4b\,x^3-a}{4x^4}$$

input
```
int(((b*x^3+a)^2)^(1/2)/x^5,x)
```

output
```
( - a - 4*b*x**3)/(4*x**4)
```

3.16 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

output
```
-1/5*a*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-1/2*b*((b*x^3+a)^2)^(1/2)/x^2/(b*
x^3+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx=-\frac{\sqrt{\left(a+bx^3\right)^2}\left(2a+5bx^3\right)}{10x^5\left(a+bx^3\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

output
```
-1/10*(Sqrt[(a + b*x^3)^2]*(2*a + 5*b*x^3))/(x^5*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^6}dx}{b(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^6}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^6}+\frac{b}{x^3}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{5x^5}-\frac{b}{2x^2}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

output
```
((-1/5*a/x^5 - b/(2*x^2))*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
 - 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(-\frac{b\,x^{3}}{2}-\frac{a}{5}\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{x^{5}\left(b\,x^{3}+a\right)}$	35
gosper	$-\frac{\left(5b\,x^{3}+2a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{10x^{5}\left(b\,x^{3}+a\right)}$	36
default	$-\frac{\left(5b\,x^{3}+2a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{10x^{5}\left(b\,x^{3}+a\right)}$	36
orering	$-\frac{\left(5b\,x^{3}+2a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{10x^{5}\left(b\,x^{3}+a\right)}$	36

input
```
int(((b*x^3+a)^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/x^5*(-1/2*b*x^3-1/5*a)/(b*x^3+a)*((b*x^3+a)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx = -\frac{5\,bx^3+2\,a}{10\,x^5}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-1/10*(5*b*x^3 + 2*a)/x^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx = -\frac{5\,bx^3+2\,a}{10\,x^5}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
-1/10*(5*b*x^3 + 2*a)/x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx = -\frac{5\,bx^3\mathrm{sgn}(bx^3+a)+2\,a\mathrm{sgn}(bx^3+a)}{10\,x^5}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="giac")
```

output
```
-1/10*(5*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^5
```

Mupad [B] (verification not implemented)

Time = 21.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx = -\frac{(5\,b\,x^3+2\,a)\,\sqrt{(b\,x^3+a)^2}}{10\,x^5\,(b\,x^3+a)}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^6,x)
```

output
```
-((2*a + 5*b*x^3)*((a + b*x^3)^2)^(1/2))/(10*x^5*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx = \frac{-5b\,x^3-2a}{10x^5}$$

input
```
int(((b*x^3+a)^2)^(1/2)/x^6,x)
```

output
```
( - 2*a - 5*b*x**3)/(10*x**5)
```

3.17 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx=-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

output
```
-1/7*a*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-1/4*b*((b*x^3+a)^2)^(1/2)/x^4/(b*
x^3+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx=-\frac{\sqrt{\left(a+bx^3\right)^2}\left(4a+7bx^3\right)}{28x^7\left(a+bx^3\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

output
```
-1/28*(Sqrt[(a + b*x^3)^2]*(4*a + 7*b*x^3))/(x^7*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b(bx^3+a)}{x^8}dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{bx^3+a}{x^8}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a}{x^8}+\frac{b}{x^5}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a}{7x^7}-\frac{b}{4x^4}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

output
```
((-1/7*a/x^7 - b/(4*x^4))*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 5.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(-\frac{b\,x^{3}}{4}-\frac{a}{7}\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{x^{7}\left(b\,x^{3}+a\right)}$	35
gosper	$-\frac{\left(7b\,x^{3}+4a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{28x^{7}\left(b\,x^{3}+a\right)}$	36
default	$-\frac{\left(7b\,x^{3}+4a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{28x^{7}\left(b\,x^{3}+a\right)}$	36
orering	$-\frac{\left(7b\,x^{3}+4a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{28x^{7}\left(b\,x^{3}+a\right)}$	36

input
```
int(((b*x^3+a)^2)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/x^7*(-1/4*b*x^3-1/7*a)/(b*x^3+a)*((b*x^3+a)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^8}\,dx = -\frac{7\,bx^3 + 4\,a}{28\,x^7}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-1/28*(7*b*x^3 + 4*a)/x^7
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^8}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**3+a)**2)**(1/2)/x**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^8}\,dx = -\frac{7\,bx^3 + 4\,a}{28\,x^7}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
-1/28*(7*b*x^3 + 4*a)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx = -\frac{7\,bx^3\text{sgn}(bx^3+a)+4\,a\text{sgn}(bx^3+a)}{28\,x^7}$$

input
```
integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="giac")
```

output
```
-1/28*(7*b*x^3*sgn(b*x^3 + a) + 4*a*sgn(b*x^3 + a))/x^7
```

Mupad [B] (verification not implemented)

Time = 21.67 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx = -\frac{(7\,b\,x^3+4\,a)\,\sqrt{(b\,x^3+a)^2}}{28\,x^7\,(b\,x^3+a)}$$

input
```
int(((a + b*x^3)^2)^(1/2)/x^8,x)
```

output
```
-((4*a + 7*b*x^3)*((a + b*x^3)^2)^(1/2))/(28*x^7*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx = \frac{-7b\,x^3-4a}{28x^7}$$

input
```
int(((b*x^3+a)^2)^(1/2)/x^8,x)
```

output
```
( - 4*a - 7*b*x**3)/(28*x**7)
```

3.18 $\int x^8(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b^3}-\frac{2a(a+bx^3)^4\sqrt{a^2+2abx^3+b^2x^6}}{15b^3}+\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^3}$$

output
```
1/12*a^2*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/b^3-2/15*a*(b*x^3+a)^4*((b*x^3+a)
^2)^(1/2)/b^3+1/18*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^9(20a^3+45a^2bx^3+36ab^2x^6+10b^3x^9)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+bx^3)^2}\right)\right)}{180\left(-a^2-abx^3+\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}$$

input
```
Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^9*(20*a^3 + 45*a^2*b*x^3 + 36*a*b^2*x^6 + 10*b^3*x^9)*(Sqrt[a^2]*b*x^3
+ a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))/(180*(-a^2 - a*b*x^3 + Sqrt[a^2]*S
qrt[(a + b*x^3)^2]))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$$
$$\Big\downarrow \text{1384}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int b^3x^8 \left(bx^3+a\right)^3 dx}{b^3 \left(a+bx^3\right)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^8 \left(bx^3+a\right)^3 dx}{a+bx^3}$$
$$\Big\downarrow \text{798}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^6 \left(bx^3+a\right)^3 dx^3}{3 \left(a+bx^3\right)}$$
$$\Big\downarrow \text{49}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(b^3x^{15}+3ab^2x^{12}+3a^2bx^9+a^3x^6\right) dx^3}{3 \left(a+bx^3\right)}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \left(\frac{a^3x^9}{3}+\frac{3}{4}a^2bx^{12}+\frac{3}{5}ab^2x^{15}+\frac{b^3x^{18}}{6}\right)}{3 \left(a+bx^3\right)}$$

input
```
Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^3*x^9)/3 + (3*a^2*b*x^12)/4 + (3*a*b^
2*x^15)/5 + (b^3*x^18)/6))/(3*(a + b*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}(b\,x^3+a)\,(b\,x^3+a)^4\,(10b^2x^6-4a\,x^3b+a^2)}{180b^3}$	42
gosper	$\frac{x^9\,(10x^9b^3+36a\,x^6b^2+45a^2x^3b+20a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{180(b\,x^3+a)^3}$	58
default	$\frac{x^9\,(10x^9b^3+36a\,x^6b^2+45a^2x^3b+20a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{180(b\,x^3+a)^3}$	58
orering	$\frac{x^9\,(10x^9b^3+36a\,x^6b^2+45a^2x^3b+20a^3)\,(b^2x^6+2a\,x^3b+a^2)^{\frac{3}{2}}}{180(b\,x^3+a)^3}$	67
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,a^3x^9}{9b\,x^3+9a}+\frac{\sqrt{(b\,x^3+a)^2}\,b\,a^2x^{12}}{4b\,x^3+4a}+\frac{\sqrt{(b\,x^3+a)^2}\,b^2a\,x^{15}}{5b\,x^3+5a}+\frac{\sqrt{(b\,x^3+a)^2}\,b^3x^{18}}{18b\,x^3+18a}$	116

input
```
int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/180*csgn(b*x^3+a)*(b*x^3+a)^4*(10*b^2*x^6-4*a*b*x^3+a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.29

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{18}\,b^3x^{18}+\frac{1}{5}\,ab^2x^{15}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{9}\,a^3x^9$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9
```

Sympy [F]

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int x^8 \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**8*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^2x^3}{12\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}x^3}{18\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^3}{12\,b^3}-\frac{7\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a}{90\,b^3}$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b
*x^3 + a^2)^(5/2)*x^3/b^2 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^3/b^3
 - 7/90*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{18}\,b^3x^{18}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,ab^2x^{15}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,a^2bx^{12}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{9}\,a^3x^9\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/18*b^3*x^18*sgn(b*x^3 + a) + 1/5*a*b^2*x^15*sgn(b*x^3 + a) + 1/4*a^2*b*x
^12*sgn(b*x^3 + a) + 1/9*a^3*x^9*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^8(a^2+2abx^3+b^2x^6)^{3/2}\,dx=\int x^8\,(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}\,dx$$

input
```
int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.31

$$\int x^8(a^2+2abx^3+b^2x^6)^{3/2}\,dx=\frac{x^9(10b^3x^9+36a\,b^2x^6+45a^2b\,x^3+20a^3)}{180}$$

input
```
int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
(x**9*(20*a**3 + 45*a**2*b*x**3 + 36*a*b**2*x**6 + 10*b**3*x**9))/180
```

3.19 $\int x^5(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = -\frac{a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b^2}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{15b^2}$$

output
```
-1/12*a*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/b^2+1/15*(b^2*x^6+2*a*b*x^3+a^2)^(
5/2)/b^2
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \frac{x^6(10a^3+20a^2bx^3+15ab^2x^6+4b^3x^9)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+bx^3)^2}\right)\right)}{60\left(-a^2-abx^3+\sqrt{a^2}\sqrt{(a+bx^3)^2}\right)}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^6*(10*a^3 + 20*a^2*b*x^3 + 15*a*b^2*x^6 + 4*b^3*x^9)*(Sqrt[a^2]*b*x^3 +
 a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))/(60*(-a^2 - a*b*x^3 + Sqrt[a^2]*Sqr
t[(a + b*x^3)^2]))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3} \int x^3 \left(b^2x^6+2abx^3+a^2\right)^{3/2} dx^3$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{3} \left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{5b^2}-\frac{a\int \left(b^2x^6+2abx^3+a^2\right)^{3/2} dx^3}{b}\right)$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{3} \left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{5b^2}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)^3 dx^3}{b^4\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{3} \left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{5b^2}-\frac{a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{4b^2}\right)$$

input
```
Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(-1/4*(a*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/b^2 + (a^2 + 2*a*b
*x^3 + b^2*x^6)^(5/2)/(5*b^2))/3
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.45

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{4}\left(-4b\,x^{3}+a\right)}{60b^{2}}$	31
gosper	$\frac{x^{6}\left(4x^{9}b^{3}+15a\,x^{6}b^{2}+20a^{2}x^{3}b+10a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{60\left(b\,x^{3}+a\right)^{3}}$	58
default	$\frac{x^{6}\left(4x^{9}b^{3}+15a\,x^{6}b^{2}+20a^{2}x^{3}b+10a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{60\left(b\,x^{3}+a\right)^{3}}$	58
orering	$\frac{x^{6}\left(4x^{9}b^{3}+15a\,x^{6}b^{2}+20a^{2}x^{3}b+10a^{3}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{3}{2}}}{60\left(b\,x^{3}+a\right)^{3}}$	67
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{3}x^{6}}{6b\,x^{3}+6a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\,a^{2}x^{9}}{3b\,x^{3}+3a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{2}a\,x^{12}}{4b\,x^{3}+4a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{3}x^{15}}{15b\,x^{3}+15a}$	116

input
```
int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*csgn(b*x^3+a)*(b*x^3+a)^4*(-4*b*x^3+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \frac{1}{15}\,b^3x^{15}+\frac{1}{4}\,ab^2x^{12}+\frac{1}{3}\,a^2bx^9+\frac{1}{6}\,a^3x^6$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6
```

Sympy [F]

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \int x^5 \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\, dx$$

input
```
integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**5*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}ax^3}{12\,b}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^2}{12\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}}{15\,b^2}$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a*x^3/b - 1/12*(b^2*x^6 + 2*a*b*x^
3 + a^2)^(3/2)*a^2/b^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{60}\left(4\,b^3x^{15}+15\,ab^2x^{12}+20\,a^2bx^9+10\,a^3x^6\right)\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/60*(4*b^3*x^15 + 15*a*b^2*x^12 + 20*a^2*b*x^9 + 10*a^3*x^6)*sgn(b*x^3 +
a)
```

Mupad [B] (verification not implemented)

Time = 20.96 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\left(-a^2+3\,a\,b\,x^3+4\,b^2\,x^6\right)}{60\,b^2}$$

input
```
int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)*(4*b^2*x^6 - a^2 + 3*a*b*x^3))/(60*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int x^5 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \frac{x^6(4b^3x^9 + 15a\,b^2x^6 + 20a^2b\,x^3 + 10a^3)}{60}$$

input
```
int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
(x**6*(10*a**3 + 20*a**2*b*x**3 + 15*a*b**2*x**6 + 4*b**3*x**9))/60
```

3.20 $\int x^2(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b}$$

output
```
1/12*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^3\sqrt{(a+bx^3)^2}(4a^3+6a^2bx^3+4ab^2x^6+b^3x^9)}{12\,(a+bx^3)}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^3*Sqrt[(a + b*x^3)^2]*(4*a^3 + 6*a^2*b*x^3 + 4*a*b^2*x^6 + b^3*x^9))/(1
2*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1690, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2 a b x^3+b^2 x^6\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3} \int \left(b^2 x^6+2 a b x^3+a^2\right)^{3/2}dx^3$$

$$\Big\downarrow \text{1079}$$

$$\frac{\sqrt{a^2+2 a b x^3+b^2 x^6} \int \left(b^2 x^3+a b\right)^3dx^3}{3 b^3 \left(a+b x^3\right)}$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(a+b x^3\right)^3 \sqrt{a^2+2 a b x^3+b^2 x^6}}{12 b}$$

input
```
Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{4}}{12 b}$	23
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}\left(b\,x^{3}+a\right)}{12 b}$	24
risch	$\frac{\left(b\,x^{3}+a\right)^{3}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{12 b}$	26
gosper	$\frac{x^{3}\left(x^{9} b^{3}+4 a\,x^{6} b^{2}+6 a^{2} x^{3} b+4 a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{12\left(b\,x^{3}+a\right)^{3}}$	57
orering	$\frac{x^{3}\left(x^{9} b^{3}+4 a\,x^{6} b^{2}+6 a^{2} x^{3} b+4 a^{3}\right)\left(b^{2} x^{6}+2 a\,x^{3} b+a^{2}\right)^{\frac{3}{2}}}{12\left(b\,x^{3}+a\right)^{3}}$	66

input
```
int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*csgn(b*x^3+a)*(b*x^3+a)^4/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{12}\,b^3x^{12}+\frac{1}{3}\,ab^2x^9+\frac{1}{2}\,a^2bx^6+\frac{1}{3}\,a^3x^3$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/12*b^3*x^12 + 1/3*a*b^2*x^9 + 1/2*a^2*b*x^6 + 1/3*a^3*x^3
```

Sympy [F]

$$\int x^2 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \int x^2 \left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**2*((a + b*x**3)**2)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(25) = 50$.

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int x^2 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \frac{1}{12} \left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}} x^3 + \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}} a}{12\,b}$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^3 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a
^2)^(3/2)*a/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int x^2 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} \, dx = \frac{1}{12} \left(2\left(bx^6 + 2\,ax^3\right)a^2 + \left(bx^6 + 2\,ax^3\right)^2 b\right) \operatorname{sgn}\left(bx^3 + a\right)$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/12*(2*(b*x^6 + 2*a*x^3)*a^2 + (b*x^6 + 2*a*x^3)^2*b)*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 20.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int x^2(a^2+2abx^3+b^2x^6)^{3/2}\,dx=\frac{(b^2\,x^3+a\,b)\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{12\,b^2}$$

input
```
int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
((a*b + b^2*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2))/(12*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int x^2(a^2+2abx^3+b^2x^6)^{3/2}\,dx=\frac{x^3(b^3x^9+4a\,b^2x^6+6a^2b\,x^3+4a^3)}{12}$$

input
```
int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
(x**3*(4*a**3 + 6*a**2*b*x**3 + 4*a*b**2*x**6 + b**3*x**9))/12
```

3.21 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x} dx$

Optimal result

Integrand size = 26, antiderivative size = 160

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x} dx = \frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{a^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
a^2*b*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+a*b^2*x^6*((b*x^3+a)^2)^(1/2)/(2*b
*x^3+2*a)+b^3*x^9*((b*x^3+a)^2)^(1/2)/(9*b*x^3+9*a)+a^3*((b*x^3+a)^2)^(1/2
)*ln(x)/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.38

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x} dx = \frac{\sqrt{(a+bx^3)^2}(bx^3(18a^2+9abx^3+2b^2x^6)+18a^3\log(x))}{18(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(b*x^3*(18*a^2 + 9*a*b*x^3 + 2*b^2*x^6) + 18*a^3*Log[
x]))/(18*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^3}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^3x^6+3ab^2x^3+3a^2b+\frac{a^3}{x^3}\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a^3\log\left(x^3\right)+3a^2bx^3+\frac{3}{2}ab^2x^6+\frac{b^3x^9}{3}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(3*a^2*b*x^3 + (3*a*b^2*x^6)/2 + (b^3*x^9
)/3 + a^3*Log[x^3]))/(3*(a + b*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}(b\,x^3+a)\,(2x^9b^3+9a\,x^6b^2+18a^2x^3b+6a^3\ln(b\,x^3)+11a^3)}{18}$	54
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{3}{2}}(2x^9b^3+9a\,x^6b^2+18a^2x^3b+18a^3\ln(x))}{18(b\,x^3+a)^3}$	57
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b\left(\frac{1}{9}b^2x^9+\frac{1}{2}ab\,x^6+a^2x^3\right)}{b\,x^3+a}+\frac{a^3\sqrt{(b\,x^3+a)^2}\,\ln(x)}{b\,x^3+a}$	73

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/18*csgn(b*x^3+a)*(2*x^9*b^3+9*a*x^6*b^2+18*a^2*x^3*b+6*a^3*ln(b*x^3)+11*
a^3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx=\frac{1}{9}\,b^3x^9+\frac{1}{2}\,ab^2x^6+a^2bx^3+a^3\log\left(x\right)$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + a^3*log(x)
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.95

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx = \frac{1}{6}\sqrt{b^2x^6+2\,abx^3+a^2}abx^3 + \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^3\log\left(2\,b^2x^3+2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{1}{2}\sqrt{b^2x^6+2\,abx^3+a^2}a^2 + \frac{1}{9}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*b)
*a^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*log(2*a*b*x
/abs(x) + 2*a^2/(x^2*abs(x))) + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2 +
1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx = \frac{1}{9}\,b^3x^9\mathrm{sgn}\left(bx^3+a\right) + \frac{1}{2}\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right) + a^2bx^3\mathrm{sgn}\left(bx^3+a\right) + a^3\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/9*b^3*x^9*sgn(b*x^3 + a) + 1/2*a*b^2*x^6*sgn(b*x^3 + a) + a^2*b*x^3*sgn(
b*x^3 + a) + a^3*log(abs(x))*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x}\,dx=\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}{x}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x}\,dx=\log(x)\,a^3+a^2b\,x^3+\frac{a\,b^2x^6}{2}+\frac{b^3x^9}{9}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x)
```

output
```
(18*log(x)*a**3 + 18*a**2*b*x**3 + 9*a*b**2*x**6 + 2*b**3*x**9)/18
```

3.22 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 161

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)} + \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/3*a^3*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+a*b^2*x^3*((b*x^3+a)^2)^(1/2)/(
b*x^3+a)+b^3*x^6*((b*x^3+a)^2)^(1/2)/(6*b*x^3+6*a)+3*a^2*b*((b*x^3+a)^2)^(
1/2)*ln(x)/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx = \frac{\sqrt{(a+bx^3)^2}(-2a^3+6ab^2x^6+b^3x^9+18a^2bx^3\log(x))}{6x^3(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-2*a^3 + 6*a*b^2*x^6 + b^3*x^9 + 18*a^2*b*x^3*Log[x]
))/(6*x^3*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^4}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^4}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^6}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^6}+\frac{3ba^2}{x^3}+3b^2a+b^3x^3\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^3}{x^3}+3a^2b\log\left(x^3\right)+3ab^2x^3+\frac{b^3x^6}{2}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^3/x^3) + 3*a*b^2*x^3 + (b^3*x^6)/2 +
 3*a^2*b*Log[x^3]))/(3*(a + b*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.37

method	result	size
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}} \left(x^{9} b^{3}+6 a\,x^{6} b^{2}+18 b\,a^{2} \ln \left(x \right) x^{3}-2 a^{3}\right)}{6 \left(b\,x^{3}+a \right)^{3} x^{3}}$	59
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a \right) \left(-\frac{x^{9} b^{3}}{2}-3 a\,x^{6} b^{2}-3 \ln \left(b\,x^{3}\right) a^{2} b\,x^{3}-\frac{5 a^{2} x^{3} b}{2}+a^{3}\right)}{3 x^{3}}$	59
risch	$\frac{\sqrt{\left(b\,x^{3}+a \right)^{2}}\, b \left(b\,x^{3}+3 a \right)^{2}}{6 b\,x^{3}+6 a}-\frac{a^{3} \sqrt{\left(b\,x^{3}+a \right)^{2}}}{3 x^{3} \left(b\,x^{3}+a \right)}+\frac{3 a^{2} b \sqrt{\left(b\,x^{3}+a \right)^{2}}\, \ln \left(x \right)}{b\,x^{3}+a}$	92

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*((b*x^3+a)^2)^(3/2)*(x^9*b^3+6*a*x^6*b^2+18*b*a^2*ln(x)*x^3-2*a^3)/(b*
x^3+a)^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^4}\,dx=\frac{b^3x^9+6\,ab^2x^6+18\,a^2bx^3\log\left(x\right)-2\,a^3}{6\,x^3}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(b^3*x^9 + 6*a*b^2*x^6 + 18*a^2*b*x^3*log(x) - 2*a^3)/x^3
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^4}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**4,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.97

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^4}\,dx = \frac{1}{2}\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^2x^3 + (-1)^{2\,b^2x^3 + 2\,ab}\,a^2b\log\left(2\,b^2x^3 + 2\,ab\right) - (-1)^{2\,abx^3 + 2\,a^2}\,a^2b\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right) + \frac{3}{2}\sqrt{b^2x^6 + 2\,abx^3 + a^2}ab - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}}{3\,x^3}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2*x^3 + (-1)^(2*b^2*x^3 + 2*a*b)*a^2
*b*log(2*b^2*x^3 + 2*a*b) - (-1)^(2*a*b*x^3 + 2*a^2)*a^2*b*log(2*a*b*x/abs
(x) + 2*a^2/(x^2*abs(x))) + 3/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b - 1/3*
(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.53

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx = \frac{1}{6}\,b^3x^6\mathrm{sgn}\left(bx^3+a\right)+ab^2x^3\mathrm{sgn}\left(bx^3+a\right) + 3\,a^2b\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{3\,a^2bx^3\mathrm{sgn}(bx^3+a)+a^3\mathrm{sgn}(bx^3+a)}{3\,x^3}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
1/6*b^3*x^6*sgn(b*x^3 + a) + a*b^2*x^3*sgn(b*x^3 + a) + 3*a^2*b*log(abs(x)
)*sgn(b*x^3 + a) - 1/3*(3*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x
^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}{x^4}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx = \frac{18\log(x)\,a^2b\,x^3-2a^3+6a\,b^2x^6+b^3x^9}{6x^3}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x)
```

output
```
(18*log(x)*a**2*b*x**3 - 2*a**3 + 6*a*b**2*x**6 + b**3*x**9)/(6*x**3)
```

3.23 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 162

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}\,dx=-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\,(a+bx^3)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\,(a+bx^3)}+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\,(a+bx^3)}+\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/6*a^3*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-a^2*b*((b*x^3+a)^2)^(1/2)/x^3/(
b*x^3+a)+b^3*x^3*((b*x^3+a)^2)^(1/2)/(3*b*x^3+3*a)+3*a*b^2*((b*x^3+a)^2)^(
1/2)*ln(x)/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.38

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}\,dx=-\frac{\sqrt{(a+bx^3)^2}\left(a^3+6a^2bx^3-2b^3x^9-18ab^2x^6\log(x)\right)}{6x^6\,(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

output
```
-1/6*(Sqrt[(a + b*x^3)^2]*(a^3 + 6*a^2*b*x^3 - 2*b^3*x^9 - 18*a*b^2*x^6*Lo
g[x]))/(x^6*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.44, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^7}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^7}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^9}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^9}+\frac{3ba^2}{x^6}+\frac{3b^2a}{x^3}+b^3\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^3}{2x^6}-\frac{3a^2b}{x^3}+3ab^2\log\left(x^3\right)+b^3x^3\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/2*a^3/x^6 - (3*a^2*b)/x^3 + b^3*x^3 +
 3*a*b^2*Log[x^3]))/(3*(a + b*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)\left(-2x^9b^3-6\ln(b\,x^3)a\,b^2x^6-2a\,x^6b^2+6a^2x^3b+a^3\right)}{6x^6}$	59
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{3}{2}}\left(2x^9b^3+18b^2a\ln(x)x^6-6a^2x^3b-a^3\right)}{6(b\,x^3+a)^3x^6}$	60
risch	$\frac{b^3x^3\sqrt{(b\,x^3+a)^2}}{3b\,x^3+3a}+\frac{\sqrt{(b\,x^3+a)^2}\,\left(-a^2x^3b-\frac{1}{6}a^3\right)}{(b\,x^3+a)x^6}+\frac{3a\,b^2\sqrt{(b\,x^3+a)^2}\,\ln(x)}{b\,x^3+a}$	97

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*csgn(b*x^3+a)*(-2*x^9*b^3-6*ln(b*x^3)*a*b^2*x^6-2*a*x^6*b^2+6*a^2*x^3
*b+a^3)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx = \frac{2\,b^3x^9+18\,ab^2x^6\log{(x)}-6\,a^2bx^3-a^3}{6\,x^6}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^9 + 18*a*b^2*x^6*log(x) - 6*a^2*b*x^3 - a^3)/x^6
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**7,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.36

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx = \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^3x^3}{2\,a} + (-1)^{2\,b^2x^3+2\,ab}ab^2\log\left(2\,b^2x^3+2\,ab\right) - (-1)^{2\,abx^3+2\,a^2}ab^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{3}{2}\sqrt{b^2x^6+2\,abx^3+a^2}b^2 + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^2}{6\,a^2} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b}{6\,ax^3} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}{6\,a^2x^6}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3*x^3/a + (-1)^(2*b^2*x^3 + 2*a*b)*a
*b^2*log(2*b^2*x^3 + 2*a*b) - (-1)^(2*a*b*x^3 + 2*a^2)*a*b^2*log(2*a*b*x/a
bs(x) + 2*a^2/(x^2*abs(x))) + 3/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2 + 1/
6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2/a^2 - 1/6*(b^2*x^6 + 2*a*b*x^3 + a
^2)^(3/2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.53

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx = \frac{1}{3}\,b^3x^3\mathrm{sgn}\left(bx^3+a\right) + 3\,ab^2\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right) - \frac{9\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+6\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+a^3\mathrm{sgn}\left(bx^3+a\right)}{6\,x^6}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
1/3*b^3*x^3*sgn(b*x^3 + a) + 3*a*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(9*a
*b^2*x^6*sgn(b*x^3 + a) + 6*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))
/x^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx=\int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}{x^7}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx=\frac{18\log(x)\,a\,b^2x^6-a^3-6a^2b\,x^3+2b^3x^9}{6x^6}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x)
```

output
```
(18*log(x)*a*b**2*x**6 - a**3 - 6*a**2*b*x**3 + 2*b**3*x**9)/(6*x**6)
```

3.24 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 161

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)} - \frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6(a+bx^3)} - \frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3(a+bx^3)} + \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/9*a^3*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-1/2*a^2*b*((b*x^3+a)^2)^(1/2)/x
^6/(b*x^3+a)-a*b^2*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+b^3*((b*x^3+a)^2)^(1/
2)*ln(x)/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.65

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = \frac{2a^3\sqrt{a^2}+9(a^2)^{3/2}bx^3+18a\sqrt{a^2}b^2x^6-2a^3\sqrt{(a+bx^3)^2}-7a^2bx^3\sqrt{(a+b}}{}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

output
```
(2*a^3*Sqrt[a^2] + 9*(a^2)^(3/2)*b*x^3 + 18*a*Sqrt[a^2]*b^2*x^6 - 2*a^3*Sq
rt[(a + b*x^3)^2] - 7*a^2*b*x^3*Sqrt[(a + b*x^3)^2] - 11*a*b^2*x^6*Sqrt[(a
 + b*x^3)^2] - 12*a*b^3*x^9*ArcTanh[(b*x^3)/(Sqrt[a^2] - Sqrt[(a + b*x^3)^
2])] - 12*Sqrt[a^2]*b^3*x^9*Log[x^3] + 6*Sqrt[a^2]*b^3*x^9*Log[a*(Sqrt[a^2
] - b*x^3 - Sqrt[(a + b*x^3)^2])] + 6*Sqrt[a^2]*b^3*x^9*Log[a*(Sqrt[a^2] +
 b*x^3 - Sqrt[(a + b*x^3)^2])])/(36*a*x^9)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^{10}}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{10}}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{12}}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^{12}}+\frac{3ba^2}{x^9}+\frac{3b^2a}{x^6}+\frac{b^3}{x^3}\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^3}{3x^9}-\frac{3a^2b}{2x^6}-\frac{3ab^2}{x^3}+b^3\log\left(x^3\right)\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/3*a^3/x^9 - (3*a^2*b)/(2*x^6) - (3*a*
b^2)/x^3 + b^3*Log[x^3]))/(3*(a + b*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.32

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(6\ln\left(b\,x^{3}\right)b^{3}x^{9}-18a\,x^{6}b^{2}-9a^{2}x^{3}b-2a^{3}\right)}{18x^{9}}$	52
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}\left(18b^{3}\ln\left(x\right)x^{9}-18a\,x^{6}b^{2}-9a^{2}x^{3}b-2a^{3}\right)}{18\left(b\,x^{3}+a\right)^{3}x^{9}}$	60
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-a\,x^{6}b^{2}-\frac{1}{2}a^{2}x^{3}b-\frac{1}{9}a^{3}\right)}{\left(b\,x^{3}+a\right)x^{9}}+\frac{b^{3}\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(x\right)}{b\,x^{3}+a}$	76

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/18*csgn(b*x^3+a)*(6*ln(b*x^3)*b^3*x^9-18*a*x^6*b^2-9*a^2*x^3*b-2*a^3)/x^
9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^{2}+2abx^{3}+b^{2}x^{6}\right)^{3/2}}{x^{10}}\,dx=\frac{18\,b^{3}x^{9}\log\left(x\right)-18\,ab^{2}x^{6}-9\,a^{2}bx^{3}-2\,a^{3}}{18\,x^{9}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
1/18*(18*b^3*x^9*log(x) - 18*a*b^2*x^6 - 9*a^2*b*x^3 - 2*a^3)/x^9
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**10,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**10, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(113) = 226.

Time = 0.04 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.57

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx &= \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^4x^3}{6\,a^2} \\ &+ \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,b^3\log\left(2\,b^2x^3+2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) \\ &+ \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^3}{2\,a} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^3}{18\,a^3} \\ &- \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^2}{6\,a^2x^3} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b}{18\,a^3x^6} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}{9\,a^2x^9}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*
a*b)*b^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^3*log(2*a
*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^
3/a - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^3/a^3 - 1/6*(b^2*x^6 + 2*a*
b*x^3 + a^2)^(3/2)*b^2/(a^2*x^3) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*
b/(a^3*x^6) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^9)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.53

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}}\,dx = b^3\log\left(|x|\right)\operatorname{sgn}\left(bx^3+a\right) - \frac{11\,b^3x^9\operatorname{sgn}(bx^3+a)+18\,ab^2x^6\operatorname{sgn}(bx^3+a)+9\,a^2bx^3\operatorname{sgn}(bx^3+a)+2\,a^3\operatorname{sgn}(bx^3+a)}{18\,x^9}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="giac")
```

output
```
b^3*log(abs(x))*sgn(b*x^3 + a) - 1/18*(11*b^3*x^9*sgn(b*x^3 + a) + 18*a*b^
2*x^6*sgn(b*x^3 + a) + 9*a^2*b*x^3*sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/
x^9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^{10}}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{10}}\,dx = \frac{18\log(x)\,b^3x^9-2a^3-9a^2b\,x^3-18a\,b^2x^6}{18x^9}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x)
```

output
```
(18*log(x)*b**3*x**9 - 2*a**3 - 9*a**2*b*x**3 - 18*a*b**2*x**6)/(18*x**9)
```

3.25 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx = -\frac{(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

output
```
-1/12*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/a/x^12
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(a^3+4a^2bx^3+6ab^2x^6+4b^3x^9)}{12x^{12}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

output
```
-1/12*(Sqrt[(a + b*x^3)^2]*(a^3 + 4*a^2*b*x^3 + 6*a*b^2*x^6 + 4*b^3*x^9))/
(x^12*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1384, 27, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{13}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^{13}}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{13}}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

output
```
-1/12*((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^12)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(2 b\,x^{3}+a\right)\left(2 b^{2} x^{6}+2 a\,x^{3} b+a^{2}\right)}{12 x^{12}}$	41
gosper	$-\frac{\left(4 x^{9} b^{3}+6 a\,x^{6} b^{2}+4 a^{2} x^{3} b+a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{12 x^{12} \left(b\,x^{3}+a\right)^{3}}$	56
default	$-\frac{\left(4 x^{9} b^{3}+6 a\,x^{6} b^{2}+4 a^{2} x^{3} b+a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{12 x^{12} \left(b\,x^{3}+a\right)^{3}}$	56
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{3} x^{9} b^{3}-\frac{1}{2} a\,x^{6} b^{2}-\frac{1}{3} a^{2} x^{3} b-\frac{1}{12} a^{3}\right)}{\left(b\,x^{3}+a\right) x^{12}}$	57
orering	$-\frac{\left(4 x^{9} b^{3}+6 a\,x^{6} b^{2}+4 a^{2} x^{3} b+a^{3}\right)\left(b^{2} x^{6}+2 a\,x^{3} b+a^{2}\right)^{\frac{3}{2}}}{12 x^{12} \left(b\,x^{3}+a\right)^{3}}$	65

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*csgn(b*x^3+a)*(2*b*x^3+a)*(2*b^2*x^6+2*a*b*x^3+a^2)/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{13}}\,dx=-\frac{4\,b^3x^9+6\,ab^2x^6+4\,a^2bx^3+a^3}{12\,x^{12}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="fricas")
```

output
```
-1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{13}}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**13,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**13, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(28) = 56$.

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.61

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{13}}\,dx = \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b^4}{12\,a^4} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b^3}{12\,a^3x^3} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^2}{12\,a^4x^6} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b}{12\,a^3x^9} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}}{12\,a^2x^{12}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="maxima")
```

output
```
1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^4/a^4 + 1/12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(3/2)*b^3/(a^3*x^3) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(
a^4*x^6) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^3*x^9) - 1/12*(b^2*
x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^12)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.66

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx = -\frac{4\,b^3x^9\mathrm{sgn}(bx^3+a)+6\,ab^2x^6\mathrm{sgn}(bx^3+a)+4\,a^2bx^3\mathrm{sgn}(bx^3+a)+a^3\mathrm{sgn}(bx^3+a)}{12\,x^{12}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="giac")
```

output
```
-1/12*(4*b^3*x^9*sgn(b*x^3 + a) + 6*a*b^2*x^6*sgn(b*x^3 + a) + 4*a^2*b*x^3
*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^12
```

Mupad [B] (verification not implemented)

Time = 20.58 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.68

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{12\,x^{12}\,(b\,x^3+a)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3\,(b\,x^3+a)} - \frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^6\,(b\,x^3+a)} - \frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^9\,(b\,x^3+a)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^13,x)
```

output
```
- (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(12*x^12*(a + b*x^3)) - (b^3*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(2*x^6*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(3*x^9*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx = \frac{-4b^3x^9-6a\,b^2x^6-4a^2b\,x^3-a^3}{12x^{12}}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x)
```

output
```
( - a**3 - 4*a**2*b*x**3 - 6*a*b**2*x**6 - 4*b**3*x**9)/(12*x**12)
```

3.26 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = -\frac{(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}} + \frac{b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}$$

output
```
-1/15*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/a/x^15+1/60*b*(b*x^3+a)^3*((b*x^3+a)
^2)^(1/2)/a^2/x^12
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(4a^3+15a^2bx^3+20ab^2x^6+10b^3x^9)}{60x^{15}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

output
```
-1/60*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 15*a^2*b*x^3 + 20*a*b^2*x^6 + 10*b^3*x
^9))/(x^15*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^{16}}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{16}}dx}{a+bx^3}$$

$$\Big\downarrow \ 798$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^{18}}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 55$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{b\int \frac{\left(bx^3+a\right)^3}{x^{15}}dx^3}{5a}-\frac{\left(a+bx^3\right)^4}{5ax^{15}}\right)}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \ 48$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{b\left(a+bx^3\right)^4}{20a^2x^{12}}-\frac{\left(a+bx^3\right)^4}{5ax^{15}}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/5*(a + b*x^3)^4/(a*x^15) + (b*(a + b*
x^3)^4)/(20*a^2*x^12)))/(3*(a + b*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$-\frac{\left(\frac{5}{2}x^{9} b^{3}+5 a \,x^{6} b^{2}+\frac{15}{4} a^{2} x^{3} b +a^{3}\right) \operatorname{csgn}\left(b \,x^{3}+a \right)}{15 x^{15}}$	44
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-\frac{1}{15} a^{3}-\frac{1}{4} a^{2} x^{3} b -\frac{1}{3} a \,x^{6} b^{2}-\frac{1}{6} x^{9} b^{3}\right)}{\left(b \,x^{3}+a \right) x^{15}}$	57
gosper	$-\frac{\left(10 x^{9} b^{3}+20 a \,x^{6} b^{2}+15 a^{2} x^{3} b +4 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{60 x^{15} \left(b \,x^{3}+a \right)^{3}}$	58
default	$-\frac{\left(10 x^{9} b^{3}+20 a \,x^{6} b^{2}+15 a^{2} x^{3} b +4 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{60 x^{15} \left(b \,x^{3}+a \right)^{3}}$	58
orering	$-\frac{\left(10 x^{9} b^{3}+20 a \,x^{6} b^{2}+15 a^{2} x^{3} b +4 a^{3}\right) \left(b^{2} x^{6}+2 a \,x^{3} b +a^{2}\right)^{\frac{3}{2}}}{60 x^{15} \left(b \,x^{3}+a \right)^{3}}$	67

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(5/2*x^9*b^3+5*a*x^6*b^2+15/4*a^2*x^3*b+a^3)*csgn(b*x^3+a)/x^15
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}\,dx=-\frac{10\,b^3x^9+20\,ab^2x^6+15\,a^2bx^3+4\,a^3}{60\,x^{15}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="fricas")
```

output
```
-1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**16,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**16, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(58) = 116.

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.13

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx &= -\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^5}{12\,a^5}\\ &\quad -\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^4}{12\,a^4x^3}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^3}{12\,a^5x^6}\\ &\quad -\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^2}{12\,a^4x^9}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b}{12\,a^3x^{12}}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}{15\,a^2x^{15}}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="maxima")
```

output
```
-1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^5/a^5 - 1/12*(b^2*x^6 + 2*a*b*x^
3 + a^2)^(3/2)*b^4/(a^4*x^3) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/
(a^5*x^6) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^4*x^9) + 1/12*(b
^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^3*x^12) - 1/15*(b^2*x^6 + 2*a*b*x^3 +
 a^2)^(5/2)/(a^2*x^15)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = -\frac{10\,b^3x^9\mathrm{sgn}(bx^3+a)+20\,ab^2x^6\mathrm{sgn}(bx^3+a)+15\,a^2bx^3\mathrm{sgn}(bx^3+a)+4\,a^3\mathrm{sgn}(bx^3+a)}{60\,x^{15}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="giac")
```

output
```
-1/60*(10*b^3*x^9*sgn(b*x^3 + a) + 20*a*b^2*x^6*sgn(b*x^3 + a) + 15*a^2*b*
x^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^15
```

Mupad [B] (verification not implemented)

Time = 20.76 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.80

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{15\,x^{15}\,(b\,x^3+a)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,(b\,x^3+a)} - \frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^9\,(b\,x^3+a)} - \frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^{12}\,(b\,x^3+a)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^16,x)
```

output
```
- (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(15*x^15*(a + b*x^3)) - (b^3*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(3*x^9*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(4*x^12*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{16}}\,dx = \frac{-10b^3x^9 - 20a\,b^2x^6 - 15a^2b\,x^3 - 4a^3}{60x^{15}}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x)
```

output
```
( - 4*a**3 - 15*a**2*b*x**3 - 20*a*b**2*x**6 - 10*b**3*x**9)/(60*x**15)
```

3.27 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{19}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{19}}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{18x^{18}(a+bx^3)} - \frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^{15}(a+bx^3)} - \frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^{12}(a+bx^3)} - \frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)}$$

output
```
-1/18*a^3*((b*x^3+a)^2)^(1/2)/x^18/(b*x^3+a)-1/5*a^2*b*((b*x^3+a)^2)^(1/2)
/x^15/(b*x^3+a)-1/4*a*b^2*((b*x^3+a)^2)^(1/2)/x^12/(b*x^3+a)-1/9*b^3*((b*x
^3+a)^2)^(1/2)/x^9/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{19}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(10a^3+36a^2bx^3+45ab^2x^6+20b^3x^9)}{180x^{18}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^19,x]
```

output
```
-1/180*(Sqrt[(a + b*x^3)^2]*(10*a^3 + 36*a^2*b*x^3 + 45*a*b^2*x^6 + 20*b^3
*x^9))/(x^18*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{19}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3(bx^3+a)^3}{x^{19}}dx}{b^3(a+bx^3)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^3}{x^{19}}dx}{a+bx^3}$$

$$\Big\downarrow \ 798$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^3}{x^{21}}dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \ 53$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^{21}}+\frac{3ba^2}{x^{18}}+\frac{3b^2a}{x^{15}}+\frac{b^3}{x^{12}}\right)dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\left(-\frac{a^3}{6x^{18}}-\frac{3a^2b}{5x^{15}}-\frac{3ab^2}{4x^{12}}-\frac{b^3}{3x^9}\right)\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^19,x]
```

output
```
((-1/6*a^3/x^18 - (3*a^2*b)/(5*x^15) - (3*a*b^2)/(4*x^12) - b^3/(3*x^9))*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.26

method	result	size
pseudoelliptic	$-\frac{\left(2x^{9}b^{3}+\frac{9}{2}a\,x^{6}b^{2}+\frac{18}{5}a^{2}x^{3}b+a^{3}\right)\operatorname{csgn}\left(b\,x^{3}+a\right)}{18x^{18}}$	44
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{18}a^{3}-\frac{1}{5}a^{2}x^{3}b-\frac{1}{4}a\,x^{6}b^{2}-\frac{1}{9}x^{9}b^{3}\right)}{\left(b\,x^{3}+a\right)x^{18}}$	57
gosper	$-\frac{\left(20x^{9}b^{3}+45a\,x^{6}b^{2}+36a^{2}x^{3}b+10a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{180x^{18}\left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(20x^{9}b^{3}+45a\,x^{6}b^{2}+36a^{2}x^{3}b+10a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{180x^{18}\left(b\,x^{3}+a\right)^{3}}$	58
orering	$-\frac{\left(20x^{9}b^{3}+45a\,x^{6}b^{2}+36a^{2}x^{3}b+10a^{3}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{3}{2}}}{180x^{18}\left(b\,x^{3}+a\right)^{3}}$	67

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/18*(2*x^9*b^3+9/2*a*x^6*b^2+18/5*a^2*x^3*b+a^3)*csgn(b*x^3+a)/x^18
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{19}}\,dx=-\frac{20\,b^3x^9+45\,ab^2x^6+36\,a^2bx^3+10\,a^3}{180\,x^{18}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^19,x, algorithm="fricas")
```

output
```
-1/180*(20*b^3*x^9 + 45*a*b^2*x^6 + 36*a^2*b*x^3 + 10*a^3)/x^18
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{19}}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{19}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**19,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**19, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.26

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{19}}\,dx = \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b^6}{12\,a^6} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b^5}{12\,a^5x^3} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^4}{12\,a^6x^6} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^3}{12\,a^5x^9} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^2}{12\,a^4x^{12}} + \frac{7\,(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b}{90\,a^3x^{15}} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}}{18\,a^2x^{18}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^19,x, algorithm="maxima")
```

output
```
1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^6/a^6 + 1/12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(3/2)*b^5/(a^5*x^3) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^4/(
a^6*x^6) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/(a^5*x^9) - 1/12*(b^
2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^4*x^12) + 7/90*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(5/2)*b/(a^3*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^
18)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{19}} \, dx = -\frac{20\, b^3 x^9 \mathrm{sgn}(bx^3 + a) + 45\, ab^2 x^6 \mathrm{sgn}(bx^3 + a) + 36\, a^2 bx^3 \mathrm{sgn}(bx^3 + a) + 10\, a^3 \mathrm{sgn}(bx^3 + a)}{180\, x^{18}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^19,x, algorithm="giac")
```

output
```
-1/180*(20*b^3*x^9*sgn(b*x^3 + a) + 45*a*b^2*x^6*sgn(b*x^3 + a) + 36*a^2*b
*x^3*sgn(b*x^3 + a) + 10*a^3*sgn(b*x^3 + a))/x^18
```

Mupad [B] (verification not implemented)

Time = 21.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{19}} \, dx = -\frac{a^3 \sqrt{a^2 + 2\, a\, b\, x^3 + b^2\, x^6}}{18\, x^{18}\, (b\, x^3 + a)} - \frac{b^3 \sqrt{a^2 + 2\, a\, b\, x^3 + b^2\, x^6}}{9\, x^9\, (b\, x^3 + a)} - \frac{a\, b^2 \sqrt{a^2 + 2\, a\, b\, x^3 + b^2\, x^6}}{4\, x^{12}\, (b\, x^3 + a)} - \frac{a^2\, b \sqrt{a^2 + 2\, a\, b\, x^3 + b^2\, x^6}}{5\, x^{15}\, (b\, x^3 + a)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^19,x)
```

output
```
- (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (b^3*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(4*x^12*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(5*x^15*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{19}} \, dx = \frac{-20b^3x^9 - 45a\,b^2x^6 - 36a^2b\,x^3 - 10a^3}{180x^{18}}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^19,x)
```

output
```
( - 10*a**3 - 36*a**2*b*x**3 - 45*a*b**2*x**6 - 20*b**3*x**9)/(180*x**18)
```

3.28 $\int x^4(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}$$

output
```
a^3*x^5*((b*x^3+a)^2)^(1/2)/(5*b*x^3+5*a)+3*a^2*b*x^8*((b*x^3+a)^2)^(1/2)/
(8*b*x^3+8*a)+3*a*b^2*x^11*((b*x^3+a)^2)^(1/2)/(11*b*x^3+11*a)+b^3*x^14*((
b*x^3+a)^2)^(1/2)/(14*b*x^3+14*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^5\sqrt{\left(a+bx^3\right)^2}\left(616a^3+1155a^2bx^3+840ab^2x^6+220b^3x^9\right)}{3080\left(a+bx^3\right)}$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^5*Sqrt[(a + b*x^3)^2]*(616*a^3 + 1155*a^2*b*x^3 + 840*a*b^2*x^6 + 220*b
^3*x^9))/(3080*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^3x^4\left(bx^3+a\right)^3dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4\left(bx^3+a\right)^3dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^3x^{13}+3ab^2x^{10}+3a^2bx^7+a^3x^4\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^3x^5}{5}+\frac{3}{8}a^2bx^8+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{14}}{14}\right)}{a+bx^3}$$

input
```
Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^3*x^5)/5 + (3*a^2*b*x^8)/8 + (3*a*b^2
*x^11)/11 + (b^3*x^14)/14))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{5} \left(220 x^{9} b^{3}+840 a \,x^{6} b^{2}+1155 a^{2} x^{3} b +616 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{3080 \left(b \,x^{3}+a \right)^{3}}$	58
default	$\frac{x^{5} \left(220 x^{9} b^{3}+840 a \,x^{6} b^{2}+1155 a^{2} x^{3} b +616 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{3080 \left(b \,x^{3}+a \right)^{3}}$	58
orering	$\frac{x^{5} \left(220 x^{9} b^{3}+840 a \,x^{6} b^{2}+1155 a^{2} x^{3} b +616 a^{3}\right) \left(b^{2} x^{6}+2 a \,x^{3} b +a^{2}\right)^{\frac{3}{2}}}{3080 \left(b \,x^{3}+a \right)^{3}}$	67
risch	$\frac{a^{3} x^{5} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{5 b \,x^{3}+5 a}+\frac{3 \sqrt{\left(b \,x^{3}+a \right)^{2}}\, b \,a^{2} x^{8}}{8 \left(b \,x^{3}+a \right)}+\frac{3 \sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{2} a \,x^{11}}{11 \left(b \,x^{3}+a \right)}+\frac{b^{3} x^{14} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{14 b \,x^{3}+14 a}$	116

input
```
int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3080*x^5*(220*b^3*x^9+840*a*b^2*x^6+1155*a^2*b*x^3+616*a^3)*((b*x^3+a)^2
)^(3/2)/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{14} b^3 x^{14} + \frac{3}{11} ab^2 x^{11} + \frac{3}{8} a^2 b x^8 + \frac{1}{5} a^3 x^5$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

Sympy [F]

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \int x^4 \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**4*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^4 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} dx = \frac{1}{14} b^3 x^{14} + \frac{3}{11} ab^2 x^{11} + \frac{3}{8} a^2 b x^8 + \frac{1}{5} a^3 x^5$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^4 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} dx = \frac{1}{14} b^3 x^{14} \mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{11} ab^2 x^{11} \mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{8} a^2 b x^8 \mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{5} a^3 x^5 \mathrm{sgn}\left(bx^3 + a\right)$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/14*b^3*x^14*sgn(b*x^3 + a) + 3/11*a*b^2*x^11*sgn(b*x^3 + a) + 3/8*a^2*b*
x^8*sgn(b*x^3 + a) + 1/5*a^3*x^5*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a^2 + 2abx^3 + b^2x^6\right)^{3/2} dx = \int x^4 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2} dx$$

input
```
int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^5(220b^3x^9+840a\,b^2x^6+1155a^2b\,x^3+616a^3)}{3080}$$

input
```
int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
(x**5*(616*a**3 + 1155*a**2*b*x**3 + 840*a*b**2*x**6 + 220*b**3*x**9))/308
0
```

3.29 $\int x^3(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}$$

output
```
a^3*x^4*((b*x^3+a)^2)^(1/2)/(4*b*x^3+4*a)+3*a^2*b*x^7*((b*x^3+a)^2)^(1/2)/
(7*b*x^3+7*a)+3*a*b^2*x^10*((b*x^3+a)^2)^(1/2)/(10*b*x^3+10*a)+b^3*x^13*((
b*x^3+a)^2)^(1/2)/(13*b*x^3+13*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^4\sqrt{\left(a+bx^3\right)^2}\left(455a^3+780a^2bx^3+546ab^2x^6+140b^3x^9\right)}{1820\left(a+bx^3\right)}$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^4*Sqrt[(a + b*x^3)^2]*(455*a^3 + 780*a^2*b*x^3 + 546*a*b^2*x^6 + 140*b^
3*x^9))/(1820*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int b^3x^3 \left(bx^3+a\right)^3dx}{b^3 \left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^3 \left(bx^3+a\right)^3dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(b^3x^{12}+3ab^2x^9+3a^2bx^6+a^3x^3\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \left(\frac{a^3x^4}{4}+\frac{3}{7}a^2bx^7+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{13}}{13}\right)}{a+bx^3}$$

input
```
Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^3*x^4)/4 + (3*a^2*b*x^7)/7 + (3*a*b^2
*x^10)/10 + (b^3*x^13)/13))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{4} \left(140 x^{9} b^{3}+546 a \,x^{6} b^{2}+780 a^{2} x^{3} b +455 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{1820 \left(b \,x^{3}+a \right)^{3}}$	58
default	$\frac{x^{4} \left(140 x^{9} b^{3}+546 a \,x^{6} b^{2}+780 a^{2} x^{3} b +455 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{1820 \left(b \,x^{3}+a \right)^{3}}$	58
orering	$\frac{x^{4} \left(140 x^{9} b^{3}+546 a \,x^{6} b^{2}+780 a^{2} x^{3} b +455 a^{3}\right) \left(b^{2} x^{6}+2 a \,x^{3} b +a^{2}\right)^{\frac{3}{2}}}{1820 \left(b \,x^{3}+a \right)^{3}}$	67
risch	$\frac{b^{3} x^{13} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{13 b \,x^{3}+13 a}+\frac{3 \sqrt{\left(b \,x^{3}+a \right)^{2}}\, a \,b^{2} x^{10}}{10 \left(b \,x^{3}+a \right)}+\frac{3 \sqrt{\left(b \,x^{3}+a \right)^{2}}\, a^{2} b \,x^{7}}{7 \left(b \,x^{3}+a \right)}+\frac{a^{3} x^{4} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{4 b \,x^{3}+4 a}$	116

input
```
int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1820*x^4*(140*b^3*x^9+546*a*b^2*x^6+780*a^2*b*x^3+455*a^3)*((b*x^3+a)^2)
^(3/2)/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{13}b^3x^{13} + \frac{3}{10}ab^2x^{10} + \frac{3}{7}a^2bx^7 + \frac{1}{4}a^3x^4$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

Sympy [F]

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \int x^3 \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**3*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{13}b^3x^{13} + \frac{3}{10}ab^2x^{10} + \frac{3}{7}a^2bx^7 + \frac{1}{4}a^3x^4$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{13}b^3x^{13}\mathrm{sgn}\left(bx^3+a\right) + \frac{3}{10}ab^2x^{10}\mathrm{sgn}\left(bx^3+a\right) + \frac{3}{7}a^2bx^7\mathrm{sgn}\left(bx^3+a\right) + \frac{1}{4}a^3x^4\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/13*b^3*x^13*sgn(b*x^3 + a) + 3/10*a*b^2*x^10*sgn(b*x^3 + a) + 3/7*a^2*b*
x^7*sgn(b*x^3 + a) + 1/4*a^3*x^4*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \int x^3 \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2} dx$$

input
```
int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{x^4(140b^3x^9+546a\,b^2x^6+780a^2b\,x^3+455a^3)}{1820}$$

input
```
int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
(x**4*(455*a**3 + 780*a**2*b*x**3 + 546*a*b**2*x**6 + 140*b**3*x**9))/1820
```

3.30 $\int x(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 167

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}$$

output
```
a^3*x^2*((b*x^3+a)^2)^(1/2)/(2*b*x^3+2*a)+3*a^2*b*x^5*((b*x^3+a)^2)^(1/2)/
(5*b*x^3+5*a)+3*a*b^2*x^8*((b*x^3+a)^2)^(1/2)/(8*b*x^3+8*a)+b^3*x^11*((b*x
^3+a)^2)^(1/2)/(11*b*x^3+11*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^2\sqrt{\left(a+bx^3\right)^2}\left(220a^3+264a^2bx^3+165ab^2x^6+40b^3x^9\right)}{440\left(a+bx^3\right)}$$

input
```
Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x^2*Sqrt[(a + b*x^3)^2]*(220*a^3 + 264*a^2*b*x^3 + 165*a*b^2*x^6 + 40*b^3
*x^9))/(440*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^3x\left(bx^3+a\right)^3dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(bx^3+a\right)^3dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^3x^{10}+3ab^2x^7+3a^2bx^4+a^3x\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^3x^2}{2}+\frac{3}{5}a^2bx^5+\frac{3}{8}ab^2x^8+\frac{b^3x^{11}}{11}\right)}{a+bx^3}$$

input
```
Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^3*x^2)/2 + (3*a^2*b*x^5)/5 + (3*a*b^2
*x^8)/8 + (b^3*x^11)/11))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{2}\left(40 x^{9} b^{3}+165 a\,x^{6} b^{2}+264 a^{2} x^{3} b +220 a^{3}\right) \left(\left(b\,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{440 \left(b\,x^{3}+a \right)^{3}}$	58
default	$\frac{x^{2}\left(40 x^{9} b^{3}+165 a\,x^{6} b^{2}+264 a^{2} x^{3} b +220 a^{3}\right) \left(\left(b\,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{440 \left(b\,x^{3}+a \right)^{3}}$	58
orering	$\frac{x^{2}\left(40 x^{9} b^{3}+165 a\,x^{6} b^{2}+264 a^{2} x^{3} b +220 a^{3}\right) \left(b^{2} x^{6}+2 a\,x^{3} b +a^{2}\right)^{\frac{3}{2}}}{440 \left(b\,x^{3}+a \right)^{3}}$	67
risch	$\frac{b^{3} x^{11} \sqrt{\left(b\,x^{3}+a \right)^{2}}}{11 b\,x^{3}+11 a}+\frac{3 \sqrt{\left(b\,x^{3}+a \right)^{2}}\, a\,b^{2} x^{8}}{8 \left(b\,x^{3}+a \right)}+\frac{3 \sqrt{\left(b\,x^{3}+a \right)^{2}}\, a^{2} b\,x^{5}}{5 \left(b\,x^{3}+a \right)}+\frac{a^{3} x^{2} \sqrt{\left(b\,x^{3}+a \right)^{2}}}{2 b\,x^{3}+2 a}$	116

input
```
int(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/440*x^2*(40*b^3*x^9+165*a*b^2*x^6+264*a^2*b*x^3+220*a^3)*((b*x^3+a)^2)^(
3/2)/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

Sympy [F]

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int x\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{11}b^3x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{8}ab^2x^8\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{5}a^2bx^5\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}a^3x^2\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/11*b^3*x^11*sgn(b*x^3 + a) + 3/8*a*b^2*x^8*sgn(b*x^3 + a) + 3/5*a^2*b*x^
5*sgn(b*x^3 + a) + 1/2*a^3*x^2*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int x\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

input
```
int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^2(40b^3x^9+165a\,b^2x^6+264a^2b\,x^3+220a^3)}{440}$$

input
```
int(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
(x**2*(220*a**3 + 264*a**2*b*x**3 + 165*a*b**2*x**6 + 40*b**3*x**9))/440
```

3.31 $\int (a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 162

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{a^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3a^2bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{3ab^2x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{b^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}$$

output
```
a^3*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3*a^2*b*x^4*((b*x^3+a)^2)^(1/2)/(4*b*x
^3+4*a)+3*a*b^2*x^7*((b*x^3+a)^2)^(1/2)/(7*b*x^3+7*a)+b^3*x^10*((b*x^3+a)^
2)^(1/2)/(10*b*x^3+10*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x\sqrt{\left(a+bx^3\right)^2}\left(140a^3+105a^2bx^3+60ab^2x^6+14b^3x^9\right)}{140\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x*Sqrt[(a + b*x^3)^2]*(140*a^3 + 105*a^2*b*x^3 + 60*a*b^2*x^6 + 14*b^3*x^
9))/(140*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.48, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1384, 747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)^3 dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 747}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^6x^9+3ab^5x^6+3a^2b^4x^3+a^3b^3\right) dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a^3b^3x+\frac{3}{4}a^2b^4x^4+\frac{3}{7}ab^5x^7+\frac{b^6x^{10}}{10}\right)}{b^3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(a^3*b^3*x + (3*a^2*b^4*x^4)/4 + (3*a*b^5
*x^7)/7 + (b^6*x^10)/10))/(b^3*(a + b*x^3))
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x\left(14x^{9}b^{3}+60a\,x^{6}b^{2}+105a^{2}x^{3}b+140a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{140\left(b\,x^{3}+a\right)^{3}}$	56
default	$\frac{x\left(14x^{9}b^{3}+60a\,x^{6}b^{2}+105a^{2}x^{3}b+140a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{140\left(b\,x^{3}+a\right)^{3}}$	56
orering	$\frac{x\left(14x^{9}b^{3}+60a\,x^{6}b^{2}+105a^{2}x^{3}b+140a^{3}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{3}{2}}}{140\left(b\,x^{3}+a\right)^{3}}$	65
risch	$\frac{b^{3}x^{10}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{10b\,x^{3}+10a}+\frac{3\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a\,b^{2}x^{7}}{7\left(b\,x^{3}+a\right)}+\frac{3\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}b\,x^{4}}{4\left(b\,x^{3}+a\right)}+\frac{a^{3}x\sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}$	113

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/140*x*(14*b^3*x^9+60*a*b^2*x^6+105*a^2*b*x^3+140*a^3)*((b*x^3+a)^2)^(3/2
)/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{10}b^3x^{10}+\frac{3}{7}ab^2x^7+\frac{3}{4}a^2bx^4+a^3x$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

Sympy [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \int \left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}} dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{1}{10}b^3x^{10}+\frac{3}{7}ab^2x^7+\frac{3}{4}a^2bx^4+a^3x$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.40

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{1}{10}b^3x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{7}ab^2x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{4}a^2bx^4\mathrm{sgn}\left(bx^3+a\right)+a^3x\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/10*b^3*x^10*sgn(b*x^3 + a) + 3/7*a*b^2*x^7*sgn(b*x^3 + a) + 3/4*a^2*b*x^
4*sgn(b*x^3 + a) + a^3*x*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int {\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.22

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x(14b^3x^9+60a\,b^2x^6+105a^2b\,x^3+140a^3)}{140}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
(x*(140*a**3 + 105*a**2*b*x**3 + 60*a*b**2*x**6 + 14*b**3*x**9))/140
```

3.32 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^2}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)} +\frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

output
```
-a^3*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+3*a^2*b*x^2*((b*x^3+a)^2)^(1/2)/(2*b*
x^3+2*a)+3*a*b^2*x^5*((b*x^3+a)^2)^(1/2)/(5*b*x^3+5*a)+b^3*x^8*((b*x^3+a)^
2)^(1/2)/(8*b*x^3+8*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{\left(a+bx^3\right)^2}\left(-40a^3+60a^2bx^3+24ab^2x^6+5b^3x^9\right)}{40x\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-40*a^3 + 60*a^2*b*x^3 + 24*a*b^2*x^6 + 5*b^3*x^9))/
(40*x*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^2}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^2}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^3x^7+3ab^2x^4+3a^2bx+\frac{a^3}{x^2}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^3}{x}+\frac{3}{2}a^2bx^2+\frac{3}{5}ab^2x^5+\frac{b^3x^8}{8}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^3/x) + (3*a^2*b*x^2)/2 + (3*a*b^2*x^
5)/5 + (b^3*x^8)/8))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$-\frac{\left(-5 x^{9} b^{3}-24 a \,x^{6} b^{2}-60 a^{2} x^{3} b +40 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{40 x \left(b \,x^{3}+a \right)^{3}}$	58
default	$-\frac{\left(-5 x^{9} b^{3}-24 a \,x^{6} b^{2}-60 a^{2} x^{3} b +40 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{40 x \left(b \,x^{3}+a \right)^{3}}$	58
orering	$-\frac{\left(-5 x^{9} b^{3}-24 a \,x^{6} b^{2}-60 a^{2} x^{3} b +40 a^{3}\right) \left(b^{2} x^{6}+2 a \,x^{3} b +a^{2}\right)^{\frac{3}{2}}}{40 x \left(b \,x^{3}+a \right)^{3}}$	67
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b \left(\frac{1}{8} b^{2} x^{8}+\frac{3}{5} a b \,x^{5}+\frac{3}{2} a^{2} x^{2}\right)}{b \,x^{3}+a}-\frac{a^{3} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{x \left(b \,x^{3}+a \right)}$	76

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(-5*b^3*x^9-24*a*b^2*x^6-60*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)^(3/2)/x/
(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^2}\,dx = \frac{5\,b^3x^9 + 24\,ab^2x^6 + 60\,a^2bx^3 - 40\,a^3}{40\,x}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^2}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**2,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^2}\,dx = \frac{5\,b^3x^9 + 24\,ab^2x^6 + 60\,a^2bx^3 - 40\,a^3}{40\,x}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^2}\,dx = \frac{1}{8}\,b^3x^8\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{5}\,ab^2x^5\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{2}\,a^2bx^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{a^3\mathrm{sgn}(bx^3 + a)}{x}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/8*b^3*x^8*sgn(b*x^3 + a) + 3/5*a*b^2*x^5*sgn(b*x^3 + a) + 3/2*a^2*b*x^2*
sgn(b*x^3 + a) - a^3*sgn(b*x^3 + a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^2}\,dx = \int \frac{{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)}^{3/2}}{x^2}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^2} \, dx = \frac{5b^3x^9 + 24a\,b^2x^6 + 60a^2b\,x^3 - 40a^3}{40x}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x)
```

output
```
( - 40*a**3 + 60*a**2*b*x**3 + 24*a*b**2*x**6 + 5*b**3*x**9)/(40*x)
```

3.33 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 163

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} + \frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)} + \frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}$$

output
```
-1/2*a^3*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+3*a^2*b*x*((b*x^3+a)^2)^(1/2)/(
b*x^3+a)+3*a*b^2*x^4*((b*x^3+a)^2)^(1/2)/(4*b*x^3+4*a)+b^3*x^7*((b*x^3+a)^
2)^(1/2)/(7*b*x^3+7*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3} dx = \frac{\sqrt{(a+bx^3)^2}(-14a^3+84a^2bx^3+21ab^2x^6+4b^3x^9)}{28x^2(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-14*a^3 + 84*a^2*b*x^3 + 21*a*b^2*x^6 + 4*b^3*x^9))/
(28*x^2*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^3}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^3}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^3x^6+3ab^2x^3+3a^2b+\frac{a^3}{x^3}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^3}{2x^2}+3a^2bx+\frac{3}{4}ab^2x^4+\frac{b^3x^7}{7}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/2*a^3/x^2 + 3*a^2*b*x + (3*a*b^2*x^4)
/4 + (b^3*x^7)/7))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.36

method	result	size
gosper	$-\frac{\left(-4x^{9}b^{3}-21a\,x^{6}b^{2}-84a^{2}x^{3}b+14a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{28x^{2}\left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(-4x^{9}b^{3}-21a\,x^{6}b^{2}-84a^{2}x^{3}b+14a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{28x^{2}\left(b\,x^{3}+a\right)^{3}}$	58
orering	$-\frac{\left(-4x^{9}b^{3}-21a\,x^{6}b^{2}-84a^{2}x^{3}b+14a^{3}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{3}{2}}}{28x^{2}\left(b\,x^{3}+a\right)^{3}}$	67
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\left(\frac{1}{7}b^{2}x^{7}+\frac{3}{4}ab\,x^{4}+3x\,a^{2}\right)}{b\,x^{3}+a}-\frac{a^{3}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2x^{2}\left(b\,x^{3}+a\right)}$	74

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/28*(-4*b^3*x^9-21*a*b^2*x^6-84*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)^(3/2)/x^
2/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3}\,dx=\frac{4\,b^3x^9+21\,ab^2x^6+84\,a^2bx^3-14\,a^3}{28\,x^2}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**3,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3}\,dx=\frac{4\,b^3x^9+21\,ab^2x^6+84\,a^2bx^3-14\,a^3}{28\,x^2}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.40

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^3} dx = \frac{1}{7}b^3x^7\text{sgn}(bx^3 + a) + \frac{3}{4}ab^2x^4\text{sgn}(bx^3 + a) + 3a^2bx\text{sgn}(bx^3 + a) - \frac{a^3\text{sgn}(bx^3 + a)}{2x^2}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/7*b^3*x^7*sgn(b*x^3 + a) + 3/4*a*b^2*x^4*sgn(b*x^3 + a) + 3*a^2*b*x*sgn(
b*x^3 + a) - 1/2*a^3*sgn(b*x^3 + a)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^3} dx = \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^3} dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^3} \, dx = \frac{4b^3x^9 + 21a\,b^2x^6 + 84a^2b\,x^3 - 14a^3}{28x^2}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x)
```

output
```
( - 14*a**3 + 84*a**2*b*x**3 + 21*a*b**2*x**6 + 4*b**3*x**9)/(28*x**2)
```

3.34 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^5} dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}$$

output
```
-1/4*a^3*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-3*a^2*b*((b*x^3+a)^2)^(1/2)/x/(
b*x^3+a)+3*a*b^2*x^2*((b*x^3+a)^2)^(1/2)/(2*b*x^3+2*a)+b^3*x^5*((b*x^3+a)^
2)^(1/2)/(5*b*x^3+5*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^5} dx = \frac{\sqrt{(a+bx^3)^2}(-5a^3-60a^2bx^3+30ab^2x^6+4b^3x^9)}{20x^4(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-5*a^3 - 60*a^2*b*x^3 + 30*a*b^2*x^6 + 4*b^3*x^9))/(
20*x^4*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^5} dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3(bx^3+a)^3}{x^5}dx}{b^3(a+bx^3)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^3}{x^5}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^3x^4+3ab^2x+\frac{3a^2b}{x^2}+\frac{a^3}{x^5}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a^3}{4x^4}-\frac{3a^2b}{x}+\frac{3}{2}ab^2x^2+\frac{b^3x^5}{5}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

output
```
((-1/4*a^3/x^4 - (3*a^2*b)/x + (3*a*b^2*x^2)/2 + (b^3*x^5)/5)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$-\frac{\left(-4 x^{9} b^{3}-30 a \,x^{6} b^{2}+60 a^{2} x^{3} b +5 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{20 x^{4} \left(b \,x^{3}+a \right)^{3}}$	58
default	$-\frac{\left(-4 x^{9} b^{3}-30 a \,x^{6} b^{2}+60 a^{2} x^{3} b +5 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{20 x^{4} \left(b \,x^{3}+a \right)^{3}}$	58
orering	$-\frac{\left(-4 x^{9} b^{3}-30 a \,x^{6} b^{2}+60 a^{2} x^{3} b +5 a^{3}\right) \left(b^{2} x^{6}+2 a \,x^{3} b +a^{2}\right)^{\frac{3}{2}}}{20 x^{4} \left(b \,x^{3}+a \right)^{3}}$	67
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{2} \left(\frac{1}{5} b \,x^{5}+\frac{3}{2} a \,x^{2}\right)}{b \,x^{3}+a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-3 a^{2} x^{3} b -\frac{1}{4} a^{3}\right)}{\left(b \,x^{3}+a \right) x^{4}}$	78

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(-4*b^3*x^9-30*a*b^2*x^6+60*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/x^4
/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^5}\,dx = \frac{4\,b^3x^9+30\,ab^2x^6-60\,a^2bx^3-5\,a^3}{20\,x^4}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^5}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**5,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^5}\,dx = \frac{4\,b^3x^9+30\,ab^2x^6-60\,a^2bx^3-5\,a^3}{20\,x^4}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^5}\, dx = \frac{1}{5}\, b^3x^5\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{2}\, ab^2x^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{12\, a^2bx^3\mathrm{sgn}(bx^3 + a) + a^3\mathrm{sgn}(bx^3 + a)}{4\, x^4}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
1/5*b^3*x^5*sgn(b*x^3 + a) + 3/2*a*b^2*x^2*sgn(b*x^3 + a) - 1/4*(12*a^2*b*
x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^5}\, dx = \int \frac{\left(a^2 + 2\, a\, b\, x^3 + b^2\, x^6\right)^{3/2}}{x^5}\, dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^5} \, dx = \frac{4b^3x^9 + 30a\,b^2x^6 - 60a^2b\,x^3 - 5a^3}{20x^4}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x)
```

output
```
( - 5*a**3 - 60*a**2*b*x**3 + 30*a*b**2*x**6 + 4*b**3*x**9)/(20*x**4)
```

3.35 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 163

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)} + \frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

output
```
-1/5*a^3*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-3/2*a^2*b*((b*x^3+a)^2)^(1/2)/x
^2/(b*x^3+a)+3*a*b^2*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+b^3*x^4*((b*x^3+a)^2)
^(1/2)/(4*b*x^3+4*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}\,dx = \frac{\sqrt{\left(a+bx^3\right)^2}\left(-4a^3-30a^2bx^3+60ab^2x^6+5b^3x^9\right)}{20x^5\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-4*a^3 - 30*a^2*b*x^3 + 60*a*b^2*x^6 + 5*b^3*x^9))/(
20*x^5*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^6}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^6}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^6}+\frac{3ba^2}{x^3}+3b^2a+b^3x^3\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{5x^5}-\frac{3a^2b}{2x^2}+3ab^2x+\frac{b^3x^4}{4}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

output
```
((-1/5*a^3/x^5 - (3*a^2*b)/(2*x^2) + 3*a*b^2*x + (b^3*x^4)/4)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.36

method	result	size
gosper	$-\frac{\left(-5x^{9}b^{3}-60a\,x^{6}b^{2}+30a^{2}x^{3}b+4a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{20x^{5}\left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(-5x^{9}b^{3}-60a\,x^{6}b^{2}+30a^{2}x^{3}b+4a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{20x^{5}\left(b\,x^{3}+a\right)^{3}}$	58
orering	$-\frac{\left(-5x^{9}b^{3}-60a\,x^{6}b^{2}+30a^{2}x^{3}b+4a^{3}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{3}{2}}}{20x^{5}\left(b\,x^{3}+a\right)^{3}}$	67
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{2}\left(\frac{1}{4}b\,x^{4}+3xa\right)}{b\,x^{3}+a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{3}{2}a^{2}x^{3}b-\frac{1}{5}a^{3}\right)}{\left(b\,x^{3}+a\right)x^{5}}$	76

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(-5*b^3*x^9-60*a*b^2*x^6+30*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^5
/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx = \frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**6,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx = \frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx = \frac{1}{4}\,b^3x^4\mathrm{sgn}\left(bx^3+a\right) + 3\,ab^2x\mathrm{sgn}\left(bx^3+a\right) - \frac{15\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a^3\mathrm{sgn}\left(bx^3+a\right)}{10\,x^5}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
1/4*b^3*x^4*sgn(b*x^3 + a) + 3*a*b^2*x*sgn(b*x^3 + a) - 1/10*(15*a^2*b*x^3
*sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^6}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^6}\,dx = \frac{5b^3x^9 + 60a\,b^2x^6 - 30a^2b\,x^3 - 4a^3}{20x^5}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x)
```

output
```
( - 4*a**3 - 30*a**2*b*x**3 + 60*a*b**2*x**6 + 5*b**3*x**9)/(20*x**5)
```

3.36 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8}\,dx = -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

output
```
-1/7*a^3*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-3/4*a^2*b*((b*x^3+a)^2)^(1/2)/x
^4/(b*x^3+a)-3*a*b^2*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+b^3*x^2*((b*x^3+a)^2)
^(1/2)/(2*b*x^3+2*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8}\,dx = -\frac{\sqrt{(a+bx^3)^2}(4a^3+21a^2bx^3+84ab^2x^6-14b^3x^9)}{28x^7(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

output
```
-1/28*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 21*a^2*b*x^3 + 84*a*b^2*x^6 - 14*b^3*x
^9))/(x^7*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^3\left(bx^3+a\right)^3}{x^8}dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^3}{x^8}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3}{x^8}+\frac{3ba^2}{x^5}+\frac{3b^2a}{x^2}+b^3x\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{7x^7}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{x}+\frac{b^3x^2}{2}\right)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

output
```
((-1/7*a^3/x^7 - (3*a^2*b)/(4*x^4) - (3*a*b^2)/x + (b^3*x^2)/2)*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 5.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$-\frac{\left(-14x^{9}b^{3}+84a\,x^{6}b^{2}+21a^{2}x^{3}b+4a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{28x^{7}\left(b\,x^{3}+a\right)^{3}}$	58
default	$-\frac{\left(-14x^{9}b^{3}+84a\,x^{6}b^{2}+21a^{2}x^{3}b+4a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{28x^{7}\left(b\,x^{3}+a\right)^{3}}$	58
orering	$-\frac{\left(-14x^{9}b^{3}+84a\,x^{6}b^{2}+21a^{2}x^{3}b+4a^{3}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{3}{2}}}{28x^{7}\left(b\,x^{3}+a\right)^{3}}$	67
risch	$\frac{b^{3}x^{2}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2b\,x^{3}+2a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-3a\,x^{6}b^{2}-\frac{3}{4}a^{2}x^{3}b-\frac{1}{7}a^{3}\right)}{\left(b\,x^{3}+a\right)x^{7}}$	78

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/28*(-14*b^3*x^9+84*a*b^2*x^6+21*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^
7/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8}\,dx = \frac{14\,b^3x^9-84\,ab^2x^6-21\,a^2bx^3-4\,a^3}{28\,x^7}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**8,x)
```

output
```
Integral(((a + b*x**3)**2)**(3/2)/x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8}\,dx = \frac{14\,b^3x^9-84\,ab^2x^6-21\,a^2bx^3-4\,a^3}{28\,x^7}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^8}\,dx = \frac{1}{2}\,b^3x^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{84\,ab^2x^6\mathrm{sgn}(bx^3 + a) + 21\,a^2bx^3\mathrm{sgn}(bx^3 + a) + 4\,a^3\mathrm{sgn}(bx^3 + a)}{28\,x^7}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
1/2*b^3*x^2*sgn(b*x^3 + a) - 1/28*(84*a*b^2*x^6*sgn(b*x^3 + a) + 21*a^2*b*
x^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^8}\,dx = \int \frac{{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)}^{3/2}}{x^8}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^8}\,dx = \frac{14b^3x^9 - 84a\,b^2x^6 - 21a^2b\,x^3 - 4a^3}{28x^7}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x)
```

output
```
( - 4*a**3 - 21*a**2*b*x**3 - 84*a*b**2*x**6 + 14*b**3*x**9)/(28*x**7)
```

3.37 $\int x^{11}(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 160

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=-\frac{a^3(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^4}+\frac{a^2(a+bx^3)^6\sqrt{a^2+2abx^3+b^2x^6}}{7b^4}-\frac{a(a+bx^3)^7\sqrt{a^2+2abx^3+b^2x^6}}{8b^4}+\frac{(a+bx^3)^8\sqrt{a^2+2abx^3+b^2x^6}}{27b^4}$$

output
```
-1/18*a^3*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^4+1/7*a^2*(b*x^3+a)^6*((b*x^3+
a)^2)^(1/2)/b^4-1/8*a*(b*x^3+a)^7*((b*x^3+a)^2)^(1/2)/b^4+1/27*(b*x^3+a)^8
*((b*x^3+a)^2)^(1/2)/b^4
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^{12}\sqrt{(a+bx^3)^2}(126a^5+504a^4bx^3+840a^3b^2x^6+720a^2b^3x^9+315ab^4x^{12}+56b^5x^{15})}{1512\,(a+bx^3)}$$

input
```
Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^12*Sqrt[(a + b*x^3)^2]*(126*a^5 + 504*a^4*b*x^3 + 840*a^3*b^2*x^6 + 720
*a^2*b^3*x^9 + 315*a*b^4*x^12 + 56*b^5*x^15))/(1512*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11} \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int b^5 x^{11} \left(bx^3+a\right)^5 dx}{b^5 \left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^{11} \left(bx^3+a\right)^5 dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^9 \left(bx^3+a\right)^5 dx^3}{3 \left(a+bx^3\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(\frac{\left(bx^3+a\right)^8}{b^3}-\frac{3a\left(bx^3+a\right)^7}{b^3}+\frac{3a^2\left(bx^3+a\right)^6}{b^3}-\frac{a^3\left(bx^3+a\right)^5}{b^3}\right) dx^3}{3 \left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \left(-\frac{a^3\left(a+bx^3\right)^6}{6b^4}+\frac{3a^2\left(a+bx^3\right)^7}{7b^4}+\frac{\left(a+bx^3\right)^9}{9b^4}-\frac{3a\left(a+bx^3\right)^8}{8b^4}\right)}{3 \left(a+bx^3\right)}$$

input
```
Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/6*(a^3*(a + b*x^3)^6)/b^4 + (3*a^2*(a
 + b*x^3)^7)/(7*b^4) - (3*a*(a + b*x^3)^8)/(8*b^4) + (a + b*x^3)^9/(9*b^4)
))/(3*(a + b*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.33

method	result
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{6}\left(-56x^{9}b^{3}+21a\,x^{6}b^{2}-6a^{2}x^{3}b+a^{3}\right)}{1512b^{4}}$
gosper	$\frac{x^{12}\left(56b^{5}x^{15}+315a\,b^{4}x^{12}+720a^{2}b^{3}x^{9}+840a^{3}b^{2}x^{6}+504a^{4}b\,x^{3}+126a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{1512\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{12}\left(56b^{5}x^{15}+315a\,b^{4}x^{12}+720a^{2}b^{3}x^{9}+840a^{3}b^{2}x^{6}+504a^{4}b\,x^{3}+126a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{1512\left(b\,x^{3}+a\right)^{5}}$
orering	$\frac{x^{12}\left(56b^{5}x^{15}+315a\,b^{4}x^{12}+720a^{2}b^{3}x^{9}+840a^{3}b^{2}x^{6}+504a^{4}b\,x^{3}+126a^{5}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{5}{2}}}{1512\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a\,b^{4}x^{24}}{24\left(b\,x^{3}+a\right)}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{5}x^{12}}{12b\,x^{3}+12a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{4}b\,x^{15}}{3b\,x^{3}+3a}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{3}b^{2}x^{18}}{9\left(b\,x^{3}+a\right)}+\frac{10\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}b^{3}x}{21\left(b\,x^{3}+a\right)}$

input
```
int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1512*csgn(b*x^3+a)*(b*x^3+a)^6*(-56*b^3*x^9+21*a*b^2*x^6-6*a^2*b*x^3+a^
3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

$$\int x^{11}\left(a^{2}+2abx^{3}+b^{2}x^{6}\right)^{5/2}\,dx=\frac{1}{27}\,b^{5}x^{27}+\frac{5}{24}\,ab^{4}x^{24}+\frac{10}{21}\,a^{2}b^{3}x^{21}+\frac{5}{9}\,a^{3}b^{2}x^{18}+\frac{1}{3}\,a^{4}bx^{15}+\frac{1}{12}\,a^{5}x^{12}$$

input
```
integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/27*b^5*x^27 + 5/24*a*b^4*x^24 + 10/21*a^2*b^3*x^21 + 5/9*a^3*b^2*x^18 +
1/3*a^4*b*x^15 + 1/12*a^5*x^12
```

Sympy [F]

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{11}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**11*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}x^6}{27\,b^2}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}a^3x^3}{18\,b^3}-\frac{11\,(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}ax^3}{216\,b^3}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}a^4}{18\,b^4}+\frac{83\,(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}a^2}{1512\,b^4}$$

input
```
integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/27*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*x^6/b^2 - 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*a^3*x^3/b^3 - 11/216*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*a*x^3
/b^3 - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^4/b^4 + 83/1512*(b^2*x^6 +
 2*a*b*x^3 + a^2)^(7/2)*a^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{27}\,b^5x^{27}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{24}\,ab^4x^{24}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{21}\,a^2b^3x^{21}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{9}\,a^3b^2x^{18}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{3}\,a^4bx^{15}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{12}\,a^5x^{12}\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/27*b^5*x^27*sgn(b*x^3 + a) + 5/24*a*b^4*x^24*sgn(b*x^3 + a) + 10/21*a^2*
b^3*x^21*sgn(b*x^3 + a) + 5/9*a^3*b^2*x^18*sgn(b*x^3 + a) + 1/3*a^4*b*x^15
*sgn(b*x^3 + a) + 1/12*a^5*x^12*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^{11}\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

input
```
int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.37

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^{12}\left(56b^5x^{15}+315a\,b^4x^{12}+720a^2b^3x^9+840a^3b^2x^6+504a^4b\,x^3+126a^5\right)}{1512}$$

input
```
int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
(x**12*(126*a**5 + 504*a**4*b*x**3 + 840*a**3*b**2*x**6 + 720*a**2*b**3*x*
*9 + 315*a*b**4*x**12 + 56*b**5*x**15))/1512
```

3.38 $\int x^8(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^2(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^3}-\frac{2a(a+bx^3)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^3}+\frac{(a+bx^3)^7\sqrt{a^2+2abx^3+b^2x^6}}{24b^3}$$

output
```
1/18*a^2*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^3-2/21*a*(b*x^3+a)^6*((b*x^3+a)
^2)^(1/2)/b^3+1/24*(b*x^3+a)^7*((b*x^3+a)^2)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^9\sqrt{(a+bx^3)^2}(56a^5+210a^4bx^3+336a^3b^2x^6+280a^2b^3x^9+120ab^4x^{12}+21b^5x^{15})}{504\,(a+bx^3)}$$

input
```
Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^9*Sqrt[(a + b*x^3)^2]*(56*a^5 + 210*a^4*b*x^3 + 336*a^3*b^2*x^6 + 280*a
^2*b^3*x^9 + 120*a*b^4*x^12 + 21*b^5*x^15))/(504*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int b^5x^8\left(bx^3+a\right)^5dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^8\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^6\left(bx^3+a\right)^5dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(\frac{\left(bx^3+a\right)^7}{b^2}-\frac{2a\left(bx^3+a\right)^6}{b^2}+\frac{a^2\left(bx^3+a\right)^5}{b^2}\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^2\left(a+bx^3\right)^6}{6b^3}+\frac{\left(a+bx^3\right)^8}{8b^3}-\frac{2a\left(a+bx^3\right)^7}{7b^3}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^2*(a + b*x^3)^6)/(6*b^3) - (2*a*(a +
b*x^3)^7)/(7*b^3) + (a + b*x^3)^8/(8*b^3)))/(3*(a + b*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

method	result
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{6}\left(21b^{2}x^{6}-6a\,x^{3}b+a^{2}\right)}{504b^{3}}$
gosper	$\frac{x^{9}\left(21b^{5}x^{15}+120a\,b^{4}x^{12}+280a^{2}b^{3}x^{9}+336a^{3}b^{2}x^{6}+210a^{4}b\,x^{3}+56a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{504\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{9}\left(21b^{5}x^{15}+120a\,b^{4}x^{12}+280a^{2}b^{3}x^{9}+336a^{3}b^{2}x^{6}+210a^{4}b\,x^{3}+56a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{504\left(b\,x^{3}+a\right)^{5}}$
orering	$\frac{x^{9}\left(21b^{5}x^{15}+120a\,b^{4}x^{12}+280a^{2}b^{3}x^{9}+336a^{3}b^{2}x^{6}+210a^{4}b\,x^{3}+56a^{5}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{5}{2}}}{504\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}b^{3}x^{18}}{9\left(b\,x^{3}+a\right)}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a\,b^{4}x^{21}}{21\left(b\,x^{3}+a\right)}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{5}x^{9}}{9b\,x^{3}+9a}+\frac{2\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{3}b^{2}x^{15}}{3\left(b\,x^{3}+a\right)}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{4}b\,x^{1}}{12\left(b\,x^{3}+a\right)}$

input
```
int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/504*csgn(b*x^3+a)*(b*x^3+a)^6*(21*b^2*x^6-6*a*b*x^3+a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.48

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{24}\,b^5x^{24}+\frac{5}{21}\,ab^4x^{21}+\frac{5}{9}\,a^2b^3x^{18}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{9}\,a^5x^9$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*x^15 + 5/
12*a^4*b*x^12 + 1/9*a^5*x^9
```

Sympy [F]

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \int x^8 \left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}} \, dx$$

input
```
integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**8*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^2x^3}{18\,b^2} + \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}x^3}{24\,b^2} + \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a^3}{18\,b^3} - \frac{3\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{7}{2}}a}{56\,b^3}$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^2*x^3/b^2 + 1/24*(b^2*x^6 + 2*a*b
*x^3 + a^2)^(7/2)*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^3/b^3
 - 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \frac{1}{24}\,b^5x^{24}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{21}\,ab^4x^{21}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{9}\,a^2b^3x^{18}\mathrm{sgn}\left(bx^3+a\right) + \frac{2}{3}\,a^3b^2x^{15}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{12}\,a^4bx^{12}\mathrm{sgn}\left(bx^3+a\right) + \frac{1}{9}\,a^5x^9\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/24*b^5*x^24*sgn(b*x^3 + a) + 5/21*a*b^4*x^21*sgn(b*x^3 + a) + 5/9*a^2*b^
3*x^18*sgn(b*x^3 + a) + 2/3*a^3*b^2*x^15*sgn(b*x^3 + a) + 5/12*a^4*b*x^12*
sgn(b*x^3 + a) + 1/9*a^5*x^9*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx=\int x^8\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2} \,d x$$

input
```
int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx=\frac{x^{9} \left(21 b^{5} x^{15}+120 a \,b^{4} x^{12}+280 a^{2} b^{3} x^{9}+336 a^{3} b^{2} x^{6}+210 a^{4} b \,x^{3}+56 a^{5}\right)}{504}$$

input
```
int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
(x**9*(56*a**5 + 210*a**4*b*x**3 + 336*a**3*b**2*x**6 + 280*a**2*b**3*x**9
 + 120*a*b**4*x**12 + 21*b**5*x**15))/504
```

3.39 $\int x^5(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=-\frac{a\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^2}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{7/2}}{21b^2}$$

output
```
-1/18*a*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^2+1/21*(b^2*x^6+2*a*b*x^3+a^2)^(
7/2)/b^2
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^6\sqrt{\left(a+bx^3\right)^2}\left(21a^5+70a^4bx^3+105a^3b^2x^6+84a^2b^3x^9+35ab^4x^{12}+6b^5x^{15}\right)}{126\left(a+bx^3\right)}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^6*Sqrt[(a + b*x^3)^2]*(21*a^5 + 70*a^4*b*x^3 + 105*a^3*b^2*x^6 + 84*a^2
*b^3*x^9 + 35*a*b^4*x^12 + 6*b^5*x^15))/(126*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^3 \left(b^2x^6+2abx^3+a^2\right)^{5/2} dx^3$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{7/2}}{7b^2}-\frac{a\int \left(b^2x^6+2abx^3+a^2\right)^{5/2} dx^3}{b}\right)$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{7/2}}{7b^2}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)^5 dx^3}{b^6\left(a+bx^3\right)}\right)$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{7/2}}{7b^2}-\frac{a\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{6b^2}\right)$$

input
```
Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(-1/6*(a*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/b^2 + (a^2 + 2*a*b
*x^3 + b^2*x^6)^(7/2)/(7*b^2))/3
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.45

method	result
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{6}\left(-6b\,x^{3}+a\right)}{126b^{2}}$
gosper	$\frac{x^{6}\left(6b^{5}x^{15}+35a\,b^{4}x^{12}+84a^{2}b^{3}x^{9}+105a^{3}b^{2}x^{6}+70a^{4}b\,x^{3}+21a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{126\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{6}\left(6b^{5}x^{15}+35a\,b^{4}x^{12}+84a^{2}b^{3}x^{9}+105a^{3}b^{2}x^{6}+70a^{4}b\,x^{3}+21a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{126\left(b\,x^{3}+a\right)^{5}}$
orering	$\frac{x^{6}\left(6b^{5}x^{15}+35a\,b^{4}x^{12}+84a^{2}b^{3}x^{9}+105a^{3}b^{2}x^{6}+70a^{4}b\,x^{3}+21a^{5}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{5}{2}}}{126\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{5}x^{6}}{6b\,x^{3}+6a}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{4}b\,x^{9}}{9\left(b\,x^{3}+a\right)}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{3}b^{2}x^{12}}{6\left(b\,x^{3}+a\right)}+\frac{2\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}b^{3}x^{15}}{3\left(b\,x^{3}+a\right)}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a\,b^{4}x^{18}}{18\left(b\,x^{3}+a\right)}$

input
```
int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/126*csgn(b*x^3+a)*(b*x^3+a)^6*(-6*b*x^3+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{21}\,b^5x^{21}+\frac{5}{18}\,ab^4x^{18}+\frac{2}{3}\,a^2b^3x^{15}+\frac{5}{6}\,a^3b^2x^{12}+\frac{5}{9}\,a^4bx^9+\frac{1}{6}\,a^5x^6$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/21*b^5*x^21 + 5/18*a*b^4*x^18 + 2/3*a^2*b^3*x^15 + 5/6*a^3*b^2*x^12 + 5/
9*a^4*b*x^9 + 1/6*a^5*x^6
```

Sympy [F]

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^5\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**5*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20

$$\int x^5 \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} \, dx = -\frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}} ax^3}{18\,b} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}} a^2}{18\,b^2} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}}{21\,b^2}$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a*x^3/b - 1/18*(b^2*x^6 + 2*a*b*x^
3 + a^2)^(5/2)*a^2/b^2 + 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int x^5 \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} \, dx = \frac{1}{126} \left(6\,b^5x^{21} + 35\,ab^4x^{18} + 84\,a^2b^3x^{15} + 105\,a^3b^2x^{12} + 70\,a^4bx^9 + 21\,a^5x^6\right) \mathrm{sgn}\left(bx^3 + a\right)$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/126*(6*b^5*x^21 + 35*a*b^4*x^18 + 84*a^2*b^3*x^15 + 105*a^3*b^2*x^12 + 7
0*a^4*b*x^9 + 21*a^5*x^6)*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x^5\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

input
```
int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^6(6b^5x^{15}+35a\,b^4x^{12}+84a^2b^3x^9+105a^3b^2x^6+70a^4b\,x^3+21a^5)}{126}$$

input
```
int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
(x**6*(21*a**5 + 70*a**4*b*x**3 + 105*a**3*b**2*x**6 + 84*a**2*b**3*x**9 +
 35*a*b**4*x**12 + 6*b**5*x**15))/126
```

3.40 $\int x^2(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int x^2(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b}$$

output
```
1/18*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 82 vs. 2(38) = 76.

Time = 1.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int x^2(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{x^3\sqrt{(a+bx^3)^2}(6a^5+15a^4bx^3+20a^3b^2x^6+15a^2b^3x^9+6ab^4x^{12}+b^5x^{15})}{18(a+bx^3)}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^3*Sqrt[(a + b*x^3)^2]*(6*a^5 + 15*a^4*b*x^3 + 20*a^3*b^2*x^6 + 15*a^2*b
^3*x^9 + 6*a*b^4*x^12 + b^5*x^15))/(18*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1690, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \left(b^2x^6+2abx^3+a^2\right)^{5/2}dx^3$$

$$\Big\downarrow \text{1079}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)^5dx^3}{3b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b}$$

input
```
Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{6}}{18b}$	23
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}\left(b\,x^{3}+a\right)}{18b}$	24
risch	$\frac{\left(b\,x^{3}+a\right)^{5}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{18b}$	26
gosper	$\frac{x^{3}\left(b^{5}x^{15}+6a\,b^{4}x^{12}+15a^{2}b^{3}x^{9}+20a^{3}b^{2}x^{6}+15a^{4}b\,x^{3}+6a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{18\left(b\,x^{3}+a\right)^{5}}$	79
orering	$\frac{x^{3}\left(b^{5}x^{15}+6a\,b^{4}x^{12}+15a^{2}b^{3}x^{9}+20a^{3}b^{2}x^{6}+15a^{4}b\,x^{3}+6a^{5}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{5}{2}}}{18\left(b\,x^{3}+a\right)^{5}}$	88

input
```
int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*csgn(b*x^3+a)*(b*x^3+a)^6/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(25) = 50$.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{1}{18}b^5x^{18}+\frac{1}{3}ab^4x^{15} + \frac{5}{6}a^2b^3x^{12}+\frac{10}{9}a^3b^2x^9+\frac{5}{6}a^4bx^6+\frac{1}{3}a^5x^3$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/18*b^5*x^18 + 1/3*a*b^4*x^15 + 5/6*a^2*b^3*x^12 + 10/9*a^3*b^2*x^9 + 5/6
*a^4*b*x^6 + 1/3*a^5*x^3
```

Sympy [F]

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \int x^2\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}} dx$$

input
```
integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**2*((a + b*x**3)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(25) = 50$.

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{1}{18}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}x^3+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a}{18\,b}$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^3 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a
^2)^(5/2)*a/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int x^2(a^2 + 2abx^3 + b^2x^6)^{5/2}\,dx = \frac{1}{18}\left(3\,(bx^6 + 2\,ax^3)a^4 + 3\,(bx^6 + 2\,ax^3)^2a^2b + (bx^6 + 2\,ax^3)^3b^2\right)\mathrm{sgn}\left(bx^3 + a\right)$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/18*(3*(b*x^6 + 2*a*x^3)*a^4 + 3*(b*x^6 + 2*a*x^3)^2*a^2*b + (b*x^6 + 2*a
*x^3)^3*b^2)*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 22.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int x^2(a^2 + 2abx^3 + b^2x^6)^{5/2}\,dx = \frac{(b^2\,x^3 + a\,b)\,\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}}{18\,b^2}$$

input
```
int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
((a*b + b^2*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2))/(18*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^3(b^5x^{15}+6a\,b^4x^{12}+15a^2b^3x^9+20a^3b^2x^6+15a^4b\,x^3+6a^5)}{18}$$

input
```
int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
(x**3*(6*a**5 + 15*a**4*b*x**3 + 20*a**3*b**2*x**6 + 15*a**2*b**3*x**9 + 6
*a*b**4*x**12 + b**5*x**15))/18
```

3.41 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx = \frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12(a+bx^3)} + \frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15(a+bx^3)} + \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
5*a^4*b*x^3*((b*x^3+a)^2)^(1/2)/(3*b*x^3+3*a)+5*a^3*b^2*x^6*((b*x^3+a)^2)^
(1/2)/(3*b*x^3+3*a)+10*a^2*b^3*x^9*((b*x^3+a)^2)^(1/2)/(9*b*x^3+9*a)+5*a*b
^4*x^12*((b*x^3+a)^2)^(1/2)/(12*b*x^3+12*a)+b^5*x^15*((b*x^3+a)^2)^(1/2)/(
15*b*x^3+15*a)+a^5*((b*x^3+a)^2)^(1/2)*ln(x)/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx = \frac{\sqrt{(a+bx^3)^2}\,(bx^3(300a^4+300a^3bx^3+200a^2b^2x^6+75ab^3x^9+12b^4x^{12})+1}{180\,(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(b*x^3*(300*a^4 + 300*a^3*b*x^3 + 200*a^2*b^2*x^6 + 7
5*a*b^3*x^9 + 12*b^4*x^12) + 180*a^5*Log[x]))/(180*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5(bx^3+a)^5}{x}dx}{b^5(a+bx^3)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x}dx}{a+bx^3}$$

$$\Big\downarrow \ 798$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^3}dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \ 49$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(b^5x^{12}+5ab^4x^9+10a^2b^3x^6+10a^3b^2x^3+5a^4b+\frac{a^5}{x^3}\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a^5\log\left(x^3\right)+5a^4bx^3+5a^3b^2x^6+\frac{10}{3}a^2b^3x^9+\frac{5}{4}ab^4x^{12}+\frac{b^5x^{15}}{5}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(5*a^4*b*x^3 + 5*a^3*b^2*x^6 + (10*a^2*b^
3*x^9)/3 + (5*a*b^4*x^12)/4 + (b^5*x^15)/5 + a^5*Log[x^3]))/(3*(a + b*x^3)
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(\frac{b^{5} x^{15}}{5}+\frac{5 a\,b^{4} x^{12}}{4}+\frac{10 a^{2} b^{3} x^{9}}{3}+5 a^{3} b^{2} x^{6}+5 a^{4} b\,x^{3}+a^{5} \ln\left(b\,x^{3}\right)+\frac{137 a^{5}}{60}\right)}{3}$	75
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}} \left(12 b^{5} x^{15}+75 a\,b^{4} x^{12}+200 a^{2} b^{3} x^{9}+300 a^{3} b^{2} x^{6}+300 a^{4} b\,x^{3}+180 a^{5} \ln\left(x\right)\right)}{180 \left(b\,x^{3}+a\right)^{5}}$	79
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b \left(\frac{1}{15} b^{4} x^{15}+\frac{5}{12} b^{3} a\,x^{12}+\frac{10}{9} a^{2} b^{2} x^{9}+\frac{5}{3} a^{3} b\,x^{6}+\frac{5}{3} a^{4} x^{3}\right)}{b\,x^{3}+a}+\frac{a^{5} \sqrt{\left(b\,x^{3}+a\right)^{2}}\, \ln\left(x\right)}{b\,x^{3}+a}$	96

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*csgn(b*x^3+a)*(1/5*b^5*x^15+5/4*a*b^4*x^12+10/3*a^2*b^3*x^9+5*a^3*b^2*
x^6+5*a^4*b*x^3+a^5*ln(b*x^3)+137/60*a^5)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x}\,dx = \frac{1}{15}\,b^5x^{15} + \frac{5}{12}\,ab^4x^{12} + \frac{10}{9}\,a^2b^3x^9 + \frac{5}{3}\,a^3b^2x^6 + \frac{5}{3}\,a^4bx^3 + a^5\log\left(x\right)$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="fricas")
```

output
```
1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3
*a^4*b*x^3 + a^5*log(x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.82

$$\begin{aligned}\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x}\,dx &= \frac{1}{6}\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^3bx^3 \\ &+ \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^5\log\left(2\,b^2x^3 + 2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^5\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right) \\ &+ \frac{1}{12}\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}abx^3 + \frac{1}{2}\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^4 \\ &+ \frac{7}{36}\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}a^2 + \frac{1}{15}\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{5}{2}}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*
b)*a^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^5*log(2*a*b
*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a
*b*x^3 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^4 + 7/36*(b^2*x^6 + 2*a*b*x
^3 + a^2)^(3/2)*a^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx = \frac{1}{15}b^5x^{15}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{12}ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right) + \frac{10}{9}a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{3}a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{3}a^4bx^3\mathrm{sgn}\left(bx^3+a\right) + a^5\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="giac")
```

output
```
1/15*b^5*x^15*sgn(b*x^3 + a) + 5/12*a*b^4*x^12*sgn(b*x^3 + a) + 10/9*a^2*b
^3*x^9*sgn(b*x^3 + a) + 5/3*a^3*b^2*x^6*sgn(b*x^3 + a) + 5/3*a^4*b*x^3*sgn
(b*x^3 + a) + a^5*log(abs(x))*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}{x}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x} \, dx = \log(x) \, a^5 + \frac{5a^4b \, x^3}{3} + \frac{5a^3b^2x^6}{3} + \frac{10a^2b^3x^9}{9} + \frac{5a \, b^4x^{12}}{12} + \frac{b^5x^{15}}{15}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x)
```

output
```
(180*log(x)*a**5 + 300*a**4*b*x**3 + 300*a**3*b**2*x**6 + 200*a**2*b**3*x*
*9 + 75*a*b**4*x**12 + 12*b**5*x**15)/180
```

3.42 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} \\ &+ \frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} \\ &+ \frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12(a+bx^3)} \\ &+ \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}\end{aligned}$$

output
```
-1/3*a^5*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+10*a^3*b^2*x^3*((b*x^3+a)^2)^(1
/2)/(3*b*x^3+3*a)+5*a^2*b^3*x^6*((b*x^3+a)^2)^(1/2)/(3*b*x^3+3*a)+5*a*b^4*
x^9*((b*x^3+a)^2)^(1/2)/(9*b*x^3+9*a)+b^5*x^12*((b*x^3+a)^2)^(1/2)/(12*b*x
^3+12*a)+5*a^4*b*((b*x^3+a)^2)^(1/2)*ln(x)/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx = \frac{\sqrt{(a+bx^3)^2}\,(-12a^5+120a^3b^2x^6+60a^2b^3x^9+20ab^4x^{12}+3b^5x^{15}+180a^4$$

$$36x^3\,(a+bx^3)$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-12*a^5 + 120*a^3*b^2*x^6 + 60*a^2*b^3*x^9 + 20*a*b^
4*x^12 + 3*b^5*x^15 + 180*a^4*b*x^3*Log[x]))/(36*x^3*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.38, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5(bx^3+a)^5}{x^4}dx}{b^5\,(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^4}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^6}dx^3}{3\,(a+bx^3)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(b^5x^9+5ab^4x^6+10a^2b^3x^3+10a^3b^2+\frac{5a^4b}{x^3}+\frac{a^5}{x^6}\right)dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{x^3}+5a^4b\log\left(x^3\right)+10a^3b^2x^3+5a^2b^3x^6+\frac{5}{3}ab^4x^9+\frac{b^5x^{12}}{4}\right)}{3(a+bx^3)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^5/x^3) + 10*a^3*b^2*x^3 + 5*a^2*b^3*
x^6 + (5*a*b^4*x^9)/3 + (b^5*x^12)/4 + 5*a^4*b*Log[x^3]))/(3*(a + b*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.32

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-\frac{b^{5} x^{15}}{4}-\frac{5 a\,b^{4} x^{12}}{3}-5 a^{2} b^{3} x^{9}-10 a^{3} b^{2} x^{6}-5 \ln \left(b\,x^{3}\right) a^{4} b\,x^{3}-\frac{77 a^{4} b\,x^{3}}{12}+a^{5}\right)}{3 x^{3}}$	81
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}} \left(3 b^{5} x^{15}+20 a\,b^{4} x^{12}+60 a^{2} b^{3} x^{9}+120 a^{3} b^{2} x^{6}+180 a^{4} b \ln \left(x \right) x^{3}-12 a^{5}\right)}{36 \left(b\,x^{3}+a\right)^{5} x^{3}}$	82
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b^{2} \left(\frac{1}{12} b^{3} x^{12}+\frac{5}{9} a\,b^{2} x^{9}+\frac{5}{3} a^{2} b\,x^{6}+\frac{10}{3} a^{3} x^{3}\right)}{b\,x^{3}+a}-\frac{a^{5} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{3 x^{3} \left(b\,x^{3}+a\right)}+\frac{5 a^{4} b \sqrt{\left(b\,x^{3}+a\right)^{2}}\, \ln \left(x \right)}{b\,x^{3}+a}$	117

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*csgn(b*x^3+a)*(-1/4*b^5*x^15-5/3*a*b^4*x^12-5*a^2*b^3*x^9-10*a^3*b^2*
x^6-5*ln(b*x^3)*a^4*b*x^3-77/12*a^4*b*x^3+a^5)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^4}\,dx=\frac{3\,b^5x^{15}+20\,ab^4x^{12}+60\,a^2b^3x^9+120\,a^3b^2x^6+180\,a^4bx^3\log\left(x\right)-12\,a^5}{36\,x^3}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
1/36*(3*b^5*x^15 + 20*a*b^4*x^12 + 60*a^2*b^3*x^9 + 120*a^3*b^2*x^6 + 180*
a^4*b*x^3*log(x) - 12*a^5)/x^3
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^4}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**4,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^4}\,dx &= \frac{5}{6}\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^2b^2x^3 \\ &+ \frac{5}{3}\,(-1)^{2\,b^2x^3 + 2\,ab}\,a^4b\log\left(2\,b^2x^3 + 2\,ab\right) \\ &- \frac{5}{3}\,(-1)^{2\,abx^3 + 2\,a^2}\,a^4b\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right) \\ &+ \frac{5}{12}\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}b^2x^3 + \frac{5}{2}\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^3b \\ &+ \frac{35}{36}\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{3}{2}}ab - \frac{\left(b^2x^6 + 2\,abx^3 + a^2\right)^{\frac{5}{2}}}{3\,x^3}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2*x^3 + 5/3*(-1)^(2*b^2*x^3 + 2*
a*b)*a^4*b*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^4*b*log
(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(
3/2)*b^2*x^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b + 35/36*(b^2*x^6
+ 2*a*b*x^3 + a^2)^(3/2)*a*b - 1/3*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.49

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^4}\,dx = \frac{1}{12}\,b^5x^{12}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{9}\,ab^4x^9\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{3}\,a^2b^3x^6\mathrm{sgn}\left(bx^3+a\right) + \frac{10}{3}\,a^3b^2x^3\mathrm{sgn}\left(bx^3+a\right) + 5\,a^4b\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right) - \frac{5\,a^4bx^3\mathrm{sgn}(bx^3+a)+a^5\mathrm{sgn}(bx^3+a)}{3\,x^3}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="giac")
```

output
```
1/12*b^5*x^12*sgn(b*x^3 + a) + 5/9*a*b^4*x^9*sgn(b*x^3 + a) + 5/3*a^2*b^3*
x^6*sgn(b*x^3 + a) + 10/3*a^3*b^2*x^3*sgn(b*x^3 + a) + 5*a^4*b*log(abs(x))
*sgn(b*x^3 + a) - 1/3*(5*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^
3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^4}\,dx = \int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^4} \,d x$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx = \frac{180\log(x)\,a^4b\,x^3-12a^5+120a^3b^2x^6+60a^2b^3x^9+20a\,b^4x^{12}+3b^5x^{15}}{36x^3}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x)
```

output
```
(180*log(x)*a**4*b*x**3 - 12*a**5 + 120*a**3*b**2*x**6 + 60*a**2*b**3*x**9
 + 20*a*b**4*x**12 + 3*b**5*x**15)/(36*x**3)
```

3.43 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)} + \frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/6*a^5*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-5/3*a^4*b*((b*x^3+a)^2)^(1/2)/x
^3/(b*x^3+a)+10*a^2*b^3*x^3*((b*x^3+a)^2)^(1/2)/(3*b*x^3+3*a)+5*a*b^4*x^6*
((b*x^3+a)^2)^(1/2)/(6*b*x^3+6*a)+b^5*x^9*((b*x^3+a)^2)^(1/2)/(9*b*x^3+9*a
)+10*a^3*b^2*((b*x^3+a)^2)^(1/2)*ln(x)/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \frac{\sqrt{(a+bx^3)^2}\,(-3a^5-30a^4bx^3+60a^2b^3x^9+15ab^4x^{12}+2b^5x^{15}+180a^3b^2x^6\log(x))}{18x^6\,(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-3*a^5 - 30*a^4*b*x^3 + 60*a^2*b^3*x^9 + 15*a*b^4*x^
12 + 2*b^5*x^15 + 180*a^3*b^2*x^6*Log[x]))/(18*x^6*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5(bx^3+a)^5}{x^7}dx}{b^5(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^7}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^9}dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^6+5ab^4x^3+10a^2b^3+\frac{10a^3b^2}{x^3}+\frac{5a^4b}{x^6}+\frac{a^5}{x^9}\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{2x^6}-\frac{5a^4b}{x^3}+10a^3b^2\log\left(x^3\right)+10a^2b^3x^3+\frac{5}{2}ab^4x^6+\frac{b^5x^9}{3}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/2*a^5/x^6 - (5*a^4*b)/x^3 + 10*a^2*b^
3*x^3 + (5*a*b^4*x^6)/2 + (b^5*x^9)/3 + 10*a^3*b^2*Log[x^3]))/(3*(a + b*x^
3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}} \left(2 b^{5} x^{15}+15 a\,b^{4} x^{12}+60 a^{2} b^{3} x^{9}+180 a^{3} b^{2} \ln \left(x \right) x^{6}-30 a^{4} b\,x^{3}-3 a^{5}\right)}{18 \left(b\,x^{3}+a \right)^{5} x^{6}}$	82
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a \right) \left(-\frac{2 b^{5} x^{15}}{3}-5 a\,b^{4} x^{12}-20 a^{2} b^{3} x^{9}-20 \ln \left(b\,x^{3}\right) a^{3} b^{2} x^{6}-\frac{47 a^{3} b^{2} x^{6}}{3}+10 a^{4} b\,x^{3}+a^{5}\right)}{6 x^{6}}$	83
risch	$\frac{\sqrt{\left(b\,x^{3}+a \right)^{2}}\, b^{3} \left(\frac{1}{9} b^{2} x^{9}+\frac{5}{6} a b\,x^{6}+\frac{10}{3} a^{2} x^{3}\right)}{b\,x^{3}+a}+\frac{\sqrt{\left(b\,x^{3}+a \right)^{2}}\, \left(-\frac{5}{3} a^{4} b\,x^{3}-\frac{1}{6} a^{5}\right)}{\left(b\,x^{3}+a \right) x^{6}}+\frac{10 a^{3} b^{2} \sqrt{\left(b\,x^{3}+a \right)^{2}}\, \ln \left(x \right)}{b\,x^{3}+a}$	119

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/18*((b*x^3+a)^2)^(5/2)*(2*b^5*x^15+15*a*b^4*x^12+60*a^2*b^3*x^9+180*a^3*
b^2*ln(x)*x^6-30*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^7}\,dx=\frac{2\,b^5x^{15}+15\,ab^4x^{12}+60\,a^2b^3x^9+180\,a^3b^2x^6\log\left(x\right)-30\,a^4bx^3-3\,a^5}{18\,x^6}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="fricas")
```

output
```
1/18*(2*b^5*x^15 + 15*a*b^4*x^12 + 60*a^2*b^3*x^9 + 180*a^3*b^2*x^6*log(x)
 - 30*a^4*b*x^3 - 3*a^5)/x^6
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^7}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**7,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.12

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \frac{5}{3}\sqrt{b^2x^6+2\,abx^3+a^2}ab^3x^3 + \frac{10}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^3b^2\log\left(2\,b^2x^3+2\,ab\right) - \frac{10}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^3b^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{5\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^3x^3}{6\,a} + 5\sqrt{b^2x^6+2\,abx^3+a^2}a^2b^2 + \frac{35}{18}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^2 + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^2}{6\,a^2} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b}{2\,ax^3} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{6\,a^2x^6}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="maxima")
```

output
```
5/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b^3*x^3 + 10/3*(-1)^(2*b^2*x^3 + 2*a
*b)*a^3*b^2*log(2*b^2*x^3 + 2*a*b) - 10/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*b^2
*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/6*(b^2*x^6 + 2*a*b*x^3 + a^2
)^(3/2)*b^3*x^3/a + 5*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2 + 35/18*(b^2
*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2 + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*
b^2/a^2 - 1/2*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a*x^3) - 1/6*(b^2*x^6 +
 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^6)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.50

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \frac{1}{9}\,b^5x^9\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{6}\,ab^4x^6\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{3}\,a^2b^3x^3\mathrm{sgn}\left(bx^3+a\right)+10\,a^3b^2\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{30\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+10\,a^4bx^3\mathrm{sgn}(bx^3+a)+a^5\mathrm{sgn}(bx^3+a)}{6\,x^6}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="giac")
```

output
```
1/9*b^5*x^9*sgn(b*x^3 + a) + 5/6*a*b^4*x^6*sgn(b*x^3 + a) + 10/3*a^2*b^3*x
^3*sgn(b*x^3 + a) + 10*a^3*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(30*a^3*b^
2*x^6*sgn(b*x^3 + a) + 10*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x
^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^7} \,d x$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \frac{180\log(x)\,a^3b^2x^6-3a^5-30a^4b\,x^3+60a^2b^3x^9+15a\,b^4x^{12}+2b^5x^{15}}{18x^6}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x)
```

output
```
(180*log(x)*a**3*b**2*x**6 - 3*a**5 - 30*a**4*b*x**3 + 60*a**2*b**3*x**9 +
 15*a*b**4*x**12 + 2*b**5*x**15)/(18*x**6)
```

3.44 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)} + \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/9*a^5*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-5/6*a^4*b*((b*x^3+a)^2)^(1/2)/x
^6/(b*x^3+a)-10/3*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+5*a*b^4*x^3*((
b*x^3+a)^2)^(1/2)/(3*b*x^3+3*a)+b^5*x^6*((b*x^3+a)^2)^(1/2)/(6*b*x^3+6*a)+
10*a^2*b^3*((b*x^3+a)^2)^(1/2)*ln(x)/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx=\frac{\sqrt{(a+bx^3)^2}(-2a^5-15a^4bx^3-60a^3b^2x^6+30ab^4x^{12}+3b^5x^{15}+180a^2b^3x^9$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-2*a^5 - 15*a^4*b*x^3 - 60*a^3*b^2*x^6 + 30*a*b^4*x^
12 + 3*b^5*x^15 + 180*a^2*b^3*x^9*Log[x]))/(18*x^9*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5(bx^3+a)^5}{x^{10}}dx}{b^5(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^{10}}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^{12}}dx^3}{3(a+bx^3)}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{12}}+\frac{5ba^4}{x^9}+\frac{10b^2a^3}{x^6}+\frac{10b^3a^2}{x^3}+5b^4a+b^5x^3\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{3x^9}-\frac{5a^4b}{2x^6}-\frac{10a^3b^2}{x^3}+10a^2b^3\log\left(x^3\right)+5ab^4x^3+\frac{b^5x^6}{2}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/3*a^5/x^9 - (5*a^4*b)/(2*x^6) - (10*a
^3*b^2)/x^3 + 5*a*b^4*x^3 + (b^5*x^6)/2 + 10*a^2*b^3*Log[x^3]))/(3*(a + b*
x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}} \left(3 b^{5} x^{15}+30 a\,b^{4} x^{12}+180 a^{2} b^{3} \ln \left(x\right) x^{9}-60 a^{3} b^{2} x^{6}-15 a^{4} b\,x^{3}-2 a^{5}\right)}{18 \left(b\,x^{3}+a\right)^{5} x^{9}}$	82
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right) \left(-\frac{3 b^{5} x^{15}}{2}-15 a\,b^{4} x^{12}-30 \ln \left(b\,x^{3}\right) a^{2} b^{3} x^{9}-\frac{27 a^{2} b^{3} x^{9}}{2}+30 a^{3} b^{2} x^{6}+\frac{15 a^{4} b\,x^{3}}{2}+a^{5}\right)}{9 x^{9}}$	83
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, b^{3} \left(b\,x^{3}+5 a\right)^{2}}{6 b\,x^{3}+6 a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\, \left(-\frac{10}{3} a^{3} b^{2} x^{6}-\frac{5}{6} a^{4} b\,x^{3}-\frac{1}{9} a^{5}\right)}{\left(b\,x^{3}+a\right) x^{9}}+\frac{10 a^{2} b^{3} \sqrt{\left(b\,x^{3}+a\right)^{2}}\, \ln \left(x\right)}{b\,x^{3}+a}$	118

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/18*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+30*a*b^4*x^12+180*a^2*b^3*ln(x)*x^9-6
0*a^3*b^2*x^6-15*a^4*b*x^3-2*a^5)/(b*x^3+a)^5/x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{10}}\,dx=\frac{3\,b^5x^{15}+30\,ab^4x^{12}+180\,a^2b^3x^9\log\left(x\right)-60\,a^3b^2x^6-15\,a^4bx^3-2\,a^5}{18\,x^9}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

output
```
1/18*(3*b^5*x^15 + 30*a*b^4*x^12 + 180*a^2*b^3*x^9*log(x) - 60*a^3*b^2*x^6
 - 15*a^4*b*x^3 - 2*a^5)/x^9
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**10,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**10, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx &= \frac{5}{3}\sqrt{b^2x^6+2\,abx^3+a^2}b^4x^3 \\ &+ \frac{10}{3}(-1)^{2\,b^2x^3+2\,ab}a^2b^3\log\left(2\,b^2x^3+2\,ab\right) \\ &- \frac{10}{3}(-1)^{2\,abx^3+2\,a^2}a^2b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{5\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^4x^3}{6\,a^2} \\ &+ 5\sqrt{b^2x^6+2\,abx^3+a^2}ab^3+\frac{35\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^3}{18\,a} \\ &+ \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^3}{18\,a^3}-\frac{11\,(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^2}{18\,a^2x^3} \\ &- \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{18\,a^3x^6}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{9\,a^2x^9}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

output
```
5/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4*x^3 + 10/3*(-1)^(2*b^2*x^3 + 2*a*b
)*a^2*b^3*log(2*b^2*x^3 + 2*a*b) - 10/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^2*b^3*l
og(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^
(3/2)*b^4*x^3/a^2 + 5*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b^3 + 35/18*(b^2*x
^6 + 2*a*b*x^3 + a^2)^(3/2)*b^3/a + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)
*b^3/a^3 - 11/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^2*x^3) - 1/18*(b
^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^6) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a
^2)^(7/2)/(a^2*x^9)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.50

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{10}} \, dx = \frac{1}{6} b^5 x^6 \mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{3} ab^4 x^3 \mathrm{sgn}\left(bx^3 + a\right) + 10\, a^2 b^3 \log\left(|x|\right) \mathrm{sgn}\left(bx^3 + a\right) - \frac{110\, a^2 b^3 x^9 \mathrm{sgn}(bx^3 + a) + 60\, a^3 b^2 x^6 \mathrm{sgn}(bx^3 + a) + 15\, a^4 b x^3 \mathrm{sgn}(bx^3 + a) + 2\, a^5 \mathrm{sgn}(bx^3 + a)}{18\, x^9}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="giac")
```

output
```
1/6*b^5*x^6*sgn(b*x^3 + a) + 5/3*a*b^4*x^3*sgn(b*x^3 + a) + 10*a^2*b^3*log
(abs(x))*sgn(b*x^3 + a) - 1/18*(110*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^
2*x^6*sgn(b*x^3 + a) + 15*a^4*b*x^3*sgn(b*x^3 + a) + 2*a^5*sgn(b*x^3 + a))
/x^9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{10}} \, dx = \int \frac{(a^2 + 2\, a\, b\, x^3 + b^2\, x^6)^{5/2}}{x^{10}} \, dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = \frac{180\log(x)\,a^2b^3x^9-2a^5-15a^4b\,x^3-60a^3b^2x^6+30a\,b^4x^{12}+3b^5x^{15}}{18x^9}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x)
```

output
```
(180*log(x)*a**2*b**3*x**9 - 2*a**5 - 15*a**4*b*x**3 - 60*a**3*b**2*x**6 +
 30*a*b**4*x**12 + 3*b**5*x**15)/(18*x**9)
```

3.45 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6(a+bx^3)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{b^5x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

output
```
-1/12*a^5*((b*x^3+a)^2)^(1/2)/x^12/(b*x^3+a)-5/9*a^4*b*((b*x^3+a)^2)^(1/2)
/x^9/(b*x^3+a)-5/3*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-10/3*a^2*b^3*
((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+b^5*x^3*((b*x^3+a)^2)^(1/2)/(3*b*x^3+3*a
)+5*a*b^4*((b*x^3+a)^2)^(1/2)*ln(x)/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = -\frac{\sqrt{(a+bx^3)^2}(3a^5+20a^4bx^3+60a^3b^2x^6+120a^2b^3x^9-12b^5x^{15}-180ab^4x^{12}\log(x))}{36x^{12}(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

output
```
-1/36*(Sqrt[(a + b*x^3)^2]*(3*a^5 + 20*a^4*b*x^3 + 60*a^3*b^2*x^6 + 120*a^
2*b^3*x^9 - 12*b^5*x^15 - 180*a*b^4*x^12*Log[x]))/(x^12*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.38, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5(bx^3+a)^5}{x^{13}}dx}{b^5(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^{13}}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^{15}}dx^3}{3\,(a+bx^3)}$$

$$\Big\downarrow \textbf{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{15}}+\frac{5ba^4}{x^{12}}+\frac{10b^2a^3}{x^9}+\frac{10b^3a^2}{x^6}+\frac{5b^4a}{x^3}+b^5\right)dx^3}{3\,(a+bx^3)}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{4x^{12}}-\frac{5a^4b}{3x^9}-\frac{5a^3b^2}{x^6}-\frac{10a^2b^3}{x^3}+5ab^4\log\left(x^3\right)+b^5x^3\right)}{3\,(a+bx^3)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/4*a^5/x^12 - (5*a^4*b)/(3*x^9) - (5*a
^3*b^2)/x^6 - (10*a^2*b^3)/x^3 + b^5*x^3 + 5*a*b^4*Log[x^3]))/(3*(a + b*x^
3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.32

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-4 b^{5} x^{15}-20 \ln\left(b\,x^{3}\right) a\,b^{4} x^{12}-4 a\,b^{4} x^{12}+40 a^{2} b^{3} x^{9}+20 a^{3} b^{2} x^{6}+\frac{20 a^{4} b\,x^{3}}{3}+a^{5}\right)}{12 x^{12}}$	81
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}\left(12 b^{5} x^{15}+180 a\,b^{4} \ln\left(x\right) x^{12}-120 a^{2} b^{3} x^{9}-60 a^{3} b^{2} x^{6}-20 a^{4} b\,x^{3}-3 a^{5}\right)}{36 \left(b\,x^{3}+a\right)^{5} x^{12}}$	82
risch	$\frac{b^{5} x^{3} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{3 b\,x^{3}+3 a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{10}{3} a^{2} b^{3} x^{9}-\frac{5}{3} a^{3} b^{2} x^{6}-\frac{5}{9} a^{4} b\,x^{3}-\frac{1}{12} a^{5}\right)}{\left(b\,x^{3}+a\right) x^{12}}+\frac{5 a\,b^{4} \sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(x\right)}{b\,x^{3}+a}$	119

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*csgn(b*x^3+a)*(-4*b^5*x^15-20*ln(b*x^3)*a*b^4*x^12-4*a*b^4*x^12+40*a
^2*b^3*x^9+20*a^3*b^2*x^6+20/3*a^4*b*x^3+a^5)/x^12
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx=\frac{12\,b^5x^{15}+180\,ab^4x^{12}\log(x)-120\,a^2b^3x^9-60\,a^3b^2x^6-20\,a^4bx^3-3\,a^5}{36\,x^{12}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="fricas")
```

output
```
1/36*(12*b^5*x^15 + 180*a*b^4*x^12*log(x) - 120*a^2*b^3*x^9 - 60*a^3*b^2*x
^6 - 20*a^4*b*x^3 - 3*a^5)/x^12
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{13}}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{13}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**13,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**13, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.36

$$\begin{aligned}\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{13}}\,dx &= \frac{5\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^5x^3}{6\,a} \\ &+ \frac{5}{3}\,(-1)^{2\,b^2x^3 + 2\,ab}\,ab^4\log\left(2\,b^2x^3 + 2\,ab\right) \\ &- \frac{5}{3}\,(-1)^{2\,abx^3 + 2\,a^2}\,ab^4\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right) + \frac{5\,(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b^5x^3}{12\,a^3} \\ &+ \frac{5}{2}\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^4 + \frac{35\,(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b^4}{36\,a^2} \\ &+ \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^4}{9\,a^4} - \frac{2\,(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^3}{9\,a^3x^3} \\ &- \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^2}{9\,a^4x^6} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b}{36\,a^3x^9} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}}{12\,a^2x^{12}}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="maxima")
```

output
```
5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5*x^3/a + 5/3*(-1)^(2*b^2*x^3 + 2*a*
b)*a*b^4*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a*b^4*log(2
*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/
2)*b^5*x^3/a^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4 + 35/36*(b^2*x^6
+ 2*a*b*x^3 + a^2)^(3/2)*b^4/a^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b
^4/a^4 - 2/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/(a^3*x^3) - 1/9*(b^2*x^
6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^6) + 1/36*(b^2*x^6 + 2*a*b*x^3 + a^2
)^(7/2)*b/(a^3*x^9) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^12)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.50

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{13}}\,dx = \frac{1}{3}\,b^5x^3\mathrm{sgn}\left(bx^3+a\right) + 5\,ab^4\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right) - \frac{125\,ab^4x^{12}\mathrm{sgn}(bx^3+a) + 120\,a^2b^3x^9\mathrm{sgn}(bx^3+a) + 60\,a^3b^2x^6\mathrm{sgn}(bx^3+a) + 20\,a^4bx^3\mathrm{sgn}(bx^3+a) + 3\,a^5}{36\,x^{12}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="giac")
```

output
```
1/3*b^5*x^3*sgn(b*x^3 + a) + 5*a*b^4*log(abs(x))*sgn(b*x^3 + a) - 1/36*(12
5*a*b^4*x^12*sgn(b*x^3 + a) + 120*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*
x^6*sgn(b*x^3 + a) + 20*a^4*b*x^3*sgn(b*x^3 + a) + 3*a^5*sgn(b*x^3 + a))/x
^12
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{13}}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^{13}}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = \frac{180\log(x)\,a\,b^4x^{12}-3a^5-20a^4b\,x^3-60a^3b^2x^6-120a^2b^3x^9+12b^5x^{15}}{36x^{12}}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x)
```

output
```
(180*log(x)*a*b**4*x**12 - 3*a**5 - 20*a**4*b*x**3 - 60*a**3*b**2*x**6 - 1
20*a**2*b**3*x**9 + 12*b**5*x**15)/(36*x**12)
```

3.46 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{16}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\begin{aligned}\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{16}}\,dx = & -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)} \\ & -\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)} \\ & -\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}\end{aligned}$$

output
```
-1/15*a^5*((b*x^3+a)^2)^(1/2)/x^15/(b*x^3+a)-5/12*a^4*b*((b*x^3+a)^2)^(1/2
)/x^12/(b*x^3+a)-10/9*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-5/3*a^2*b^
3*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-5/3*a*b^4*((b*x^3+a)^2)^(1/2)/x^3/(b*x
^3+a)+b^5*((b*x^3+a)^2)^(1/2)*ln(x)/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = \frac{1}{360}\left(-\frac{\sqrt{(a+bx^3)^2}(12a^4+63a^3bx^3+137a^2b^2x^6+163ab^3x^9+137b^4x^{12})}{x^{15}} + \frac{\sqrt{a^2}(12a^4+75a^3bx^3+200a^2b^2x^6+300ab^3x^9+300b^4x^{12})}{x^{15}} - 120b^5\operatorname{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+bx^3)^2}}\right) - \frac{120\sqrt{a^2}b^5\log(x^3)}{a} + \frac{60\sqrt{a^2}b^5\log\left(a\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)\right)}{a} + \frac{60\sqrt{a^2}b^5\log\left(a\left(\sqrt{a^2}+bx^3-\sqrt{(a+bx^3)^2}\right)\right)}{a}\right)$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

output
```
(-((Sqrt[(a + b*x^3)^2]*(12*a^4 + 63*a^3*b*x^3 + 137*a^2*b^2*x^6 + 163*a*b
^3*x^9 + 137*b^4*x^12))/x^15) + (Sqrt[a^2]*(12*a^4 + 75*a^3*b*x^3 + 200*a^
2*b^2*x^6 + 300*a*b^3*x^9 + 300*b^4*x^12))/x^15 - 120*b^5*ArcTanh[(b*x^3)/
(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])] - (120*Sqrt[a^2]*b^5*Log[x^3])/a + (60*
Sqrt[a^2]*b^5*Log[a*(Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2])])/a + (60*Sq
rt[a^2]*b^5*Log[a*(Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2])])/a)/360
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{16}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{16}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{16}}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{18}}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5}{x^{18}}+\frac{5ba^4}{x^{15}}+\frac{10b^2a^3}{x^{12}}+\frac{10b^3a^2}{x^9}+\frac{5b^4a}{x^6}+\frac{b^5}{x^3}\right)dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{5x^{15}}-\frac{5a^4b}{4x^{12}}-\frac{10a^3b^2}{3x^9}-\frac{5a^2b^3}{x^6}-\frac{5ab^4}{x^3}+b^5\log\left(x^3\right)\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/5*a^5/x^15 - (5*a^4*b)/(4*x^12) - (10
*a^3*b^2)/(3*x^9) - (5*a^2*b^3)/x^6 - (5*a*b^4)/x^3 + b^5*Log[x^3]))/(3*(a
 + b*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.29

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-5\ln\left(b\,x^{3}\right)b^{5}x^{15}+a\left(25b^{4}x^{12}+25a\,b^{3}x^{9}+\frac{50}{3}b^{2}x^{6}a^{2}+\frac{25}{4}b\,x^{3}a^{3}+a^{4}\right)\right)}{15x^{15}}$	72
default	$\frac{\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}\left(180b^{5}\ln\left(x\right)x^{15}-300a\,b^{4}x^{12}-300a^{2}b^{3}x^{9}-200a^{3}b^{2}x^{6}-75a^{4}b\,x^{3}-12a^{5}\right)}{180\left(b\,x^{3}+a\right)^{5}x^{15}}$	82
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{15}a^{5}-\frac{5}{12}a^{4}b\,x^{3}-\frac{10}{9}a^{3}b^{2}x^{6}-\frac{5}{3}a^{2}b^{3}x^{9}-\frac{5}{3}a\,b^{4}x^{12}\right)}{\left(b\,x^{3}+a\right)x^{15}}+\frac{b^{5}\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(x\right)}{b\,x^{3}+a}$	98

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*csgn(b*x^3+a)*(-5*ln(b*x^3)*b^5*x^15+a*(25*b^4*x^12+25*a*b^3*x^9+50/
3*b^2*x^6*a^2+25/4*b*x^3*a^3+a^4))/x^15
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = \frac{180\,b^5x^{15}\log{(x)} - 300\,ab^4x^{12} - 300\,a^2b^3x^9 - 200\,a^3b^2x^6 - 75\,a^4bx^3 - 12\,a^5}{180\,x^{15}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="fricas")
```

output
```
1/180*(180*b^5*x^15*log(x) - 300*a*b^4*x^12 - 300*a^2*b^3*x^9 - 200*a^3*b^
2*x^6 - 75*a^4*b*x^3 - 12*a^5)/x^15
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = \int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{16}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**16,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**16, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(175) = 350$.

Time = 0.05 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.49

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx = \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^6x^3}{6\,a^2} + \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,b^5\log\left(2\,b^2x^3+2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,b^5\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right) + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^6x^3}{12\,a^4} + \frac{\sqrt{b^2x^6+2\,abx^3+a^2}b^5}{2\,a} + \frac{7\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^5}{36\,a^3} - \frac{2\,(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^5}{45\,a^5} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^4}{9\,a^4x^3} + \frac{2\,(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^3}{45\,a^5x^6} - \frac{11\,(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^2}{180\,a^4x^9} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{20\,a^3x^{12}} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{15\,a^2x^{15}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="maxima")
```

output
```
1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^6*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*
a*b)*b^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^5*log(2*a
*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)
*b^6*x^3/a^4 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5/a + 7/36*(b^2*x^6 +
 2*a*b*x^3 + a^2)^(3/2)*b^5/a^3 - 2/45*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b
^5/a^5 - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^4/(a^4*x^3) + 2/45*(b^2*x
^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^6) - 11/180*(b^2*x^6 + 2*a*b*x^3 +
a^2)^(7/2)*b^2/(a^4*x^9) + 1/20*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x
^12) - 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^15)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.49

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{16}}\,dx = b^5 \log(|x|)\,\mathrm{sgn}(bx^3 + a) - \frac{137\,b^5x^{15}\mathrm{sgn}(bx^3 + a) + 300\,ab^4x^{12}\mathrm{sgn}(bx^3 + a) + 300\,a^2b^3x^9\mathrm{sgn}(bx^3 + a) + 200\,a^3b^2x^6\mathrm{sgn}(bx^3 + a) + 7}{180\,x^{15}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="giac")
```

output
```
b^5*log(abs(x))*sgn(b*x^3 + a) - 1/180*(137*b^5*x^15*sgn(b*x^3 + a) + 300*
a*b^4*x^12*sgn(b*x^3 + a) + 300*a^2*b^3*x^9*sgn(b*x^3 + a) + 200*a^3*b^2*x
^6*sgn(b*x^3 + a) + 75*a^4*b*x^3*sgn(b*x^3 + a) + 12*a^5*sgn(b*x^3 + a))/x
^15
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{16}}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}}{x^{16}}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{16}}\,dx = \frac{180\log(x)\,b^5x^{15} - 12a^5 - 75a^4b\,x^3 - 200a^3b^2x^6 - 300a^2b^3x^9 - 300a\,b^4x^{12}}{180x^{15}}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x)
```

output
```
(180*log(x)*b**5*x**15 - 12*a**5 - 75*a**4*b*x**3 - 200*a**3*b**2*x**6 - 3
00*a**2*b**3*x**9 - 300*a*b**4*x**12)/(180*x**15)
```

3.47 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{19}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{19}}\,dx=-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

output
```
-1/18*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^18
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.98

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{19}}\,dx = -\frac{\sqrt{\left(a+bx^3\right)^2}\left(a^5+6a^4bx^3+15a^3b^2x^6+20a^2b^3x^9+15ab^4x^{12}+6b^5x^{15}\right)}{18x^{18}\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

output
```
-1/18*(Sqrt[(a + b*x^3)^2]*(a^5 + 6*a^4*b*x^3 + 15*a^3*b^2*x^6 + 20*a^2*b^
3*x^9 + 15*a*b^4*x^12 + 6*b^5*x^15))/(x^18*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1384, 27, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{19}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{19}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{19}}dx}{a+bx^3}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

output
```
-1/18*((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^18)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.65 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(2b\,x^{3}+a\right)\left(3b^{2}x^{6}+3a\,x^{3}b+a^{2}\right)\left(b^{2}x^{6}+a\,x^{3}b+a^{2}\right)}{18x^{18}}$	58
gosper	$-\frac{\left(6b^{5}x^{15}+15a\,b^{4}x^{12}+20a^{2}b^{3}x^{9}+15a^{3}b^{2}x^{6}+6a^{4}b\,x^{3}+a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{18x^{18}\left(b\,x^{3}+a\right)^{5}}$	78
default	$-\frac{\left(6b^{5}x^{15}+15a\,b^{4}x^{12}+20a^{2}b^{3}x^{9}+15a^{3}b^{2}x^{6}+6a^{4}b\,x^{3}+a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{18x^{18}\left(b\,x^{3}+a\right)^{5}}$	78
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{18}a^{5}-\frac{1}{3}a^{4}b\,x^{3}-\frac{5}{6}a^{3}b^{2}x^{6}-\frac{10}{9}a^{2}b^{3}x^{9}-\frac{5}{6}a\,b^{4}x^{12}-\frac{1}{3}b^{5}x^{15}\right)}{\left(b\,x^{3}+a\right)x^{18}}$	79
orering	$-\frac{\left(6b^{5}x^{15}+15a\,b^{4}x^{12}+20a^{2}b^{3}x^{9}+15a^{3}b^{2}x^{6}+6a^{4}b\,x^{3}+a^{5}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{5}{2}}}{18x^{18}\left(b\,x^{3}+a\right)^{5}}$	87

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/18*csgn(b*x^3+a)*(2*b*x^3+a)*(3*b^2*x^6+3*a*b*x^3+a^2)*(b^2*x^6+a*b*x^3
+a^2)/x^18
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(28) = 56.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx = -\frac{6\,b^5x^{15}+15\,ab^4x^{12}+20\,a^2b^3x^9+15\,a^3b^2x^6+6\,a^4bx^3+a^5}{18\,x^{18}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="fricas")
```

output
```
-1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^
4*b*x^3 + a^5)/x^18
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{19}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**19,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**19, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(28) = 56.

Time = 0.05 (sec) , antiderivative size = 210, normalized size of antiderivative = 5.12

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx = \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^6}{18\,a^6} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^5}{18\,a^5x^3} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^4}{18\,a^6x^6} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^3}{18\,a^5x^9} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^2}{18\,a^4x^{12}} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{18\,a^3x^{15}} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{18\,a^2x^{18}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="maxima")
```

output
```
1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^6/a^6 + 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*b^5/(a^5*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(
a^6*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^9) - 1/18*(b^
2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(7/2)*b/(a^3*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^
18)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(28) = 56$.

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.59

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{19}}\,dx = -\frac{6\,b^5x^{15}\text{sgn}(bx^3+a) + 15\,ab^4x^{12}\text{sgn}(bx^3+a) + 20\,a^2b^3x^9\text{sgn}(bx^3+a) + 15\,a^3b^2x^6\text{sgn}(bx^3+a) + 6\,a^4bx^3}{18\,x^{18}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="giac")
```

output
```
-1/18*(6*b^5*x^15*sgn(b*x^3 + a) + 15*a*b^4*x^12*sgn(b*x^3 + a) + 20*a^2*b
^3*x^9*sgn(b*x^3 + a) + 15*a^3*b^2*x^6*sgn(b*x^3 + a) + 6*a^4*b*x^3*sgn(b*
x^3 + a) + a^5*sgn(b*x^3 + a))/x^18
```

Mupad [B] (verification not implemented)

Time = 19.37 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.63

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx = -\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{18\,x^{18}\,\left(b\,x^3+a\right)} - \frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3\,\left(b\,x^3+a\right)} - \frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,\left(b\,x^3+a\right)} - \frac{a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^{15}\,\left(b\,x^3+a\right)} - \frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\,\left(b\,x^3+a\right)} - \frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^{12}\,\left(b\,x^3+a\right)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^19,x)
```

output
```
- (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (b^5*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*
x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (a^4*b*(a^2 + b^2*x^6 + 2*a*
b*x^3)^(1/2))/(3*x^15*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^
3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(6*x^12*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx = \frac{-6b^5x^{15}-15a\,b^4x^{12}-20a^2b^3x^9-15a^3b^2x^6-6a^4b\,x^3-a^5}{18x^{18}}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x)
```

output
```
( - a**5 - 6*a**4*b*x**3 - 15*a**3*b**2*x**6 - 20*a**2*b**3*x**9 - 15*a*b*
*4*x**12 - 6*b**5*x**15)/(18*x**18)
```

3.48 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx = -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}$$

output
```
-1/21*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^21+1/126*b*(b*x^3+a)^5*((b*x^3+a
)^2)^(1/2)/a^2/x^18
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx = -\frac{\sqrt{\left(a+bx^3\right)^2}\left(6a^5+35a^4bx^3+84a^3b^2x^6+105a^2b^3x^9+70ab^4x^{12}+21b^5x^{15}\right)}{126x^{21}\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

output
```
-1/126*(Sqrt[(a + b*x^3)^2]*(6*a^5 + 35*a^4*b*x^3 + 84*a^3*b^2*x^6 + 105*a
^2*b^3*x^9 + 70*a*b^4*x^12 + 21*b^5*x^15))/(x^21*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{22}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{22}}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{24}}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{55}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{b\int \frac{\left(bx^3+a\right)^5}{x^{21}}dx^3}{7a}-\frac{\left(a+bx^3\right)^6}{7ax^{21}}\right)}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{48}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{b\left(a+bx^3\right)^6}{42a^2x^{18}}-\frac{\left(a+bx^3\right)^6}{7ax^{21}}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/7*(a + b*x^3)^6/(a*x^21) + (b*(a + b*
x^3)^6)/(42*a^2*x^18)))/(3*(a + b*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
 - 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(21 b^{5} x^{15}+70 a\,b^{4} x^{12}+105 a^{2} b^{3} x^{9}+84 a^{3} b^{2} x^{6}+35 a^{4} b\,x^{3}+6 a^{5}\right)}{126 x^{21}}$	68
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{21} a^{5}-\frac{5}{18} a^{4} b\,x^{3}-\frac{2}{3} a^{3} b^{2} x^{6}-\frac{5}{6} a^{2} b^{3} x^{9}-\frac{5}{9} a\,b^{4} x^{12}-\frac{1}{6} b^{5} x^{15}\right)}{\left(b\,x^{3}+a\right) x^{21}}$	79
gosper	$-\frac{\left(21 b^{5} x^{15}+70 a\,b^{4} x^{12}+105 a^{2} b^{3} x^{9}+84 a^{3} b^{2} x^{6}+35 a^{4} b\,x^{3}+6 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{126 x^{21} \left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(21 b^{5} x^{15}+70 a\,b^{4} x^{12}+105 a^{2} b^{3} x^{9}+84 a^{3} b^{2} x^{6}+35 a^{4} b\,x^{3}+6 a^{5}\right) \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{126 x^{21} \left(b\,x^{3}+a\right)^{5}}$	80
orering	$-\frac{\left(21 b^{5} x^{15}+70 a\,b^{4} x^{12}+105 a^{2} b^{3} x^{9}+84 a^{3} b^{2} x^{6}+35 a^{4} b\,x^{3}+6 a^{5}\right) \left(b^{2} x^{6}+2 a\,x^{3} b+a^{2}\right)^{\frac{5}{2}}}{126 x^{21} \left(b\,x^{3}+a\right)^{5}}$	89

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x,method=_RETURNVERBOSE)
```

output
```
-1/126*csgn(b*x^3+a)*(21*b^5*x^15+70*a*b^4*x^12+105*a^2*b^3*x^9+84*a^3*b^2
*x^6+35*a^4*b*x^3+6*a^5)/x^21
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx = -\frac{21\,b^5x^{15}+70\,ab^4x^{12}+105\,a^2b^3x^9+84\,a^3b^2x^6+35\,a^4bx^3+6\,a^5}{126\,x^{21}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="fricas")
```

output
```
-1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 3
5*a^4*b*x^3 + 6*a^5)/x^21
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{22}}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{22}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**22,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**22, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(58) = 116.

Time = 0.05 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.87

$$\begin{aligned}\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{22}}\,dx = &-\frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^7}{18\,a^7} \\ &- \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^6}{18\,a^6x^3} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^5}{18\,a^7x^6} \\ &- \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^4}{18\,a^6x^9} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^3}{18\,a^5x^{12}} \\ &- \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^2}{18\,a^4x^{15}} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b}{18\,a^3x^{18}} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}}{21\,a^2x^{21}}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="maxima")
```

output
```
-1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^7/a^7 - 1/18*(b^2*x^6 + 2*a*b*x^
3 + a^2)^(5/2)*b^6/(a^6*x^3) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/
(a^7*x^6) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^9) + 1/18*(b
^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^12) - 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(7/2)*b^2/(a^4*x^15) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a
^3*x^18) - 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^21)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{22}}\,dx = -\frac{21\,b^5x^{15}\mathrm{sgn}(bx^3 + a) + 70\,ab^4x^{12}\mathrm{sgn}(bx^3 + a) + 105\,a^2b^3x^9\mathrm{sgn}(bx^3 + a) + 84\,a^3b^2x^6\mathrm{sgn}(bx^3 + a) + 35\,a^4}{126\,x^{21}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="giac")
```

output
```
-1/126*(21*b^5*x^15*sgn(b*x^3 + a) + 70*a*b^4*x^12*sgn(b*x^3 + a) + 105*a^
2*b^3*x^9*sgn(b*x^3 + a) + 84*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sg
n(b*x^3 + a) + 6*a^5*sgn(b*x^3 + a))/x^21
```

Mupad [B] (verification not implemented)

Time = 19.51 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.75

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{22}}\,dx = -\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{21\,x^{21}\,\left(b\,x^3+a\right)} - \frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,\left(b\,x^3+a\right)} - \frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\,\left(b\,x^3+a\right)} - \frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{18\,x^{18}\,\left(b\,x^3+a\right)} - \frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^{12}\,\left(b\,x^3+a\right)} - \frac{2\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^{15}\,\left(b\,x^3+a\right)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^22,x)
```

output
```
- (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(21*x^21*(a + b*x^3)) - (b^5*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*
x^6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*
a*b*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(6*x^12*(a + b*x^3)) - (2*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^
(1/2))/(3*x^15*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{22}}\,dx = \frac{-21b^5x^{15} - 70a\,b^4x^{12} - 105a^2b^3x^9 - 84a^3b^2x^6 - 35a^4b\,x^3 - 6a^5}{126x^{21}}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x)
```

output
```
( - 6*a**5 - 35*a**4*b*x**3 - 84*a**3*b**2*x**6 - 105*a**2*b**3*x**9 - 70*
a*b**4*x**12 - 21*b**5*x**15)/(126*x**21)
```

3.49 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 128

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}\,dx = -\frac{\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} + \frac{b\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}} - \frac{b^2\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{504a^3x^{18}}$$

output
```
-1/24*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^24+1/84*b*(b*x^3+a)^5*((b*x^3+a)
^2)^(1/2)/a^2/x^21-1/504*b^2*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a^3/x^18
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}\,dx = -\frac{\sqrt{\left(a+bx^3\right)^2}\left(21a^5+120a^4bx^3+280a^3b^2x^6+336a^2b^3x^9+210ab^4x^{12}+56b^5x^{15}\right)}{504x^{24}\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

output
```
-1/504*(Sqrt[(a + b*x^3)^2]*(21*a^5 + 120*a^4*b*x^3 + 280*a^3*b^2*x^6 + 33
6*a^2*b^3*x^9 + 210*a*b^4*x^12 + 56*b^5*x^15))/(x^24*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 27, 798, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{25}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^{25}}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{25}}dx}{a+bx^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^{27}}dx^3}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{55}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{b\int \frac{\left(bx^3+a\right)^5}{x^{24}}dx^3}{4a}-\frac{\left(a+bx^3\right)^6}{8ax^{24}}\right)}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{55}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{b\left(-\frac{b\int \frac{\left(bx^3+a\right)^5}{x^{21}}dx^3}{7a}-\frac{\left(a+bx^3\right)^6}{7ax^{21}}\right)}{4a}-\frac{\left(a+bx^3\right)^6}{8ax^{24}}\right)}{3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{48}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{b\left(\frac{b\left(a+bx^3\right)^6}{42a^2x^{18}}-\frac{\left(a+bx^3\right)^6}{7ax^{21}}\right)}{4a}-\frac{\left(a+bx^3\right)^6}{8ax^{24}}\right)}{3\left(a+bx^3\right)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/8*(a + b*x^3)^6/(a*x^24) - (b*(-1/7*(
a + b*x^3)^6/(a*x^21) + (b*(a + b*x^3)^6)/(42*a^2*x^18)))/(4*a)))/(3*(a +
b*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 2.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$-\frac{\left(\frac{8}{3} b^{5} x^{15}+10 a \,b^{4} x^{12}+16 a^{2} b^{3} x^{9}+\frac{40}{3} a^{3} b^{2} x^{6}+\frac{40}{7} a^{4} b \,x^{3}+a^{5}\right) \operatorname{csgn}\left(b \,x^{3}+a \right)}{24 x^{24}}$	66
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-\frac{1}{24} a^{5}-\frac{5}{9} a^{3} b^{2} x^{6}-\frac{2}{3} a^{2} b^{3} x^{9}-\frac{5}{12} a \,b^{4} x^{12}-\frac{1}{9} b^{5} x^{15}-\frac{5}{21} a^{4} b \,x^{3}\right)}{\left(b \,x^{3}+a \right) x^{24}}$	79
gosper	$-\frac{\left(56 b^{5} x^{15}+210 a \,b^{4} x^{12}+336 a^{2} b^{3} x^{9}+280 a^{3} b^{2} x^{6}+120 a^{4} b \,x^{3}+21 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{504 x^{24} \left(b \,x^{3}+a \right)^{5}}$	80
default	$-\frac{\left(56 b^{5} x^{15}+210 a \,b^{4} x^{12}+336 a^{2} b^{3} x^{9}+280 a^{3} b^{2} x^{6}+120 a^{4} b \,x^{3}+21 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{504 x^{24} \left(b \,x^{3}+a \right)^{5}}$	80
orering	$-\frac{\left(56 b^{5} x^{15}+210 a \,b^{4} x^{12}+336 a^{2} b^{3} x^{9}+280 a^{3} b^{2} x^{6}+120 a^{4} b \,x^{3}+21 a^{5}\right) \left(b^{2} x^{6}+2 a \,x^{3} b +a^{2}\right)^{\frac{5}{2}}}{504 x^{24} \left(b \,x^{3}+a \right)^{5}}$	89

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(8/3*b^5*x^15+10*a*b^4*x^12+16*a^2*b^3*x^9+40/3*a^3*b^2*x^6+40/7*a^4
*b*x^3+a^5)*csgn(b*x^3+a)/x^24
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx = -\frac{56\,b^5x^{15}+210\,ab^4x^{12}+336\,a^2b^3x^9+280\,a^3b^2x^6+120\,a^4bx^3+21\,a^5}{504\,x^{24}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="fricas")
```

output
```
-1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 +
 120*a^4*b*x^3 + 21*a^5)/x^24
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{25}}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**25,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**25, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(89) = 178$.

Time = 0.05 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.12

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{25}}\,dx = \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^8}{18\,a^8} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^7}{18\,a^7x^3} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^6}{18\,a^8x^6} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^5}{18\,a^7x^9} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^4}{18\,a^6x^{12}} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^3}{18\,a^5x^{15}} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^2}{18\,a^4x^{18}} + \frac{3\,(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b}{56\,a^3x^{21}} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}}{24\,a^2x^{24}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="maxima")
```

output
```
1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^8/a^8 + 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*b^7/(a^7*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^6/(
a^8*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/(a^7*x^9) - 1/18*(b^
2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(7/2)*b^3/(a^5*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(
a^4*x^18) + 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^21) - 1/24*(b^
2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^24)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{25}}\,dx = -\frac{56\,b^5x^{15}\mathrm{sgn}(bx^3 + a) + 210\,ab^4x^{12}\mathrm{sgn}(bx^3 + a) + 336\,a^2b^3x^9\mathrm{sgn}(bx^3 + a) + 280\,a^3b^2x^6\mathrm{sgn}(bx^3 + a) + 12}{504\,x^{24}}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="giac")
```

output
```
-1/504*(56*b^5*x^15*sgn(b*x^3 + a) + 210*a*b^4*x^12*sgn(b*x^3 + a) + 336*a
^2*b^3*x^9*sgn(b*x^3 + a) + 280*a^3*b^2*x^6*sgn(b*x^3 + a) + 120*a^4*b*x^3
*sgn(b*x^3 + a) + 21*a^5*sgn(b*x^3 + a))/x^24
```

Mupad [B] (verification not implemented)

Time = 19.60 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.80

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx = -\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{24\,x^{24}\,(b\,x^3+a)} - \frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\,(b\,x^3+a)} - \frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{12\,x^{12}\,(b\,x^3+a)} - \frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{21\,x^{21}\,(b\,x^3+a)} - \frac{2\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^{15}\,(b\,x^3+a)} - \frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^{18}\,(b\,x^3+a)}$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^25,x)
```

output
```
- (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(24*x^24*(a + b*x^3)) - (b^5*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*
x^6 + 2*a*b*x^3)^(1/2))/(12*x^12*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 +
2*a*b*x^3)^(1/2))/(21*x^21*(a + b*x^3)) - (2*a^2*b^3*(a^2 + b^2*x^6 + 2*a*
b*x^3)^(1/2))/(3*x^15*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(9*x^18*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx = \frac{-56b^5x^{15}-210a\,b^4x^{12}-336a^2b^3x^9-280a^3b^2x^6-120a^4b\,x^3-21a^5}{504x^{24}}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x)
```

output
```
( - 21*a**5 - 120*a**4*b*x**3 - 280*a**3*b**2*x**6 - 336*a**2*b**3*x**9 -
210*a*b**4*x**12 - 56*b**5*x**15)/(504*x**24)
```

3.50 $\int x^4(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5a^4bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^2b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{b^5x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{20\left(a+bx^3\right)}$$

output
```
a^5*x^5*((b*x^3+a)^2)^(1/2)/(5*b*x^3+5*a)+5*a^4*b*x^8*((b*x^3+a)^2)^(1/2)/
(8*b*x^3+8*a)+10*a^3*b^2*x^11*((b*x^3+a)^2)^(1/2)/(11*b*x^3+11*a)+5*a^2*b^
3*x^14*((b*x^3+a)^2)^(1/2)/(7*b*x^3+7*a)+5*a*b^4*x^17*((b*x^3+a)^2)^(1/2)/
(17*b*x^3+17*a)+b^5*x^20*((b*x^3+a)^2)^(1/2)/(20*b*x^3+20*a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^5\sqrt{(a+bx^3)^2}(10472a^5+32725a^4bx^3+47600a^3b^2x^6+37400a^2b^3x^9+15400ab^4x^{12}+261}{52360\,(a+bx^3)}$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^5*Sqrt[(a + b*x^3)^2]*(10472*a^5 + 32725*a^4*b*x^3 + 47600*a^3*b^2*x^6
+ 37400*a^2*b^3*x^9 + 15400*a*b^4*x^12 + 2618*b^5*x^15))/(52360*(a + b*x^3
))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^5x^4\left(bx^3+a\right)^5dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \; 802$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(b^5x^{19}+5ab^4x^{16}+10a^2b^3x^{13}+10a^3b^2x^{10}+5a^4bx^7+a^5x^4\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5x^5}{5}+\frac{5}{8}a^4bx^8+\frac{10}{11}a^3b^2x^{11}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{20}}{20}\right)}{a+bx^3}$$

input
```
Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*x^5)/5 + (5*a^4*b*x^8)/8 + (10*a^3*
b^2*x^11)/11 + (5*a^2*b^3*x^14)/7 + (5*a*b^4*x^17)/17 + (b^5*x^20)/20))/(a
 + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{5}\left(2618b^{5}x^{15}+15400a\,b^{4}x^{12}+37400a^{2}b^{3}x^{9}+47600a^{3}b^{2}x^{6}+32725a^{4}b\,x^{3}+10472a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{52360\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{5}\left(2618b^{5}x^{15}+15400a\,b^{4}x^{12}+37400a^{2}b^{3}x^{9}+47600a^{3}b^{2}x^{6}+32725a^{4}b\,x^{3}+10472a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{52360\left(b\,x^{3}+a\right)^{5}}$
orering	$\frac{x^{5}\left(2618b^{5}x^{15}+15400a\,b^{4}x^{12}+37400a^{2}b^{3}x^{9}+47600a^{3}b^{2}x^{6}+32725a^{4}b\,x^{3}+10472a^{5}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{5}{2}}}{52360\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{a^{5}x^{5}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{5b\,x^{3}+5a}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{4}b\,x^{8}}{8\left(b\,x^{3}+a\right)}+\frac{10\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{3}b^{2}x^{11}}{11\left(b\,x^{3}+a\right)}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}b^{3}x^{14}}{7\left(b\,x^{3}+a\right)}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a\,b^{4}x^{17}}{17\left(b\,x^{3}+a\right)}+\frac{b^{5}}{}$

input
```
int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/52360*x^5*(2618*b^5*x^15+15400*a*b^4*x^12+37400*a^2*b^3*x^9+47600*a^3*b^
2*x^6+32725*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 +
5/8*a^4*b*x^8 + 1/5*a^5*x^5
```

Sympy [F]

$$\int x^4(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\int x^4\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**4*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^4(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 +
5/8*a^4*b*x^8 + 1/5*a^5*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^4(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{1}{20}\,b^5x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{17}\,ab^4x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,a^2b^3x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{11}\,a^3b^2x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,a^4bx^8\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,a^5x^5\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/20*b^5*x^20*sgn(b*x^3 + a) + 5/17*a*b^4*x^17*sgn(b*x^3 + a) + 5/7*a^2*b^
3*x^14*sgn(b*x^3 + a) + 10/11*a^3*b^2*x^11*sgn(b*x^3 + a) + 5/8*a^4*b*x^8*
sgn(b*x^3 + a) + 1/5*a^5*x^5*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \int x^4 \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2} dx$$

input
```
int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{x^5(2618b^5x^{15}+15400a\,b^4x^{12}+37400a^2b^3x^9+47600a^3b^2x^6+32725a^4b\,x^3+10472a^5)}{52360}$$

input
```
int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
(x**5*(10472*a**5 + 32725*a**4*b*x**3 + 47600*a**3*b**2*x**6 + 37400*a**2*
b**3*x**9 + 15400*a*b**4*x**12 + 2618*b**5*x**15))/52360
```

3.51 $\int x^3(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\left(a+bx^3\right)}$$

output

```
a^5*x^4*((b*x^3+a)^2)^(1/2)/(4*b*x^3+4*a)+5*a^4*b*x^7*((b*x^3+a)^2)^(1/2)/
(7*b*x^3+7*a)+a^3*b^2*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10*a^2*b^3*x^13*(
(b*x^3+a)^2)^(1/2)/(13*b*x^3+13*a)+5*a*b^4*x^16*((b*x^3+a)^2)^(1/2)/(16*b*
x^3+16*a)+b^5*x^19*((b*x^3+a)^2)^(1/2)/(19*b*x^3+19*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^3(a^2+2abx^3+b^2x^6)^{5/2}\,dx=\frac{x^4\sqrt{(a+bx^3)^2}(6916a^5+19760a^4bx^3+27664a^3b^2x^6+21280a^2b^3x^9+8645ab^4x^{12}+1456b}{27664\,(a+bx^3)}$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^4*Sqrt[(a + b*x^3)^2]*(6916*a^5 + 19760*a^4*b*x^3 + 27664*a^3*b^2*x^6 +
 21280*a^2*b^3*x^9 + 8645*a*b^4*x^12 + 1456*b^5*x^15))/(27664*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^5x^3\left(bx^3+a\right)^5dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{18}+5ab^4x^{15}+10a^2b^3x^{12}+10a^3b^2x^9+5a^4bx^6+a^5x^3\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5x^4}{4}+\frac{5}{7}a^4bx^7+a^3b^2x^{10}+\frac{10}{13}a^2b^3x^{13}+\frac{5}{16}ab^4x^{16}+\frac{b^5x^{19}}{19}\right)}{a+bx^3}$$

input
```
Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*x^4)/4 + (5*a^4*b*x^7)/7 + a^3*b^2*
x^10 + (10*a^2*b^3*x^13)/13 + (5*a*b^4*x^16)/16 + (b^5*x^19)/19))/(a + b*x
^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result
gosper	$\frac{x^{4}\left(1456 b^{5} x^{15}+8645 a\,b^{4} x^{12}+21280 a^{2} b^{3} x^{9}+27664 a^{3} b^{2} x^{6}+19760 a^{4} b\,x^{3}+6916 a^{5}\right) \left(\left(b\,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{27664 \left(b\,x^{3}+a \right)^{5}}$
default	$\frac{x^{4}\left(1456 b^{5} x^{15}+8645 a\,b^{4} x^{12}+21280 a^{2} b^{3} x^{9}+27664 a^{3} b^{2} x^{6}+19760 a^{4} b\,x^{3}+6916 a^{5}\right) \left(\left(b\,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{27664 \left(b\,x^{3}+a \right)^{5}}$
orering	$\frac{x^{4}\left(1456 b^{5} x^{15}+8645 a\,b^{4} x^{12}+21280 a^{2} b^{3} x^{9}+27664 a^{3} b^{2} x^{6}+19760 a^{4} b\,x^{3}+6916 a^{5}\right) \left(b^{2} x^{6}+2 a\,x^{3} b+a^{2}\right)^{\frac{5}{2}}}{27664 \left(b\,x^{3}+a \right)^{5}}$
risch	$\frac{a^{5} x^{4} \sqrt{\left(b\,x^{3}+a \right)^{2}}}{4 b\,x^{3}+4 a}+\frac{5 \sqrt{\left(b\,x^{3}+a \right)^{2}}\, a^{4} b\,x^{7}}{7 \left(b\,x^{3}+a \right)}+\frac{a^{3} b^{2} x^{10} \sqrt{\left(b\,x^{3}+a \right)^{2}}}{b\,x^{3}+a}+\frac{10 \sqrt{\left(b\,x^{3}+a \right)^{2}}\, a^{2} b^{3} x^{13}}{13 \left(b\,x^{3}+a \right)}+\frac{5 \sqrt{\left(b\,x^{3}+a \right)^{2}}\, a\,b^{4} x^{16}}{16 \left(b\,x^{3}+a \right)}+\frac{b^{5} x^{1}}{19}$

input
```
int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/27664*x^4*(1456*b^5*x^15+8645*a*b^4*x^12+21280*a^2*b^3*x^9+27664*a^3*b^2
*x^6+19760*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*
a^4*b*x^7 + 1/4*a^5*x^4
```

Sympy [F]

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \int x^3 \left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}} dx$$

input
```
integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**3*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{1}{19}b^5x^{19} + \frac{5}{16}ab^4x^{16} + \frac{10}{13}a^2b^3x^{13} + a^3b^2x^{10} + \frac{5}{7}a^4bx^7 + \frac{1}{4}a^5x^4$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*
a^4*b*x^7 + 1/4*a^5*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{1}{19}b^5x^{19}\operatorname{sgn}\left(bx^3+a\right) + \frac{5}{16}ab^4x^{16}\operatorname{sgn}\left(bx^3+a\right) + \frac{10}{13}a^2b^3x^{13}\operatorname{sgn}\left(bx^3+a\right) + a^3b^2x^{10}\operatorname{sgn}\left(bx^3+a\right) + \frac{5}{7}a^4bx^7\operatorname{sgn}\left(bx^3+a\right) + \frac{1}{4}a^5x^4\operatorname{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/19*b^5*x^19*sgn(b*x^3 + a) + 5/16*a*b^4*x^16*sgn(b*x^3 + a) + 10/13*a^2*
b^3*x^13*sgn(b*x^3 + a) + a^3*b^2*x^10*sgn(b*x^3 + a) + 5/7*a^4*b*x^7*sgn(
b*x^3 + a) + 1/4*a^5*x^4*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \int x^3 \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2} dx$$

input
```
int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{x^4(1456b^5x^{15}+8645a\,b^4x^{12}+21280a^2b^3x^9+27664a^3b^2x^6+19760a^4b\,x^3+6916a^5)}{27664}$$

input
```
int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
(x**4*(6916*a**5 + 19760*a**4*b*x**3 + 27664*a**3*b**2*x**6 + 21280*a**2*b
**3*x**9 + 8645*a*b**4*x**12 + 1456*b**5*x**15))/27664
```

3.52 $\int x(a^2 + 2abx^3 + b^2x^6)^{5/2}\, dx$

Optimal result

Integrand size = 24, antiderivative size = 252

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}$$

output
```
a^5*x^2*((b*x^3+a)^2)^(1/2)/(2*b*x^3+2*a)+a^4*b*x^5*((b*x^3+a)^2)^(1/2)/(b
*x^3+a)+5*a^3*b^2*x^8*((b*x^3+a)^2)^(1/2)/(4*b*x^3+4*a)+10*a^2*b^3*x^11*((
b*x^3+a)^2)^(1/2)/(11*b*x^3+11*a)+5*a*b^4*x^14*((b*x^3+a)^2)^(1/2)/(14*b*x
^3+14*a)+b^5*x^17*((b*x^3+a)^2)^(1/2)/(17*b*x^3+17*a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^2\sqrt{(a+bx^3)^2}(2618a^5+5236a^4bx^3+6545a^3b^2x^6+4760a^2b^3x^9+1870ab^4x^{12}+308b^5x^{15})}{5236\,(a+bx^3)}$$

input
```
Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^2*Sqrt[(a + b*x^3)^2]*(2618*a^5 + 5236*a^4*b*x^3 + 6545*a^3*b^2*x^6 + 4
760*a^2*b^3*x^9 + 1870*a*b^4*x^12 + 308*b^5*x^15))/(5236*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^5x\left(bx^3+a\right)^5dx}{b^5\,(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(bx^3+a\right)^5dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{16}+5ab^4x^{13}+10a^2b^3x^{10}+10a^3b^2x^7+5a^4bx^4+a^5x\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5x^2}{2}+a^4bx^5+\frac{5}{4}a^3b^2x^8+\frac{10}{11}a^2b^3x^{11}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{17}}{17}\right)}{a+bx^3}$$

input
```
Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*x^2)/2 + a^4*b*x^5 + (5*a^3*b^2*x^8
)/4 + (10*a^2*b^3*x^11)/11 + (5*a*b^4*x^14)/14 + (b^5*x^17)/17))/(a + b*x^
3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result
gosper	$\frac{x^{2}\left(308 b^{5} x^{15}+1870 a\,b^{4} x^{12}+4760 a^{2} b^{3} x^{9}+6545 a^{3} b^{2} x^{6}+5236 a^{4} b\,x^{3}+2618 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{5236 \left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{2}\left(308 b^{5} x^{15}+1870 a\,b^{4} x^{12}+4760 a^{2} b^{3} x^{9}+6545 a^{3} b^{2} x^{6}+5236 a^{4} b\,x^{3}+2618 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{5236 \left(b\,x^{3}+a\right)^{5}}$
orering	$\frac{x^{2}\left(308 b^{5} x^{15}+1870 a\,b^{4} x^{12}+4760 a^{2} b^{3} x^{9}+6545 a^{3} b^{2} x^{6}+5236 a^{4} b\,x^{3}+2618 a^{5}\right)\left(b^{2} x^{6}+2 a\,x^{3} b+a^{2}\right)^{\frac{5}{2}}}{5236 \left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{a^{5} x^{2} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{2 b\,x^{3}+2 a}+\frac{a^{4} b\,x^{5} \sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}+\frac{5 \sqrt{\left(b\,x^{3}+a\right)^{2}}\, a^{3} b^{2} x^{8}}{4 \left(b\,x^{3}+a\right)}+\frac{10 \sqrt{\left(b\,x^{3}+a\right)^{2}}\, a^{2} b^{3} x^{11}}{11 \left(b\,x^{3}+a\right)}+\frac{5 \sqrt{\left(b\,x^{3}+a\right)^{2}}\, a\,b^{4} x^{14}}{14 \left(b\,x^{3}+a\right)}+\frac{b^{5} x^{17}}{17}$

input
```
int(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5236*x^2*(308*b^5*x^15+1870*a*b^4*x^12+4760*a^2*b^3*x^9+6545*a^3*b^2*x^6
+5236*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a
^4*b*x^5 + 1/2*a^5*x^2
```

Sympy [F]

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a
^4*b*x^5 + 1/2*a^5*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{1}{17}\,b^5x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{14}\,ab^4x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{11}\,a^2b^3x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,a^3b^2x^8\mathrm{sgn}\left(bx^3+a\right)+a^4bx^5\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^5x^2\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/17*b^5*x^17*sgn(b*x^3 + a) + 5/14*a*b^4*x^14*sgn(b*x^3 + a) + 10/11*a^2*
b^3*x^11*sgn(b*x^3 + a) + 5/4*a^3*b^2*x^8*sgn(b*x^3 + a) + a^4*b*x^5*sgn(b
*x^3 + a) + 1/2*a^5*x^2*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int x\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

input
```
int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x^2(308b^5x^{15}+1870a\,b^4x^{12}+4760a^2b^3x^9+6545a^3b^2x^6+5236a^4b\,x^3+2618a^5)}{5236}$$

input
```
int(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
(x**2*(2618*a**5 + 5236*a**4*b*x**3 + 6545*a**3*b**2*x**6 + 4760*a**2*b**3
*x**9 + 1870*a*b**4*x**12 + 308*b**5*x**15))/5236
```

3.53 $\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal result

Integrand size = 22, antiderivative size = 247

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{a^5x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5a^4bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)} + \frac{10a^3b^2x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)} + \frac{a^2b^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5ab^4x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)} + \frac{b^5x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}$$

output

```
a^5*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5*a^4*b*x^4*((b*x^3+a)^2)^(1/2)/(4*b*x
^3+4*a)+10*a^3*b^2*x^7*((b*x^3+a)^2)^(1/2)/(7*b*x^3+7*a)+a^2*b^3*x^10*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)+5*a*b^4*x^13*((b*x^3+a)^2)^(1/2)/(13*b*x^3+13*a)
+b^5*x^16*((b*x^3+a)^2)^(1/2)/(16*b*x^3+16*a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.33

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x\sqrt{(a+bx^3)^2}\left(1456a^5+1820a^4bx^3+2080a^3b^2x^6+1456a^2b^3x^9+560ab^4x^{12}+91b^5x^{15}\right)}{1456\,(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x*Sqrt[(a + b*x^3)^2]*(1456*a^5 + 1820*a^4*b*x^3 + 2080*a^3*b^2*x^6 + 145
6*a^2*b^3*x^9 + 560*a*b^4*x^12 + 91*b^5*x^15))/(1456*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.41, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1384, 747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^2x^3+ab\right)^5dx}{b^5\,(a+bx^3)}$$

$$\Big\downarrow \text{747}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^{10}x^{15}+5ab^9x^{12}+10a^2b^8x^9+10a^3b^7x^6+5a^4b^6x^3+a^5b^5\right)dx}{b^5\,(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(a^5b^5x+\frac{5}{4}a^4b^6x^4+\frac{10}{7}a^3b^7x^7+a^2b^8x^{10}+\frac{5}{13}ab^9x^{13}+\frac{b^{10}x^{16}}{16}\right)}{b^5\,(a+bx^3)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(a^5*b^5*x + (5*a^4*b^6*x^4)/4 + (10*a^3*
b^7*x^7)/7 + a^2*b^8*x^10 + (5*a*b^9*x^13)/13 + (b^10*x^16)/16))/(b^5*(a +
 b*x^3))
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.32

method	result
gosper	$\frac{x\left(91b^5x^{15}+560a\,b^4x^{12}+1456a^2b^3x^9+2080a^3b^2x^6+1820a^4b\,x^3+1456a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{1456\left(b\,x^3+a\right)^5}$
default	$\frac{x\left(91b^5x^{15}+560a\,b^4x^{12}+1456a^2b^3x^9+2080a^3b^2x^6+1820a^4b\,x^3+1456a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{1456\left(b\,x^3+a\right)^5}$
orering	$\frac{x\left(91b^5x^{15}+560a\,b^4x^{12}+1456a^2b^3x^9+2080a^3b^2x^6+1820a^4b\,x^3+1456a^5\right)\left(b^2x^6+2a\,x^3b+a^2\right)^{\frac{5}{2}}}{1456\left(b\,x^3+a\right)^5}$
risch	$\frac{a^5x\sqrt{\left(b\,x^3+a\right)^2}}{b\,x^3+a}+\frac{5\sqrt{\left(b\,x^3+a\right)^2}\,a^4b\,x^4}{4\left(b\,x^3+a\right)}+\frac{10\sqrt{\left(b\,x^3+a\right)^2}\,a^3b^2x^7}{7\left(b\,x^3+a\right)}+\frac{a^2b^3x^{10}\sqrt{\left(b\,x^3+a\right)^2}}{b\,x^3+a}+\frac{5\sqrt{\left(b\,x^3+a\right)^2}\,a\,b^4x^{13}}{13\left(b\,x^3+a\right)}+\frac{b^5x^{16}}{16b}$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/1456*x*(91*b^5*x^15+560*a*b^4*x^12+1456*a^2*b^3*x^9+2080*a^3*b^2*x^6+182
0*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.21

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{1}{16}b^5x^{16}+\frac{5}{13}ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^4bx^4+a^5x$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^
4*b*x^4 + a^5*x
```

Sympy [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \int \left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}\, dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.21

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{1}{16} b^5x^{16} + \frac{5}{13} ab^4x^{13} + a^2b^3x^{10} + \frac{10}{7} a^3b^2x^7 + \frac{5}{4} a^4bx^4 + a^5x$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^
4*b*x^4 + a^5*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.41

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{1}{16} b^5x^{16}\text{sgn}\left(bx^3+a\right) + \frac{5}{13} ab^4x^{13}\text{sgn}\left(bx^3+a\right) + a^2b^3x^{10}\text{sgn}\left(bx^3+a\right) + \frac{10}{7} a^3b^2x^7\text{sgn}\left(bx^3+a\right) + \frac{5}{4} a^4bx^4\text{sgn}\left(bx^3+a\right) + a^5x\text{sgn}\left(bx^3+a\right)$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/16*b^5*x^16*sgn(b*x^3 + a) + 5/13*a*b^4*x^13*sgn(b*x^3 + a) + a^2*b^3*x^
10*sgn(b*x^3 + a) + 10/7*a^3*b^2*x^7*sgn(b*x^3 + a) + 5/4*a^4*b*x^4*sgn(b*
x^3 + a) + a^5*x*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\int \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{x(91b^5x^{15}+560a\,b^4x^{12}+1456a^2b^3x^9+2080a^3b^2x^6+1820a^4b\,x^3+1456a^5)}{1456}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
(x*(1456*a**5 + 1820*a**4*b*x**3 + 2080*a**3*b**2*x**6 + 1456*a**2*b**3*x**
*9 + 560*a*b**4*x**12 + 91*b**5*x**15))/1456
```

3.54 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2} dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{2a^3b^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)} + \frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)} + \frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)}$$

output
```
-a^5*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+5*a^4*b*x^2*((b*x^3+a)^2)^(1/2)/(2*b*
x^3+2*a)+2*a^3*b^2*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5*a^2*b^3*x^8*((b*x^3
+a)^2)^(1/2)/(4*b*x^3+4*a)+5*a*b^4*x^11*((b*x^3+a)^2)^(1/2)/(11*b*x^3+11*a
)+b^5*x^14*((b*x^3+a)^2)^(1/2)/(14*b*x^3+14*a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx=\frac{\sqrt{\left(a+bx^3\right)^2}\left(-308a^5+770a^4bx^3+616a^3b^2x^6+385a^2b^3x^9+140ab^4x^{12}+\right.}{308x\left(a+bx^3\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-308*a^5 + 770*a^4*b*x^3 + 616*a^3*b^2*x^6 + 385*a^2
*b^3*x^9 + 140*a*b^4*x^12 + 22*b^5*x^15))/(308*x*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5\left(bx^3+a\right)^5}{x^2}dx}{b^5\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(bx^3+a\right)^5}{x^2}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{13}+5ab^4x^{10}+10a^2b^3x^7+10a^3b^2x^4+5a^4bx+\frac{a^5}{x^2}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{x}+\frac{5}{2}a^4bx^2+2a^3b^2x^5+\frac{5}{4}a^2b^3x^8+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{14}}{14}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^5/x) + (5*a^4*b*x^2)/2 + 2*a^3*b^2*x
^5 + (5*a^2*b^3*x^8)/4 + (5*a*b^4*x^11)/11 + (b^5*x^14)/14))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{(-22b^5x^{15}-140a\,b^4x^{12}-385a^2b^3x^9-616a^3b^2x^6-770a^4b\,x^3+308a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{308x(b\,x^3+a)^5}$	80
default	$-\frac{(-22b^5x^{15}-140a\,b^4x^{12}-385a^2b^3x^9-616a^3b^2x^6-770a^4b\,x^3+308a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{308x(b\,x^3+a)^5}$	80
orering	$-\frac{(-22b^5x^{15}-140a\,b^4x^{12}-385a^2b^3x^9-616a^3b^2x^6-770a^4b\,x^3+308a^5)(b^2x^6+2a\,x^3b+a^2)^{\frac{5}{2}}}{308x(b\,x^3+a)^5}$	89
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b(\frac{1}{14}b^4x^{14}+\frac{5}{11}b^3a\,x^{11}+\frac{5}{4}a^2b^2x^8+2a^3b\,x^5+\frac{5}{2}a^4x^2)}{b\,x^3+a}-\frac{a^5\sqrt{(b\,x^3+a)^2}}{x(b\,x^3+a)}$	98

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/308*(-22*b^5*x^15-140*a*b^4*x^12-385*a^2*b^3*x^9-616*a^3*b^2*x^6-770*a^
4*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2}\,dx=\frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 +
770*a^4*b*x^3 - 308*a^5)/x
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2}\,dx = \int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**2,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2}\,dx = \frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 +
770*a^4*b*x^3 - 308*a^5)/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2}\,dx = \frac{1}{14}\,b^5x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{11}\,ab^4x^{11}\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{4}\,a^2b^3x^8\mathrm{sgn}\left(bx^3+a\right)+2\,a^3b^2x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^4bx^2\mathrm{sgn}\left(bx^3+a\right) - \frac{a^5\mathrm{sgn}(bx^3+a)}{x}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="giac")
```

output
```
1/14*b^5*x^14*sgn(b*x^3 + a) + 5/11*a*b^4*x^11*sgn(b*x^3 + a) + 5/4*a^2*b^
3*x^8*sgn(b*x^3 + a) + 2*a^3*b^2*x^5*sgn(b*x^3 + a) + 5/2*a^4*b*x^2*sgn(b*
x^3 + a) - a^5*sgn(b*x^3 + a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^2}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}}{x^2}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^2}\,dx = \frac{22b^5x^{15} + 140a\,b^4x^{12} + 385a^2b^3x^9 + 616a^3b^2x^6 + 770a^4b\,x^3 - 308a^5}{308x}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x)
```

output
```
( - 308*a**5 + 770*a**4*b*x**3 + 616*a**3*b**2*x**6 + 385*a**2*b**3*x**9 +
 140*a*b**4*x**12 + 22*b**5*x**15)/(308*x)
```

3.55 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} + \frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)} + \frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}$$

output
```
-1/2*a^5*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+5*a^4*b*x*((b*x^3+a)^2)^(1/2)/(
b*x^3+a)+5*a^3*b^2*x^4*((b*x^3+a)^2)^(1/2)/(2*b*x^3+2*a)+10*a^2*b^3*x^7*((
b*x^3+a)^2)^(1/2)/(7*b*x^3+7*a)+a*b^4*x^10*((b*x^3+a)^2)^(1/2)/(2*b*x^3+2*
a)+b^5*x^13*((b*x^3+a)^2)^(1/2)/(13*b*x^3+13*a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx = \frac{\sqrt{(a+bx^3)^2}(-91a^5+910a^4bx^3+455a^3b^2x^6+260a^2b^3x^9+91ab^4x^{12}+14}{182x^2(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-91*a^5 + 910*a^4*b*x^3 + 455*a^3*b^2*x^6 + 260*a^2*
b^3*x^9 + 91*a*b^4*x^12 + 14*b^5*x^15))/(182*x^2*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5(bx^3+a)^5}{x^3}dx}{b^5(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^3}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^{12}+5ab^4x^9+10a^2b^3x^6+10a^3b^2x^3+5a^4b+\frac{a^5}{x^3}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{2x^2}+5a^4bx+\frac{5}{2}a^3b^2x^4+\frac{10}{7}a^2b^3x^7+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{13}}{13}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/2*a^5/x^2 + 5*a^4*b*x + (5*a^3*b^2*x^
4)/2 + (10*a^2*b^3*x^7)/7 + (a*b^4*x^10)/2 + (b^5*x^13)/13))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-14b^{5}x^{15}-91a\,b^{4}x^{12}-260a^{2}b^{3}x^{9}-455a^{3}b^{2}x^{6}-910a^{4}b\,x^{3}+91a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{182x^{2}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(-14b^{5}x^{15}-91a\,b^{4}x^{12}-260a^{2}b^{3}x^{9}-455a^{3}b^{2}x^{6}-910a^{4}b\,x^{3}+91a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{182x^{2}\left(b\,x^{3}+a\right)^{5}}$	80
orering	$-\frac{\left(-14b^{5}x^{15}-91a\,b^{4}x^{12}-260a^{2}b^{3}x^{9}-455a^{3}b^{2}x^{6}-910a^{4}b\,x^{3}+91a^{5}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{5}{2}}}{182x^{2}\left(b\,x^{3}+a\right)^{5}}$	89
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\left(\frac{1}{13}b^{4}x^{13}+\frac{1}{2}a\,b^{3}x^{10}+\frac{10}{7}b^{2}x^{7}a^{2}+\frac{5}{2}b\,x^{4}a^{3}+5a^{4}x\right)}{b\,x^{3}+a}-\frac{a^{5}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2x^{2}\left(b\,x^{3}+a\right)}$	96

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/182*(-14*b^5*x^15-91*a*b^4*x^12-260*a^2*b^3*x^9-455*a^3*b^2*x^6-910*a^4
*b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^2/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^3}\,dx=\frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 9
10*a^4*b*x^3 - 91*a^5)/x^2
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^3}\, dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^3}\, dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**3,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^3}\, dx = \frac{14\, b^5x^{15} + 91\, ab^4x^{12} + 260\, a^2b^3x^9 + 455\, a^3b^2x^6 + 910\, a^4bx^3 - 91\, a^5}{182\, x^2}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 9
10*a^4*b*x^3 - 91*a^5)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^3}\, dx = \frac{1}{13}\, b^5x^{13}\mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{2}\, ab^4x^{10}\mathrm{sgn}\left(bx^3 + a\right) + \frac{10}{7}\, a^2b^3x^7\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{2}\, a^3b^2x^4\mathrm{sgn}\left(bx^3 + a\right) + 5\, a^4bx\mathrm{sgn}\left(bx^3 + a\right) - \frac{a^5\mathrm{sgn}\left(bx^3 + a\right)}{2\, x^2}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="giac")
```

output
```
1/13*b^5*x^13*sgn(b*x^3 + a) + 1/2*a*b^4*x^10*sgn(b*x^3 + a) + 10/7*a^2*b^
3*x^7*sgn(b*x^3 + a) + 5/2*a^3*b^2*x^4*sgn(b*x^3 + a) + 5*a^4*b*x*sgn(b*x^
3 + a) - 1/2*a^5*sgn(b*x^3 + a)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^3}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}}{x^3}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^3}\,dx = \frac{14b^5x^{15} + 91a\,b^4x^{12} + 260a^2b^3x^9 + 455a^3b^2x^6 + 910a^4b\,x^3 - 91a^5}{182x^2}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x)
```

output
```
( - 91*a**5 + 910*a**4*b*x**3 + 455*a**3*b**2*x**6 + 260*a**2*b**3*x**9 +
91*a*b**4*x**12 + 14*b**5*x**15)/(182*x**2)
```

3.56 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^5} dx$

Optimal result

Integrand size = 26, antiderivative size = 249

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^5} dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)} + \frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}$$

output

```
-1/4*a^5*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-5*a^4*b*((b*x^3+a)^2)^(1/2)/x/(
b*x^3+a)+5*a^3*b^2*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+2*a^2*b^3*x^5*((b*x^3
+a)^2)^(1/2)/(b*x^3+a)+5*a*b^4*x^8*((b*x^3+a)^2)^(1/2)/(8*b*x^3+8*a)+b^5*x
^11*((b*x^3+a)^2)^(1/2)/(11*b*x^3+11*a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^5} \, dx = \frac{\sqrt{(a + bx^3)^2}(-22a^5 - 440a^4bx^3 + 440a^3b^2x^6 + 176a^2b^3x^9 + 55ab^4x^{12} + 8b}{88x^4\,(a + bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-22*a^5 - 440*a^4*b*x^3 + 440*a^3*b^2*x^6 + 176*a^2*
b^3*x^9 + 55*a*b^4*x^12 + 8*b^5*x^15))/(88*x^4*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^5} \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{b^5(bx^3+a)^5}{x^5}dx}{b^5\,(a + bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(bx^3+a)^5}{x^5}dx}{a + bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(b^5x^{10} + 5ab^4x^7 + 10a^2b^3x^4 + 10a^3b^2x + \frac{5a^4b}{x^2} + \frac{a^5}{x^5}\right) dx}{a + bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{4x^4}-\frac{5a^4b}{x}+5a^3b^2x^2+2a^2b^3x^5+\frac{5}{8}ab^4x^8+\frac{b^5x^{11}}{11}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/4*a^5/x^4 - (5*a^4*b)/x + 5*a^3*b^2*x
^2 + 2*a^2*b^3*x^5 + (5*a*b^4*x^8)/8 + (b^5*x^11)/11))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-8 b^{5} x^{15}-55 a \,b^{4} x^{12}-176 a^{2} b^{3} x^{9}-440 a^{3} b^{2} x^{6}+440 a^{4} b \,x^{3}+22 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{88 x^{4} \left(b \,x^{3}+a \right)^{5}}$	80
default	$-\frac{\left(-8 b^{5} x^{15}-55 a \,b^{4} x^{12}-176 a^{2} b^{3} x^{9}-440 a^{3} b^{2} x^{6}+440 a^{4} b \,x^{3}+22 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{88 x^{4} \left(b \,x^{3}+a \right)^{5}}$	80
orering	$-\frac{\left(-8 b^{5} x^{15}-55 a \,b^{4} x^{12}-176 a^{2} b^{3} x^{9}-440 a^{3} b^{2} x^{6}+440 a^{4} b \,x^{3}+22 a^{5}\right) \left(b^{2} x^{6}+2 a \,x^{3} b +a^{2}\right)^{\frac{5}{2}}}{88 x^{4} \left(b \,x^{3}+a \right)^{5}}$	89
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{2} \left(\frac{1}{11} b^{3} x^{11}+\frac{5}{8} a \,b^{2} x^{8}+2 a^{2} b \,x^{5}+5 a^{3} x^{2}\right)}{b \,x^{3}+a}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-5 a^{4} b \,x^{3}-\frac{1}{4} a^{5}\right)}{\left(b \,x^{3}+a \right) x^{4}}$	100

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/88*(-8*b^5*x^15-55*a*b^4*x^12-176*a^2*b^3*x^9-440*a^3*b^2*x^6+440*a^4*b
*x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^4/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}\,dx = \frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440
*a^4*b*x^3 - 22*a^5)/x^4
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^5} \, dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^5} \, dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**5,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^5} \, dx = \frac{8\,b^5x^{15} + 55\,ab^4x^{12} + 176\,a^2b^3x^9 + 440\,a^3b^2x^6 - 440\,a^4bx^3 - 22\,a^5}{88\,x^4}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440
*a^4*b*x^3 - 22*a^5)/x^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.43

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^5} \, dx = \frac{1}{11}\,b^5x^{11}\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{8}\,ab^4x^8\mathrm{sgn}\left(bx^3 + a\right) + 2\,a^2b^3x^5\mathrm{sgn}\left(bx^3 + a\right) + 5\,a^3b^2x^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{20\,a^4bx^3\mathrm{sgn}(bx^3 + a) + a^5\mathrm{sgn}(bx^3 + a)}{4\,x^4}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="giac")
```

output
```
1/11*b^5*x^11*sgn(b*x^3 + a) + 5/8*a*b^4*x^8*sgn(b*x^3 + a) + 2*a^2*b^3*x^
5*sgn(b*x^3 + a) + 5*a^3*b^2*x^2*sgn(b*x^3 + a) - 1/4*(20*a^4*b*x^3*sgn(b*
x^3 + a) + a^5*sgn(b*x^3 + a))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^5} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}}{x^5} \, dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^5} \, dx = \frac{8b^5x^{15} + 55a\,b^4x^{12} + 176a^2b^3x^9 + 440a^3b^2x^6 - 440a^4b\,x^3 - 22a^5}{88x^4}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x)
```

output
```
( - 22*a**5 - 440*a**4*b*x**3 + 440*a**3*b**2*x**6 + 176*a**2*b**3*x**9 +
55*a*b**4*x**12 + 8*b**5*x**15)/(88*x**4)
```

3.57 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10(a+bx^3)}$$

output
```
-1/5*a^5*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-5/2*a^4*b*((b*x^3+a)^2)^(1/2)/x
^2/(b*x^3+a)+10*a^3*b^2*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5*a^2*b^3*x^4*((b*
x^3+a)^2)^(1/2)/(2*b*x^3+2*a)+5*a*b^4*x^7*((b*x^3+a)^2)^(1/2)/(7*b*x^3+7*a
)+b^5*x^10*((b*x^3+a)^2)^(1/2)/(10*b*x^3+10*a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx = \frac{\sqrt{(a+bx^3)^2}(-14a^5-175a^4bx^3+700a^3b^2x^6+175a^2b^3x^9+50ab^4x^{12}+7b}{70x^5(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-14*a^5 - 175*a^4*b*x^3 + 700*a^3*b^2*x^6 + 175*a^2*
b^3*x^9 + 50*a*b^4*x^12 + 7*b^5*x^15))/(70*x^5*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5(bx^3+a)^5}{x^6}dx}{b^5(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^6}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^9+5ab^4x^6+10a^2b^3x^3+10a^3b^2+\frac{5a^4b}{x^3}+\frac{a^5}{x^6}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{5x^5}-\frac{5a^4b}{2x^2}+10a^3b^2x+\frac{5}{2}a^2b^3x^4+\frac{5}{7}ab^4x^7+\frac{b^5x^{10}}{10}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/5*a^5/x^5 - (5*a^4*b)/(2*x^2) + 10*a^
3*b^2*x + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^7)/7 + (b^5*x^10)/10))/(a + b*x^3
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
 - 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{(-7b^5x^{15}-50a\,b^4x^{12}-175a^2b^3x^9-700a^3b^2x^6+175a^4b\,x^3+14a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{70x^5(b\,x^3+a)^5}$	80
default	$-\frac{(-7b^5x^{15}-50a\,b^4x^{12}-175a^2b^3x^9-700a^3b^2x^6+175a^4b\,x^3+14a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{70x^5(b\,x^3+a)^5}$	80
orering	$-\frac{(-7b^5x^{15}-50a\,b^4x^{12}-175a^2b^3x^9-700a^3b^2x^6+175a^4b\,x^3+14a^5)(b^2x^6+2a\,x^3b+a^2)^{\frac{5}{2}}}{70x^5(b\,x^3+a)^5}$	89
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b^2(\frac{1}{10}b^3x^{10}+\frac{5}{7}a\,b^2x^7+\frac{5}{2}a^2b\,x^4+10a^3x)}{b\,x^3+a}+\frac{\sqrt{(b\,x^3+a)^2}\,(-\frac{5}{2}a^4b\,x^3-\frac{1}{5}a^5)}{(b\,x^3+a)x^5}$	98

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/70*(-7*b^5*x^15-50*a*b^4*x^12-175*a^2*b^3*x^9-700*a^3*b^2*x^6+175*a^4*b
*x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/x^5/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx = \frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175
*a^4*b*x^3 - 14*a^5)/x^5
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^6}\,dx = \int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**6,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^6}\,dx = \frac{7\,b^5x^{15} + 50\,ab^4x^{12} + 175\,a^2b^3x^9 + 700\,a^3b^2x^6 - 175\,a^4bx^3 - 14\,a^5}{70\,x^5}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175
*a^4*b*x^3 - 14*a^5)/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.42

$$\begin{aligned}\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^6}\,dx &= \frac{1}{10}\,b^5x^{10}\mathrm{sgn}\left(bx^3 + a\right) \\ &+ \frac{5}{7}\,ab^4x^7\mathrm{sgn}\left(bx^3 + a\right) + \frac{5}{2}\,a^2b^3x^4\mathrm{sgn}\left(bx^3 + a\right) \\ &+ 10\,a^3b^2x\mathrm{sgn}\left(bx^3 + a\right) - \frac{25\,a^4bx^3\mathrm{sgn}(bx^3 + a) + 2\,a^5\mathrm{sgn}(bx^3 + a)}{10\,x^5}\end{aligned}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="giac")
```

output
```
1/10*b^5*x^10*sgn(b*x^3 + a) + 5/7*a*b^4*x^7*sgn(b*x^3 + a) + 5/2*a^2*b^3*
x^4*sgn(b*x^3 + a) + 10*a^3*b^2*x*sgn(b*x^3 + a) - 1/10*(25*a^4*b*x^3*sgn(
b*x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx=\int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}{x^6}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx=\frac{7b^5x^{15}+50a\,b^4x^{12}+175a^2b^3x^9+700a^3b^2x^6-175a^4b\,x^3-14a^5}{70x^5}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x)
```

output
```
( - 14*a**5 - 175*a**4*b*x**3 + 700*a**3*b**2*x**6 + 175*a**2*b**3*x**9 +
50*a*b**4*x**12 + 7*b**5*x**15)/(70*x**5)
```

3.58 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^8}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 248

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^8}\,dx = -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

output

```
-1/7*a^5*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-5/4*a^4*b*((b*x^3+a)^2)^(1/2)/x
^4/(b*x^3+a)-10*a^3*b^2*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+5*a^2*b^3*x^2*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)+a*b^4*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+b^5*x^8*
((b*x^3+a)^2)^(1/2)/(8*b*x^3+8*a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx = \frac{\sqrt{(a+bx^3)^2}(-8a^5-70a^4bx^3-560a^3b^2x^6+280a^2b^3x^9+56ab^4x^{12}+7b^5x}{56x^7\,(a+bx^3)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

output
```
(Sqrt[(a + b*x^3)^2]*(-8*a^5 - 70*a^4*b*x^3 - 560*a^3*b^2*x^6 + 280*a^2*b^
3*x^9 + 56*a*b^4*x^12 + 7*b^5*x^15))/(56*x^7*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{b^5(bx^3+a)^5}{x^8}dx}{b^5\,(a+bx^3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(bx^3+a)^5}{x^8}dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^5x^7+5ab^4x^4+10a^2b^3x+\frac{10a^3b^2}{x^2}+\frac{5a^4b}{x^5}+\frac{a^5}{x^8}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(-\frac{a^5}{7x^7}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{x}+5a^2b^3x^2+ab^4x^5+\frac{b^5x^8}{8}\right)}{a+bx^3}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-1/7*a^5/x^7 - (5*a^4*b)/(4*x^4) - (10*a
^3*b^2)/x + 5*a^2*b^3*x^2 + a*b^4*x^5 + (b^5*x^8)/8))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 5.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-7b^{5}x^{15}-56a\,b^{4}x^{12}-280a^{2}b^{3}x^{9}+560a^{3}b^{2}x^{6}+70a^{4}b\,x^{3}+8a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{56x^{7}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(-7b^{5}x^{15}-56a\,b^{4}x^{12}-280a^{2}b^{3}x^{9}+560a^{3}b^{2}x^{6}+70a^{4}b\,x^{3}+8a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{56x^{7}\left(b\,x^{3}+a\right)^{5}}$	80
orering	$-\frac{\left(-7b^{5}x^{15}-56a\,b^{4}x^{12}-280a^{2}b^{3}x^{9}+560a^{3}b^{2}x^{6}+70a^{4}b\,x^{3}+8a^{5}\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{\frac{5}{2}}}{56x^{7}\left(b\,x^{3}+a\right)^{5}}$	89
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{3}\left(\frac{1}{8}b^{2}x^{8}+ab\,x^{5}+5a^{2}x^{2}\right)}{b\,x^{3}+a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\left(-10a^{3}b^{2}x^{6}-\frac{5}{4}a^{4}b\,x^{3}-\frac{1}{7}a^{5}\right)}{\left(b\,x^{3}+a\right)x^{7}}$	99

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/56*(-7*b^5*x^15-56*a*b^4*x^12-280*a^2*b^3*x^9+560*a^3*b^2*x^6+70*a^4*b*
x^3+8*a^5)*((b*x^3+a)^2)^(5/2)/x^7/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^8}\,dx=\frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

output
```
1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*
a^4*b*x^3 - 8*a^5)/x^7
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx=\int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^8}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**8,x)
```

output
```
Integral(((a + b*x**3)**2)**(5/2)/x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx=\frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

output
```
1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*
a^4*b*x^3 - 8*a^5)/x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx=\frac{1}{8}\,b^5x^8\mathrm{sgn}\left(bx^3+a\right)+ab^4x^5\mathrm{sgn}\left(bx^3+a\right)+5\,a^2b^3x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{280\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+35\,a^4bx^3\mathrm{sgn}(bx^3+a)+4\,a^5\mathrm{sgn}(bx^3+a)}{28\,x^7}$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="giac")
```

output
```
1/8*b^5*x^8*sgn(b*x^3 + a) + a*b^4*x^5*sgn(b*x^3 + a) + 5*a^2*b^3*x^2*sgn(
b*x^3 + a) - 1/28*(280*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sgn(b*x^3
 + a) + 4*a^5*sgn(b*x^3 + a))/x^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^8}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}}{x^8}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^8}\,dx = \frac{7b^5x^{15} + 56a\,b^4x^{12} + 280a^2b^3x^9 - 560a^3b^2x^6 - 70a^4b\,x^3 - 8a^5}{56x^7}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x)
```

output
```
( - 8*a**5 - 70*a**4*b*x**3 - 560*a**3*b**2*x**6 + 280*a**2*b**3*x**9 + 56
*a*b**4*x**12 + 7*b**5*x**15)/(56*x**7)
```

3.59 $\int \frac{x^{11}}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 169

$$\int \frac{x^{11}}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{a^2x^3(a+bx^3)}{3b^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{ax^6(a+bx^3)}{6b^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^9(a+bx^3)}{9b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^3(a+bx^3)\log\left(a+bx^3\right)}{3b^4\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/3*a^2*x^3*(b*x^3+a)/b^3/((b*x^3+a)^2)^(1/2)-1/6*a*x^6*(b*x^3+a)/b^2/((b*
x^3+a)^2)^(1/2)+1/9*x^9*(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)-1/3*a^3*(b*x^3+a)*
ln(b*x^3+a)/b^4/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int \frac{x^{11}}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(a+bx^3)\left(bx^3(6a^2-3abx^3+2b^2x^6)-6a^3\log\left(a+bx^3\right)\right)}{18b^4\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[x^11/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(b*x^3*(6*a^2 - 3*a*b*x^3 + 2*b^2*x^6) - 6*a^3*Log[a + b*x^3]
))/(18*b^4*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt{a^2+2abx^3+b^2x^6}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b(a+bx^3)\int \frac{x^{11}}{b(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^3)\int \frac{x^{11}}{bx^3+a}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{(a+bx^3)\int \frac{x^9}{bx^3+a}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{(a+bx^3)\int \left(\frac{x^6}{b}-\frac{ax^3}{b^2}-\frac{a^3}{b^3(bx^3+a)}+\frac{a^2}{b^3}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx^3)\left(-\frac{a^3\log(a+bx^3)}{b^4}+\frac{a^2x^3}{b^3}-\frac{ax^6}{2b^2}+\frac{x^9}{3b}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^11/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*((a^2*x^3)/b^3 - (a*x^6)/(2*b^2) + x^9/(3*b) - (a^3*Log[a + b
*x^3])/b^4))/(3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.38

method	result	size
default	$-\frac{(b\,x^3+a)\left(-2x^9b^3+3a\,x^6b^2-6a^2x^3b+6a^3\ln\left(b\,x^3+a\right)\right)}{18\sqrt{(b\,x^3+a)^2}\,b^4}$	64
pseudoelliptic	$\frac{-6a^3\,\mathrm{csgn}\left(b\,x^3+a\right)\ln\left(b\,x^3+a\right)+\left(2x^9b^3-3a\,x^6b^2+6a^2x^3b+11a^3\right)\mathrm{csgn}\left(b\,x^3+a\right)}{18b^4}$	69
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(\frac{1}{9}b^2x^9-\frac{1}{6}ab\,x^6+\frac{1}{3}a^2x^3\right)}{(b\,x^3+a)b^3}-\frac{\sqrt{(b\,x^3+a)^2}\,a^3\ln\left(b\,x^3+a\right)}{3(b\,x^3+a)b^4}$	86

input
```
int(x^11/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*(b*x^3+a)*(-2*x^9*b^3+3*a*x^6*b^2-6*a^2*x^3*b+6*a^3*ln(b*x^3+a))/((b
*x^3+a)^2)^(1/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.27

$$\int \frac{x^{11}}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{2\,b^3x^9-3\,ab^2x^6+6\,a^2bx^3-6\,a^3\log\left(bx^3+a\right)}{18\,b^4}$$

input
```
integrate(x^11/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/18*(2*b^3*x^9 - 3*a*b^2*x^6 + 6*a^2*b*x^3 - 6*a^3*log(b*x^3 + a))/b^4
```

Sympy [F]

$$\int \frac{x^{11}}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x^{11}}{\sqrt{\left(a+bx^3\right)^2}}\,dx$$

input
```
integrate(x**11/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x**11/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.56

$$\int \frac{x^{11}}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{5\,ax^6}{18\,b^2} + \frac{\sqrt{b^2x^6+2\,abx^3+a^2}x^6}{9\,b^2} + \frac{5\,a^2x^3}{9\,b^3} - \frac{a^3\log\left(x^3+\frac{a}{b}\right)}{3\,b^4} - \frac{2\,\sqrt{b^2x^6+2\,abx^3+a^2}a^2}{9\,b^4}$$

input
```
integrate(x^11/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-5/18*a*x^6/b^2 + 1/9*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*x^6/b^2 + 5/9*a^2*x^
3/b^3 - 1/3*a^3*log(x^3 + a/b)/b^4 - 2/9*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a
^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.47

$$\int \frac{x^{11}}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{a^3\log\left(|bx^3+a|\right)\mathrm{sgn}(bx^3+a)}{3\,b^4} + \frac{2\,b^2x^9\mathrm{sgn}(bx^3+a) - 3\,abx^6\mathrm{sgn}(bx^3+a) + 6\,a^2x^3\mathrm{sgn}(bx^3+a)}{18\,b^3}$$

input
```
integrate(x^11/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/3*a^3*log(abs(b*x^3 + a))*sgn(b*x^3 + a)/b^4 + 1/18*(2*b^2*x^9*sgn(b*x^
3 + a) - 3*a*b*x^6*sgn(b*x^3 + a) + 6*a^2*x^3*sgn(b*x^3 + a))/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{x^{11}}{\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

input
```
int(x^11/((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x^11/((a + b*x^3)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.43

$$\int \frac{x^{11}}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{-6\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^3-6\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a^3+6a^2b\,x^3-3a\,b^2x^6+2b^3x^9}{18b^4}$$

input
```
int(x^11/((b*x^3+a)^2)^(1/2),x)
```

output
```
( - 6*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3 - 6*log(a**
(1/3) + b**(1/3)*x)*a**3 + 6*a**2*b*x**3 - 3*a*b**2*x**6 + 2*b**3*x**9)/(1
8*b**4)
```

3.60 $\int \frac{x^8}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 127

$$\int \frac{x^8}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{ax^3(a+bx^3)}{3b^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^6(a+bx^3)}{6b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^2(a+bx^3)\log\left(a+bx^3\right)}{3b^3\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
-1/3*a*x^3*(b*x^3+a)/b^2/((b*x^3+a)^2)^(1/2)+1/6*x^6*(b*x^3+a)/b/((b*x^3+a
)^2)^(1/2)+1/3*a^2*(b*x^3+a)*ln(b*x^3+a)/b^3/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.31

$$\int \frac{x^8}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{\frac{bx^3(-2a+bx^3)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{(a+bx^3)^2}\right)\right)}{a^2+abx^3-\sqrt{a^2}\sqrt{(a+bx^3)^2}}-2a^2\log\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)+2a^2\log\left(b^3\left(\sqrt{a^2}+b\right.\right.}{6b^3}$$

input
```
Integrate[x^8/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
-1/6*((b*x^3*(-2*a + b*x^3)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*
x^3)^2])))/(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]) - 2*a^2*Log[Sqr
t[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] + 2*a^2*Log[b^3*(Sqrt[a^2] + b*x^3 -
 Sqrt[(a + b*x^3)^2])])/b^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{a^2+2abx^3+b^2x^6}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^3)\int \frac{x^8}{b(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{x^8}{bx^3+a}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{(a+bx^3)\int \frac{x^6}{bx^3+a}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx^3)\int \left(\frac{x^3}{b}+\frac{a^2}{b^2(bx^3+a)}-\frac{a}{b^2}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^3)\left(\frac{a^2\log(a+bx^3)}{b^3}-\frac{ax^3}{b^2}+\frac{x^6}{2b}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^8/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(-((a*x^3)/b^2) + x^6/(2*b) + (a^2*Log[a + b*x^3])/b^3))/(3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.37

method	result	size
pseudoelliptic	$\frac{\left(b^{2} x^{6}-2 a \,x^{3} b +2 a^{2} \ln \left(b \,x^{3}+a \right)-3 a^{2}\right) \operatorname{csgn}\left(b \,x^{3}+a \right)}{6 b^{3}}$	47
default	$\frac{\left(b \,x^{3}+a \right) \left(b^{2} x^{6}-2 a \,x^{3} b +2 a^{2} \ln \left(b \,x^{3}+a \right)\right)}{6 \sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{3}}$	52
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-b \,x^{3}+a \right)^{2}}{6 \left(b \,x^{3}+a \right) b^{3}}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, a^{2} \ln \left(b \,x^{3}+a \right)}{3 \left(b \,x^{3}+a \right) b^{3}}$	73

input
```
int(x^8/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(b^2*x^6-2*a*x^3*b+2*a^2*ln(b*x^3+a)-3*a^2)*csgn(b*x^3+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{x^8}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{b^{2} x^{6}-2\,abx^{3}+2\,a^{2}\log\left(bx^{3}+a\right)}{6\,b^{3}}$$

input
```
integrate(x^8/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(b^2*x^6 - 2*a*b*x^3 + 2*a^2*log(b*x^3 + a))/b^3
```

Sympy [F]

$$\int \frac{x^8}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{x^8}{\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(x**8/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x**8/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.28

$$\int \frac{x^8}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{x^6}{6\,b}-\frac{ax^3}{3\,b^2}+\frac{a^2\log\left(x^3+\frac{a}{b}\right)}{3\,b^3}$$

input
```
integrate(x^8/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*x^6/b - 1/3*a*x^3/b^2 + 1/3*a^2*log(x^3 + a/b)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{x^8}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{a^2\log\left(|bx^3+a|\right)\mathrm{sgn}(bx^3+a)}{3\,b^3}+\frac{bx^6\mathrm{sgn}(bx^3+a)-2\,ax^3\mathrm{sgn}(bx^3+a)}{6\,b^2}$$

input
```
integrate(x^8/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*a^2*log(abs(b*x^3 + a))*sgn(b*x^3 + a)/b^3 + 1/6*(b*x^6*sgn(b*x^3 + a)
 - 2*a*x^3*sgn(b*x^3 + a))/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{x^8}{\sqrt{\left(b\,x^3+a\right)^2}}\,dx$$

input
```
int(x^8/((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x^8/((a + b*x^3)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.48

$$\int \frac{x^8}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a^2-2ab\,x^3+b^2x^6}{6b^3}$$

input
```
int(x^8/((b*x^3+a)^2)^(1/2),x)
```

output
```
(2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 + 2*log(a**(1/
3) + b**(1/3)*x)*a**2 - 2*a*b*x**3 + b**2*x**6)/(6*b**3)
```

3.61 $\int \frac{x^5}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{x^5}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}}{3b^2} - \frac{a(a+bx^3)\log(a+bx^3)}{3b^2\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/3*((b*x^3+a)^2)^(1/2)/b^2-1/3*a*(b*x^3+a)*ln(b*x^3+a)/b^2/((b*x^3+a)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.51

$$\int \frac{x^5}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx = \frac{\frac{bx^3\left(-\sqrt{a^2}(a+bx^3)+a\sqrt{(a+bx^3)^2}\right)}{a^2+abx^3-\sqrt{a^2}\sqrt{(a+bx^3)^2}} + 2a\text{arctanh}\left(\frac{bx^3}{\sqrt{a^2}-\sqrt{(a+bx^3)^2}}\right)}{3b^2}$$

input
```
Integrate[x^5/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((b*x^3*(-(Sqrt[a^2]*(a + b*x^3)) + a*Sqrt[(a + b*x^3)^2]))/(a^2 + a*b*x^3
 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]) + 2*a*ArcTanh[(b*x^3)/(Sqrt[a^2] - Sqrt[
(a + b*x^3)^2])])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1693, 1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt{b^2x^6+2abx^3+a^2}}dx^3$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{3}\left(\frac{\sqrt{a^2+2abx^3+b^2x^6}}{b^2}-\frac{a\int \frac{1}{\sqrt{b^2x^6+2abx^3+a^2}}dx^3}{b}\right)$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{3}\left(\frac{\sqrt{a^2+2abx^3+b^2x^6}}{b^2}-\frac{a(a+bx^3)\int \frac{1}{b^2x^3+ab}dx^3}{\sqrt{a^2+2abx^3+b^2x^6}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{\sqrt{a^2+2abx^3+b^2x^6}}{b^2}-\frac{a(a+bx^3)\log(a+bx^3)}{b^2\sqrt{a^2+2abx^3+b^2x^6}}\right)$$

input
```
Int[x^5/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/b^2 - (a*(a + b*x^3)*Log[a + b*x^3])/(b^2
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.45

method	result	size
pseudoelliptic	$-\frac{\left(a \ln \left(b \,x^{3}+a \right)-b \,x^{3}-a \right) \operatorname{csgn}\left(b \,x^{3}+a \right)}{3 b^{2}}$	34
default	$-\frac{\left(b \,x^{3}+a \right) \left(-b \,x^{3}+a \ln \left(b \,x^{3}+a \right)\right)}{3 \sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{2}}$	41
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, x^{3}}{3 \left(b \,x^{3}+a \right) b}-\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, a \ln \left(b \,x^{3}+a \right)}{3 \left(b \,x^{3}+a \right) b^{2}}$	64

input
```
int(x^5/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^2*(a*ln(b*x^3+a)-b*x^3-a)*csgn(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.29

$$\int \frac{x^5}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{bx^3-a\log\left(bx^3+a\right)}{3\,b^2}$$

input
```
integrate(x^5/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(b*x^3 - a*log(b*x^3 + a))/b^2
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{x^5}{\sqrt{\left(a+bx^3\right)^2}}\,dx$$

input
```
integrate(x**5/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x**5/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.56

$$\int \frac{x^5}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=-\frac{a\log\left(x^3+\frac{a}{b}\right)}{3\,b^2}+\frac{\sqrt{b^2x^6+2\,abx^3+a^2}}{3\,b^2}$$

input
```
integrate(x^5/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*a*log(x^3 + a/b)/b^2 + 1/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.44

$$\int \frac{x^5}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{1}{3}\left(\frac{x^3}{b} - \frac{a\log\left(|bx^3+a|\right)}{b^2}\right)\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(x^5/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*(x^3/b - a*log(abs(b*x^3 + a))/b^2)*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 19.85 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,b^2} - \frac{a\,b\,\ln\left(a\,b+\sqrt{{\left(b\,x^3+a\right)}^2}\,\sqrt{b^2}+b^2\,x^3\right)}{3\,{\left(b^2\right)}^{3/2}}$$

input
```
int(x^5/((a + b*x^3)^2)^(1/2),x)
```

output
```
(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*b^2) - (a*b*log(a*b + ((a + b*x^3)^2)
^(1/2)*(b^2)^(1/2) + b^2*x^3))/(3*(b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

$$\int \frac{x^5}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a-\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a+b\,x^3}{3b^2}$$

input
```
int(x^5/((b*x^3+a)^2)^(1/2),x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a - log(a**(1/3) +
 b**(1/3)*x)*a + b*x**3)/(3*b**2)
```

3.62 $\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/3*(b*x^3+a)*ln(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{2\text{arctanh}\left(\frac{\frac{\sqrt{a^2}}{b}-\frac{\sqrt{a^2+2abx^3+b^2x^6}}{b}}{x^3}\right)}{3b}$$

input
```
Integrate[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(-2*ArcTanh[(Sqrt[a^2]/b - Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/b)/x^3])/(3*b)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1690, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}dx$$

$$\Big\downarrow \text{ 1690}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{b^2x^6+2abx^3+a^2}}dx^3$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{b(a+bx^3)\int \frac{1}{b^2x^3+ab}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

method	result	size
pseudoelliptic	$\frac{\ln(b\,x^3+a)\,\mathrm{csgn}(b\,x^3+a)}{3b}$	22
default	$\frac{(b\,x^3+a)\ln(b\,x^3+a)}{3b\sqrt{(b\,x^3+a)^2}}$	32
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\ln(b\,x^3+a)}{3(b\,x^3+a)b}$	34

input
```
int(x^2/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(b*x^3+a)/b*csgn(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{\log\left(bx^3+a\right)}{3\,b}$$

input
```
integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*log(b*x^3 + a)/b
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{x^2}{\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(x**2/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x**2/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.34

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{\log\left(x^3+\frac{a}{b}\right)}{3\,b}$$

input
```
integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*log(x^3 + a/b)/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{\log\left(|bx^3+a|\right)\mathrm{sgn}(bx^3+a)}{3\,b}$$

input
```
integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*log(abs(b*x^3 + a))*sgn(b*x^3 + a)/b
```

Mupad [B] (verification not implemented)

Time = 19.95 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\ln\left(b^2\,x^3+a\,b\right)\,\mathrm{sign}(2\,b^2\,x^3+2\,a\,b)}{3\,\sqrt{b^2}}$$

input
```
int(x^2/((a + b*x^3)^2)^(1/2),x)
```

output
```
(log(a*b + b^2*x^3)*sign(2*a*b + 2*b^2*x^3))/(3*(b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b}$$

input
```
int(x^2/((b*x^3+a)^2)^(1/2),x)
```

output
```
(log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) + log(a**(1/3) + b**(
1/3)*x))/(3*b)
```

3.63 $\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{(a+bx^3)\log(x)}{a\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log(a+bx^3)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
(b*x^3+a)*ln(x)/a/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a/((b*x^3+
a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$=\frac{-2a\log(x^3)+\left(a-\sqrt{a^2}\right)\log\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)+a\log\left(\sqrt{a^2}+bx^3-\sqrt{(a+bx^3)^2}\right)+\sqrt{a}}{6a\sqrt{a^2}}$$

input
```
Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
(-2*a*Log[x^3] + (a - Sqrt[a^2])*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^
2]] + a*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] + Sqrt[a^2]*Log[a*(Sq
rt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2])])/(6*a*Sqrt[a^2])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1384, 27, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b\left(a+bx^3\right)\int \frac{1}{bx(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^3(bx^3+a)}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{47}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\int \frac{1}{x^3}dx^3}{a}-\frac{b\int \frac{1}{bx^3+a}dx^3}{a}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{14}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\log\left(x^3\right)}{a}-\frac{b\int \frac{1}{bx^3+a}dx^3}{a}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{16}$$

$$\frac{(a + bx^3)\left(\frac{\log(x^3)}{a} - \frac{\log(a+bx^3)}{a}\right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

input
```
Int[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
((a + b*x^3)*(Log[x^3]/a - Log[a + b*x^3]/a))/(3*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

method	result	size
pseudoelliptic	$\frac{\left(-\ln \left(b\,x^{3}+a \right)+\ln \left(b\,x^{3}\right)\right) \operatorname{csgn}\left(b\,x^{3}+a \right)}{3a}$	31
default	$-\frac{\left(b\,x^{3}+a \right) \left(\ln \left(b\,x^{3}+a \right)-3 \ln \left(x \right)\right)}{3 \sqrt{\left(b\,x^{3}+a \right)^{2}}\, a}$	37
risch	$-\frac{\sqrt{\left(b\,x^{3}+a \right)^{2}}\, \ln \left(b\,x^{3}+a \right)}{3 \left(b\,x^{3}+a \right) a}+\frac{\sqrt{\left(b\,x^{3}+a \right)^{2}}\, \ln \left(x \right)}{\left(b\,x^{3}+a \right) a}$	61

input
```
int(1/x/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-ln(b*x^3+a)+ln(b*x^3))*csgn(b*x^3+a)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.22

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{\log\left(bx^3+a\right)-3\,\log\left(x\right)}{3\,a}$$

input
```
integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(log(b*x^3 + a) - 3*log(x))/a
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \int \frac{1}{x\sqrt{(a + bx^3)^2}}\,dx$$

input
```
integrate(1/x/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a + b*x**3)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.54

$$\int \frac{1}{x\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = -\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right)}{3\,a}$$

input
```
integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.40

$$\int \frac{1}{x\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = -\frac{1}{3}\left(\frac{\log\left(|bx^3 + a|\right)}{a} - \frac{3\,\log\left(|x|\right)}{a}\right)\mathrm{sgn}\left(bx^3 + a\right)$$

input
```
integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/3*(log(abs(b*x^3 + a))/a - 3*log(abs(x))/a)*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 20.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)}{3\,\sqrt{a^2}}$$

input
```
int(1/(x*((a + b*x^3)^2)^(1/2)),x)
```

output
```
-log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)/
(3*(a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.56

$$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)+3\log(x)}{3a}$$

input
```
int(1/x/((b*x^3+a)^2)^(1/2),x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - log(a**(1/3) + b
**(1/3)*x) + 3*log(x))/(3*a)
```

3.64 $\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 125

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{-a-bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b(a+bx^3)\log(x)}{a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b(a+bx^3)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/3*(-b*x^3-a)/a/x^3/((b*x^3+a)^2)^(1/2)-b*(b*x^3+a)*ln(x)/a^2/((b*x^3+a)^
2)^(1/2)+1/3*b*(b*x^3+a)*ln(b*x^3+a)/a^2/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{a^2-\sqrt{a^2}\sqrt{(a+bx^3)^2}+2abx^3\log\left(x^3\right)+\left(-a+\sqrt{a^2}\right)bx^3\log\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)-abx^3\log}{6\left(a^2\right)^{3/2}x^3}$$

input
```
Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
(a^2 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2] + 2*a*b*x^3*Log[x^3] + (-a + Sqrt[a^2
])*b*x^3*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] - a*b*x^3*Log[Sqrt[a
^2] + b*x^3 - Sqrt[(a + b*x^3)^2]] - Sqrt[a^2]*b*x^3*Log[Sqrt[a^2] + b*x^3
 - Sqrt[(a + b*x^3)^2]])/(6*(a^2)^(3/2)*x^3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = $\{1384, 27, 798, 54, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^3)\int \frac{1}{bx^4(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{1}{x^4(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{(a+bx^3)\int \frac{1}{x^6(bx^3+a)}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx^3)\int \left(\frac{b^2}{a^2(bx^3+a)}-\frac{b}{a^2x^3}+\frac{1}{ax^6}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^3)\left(-\frac{b\log(x^3)}{a^2}+\frac{b\log(a+bx^3)}{a^2}-\frac{1}{ax^3}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
((a + b*x^3)*(-(1/(a*x^3)) - (b*Log[x^3])/a^2 + (b*Log[a + b*x^3])/a^2))/(
3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$-\frac{\left(\ln \left(b \,x^{3}\right) b \,x^{3}-b \ln \left(b \,x^{3}+a \right) x^{3}+a \right) \operatorname{csgn}\left(b \,x^{3}+a \right)}{3 a^{2} x^{3}}$	44
default	$\frac{\left(b \,x^{3}+a \right) \left(b \ln \left(b \,x^{3}+a \right) x^{3}-3 b \ln \left(x \right) x^{3}-a \right)}{3 \sqrt{\left(b \,x^{3}+a \right)^{2}}\, a^{2} x^{3}}$	52
risch	$-\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}}{3 \left(b \,x^{3}+a \right) a \,x^{3}}-\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b \ln \left(x \right)}{\left(b \,x^{3}+a \right) a^{2}}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b \ln \left(-b \,x^{3}-a \right)}{3 \left(b \,x^{3}+a \right) a^{2}}$	95

input
```
int(1/x^4/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(ln(b*x^3)*b*x^3-b*ln(b*x^3+a)*x^3+a)*csgn(b*x^3+a)/a^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{bx^{3}\log\left(bx^{3}+a\right)-3\,bx^{3}\log\left(x\right)-a}{3\,a^{2}x^{3}}$$

input
```
integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(b*x^3*log(b*x^3 + a) - 3*b*x^3*log(x) - a)/(a^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{1}{x^4\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(1/x**4/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt((a + b*x**3)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^2}-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}}{3\,a^2x^3}$$

input
```
integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^
2 - 1/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)/(a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{1}{3}\left(\frac{b\log\left(|bx^3+a|\right)}{a^2}-\frac{3\,b\log\left(|x|\right)}{a^2}+\frac{bx^3-a}{a^2x^3}\right)\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*(b*log(abs(b*x^3 + a))/a^2 - 3*b*log(abs(x))/a^2 + (b*x^3 - a)/(a^2*x^
3))*sgn(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 20.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,a\,x^3}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}\right)}{3\,{\left(a^2\right)}^{3/2}}-\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,a^2\,x^3}$$

input
```
int(1/(x^4*((a + b*x^3)^2)^(1/2)),x)
```

output
```
(a*b*atanh((a^2 + a*b*x^3)/((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))
))/(3*(a^2)^(3/2)) - (a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*a^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b\,x^3+\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)b\,x^3-3\log(x)\,b\,x^3-a}{3a^2x^3}$$

input
```
int(1/x^4/((b*x^3+a)^2)^(1/2),x)
```

output
```
(log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 + log(a**(1/3)
 + b**(1/3)*x)*b*x**3 - 3*log(x)*b*x**3 - a)/(3*a**2*x**3)
```

3.65 $\int \frac{1}{x^7\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{1}{x^7\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{a+bx^3}{6ax^6\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b(a+bx^3)}{3a^2x^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^2(a+bx^3)\log(x)}{a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b^2(a+bx^3)\log\left(a+bx^3\right)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
-1/6*(b*x^3+a)/a/x^6/((b*x^3+a)^2)^(1/2)+1/3*b*(b*x^3+a)/a^2/x^3/((b*x^3+a
)^2)^(1/2)+b^2*(b*x^3+a)*ln(x)/a^3/((b*x^3+a)^2)^(1/2)-1/3*b^2*(b*x^3+a)*l
n(b*x^3+a)/a^3/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^7\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\frac{a^3(a-2bx^3)}{\sqrt{a^2}x^6}-\frac{a(a-3bx^3)\sqrt{(a+bx^3)^2}}{x^6}-4\sqrt{a^2}b^2\log\left(x^3\right)+2\left(-a+\sqrt{a^2}\right)b^2\log\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)+}{12a^4}$$

input
```
Integrate[1/(x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
((a^3*(a - 2*b*x^3))/(Sqrt[a^2]*x^6) - (a*(a - 3*b*x^3)*Sqrt[(a + b*x^3)^2
])/x^6 - 4*Sqrt[a^2]*b^2*Log[x^3] + 2*(-a + Sqrt[a^2])*b^2*Log[Sqrt[a^2] -
 b*x^3 - Sqrt[(a + b*x^3)^2]] + 2*(a + Sqrt[a^2])*b^2*Log[Sqrt[a^2] + b*x^
3 - Sqrt[(a + b*x^3)^2]])/(12*a^4)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{a^2+2abx^3+b^2x^6}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^3)\int \frac{1}{bx^7(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{1}{x^7(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{(a+bx^3)\int \frac{1}{x^9(bx^3+a)}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx^3)\int \left(-\frac{b^3}{a^3(bx^3+a)}+\frac{b^2}{a^3x^3}-\frac{b}{a^2x^6}+\frac{1}{ax^9}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^3)\left(\frac{b^2\log(x^3)}{a^3}-\frac{b^2\log(a+bx^3)}{a^3}+\frac{b}{a^2x^3}-\frac{1}{2ax^6}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
((a + b*x^3)*(-1/2*1/(a*x^6) + b/(a^2*x^3) + (b^2*Log[x^3])/a^3 - (b^2*Log
[a + b*x^3])/a^3))/(3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$-\frac{\left(2 b^{2} \ln \left(b \,x^{3}+a \right) x^{6}-2 \ln \left(b \,x^{3}\right) b^{2} x^{6}-2 a \,x^{3} b +a^{2}\right) \operatorname{csgn}\left(b \,x^{3}+a \right)}{6 a^{3} x^{6}}$	58
default	$-\frac{\left(b \,x^{3}+a \right) \left(2 b^{2} \ln \left(b \,x^{3}+a \right) x^{6}-6 b^{2} \ln \left(x \right) x^{6}-2 a \,x^{3} b +a^{2}\right)}{6 \sqrt{\left(b \,x^{3}+a \right)^{2}}\, a^{3} x^{6}}$	64
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(\frac{b \,x^{3}}{3 a^{2}}-\frac{1}{6 a}\right)}{\left(b \,x^{3}+a \right) x^{6}}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{2} \ln \left(x \right)}{\left(b \,x^{3}+a \right) a^{3}}-\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{2} \ln \left(b \,x^{3}+a \right)}{3 \left(b \,x^{3}+a \right) a^{3}}$	106

input
```
int(1/x^7/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*b^2*ln(b*x^3+a)*x^6-2*ln(b*x^3)*b^2*x^6-2*a*x^3*b+a^2)*csgn(b*x^3+
a)/a^3/x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^7\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{2\,b^2x^6\log\left(bx^3+a\right)-6\,b^2x^6\log\left(x\right)-2\,abx^3+a^2}{6\,a^3x^6}$$

input
```
integrate(1/x^7/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(2*b^2*x^6*log(b*x^3 + a) - 6*b^2*x^6*log(x) - 2*a*b*x^3 + a^2)/(a^3*
x^6)
```

Sympy [F]

$$\int \frac{1}{x^7\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{1}{x^7\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(1/x**7/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**7*sqrt((a + b*x**3)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^7\sqrt{a^2+2abx^3+b^2x^6}}\,dx=-\frac{(-1)^{2\,abx^3+2\,a^2}\,b^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^3}+\frac{\sqrt{b^2x^6+2\,abx^3+a^2}b}{2\,a^3x^3}-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}}{6\,a^2x^6}$$

input
```
integrate(1/x^7/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^2*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))
/a^3 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b/(a^3*x^3) - 1/6*sqrt(b^2*x^6
+ 2*a*b*x^3 + a^2)/(a^2*x^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^7\sqrt{a^2+2abx^3+b^2x^6}}\,dx=-\frac{1}{6}\left(\frac{2\,b^2\log\left(|bx^3+a|\right)}{a^3}-\frac{6\,b^2\log\left(|x|\right)}{a^3}+\frac{3\,b^2x^6-2\,abx^3+a^2}{a^3x^6}\right)\mathrm{sgn}\left(bx^3+a\right)$$

input
```
integrate(1/x^7/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/6*(2*b^2*log(abs(b*x^3 + a))/a^3 - 6*b^2*log(abs(x))/a^3 + (3*b^2*x^6 -
 2*a*b*x^3 + a^2)/(a^3*x^6))*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{1}{x^7\,\sqrt{(b\,x^3+a)^2}}\,dx$$

input
```
int(1/(x^7*((a + b*x^3)^2)^(1/2)),x)
```

output
```
int(1/(x^7*((a + b*x^3)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^7\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$=\frac{-2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b^2x^6-2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)b^2x^6+6\log(x)\,b^2x^6-a^2+2ab\,x^3}{6a^3x^6}$$

input
```
int(1/x^7/((b*x^3+a)^2)^(1/2),x)
```

output
```
( - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*x**6 - 2*lo
g(a**(1/3) + b**(1/3)*x)*b**2*x**6 + 6*log(x)*b**2*x**6 - a**2 + 2*a*b*x**
3)/(6*a**3*x**6)
```

3.66 $\int \frac{1}{x^{10}\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 208

$$\begin{aligned}\int \frac{1}{x^{10}\sqrt{a^2+2abx^3+b^2x^6}}\,dx = &-\frac{a+bx^3}{9ax^9\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b(a+bx^3)}{6a^2x^6\sqrt{a^2+2abx^3+b^2x^6}}\\ &-\frac{b^2(a+bx^3)}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}\\ &-\frac{b^3(a+bx^3)\log(x)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^3(a+bx^3)\log(a+bx^3)}{3a^4\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output

```
-1/9*(b*x^3+a)/a/x^9/((b*x^3+a)^2)^(1/2)+1/6*b*(b*x^3+a)/a^2/x^6/((b*x^3+a
)^2)^(1/2)-1/3*b^2*(b*x^3+a)/a^3/x^3/((b*x^3+a)^2)^(1/2)-b^3*(b*x^3+a)*ln(
x)/a^4/((b*x^3+a)^2)^(1/2)+1/3*b^3*(b*x^3+a)*ln(b*x^3+a)/a^4/((b*x^3+a)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{10}\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{1}{36}\left(\frac{\sqrt{(a+bx^3)^2}(-2a^2+5abx^3-11b^2x^6)}{a^4x^9} + \frac{2a^2-3abx^3+6b^2x^6}{(a^2)^{3/2}x^9} + \frac{12\sqrt{a^2}b^3\log(x^3)}{a^5} - \frac{6\left(-a+\sqrt{a^2}\right)b^3\log\left(\sqrt{a^2}-bx^3-\sqrt{(a+bx^3)^2}\right)}{a^5} - \frac{6\left(a+\sqrt{a^2}\right)b^3\log\left(\sqrt{a^2}+bx^3-\sqrt{(a+bx^3)^2}\right)}{a^5}\right)$$

input
```
Integrate[1/(x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
((Sqrt[(a + b*x^3)^2]*(-2*a^2 + 5*a*b*x^3 - 11*b^2*x^6))/(a^4*x^9) + (2*a^
2 - 3*a*b*x^3 + 6*b^2*x^6)/((a^2)^(3/2)*x^9) + (12*Sqrt[a^2]*b^3*Log[x^3])
/a^5 - (6*(-a + Sqrt[a^2])*b^3*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]
])/a^5 - (6*(a + Sqrt[a^2])*b^3*Log[Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2
]])/a^5)/36
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^3)\int \frac{1}{bx^{10}(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{1}{x^{10}(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{(a+bx^3)\int \frac{1}{x^{12}(bx^3+a)}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx^3)\int \left(\frac{b^4}{a^4(bx^3+a)}-\frac{b^3}{a^4x^3}+\frac{b^2}{a^3x^6}-\frac{b}{a^2x^9}+\frac{1}{ax^{12}}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^3)\left(-\frac{b^3\log(x^3)}{a^4}+\frac{b^3\log(a+bx^3)}{a^4}-\frac{b^2}{a^3x^3}+\frac{b}{2a^2x^6}-\frac{1}{3ax^9}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
((a + b*x^3)*(-1/3*1/(a*x^9) + b/(2*a^2*x^6) - b^2/(a^3*x^3) - (b^3*Log[x^
3])/a^4 + (b^3*Log[a + b*x^3])/a^4))/(3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.34

method	result	size
pseudoelliptic	$\frac{\left(6 b^{3} \ln \left(b \,x^{3}+a \right) x^{9}-6 \ln \left(b \,x^{3}\right) b^{3} x^{9}-6 a \,x^{6} b^{2}+3 a^{2} x^{3} b -2 a^{3}\right) \operatorname{csgn}\left(b \,x^{3}+a \right)}{18 a^{4} x^{9}}$	71
default	$\frac{\left(b \,x^{3}+a \right) \left(6 b^{3} \ln \left(b \,x^{3}+a \right) x^{9}-18 b^{3} \ln \left(x \right) x^{9}-6 a \,x^{6} b^{2}+3 a^{2} x^{3} b -2 a^{3}\right)}{18 \sqrt{\left(b \,x^{3}+a \right)^{2}}\, a^{4} x^{9}}$	77
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-\frac{b^{2} x^{6}}{3 a^{3}}+\frac{b \,x^{3}}{6 a^{2}}-\frac{1}{9 a}\right)}{\left(b \,x^{3}+a \right) x^{9}}-\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{3} \ln \left(x \right)}{\left(b \,x^{3}+a \right) a^{4}}+\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, b^{3} \ln \left(-b \,x^{3}-a \right)}{3 \left(b \,x^{3}+a \right) a^{4}}$	121

input
```
int(1/x^10/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(6*b^3*ln(b*x^3+a)*x^9-6*ln(b*x^3)*b^3*x^9-6*a*x^6*b^2+3*a^2*x^3*b-2*
a^3)*csgn(b*x^3+a)/a^4/x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^{10}\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{6\,b^3x^9\log\left(bx^3+a\right)-18\,b^3x^9\log\left(x\right)-6\,ab^2x^6+3\,a^2bx^3-2\,a^3}{18\,a^4x^9}$$

input
```
integrate(1/x^10/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/18*(6*b^3*x^9*log(b*x^3 + a) - 18*b^3*x^9*log(x) - 6*a*b^2*x^6 + 3*a^2*b
*x^3 - 2*a^3)/(a^4*x^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**10/((b*x**3+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^{10}\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(-1\right)^{2\,abx^3+2\,a^2}b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^4} - \frac{11\,\sqrt{b^2x^6+2\,abx^3+a^2}b^2}{18\,a^4x^3} + \frac{5\,\sqrt{b^2x^6+2\,abx^3+a^2}b}{18\,a^3x^6} - \frac{\sqrt{b^2x^6+2\,abx^3+a^2}}{9\,a^2x^9}$$

input
```
integrate(1/x^10/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^3*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/
a^4 - 11/18*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2/(a^4*x^3) + 5/18*sqrt(b^2*
x^6 + 2*a*b*x^3 + a^2)*b/(a^3*x^6) - 1/9*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)/(
a^2*x^9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^{10}\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{1}{18}\left(\frac{6\,b^3\log\left(|bx^3+a|\right)}{a^4}-\frac{18\,b^3\log\left(|x|\right)}{a^4}+\frac{11\,b^3x^9-6\,ab^2x^6+3\,a^2bx^3-2\,a^3}{a^4x^9}\right)\operatorname{sgn}\left(bx^3+a\right)$$

input
```
integrate(1/x^10/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/18*(6*b^3*log(abs(b*x^3 + a))/a^4 - 18*b^3*log(abs(x))/a^4 + (11*b^3*x^9
 - 6*a*b^2*x^6 + 3*a^2*b*x^3 - 2*a^3)/(a^4*x^9))*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{x^{10}\,\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

input
```
int(1/(x^10*((a + b*x^3)^2)^(1/2)),x)
```

output
```
int(1/(x^10*((a + b*x^3)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^{10}\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{6\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b^3x^9+6\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)b^3x^9-18\log(x)\,b^3x^9-2a^3+3a^2b\,x^3-6a\,b^2x^6}{18a^4x^9}$$

input
```
int(1/x^10/((b*x^3+a)^2)^(1/2),x)
```

output
```
(6*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**3*x**9 + 6*log(a
**(1/3) + b**(1/3)*x)*b**3*x**9 - 18*log(x)*b**3*x**9 - 2*a**3 + 3*a**2*b*
x**3 - 6*a*b**2*x**6)/(18*a**4*x**9)
```

3.67 $\int \frac{x^6}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 277

$$\begin{aligned}\int \frac{x^6}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= -\frac{ax(a+bx^3)}{b^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^4(a+bx^3)}{4b\sqrt{a^2+2abx^3+b^2x^6}} \\ &\quad -\frac{a^{4/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &\quad +\frac{a^{4/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &\quad -\frac{a^{4/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output

```
-a*x*(b*x^3+a)/b^2/((b*x^3+a)^2)^(1/2)+1/4*x^4*(b*x^3+a)/b/((b*x^3+a)^2)^(
1/2)-1/3*a^(4/3)*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3
))*3^(1/2)/b^(7/3)/((b*x^3+a)^2)^(1/2)+1/3*a^(4/3)*(b*x^3+a)*ln(a^(1/3)+b^
(1/3)*x)/b^(7/3)/((b*x^3+a)^2)^(1/2)-1/6*a^(4/3)*(b*x^3+a)*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(7/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.51

$$\int \frac{x^6}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$= \frac{(a+bx^3)\left(-12a\sqrt[3]{b}x+3b^{4/3}x^4-4\sqrt{3}a^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2a^{4/3}\log\left(a^{2}\right.\right.}{12b^{7/3}\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[x^6/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(-12*a*b^(1/3)*x + 3*b^(4/3)*x^4 - 4*Sqrt[3]*a^(4/3)*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[a^(1/3) + b^(1/3)*x] -
 2*a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(12*b^(7/3)*Sq
rt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b(a+bx^3)\int \frac{x^6}{b(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^3)\int \frac{x^6}{bx^3+a}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 831

$$\frac{(a+bx^3)\int \left(\frac{x^3}{b}+\frac{a^2}{b^2(bx^3+a)}-\frac{a}{b^2}\right)dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 2009

$$\frac{(a+bx^3)\left(-\frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}}-\frac{ax}{b^2}+\frac{x^4}{4b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^6/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(-((a*x)/b^2) + x^4/(4*b) - (a^(4/3)*ArcTan[(a^(1/3) - 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) + (a^(4/3)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(7/3)) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(6*b^(7/3))))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{1}{4}b\,x^{4}-xa\right)}{\left(b\,x^{3}+a\right)b^{2}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}\left(\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{3\left(b\,x^{3}+a\right)b^{3}}$
default	$\frac{\left(b\,x^{3}+a\right)\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}x^{4}-12\left(\frac{a}{b}\right)^{\frac{2}{3}}abx-4\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}\,a^{2}+4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)a^{2}-2\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)a^{2}\right)}{12\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$

input
```
int(x^6/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(1/4*b*x^4-x*a)/b^2+1/3*((b*x^3+a)^2)^(1/2)/
(b*x^3+a)/b^3*a^2*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.39

$$\int \frac{x^6}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{3\,bx^4+4\,\sqrt{3}a\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)-2\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+4\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right.\right.}{12\,b^2}$$

input
```
integrate(x^6/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(3*b*x^4 + 4*sqrt(3)*a*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2
/3) - sqrt(3)*a)/a) - 2*a*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
)) + 4*a*(a/b)^(1/3)*log(x + (a/b)^(1/3)) - 12*a*x)/b^2
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x^6}{\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(x**6/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x**6/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.44

$$\int \frac{x^6}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{a^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{a^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{bx^4-4\,ax}{4\,b^2}$$

input
```
integrate(x^6/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(
a/b)^(2/3)) - 1/6*a^2*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2
/3)) + 1/3*a^2*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 1/4*(b*x^4 - 4*a*x
)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.61

$$\int \frac{x^6}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\mathrm{sgn}(bx^3+a)}{3\,b^2} + \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\mathrm{sgn}(bx^3+a)}{3\,b^3} + \frac{(-ab^2)^{\frac{1}{3}}\,a\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\mathrm{sgn}(bx^3+a)}{6\,b^3} + \frac{b^3x^4\mathrm{sgn}(bx^3+a)-4\,ab^2x\mathrm{sgn}(bx^3+a)}{4\,b^4}$$

input
```
integrate(x^6/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/3*a*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*x^3 + a)/b^2 + 1/3*sq
rt(3)*(-a*b^2)^(1/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))*sgn(b*x^3 + a)/b^3 + 1/6*(-a*b^2)^(1/3)*a*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))*sgn(b*x^3 + a)/b^3 + 1/4*(b^3*x^4*sgn(b*x^3 + a) - 4*a*b^2*x*s
gn(b*x^3 + a))/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x^6}{\sqrt{(b\,x^3+a)^2}}\,dx$$

input
```
int(x^6/((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x^6/((a + b*x^3)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.32

$$\int \frac{x^6}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{-4a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-2a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+4a^{\frac{4}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)-12b^{\frac{1}{3}}ax+3b^{\frac{4}{3}}x^4}{12b^{\frac{7}{3}}}$$

input
```
int(x^6/((b*x^3+a)^2)^(1/2),x)
```

output
```
( - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a - 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a + 4*a
**(1/3)*log(a**(1/3) + b**(1/3)*x)*a - 12*b**(1/3)*a*x + 3*b**(1/3)*b*x**4
)/(12*b**(1/3)*b**2)
```

3.68 $\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 240

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}} \, dx = \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/2*x^2*(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)+1/3*a^(2/3)*(b*x^3+a)*arctan(1/3*(
a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/b^(5/3)/((b*x^3+a)^2)^(1/2)+
1/3*a^(2/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/b^(5/3)/((b*x^3+a)^2)^(1/2)-1/
6*a^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)/((b*
x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.55

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(a+bx^3)\left(3b^{2/3}x^2+2\sqrt{3}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.\right.}{6b^{5/3}\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x] - a^(2/3)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(5/3)*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.68, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1384, 27, 843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b(a+bx^3)\int \frac{x^4}{b(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^3)\int \frac{x^4}{bx^3+a}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{(a+bx^3)\left(\frac{x^2}{2b}-\frac{a\int \frac{x}{bx^3+a}dx}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{(a+bx^3)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{(a+bx^3)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{(a+bx^3)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 25$$

$$\frac{(a+bx^3)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^3)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\left(a+bx^3\right)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 217

$$\frac{\left(a+bx^3\right)\left(\frac{x^2}{2b}-\frac{a\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1103

$$\frac{(a+bx^3)\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(x^2/(2*b) - (a*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/
3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) +
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(
1/3))))/b))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.32

method	result	size
risch	$\frac{x^{2}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2\left(b\,x^{3}+a\right)b}-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a\left(\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{3\left(b\,x^{3}+a\right)b^{2}}$	77
default	$\frac{\left(b\,x^{3}+a\right)\left(3x^{2}b\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)a-\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)a\right)}{6\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	113

input
```
int(x^4/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b-1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/
b^2*a*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{3\,x^{2}-2\,\sqrt{3}\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)-\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(ax^{2}-bx\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}+a\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\right)+2\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(\right.}{6\,b}$$

input
```
integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*x^2 - 2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)
^(1/3) - sqrt(3)*a)/a) - (a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) +
 a*(a^2/b^2)^(1/3)) + 2*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x^4}{\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(x**4/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x**4/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.45

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{x^2}{2\,b} - \frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*x^2/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(b^2*(a/b)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*
(a/b)^(1/3)) + 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.61

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{x^2\operatorname{sgn}(bx^3+a)}{2\,b} + \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sgn}(bx^3+a)}{3\,b} + \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sgn}(bx^3+a)}{3\,b^3} - \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sgn}(bx^3+a)}{6\,b^3}$$

input
```
integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*x^2*sgn(b*x^3 + a)/b + 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn
(b*x^3 + a)/b + 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^3 - 1/6*(-a*b^2)^(2/3)*log(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x^4}{\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

input
```
int(x^4/((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x^4/((a + b*x^3)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.34

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a+3b^{\frac{2}{3}}a^{\frac{1}{3}}x^2-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a}{6b^{\frac{5}{3}}a^{\frac{1}{3}}}$$

input
```
int(x^4/((b*x^3+a)^2)^(1/2),x)
```

output
```
(2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a + 3*b**(2/
3)*a**(1/3)*x**2 - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a +
 2*log(a**(1/3) + b**(1/3)*x)*a)/(6*b**(2/3)*a**(1/3)*b)
```

3.69 $\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 235

$$\begin{aligned}\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}\\ &\quad-\frac{\sqrt[3]{a}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}\\ &\quad+\frac{\sqrt[3]{a}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output

```
x*(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)+1/3*a^(1/3)*(b*x^3+a)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/b^(4/3)/((b*x^3+a)^2)^(1/2)-1/3*a^
(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/b^(4/3)/((b*x^3+a)^2)^(1/2)+1/6*a^(1
/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(4/3)/((b*x^3+a)
^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.54

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(a+bx^3)\left(6\sqrt[3]{b}x+2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2}\right.\right.}{6b^{4/3}\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(4/3)*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1384, 27, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b(a+bx^3)\int \frac{x^3}{b(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^3)\int \frac{x^3}{bx^3+a}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 843$$

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^3+a}dx}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 750$$

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 16$$

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 1142$$

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 25$$

$$\frac{\left(a+bx^3\right)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(a+bx^3\right)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1082$$

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 217

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1103

$$\frac{(a+bx^3)\left(\frac{x}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(x/b - (a*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-(
(Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b))/Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.81 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{x\sqrt{(b\,x^3+a)^2}}{(b\,x^3+a)b}-\frac{\sqrt{(b\,x^3+a)^2}\,a\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{3(b\,x^3+a)b^2}$	74
default	$\frac{(b\,x^3+a)\left(6xb\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a-2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)a+\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)a\right)}{6\sqrt{(b\,x^3+a)^2}\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	110

input
```
int(x^3/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b-1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b^2*a*
sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.45

$$\int\frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$=\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right.\right.}{6b}$$

input
```
integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(
3)*a)/a) - (-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b
)^(1/3)*log(x - (-a/b)^(1/3)) + 6*x)/b
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x^3}{\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(x**3/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.45

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{x}{b} - \frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
x/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b
^2*(a/b)^(2/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^
(2/3)) - 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sgn}(bx^3+a)}{3\,b} + \frac{x\operatorname{sgn}(bx^3+a)}{b} - \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sgn}(bx^3+a)}{3\,b^2} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sgn}(bx^3+a)}{6\,b^2}$$

input
```
integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*x^3 + a)/b + x*sgn(b*x^3
 + a)/b - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^2 - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x^3}{\sqrt{(b\,x^3+a)^2}}\,dx$$

input
```
int(x^3/((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x^3/((a + b*x^3)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.33

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-2a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)+6b^{\frac{1}{3}}x}{6b^{\frac{4}{3}}}$$

input
```
int(x^3/((b*x^3+a)^2)^(1/2),x)
```

output
```
(2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))) + a
**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - 2*a**(1/3)*l
og(a**(1/3) + b**(1/3)*x) + 6*b**(1/3)*x)/(6*b**(1/3)*b)
```

3.70 $\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 202

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
-1/3*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a
^(1/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(
1/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)+1/6*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/a^(1/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.54

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(a+bx^3)\left(-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6\sqrt[3]{a}b^{2/3}\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*L
og[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))
/(6*a^(1/3)*b^(2/3)*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.72, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1384, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b(a+bx^3)\int \frac{x}{b(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^3)\int \frac{x}{bx^3+a}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 821

$$\frac{(a+bx^3)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 16

$$\frac{(a+bx^3)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1142

$$\frac{(a+bx^3)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 25

$$\frac{(a+bx^3)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 27

$$\frac{(a+bx^3)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1082

$$\frac{(a+bx^3)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 217$$

$$\frac{(a+bx^3)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1103$$

$$\frac{(a+bx^3)\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((a + b*x^3)*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3
]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{3\left(b\,x^{3}+a\right)b}$	47
default	$-\frac{\left(b\,x^{3}+a\right)\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	97

input
```
int(x/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.50

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\,(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x}{bx^3+a}\right)+(-ab^2)^{\frac{2}{3}}\log\left(b^2x\right.}{6\,ab^2}\right.$$

input
```
integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt
(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3
)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*log(b^2*x^2 + (-a
*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1
/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/
3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*lo
g(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*
x - (-a*b^2)^(1/3)))/(a*b^2)]
```

Sympy [F]

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{x}{\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(x/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(x/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.49

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(
1/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 1/3*l
og(x + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.61

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sgn}(bx^3+a)}{3\,\left(-ab^2\right)^{\frac{1}{3}}} - \frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sgn}(bx^3+a)}{6\,\left(-ab^2\right)^{\frac{1}{3}}} - \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sgn}(bx^3+a)}{3\,a}$$

input
```
integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))*sgn(b*x^
3 + a)/(-a*b^2)^(1/3) - 1/6*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))*sgn(b
*x^3 + a)/(-a*b^2)^(1/3) - 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn
(b*x^3 + a)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{x}{\sqrt{\left(b\,x^3+a\right)^2}}\,dx$$

input
```
int(x/((a + b*x^3)^2)^(1/2),x)
```

output
```
int(x/((a + b*x^3)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.32

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{6b^{\frac{2}{3}}a^{\frac{1}{3}}}$$

input
```
int(x/((b*x^3+a)^2)^(1/2),x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))) + log(a**
(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - 2*log(a**(1/3) + b**(1/3)*x
))/(6*b**(2/3)*a**(1/3))
```

3.71 $\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 202

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
-1/3*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a
^(2/3)/b^(1/3)/((b*x^3+a)^2)^(1/2)+1/3*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(
2/3)/b^(1/3)/((b*x^3+a)^2)^(1/2)-1/6*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{(a+bx^3)\left(2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6a^{2/3}\sqrt[3]{b}\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
-1/6*((a + b*x^3)*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] -
 2*Log[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2]))/(a^(2/3)*b^(1/3)*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.71, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1384, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b(a+bx^3)\int \frac{1}{b^2x^3+ab}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{b\left(a+bx^3\right)\left(\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}b^{2/3}}+\frac{\int \frac{1}{b^{2/3}x+\sqrt[3]{a}\sqrt[3]{b}}dx}{3a^{2/3}b^{2/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 16$$

$$\frac{b\left(a+bx^3\right)\left(\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(a+bx^3\right)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 1142$$

$$\frac{b\left(a+bx^3\right)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx-\frac{\int -\frac{b\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{2b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 25$$

$$\frac{b\left(a+bx^3\right)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{\int \frac{b\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{2b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 27$$

$$\frac{b(a+bx^3)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1082

$$\frac{b(a+bx^3)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 217

$$\frac{b(a+bx^3)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1103

$$\frac{b(a+bx^3)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2b}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input

```
Int[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(b*(a + b*x^3)*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(4/3)) + (-((Sqrt[3]
*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b) - Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2]/(2*b))/(3*a^(2/3)*b^(1/3))))/Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{3\left(b\,x^{3}+a\right)b}$	47
default	$\frac{\left(b\,x^{3}+a\right)\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	97

input
```
int(1/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/b*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a
))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-(a^2b)^{\frac{2}{3}}\log\left(abx^2-\right.}{6\,a^2b}\right.$$

input
```
integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1
/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*log(a*b*x^2 - (a^2*
b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3))
)/(a^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2
/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log
(a*b*x + (a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(1/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(1/sqrt((a + b*x**3)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(
2/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*l
og(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{1}{6}\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a} - \frac{2\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right.\right.}{ab} \right. + a)$$

input
```
integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/6*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*b^2)^(1/
3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - (-a*b^2)^
(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b))*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{1}{\sqrt{\left(b\,x^3+a\right)^2}}\,dx$$

input
```
int(1/((a + b*x^3)^2)^(1/2),x)
```

output
```
int(1/((a + b*x^3)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{6a^{\frac{2}{3}}b^{\frac{1}{3}}}$$

input
```
int(1/((b*x^3+a)^2)^(1/2),x)
```

output
```
(a**(1/3)*( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
 - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) + 2*log(a**(1/3) +
b**(1/3)*x)))/(6*b**(1/3)*a)
```

3.72 $\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 238

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
-(b*x^3+a)/a/x/((b*x^3+a)^2)^(1/2)+1/3*b^(1/3)*(b*x^3+a)*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)/((b*x^3+a)^2)^(1/2)+1/3*b
^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/((b*x^3+a)^2)^(1/2)-1/6*b^(
1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/((b*x^3+a
)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{(a+bx^3)\left(6\sqrt[3]{a}-2\sqrt{3}\sqrt[3]{b}x\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{b}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\sqrt[3]{b}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6a^{4/3}x\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
-1/6*((a + b*x^3)*(6*a^(1/3) - 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*x*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*x*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(4/3)*x*Sqrt[(a + b*x^3)
^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.68, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1384, 27, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b(a+bx^3)\int \frac{1}{bx^2(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^2\left(bx^3+a\right)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 847

$$\frac{\left(a+bx^3\right)\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 821

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 16

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1142

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 25$$

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 217

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1103

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
((a + b*x^3)*(-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2
/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) +
 Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^
(1/3))))/a))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.39

method	result	size
risch	$-\frac{\sqrt{(b\,x^3+a)^2}}{(b\,x^3+a)ax}+\frac{\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^4\_Z^3-b\right)}\_R\ln\left(\left(-4\_R^3a^4+3b\right)x-a^3\_R^2\right)\right)}{3b\,x^3+3a}$	93
default	$-\frac{(b\,x^3+a)\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)x-2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)x+\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x+6\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\sqrt{(b\,x^3+a)^2}\left(\frac{a}{b}\right)^{\frac{1}{3}}ax}$	111

input
```
int(1/x^2/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a/x+1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)*sum(_
R*ln((-4*_R^3*a^4+3*b)*x-a^3*_R^2),_R=RootOf(_Z^3*a^4-b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{2\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\right.}{6\,ax}$$

input
```
integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(
3)) + x*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*x*(b/
a)^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6)/(a*x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{x^2\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(1/x**2/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((a + b*x**3)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{1}{ax}$$

input
```
integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^
(1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) + 1/3*
log(x + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 1/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$=\frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}+\frac{2\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b}-\frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)\right.}{a^2b}\right.$$
$$\left.+a\right)$$

input
```
integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*(2*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(-a*b^2)^
(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - (-a*
b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 6/(a*x))*sgn
(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{1}{x^2\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

input
```
int(1/(x^2*((a + b*x^3)^2)^(1/2)),x)
```

output
```
int(1/(x^2*((a + b*x^3)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$= \frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)bx-6b^{\frac{2}{3}}a^{\frac{1}{3}}-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)bx+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)bx}{6b^{\frac{2}{3}}a^{\frac{4}{3}}x}$$

input
```
int(1/x^2/((b*x^3+a)^2)^(1/2),x)
```

output
```
(2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x - 6*b**(
2/3)*a**(1/3) - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x +
2*log(a**(1/3) + b**(1/3)*x)*b*x)/(6*b**(2/3)*a**(1/3)*a*x)
```

3.73 $\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 243

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{-a-bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}}\\ &+ \frac{b^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\\ &- \frac{b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\\ &+ \frac{b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
1/2*(-b*x^3-a)/a/x^2/((b*x^3+a)^2)^(1/2)+1/3*b^(2/3)*(b*x^3+a)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/((b*x^3+a)^2)^(1/2)
-1/3*b^(2/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/((b*x^3+a)^2)^(1/2)+1
/6*b^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/((b
*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx =$$

$$-\frac{(a+bx^3)\left(3a^{2/3}-2\sqrt{3}b^{2/3}x^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2b^{2/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\right.\right.}{6a^{5/3}x^2\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
-1/6*((a + b*x^3)*(3*a^(2/3) - 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]] + 2*b^(2/3)*x^2*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*
x^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(5/3)*x^2*Sqrt[(a
+ b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.65, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1384, 27, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^3)\int \frac{1}{bx^3(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{1}{x^3(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 847$$

$$\frac{(a+bx^3)\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 750$$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 16$$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1142$$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 25$$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{217}$$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(a+bx^3)\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
((a + b*x^3)*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(
1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3))
- Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3)))
)/a))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.86 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.39

method	result	size
risch	$-\frac{\sqrt{(b\,x^3+a)^2}}{2(b\,x^3+a)a\,x^2}+\frac{\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^3+b^2\right)}\_R\ln\left(\left(-4\_R^3a^5-3b^2\right)x-a^2b\_R\right)\right)}{3b\,x^3+3a}$	94
default	$\frac{(b\,x^3+a)\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)x^2+\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x^2-2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)x^2-3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\sqrt{(b\,x^3+a)^2}\,a\,x^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	117

input
```
int(1/x^3/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a/x^2+1/3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
*sum(_R*ln((-4*_R^3*a^5-3*b^2)*x-a^2*b*_R),_R=RootOf(_Z^3*a^5+b^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$= \frac{2\sqrt{3}x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2x^2}{6ax^2}$$

input
```
integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*x^2*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(
2/3) - sqrt(3)*b)/b) - x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)
^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x^2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/
a^2)^(1/3)) - 3)/(a*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{1}{x^3\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(1/x**3/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt((a + b*x**3)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{1}{2\,ax^2}$$

input
```
integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^
(2/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*
log(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2} - \frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2} - \frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)\right.}{a^2}\right. + a)$$

input
```
integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*(2*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 2*sqrt(3)*(-a*b^2)^
(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - (-a*b^2)
^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/(a*x^2))*sgn(b*x^3
 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{x^3\,\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

input
```
int(1/(x^3*((a + b*x^3)^2)^(1/2)),x)
```

output
```
int(1/(x^3*((a + b*x^3)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^2+a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b\,x^2-2a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)b\,x^2-3b^{\frac{1}{3}}a}{6b^{\frac{1}{3}}a^2x^2}$$

input

```
int(1/x^3/((b*x^3+a)^2)^(1/2),x)
```

output

```
(2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x
**2 + a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**2
- 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b*x**2 - 3*b**(1/3)*a)/(6*b**(1/3)
*a**2*x**2)
```

3.74 $\int \frac{1}{x^5\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 278

$$\int \frac{1}{x^5\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{a+bx^3}{4ax^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b(a+bx^3)}{a^2x\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{4/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{4/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{4/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
-1/4*(b*x^3+a)/a/x^4/((b*x^3+a)^2)^(1/2)+b*(b*x^3+a)/a^2/x/((b*x^3+a)^2)^(
1/2)-1/3*b^(4/3)*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3
))*3^(1/2)/a^(7/3)/((b*x^3+a)^2)^(1/2)-1/3*b^(4/3)*(b*x^3+a)*ln(a^(1/3)+b^
(1/3)*x)/a^(7/3)/((b*x^3+a)^2)^(1/2)+1/6*b^(4/3)*(b*x^3+a)*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^5\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{(a+bx^3)\left(3a^{4/3}-12\sqrt[3]{a}bx^3+4\sqrt{3}b^{4/3}x^4\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4b^{4/3}x^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2b^{4/3}x^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{12a^{7/3}x^4\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[1/(x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
-1/12*((a + b*x^3)*(3*a^(4/3) - 12*a^(1/3)*b*x^3 + 4*Sqrt[3]*b^(4/3)*x^4*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(4/3)*x^4*Log[a^(1/3) + b
^(1/3)*x] - 2*b^(4/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))
/(a^(7/3)*x^4*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.64, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1384, 27, 847, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^3)\int \frac{1}{bx^5(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{1}{x^5(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 847$$

$$\frac{(a+bx^3)\left(-\frac{b\int \frac{1}{x^2(bx^3+a)}dx}{a}-\frac{1}{4ax^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 847$$

$$\frac{(a+bx^3)\left(-\frac{b\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 821$$

$$\frac{(a+bx^3)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 16$$

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1142

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 25

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1082

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 217

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(-\frac{b\left(\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1103

$$\frac{\left(a+bx^3\right)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{4ax^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

output
```
((a + b*x^3)*(-1/4*1/(a*x^4) - (b*(-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(
1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^
(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/a))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.62 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.38

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{b\,x^{3}}{a^{2}}-\frac{1}{4a}\right)}{\left(b\,x^{3}+a\right)x^{4}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\operatorname{RootOf}\left(a^{7}\_Z^{3}+b^{4}\right)}\_R\ln\left(\left(-4\_R^{3}a^{7}-3b^{4}\right)x+a^{5}b\_R^{2}\right)\right)}{3b\,x^{3}+3a}$
default	$-\frac{\left(b\,x^{3}+a\right)\left(4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b\,x^{4}+4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b\,x^{4}-2\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b\,x^{4}-12b\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{3}+3\left(\frac{a}{b}\right)^{\frac{1}{3}}a\right)}{12\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}x^{4}}$

input
```
int(1/x^5/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)*(1/a^2*b*x^3-1/4/a)/x^4+1/3*((b*x^3+a)^2)^(1
/2)/(b*x^3+a)*sum(_R*ln((-4*_R^3*a^7-3*b^4)*x+a^5*b*_R^2),_R=RootOf(_Z^3*a
^7+b^4))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^5\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$= \frac{4\sqrt{3}bx^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\,bx^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+4\,bx^4\left(-\right.}{12\,a^2x^4}$$

input
```
integrate(1/x^5/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(4*sqrt(3)*b*x^4*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3
*sqrt(3)) - 2*b*x^4*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(
1/3)) + 4*b*x^4*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) + 12*b*x^3 - 3*a)/(
a^2*x^4)
```

Sympy [F]

$$\int \frac{1}{x^5\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{x^5\sqrt{(a+bx^3)^2}}\,dx$$

input
```
integrate(1/x**5/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**5*sqrt((a + b*x**3)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^5\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{b\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{b\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{4\,bx^3-a}{4\,a^2x^4}$$

input
```
integrate(1/x^5/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/
b)^(1/3)) + 1/6*b*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(1/3))
 - 1/3*b*log(x + (a/b)^(1/3))/(a^2*(a/b)^(1/3)) + 1/4*(4*b*x^3 - a)/(a^2*x
^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^5\sqrt{a^2+2abx^3+b^2x^6}}\,dx = -\frac{1}{12}\left(\frac{4\,b^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^3} + \frac{4\,\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^3} - \frac{2\,(-ab^2)^{\frac{2}{3}}\log\left(x^2+\right.}{a^3} \right. \ldots + a)$$

input
```
integrate(1/x^5/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/12*(4*b^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 4*sqrt(3)*(-a*b
^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3 - 2*(-
a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^3 - 3*(4*b*x^3 - a
)/(a^2*x^4))*sgn(b*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{1}{x^5\,\sqrt{(b\,x^3+a)^2}}\,dx$$

input
```
int(1/(x^5*((a + b*x^3)^2)^(1/2)),x)
```

output
```
int(1/(x^5*((a + b*x^3)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^5\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$
$$= \frac{-4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b^2x^4 - 3b^{\frac{2}{3}}a^{\frac{4}{3}} + 12b^{\frac{5}{3}}a^{\frac{1}{3}}x^3 + 2\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right) b^2x^4 - 4\log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) b}{12b^{\frac{2}{3}}a^{\frac{7}{3}}x^4}$$

input
```
int(1/x^5/((b*x^3+a)^2)^(1/2),x)
```

output
```
( - 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**2*x**4
 - 3*b**(2/3)*a**(1/3)*a + 12*b**(2/3)*a**(1/3)*b*x**3 + 2*log(a**(2/3) -
b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*x**4 - 4*log(a**(1/3) + b**(1/3)
*x)*b**2*x**4)/(12*b**(2/3)*a**(1/3)*a**2*x**4)
```

3.75 $\int \frac{x^{14}}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 196

$$\int \frac{x^{14}}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{4a^3}{3b^5\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^4}{6b^5\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{ax^3(a+bx^3)}{b^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^6(a+bx^3)}{6b^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{2a^2(a+bx^3)\log\left(a+bx^3\right)}{b^5\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
4/3*a^3/b^5/((b*x^3+a)^2)^(1/2)-1/6*a^4/b^5/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-
a*x^3*(b*x^3+a)/b^4/((b*x^3+a)^2)^(1/2)+1/6*x^6*(b*x^3+a)/b^3/((b*x^3+a)^2
)^(1/2)+2*a^2*(b*x^3+a)*ln(b*x^3+a)/b^5/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.47

$$\int \frac{x^{14}}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{7a^4+2a^3bx^3-11a^2b^2x^6-4ab^3x^9+b^4x^{12}+12a^2(a+bx^3)^2\log\left(a+bx^3\right)}{6b^5\left(a+bx^3\right)\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[x^14/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(7*a^4 + 2*a^3*b*x^3 - 11*a^2*b^2*x^6 - 4*a*b^3*x^9 + b^4*x^12 + 12*a^2*(a
 + b*x^3)^2*Log[a + b*x^3])/(6*b^5*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.54, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(a^2+2abx^3+b^2x^6)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^3)\int \frac{x^{14}}{b^3(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{x^{14}}{(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{(a+bx^3)\int \frac{x^{12}}{(bx^3+a)^3}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx^3)\int \left(\frac{a^4}{b^4(bx^3+a)^3}-\frac{4a^3}{b^4(bx^3+a)^2}+\frac{6a^2}{b^4(bx^3+a)}-\frac{3a}{b^4}+\frac{x^3}{b^3}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^3)\left(-\frac{a^4}{2b^5(a+bx^3)^2}+\frac{4a^3}{b^5(a+bx^3)}+\frac{6a^2\log(a+bx^3)}{b^5}-\frac{3ax^3}{b^4}+\frac{x^6}{2b^3}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^14/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((a + b*x^3)*((-3*a*x^3)/b^4 + x^6/(2*b^3) - a^4/(2*b^5*(a + b*x^3)^2) + (
4*a^3)/(b^5*(a + b*x^3)) + (6*a^2*Log[a + b*x^3])/b^5))/(3*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.39

method	result	s
pseudoelliptic	$\frac{2\,\mathrm{csgn}\left(b\,x^{3}+a\right)\left(a^{2}\ln\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{2}-\left(\frac{b\,x^{3}}{2}+a\right)\left(-\frac{1}{6}b^{2}x^{6}+a\,x^{3}b+a^{2}\right)b\,x^{3}\right)}{b^{5}\left(b\,x^{3}+a\right)^{2}}$	7
default	$\frac{\left(b^{4}x^{12}-4a\,b^{3}x^{9}+12\ln\left(b\,x^{3}+a\right)a^{2}b^{2}x^{6}-2b^{2}x^{6}a^{2}+24\ln\left(b\,x^{3}+a\right)a^{3}b\,x^{3}+20b\,x^{3}a^{3}+12\ln\left(b\,x^{3}+a\right)a^{4}+16a^{4}\right)\left(b\,x^{3}+a\right)}{6b^{5}\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$	1
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-b\,x^{3}+3a\right)^{2}}{6\left(b\,x^{3}+a\right)b^{5}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{4a^{3}x^{3}}{3}+\frac{7a^{4}}{6b}\right)}{\left(b\,x^{3}+a\right)^{3}b^{4}}+\frac{2\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}\ln\left(b\,x^{3}+a\right)}{\left(b\,x^{3}+a\right)b^{5}}$	1

input
```
int(x^14/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*csgn(b*x^3+a)*(a^2*ln(b*x^3+a)*(b*x^3+a)^2-(1/2*b*x^3+a)*(-1/6*b^2*x^6+a
*x^3*b+a^2)*b*x^3)/b^5/(b*x^3+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.53

$$\int \frac{x^{14}}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=\frac{b^4x^{12}-4\,ab^3x^9-11\,a^2b^2x^6+2\,a^3bx^3+7\,a^4+12\,\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)\mathrm{l}}{6\,\left(b^7x^6+2\,ab^6x^3+a^2b^5\right)}$$

input
```
integrate(x^14/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(b^4*x^12 - 4*a*b^3*x^9 - 11*a^2*b^2*x^6 + 2*a^3*b*x^3 + 7*a^4 + 12*(a
^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*log(b*x^3 + a))/(b^7*x^6 + 2*a*b^6*x^3 + a
^2*b^5)
```

Sympy [F]

$$\int \frac{x^{14}}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^{14}}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**14/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**14/((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.74

$$\int \frac{x^{14}}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{x^9}{6\sqrt{b^2x^6+2\,abx^3+a^2}b^2} - \frac{5\,ax^6}{6\sqrt{b^2x^6+2\,abx^3+a^2}b^3} + \frac{4\,a^3x^3}{\left(x^3+\frac{a}{b}\right)^2b^6} + \frac{2\,a^2\log\left(x^3+\frac{a}{b}\right)}{b^5} - \frac{5\,a^3}{3\sqrt{b^2x^6+2\,abx^3+a^2}b^5} + \frac{23\,a^4}{6\left(x^3+\frac{a}{b}\right)^2b^7}$$

input
```
integrate(x^14/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/6*x^9/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2) - 5/6*a*x^6/(sqrt(b^2*x^6 +
2*a*b*x^3 + a^2)*b^3) + 4*a^3*x^3/((x^3 + a/b)^2*b^6) + 2*a^2*log(x^3 + a/
b)/b^5 - 5/3*a^3/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5) + 23/6*a^4/((x^3 +
a/b)^2*b^7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.59

$$\int \frac{x^{14}}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{2\,a^2\log\left(|bx^3+a|\right)}{b^5\mathrm{sgn}\left(bx^3+a\right)} + \frac{b^3x^6\mathrm{sgn}(bx^3+a) - 6\,ab^2x^3\mathrm{sgn}(bx^3+a)}{6\,b^6} - \frac{18\,a^2b^2x^6+28\,a^3bx^3+11\,a^4}{6\,(bx^3+a)^2b^5\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(x^14/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
2*a^2*log(abs(b*x^3 + a))/(b^5*sgn(b*x^3 + a)) + 1/6*(b^3*x^6*sgn(b*x^3 +
a) - 6*a*b^2*x^3*sgn(b*x^3 + a))/b^6 - 1/6*(18*a^2*b^2*x^6 + 28*a^3*b*x^3
+ 11*a^4)/((b*x^3 + a)^2*b^5*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^{14}}{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}\,dx$$

input
```
int(x^14/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^14/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.03

$$\int \frac{x^{14}}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{12\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^4+24\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^3b\,x^3+12\,\mathrm{lo}}{}$$

input
```
int(x^14/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
(12*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**4 + 24*log(a**(
2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b*x**3 + 12*log(a**(2/3)
- b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**2*x**6 + 12*log(a**(1/3) +
b**(1/3)*x)*a**4 + 24*log(a**(1/3) + b**(1/3)*x)*a**3*b*x**3 + 12*log(a**(
1/3) + b**(1/3)*x)*a**2*b**2*x**6 + 6*a**4 - 12*a**2*b**2*x**6 - 4*a*b**3*
x**9 + b**4*x**12)/(6*b**5*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.76 $\int \frac{x^{11}}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 154

$$\begin{aligned}\int \frac{x^{11}}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx &= -\frac{a^2}{b^4\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{a^3}{6b^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{x^3(a+bx^3)}{3b^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a(a+bx^3)\log\left(a+bx^3\right)}{b^4\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
-a^2/b^4/((b*x^3+a)^2)^(1/2)+1/6*a^3/b^4/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+1/3
*x^3*(b*x^3+a)/b^3/((b*x^3+a)^2)^(1/2)-a*(b*x^3+a)*ln(b*x^3+a)/b^4/((b*x^3
+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.53

$$\int \frac{x^{11}}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{-5a^3-4a^2bx^3+4ab^2x^6+2b^3x^9-6a\left(a+bx^3\right)^2\log\left(a+bx^3\right)}{6b^4\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

input
```
Integrate[x^11/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(-5*a^3 - 4*a^2*b*x^3 + 4*a*b^2*x^6 + 2*b^3*x^9 - 6*a*(a + b*x^3)^2*Log[a
+ b*x^3])/(6*b^4*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^3\left(a+bx^3\right)\int \frac{x^{11}}{b^3\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(a+bx^3\right)\int \frac{x^{11}}{\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\left(a+bx^3\right)\int \frac{x^9}{\left(bx^3+a\right)^3}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\left(a+bx^3\right)\int \left(-\frac{a^3}{b^3\left(bx^3+a\right)^3}+\frac{3a^2}{b^3\left(bx^3+a\right)^2}-\frac{3a}{b^3\left(bx^3+a\right)}+\frac{1}{b^3}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(a+bx^3\right)\left(\frac{a^3}{2b^4\left(a+bx^3\right)^2}-\frac{3a^2}{b^4\left(a+bx^3\right)}-\frac{3a\log\left(a+bx^3\right)}{b^4}+\frac{x^3}{b^3}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^11/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((a + b*x^3)*(x^3/b^3 + a^3/(2*b^4*(a + b*x^3)^2) - (3*a^2)/(b^4*(a + b*x^
3)) - (3*a*Log[a + b*x^3])/b^4))/(3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.42

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(\ln\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{2}a-\frac{x^{9}b^{3}}{3}-a\,x^{6}b^{2}+\frac{a^{3}}{2}\right)}{b^{4}\left(b\,x^{3}+a\right)^{2}}$	65
default	$-\frac{\left(-2x^{9}b^{3}+6\ln\left(b\,x^{3}+a\right)a\,b^{2}x^{6}-4a\,x^{6}b^{2}+12\ln\left(b\,x^{3}+a\right)a^{2}b\,x^{3}+4a^{2}x^{3}b+6a^{3}\ln\left(b\,x^{3}+a\right)+5a^{3}\right)\left(b\,x^{3}+a\right)}{6b^{4}\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$	103
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,x^{3}}{3\left(b\,x^{3}+a\right)b^{3}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-a^{2}x^{3}-\frac{5a^{3}}{6b}\right)}{\left(b\,x^{3}+a\right)^{3}b^{3}}-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a\ln\left(b\,x^{3}+a\right)}{\left(b\,x^{3}+a\right)b^{4}}$	105

input
```
int(x^11/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-csgn(b*x^3+a)*(ln(b*x^3+a)*(b*x^3+a)^2*a-1/3*x^9*b^3-a*x^6*b^2+1/2*a^3)/b
^4/(b*x^3+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.59

$$\int \frac{x^{11}}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=\frac{2\,b^3x^9+4\,ab^2x^6-4\,a^2bx^3-5\,a^3-6\left(ab^2x^6+2\,a^2bx^3+a^3\right)\log\left(bx^3+a\right)}{6\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}$$

input
```
integrate(x^11/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^9 + 4*a*b^2*x^6 - 4*a^2*b*x^3 - 5*a^3 - 6*(a*b^2*x^6 + 2*a^2*
b*x^3 + a^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [F]

$$\int \frac{x^{11}}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{x^{11}}{\left((a + bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**11/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**11/((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.74

$$\int \frac{x^{11}}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \frac{x^6}{3\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^2} - \frac{2\,a^2x^3}{\left(x^3 + \frac{a}{b}\right)^2 b^5} - \frac{a\log\left(x^3 + \frac{a}{b}\right)}{b^4} + \frac{2\,a^2}{3\sqrt{b^2x^6 + 2\,abx^3 + a^2}b^4} - \frac{11\,a^3}{6\left(x^3 + \frac{a}{b}\right)^2 b^6}$$

input
```
integrate(x^11/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*x^6/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2) - 2*a^2*x^3/((x^3 + a/b)^2*b
^5) - a*log(x^3 + a/b)/b^4 + 2/3*a^2/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4)
 - 11/6*a^3/((x^3 + a/b)^2*b^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.60

$$\int \frac{x^{11}}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \frac{x^3}{3\,b^3\mathrm{sgn}\left(bx^3 + a\right)} - \frac{a\log\left(|bx^3 + a|\right)}{b^4\mathrm{sgn}\left(bx^3 + a\right)} + \frac{9\,ab^2x^6 + 12\,a^2bx^3 + 4\,a^3}{6\,(bx^3 + a)^2 b^4\mathrm{sgn}\left(bx^3 + a\right)}$$

input
```
integrate(x^11/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/3*x^3/(b^3*sgn(b*x^3 + a)) - a*log(abs(b*x^3 + a))/(b^4*sgn(b*x^3 + a))
+ 1/6*(9*a*b^2*x^6 + 12*a^2*b*x^3 + 4*a^3)/((b*x^3 + a)^2*b^4*sgn(b*x^3 +
a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{x^{11}}{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{3/2}}\,dx$$

input
```
int(x^11/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^11/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21

$$\int \frac{x^{11}}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \frac{-6\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)a^3 - 12\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)a^2b\,x^3 - 6\log}{}$$

input
```
int(x^11/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
( - 6*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3 - 12*log(a*
*(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b*x**3 - 6*log(a**(2/3)
 - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**2*x**6 - 6*log(a**(1/3) + b**
(1/3)*x)*a**3 - 12*log(a**(1/3) + b**(1/3)*x)*a**2*b*x**3 - 6*log(a**(1/3)
 + b**(1/3)*x)*a*b**2*x**6 - 3*a**3 + 6*a*b**2*x**6 + 2*b**3*x**9)/(6*b**4
*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.77 $\int \frac{x^8}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 116

$$\int \frac{x^8}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{2a}{3b^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^2}{6b^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3b^3\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
2/3*a/b^3/((b*x^3+a)^2)^(1/2)-1/6*a^2/b^3/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+1/
3*(b*x^3+a)*ln(b*x^3+a)/b^3/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.66

$$\int \frac{x^8}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\frac{bx^3\left(a\sqrt{\left(a+bx^3\right)^2}\left(-2a^2-abx^3+b^2x^6\right)+\sqrt{a^2}\left(2a^3+3a^2bx^3+b^3x^9\right)\right)}{a^2\left(a+bx^3\right)\left(a^2+abx^3-\sqrt{a^2}\sqrt{\left(a+bx^3\right)^2}\right)}+2\log\left(\sqrt{a^2}-bx^3-\sqrt{}\right.}{6b^3}$$

input
```
Integrate[x^8/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((b*x^3*(a*Sqrt[(a + b*x^3)^2]*(-2*a^2 - a*b*x^3 + b^2*x^6) + Sqrt[a^2]*(2
*a^3 + 3*a^2*b*x^3 + b^3*x^9)))/(a^2*(a + b*x^3)*(a^2 + a*b*x^3 - Sqrt[a^2
]*Sqrt[(a + b*x^3)^2])) + 2*Log[Sqrt[a^2] - b*x^3 - Sqrt[(a + b*x^3)^2]] -
 2*Log[b^3*(Sqrt[a^2] + b*x^3 - Sqrt[(a + b*x^3)^2])])/(6*b^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^3)\int \frac{x^8}{b^3(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{x^8}{(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{(a+bx^3)\int \frac{x^6}{(bx^3+a)^3}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx^3)\int \left(\frac{a^2}{b^2(bx^3+a)^3}-\frac{2a}{b^2(bx^3+a)^2}+\frac{1}{b^2(bx^3+a)}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^3)\left(-\frac{a^2}{2b^3(a+bx^3)^2}+\frac{2a}{b^3(a+bx^3)}+\frac{\log(a+bx^3)}{b^3}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^8/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((a + b*x^3)*(-1/2*a^2/(b^3*(a + b*x^3)^2) + (2*a)/(b^3*(a + b*x^3)) + Log
[a + b*x^3]/b^3))/(3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(\ln\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{2}+2 a\,x^{3} b+\frac{3 a^{2}}{2}\right)}{3 b^{3}\left(b\,x^{3}+a\right)^{2}}$	54
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{2 a\,x^{3}}{3 b^{2}}+\frac{a^{2}}{2 b^{3}}\right)}{\left(b\,x^{3}+a\right)^{3}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(b\,x^{3}+a\right)}{3\left(b\,x^{3}+a\right) b^{3}}$	74
default	$\frac{\left(2 b^{2} \ln\left(b\,x^{3}+a\right) x^{6}+4 \ln\left(b\,x^{3}+a\right) a b\,x^{3}+4 a\,x^{3} b+2 a^{2} \ln\left(b\,x^{3}+a\right)+3 a^{2}\right)\left(b\,x^{3}+a\right)}{6 b^{3}\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$	81

input
```
int(x^8/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*csgn(b*x^3+a)*(ln(b*x^3+a)*(b*x^3+a)^2+2*a*x^3*b+3/2*a^2)/b^3/(b*x^3+a
)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59

$$\int \frac{x^8}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\frac{4\,abx^3+3\,a^2+2\,(b^2x^6+2\,abx^3+a^2)\log\left(bx^3+a\right)}{6\,(b^5x^6+2\,ab^4x^3+a^2b^3)}$$

input
```
integrate(x^8/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(4*a*b*x^3 + 3*a^2 + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3 + a))/(b^
5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

Sympy [F]

$$\int \frac{x^8}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\int \frac{x^8}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**8/((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

$$\int \frac{x^8}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\frac{2\,ax^3}{3\left(x^3+\frac{a}{b}\right)^2b^4}+\frac{\log\left(x^3+\frac{a}{b}\right)}{3\,b^3}+\frac{a^2}{2\left(x^3+\frac{a}{b}\right)^2b^5}$$

input
```
integrate(x^8/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
2/3*a*x^3/((x^3 + a/b)^2*b^4) + 1/3*log(x^3 + a/b)/b^3 + 1/2*a^2/((x^3 + a
/b)^2*b^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.53

$$\int \frac{x^8}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\frac{\log\left(|bx^3+a|\right)}{3\,b^3\mathrm{sgn}\left(bx^3+a\right)}-\frac{3\,bx^6+2\,ax^3}{6\,(bx^3+a)^2b^2\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(x^8/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/3*log(abs(b*x^3 + a))/(b^3*sgn(b*x^3 + a)) - 1/6*(3*b*x^6 + 2*a*x^3)/((b
*x^3 + a)^2*b^2*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\int \frac{x^8}{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}\,dx$$

input
```
int(x^8/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^8/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.47

$$\int \frac{x^8}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx=\frac{2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2+4\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)ab\,x^3+2\log\left(a\right.}{}$$

input
```
int(x^8/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
(2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 + 4*log(a**(2/
3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b*x**3 + 2*log(a**(2/3) - b**(
1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*x**6 + 2*log(a**(1/3) + b**(1/3)*x)*
a**2 + 4*log(a**(1/3) + b**(1/3)*x)*a*b*x**3 + 2*log(a**(1/3) + b**(1/3)*x
)*b**2*x**6 + a**2 - 2*b**2*x**6)/(6*b**3*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.78 $\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=\frac{x^6}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/6*x^6/a/(b*x^3+a)/((b*x^3+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 119 vs. 2(41) = 82.

Time = 0.50 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.90

$$\int \frac{x^5}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = -\frac{x^6\left(a^3+ab^2x^6-a\sqrt{a^2}\sqrt{\left(a+bx^3\right)^2}+\sqrt{a^2}bx^3\sqrt{\left(a+bx^3\right)^2}\right)}{6a^3\left(a+bx^3\right)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{\left(a+bx^3\right)^2}\right)\right)}$$

input
```
Integrate[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
-1/6*(x^6*(a^3 + a*b^2*x^6 - a*Sqrt[a^2]*Sqrt[(a + b*x^3)^2] + Sqrt[a^2]*b
*x^3*Sqrt[(a + b*x^3)^2]))/(a^3*(a + b*x^3)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2
] - Sqrt[(a + b*x^3)^2])))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1693, 1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^3}{(b^2x^6+2abx^3+a^2)^{3/2}}dx^3$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{3}\left(-\frac{a\int \frac{1}{(b^2x^6+2abx^3+a^2)^{3/2}}dx^3}{b}-\frac{1}{b^2\sqrt{a^2+2abx^3+b^2x^6}}\right)$$

$$\Big\downarrow \text{1078}$$

$$\frac{1}{3}\left(\frac{a}{2b^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{b^2\sqrt{a^2+2abx^3+b^2x^6}}\right)$$

input
```
Int[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(-(1/(b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + a/(2*b^2*(a + b*x^3)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]))/3
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$-\frac{\left(2 b \,x^{3}+a \right) \operatorname{csgn}\left(b \,x^{3}+a \right)}{6 b^{2} \left(b \,x^{3}+a \right)^{2}}$	31
gosper	$-\frac{\left(b \,x^{3}+a \right) \left(2 b \,x^{3}+a \right)}{6 b^{2} \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}$	32
default	$-\frac{\left(b \,x^{3}+a \right) \left(2 b \,x^{3}+a \right)}{6 b^{2} \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}$	32
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-\frac{x^{3}}{3 b}-\frac{a}{6 b^{2}}\right)}{\left(b \,x^{3}+a \right)^{3}}$	37
orering	$-\frac{\left(2 b \,x^{3}+a \right) \left(b \,x^{3}+a \right)}{6 b^{2} \left(b^{2} x^{6}+2 a \,x^{3} b +a^{2}\right)^{\frac{3}{2}}}$	41

input
```
int(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*b*x^3+a)*csgn(b*x^3+a)/b^2/(b*x^3+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{2\,bx^3+a}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

input
```
integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Sympy [F]

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^5}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**5/((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{1}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}b^2} + \frac{a}{6\,\left(x^3+\frac{a}{b}\right)^2b^4}$$

input
```
integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2) + 1/6*a/((x^3 + a/b)^2*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{2\,bx^3+a}{6\,(bx^3+a)^2b^2\mathrm{sgn}\,(bx^3+a)}$$

input
```
integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/6*(2*b*x^3 + a)/((b*x^3 + a)^2*b^2*sgn(b*x^3 + a))
```

Mupad [B] (verification not implemented)

Time = 20.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{(2\,b\,x^3+a)\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,b^2\,(b\,x^3+a)^3}$$

input
```
int(x^5/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
-((a + 2*b*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*b^2*(a + b*x^3)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{x^6}{6a\,(b^2x^6+2ab\,x^3+a^2)}$$

input
```
int(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
x**6/(6*a*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.79 $\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \, dx=-\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
-1/6/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(38) = 76.

Time = 0.55 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.76

$$\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}} \, dx = -\frac{x^3\left(2a^4+a^3bx^3-ab^3x^9+a\sqrt{a^2}bx^3\sqrt{\left(a+bx^3\right)^2}-\sqrt{a^2}\sqrt{\left(a+bx^3\right)^2}\left(2a^2+b^2x^6\right)\right)}{6a^4\left(a+bx^3\right)\left(\sqrt{a^2}bx^3+a\left(\sqrt{a^2}-\sqrt{\left(a+bx^3\right)^2}\right)\right)}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
-1/6*(x^3*(2*a^4 + a^3*b*x^3 - a*b^3*x^9 + a*Sqrt[a^2]*b*x^3*Sqrt[(a + b*x
^3)^2] - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]*(2*a^2 + b^2*x^6)))/(a^4*(a + b*x^3
)*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^2])))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1690, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$$

$$\Big\downarrow \; 1690$$

$$\frac{1}{3}\int \frac{1}{(b^2x^6+2abx^3+a^2)^{3/2}}dx^3$$

$$\Big\downarrow \; 1078$$

$$-\frac{1}{6b\,(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
-1/6*1/(b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)}{6\left(b\,x^{3}+a\right)^{2} b}$	23
gosper	$-\frac{b\,x^{3}+a}{6 b \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$	24
default	$-\frac{b\,x^{3}+a}{6 b \left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$	24
risch	$-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}}{6\left(b\,x^{3}+a\right)^{3} b}$	26
orering	$-\frac{b\,x^{3}+a}{6 b \left(b^{2} x^{6}+2 a\,x^{3} b+a^{2}\right)^{\frac{3}{2}}}$	33

input
```
int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(b*x^3+a)^2/b*csgn(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{1}{6\,(b^3x^6+2\,ab^2x^3+a^2b)}$$

input
```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/6/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
```

Sympy [F]

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^2}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**2/((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.42

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{1}{6\left(x^3+\frac{a}{b}\right)^2b^3}$$

input
```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/6/((x^3 + a/b)^2*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{1}{6\,(bx^3+a)^2 b\mathrm{sgn}\,(bx^3+a)}$$

input
```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/6/((b*x^3 + a)^2*b*sgn(b*x^3 + a))
```

Mupad [B] (verification not implemented)

Time = 20.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,b\,(b\,x^3+a)^3}$$

input
```
int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
-(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(6*b*(a + b*x^3)^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{1}{6b\,(b^2x^6+2ab\,x^3+a^2)}$$

input
```
int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
( - 1)/(6*b*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.80 $\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(a+bx^3\right)\log(x)}{a^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/3/a^2/((b*x^3+a)^2)^(1/2)+1/6/a/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+(b*x^3+a)*
ln(x)/a^3/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a^3/((b*x^3+a)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.50

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{a(3a+2bx^3)+6(a+bx^3)^2\log(x)-2(a+bx^3)^2\log\left(a+bx^3\right)}{6a^3\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
(a*(3*a + 2*b*x^3) + 6*(a + b*x^3)^2*Log[x] - 2*(a + b*x^3)^2*Log[a + b*x^
3])/(6*a^3*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.57, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3\left(a+bx^3\right)\int \frac{1}{b^3x\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^3\left(bx^3+a\right)^3}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{54}$$

$$\frac{\left(a+bx^3\right)\int \left(-\frac{b}{a^3\left(bx^3+a\right)}-\frac{b}{a^2\left(bx^3+a\right)^2}-\frac{b}{a\left(bx^3+a\right)^3}+\frac{1}{a^3x^3}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a+bx^3\right)\left(-\frac{\log\left(a+bx^3\right)}{a^3}+\frac{\log\left(x^3\right)}{a^3}+\frac{1}{a^2\left(a+bx^3\right)}+\frac{1}{2a\left(a+bx^3\right)^2}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
((a + b*x^3)*(1/(2*a*(a + b*x^3)^2) + 1/(a^2*(a + b*x^3)) + Log[x^3]/a^3 -
 Log[a + b*x^3]/a^3))/(3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.48

method	result
pseudoelliptic	$\frac{\left(-\ln\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{2}+\ln\left(b\,x^{3}\right)\left(b\,x^{3}+a\right)^{2}+a\,x^{3}b+\frac{3a^{2}}{2}\right)\operatorname{csgn}\left(b\,x^{3}+a\right)}{3\left(b\,x^{3}+a\right)^{2}a^{3}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{b\,x^{3}}{3a^{2}}+\frac{1}{2a}\right)}{\left(b\,x^{3}+a\right)^{3}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(x\right)}{\left(b\,x^{3}+a\right)a^{3}}-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(b\,x^{3}+a\right)}{3\left(b\,x^{3}+a\right)a^{3}}$
default	$-\frac{\left(2b^{2}\ln\left(b\,x^{3}+a\right)x^{6}-6b^{2}\ln\left(x\right)x^{6}+4\ln\left(b\,x^{3}+a\right)ab\,x^{3}-12\ln\left(x\right)ab\,x^{3}-2a\,x^{3}b+2a^{2}\ln\left(b\,x^{3}+a\right)-6\ln\left(x\right)a^{2}-3a^{2}\right)\left(b\,x^{3}+a\right)}{6a^{3}\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$

input
```
int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-ln(b*x^3+a)*(b*x^3+a)^2+ln(b*x^3)*(b*x^3+a)^2+a*x^3*b+3/2*a^2)*csgn(
b*x^3+a)/(b*x^3+a)^2/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=\frac{2\,abx^3+3\,a^2-2\,(b^2x^6+2\,abx^3+a^2)\log\left(bx^3+a\right)+6\,(b^2x^6+2\,abx^3+a}{6\,(a^3b^2x^6+2\,a^4bx^3+a^5)}$$

input
```
integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(2*a*b*x^3 + 3*a^2 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3 + a) + 6*
(b^2*x^6 + 2*a*b*x^3 + a^2)*log(x))/(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)
```

Sympy [F]

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=\int \frac{1}{x\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(1/(x*((a + b*x**3)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = -\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^3} + \frac{1}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}a^2} + \frac{1}{6\left(x^3+\frac{a}{b}\right)^2ab^2}$$

input
```
integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^3
 + 1/3/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2) + 1/6/((x^3 + a/b)^2*a*b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.59

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = -\frac{\log\left(|bx^3+a|\right)}{3\,a^3\mathrm{sgn}\left(bx^3+a\right)} + \frac{\log\left(|x|\right)}{a^3\mathrm{sgn}\left(bx^3+a\right)} + \frac{3\,b^2x^6+8\,abx^3+6\,a^2}{6\left(bx^3+a\right)^2a^3\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/3*log(abs(b*x^3 + a))/(a^3*sgn(b*x^3 + a)) + log(abs(x))/(a^3*sgn(b*x^3
 + a)) + 1/6*(3*b^2*x^6 + 8*a*b*x^3 + 6*a^2)/((b*x^3 + a)^2*a^3*sgn(b*x^3
+ a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{1}{x\,(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}\,dx$$

input
```
int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

output
```
int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.35

$$\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{-2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2-4\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)ab\,x^3-2\,\mathrm{lo}}{}$$

input
```
int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
( - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 - 4*log(a**
(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b*x**3 - 2*log(a**(2/3) - b
**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*x**6 - 2*log(a**(1/3) + b**(1/3)*
x)*a**2 - 4*log(a**(1/3) + b**(1/3)*x)*a*b*x**3 - 2*log(a**(1/3) + b**(1/3
)*x)*b**2*x**6 + 6*log(x)*a**2 + 12*log(x)*a*b*x**3 + 6*log(x)*b**2*x**6 +
 2*a**2 - b**2*x**6)/(6*a**3*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.81 $\int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 188

$$\begin{aligned}\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx &= -\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}\\ &\quad -\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}\\ &\quad -\frac{3b(a+bx^3)\log(x)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b(a+bx^3)\log\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
-2/3*b/a^3/((b*x^3+a)^2)^(1/2)-1/6*b/a^2/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-1/3
*(b*x^3+a)/a^3/x^3/((b*x^3+a)^2)^(1/2)-3*b*(b*x^3+a)*ln(x)/a^4/((b*x^3+a)^
2)^(1/2)+b*(b*x^3+a)*ln(b*x^3+a)/a^4/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{-a(2a^2+9abx^3+6b^2x^6)-18bx^3(a+bx^3)^2\log(x)+6bx^3(a+bx^3)^2\log}{6a^4x^3\left(a+bx^3\right)\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
(-(a*(2*a^2 + 9*a*b*x^3 + 6*b^2*x^6)) - 18*b*x^3*(a + b*x^3)^2*Log[x] + 6*
b*x^3*(a + b*x^3)^2*Log[a + b*x^3])/(6*a^4*x^3*(a + b*x^3)*Sqrt[(a + b*x^3
)^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3 \left(a+bx^3\right) \int \frac{1}{b^3x^4 \left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right) \int \frac{1}{x^4 \left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\left(a+bx^3\right) \int \frac{1}{x^6 \left(bx^3+a\right)^3}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{54}$$

$$\frac{\left(a+bx^3\right) \int \left(\frac{3b^2}{a^4 \left(bx^3+a\right)}+\frac{2b^2}{a^3 \left(bx^3+a\right)^2}+\frac{b^2}{a^2 \left(bx^3+a\right)^3}-\frac{3b}{a^4x^3}+\frac{1}{a^3x^6}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a+bx^3\right) \left(-\frac{3b\log \left(x^3\right)}{a^4}+\frac{3b\log \left(a+bx^3\right)}{a^4}-\frac{2b}{a^3 \left(a+bx^3\right)}-\frac{1}{a^3x^3}-\frac{b}{2a^2 \left(a+bx^3\right)^2}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
((a + b*x^3)*(-(1/(a^3*x^3)) - b/(2*a^2*(a + b*x^3)^2) - (2*b)/(a^3*(a + b
*x^3)) - (3*b*Log[x^3])/a^4 + (3*b*Log[a + b*x^3])/a^4))/(3*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.49

method	result
pseudoelliptic	$-\frac{\left(-3b\,x^{3}\left(b\,x^{3}+a\right)^{2}\ln\left(b\,x^{3}+a\right)+3b\,x^{3}\left(b\,x^{3}+a\right)^{2}\ln\left(b\,x^{3}\right)+a\left(3b^{2}x^{6}+\frac{9}{2}a\,x^{3}b+a^{2}\right)\right)\operatorname{csgn}\left(b\,x^{3}+a\right)}{3\left(b\,x^{3}+a\right)^{2}a^{4}x^{3}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\left(-\frac{b^{2}x^{6}}{a^{3}}-\frac{3b\,x^{3}}{2a^{2}}-\frac{1}{3a}\right)}{\left(b\,x^{3}+a\right)^{3}x^{3}}-\frac{3\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\ln\left(x\right)}{\left(b\,x^{3}+a\right)a^{4}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\ln\left(-b\,x^{3}-a\right)}{\left(b\,x^{3}+a\right)a^{4}}$
default	$\frac{(6b^{3}\ln\left(b\,x^{3}+a\right)x^{9}-18b^{3}\ln\left(x\right)x^{9}+12\ln\left(b\,x^{3}+a\right)a\,b^{2}x^{6}-36b^{2}a\ln\left(x\right)x^{6}-6a\,x^{6}b^{2}+6\ln\left(b\,x^{3}+a\right)a^{2}b\,x^{3}-18b\,a^{2}\ln\left(x\right)x^{3}-9a^{2}}{6x^{3}a^{4}\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$

input
```
int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*b*x^3*(b*x^3+a)^2*ln(b*x^3+a)+3*b*x^3*(b*x^3+a)^2*ln(b*x^3)+a*(3*
b^2*x^6+9/2*a*x^3*b+a^2))*csgn(b*x^3+a)/(b*x^3+a)^2/a^4/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx =$$
$$-\frac{6\,ab^{2}x^{6}+9\,a^{2}bx^{3}+2\,a^{3}-6\left(b^{3}x^{9}+2\,ab^{2}x^{6}+a^{2}bx^{3}\right)\log\left(bx^{3}+a\right)+18\left(b^{3}x^{9}+2\,ab^{2}x^{6}+a^{2}bx^{3}\right)\log\left(x\right)}{6\left(a^{4}b^{2}x^{9}+2\,a^{5}bx^{6}+a^{6}x^{3}\right)}$$

input
```
integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(6*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3 - 6*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b
*x^3)*log(b*x^3 + a) + 18*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*x^3)*log(x))/(a^4
*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \int \frac{1}{x^4 \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(1/(x**4*((a + b*x**3)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{a^4} - \frac{b}{\sqrt{b^2x^6+2\,abx^3+a^2}a^3} - \frac{1}{6\left(x^3+\frac{a}{b}\right)^2 a^2 b} - \frac{1}{3\sqrt{b^2x^6+2\,abx^3+a^2}a^2x^3}$$

input
```
integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^4 -
b/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3) - 1/6/((x^3 + a/b)^2*a^2*b) - 1/3/
(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{b\log\left(|bx^3+a|\right)}{a^4\mathrm{sgn}\left(bx^3+a\right)} - \frac{3\,b\log\left(|x|\right)}{a^4\mathrm{sgn}\left(bx^3+a\right)} - \frac{9\,b^3x^6+22\,ab^2x^3+14\,a^2b}{6\left(bx^3+a\right)^2a^4\mathrm{sgn}\left(bx^3+a\right)} + \frac{3\,bx^3-a}{3\,a^4x^3\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
b*log(abs(b*x^3 + a))/(a^4*sgn(b*x^3 + a)) - 3*b*log(abs(x))/(a^4*sgn(b*x^
3 + a)) - 1/6*(9*b^3*x^6 + 22*a*b^2*x^3 + 14*a^2*b)/((b*x^3 + a)^2*a^4*sgn
(b*x^3 + a)) + 1/3*(3*b*x^3 - a)/(a^4*x^3*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{1}{x^4 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

output
```
int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \frac{6\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)a^2b\,x^3 + 12\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)a\,b^2x^6 +}{}$$

input
```
int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
(6*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b*x**3 + 12*lo
g(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**2*x**6 + 6*log(a**(
2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**3*x**9 + 6*log(a**(1/3) + b
**(1/3)*x)*a**2*b*x**3 + 12*log(a**(1/3) + b**(1/3)*x)*a*b**2*x**6 + 6*log
(a**(1/3) + b**(1/3)*x)*b**3*x**9 - 18*log(x)*a**2*b*x**3 - 36*log(x)*a*b*
*2*x**6 - 18*log(x)*b**3*x**9 - 2*a**3 - 6*a**2*b*x**3 + 3*b**3*x**9)/(6*a
**4*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.82 $\int \frac{1}{x^7(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 231

$$\int \frac{1}{x^7\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{b^2}{a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^2}{6a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a+bx^3}{6a^3x^6\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b(a+bx^3)}{a^4x^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{6b^2(a+bx^3)\log(x)}{a^5\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2b^2(a+bx^3)\log\left(a+bx^3\right)}{a^5\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
b^2/a^4/((b*x^3+a)^2)^(1/2)+1/6*b^2/a^3/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-1/6*
(b*x^3+a)/a^3/x^6/((b*x^3+a)^2)^(1/2)+b*(b*x^3+a)/a^4/x^3/((b*x^3+a)^2)^(1
/2)+6*b^2*(b*x^3+a)*ln(x)/a^5/((b*x^3+a)^2)^(1/2)-2*b^2*(b*x^3+a)*ln(b*x^3
+a)/a^5/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^7 (a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{a(-a^3+4a^2bx^3+18ab^2x^6+12b^3x^9)+36b^2x^6(a+bx^3)^2\log(x)-12b^2x}{6a^5x^6(a+bx^3)\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[1/(x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
(a*(-a^3 + 4*a^2*b*x^3 + 18*a*b^2*x^6 + 12*b^3*x^9) + 36*b^2*x^6*(a + b*x^
3)^2*Log[x] - 12*b^2*x^6*(a + b*x^3)^2*Log[a + b*x^3])/(6*a^5*x^6*(a + b*x
^3)*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.51, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a^2+2abx^3+b^2x^6)^{3/2}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b^3(a+bx^3)\int \frac{1}{b^3x^7(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^3)\int \frac{1}{x^7(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 798$$

$$\frac{(a+bx^3)\int \frac{1}{x^9(bx^3+a)^3}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx^3)\int \left(-\frac{6b^3}{a^5(bx^3+a)}-\frac{3b^3}{a^4(bx^3+a)^2}-\frac{b^3}{a^3(bx^3+a)^3}+\frac{6b^2}{a^5x^3}-\frac{3b}{a^4x^6}+\frac{1}{a^3x^9}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^3)\left(\frac{6b^2\log(x^3)}{a^5}-\frac{6b^2\log(a+bx^3)}{a^5}+\frac{3b^2}{a^4(a+bx^3)}+\frac{3b}{a^4x^3}+\frac{b^2}{2a^3(a+bx^3)^2}-\frac{1}{2a^3x^6}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
((a + b*x^3)*(-1/2*1/(a^3*x^6) + (3*b)/(a^4*x^3) + b^2/(2*a^3*(a + b*x^3)^
2) + (3*b^2)/(a^4*(a + b*x^3)) + (6*b^2*Log[x^3])/a^5 - (6*b^2*Log[a + b*x
^3])/a^5))/(3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.45

method	result
pseudoelliptic	$-\frac{\left(12b^{2}x^{6}\left(b\,x^{3}+a\right)^{2}\ln\left(b\,x^{3}+a\right)-12b^{2}x^{6}\left(b\,x^{3}+a\right)^{2}\ln\left(b\,x^{3}\right)+a\left(2b\,x^{3}+a\right)\left(-6b^{2}x^{6}-6a\,x^{3}b+a^{2}\right)\right)\operatorname{csgn}\left(b\,x^{3}+a\right)}{6\left(b\,x^{3}+a\right)^{2}a^{5}x^{6}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{2b^{3}x^{9}}{a^{4}}+\frac{3b^{2}x^{6}}{a^{3}}+\frac{2b\,x^{3}}{3a^{2}}-\frac{1}{6a}\right)}{\left(b\,x^{3}+a\right)^{3}x^{6}}+\frac{6\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{2}\ln\left(x\right)}{\left(b\,x^{3}+a\right)a^{5}}-\frac{2\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{2}\ln\left(b\,x^{3}+a\right)}{\left(b\,x^{3}+a\right)a^{5}}$
default	$-\frac{(12\ln\left(b\,x^{3}+a\right)b^{4}x^{12}-36\ln\left(x\right)b^{4}x^{12}+24\ln\left(b\,x^{3}+a\right)a\,b^{3}x^{9}-72\ln\left(x\right)a\,b^{3}x^{9}-12a\,b^{3}x^{9}+12\ln\left(b\,x^{3}+a\right)a^{2}b^{2}x^{6}-36\ln\left(x\right)a^{2}}{6x^{6}a^{5}\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}$

input
```
int(1/x^7/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(12*b^2*x^6*(b*x^3+a)^2*ln(b*x^3+a)-12*b^2*x^6*(b*x^3+a)^2*ln(b*x^3)+
a*(2*b*x^3+a)*(-6*b^2*x^6-6*a*b*x^3+a^2))*csgn(b*x^3+a)/(b*x^3+a)^2/a^5/x^
6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{7}\left(a^{2}+2abx^{3}+b^{2}x^{6}\right)^{3/2}}\,dx=\frac{12\,ab^{3}x^{9}+18\,a^{2}b^{2}x^{6}+4\,a^{3}bx^{3}-a^{4}-12\left(b^{4}x^{12}+2\,ab^{3}x^{9}+a^{2}b^{2}x^{6}\right)\log\left(\right.}{6\left(a^{5}b^{2}x^{12}+2\,a^{6}bx^{9}+a^{7}x\right.}$$

input
```
integrate(1/x^7/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(12*a*b^3*x^9 + 18*a^2*b^2*x^6 + 4*a^3*b*x^3 - a^4 - 12*(b^4*x^12 + 2*
a*b^3*x^9 + a^2*b^2*x^6)*log(b*x^3 + a) + 36*(b^4*x^12 + 2*a*b^3*x^9 + a^2
*b^2*x^6)*log(x))/(a^5*b^2*x^12 + 2*a^6*b*x^9 + a^7*x^6)
```

Sympy [F]

$$\int \frac{1}{x^7 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{1}{x^7 \left((a + bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**7/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(1/(x**7*((a + b*x**3)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^7 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = -\frac{2\,(-1)^{2\,abx^3+2\,a^2}\,b^2\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right)}{a^5} + \frac{2\,b^2}{\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^4} + \frac{1}{6\,\left(x^3 + \frac{a}{b}\right)^2 a^3} + \frac{5\,b}{6\,\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^3x^3} - \frac{1}{6\,\sqrt{b^2x^6 + 2\,abx^3 + a^2}a^2x^6}$$

input
```
integrate(1/x^7/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-2*(-1)^(2*a*b*x^3 + 2*a^2)*b^2*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a
^5 + 2*b^2/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^4) + 1/6/((x^3 + a/b)^2*a^3)
 + 5/6*b/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*x^3) - 1/6/(sqrt(b^2*x^6 + 2
*a*b*x^3 + a^2)*a^2*x^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^7 (a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{2\,b^2\log\left(|bx^3+a|\right)}{a^5\mathrm{sgn}\left(bx^3+a\right)} + \frac{6\,b^2\log\left(|x|\right)}{a^5\mathrm{sgn}\left(bx^3+a\right)} + \frac{12\,b^3x^9+18\,ab^2x^6+4\,a^2bx^3-a^3}{6\,(bx^6+ax^3)^2a^4\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/x^7/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-2*b^2*log(abs(b*x^3 + a))/(a^5*sgn(b*x^3 + a)) + 6*b^2*log(abs(x))/(a^5*s
gn(b*x^3 + a)) + 1/6*(12*b^3*x^9 + 18*a*b^2*x^6 + 4*a^2*b*x^3 - a^3)/((b*x
^6 + a*x^3)^2*a^4*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{1}{x^7\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}\,dx$$

input
```
int(1/(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

output
```
int(1/(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^7 (a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{-12\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2b^2x^6-24\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a\,b^3}{}$$

input
```
int(1/x^7/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
( - 12*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**2*x**6
- 24*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**3*x**9 - 12*
log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**4*x**12 - 12*log(a*
*(1/3) + b**(1/3)*x)*a**2*b**2*x**6 - 24*log(a**(1/3) + b**(1/3)*x)*a*b**3
*x**9 - 12*log(a**(1/3) + b**(1/3)*x)*b**4*x**12 + 36*log(x)*a**2*b**2*x**
6 + 72*log(x)*a*b**3*x**9 + 36*log(x)*b**4*x**12 - a**4 + 4*a**3*b*x**3 +
12*a**2*b**2*x**6 - 6*b**4*x**12)/(6*a**5*x**6*(a**2 + 2*a*b*x**3 + b**2*x
**6))
```

3.83 $\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 275

$$\begin{aligned}\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx &= -\frac{2x}{9b^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{x^4}{6b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{2(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
-2/9*x/b^2/((b*x^3+a)^2)^(1/2)-1/6*x^4/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-2/2
7*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(2
/3)/b^(7/3)/((b*x^3+a)^2)^(1/2)+2/27*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/
3)/b^(7/3)/((b*x^3+a)^2)^(1/2)-1/27*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/a^(2/3)/b^(7/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.85

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{-12a^{5/3}\sqrt[3]{b}x - 21a^{2/3}b^{4/3}x^4 - 4\sqrt{3}(a+bx^3)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4(a+}{}$$

input
```
Integrate[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(-12*a^(5/3)*b^(1/3)*x - 21*a^(2/3)*b^(4/3)*x^4 - 4*Sqrt[3]*(a + b*x^3)^2*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*(a + b*x^3)^2*Log[a^(1/3)
+ b^(1/3)*x] - 2*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 4*a*
b*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b^2*x^6*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(2/3)*b^(7/3)*(a + b*x^3)*Sqr
t[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.70, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1384, 27, 817, 817, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^3)\int \frac{x^6}{b^3(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right)\int \frac{x^6}{\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 817$$

$$\frac{\left(a+bx^3\right)\left(\frac{2\int \frac{x^3}{\left(bx^3+a\right)^2}dx}{3b}-\frac{x^4}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 817$$

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\int \frac{1}{bx^3+a}dx}{3b}-\frac{x}{3b\left(a+bx^3\right)}\right)}{3b}-\frac{x^4}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 750$$

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3b}-\frac{x}{3b\left(a+bx^3\right)}\right)}{3b}-\frac{x^4}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 16$$

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b\left(a+bx^3\right)}\right)}{3b}-\frac{x^4}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 1142$$

$$\frac{(a+bx^3)\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b\left(a+bx^3\right)}\right)}{3b}-\frac{x^4}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 25$$

$$\frac{(a+bx^3)\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b\left(a+bx^3\right)}\right)}{3b}-\frac{x^4}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx^3)\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b\left(a+bx^3\right)}\right)}{3b}-\frac{x^4}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1082

$$\frac{(a+bx^3)\left(\frac{2\left(\frac{\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b\left(a+bx^3\right)}\right)}{3b}-\frac{x^4}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 217

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b\left(a+bx^3\right)}\right)}{3b}-\frac{x^4}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1103$$

$$\frac{(a+bx^3)\left(\frac{2\left(\frac{\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3b}-\frac{x^4}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((a + b*x^3)*(-1/6*x^4/(b*(a + b*x^3)^2) + (2*(-1/3*x/(b*(a + b*x^3)) + (L
og[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3)))/(3*b)))/(3*b)))/Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.98 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{7 x^{4}}{18 b}-\frac{2 a x}{9 b^{2}}\right)}{\left(b\,x^{3}+a\right)^{3}}+\frac{2 \sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{27 \left(b\,x^{3}+a\right) b^{3}}$
default	$-\frac{\left(4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) b^{2}x^{6}-4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) b^{2}x^{6}+2\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) b^{2}x^{6}+21\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}x^{4}+8\sqrt{3}\,\arctan\left(\right.\right.}{}$

input
```
int(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(-7/18/b*x^4-2/9*a/b^2*x)+2/27*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)/b^3*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.83

$$\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \left[-\frac{21\,a^2b^2x^4+12\,a^3bx-6\,\sqrt{\frac{1}{3}}(ab^3x^6+2\,a^2b^2x^3+a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,ab\cdots}{\cdots}\right.\cdots}{\cdots}\right.$$

$$-\frac{21\,a^2b^2x^4+12\,a^3bx-12\,\sqrt{\frac{1}{3}}(ab^3x^6+2\,a^2b^2x^3+a^3b)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)+\cdots}{54\,(a^2b^5x^6\cdots}$$

input
```
integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/54*(21*a^2*b^2*x^4 + 12*a^3*b*x - 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x
^3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x -
a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a
^2*b)^(1/3)/b))/(b*x^3 + a)) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)
*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*(b^2*x^6 + 2*a*b*x^3
 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^5*x^6 + 2*a^3*b^4
*x^3 + a^4*b^3), -1/54*(21*a^2*b^2*x^4 + 12*a^3*b*x - 12*sqrt(1/3)*(a*b^3*
x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^
2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/
3)*a) - 4*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2
/3)))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [F]

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^6}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**6/((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.50

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{7\,bx^4+4\,ax}{18\,(b^4x^6+2\,ab^3x^3+a^2b^2)} + \frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/18*(7*b*x^4 + 4*a*x)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2) + 2/27*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 1/2
7*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 2/27*log(x +
(a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.64

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,b\text{sgn}\,(bx^3+a)} - \frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^2\text{sgn}\,(bx^3+a)} + \frac{2\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\text{sgn}\,(bx^3+a)} - \frac{7\,bx^4+4\,ax}{18\,(bx^3+a)^2b^2\text{sgn}\,(bx^3+a)}$$

input
```
integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/27*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b*sgn(b*x^3
 + a)) - 2/27*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*sgn(b*x^3 + a
)) + 2/27*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/(a*b^3*sgn(b*x^3 + a)) - 1/18*(7*b*x^4 + 4*a*x)/((b*x^3 + a)^
2*b^2*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^6}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

input
```
int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.01

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{-4a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-8a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3-4a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-}{a^{\frac{1}{3}}}\right.}{}$$

input
```
int(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
( - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a**2 - 8*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)
))*a*b*x**3 - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*
sqrt(3)))*b**2*x**6 - 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(
2/3)*x**2)*a**2 - 4*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)
*x**2)*a*b*x**3 - 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)
*x**2)*b**2*x**6 + 4*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**2 + 8*a**(1/3)
*log(a**(1/3) + b**(1/3)*x)*a*b*x**3 + 4*a**(1/3)*log(a**(1/3) + b**(1/3)*
x)*b**2*x**6 - 12*b**(1/3)*a**2*x - 21*b**(1/3)*a*b*x**4)/(54*b**(1/3)*a*b
**2*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.84 $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 280

$$\begin{aligned}\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx &= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{x^2}{6b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
1/9*x^2/a/b/((b*x^3+a)^2)^(1/2)-1/6*x^2/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-1/
27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(
4/3)/b^(5/3)/((b*x^3+a)^2)^(1/2)-1/27*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(4
/3)/b^(5/3)/((b*x^3+a)^2)^(1/2)+1/54*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/a^(4/3)/b^(5/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{-3a^{4/3}b^{2/3}x^2+6\sqrt[3]{a}b^{5/3}x^5-2\sqrt{3}(a+bx^3)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2(a+bx$$

input
```
Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(-3*a^(4/3)*b^(2/3)*x^2 + 6*a^(1/3)*b^(5/3)*x^5 - 2*Sqrt[3]*(a + b*x^3)^2*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(a + b*x^3)^2*Log[a^(1/3)
+ b^(1/3)*x] + a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*b*
x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + b^2*x^6*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(5/3)*(a + b*x^3)*Sqrt[(a
 + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.71, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1384, 27, 817, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^3(a+bx^3)\int \frac{x^4}{b^3(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(a+bx^3\right)\int \frac{x^4}{\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \textbf{817}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\int \frac{x}{\left(bx^3+a\right)^2}dx}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \textbf{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{\int \frac{x}{bx^3+a}dx}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \textbf{821}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \textbf{16}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \textbf{1142}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 25

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 27

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1082

$$\frac{(a+bx^3)\left(\frac{\frac{\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 217

$$\frac{(a+bx^3)\left(\frac{\frac{\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 1103

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}}{3b}-\frac{x^2}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((a + b*x^3)*(-1/6*x^2/(b*(a + b*x^3)^2) + (x^2/(3*a*(a + b*x^3)) + (-1/3*
Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3)))/(3*a))/(3*b)))/Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(\frac{x^5}{9a}-\frac{x^2}{18b}\right)}{(b x^3+a)^3}+\frac{\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\ln(x-\_R)}{\_R}\right)}{27(b x^3+a)b^2 a}$
default	$-\frac{\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^6+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^6-\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^6-6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^5+4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\right.}{}\right.\right.}{}$

input
```
int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(1/9/a*x^5-1/18*x^2/b)+1/27*((b*x^3+a)^2)^
(1/2)/(b*x^3+a)/b^2/a*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.83

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \left[\frac{6\,ab^3x^5-3\,a^2b^2x^2+3\,\sqrt{\frac{1}{3}}(ab^3x^6+2\,a^2b^2x^3+a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-a}{}\right.}{}\right.$$

input
```
integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^
3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*
x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-
a*b^2)^(2/3)*x)/(b*x^3 + a)) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*
log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 + 2*a*b*x^
3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2*a^3*b^
4*x^3 + a^4*b^3), 1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x
^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*
x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^6 + 2*a*b*x^3 + a^
2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(
b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*
b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [F]

$$\int \frac{x^4}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{x^4}{\left((a + bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**4/((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \frac{2\,bx^5 - ax^2}{18\,(ab^3x^6 + 2\,a^2b^2x^3 + a^3b)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/18*(2*b*x^5 - a*x^2)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) +
1/54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) - 1/27*log
(x + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.66

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{1}{3}}\,ab\mathrm{sgn}\left(bx^3+a\right)} - \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b\mathrm{sgn}\left(bx^3+a\right)} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3\mathrm{sgn}\left(bx^3+a\right)} + \frac{2\,bx^5-ax^2}{18\left(bx^3+a\right)^2ab\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/54*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b*sgn(b*x
^3 + a)) - 1/27*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*sgn(b*x^3 +
 a)) - 1/27*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))/(a^2*b^3*sgn(b*x^3 + a)) + 1/18*(2*b*x^5 - a*x^2)/((b*x^3 +
 a)^2*a*b*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^4}{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}\,dx$$

input
```
int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.95

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2-4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)ab\,x^3-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right.}{}$$

input
```
int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2 - 4*
sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b*x**3 - 2*sq
rt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**2*x**6 - 3*b**
(2/3)*a**(1/3)*a*x**2 + 6*b**(2/3)*a**(1/3)*b*x**5 + log(a**(2/3) - b**(1/
3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 + 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x
 + b**(2/3)*x**2)*a*b*x**3 + log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)
*x**2)*b**2*x**6 - 2*log(a**(1/3) + b**(1/3)*x)*a**2 - 4*log(a**(1/3) + b*
*(1/3)*x)*a*b*x**3 - 2*log(a**(1/3) + b**(1/3)*x)*b**2*x**6)/(54*b**(2/3)*
a**(1/3)*a*b*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.85 $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 276

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{6b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
1/18*x/a/b/((b*x^3+a)^2)^(1/2)-1/6*x/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-1/27*
(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3
)/b^(4/3)/((b*x^3+a)^2)^(1/2)+1/27*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)
/b^(4/3)/((b*x^3+a)^2)^(1/2)-1/54*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/a^(5/3)/b^(4/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{-6a^{5/3}\sqrt[3]{b}x+3a^{2/3}b^{4/3}x^4-2\sqrt{3}\left(a+bx^3\right)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2(a+bx^3}{}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(-6*a^(5/3)*b^(1/3)*x + 3*a^(2/3)*b^(4/3)*x^4 - 2*Sqrt[3]*(a + b*x^3)^2*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(a + b*x^3)^2*Log[a^(1/3) +
b^(1/3)*x] - a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*b*x^
3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - b^2*x^6*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(4/3)*(a + b*x^3)*Sqrt[(a +
 b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.69, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1384, 27, 817, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3\left(a+bx^3\right)\int \frac{x^3}{b^3\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{x^3}{(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 817$$

$$\frac{(a+bx^3)\left(\frac{\int \frac{1}{(bx^3+a)^2}dx}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 749$$

$$\frac{(a+bx^3)\left(\frac{\frac{2\int \frac{1}{bx^3+a}dx}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 750$$

$$\frac{(a+bx^3)\left(\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 16$$

$$\frac{(a+bx^3)\left(\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1142$$

$$\frac{(a+bx^3)\left(\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{25}$$

$$\frac{(a+bx^3)\left(\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\left(\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 1082

$$\frac{(a+bx^3)\left(\frac{\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 217

$$\frac{(a+bx^3)\left(\frac{\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(a+bx^3)\left(\frac{\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}}{6b}-\frac{x}{6b(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((a + b*x^3)*(-1/6*x/(b*(a + b*x^3)^2) + (x/(3*a*(a + b*x^3)) + (2*(Log[a^
(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(3*a))/(6*b)))/Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.86 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{x^{4}}{18a}-\frac{x}{9b}\right)}{\left(b\,x^{3}+a\right)^{3}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{27\left(b\,x^{3}+a\right)b^{2}a}$
default	$-\frac{\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{2}x^{6}-2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{2}x^{6}+\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{2}x^{6}-3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}x^{4}+4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(\cdots}{\cdots}\right.\right.}{\cdots}$

input
```
int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(1/18/a*x^4-1/9*x/b)+1/27*((b*x^3+a)^2)^(1
/2)/(b*x^3+a)/b^2/a*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.82

$$\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \left[\frac{3\,a^2b^2x^4-6\,a^3bx+3\,\sqrt{\frac{1}{3}}\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^3-3\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3
+ a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2
 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^
2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3
+ a^5*b^2), 1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a
^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3
)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^6 + 2*a*b*x^3 +
 a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(
b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*
b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [F]

$$\int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{x^3}{\left((a + bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x**3/((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \frac{bx^4 - 2\,ax}{18\,(ab^3x^6 + 2\,a^2b^2x^3 + a^3b)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/18*(b*x^4 - 2*a*x)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/
54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/27*log(x
 + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a\text{sgn}\left(bx^3+a\right)} - \frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a\text{sgn}\left(bx^3+a\right)} - \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b\text{sgn}\left(bx^3+a\right)} + \frac{bx^4-2\,ax}{18\left(bx^3+a\right)^2ab\text{sgn}\left(bx^3+a\right)}$$

input
```
integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b
^2)^(2/3)*a*sgn(b*x^3 + a)) - 1/54*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/((-a*b^2)^(2/3)*a*sgn(b*x^3 + a)) - 1/27*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b*sgn(b*x^3 + a)) + 1/18*(b*x^4 - 2*a*x)/((b*x^3 + a)^2*a*b*
sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x^3}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

input
```
int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{-2a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-4a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3-2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-}{a^{\frac{1}{3}}}\right.}{}$$

input
```
int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a**2 - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)
))*a*b*x**3 - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*
sqrt(3)))*b**2*x**6 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/
3)*x**2)*a**2 - 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x
**2)*a*b*x**3 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**
2)*b**2*x**6 + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**2 + 4*a**(1/3)*log
(a**(1/3) + b**(1/3)*x)*a*b*x**3 + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b
**2*x**6 - 6*b**(1/3)*a**2*x + 3*b**(1/3)*a*b*x**4)/(54*b**(1/3)*a**2*b*(a
**2 + 2*a*b*x**3 + b**2*x**6))
```

3.86 $\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 277

$$\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output

```
2/9*x^2/a^2/((b*x^3+a)^2)^(1/2)+1/6*x^2/a/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-2/
27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(
7/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)-2/27*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(7
/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)+1/27*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/a^(7/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.86

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{21a^{4/3}b^{2/3}x^2+12\sqrt[3]{a}b^{5/3}x^5-4\sqrt{3}(a+bx^3)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-4(a+b$$

input
```
Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(21*a^(4/3)*b^(2/3)*x^2 + 12*a^(1/3)*b^(5/3)*x^5 - 4*Sqrt[3]*(a + b*x^3)^2
*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*(a + b*x^3)^2*Log[a^(1/3)
 + b^(1/3)*x] + 2*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 4*a
*b*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^2*x^6*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(2/3)*(a + b*x^3)*Sq
rt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.72, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1384, 27, 819, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^3)\int \frac{x}{b^3(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right)\int \frac{x}{\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{2\int \frac{x}{\left(bx^3+a\right)^2}dx}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\int \frac{x}{bx^3+a}dx}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{821}$$

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 1142

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 25

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 27

$$\frac{\left(a+bx^3\right)\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 1082

$$\frac{(a+bx^3)\left(\frac{2\left(\frac{\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{217}$$

$$\frac{(a+bx^3)\left(\frac{2\left(\frac{\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1103$$

$$\frac{(a+bx^3)\left(\frac{2\left(\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((a + b*x^3)*(x^2/(6*a*(a + b*x^3)^2) + (2*(x^2/(3*a*(a + b*x^3)) + (-1/3*
Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3)))/(3*a)))/(3*a)))/Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(\frac{2b\,x^5}{9a^2}+\frac{7x^2}{18a}\right)}{(b\,x^3+a)^3}+\frac{2\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\ln(x-\_R)}{\_R}\right)}{27(b\,x^3+a)b\,a^2}$
default	$\frac{\left(-4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^6-4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^6+2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^6+12\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^5-8\sqrt{3}\,\arctan\left(\right.\right.}{}$

input
```
int(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(2/9/a^2*b*x^5+7/18/a*x^2)+2/27*((b*x^3+a)
^2)^(1/2)/(b*x^3+a)/b/a^2*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.86

$$\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \left[\frac{12\,ab^3x^5+21\,a^2b^2x^2+6\,\sqrt{\frac{1}{3}}(ab^3x^6+2\,a^2b^2x^3+a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3}{\ldots}\right.}{\ldots}\right.$$

input
```
integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*
x^3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*
b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*
(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2
/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b^2*x^6 + 2*a*
b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^4*x^6 + 2*a^
4*b^3*x^3 + a^5*b^2), 1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 12*sqrt(1/3)*(
a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3
)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^6 + 2*a*b
*x^3 + a^2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/
3)) - 4*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3
)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [F]

$$\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \int \frac{x}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(x/((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.53

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{4\,bx^5+7\,ax^2}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)} + \frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/18*(4*b*x^5 + 7*a*x^2)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 2/27*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/3)) +
1/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(1/3)) - 2/27*log
(x + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.64

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,(-ab^2)^{\frac{1}{3}}\,a^2\mathrm{sgn}\left(bx^3+a\right)} - \frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3\mathrm{sgn}\left(bx^3+a\right)} - \frac{2\,\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2\mathrm{sgn}\left(bx^3+a\right)} + \frac{4\,bx^5+7\,ax^2}{18\,(bx^3+a)^2a^2\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/27*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*sgn(b*x
^3 + a)) - 2/27*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*sgn(b*x^3 + a
)) - 2/27*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/(a^3*b^2*sgn(b*x^3 + a)) + 1/18*(4*b*x^5 + 7*a*x^2)/((b*x^3 +
 a)^2*a^2*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{x}{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}\,dx$$

input
```
int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.96

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{-4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2-8\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)ab\,x^3-4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right.}{}$$

input
```
int(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
( - 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2 - 8*
sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b*x**3 - 4*sq
rt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**2*x**6 + 21*b*
*(2/3)*a**(1/3)*a*x**2 + 12*b**(2/3)*a**(1/3)*b*x**5 + 2*log(a**(2/3) - b*
*(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 + 4*log(a**(2/3) - b**(1/3)*a**(1/
3)*x + b**(2/3)*x**2)*a*b*x**3 + 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b*
*(2/3)*x**2)*b**2*x**6 - 4*log(a**(1/3) + b**(1/3)*x)*a**2 - 8*log(a**(1/3
) + b**(1/3)*x)*a*b*x**3 - 4*log(a**(1/3) + b**(1/3)*x)*b**2*x**6)/(54*b**
(2/3)*a**(1/3)*a**2*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.87 $\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 273

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{5x}{18a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
5/18*x/a^2/((b*x^3+a)^2)^(1/2)+1/6*x/a/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-5/27*
(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3
)/b^(1/3)/((b*x^3+a)^2)^(1/2)+5/27*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)
/b^(1/3)/((b*x^3+a)^2)^(1/2)-5/54*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/a^(8/3)/b^(1/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{24a^{5/3}\sqrt[3]{b}x+15a^{2/3}b^{4/3}x^4-10\sqrt{3}(a+bx^3)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10(a+}{}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

output
```
(24*a^(5/3)*b^(1/3)*x + 15*a^(2/3)*b^(4/3)*x^4 - 10*Sqrt[3]*(a + b*x^3)^2*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(a + b*x^3)^2*Log[a^(1/3)
 + b^(1/3)*x] - 5*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 10*
a*b*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 5*b^2*x^6*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a + b*x^3)*S
qrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.76, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {1384, 749, 749, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^3)\int \frac{1}{(b^2x^3+ab)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{749}$$

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\int \frac{1}{\left(b^2x^3+ab\right)^2}dx}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\int \frac{1}{b^2x^3+ab}dx}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}b^{2/3}}+\frac{\int \frac{1}{b^{2/3}x+\sqrt[3]{a}\sqrt[3]{b}}dx}{3a^{2/3}b^{2/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1142$$

$$\frac{b^3(a+bx^3)\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx-\frac{\int-\frac{b\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{2b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{\cdots}{6ab^3(a\cdots}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\big\downarrow$ 25

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{\int\frac{b\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{2b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3(a+\cdots}\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 27

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3(a\cdots}\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1082

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{}{6ab^3(a}\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 217$$

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1103

$$\frac{b^3\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2b}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

output
```
(b^3*(a + b*x^3)*(x/(6*a*b^3*(a + b*x^3)^2) + (5*(x/(3*a*b^2*(a + b*x^3))
+ (2*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(4/3)) + (-((Sqrt[3]*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2]/(2*b))/(3*a^(2/3)*b^(1/3))))/(3*a*b)))/(6*a*b)))/Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(\frac{5b\,x^4}{18a^2}+\frac{4x}{9a}\right)}{(b\,x^3+a)^3}+\frac{5\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{27(b\,x^3+a)b\,a^2}$
default	$\frac{\left(-10\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^6+10\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^6-5\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^6+15\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2x^4-20\sqrt{3}\,\arctan\left(\right.}{}$

input
```
int(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(5/18/a^2*b*x^4+4/9/a*x)+5/27*((b*x^3+a)^2
)^(1/2)/(b*x^3+a)/b/a^2*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.83

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \left[\frac{15\,a^2b^2x^4+24\,a^3bx+15\,\sqrt{\frac{1}{3}}(ab^3x^6+2\,a^2b^2x^3+a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx}{}\right.}{}\right.$$

input
```
integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 15*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x
^3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x -
a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a
^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)
*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(b^2*x^6 + 2*a*b*x^
3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^3*x^6 + 2*a^5*b^
2*x^3 + a^6*b), 1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 30*sqrt(1/3)*(a*b^3*x^
6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^6 + 2*
a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) + 10*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/
3)))/(a^4*b^3*x^6 + 2*a^5*b^2*x^3 + a^6*b)]
```

Sympy [F]

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{1}{(a^2+2abx^3+b^2x^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.53

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{5\,bx^4+8\,ax}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)} + \frac{5\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/18*(5*b*x^4 + 8*a*x)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 5/27*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) - 5/
54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) + 5/27*log(x
 + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = -\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3\mathrm{sgn}\left(bx^3+a\right)} + \frac{5\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\mathrm{sgn}\left(bx^3+a\right)} + \frac{5\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\mathrm{sgn}\left(bx^3+a\right)} + \frac{5\,bx^4+8\,ax}{18\left(bx^3+a\right)^2a^2\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
-5/27*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*sgn(b*x^3 + a)) + 5/27*
sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(a^3*b*sgn(b*x^3 + a)) + 5/54*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/(a^3*b*sgn(b*x^3 + a)) + 1/18*(5*b*x^4 + 8*a*x)/((b*x^3 + a
)^2*a^2*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} \, dx = \int \frac{1}{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{3/2}} \, dx$$

input
```
int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} \, dx = \frac{-10a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) - 20a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b\,x^3 - 10a^{\frac{1}{3}}\sqrt{3}\,atan\Big(}{}$$

input
```
int(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output

```
( - 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
*a**2 - 20*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(
3)))*a*b*x**3 - 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/
3)*sqrt(3)))*b**2*x**6 - 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b
**(2/3)*x**2)*a**2 - 10*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(
2/3)*x**2)*a*b*x**3 - 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(
2/3)*x**2)*b**2*x**6 + 10*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**2 + 20*a*
*(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b*x**3 + 10*a**(1/3)*log(a**(1/3) + b*
*(1/3)*x)*b**2*x**6 + 24*b**(1/3)*a**2*x + 15*b**(1/3)*a*b*x**4)/(54*b**(1
/3)*a**3*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.88 $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 316

$$\begin{aligned}\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx &= -\frac{5bx^2}{9a^3\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{bx^2}{6a^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a+bx^3}{a^3x\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{14\sqrt[3]{b}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{14\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{7\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
-5/9*b*x^2/a^3/((b*x^3+a)^2)^(1/2)-1/6*b*x^2/a^2/(b*x^3+a)/((b*x^3+a)^2)^(
1/2)-(b*x^3+a)/a^3/x/((b*x^3+a)^2)^(1/2)+14/27*b^(1/3)*(b*x^3+a)*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(10/3)/((b*x^3+a)^2)^(1
/2)+14/27*b^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/((b*x^3+a)^2)^(
1/2)-7/27*b^(1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1
0/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{-54a^{7/3}-147a^{4/3}bx^3-84\sqrt[3]{a}b^2x^6+28\sqrt{3}\sqrt[3]{b}x\left(a+bx^3\right)^2\arctan\left(\frac{1-\frac{2\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}}{\sqrt{\cdots}}\right.}{\cdots}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
(-54*a^(7/3) - 147*a^(4/3)*b*x^3 - 84*a^(1/3)*b^2*x^6 + 28*Sqrt[3]*b^(1/3)
*x*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 28*b^(1/3)*
x*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] - 14*a^2*b^(1/3)*x*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 28*a*b^(4/3)*x^4*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2] - 14*b^(7/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(10/3)*x*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.68, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1384, 27, 819, 819, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3\left(a+bx^3\right)\int \frac{1}{b^3x^2\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{1}{x^2(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{(a+bx^3)\left(\frac{7\int \frac{1}{x^2\left(bx^3+a\right)^2}dx}{6a}+\frac{1}{6ax(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{(a+bx^3)\left(\frac{7\left(\frac{4\int \frac{1}{x^2\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{847}$$

$$\frac{(a+bx^3)\left(\frac{7\left(\frac{4\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{821}$$

$$\frac{\left(a+bx^3\right)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 16$$

$$\frac{\left(a+bx^3\right)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1142

$$\left(a+bx^3\right)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\cdots\right.$$

↓ 25

$$\left(a+bx^3\right)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{}{6a\cdots}\right.$$

$\downarrow$ 27

$$\frac{(a+bx^3)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+6 \right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1082

$$\left(a+bx^3\right)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\right.$$

$\Big\downarrow$ 217

$$(a+bx^3)\quad \frac{7\left(\frac{4\left(-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax(a+bx^3)}$$

↓ 1103

$$(a+bx^3)\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax(a+bx^3)^2}\right.$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
((a + b*x^3)*(1/(6*a*x*(a + b*x^3)^2) + (7*(1/(3*a*x*(a + b*x^3)) + (4*(-(
1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[
3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/(3*
a)))/(6*a)))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(-\frac{14b^2x^6}{9a^3}-\frac{49b\,x^3}{18a^2}-\frac{1}{a}\right)}{(b\,x^3+a)^3 x}+\frac{14\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}\_Z^3-b\right)}\_R\ln\left(\left(-4\_R^3a^{10}+3b\right)x-a^7\_R^2\right)\right)}{27(b\,x^3+a)}$
default	$-\frac{\left(-28\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^7-28\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^7+14\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^7+84\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^6-56\sqrt{3}\,\mathrm{arct}\right.}{}$

input
```
int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(-14/9/a^3*b^2*x^6-49/18/a^2*b*x^3-1/a)/x+
14/27*((b*x^3+a)^2)^(1/2)/(b*x^3+a)*sum(_R*ln((-4*_R^3*a^10+3*b)*x-a^7*_R^
2),_R=RootOf(_Z^3*a^10-b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx =$$
$$-\frac{84\,b^2x^6+147\,abx^3+28\,\sqrt{3}(b^2x^7+2\,abx^4+a^2x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+14\,(b^2x^7+2\,abx^4}{54\,(a^3b^2x^7+2\,a}$$

input
```
integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/54*(84*b^2*x^6 + 147*a*b*x^3 + 28*sqrt(3)*(b^2*x^7 + 2*a*b*x^4 + a^2*x)
*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^7
 + 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1
/3)) - 28*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3
)) + 54*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(1/(x**2*((a + b*x**3)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=-\frac{28\,b^2x^6+49\,abx^3+18\,a^2}{18\left(a^3b^2x^7+2\,a^4bx^4+a^5x\right)}-\frac{14\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/18*(28*b^2*x^6 + 49*a*b*x^3 + 18*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*
x) - 14/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^
3*(a/b)^(1/3)) - 7/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(1
/3)) + 14/27*log(x + (a/b)^(1/3))/(a^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\, dx = \frac{14\, b\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\, a^4 \mathrm{sgn}\left(bx^3 + a\right)} + \frac{14\,\sqrt{3}(-ab^2)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\, a^4 b \mathrm{sgn}\left(bx^3 + a\right)} - \frac{7\,(-ab^2)^{\frac{2}{3}} \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\, a^4 b \mathrm{sgn}\left(bx^3 + a\right)} - \frac{10\, b^2x^5 + 13\, abx^2}{18\,(bx^3 + a)^2 a^3 \mathrm{sgn}\left(bx^3 + a\right)} - \frac{1}{a^3 x \mathrm{sgn}\left(bx^3 + a\right)}$$

input
```
integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
14/27*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*sgn(b*x^3 + a)) + 14/
27*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/(a^4*b*sgn(b*x^3 + a)) - 7/27*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/(a^4*b*sgn(b*x^3 + a)) - 1/18*(10*b^2*x^5 + 13*a*b*x^2)/
((b*x^3 + a)^2*a^3*sgn(b*x^3 + a)) - 1/(a^3*x*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{1}{x^2 \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{28\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2bx+56\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a\,b^2x^4+28\sqrt{3}\,atan\Big(}{}$$

input
```
int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
(28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b*x +
56*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**2*x**4
+ 28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**3*x**7
- 54*b**(2/3)*a**(1/3)*a**2 - 147*b**(2/3)*a**(1/3)*a*b*x**3 - 84*b**(2/3)
*a**(1/3)*b**2*x**6 - 14*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**
2)*a**2*b*x - 28*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**
2*x**4 - 14*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**3*x**7
+ 28*log(a**(1/3) + b**(1/3)*x)*a**2*b*x + 56*log(a**(1/3) + b**(1/3)*x)*a
*b**2*x**4 + 28*log(a**(1/3) + b**(1/3)*x)*b**3*x**7)/(54*b**(2/3)*a**(1/3
)*a**3*x*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.89 $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 314

$$\begin{aligned}\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}}\,dx &= -\frac{11bx}{18a^3\sqrt{a^2+2abx^3+b^2x^6}} \\ &\quad -\frac{bx}{6a^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{2a^3x^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &\quad +\frac{20b^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{20b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &\quad +\frac{10b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
-11/18*b*x/a^3/((b*x^3+a)^2)^(1/2)-1/6*b*x/a^2/(b*x^3+a)/((b*x^3+a)^2)^(1/
2)-1/2*(b*x^3+a)/a^3/x^2/((b*x^3+a)^2)^(1/2)+20/27*b^(2/3)*(b*x^3+a)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(11/3)/((b*x^3+a)^2
)^(1/2)-20/27*b^(2/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/((b*x^3+a)^
2)^(1/2)+10/27*b^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/a^(11/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{-27a^{8/3}-96a^{5/3}bx^3-60a^{2/3}b^2x^6+40\sqrt{3}b^{2/3}x^2\left(a+bx^3\right)^2\arctan\left(\frac{1-\frac{2}{\ }}{\ }\right.}{}$$

input
```
Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
(-27*a^(8/3) - 96*a^(5/3)*b*x^3 - 60*a^(2/3)*b^2*x^6 + 40*Sqrt[3]*b^(2/3)*
x^2*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 40*b^(2/3)
*x^2*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] + 20*a^2*b^(2/3)*x^2*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 40*a*b^(5/3)*x^5*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2] + 20*b^(8/3)*x^8*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^(11/3)*x^2*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.68, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1384, 27, 819, 819, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3\left(a+bx^3\right)\int \frac{1}{b^3x^3\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{1}{x^3(bx^3+a)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{(a+bx^3)\left(\frac{4\int \frac{1}{x^3\left(bx^3+a\right)^2}dx}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{(a+bx^3)\left(\frac{4\left(\frac{5\int \frac{1}{x^3\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{847}$$

$$\frac{(a+bx^3)\left(\frac{4\left(\frac{5\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{750}$$

$$\frac{(a+bx^3)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 16

$$\frac{\left(a+bx^3\right)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\downarrow 1142$$

$$\left(a+bx^3\right)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}\right.$$

↓ 25

$$\left(a+bx^3\right)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\right.$$

$$\Big\downarrow \; 27$$

$$\frac{\left(a+bx^3\right)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1082$$

$$(a+bx^3)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}\right.$$

↓ 217

$$(a+bx^3)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2(a+b$$

↓ 1103

$$\left(a+bx^3\right)\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2(a+\right.$$

input
```
Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

output
```
((a + b*x^3)*(1/(6*a*x^2*(a + b*x^3)^2) + (4*(1/(3*a*x^2*(a + b*x^3)) + (5
*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((
Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/a))/(3*a))
)/(3*a)))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.83 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.37

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(-\frac{10 b^2 x^6}{9 a^3}-\frac{16 b x^3}{9 a^2}-\frac{1}{2 a}\right)}{(b x^3+a)^3 x^2}+\frac{20\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\operatorname{RootOf}(a^{11}\_Z^3+b^2)}\_R\ln\left(\left(-4\_R^3 a^{11}-3b^2\right)x-a^4 b\_R\right)\right)}{27(b x^3+a)}$
default	$-\frac{\left(-40\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^8+40\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^8-20\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^8+60\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2x^6-80\sqrt{3}\operatorname{arct}\right.}{}$

input
```
int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^3*(-10/9/a^3*b^2*x^6-16/9/a^2*b*x^3-1/2/a)/x
^2+20/27*((b*x^3+a)^2)^(1/2)/(b*x^3+a)*sum(_R*ln((-4*_R^3*a^11-3*b^2)*x-a^
4*b*_R),_R=RootOf(_Z^3*a^11+b^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx =$$

$$-\frac{60\,b^2x^6+96\,abx^3-40\,\sqrt{3}(b^2x^8+2\,abx^5+a^2x^2)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)+20\,(b^2x^8+2\,abx}{54\,(}$$

input
```
integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/54*(60*b^2*x^6 + 96*a*b*x^3 - 40*sqrt(3)*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2
)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)
/b) + 20*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*
b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 40*(b^2*x^8 + 2*a*b*x^5 + a
^2*x^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 27*a^2)/(a^3*b^2*
x^8 + 2*a^4*b*x^5 + a^5*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{1}{x^3 \left((a + bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral(1/(x**3*((a + b*x**3)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = -\frac{20\,b^2x^6 + 32\,abx^3 + 9\,a^2}{18\,(a^3b^2x^8 + 2\,a^4bx^5 + a^5x^2)} - \frac{20\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{10\,\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{20\,\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/18*(20*b^2*x^6 + 32*a*b*x^3 + 9*a^2)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x
^2) - 20/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a
^3*(a/b)^(2/3)) + 10/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^
(2/3)) - 20/27*log(x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \frac{20\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4\mathrm{sgn}\left(bx^3 + a\right)} - \frac{20\,\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4\mathrm{sgn}\left(bx^3 + a\right)} - \frac{10\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4\mathrm{sgn}\left(bx^3 + a\right)} - \frac{20\,b^2x^6 + 32\,abx^3 + 9\,a^2}{18\left(bx^4 + ax\right)^2a^3\mathrm{sgn}\left(bx^3 + a\right)}$$

input
```
integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
20/27*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*sgn(b*x^3 + a)) - 20/
27*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/(a^4*sgn(b*x^3 + a)) - 10/27*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/(a^4*sgn(b*x^3 + a)) - 1/18*(20*b^2*x^6 + 32*a*b*x^3 + 9*
a^2)/((b*x^4 + a*x)^2*a^3*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{1}{x^3\,{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)}^{3/2}}\,dx$$

input
```
int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

output
```
int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \frac{40a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b\,x^2 + 80a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b^2x^5 + 40a^{\frac{1}{3}}\sqrt{3}\,a}{}$$

input
```
int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
(40*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*
*2*b*x**2 + 80*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*a*b**2*x**5 + 40*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/
(a**(1/3)*sqrt(3)))*b**3*x**8 + 20*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/
3)*x + b**(2/3)*x**2)*a**2*b*x**2 + 40*a**(1/3)*log(a**(2/3) - b**(1/3)*a*
*(1/3)*x + b**(2/3)*x**2)*a*b**2*x**5 + 20*a**(1/3)*log(a**(2/3) - b**(1/3
)*a**(1/3)*x + b**(2/3)*x**2)*b**3*x**8 - 40*a**(1/3)*log(a**(1/3) + b**(1
/3)*x)*a**2*b*x**2 - 80*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b**2*x**5 -
40*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b**3*x**8 - 27*b**(1/3)*a**3 - 96*b
**(1/3)*a**2*b*x**3 - 60*b**(1/3)*a*b**2*x**6)/(54*b**(1/3)*a**4*x**2*(a**
2 + 2*a*b*x**3 + b**2*x**6))
```

3.90 $\int \frac{x^8}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{x^8}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = -\frac{a^2}{12b^3\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{2a}{9b^3\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{6b^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
-1/12*a^2/b^3/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)+2/9*a/b^3/(b*x^3+a)^2/((b*x^
3+a)^2)^(1/2)-1/6/b^3/(b*x^3+a)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.73

$$\int \frac{x^8}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{x^9\left(-4a^6-a^5bx^3+4a^4\sqrt{a^2}\sqrt{\left(a+bx^3\right)^2}-3a^3\sqrt{a^2}bx^3\sqrt{\left(a+bx^3\right)^2}+3\sqrt{a\cdots}\right.}{36a^6\left(a+bx^3\right)^3\left(\sqrt{a^2}bx^3+a\left(\cdots\right.\right.}$$

input
```
Integrate[x^8/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x^9*(-4*a^6 - a^5*b*x^3 + 4*a^4*Sqrt[a^2]*Sqrt[(a + b*x^3)^2] - 3*a^3*Sqr
t[a^2]*b*x^3*Sqrt[(a + b*x^3)^2] + 3*Sqrt[a^2]*b^2*x^6*Sqrt[(a + b*x^3)^2]
*(a^2 + b^2*x^6) + 3*a*b^3*x^9*(b^2*x^6 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2])))
/(36*a^6*(a + b*x^3)^3*(Sqrt[a^2]*b*x^3 + a*(Sqrt[a^2] - Sqrt[(a + b*x^3)^
2])))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a^2+2abx^3+b^2x^6)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5(a+bx^3)\int \frac{x^8}{b^5(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{x^8}{(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{(a+bx^3)\int \frac{x^6}{(bx^3+a)^5}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{53}$$

$$\frac{(a+bx^3)\int \left(\frac{a^2}{b^2(bx^3+a)^5}-\frac{2a}{b^2(bx^3+a)^4}+\frac{1}{b^2(bx^3+a)^3}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^3)\left(-\frac{a^2}{4b^3(a+bx^3)^4}+\frac{2a}{3b^3(a+bx^3)^3}-\frac{1}{2b^3(a+bx^3)^2}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^8/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(-1/4*a^2/(b^3*(a + b*x^3)^4) + (2*a)/(3*b^3*(a + b*x^3)^3) -
 1/(2*b^3*(a + b*x^3)^2)))/(3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$-\frac{(6b^2x^6+4a\,x^3b+a^2)\,\mathrm{csgn}(b\,x^3+a)}{36(b\,x^3+a)^4b^3}$	42
gosper	$-\frac{(b\,x^3+a)(6b^2x^6+4a\,x^3b+a^2)}{36b^3\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}$	43
default	$-\frac{(b\,x^3+a)(6b^2x^6+4a\,x^3b+a^2)}{36b^3\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}$	43
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(-\frac{x^6}{6b}-\frac{a\,x^3}{9b^2}-\frac{a^2}{36b^3}\right)}{(b\,x^3+a)^5}$	48
orering	$-\frac{(b\,x^3+a)(6b^2x^6+4a\,x^3b+a^2)}{36b^3(b^2x^6+2a\,x^3b+a^2)^{\frac{5}{2}}}$	52

input
```
int(x^8/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/36/(b*x^3+a)^4*(6*b^2*x^6+4*a*b*x^3+a^2)/b^3*csgn(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.58

$$\int \frac{x^8}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{6\,b^2x^6+4\,abx^3+a^2}{36\,(b^7x^{12}+4\,ab^6x^9+6\,a^2b^5x^6+4\,a^3b^4x^3+a^4b^3)}$$

input
```
integrate(x^8/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/36*(6*b^2*x^6 + 4*a*b*x^3 + a^2)/(b^7*x^12 + 4*a*b^6*x^9 + 6*a^2*b^5*x^
6 + 4*a^3*b^4*x^3 + a^4*b^3)
```

Sympy [F]

$$\int \frac{x^8}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x^8}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**8/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**8/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.45

$$\int \frac{x^8}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{6\left(x^3+\frac{a}{b}\right)^2b^5} + \frac{2\,a}{9\left(x^3+\frac{a}{b}\right)^3b^6} - \frac{a^2}{12\left(x^3+\frac{a}{b}\right)^4b^7}$$

input
```
integrate(x^8/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/6/((x^3 + a/b)^2*b^5) + 2/9*a/((x^3 + a/b)^3*b^6) - 1/12*a^2/((x^3 + a/
b)^4*b^7)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.36

$$\int \frac{x^8}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{6\,b^2x^6+4\,abx^3+a^2}{36\,(bx^3+a)^4b^3\mathrm{sgn}\,(bx^3+a)}$$

input
```
integrate(x^8/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/36*(6*b^2*x^6 + 4*a*b*x^3 + a^2)/((b*x^3 + a)^4*b^3*sgn(b*x^3 + a))
```

Mupad [B] (verification not implemented)

Time = 18.72 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.45

$$\int \frac{x^8}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}\,\left(a^2+4\,a\,b\,x^3+6\,b^2\,x^6\right)}{36\,b^3\,{\left(b\,x^3+a\right)}^5}$$

input
```
int(x^8/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
-((a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)*(a^2 + 6*b^2*x^6 + 4*a*b*x^3))/(36*b^3
*(a + b*x^3)^5)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.57

$$\int \frac{x^8}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{-6b^2x^6-4ab\,x^3-a^2}{36b^3\left(b^4x^{12}+4a\,b^3x^9+6a^2b^2x^6+4a^3b\,x^3+a^4\right)}$$

input
```
int(x^8/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
( - a**2 - 4*a*b*x**3 - 6*b**2*x**6)/(36*b**3*(a**4 + 4*a**3*b*x**3 + 6*a*
*2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**12))
```

3.91 $\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{9b^2(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{a}{12b^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
-1/9/b^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)+1/12*a/b^2/(b*x^3+a)^3/((b*x^3+a)^2
)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(69) = 138.

Time = 0.91 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.36

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{x^6\left(3\sqrt{a^2}b^6x^{18}+3a^3b^3x^9\sqrt{(a+bx^3)^2}-3a^2b^4x^{12}\sqrt{(a+bx^3)^2}+3ab^5x^{15}\sqrt{(a+bx^3)^2}+a^4b^2x^6\left(\sqrt{a^2}-3\right.\right.}{36a^7(a+bx^3)^3\left(a^2+abx^3-\sqrt{a^2}\sqrt{\right.}}$$

input
```
Integrate[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
-1/36*(x^6*(3*Sqrt[a^2]*b^6*x^18 + 3*a^3*b^3*x^9*Sqrt[(a + b*x^3)^2] - 3*a
^2*b^4*x^12*Sqrt[(a + b*x^3)^2] + 3*a*b^5*x^15*Sqrt[(a + b*x^3)^2] + a^4*b
^2*x^6*(Sqrt[a^2] - 3*Sqrt[(a + b*x^3)^2]) + 6*a^6*(Sqrt[a^2] - Sqrt[(a +
b*x^3)^2]) + 2*a^5*b*x^3*(2*Sqrt[a^2] + Sqrt[(a + b*x^3)^2])))/(a^7*(a + b
*x^3)^3*(a^2 + a*b*x^3 - Sqrt[a^2]*Sqrt[(a + b*x^3)^2]))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1693, 1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^3}{(b^2x^6+2abx^3+a^2)^{5/2}}dx^3$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{3}\left(-\frac{a\int \frac{1}{(b^2x^6+2abx^3+a^2)^{5/2}}dx^3}{b}-\frac{1}{3b^2(a^2+2abx^3+b^2x^6)^{3/2}}\right)$$

$$\Big\downarrow \text{1078}$$

$$\frac{1}{3}\left(\frac{a}{4b^2(a+bx^3)(a^2+2abx^3+b^2x^6)^{3/2}}-\frac{1}{3b^2(a^2+2abx^3+b^2x^6)^{3/2}}\right)$$

input
```
Int[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(-1/3*1/(b^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + a/(4*b^2*(a + b*x^3)*(a^
2 + 2*a*b*x^3 + b^2*x^6)^(3/2)))/3
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.45

method	result	size
pseudoelliptic	$-\frac{\left(4 b \,x^{3}+a \right) \operatorname{csgn}\left(b \,x^{3}+a \right)}{36 b^{2} \left(b \,x^{3}+a \right)^{4}}$	31
gosper	$-\frac{\left(b \,x^{3}+a \right) \left(4 b \,x^{3}+a \right)}{36 b^{2} \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}$	32
default	$-\frac{\left(b \,x^{3}+a \right) \left(4 b \,x^{3}+a \right)}{36 b^{2} \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}$	32
risch	$\frac{\sqrt{\left(b \,x^{3}+a \right)^{2}}\, \left(-\frac{x^{3}}{9 b}-\frac{a}{36 b^{2}}\right)}{\left(b \,x^{3}+a \right)^{5}}$	37
orering	$-\frac{\left(4 b \,x^{3}+a \right) \left(b \,x^{3}+a \right)}{36 b^{2} \left(b^{2} x^{6}+2 a \,x^{3} b +a^{2}\right)^{\frac{5}{2}}}$	41

input
```
int(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/36*(4*b*x^3+a)*csgn(b*x^3+a)/b^2/(b*x^3+a)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{4\,bx^3+a}{36\,(b^6x^{12}+4\,ab^5x^9+6\,a^2b^4x^6+4\,a^3b^3x^3+a^4b^2)}$$

input
```
integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/36*(4*b*x^3 + a)/(b^6*x^12 + 4*a*b^5*x^9 + 6*a^2*b^4*x^6 + 4*a^3*b^3*x^
3 + a^4*b^2)
```

Sympy [F]

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x^5}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**5/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{9\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^2} + \frac{a}{12\left(x^3+\frac{a}{b}\right)^4 b^6}$$

input
```
integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2) + 1/12*a/((x^3 + a/b)^4*b^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{4\,bx^3+a}{36\,(bx^3+a)^4b^2\mathrm{sgn}\,(bx^3+a)}$$

input
```
integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/36*(4*b*x^3 + a)/((b*x^3 + a)^4*b^2*sgn(b*x^3 + a))
```

Mupad [B] (verification not implemented)

Time = 19.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{(4\,b\,x^3+a)\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{36\,b^2\,(b\,x^3+a)^5}$$

input
```
int(x^5/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
-((a + 4*b*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(36*b^2*(a + b*x^3)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \frac{-4b\,x^3 - a}{36b^2\left(b^4x^{12} + 4a\,b^3x^9 + 6a^2b^2x^6 + 4a^3b\,x^3 + a^4\right)}$$

input
```
int(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
( - a - 4*b*x**3)/(36*b**2*(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*
b**3*x**9 + b**4*x**12))
```

3.92 $\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{12b\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
-1/12/b/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 267 vs. 2(38) = 76.

Time = 0.92 (sec) , antiderivative size = 267, normalized size of antiderivative = 7.03

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{x^3\left(-\sqrt{a^2}b^7x^{21}-a^3b^4x^{12}\sqrt{(a+bx^3)^2}+a^2b^5x^{15}\sqrt{(a+bx^3)^2}-ab^6x^{18}\sqrt{(a+bx^3)^2}+4a^7\left(\sqrt{a^2}-\sqrt{(a\right.\right.}{12a^8\,(a+bx^3)^3\left(a^2\right.}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
-1/12*(x^3*(-(Sqrt[a^2]*b^7*x^21) - a^3*b^4*x^12*Sqrt[(a + b*x^3)^2] + a^2
*b^5*x^15*Sqrt[(a + b*x^3)^2] - a*b^6*x^18*Sqrt[(a + b*x^3)^2] + 4*a^7*(Sq
rt[a^2] - Sqrt[(a + b*x^3)^2]) + 2*a^5*b^2*x^6*(2*Sqrt[a^2] - Sqrt[(a + b*
x^3)^2]) + 2*a^6*b*x^3*(3*Sqrt[a^2] - Sqrt[(a + b*x^3)^2]) + a^4*b^3*x^9*(
Sqrt[a^2] + Sqrt[(a + b*x^3)^2])))/(a^8*(a + b*x^3)^3*(a^2 + a*b*x^3 - Sqr
t[a^2]*Sqrt[(a + b*x^3)^2]))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{1690, 1078\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}dx$$
$$\Big\downarrow \; 1690$$
$$\frac{1}{3}\int \frac{1}{(b^2x^6+2abx^3+a^2)^{5/2}}dx^3$$
$$\Big\downarrow \; 1078$$
$$-\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

input
```
Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
-1/12*1/(b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^3+a)}{12(b\,x^3+a)^4 b}$	23
gosper	$-\frac{b\,x^3+a}{12b\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}$	24
default	$-\frac{b\,x^3+a}{12b\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}$	24
risch	$-\frac{\sqrt{(b\,x^3+a)^2}}{12(b\,x^3+a)^5 b}$	26
orering	$-\frac{b\,x^3+a}{12b(b^2x^6+2a\,x^3b+a^2)^{\frac{5}{2}}}$	33

input
```
int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/(b*x^3+a)^4/b*csgn(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{x^2}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = -\frac{1}{12\,(b^5x^{12} + 4\,ab^4x^9 + 6\,a^2b^3x^6 + 4\,a^3b^2x^3 + a^4b)}$$

input
```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12/(b^5*x^12 + 4*a*b^4*x^9 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^3 + a^4*b)
```

Sympy [F]

$$\int \frac{x^2}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \int \frac{x^2}{\left((a + bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**2/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.42

$$\int \frac{x^2}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = -\frac{1}{12\,\left(x^3 + \frac{a}{b}\right)^4 b^5}$$

input
```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/12/((x^3 + a/b)^4*b^5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{12\,(bx^3+a)^4 b\,\mathrm{sgn}\,(bx^3+a)}$$

input
```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/12/((b*x^3 + a)^4*b*sgn(b*x^3 + a))
```

Mupad [B] (verification not implemented)

Time = 18.84 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{12\,b\,(b\,x^3+a)^5}$$

input
```
int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
-(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(12*b*(a + b*x^3)^5)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{1}{12b\,(b^4x^{12}+4a\,b^3x^9+6a^2b^2x^6+4a^3b\,x^3+a^4)}$$

input
```
int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
( - 1)/(12*b*(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9 + b*
*4*x**12))
```

3.93 $\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 223

$$\begin{aligned}\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{\left(a+bx^3\right)\log(x)}{a^5\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
1/3/a^4/((b*x^3+a)^2)^(1/2)+1/12/a/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)+1/9/a^2
/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/6/a^3/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+(b*
x^3+a)*ln(x)/a^5/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a^5/((b*x^3
+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.43

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\frac{a(25a^3+52a^2bx^3+42ab^2x^6+12b^3x^9)+36(a+bx^3)^4\log(x)-12(a+b}{36a^5\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
(a*(25*a^3 + 52*a^2*b*x^3 + 42*a*b^2*x^6 + 12*b^3*x^9) + 36*(a + b*x^3)^4*
Log[x] - 12*(a + b*x^3)^4*Log[a + b*x^3])/(36*a^5*(a + b*x^3)^3*Sqrt[(a +
b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{b^5\left(a+bx^3\right)\int \frac{1}{b^5x\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 798$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^3\left(bx^3+a\right)^5}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx^3)\int\left(-\frac{b}{a^5(bx^3+a)}-\frac{b}{a^4(bx^3+a)^2}-\frac{b}{a^3(bx^3+a)^3}-\frac{b}{a^2(bx^3+a)^4}-\frac{b}{a(bx^3+a)^5}+\frac{1}{a^5x^3}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^3)\left(-\frac{\log(a+bx^3)}{a^5}+\frac{\log(x^3)}{a^5}+\frac{1}{a^4(a+bx^3)}+\frac{1}{2a^3(a+bx^3)^2}+\frac{1}{3a^2(a+bx^3)^3}+\frac{1}{4a(a+bx^3)^4}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
((a + b*x^3)*(1/(4*a*(a + b*x^3)^4) + 1/(3*a^2*(a + b*x^3)^3) + 1/(2*a^3*(
a + b*x^3)^2) + 1/(a^4*(a + b*x^3)) + Log[x^3]/a^5 - Log[a + b*x^3]/a^5))/
(3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.41

method	result
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-\ln\left(b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{4}+\ln\left(b\,x^{3}\right)\left(b\,x^{3}+a\right)^{4}+a\,b^{3} x^{9}+\frac{7 b^{2} x^{6} a^{2}}{2}+\frac{13 b\,x^{3} a^{3}}{3}+\frac{25 a^{4}}{12}\right)}{3\left(b\,x^{3}+a\right)^{4} a^{5}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{b^{3} x^{9}}{3 a^{4}}+\frac{7 b^{2} x^{6}}{6 a^{3}}+\frac{13 b\,x^{3}}{9 a^{2}}+\frac{25}{36 a}\right)}{\left(b\,x^{3}+a\right)^{5}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(x\right)}{\left(b\,x^{3}+a\right) a^{5}}-\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\ln\left(b\,x^{3}+a\right)}{3\left(b\,x^{3}+a\right) a^{5}}$
default	$-\frac{(12\ln(b\,x^{3}+a) b^{4} x^{12}-36\ln(x) b^{4} x^{12}+48\ln(b\,x^{3}+a) a\,b^{3} x^{9}-144\ln(x) a\,b^{3} x^{9}-12 a\,b^{3} x^{9}+72\ln(b\,x^{3}+a) a^{2} b^{2} x^{6}-216\ln(x)}{36 a^{5}\left((b\,x^{3}\right.}$

input
```
int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*csgn(b*x^3+a)*(-ln(b*x^3+a)*(b*x^3+a)^4+ln(b*x^3)*(b*x^3+a)^4+a*b^3*x^
9+7/2*b^2*x^6*a^2+13/3*b*x^3*a^3+25/12*a^4)/(b*x^3+a)^4/a^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\left(a^{2}+2abx^{3}+b^{2}x^{6}\right)^{5/2}}\,dx=\frac{12\,ab^{3}x^{9}+42\,a^{2}b^{2}x^{6}+52\,a^{3}bx^{3}+25\,a^{4}-12\left(b^{4}x^{12}+4\,ab^{3}x^{9}+6\,a^{2}b^{2}x^{6}\right.}{36\left(a^{5}b^{4}x^{12}+4\,a^{6}b\right.}$$

input
```
integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/36*(12*a*b^3*x^9 + 42*a^2*b^2*x^6 + 52*a^3*b*x^3 + 25*a^4 - 12*(b^4*x^12
 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(b*x^3 + a) + 36*(b
^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(x))/(a^5*b^
4*x^12 + 4*a^6*b^3*x^9 + 6*a^7*b^2*x^6 + 4*a^8*b*x^3 + a^9)
```

Sympy [F]

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\int \frac{1}{x\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(1/(x*((a + b*x**3)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.59

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=-\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^5}+\frac{1}{9\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^2}+\frac{1}{3\sqrt{b^2x^6+2\,abx^3+a^2}a^4}+\frac{1}{6\left(x^3+\frac{a}{b}\right)^2a^3b^2}+\frac{1}{12\left(x^3+\frac{a}{b}\right)^4ab^4}$$

input
```
integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^5
 + 1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2) + 1/3/(sqrt(b^2*x^6 + 2*a*b
*x^3 + a^2)*a^4) + 1/6/((x^3 + a/b)^2*a^3*b^2) + 1/12/((x^3 + a/b)^4*a*b^4
)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.49

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = -\frac{\log\left(|bx^3+a|\right)}{3\,a^5\mathrm{sgn}\left(bx^3+a\right)}+\frac{\log\left(|x|\right)}{a^5\mathrm{sgn}\left(bx^3+a\right)} + \frac{25\,b^4x^{12}+112\,ab^3x^9+192\,a^2b^2x^6+152\,a^3bx^3+50\,a^4}{36\left(bx^3+a\right)^4a^5\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*log(abs(b*x^3 + a))/(a^5*sgn(b*x^3 + a)) + log(abs(x))/(a^5*sgn(b*x^3
 + a)) + 1/36*(25*b^4*x^12 + 112*a*b^3*x^9 + 192*a^2*b^2*x^6 + 152*a^3*b*x
^3 + 50*a^4)/((b*x^3 + a)^4*a^5*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \int \frac{1}{x\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}} \,d x$$

input
```
int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

output
```
int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{-12\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^4-48\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^3b\,x^3-}{}$$

input
```
int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
( - 12*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**4 - 48*log(a
**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b*x**3 - 72*log(a**(2/
3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**2*x**6 - 48*log(a**(2/3)
 - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**3*x**9 - 12*log(a**(2/3) - b*
*(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**4*x**12 - 12*log(a**(1/3) + b**(1/3)
*x)*a**4 - 48*log(a**(1/3) + b**(1/3)*x)*a**3*b*x**3 - 72*log(a**(1/3) + b
**(1/3)*x)*a**2*b**2*x**6 - 48*log(a**(1/3) + b**(1/3)*x)*a*b**3*x**9 - 12
*log(a**(1/3) + b**(1/3)*x)*b**4*x**12 + 36*log(x)*a**4 + 144*log(x)*a**3*
b*x**3 + 216*log(x)*a**2*b**2*x**6 + 144*log(x)*a*b**3*x**9 + 36*log(x)*b*
*4*x**12 + 22*a**4 + 40*a**3*b*x**3 + 24*a**2*b**2*x**6 - 3*b**4*x**12)/(3
6*a**5*(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**
12))
```

3.94 $\int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 269

$$\begin{aligned}\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= -\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}\\ &\quad -\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{9a^3\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}\\ &\quad -\frac{b}{2a^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^5x^3\sqrt{a^2+2abx^3+b^2x^6}}\\ &\quad -\frac{5b(a+bx^3)\log(x)}{a^6\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5b(a+bx^3)\log\left(a+bx^3\right)}{3a^6\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output

```
-4/3*b/a^5/((b*x^3+a)^2)^(1/2)-1/12*b/a^2/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)-
2/9*b/a^3/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)-1/2*b/a^4/(b*x^3+a)/((b*x^3+a)^2
)^(1/2)-1/3*(b*x^3+a)/a^5/x^3/((b*x^3+a)^2)^(1/2)-5*b*(b*x^3+a)*ln(x)/a^6/
((b*x^3+a)^2)^(1/2)+5/3*b*(b*x^3+a)*ln(b*x^3+a)/a^6/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{-a(12a^4+125a^3bx^3+260a^2b^2x^6+210ab^3x^9+60b^4x^{12})-180bx^3(a+}{36a^6x^3\left(a+bx^3\right)^3\sqrt{(a+bx^3)}}$$

input
```
Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
(-(a*(12*a^4 + 125*a^3*b*x^3 + 260*a^2*b^2*x^6 + 210*a*b^3*x^9 + 60*b^4*x^
12)) - 180*b*x^3*(a + b*x^3)^4*Log[x] + 60*b*x^3*(a + b*x^3)^4*Log[a + b*x
^3])/(36*a^6*x^3*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.49, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5\left(a+bx^3\right)\int \frac{1}{b^5x^4\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^4\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^6\left(bx^3+a\right)^5}dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx^3)\int\left(\frac{5b^2}{a^6(bx^3+a)}+\frac{4b^2}{a^5(bx^3+a)^2}+\frac{3b^2}{a^4(bx^3+a)^3}+\frac{2b^2}{a^3(bx^3+a)^4}+\frac{b^2}{a^2(bx^3+a)^5}-\frac{5b}{a^6x^3}+\frac{1}{a^5x^6}\right)dx^3}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^3)\left(-\frac{5b\log(x^3)}{a^6}+\frac{5b\log(a+bx^3)}{a^6}-\frac{4b}{a^5(a+bx^3)}-\frac{1}{a^5x^3}-\frac{3b}{2a^4(a+bx^3)^2}-\frac{2b}{3a^3(a+bx^3)^3}-\frac{b}{4a^2(a+bx^3)^4}\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
((a + b*x^3)*(-(1/(a^5*x^3)) - b/(4*a^2*(a + b*x^3)^4) - (2*b)/(3*a^3*(a +
 b*x^3)^3) - (3*b)/(2*a^4*(a + b*x^3)^2) - (4*b)/(a^5*(a + b*x^3)) - (5*b*
Log[x^3])/a^6 + (5*b*Log[a + b*x^3])/a^6))/(3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.42

method	result
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{3}+a\right)\left(-5b\,x^{3}\left(b\,x^{3}+a\right)^{4}\ln\left(b\,x^{3}+a\right)+5b\,x^{3}\left(b\,x^{3}+a\right)^{4}\ln\left(b\,x^{3}\right)+a\left(5b^{4}x^{12}+\frac{35}{2}a\,b^{3}x^{9}+\frac{65}{3}b^{2}x^{6}a^{2}+\frac{125}{12}b\,x^{3}a^{3}+a^{4}\right.\right.}{3\left(b\,x^{3}+a\right)^{4}a^{6}x^{3}}$
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\left(-\frac{1}{3a}-\frac{125b\,x^{3}}{36a^{2}}-\frac{65b^{2}x^{6}}{9a^{3}}-\frac{35b^{3}x^{9}}{6a^{4}}-\frac{5b^{4}x^{12}}{3a^{5}}\right)}{\left(b\,x^{3}+a\right)^{5}x^{3}}-\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\ln\left(x\right)}{\left(b\,x^{3}+a\right)a^{6}}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\ln\left(-b\,x^{3}-a\right)}{3\left(b\,x^{3}+a\right)a^{6}}$
default	$\frac{\left(60\ln\left(b\,x^{3}+a\right)b^{5}x^{15}-180b^{5}\ln\left(x\right)x^{15}+240\ln\left(b\,x^{3}+a\right)a\,b^{4}x^{12}-720a\,b^{4}\ln\left(x\right)x^{12}-60a\,b^{4}x^{12}+360\ln\left(b\,x^{3}+a\right)a^{2}b^{3}x^{9}-1080\right.}{}$

input
```
int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*csgn(b*x^3+a)*(-5*b*x^3*(b*x^3+a)^4*ln(b*x^3+a)+5*b*x^3*(b*x^3+a)^4*l
n(b*x^3)+a*(5*b^4*x^12+35/2*a*b^3*x^9+65/3*b^2*x^6*a^2+125/12*b*x^3*a^3+a^
4))/(b*x^3+a)^4/a^6/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx =$$
$$-\frac{60\,ab^4x^{12} + 210\,a^2b^3x^9 + 260\,a^3b^2x^6 + 125\,a^4bx^3 + 12\,a^5 - 60\,(b^5x^{15} + 4\,ab^4x^{12} + 6\,a^2b^3x^9 + 4\,a^3b^2x^6 + a}{36\,(a^6b^4x^{15} + 4\,a^7b^3x^{12} + 6\,a^8b^2x^9 + 4}$$

input
```
integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/36*(60*a*b^4*x^12 + 210*a^2*b^3*x^9 + 260*a^3*b^2*x^6 + 125*a^4*b*x^3 +
 12*a^5 - 60*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4*a^3*b^2*x^6 + a^
4*b*x^3)*log(b*x^3 + a) + 180*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4
*a^3*b^2*x^6 + a^4*b*x^3)*log(x))/(a^6*b^4*x^15 + 4*a^7*b^3*x^12 + 6*a^8*b
^2*x^9 + 4*a^9*b*x^6 + a^10*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \int \frac{1}{x^4 \left((a + bx^3)^2\right)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(1/(x**4*((a + b*x**3)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \frac{5 (-1)^{2 abx^3 + 2 a^2} b \log \left(\frac{2 abx}{|x|} + \frac{2 a^2}{x^2 |x|} \right)}{3 a^6} - \frac{5 b}{9 (b^2x^6 + 2 abx^3 + a^2)^{\frac{3}{2}} a^3} - \frac{5 b}{3 \sqrt{b^2x^6 + 2 abx^3 + a^2} a^5} - \frac{1}{3 (b^2x^6 + 2 abx^3 + a^2)^{\frac{3}{2}} a^2 x^3} - \frac{5}{6 \left(x^3 + \frac{a}{b}\right)^2 a^4 b} - \frac{1}{12 \left(x^3 + \frac{a}{b}\right)^4 a^2 b^3}$$

input
```
integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
5/3*(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^
6 - 5/9*b/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^3) - 5/3*b/(sqrt(b^2*x^6 +
2*a*b*x^3 + a^2)*a^5) - 1/3/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2*x^3) -
5/6/((x^3 + a/b)^2*a^4*b) - 1/12/((x^3 + a/b)^4*a^2*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \frac{5 b \log (|bx^3 + a|)}{3 a^6 \mathrm{sgn}(bx^3 + a)} - \frac{5 b \log (|x|)}{a^6 \mathrm{sgn}(bx^3 + a)} + \frac{5 bx^3 - a}{3 a^6 x^3 \mathrm{sgn}(bx^3 + a)} - \frac{125 b^5 x^{12} + 548 ab^4 x^9 + 912 a^2 b^3 x^6 + 688 a^3 b^2 x^3 + 202 a^4 b}{36 (bx^3 + a)^4 a^6 \mathrm{sgn}(bx^3 + a)}$$

input
```
integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
5/3*b*log(abs(b*x^3 + a))/(a^6*sgn(b*x^3 + a)) - 5*b*log(abs(x))/(a^6*sgn(
b*x^3 + a)) + 1/3*(5*b*x^3 - a)/(a^6*x^3*sgn(b*x^3 + a)) - 1/36*(125*b^5*x
^12 + 548*a*b^4*x^9 + 912*a^2*b^3*x^6 + 688*a^3*b^2*x^3 + 202*a^4*b)/((b*x
^3 + a)^4*a^6*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \int \frac{1}{x^4 (a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}} \, dx$$

input
```
int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

output
```
int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \frac{60 \log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}} a^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2\right) a^4 b\, x^3 + 240 \log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}} a^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2\right) a^3 b^2 x}{}$$

input
```
int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
(60*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**4*b*x**3 + 240*
log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b**2*x**6 + 360*l
og(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**3*x**9 + 240*lo
g(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**4*x**12 + 60*log(a*
*(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**5*x**15 + 60*log(a**(1/3)
 + b**(1/3)*x)*a**4*b*x**3 + 240*log(a**(1/3) + b**(1/3)*x)*a**3*b**2*x**6
 + 360*log(a**(1/3) + b**(1/3)*x)*a**2*b**3*x**9 + 240*log(a**(1/3) + b**(
1/3)*x)*a*b**4*x**12 + 60*log(a**(1/3) + b**(1/3)*x)*b**5*x**15 - 180*log(
x)*a**4*b*x**3 - 720*log(x)*a**3*b**2*x**6 - 1080*log(x)*a**2*b**3*x**9 -
720*log(x)*a*b**4*x**12 - 180*log(x)*b**5*x**15 - 12*a**5 - 110*a**4*b*x**
3 - 200*a**3*b**2*x**6 - 120*a**2*b**3*x**9 + 15*b**5*x**15)/(36*a**6*x**3
*(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**12))
```

3.95 $\int \frac{x^7}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 365

$$\begin{aligned}\int \frac{x^7}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{5x^2}{243a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{x^5}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5x^2}{108b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{5x^2}{324ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{7/3}b^{8/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{5(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{7/3}b^{8/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{7/3}b^{8/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
5/243*x^2/a^2/b^2/((b*x^3+a)^2)^(1/2)-1/12*x^5/b/(b*x^3+a)^3/((b*x^3+a)^2)
^(1/2)-5/108*x^2/b^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+5/324*x^2/a/b^2/(b*x^
3+a)/((b*x^3+a)^2)^(1/2)-5/729*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*
3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)/b^(8/3)/((b*x^3+a)^2)^(1/2)-5/729*(b*x^3+
a)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(8/3)/((b*x^3+a)^2)^(1/2)+5/1458*(b*x^3
+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(8/3)/((b*x^3+a)^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.62

$$\int \frac{x^7}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(243ab^{2/3}x^2-378b^{2/3}x^2(a+bx^3)+\frac{45b^{2/3}x^2(a+bx^3)^2}{a}+\frac{60b^{2/3}x^2(a+\cdots}{a^2}\right.}{}$$

input
```
Integrate[x^7/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(243*a*b^(2/3)*x^2 - 378*b^(2/3)*x^2*(a + b*x^3) + (45*b^(2/3
)*x^2*(a + b*x^3)^2)/a + (60*b^(2/3)*x^2*(a + b*x^3)^3)/a^2 + (20*Sqrt[3]*
(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(7/3)
- (20*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/a^(7/3) + (10*(a + b*x^3)^4*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(7/3)))/(2916*b^(8/3)*((
a + b*x^3)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.70, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1384, 27, 817, 817, 819, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{b^5(a+bx^3)\int \frac{x^7}{b^5(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx^3)\int \frac{x^7}{(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 817$$

$$\frac{(a+bx^3)\left(\frac{5\int \frac{x^4}{\left(bx^3+a\right)^4}dx}{12b}-\frac{x^5}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 817$$

$$\frac{(a+bx^3)\left(\frac{5\left(\frac{2\int \frac{x}{\left(bx^3+a\right)^3}dx}{9b}-\frac{x^2}{9b\left(a+bx^3\right)^3}\right)}{12b}-\frac{x^5}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 819$$

$$\frac{(a+bx^3)\left(\frac{5\left(\frac{2\left(\frac{2\int \frac{x}{\left(bx^3+a\right)^2}dx}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9b}-\frac{x^2}{9b\left(a+bx^3\right)^3}\right)}{12b}-\frac{x^5}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 819$$

$$\frac{(a+bx^3)\left(\frac{5\left(\frac{2\left(\frac{2\left(\frac{\int \frac{x}{bx^3+a}\,dx}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9b}-\frac{x^2}{9b\left(a+bx^3\right)^3}\right)}{12b}-\frac{x^5}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 821

$$\frac{\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{2\left(\frac{\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9b}-\frac{x^2}{9b\left(a+bx^3\right)^3}\right)}{12b}-\frac{x^5}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

↓ 16

$$\frac{\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9b}-\frac{x^2}{9b\left(a+bx^3\right)^3}\right)}{12b}-\frac{x^5}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

↓ 1142

$$\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{2\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9b}-\cdots}{12b}\right.$$

$\downarrow$ 25

$$\left(a+bx^3\right)\left(\frac{5\left(\frac{2\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9b}-\cdots\right)}{12b}\cdots\right.$$

↓ 27

$$\frac{(a+bx^3)\left(\frac{5\left(\frac{2\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9b}-\cdots}{12b}\cdots}{\sqrt{a^2+2abx^3+b^2x^6}}$$

↓ 1082

$$(a+bx^3)\left(\frac{5\left(\frac{2\left(\frac{2\left(\frac{\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}\,d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9b}}{12b}\right.\right.$$

↓ 217

$$(a+bx^3)\left|\frac{5\left(\frac{2\left(\frac{2\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9b}-\frac{x^2}{9b\left(a+bx^3\right.}}{12b}\right.$$

↓ 1103

$$\left(a+bx^3\right) \left| \frac{\left(5\,\frac{\left(2\,\frac{\left(2\,\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\,\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9b}-\frac{x^2}{9b\left(a+bx^3\right)^3} \cdots\right)}{12b} \right.$$

input
```
Int[x^7/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(-1/12*x^5/(b*(a + b*x^3)^4) + (5*(-1/9*x^2/(b*(a + b*x^3)^3)
 + (2*(x^2/(6*a*(a + b*x^3)^2) + (2*(x^2/(3*a*(a + b*x^3)) + (-1/3*Log[a^(
1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(3*a)))/(3*a)))/(9*b)))/(12*b)))/Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.62 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.29

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(\frac{5 b x^{11}}{243 a^2}+\frac{25 x^8}{324 a}-\frac{x^5}{27 b}-\frac{5 a x^2}{486 b^2}\right)}{(b x^3+a)^5}+\frac{5\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\ln(x-\_R)}{\_R}\right)}{729(b x^3+a)a^2 b^3}$
default	$-\frac{\left(20\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}+20\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}-10\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}-60\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4x^{11}+80\sqrt{3}\text{ ar}}{}$

input
```
int(x^7/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(5/243/a^2*b*x^11+25/324/a*x^8-1/27/b*x^5-
5/486*a/b^2*x^2)+5/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a^2/b^3*sum(1/_R*ln(x
-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 734, normalized size of antiderivative = 2.01

$$\int \frac{x^7}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^7/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/2916*(60*a*b^5*x^11 + 225*a^2*b^4*x^8 - 108*a^3*b^3*x^5 - 30*a^4*b^2*x^
2 + 30*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x
^3 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b
*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(
-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 10*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6
 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-
a*b^2)^(2/3)) - 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 +
 a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^8*x^12 + 4*a^4*b^7*
x^9 + 6*a^5*b^6*x^6 + 4*a^6*b^5*x^3 + a^7*b^4), 1/2916*(60*a*b^5*x^11 + 22
5*a^2*b^4*x^8 - 108*a^3*b^3*x^5 - 30*a^4*b^2*x^2 + 60*sqrt(1/3)*(a*b^5*x^1
2 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2)^
(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)
/b) + 10*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 20*(b^4*x^
12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b
*x - (-a*b^2)^(1/3)))/(a^3*b^8*x^12 + 4*a^4*b^7*x^9 + 6*a^5*b^6*x^6 + 4*a^
6*b^5*x^3 + a^7*b^4)]
```

Sympy [F]

$$\int \frac{x^7}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \int \frac{x^7}{\left((a + bx^3)^2\right)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x**7/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**7/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.54

$$\int \frac{x^7}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{20\,b^3x^{11}+75\,ab^2x^8-36\,a^2bx^5-10\,a^3x^2}{972\,(a^2b^6x^{12}+4\,a^3b^5x^9+6\,a^4b^4x^6+4\,a^5b^3x^3+a^6b^2)} + \frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{5\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^7/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/972*(20*b^3*x^11 + 75*a*b^2*x^8 - 36*a^2*b*x^5 - 10*a^3*x^2)/(a^2*b^6*x^
12 + 4*a^3*b^5*x^9 + 6*a^4*b^4*x^6 + 4*a^5*b^3*x^3 + a^6*b^2) + 5/729*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^(1/
3)) + 5/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(1/3))
- 5/729*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.57

$$\int \frac{x^7}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,(-ab^2)^{\frac{1}{3}}\,a^2b^2\mathrm{sgn}\,(bx^3+a)} - \frac{5\,\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,(-ab^2)^{\frac{1}{3}}\,a^2b^2\mathrm{sgn}\,(bx^3+a)} - \frac{5\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^3b^2\mathrm{sgn}\,(bx^3+a)} + \frac{20\,b^3x^{11}+75\,ab^2x^8-36\,a^2bx^5-10\,a^3x^2}{972\,(bx^3+a)^4a^2b^2\mathrm{sgn}\,(bx^3+a)}$$

input
```
integrate(x^7/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
5/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b
^2)^(1/3)*a^2*b^2*sgn(b*x^3 + a)) - 5/1458*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3))/((-a*b^2)^(1/3)*a^2*b^2*sgn(b*x^3 + a)) - 5/729*(-a/b)^(2/3)*log
(abs(x - (-a/b)^(1/3)))/(a^3*b^2*sgn(b*x^3 + a)) + 1/972*(20*b^3*x^11 + 75
*a*b^2*x^8 - 36*a^2*b*x^5 - 10*a^3*x^2)/((b*x^3 + a)^4*a^2*b^2*sgn(b*x^3 +
 a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\int \frac{x^7}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

input
```
int(x^7/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^7/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.32

$$\int \frac{x^7}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\frac{-20\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^4-80\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^3b\,x^3-120\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}\cdots}{\cdots}\right.}{\cdots}$$

input
```
int(x^7/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output

```
( - 20*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**4 - 8
0*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**3*b*x**3 -
 120*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b**2*
x**6 - 80*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**
3*x**9 - 20*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**
4*x**12 - 30*b**(2/3)*a**(1/3)*a**3*x**2 - 108*b**(2/3)*a**(1/3)*a**2*b*x*
*5 + 225*b**(2/3)*a**(1/3)*a*b**2*x**8 + 60*b**(2/3)*a**(1/3)*b**3*x**11 +
 10*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**4 + 40*log(a**(
2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b*x**3 + 60*log(a**(2/3)
- b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**2*x**6 + 40*log(a**(2/3) -
b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**3*x**9 + 10*log(a**(2/3) - b**(1
/3)*a**(1/3)*x + b**(2/3)*x**2)*b**4*x**12 - 20*log(a**(1/3) + b**(1/3)*x)
*a**4 - 80*log(a**(1/3) + b**(1/3)*x)*a**3*b*x**3 - 120*log(a**(1/3) + b**
(1/3)*x)*a**2*b**2*x**6 - 80*log(a**(1/3) + b**(1/3)*x)*a*b**3*x**9 - 20*l
og(a**(1/3) + b**(1/3)*x)*b**4*x**12)/(2916*b**(2/3)*a**(1/3)*a**2*b**2*(a
**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**12))
```

3.96 $\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 359

$$\begin{aligned}\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{x}{162ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{5(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
5/486*x/a^2/b^2/((b*x^3+a)^2)^(1/2)-1/12*x^4/b/(b*x^3+a)^3/((b*x^3+a)^2)^(
1/2)-1/27*x/b^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/162*x/a/b^2/(b*x^3+a)/((
b*x^3+a)^2)^(1/2)-5/729*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)
/a^(1/3))*3^(1/2)/a^(8/3)/b^(7/3)/((b*x^3+a)^2)^(1/2)+5/729*(b*x^3+a)*ln(a
^(1/3)+b^(1/3)*x)/a^(8/3)/b^(7/3)/((b*x^3+a)^2)^(1/2)-5/1458*(b*x^3+a)*ln(
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(7/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.61

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(243a\sqrt[3]{b}x-351\sqrt[3]{b}x(a+bx^3)+\frac{18\sqrt[3]{b}x(a+bx^3)^2}{a}+\frac{30\sqrt[3]{b}x(a+bx^3)^3}{a^2}\right.}{}$$

input
```
Integrate[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(243*a*b^(1/3)*x - 351*b^(1/3)*x*(a + b*x^3) + (18*b^(1/3)*x*
(a + b*x^3)^2)/a + (30*b^(1/3)*x*(a + b*x^3)^3)/a^2 + (20*Sqrt[3]*(a + b*x
^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(8/3) + (20*(a
 + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) - (10*(a + b*x^3)^4*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(8/3)))/(2916*b^(7/3)*((a + b*x^
3)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.68, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1384, 27, 817, 817, 749, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5(a+bx^3)\int \frac{x^6}{b^5(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{x^6}{(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(a+bx^3)\left(\frac{\int \frac{x^3}{\left(bx^3+a\right)^4}dx}{3b}-\frac{x^4}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(a+bx^3)\left(\frac{\frac{\int \frac{1}{\left(bx^3+a\right)^3}dx}{9b}-\frac{x}{9b\left(a+bx^3\right)^3}}{3b}-\frac{x^4}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(a+bx^3)\left(\frac{\frac{\frac{5\int \frac{1}{\left(bx^3+a\right)^2}dx}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{x}{9b\left(a+bx^3\right)^3}}{3b}-\frac{x^4}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(a+bx^3)\left(\frac{\frac{\frac{5\left(\frac{2\int \frac{1}{bx^3+a}dx}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{x}{9b\left(a+bx^3\right)^3}}{3b}-\frac{x^4}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{750}$$

$$\frac{(a+bx^3)\left(\frac{\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{x}{9b\left(a+bx^3\right)^3}}{3b}-\frac{x^4}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 16$$

$$\frac{(a+bx^3)\left(\frac{\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{x}{9b\left(a+bx^3\right)^3}}{3b}-\frac{x^4}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1142

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{\cdots}{9b\cdots}}{3b}\cdots\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 25

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{}{9b(}}{3b}\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 27

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{\cdots}{9b\cdots}}{3b}\cdots\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1082

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\cdots}{3b}\cdots\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 217$$

$$\frac{(a+bx^3)\left(\frac{\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}}{9b}-\frac{x}{9b\left(a+bx^3\right)^3}}{3b}-\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 1103

$$\frac{(a+bx^3)\left(\frac{\frac{\frac{5\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6a}+\frac{x}{6a(a+bx^3)^2}}{9b}-\frac{x}{9b(a+bx^3)^3}}{3b}\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(-1/12*x^4/(b*(a + b*x^3)^4) + (-1/9*x/(b*(a + b*x^3)^3) + (x
/(6*a*(a + b*x^3)^2) + (5*(x/(3*a*(a + b*x^3)) + (2*(Log[a^(1/3) + b^(1/3)
*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1
/3)))/(3*a^(2/3))))/(3*a)))/(6*a))/(9*b))/(3*b)))/Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.00 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.29

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(\frac{5b x^{10}}{486a^2}+\frac{x^7}{27a}-\frac{25x^4}{324b}-\frac{5ax}{243b^2}\right)}{(b x^3+a)^5}+\frac{5\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\ln(x-\_R)}{\_R^2}\right)}{729(b x^3+a)a^2b^3}$
default	$-\frac{\left(20\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}-20\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}+10\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}-30\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x^{10}+80\sqrt{3}\ \mathrm{ar}}{}$

input
```
int(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(5/486/a^2*b*x^10+1/27/a*x^7-25/324/b*x^4-
5/243*a/b^2*x)+5/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a^2/b^3*sum(1/_R^2*ln(x
-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 723, normalized size of antiderivative = 2.01

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7 - 225*a^4*b^2*x^4 - 60*a^5*b*x
+ 30*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3
 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2
*b)^(1/3)/b))/(b*x^3 + a)) - 10*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 +
4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(
1/3)*a) + 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*
(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 +
6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(30*a^2*b^4*x^10 + 108*a^
3*b^3*x^7 - 225*a^4*b^2*x^4 - 60*a^5*b*x + 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^
2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b)*a
rctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b
)/a^2) - 10*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(
a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 20*(b^4*x^
12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*
b*x + (a^2*b)^(2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^
7*b^4*x^3 + a^8*b^3)]
```

Sympy [F]

$$\int \frac{x^6}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \int \frac{x^6}{\left((a + bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**6/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**6/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.54

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{10\,b^3x^{10}+36\,ab^2x^7-75\,a^2bx^4-20\,a^3x}{972\,(a^2b^6x^{12}+4\,a^3b^5x^9+6\,a^4b^4x^6+4\,a^5b^3x^3+a^6b^2)} + \frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/972*(10*b^3*x^10 + 36*a*b^2*x^7 - 75*a^2*b*x^4 - 20*a^3*x)/(a^2*b^6*x^12
 + 4*a^3*b^5*x^9 + 6*a^4*b^4*x^6 + 4*a^5*b^3*x^3 + a^6*b^2) + 5/729*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3)
) - 5/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(2/3)) +
5/729*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.57

$$\int \frac{x^6}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{5\,\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,\left(-ab^2\right)^{\frac{2}{3}}a^2b\mathrm{sgn}\left(bx^3+a\right)} - \frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^3b^2\mathrm{sgn}\left(bx^3+a\right)} + \frac{5\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^3b^3\mathrm{sgn}\left(bx^3+a\right)} + \frac{10\,b^3x^{10}+36\,ab^2x^7-75\,a^2bx^4-20\,a^3x}{972\,(bx^3+a)^4a^2b^2\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-5/1458*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b*sgn
(b*x^3 + a)) - 5/729*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^2*sgn(
b*x^3 + a)) + 5/729*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/(a^3*b^3*sgn(b*x^3 + a)) + 1/972*(10*b^3*x^10 + 36*
a*b^2*x^7 - 75*a^2*b*x^4 - 20*a^3*x)/((b*x^3 + a)^4*a^2*b^2*sgn(b*x^3 + a)
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\int \frac{x^6}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

input
```
int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.37

$$\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output

```
( - 20*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
*a**4 - 80*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(
3)))*a**3*b*x**3 - 120*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a*
*(1/3)*sqrt(3)))*a**2*b**2*x**6 - 80*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b
**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**3*x**9 - 20*a**(1/3)*sqrt(3)*atan((a**
(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**4*x**12 - 10*a**(1/3)*log(a**
(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**4 - 40*a**(1/3)*log(a**(2/
3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b*x**3 - 60*a**(1/3)*log(a*
*(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**2*x**6 - 40*a**(1/3)
*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**3*x**9 - 10*a**(
1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**4*x**12 + 20*a
**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**4 + 80*a**(1/3)*log(a**(1/3) + b**(1
/3)*x)*a**3*b*x**3 + 120*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**2*b**2*x**
6 + 80*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b**3*x**9 + 20*a**(1/3)*log(a
**(1/3) + b**(1/3)*x)*b**4*x**12 - 60*b**(1/3)*a**4*x - 225*b**(1/3)*a**3*
b*x**4 + 108*b**(1/3)*a**2*b**2*x**7 + 30*b**(1/3)*a*b**3*x**10)/(2916*b**
(1/3)*a**3*b**2*(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9 +
 b**4*x**12))
```

3.97 $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 368

$$\begin{aligned}\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx &= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{x^2}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{7x^2}{324a^2b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{7(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
7/243*x^2/a^3/b/((b*x^3+a)^2)^(1/2)-1/12*x^2/b/(b*x^3+a)^3/((b*x^3+a)^2)^(
1/2)+1/54*x^2/a/b/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+7/324*x^2/a^2/b/(b*x^3+a
)/((b*x^3+a)^2)^(1/2)-7/729*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(
1/2)/a^(1/3))*3^(1/2)/a^(10/3)/b^(5/3)/((b*x^3+a)^2)^(1/2)-7/729*(b*x^3+a)
*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(5/3)/((b*x^3+a)^2)^(1/2)+7/1458*(b*x^3+
a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(5/3)/((b*x^3+a)^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.62

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{(a+bx^3)\left(-243a^{10/3}b^{2/3}x^2+54a^{7/3}b^{2/3}x^2(a+bx^3)+63a^{4/3}b^{2/3}x^2(a+bx\right.}{}$$

input
```
Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(-243*a^(10/3)*b^(2/3)*x^2 + 54*a^(7/3)*b^(2/3)*x^2*(a + b*x^
3) + 63*a^(4/3)*b^(2/3)*x^2*(a + b*x^3)^2 + 84*a^(1/3)*b^(2/3)*x^2*(a + b*
x^3)^3 + 28*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]
*a^(1/3))] - 28*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] + 14*(a + b*x^3)^4*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(10/3)*b^(5/3)*((
a + b*x^3)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.69, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1384, 27, 817, 819, 819, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{b^5\left(a+bx^3\right)\int \frac{x^4}{b^5\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a+bx^3\right)\int \frac{x^4}{\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{817}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\int \frac{x}{\left(bx^3+a\right)^4}dx}{6b}-\frac{x^2}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{7\int \frac{x}{\left(bx^3+a\right)^3}dx}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}}{6b}-\frac{x^2}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{7\left(\frac{2\int \frac{x}{\left(bx^3+a\right)^2}dx}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}}{6b}-\frac{x^2}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{7\left(\frac{2\left(\frac{\int \frac{x}{bx^3+a}dx}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}}{6b}-\frac{x^2}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{821}$$

$$\frac{(a+bx^3)\left(\frac{7\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}}{6b}-\frac{x^2}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 16$$

$$\frac{(a+bx^3)\left(\frac{\frac{7\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}}{6b}-\frac{x^2}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1142$$

$$\frac{(a+bx^3)\left(\frac{\frac{7\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{\cdots}{9a\cdots}}{6b}\cdots\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 25

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{7\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{\cdots}{9a\left(\cdots\right.}}{6b}\cdots\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 27

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{7\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{\cdots}{9a\cdots}}{6b}\cdots\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{7\left(\frac{2\left(\frac{\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+}{6b}}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 217

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{7\left(\frac{2\left(\frac{\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}}{6b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

↓ 1103

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{7\left(\frac{2\left(\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}}{6b}-\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(-1/12*x^2/(b*(a + b*x^3)^4) + (x^2/(9*a*(a + b*x^3)^3) + (7*
(x^2/(6*a*(a + b*x^3)^2) + (2*(x^2/(3*a*(a + b*x^3)) + (-1/3*Log[a^(1/3) +
 b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(
2*b^(1/3)))/(3*a^(1/3)*b^(1/3)))/(3*a))/(3*a)))/(9*a))/(6*b)))/Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{7 b^{2} x^{11}}{243 a^{3}}+\frac{35 b\,x^{8}}{324 a^{2}}+\frac{4 x^{5}}{27 a}-\frac{7 x^{2}}{486 b}\right)}{\left(b\,x^{3}+a\right)^{5}}+\frac{7\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R =\operatorname{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{729\left(b\,x^{3}+a\right) a^{3} b^{2}}$
default	$-\frac{\left(28\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2 x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) b^{4} x^{12}+28\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) b^{4} x^{12}-14\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) b^{4} x^{12}-84\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{4} x^{11}+112\sqrt{3}\,a}{}$

input
```
int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(7/243/a^3*b^2*x^11+35/324/a^2*b*x^8+4/27/
a*x^5-7/486*x^2/b)+7/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a^3/b^2*sum(1/_R*ln
(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.99

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/2916*(84*a*b^5*x^11 + 315*a^2*b^4*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^
2 + 42*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x
^3 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b
*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(
-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 14*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6
 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-
a*b^2)^(2/3)) - 28*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 +
 a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*
x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(84*a*b^5*x^11 + 31
5*a^2*b^4*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^2 + 84*sqrt(1/3)*(a*b^5*x^1
2 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2)^
(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)
/b) + 14*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^4*x^
12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b
*x - (-a*b^2)^(1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^
7*b^4*x^3 + a^8*b^3)]
```

Sympy [F]

$$\int \frac{x^4}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \int \frac{x^4}{\left((a + bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**4/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{28\,b^3x^{11}+105\,ab^2x^8+144\,a^2bx^5-14\,a^3x^2}{972\,(a^3b^5x^{12}+4\,a^4b^4x^9+6\,a^5b^3x^6+4\,a^6b^2x^3+a^7b)} + \frac{7\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^3b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{7\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^3b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{7\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^3b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/972*(28*b^3*x^11 + 105*a*b^2*x^8 + 144*a^2*b*x^5 - 14*a^3*x^2)/(a^3*b^5*
x^12 + 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 7/729*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(1/
3)) + 7/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(1/3))
- 7/729*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.56

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{7\,\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,\left(-ab^2\right)^{\frac{1}{3}}\,a^3b\mathrm{sgn}\left(bx^3+a\right)} - \frac{7\,\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^4b\mathrm{sgn}\left(bx^3+a\right)} - \frac{7\,\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^4b^3\mathrm{sgn}\left(bx^3+a\right)} + \frac{28\,b^3x^{11}+105\,ab^2x^8+144\,a^2bx^5-14\,a^3x^2}{972\,\left(bx^3+a\right)^4a^3b\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-7/1458*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^3*b*sgn
(b*x^3 + a)) - 7/729*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b*sgn(b*
x^3 + a)) - 7/729*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))/(a^4*b^3*sgn(b*x^3 + a)) + 1/972*(28*b^3*x^11 + 105*a
*b^2*x^8 + 144*a^2*b*x^5 - 14*a^3*x^2)/((b*x^3 + a)^4*a^3*b*sgn(b*x^3 + a)
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\int \frac{x^4}{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}\,dx$$

input
```
int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.31

$$\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\frac{-28\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^4-112\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^3b\,x^3-168\sqrt{3}\,atan\left(\cdots\right.}{\cdots}$$

input
```
int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
( - 28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**4 - 1
12*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**3*b*x**3
- 168*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b**2
*x**6 - 112*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b
**3*x**9 - 28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b
**4*x**12 - 42*b**(2/3)*a**(1/3)*a**3*x**2 + 432*b**(2/3)*a**(1/3)*a**2*b*
x**5 + 315*b**(2/3)*a**(1/3)*a*b**2*x**8 + 84*b**(2/3)*a**(1/3)*b**3*x**11
 + 14*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**4 + 56*log(a*
*(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b*x**3 + 84*log(a**(2/3
) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**2*x**6 + 56*log(a**(2/3)
- b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**3*x**9 + 14*log(a**(2/3) - b**
(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**4*x**12 - 28*log(a**(1/3) + b**(1/3)*
x)*a**4 - 112*log(a**(1/3) + b**(1/3)*x)*a**3*b*x**3 - 168*log(a**(1/3) +
b**(1/3)*x)*a**2*b**2*x**6 - 112*log(a**(1/3) + b**(1/3)*x)*a*b**3*x**9 -
28*log(a**(1/3) + b**(1/3)*x)*b**4*x**12)/(2916*b**(2/3)*a**(1/3)*a**3*b*(
a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**12))
```

3.98 $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 360

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{108ab(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{81a^2b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{10(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

output

```
5/243*x/a^3/b/((b*x^3+a)^2)^(1/2)-1/12*x/b/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)
+1/108*x/a/b/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/81*x/a^2/b/(b*x^3+a)/((b*x^
3+a)^2)^(1/2)-10/729*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^
(1/3))*3^(1/2)/a^(11/3)/b^(4/3)/((b*x^3+a)^2)^(1/2)+10/729*(b*x^3+a)*ln(a^
(1/3)+b^(1/3)*x)/a^(11/3)/b^(4/3)/((b*x^3+a)^2)^(1/2)-5/729*(b*x^3+a)*ln(a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(4/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(-243a^{11/3}\sqrt[3]{b}x+27a^{8/3}\sqrt[3]{b}x(a+bx^3)+36a^{5/3}\sqrt[3]{b}x(a+bx^3)^2+\right.}{}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(-243*a^(11/3)*b^(1/3)*x + 27*a^(8/3)*b^(1/3)*x*(a + b*x^3) +
 36*a^(5/3)*b^(1/3)*x*(a + b*x^3)^2 + 60*a^(2/3)*b^(1/3)*x*(a + b*x^3)^3 +
 40*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)] + 40*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] - 20*(a + b*x^3)^4*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(11/3)*b^(4/3)*((a + b*x^
3)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.67, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1384, 27, 817, 749, 749, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5(a+bx^3)\int \frac{x^3}{b^5(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{x^3}{(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{817}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\int \frac{1}{\left(bx^3+a\right)^4}dx}{12b}-\frac{x}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{749}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{8\int \frac{1}{\left(bx^3+a\right)^3}dx}{9a}+\frac{x}{9a\left(a+bx^3\right)^3}}{12b}-\frac{x}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{749}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{8\left(\frac{5\int \frac{1}{\left(bx^3+a\right)^2}dx}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x}{9a\left(a+bx^3\right)^3}}{12b}-\frac{x}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{749}$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{8\left(\frac{5\left(\frac{2\int \frac{1}{bx^3+a}dx}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x}{9a\left(a+bx^3\right)^3}}{12b}-\frac{x}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{750}$$

$$\frac{(a+bx^3)\left(\frac{\frac{8\left(\frac{5\left(\frac{2\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x}{9a\left(a+bx^3\right)^3}}{12b}-\frac{x}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 16$$

$$\frac{\left(a+bx^3\right)\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x}{9a\left(a+bx^3\right)^3}}{12b}-\frac{x}{12b(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\downarrow$ 1142

$$\left(a+bx^3\right)\left(\frac{\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}}{12b}\right.$$

↓ 25

$$\left(a+bx^3\right)\left(\frac{\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+}{12b}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{\left(a+bx^3\right)\left(\frac{\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}}{12b}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 1082$$

$$\left(a+bx^3\right) \left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2} \cdots\right)}{9a} \cdots}{12b} \cdots \right.$$

↓ 217

$$\left(a+bx^3\right)\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x}{9a\left(a+bx^3\right)}}{12b}\right.$$

↓ 1103

$$\left(a+bx^3\right)\left(\frac{\frac{8\left(\frac{5\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x}{9a\left(a+bx\ldots\right.}}{12b}\right.$$

input
```
Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(-1/12*x/(b*(a + b*x^3)^4) + (x/(9*a*(a + b*x^3)^3) + (8*(x/(
6*a*(a + b*x^3)^2) + (5*(x/(3*a*(a + b*x^3)) + (2*(Log[a^(1/3) + b^(1/3)*x
]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3
)))/(3*a^(2/3))))/(3*a)))/(6*a)))/(9*a))/(12*b)))/Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.79 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(\frac{5b^2x^{10}}{243a^3}+\frac{2b\,x^7}{27a^2}+\frac{31x^4}{324a}-\frac{10x}{243b}\right)}{(b\,x^3+a)^5}+\frac{10\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{729(b\,x^3+a)a^3b^2}$
default	$-\frac{\left(40\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}-40\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}+20\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}-60\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x^{10}+160\sqrt{3}\,a}{}$

input
```
int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(5/243/a^3*b^2*x^10+2/27/a^2*b*x^7+31/324/
a*x^4-10/243*x/b)+10/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a^3/b^2*sum(1/_R^2*
ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 723, normalized size of antiderivative = 2.01

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^7 + 279*a^4*b^2*x^4 - 120*a^5*b*x
 + 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^
3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a
^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^
2*b)^(1/3)/b))/(b*x^3 + a)) - 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 +
 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 40*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)
*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 +
 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(60*a^2*b^4*x^10 + 216*a
^3*b^3*x^7 + 279*a^4*b^2*x^4 - 120*a^5*b*x + 120*sqrt(1/3)*(a*b^5*x^12 + 4
*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b
)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3
)/b)/a^2) - 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4
)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^4
*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log
(a*b*x + (a^2*b)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4
*a^8*b^3*x^3 + a^9*b^2)]
```

Sympy [F]

$$\int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \int \frac{x^3}{\left((a + bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x**3/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.54

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{20\,b^3x^{10}+72\,ab^2x^7+93\,a^2bx^4-40\,a^3x}{972\,(a^3b^5x^{12}+4\,a^4b^4x^9+6\,a^5b^3x^6+4\,a^6b^2x^3+a^7b)} + \frac{10\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{10\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/972*(20*b^3*x^10 + 72*a*b^2*x^7 + 93*a^2*b*x^4 - 40*a^3*x)/(a^3*b^5*x^12
 + 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 10/729*sqrt(3)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3))
 - 5/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) + 10
/729*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.55

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{10\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,(-ab^2)^{\frac{2}{3}}\,a^3\mathrm{sgn}\left(bx^3+a\right)} - \frac{5\,\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,(-ab^2)^{\frac{2}{3}}\,a^3\mathrm{sgn}\left(bx^3+a\right)} - \frac{10\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^4b\mathrm{sgn}\left(bx^3+a\right)} + \frac{20\,b^3x^{10}+72\,ab^2x^7+93\,a^2bx^4-40\,a^3x}{972\,(bx^3+a)^4a^3b\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-10/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a
*b^2)^(2/3)*a^3*sgn(b*x^3 + a)) - 5/729*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^
(2/3))/((-a*b^2)^(2/3)*a^3*sgn(b*x^3 + a)) - 10/729*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a^4*b*sgn(b*x^3 + a)) + 1/972*(20*b^3*x^10 + 72*a*b^2*x
^7 + 93*a^2*b*x^4 - 40*a^3*x)/((b*x^3 + a)^4*a^3*b*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\int \frac{x^3}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

input
```
int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.37

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
( - 40*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
*a**4 - 160*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt
(3)))*a**3*b*x**3 - 240*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a
**(1/3)*sqrt(3)))*a**2*b**2*x**6 - 160*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2
*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**3*x**9 - 40*a**(1/3)*sqrt(3)*atan((a
**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**4*x**12 - 20*a**(1/3)*log(a
**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**4 - 80*a**(1/3)*log(a**(
2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b*x**3 - 120*a**(1/3)*log
(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**2*x**6 - 80*a**(1
/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**3*x**9 - 20*a
**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**4*x**12 + 4
0*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**4 + 160*a**(1/3)*log(a**(1/3) + b
**(1/3)*x)*a**3*b*x**3 + 240*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**2*b**2
*x**6 + 160*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b**3*x**9 + 40*a**(1/3)*
log(a**(1/3) + b**(1/3)*x)*b**4*x**12 - 120*b**(1/3)*a**4*x + 279*b**(1/3)
*a**3*b*x**4 + 216*b**(1/3)*a**2*b**2*x**7 + 60*b**(1/3)*a*b**3*x**10)/(29
16*b**(1/3)*a**4*b*(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**
9 + b**4*x**12))
```

3.99 $\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 359

$$\begin{aligned}\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx &= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{x^2}{12a(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x^2}{54a^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{35x^2}{324a^3(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{35(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{35(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{35(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
35/243*x^2/a^4/((b*x^3+a)^2)^(1/2)+1/12*x^2/a/(b*x^3+a)^3/((b*x^3+a)^2)^(1
/2)+5/54*x^2/a^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+35/324*x^2/a^3/(b*x^3+a)/
((b*x^3+a)^2)^(1/2)-35/729*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1
/2)/a^(1/3))*3^(1/2)/a^(13/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)-35/729*(b*x^3+a)
*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)+35/1458*(b*x^3
+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(13/3)/b^(2/3)/((b*x^3+a)^
2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.61

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(243a^{10/3}x^2+270a^{7/3}x^2(a+bx^3)+315a^{4/3}x^2(a+bx^3)^2+420\cdots\right.}{\cdots}$$

input
```
Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(243*a^(10/3)*x^2 + 270*a^(7/3)*x^2*(a + b*x^3) + 315*a^(4/3)
*x^2*(a + b*x^3)^2 + 420*a^(1/3)*x^2*(a + b*x^3)^3 + (140*Sqrt[3]*(a + b*x
^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(2/3) - (140*(
a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (70*(a + b*x^3)^4*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3)))/(2916*a^(13/3)*((a + b*
x^3)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.71, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {1384, 27, 819, 819, 819, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5(a+bx^3)\int \frac{x}{b^5(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right)\int \frac{x}{\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{5\int \frac{x}{\left(bx^3+a\right)^4}dx}{6a}+\frac{x^2}{12a(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{5\left(\frac{7\int \frac{x}{\left(bx^3+a\right)^3}dx}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}\right)}{6a}+\frac{x^2}{12a(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{5\left(\frac{7\left(\frac{2\int \frac{x}{\left(bx^3+a\right)^2}dx}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}\right)}{6a}+\frac{x^2}{12a(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\int \frac{x}{bx^3+a}dx}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}\right)}{6a}+\frac{x^2}{12a(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 821

$$\frac{(a+bx^3)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}\right)}{6a}+\frac{x^2}{12a(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

↓ 16

$$\frac{(a+bx^3)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}\right)}{6a}+\frac{x^2}{12a(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

↓ 1142

$$(a+bx^3)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}-\cdots\right)}{6a}\cdots\right.$$

$\Big\downarrow$ 25

$$\left(a+bx^3\right)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\right.}{6a}\right.$$

$\downarrow$ 27

$$\frac{\left(a+bx^3\right)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}-}{6a}\right.}{\sqrt{a^2+2abx^3+b^2x^6}}$$

↓ 1082

$$\left(a+bx^3\right)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}\,d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}\right.\right.\right.}{6a}\right.$$

↓ 217

$$(a+bx^3)\ \left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right.}}{6a}\right.$$

↓ 1103

$$(a+bx^3)\left(\frac{5\left(\frac{7\left(\frac{2\left(\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}\right)}{9a}+\frac{x^2}{9a\left(a+bx^3\right)^3}\right)}{6a}\right.$$

input
```
Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((a + b*x^3)*(x^2/(12*a*(a + b*x^3)^4) + (5*(x^2/(9*a*(a + b*x^3)^3) + (7*
(x^2/(6*a*(a + b*x^3)^2) + (2*(x^2/(3*a*(a + b*x^3)) + (-1/3*Log[a^(1/3) +
 b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(
2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(3*a)))/(3*a)))/(9*a)))/(6*a)))/Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(\frac{35 b^3 x^{11}}{243 a^4}+\frac{175 b^2 x^8}{324 a^3}+\frac{20 b\,x^5}{27 a^2}+\frac{104 x^2}{243 a}\right)}{\left(b\,x^3+a\right)^5}+\frac{35\sqrt{\left(b\,x^3+a\right)^2}\,\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{729\left(b\,x^3+a\right)a^4 b}$
default	$\frac{\left(-140\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4 x^{12}-140\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4 x^{12}+70\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4 x^{12}+420\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4 x^{11}-560\sqrt{}\right.}{}$

input
```
int(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(35/243/a^4*b^3*x^11+175/324/a^3*b^2*x^8+2
0/27/a^2*b*x^5+104/243/a*x^2)+35/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a^4/b*s
um(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 734, normalized size of antiderivative = 2.04

$$\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/2916*(420*a*b^5*x^11 + 1575*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b
^2*x^2 + 210*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4
*b^2*x^3 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3
)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a)
 - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b
^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*
x + (-a*b^2)^(2/3)) - 140*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*
b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a
^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(420*a*b^5*x
^11 + 1575*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^2*x^2 + 420*sqrt(1/
3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sq
rt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*
b^2)^(1/3)/a)/b) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^
3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3))
 - 140*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^
2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*
b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2)]
```

Sympy [F]

$$\int \frac{x}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \int \frac{x}{\left((a + bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(x/((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.53

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{140\,b^3x^{11}+525\,ab^2x^8+720\,a^2bx^5+416\,a^3x^2}{972\,(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8)} + \frac{35\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^4b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{35\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^4b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{35\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^4b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input

```
integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output

```
1/972*(140*b^3*x^11 + 525*a*b^2*x^8 + 720*a^2*b*x^5 + 416*a^3*x^2)/(a^4*b^
4*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 35/729*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(1/3))
 + 35/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(1/3)) - 35
/729*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.55

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{35\,\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,(-ab^2)^{\frac{1}{3}}\,a^4\mathrm{sgn}\,(bx^3+a)} - \frac{35\,\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^5\mathrm{sgn}\,(bx^3+a)} - \frac{35\,\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^5b^2\mathrm{sgn}\,(bx^3+a)} + \frac{140\,b^3x^{11}+525\,ab^2x^8+720\,a^2bx^5+416\,a^3x^2}{972\,(bx^3+a)^4a^4\mathrm{sgn}\,(bx^3+a)}$$

input

```
integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-35/1458*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^4*sgn(
b*x^3 + a)) - 35/729*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*sgn(b*x^
3 + a)) - 35/729*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/(a^5*b^2*sgn(b*x^3 + a)) + 1/972*(140*b^3*x^11 + 525*a
*b^2*x^8 + 720*a^2*b*x^5 + 416*a^3*x^2)/((b*x^3 + a)^4*a^4*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{x}{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}\,dx$$

input
```
int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.34

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{-140\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^4-560\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^3b\,x^3-840\sqrt{3}\,atan\left(\right.}{}$$

input
```
int(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
( - 140*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**4 -
560*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**3*b*x**3
 - 840*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b**
2*x**6 - 560*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*
b**3*x**9 - 140*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
*b**4*x**12 + 1248*b**(2/3)*a**(1/3)*a**3*x**2 + 2160*b**(2/3)*a**(1/3)*a*
*2*b*x**5 + 1575*b**(2/3)*a**(1/3)*a*b**2*x**8 + 420*b**(2/3)*a**(1/3)*b**
3*x**11 + 70*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**4 + 28
0*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b*x**3 + 420*lo
g(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**2*x**6 + 280*log
(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**3*x**9 + 70*log(a**(
2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**4*x**12 - 140*log(a**(1/3)
+ b**(1/3)*x)*a**4 - 560*log(a**(1/3) + b**(1/3)*x)*a**3*b*x**3 - 840*log(
a**(1/3) + b**(1/3)*x)*a**2*b**2*x**6 - 560*log(a**(1/3) + b**(1/3)*x)*a*b
**3*x**9 - 140*log(a**(1/3) + b**(1/3)*x)*b**4*x**12)/(2916*b**(2/3)*a**(1
/3)*a**4*(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x
**12))
```

3.100 $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 351

$$\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{55x}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{11x}{108a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{11x}{81a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{110(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{14/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{110(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{14/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{55(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{14/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
55/243*x/a^4/((b*x^3+a)^2)^(1/2)+1/12*x/a/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)+
11/108*x/a^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+11/81*x/a^3/(b*x^3+a)/((b*x^3
+a)^2)^(1/2)-110/729*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^
(1/3))*3^(1/2)/a^(14/3)/b^(1/3)/((b*x^3+a)^2)^(1/2)+110/729*(b*x^3+a)*ln(a
^(1/3)+b^(1/3)*x)/a^(14/3)/b^(1/3)/((b*x^3+a)^2)^(1/2)-55/729*(b*x^3+a)*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)/b^(1/3)/((b*x^3+a)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.60

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(243a^{11/3}x+297a^{8/3}x(a+bx^3)+396a^{5/3}x(a+bx^3)^2+660a^{2/3}\right.}{}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

output
```
((a + b*x^3)*(243*a^(11/3)*x + 297*a^(8/3)*x*(a + b*x^3) + 396*a^(5/3)*x*(
a + b*x^3)^2 + 660*a^(2/3)*x*(a + b*x^3)^3 + (440*Sqrt[3]*(a + b*x^3)^4*Ar
cTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(1/3) + (440*(a + b*x^
3)^4*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (220*(a + b*x^3)^4*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3)))/(2916*a^(14/3)*((a + b*x^3)^2)
^(5/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.77, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {1384, 749, 749, 749, 749, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b^5(a+bx^3)\int \frac{1}{(b^2x^3+ab)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 749$$

$$\frac{b^5(a+bx^3)\left(\frac{11\int\frac{1}{(b^2x^3+ab)^4}dx}{12ab}+\frac{x}{12ab^5(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 749

$$\frac{b^5(a+bx^3)\left(\frac{11\left(\frac{8\int\frac{1}{(b^2x^3+ab)^3}dx}{9ab}+\frac{x}{9ab^4(a+bx^3)^3}\right)}{12ab}+\frac{x}{12ab^5(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 749

$$\frac{b^5(a+bx^3)\left(\frac{11\left(\frac{8\left(\frac{5\int\frac{1}{(b^2x^3+ab)^2}dx}{6ab}+\frac{x}{6ab^3(a+bx^3)^2}\right)}{9ab}+\frac{x}{9ab^4(a+bx^3)^3}\right)}{12ab}+\frac{x}{12ab^5(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$\Big\downarrow$ 749

$$\frac{b^5(a+bx^3)\left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\int\frac{1}{b^2x^3+ab}dx}{3ab}+\frac{x}{3ab^2(a+bx^3)}\right)}{6ab}+\frac{x}{6ab^3(a+bx^3)^2}\right)}{9ab}+\frac{x}{9ab^4(a+bx^3)^3}\right)}{12ab}+\frac{x}{12ab^5(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

↓ 750

$$b^5\left(a+bx^3\right)\left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\int\frac{\sqrt[3]{b}\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}b^{2/3}}+\frac{\int\frac{1}{b^{2/3}x+\sqrt[3]{a}\sqrt[3]{b}}dx}{3a^{2/3}b^{2/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{9ab}+\frac{x}{9ab^4\left(a+bx^3\right)^3}\right)}{12ab}+\cdots\right.$$

↓ 16

$$b^5\left(a+bx^3\right)\left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\int\frac{\sqrt[3]{b}\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{9ab}+\frac{x}{9ab^4\left(a+bx^3\right)^3}\right)}{12ab}+\frac{}{12ab}\right.$$

$\Big\downarrow$ 27

$$b^5\left(a+bx^3\right) \quad \left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{9ab}+\frac{x}{9ab^4\left(a+bx^3\right)^3}\right)}{12ab}+\frac{\ldots}{12a\ldots}\right.$$

↓ 1142

$$b^5\left(a+bx^3\right)\left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx-\frac{\int-\frac{b\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{2b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{\cdots}{6ab}\right)}{9ab}\cdots\right)}{12ab}\cdots\right.$$

$\downarrow$ 25

$$
b^5\left(a+bx^3\right) \left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{\int \frac{b\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{2b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} \right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)} \right)}{6ab}+\frac{\cdots}{6ab^3} \cdots}{9ab} \cdots}{12ab} \cdots
$$

↓ 27

$$b^5\left(a+bx^3\right) \quad \left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{\cdots}{6ab}\right)}{9ab}\cdots\right)}{12ab}\cdots \right.$$

↓ 1082

$$\left(\frac{11\left(\frac{8\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}\,dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}\,d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{\cdots}{6a\cdots}\right)}{9ab}\cdots\right.$$

↓ 217

$$\left(11\,\frac{\left(8\,\frac{\left(5\,\frac{\left(2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{4/3}x^2-\sqrt[3]{a}bx+a^{2/3}b^{2/3}}\,dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}\right)\right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)}\right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2}\right)}{9ab}\right.$$

↓ 1103

$$\left(11 \left(\frac{8 \left(\frac{5 \left(\frac{2 \left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2b}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} \right)}{3ab}+\frac{x}{3ab^2\left(a+bx^3\right)} \right)}{6ab}+\frac{x}{6ab^3\left(a+bx^3\right)^2} \right)}{9ab} \right.\right.$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

output
```
(b^5*(a + b*x^3)*(x/(12*a*b^5*(a + b*x^3)^4) + (11*(x/(9*a*b^4*(a + b*x^3)
^3) + (8*(x/(6*a*b^3*(a + b*x^3)^2) + (5*(x/(3*a*b^2*(a + b*x^3)) + (2*(Lo
g[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(4/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]])/b) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2]/(2*b))/(3*a^(2/3)*b^(1/3))))/(3*a*b)))/(6*a*b)))/(9*a*b)))/(12*a*b))
)/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(\frac{55b^3x^{10}}{243a^4}+\frac{22b^2x^7}{27a^3}+\frac{341b\,x^4}{324a^2}+\frac{133x}{243a}\right)}{(b\,x^3+a)^5}+\frac{110\sqrt{(b\,x^3+a)^2}\,\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{729(b\,x^3+a)a^4b}$
default	$\frac{\left(-440\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}+440\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}-220\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}+660\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x^{10}-1760\right.}{}$

input
```
int(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(55/243/a^4*b^3*x^10+22/27/a^3*b^2*x^7+341
/324/a^2*b*x^4+133/243/a*x)+110/729*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/a^4/b*su
m(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 719, normalized size of antiderivative = 2.05

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/2916*(660*a^2*b^4*x^10 + 2376*a^3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5
*b*x + 660*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b
^2*x^3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*
x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt
(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2
*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a
^2*b)^(1/3)*a) + 440*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3
 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^
4*x^9 + 6*a^8*b^3*x^6 + 4*a^9*b^2*x^3 + a^10*b), 1/2916*(660*a^2*b^4*x^10
+ 2376*a^3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*b*x + 1320*sqrt(1/3)*(a*b
^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2
*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((
a^2*b)^(1/3)/b)/a^2) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3
*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*
a) + 440*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2
*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^4*x^9 + 6*a^
8*b^3*x^6 + 4*a^9*b^2*x^3 + a^10*b)]
```

Sympy [F]

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.54

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \frac{220\,b^3x^{10} + 792\,ab^2x^7 + 1023\,a^2bx^4 + 532\,a^3x}{972\,(a^4b^4x^{12} + 4\,a^5b^3x^9 + 6\,a^6b^2x^6 + 4\,a^7bx^3 + a^8)} + \frac{110\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{55\,\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{110\,\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/972*(220*b^3*x^10 + 792*a*b^2*x^7 + 1023*a^2*b*x^4 + 532*a^3*x)/(a^4*b^4
*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 110/729*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
 - 55/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) + 110
/729*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = -\frac{110\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^5\mathrm{sgn}\left(bx^3+a\right)} + \frac{110\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^5b\mathrm{sgn}\left(bx^3+a\right)} + \frac{55\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^5b\mathrm{sgn}\left(bx^3+a\right)} + \frac{220\,b^3x^{10}+792\,ab^2x^7+1023\,a^2bx^4+532\,a^3x}{972\left(bx^3+a\right)^4a^4\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
-110/729*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*sgn(b*x^3 + a)) + 11
0/729*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b
)^(1/3))/(a^5*b*sgn(b*x^3 + a)) + 55/729*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/(a^5*b*sgn(b*x^3 + a)) + 1/972*(220*b^3*x^10 + 792*
a*b^2*x^7 + 1023*a^2*b*x^4 + 532*a^3*x)/((b*x^3 + a)^4*a^4*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{1}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.40

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
( - 440*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))
)*a**4 - 1760*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sq
rt(3)))*a**3*b*x**3 - 2640*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)
/(a**(1/3)*sqrt(3)))*a**2*b**2*x**6 - 1760*a**(1/3)*sqrt(3)*atan((a**(1/3)
 - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**3*x**9 - 440*a**(1/3)*sqrt(3)*at
an((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**4*x**12 - 220*a**(1/3)
*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**4 - 880*a**(1/3)*l
og(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b*x**3 - 1320*a**(
1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**2*x**6 -
880*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**3*x*
*9 - 220*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**4
*x**12 + 440*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**4 + 1760*a**(1/3)*log(
a**(1/3) + b**(1/3)*x)*a**3*b*x**3 + 2640*a**(1/3)*log(a**(1/3) + b**(1/3)
*x)*a**2*b**2*x**6 + 1760*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b**3*x**9
+ 440*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b**4*x**12 + 1596*b**(1/3)*a**4*
x + 3069*b**(1/3)*a**3*b*x**4 + 2376*b**(1/3)*a**2*b**2*x**7 + 660*b**(1/3
)*a*b**3*x**10)/(2916*b**(1/3)*a**5*(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x*
*6 + 4*a*b**3*x**9 + b**4*x**12))
```

3.101 $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 400

$$\begin{aligned}\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}}\,dx &= -\frac{212bx^2}{243a^5\sqrt{a^2+2abx^3+b^2x^6}} \\ &\quad -\frac{bx^2}{12a^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{11bx^2}{54a^3(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &\quad -\frac{131bx^2}{324a^4(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{a^5x\sqrt{a^2+2abx^3+b^2x^6}} \\ &\quad +\frac{455\sqrt[3]{b}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{455\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &\quad -\frac{455\sqrt[3]{b}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output

```
-212/243*b*x^2/a^5/((b*x^3+a)^2)^(1/2)-1/12*b*x^2/a^2/(b*x^3+a)^3/((b*x^3+
a)^2)^(1/2)-11/54*b*x^2/a^3/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)-131/324*b*x^2/
a^4/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-(b*x^3+a)/a^5/x/((b*x^3+a)^2)^(1/2)+455/
729*b^(1/3)*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^
(1/2)/a^(16/3)/((b*x^3+a)^2)^(1/2)+455/729*b^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^
(1/3)*x)/a^(16/3)/((b*x^3+a)^2)^(1/2)-455/1458*b^(1/3)*(b*x^3+a)*ln(a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(16/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{(a+bx^3)\left(-243a^{10/3}bx^2-594a^{7/3}bx^2(a+bx^3)-1179a^{4/3}bx^2(a+bx^3)\right.}{}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
((a + b*x^3)*(-243*a^(10/3)*b*x^2 - 594*a^(7/3)*b*x^2*(a + b*x^3) - 1179*a
^(4/3)*b*x^2*(a + b*x^3)^2 - 2544*a^(1/3)*b*x^2*(a + b*x^3)^3 - (2916*a^(1
/3)*(a + b*x^3)^4)/x - 1820*Sqrt[3]*b^(1/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3)
 + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 1820*b^(1/3)*(a + b*x^3)^4*Log[a^(1/3
) + b^(1/3)*x] - 910*b^(1/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2]))/(2916*a^(16/3)*((a + b*x^3)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.67, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {1384, 27, 819, 819, 819, 819, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^5\left(a+bx^3\right)\int \frac{1}{b^5x^2\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(a+bx^3\right)\int \frac{1}{x^2\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{13\int \frac{1}{x^2\left(bx^3+a\right)^4}dx}{12a}+\frac{1}{12ax(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{13\left(\frac{10\int \frac{1}{x^2\left(bx^3+a\right)^3}dx}{9a}+\frac{1}{9ax\left(a+bx^3\right)^3}\right)}{12a}+\frac{1}{12ax(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{13\left(\frac{10\left(\frac{7\int \frac{1}{x^2\left(bx^3+a\right)^2}dx}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax\left(a+bx^3\right)^3}\right)}{12a}+\frac{1}{12ax(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{13\left(\frac{10\left(\frac{7\left(\frac{4\int\frac{1}{x^2\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax\left(a+bx^3\right)^3}\right)}{12a}+\frac{1}{12ax(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{(a+bx^3)\left(\frac{13\left(\frac{10\left(\frac{7\left(\frac{4\left(-\frac{b\int\frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax\left(a+bx^3\right)^3}\right)}{12a}+\frac{1}{12ax(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 821}$$

$$\Bigg(13\left(\frac{10\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax\left(a+bx^3\right)^3}\right)$$

$$\Big\downarrow 16$$

$$\left(13\left(\frac{\left(10\,\frac{\left(7\,\frac{\left(4\,-\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax\left(a+bx^3\right)^3}\right.\right.$$

↓ 1142

$$\left(\left(\left(10\,\frac{\left(7\,\frac{\left(4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}}{6a}$$

↓ 25

$$\left(\left(10\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}\right.\right.\right.$$

↓ 27

$$13\left(\frac{10\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}}{6a}}{9a}$$

↓ 1082

$$\left(\left(10\left(7\left(4\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}\,d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right.}\right.}{6a}\right.\right.\right.$$

↓ 217

$$10\left(\frac{7\left(\frac{4\left(-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}\right)}{6a}+\frac{1}{6ax\left(a-\cdots\right.}$$

↓ 1103

$$4 \quad -\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

$$7 \quad \frac{(\ldots)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}$$

$$10 \quad \frac{(\ldots)}{6a}+\frac{1}{6ax\left(a+b\right.}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
((a + b*x^3)*(1/(12*a*x*(a + b*x^3)^4) + (13*(1/(9*a*x*(a + b*x^3)^3) + (1
0*(1/(6*a*x*(a + b*x^3)^2) + (7*(1/(3*a*x*(a + b*x^3)) + (4*(-(1/(a*x)) -
(b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/(3*a)))/(6*a))
)/(9*a)))/(12*a)))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.57 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(-\frac{455 b^4 x^{12}}{243 a^5}-\frac{2275 b^3 x^9}{324 a^4}-\frac{260 b^2 x^6}{27 a^3}-\frac{1352 b x^3}{243 a^2}-\frac{1}{a}\right)}{(b x^3+a)^5 x}+\frac{455\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^{16}\_Z^3-b\right)}\_R\ln\left(\left(-4\_R^3 a^1\right.\right.\right.}{729(b x^3+a)}$
default	$-\frac{\left(-1820\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{13}-1820\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{13}+910\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{13}+5460\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4x^{12}\right.}{}$

input
```
int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(-455/243/a^5*b^4*x^12-2275/324/a^4*b^3*x^
9-260/27/a^3*b^2*x^6-1352/243/a^2*b*x^3-1/a)/x+455/729*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)*sum(_R*ln((-4*_R^3*a^16+3*b)*x-a^11*_R^2),_R=RootOf(_Z^3*a^16-b
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx =$$

$$-\frac{5460\,b^4x^{12} + 20475\,ab^3x^9 + 28080\,a^2b^2x^6 + 16224\,a^3bx^3 + 2916\,a^4 + 1820\,\sqrt{3}(b^4x^{13} + 4\,ab^3x^{10} + 6\,a^2b^2x^7}{}$$

input
```
integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/2916*(5460*b^4*x^12 + 20475*a*b^3*x^9 + 28080*a^2*b^2*x^6 + 16224*a^3*b
*x^3 + 2916*a^4 + 1820*sqrt(3)*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 +
4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sq
rt(3)) + 910*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*
x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 1820*(b^4*x^
13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*log(b
*x + a*(b/a)^(2/3)))/(a^5*b^4*x^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^
8*b*x^4 + a^9*x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2}}\, dx = \int \frac{1}{x^2 \left((a + bx^3)^2\right)^{\frac{5}{2}}}\, dx$$

input
```
integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(1/(x**2*((a + b*x**3)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2}}\, dx = -\frac{1820\, b^4x^{12} + 6825\, ab^3x^9 + 9360\, a^2b^2x^6 + 5408\, a^3bx^3 + 972\, a^4}{972\, (a^5b^4x^{13} + 4\, a^6b^3x^{10} + 6\, a^7b^2x^7 + 4\, a^8bx^4 + a^9x)} - \frac{455\, \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2\, x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\, \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\, a^5 \left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{455\, \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\, a^5 \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{455\, \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\, a^5 \left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/972*(1820*b^4*x^12 + 6825*a*b^3*x^9 + 9360*a^2*b^2*x^6 + 5408*a^3*b*x^3
 + 972*a^4)/(a^5*b^4*x^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^8*b*x^4 +
 a^9*x) - 455/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(a^5*(a/b)^(1/3)) - 455/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
^5*(a/b)^(1/3)) + 455/729*log(x + (a/b)^(1/3))/(a^5*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{455\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^6\mathrm{sgn}\left(bx^3+a\right)} + \frac{455\,\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^6b\mathrm{sgn}\left(bx^3+a\right)} - \frac{455\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^6b\mathrm{sgn}\left(bx^3+a\right)} - \frac{1}{a^5x\mathrm{sgn}\left(bx^3+a\right)} - \frac{848\,b^4x^{11}+2937\,ab^3x^8+3528\,a^2b^2x^5+1520\,a^3bx^2}{972\left(bx^3+a\right)^4a^5\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
455/729*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^6*sgn(b*x^3 + a)) + 4
55/729*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(a^6*b*sgn(b*x^3 + a)) - 455/1458*(-a*b^2)^(2/3)*log(x^2 + x*(-a
/b)^(1/3) + (-a/b)^(2/3))/(a^6*b*sgn(b*x^3 + a)) - 1/(a^5*x*sgn(b*x^3 + a)
) - 1/972*(848*b^4*x^11 + 2937*a*b^3*x^8 + 3528*a^2*b^2*x^5 + 1520*a^3*b*x
^2)/((b*x^3 + a)^4*a^5*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \int \frac{1}{x^2 (a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}} \, dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2}} \, dx = \text{Too large to display}$$

input
```
int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
(1820*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**4*b*x
+ 7280*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**3*b**
2*x**4 + 10920*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a**2*b**3*x**7 + 7280*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqr
t(3)))*a*b**4*x**10 + 1820*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3
)*sqrt(3)))*b**5*x**13 - 2916*b**(2/3)*a**(1/3)*a**4 - 16224*b**(2/3)*a**(
1/3)*a**3*b*x**3 - 28080*b**(2/3)*a**(1/3)*a**2*b**2*x**6 - 20475*b**(2/3)
*a**(1/3)*a*b**3*x**9 - 5460*b**(2/3)*a**(1/3)*b**4*x**12 - 910*log(a**(2/
3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**4*b*x - 3640*log(a**(2/3) - b
**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b**2*x**4 - 5460*log(a**(2/3) - b
**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**3*x**7 - 3640*log(a**(2/3) - b
**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**4*x**10 - 910*log(a**(2/3) - b**(
1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**5*x**13 + 1820*log(a**(1/3) + b**(1/3)
*x)*a**4*b*x + 7280*log(a**(1/3) + b**(1/3)*x)*a**3*b**2*x**4 + 10920*log(
a**(1/3) + b**(1/3)*x)*a**2*b**3*x**7 + 7280*log(a**(1/3) + b**(1/3)*x)*a*
b**4*x**10 + 1820*log(a**(1/3) + b**(1/3)*x)*b**5*x**13)/(2916*b**(2/3)*a*
*(1/3)*a**5*x*(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9 + b
**4*x**12))
```

3.102 $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 394

$$\begin{aligned}\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx &= -\frac{527bx}{486a^5\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{bx}{12a^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{23bx}{108a^3(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &- \frac{73bx}{162a^4(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a+bx^3}{2a^5x^2\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{770b^{2/3}(a+bx^3)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{770b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}} \\ &+ \frac{385b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

output
```
-527/486*b*x/a^5/((b*x^3+a)^2)^(1/2)-1/12*b*x/a^2/(b*x^3+a)^3/((b*x^3+a)^2
)^(1/2)-23/108*b*x/a^3/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)-73/162*b*x/a^4/(b*x
^3+a)/((b*x^3+a)^2)^(1/2)-1/2*(b*x^3+a)/a^5/x^2/((b*x^3+a)^2)^(1/2)+770/72
9*b^(2/3)*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1
/2)/a^(17/3)/((b*x^3+a)^2)^(1/2)-770/729*b^(2/3)*(b*x^3+a)*ln(a^(1/3)+b^(1
/3)*x)/a^(17/3)/((b*x^3+a)^2)^(1/2)+385/729*b^(2/3)*(b*x^3+a)*ln(a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(17/3)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \frac{(a+bx^3)\left(-243a^{11/3}bx - 621a^{8/3}bx(a+bx^3) - 1314a^{5/3}bx(a+bx^3)^2 - \right.}{}$$

input
```
Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
((a + b*x^3)*(-243*a^(11/3)*b*x - 621*a^(8/3)*b*x*(a + b*x^3) - 1314*a^(5/
3)*b*x*(a + b*x^3)^2 - 3162*a^(2/3)*b*x*(a + b*x^3)^3 - (1458*a^(2/3)*(a +
 b*x^3)^4)/x^2 - 3080*Sqrt[3]*b^(2/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))] - 3080*b^(2/3)*(a + b*x^3)^4*Log[a^(1/3) + b^
(1/3)*x] + 1540*b^(2/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2]))/(2916*a^(17/3)*((a + b*x^3)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.68, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {1384, 27, 819, 819, 819, 819, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b^5 (a+bx^3) \int \frac{1}{b^5x^3(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^3) \int \frac{1}{x^3(bx^3+a)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(a+bx^3)\left(\frac{7\int \frac{1}{x^3(bx^3+a)^4}dx}{6a}+\frac{1}{12ax^2(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(a+bx^3)\left(\frac{7\left(\frac{11\int \frac{1}{x^3(bx^3+a)^3}dx}{9a}+\frac{1}{9ax^2(a+bx^3)^3}\right)}{6a}+\frac{1}{12ax^2(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(a+bx^3)\left(\frac{7\left(\frac{11\left(\frac{4\int \frac{1}{x^3(bx^3+a)^2}dx}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{9a}+\frac{1}{9ax^2(a+bx^3)^3}\right)}{6a}+\frac{1}{12ax^2(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{\left(a+bx^3\right)\left(\frac{7\left(\frac{11\left(\frac{4\left(\frac{5\int \frac{1}{x^3\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax^2\left(a+bx^3\right)^3}\right)}{6a}+\frac{1}{12ax^2(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \; 847$$

$$\frac{(a+bx^3)\left(\frac{7\left(\frac{11\left(\frac{4\left(\frac{5\left(-\frac{b\int\frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2(a+bx^3)}\right)}{3a}+\frac{1}{6ax^2(a+bx^3)^2}\right)}{9a}+\frac{1}{9ax^2(a+bx^3)^3}\right)}{6a}+\frac{1}{12ax^2(a+bx^3)^4}\right)}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{ 750}$$

$$\left(7\left(\frac{11\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax^2\left(a+bx^3\right)^3}\right.\right.$$

$\downarrow$ 16

$$\left(\left(7\left(\frac{11\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{1}{6ax^2\left(a+bx^3\right)^2}\right)}{9a}+\frac{1}{9ax^2\left(a+bx^3\right)^3}\right)\right.\right.$$

↓ 1142

$$11\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx\right.}}{3a}\right.$$

$\downarrow$ 25

$$\left(\left(\frac{11\left(\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}\right.\right.\right.$$

$\downarrow$ 27

$$7\,\frac{11\,\frac{4\,\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\,dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx\right.}}{3a}}{9a}$$

↓ 1082

$$\left(\left(11\frac{\left(4\frac{\left(5\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+b\right.}}{3a}\right.\right.$$

↓ 217

$$\left(\left(\left(11\left(4\left(5\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}\right)}{3a}+\frac{}{6ax^2\left(\right.}\right.\right.\right.$$

↓ 1103

5 $$-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

4 $$\frac{\left(\ \cdots\ \right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}$$

11 $$\frac{\left(\ \cdots\ \right)}{3a}+\frac{\cdots}{6ax^2}$$

input
```
Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

output
```
((a + b*x^3)*(1/(12*a*x^2*(a + b*x^3)^4) + (7*(1/(9*a*x^2*(a + b*x^3)^3) +
 (11*(1/(6*a*x^2*(a + b*x^3)^2) + (4*(1/(3*a*x^2*(a + b*x^3)) + (5*(-1/2*1
/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/a))/(3*a)))/(3*a))
)/(9*a)))/(6*a)))/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.82 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(-\frac{385 b^4 x^{12}}{243 a^5}-\frac{154 b^3 x^9}{27 a^4}-\frac{2387 b^2 x^6}{324 a^3}-\frac{931 b x^3}{243 a^2}-\frac{1}{2 a}\right)}{(b x^3+a)^5 x^2}+\frac{770\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^{17}\_Z^3+b^2\right)}\_R\ln\left(\left(-4\_R^3 a\right.\right.\right.}{729(b x^3+a)}$
default	$-\frac{\left(-3080\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{14}+3080\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{14}-1540\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{14}+4620\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x^{1}\right.}{}$

input
```
int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^3+a)^2)^(1/2)/(b*x^3+a)^5*(-385/243/a^5*b^4*x^12-154/27/a^4*b^3*x^9-
2387/324/a^3*b^2*x^6-931/243/a^2*b*x^3-1/2/a)/x^2+770/729*((b*x^3+a)^2)^(1
/2)/(b*x^3+a)*sum(_R*ln((-4*_R^3*a^17-3*b^2)*x-a^6*b*_R),_R=RootOf(_Z^3*a^
17+b^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx =$$

$$-\frac{4620\,b^4x^{12}+16632\,ab^3x^9+21483\,a^2b^2x^6+11172\,a^3bx^3+1458\,a^4-3080\,\sqrt{3}(b^4x^{14}+4\,ab^3x^{11}+6\,a^2b^2x^{8}}{}$$

input
```
integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/2916*(4620*b^4*x^12 + 16632*a*b^3*x^9 + 21483*a^2*b^2*x^6 + 11172*a^3*b
*x^3 + 1458*a^4 - 3080*sqrt(3)*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 +
4*a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^
2)^(2/3) - sqrt(3)*b)/b) + 1540*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 +
 4*a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1
/3) + a^2*(-b^2/a^2)^(2/3)) - 3080*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^
8 + 4*a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))
/(a^5*b^4*x^14 + 4*a^6*b^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 + a^9*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx=\int \frac{1}{x^3\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral(1/(x**3*((a + b*x**3)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.49

$$\begin{aligned}\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \\ &-\frac{1540\,b^4x^{12}+5544\,ab^3x^9+7161\,a^2b^2x^6+3724\,a^3bx^3+486\,a^4}{972\,\left(a^5b^4x^{14}+4\,a^6b^3x^{11}+6\,a^7b^2x^8+4\,a^8bx^5+a^9x^2\right)} \\ &-\frac{770\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &+\frac{385\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{770\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

input
```
integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/972*(1540*b^4*x^12 + 5544*a*b^3*x^9 + 7161*a^2*b^2*x^6 + 3724*a^3*b*x^3
 + 486*a^4)/(a^5*b^4*x^14 + 4*a^6*b^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 +
 a^9*x^2) - 770/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/(a^5*(a/b)^(2/3)) + 385/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(
a^5*(a/b)^(2/3)) - 770/729*log(x + (a/b)^(1/3))/(a^5*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{770\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^6\mathrm{sgn}\left(bx^3+a\right)} - \frac{770\,\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^6\mathrm{sgn}\left(bx^3+a\right)} - \frac{385\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^6\mathrm{sgn}\left(bx^3+a\right)} - \frac{1}{2\,a^5x^2\mathrm{sgn}\left(bx^3+a\right)} - \frac{1054\,b^4x^{10}+3600\,ab^3x^7+4245\,a^2b^2x^4+1780\,a^3bx}{972\left(bx^3+a\right)^4a^5\mathrm{sgn}\left(bx^3+a\right)}$$

input
```
integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
770/729*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^6*sgn(b*x^3 + a)) - 7
70/729*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(a^6*sgn(b*x^3 + a)) - 385/729*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/(a^6*sgn(b*x^3 + a)) - 1/2/(a^5*x^2*sgn(b*x^3 + a))
 - 1/972*(1054*b^4*x^10 + 3600*a*b^3*x^7 + 4245*a^2*b^2*x^4 + 1780*a^3*b*x
)/((b*x^3 + a)^4*a^5*sgn(b*x^3 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{1}{x^3\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

input
```
int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

output
```
int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^3 (a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
(3080*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a**4*b*x**2 + 12320*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1
/3)*sqrt(3)))*a**3*b**2*x**5 + 18480*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b
**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b**3*x**8 + 12320*a**(1/3)*sqrt(3)*ata
n((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**4*x**11 + 3080*a**(1/
3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**5*x**14 +
 1540*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**4*b*
x**2 + 6160*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a
**3*b**2*x**5 + 9240*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3
)*x**2)*a**2*b**3*x**8 + 6160*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x
+ b**(2/3)*x**2)*a*b**4*x**11 + 1540*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(
1/3)*x + b**(2/3)*x**2)*b**5*x**14 - 3080*a**(1/3)*log(a**(1/3) + b**(1/3)
*x)*a**4*b*x**2 - 12320*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**3*b**2*x**5
 - 18480*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**2*b**3*x**8 - 12320*a**(1/
3)*log(a**(1/3) + b**(1/3)*x)*a*b**4*x**11 - 3080*a**(1/3)*log(a**(1/3) +
b**(1/3)*x)*b**5*x**14 - 1458*b**(1/3)*a**5 - 11172*b**(1/3)*a**4*b*x**3 -
 21483*b**(1/3)*a**3*b**2*x**6 - 16632*b**(1/3)*a**2*b**3*x**9 - 4620*b**(
1/3)*a*b**4*x**12)/(2916*b**(1/3)*a**6*x**2*(a**4 + 4*a**3*b*x**3 + 6*a**2
*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**12))
```

3.103 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx$

Optimal result

Integrand size = 28, antiderivative size = 313

$$\begin{aligned}\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx &= \frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\,(a+bx^3)} \\ &+ \frac{5a^4b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\,(a+bx^3)} + \frac{10a^3b^2(dx)^{7+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^7(7+m)\,(a+bx^3)} \\ &+ \frac{10a^2b^3(dx)^{10+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{10}(10+m)\,(a+bx^3)} \\ &+ \frac{5ab^4(dx)^{13+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{13}(13+m)\,(a+bx^3)} + \frac{b^5(dx)^{16+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{16}(16+m)\,(a+bx^3)}\end{aligned}$$

output
```
a^5*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+5*a^4*b*(d*x)^(4+m)*
((b*x^3+a)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)+10*a^3*b^2*(d*x)^(7+m)*((b*x^3+a)^
2)^(1/2)/d^7/(7+m)/(b*x^3+a)+10*a^2*b^3*(d*x)^(10+m)*((b*x^3+a)^2)^(1/2)/d
^10/(10+m)/(b*x^3+a)+5*a*b^4*(d*x)^(13+m)*((b*x^3+a)^2)^(1/2)/d^13/(13+m)/
(b*x^3+a)+b^5*(d*x)^(16+m)*((b*x^3+a)^2)^(1/2)/d^16/(16+m)/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.35

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{x(dx)^m \left(\left(a+bx^3\right)^2\right)^{5/2} \left(\frac{a^5}{1+m}+\frac{5a^4bx^3}{4+m}+\frac{10a^3b^2x^6}{7+m}+\frac{10a^2b^3x^9}{10+m}+\frac{5ab^4x^{12}}{13+m}+\frac{b^5x^{15}}{16+m}\right)}{\left(a+bx^3\right)^5}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x*(d*x)^m*((a + b*x^3)^2)^(5/2)*(a^5/(1 + m) + (5*a^4*b*x^3)/(4 + m) + (1
0*a^3*b^2*x^6)/(7 + m) + (10*a^2*b^3*x^9)/(10 + m) + (5*a*b^4*x^12)/(13 +
m) + (b^5*x^15)/(16 + m)))/(a + b*x^3)^5
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2} (dx)^m \, dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int b^5 (dx)^m \left(bx^3+a\right)^5 dx}{b^5 \left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6} \int (dx)^m \left(bx^3+a\right)^5 dx}{a+bx^3}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5(dx)^m+\frac{5a^4b(dx)^{m+3}}{d^3}+\frac{10a^3b^2(dx)^{m+6}}{d^6}+\frac{10a^2b^3(dx)^{m+9}}{d^9}+\frac{5ab^4(dx)^{m+12}}{d^{12}}+\frac{b^5(dx)^{m+15}}{d^{15}}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^5(dx)^{m+1}}{d(m+1)}+\frac{5a^4b(dx)^{m+4}}{d^4(m+4)}+\frac{10a^3b^2(dx)^{m+7}}{d^7(m+7)}+\frac{10a^2b^3(dx)^{m+10}}{d^{10}(m+10)}+\frac{5ab^4(dx)^{m+13}}{d^{13}(m+13)}+\frac{b^5(dx)^{m+16}}{d^{16}(m+16)}\right)}{a+bx^3}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^5*(d*x)^(1 + m))/(d*(1 + m)) + (5*a^4
*b*(d*x)^(4 + m))/(d^4*(4 + m)) + (10*a^3*b^2*(d*x)^(7 + m))/(d^7*(7 + m))
 + (10*a^2*b^3*(d*x)^(10 + m))/(d^10*(10 + m)) + (5*a*b^4*(d*x)^(13 + m))/
(d^13*(13 + m)) + (b^5*(d*x)^(16 + m))/(d^16*(16 + m))))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
 - 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.45

method	result
gosper	$\frac{x\left(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}+5714m\,x^{15}b^5+2555a\,b^4m^3x^{12}+3640b^5x^{15}+\cdots\right.}{}$
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}+5714m\,x^{15}b^5+2555a\,b^4m^3x^{12}+\cdots\right.}{}$
orering	$\frac{\left(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}+5714m\,x^{15}b^5+2555a\,b^4m^3x^{12}+3640b^5x^{15}+\cdots\right.}{}$

input
```
int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b^5*m^5*x^15+35*b^5*m^4*x^15+445*b^5*m^3*x^15+5*a*b^4*m^5*x^12+2485*b^5
*m^2*x^15+190*a*b^4*m^4*x^12+5714*b^5*m*x^15+2555*a*b^4*m^3*x^12+3640*b^5*
x^15+10*a^2*b^3*m^5*x^9+14810*a*b^4*m^2*x^12+410*a^2*b^3*m^4*x^9+34840*a*b
^4*m*x^12+5950*a^2*b^3*m^3*x^9+22400*a*b^4*x^12+10*a^3*b^2*m^5*x^6+36550*a
^2*b^3*m^2*x^9+440*a^3*b^2*m^4*x^6+89240*a^2*b^3*m*x^9+6970*a^3*b^2*m^3*x^
6+58240*a^2*b^3*x^9+5*a^4*b*m^5*x^3+47260*a^3*b^2*m^2*x^6+235*a^4*b*m^4*x^
3+123920*a^3*b^2*m*x^6+4085*a^4*b*m^3*x^3+83200*a^3*b^2*x^6+a^5*m^5+31685*
a^4*b*m^2*x^3+50*a^5*m^4+100630*a^4*b*m*x^3+955*a^5*m^3+72800*a^4*b*x^3+86
50*a^5*m^2+36824*a^5*m+58240*a^5)*(d*x)^m*((b*x^3+a)^2)^(5/2)/(1+m)/(4+m)/
(7+m)/(10+m)/(13+m)/(16+m)/(b*x^3+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.18

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx = \frac{\left(\left(b^5m^5+35\,b^5m^4+445\,b^5m^3+2485\,b^5m^2+5714\,b^5m+3640\,b^5\right)x^{16}+5\left(ab^4m^5+38\,ab^4m\cdots\right.\right.}{}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
((b^5*m^5 + 35*b^5*m^4 + 445*b^5*m^3 + 2485*b^5*m^2 + 5714*b^5*m + 3640*b^
5)*x^16 + 5*(a*b^4*m^5 + 38*a*b^4*m^4 + 511*a*b^4*m^3 + 2962*a*b^4*m^2 + 6
968*a*b^4*m + 4480*a*b^4)*x^13 + 10*(a^2*b^3*m^5 + 41*a^2*b^3*m^4 + 595*a^
2*b^3*m^3 + 3655*a^2*b^3*m^2 + 8924*a^2*b^3*m + 5824*a^2*b^3)*x^10 + 10*(a
^3*b^2*m^5 + 44*a^3*b^2*m^4 + 697*a^3*b^2*m^3 + 4726*a^3*b^2*m^2 + 12392*a
^3*b^2*m + 8320*a^3*b^2)*x^7 + 5*(a^4*b*m^5 + 47*a^4*b*m^4 + 817*a^4*b*m^3
 + 6337*a^4*b*m^2 + 20126*a^4*b*m + 14560*a^4*b)*x^4 + (a^5*m^5 + 50*a^5*m
^4 + 955*a^5*m^3 + 8650*a^5*m^2 + 36824*a^5*m + 58240*a^5)*x)*(d*x)^m/(m^6
 + 51*m^5 + 1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

Sympy [F]

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} dx = \int (dx)^m \left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}} dx$$

input
```
integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral((d*x)**m*((a + b*x**3)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.78

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} dx = \frac{((m^5 + 35\,m^4 + 445\,m^3 + 2485\,m^2 + 5714\,m + 3640)b^5d^mx^{16} + 5\,(m^5 + 38\,m^4 + 511\,m^3 +}{}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
((m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)*b^5*d^m*x^16 + 5*(m^5
 + 38*m^4 + 511*m^3 + 2962*m^2 + 6968*m + 4480)*a*b^4*d^m*x^13 + 10*(m^5 +
 41*m^4 + 595*m^3 + 3655*m^2 + 8924*m + 5824)*a^2*b^3*d^m*x^10 + 10*(m^5 +
 44*m^4 + 697*m^3 + 4726*m^2 + 12392*m + 8320)*a^3*b^2*d^m*x^7 + 5*(m^5 +
47*m^4 + 817*m^3 + 6337*m^2 + 20126*m + 14560)*a^4*b*d^m*x^4 + (m^5 + 50*m
^4 + 955*m^3 + 8650*m^2 + 36824*m + 58240)*a^5*d^m*x)*x^m/(m^6 + 51*m^5 +
1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(247) = 494$.

Time = 0.16 (sec) , antiderivative size = 900, normalized size of antiderivative = 2.88

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
((d*x)^m*b^5*m^5*x^16*sgn(b*x^3 + a) + 35*(d*x)^m*b^5*m^4*x^16*sgn(b*x^3 +
 a) + 445*(d*x)^m*b^5*m^3*x^16*sgn(b*x^3 + a) + 5*(d*x)^m*a*b^4*m^5*x^13*s
gn(b*x^3 + a) + 2485*(d*x)^m*b^5*m^2*x^16*sgn(b*x^3 + a) + 190*(d*x)^m*a*b
^4*m^4*x^13*sgn(b*x^3 + a) + 5714*(d*x)^m*b^5*m*x^16*sgn(b*x^3 + a) + 2555
*(d*x)^m*a*b^4*m^3*x^13*sgn(b*x^3 + a) + 3640*(d*x)^m*b^5*x^16*sgn(b*x^3 +
 a) + 10*(d*x)^m*a^2*b^3*m^5*x^10*sgn(b*x^3 + a) + 14810*(d*x)^m*a*b^4*m^2
*x^13*sgn(b*x^3 + a) + 410*(d*x)^m*a^2*b^3*m^4*x^10*sgn(b*x^3 + a) + 34840
*(d*x)^m*a*b^4*m*x^13*sgn(b*x^3 + a) + 5950*(d*x)^m*a^2*b^3*m^3*x^10*sgn(b
*x^3 + a) + 22400*(d*x)^m*a*b^4*x^13*sgn(b*x^3 + a) + 10*(d*x)^m*a^3*b^2*m
^5*x^7*sgn(b*x^3 + a) + 36550*(d*x)^m*a^2*b^3*m^2*x^10*sgn(b*x^3 + a) + 44
0*(d*x)^m*a^3*b^2*m^4*x^7*sgn(b*x^3 + a) + 89240*(d*x)^m*a^2*b^3*m*x^10*sg
n(b*x^3 + a) + 6970*(d*x)^m*a^3*b^2*m^3*x^7*sgn(b*x^3 + a) + 58240*(d*x)^m
*a^2*b^3*x^10*sgn(b*x^3 + a) + 5*(d*x)^m*a^4*b*m^5*x^4*sgn(b*x^3 + a) + 47
260*(d*x)^m*a^3*b^2*m^2*x^7*sgn(b*x^3 + a) + 235*(d*x)^m*a^4*b*m^4*x^4*sgn
(b*x^3 + a) + 123920*(d*x)^m*a^3*b^2*m*x^7*sgn(b*x^3 + a) + 4085*(d*x)^m*a
^4*b*m^3*x^4*sgn(b*x^3 + a) + 83200*(d*x)^m*a^3*b^2*x^7*sgn(b*x^3 + a) + (
d*x)^m*a^5*m^5*x*sgn(b*x^3 + a) + 31685*(d*x)^m*a^4*b*m^2*x^4*sgn(b*x^3 +
a) + 50*(d*x)^m*a^5*m^4*x*sgn(b*x^3 + a) + 100630*(d*x)^m*a^4*b*m*x^4*sgn(
b*x^3 + a) + 955*(d*x)^m*a^5*m^3*x*sgn(b*x^3 + a) + 72800*(d*x)^m*a^4*b*x^
4*sgn(b*x^3 + a) + 8650*(d*x)^m*a^5*m^2*x*sgn(b*x^3 + a) + 36824*(d*x)^...
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx = \int (d\,x)^m\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2} \,dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{x^m d^m x(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}}{}$$

input
```
int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
(x**m*d**m*x*(a**5*m**5 + 50*a**5*m**4 + 955*a**5*m**3 + 8650*a**5*m**2 +
36824*a**5*m + 58240*a**5 + 5*a**4*b*m**5*x**3 + 235*a**4*b*m**4*x**3 + 40
85*a**4*b*m**3*x**3 + 31685*a**4*b*m**2*x**3 + 100630*a**4*b*m*x**3 + 7280
0*a**4*b*x**3 + 10*a**3*b**2*m**5*x**6 + 440*a**3*b**2*m**4*x**6 + 6970*a*
*3*b**2*m**3*x**6 + 47260*a**3*b**2*m**2*x**6 + 123920*a**3*b**2*m*x**6 +
83200*a**3*b**2*x**6 + 10*a**2*b**3*m**5*x**9 + 410*a**2*b**3*m**4*x**9 +
5950*a**2*b**3*m**3*x**9 + 36550*a**2*b**3*m**2*x**9 + 89240*a**2*b**3*m*x
**9 + 58240*a**2*b**3*x**9 + 5*a*b**4*m**5*x**12 + 190*a*b**4*m**4*x**12 +
 2555*a*b**4*m**3*x**12 + 14810*a*b**4*m**2*x**12 + 34840*a*b**4*m*x**12 +
 22400*a*b**4*x**12 + b**5*m**5*x**15 + 35*b**5*m**4*x**15 + 445*b**5*m**3
*x**15 + 2485*b**5*m**2*x**15 + 5714*b**5*m*x**15 + 3640*b**5*x**15))/(m**
6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240)
```

3.104 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 205

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2} \, dx = \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\,(a+bx^3)} + \frac{3a^2b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\,(a+bx^3)} + \frac{3ab^2(dx)^{7+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^7(7+m)\,(a+bx^3)} + \frac{b^3(dx)^{10+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^{10}(10+m)\,(a+bx^3)}$$

output
```
a^3*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+3*a^2*b*(d*x)^(4+m)*
((b*x^3+a)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)+3*a*b^2*(d*x)^(7+m)*((b*x^3+a)^2)^
(1/2)/d^7/(7+m)/(b*x^3+a)+b^3*(d*x)^(10+m)*((b*x^3+a)^2)^(1/2)/d^10/(10+m)
/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{x(dx)^m\sqrt{(a+bx^3)^2}\left(a^3(280+138m+21m^2+m^3)+3a^2b(70+87m+18m^2+m^3)\,x^3+3\cdots\right.}{(1+m)(4+m)(7+m)(10+m)\cdots}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a^3*(280 + 138*m + 21*m^2 + m^3) + 3*a^2*b
*(70 + 87*m + 18*m^2 + m^3)*x^3 + 3*a*b^2*(40 + 54*m + 15*m^2 + m^3)*x^6 +
 b^3*(28 + 39*m + 12*m^2 + m^3)*x^9))/((1 + m)*(4 + m)*(7 + m)*(10 + m)*(a
 + b*x^3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2} (dx)^m \, dx$$

$$\Big\downarrow \; 1384$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b^3(dx)^m \left(bx^3+a\right)^3 dx}{b^3\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m \left(bx^3+a\right)^3 dx}{a+bx^3}$$

$$\Big\downarrow \; 802$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^3(dx)^m+\frac{3a^2b(dx)^{m+3}}{d^3}+\frac{3ab^2(dx)^{m+6}}{d^6}+\frac{b^3(dx)^{m+9}}{d^9}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2b(dx)^{m+4}}{d^4(m+4)}+\frac{3ab^2(dx)^{m+7}}{d^7(m+7)}+\frac{b^3(dx)^{m+10}}{d^{10}(m+10)}\right)}{a+bx^3}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a^3*(d*x)^(1 + m))/(d*(1 + m)) + (3*a^2
*b*(d*x)^(4 + m))/(d^4*(4 + m)) + (3*a*b^2*(d*x)^(7 + m))/(d^7*(7 + m)) +
(b^3*(d*x)^(10 + m))/(d^10*(10 + m))))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.97

method	result
gosper	$\frac{x(b^3m^3x^9+12b^3m^2x^9+39m\,x^9b^3+3a\,b^2m^3x^6+28x^9b^3+45a\,b^2m^2x^6+162m\,x^6b^2a+3a^2b\,m^3x^3+120a\,x^6b^2+54a^2b\,m^2x^3+261m\,x^3}{(10+m)(7+m)(4+m)(1+m)(b\,x^3+a)^3}$
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,(b^3m^3x^9+12b^3m^2x^9+39m\,x^9b^3+3a\,b^2m^3x^6+28x^9b^3+45a\,b^2m^2x^6+162m\,x^6b^2a+3a^2b\,m^3x^3+120a\,x^6b^2+54a^2b\,m^2}{(b\,x^3+a)(10+m)(7+m)(4+m)(1+m)}$
orering	$\frac{(b^3m^3x^9+12b^3m^2x^9+39m\,x^9b^3+3a\,b^2m^3x^6+28x^9b^3+45a\,b^2m^2x^6+162m\,x^6b^2a+3a^2b\,m^3x^3+120a\,x^6b^2+54a^2b\,m^2x^3+261m\,x^3}{(10+m)(7+m)(4+m)(1+m)(b\,x^3+a)^3}$

input
```
int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b^3*m^3*x^9+12*b^3*m^2*x^9+39*b^3*m*x^9+3*a*b^2*m^3*x^6+28*b^3*x^9+45*a
*b^2*m^2*x^6+162*a*b^2*m*x^6+3*a^2*b*m^3*x^3+120*a*b^2*x^6+54*a^2*b*m^2*x^
3+261*a^2*b*m*x^3+a^3*m^3+210*a^2*b*x^3+21*a^3*m^2+138*a^3*m+280*a^3)*(d*x
)^m*((b*x^3+a)^2)^(3/2)/(10+m)/(7+m)/(4+m)/(1+m)/(b*x^3+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \frac{((b^3m^3+12\,b^3m^2+39\,b^3m+28\,b^3)x^{10}+3\,(ab^2m^3+15\,ab^2m^2+54\,ab^2m+40\,ab^2)x^7+3\,(}{m^4+22\,m^3+159}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
((b^3*m^3 + 12*b^3*m^2 + 39*b^3*m + 28*b^3)*x^10 + 3*(a*b^2*m^3 + 15*a*b^2
*m^2 + 54*a*b^2*m + 40*a*b^2)*x^7 + 3*(a^2*b*m^3 + 18*a^2*b*m^2 + 87*a^2*b
*m + 70*a^2*b)*x^4 + (a^3*m^3 + 21*a^3*m^2 + 138*a^3*m + 280*a^3)*x)*(d*x)
^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Sympy [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \int (dx)^m \left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral((d*x)**m*((a + b*x**3)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.58

$$\int (dx)^m \left(a^2+2abx^3 +b^2x^6\right)^{3/2} dx = \frac{((m^3+12\,m^2+39\,m+28)b^3d^mx^{10}+3\,(m^3+15\,m^2+54\,m+40)ab^2d^mx^7+3\,(m^3+18\,m}{m^4+22\,m^3+159\,m^2+418\,m}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
((m^3 + 12*m^2 + 39*m + 28)*b^3*d^m*x^10 + 3*(m^3 + 15*m^2 + 54*m + 40)*a*
b^2*d^m*x^7 + 3*(m^3 + 18*m^2 + 87*m + 70)*a^2*b*d^m*x^4 + (m^3 + 21*m^2 +
 138*m + 280)*a^3*d^m*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(161) = 322.

Time = 0.14 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.87

$$\int (dx)^m \left(a^2+2abx^3 +b^2x^6\right)^{3/2} dx = \frac{(dx)^m\,b^3m^3x^{10}\mathrm{sgn}(bx^3+a)+12\,(dx)^m\,b^3m^2x^{10}\mathrm{sgn}(bx^3+a)+39\,(dx)^m\,b^3mx^{10}\mathrm{sgn}(bx^3+}{}$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
((d*x)^m*b^3*m^3*x^10*sgn(b*x^3 + a) + 12*(d*x)^m*b^3*m^2*x^10*sgn(b*x^3 +
 a) + 39*(d*x)^m*b^3*m*x^10*sgn(b*x^3 + a) + 3*(d*x)^m*a*b^2*m^3*x^7*sgn(b
*x^3 + a) + 28*(d*x)^m*b^3*x^10*sgn(b*x^3 + a) + 45*(d*x)^m*a*b^2*m^2*x^7*
sgn(b*x^3 + a) + 162*(d*x)^m*a*b^2*m*x^7*sgn(b*x^3 + a) + 3*(d*x)^m*a^2*b*
m^3*x^4*sgn(b*x^3 + a) + 120*(d*x)^m*a*b^2*x^7*sgn(b*x^3 + a) + 54*(d*x)^m
*a^2*b*m^2*x^4*sgn(b*x^3 + a) + 261*(d*x)^m*a^2*b*m*x^4*sgn(b*x^3 + a) + (
d*x)^m*a^3*m^3*x*sgn(b*x^3 + a) + 210*(d*x)^m*a^2*b*x^4*sgn(b*x^3 + a) + 2
1*(d*x)^m*a^3*m^2*x*sgn(b*x^3 + a) + 138*(d*x)^m*a^3*m*x*sgn(b*x^3 + a) +
280*(d*x)^m*a^3*x*sgn(b*x^3 + a))/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\int (d\,x)^m\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.87

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{x^m d^m x(b^3m^3x^9+12b^3m^2x^9+39b^3m\,x^9+3a\,b^2m^3x^6+28b^3x^9+45a\,b^2m^2x^6+162a\,b^2m\,x}{m^4+22m}$$

input
```
int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
(x**m*d**m*x*(a**3*m**3 + 21*a**3*m**2 + 138*a**3*m + 280*a**3 + 3*a**2*b*
m**3*x**3 + 54*a**2*b*m**2*x**3 + 261*a**2*b*m*x**3 + 210*a**2*b*x**3 + 3*
a*b**2*m**3*x**6 + 45*a*b**2*m**2*x**6 + 162*a*b**2*m*x**6 + 120*a*b**2*x*
*6 + b**3*m**3*x**9 + 12*b**3*m**2*x**9 + 39*b**3*m*x**9 + 28*b**3*x**9))/
(m**4 + 22*m**3 + 159*m**2 + 418*m + 280)
```

3.105 $\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 97

$$\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{a(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\,(a+bx^3)} + \frac{b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\,(a+bx^3)}$$

output
```
a*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+b*(d*x)^(4+m)*((b*x^3+
a)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.55

$$\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\,dx = \frac{x(dx)^m\sqrt{(a+bx^3)^2}(a(4+m)+b(1+m)x^3)}{(1+m)(4+m)\,(a+bx^3)}$$

input
```
Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a*(4 + m) + b*(1 + m)*x^3))/((1 + m)*(4 +
m)*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx^3+b^2x^6}(dx)^m\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int b(dx)^m\left(bx^3+a\right)dx}{b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m\left(bx^3+a\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a(dx)^m+\frac{b(dx)^{m+3}}{d^3}\right)dx}{a+bx^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\left(\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}\right)}{a+bx^3}$$

input
```
Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*((a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)
^(4 + m))/(d^4*(4 + m))))/(a + b*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{x\left(bm\,x^{3}+b\,x^{3}+am+4a\right)\left(dx\right)^{m}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{\left(4+m\right)\left(1+m\right)\left(b\,x^{3}+a\right)}$	56
risch	$\frac{x\left(bm\,x^{3}+b\,x^{3}+am+4a\right)\left(dx\right)^{m}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{\left(4+m\right)\left(1+m\right)\left(b\,x^{3}+a\right)}$	56
orering	$\frac{x\left(bm\,x^{3}+b\,x^{3}+am+4a\right)\left(dx\right)^{m}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{\left(4+m\right)\left(1+m\right)\left(b\,x^{3}+a\right)}$	56

input
```
int((d*x)^m*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b*m*x^3+b*x^3+a*m+4*a)*(d*x)^m*((b*x^3+a)^2)^(1/2)/(4+m)/(1+m)/(b*x^3+a
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

$$\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{((bm+b)x^4+(am+4\,a)x)(dx)^m}{m^2+5\,m+4}$$

input
```
integrate((d*x)^m*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
((b*m + b)*x^4 + (a*m + 4*a)*x)*(d*x)^m/(m^2 + 5*m + 4)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx=\text{Timed out}$$

input
```
integrate((d*x)**m*((b*x**3+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

$$\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{(bd^m(m+1)x^4+ad^m(m+4)x)x^m}{m^2+5\,m+4}$$

input
```
integrate((d*x)^m*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
(b*d^m*(m + 1)*x^4 + a*d^m*(m + 4)*x)*x^m/(m^2 + 5*m + 4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx$$
$$=\frac{(dx)^m\,bmx^4\mathrm{sgn}(bx^3+a)+(dx)^m\,bx^4\mathrm{sgn}(bx^3+a)+(dx)^m\,amx\mathrm{sgn}(bx^3+a)+4\,(dx)^m\,ax\mathrm{sgn}(bx^3+a)}{m^2+5\,m+4}$$

input
```
integrate((d*x)^m*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
((d*x)^m*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m*b*x^4*sgn(b*x^3 + a) + (d*x)^m*a
*m*x*sgn(b*x^3 + a) + 4*(d*x)^m*a*x*sgn(b*x^3 + a))/(m^2 + 5*m + 4)
```

Mupad [B] (verification not implemented)

Time = 19.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.57

$$\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{x\,(d\,x)^m\,\sqrt{{(b\,x^3+a)}^2}\,(4\,a+a\,m+b\,x^3+b\,m\,x^3)}{(b\,x^3+a)\,\left(m^2+5\,m+4\right)}$$

input
```
int((d*x)^m*((a + b*x^3)^2)^(1/2),x)
```

output
```
(x*(d*x)^m*((a + b*x^3)^2)^(1/2)*(4*a + a*m + b*x^3 + b*m*x^3))/((a + b*x^
3)*(5*m + m^2 + 4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.37

$$\int (dx)^m\sqrt{a^2+2abx^3+b^2x^6}\,dx=\frac{x^m d^m x(bm\,x^3+b\,x^3+am+4a)}{m^2+5m+4}$$

input
```
int((d*x)^m*((b*x^3+a)^2)^(1/2),x)
```

output
```
(x**m*d**m*x*(a*m + 4*a + b*m*x**3 + b*x**3))/(m**2 + 5*m + 4)
```

3.106 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\frac{(dx)^{1+m}\,(a+bx^3)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
(d*x)^(1+m)*(b*x^3+a)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a/d/(
1+m)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx =\frac{x(dx)^m\,(a+bx^3)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{a(1+m)\sqrt{(a+bx^3)^2}}$$

input
```
Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
(x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, 1 + (1 + m)/3, -((b
*x^3)/a)])/(a*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1384, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^3)\int \frac{(dx)^m}{b(bx^3+a)}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^3)\int \frac{(dx)^m}{bx^3+a}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{888}$$

$$\frac{(a+bx^3)(dx)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

output
```
((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b
*x^3)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{(dx)^m}{\sqrt{(b\,x^3+a)^2}}dx$$

input
```
int((d*x)^m/((b*x^3+a)^2)^(1/2),x)
```

output
```
int((d*x)^m/((b*x^3+a)^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \int \frac{(dx)^m}{\sqrt{(bx^3+a)^2}}\,dx$$

input
```
integrate((d*x)^m/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \int \frac{(dx)^m}{\sqrt{(a + bx^3)^2}}\,dx$$

input
```
integrate((d*x)**m/((b*x**3+a)**2)**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt((a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \int \frac{(dx)^m}{\sqrt{(bx^3 + a)^2}}\,dx$$

input
```
integrate((d*x)^m/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/sqrt((b*x^3 + a)^2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\,dx = \int \frac{(dx)^m}{\sqrt{(bx^3 + a)^2}}\,dx$$

input
```
integrate((d*x)^m/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/sqrt((b*x^3 + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=\int \frac{(d\,x)^m}{\sqrt{(b\,x^3+a)^2}}\,dx$$

input
```
int((d*x)^m/((a + b*x^3)^2)^(1/2),x)
```

output
```
int((d*x)^m/((a + b*x^3)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx=d^m\left(\int \frac{x^m}{b\,x^3+a}dx\right)$$

input
```
int((d*x)^m/((b*x^3+a)^2)^(1/2),x)
```

output
```
d**m*int(x**m/(a + b*x**3),x)
```

3.107 $\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=\frac{(dx)^{1+m}\left(a+bx^3\right)\text{Hypergeometric2F1}\left(3,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
(d*x)^(1+m)*(b*x^3+a)*hypergeom([3, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^3/d
/(1+m)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx=\frac{x(dx)^m\left(a+bx^3\right)\text{Hypergeometric2F1}\left(3,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^3(1+m)\sqrt{\left(a+bx^3\right)^2}}$$

input
```
Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
(x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3
)/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1384, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3\left(a+bx^3\right)\int \frac{(dx)^m}{b^3\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^3\right)\int \frac{(dx)^m}{\left(bx^3+a\right)^3}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\left(a+bx^3\right)(dx)^{m+1}\text{Hypergeometric2F1}\left(3,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

output
```
((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b
*x^3)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{(dx)^m}{(b^2x^6 + 2a\,x^3b + a^2)^{\frac{3}{2}}}dx$$

input
```
int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^3 + b^2x^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m/(b^4*x^12 + 4*a*b^3*x^9 +
 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{(dx)^m}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

output
```
Integral((d*x)**m/((a + b*x**3)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \int \frac{(d\,x)^m}{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{3/2}}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = d^m\left(\int \frac{x^m}{b^3x^9+3a\,b^2x^6+3a^2b\,x^3+a^3}dx\right)$$

input
```
int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

output
```
d**m*int(x**m/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)
```

3.108 $\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}} dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}} \, dx=\frac{(dx)^{1+m}\left(a+bx^3\right)\text{Hypergeometric2F1}\left(5,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^5d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

output
```
(d*x)^(1+m)*(b*x^3+a)*hypergeom([5, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^5/d
/(1+m)/((b*x^3+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}} \, dx=\frac{x(dx)^m\left(a+bx^3\right)\text{Hypergeometric2F1}\left(5,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^5(1+m)\sqrt{\left(a+bx^3\right)^2}}$$

input
```
Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
(x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3, -((b*x^3
)/a)])/(a^5*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1384, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{b^5\left(a+bx^3\right)\int \frac{(dx)^m}{b^5\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a+bx^3\right)\int \frac{(dx)^m}{\left(bx^3+a\right)^5}dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$\Big\downarrow \ 888$$

$$\frac{\left(a+bx^3\right)(dx)^{m+1}\,\text{Hypergeometric2F1}\left(5,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

input
```
Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

output
```
((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3, -((b
*x^3)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{(dx)^m}{(b^2x^6 + 2a\,x^3b + a^2)^{\frac{5}{2}}}dx$$

input
```
int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\,dx = \int \frac{(dx)^m}{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m/(b^6*x^18 + 6*a*b^5*x^15
+ 15*a^2*b^4*x^12 + 20*a^3*b^3*x^9 + 15*a^4*b^2*x^6 + 6*a^5*b*x^3 + a^6),
x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\, dx = \int \frac{(dx)^m}{\left((a + bx^3)^2\right)^{\frac{5}{2}}}\, dx$$

input
```
integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

output
```
Integral((d*x)**m/((a + b*x**3)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\, dx = \int \frac{(dx)^m}{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}}\, dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^3 + b^2x^6)^{5/2}}\, dx = \int \frac{(dx)^m}{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}}\, dx$$

input
```
integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \int \frac{(d\,x)^m}{(a^2+2\,a\,b\,x^3+b^2\,x^6)^{5/2}}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = d^m\left(\int \frac{x^m}{b^5x^{15}+5a\,b^4x^{12}+10a^2b^3x^9+10a^3b^2x^6+5a^4b\,x^3+a^5}dx\right)$$

input
```
int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

output
```
d**m*int(x**m/(a**5 + 5*a**4*b*x**3 + 10*a**3*b**2*x**6 + 10*a**2*b**3*x**
9 + 5*a*b**4*x**12 + b**5*x**15),x)
```

3.109 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 77

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \frac{(dx)^{1+m}\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(\frac{1+m}{3},-2p,\frac{4+m}{3},-\frac{bx^3}{a}\right)}{d(1+m)}$$

output
```
(d*x)^(1+m)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-2*p, 1/3+1/3*m],[4/3+1/3
*m],-b*x^3/a)/d/(1+m)/((1+b*x^3/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \frac{x(dx)^m \left(\left(a+bx^3\right)^2\right)^p \left(1+\frac{bx^3}{a}\right)^{-2p} \text{Hypergeometric2F1}\left(\frac{1+m}{3},-2p,1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{1+m}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x*(d*x)^m*((a + b*x^3)^2)^p*Hypergeometric2F1[(1 + m)/3, -2*p, 1 + (1 + m
)/3, -((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx$$

$$\Big\downarrow \ 1385$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \int (dx)^m \left(\frac{bx^3}{a}+1\right)^{2p} dx$$

$$\Big\downarrow \ 888$$

$$\frac{(dx)^{m+1} \left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(\frac{m+1}{3},-2p,\frac{m+4}{3},-\frac{bx^3}{a}\right)}{d(m+1)}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((d*x)^(1 + m)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[(1 + m)/3,
-2*p, (4 + m)/3, -((b*x^3)/a)])/(d*(1 + m)*(1 + (b*x^3)/a)^(2*p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int (dx)^m \left(b^2x^6 + 2a\,x^3b + a^2\right)^p dx$$

input
```
int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Fricas [F]

$$\int (dx)^m \left(a^2 + 2abx^3 + b^2x^6\right)^p \,dx = \int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p (dx)^m \,dx$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int (dx)^m \left(\left(a+bx^3\right)^2\right)^p\,dx$$

input
```
integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Integral((d*x)**m*((a + b*x**3)**2)**p, x)
```

Maxima [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^2x^6+2\,abx^3+a^2\right)^p(dx)^m\,dx$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^2x^6+2\,abx^3+a^2\right)^p(dx)^m\,dx$$

input
```
integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \int (d\,x)^m \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p \,dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

Reduce [F]

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx$$
$$= \frac{d^m \left(x^m (b^2x^6+2ab\,x^3+a^2)^p\,x + 6\left(\int \frac{x^m (b^2x^6+2ab\,x^3+a^2)^p}{bm\,x^3+6bp\,x^3+b\,x^3+am+6ap+a}dx\right) amp + 36\left(\int \frac{x^m (b^2x^6+2ab\,x^3+a^2)^p}{bm\,x^3+6bp\,x^3+b\,x^3+am+6ap+}\right.\right.}{m+6p+1}$$

input
```
int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
(d**m*(x**m*(a**2 + 2*a*b*x**3 + b**2*x**6)**p*x + 6*int((x**m*(a**2 + 2*a
*b*x**3 + b**2*x**6)**p)/(a*m + 6*a*p + a + b*m*x**3 + 6*b*p*x**3 + b*x**3
),x)*a*m*p + 36*int((x**m*(a**2 + 2*a*b*x**3 + b**2*x**6)**p)/(a*m + 6*a*p
 + a + b*m*x**3 + 6*b*p*x**3 + b*x**3),x)*a*p**2 + 6*int((x**m*(a**2 + 2*a
*b*x**3 + b**2*x**6)**p)/(a*m + 6*a*p + a + b*m*x**3 + 6*b*p*x**3 + b*x**3
),x)*a*p))/(m + 6*p + 1)
```

3.110 $\int x^{11}(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx=-\frac{a^3(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^4(1+2p)}+\frac{a^2(a+bx^3)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(1+p)}-\frac{a(a+bx^3)^3\left(a^2+2abx^3+b^2x^6\right)^p}{b^4(3+2p)}+\frac{(a+bx^3)^4\left(a^2+2abx^3+b^2x^6\right)^p}{6b^4(2+p)}$$

output
```
-1/3*a^3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(1+2*p)+1/2*a^2*(b*x^3+a)
^2*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(p+1)-a*(b*x^3+a)^3*(b^2*x^6+2*a*b*x^3+a^
2)^p/b^4/(3+2*p)+1/6*(b*x^3+a)^4*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(2+p)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.64

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$
$$=\frac{(a+bx^3)\left((a+bx^3)^2\right)^p\left(-3a^3+3a^2b(1+2p)x^3-3ab^2(1+3p+2p^2)\,x^6+b^3(3+11p+12p^2+4p^3)\,x^9\right)}{6b^4(1+p)(2+p)(1+2p)(3+2p)}$$

input
```
Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a + b*x^3)*((a + b*x^3)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^3 - 3*a*b^2*(
1 + 3*p + 2*p^2)*x^6 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x^9))/(6*b^4*(1 + p
)*(2 + p)*(1 + 2*p)*(3 + 2*p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1385, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\int x^{11}\left(\frac{bx^3}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\int x^9\left(\frac{bx^3}{a}+1\right)^{2p}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\int\left(-\frac{a^3\left(\frac{bx^3}{a}+1\right)^{2p}}{b^3}+\frac{3a^3\left(\frac{bx^3}{a}+1\right)^{2p+1}}{b^3}-\frac{3a^3\left(\frac{bx^3}{a}+1\right)^{2p+2}}{b^3}+\frac{a^3\left(\frac{bx^3}{a}\right.}{}\right.$$

$\downarrow$ 2009

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\left(\frac{3a^4\left(\frac{bx^3}{a}+1\right)^{2(p+1)}}{2b^4(p+1)}+\frac{a^4\left(\frac{bx^3}{a}+1\right)^{2(p+2)}}{2b^4(p+2)}-\frac{a^4\left(\frac{bx^3}{a}+1\right)^{2p+1}}{b^4(2p+1)}-\frac{3a^4\left(\frac{bx^3}{a}\right.}{b^4(2}\right.$$

input
```
Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a^2 + 2*a*b*x^3 + b^2*x^6)^p*((3*a^4*(1 + (b*x^3)/a)^(2*(1 + p)))/(2*b^4
*(1 + p)) + (a^4*(1 + (b*x^3)/a)^(2*(2 + p)))/(2*b^4*(2 + p)) - (a^4*(1 +
(b*x^3)/a)^(1 + 2*p))/(b^4*(1 + 2*p)) - (3*a^4*(1 + (b*x^3)/a)^(3 + 2*p))/
(b^4*(3 + 2*p))))/(3*(1 + (b*x^3)/a)^(2*p))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.87

method	result
gosper	$-\frac{(b^2x^6+2a\,x^3b+a^2)^p(-4b^3p^3x^9-12b^3p^2x^9-11b^3p\,x^9-3x^9b^3+6a\,b^2p^2x^6+9a\,b^2p\,x^6+3a\,x^6b^2-6a^2bp\,x^3-3a^2x^3b+3a^3)(b}{6b^4(4p^4+20p^3+35p^2+25p+6)}$
orering	$-\frac{(b^2x^6+2a\,x^3b+a^2)^p(-4b^3p^3x^9-12b^3p^2x^9-11b^3p\,x^9-3x^9b^3+6a\,b^2p^2x^6+9a\,b^2p\,x^6+3a\,x^6b^2-6a^2bp\,x^3-3a^2x^3b+3a^3)(b}{6b^4(4p^4+20p^3+35p^2+25p+6)}$
risch	$-\frac{(-4b^4p^3x^{12}-12b^4p^2x^{12}-11b^4p\,x^{12}-4a\,b^3p^3x^9-3b^4x^{12}-6a\,b^3p^2x^9-2ap\,x^9b^3+6a^2b^2p^2x^6+3a^2p\,x^6b^2-6a^3p\,x^3b+3a^4)(}{6(3+2p)(2+p)(p+1)(1+2p)b^4}$
parallelrisch	$\frac{4x^{12}(b^2x^6+2a\,x^3b+a^2)^p b^4p^3+12x^{12}(b^2x^6+2a\,x^3b+a^2)^p b^4p^2+11x^{12}(b^2x^6+2a\,x^3b+a^2)^p b^4p+3x^{12}(b^2x^6+2a\,x^3b+a^2)^p b^4+}{}$

input
```
int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b^2*x^6+2*a*b*x^3+a^2)^p*(-4*b^3*p^3*x^9-12*b^3*p^2*x^9-11*b^3*p*x^9
-3*b^3*x^9+6*a*b^2*p^2*x^6+9*a*b^2*p*x^6+3*a*b^2*x^6-6*a^2*b*p*x^3-3*a^2*b
*x^3+3*a^3)*(b*x^3+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$
$$=\frac{((4\,b^4p^3+12\,b^4p^2+11\,b^4p+3\,b^4)x^{12}+2\,(2\,ab^3p^3+3\,ab^3p^2+ab^3p)x^9+6\,a^3bpx^3-3\,(2\,a^2b^2p^2+a^2b^2p)x}{6\,(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4)}$$

input
```
integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
1/6*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^12 + 2*(2*a*b^3*p^3 + 3
*a*b^3*p^2 + a*b^3*p)*x^9 + 6*a^3*b*p*x^3 - 3*(2*a^2*b^2*p^2 + a^2*b^2*p)*
x^6 - 3*a^4)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^4*p^4 + 20*b^4*p^3 + 35*b^
4*p^2 + 25*b^4*p + 6*b^4)
```

Sympy [F]

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \text{Too large to display}$$

input
```
integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Piecewise((x**12*(a**2)**p/12, Eq(b, 0)), (6*a**3*log(x - (-a/b)**(1/3))/(
18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 6*a**3
*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**3*b**4 + 54*a**2
*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 12*a**3*log(2)/(18*a**3*b**4
 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 11*a**3/(18*a**3*b
**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a**2*b*x**3*
log(x - (-a/b)**(1/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6
+ 18*b**7*x**9) + 18*a**2*b*x**3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)
**(2/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9
) - 36*a**2*b*x**3*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x*
*6 + 18*b**7*x**9) + 27*a**2*b*x**3/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54
*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*x**6*log(x - (-a/b)**(1/3))/(18*a
**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*
x**6*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**3*b**4 + 54*
a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 36*a*b**2*x**6*log(2)/(1
8*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b*
*2*x**6/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9)
 + 6*b**3*x**9*log(x - (-a/b)**(1/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 +
54*a*b**6*x**6 + 18*b**7*x**9) + 6*b**3*x**9*log(4*x**2 + 4*x*(-a/b)**(1/3
) + 4*(-a/b)**(2/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.67

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$
$$=\frac{((4\,p^3+12\,p^2+11\,p+3)b^4x^{12}+2\,(2\,p^3+3\,p^2+p)ab^3x^9-3\,(2\,p^2+p)a^2b^2x^6+6\,a^3bpx^3-3\,a^4)(bx^3+}{6\,(4\,p^4+20\,p^3+35\,p^2+25\,p+6)b^4}$$

input
```
integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
1/6*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^12 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^
9 - 3*(2*p^2 + p)*a^2*b^2*x^6 + 6*a^3*b*p*x^3 - 3*a^4)*(b*x^3 + a)^(2*p)/(
(4*p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(166) = 332.

Time = 0.14 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.18

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$
$$=\frac{4\,(b^2x^6+2\,abx^3+a^2)^pb^4p^3x^{12}+12\,(b^2x^6+2\,abx^3+a^2)^pb^4p^2x^{12}+11\,(b^2x^6+2\,abx^3+a^2)^pb^4px^{12}+4\,(b}{}$$

input
```
integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
1/6*(4*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p^3*x^12 + 12*(b^2*x^6 + 2*a*b*x^
3 + a^2)^p*b^4*p^2*x^12 + 11*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p*x^12 + 4*
(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^3*x^9 + 3*(b^2*x^6 + 2*a*b*x^3 + a^2
)^p*b^4*x^12 + 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^2*x^9 + 2*(b^2*x^6
+ 2*a*b*x^3 + a^2)^p*a*b^3*p*x^9 - 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2
*p^2*x^6 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2*p*x^6 + 6*(b^2*x^6 + 2*
a*b*x^3 + a^2)^p*a^3*b*p*x^3 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^4)/(4*b^4
*p^4 + 20*b^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

Mupad [B] (verification not implemented)

Time = 18.92 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.20

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\,\left(\frac{x^{12}\,\left(4\,p^3+12\,p^2+11\,p+3\right)}{6\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)} - \frac{a^4}{2\,b^4\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)} + \frac{a^3\,p\,x^3}{b^3\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)} + \frac{a\,p\,x^9\,\left(2\,p^2+3\,p+1\right)}{3\,b\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)} - \frac{a^2\,p\,x^6\,\left(2\,p+1\right)}{2\,b^2\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}\right)$$

input
```
int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
(a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^12*(11*p + 12*p^2 + 4*p^3 + 3))/(6*(25*p
 + 35*p^2 + 20*p^3 + 4*p^4 + 6)) - a^4/(2*b^4*(25*p + 35*p^2 + 20*p^3 + 4*
p^4 + 6)) + (a^3*p*x^3)/(b^3*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) + (a*p*
x^9*(3*p + 2*p^2 + 1))/(3*b*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) - (a^2*p
*x^6*(2*p + 1))/(2*b^2*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.94

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p\left(4b^4p^3x^{12}+12b^4p^2x^{12}+11b^4p\,x^{12}+4a\,b^3p^3x^9+3b^4x^{12}+6a\,b^3p^2x^9+2a\,b^3p\,x^9-\cdots\right)}{6b^4\left(4p^4+20p^3+35p^2+25p+6\right)}$$

input
```
int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x**3 + b**2*x**6)**p*( - 3*a**4 + 6*a**3*b*p*x**3 - 6*a**2*
b**2*p**2*x**6 - 3*a**2*b**2*p*x**6 + 4*a*b**3*p**3*x**9 + 6*a*b**3*p**2*x
**9 + 2*a*b**3*p*x**9 + 4*b**4*p**3*x**12 + 12*b**4*p**2*x**12 + 11*b**4*p
*x**12 + 3*b**4*x**12))/(6*b**4*(4*p**4 + 20*p**3 + 35*p**2 + 25*p + 6))
```

3.111 $\int x^8(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{a^2(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(1+2p)}-\frac{a\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(1+p)}+\frac{\left(a+bx^3\right)^3\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(3+2p)}$$

output
```
1/3*a^2*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(1+2*p)-1/3*a*(b*x^3+a)^2*
(b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(p+1)+1/3*(b*x^3+a)^3*(b^2*x^6+2*a*b*x^3+a^2
)^p/b^3/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.59

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(a+bx^3)\left(\left(a+bx^3\right)^2\right)^p\left(a^2-ab(1+2p)x^3+b^2\left(1+3p+2p^2\right)x^6\right)}{3b^3(1+p)(1+2p)(3+2p)}$$

input
```
Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a + b*x^3)*((a + b*x^3)^2)^p*(a^2 - a*b*(1 + 2*p)*x^3 + b^2*(1 + 3*p + 2
*p^2)*x^6))/(3*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1385, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \int x^8 \left(\frac{bx^3}{a}+1\right)^{2p} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \int x^6 \left(\frac{bx^3}{a}+1\right)^{2p} dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \int \left(\frac{a^2\left(\frac{bx^3}{a}+1\right)^{2p}}{b^2}-\frac{2a^2\left(\frac{bx^3}{a}+1\right)^{2p+1}}{b^2}+\frac{a^2\left(\frac{bx^3}{a}+1\right)^{2p+2}}{b^2}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \left(-\frac{a^3\left(\frac{bx^3}{a}+1\right)^{2(p+1)}}{b^3(p+1)}+\frac{a^3\left(\frac{bx^3}{a}+1\right)^{2p+1}}{b^3(2p+1)}+\frac{a^3\left(\frac{bx^3}{a}+1\right)^{2p+3}}{b^3(2p+3)}\right)$$

input
```
Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a^2 + 2*a*b*x^3 + b^2*x^6)^p*(-((a^3*(1 + (b*x^3)/a)^(2*(1 + p)))/(b^3*(
1 + p))) + (a^3*(1 + (b*x^3)/a)^(1 + 2*p))/(b^3*(1 + 2*p)) + (a^3*(1 + (b*
x^3)/a)^(3 + 2*p))/(b^3*(3 + 2*p))))/(3*(1 + (b*x^3)/a)^(2*p))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.74

method	result
gosper	$\frac{(b\,x^3+a)\,(2b^2p^2x^6+3b^2p\,x^6+b^2x^6-2abp\,x^3-a\,x^3b+a^2)\,(b^2x^6+2a\,x^3b+a^2)^p}{3b^3(4p^3+12p^2+11p+3)}$
orering	$\frac{(b\,x^3+a)\,(2b^2p^2x^6+3b^2p\,x^6+b^2x^6-2abp\,x^3-a\,x^3b+a^2)\,(b^2x^6+2a\,x^3b+a^2)^p}{3b^3(4p^3+12p^2+11p+3)}$
risch	$\frac{(2b^3p^2x^9+3b^3p\,x^9+x^9b^3+2a\,b^2p^2x^6+a\,b^2p\,x^6-2a^2bp\,x^3+a^3)\left((b\,x^3+a)^2\right)^p}{3(p+1)(3+2p)(1+2p)b^3}$
parallelrisch	$\frac{2x^9(b^2x^6+2a\,x^3b+a^2)^p a\,b^3p^2+3x^9(b^2x^6+2a\,x^3b+a^2)^p a\,b^3p+x^9(b^2x^6+2a\,x^3b+a^2)^p a\,b^3+2x^6(b^2x^6+2a\,x^3b+a^2)^p a^2b^2p^2+}{3(3+2p)(p+1)a(1+2p)b^3}$

input
```
int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^3+a)*(2*b^2*p^2*x^6+3*b^2*p*x^6+b^2*x^6-2*a*b*p*x^3-a*b*x^3+a^2)*
(b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.83

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$
$$=\frac{((2\,b^3p^2+3\,b^3p+b^3)x^9-2\,a^2bpx^3+(2\,ab^2p^2+ab^2p)x^6+a^3)(b^2x^6+2\,abx^3+a^2)^p}{3\,(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3)}$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
1/3*((2*b^3*p^2 + 3*b^3*p + b^3)*x^9 - 2*a^2*b*p*x^3 + (2*a*b^2*p^2 + a*b^
2*p)*x^6 + a^3)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11
*b^3*p + 3*b^3)
```

Sympy [F]

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p\,dx = \begin{cases} \frac{x^9(a^2)^p}{9} \\ \int \frac{x^8}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx \\ -\frac{2a^2\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^3+3b^4x^3} - \frac{2a^2\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^3+3b^4x^3} - \frac{2a^2}{3ab^3+3b^4x^3} + \frac{4a^2\log(2)}{3ab^3+3b^4x^3} - \frac{2abx^3\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^3+3b^4x^3} - \frac{2abx}{} \\ \int \frac{x^8}{\sqrt{(a+bx^3)^2}}\,dx \\ \frac{a^3(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} - \frac{2a^2bpx^3(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} + \frac{2ab^2p^2x^6(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} + \frac{ab^2px^6(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} + \frac{2b^3p^2x}{12b^3p^3} \end{cases}$$

input

```
integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output

```
Piecewise((x**9*(a**2)**p/9, Eq(b, 0)), (Integral(x**8/((a + b*x**3)**2)**
(3/2), x), Eq(p, -3/2)), (-2*a**2*log(x - (-a/b)**(1/3))/(3*a*b**3 + 3*b**
4*x**3) - 2*a**2*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b*
*3 + 3*b**4*x**3) - 2*a**2/(3*a*b**3 + 3*b**4*x**3) + 4*a**2*log(2)/(3*a*b
**3 + 3*b**4*x**3) - 2*a*b*x**3*log(x - (-a/b)**(1/3))/(3*a*b**3 + 3*b**4*
x**3) - 2*a*b*x**3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*
b**3 + 3*b**4*x**3) + 4*a*b*x**3*log(2)/(3*a*b**3 + 3*b**4*x**3) + b**2*x*
*6/(3*a*b**3 + 3*b**4*x**3), Eq(p, -1)), (Integral(x**8/sqrt((a + b*x**3)*
*2), x), Eq(p, -1/2)), (a**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p
**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) - 2*a**2*b*p*x**3*(a**2 + 2*a*b*x
**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + 2
*a*b**2*p**2*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b*
*3*p**2 + 33*b**3*p + 9*b**3) + a*b**2*p*x**6*(a**2 + 2*a*b*x**3 + b**2*x*
*6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + 2*b**3*p**2*x*
*9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b*
*3*p + 9*b**3) + 3*b**3*p*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3
*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + b**3*x**9*(a**2 + 2*a*b*x**3
+ b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.61

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \frac{\left((2\,p^2+3\,p+1)b^3x^9+(2\,p^2+p)ab^2x^6-2\,a^2bpx^3+a^3\right)(bx^3+a)^{2\,p}}{3\,(4\,p^3+12\,p^2+11\,p+3)b^3}$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
1/3*((2*p^2 + 3*p + 1)*b^3*x^9 + (2*p^2 + p)*a*b^2*x^6 - 2*a^2*b*p*x^3 + a
^3)*(b*x^3 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.81

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \frac{2\,(b^2x^6+2\,abx^3+a^2)^p b^3p^2x^9+3\,(b^2x^6+2\,abx^3+a^2)^p b^3px^9+(b^2x^6+2\,abx^3+a^2)^p b^3x^9+2\,(b^2x^6+2\,a}{3\,(4\,b^3p^3+12\,b}$$

input
```
integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
1/3*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*p^2*x^9 + 3*(b^2*x^6 + 2*a*b*x^3
+ a^2)^p*b^3*p*x^9 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*x^9 + 2*(b^2*x^6 +
2*a*b*x^3 + a^2)^p*a*b^2*p^2*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^2*p*x
^6 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b*p*x^3 + (b^2*x^6 + 2*a*b*x^3 +
a^2)^p*a^3)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Mupad [B] (verification not implemented)

Time = 19.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p \left(\frac{x^9\left(\frac{2\,p^2}{3}+p+\frac{1}{3}\right)}{4\,p^3+12\,p^2+11\,p+3} + \frac{a^3}{3\,b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)} - \frac{2\,a^2\,p\,x^3}{3\,b^2\,\left(4\,p^3+12\,p^2+11\,p+3\right)} + \frac{a\,p\,x^6\,(2\,p+1)}{3\,b\,\left(4\,p^3+12\,p^2+11\,p+3\right)}\right)$$

input
```
int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
(a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^9*(p + (2*p^2)/3 + 1/3))/(11*p + 12*p^2
+ 4*p^3 + 3) + a^3/(3*b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (2*a^2*p*x^3)/(3*
b^2*(11*p + 12*p^2 + 4*p^3 + 3)) + (a*p*x^6*(2*p + 1))/(3*b*(11*p + 12*p^2
 + 4*p^3 + 3)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.80

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p \left(2b^3p^2x^9+3b^3p\,x^9+b^3x^9+2a\,b^2p^2x^6+a\,b^2p\,x^6-2a^2bp\,x^3+a^3\right)}{3b^3\left(4p^3+12p^2+11p+3\right)}$$

input
```
int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x**3 + b**2*x**6)**p*(a**3 - 2*a**2*b*p*x**3 + 2*a*b**2*p**
2*x**6 + a*b**2*p*x**6 + 2*b**3*p**2*x**9 + 3*b**3*p*x**9 + b**3*x**9))/(3
*b**3*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.112 $\int x^5(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx=-\frac{a(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(1+2p)}+\frac{\left(a^2+2abx^3+b^2x^6\right)^{1+p}}{6b^2(1+p)}$$

output
```
-1/3*a*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^2/(1+2*p)+1/6*(b^2*x^6+2*a*b*
x^3+a^2)^(p+1)/b^2/(p+1)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(a+bx^3)\left((a+bx^3)^2\right)^p(-a+b(1+2p)x^3)}{6b^2(1+p)(1+2p)}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a + b*x^3)*((a + b*x^3)^2)^p*(-a + b*(1 + 2*p)*x^3))/(6*b^2*(1 + p)*(1 +
 2*p))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a^2+2abx^3+b^2x^6\right)^p dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^3 \left(b^2x^6+2abx^3+a^2\right)^p dx^3$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{p+1}}{2b^2(p+1)}-\frac{a\int \left(b^2x^6+2abx^3+a^2\right)^p dx^3}{b}\right)$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{p+1}}{2b^2(p+1)}-\frac{a\left(ab+b^2x^3\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\int \left(b^2x^3+ab\right)^{2p} dx^3}{b}\right)$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{3}\left(\frac{\left(a^2+2abx^3+b^2x^6\right)^{p+1}}{2b^2(p+1)}-\frac{a\left(ab+b^2x^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{b^3(2p+1)}\right)$$

input
```
Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(-((a*(a*b + b^2*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^3*(1 + 2*p))) + (a
^2 + 2*a*b*x^3 + b^2*x^6)^(1 + p)/(2*b^2*(1 + p)))/3
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{\left(-2 b^{2} p\, x^{6}-b^{2} x^{6}-2 a b p\, x^{3}+a^{2}\right) \left(\left(b\, x^{3}+a \right)^{2}\right)^{p}}{6 b^{2} \left(p +1\right) \left(1+2 p \right)}$	58
gosper	$-\frac{\left(b^{2} x^{6}+2 a\, x^{3} b +a^{2}\right)^{p} \left(-2 p b\, x^{3}-b\, x^{3}+a \right) \left(b\, x^{3}+a \right)}{6 b^{2} \left(2 p^{2}+3 p +1\right)}$	60
orering	$-\frac{\left(b^{2} x^{6}+2 a\, x^{3} b +a^{2}\right)^{p} \left(-2 p b\, x^{3}-b\, x^{3}+a \right) \left(b\, x^{3}+a \right)}{6 b^{2} \left(2 p^{2}+3 p +1\right)}$	60
norman	$\frac{x^{6} {\mathrm e}^{p \ln \left(b^{2} x^{6}+2 a\, x^{3} b +a^{2}\right)}}{6 p +6}-\frac{a^{2} {\mathrm e}^{p \ln \left(b^{2} x^{6}+2 a\, x^{3} b +a^{2}\right)}}{6 b^{2} \left(2 p^{2}+3 p +1\right)}+\frac{p a\, x^{3} {\mathrm e}^{p \ln \left(b^{2} x^{6}+2 a\, x^{3} b +a^{2}\right)}}{3 b \left(2 p^{2}+3 p +1\right)}$	120
parallelrisch	$\frac{2 x^{6} \left(b^{2} x^{6}+2 a\, x^{3} b +a^{2}\right)^{p} b^{2} p +x^{6} \left(b^{2} x^{6}+2 a\, x^{3} b +a^{2}\right)^{p} b^{2}+2 x^{3} \left(b^{2} x^{6}+2 a\, x^{3} b +a^{2}\right)^{p} a b p -\left(b^{2} x^{6}+2 a\, x^{3} b +a^{2}\right)^{p} a^{2}}{6 b^{2} \left(2 p^{2}+3 p +1\right)}$	128

input
```
int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-2*b^2*p*x^6-b^2*x^6-2*a*b*p*x^3+a^2)/b^2/(p+1)/(1+2*p)*((b*x^3+a)^2
)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int x^5 (a^2 + 2abx^3 + b^2x^6)^p \, dx = \frac{((2\,b^2p + b^2)x^6 + 2\,abpx^3 - a^2)(b^2x^6 + 2\,abx^3 + a^2)^p}{6\,(2\,b^2p^2 + 3\,b^2p + b^2)}$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
1/6*((2*b^2*p + b^2)*x^6 + 2*a*b*p*x^3 - a^2)*(b^2*x^6 + 2*a*b*x^3 + a^2)^
p/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Sympy [F]

$$\int x^5 (a^2 + 2abx^3 + b^2x^6)^p \, dx$$

$$= \begin{cases} \frac{x^6(a^2)^p}{6} \\ \frac{a\log\left(x - \sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x^3} + \frac{a\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2+3b^3x^3} - \frac{2a\log(2)}{3ab^2+3b^3x^3} + \frac{a}{3ab^2+3b^3x^3} + \frac{bx^3\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x^3} + \frac{bx^3\log\left(4x^2+\right.}{3a} \\ \int \frac{x^5}{\sqrt{(a+bx^3)^2}}\,dx \\ -\frac{a^2(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2} + \frac{2abpx^3(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2} + \frac{2b^2px^6(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2} + \frac{b^2x^6(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2} \end{cases}$$

input
```
integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Piecewise((x**6*(a**2)**p/6, Eq(b, 0)), (a*log(x - (-a/b)**(1/3))/(3*a*b**
2 + 3*b**3*x**3) + a*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*
a*b**2 + 3*b**3*x**3) - 2*a*log(2)/(3*a*b**2 + 3*b**3*x**3) + a/(3*a*b**2
+ 3*b**3*x**3) + b*x**3*log(x - (-a/b)**(1/3))/(3*a*b**2 + 3*b**3*x**3) +
b*x**3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**2 + 3*b**
3*x**3) - 2*b*x**3*log(2)/(3*a*b**2 + 3*b**3*x**3), Eq(p, -1)), (Integral(
x**5/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (-a**2*(a**2 + 2*a*b*x**3 +
b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + 2*a*b*p*x**3*(a**2 + 2
*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + 2*b**2*p*x
**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2)
 + b**2*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p
+ 6*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{\left(b^2(2\,p+1)x^6+2\,abpx^3-a^2\right)\left(bx^3+a\right)^{2\,p}}{6\,(2\,p^2+3\,p+1)b^2}$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
1/6*(b^2*(2*p + 1)*x^6 + 2*a*b*p*x^3 - a^2)*(b*x^3 + a)^(2*p)/((2*p^2 + 3*
p + 1)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.71

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$
$$=\frac{2\left(b^2x^6+2\,abx^3+a^2\right)^pb^2px^6+\left(b^2x^6+2\,abx^3+a^2\right)^pb^2x^6+2\left(b^2x^6+2\,abx^3+a^2\right)^pabpx^3-\left(b^2x^6+2\,ab\right.}{6\,(2\,b^2p^2+3\,b^2p+b^2)}$$

input
```
integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
1/6*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^2*p*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^
2)^p*b^2*x^6 + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b*p*x^3 - (b^2*x^6 + 2*a*
b*x^3 + a^2)^p*a^2)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Mupad [B] (verification not implemented)

Time = 19.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\left(\frac{x^6\,(2\,p+1)}{6\,\left(2\,p^2+3\,p+1\right)}-\frac{a^2}{6\,b^2\,\left(2\,p^2+3\,p+1\right)}+\frac{a\,p\,x^3}{3\,b\,\left(2\,p^2+3\,p+1\right)}\right)$$

input
```
int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
(a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^6*(2*p + 1))/(6*(3*p + 2*p^2 + 1)) - a^2
/(6*b^2*(3*p + 2*p^2 + 1)) + (a*p*x^3)/(3*b*(3*p + 2*p^2 + 1)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p\left(2b^2p\,x^6+b^2x^6+2abp\,x^3-a^2\right)}{6b^2\left(2p^2+3p+1\right)}$$

input
```
int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x**3 + b**2*x**6)**p*( - a**2 + 2*a*b*p*x**3 + 2*b**2*p*x**
6 + b**2*x**6))/(6*b**2*(2*p**2 + 3*p + 1))
```

3.113 $\int x^4(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{1}{5}x^5\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\text{Hypergeometric2F1}\left(\frac{5}{3},-2p,\frac{8}{3},-\frac{bx^3}{a}\right)$$

output
```
1/5*x^5*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([5/3, -2*p],[8/3],-b*x^3/a)/((
1+b*x^3/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{1}{5}x^5\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{5}{3},-2p,\frac{8}{3},-\frac{bx^3}{a}\right)$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x^5*((a + b*x^3)^2)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*x^3)/a)])/(5
*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a^2+2 a b x^3+b^2 x^6\right)^p \, dx$$

$$\Big\downarrow \ 1385$$

$$\left(\frac{b x^3}{a}+1\right)^{-2 p} \left(a^2+2 a b x^3+b^2 x^6\right)^p \int x^4 \left(\frac{b x^3}{a}+1\right)^{2 p} dx$$

$$\Big\downarrow \ 888$$

$$\frac{1}{5} x^5 \left(\frac{b x^3}{a}+1\right)^{-2 p} \left(a^2+2 a b x^3+b^2 x^6\right)^p \text{Hypergeometric2F1}\left(\frac{5}{3},-2 p,\frac{8}{3},-\frac{b x^3}{a}\right)$$

input
```
Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*
x^3)/a)])/(5*(1 + (b*x^3)/a)^(2*p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int x^4 (b^2x^6 + 2a\,x^3b + a^2)^p\,dx$$

input
```
int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Fricas [F]

$$\int x^4(a^2 + 2abx^3 + b^2x^6)^p\ dx = \int (b^2x^6 + 2\,abx^3 + a^2)^p x^4\,dx$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

Sympy [F]

$$\int x^4 \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx = \int x^4 \left(\left(a + bx^3\right)^2\right)^p \, dx$$

input
```
integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Integral(x**4*((a + b*x**3)**2)**p, x)
```

Maxima [F]

$$\int x^4 \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx = \int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^4 \, dx$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

Giac [F]

$$\int x^4 \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx = \int \left(b^2x^6 + 2\,abx^3 + a^2\right)^p x^4 \, dx$$

input
```
integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \int x^4 \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p dx$$

input
```
int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

Reduce [F]

$$\int x^4 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$$
$$= \frac{3(b^2x^6+2ab\,x^3+a^2)^p\,ap\,x^2+3(b^2x^6+2ab\,x^3+a^2)^p\,bp\,x^5+(b^2x^6+2ab\,x^3+a^2)^p\,b\,x^5-108\Big(\int \frac{}{18b\,p^2x^3\text{-}}}{}$$

input
```
int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
(3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p*a*p*x**2 + 3*(a**2 + 2*a*b*x**3 + b*
*2*x**6)**p*b*p*x**5 + (a**2 + 2*a*b*x**3 + b**2*x**6)**p*b*x**5 - 108*int
(((a**2 + 2*a*b*x**3 + b**2*x**6)**p*x)/(18*a*p**2 + 21*a*p + 5*a + 18*b*p
**2*x**3 + 21*b*p*x**3 + 5*b*x**3),x)*a**2*p**3 - 126*int(((a**2 + 2*a*b*x
**3 + b**2*x**6)**p*x)/(18*a*p**2 + 21*a*p + 5*a + 18*b*p**2*x**3 + 21*b*p
*x**3 + 5*b*x**3),x)*a**2*p**2 - 30*int(((a**2 + 2*a*b*x**3 + b**2*x**6)**
p*x)/(18*a*p**2 + 21*a*p + 5*a + 18*b*p**2*x**3 + 21*b*p*x**3 + 5*b*x**3),
x)*a**2*p)/(b*(18*p**2 + 21*p + 5))
```

3.114 $\int x^3(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int x^3(a^2+2abx^3+b^2x^6)^p\,dx=\frac{1}{4}x^4\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\,\text{Hypergeometric2F1}\left(\frac{4}{3},-2p,\frac{7}{3},-\frac{bx^3}{a}\right)$$

output
```
1/4*x^4*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([4/3, -2*p],[7/3],-b*x^3/a)/((
1+b*x^3/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x^3(a^2+2abx^3+b^2x^6)^p\,dx=\frac{1}{4}x^4\left((a+bx^3)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{4}{3},-2p,\frac{7}{3},-\frac{bx^3}{a}\right)$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x^4*((a + b*x^3)^2)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*x^3)/a)])/(4
*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^p dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \int x^3 \left(\frac{bx^3}{a}+1\right)^{2p} dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{4}x^4 \left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(\frac{4}{3},-2p,\frac{7}{3},-\frac{bx^3}{a}\right)$$

input
```
Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*
x^3)/a)])/(4*(1 + (b*x^3)/a)^(2*p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int x^{3}\left(b^{2} x^{6}+2 a\,x^{3} b +a^{2}\right)^{p}d x$$

input
```
int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Fricas [F]

$$\int x^3 \left(a^2+2 a b x^3+b^2 x^6\right)^p \, dx=\int { {\left(b^{2} x^{6} + 2 \, a b x^{3} + a^{2}\right)}^{p} x^{3} \,d x }$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

Sympy [F]

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \int x^3 \left(\left(a+bx^3\right)^2\right)^p \, dx$$

input
```
integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Integral(x**3*((a + b*x**3)**2)**p, x)
```

Maxima [F]

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \int \left(b^2x^6+2\,abx^3+a^2\right)^p x^3 \, dx$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

Giac [F]

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^p \, dx = \int \left(b^2x^6+2\,abx^3+a^2\right)^p x^3 \, dx$$

input
```
integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^p \, dx=\int x^3\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^p \,d x$$

input
```
int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

Reduce [F]

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^p \, dx$$
$$=\frac{6(b^2x^6+2ab\,x^3+a^2)^p\,apx+6(b^2x^6+2ab\,x^3+a^2)^p\,bp\,x^4+(b^2x^6+2ab\,x^3+a^2)^p\,b\,x^4-108\Big(\int \frac{}{18b\,p^2x^3+1}$$

input
```
int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
(6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p*a*p*x + 6*(a**2 + 2*a*b*x**3 + b**2*
x**6)**p*b*p*x**4 + (a**2 + 2*a*b*x**3 + b**2*x**6)**p*b*x**4 - 108*int((a
**2 + 2*a*b*x**3 + b**2*x**6)**p/(18*a*p**2 + 15*a*p + 2*a + 18*b*p**2*x**
3 + 15*b*p*x**3 + 2*b*x**3),x)*a**2*p**3 - 90*int((a**2 + 2*a*b*x**3 + b**
2*x**6)**p/(18*a*p**2 + 15*a*p + 2*a + 18*b*p**2*x**3 + 15*b*p*x**3 + 2*b*
x**3),x)*a**2*p**2 - 12*int((a**2 + 2*a*b*x**3 + b**2*x**6)**p/(18*a*p**2
+ 15*a*p + 2*a + 18*b*p**2*x**3 + 15*b*p*x**3 + 2*b*x**3),x)*a**2*p)/(2*b*
(18*p**2 + 15*p + 2))
```

3.115 $\int x^2(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 41

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b(1+2p)}$$

output
```
1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(a+bx^3)\left((a+bx^3)^2\right)^p}{3b(1+2p)}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a + b*x^3)*((a + b*x^3)^2)^p)/(3*b*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1690, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx^3+b^2x^6\right)^p dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \left(b^2x^6+2abx^3+a^2\right)^p dx^3$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{3}\left(ab+b^2x^3\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p \int \left(b^2x^3+ab\right)^{2p} dx^3$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(ab+b^2x^3\right)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(2p+1)}$$

input
```
Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
((a*b + b^2*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^2*(1 + 2*p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\left(b\,x^{3}+a\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{p}}{3b\left(1+2p\right)}$	31
gosper	$\frac{\left(b\,x^{3}+a\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{p}}{3b\left(1+2p\right)}$	40
orering	$\frac{\left(b\,x^{3}+a\right)\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{p}}{3b\left(1+2p\right)}$	40
parallelrisch	$\frac{x^{3}\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{p}ab+\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)^{p}a^{2}}{3ab\left(1+2p\right)}$	67
norman	$\frac{x^{3}{\mathrm e}^{p\ln\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)}}{6p+3}+\frac{a\,{\mathrm e}^{p\ln\left(b^{2}x^{6}+2a\,x^{3}b+a^{2}\right)}}{3b\left(1+2p\right)}$	71

input
```
int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^3+a)/b/(1+2*p)*((b*x^3+a)^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\frac{(bx^3+a)(b^2x^6+2\,abx^3+a^2)^p}{3\,(2\,bp+b)}$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
1/3*(b*x^3 + a)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(2*b*p + b)
```

Sympy [F]

$$\int x^2(a^2+2abx^3+b^2x^6)^p\,dx$$
$$= \begin{cases} \frac{x^3}{3\sqrt{a^2}} & \text{for } b=0 \wedge p=-\frac{1}{2} \\ \frac{x^3(a^2)^p}{3} & \text{for } b=0 \\ \int \frac{x^2}{\sqrt{(a+bx^3)^2}}\,dx & \text{for } p=-\frac{1}{2} \\ \frac{a(a^2+2abx^3+b^2x^6)^p}{6bp+3b} + \frac{bx^3(a^2+2abx^3+b^2x^6)^p}{6bp+3b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Piecewise((x**3/(3*sqrt(a**2)), Eq(b, 0) & Eq(p, -1/2)), (x**3*(a**2)**p/3
, Eq(b, 0)), (Integral(x**2/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (a*(a
**2 + 2*a*b*x**3 + b**2*x**6)**p/(6*b*p + 3*b) + b*x**3*(a**2 + 2*a*b*x**3
 + b**2*x**6)**p/(6*b*p + 3*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int x^2(a^2+2abx^3+b^2x^6)^p\,dx = \frac{(bx^3+a)(bx^3+a)^{2p}}{3b(2p+1)}$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
1/3*(b*x^3 + a)*(b*x^3 + a)^(2*p)/(b*(2*p + 1))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int x^2 \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx = \frac{(b^2x^6 + 2\,abx^3 + a^2)^p bx^3 + (b^2x^6 + 2\,abx^3 + a^2)^p a}{3\,(2\,bp + b)}$$

input
```
integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
1/3*((b^2*x^6 + 2*a*b*x^3 + a^2)^p*b*x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a
)/(2*b*p + b)
```

Mupad [B] (verification not implemented)

Time = 18.93 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int x^2 \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx = \left(\frac{x^3}{3\,(2\,p+1)} + \frac{a}{3\,b\,(2\,p+1)}\right)\,\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^p$$

input
```
int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
(x^3/(3*(2*p + 1)) + a/(3*b*(2*p + 1)))*(a^2 + b^2*x^6 + 2*a*b*x^3)^p
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int x^2 \left(a^2 + 2abx^3 + b^2x^6\right)^p \, dx = \frac{(b^2x^6 + 2ab\,x^3 + a^2)^p\,(b\,x^3 + a)}{3b\,(2p+1)}$$

input
```
int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x**3 + b**2*x**6)**p*(a + b*x**3))/(3*b*(2*p + 1))
```

3.116 $\int x(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \frac{x^2\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(1,\frac{5}{3}+2p,\frac{5}{3},-\frac{bx^3}{a}\right)}{2a}$$

output
```
1/2*x^2*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 5/3+2*p],[5/3],-
b*x^3/a)/a
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \frac{1}{2}x^2\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{2}{3},-2p,\frac{5}{3},-\frac{bx^3}{a}\right)$$

input
```
Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x^2*((a + b*x^3)^2)^p*Hypergeometric2F1[2/3, -2*p, 5/3, -((b*x^3)/a)])/(2
*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\int x\left(\frac{bx^3}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}x^2\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\text{Hypergeometric2F1}\left(\frac{2}{3},-2p,\frac{5}{3},-\frac{bx^3}{a}\right)$$

input
```
Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2/3, -2*p, 5/3, -((b*
x^3)/a)])/(2*(1 + (b*x^3)/a)^(2*p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int x\left(b^{2} x^{6}+2 a \,x^{3} b +a^{2}\right)^{p} d x$$

input
```
int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Fricas [F]

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^{2} x^{6}+2 \, a b x^{3}+a^{2}\right)^{p} x \,dx$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

Sympy [F]

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int x\left(\left(a+bx^3\right)^2\right)^p\,dx$$

input
```
integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Integral(x*((a + b*x**3)**2)**p, x)
```

Maxima [F]

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^2x^6+2\,abx^3+a^2\right)^p x\,dx$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

Giac [F]

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^2x^6+2\,abx^3+a^2\right)^p x\,dx$$

input
```
integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int x\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\,dx$$

input
```
int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

Reduce [F]

$$\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx$$
$$=\frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p x^2+18\left(\int \frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p x}{3bp\,x^3+b\,x^3+3ap+a}dx\right)a\,p^2+6\left(\int \frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p x}{3bp\,x^3+b\,x^3+3ap+a}dx\right)ap}{6p+2}$$

input
```
int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x**3 + b**2*x**6)**p*x**2 + 18*int(((a**2 + 2*a*b*x**3 + b*
*2*x**6)**p*x)/(3*a*p + a + 3*b*p*x**3 + b*x**3),x)*a*p**2 + 6*int(((a**2
+ 2*a*b*x**3 + b**2*x**6)**p*x)/(3*a*p + a + 3*b*p*x**3 + b*x**3),x)*a*p)/
(2*(3*p + 1))
```

3.117 $\int (a^2+2abx^3+b^2x^6)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int (a^2+2abx^3+b^2x^6)^p \, dx = \frac{x(a+bx^3)(a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(1,\frac{4}{3}+2p,\frac{4}{3},-\frac{bx^3}{a}\right)}{a}$$

output
```
x*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 4/3+2*p],[4/3],-b*x^3/
a)/a
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.98

$$\int (a^2+2abx^3+b^2x^6)^p \, dx = \frac{4^{-p}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-2p}\left(\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}\right)^{-2p}\left((a+bx^3)^2\right)^p \text{AppellF1}\left(1+2p,-2\right.}{\sqrt[3]{b}(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*((a + b*x^3)^2)^p*AppellF1[1 + 2*p, -2*p
, -2*p, 2*(1 + p), -(((-1)^(2/3)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/((1 + (
-1)^(1/3))*a^(1/3))), (I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqr
t[3])])/(4^p*b^(1/3)*(1 + 2*p)*((a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3)))^(2*p)*((I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^(
2*p))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1385, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^3+b^2x^6\right)^p \, dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \int \left(\frac{bx^3}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{778}$$

$$x\left(\frac{bx^3}{a}+1\right)^{-2p} \left(a^2+2abx^3+b^2x^6\right)^p \text{Hypergeometric2F1}\left(\frac{1}{3},-2p,\frac{4}{3},-\frac{bx^3}{a}\right)$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

output
```
(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1/3, -2*p, 4/3, -((b*x^
3)/a)])/(1 + (b*x^3)/a)^(2*p)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \left(b^{2} x^{6}+2 a \,x^{3} b +a^{2}\right)^{p}d x$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Fricas [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^{2} x^{6}+2 \, a b x^{3}+a^{2}\right)^{p}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

Sympy [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

output
```
Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**p, x)
```

Maxima [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^2x^6+2\,abx^3+a^2\right)^p\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

Giac [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(b^2x^6+2\,abx^3+a^2\right)^p\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx=\int \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

Reduce [F]

$$\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx$$
$$=\frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p x+36\left(\int \frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p}{6bp\,x^3+b\,x^3+6ap+a}dx\right)a\,p^2+6\left(\int \frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p}{6bp\,x^3+b\,x^3+6ap+a}dx\right)ap}{6p+1}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x**3 + b**2*x**6)**p*x + 36*int((a**2 + 2*a*b*x**3 + b**2*x
**6)**p/(6*a*p + a + 6*b*p*x**3 + b*x**3),x)*a*p**2 + 6*int((a**2 + 2*a*b*
x**3 + b**2*x**6)**p/(6*a*p + a + 6*b*p*x**3 + b*x**3),x)*a*p)/(6*p + 1)
```

3.118 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x}\,dx = -\frac{(a+bx^3)\,(a^2+2abx^3+b^2x^6)^p\,\text{Hypergeometric2F1}\left(1,1+2p,2(1+p),1+\frac{bx^3}{a}\right)}{3a(1+2p)}$$

output
```
-1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 1+2*p],[2*p+2],1+b*
x^3/a)/a/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x}\,dx = -\frac{(a+bx^3)\left((a+bx^3)^2\right)^p\,\text{Hypergeometric2F1}\left(1,1+2p,2+2p,1+\frac{bx^3}{a}\right)}{3a(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
```

output
```
-1/3*((a + b*x^3)*((a + b*x^3)^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 + 2*p,
 1 + (b*x^3)/a])/(a*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1385, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x}dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x^3}dx^3$$

$$\Big\downarrow \text{75}$$

$$-\frac{\left(\frac{bx^3}{a}+1\right)(a^2+2abx^3+b^2x^6)^p\,\text{Hypergeometric2F1}\left(1,2p+1,2(p+1),\frac{bx^3}{a}+1\right)}{3(2p+1)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
```

output
```
-1/3*((1 + (b*x^3)/a)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1, 1
 + 2*p, 2*(1 + p), 1 + (b*x^3)/a])/(1 + 2*p)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{(b^2x^6 + 2a\,x^3b + a^2)^p}{x}dx$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
```

output
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x}\,dx=\int \frac{\left((a+bx^3)^2\right)^p}{x}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x,x)
```

output
```
Integral(((a + b*x**3)**2)**p/x, x)
```

Maxima [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x}\,dx=\int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

Giac [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x}\,dx=\int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^p}{x}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x, x)
```

Reduce [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x}\,dx = \frac{(b^2x^6+2ab\,x^3+a^2)^p+6\left(\int \frac{(b^2x^6+2ab\,x^3+a^2)^p}{b\,x^4+ax}dx\right)ap}{6p}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
```

output
```
((a**2 + 2*a*b*x**3 + b**2*x**6)**p + 6*int((a**2 + 2*a*b*x**3 + b**2*x**6
)**p/(a*x + b*x**4),x)*a*p)/(6*p)
```

3.119 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx = -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p \,\text{Hypergeometric2F1}\left(-\frac{1}{3},-2p,\frac{2}{3},-\frac{bx^3}{a}\right)}{x}$$

output
```
-(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-1/3, -2*p],[2/3],-b*x^3/a)/x/((1+b*
x^3/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx = -\frac{\left((a+bx^3)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(-\frac{1}{3},-2p,\frac{2}{3},-\frac{bx^3}{a}\right)}{x}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
```

output
```
-((((a + b*x^3)^2)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x^3)/a)])/(x*
(1 + (b*x^3)/a)^(2*p)))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1385, 888\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x^2}dx$$

$$\Big\downarrow \text{888}$$

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(-\frac{1}{3},-2p,\frac{2}{3},-\frac{bx^3}{a}\right)}{x}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
```

output
```
-(((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x
^3)/a)])/(x*(1 + (b*x^3)/a)^(2*p)))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{(b^2x^6 + 2a\,x^3b + a^2)^p}{x^2}dx$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
```

output
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^2}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^2}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx = \int \frac{\left((a+bx^3)^2\right)^p}{x^2}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**2,x)
```

output
```
Integral(((a + b*x**3)**2)**p/x**2, x)
```

Maxima [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx = \int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^2}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx = \int \frac{(b^2x^6+2\,abx^3+a^2)^p}{x^2}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx=\int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^p}{x^2}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^2,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^2, x)
```

Reduce [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx$$
$$=\frac{(b^2x^6+2ab\,x^3+a^2)^p+36\left(\int \frac{(b^2x^6+2ab\,x^3+a^2)^p}{6bp\,x^5-b\,x^5+6ap\,x^2-a\,x^2}dx\right)a\,p^2x-6\left(\int \frac{(b^2x^6+2ab\,x^3+a^2)^p}{6bp\,x^5-b\,x^5+6ap\,x^2-a\,x^2}dx\right)apx}{x\,(6p-1)}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
```

output
```
((a**2 + 2*a*b*x**3 + b**2*x**6)**p + 36*int((a**2 + 2*a*b*x**3 + b**2*x**
6)**p/(6*a*p*x**2 - a*x**2 + 6*b*p*x**5 - b*x**5),x)*a*p**2*x - 6*int((a**
2 + 2*a*b*x**3 + b**2*x**6)**p/(6*a*p*x**2 - a*x**2 + 6*b*p*x**5 - b*x**5)
,x)*a*p*x)/(x*(6*p - 1))
```

3.120 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3} dx = -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(-\frac{2}{3},-2p,\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2}$$

output
```
-1/2*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-2/3, -2*p],[1/3],-b*x^3/a)/x^2/
((1+b*x^3/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3} dx = -\frac{\left((a+bx^3)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p} \text{Hypergeometric2F1}\left(-\frac{2}{3},-2p,\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
```

output
```
-1/2*(((a + b*x^3)^2)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((b*x^3)/a)])/
(x^2*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3}\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x^3}dx$$

$$\Big\downarrow \text{888}$$

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-2p,\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
```

output
```
-1/2*((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((
b*x^3)/a)])/(x^2*(1 + (b*x^3)/a)^(2*p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{(b^2x^6 + 2a\,x^3b + a^2)^p}{x^3}dx$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
```

output
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^3}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^3}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^3}\,dx = \int \frac{\left((a + bx^3)^2\right)^p}{x^3}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**3,x)
```

output
```
Integral(((a + b*x**3)**2)**p/x**3, x)
```

Maxima [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^3}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^3}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

Giac [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^3}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^3}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^3} dx = \int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^p}{x^3} dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^3,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^3, x)
```

Reduce [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^3} dx = \frac{(b^2x^6 + 2ab\,x^3 + a^2)^p + 18\left(\int \frac{(b^2x^6+2ab\,x^3+a^2)^p}{3bp\,x^6-b\,x^6+3ap\,x^3-a\,x^3}dx\right) a\,p^2x^2 - 6\left(\int \frac{(b^2x^6+2ab\,x^3+a^2)^p}{3bp\,x^6-b\,x^6+3ap\,x^3-a\,x^3}dx\right) ap\,x^2}{2x^2\,(3p-1)}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
```

output
```
((a**2 + 2*a*b*x**3 + b**2*x**6)**p + 18*int((a**2 + 2*a*b*x**3 + b**2*x**
6)**p/(3*a*p*x**3 - a*x**3 + 3*b*p*x**6 - b*x**6),x)*a*p**2*x**2 - 6*int((
a**2 + 2*a*b*x**3 + b**2*x**6)**p/(3*a*p*x**3 - a*x**3 + 3*b*p*x**6 - b*x*
*6),x)*a*p*x**2)/(2*x**2*(3*p - 1))
```

3.121 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx = \frac{b(a+bx^3)(a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(2,1+2p,2(1+p),1+\frac{bx^3}{a}\right)}{3a^2(1+2p)}$$

output
```
1/3*b*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([2, 1+2*p],[2*p+2],1+b
*x^3/a)/a^2/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx = \frac{b(a+bx^3)\left((a+bx^3)^2\right)^p \text{Hypergeometric2F1}\left(2,1+2p,2+2p,1+\frac{bx^3}{a}\right)}{3a^2(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]
```

output
```
(b*(a + b*x^3)*((a + b*x^3)^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2 + 2*p, 1
+ (b*x^3)/a])/(3*a^2*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1385, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}dx$$

$$\Big\downarrow \text{ 1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x^4}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x^6}dx^3$$

$$\Big\downarrow \text{ 75}$$

$$\frac{b\left(\frac{bx^3}{a}+1\right)(a^2+2abx^3+b^2x^6)^p\,\text{Hypergeometric2F1}\left(2,2p+1,2(p+1),\frac{bx^3}{a}+1\right)}{3a(2p+1)}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]
```

output
```
(b*(1 + (b*x^3)/a)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2, 1 +
2*p, 2*(1 + p), 1 + (b*x^3)/a])/(3*a*(1 + 2*p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{(b^2x^6 + 2a\,x^3b + a^2)^p}{x^4}dx$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

output
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^4}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^4}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^4} \, dx = \int \frac{\left((a + bx^3)^2\right)^p}{x^4} \, dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**4,x)
```

output
```
Integral(((a + b*x**3)**2)**p/x**4, x)
```

Maxima [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^4} \, dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^4} \, dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

Giac [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^4} \, dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^4} \, dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx=\int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^p}{x^4}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^4,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^4, x)
```

Reduce [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx=\frac{-(b^2x^6+2ab\,x^3+a^2)^p+6\left(\int \frac{(b^2x^6+2ab\,x^3+a^2)^p}{b\,x^4+ax}dx\right)bp\,x^3}{3x^3}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

output
```
( - (a**2 + 2*a*b*x**3 + b**2*x**6)**p + 6*int((a**2 + 2*a*b*x**3 + b**2*x
**6)**p/(a*x + b*x**4),x)*b*p*x**3)/(3*x**3)
```

3.122 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx = -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p \operatorname{Hypergeometric2F1}\left(-\frac{4}{3},-2p,-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4}$$

output
```
-1/4*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-4/3, -2*p],[-1/3],-b*x^3/a)/x^4
/((1+b*x^3/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx = -\frac{\left((a+bx^3)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p} \operatorname{Hypergeometric2F1}\left(-\frac{4}{3},-2p,-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4}$$

input
```
Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
```

output
```
-1/4*(((a + b*x^3)^2)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -((b*x^3)/a)])
/(x^4*(1 + (b*x^3)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\int \frac{\left(\frac{bx^3}{a}+1\right)^{2p}}{x^5}dx$$

$$\Big\downarrow \text{888}$$

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p \text{Hypergeometric2F1}\left(-\frac{4}{3},-2p,-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4}$$

input
```
Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
```

output
```
-1/4*((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -(
(b*x^3)/a)])/(x^4*(1 + (b*x^3)/a)^(2*p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{(b^2x^6 + 2a\,x^3b + a^2)^p}{x^5}dx$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
```

output
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^5}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^5}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^5}\,dx = \int \frac{\left((a + bx^3)^2\right)^p}{x^5}\,dx$$

input
```
integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**5,x)
```

output
```
Integral(((a + b*x**3)**2)**p/x**5, x)
```

Maxima [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^5}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^5}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="maxima")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

Giac [F]

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^p}{x^5}\,dx = \int \frac{(b^2x^6 + 2\,abx^3 + a^2)^p}{x^5}\,dx$$

input
```
integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="giac")
```

output
```
integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx = \int \frac{(a^2+2\,a\,b\,x^3+b^2\,x^6)^p}{x^5}\,dx$$

input
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^5,x)
```

output
```
int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^5, x)
```

Reduce [F]

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx = \frac{(b^2x^6+2ab\,x^3+a^2)^p+18\left(\int \frac{(b^2x^6+2ab\,x^3+a^2)^p}{3bp\,x^8-2b\,x^8+3ap\,x^5-2a\,x^5}dx\right)a\,p^2x^4-12\left(\int \frac{(b^2x^6+2ab\,x^3+a^2)^p}{3bp\,x^8-2b\,x^8+3ap\,x^5-2a\,x^5}dx\right)ap\,x^4}{2x^4\,(3p-2)}$$

input
```
int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
```

output
```
((a**2 + 2*a*b*x**3 + b**2*x**6)**p + 18*int((a**2 + 2*a*b*x**3 + b**2*x**
6)**p/(3*a*p*x**5 - 2*a*x**5 + 3*b*p*x**8 - 2*b*x**8),x)*a*p**2*x**4 - 12*
int((a**2 + 2*a*b*x**3 + b**2*x**6)**p/(3*a*p*x**5 - 2*a*x**5 + 3*b*p*x**8
 - 2*b*x**8),x)*a*p*x**4)/(2*x**4*(3*p - 2))
```

3.123 $\int \frac{x^8}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx=\frac{x^3}{3c}-\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}$$

output
```
1/3*x^3/c-1/3*(-2*a*c+b^2)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4
*a*c+b^2)^(1/2)-1/6*b*ln(c*x^6+b*x^3+a)/c^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx=\frac{2cx^3+\frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log\left(a+bx^3+cx^6\right)}{6c^2}$$

input
```
Integrate[x^8/(a + b*x^3 + c*x^6),x]
```

output
```
(2*c*x^3 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] - b*Log[a + b*x^3 + c*x^6])/(6*c^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{a+bx^3+cx^6} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^6}{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{1143}$$

$$\frac{1}{3}\int \left(\frac{1}{c}-\frac{bx^3+a}{c\,(cx^6+bx^3+a)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{2c^2}+\frac{x^3}{c}\right)$$

input
```
Int[x^8/(a + b*x^3 + c*x^6),x]
```

output
```
(x^3/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqr
t[b^2 - 4*a*c]) - (b*Log[a + b*x^3 + c*x^6])/(2*c^2))/3
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

method	result
default	$\frac{x^3}{3c}+\frac{-\frac{b\ln\left(c\,x^6+b\,x^3+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{3c}$
risch	$\frac{x^3}{3c}-\frac{2\ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,b\right)x^3+2\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,a\right)ab}{3c(4ac-b^2)}+\frac{\ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{}\right.\right.}{}$

input
```
int(x^8/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/c+1/3/c*(-1/2*b/c*ln(c*x^6+b*x^3+a)+2*(-a+1/2*b^2/c)/(4*a*c-b^2)^(
1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.14

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx = \left[\frac{2\,(b^2c-4\,ac^2)x^3-(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-(b^3-4\,abc)\log\left(cx\right.}{6\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(2*(b^2*c - 4*a*c^2)*x^3 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 +
 b*x^3 + a)) - (b^3 - 4*a*b*c)*log(c*x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3)
, 1/6*(2*(b^2*c - 4*a*c^2)*x^3 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*
x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(75) = 150$.

Time = 1.70 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.90

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx = \left(-\frac{b}{6c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)\log\left(x^3 + \frac{-ab-12ac^2\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)+3b^2c\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}}{6c^2\cdot(4}\right.}{2ac-b^2}\right.$$
$$+\left(-\frac{b}{6c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)\log\left(x^3 + \frac{-ab-12ac^2\left(-\frac{b}{6c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)+3b^2c\left(-\frac{b}{6c^2}+\frac{\sqrt{-4ac+b^2}}{6c^2\cdot(4}\right.}{2ac-b^2}\right.$$
$$+\frac{x^3}{3c}$$

input
```
integrate(x**8/(c*x**6+b*x**3+a),x)
```

output
```
(-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))
*log(x**3 + (-a*b - 12*a*c**2*(-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c -
b**2)/(6*c**2*(4*a*c - b**2))) + 3*b**2*c*(-b/(6*c**2) - sqrt(-4*a*c + b**
2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(6*c**2)
 + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))*log(x**3 +
(-a*b - 12*a*c**2*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**
2*(4*a*c - b**2))) + 3*b**2*c*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c -
b**2)/(6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**3/(3*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx = \frac{x^3}{3\,c} - \frac{b\log\left(cx^6+bx^3+a\right)}{6\,c^2} + \frac{(b^2-2\,ac)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/3*x^3/c - 1/6*b*log(c*x^6 + b*x^3 + a)/c^2 + 1/3*(b^2 - 2*a*c)*arctan((2
*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 20.37 (sec) , antiderivative size = 1758, normalized size of antiderivative = 21.70

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int(x^8/(a + b*x^3 + c*x^6),x)
```

output
```
x^3/(3*c) + (log(a + b*x^3 + c*x^6)*(3*b^3 - 12*a*b*c))/(2*(36*a*c^3 - 9*b
^2*c^2)) + (atan((4*c^3*x^3*(4*a*c - b^2)^(3/2)*((b*((b^5 + a^2*b*c^2 - 2*
a*b^3*c)/c^3 + ((3*b^3 - 12*a*b*c)*((6*a^2*c^4 + 12*b^4*c^2 - 18*a*b^2*c^3
)/c^3 + ((3*b^3 - 12*a*b*c)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(
3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2))))/(
2*(36*a*c^3 - 9*b^2*c^2)) - ((((2*a*c - b^2)*((45*b^3*c^4 - 36*a*b*c^5)/c^
3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(6*c^2*(4*a*c
 - b^2)^(1/2)) + (9*b^2*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^
2)^(1/2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2
)) - (3*b^2*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(4*c*(4*a*c - b^2)*(36*a*c
^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*(((2*a*
c - b^2)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/
(36*a*c^3 - 9*b^2*c^2)))/(6*c^2*(4*a*c - b^2)^(1/2)) + (9*b^2*c*(3*b^3 - 1
2*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2))))/(
2*(36*a*c^3 - 9*b^2*c^2)) - (b^2*(2*a*c - b^2)^3)/(4*c^3*(4*a*c - b^2)^(3/
2)) + (((6*a^2*c^4 + 12*b^4*c^2 - 18*a*b^2*c^3)/c^3 + ((3*b^3 - 12*a*b*c)*
((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3
 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c -
 b^2)^(1/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))))/(b^6 - 8*a^3*c^3 + 12*a^2*b
^2*c^2 - 6*a*b^4*c) - (c^2*(2*a*c - b^2)*(4*a*c - b^2)*(((3*b^3 - 12*a*...
```

Reduce [F]

$$\int \frac{x^8}{a+bx^3+cx^6}\,dx = \frac{-6\left(\int \frac{x^2}{c\,x^6+b\,x^3+a}dx\right)ac+3\left(\int \frac{x^2}{c\,x^6+b\,x^3+a}dx\right)b^2-\log(c\,x^6+b\,x^3+a)\,b+2c\,x^3}{6c^2}$$

input
```
int(x^8/(c*x^6+b*x^3+a),x)
```

output
```
( - 6*int(x**2/(a + b*x**3 + c*x**6),x)*a*c + 3*int(x**2/(a + b*x**3 + c*x
**6),x)*b**2 - log(a + b*x**3 + c*x**6)*b + 2*c*x**3)/(6*c**2)
```

3.124 $\int \frac{x^5}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

output
```
1/3*b*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)+1/6*ln(
c*x^6+b*x^3+a)/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx=\frac{-\frac{2b\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+bx^3+cx^6\right)}{6c}$$

input
```
Integrate[x^5/(a + b*x^3 + c*x^6),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[
a + b*x^3 + c*x^6])/(6*c)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1693, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+bx^3+cx^6}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^3}{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3}{2c}-\frac{b\int \frac{1}{cx^6+bx^3+a}dx^3}{2c}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{b\int \frac{1}{-x^6+b^2-4ac}d(2cx^3+b)}{c}+\frac{\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3}{2c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3}{2c}+\frac{b\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\frac{b\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^3+cx^6\right)}{2c}\right)$$

input
```
Int[x^5/(a + b*x^3 + c*x^6),x]
```

output
```
((b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^3 + c*x^6]/(2*c))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
default	$\frac{\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{6c}-\frac{b\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{3c\sqrt{4ac-b^{2}}}$
risch	$\frac{2\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,b\right)x^{3}+2\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,a\right)a}{3\left(4ac-b^{2}\right)}-\frac{\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,b\right)x^{3}+2\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,a\right)b^{2}}{6c\left(4ac-b^{2}\right)}$

input
```
int(x^5/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(c*x^6+b*x^3+a)/c-1/3*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*
c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.13

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx$$
$$= \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+(b^2-4\,ac)\log\left(cx^6+bx^3+a\right)}{6\,(b^2c-4\,ac^2)}, \frac{2\,\sqrt{-b^2+4\,ac}b}{}\right.$$

input
```
integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*
x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) + (b^2 - 4*a*c)*log(c*x^6
 + b*x^3 + a))/(b^2*c - 4*a*c^2), 1/6*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c
*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^6 + b*
x^3 + a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54) = 108$.

Time = 0.85 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.54

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{6c\left(4ac-b^2\right)} + \frac{1}{6c}\right)\log\left(x^3 + \frac{-12ac\left(-\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)+2a+3b^2\left(-\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{6c\left(4ac-b^2\right)} + \frac{1}{6c}\right)\log\left(x^3 + \frac{-12ac\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)+2a+3b^2\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)}+\frac{1}{6c}\right)}{b}\right)$$

input
```
integrate(x**5/(c*x**6+b*x**3+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*
c*(-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(
-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b) + (b*sqrt(-4*a*
c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c*(b*sqrt(-4*a
*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(b*sqrt(-4*a*c +
 b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = -\frac{b\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c} + \frac{\log\left(cx^6+bx^3+a\right)}{6\,c}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1
/6*log(c*x^6 + b*x^3 + a)/c
```

Mupad [B] (verification not implemented)

Time = 20.18 (sec) , antiderivative size = 1199, normalized size of antiderivative = 19.03

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int(x^5/(a + b*x^3 + c*x^6),x)
```

output
```
(log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) + (b*at
an((4*x^3*((b*(b^2 - ((12*b^2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b
^2))/(36*a*c^2 - 9*b^2*c))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)))*(12
*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(45*b^2*c^2 - (27*b^2*c^3
*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b
^3*c^2*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(6
*c*(4*a*c - b^2)^(1/2)) + (3*b^4*c*(12*a*c - 3*b^2))/(4*(36*a*c^2 - 9*b^2*
c)*(4*a*c - b^2))))/(4*a^2*c) + ((2*a*c - b^2)*(b^5/(4*(4*a*c - b^2)^(3/2)
) + ((12*a*c - 3*b^2)*((b*(45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*
a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b^3*c^2*(12*a*c - 3*b^2)
)/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c))
 - (b*(12*b^2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*a*c^2 -
9*b^2*c))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c))))/(6*c*(4*a*c - b^2)^
(1/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^(3/2))/b^3 + ((4*a*c
 - b^2)^(3/2)*(a*b + ((((72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*
c^2 - 9*b^2*c))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - 15*a*b*c)*(12
*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(72*a*b*c^2 - (54*a*b*c^3
*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*a
*b^2*c^2*(12*a*c - 3*b^2))/((36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(6
*c*(4*a*c - b^2)^(1/2)) + (3*a*b^3*c*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9...
```

Reduce [F]

$$\int \frac{x^5}{a+bx^3+cx^6}\,dx = \frac{-3\left(\int \frac{x^2}{c\,x^6+b\,x^3+a}dx\right)b+\log(c\,x^6+b\,x^3+a)}{6c}$$

input
```
int(x^5/(c*x^6+b*x^3+a),x)
```

output
```
( - 3*int(x**2/(a + b*x**3 + c*x**6),x)*b + log(a + b*x**3 + c*x**6))/(6*c
)
```

3.125 $\int \frac{x^2}{a+bx^3+cx^6} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{x^2}{a+bx^3+cx^6} \, dx = -\frac{2\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

output
```
-2/3*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+bx^3+cx^6} \, dx = \frac{2\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{3\sqrt{-b^2+4ac}}$$

input
```
Integrate[x^2/(a + b*x^3 + c*x^6),x]
```

output
```
(2*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/(3*Sqrt[-b^2 + 4*a*c])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1690, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bx^3+cx^6} \, dx$$

$$\Big\downarrow \text{ 1690}$$

$$\frac{1}{3} \int \frac{1}{cx^6+bx^3+a} dx^3$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{2}{3} \int \frac{1}{-x^6+b^2-4ac} d\left(2cx^3+b\right)$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{2\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

input
```
Int[x^2/(a + b*x^3 + c*x^6),x]
```

output
```
(-2*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*Sqrt[b^2 - 4*a*c])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{3\sqrt{4ac-b^{2}}}$	37
risch	$-\frac{\ln\left(\left(-b+\sqrt{-4ac+b^{2}}\right)x^{3}-2a\right)}{3\sqrt{-4ac+b^{2}}}+\frac{\ln\left(\left(b+\sqrt{-4ac+b^{2}}\right)x^{3}+2a\right)}{3\sqrt{-4ac+b^{2}}}$	70

input
```
int(x^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.39

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = \left[\frac{\log\left(\frac{2\,c^{2}x^{6}+2\,bcx^{3}+b^{2}-2\,ac-\left(2\,cx^{3}+b\right)\sqrt{b^{2}-4\,ac}}{cx^{6}+bx^{3}+a}\right)}{3\,\sqrt{b^{2}-4\,ac}},\; -\frac{2\,\sqrt{-b^{2}+4\,ac}\arctan\left(-\frac{\left(2\,cx^{3}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)}{3\,\left(b^{2}-4\,ac\right)}\right]$$

input
```
integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/3*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4
*a*c))/(c*x^6 + b*x^3 + a))/sqrt(b^2 - 4*a*c), -2/3*sqrt(-b^2 + 4*a*c)*arc
tan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(37) = 74$.

Time = 0.39 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.45

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = -\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3} + \frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3}$$

input
```
integrate(x**2/(c*x**6+b*x**3+a),x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x**3 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2
*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/3 + sqrt(-1/(4*a*c - b**2))*log(x**3
+ (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c)
)/3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx=\frac{2\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\sqrt{-b^2+4\,ac}}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
2/3*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 19.92 (sec) , antiderivative size = 174, normalized size of antiderivative = 4.58

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\frac{x^3\,{\left(4\,a\,c-b^2\right)}^4}{2}+a\,b\,{\left(4\,a\,c-b^2\right)}^3+a\,b^3\,{\left(4\,a\,c-b^2\right)}^2+b^2\,x^3\,{\left(4\,a\,c-b^2\right)}^3+\frac{b^4\,x^3\,{\left(4\,a\,c-b^2\right)}^2}{2}}{b^2\,\left(32\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}-4\,a^2\,b^2\,c\,\sqrt{4\,a\,c-b^2}\right)-64\,a^4\,c^3\,\sqrt{4\,a\,c-b^2}}\right)}{3\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^2/(a + b*x^3 + c*x^6),x)
```

output
```
-(2*atan(((x^3*(4*a*c - b^2)^4)/2 + a*b*(4*a*c - b^2)^3 + a*b^3*(4*a*c - b
^2)^2 + b^2*x^3*(4*a*c - b^2)^3 + (b^4*x^3*(4*a*c - b^2)^2)/2)/(b^2*(32*a^
3*c^2*(4*a*c - b^2)^(1/2) - 4*a^2*b^2*c*(4*a*c - b^2)^(1/2)) - 64*a^4*c^3*
(4*a*c - b^2)^(1/2))))/(3*(4*a*c - b^2)^(1/2))
```

Reduce [F]

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = \int \frac{x^2}{c\,x^6+b\,x^3+a}dx$$

input
```
int(x^2/(c*x^6+b*x^3+a),x)
```

output
```
int(x**2/(a + b*x**3 + c*x**6),x)
```

3.126 $\int \frac{1}{x(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{1}{x(a+bx^3+cx^6)}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}$$

output
```
1/3*b*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)+ln(x)/a
-1/6*ln(c*x^6+b*x^3+a)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^3+cx^6)}\,dx=\frac{\log(x)}{a}-\frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^3}{b+2c\#1^3}\&\right]}{3a}$$

input
```
Integrate[1/(x*(a + b*x^3 + c*x^6)),x]
```

output
```
Log[x]/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1]*
#1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1693, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^3+cx^6)} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^3 (cx^6+bx^3+a)}dx^3$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{3}\left(\frac{\int -\frac{cx^3+b}{cx^6+bx^3+a}dx^3}{a}+\frac{\log (x^3)}{a}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{\log (x^3)}{a}-\frac{\int \frac{cx^3+b}{cx^6+bx^3+a}dx^3}{a}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{\log (x^3)}{a}-\frac{\frac{1}{2}b\int \frac{1}{cx^6+bx^3+a}dx^3+\frac{1}{2}\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3}{a}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{\log (x^3)}{a}-\frac{\frac{1}{2}\int \frac{2cx^3+b}{cx^6+bx^3+a}dx^3-b\int \frac{1}{-x^6+b^2-4ac}d(2cx^3+b)}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\log(x^3)}{a}-\frac{\frac{1}{2}\int\frac{2cx^3+b}{cx^6+bx^3+a}dx^3-\frac{b\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\frac{\log(x^3)}{a}-\frac{\frac{1}{2}\log(a+bx^3+cx^6)-\frac{b\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

input
```
Int[1/(x*(a + b*x^3 + c*x^6)),x]
```

output
```
(Log[x^3]/a - (-((b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4
*a*c]) + Log[a + b*x^3 + c*x^6]/2)/a)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{\frac{\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{2}+\frac{b\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{3a}+\frac{\ln\left(x\right)}{a}$	65
risch	$\frac{\ln\left(x\right)}{a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4a^{2}c-b^{2}a\right)\_Z^{2}+\left(4ac-b^{2}\right)\_Z+c\right)}\_R\ln\left(\left(\left(-14ac+4b^{2}\right)\_R-7c\right)x^{3}+\_Rab-3b\right)\right)}{3}$	76

input
```
int(1/x/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a*(1/2*ln(c*x^6+b*x^3+a)+b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*
c-b^2)^(1/2)))+ln(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.23

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^6+bx^3+a\right)+6\left(b^2-4\,ac\right)\mathrm{lo}}{6\left(ab^2-4\,a^2c\right)}\right.$$

input
```
integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*
x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^2 - 4*a*c)*log(c*x^6
 + b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/6*(2*sqrt(-b^
2 + 4*a*c)*b*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^
2 - 4*a*c)*log(c*x^6 + b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2
*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.

Time = 4.25 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.67

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)} - \frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)+3ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{6a\left(4ac-b^2\right)} - \frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)+3ab^2\left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right) + \frac{\log\left(x\right)}{a}$$

input
```
integrate(1/x/(c*x**6+b*x**3+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-12*a*
*2*c*(-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2*(-
b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b*c
)) + (b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-1
2*a**2*c*(b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2
*(b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b
*c)) + log(x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx = -\frac{b\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\sqrt{-b^2+4\,aca}} - \frac{\log\left(cx^6+bx^3+a\right)}{6\,a} + \frac{\log\left(|x|\right)}{a}$$

input
```
integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1
/6*log(c*x^6 + b*x^3 + a)/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 20.84 (sec) , antiderivative size = 1362, normalized size of antiderivative = 19.74

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^3 + c*x^6)),x)
```

output
```
log(x)/a + (log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*
c)) - (b*atan((3*(4*a*c - b^2)^2*(4*b^4 + 7*a^2*c^2 - 15*a*b^2*c)*((b^3*(2
7*b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(21
6*a^3*(4*a*c - b^2)^(3/2)) + (9*b^4*c^3*(12*a*c - 3*b^2)^3)/(16*(9*a*b^2 -
 36*a^2*c)^3*(4*a*c - b^2)^(1/2)) - (3*b^6*c^3*(12*a*c - 3*b^2))/(16*a^2*(
9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(3/2)) - (b*(12*a*c - 3*b^2)^2*(27*b^3*c
^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(8*a*(9*a*
b^2 - 36*a^2*c)^2*(4*a*c - b^2)^(1/2))))/(b^3*c^6*(49*a*c - 12*b^2)) - (3*
(4*a*c - b^2)^(3/2)*(4*b^5 + 29*a^2*b*c^2 - 23*a*b^3*c)*(((12*a*c - 3*b^2)
^3*(27*b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))
)/(8*(9*a*b^2 - 36*a^2*c)^3) - (b^7*c^3)/(48*a^3*(4*a*c - b^2)^2) - (b^2*(
12*a*c - 3*b^2)*(27*b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2
- 36*a^2*c))))/(24*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)) + (9*b^5*c^3*(1
2*a*c - 3*b^2)^2)/(16*a*(9*a*b^2 - 36*a^2*c)^2*(4*a*c - b^2))))/(b^3*c^6*(
49*a*c - 12*b^2)) + (48*a^4*x^3*(((4*b^4 + 7*a^2*c^2 - 15*a*b^2*c)*((b^3*(
63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2))/(2*(9*a*b^2
- 36*a^2*c))))/(216*a^3*(4*a*c - b^2)^(3/2)) + (b*(108*b^4*c^3 - 378*a*b^2
*c^4)*(12*a*c - 3*b^2)^3)/(48*a*(9*a*b^2 - 36*a^2*c)^3*(4*a*c - b^2)^(1/2)
) - (b^3*(108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2))/(144*a^3*(9*a*b^2
 - 36*a^2*c)*(4*a*c - b^2)^(3/2)) - (b*(63*b^2*c^4 - ((108*b^4*c^3 - 37...
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx=\int \frac{1}{c\,x^7+b\,x^4+ax}dx$$

input
```
int(1/x/(c*x^6+b*x^3+a),x)
```

output
```
int(1/(a*x + b*x**4 + c*x**7),x)
```

3.127 $\int \frac{1}{x^4(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{1}{x^4(a+bx^3+cx^6)}\,dx = -\frac{1}{3ax^3}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} -\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}$$

output
```
-1/3/a/x^3-1/3*(-2*a*c+b^2)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/a^2/(-
4*a*c+b^2)^(1/2)-b*ln(x)/a^2+1/6*b*ln(c*x^6+b*x^3+a)/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{1}{x^4(a+bx^3+cx^6)}\,dx &= -\frac{1}{3ax^3}-\frac{b\log(x)}{a^2} \\ &\quad+\frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{b^2\log(x-\#1)-ac\log(x-\#1)+bc\log(x-\#1)\#1^3}{b+2c\#1^3}\&\right]}{3a^2}\end{aligned}$$

input
```
Integrate[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/3*1/(a*x^3) - (b*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6 & , (b^2*Log
[x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a
^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3+cx^6)} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^6 (cx^6+bx^3+a)}dx^3$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{3}\left(\frac{\int -\frac{cx^3+b}{x^3(cx^6+bx^3+a)}dx^3}{a}-\frac{1}{ax^3}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(-\frac{\int \frac{cx^3+b}{x^3(cx^6+bx^3+a)}dx^3}{a}-\frac{1}{ax^3}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{3}\left(-\frac{\int \left(\frac{b}{ax^3}+\frac{-bcx^3-b^2+ac}{a(cx^6+bx^3+a)}\right)dx^3}{a}-\frac{1}{ax^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log(a+bx^3+cx^6)}{2a}+\frac{b\log(x^3)}{a}}{a}-\frac{1}{ax^3}\right)$$

input
```
Int[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

output
```
(-(1/(a*x^3)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/
(a*Sqrt[b^2 - 4*a*c]) + (b*Log[x^3])/a - (b*Log[a + b*x^3 + c*x^6])/(2*a))
/a)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
default	$-\frac{-\frac{b\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{2}+\frac{2\left(ac-\frac{b^{2}}{2}\right)\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{3a^{2}}-\frac{1}{3a\,x^{3}}-\frac{b\ln\left(x\right)}{a^{2}}$
risch	$-\frac{1}{3a\,x^{3}}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(-4abc+b^{3}\right)\_Z+c^{2}\right)}\_R\ln\left(\left(\left(-14a^{3}c+4a^{2}b^{2}\right)\_R^{2}+6\_Rabc-3c^{2}\right)x^{3}\right.\right.}{3}$

input
```
int(1/x^4/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2*(-1/2*b*ln(c*x^6+b*x^3+a)+2*(a*c-1/2*b^2)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))-1/3/a/x^3-b*ln(x)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.29

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)}\,dx$$

$$=\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x^3\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-(b^3-4\,abc)x^3\log\left(cx^6+bx^3+a\right)}{6\,(a^2b^2-4\,a^3c)x^3}\right.$$

$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x^3\arctan\left(-\frac{(2\,cx^3+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^3-4\,abc)x^3\log\left(cx^6+bx^3+a\right)+6\,(b^3-}{6\,(a^2b^2-4\,a^3c)x^3}$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/6*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b*c*x^3 + b^
2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^3 -
 4*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*
b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3), -1/6*(2*(b^2 - 2*a*c)*sqrt(-b^2
+ 4*a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^
3 - 4*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2
*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(85) = 170$.

Time = 86.39 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.88

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)}\,dx = \left(\frac{b}{6a^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6a^2\cdot(4ac-b^2)}\right)\log\left(x^3 + \frac{-12a^3c\left(\frac{b}{6a^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6a^2\cdot(4ac-b^2)}\right) + 3a^2b^2\left(\frac{b}{6a^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2}{6a^2\cdot(4ac-b^2)}\right.}{2ac^2-b^2c}\right.$$
$$+ \left(\frac{b}{6a^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6a^2\cdot(4ac-b^2)}\right)\log\left(x^3 + \frac{-12a^3c\left(\frac{b}{6a^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6a^2\cdot(4ac-b^2)}\right) + 3a^2b^2\left(\frac{b}{6a^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6a^2\cdot(4ac-b^2)}\right.}{2ac^2-b^2c}\right.$$
$$- \frac{1}{3ax^3} - \frac{b\log(x)}{a^2}$$

input
```
integrate(1/x**4/(c*x**6+b*x**3+a),x)
```

output
```
(b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a**2*(4*a*c - b**2)))*
log(x**3 + (-12*a**3*c*(b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6
*a**2*(4*a*c - b**2))) + 3*a**2*b**2*(b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*
a*c - b**2)/(6*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)
) + (b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a**2*(4*a*c - b**2
)))*log(x**3 + (-12*a**3*c*(b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2
)/(6*a**2*(4*a*c - b**2))) + 3*a**2*b**2*(b/(6*a**2) + sqrt(-4*a*c + b**2)
*(2*a*c - b**2)/(6*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**
2*c)) - 1/(3*a*x**3) - b*log(x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)} \, dx = \frac{b \log (cx^6 + bx^3 + a)}{6 a^2} - \frac{b \log (|x|)}{a^2} + \frac{(b^2 - 2 ac) \arctan \left(\frac{2 cx^3 + b}{\sqrt{-b^2 + 4 ac}}\right)}{3 \sqrt{-b^2 + 4 ac} a^2} + \frac{bx^3 - a}{3 a^2 x^3}$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/6*b*log(c*x^6 + b*x^3 + a)/a^2 - b*log(abs(x))/a^2 + 1/3*(b^2 - 2*a*c)*a
rctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/3*(b*
x^3 - a)/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 20.24 (sec) , antiderivative size = 4281, normalized size of antiderivative = 48.10

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^3 + c*x^6)),x)
```

output
```
(atan((48*a^8*x^3*(((4*b^5 + 9*a^2*b*c^2 - 16*a*b^3*c)*(((3*b^3 - 12*a*b*c
)*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a
^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a
^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*
a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*(4*a*c - b^2)^(1/2
)*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) + (((42*a^3*c^6 + 3
3*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)
/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36
*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(
4*a*c - b^2)^(1/2))))/(2*(36*a^3*c - 9*a^2*b^2)) - ((((((2*a*c - b^2)*((25
2*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 -
 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(
1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c
^4))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(
6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2*(108*a^4*
b^4*c^3 - 378*a^5*b^2*c^4))/(72*a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))
*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*(((3*b^3 - 12
*a*b*c)*((42*a^3*c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4
*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*
a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2...
```

Reduce [F]

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)}\, dx = \int \frac{1}{c\, x^{10} + b\, x^7 + a\, x^4} dx$$

input
```
int(1/x^4/(c*x^6+b*x^3+a),x)
```

output
```
int(1/(a*x**4 + b*x**7 + c*x**10),x)
```

3.128 $\int \frac{x^7}{a+bx^3+cx^6} dx$

Optimal result

Integrand size = 18, antiderivative size = 636

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x^2/c+1/6*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/3)*
c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/c^(5/3)/(
b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan
(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)
*3^(1/2)/c^(5/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)+1/6*(b-(-2*a*c+b^2)/(-4*a*c+
b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/c^(
5/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*
ln((b+(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/c^(5/3)/(b+(-4*
a*c+b^2)^(1/2))^(1/3)-1/12*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a
*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3
)*c^(2/3)*x^2)*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/12*(b+(-2*a*
c+b^2)/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)
*(b+(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(1/3)/c^(5/3)/(b+(-
4*a*c+b^2)^(1/2))^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \frac{3x^2-2\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{6c}$$

input
```
Integrate[x^7/(a + b*x^3 + c*x^6),x]
```

output
```
(3*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*
#1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 546, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1703, 27, 1834, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{a+bx^3+cx^6}dx$$

$$\Big\downarrow \text{1703}$$

$$\frac{x^2}{2c}-\frac{\int \frac{2x(bx^3+a)}{cx^6+bx^3+a}dx}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2}{2c}-\frac{\int \frac{x(bx^3+a)}{cx^6+bx^3+a}dx}{c}$$

$$\Big\downarrow \text{1834}$$

$$\frac{x^2}{2c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{2x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2}{2c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{c}$$

$$\Big\downarrow \text{821}$$

$$\frac{x^2}{2c}-$$

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)+\Big(\frac{b}{\sqrt{}}}{c}$$

$$\Big\downarrow \text{16}$$

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)+\Big(}{c}$$

$$\frac{x^2}{2c}-$$

$\downarrow$ 1142

$$\frac{x^2}{2c}-$$

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\int -\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b\cdots}\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\cdots}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.}{}$$

$\downarrow$ 25

$$\frac{x^2}{2c}-$$

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\int \frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b-\cdots}\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b\cdots}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.}{}$$

$\downarrow$ 27

$$\frac{x^2}{2c}-\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{b-\sqrt{\ldots}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\ldots}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$\Big\downarrow$ 1082

$$\frac{x^2}{2c}-\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}\ldots}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+(\ldots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$\Big\downarrow$ 217

$$\frac{x^2}{2c}-\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\ldots\right.}{3\ldots}\right.$$

$$\Big\downarrow \text{1103}$$

$$\frac{x^2}{2c}-\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\cdots\right.}{3\cdots}\right.$$

input
```
Int[x^7/(a + b*x^3 + c*x^6),x]
```

output
```
x^2/(2*c) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b - Sqrt[b^2
 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*a
*c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2
- 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b - Sqrt[b^2 - 4*a*c]
)^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3
)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/
3))) + (b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b + Sqrt[b^2 - 4*a
*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(
1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*
c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b + Sqrt[b^2 - 4*a*c])^(2/3
) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]
/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))))/
c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{x^2}{2c} - \frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6 c+b\_Z^3+a\right)} \frac{\left(\_R^4 b+\_R a\right)\ln\left(x-\_R\right)}{2\_R^5 c+\_R^2 b}}{3c}$	61
risch	$\frac{x^2}{2c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6 c+b\_Z^3+a\right)} \frac{\left(-\_R^4 b-\_R a\right)\ln\left(x-\_R\right)}{2\_R^5 c+\_R^2 b}}{3c}$	63

input
```
int(x^7/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c-1/3/c*sum((_R^4*b+_R*a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^
6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3402 vs. 2(498) = 996.

Time = 0.28 (sec) , antiderivative size = 3402, normalized size of antiderivative = 5.35

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 10.75 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.44

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx$$
$$= \text{RootSum}\left(t^6\cdot\left(46656a^3c^8-34992a^2b^2c^7+8748ab^4c^6-729b^6c^5\right)+t^3\cdot\left(432a^4c^4-1512a^3b^2c^3+1107a\right.\right.$$
$$+\frac{x^2}{2c}$$

input
```
integrate(x**7/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**3*c**8 - 34992*a**2*b**2*c**7 + 8748*a*b**4*c**6 -
 729*b**6*c**5) + _t**3*(432*a**4*c**4 - 1512*a**3*b**2*c**3 + 1107*a**2*b
**4*c**2 - 297*a*b**6*c + 27*b**8) + a**5, Lambda(_t, _t*log(x + (-15552*_
t**5*a**4*c**9 + 27216*_t**5*a**3*b**2*c**8 - 14580*_t**5*a**2*b**4*c**7 +
 3159*_t**5*a*b**6*c**6 - 243*_t**5*b**8*c**5 - 72*_t**2*a**5*c**5 + 594*_
t**2*a**4*b**2*c**4 - 864*_t**2*a**3*b**4*c**3 + 468*_t**2*a**2*b**6*c**2
- 108*_t**2*a*b**8*c + 9*_t**2*b**10)/(5*a**5*b*c**2 - 5*a**4*b**3*c + a**
3*b**5)))) + x**2/(2*c)
```

Maxima [F]

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \int \frac{x^7}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
1/2*x^2/c - integrate((b*x^4 + a*x)/(c*x^6 + b*x^3 + a), x)/c
```

Giac [F]

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \int \frac{x^7}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^7/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 30.87 (sec) , antiderivative size = 4069, normalized size of antiderivative = 6.40

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int(x^7/(a + b*x^3 + c*x^6),x)
```

output
```
log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(
1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^
(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*
c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)
^3))^(2/3))/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2
*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1
/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/6
- (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16
*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
- 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c -
 b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/18 + (a^4*x*(a*c - b^2))/c^2
)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56
*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3
*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48
*a^2*b^2*c^7)))^(1/3) + log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(b^4 + 8*a^2*c^
2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 -
 b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6
*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(
1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c
- b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*...
```

Reduce [F]

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \frac{-2\left(\int \frac{x^4}{c\,x^6+b\,x^3+a}dx\right)b-2\left(\int \frac{x}{c\,x^6+b\,x^3+a}dx\right)a+x^2}{2c}$$

input
```
int(x^7/(c*x^6+b*x^3+a),x)
```

output
```
( - 2*int(x**4/(a + b*x**3 + c*x**6),x)*b - 2*int(x/(a + b*x**3 + c*x**6),
x)*a + x**2)/(2*c)
```

3.129 $\int \frac{x^6}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 631

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

output
```
x/c+1/6*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/3)*c^(1/3
)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)/c^(4/3)/(b-(-4*
a*c+b^2)^(1/2))^(2/3)+1/6*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(
1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(2/3)*3^(1/
2)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)-1/6*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(
1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(2/3)/c^(4/3)/(
b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*ln((b+
(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^
2)^(1/2))^(2/3)+1/12*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2
)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2
/3)*x^2)*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*(b+(-2*a*c+b^2)
/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b+(-
4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(2/3)/c^(4/3)/(b+(-4*a*c+
b^2)^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx = \frac{x}{c} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

input
```
Integrate[x^6/(a + b*x^3 + c*x^6),x]
```

output
```
x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)
/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 521, normalized size of antiderivative = 0.83, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {1703, 1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{a+bx^3+cx^6}dx$$

$$\Big\downarrow \text{1703}$$

$$\frac{x}{c}-\frac{\int \frac{bx^3+a}{cx^6+bx^3+a}dx}{c}$$

$$\Big\downarrow \text{1752}$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \text{750}$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{}$$

$$\Big\downarrow \text{16}$$

$$\frac{x}{c}-$$

$$\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int\frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\cdots\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.$$

$\downarrow$ 27

$$\frac{x}{c}-$$

$$\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{\cdots}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.$$

$\downarrow$ 1142

$$\frac{x}{c}-$$

$$\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\int-\frac{\sqrt[3]{\cdots}}{2c^{2/3}x^2-2^{2/3}\cdots}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.$$

$\downarrow$ 25

$$\frac{\frac{x}{c}-\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int\frac{\sqrt[3]{c}\cdots}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{\cdots}}}{\cdots}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\cdots}$$

$\downarrow$ 27

$$\frac{\frac{x}{c}-\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{\cdots}{2c^{2/3}x^2-2^{2/3}\cdots}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\cdots}$$

$\downarrow$ 1082

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\left(\frac{1}{4}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d}{2\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}$$

$\Big\downarrow$ 217

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\left(\frac{1}{4}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}$$

$$\Big\downarrow \text{1103}$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right.}{4\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}$$

input
```
Int[x^6/(a + b*x^3 + c*x^6),x]
```

output
```
x/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b - Sqrt[b^2
- 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(
2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - S
qrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b - Sqrt[b^2 - 4*a*c])^(
2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x
^2]/(4*c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/2 + ((b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/
3)*c^(1/3)*x])/(3*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/
2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3)
)/Sqrt[3]])/c^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*
(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b
 + Sqrt[b^2 - 4*a*c])^(2/3))))/2)/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+b\_Z^{3}+a\right)}\frac{\left(-\_R^{3}b-a\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+b\_Z^{3}+a\right)}\frac{\left(-\_R^{3}b-a\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	59

input
```
int(x^6/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3
*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2882 vs. $2(495) = 990$.

Time = 0.19 (sec) , antiderivative size = 2882, normalized size of antiderivative = 4.57

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*((1/2)^(1/3)*(sqrt(-3)*c + c)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*
c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))
^(1/3)*log(4*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*
b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 + sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^
2*c^2 - 8*a^3*c^3) - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6 + sqrt(-3)*(b^5
*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 -
 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9
+ 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - (1/2)^(1/
3)*(sqrt(-3)*c - c)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*
c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(4*(a
*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*
b^2*c^2 - 8*a^3*c^3 - sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c
^3) - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(-3)*(b^5*c^4 - 8*a*b^3*
c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c
^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))
*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*...
```

Sympy [A] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.31

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx$$
$$= \text{RootSum}\Big(t^6\cdot\left(46656a^3c^7-34992a^2b^2c^6+8748ab^4c^5-729b^6c^4\right)+t^3\cdot\left(864a^3bc^3-864a^2b^3c^2+270ab^5\right.$$
$$+\frac{x}{c}$$

input
```
integrate(x**6/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
 729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5
*c - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*
_t**4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2
*c**2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)
))) + x/c
```

Maxima [F]

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx = \int \frac{x^6}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

Giac [F]

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx=\int \frac{x^6}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^6/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 22.47 (sec) , antiderivative size = 2280, normalized size of antiderivative = 3.61

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx=\text{Too large to display}$$

input
```
int(x^6/(a + b*x^3 + c*x^6),x)
```

output
```
log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a
*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^
4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)
^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*(-(b^4*(-(
4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/
(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + x/c +
 log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*((b^7 + b^4*
(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^
4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)
^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*((b^7 + b^
4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(
4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(
54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + log((3
*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i - 1)*((
b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2
*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^
(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-...
```

Reduce [F]

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx = \frac{-\left(\int \frac{x^3}{c\,x^6+b\,x^3+a}dx\right)b-\left(\int \frac{1}{c\,x^6+b\,x^3+a}dx\right)a+x}{c}$$

input
```
int(x^6/(c*x^6+b*x^3+a),x)
```

output
```
( - int(x**3/(a + b*x**3 + c*x**6),x)*b - int(1/(a + b*x**3 + c*x**6),x)*a
 + x)/c
```

3.130 $\int \frac{x^4}{a+bx^3+cx^6} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 558

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}} + \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} + \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$

output
```
1/6*(b-(-4*a*c+b^2)^(1/2))^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*
a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/c^(2/3)/(-4*a*c+b^2)^(1/2)
-1/6*(b+(-4*a*c+b^2)^(1/2))^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4
*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/c^(2/3)/(-4*a*c+b^2)^(1/2
)+1/6*(b-(-4*a*c+b^2)^(1/2))^(2/3)*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)
*c^(1/3)*x)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)-1/6*(b+(-4*a*c+b^2)^(1/2))^
(2/3)*ln((b+(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/c^(2/3)/(
-4*a*c+b^2)^(1/2)-1/12*(b-(-4*a*c+b^2)^(1/2))^(2/3)*ln((b-(-4*a*c+b^2)^(1/
2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x
^2)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)+1/12*(b+(-4*a*c+b^2)^(1/2))^(2/3)*l
n((b+(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b+(-4*a*c+b^2)^(1/2))^(1/3
)*x+2^(2/3)*c^(2/3)*x^2)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.08

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)\#1^2}{b+2c\#1^3}\&\right]$$

input
```
Integrate[x^4/(a + b*x^3 + c*x^6),x]
```

output
```
RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1^2)/(b + 2*c*#1^3) & ]/3
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 516, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1710, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{2x}{2cx^3+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{x}{2cx^3+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{821}$$

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$$\Big\downarrow \text{16}$$

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$\downarrow$ 1142

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{2}}{2^{2/3}c^{2/3}x^2-}}{}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{2}}{2^{2/3}c^{2/3}x^2-}}{}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$\downarrow$ 25

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int\frac{\sqrt[3]{2}\sqrt[3]{}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{}}}{}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int\frac{\sqrt[3]{2}\sqrt[3]{}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{}}}{}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$\downarrow$ 27

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int\frac{}{2^{2/3}c^{2/3}x^2-}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int\frac{}{2^{2/3}c^{2/3}x^2-}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$\downarrow$ 1082

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}-}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-}}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b+\sqrt{b^2-4ac}-}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-}}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$\downarrow$ 217

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$\downarrow$ 1103

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

input
```
Int[x^4/(a + b*x^3 + c*x^6),x]
```

output
```
(1 - b/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3
)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]
*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]]
)/(2^(1/3)*c^(1/3))) + Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)
*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)
))/(3*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3))) + (1 + b/Sqrt[b^2 -
4*a*c])*(-1/3*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2
/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^
(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3)
)) + Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4
*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1
/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+b\_Z^{3}+a\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+b\_Z^{3}+a\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	43

input
```
int(x^4/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R^4/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2314 vs. 2(421) = 842.

Time = 0.11 (sec) , antiderivative size = 2314, normalized size of antiderivative = 4.15

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*
c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b
)/(b^2*c^2 - 4*a*c^3))^(1/3)*log((1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^
2 + sqrt(-3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2) - (b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5 + sqrt(-3)*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5))*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4
*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4
*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^
7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 4*(a*b^2 - 2*a^2*c)*x) - 1/6*(1/2)^(
1/3)*(sqrt(-3) + 1)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c
^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2
- 4*a*c^3))^(1/3)*log((1/2)^(2/3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2 - sqrt(-3
)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2) - (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5 - sqrt(-3)*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a
^2*b^2*c^6 - 64*a^3*c^7)))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c +
4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b
^2*c^2 - 4*a*c^3))^(2/3) - 4*(a*b^2 - 2*a^2*c)*x) + 1/6*(1/2)^(1/3)*(sqrt(
-3) - 1)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4
 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3...
```

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.31

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx$$
$$= \text{RootSum}\left(t^6\cdot\left(46656a^3c^5-34992a^2b^2c^4+8748ab^4c^3-729b^6c^2\right)+t^3\left(-432a^2bc^2+216ab^3c-27b^5\right)+\right.$$

input
```
integrate(x**4/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**3*c**5 - 34992*a**2*b**2*c**4 + 8748*a*b**4*c**3 -
 729*b**6*c**2) + _t**3*(-432*a**2*b*c**2 + 216*a*b**3*c - 27*b**5) + a**2
, Lambda(_t, _t*log(x + (15552*_t**5*a**3*c**5 - 11664*_t**5*a**2*b**2*c**
4 + 2916*_t**5*a*b**4*c**3 - 243*_t**5*b**6*c**2 - 108*_t**2*a**2*b*c**2 +
 63*_t**2*a*b**3*c - 9*_t**2*b**5)/(2*a**2*c - a*b**2))))
```

Maxima [F]

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx=\int \frac{x^4}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx=\int \frac{x^4}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 26.47 (sec) , antiderivative size = 2695, normalized size of antiderivative = 4.83

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx=\text{Too large to display}$$

input
```
int(x^4/(a + b*x^3 + c*x^6),x)
```

output
```
log((2^(1/3)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3
*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*
c^3 - (2^(2/3)*(54*a^2*c^3*x*(4*a*c - b^2) - (27*2^(1/3)*a*b*c^3*(4*a*c -
b^2)^2*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2
*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/2)*((b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b
^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/6 - 45*a^2*b^2*c^2 + 9*a*b^4*c
))/18 + a^2*b*c*x)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*
a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) + log((2^(1/3)*((b^5 - b^2*(-(4*a*c -
b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))
/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3 - (2^(2/3)*(54*a^2*c^3*x*(4*a*c
- b^2) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*((b^5 - b^2*(-(4*a*c - b^2)^3
)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*
(4*a*c - b^2)^3))^(2/3))/2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*
b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))
^(1/3))/6 - 45*a^2*b^2*c^2 + 9*a*b^4*c))/18 + a^2*b*c*x)*((b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^
(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3)
- log((2^(1/3)*(3^(1/2)*1i - 1)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + ...
```

Reduce [F]

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = \int \frac{x^4}{c\,x^6+b\,x^3+a}dx$$

input
```
int(x^4/(c*x^6+b*x^3+a),x)
```

output
```
int(x**4/(a + b*x**3 + c*x**6),x)
```

3.131 $\int \frac{x^3}{a+bx^3+cx^6} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 558

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

output
```
1/6*(b-(-4*a*c+b^2)^(1/2))^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*
a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)/c^(1/3)/(-4*a*c+b^2)^(1/2)
-1/6*(b+(-4*a*c+b^2)^(1/2))^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4
*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)/c^(1/3)/(-4*a*c+b^2)^(1/2
)-1/6*(b-(-4*a*c+b^2)^(1/2))^(1/3)*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)
*c^(1/3)*x)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)+1/6*(b+(-4*a*c+b^2)^(1/2))^
(1/3)*ln((b+(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(2/3)/c^(1/3)/(
-4*a*c+b^2)^(1/2)+1/12*(b-(-4*a*c+b^2)^(1/2))^(1/3)*ln((b-(-4*a*c+b^2)^(1/
2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x
^2)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)-1/12*(b+(-4*a*c+b^2)^(1/2))^(1/3)*l
n((b+(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b+(-4*a*c+b^2)^(1/2))^(1/3
)*x+2^(2/3)*c^(2/3)*x^2)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.08

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx=\frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)\#1}{b+2c\#1^3}\&\right]$$

input
```
Integrate[x^3/(a + b*x^3 + c*x^6),x]
```

output
```
RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1)/(b + 2*c*#1^3) & ]/3
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1710, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \; 1710$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+$$
$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx$$

$$\Big\downarrow \; 750$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-}}}{\sqrt[3]{2}}}}{3\left(b-\sqrt{b^2-4ac}\right)^{2}}\right.$$
$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b+\sqrt{b^2-}}}{\sqrt[3]{2}}}}{3\left(\sqrt{b^2-4ac}+b\right)^{2}}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4a\cdots}}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4\cdots}\right.}\cdots\right.$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+\cdots}\right.}{3\sqrt[3]{c}\left(\sqrt{b^2-4ac}\cdots\right.}\cdots\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2\;2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-\cdots}}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-\cdots}\right.}\cdots\right.$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\;2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}\cdots}\right.}{3\sqrt[3]{c}\left(\sqrt{b^2-4a\cdots}\right.}\cdots\right.$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int-\cdots}{2c^{2/\cdots}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int-\cdots}{2c^{2/\cdots}}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

$\downarrow$ 25

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int}{2c^{2/3}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int}{2c^{2/3}}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

$\downarrow$ 27

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int -\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int -\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

$\downarrow$ 1082

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\cdots}{\sqrt[3]{b-\sqrt{b^2-\cdots}}}\right.\cdots}\cdots}{\cdots}\right.\cdots}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\right.$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\cdots}{\sqrt[3]{b+\sqrt{b^2-\cdots}}}\right.\cdots}\cdots}{\cdots}\right.\cdots}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}\cdots}\right.$$

$\downarrow$ 217

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\frac{2\,2^{2/3}\left(\frac{1}{4}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\cdots}{\sqrt[3]{b-\sqrt{\cdots}}}}{\sqrt{\cdots}}\right)}{2\sqrt[3]{c}}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\frac{2\,2^{2/3}\left(\frac{1}{4}\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\cdots}{\sqrt[3]{\sqrt{b^2\cdots}}}}{\sqrt{\cdots}}\right)}{2\sqrt[3]{c}}\cdots\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\cdots$$

↓ 1103

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/}\right.}{4\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/}\right.}{4\sqrt[3]{c}}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

input
```
Int[x^3/(a + b*x^3 + c*x^6),x]
```

output
```
((1 - b/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2
^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*
(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(
1/3))/Sqrt[3]])/c^(1/3) - Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1
/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(
3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/2 + ((1 + b/Sqrt[b^2 - 4*a*c])*((2^(2/3
)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/
3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b +
Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x
 + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b + Sqrt[b^2 - 4*a*c])^(2/3))))/
2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+b\_Z^{3}+a\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	43
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+b\_Z^{3}+a\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	43

input
```
int(x^3/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R^3/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1542 vs. 2(421) = 842.

Time = 0.09 (sec) , antiderivative size = 1542, normalized size of antiderivative = 2.76

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)*l
og(-(1/2)^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3 + sqrt(-3)*(b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3))*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3) + 2*b*x) + 1/6*(1/2
)^(1/3)*(sqrt(-3) - 1)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)*log(-(1/2)
^(1/3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3 - sqrt(-3)*(b^4*c - 8*a*b^2*c^2 +
 16*a^2*c^3))*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5))*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3) + 2*b*x) - 1/6*(1/2)^(1/3)*(
sqrt(-3) + 1)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3)*log((1/2)^(1/3)*(b
^4*c - 8*a*b^2*c^2 + 16*a^2*c^3 + sqrt(-3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3))*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(-((
b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3) + 2*b*x) + 1/6*(1/2)^(1/3)*(sqrt(-3)
 - 1)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3)*log((1/2)^(1/3)*(b^4*c ...
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.22

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx$$
$$=\operatorname{RootSum}\left(t^6\cdot\left(46656a^3c^4-34992a^2b^2c^3+8748ab^4c^2-729b^6c\right)+t^3\cdot\left(432a^2c^2-216ab^2c+27b^4\right)+a,\right.$$

input
```
integrate(x**3/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**3*c**4 - 34992*a**2*b**2*c**3 + 8748*a*b**4*c**2 -
 729*b**6*c) + _t**3*(432*a**2*c**2 - 216*a*b**2*c + 27*b**4) + a, Lambda(
_t, _t*log(x + (2592*_t**4*a**2*c**3 - 1296*_t**4*a*b**2*c**2 + 162*_t**4*
b**4*c + 12*_t*a*c - 3*_t*b**2)/b)))
```

Maxima [F]

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx=\int \frac{x^3}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx=\int \frac{x^3}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 26.25 (sec) , antiderivative size = 2129, normalized size of antiderivative = 3.82

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int(x^3/(a + b*x^3 + c*x^6),x)
```

output
```
log((2^(2/3)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)
/(c*(4*a*c - b^2)^3))^(1/3)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (9*2^(1/3)*a*c^3
*(4*a*c - b^2)^2*(x - (2^(2/3)*b*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*
a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/2)*(-(b*(-(4*a*c - b^2)^3
)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/2))/6
+ 3*a*c^2*x*(2*a*c - b^2))*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2
 - 8*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(
1/3) + log((2^(2/3)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (9*2^(1/3)*a*c^3*(x - (2
^(2/3)*b*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(
4*a*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) -
b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/2)*((b*(-(4*a*c
- b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3)
)/6 + 3*a*c^2*x*(2*a*c - b^2))*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^
2*c^2 + 8*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3
)))^(1/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*(36*a^2*b*c^3 - 9*a*b^3*c^2 + (2
^(1/3)*(3^(1/2)*1i + 1)*(81*a*c^3*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*
(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*
a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b*(-(4*a*c - b^2)^3
)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/36)*(-
(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c -...
```

Reduce [F]

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = \int \frac{x^3}{c\,x^6+b\,x^3+a}dx$$

input
```
int(x^3/(c*x^6+b*x^3+a),x)
```

output
```
int(x**3/(a + b*x**3 + c*x**6),x)
```

3.132 $\int \frac{x}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 558

$$\int \frac{x}{a+bx^3+cx^6}\,dx$$
$$= -\frac{\sqrt[3]{2}\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt[3]{c}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt[3]{c}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

output
```
-1/3*2^(1/3)*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/
2))^(1/3))*3^(1/2))*3^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3
)+1/3*2^(1/3)*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1
/2))^(1/3))*3^(1/2))*3^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
3)-1/3*2^(1/3)*c^(1/3)*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)/
(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/3*2^(1/3)*c^(1/3)*ln((b+
(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*ln((b-(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(
1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(1/3)/(-4*a*c+b
^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/6*c^(1/3)*ln((b+(-4*a*c+b^2)^(1/2
))^(2/3)-2^(1/3)*c^(1/3)*(b+(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^
2)*2^(1/3)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.08

$$\int \frac{x}{a+bx^3+cx^6}\,dx=\frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)}{b\#1+2c\#1^4}\&\right]$$

input
```
Integrate[x/(a + b*x^3 + c*x^6),x]
```

output
```
RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1 + 2*c*#1^4) & ]/3
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 511, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1711, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{c\int \frac{2x}{2cx^3+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{2x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2c\int \frac{x}{2cx^3+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{821}$$

$$\frac{2c\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 16$$

$$\frac{2c\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 1142$$

$$\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b-\sqrt{b^2-4ac}}\right.$$

$$\int -\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b-\sqrt{b^2-4ac}}\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4}}}$$

$$\frac{2c\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\cdots}{2\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}}$$

$$\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\right.$$

$$\int -\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4}}}$$

$$\frac{2c\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\cdots}{2\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 25$$

$$2c\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b-\sqrt{b^2-4ac}}-\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4a}}}}{2\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$
$$\overline{\sqrt{b^2-4ac}}$$

$$2c\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{b+\sqrt{b^2-4ac}}-\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4a}}}}{2\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$
$$\overline{\sqrt{b^2-4ac}}$$

$\downarrow$ 27

$$2c\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4}}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$
$$\overline{\sqrt{b^2-4ac}}$$

$$2c\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}-2\sqrt[3]{}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4}}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$
$$\overline{\sqrt{b^2-4ac}}$$

$\downarrow$ 1082

$$\frac{2c\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}d}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}}$$

$$\frac{2c\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}d}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.}{\sqrt{b^2-4ac}}$$

$\downarrow$ 217

$$2c\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-\cdots}}\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\cdots}}\right.$$
$$\overline{\sqrt{b^2-4ac}}$$

$$2c\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\log\left(\sqrt[3]{\sqrt{b^2-4ac\cdots}}\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{\cdots}}}\right.$$
$$\overline{\sqrt{b^2-4ac}}$$

$\downarrow$ 1103

$$\frac{2c\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-}\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-}}\right.}{\sqrt{b^2-4ac}}$$

$$\frac{2c\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\log\left(\sqrt[3]{\sqrt{b^2-4ac}\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b}}}\right.}{\sqrt{b^2-4ac}}$$

input
```
Int[x/(a + b*x^3 + c*x^6),x]
```

output
```
(2*c*(-1/3*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)
*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/
3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3)))
+ Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*
c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)
*(b - Sqrt[b^2 - 4*a*c])^(1/3))))/Sqrt[b^2 - 4*a*c] - (2*c*(-1/3*Log[(b +
Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b + Sqrt[b
^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + S
qrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3
)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a
*c])^(1/3))))/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6} c+b \_Z^{3}+a\right)}\frac{\_R \ln\left(x-\_R\right)}{2 \_R^{5} c+\_R^{2} b}\right)}{3}$	41
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6} c+b \_Z^{3}+a\right)}\frac{\_R \ln\left(x-\_R\right)}{2 \_R^{5} c+\_R^{2} b}\right)}{3}$	41

input
```
int(x/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(421) = 842$.

Time = 0.10 (sec) , antiderivative size = 1798, normalized size of antiderivative = 3.22

$$\int \frac{x}{a + bx^3 + cx^6} \, dx = \text{Too large to display}$$

input
```
integrate(x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*l
og(4*b*c*x - (1/2)^(2/3)*(b^4 - 4*a*b^2*c + sqrt(-3)*(b^4 - 4*a*b^2*c) - (
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3 + sqrt(-3)*(a*b^6 - 12*
a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3
)) - 1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1
/3)*log(4*b*c*x - (1/2)^(2/3)*(b^4 - 4*a*b^2*c - sqrt(-3)*(b^4 - 4*a*b^2*c
) - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3 - sqrt(-3)*(a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*sqrt(b^2/(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))
^(2/3)) + 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2
*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c)
)^(1/3)*log(4*b*c*x - (1/2)^(2/3)*(b^4 - 4*a*b^2*c + sqrt(-3)*(b^4 - 4*a*b
^2*c) + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3 + sqrt(-3)*(a*
b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*sqrt(b^2/(a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2...
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.28

$$\int \frac{x}{a+bx^3+cx^6}\,dx$$
$$=\operatorname{RootSum}\left(t^6\cdot\left(46656a^4c^3-34992a^3b^2c^2+8748a^2b^4c-729ab^6\right)+t^3\left(-432a^2c^2+216ab^2c-27b^4\right)+c\right.$$

input
```
integrate(x/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**4*c**3 - 34992*a**3*b**2*c**2 + 8748*a**2*b**4*c -
 729*a*b**6) + _t**3*(-432*a**2*c**2 + 216*a*b**2*c - 27*b**4) + c, Lambda
(_t, _t*log(x + (-15552*_t**5*a**4*c**3 + 11664*_t**5*a**3*b**2*c**2 - 291
6*_t**5*a**2*b**4*c + 243*_t**5*a*b**6 + 72*_t**2*a**2*c**2 - 54*_t**2*a*b
**2*c + 9*_t**2*b**4)/(b*c))))
```

Maxima [F]

$$\int \frac{x}{a+bx^3+cx^6}\,dx=\int \frac{x}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(x/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x}{a+bx^3+cx^6}\,dx=\int \frac{x}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 23.89 (sec) , antiderivative size = 1543, normalized size of antiderivative = 2.77

$$\int \frac{x}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int(x/(a + b*x^3 + c*x^6),x)
```

output
```
log(c^4*x - ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3
*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2
*c)/(a*(4*a*c - b^2)^3))^(2/3))/2)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*
a^2*c^2 - 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*(-(b*(-(4*a*c - b^2)^3)^(1/2
) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c +
 48*a^3*b^2*c^2)))^(1/3) + log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b
^2*c) + (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2)
- b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/2)*(b*(-(4*a*c
 - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*
((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(54*(a*b^6 -
64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/3) - log(c^4*x - ((27*c^3
*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4
*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)
/(a*(4*a*c - b^2)^3))^(2/3))/4)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2
*c^2 - 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*((3^(1/2)*1i)/2 + 1/2)*(-(b*(-(
4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*
c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/3) + log(c^4*x - ((27*c^3*x*(b^4
 + 8*a^2*c^2 - 6*a*b^2*c) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c -
b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(a*(4*
a*c - b^2)^3))^(2/3))/4)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2...
```

Reduce [F]

$$\int \frac{x}{a+bx^3+cx^6}\,dx = \int \frac{x}{c\,x^6+b\,x^3+a}dx$$

input
```
int(x/(c*x^6+b*x^3+a),x)
```

output
```
int(x/(a + b*x**3 + c*x**6),x)
```

3.133 $\int \frac{1}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 558

$$
\begin{aligned}
\int \frac{1}{a+bx^3+cx^6}\,dx &= -\frac{2^{2/3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \\
&\quad+\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \\
&\quad-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \\
&\quad-\frac{c^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \\
&\quad+\frac{c^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}
\end{aligned}
$$

output
```
-1/3*2^(2/3)*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/
2))^(1/3))*3^(1/2))*3^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3
)+1/3*2^(2/3)*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1
/2))^(1/3))*3^(1/2))*3^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/
3)+1/3*2^(2/3)*c^(2/3)*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)/
(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/3*2^(2/3)*c^(2/3)*ln((b+
(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(2/3)-1/6*c^(2/3)*ln((b-(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(
1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(2/3)/(-4*a*c+b
^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*ln((b+(-4*a*c+b^2)^(1/2
))^(2/3)-2^(1/3)*c^(1/3)*(b+(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^
2)*2^(2/3)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.08

$$\int \frac{1}{a+bx^3+cx^6}\,dx=\frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)}{b\#1^2+2c\#1^5}\&\right]$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(-1),x]
```

output
```
RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1^2 + 2*c*#1^5) & ]/3
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {1685, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \; 1685$$

$$\frac{c\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 750$$

$$\frac{c\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 16$$

$$\frac{c\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}$$

$\big\downarrow$ 27

$$\frac{c\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}$$

$\big\downarrow$ 1142

$$\frac{c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int-\frac{\sqrt[3]{c}\left(2^{2/3}\sqrt[3]{b-\sqrt{b}\cdots}\right.}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2\cdots}}}}{4\sqrt[3]{c}\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\sqrt{b^2-4ac}}$$

$$\frac{c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}\,x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int-\frac{\sqrt[3]{c}\left(2^{2/3}\sqrt[3]{b+\sqrt{b}\cdots}\right.}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2\cdots}}}}{4\sqrt[3]{c}\cdots}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.}{\sqrt{b^2-4ac}}$$

$\downarrow$ 25

$$\frac{c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int\frac{\sqrt[3]{c}\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2 \cdots}}\right.}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-\cdots}}}}{4\sqrt[3]{c}\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\sqrt{b^2-4ac}\cdots}$$

$$\frac{c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}\,x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int\frac{\sqrt[3]{c}\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2 \cdots}}\right.}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-\cdots}}}}{4\sqrt[3]{c}\cdots}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\cdots\right.}{\sqrt{b^2-4ac}\cdots}$$

$\downarrow$ 27

$$c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2}}}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2}}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\overline{\sqrt{b^2-4ac}}$$

$$c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2}}}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2}}}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

$$\overline{\sqrt{b^2-4ac}}$$

$\downarrow$ 1082

$$c\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\cdots}{\sqrt[3]{b-\sqrt{b^2\cdots}}}\right.\cdots}{2\sqrt[3]{c}}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.$$

$$\overline{\qquad\qquad\sqrt{b^2-4ac}\qquad\qquad}$$

$$c\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\cdots}{\sqrt[3]{b+\sqrt{b^2\cdots}}}\right.\cdots}{2\sqrt[3]{c}}\cdots\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\cdots\right.$$

$$\overline{\qquad\qquad\sqrt{b^2-4ac}\qquad\qquad}$$

↓ 217

$$c\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\ldots}\right.}{3\ldots}\right.$$

$$\overline{\sqrt{b^2-4ac}}$$

$$c\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\right)}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\ldots}\right.}{3\ldots}\right.$$

$$\overline{\sqrt{b^2-4ac}}$$

$$\downarrow \text{1103}$$

$$c\left(\frac{2\ 2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{4\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\ldots}\right.}{3\ldots}\right.$$

$$\overline{\sqrt{b^2-4ac}}$$

$$c\left(\frac{2\ 2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{4\sqrt[3]{c}}\right)}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\ldots}\right.}{3\ldots}\right.$$

$$\overline{\sqrt{b^2-4ac}}$$

input
```
Int[(a + b*x^3 + c*x^6)^(-1),x]
```

output
```
(c*((2^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^
(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1
 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3)
- Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*
c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c]
)^(2/3))))/Sqrt[b^2 - 4*a*c] - (c*((2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1
/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2
^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*
a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/
3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/
3))))/(3*(b + Sqrt[b^2 - 4*a*c])^(2/3))))/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6c+b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}\right)}{3}$	40
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6c+b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}\right)}{3}$	40

input
```
int(1/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(1/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2206 vs. $2(421) = 842$.

Time = 0.13 (sec) , antiderivative size = 2206, normalized size of antiderivative = 3.95

$$\int \frac{1}{a + bx^3 + cx^6} \, dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*
c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b
)/(a^2*b^2 - 4*a^3*c))^(1/3)*log(-4*(b^2*c - 2*a*c^2)*x - (1/2)^(1/3)*(b^4
 - 6*a*b^2*c + 8*a^2*c^2 + sqrt(-3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2) - (a^2*b
^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt(-3)*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4
*b*c^2))*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a
^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4
*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^
2*b^2 - 4*a^3*c))^(1/3)) + 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(((a^2*b^2 - 4*a
^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)*log(-4*(b^2*c - 2*a
*c^2)*x - (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2 - sqrt(-3)*(b^4 - 6*a*b
^2*c + 8*a^2*c^2) - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 - sqrt(-3)*(a^2*
b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2))*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4
*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c)
*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c
^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)) - 1/6*(1/2)^(1/3)*(sqrt
(-3) + 1)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b
^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c)
)^(1/3)*log(-4*(b^2*c - 2*a*c^2)*x - (1/2)^(1/3)*(b^4 - 6*a*b^2*c + 8*a...
```

Sympy [A] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.28

$$\int \frac{1}{a+bx^3+cx^6}\,dx$$
$$=\operatorname{RootSum}\left(t^6\cdot\left(46656a^5c^3-34992a^4b^2c^2+8748a^3b^4c-729a^2b^6\right)+t^3\cdot\left(432a^2bc^2-216ab^3c+27b^5\right)+\right.$$

input
```
integrate(1/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**5*c**3 - 34992*a**4*b**2*c**2 + 8748*a**3*b**4*c -
 729*a**2*b**6) + _t**3*(432*a**2*b*c**2 - 216*a*b**3*c + 27*b**5) + c**2,
 Lambda(_t, _t*log(x + (-1296*_t**4*a**4*b*c**2 + 648*_t**4*a**3*b**3*c -
81*_t**4*a**2*b**5 + 12*_t*a**2*c**2 - 15*_t*a*b**2*c + 3*_t*b**4)/(2*a*c*
*2 - b**2*c))))
```

Maxima [F]

$$\int \frac{1}{a+bx^3+cx^6}\,dx=\int \frac{1}{cx^6+bx^3+a}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(1/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{1}{a+bx^3+cx^6}\,dx=\int \frac{1}{cx^6+bx^3+a}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(1/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 26.98 (sec) , antiderivative size = 2597, normalized size of antiderivative = 4.65

$$\int \frac{1}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*x^3 + c*x^6),x)
```

output
```
log(6*c^5*x + (2^(2/3)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^
2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/
3)*(36*a*c^5 - 9*b^2*c^4 + (9*2^(1/3)*b*c^3*(x + (2^(2/3)*a*(-(b^5 + b^2*(
-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)
^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*(-(b^5 + b^2*(
-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)
^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(2/3))/2))/6)*((b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(5
4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c
^5*x + (2^(2/3)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a
*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(36*
a*c^5 - 9*b^2*c^4 + (9*2^(1/3)*b*c^3*(x + (2^(2/3)*a*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/
2))/(a^2*(4*a*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/
2))/(a^2*(4*a*c - b^2)^3))^(2/3))/2))/6)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*
b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*x -
(2^(2/3)*(3^(1/2)*1i - 1)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3...
```

Reduce [F]

$$\int \frac{1}{a+bx^3+cx^6}\,dx=\int \frac{1}{c\,x^6+b\,x^3+a}dx$$

input
```
int(1/(c*x^6+b*x^3+a),x)
```

output
```
int(1/(a + b*x**3 + c*x**6),x)
```

3.134 $\int \frac{1}{x^2(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 610

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx=-\frac{1}{ax}+\frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

output
```
-1/a/x+1/6*c^(1/3)*(1+b/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/3)*c^(1/3
)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/a/(b-(-4*a*c+b^
2)^(1/2))^(1/3)+1/6*c^(1/3)*(1-b/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/
3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/a/(b+(
-4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*(1+b/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*
c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/a/(b-(-4*a*c+b^2)^(1/2))^(1
/3)+1/6*c^(1/3)*(1-b/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(1/3)+2
^(1/3)*c^(1/3)*x)*2^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/12*c^(1/3)*(1+b
/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-
4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(1/3)/a/(b-(-4*a*c+b^2)^(
1/2))^(1/3)-1/12*c^(1/3)*(1-b/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2)
)^(2/3)-2^(1/3)*c^(1/3)*(b+(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2
)*2^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)} \, dx = -\frac{1}{ax} - \frac{\text{RootSum}\left[a + b\#1^3 + c\#1^6\&, \frac{b\log(x-\#1)+c\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{3a}$$

input
```
Integrate[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

output
```
-(1/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1
]*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 532, normalized size of antiderivative = 0.87, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1704, 25, 1834, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{\int -\frac{x\left(cx^3+b\right)}{cx^6+bx^3+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{x\left(cx^3+b\right)}{cx^6+bx^3+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{1834}$$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{2x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{x}{2cx^3+b-\sqrt{b^2-4ac}}dx+c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{821}$$

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)+\cdots}{a}$$

$$\frac{1}{ax}$$

$$\Big\downarrow \text{16}$$

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)\cdots}{a\cdots}$$

$$\frac{1}{ax}$$

$\downarrow$ 1142

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\int -\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{\cdots}\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\cdots}\cdots}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\cdots\right.}{\cdots}$$

$$\frac{1}{ax}$$

$\downarrow$ 25

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\int \frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{\cdots}\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\cdots}\cdots}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\cdots\right.}{\cdots}$$

$$\frac{1}{ax}$$

$\downarrow$ 27

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{b \cdots}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{\cdots}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\cdots\right.}{\cdots}$$

$$\frac{1}{ax}$$

$\big\downarrow$ 1082

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\cdots}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\cdots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\cdots\right.}{\cdots}$$

$$\frac{1}{ax}$$

$\big\downarrow$ 217

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log}{}\right)}{}$$

$$\frac{1}{ax}$$

$\Big\downarrow$ 1103

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log}{}\right)}{}$$

$$\frac{1}{ax}$$

input

```
Int[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

output
```
-(1/(a*x)) - (c*(1 + b/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b - Sqrt[b^2 - 4*a*c]
)^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3
)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])
^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) -
 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2
*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3))) + c*
(1 - b/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3
)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]
*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]]
)/(2^(1/3)*c^(1/3))) + Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)
*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)
))/(3*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.10

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+b\_Z^{3}+a\right)}\frac{\left(\_R^{4}c+\_Rb\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3a}-\frac{1}{ax}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^{7}c^{3}-48b^{2}c^{2}a^{6}+12b^{4}c\,a^{5}-a^{4}b^{6}\right)\_Z^{6}+\left(-32a^{3}b\,c^{3}+32a^{2}b^{3}c^{2}-10a\,b^{5}c+b^{7}\right)\_Z^{3}+c^{4}\right)}\_R\ln\left(\left(\left(224a^{7}c^{3}-1\right.\right.\right.$

input
```
int(1/x^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a*sum((_R^4*c+_R*b)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*
b+a))-1/a/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. 2(471) = 942.

Time = 0.21 (sec) , antiderivative size = 3225, normalized size of antiderivative = 5.29

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx$$
$$=\mathrm{RootSum}\left(t^{6}\cdot\left(46656a^{7}c^{3}-34992a^{6}b^{2}c^{2}+8748a^{5}b^{4}c-729a^{4}b^{6}\right)+t^{3}\left(-864a^{3}bc^{3}+864a^{2}b^{3}c^{2}-270ab\right.\right.$$
$$-\frac{1}{ax}$$

input
```
integrate(1/x**2/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**7*c**3 - 34992*a**6*b**2*c**2 + 8748*a**5*b**4*c -
 729*a**4*b**6) + _t**3*(-864*a**3*b*c**3 + 864*a**2*b**3*c**2 - 270*a*b**
5*c + 27*b**7) + c**4, Lambda(_t, _t*log(x + (-15552*_t**5*a**8*c**4 + 272
16*_t**5*a**7*b**2*c**3 - 14580*_t**5*a**6*b**4*c**2 + 3159*_t**5*a**5*b**
6*c - 243*_t**5*a**4*b**8 + 252*_t**2*a**4*b*c**4 - 567*_t**2*a**3*b**3*c*
*3 + 378*_t**2*a**2*b**5*c**2 - 99*_t**2*a*b**7*c + 9*_t**2*b**9)/(2*a**2*
c**5 - 4*a*b**2*c**4 + b**4*c**3)))) - 1/(a*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((c*x^4 + b*x)/(c*x^6 + b*x^3 + a), x)/a - 1/(a*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 25.46 (sec) , antiderivative size = 2978, normalized size of antiderivative = 4.88

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)} dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^3 + c*x^6)),x)
```

output
```
log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(27*a^7*c^3*x*(
b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(4*
a*c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2
*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(
4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^7 + b^4*(-(4*
a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*
a*c - b^2)^3))^(1/3))/6)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c
^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*
a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c
 + 48*a^6*b^2*c^2)))^(1/3) + log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c
^5 - (2^(2/3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c)
 + (27*2^(1/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) -
 b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
+ 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^
(2/3))/2)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3
*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(1/3))/6)*(-(b^4*(-(4*a*c - b^2)
^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2
)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*...
```

Reduce [F]

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)} dx = \frac{-\left(\int \frac{x^4}{c\,x^6+b\,x^3+a}dx\right) cx - \left(\int \frac{x}{c\,x^6+b\,x^3+a}dx\right) bx - 1}{ax}$$

input
```
int(1/x^2/(c*x^6+b*x^3+a),x)
```

output
```
( - (int(x**4/(a + b*x**3 + c*x**6),x)*c*x + int(x/(a + b*x**3 + c*x**6),x
)*b*x + 1))/(a*x)
```

3.135 $\int \frac{1}{x^3(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 612

$$\int \frac{1}{x^3(a+bx^3+cx^6)}\,dx = -\frac{1}{2ax^2} + \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

output
```
-1/2/a/x^2+1/6*c^(2/3)*(1+b/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/3)*c^
(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)/a/(b-(-4*a*
c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*(1-b/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2
^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)/a/
(b+(-4*a*c+b^2)^(1/2))^(2/3)-1/6*c^(2/3)*(1+b/(-4*a*c+b^2)^(1/2))*ln((b-(-
4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2)
)^(2/3)-1/6*c^(2/3)*(1-b/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(1/
3)+2^(1/3)*c^(1/3)*x)*2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/12*c^(2/3)*
(1+b/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(
b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(2/3)/a/(b-(-4*a*c+b^
2)^(1/2))^(2/3)+1/12*c^(2/3)*(1-b/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(
1/2))^(2/3)-2^(1/3)*c^(1/3)*(b+(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)
*x^2)*2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)} \, dx = -\frac{1}{2ax^2} - \frac{\text{RootSum}\left[a + b\#1^3 + c\#1^6\&, \frac{b\log(x-\#1)+c\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3a}$$

input
```
Integrate[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/2*1/(a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x
- #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 514, normalized size of antiderivative = 0.84, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1704, 27, 1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)}dx$$

$$\Big\downarrow \ 1704$$

$$\frac{\int -\frac{2\left(cx^3+b\right)}{cx^6+bx^3+a}dx}{2a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{cx^3+b}{cx^6+bx^3+a}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \ 1752$$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \ 750$$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}d}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}$$

$$\frac{1}{2ax^2}$$

$$\Big\downarrow \ 16$$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2^{2/3}\int\frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\frac{1}{2ax^2}}$$

$\downarrow$ 27

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\ 2^{2/3}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\frac{1}{2ax^2}}$$

$\downarrow$ 1142

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\int-\frac{}{2c^{2/3}x^2-}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\frac{1}{2ax^2}}$$

$\downarrow$ 25

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int}{2c^{2/3}x^2-2^{2}\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\frac{1}{2ax^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{}{2c^{2/3}x^2-\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\frac{1}{2ax^2}}$$

$$\Big\downarrow \; 1082$$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2\cdots}}{2\sqrt[3]{\cdots}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$$\frac{1}{2ax^2}$$

$\downarrow$ 217

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\ 2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4a}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{}$$

$$\frac{1}{2ax^2}$$

$\downarrow$ 1103

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/}\right.}{4\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\frac{1}{2ax^2}}$$

input
```
Int[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/2*1/(a*x^2) - ((c*(1 + b/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b - Sqrt[b^2
 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^
(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b -
Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b - Sqrt[b^2 - 4*a*c])^
(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*
x^2]/(4*c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/2 + (c*(1 - b/Sqrt[
b^2 - 4*a*c])*((2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3
)*x])/(3*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3
]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]
])/c^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b + Sqrt[b
^2 - 4*a*c])^(2/3))))/2)/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.10

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+b\_Z^{3}+a\right)}\frac{\left(-\_R^{3}c-b\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3a}-\frac{1}{2a\,x^{2}}$
risch	$-\frac{1}{2a\,x^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64c^{3}a^{8}-48a^{7}b^{2}c^{2}+12a^{6}b^{4}c-a^{5}b^{6}\right)\_Z^{6}+\left(-16c^{4}a^{4}+56b^{2}c^{3}a^{3}-41b^{4}c^{2}a^{2}+11b^{6}ca-b^{8}\right)\_Z^{3}+c^{5}\right)}\_R\ln\left(\left(\left(2\right.\right.\right.}{}$

input
```
int(1/x^3/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*sum((-_R^3*c-b)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a
))-1/2/a/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. $2(471) = 942$.

Time = 0.22 (sec) , antiderivative size = 3225, normalized size of antiderivative = 5.27

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)}\, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 31.65 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)}\, dx$$
$$= \text{RootSum}\left(t^6 \cdot \left(46656a^8c^3 - 34992a^7b^2c^2 + 8748a^6b^4c - 729a^5b^6\right) + t^3\left(-432a^4c^4 + 1512a^3b^2c^3 - 1107a\right.\right.$$
$$- \frac{1}{2ax^2}$$

input
```
integrate(1/x**3/(c*x**6+b*x**3+a),x)
```

output
```
RootSum(_t**6*(46656*a**8*c**3 - 34992*a**7*b**2*c**2 + 8748*a**6*b**4*c -
 729*a**5*b**6) + _t**3*(-432*a**4*c**4 + 1512*a**3*b**2*c**3 - 1107*a**2*
b**4*c**2 + 297*a*b**6*c - 27*b**8) + c**5, Lambda(_t, _t*log(x + (-2592*_
t**4*a**8*c**3 + 2592*_t**4*a**7*b**2*c**2 - 810*_t**4*a**6*b**4*c + 81*_t
**4*a**5*b**6 + 12*_t*a**4*c**4 - 75*_t*a**3*b**2*c**3 + 78*_t*a**2*b**4*c
**2 - 27*_t*a*b**6*c + 3*_t*b**8)/(5*a**2*b*c**4 - 5*a*b**3*c**3 + b**5*c*
*2)))) - 1/(2*a*x**2)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)x^3} \, dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((c*x^3 + b)/(c*x^6 + b*x^3 + a), x)/a - 1/2/(a*x^2)
```

Giac [F]

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)x^3} \, dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 29.85 (sec) , antiderivative size = 4063, normalized size of antiderivative = 6.64

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^3 + c*x^6)),x)
```

output
```
log((2^(2/3)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^
4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2)
 - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(72*a^
8*b*c^6 + (2^(1/3)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)
*a^10*b*c^3*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1
/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c
- b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3
))^(1/3))/2)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^
4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2)
 - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/18 +
9*a^6*b^5*c^4 - 54*a^7*b^3*c^5))/6 - 3*a^6*c^6*x*(2*a*c - b^2))*(-(b^8 + 1
6*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c
- b^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
)))^(1/3) + log((2^(2/3)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2)
 + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b
^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^
(1/3)*(72*a^8*b*c^6 + (2^(1/3)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 +
 (81*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c
- b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*...
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^3+cx^6)}\,dx = \frac{-2\left(\int \frac{x^3}{c\,x^6+b\,x^3+a}dx\right)c\,x^2-2\left(\int \frac{1}{c\,x^6+b\,x^3+a}dx\right)b\,x^2-1}{2a\,x^2}$$

input
```
int(1/x^3/(c*x^6+b*x^3+a),x)
```

output
```
( - 2*int(x**3/(a + b*x**3 + c*x**6),x)*c*x**2 - 2*int(1/(a + b*x**3 + c*x
**6),x)*b*x**2 - 1)/(2*a*x**2)
```

3.136 $\int \frac{x^{11}}{3+4x^3+x^6} dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{x^{11}}{3+4x^3+x^6} dx = -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\log\left(1+x^3\right)+\frac{9}{2}\log\left(3+x^3\right)$$

output
```
-4/3*x^3+1/6*x^6-1/6*ln(x^3+1)+9/2*ln(x^3+3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{3+4x^3+x^6} dx = -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\log\left(1+x^3\right)+\frac{9}{2}\log\left(3+x^3\right)$$

input
```
Integrate[x^11/(3 + 4*x^3 + x^6),x]
```

output
```
(-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^9}{x^6+4x^3+3}dx^3$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{3}\int \left(x^3-\frac{1}{2(x^3+1)}+\frac{27}{2(x^3+3)}-4\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{x^6}{2}-4x^3-\frac{1}{2}\log\left(x^3+1\right)+\frac{27}{2}\log\left(x^3+3\right)\right)$$

input
```
Int[x^11/(3 + 4*x^3 + x^6),x]
```

output
```
(-4*x^3 + x^6/2 - Log[1 + x^3]/2 + (27*Log[3 + x^3])/2)/3
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{\ln(x^3+1)}{6}+\frac{9\ln(x^3+3)}{2}$	28
risch	$\frac{x^6}{6}-\frac{4x^3}{3}+\frac{8}{3}-\frac{\ln(x^3+1)}{6}+\frac{9\ln(x^3+3)}{2}$	29
norman	$-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{\ln(x+1)}{6}+\frac{9\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	37
parallelrisch	$-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{\ln(x+1)}{6}+\frac{9\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	37

input
```
int(x^11/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-4/3*x^3+1/6*x^6-1/6*ln(x^3+1)+9/2*ln(x^3+3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx=\frac{1}{6}\,x^6-\frac{4}{3}\,x^3+\frac{9}{2}\,\log\left(x^3+3\right)-\frac{1}{6}\,\log\left(x^3+1\right)$$

input
```
integrate(x^11/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = \frac{x^6}{6} - \frac{4x^3}{3} - \frac{\log(x^3+1)}{6} + \frac{9\log(x^3+3)}{2}$$

input
```
integrate(x**11/(x**6+4*x**3+3),x)
```

output
```
x**6/6 - 4*x**3/3 - log(x**3 + 1)/6 + 9*log(x**3 + 3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = \frac{1}{6}\,x^6 - \frac{4}{3}\,x^3 + \frac{9}{2}\,\log\left(x^3+3\right) - \frac{1}{6}\,\log\left(x^3+1\right)$$

input
```
integrate(x^11/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = \frac{1}{6}\,x^6 - \frac{4}{3}\,x^3 + \frac{9}{2}\,\log\left(|x^3+3|\right) - \frac{1}{6}\,\log\left(|x^3+1|\right)$$

input
```
integrate(x^11/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/6*x^6 - 4/3*x^3 + 9/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = \frac{9\,\ln\left(x^3+3\right)}{2} - \frac{\ln\left(x^3+1\right)}{6} - \frac{4\,x^3}{3} + \frac{x^6}{6}$$

input
```
int(x^11/(4*x^3 + x^6 + 3),x)
```

output
```
(9*log(x^3 + 3))/2 - log(x^3 + 1)/6 - (4*x^3)/3 + x^6/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

$$\int \frac{x^{11}}{3+4x^3+x^6}\,dx = -\frac{\log(x^2-x+1)}{6} + \frac{9\log\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{2} + \frac{9\log\left(3^{\frac{1}{3}}+x\right)}{2} - \frac{\log(x+1)}{6} + \frac{x^6}{6} - \frac{4x^3}{3}$$

input
```
int(x^11/(x^6+4*x^3+3),x)
```

output
```
( - log(x**2 - x + 1) + 27*log(3**(2/3) - 3**(1/3)*x + x**2) + 27*log(3**(
1/3) + x) - log(x + 1) + x**6 - 8*x**3)/6
```

3.137 $\int \frac{x^8}{3+4x^3+x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 28

$$\int \frac{x^8}{3+4x^3+x^6} \, dx = \frac{x^3}{3} + \frac{1}{6} \log\left(1+x^3\right) - \frac{3}{2} \log\left(3+x^3\right)$$

output
```
1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{3+4x^3+x^6} \, dx = \frac{x^3}{3} + \frac{1}{6} \log\left(1+x^3\right) - \frac{3}{2} \log\left(3+x^3\right)$$

input
```
Integrate[x^8/(3 + 4*x^3 + x^6),x]
```

output
```
x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^6+4x^3+3} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^6}{x^6+4x^3+3}dx^3$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{3}\int \left(-\frac{9}{2\left(x^3+3\right)}+1+\frac{1}{2\left(x^3+1\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(x^3+\frac{1}{2}\log \left(x^3+1\right)-\frac{9}{2}\log \left(x^3+3\right)\right)$$

input
```
Int[x^8/(3 + 4*x^3 + x^6),x]
```

output
```
(x^3 + Log[1 + x^3]/2 - (9*Log[3 + x^3])/2)/3
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^{3}}{3}+\frac{\ln\left(x^{3}+1\right)}{6}-\frac{3\ln\left(x^{3}+3\right)}{2}$	23
risch	$\frac{x^{3}}{3}+\frac{\ln\left(x^{3}+1\right)}{6}-\frac{3\ln\left(x^{3}+3\right)}{2}$	23
norman	$\frac{x^{3}}{3}+\frac{\ln\left(x+1\right)}{6}-\frac{3\ln\left(x^{3}+3\right)}{2}+\frac{\ln\left(x^{2}-x+1\right)}{6}$	32
parallelrisch	$\frac{x^{3}}{3}+\frac{\ln\left(x+1\right)}{6}-\frac{3\ln\left(x^{3}+3\right)}{2}+\frac{\ln\left(x^{2}-x+1\right)}{6}$	32

input
```
int(x^8/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{3+4x^3+x^6}\,dx=\frac{1}{3}\,x^{3}-\frac{3}{2}\,\log\left(x^{3}+3\right)+\frac{1}{6}\,\log\left(x^{3}+1\right)$$

input
```
integrate(x^8/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{x^3}{3} + \frac{\log(x^3+1)}{6} - \frac{3\log(x^3+3)}{2}$$

input
```
integrate(x**8/(x**6+4*x**3+3),x)
```

output
```
x**3/3 + log(x**3 + 1)/6 - 3*log(x**3 + 3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{1}{3}x^3 - \frac{3}{2}\log\left(x^3+3\right) + \frac{1}{6}\log\left(x^3+1\right)$$

input
```
integrate(x^8/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{1}{3}x^3 - \frac{3}{2}\log\left(\left|x^3+3\right|\right) + \frac{1}{6}\log\left(\left|x^3+1\right|\right)$$

input
```
integrate(x^8/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/3*x^3 - 3/2*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{\ln (x^3+1)}{6} - \frac{3\ \ln (x^3+3)}{2} + \frac{x^3}{3}$$

input
```
int(x^8/(4*x^3 + x^6 + 3),x)
```

output
```
log(x^3 + 1)/6 - (3*log(x^3 + 3))/2 + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{x^8}{3+4x^3+x^6}\,dx = \frac{\log(x^2-x+1)}{6} - \frac{3\log\left(3^{\frac{2}{3}} - 3^{\frac{1}{3}}x + x^2\right)}{2} - \frac{3\log\left(3^{\frac{1}{3}}+x\right)}{2} + \frac{\log(x+1)}{6} + \frac{x^3}{3}$$

input
```
int(x^8/(x^6+4*x^3+3),x)
```

output
```
(log(x**2 - x + 1) - 9*log(3**(2/3) - 3**(1/3)*x + x**2) - 9*log(3**(1/3)
+ x) + log(x + 1) + 2*x**3)/6
```

3.138 $\int \frac{x^5}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = -\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{2}\log\left(3+x^3\right)$$

output
```
-1/6*ln(x^3+1)+1/2*ln(x^3+3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = -\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{2}\log\left(3+x^3\right)$$

input
```
Integrate[x^5/(3 + 4*x^3 + x^6),x]
```

output
```
-1/6*Log[1 + x^3] + Log[3 + x^3]/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^6+4x^3+3} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^3}{x^6+4x^3+3}dx^3$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{3}\int \left(\frac{3}{2\left(x^3+3\right)}-\frac{1}{2\left(x^3+1\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3}{2}\log\left(x^3+3\right)-\frac{1}{2}\log\left(x^3+1\right)\right)$$

input
```
Int[x^5/(3 + 4*x^3 + x^6),x]
```

output
```
(-1/2*Log[1 + x^3] + (3*Log[3 + x^3])/2)/3
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{2}$	18
risch	$-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{2}$	18
norman	$-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	27
parallelrisch	$-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	27

input
```
int(x^5/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(x^3+1)+1/2*ln(x^3+3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{3+4x^3+x^6}\,dx=\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

input
```
integrate(x^5/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = -\frac{\log\left(x^3+1\right)}{6} + \frac{\log\left(x^3+3\right)}{2}$$

input
```
integrate(x**5/(x**6+4*x**3+3),x)
```

output
```
-log(x**3 + 1)/6 + log(x**3 + 3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = \frac{1}{2}\log\left(x^3+3\right) - \frac{1}{6}\log\left(x^3+1\right)$$

input
```
integrate(x^5/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = \frac{1}{2}\log\left(\left|x^3+3\right|\right) - \frac{1}{6}\log\left(\left|x^3+1\right|\right)$$

input
```
integrate(x^5/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = \frac{\ln(x^3+3)}{2} - \frac{\ln(x^3+1)}{6}$$

input
```
int(x^5/(4*x^3 + x^6 + 3),x)
```

output
```
log(x^3 + 3)/2 - log(x^3 + 1)/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = -\frac{\log(x^2-x+1)}{6} + \frac{\log\left(3^{\frac{2}{3}} - 3^{\frac{1}{3}}x + x^2\right)}{2} + \frac{\log\left(3^{\frac{1}{3}} + x\right)}{2} - \frac{\log(x+1)}{6}$$

input
```
int(x^5/(x^6+4*x^3+3),x)
```

output
```
( - log(x**2 - x + 1) + 3*log(3**(2/3) - 3**(1/3)*x + x**2) + 3*log(3**(1/
3) + x) - log(x + 1))/6
```

3.139 $\int \frac{x^2}{3+4x^3+x^6} dx$

Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{x^2}{3+4x^3+x^6} dx = -\frac{1}{3}\text{arctanh}(2+x^3)$$

output
```
-1/3*arctanh(x^3+2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{x^2}{3+4x^3+x^6} dx = \frac{1}{6}\log(1+x^3) - \frac{1}{6}\log(3+x^3)$$

input
```
Integrate[x^2/(3 + 4*x^3 + x^6),x]
```

output
```
Log[1 + x^3]/6 - Log[3 + x^3]/6
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.50, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1690, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^6+4x^3+3} \, dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \frac{1}{x^6+4x^3+3}dx^3$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{3}\int \left(\frac{1}{2\left(x^3+1\right)}-\frac{1}{2\left(x^3+3\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{2}\log\left(x^3+1\right)-\frac{1}{2}\log\left(x^3+3\right)\right)$$

input
```
Int[x^2/(3 + 4*x^3 + x^6),x]
```

output
```
(Log[1 + x^3]/2 - Log[3 + x^3]/2)/3
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

method	result	size
default	$\frac{\ln(x^3+1)}{6} - \frac{\ln(x^3+3)}{6}$	18
risch	$\frac{\ln(x^3+1)}{6} - \frac{\ln(x^3+3)}{6}$	18
norman	$\frac{\ln(x+1)}{6} - \frac{\ln(x^3+3)}{6} + \frac{\ln(x^2-x+1)}{6}$	27
parallelrisch	$\frac{\ln(x+1)}{6} - \frac{\ln(x^3+3)}{6} + \frac{\ln(x^2-x+1)}{6}$	27

input
```
int(x^2/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(x^3+1)-1/6*ln(x^3+3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = -\frac{1}{6}\log\left(x^3+3\right) + \frac{1}{6}\log\left(x^3+1\right)$$

input
```
integrate(x^2/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
-1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = \frac{\log(x^3+1)}{6} - \frac{\log(x^3+3)}{6}$$

input
```
integrate(x**2/(x**6+4*x**3+3),x)
```

output
```
log(x**3 + 1)/6 - log(x**3 + 3)/6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = -\frac{1}{6}\log\left(x^3+3\right) + \frac{1}{6}\log\left(x^3+1\right)$$

input
```
integrate(x^2/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = -\frac{1}{6}\log\left(\left|x^3+3\right|\right) + \frac{1}{6}\log\left(\left|x^3+1\right|\right)$$

input
```
integrate(x^2/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/6*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

Mupad [B] (verification not implemented)

Time = 20.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = \frac{\operatorname{atanh}\left(\frac{9}{2\,(8\,x^3+6)}+\frac{5}{4}\right)}{3}$$

input
```
int(x^2/(4*x^3 + x^6 + 3),x)
```

output
```
atanh(9/(2*(8*x^3 + 6)) + 5/4)/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 4.20

$$\int \frac{x^2}{3+4x^3+x^6}\,dx = \frac{\log(x^2-x+1)}{6} - \frac{\log\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{6} - \frac{\log\left(3^{\frac{1}{3}}+x\right)}{6} + \frac{\log(x+1)}{6}$$

input
```
int(x^2/(x^6+4*x^3+3),x)
```

output
```
(log(x**2 - x + 1) - log(3**(2/3) - 3**(1/3)*x + x**2) - log(3**(1/3) + x)
 + log(x + 1))/6
```

3.140 $\int \frac{1}{x(3+4x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{x(3+4x^3+x^6)}\,dx = \frac{\log(x)}{3} - \frac{1}{6}\log\left(1+x^3\right) + \frac{1}{18}\log\left(3+x^3\right)$$

output
```
1/3*ln(x)-1/6*ln(x^3+1)+1/18*ln(x^3+3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(3+4x^3+x^6)}\,dx = \frac{\log(x)}{3} - \frac{1}{6}\log\left(1+x^3\right) + \frac{1}{18}\log\left(3+x^3\right)$$

input
```
Integrate[1/(x*(3 + 4*x^3 + x^6)),x]
```

output
```
Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(x^6+4x^3+3\right)}dx$$

$$\Big\downarrow \ 1693$$

$$\frac{1}{3}\int \frac{1}{x^3\left(x^6+4x^3+3\right)}dx^3$$

$$\Big\downarrow \ 1141$$

$$\frac{1}{3}\int \left(\frac{1}{6\left(x^3+3\right)}+\frac{1}{3x^3}-\frac{1}{2\left(x^3+1\right)}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(\frac{\log\left(x^3\right)}{3}-\frac{1}{2}\log\left(x^3+1\right)+\frac{1}{6}\log\left(x^3+3\right)\right)$$

input
```
Int[1/(x*(3 + 4*x^3 + x^6)),x]
```

output
```
(Log[x^3]/3 - Log[1 + x^3]/2 + Log[3 + x^3]/6)/3
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{\ln(x)}{3}-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{18}$	22
default	$-\frac{\ln(x^2-x+1)}{6}+\frac{\ln(x^3+3)}{18}-\frac{\ln(x+1)}{6}+\frac{\ln(x)}{3}$	31
norman	$-\frac{\ln(x^2-x+1)}{6}+\frac{\ln(x^3+3)}{18}-\frac{\ln(x+1)}{6}+\frac{\ln(x)}{3}$	31
parallelrisch	$-\frac{\ln(x^2-x+1)}{6}+\frac{\ln(x^3+3)}{18}-\frac{\ln(x+1)}{6}+\frac{\ln(x)}{3}$	31

input
```
int(1/x/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x)-1/6*ln(x^3+1)+1/18*ln(x^3+3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\left(3+4x^3+x^6\right)}\,dx=\frac{1}{18}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{3}\log\left(x\right)$$

input
```
integrate(1/x/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{x(3+4x^3+x^6)}\,dx = \frac{\log(x)}{3} - \frac{\log(x^3+1)}{6} + \frac{\log(x^3+3)}{18}$$

input
```
integrate(1/x/(x**6+4*x**3+3),x)
```

output
```
log(x)/3 - log(x**3 + 1)/6 + log(x**3 + 3)/18
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(3+4x^3+x^6)}\,dx = \frac{1}{18}\log\left(x^3+3\right) - \frac{1}{6}\log\left(x^3+1\right) + \frac{1}{9}\log\left(x^3\right)$$

input
```
integrate(1/x/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/9*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(3+4x^3+x^6)}\,dx = \frac{1}{18}\log\left(|x^3+3|\right) - \frac{1}{6}\log\left(|x^3+1|\right) + \frac{1}{3}\log\left(|x|\right)$$

input
```
integrate(1/x/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/18*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1)) + 1/3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\,(3+4x^3+x^6)}\,dx = \frac{\ln\left(x^3+3\right)}{18} - \frac{\ln\left(x^3+1\right)}{6} + \frac{\ln\left(x\right)}{3}$$

input
```
int(1/(x*(4*x^3 + x^6 + 3)),x)
```

output
```
log(x^3 + 3)/18 - log(x^3 + 1)/6 + log(x)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \frac{1}{x\,(3+4x^3+x^6)}\,dx = -\frac{\log(x^2-x+1)}{6} + \frac{\log\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{18} + \frac{\log\left(3^{\frac{1}{3}}+x\right)}{18} - \frac{\log(x+1)}{6} + \frac{\log(x)}{3}$$

input
```
int(1/x/(x^6+4*x^3+3),x)
```

output
```
( - 3*log(x**2 - x + 1) + log(3**(2/3) - 3**(1/3)*x + x**2) + log(3**(1/3)
 + x) - 3*log(x + 1) + 6*log(x))/18
```

3.141 $\int \frac{1}{x^4(3+4x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{1}{x^4\left(3+4x^3+x^6\right)}\,dx = -\frac{1}{9x^3}-\frac{4\log(x)}{9}+\frac{1}{6}\log\left(1+x^3\right)-\frac{1}{54}\log\left(3+x^3\right)$$

output
```
-1/9/x^3-4/9*ln(x)+1/6*ln(x^3+1)-1/54*ln(x^3+3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(3+4x^3+x^6\right)}\,dx = -\frac{1}{9x^3}-\frac{4\log(x)}{9}+\frac{1}{6}\log\left(1+x^3\right)-\frac{1}{54}\log\left(3+x^3\right)$$

input
```
Integrate[1/(x^4*(3 + 4*x^3 + x^6)),x]
```

output
```
-1/9*1/x^3 - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^6+4x^3+3)} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3} \int \frac{1}{x^6 (x^6+4x^3+3)}dx^3$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{3} \int \left(-\frac{1}{18 (x^3+3)}-\frac{4}{9x^3}+\frac{1}{3x^6}+\frac{1}{2 (x^3+1)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{1}{3x^3}-\frac{4}{9}\log \left(x^3\right)+\frac{1}{2}\log \left(x^3+1\right)-\frac{1}{18}\log \left(x^3+3\right)\right)$$

input
```
Int[1/(x^4*(3 + 4*x^3 + x^6)),x]
```

output
```
(-1/3*1/x^3 - (4*Log[x^3])/9 + Log[1 + x^3]/2 - Log[3 + x^3]/18)/3
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{1}{9x^{3}}-\frac{4\ln(x)}{9}+\frac{\ln(x^{3}+1)}{6}-\frac{\ln(x^{3}+3)}{54}$	27
default	$\frac{\ln(x^{2}-x+1)}{6}-\frac{\ln(x^{3}+3)}{54}+\frac{\ln(x+1)}{6}-\frac{1}{9x^{3}}-\frac{4\ln(x)}{9}$	36
norman	$\frac{\ln(x^{2}-x+1)}{6}-\frac{\ln(x^{3}+3)}{54}+\frac{\ln(x+1)}{6}-\frac{1}{9x^{3}}-\frac{4\ln(x)}{9}$	36
parallelrisch	$-\frac{24\ln(x)x^{3}-9\ln(x+1)x^{3}+\ln(x^{3}+3)x^{3}-9\ln(x^{2}-x+1)x^{3}+6}{54x^{3}}$	48

input
```
int(1/x^4/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-1/9/x^3-4/9*ln(x)+1/6*ln(x^3+1)-1/54*ln(x^3+3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4(3+4x^3+x^6)}\,dx=-\frac{x^3\log\left(x^3+3\right)-9\,x^3\log\left(x^3+1\right)+24\,x^3\log\left(x\right)+6}{54\,x^3}$$

input
```
integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
-1/54*(x^3*log(x^3 + 3) - 9*x^3*log(x^3 + 1) + 24*x^3*log(x) + 6)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4 (3+4x^3+x^6)}\,dx = -\frac{4\log(x)}{9} + \frac{\log(x^3+1)}{6} - \frac{\log(x^3+3)}{54} - \frac{1}{9x^3}$$

input
```
integrate(1/x**4/(x**6+4*x**3+3),x)
```

output
```
-4*log(x)/9 + log(x**3 + 1)/6 - log(x**3 + 3)/54 - 1/(9*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4 (3+4x^3+x^6)}\,dx = -\frac{1}{9\,x^3} - \frac{1}{54}\log\left(x^3+3\right) + \frac{1}{6}\log\left(x^3+1\right) - \frac{4}{27}\log\left(x^3\right)$$

input
```
integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/9/x^3 - 1/54*log(x^3 + 3) + 1/6*log(x^3 + 1) - 4/27*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4 (3+4x^3+x^6)}\,dx = \frac{4\,x^3-3}{27\,x^3} - \frac{1}{54}\log\left(|x^3+3|\right) + \frac{1}{6}\log\left(|x^3+1|\right) - \frac{4}{9}\log\left(|x|\right)$$

input
```
integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/27*(4*x^3 - 3)/x^3 - 1/54*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1)) - 4/
9*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 19.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4 (3 + 4x^3 + x^6)} dx = \frac{\ln(x^3+1)}{6} - \frac{\ln(x^3+3)}{54} - \frac{4 \ln(x)}{9} - \frac{1}{9 x^3}$$

input
```
int(1/(x^4*(4*x^3 + x^6 + 3)),x)
```

output
```
log(x^3 + 1)/6 - log(x^3 + 3)/54 - (4*log(x))/9 - 1/(9*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int \frac{1}{x^4 (3 + 4x^3 + x^6)} dx$$
$$= \frac{9 \log(x^2 - x + 1) x^3 - \log\left(3^{\frac{2}{3}} - 3^{\frac{1}{3}} x + x^2\right) x^3 - \log\left(3^{\frac{1}{3}} + x\right) x^3 + 9 \log(x+1) x^3 - 24 \log(x) x^3 - 6}{54x^3}$$

input
```
int(1/x^4/(x^6+4*x^3+3),x)
```

output
```
(9*log(x**2 - x + 1)*x**3 - log(3**(2/3) - 3**(1/3)*x + x**2)*x**3 - log(3
**(1/3) + x)*x**3 + 9*log(x + 1)*x**3 - 24*log(x)*x**3 - 6)/(54*x**3)
```

3.142 $\int \frac{1}{x^7(3+4x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx = -\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\log(x)}{27}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{162}\log\left(3+x^3\right)$$

output
```
-1/18/x^6+4/27/x^3+13/27*ln(x)-1/6*ln(x^3+1)+1/162*ln(x^3+3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx = -\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\log(x)}{27}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{162}\log\left(3+x^3\right)$$

input
```
Integrate[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

output
```
-1/18*1/x^6 + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/
162
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (x^6+4x^3+3)} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3} \int \frac{1}{x^9 (x^6+4x^3+3)}dx^3$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{3} \int \left(\frac{1}{54 (x^3+3)}+\frac{13}{27x^3}-\frac{4}{9x^6}+\frac{1}{3x^9}-\frac{1}{2 (x^3+1)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{1}{6x^6}+\frac{4}{9x^3}+\frac{13\log (x^3)}{27}-\frac{1}{2}\log (x^3+1)+\frac{1}{54}\log (x^3+3)\right)$$

input
```
Int[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

output
```
(-1/6*1/x^6 + 4/(9*x^3) + (13*Log[x^3])/27 - Log[1 + x^3]/2 + Log[3 + x^3]
/54)/3
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{-\frac{1}{18}+\frac{4x^3}{27}}{x^6}+\frac{13\ln(x)}{27}-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{162}$	33
default	$-\frac{\ln(x^2-x+1)}{6}+\frac{\ln(x^3+3)}{162}-\frac{\ln(x+1)}{6}-\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\ln(x)}{27}$	41
norman	$\frac{-\frac{1}{18}+\frac{4x^3}{27}}{x^6}+\frac{13\ln(x)}{27}-\frac{\ln(x+1)}{6}+\frac{\ln(x^3+3)}{162}-\frac{\ln(x^2-x+1)}{6}$	42
parallelrisch	$\frac{78\ln(x)x^6-27\ln(x+1)x^6+\ln(x^3+3)x^6-27\ln(x^2-x+1)x^6-9+24x^3}{162x^6}$	53

input
```
int(1/x^7/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
(-1/18+4/27*x^3)/x^6+13/27*ln(x)-1/6*ln(x^3+1)+1/162*ln(x^3+3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx = \frac{x^6\log(x^3+3)-27\,x^6\log(x^3+1)+78\,x^6\log(x)+24\,x^3-9}{162\,x^6}$$

input
```
integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/162*(x^6*log(x^3 + 3) - 27*x^6*log(x^3 + 1) + 78*x^6*log(x) + 24*x^3 - 9
)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx = \frac{13\log(x)}{27} - \frac{\log(x^3+1)}{6} + \frac{\log(x^3+3)}{162} + \frac{8x^3-3}{54x^6}$$

input
```
integrate(1/x**7/(x**6+4*x**3+3),x)
```

output
```
13*log(x)/27 - log(x**3 + 1)/6 + log(x**3 + 3)/162 + (8*x**3 - 3)/(54*x**6
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx = \frac{8\,x^3-3}{54\,x^6} + \frac{1}{162}\log\left(x^3+3\right) - \frac{1}{6}\log\left(x^3+1\right) + \frac{13}{81}\log\left(x^3\right)$$

input
```
integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/54*(8*x^3 - 3)/x^6 + 1/162*log(x^3 + 3) - 1/6*log(x^3 + 1) + 13/81*log(x
^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^7(3+4x^3+x^6)}\,dx = -\frac{13\,x^6-8\,x^3+3}{54\,x^6} + \frac{1}{162}\log\left(\left|x^3+3\right|\right) - \frac{1}{6}\log\left(\left|x^3+1\right|\right) + \frac{13}{27}\log\left(|x|\right)$$

input
```
integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/54*(13*x^6 - 8*x^3 + 3)/x^6 + 1/162*log(abs(x^3 + 3)) - 1/6*log(abs(x^3
 + 1)) + 13/27*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7 (3+4x^3+x^6)} \, dx = \frac{\ln(x^3+3)}{162} - \frac{\ln(x^3+1)}{6} + \frac{13\ln(x)}{27} + \frac{\frac{4x^3}{27} - \frac{1}{18}}{x^6}$$

input
```
int(1/(x^7*(4*x^3 + x^6 + 3)),x)
```

output
```
log(x^3 + 3)/162 - log(x^3 + 1)/6 + (13*log(x))/27 + ((4*x^3)/27 - 1/18)/x
^6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.71

$$\int \frac{1}{x^7 (3+4x^3+x^6)} \, dx = \frac{-27\log(x^2-x+1)x^6 + \log\left(3^{\frac{2}{3}} - 3^{\frac{1}{3}}x + x^2\right)x^6 + \log\left(3^{\frac{1}{3}} + x\right)x^6 - 27\log(x+1)x^6 + 78\log(x)x^6 +}{162x^6}$$

input
```
int(1/x^7/(x^6+4*x^3+3),x)
```

output
```
( - 27*log(x**2 - x + 1)*x**6 + log(3**(2/3) - 3**(1/3)*x + x**2)*x**6 + l
og(3**(1/3) + x)*x**6 - 27*log(x + 1)*x**6 + 78*log(x)*x**6 + 24*x**3 - 9)
/(162*x**6)
```

3.143 $\int \frac{x^{10}}{3+4x^3+x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 124

$$\int \frac{x^{10}}{3+4x^3+x^6} \, dx = -2x^2+\frac{x^5}{5}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{9}{2}\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right) - \frac{1}{12}\log\left(1-x+x^2\right)+\frac{3}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

output
```
-2*x^2+1/5*x^5+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-9/2*3^(1/6)*arctan(
1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-3/2*3^(2/3)*ln(3^(1/3)+x)-1/12*ln(x
^2-x+1)+3/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.95

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{1}{60}\left(-120x^2+12x^5 - 270\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) - 10\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 10\log(1+x) - 90\,3^{2/3}\log\left(3+3^{2/3}x\right) - 5\log\left(1-x+x^2\right) + 45\,3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

input
```
Integrate[x^10/(3 + 4*x^3 + x^6),x]
```

output
```
(-120*x^2 + 12*x^5 - 270*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 10*Sqrt
[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Log[1 + x] - 90*3^(2/3)*Log[3 + 3^(2/3
)*x] - 5*Log[1 - x + x^2] + 45*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/6
0
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {1703, 27, 1826, 27, 1834, 821, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{x^6+4x^3+3}\,dx$$

$$\Big\downarrow \text{1703}$$

$$\frac{x^5}{5}-\frac{1}{5}\int \frac{5x^4\left(4x^3+3\right)}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^5}{5}-\int \frac{x^4\left(4x^3+3\right)}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{ 1826}$$

$$\frac{1}{2}\int \frac{2x\left(13x^3+12\right)}{x^6+4x^3+3}dx+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x\left(13x^3+12\right)}{x^6+4x^3+3}dx+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{ 1834}$$

$$-\frac{1}{2}\int \frac{x}{x^3+1}dx+\frac{27}{2}\int \frac{x}{x^3+3}dx+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{ 821}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{x+1}dx-\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx\right)+\frac{27}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}\left(\frac{1}{3}\log(x+1)-\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx\right)+\frac{27}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int\frac{1}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{3\int\frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{217}$$

$$\frac{27}{2}\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int\frac{1}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{1083}$$

$$\frac{27}{2}\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx+3\int\frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{3}\log(x+1)\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^5}{5}-2x^2$$

input
```
Int[x^10/(3 + 4*x^3 + x^6),x]
```

output
```
-2*x^2 + x^5/5 + (Log[1 + x]/3 + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]]) -
Log[1 - x + x^2]/2)/3)/2 + (27*(-1/3*Log[3^(1/3) + x]/3^(1/3) + (-(Sqrt[3]
*ArcTan[(1 - (2*x)/3^(1/3))/Sqrt[3]]) + Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/
(3*3^(1/3))))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

method	result
risch	$\frac{x^5}{5}-2x^2+\frac{\ln(x+1)}{6}-\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3+9\right)}\_R\ln\left(\_R^2+3x\right)\right)}{2}$
default	$\frac{x^5}{5}-2x^2-\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{3\,3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}+\frac{3\,3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}+\frac{9\,3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(2\right.}{}\right.}{2}$

input
```
int(x^10/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5-2*x^2+1/6*ln(x+1)-1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(2/3*(x-1/2)*
3^(1/2))+3/2*sum(_R*ln(_R^2+3*x),_R=RootOf(_Z^3+9))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.84

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{1}{5}\,x^5-2\,x^2+\frac{3}{2}\cdot 9^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{2}{9}\cdot 9^{\frac{1}{3}}\sqrt{3}x-\frac{1}{3}\,\sqrt{3}\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{3}{4}\cdot 9^{\frac{1}{3}}\log\left(3\,x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right) - \frac{3}{2}\cdot 9^{\frac{1}{3}}\log\left(3\,x+9^{\frac{2}{3}}\right)-\frac{1}{12}\,\log\left(x^2-x+1\right)+\frac{1}{6}\,\log\left(x+1\right)$$

input
```
integrate(x^10/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/5*x^5 - 2*x^2 + 3/2*9^(1/3)*sqrt(3)*arctan(2/9*9^(1/3)*sqrt(3)*x - 1/3*s
qrt(3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/4*9^(1/3)*log(3*x^
2 - 9^(2/3)*x + 3*9^(1/3)) - 3/2*9^(1/3)*log(3*x + 9^(2/3)) - 1/12*log(x^2
 - x + 1) + 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{x^5}{5} - 2x^2 + \frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{3872\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{3281} + \frac{3188648\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{88587}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{3188648\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{88587} + \frac{3872\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{3281}\right) + \mathrm{RootSum}\left(8t^3+243, \left(t \mapsto t\log\left(\frac{3872t^5}{3281} + \frac{3188648t^2}{88587} + x\right)\right)\right)$$

input
```
integrate(x**10/(x**6+4*x**3+3),x)
```

output
```
x**5/5 - 2*x**2 + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 3872*(-1/1
2 - sqrt(3)*I/12)**5/3281 + 3188648*(-1/12 - sqrt(3)*I/12)**2/88587) + (-1
/12 + sqrt(3)*I/12)*log(x + 3188648*(-1/12 + sqrt(3)*I/12)**2/88587 + 3872
*(-1/12 + sqrt(3)*I/12)**5/3281) + RootSum(8*_t**3 + 243, Lambda(_t, _t*lo
g(3872*_t**5/3281 + 3188648*_t**2/88587 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{1}{5}\,x^5 - 2\,x^2 + \frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{3}{2} \cdot 3^{\frac{2}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{9}{2} \cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{12}\,\log\left(x^2-x+1\right) + \frac{1}{6}\,\log\left(x+1\right)$$

input
```
integrate(x^10/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)
*log(x + 3^(1/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3^(1/6
)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x
+ 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{1}{5}\,x^5 - 2\,x^2 + \frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{3}{2} \cdot 3^{\frac{2}{3}}\log\left(\left|x + 3^{\frac{1}{3}}\right|\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{9}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{12}\,\log\left(x^2-x+1\right) + \frac{1}{6}\,\log\left(|x+1|\right)$$

input
```
integrate(x^10/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)
*log(abs(x + 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3
^(1/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*l
og(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = \frac{\ln(x+1)}{6} - \frac{3\,3^{2/3}\ln\left(x+3^{1/3}\right)}{2} + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - 2\,x^2 + \frac{x^5}{5} - \frac{3\,(-1)^{1/3}\ln\left(x-\frac{(-1)^{1/3}\,3^{1/3}}{2}-\frac{(-1)^{1/6}\,3^{5/6}}{2}+\frac{3^{1/3}}{2}\right)\left(3^{2/3}+3^{1/6}\,3\mathrm{i}\right)}{4} + \frac{3\,(-1)^{1/3}\,3^{2/3}\ln\left(x+(-1)^{2/3}\,3^{1/3}\right)}{2}$$

input
```
int(x^10/(4*x^3 + x^6 + 3),x)
```

output
```
log(x + 1)/6 - (3*3^(2/3)*log(x + 3^(1/3)))/2 + log(x - (3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/12 + 1/12) - 2*x^2 + x^5/5 - (3*(-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2
 - ((-1)^(1/6)*3^(5/6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/4 + (3*(-1)
^(1/3)*3^(2/3)*log(x + (-1)^(2/3)*3^(1/3)))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \frac{x^{10}}{3+4x^3+x^6}\,dx = -\frac{9\,3^{\frac{1}{6}}\,atan\left(\frac{\left(3^{\frac{1}{3}}-2x\right)3^{\frac{1}{6}}}{3}\right)}{2} - \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} + \frac{3\,3^{\frac{2}{3}}\log\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4} - \frac{3\,3^{\frac{2}{3}}\log\left(3^{\frac{1}{3}}+x\right)}{2} - \frac{\log(x^2-x+1)}{12} + \frac{\log(x+1)}{6} + \frac{x^5}{5} - 2x^2$$

input
```
int(x^10/(x^6+4*x^3+3),x)
```

output

```
( - 270*3**(1/6)*atan((3**(1/3) - 2*x)/(3**(2/3)*3**(1/6))) - 10*sqrt(3)*a
tan((2*x - 1)/sqrt(3)) + 45*3**(2/3)*log(3**(2/3) - 3**(1/3)*x + x**2) - 9
0*3**(2/3)*log(3**(1/3) + x) - 5*log(x**2 - x + 1) + 10*log(x + 1) + 12*x*
*5 - 120*x**2)/60
```

3.144 $\int \frac{x^9}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 122

$$\int \frac{x^9}{3+4x^3+x^6}\,dx=-4x+\frac{x^4}{4}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{3}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

output
```
-4*x+1/4*x^4+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-3/2*3^(5/6)*arctan(1/
3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+3/2*3^(1/3)*ln(3^(1/3)+x)+1/12*ln(x^2
-x+1)-3/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \frac{x^9}{3+4x^3+x^6}\,dx=\frac{1}{12}\Bigg(-48x+3x^4-18\,3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\log(1+x)+18\sqrt[3]{3}\log\left(3+3^{2/3}x\right)+\log\left(1-x+\right.$$

input
```
Integrate[x^9/(3 + 4*x^3 + x^6),x]
```

output
```
(-48*x + 3*x^4 - 18*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*Ar
cTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 18*3^(1/3)*Log[3 + 3^(2/3)*x] +
Log[1 - x + x^2] - 9*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {1703, 27, 1826, 1752, 750, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1703}$$

$$\frac{x^4}{4}-\frac{1}{4}\int \frac{4x^3\left(4x^3+3\right)}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4}{4}-\int \frac{x^3\left(4x^3+3\right)}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1826}$$

$$\int \frac{13x^3+12}{x^6+4x^3+3}dx+\frac{x^4}{4}-4x$$

$$\Big\downarrow \text{1752}$$

$$-\frac{1}{2}\int \frac{1}{x^3+1}dx+\frac{27}{2}\int \frac{1}{x^3+3}dx+\frac{x^4}{4}-4x$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx\right)+\frac{27}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{27}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \; 217$$

$$\frac{27}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \; 1083$$

$$\frac{27}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+$$
$$\frac{1}{2}\left(\frac{1}{3}\left(3\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(x+1)\right)+$$
$$\frac{27}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(x+1)\right)+$$
$$\frac{27}{2}\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{x^4}{4}-4x$$

input
```
Int[x^9/(3 + 4*x^3 + x^6),x]
```

output
```
-4*x + x^4/4 + (-1/3*Log[1 + x] + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]]) +
 Log[1 - x + x^2]/2)/3)/2 + (27*(Log[3^(1/3) + x]/(3*3^(2/3)) + (-(Sqrt[3]
*ArcTan[(1 - (2*x)/3^(1/3))/Sqrt[3]]) - Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/
(3*3^(2/3))))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.48

method	result
risch	$\frac{x^4}{4} - 4x + \frac{3\left(\sum_{\_R=\operatorname{RootOf}(\_Z^3-3)} \_R \ln(x+\_R)\right)}{2} - \frac{\ln(x+1)}{6} + \frac{\ln(x^2-x+1)}{12} - \frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$
default	$\frac{x^4}{4} - 4x + \frac{\ln(x^2-x+1)}{12} - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{3\,3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2} - \frac{3\,3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4} + \frac{3\,3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(2\,3\ldots\right)}{3\ldots}\right)}{2}$

input
```
int(x^9/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4-4*x+3/2*sum(_R*ln(x+_R),_R=RootOf(_Z^3-3))-1/6*ln(x+1)+1/12*ln(x^2
-x+1)-1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{1}{4}\,x^4 + \frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{2}{3}\cdot 3^{\frac{1}{6}}x - \frac{1}{3}\sqrt{3}\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - 4\,x + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(x^9/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/4*x^4 + 3/2*3^(5/6)*arctan(2/3*3^(1/6)*x - 1/3*sqrt(3)) - 1/6*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) +
 3/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x +
1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.06

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{x^4}{4} - 4x - \frac{\log(x+1)}{6} + \left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{9841}{19692} - \frac{9841\sqrt{3}i}{19692} + \frac{360\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{547}\right) + \left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{9841}{19692} + \frac{360\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{547} + \frac{9841\sqrt{3}i}{19692}\right) + \operatorname{RootSum}\left(8t^3-81, \left(t \mapsto t\log\left(\frac{360t^4}{547} - \frac{9841t}{1641} + x\right)\right)\right)$$

input
```
integrate(x**9/(x**6+4*x**3+3),x)
```

output
```
x**4/4 - 4*x - log(x + 1)/6 + (1/12 + sqrt(3)*I/12)*log(x - 9841/19692 - 9
841*sqrt(3)*I/19692 + 360*(1/12 + sqrt(3)*I/12)**4/547) + (1/12 - sqrt(3)*
I/12)*log(x - 9841/19692 + 360*(1/12 - sqrt(3)*I/12)**4/547 + 9841*sqrt(3)
*I/19692) + RootSum(8*_t**3 - 81, Lambda(_t, _t*log(360*_t**4/547 - 9841*_
t/1641 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{1}{4}x^4 + \frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - 4x + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log(x+1)$$

input
```
integrate(x^9/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) +
 3/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x +
1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.77

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{1}{4}x^4 + \frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) + \frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) - 4x + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(x^9/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) +
 3/2*3^(1/3)*log(abs(x + 3^(1/3))) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log
(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 19.58 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{3\,3^{1/3}\,\ln\left(x+3^{1/3}\right)}{2} - \frac{\ln\left(x+1\right)}{6} - 4\,x + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \frac{x^4}{4} - \ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3\,3^{1/3}}{4}+\frac{3^{5/6}\,3\mathrm{i}}{4}\right) + 3^{1/3}\,\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)$$

input
```
int(x^9/(4*x^3 + x^6 + 3),x)
```

output
```
(3*3^(1/3)*log(x + 3^(1/3)))/2 - log(x + 1)/6 - 4*x + log(x - (3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/12 - 1/12) + x^4/4 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*((3*3^(1/3
))/4 + (3^(5/6)*3i)/4) + 3^(1/3)*log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*((3^(
1/2)*3i)/4 - 3/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.73

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = -\frac{3\,3^{\frac{5}{6}}\,atan\left(\frac{\left(3^{\frac{1}{3}}-2x\right)3^{\frac{1}{6}}}{3}\right)}{2} - \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} - \frac{3\,3^{\frac{1}{3}}\log\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4} + \frac{3\,3^{\frac{1}{3}}\log\left(3^{\frac{1}{3}}+x\right)}{2} + \frac{\log(x^2-x+1)}{12} - \frac{\log(x+1)}{6} + \frac{x^4}{4} - 4x$$

input
```
int(x^9/(x^6+4*x^3+3),x)
```

output
```
( - 18*3**(2/3)*3**(1/6)*atan((3**(1/3) - 2*x)/(3**(2/3)*3**(1/6))) - 2*sq
rt(3)*atan((2*x - 1)/sqrt(3)) - 9*3**(1/3)*log(3**(2/3) - 3**(1/3)*x + x**
2) + 18*3**(1/3)*log(3**(1/3) + x) + log(x**2 - x + 1) - 2*log(x + 1) + 3*
x**4 - 48*x)/12
```

3.145 $\int \frac{x^7}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 119

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{x^2}{2}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{3}{2}\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right) + \frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

output
```
1/2*x^2-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+3/2*3^(1/6)*arctan(1/3*(3^
(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/2*3^(2/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)
-1/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{1}{12}\left(6x^2+18\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\log(1+x)+6\,3^{2/3}\log\left(3+3^{2/3}x\right)+\log\left(1-x+x^2\right)-3\,3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

input
```
Integrate[x^7/(3 + 4*x^3 + x^6),x]
```

output
```
(6*x^2 + 18*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1
 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 6*3^(2/3)*Log[3 + 3^(2/3)*x] + Log[1 - x
 + x^2] - 3*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {1703, 27, 1834, 821, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^6+4x^3+3}\,dx$$

$$\Big\downarrow \text{1703}$$

$$\frac{x^2}{2}-\frac{1}{2}\int \frac{2x\left(4x^3+3\right)}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2}{2}-\int \frac{x\left(4x^3+3\right)}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1834}$$

$$\frac{1}{2}\int \frac{x}{x^3+1}dx-\frac{9}{2}\int \frac{x}{x^3+3}dx+\frac{x^2}{2}$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx\right)-\frac{9}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{9}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{x^2}{2}$$

$$\Big\downarrow \ 1083$$

$$-\frac{9}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)-\frac{1}{3}\log(x+1)\right)+\frac{x^2}{2}$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2-x+1\right)\right)-\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+\frac{x^2}{2}$$

input
```
Int[x^7/(3 + 4*x^3 + x^6),x]
```

output
```
x^2/2 + (-1/3*Log[1 + x] + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + Log[1 - x
 + x^2]/2)/3)/2 - (9*(-1/3*Log[3^(1/3) + x]/3^(1/3) + (-(Sqrt[3]*ArcTan[(1
 - (2*x)/3^(1/3))/Sqrt[3]]) + Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(1/3)
)))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

method	result
risch	$\frac{x^2}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3-9\right)}\_R\ln\left(\_R^2+3x\right)\right)}{2}-\frac{\ln(x+1)}{6}+\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$
default	$\frac{x^2}{2}+\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}-\frac{3\,3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}$

input
```
int(x^7/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/2*sum(_R*ln(_R^2+3*x),_R=RootOf(_Z^3-9))-1/6*ln(x+1)+1/12*ln(x^2
-x+1)+1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\cdot 9^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{2}{9}\cdot 9^{\frac{1}{3}}\sqrt{3}x - \frac{1}{3}\,\sqrt{3}\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{4}\cdot 9^{\frac{1}{3}}\log\left(3\,x^2 - 9^{\frac{2}{3}}x + 3\cdot 9^{\frac{1}{3}}\right) + \frac{1}{2}\cdot 9^{\frac{1}{3}}\log\left(3\,x + 9^{\frac{2}{3}}\right) + \frac{1}{12}\,\log\left(x^2 - x + 1\right) - \frac{1}{6}\,\log\left(x+1\right)$$

input
```
integrate(x^7/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/2*9^(1/3)*sqrt(3)*arctan(2/9*9^(1/3)*sqrt(3)*x - 1/3*sqrt(3))
+ 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*9^(1/3)*log(3*x^2 - 9^(2
/3)*x + 3*9^(1/3)) + 1/2*9^(1/3)*log(3*x + 9^(2/3)) + 1/12*log(x^2 - x + 1
) - 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{x^2}{2} - \frac{\log(x+1)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{6562\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{183} - \frac{1872\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{61}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{1872\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{61} + \frac{6562\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{183}\right) + \operatorname{RootSum}\left(8t^3 - 9, \left(t \mapsto t\log\left(-\frac{1872t^5}{61} + \frac{6562t^2}{183} + x\right)\right)\right)$$

input
```
integrate(x**7/(x**6+4*x**3+3),x)
```

output
```
x**2/2 - log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 6562*(1/12 - sqrt(3)
*I/12)**2/183 - 1872*(1/12 - sqrt(3)*I/12)**5/61) + (1/12 + sqrt(3)*I/12)*
log(x - 1872*(1/12 + sqrt(3)*I/12)**5/61 + 6562*(1/12 + sqrt(3)*I/12)**2/1
83) + RootSum(8*_t**3 - 9, Lambda(_t, _t*log(-1872*_t**5/61 + 6562*_t**2/1
83 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{1}{2}x^2 - \frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(x + 3^{\frac{1}{3}}\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{12}\log\left(x^2 - x + 1\right) - \frac{1}{6}\log(x+1)$$

input
```
integrate(x^7/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3)*log(x +
 3^(1/3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/2*3^(1/6)*arctan
(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{1}{2}\,x^2 - \frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(\left|x + 3^{\frac{1}{3}}\right|\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{12}\log\left(x^2 - x + 1\right) - \frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(x^7/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3)*log(abs
(x + 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/2*3^(1/6)*a
rctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x
 + 1))
```

Mupad [B] (verification not implemented)

Time = 19.71 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{3^{2/3}\,\ln\left(x+3^{1/3}\right)}{2} - \frac{\ln\left(x+1\right)}{6} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \frac{x^2}{2} - \ln\left(x - \frac{3^{1/3}}{2} - \frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{4} - \frac{3^{1/6}\,3\mathrm{i}}{4}\right) - \ln\left(x - \frac{3^{1/3}}{2} + \frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{4} + \frac{3^{1/6}\,3\mathrm{i}}{4}\right)$$

input
```
int(x^7/(4*x^3 + x^6 + 3),x)
```

output
```
(3^(2/3)*log(x + 3^(1/3)))/2 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 + 1/12) + x^2/2 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(2/3)/4 - (3^(1
/6)*3i)/4) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/3)/4 + (3^(1/6)*3i)
/4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.72

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{3\,3^{\frac{1}{6}}\,atan\left(\frac{\left(3^{\frac{1}{3}}-2x\right)3^{\frac{1}{6}}}{3}\right)}{2}+\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6}-\frac{3^{\frac{2}{3}}\log\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}+\frac{3^{\frac{2}{3}}\log\left(3^{\frac{1}{3}}+x\right)}{2}+\frac{\log(x^2-x+1)}{12}-\frac{\log(x+1)}{6}+\frac{x^2}{2}$$

input
```
int(x^7/(x^6+4*x^3+3),x)
```

output
```
(18*3**(1/6)*atan((3**(1/3) - 2*x)/(3**(2/3)*3**(1/6))) + 2*sqrt(3)*atan((
2*x - 1)/sqrt(3)) - 3*3**(2/3)*log(3**(2/3) - 3**(1/3)*x + x**2) + 6*3**(2
/3)*log(3**(1/3) + x) + log(x**2 - x + 1) - 2*log(x + 1) + 6*x**2)/12
```

3.146 $\int \frac{x^6}{3+4x^3+x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{x^6}{3+4x^3+x^6} \, dx = x-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} +\frac{1}{2}3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right) -\frac{1}{12}\log\left(1-x+x^2\right)+\frac{1}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)$$

output
```
x-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/2*3^(5/6)*arctan(1/3*(3^(1/3)-
2*x)*3^(1/6))+1/6*ln(1+x)-1/2*3^(1/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/4*3
^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

$$\int \frac{x^6}{3+4x^3+x^6} \, dx = \frac{1}{12}\left(12x +6\,3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+2\log(1+x)-6\sqrt[3]{3}\log\left(3+3^{2/3}x\right)-\log\left(1-x+x\right.\right.$$

input
```
Integrate[x^6/(3 + 4*x^3 + x^6),x]
```

output
```
(12*x + 6*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1 +
 2*x)/Sqrt[3]] + 2*Log[1 + x] - 6*3^(1/3)*Log[3 + 3^(2/3)*x] - Log[1 - x +
 x^2] + 3*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1703, 1752, 750, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1703}$$

$$x-\int \frac{4x^3+3}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1752}$$

$$\frac{1}{2}\int \frac{1}{x^3+1}dx-\frac{9}{2}\int \frac{1}{x^3+3}dx+x$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\int \frac{1}{x+1}dx\right)-\frac{9}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\ 3^{2/3}}\right)+x$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+x$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+x$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+x$$

$\Big\downarrow$ 1082

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+x$$

$\Big\downarrow$ 217

$$-\frac{9}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+x$$

$\Big\downarrow$ 1083

$$-\frac{9}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{3}\log(x+1)\right)+x$$

$\Big\downarrow$ 217

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+x$$

$\Big\downarrow$ 1103

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{3}\log(x+1)\right)-\frac{9}{2}\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+x$$

input
```
Int[x^6/(3 + 4*x^3 + x^6),x]
```

output
```
x + (Log[1 + x]/3 + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - Log[1 - x + x^2]
/2)/3)/2 - (9*(Log[3^(1/3) + x]/(3*3^(2/3)) + (-(Sqrt[3]*ArcTan[(1 - (2*x)
/3^(1/3))/Sqrt[3]]) - Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(2/3))))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

method	result
risch	$x-\frac{\ln\left(4x^{2}-4x+4\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3}+3\right)}\_R\ln\left(x-\_R\right)\right)}{2}+\frac{\ln(x+1)}{6}$
default	$x-\frac{\ln\left(x^{2}-x+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^{2}\right)}{4}-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}+$

input
```
int(x^6/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
x-1/12*ln(4*x^2-4*x+4)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/2*sum(_R*
ln(x-_R),_R=RootOf(_Z^3+3))+1/6*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = -\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{2}{3}\cdot 3^{\frac{1}{6}}x-\frac{1}{3}\sqrt{3}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$
$$+\frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)$$
$$+x-\frac{1}{12}\log\left(x^{2}-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
-1/2*3^(5/6)*arctan(2/3*3^(1/6)*x - 1/3*sqrt(3)) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1
/3)*log(x + 3^(1/3)) + x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = x + \frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{121}{246} - \frac{121\sqrt{3}i}{246} + \frac{864\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^4}{41}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{121}{246} + \frac{864\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^4}{41} + \frac{121\sqrt{3}i}{246}\right) + \operatorname{RootSum}\left(8t^3+3, \left(t \mapsto t\log\left(\frac{864t^4}{41} + \frac{242t}{41} + x\right)\right)\right)$$

input
```
integrate(x**6/(x**6+4*x**3+3),x)
```

output
```
x + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 121/246 - 121*sqrt(3)*I/
246 + 864*(-1/12 - sqrt(3)*I/12)**4/41) + (-1/12 + sqrt(3)*I/12)*log(x - 1
21/246 + 864*(-1/12 + sqrt(3)*I/12)**4/41 + 121*sqrt(3)*I/246) + RootSum(8
*_t**3 + 3, Lambda(_t, _t*log(864*_t**4/41 + 242*_t/41 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = -\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) - \frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) + x - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1
/3)*log(x + 3^(1/3)) + x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = -\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) - \frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) + x - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1
/3)*log(abs(x + 3^(1/3))) + x - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1)
)
```

Mupad [B] (verification not implemented)

Time = 19.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = x+\frac{\ln(x+1)}{6}-\frac{3^{1/3}\,\ln\left(x+3^{1/3}\right)}{2} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{4}-\frac{3^{5/6}\,1\mathrm{i}}{4}\right)+\frac{(-1)^{1/3}\,3^{1/3}\,\ln\left(x-(-1)^{1/3}\,3^{1/3}\right)}{2}$$

input
```
int(x^6/(4*x^3 + x^6 + 3),x)
```

output
```
x + log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/2 - log(x - (3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/12 - 1/12) + log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/4 - (3^(5/6)*
1i)/4) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = \frac{3^{\frac{5}{6}}\,atan\left(\frac{\left(3^{\frac{1}{3}}-2x\right)3^{\frac{1}{6}}}{3}\right)}{2}+\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6}+\frac{3^{\frac{1}{3}}\log\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4} - \frac{3^{\frac{1}{3}}\log\left(3^{\frac{1}{3}}+x\right)}{2}-\frac{\log(x^2-x+1)}{12}+\frac{\log(x+1)}{6}+x$$

input
```
int(x^6/(x^6+4*x^3+3),x)
```

output
```
(6*3**(2/3)*3**(1/6)*atan((3**(1/3) - 2*x)/(3**(2/3)*3**(1/6))) + 2*sqrt(3
)*atan((2*x - 1)/sqrt(3)) + 3*3**(1/3)*log(3**(2/3) - 3**(1/3)*x + x**2) -
 6*3**(1/3)*log(3**(1/3) + x) - log(x**2 - x + 1) + 2*log(x + 1) + 12*x)/1
2
```

3.147 $\int \frac{x^4}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \frac{x^4}{3+4x^3+x^6}\,dx=\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\sqrt[3]{3}}$$

output
```
1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/2*3^(1/6)*arctan(1/3*(3^(1/3)-2*
x)*3^(1/6))+1/6*ln(1+x)-1/6*3^(2/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/12*3^
(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{3+4x^3+x^6}\,dx =\frac{1}{12}\left(-6\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+2\log(1+x)-2\ 3^{2/3}\log\left(3+3^{2/3}x\right)-\log\left(1-x+x^2\right)+3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

input
```
Integrate[x^4/(3 + 4*x^3 + x^6),x]
```

output
```
(-6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/
Sqrt[3]] + 2*Log[1 + x] - 2*3^(2/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x^2]
+ 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1710, 821, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^6+4x^3+3} \, dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{3}{2}\int \frac{x}{x^3+3}dx-\frac{1}{2}\int \frac{x}{x^3+1}dx$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{x+1}dx-\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx\right)+\frac{3}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\log(x+1)-\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx\right)+\frac{3}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+$$
$$\frac{3}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+$$
$$\frac{3}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)$$

$\Big\downarrow$ 1082

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+$$
$$\frac{3}{2}\left(\frac{3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)$$

$\Big\downarrow$ 217

$$\frac{3}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+$$
$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)$$

$\Big\downarrow$ 1083

$$\frac{3}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)+$$
$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{3}\log(x+1)\right)$$

$\Big\downarrow$ 217

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)$$

input
```
Int[x^4/(3 + 4*x^3 + x^6),x]
```

output
```
(Log[1 + x]/3 + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]]) - Log[1 - x + x^2]/
2)/3)/2 + (3*(-1/3*Log[3^(1/3) + x]/3^(1/3) + (-(Sqrt[3]*ArcTan[(1 - (2*x)
/3^(1/3))/Sqrt[3]]) + Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(1/3))))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.54

method	result
risch	$-\frac{\ln(4x^2-4x+4)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(x+1)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(3\_Z^3+1)}\_R\ln\left(3\_R^2+x\right)\right)}{2}$
default	$-\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{12}+\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}+\ln($

input
```
int(x^4/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-1/12*ln(4*x^2-4*x+4)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*ln(x+1)+
1/2*sum(_R*ln(3*_R^2+x),_R=RootOf(3*_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{2}\cdot 3^{\frac{1}{6}}\arctan\left(-\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(x^4/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(x + 3^(1/3))
 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/2*3^(1/6)*arctan(-1/3*3^(
1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.20

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{2592\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{5} + \frac{168\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{5}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{168\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{5} + \frac{2592\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{5}\right) + \operatorname{RootSum}\left(24t^3+1,\left(t\mapsto t\log\left(\frac{2592t^5}{5}+\frac{168t^2}{5}+x\right)\right)\right)$$

input
```
integrate(x**4/(x**6+4*x**3+3),x)
```

output
```
log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 2592*(-1/12 - sqrt(3)*I/12)*
*5/5 + 168*(-1/12 - sqrt(3)*I/12)**2/5) + (-1/12 + sqrt(3)*I/12)*log(x + 1
68*(-1/12 + sqrt(3)*I/12)**2/5 + 2592*(-1/12 + sqrt(3)*I/12)**5/5) + RootS
um(24*_t**3 + 1, Lambda(_t, _t*log(2592*_t**5/5 + 168*_t**2/5 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2} \cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(x^4/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(x + 3^(1/3))
 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*arctan(1/3*3^(1
/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) - \frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(x^4/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(abs(x + 3^(1
/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*arctan(1/3
*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = \frac{\ln(x+1)}{6} - \frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{6} + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \frac{(-1)^{1/3}\ln\left(x-\frac{(-1)^{1/3}3^{1/3}}{2}-\frac{(-1)^{1/6}3^{5/6}}{2}+\frac{3^{1/3}}{2}\right)\left(3^{2/3}+3^{1/6}\,3\mathrm{i}\right)}{12} + \frac{(-1)^{1/3}3^{2/3}\ln\left(x+(-1)^{2/3}3^{1/3}\right)}{6}$$

input
```
int(x^4/(4*x^3 + x^6 + 3),x)
```

output
```
log(x + 1)/6 - (3^(2/3)*log(x + 3^(1/3)))/6 + log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 + 1/12) - ((-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 - ((-1)^(1/6)*3^(5/
6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/12 + ((-1)^(1/3)*3^(2/3)*log(x
+ (-1)^(2/3)*3^(1/3)))/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = -\frac{3^{\frac{1}{6}}\,atan\left(\frac{\left(3^{\frac{1}{3}}-2x\right)3^{\frac{1}{6}}}{3}\right)}{2} - \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} + \frac{3^{\frac{2}{3}}\log\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{12} - \frac{3^{\frac{2}{3}}\log\left(3^{\frac{1}{3}}+x\right)}{6} - \frac{\log(x^2-x+1)}{12} + \frac{\log(x+1)}{6}$$

input
```
int(x^4/(x^6+4*x^3+3),x)
```

output
```
( - 6*3**(1/6)*atan((3**(1/3) - 2*x)/(3**(2/3)*3**(1/6))) - 2*sqrt(3)*atan
((2*x - 1)/sqrt(3)) + 3**(2/3)*log(3**(2/3) - 3**(1/3)*x + x**2) - 2*3**(2
/3)*log(3**(1/3) + x) - log(x**2 - x + 1) + 2*log(x + 1))/12
```

3.148 $\int \frac{x^3}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \frac{x^3}{3+4x^3+x^6}\,dx=\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{2\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\ 3^{2/3}}$$

output
```
1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/6*3^(5/6)*arctan(1/3*(3^(1/3)-2*
x)*3^(1/6))-1/6*ln(1+x)+1/6*3^(1/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/12*3^
(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{3+4x^3+x^6}\,dx$$
$$=\frac{1}{12}\left(-23^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\log(1+x)+2\sqrt[3]{3}\log\left(3+3^{2/3}x\right)+\log\left(1-x\right.\right.$$

input
```
Integrate[x^3/(3 + 4*x^3 + x^6),x]
```

output
```
(-2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/
Sqrt[3]] - 2*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + Log[1 - x + x^2]
- 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1710, 750, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^6+4x^3+3}\,dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{3}{2}\int \frac{1}{x^3+3}dx-\frac{1}{2}\int \frac{1}{x^3+1}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx\right)+\frac{3}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \; 217$$

$$\frac{3}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)$$

$$\Big\downarrow \; 1083$$

$$\frac{3}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(3\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

input
```
Int[x^3/(3 + 4*x^3 + x^6),x]
```

output
```
(-1/3*Log[1 + x] + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]]) + Log[1 - x + x^
2]/2)/3)/2 + (3*(Log[3^(1/3) + x]/(3*3^(2/3)) + (-(Sqrt[3]*ArcTan[(1 - (2*
x)/3^(1/3))/Sqrt[3]]) - Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(2/3))))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

method	result
risch	$-\frac{\ln(x+1)}{6}+\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^3-1\right)}\_R\ln\left(x+3\_R\right)\right)}{2}$
default	$\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{6}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{12}+\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{6}-\frac{\ln(x-}{6}$

input
```
int(x^3/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(x+1)+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))+1/2*
sum(_R*ln(x+3*_R),_R=RootOf(9*_Z^3-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = \frac{1}{2}\cdot 9^{\frac{1}{6}}\sqrt{\frac{1}{3}}\arctan\left(\frac{1}{9}\cdot 9^{\frac{1}{6}}\sqrt{\frac{1}{3}}\left(2\cdot 9^{\frac{2}{3}}x-3\cdot 9^{\frac{1}{3}}\right)\right) - \frac{1}{36}\cdot 9^{\frac{2}{3}}\log\left(3\,x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+\frac{1}{18} \cdot 9^{\frac{2}{3}}\log\left(3\,x+9^{\frac{2}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/2*9^(1/6)*sqrt(1/3)*arctan(1/9*9^(1/6)*sqrt(1/3)*(2*9^(2/3)*x - 3*9^(1/3
))) - 1/36*9^(2/3)*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 1/18*9^(2/3)*log(3
*x + 9^(2/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/12*log(x^2 -
 x + 1) - 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = -\frac{\log(x+1)}{6} + \left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4+\frac{\sqrt{3}i}{4}\right) + \left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4-\frac{\sqrt{3}i}{4}\right) + \operatorname{RootSum}\left(72t^3-1,\left(t\mapsto t\log\left(648t^4-3t+x\right)\right)\right)$$

input
```
integrate(x**3/(x**6+4*x**3+3),x)
```

output
```
-log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 - sqrt(3)*I/
12)**4 + sqrt(3)*I/4) + (1/12 + sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 + sq
rt(3)*I/12)**4 - sqrt(3)*I/4) + RootSum(72*_t**3 - 1, Lambda(_t, _t*log(64
8*_t**4 - 3*_t + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = \frac{1}{6}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/6*3^(1
/3)*log(x + 3^(1/3)) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = \frac{1}{6}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) + \frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/6*3^(1
/3)*log(abs(x + 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 19.45 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = \frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{6}-\frac{\ln\left(x+1\right)}{6} + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{12}+\frac{3^{5/6}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{12}-\frac{3^{5/6}\,1\mathrm{i}}{12}\right)$$

input
```
int(x^3/(4*x^3 + x^6 + 3),x)
```

output
```
(3^(1/3)*log(x + 3^(1/3)))/6 - log(x + 1)/6 + log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 - 1/12) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(1/3)/12 + (3^(5/6)*1i)
/12) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/12 - (3^(5/6)*1i)/12)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = -\frac{3^{\frac{5}{6}}\,atan\left(\frac{\left(3^{\frac{1}{3}}-2x\right)3^{\frac{1}{6}}}{3}\right)}{6}-\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6}-\frac{3^{\frac{1}{3}}\log\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{12}+\frac{3^{\frac{1}{3}}\log\left(3^{\frac{1}{3}}+x\right)}{6}+\frac{\log(x^2-x+1)}{12}-\frac{\log(x+1)}{6}$$

input
```
int(x^3/(x^6+4*x^3+3),x)
```

output
```
( - 2*3**(2/3)*3**(1/6)*atan((3**(1/3) - 2*x)/(3**(2/3)*3**(1/6))) - 2*sqr
t(3)*atan((2*x - 1)/sqrt(3)) - 3**(1/3)*log(3**(2/3) - 3**(1/3)*x + x**2)
+ 2*3**(1/3)*log(3**(1/3) + x) + log(x**2 - x + 1) - 2*log(x + 1))/12
```

3.149 $\int \frac{x}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 112

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}} - \frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\sqrt[3]{3}}$$

output
```
-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*3^(1/6)*arctan(1/3*(3^(1/3)-2
*x)*3^(1/6))-1/6*ln(1+x)+1/18*3^(2/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/36*
3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int \frac{x}{3+4x^3+x^6}\,dx = \frac{1}{36}\left(6\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + 6\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 6\log(1+x) + 2\ 3^{2/3}\log\left(3+3^{2/3}x\right) + 3\log\left(1-x+x^2\right) - 3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

input
```
Integrate[x/(3 + 4*x^3 + x^6),x]
```

output
```
(6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] - 6*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] + 3*Log[1 - x + x^2]
 - 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {1711, 821, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{1}{2}\int \frac{x}{x^3+1}dx-\frac{1}{2}\int \frac{x}{x^3+3}dx$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx\right)+\frac{1}{2}\left(\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\sqrt[3]{3}}-\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)-\frac{1}{3}\log(x+1)\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2-x+1\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}\right)$$

input
```
Int[x/(3 + 4*x^3 + x^6),x]
```

output
```
(-1/3*Log[1 + x] + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + Log[1 - x + x^2]/
2)/3)/2 + (Log[3^(1/3) + x]/(3*3^(1/3)) - (-(Sqrt[3]*ArcTan[(1 - (2*x)/3^(
1/3))/Sqrt[3]]) + Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(1/3)))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.54

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^3-1\right)}\_R\ln\left(3\_R^2+x\right)\right)}{6}+\frac{\ln(4x^2-4x+4)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x+1)}{6}$
default	$\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{18}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{36}-\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{6}-\frac{\ln(x-}{6}$

input

```
int(x/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output

```
1/6*sum(_R*ln(3*_R^2+x),_R=RootOf(3*_Z^3-1))+1/12*ln(4*x^2-4*x+4)+1/6*3^(1
/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/6*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(-\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

input

```
integrate(x/(x^6+4*x^3+3),x, algorithm="fricas")
```

output

```
-1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3
)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*3^(1/6)*arctan(-1/3*3
^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

$$\int \frac{x}{3+4x^3+x^6} dx = -\frac{\log(x+1)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x+90\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2+11664\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5\right) + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x+11664\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5+90\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2\right) + \operatorname{RootSum}\left(648t^3-1, \left(t \mapsto t\log\left(11664t^5+90t^2+x\right)\right)\right)$$

input
```
integrate(x/(x**6+4*x**3+3),x)
```

output
```
-log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 90*(1/12 - sqrt(3)*I/12)**2
+ 11664*(1/12 - sqrt(3)*I/12)**5) + (1/12 + sqrt(3)*I/12)*log(x + 11664*(1
/12 + sqrt(3)*I/12)**5 + 90*(1/12 + sqrt(3)*I/12)**2) + RootSum(648*_t**3
- 1, Lambda(_t, _t*log(11664*_t**5 + 90*_t**2 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{x}{3+4x^3+x^6} dx = -\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) + \frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log(x+1)$$

input
```
integrate(x/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3
)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(1/6)*arctan(1/3*3^
(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{x}{3+4x^3+x^6}\,dx = -\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(x/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(abs(x + 3^
(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(1/6)*arctan(1
/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 19.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01

$$\int \frac{x}{3+4x^3+x^6}\,dx = \frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{18}-\frac{\ln\left(x+1\right)}{6}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{36}-\frac{3^{1/6}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{36}+\frac{3^{1/6}\,1\mathrm{i}}{12}\right)$$

input
```
int(x/(4*x^3 + x^6 + 3),x)
```

output
```
(3^(2/3)*log(x + 3^(1/3)))/18 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
12 + 1/12) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(2/3)/36 - (3^(1/6)*1i
)/12) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/3)/36 + (3^(1/6)*1i)/12)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int \frac{x}{3+4x^3+x^6}\,dx = \frac{3^{\frac{1}{6}}\,atan\left(\frac{\left(3^{\frac{1}{3}}-2x\right)3^{\frac{1}{6}}}{3}\right)}{6} + \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} - \frac{3^{\frac{2}{3}}\log\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{36} + \frac{3^{\frac{2}{3}}\log\left(3^{\frac{1}{3}}+x\right)}{18} + \frac{\log(x^2-x+1)}{12} - \frac{\log(x+1)}{6}$$

input
```
int(x/(x^6+4*x^3+3),x)
```

output
```
(6*3**(1/6)*atan((3**(1/3) - 2*x)/(3**(2/3)*3**(1/6))) + 6*sqrt(3)*atan((2
*x - 1)/sqrt(3)) - 3**(2/3)*log(3**(2/3) - 3**(1/3)*x + x**2) + 2*3**(2/3)
*log(3**(1/3) + x) + 3*log(x**2 - x + 1) - 6*log(x + 1))/36
```

3.150 $\int \frac{1}{3+4x^3+x^6}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 112

$$\int \frac{1}{3+4x^3+x^6}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}+\frac{1}{6}\log(1+x) -\frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{12\ 3^{2/3}}$$

output
```
-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/18*3^(5/6)*arctan(1/3*(3^(1/3)-
2*x)*3^(1/6))+1/6*ln(1+x)-1/18*3^(1/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/36
*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{1}{3+4x^3+x^6}\,dx = \frac{1}{36}\left(2\,3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+6\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+6\log(1+x)-2\sqrt[3]{3}\log\left(3+3^{2/3}x\right)-3\log\left(1-x\right.\right.$$

input
```
Integrate[(3 + 4*x^3 + x^6)^(-1),x]
```

output
```
(2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] + 6*Log[1 + x] - 2*3^(1/3)*Log[3 + 3^(2/3)*x] - 3*Log[1 - x + x^2]
 + 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {1685, 750, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6+4x^3+3}dx$$

$$\Big\downarrow \text{1685}$$

$$\frac{1}{2}\int \frac{1}{x^3+1}dx-\frac{1}{2}\int \frac{1}{x^3+3}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\int \frac{1}{x+1}dx\right)+\frac{1}{2}\left(-\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}-\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{3}\log(x+1)\right)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log(x+1)\right)+$$
$$\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

$\downarrow$ 1103

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{3}\log(x+1)\right)+$$
$$\frac{1}{2}\left(-\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)$$

input
```
Int[(3 + 4*x^3 + x^6)^(-1),x]
```

output
```
(Log[1 + x]/3 + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - Log[1 - x + x^2]/2)/
3)/2 + (-1/3*Log[3^(1/3) + x]/3^(2/3) - (-(Sqrt[3]*ArcTan[(1 - (2*x)/3^(1/
3))/Sqrt[3]]) - Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(2/3)))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^3+1\right)}\_R\ln\left(x-3\_R\right)\right)}{6}-\frac{\ln\left(4x^2-4x+4\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(x+1)}{6}$
default	$-\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{18}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{36}-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{18}+\ln($

input
```
int(1/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(_R*ln(x-3*_R),_R=RootOf(9*_Z^3+1))-1/12*ln(4*x^2-4*x+4)+1/6*3^(1/2
)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{1}{3+4x^3+x^6}\,dx = -\frac{1}{6}\cdot 9^{\frac{1}{6}}\sqrt{\frac{1}{3}}\arctan\left(\frac{1}{9}\cdot 9^{\frac{1}{6}}\sqrt{\frac{1}{3}}\left(2\cdot 9^{\frac{2}{3}}x-3\cdot 9^{\frac{1}{3}}\right)\right) + \frac{1}{108}\cdot 9^{\frac{2}{3}}\log\left(3\,x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)-\frac{1}{54} \cdot 9^{\frac{2}{3}}\log\left(3\,x+9^{\frac{2}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(1/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
-1/6*9^(1/6)*sqrt(1/3)*arctan(1/9*9^(1/6)*sqrt(1/3)*(2*9^(2/3)*x - 3*9^(1/
3))) + 1/108*9^(2/3)*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) - 1/54*9^(2/3)*log
(3*x + 9^(2/3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/12*log(x^2
 - x + 1) + 1/6*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{1}{3+4x^3+x^6}\,dx = \frac{\log{(x+1)}}{6} + \left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log{\left(x+\frac{13}{10}-\frac{13\sqrt{3}i}{10}+\frac{23328\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{5}\right)} + \left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log{\left(x+\frac{13}{10}+\frac{23328\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{5}+\frac{13\sqrt{3}i}{10}\right)} + \operatorname{RootSum}{\left(1944t^3+1,\left(t\mapsto t\log{\left(\frac{23328t^4}{5}-\frac{78t}{5}+x\right)}\right)\right)}$$

input
```
integrate(1/(x**6+4*x**3+3),x)
```

output
```
log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 13/10 - 13*sqrt(3)*I/10 + 23
328*(-1/12 + sqrt(3)*I/12)**4/5) + (-1/12 - sqrt(3)*I/12)*log(x + 13/10 +
23328*(-1/12 - sqrt(3)*I/12)**4/5 + 13*sqrt(3)*I/10) + RootSum(1944*_t**3
+ 1, Lambda(_t, _t*log(23328*_t**4/5 - 78*_t/5 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{1}{3+4x^3+x^6}\,dx = -\frac{1}{18}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{36}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) - \frac{1}{18}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(1/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3
*sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/18*3
^(1/3)*log(x + 3^(1/3)) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{1}{3+4x^3+x^6}\,dx = -\frac{1}{18}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{36}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) - \frac{1}{18}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(1/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3
*sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/18*3
^(1/3)*log(abs(x + 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 19.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int \frac{1}{3+4x^3+x^6}\,dx = \frac{\ln(x+1)}{6} - \frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{18} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \frac{(-1)^{1/3}\,3^{1/3}\ln\left(x-(-1)^{1/3}\,3^{1/3}\right)}{18} - \frac{(-1)^{1/3}\ln\left(x+\frac{(-1)^{1/3}\,3^{1/3}}{2}+\frac{(-1)^{1/3}\,3^{5/6}\,1\mathrm{i}}{2}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(1/(4*x^3 + x^6 + 3),x)
```

output
```
log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/18 - log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
12 - 1/12) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/18 - ((-1)^(
1/3)*log(x + ((-1)^(1/3)*3^(1/3))/2 + ((-1)^(1/3)*3^(5/6)*1i)/2)*(3^(1/3)
+ 3^(5/6)*1i))/36
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int \frac{1}{3+4x^3+x^6}\,dx = \frac{3^{\frac{5}{6}}\,atan\left(\frac{\left(3^{\frac{1}{3}}-2x\right)3^{\frac{1}{6}}}{3}\right)}{18} + \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} + \frac{3^{\frac{1}{3}}\log\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{36} - \frac{3^{\frac{1}{3}}\log\left(3^{\frac{1}{3}}+x\right)}{18} - \frac{\log(x^2-x+1)}{12} + \frac{\log(x+1)}{6}$$

input
```
int(1/(x^6+4*x^3+3),x)
```

output
```
(2*3**(2/3)*3**(1/6)*atan((3**(1/3) - 2*x)/(3**(2/3)*3**(1/6))) + 6*sqrt(3
)*atan((2*x - 1)/sqrt(3)) + 3**(1/3)*log(3**(2/3) - 3**(1/3)*x + x**2) - 2
*3**(1/3)*log(3**(1/3) + x) - 3*log(x**2 - x + 1) + 6*log(x + 1))/36
```

3.151 $\int \frac{1}{x^2(3+4x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 119

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = -\frac{1}{3x}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\ 3^{5/6}}+\frac{1}{6}\log(1+x) -\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\sqrt[3]{3}}$$

output
```
-1/3/x+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/18*3^(1/6)*arctan(1/3*(3^
(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/54*3^(2/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1
)+1/108*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = -\frac{36+6\sqrt[6]{3}x\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+18\sqrt{3}x\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-18x\log(1+x)+2\ 3^{2/3}x\log\left(3+3^{2/3}x\right)+9}{108x}$$

input
```
Integrate[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

output
```
-1/108*(36 + 6*3^(1/6)*x*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 18*Sqrt[3]*x*Ar
cTan[(-1 + 2*x)/Sqrt[3]] - 18*x*Log[1 + x] + 2*3^(2/3)*x*Log[3 + 3^(2/3)*x
] + 9*x*Log[1 - x + x^2] - 3^(2/3)*x*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/x
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {1704, 25, 1834, 821, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^6+4x^3+3)}dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{3}\int -\frac{x(x^3+4)}{x^6+4x^3+3}dx-\frac{1}{3x}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}\int \frac{x(x^3+4)}{x^6+4x^3+3}dx-\frac{1}{3x}$$

$$\Big\downarrow \text{1834}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{x}{x^3+3}dx-\frac{3}{2}\int \frac{x}{x^3+1}dx\right)-\frac{1}{3x}$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx\right)\right)-\frac{1}{3x}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)\right)-\frac{1}{3x}$$

$\big\downarrow$ 1142

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int -\right.\right.\right.$$

$$\frac{1}{3x}$$

$\big\downarrow$ 25

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2\ \cdots}\right.\right.\right.$$

$$\frac{1}{3x}$$

$\big\downarrow$ 1082

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\cdots\right.\right.\right.$$

$$\frac{1}{3x}$$

$\big\downarrow$ 217

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2\ \cdots}\right.\right.\right.$$

$$\frac{1}{3x}$$

$\big\downarrow$ 1083

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-3\int\frac{}{-\cdots}\right.\right.\right.$$

$$\frac{1}{3x}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\int\frac{}{x\cdots}\right.\right.\right.$$

$$\frac{1}{3x}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{1}{2}\mathrm{l}\cdots\right.\right.\right.$$

$$\frac{1}{3x}$$

input
```
Int[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

output
```
-1/3*1/x + ((-3*(-1/3*Log[1 + x] + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + L
og[1 - x + x^2]/2)/3))/2 + (-1/3*Log[3^(1/3) + x]/3^(1/3) + (-(Sqrt[3]*Arc
Tan[(1 - (2*x)/3^(1/3))/Sqrt[3]]) + Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3
^(1/3)))/2)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{1}{3x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^{3}+1\right)}\_R\ln\left(3\_R^{2}+x\right)\right)}{18}-\frac{\ln\left(4x^{2}-4x+4\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(x+1)}{6}$
default	$-\frac{\ln\left(x^{2}-x+1\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{54}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^{2}\right)}{108}+\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{18}+\ln($

input
```
int(1/x^2/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x+1/18*sum(_R*ln(3*_R^2+x),_R=RootOf(3*_Z^3+1))-1/12*ln(4*x^2-4*x+4)-
1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = \frac{3^{\frac{2}{3}}x\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-2\cdot 3^{\frac{2}{3}}x\log\left(x+3^{\frac{1}{3}}\right)-18\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6\cdot 3^{\frac{1}{6}}x\arctan\left(-\frac{1}{3}\right.}{108x}$$

input
```
integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/108*(3^(2/3)*x*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 2*3^(2/3)*x*log(x + 3^(1
/3)) - 18*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*x*arctan(-1/
3*3^(1/6)*(2*x - 3^(1/3))) - 9*x*log(x^2 - x + 1) + 18*x*log(x + 1) - 36)/
x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{1}{x^2(3+4x^3+x^6)}\,dx &= \frac{\log(x+1)}{6} \\ &+ \left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{8188128\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{41}+\frac{39384\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{41}\right) \\ &+ \left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{39384\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{41}-\frac{8188128\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{41}\right) \\ &+ \operatorname{RootSum}\left(17496t^3+1,\left(t\mapsto t\log\left(-\frac{8188128t^5}{41}+\frac{39384t^2}{41}+x\right)\right)\right)-\frac{1}{3x}\end{aligned}$$

input
```
integrate(1/x**2/(x**6+4*x**3+3),x)
```

output
```
log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 8188128*(-1/12 - sqrt(3)*I/1
2)**5/41 + 39384*(-1/12 - sqrt(3)*I/12)**2/41) + (-1/12 + sqrt(3)*I/12)*lo
g(x + 39384*(-1/12 + sqrt(3)*I/12)**2/41 - 8188128*(-1/12 + sqrt(3)*I/12)*
*5/41) + RootSum(17496*_t**3 + 1, Lambda(_t, _t*log(-8188128*_t**5/41 + 39
384*_t**2/41 + x))) - 1/(3*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = \frac{1}{108}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{18}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{3x}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(x + 3^(1/3
)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^(1/6)*arctan(1/3*3
^(1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = \frac{1}{108}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{18}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{3x}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(abs(x + 3^
(1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^(1/6)*arctan(
1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2 - x + 1) + 1/6*log(abs
(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = \frac{\ln(x+1)}{6} - \frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{54} + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \frac{1}{3\,x} - \frac{(-1)^{1/3}\ln\left(x-\frac{(-1)^{1/3}\,3^{1/3}}{2}-\frac{(-1)^{1/6}\,3^{5/6}}{2}+\frac{3^{1/3}}{2}\right)\left(3^{2/3}+3^{1/6}\,3\mathrm{i}\right)}{108} + \frac{(-1)^{1/3}\,3^{2/3}\ln\left(x+(-1)^{2/3}\,3^{1/3}\right)}{54}$$

input
```
int(1/(x^2*(4*x^3 + x^6 + 3)),x)
```

output
```
log(x + 1)/6 - (3^(2/3)*log(x + 3^(1/3)))/54 + log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
12 + 1/12) - 1/(3*x) - ((-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 - ((-1)^
(1/6)*3^(5/6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/108 + ((-1)^(1/3)*3^
(2/3)*log(x + (-1)^(2/3)*3^(1/3)))/54
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = \frac{\left(-2\sqrt{3}\,atan\left(\frac{\left(3^{\frac{1}{3}}-2x\right)3^{\frac{1}{6}}}{3}\right)x-6\sqrt{3}\,3^{\frac{1}{3}}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x-3\,3^{\frac{1}{3}}\log(x^2-x+1)\,x+6\,3^{\frac{1}{3}}\log(x+1)\,x-12\,3^{\frac{1}{3}}}{108x}$$

input
```
int(1/x^2/(x^6+4*x^3+3),x)
```

output
```
( - 2*3**(1/3)*3**(1/6)*atan((3**(1/3) - 2*x)/(3**(2/3)*3**(1/6)))*x - 6*s
qrt(3)*3**(1/3)*atan((2*x - 1)/sqrt(3))*x - 3*3**(1/3)*log(x**2 - x + 1)*x
 + 6*3**(1/3)*log(x + 1)*x - 12*3**(1/3) + log(3**(2/3) - 3**(1/3)*x + x**
2)*x - 2*log(3**(1/3) + x)*x)/(36*3**(1/3)*x)
```

3.152 $\int \frac{1}{x^3(3+4x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 119

$$\int \frac{1}{x^3(3+4x^3+x^6)}\,dx = -\frac{1}{6x^2}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{36\ 3^{2/3}}$$

output
```
-1/6/x^2+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/54*3^(5/6)*arctan(1/3*(
3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/54*3^(1/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x
+1)-1/108*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3(3+4x^3+x^6)}\,dx = \frac{1}{108}\left(-\frac{18}{x^2}\right.$$
$$-2\,3^{5/6}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-18\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-18\log(1+x)+2\sqrt[3]{3}\log\left(3+3^{2/3}x\right)+9\log\left(1-x\right.$$

input
```
Integrate[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

output
```
(-18/x^2 - 2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 18*Sqrt[3]*ArcTan[(
-1 + 2*x)/Sqrt[3]] - 18*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + 9*Log[
1 - x + x^2] - 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/108
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {1704, 27, 1752, 750, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(x^6+4x^3+3)}dx$$
$$\Big\downarrow \; 1704$$
$$\frac{1}{6}\int -\frac{2(x^3+4)}{x^6+4x^3+3}dx-\frac{1}{6x^2}$$
$$\Big\downarrow \; 27$$
$$-\frac{1}{3}\int \frac{x^3+4}{x^6+4x^3+3}dx-\frac{1}{6x^2}$$
$$\Big\downarrow \; 1752$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^3+3}dx-\frac{3}{2}\int \frac{1}{x^3+1}dx\right)-\frac{1}{6x^2}$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\int \frac{1}{x+1}dx\right)\right)-\frac{1}{6x^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\log(x+1)\right)\right)-\frac{1}{6x^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int -\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\right.\right.\right.$$

$$\frac{1}{6x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1}{x^2-}\right.\right.\right.$$

$$\frac{1}{6x^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\right.\right.\right.$$

$$\frac{1}{6x^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx+\frac{1}{2}\int\frac{1-}{x^2-}\right.\right.\right.$$

$$\frac{1}{6x^2}$$

$\Big\downarrow$ 1083

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-3\int\frac{}{-(2x}\right.\right.\right.$$

$$\frac{1}{6x^2}$$

$\Big\downarrow$ 217

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx+\sqrt{3}\arctan\right.\right.\right.$$

$$\frac{1}{6x^2}$$

$\Big\downarrow$ 1103

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{3\ 3^{2/3}}+\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)-\frac{3}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\right.\right.\right.$$

$$\frac{1}{6x^2}$$

input
```
Int[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

output
```
-1/6*1/x^2 + ((-3*(Log[1 + x]/3 + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - Lo
g[1 - x + x^2]/2)/3))/2 + (Log[3^(1/3) + x]/(3*3^(2/3)) + (-(Sqrt[3]*ArcTa
n[(1 - (2*x)/3^(1/3))/Sqrt[3]]) - Log[3^(2/3) - 3^(1/3)*x + x^2]/2)/(3*3^(
2/3)))/2)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

method	result
risch	$-\frac{1}{6x^{2}}-\frac{\ln\left(x+1\right)}{6}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^{3}-1\right)}\_R\ln\left(x+3\_R\right)\right)}{18}+\frac{\ln\left(x^{2}-x+1\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$
default	$\frac{\ln\left(x^{2}-x+1\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{54}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^{2}\right)}{108}+\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{54}-\frac{\ln(x-}{6}$

input
```
int(1/x^3/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^2-1/6*ln(x+1)+1/18*sum(_R*ln(x+3*_R),_R=RootOf(9*_Z^3-1))+1/12*ln(x
^2-x+1)-1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\left(3+4x^3+x^6\right)}\,dx = \frac{18\cdot 9^{\frac{1}{6}}\sqrt{\frac{1}{3}}x^2\arctan\left(\frac{1}{9}\cdot 9^{\frac{1}{6}}\sqrt{\frac{1}{3}}\left(2\cdot 9^{\frac{2}{3}}x-3\cdot 9^{\frac{1}{3}}\right)\right)-9^{\frac{2}{3}}x^2\log\left(3\,x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+2\cdot 9^{\frac{2}{3}}x^2\log\left(3\,x+\cdots\right)}{324\,x^2}$$

input
```
integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
1/324*(18*9^(1/6)*sqrt(1/3)*x^2*arctan(1/9*9^(1/6)*sqrt(1/3)*(2*9^(2/3)*x
- 3*9^(1/3))) - 9^(2/3)*x^2*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 2*9^(2/3)
*x^2*log(3*x + 9^(2/3)) - 54*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x - 1)) + 2
7*x^2*log(x^2 - x + 1) - 54*x^2*log(x + 1) - 54)/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3(3+4x^3+x^6)}\,dx = -\frac{\log(x+1)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{1093}{244} - \frac{1093\sqrt{3}i}{244} + \frac{787320\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^4}{61}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{1093}{244} + \frac{787320\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^4}{61} + \frac{1093\sqrt{3}i}{244}\right) + \operatorname{RootSum}\left(52488t^3 - 1, \left(t \mapsto t\log\left(\frac{787320t^4}{61} + \frac{3279t}{61} + x\right)\right)\right) - \frac{1}{6x^2}$$

input
```
integrate(1/x**3/(x**6+4*x**3+3),x)
```

output
```
-log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 1093/244 - 1093*sqrt(3)*I/24
4 + 787320*(1/12 - sqrt(3)*I/12)**4/61) + (1/12 + sqrt(3)*I/12)*log(x + 10
93/244 + 787320*(1/12 + sqrt(3)*I/12)**4/61 + 1093*sqrt(3)*I/244) + RootSu
m(52488*_t**3 - 1, Lambda(_t, _t*log(787320*_t**4/61 + 3279*_t/61 + x))) -
 1/(6*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3(3+4x^3+x^6)}\,dx = \frac{1}{54}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{1}{108}\cdot 3^{\frac{1}{3}}\log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{54}\cdot 3^{\frac{1}{3}}\log\left(x + 3^{\frac{1}{3}}\right) - \frac{1}{6\,x^2} + \frac{1}{12}\log\left(x^2 - x + 1\right) - \frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3
^(1/3)*log(x + 3^(1/3)) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3 (3 + 4x^3 + x^6)} \, dx = \frac{1}{54} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2\,x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x - 1)\right) - \frac{1}{108} \cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{54} \cdot 3^{\frac{1}{3}} \log\left(\left|x + 3^{\frac{1}{3}}\right|\right) - \frac{1}{6\,x^2} + \frac{1}{12}\,\log\left(x^2 - x + 1\right) - \frac{1}{6}\,\log\left(|x + 1|\right)$$

input
```
integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3
^(1/3)*log(abs(x + 3^(1/3))) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(a
bs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 19.77 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3 (3+4x^3+x^6)}\,dx = \frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{54} - \frac{\ln(x+1)}{6} + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \frac{1}{6\,x^2} - \ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{108}+\frac{3^{5/6}\,1\mathrm{i}}{108}\right) - \ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{108}-\frac{3^{5/6}\,1\mathrm{i}}{108}\right)$$

input
```
int(1/(x^3*(4*x^3 + x^6 + 3)),x)
```

output
```
(3^(1/3)*log(x + 3^(1/3)))/54 - log(x + 1)/6 + log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
12 - 1/12) - 1/(6*x^2) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(1/3)/108
+ (3^(5/6)*1i)/108) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/108 - (
3^(5/6)*1i)/108)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^3 (3+4x^3+x^6)}\,dx = \frac{\left(-2\sqrt{3}\,atan\left(\frac{\left(3^{\frac{1}{3}}-2x\right)3^{\frac{1}{6}}}{3}\right)x^2 - 6\sqrt{3}\,3^{\frac{2}{3}}atan\left(\frac{2x-1}{\sqrt{3}}\right)x^2 + 3\,3^{\frac{2}{3}}\log(x^2-x+1)\,x^2 - 6\,3^{\frac{2}{3}}\log(x+1)\,x^2 - 6}{108x^2}$$

input
```
int(1/x^3/(x^6+4*x^3+3),x)
```

output
```
( - 2*3**(1/3)*3**(1/6)*atan((3**(1/3) - 2*x)/(3**(2/3)*3**(1/6)))*x**2 -
6*sqrt(3)*3**(2/3)*atan((2*x - 1)/sqrt(3))*x**2 + 3*3**(2/3)*log(x**2 - x
+ 1)*x**2 - 6*3**(2/3)*log(x + 1)*x**2 - 6*3**(2/3) - log(3**(2/3) - 3**(1
/3)*x + x**2)*x**2 + 2*log(3**(1/3) + x)*x**2)/(36*3**(2/3)*x**2)
```

3.153 $\int \frac{1}{x^5(3+4x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{1}{x^5(3+4x^3+x^6)}\,dx = -\frac{1}{12x^4}+\frac{4}{9x}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\ 3^{5/6}}-\frac{1}{6}\log(1+x) +\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{3}}$$

output

```
-1/12/x^4+4/9/x-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/54*3^(1/6)*arcta
n(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/162*3^(2/3)*ln(3^(1/3)+x)+1/12*
ln(x^2-x+1)-1/324*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5(3+4x^3+x^6)}\,dx = \frac{1}{324}\left(-\frac{27}{x^4}+\frac{144}{x} + 6\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+54\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-54\log(1+x) + 2\,3^{2/3}\log\left(3+3^{2/3}x\right)+27\log\left(1-x+x^2\right)-3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

input
```
Integrate[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

output
```
(-27/x^4 + 144/x + 6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 54*Sqrt[3]*
ArcTan[(-1 + 2*x)/Sqrt[3]] - 54*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x]
+ 27*Log[1 - x + x^2] - 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/324
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {1704, 27, 1828, 1834, 821, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5(x^6+4x^3+3)}\,dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{12}\int -\frac{4(x^3+4)}{x^2(x^6+4x^3+3)}dx-\frac{1}{12x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\int \frac{x^3+4}{x^2\left(x^6+4x^3+3\right)}dx-\frac{1}{12x^4}$$

$\downarrow$ 1828

$$\frac{1}{3}\left(\frac{1}{3}\int \frac{x\left(4x^3+13\right)}{x^6+4x^3+3}dx+\frac{4}{3x}\right)-\frac{1}{12x^4}$$

$\downarrow$ 1834

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\int \frac{x}{x^3+1}dx-\frac{1}{2}\int \frac{x}{x^3+3}dx\right)+\frac{4}{3x}\right)-\frac{1}{12x^4}$$

$\downarrow$ 821

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx\right)+\frac{1}{2}\left(\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\sqrt[3]{3}}-\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}\right)\right)+\frac{4}{3x}\right)-\frac{1}{12x^4}$$

$\downarrow$ 16

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\int \frac{x+\sqrt[3]{3}}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\sqrt[3]{3}}\right)\right)+\frac{4}{3x}\right)-\frac{1}{12x^4}$$

$\downarrow$ 1142

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{}{x^2-\sqrt[3]{3}}}{}\right.\right.\right.$$

$$\frac{1}{12x^4}$$

$\downarrow$ 25

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\frac{3}{2}\sqrt[3]{3}\int\frac{1}{x^2-\sqrt[3]{3}x}\right.\right.\right.\right.$$

$$\frac{1}{12x^4}$$

$$\downarrow 1082$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{3\int\frac{1}{-\left(1-\frac{2x}{\sqrt[3]{3}}\right)^2}}{}\right.\right.\right.$$

$$\frac{1}{12x^4}$$

$$\downarrow 217$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}\right)+\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\right.\right.\right.\right.$$

$$\frac{1}{12x^4}$$

$$\downarrow 1083$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\sqrt[3]{3}}\right)+\frac{9}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-3\int\right.\right.\right.\right.$$

$$\frac{1}{12x^4}$$

$$\downarrow 217$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{-\frac{1}{2}\int\frac{\sqrt[3]{3}}{x^2-\sqrt[3]{3}}}{}\right.\right.\right.$$

$$\frac{1}{12x^4}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2-x+1\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\frac{1}{2}\log\left(x^2-\sqrt{}\right.}{}\right.\right.\right.$$

$$\frac{1}{12x^4}$$

input
```
Int[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

output
```
-1/12*1/x^4 + (4/(3*x) + ((9*(-1/3*Log[1 + x] + (Sqrt[3]*ArcTan[(-1 + 2*x)
/Sqrt[3]] + Log[1 - x + x^2]/2)/3))/2 + (Log[3^(1/3) + x]/(3*3^(1/3)) - (-
(Sqrt[3]*ArcTan[(1 - (2*x)/3^(1/3))/Sqrt[3]]) + Log[3^(2/3) - 3^(1/3)*x +
x^2]/2)/(3*3^(1/3)))/2)/3)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

method	result
risch	$\frac{\frac{4x^3}{9}-\frac{1}{12}}{x^4}+\frac{\ln(4x^2-4x+4)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x+1)}{6}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^3-1\right)}\_R\ln\left(3\_R^2+x\right)\right)}{54}$
default	$\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{162}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{324}-\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{54}-\frac{\ln(x-}{6}$

input
```
int(1/x^5/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
(4/9*x^3-1/12)/x^4+1/12*ln(4*x^2-4*x+4)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(
1/2))-1/6*ln(x+1)+1/54*sum(_R*ln(3*_R^2+x),_R=RootOf(3*_Z^3-1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5(3+4x^3+x^6)}\,dx = -\frac{3^{\frac{2}{3}}x^4\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-2\cdot 3^{\frac{2}{3}}x^4\log\left(x+3^{\frac{1}{3}}\right)-54\sqrt{3}x^4\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6\cdot 3^{\frac{1}{6}}x^4\operatorname{arcta}}{324x^4}$$

input
```
integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
-1/324*(3^(2/3)*x^4*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 2*3^(2/3)*x^4*log(x +
 3^(1/3)) - 54*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*x^4*a
rctan(-1/3*3^(1/6)*(2*x - 3^(1/3))) - 27*x^4*log(x^2 - x + 1) + 54*x^4*log
(x + 1) - 144*x^3 + 27)/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{1}{x^5(3+4x^3+x^6)}\,dx &= -\frac{\log(x+1)}{6} \\ &+ \left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{4782978\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{547}+\frac{1028869776\left(\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{547}\right) \\ &+ \left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{1028869776\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{547}+\frac{4782978\left(\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{547}\right) \\ &+ \operatorname{RootSum}\left(472392t^3-1,\left(t\mapsto t\log\left(\frac{1028869776t^5}{547}+\frac{4782978t^2}{547}+x\right)\right)\right) \\ &+ \frac{16x^3-3}{36x^4}\end{aligned}$$

input
```
integrate(1/x**5/(x**6+4*x**3+3),x)
```

output
```
-log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 4782978*(1/12 - sqrt(3)*I/12
)**2/547 + 1028869776*(1/12 - sqrt(3)*I/12)**5/547) + (1/12 + sqrt(3)*I/12
)*log(x + 1028869776*(1/12 + sqrt(3)*I/12)**5/547 + 4782978*(1/12 + sqrt(3
)*I/12)**2/547) + RootSum(472392*_t**3 - 1, Lambda(_t, _t*log(1028869776*_
t**5/547 + 4782978*_t**2/547 + x))) + (16*x**3 - 3)/(36*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^5(3+4x^3+x^6)}\,dx = -\frac{1}{324}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{162}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{54}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{16x^3-3}{36x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(x + 3^(1
/3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arctan(1/3
*3^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x + 1)
- 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^5(3+4x^3+x^6)}\,dx = -\frac{1}{324}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{162}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{54}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{16x^3-3}{36x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(abs(x +
3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arcta
n(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x
+ 1) - 1/6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 19.93 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^5(3+4x^3+x^6)}\,dx = \frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{162}-\frac{\ln(x+1)}{6}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{\frac{4x^3}{9}-\frac{1}{12}}{x^4}-\ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{324}-\frac{3^{1/6}\,1\mathrm{i}}{108}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{324}+\frac{3^{1/6}\,1\mathrm{i}}{108}\right)$$

input
```
int(1/(x^5*(4*x^3 + x^6 + 3)),x)
```

output
```
(3^(2/3)*log(x + 3^(1/3)))/162 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/12 + 1/12) + ((4*x^3)/9 - 1/12)/x^4 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)
*(3^(2/3)/324 - (3^(1/6)*1i)/108) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3
^(2/3)/324 + (3^(1/6)*1i)/108)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^5 (3 + 4x^3 + x^6)} \, dx$$
$$= \frac{\left(2\sqrt{3}\, atan\left(\frac{\left(3^{\frac{1}{3}} - 2x\right) 3^{\frac{1}{6}}}{3}\right) x^4 + 18\sqrt{3}\, 3^{\frac{1}{3}} atan\left(\frac{2x-1}{\sqrt{3}}\right) x^4 + 9\, 3^{\frac{1}{3}} \log(x^2 - x + 1)\, x^4 - 18\, 3^{\frac{1}{3}} \log(x+1)\, x^4 + 4}{324x^4}$$

input
```
int(1/x^5/(x^6+4*x^3+3),x)
```

output
```
(2*3**(1/3)*3**(1/6)*atan((3**(1/3) - 2*x)/(3**(2/3)*3**(1/6)))*x**4 + 18*
sqrt(3)*3**(1/3)*atan((2*x - 1)/sqrt(3))*x**4 + 9*3**(1/3)*log(x**2 - x +
1)*x**4 - 18*3**(1/3)*log(x + 1)*x**4 + 48*3**(1/3)*x**3 - 9*3**(1/3) - lo
g(3**(2/3) - 3**(1/3)*x + x**2)*x**4 + 2*log(3**(1/3) + x)*x**4)/(108*3**(
1/3)*x**4)
```

3.154 $\int \frac{1}{x^6(3+4x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{1}{x^6\left(3+4x^3+x^6\right)}\,dx=-\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{108\ 3^{2/3}}$$

output
```
-1/15/x^5+2/9/x^2-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/162*3^(5/6)*ar
ctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/162*3^(1/3)*ln(3^(1/3)+x)-1/
12*ln(x^2-x+1)+1/324*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^6 (3 + 4x^3 + x^6)} dx$$
$$= \frac{-\frac{108}{x^5} + \frac{360}{x^2} + 10\, 3^{5/6} \arctan\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + 270\sqrt{3} \arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 270 \log(1+x) - 10\sqrt[3]{3} \log\left(3 + 3^{2/3}x\right) \cdots}{1620}$$

input
```
Integrate[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

output
```
(-108/x^5 + 360/x^2 + 10*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 270*Sqr
t[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 270*Log[1 + x] - 10*3^(1/3)*Log[3 + 3^(2
/3)*x] - 135*Log[1 - x + x^2] + 5*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2]
)/1620
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {1704, 27, 1828, 27, 1752, 750, 16, 1142, 25, 1082, 217, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (x^6 + 4x^3 + 3)} dx$$

$$\Big\downarrow \text{ 1704}$$

$$\frac{1}{15} \int -\frac{5(x^3+4)}{x^3 (x^6 + 4x^3 + 3)} dx - \frac{1}{15x^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{3} \int \frac{x^3+4}{x^3 (x^6 + 4x^3 + 3)} dx - \frac{1}{15x^5}$$

$$\Big\downarrow \text{ 1828}$$

$$\frac{1}{3}\left(\frac{1}{6}\int \frac{2\left(4x^3+13\right)}{x^6+4x^3+3}dx+\frac{2}{3x^2}\right)-\frac{1}{15x^5}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{1}{3}\left(\frac{1}{3}\int \frac{4x^3+13}{x^6+4x^3+3}dx+\frac{2}{3x^2}\right)-\frac{1}{15x^5}$$

$$\Big\downarrow \ \textbf{1752}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\int \frac{1}{x^3+1}dx-\frac{1}{2}\int \frac{1}{x^3+3}dx\right)+\frac{2}{3x^2}\right)-\frac{1}{15x^5}$$

$$\Big\downarrow \ \textbf{750}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\int \frac{1}{x+1}dx\right)+\frac{1}{2}\left(-\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}-\frac{\int \frac{1}{x+\sqrt[3]{3}}dx}{3\ 3^{2/3}}\right)\right)+\frac{2}{3x^2}\right)-\frac{1}{15x^5}$$

$$\Big\downarrow \ \textbf{16}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\int \frac{2\sqrt[3]{3}-x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)\right)+\frac{2}{3x^2}\right)-\frac{1}{15x^5}$$

$$\Big\downarrow \ \textbf{1142}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{3}{2}\sqrt[3]{3}\int \frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\frac{1}{2}\int}{3\ 3^{2/3}}\right.\right.\right.$$

$$\frac{1}{15x^5}$$

$$\Big\downarrow \ \textbf{25}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx+\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{3}{2}\sqrt[3]{3}\int\frac{1}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+\frac{1}{2}\int\cdots}{3\ 3^{2/3}}\cdots\right.\right.\right.$$

$$\frac{1}{15x^5}$$

$\big\downarrow$ 1082

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx+\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx+3\int\frac{\cdots}{-\left(1\cdots\right.}}{3\ 3^{2/3}}\cdots\right.\right.\right.$$

$$\frac{1}{15x^5}$$

$\big\downarrow$ 217

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{2}\left(-\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{9}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx+\frac{1}{2}\int\cdots\right.\right.\right.\right.$$

$$\frac{1}{15x^5}$$

$\big\downarrow$ 1083

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{2}\left(-\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)}{3\ 3^{2/3}}-\frac{\log\left(x+\sqrt[3]{3}\right)}{3\ 3^{2/3}}\right)+\frac{9}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-3\int\cdots\right.\right.\right.\right.$$

$$\frac{1}{15x^5}$$

$\big\downarrow$ 217

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx+\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\int\frac{\sqrt[3]{3}-2x}{x^2-\sqrt[3]{3}x+3^{2/3}}dx-\sqrt{3}\text{ar}}{3\ 3^{2/3}}\right.\right.\right.$$

$$\frac{1}{15x^5}$$

$\downarrow$ 1103

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{9}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{3}\log(x+1)\right)+\frac{1}{2}\left(-\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{3}}}{\sqrt{3}}\right)-\frac{1}{2}\text{l}}{3\ 3^{2/3}}\right.\right.\right.$$

$$\frac{1}{15x^5}$$

input
```
Int[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

output
```
-1/15*1/x^5 + (2/(3*x^2) + ((9*(Log[1 + x]/3 + (Sqrt[3]*ArcTan[(-1 + 2*x)/
Sqrt[3]] - Log[1 - x + x^2]/2)/3))/2 + (-1/3*Log[3^(1/3) + x]/3^(2/3) - (-
(Sqrt[3]*ArcTan[(1 - (2*x)/3^(1/3))/Sqrt[3]]) - Log[3^(2/3) - 3^(1/3)*x +
x^2]/2)/(3*3^(2/3)))/2)/3)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52

method	result
risch	$\frac{\frac{2x^3}{9}-\frac{1}{15}}{x^5}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^3+1\right)}\_R\ln\left(x-3\_R\right)\right)}{54}-\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\ln\left(x+1\right)}{6}$
default	$-\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{162}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{324}-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{162}+\ln($

input
```
int(1/x^6/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

output
```
(2/9*x^3-1/15)/x^5+1/54*sum(_R*ln(x-3*_R),_R=RootOf(9*_Z^3+1))-1/12*ln(x^2
-x+1)+1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))+1/6*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^6 (3+4x^3+x^6)} dx = -\frac{90 \cdot 9^{\frac{1}{6}} \sqrt{\frac{1}{3}} x^5 \arctan\left(\frac{1}{9} \cdot 9^{\frac{1}{6}} \sqrt{\frac{1}{3}} \left(2 \cdot 9^{\frac{2}{3}} x - 3 \cdot 9^{\frac{1}{3}}\right)\right) - 5 \cdot 9^{\frac{2}{3}} x^5 \log\left(3 x^2 - 9^{\frac{2}{3}} x + 3 \cdot 9^{\frac{1}{3}}\right) + 10 \cdot 9^{\frac{2}{3}} x^5 \log}{}$$

input
```
integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

output
```
-1/4860*(90*9^(1/6)*sqrt(1/3)*x^5*arctan(1/9*9^(1/6)*sqrt(1/3)*(2*9^(2/3)*
x - 3*9^(1/3))) - 5*9^(2/3)*x^5*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 10*9^
(2/3)*x^5*log(3*x + 9^(2/3)) - 810*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x - 1
)) + 405*x^5*log(x^2 - x + 1) - 810*x^5*log(x + 1) - 1080*x^3 + 324)/x^5
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08

$$\begin{aligned}
\int \frac{1}{x^6 (3+4x^3+x^6)} dx &= \frac{\log(x+1)}{6} \\
&+ \left(-\frac{1}{12} + \frac{\sqrt{3} i}{12}\right) \log\left(x + \frac{88573}{6562} - \frac{88573\sqrt{3} i}{6562} + \frac{119042784\left(-\frac{1}{12} + \frac{\sqrt{3} i}{12}\right)^4}{3281}\right) \\
&+ \left(-\frac{1}{12} - \frac{\sqrt{3} i}{12}\right) \log\left(x + \frac{88573}{6562} + \frac{119042784\left(-\frac{1}{12} - \frac{\sqrt{3} i}{12}\right)^4}{3281} + \frac{88573\sqrt{3} i}{6562}\right) \\
&+ \operatorname{RootSum}\left(1417176t^3 + 1, \left(t \mapsto t \log\left(\frac{119042784t^4}{3281} - \frac{531438t}{3281} + x\right)\right)\right) \\
&+ \frac{10x^3 - 3}{45x^5}
\end{aligned}$$

input
```
integrate(1/x**6/(x**6+4*x**3+3),x)
```

output
```
log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 88573/6562 - 88573*sqrt(3)*I
/6562 + 119042784*(-1/12 + sqrt(3)*I/12)**4/3281) + (-1/12 - sqrt(3)*I/12)
*log(x + 88573/6562 + 119042784*(-1/12 - sqrt(3)*I/12)**4/3281 + 88573*sqr
t(3)*I/6562) + RootSum(1417176*_t**3 + 1, Lambda(_t, _t*log(119042784*_t**
4/3281 - 531438*_t/3281 + x))) + (10*x**3 - 3)/(45*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^6\left(3+4x^3+x^6\right)}\,dx = -\frac{1}{162}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{324}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) - \frac{1}{162}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) + \frac{10\,x^3-3}{45\,x^5} - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log\left(x+1\right)$$

input
```
integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

output
```
-1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/16
2*3^(1/3)*log(x + 3^(1/3)) + 1/45*(10*x^3 - 3)/x^5 - 1/12*log(x^2 - x + 1)
 + 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^6(3+4x^3+x^6)}\,dx = -\frac{1}{162}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{324}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) - \frac{1}{162}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right) + \frac{10x^3-3}{45x^5} - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log\left(|x+1|\right)$$

input
```
integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

output
```
-1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/16
2*3^(1/3)*log(abs(x + 3^(1/3))) + 1/45*(10*x^3 - 3)/x^5 - 1/12*log(x^2 - x
 + 1) + 1/6*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 19.78 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^6(3+4x^3+x^6)}\,dx = \frac{\ln(x+1)}{6} - \frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{162} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \frac{\frac{2x^3}{9}-\frac{1}{15}}{x^5} + \frac{(-1)^{1/3}\,3^{1/3}\ln\left(x-(-1)^{1/3}\,3^{1/3}\right)}{162} - \frac{(-1)^{1/3}\ln\left(x+\frac{(-1)^{1/3}\,3^{1/3}}{2}+\frac{(-1)^{1/3}\,3^{5/6}\,1\mathrm{i}}{2}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{324}$$

input
```
int(1/(x^6*(4*x^3 + x^6 + 3)),x)
```

output
```
log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/162 - log(x - (3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/12 - 1/12) + ((2*x^3)/9 - 1/15)/x^5 + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1
/3)*3^(1/3)))/162 - ((-1)^(1/3)*log(x + ((-1)^(1/3)*3^(1/3))/2 + ((-1)^(1/
3)*3^(5/6)*1i)/2)*(3^(1/3) + 3^(5/6)*1i))/324
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^6(3+4x^3+x^6)}\,dx$$
$$= \frac{\left(10\sqrt{3}\,atan\left(\frac{\left(3^{\frac{1}{3}}-2x\right)3^{\frac{1}{6}}}{3}\right)x^5+90\sqrt{3}\,3^{\frac{2}{3}}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^5-45\,3^{\frac{2}{3}}\log(x^2-x+1)\,x^5+90\,3^{\frac{2}{3}}\log(x+1)\,x^5 -}{1620x^5}$$

input
```
int(1/x^6/(x^6+4*x^3+3),x)
```

output
```
(10*3**(1/3)*3**(1/6)*atan((3**(1/3) - 2*x)/(3**(2/3)*3**(1/6)))*x**5 + 90
*sqrt(3)*3**(2/3)*atan((2*x - 1)/sqrt(3))*x**5 - 45*3**(2/3)*log(x**2 - x
+ 1)*x**5 + 90*3**(2/3)*log(x + 1)*x**5 + 120*3**(2/3)*x**3 - 36*3**(2/3)
+ 5*log(3**(2/3) - 3**(1/3)*x + x**2)*x**5 - 10*log(3**(1/3) + x)*x**5)/(5
40*3**(2/3)*x**5)
```

3.155 $\int \frac{x^6}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 412

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x+\frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

output
```
x+1/6*(I-3^(1/2))*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*3^(1/2))*2^
(2/3)/(1-I*3^(1/2))^(2/3)-1/6*(3^(1/2)+I)*arctan(1/3*(1+2*x/(1/2+1/2*I*3^(
1/2))^(1/3))*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/18*(3-I*3^(1/2))*ln((1
-I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/18*(3+I*3^(1/2)
)*ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/36*(3-I*
3^(1/2))*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(2/
3)/(1-I*3^(1/2))^(2/3)-1/36*(3+I*3^(1/2))*ln((1+I*3^(1/2))^(2/3)+(2+2*I*3^
(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.14

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x+\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1^2+2\#1^5}\&\right]$$

input
```
Integrate[x^6/(1 - x^3 + x^6),x]
```

output
```
x + RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 +
 2*#1^5) & ]/3
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1703, 1752, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{1703}$$

$$x-\int \frac{1-x^3}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{1752}$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx+\frac{1}{6}\left(3-i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx+x$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)+x$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)+x$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)+x$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int \text{—} \; x^2+}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int \text{—} \; x^2+}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$x$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$x$$

$$\downarrow 217$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{\cdots}{\sqrt[3]{\frac{1}{2}\left(1\cdots\right.}}}{\sqrt{\cdots}}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{\cdots}{\sqrt[3]{\frac{1}{2}\left(1\cdots\right.}}}{\sqrt{\cdots}}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

x

$\Big\downarrow$ 1103

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right.}\cdots\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right.}\cdots\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

x

input
```
Int[x^6/(1 - x^3 + x^6),x]
```

output
```
x + ((3 - I*Sqrt[3])*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 - I*Sq
rt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/S
qrt[3]] + Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3
)*x^2]/2)/(3*((1 - I*Sqrt[3])/2)^(2/3))))/6 + ((3 + I*Sqrt[3])*(Log[(1 + I
*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcT
an[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sqrt[3])^(2
/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[3])/2)
^(2/3))))/6
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.11

method	result	size
default	$x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(\_R^{3}-1\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44
risch	$x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(\_R^{3}-1\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44

input
```
int(x^6/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
x+1/3*sum((_R^3-1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.62

$$\int \frac{x^6}{1-x^3+x^6}\,dx = -\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}(\sqrt{-3}+1)\log\left(-3\left(\sqrt{-\frac{1}{3}}(\sqrt{-3}+1)-\sqrt{-3}-1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+4x\right) + \frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}(\sqrt{-3}-1)\log\left(3\left(\sqrt{-\frac{1}{3}}(\sqrt{-3}-1)-\sqrt{-3}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+4x\right) - \frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}(\sqrt{-3}+1)\log\left(3\left(\sqrt{-\frac{1}{3}}(\sqrt{-3}+1)+\sqrt{-3}+1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+4x\right) + \frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}(\sqrt{-3}-1)\log\left(-3\left(\sqrt{-\frac{1}{3}}(\sqrt{-3}-1)+\sqrt{-3}-1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+4x\right) + \frac{1}{3}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(3\left(\sqrt{-\frac{1}{3}}-1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+2x\right) + \frac{1}{3}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(-3\left(\sqrt{-\frac{1}{3}}+1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+2x\right)+x$$

input
```
integrate(x^6/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/6*(1/6*sqrt(-1/3) + 1/6)^(1/3)*(sqrt(-3) + 1)*log(-3*(sqrt(-1/3)*(sqrt(
-3) + 1) - sqrt(-3) - 1)*(1/6*sqrt(-1/3) + 1/6)^(1/3) + 4*x) + 1/6*(1/6*sq
rt(-1/3) + 1/6)^(1/3)*(sqrt(-3) - 1)*log(3*(sqrt(-1/3)*(sqrt(-3) - 1) - sq
rt(-3) + 1)*(1/6*sqrt(-1/3) + 1/6)^(1/3) + 4*x) - 1/6*(-1/6*sqrt(-1/3) + 1
/6)^(1/3)*(sqrt(-3) + 1)*log(3*(sqrt(-1/3)*(sqrt(-3) + 1) + sqrt(-3) + 1)*
(-1/6*sqrt(-1/3) + 1/6)^(1/3) + 4*x) + 1/6*(-1/6*sqrt(-1/3) + 1/6)^(1/3)*(
sqrt(-3) - 1)*log(-3*(sqrt(-1/3)*(sqrt(-3) - 1) + sqrt(-3) - 1)*(-1/6*sqrt
(-1/3) + 1/6)^(1/3) + 4*x) + 1/3*(1/6*sqrt(-1/3) + 1/6)^(1/3)*log(3*(sqrt(
-1/3) - 1)*(1/6*sqrt(-1/3) + 1/6)^(1/3) + 2*x) + 1/3*(-1/6*sqrt(-1/3) + 1/
6)^(1/3)*log(-3*(sqrt(-1/3) + 1)*(-1/6*sqrt(-1/3) + 1/6)^(1/3) + 2*x) + x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{x^6}{1-x^3+x^6} \, dx = x + \mathrm{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log \left(729t^4 - 9t + x\right)\right)\right)$$

input
```
integrate(x**6/(x**6-x**3+1),x)
```

output
```
x + RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_
t + x)))
```

Maxima [F]

$$\int \frac{x^6}{1-x^3+x^6} \, dx = \int \frac{x^6}{x^6 - x^3 + 1} \, dx$$

input
```
integrate(x^6/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
x + integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(256) = 512.

Time = 0.15 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.56

$$\int \frac{x^6}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt
(3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi
)^3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*
cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos
(2/9*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4
 + 4*cos(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*c
os(2/9*pi) + 2*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x
)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sq
rt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^
3*sin(1/9*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*si
n(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3
) + 1/2)*sin(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqr
t(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*p
i)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) - 2*cos(4/9*pi))*log((-I*sqrt
(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(2/9*pi)^
3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*co
s(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(
2/9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(4
*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 +
 cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 - 2*sqrt...
```

Mupad [B] (verification not implemented)

Time = 19.86 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x+\frac{\ln\left(x+\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+\frac{\ln\left(x-\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^3}{16}-27\right)}{108}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^3}{16}+27\right)}{108}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(x^6/(x^6 - x^3 + 1),x)
```

output
```
x + (log(x + (((3^(1/2)*9i)/2 - 27/2)*(3^(1/2)*12i + 36)^(1/3))/54)*(3^(1/
2)*12i + 36)^(1/3))/18 + (log(x - (((3^(1/2)*9i)/2 + 27/2)*(36 - 3^(1/2)*1
2i)^(1/3))/54)*(36 - 3^(1/2)*12i)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*(3
 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3)
- 3^(5/6)*1i)^3)/16 - 27))/108)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*
1i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5
/6)*1i)*((3*(3^(1/2)*1i + 3)*(3^(1/3) + 3^(5/6)*1i)^3)/16 + 27))/108)*(3^(
1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*
3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^
(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1
i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

Reduce [F]

$$\int \frac{x^6}{1-x^3+x^6}\,dx=\int \frac{x^3}{x^6-x^3+1}dx-\left(\int \frac{1}{x^6-x^3+1}dx\right)+x$$

input
```
int(x^6/(x^6-x^3+1),x)
```

output
```
int(x**3/(x**6 - x**3 + 1),x) - int(1/(x**6 - x**3 + 1),x) + x
```

3.156 $\int \frac{x^5}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{x^5}{1-x^3+x^6} \, dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)+1/6*ln(x^6-x^3+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{1-x^3+x^6} \, dx = \frac{\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{6}\log\left(1-x^3+x^6\right)$$

input
```
Integrate[x^5/(1 - x^3 + x^6),x]
```

output
```
ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1693, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^6-x^3+1} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3} \int \frac{x^3}{x^6-x^3+1}dx^3$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3} \left(\frac{1}{2} \int \frac{1}{x^6-x^3+1}dx^3+\frac{1}{2} \int -\frac{1-2 x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3} \left(\frac{1}{2} \int \frac{1}{x^6-x^3+1}dx^3-\frac{1}{2} \int \frac{1-2 x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3} \left(-\int \frac{1}{-x^6-3}d\left(2 x^3-1\right)-\frac{1}{2} \int \frac{1-2 x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3} \left(\frac{\arctan \left(\frac{2 x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2} \int \frac{1-2 x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3} \left(\frac{\arctan \left(\frac{2 x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2} \log \left(x^6-x^3+1\right)\right)$$

input

```
Int[x^5/(1 - x^3 + x^6),x]
```

output
```
(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/2)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	33
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}+\frac{\ln(4x^6-4x^3+4)}{6}$	35

input
```
int(x^5/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^5/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{1-x^3+x^6}\,dx = \frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**5/(x**6-x**3+1),x)
```

output
```
log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^5/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^5/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{1-x^3+x^6}\,dx = \frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

input
```
int(x^5/(x^6 - x^3 + 1),x)
```

output
```
log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

Reduce [F]

$$\int \frac{x^5}{1-x^3+x^6}\,dx=\frac{\left(\int \frac{x^2}{x^6-x^3+1}dx\right)}{2}+\frac{\log(x^6-x^3+1)}{6}$$

input
```
int(x^5/(x^6-x^3+1),x)
```

output
```
(3*int(x**2/(x**6 - x**3 + 1),x) + log(x**6 - x**3 + 1))/6
```

3.157 $\int \frac{x^4}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 411

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

output
```
1/6*(3^(1/2)+I)*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*3^(1/2))*2^(1
/3)/(1-I*3^(1/2))^(1/3)-1/6*(I-3^(1/2))*arctan(1/3*(1+2*x/(1/2+1/2*I*3^(1/
2))^(1/3))*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/18*(3+I*3^(1/2))*ln((1-I
*3^(1/2))^(1/3)-2^(1/3)*x)*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/18*(3-I*3^(1/2))*
ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/36*(3+I*3^
(1/2))*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(1/3)
/(1-I*3^(1/2))^(1/3)-1/36*(3-I*3^(1/2))*ln((1+I*3^(1/2))^(2/3)+(2+2*I*3^(1
/2))^(1/3)*x+2^(2/3)*x^2)*2^(1/3)/(1+I*3^(1/2))^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.10

$$\int \frac{x^4}{1-x^3+x^6}\,dx=\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^3}\&\right]$$

input
```
Integrate[x^4/(1 - x^3 + x^6),x]
```

output
```
RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]/3
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1710, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx+\frac{1}{6}\left(3-i\sqrt{3}\right)\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx$$

$$\Big\downarrow 27$$

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\int\frac{x}{-2x^3-i\sqrt{3}+1}dx-\frac{1}{3}\left(3-i\sqrt{3}\right)\int\frac{x}{-2x^3+i\sqrt{3}+1}dx$$

$$\Big\downarrow 821$$

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\left(\frac{\int\frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int\frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)-$$
$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(\frac{\int\frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int\frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)$$

$$\Big\downarrow 16$$

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\int\frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)-$$
$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\int\frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)$$

$$\Big\downarrow 1142$$

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\right.$$

$\downarrow$ 1082

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\right.$$

$\downarrow$ 217

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-\right.}{3\;2^{2/3}\sqrt[3]{1-i\sqrt{}}}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\right.}{3\;2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right.$$

$\downarrow$ 1103

$$-\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{\vphantom{1}}\right.}{3\ 2^{2/3}\sqrt[3]{1-}}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1}\right.}{3\ 2^{2/3}\sqrt[3]{1+i}}\right.$$

input
```
Int[x^4/(1 - x^3 + x^6),x]
```

output
```
-1/3*((3 + I*Sqrt[3])*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3
)*(1 - I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)
^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3]))^(1/3)))) - ((
3 - I*Sqrt[3])*(-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1 +
I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))
/Sqrt[3]])/2^(1/3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)
*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 + I*Sqrt[3]))^(1/3))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	40

input
```
int(x^4/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R^4/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.64

$$\int \frac{x^4}{1-x^3+x^6}\,dx$$

$$=\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(-3\left(3\sqrt{-\frac{1}{3}}\left(\sqrt{-3}+1\right)+\sqrt{-3}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{2}{3}}+4\,x\right)$$
$$-\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(3\left(3\sqrt{-\frac{1}{3}}\left(\sqrt{-3}-1\right)+\sqrt{-3}-1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{2}{3}}+4\,x\right)$$
$$+\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(3\left(3\sqrt{-\frac{1}{3}}\left(\sqrt{-3}+1\right)-\sqrt{-3}-1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{2}{3}}+4\,x\right)$$
$$-\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(-3\left(3\sqrt{-\frac{1}{3}}\left(\sqrt{-3}-1\right)-\sqrt{-3}+1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{2}{3}}+4\,x\right)+\frac{1}{3}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\log\left(3\left(3\sqrt{-\frac{1}{3}}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{2}{3}}+2\,x\right)$$
$$+\frac{1}{3}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\log\left(-3\left(3\sqrt{-\frac{1}{3}}-1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{2}{3}}+2\,x\right)$$

input
```
integrate(x^4/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/6*(1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) - 1)*log(-3*(3*sqrt(-1/3)*(sqrt
(-3) + 1) + sqrt(-3) + 1)*(1/6*sqrt(-1/3) - 1/6)^(2/3) + 4*x) - 1/6*(1/6*s
qrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) + 1)*log(3*(3*sqrt(-1/3)*(sqrt(-3) - 1) +
 sqrt(-3) - 1)*(1/6*sqrt(-1/3) - 1/6)^(2/3) + 4*x) + 1/6*(-1/6*sqrt(-1/3)
- 1/6)^(1/3)*(sqrt(-3) - 1)*log(3*(3*sqrt(-1/3)*(sqrt(-3) + 1) - sqrt(-3)
- 1)*(-1/6*sqrt(-1/3) - 1/6)^(2/3) + 4*x) - 1/6*(-1/6*sqrt(-1/3) - 1/6)^(1
/3)*(sqrt(-3) + 1)*log(-3*(3*sqrt(-1/3)*(sqrt(-3) - 1) - sqrt(-3) + 1)*(-1
/6*sqrt(-1/3) - 1/6)^(2/3) + 4*x) + 1/3*(1/6*sqrt(-1/3) - 1/6)^(1/3)*log(3
*(3*sqrt(-1/3) + 1)*(1/6*sqrt(-1/3) - 1/6)^(2/3) + 2*x) + 1/3*(-1/6*sqrt(-
1/3) - 1/6)^(1/3)*log(-3*(3*sqrt(-1/3) - 1)*(-1/6*sqrt(-1/3) - 1/6)^(2/3)
+ 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(6561t^5+54t^2+x\right)\right)\right)$$

input
```
integrate(x**4/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t*
*2 + x)))
```

Maxima [F]

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \int \frac{x^4}{x^6-x^3+1}\,dx$$

input
```
integrate(x^4/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^4/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(255) = 510$.

Time = 0.16 (sec) , antiderivative size = 827, normalized size of antiderivative = 2.01

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 1
0*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*co
s(4/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt
(3)*sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1
)*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(2*sqrt(3)
*cos(2/9*pi)^5 - 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2
/9*pi)*sin(2/9*pi)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin
(2/9*pi)^3 - 2*sin(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)
^2 - 2*cos(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) +
 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^5
- 20*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*
pi)^4 + 10*cos(1/9*pi)^4*sin(1/9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*
sin(1/9*pi)^5 - sqrt(3)*cos(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*
pi)*sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*
sqrt(3) + 1/2)*sin(1/9*pi))) - 1/18*(10*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi)
- 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqrt(3)*sin(4/9*pi)^5 + 2*cos
(4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*cos(4/9*pi)*sin(4/9*pi)^4
 + 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi)^2 - sin(4/9*pi)^2)*log(
(-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(10*sqrt(3)*cos
(2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + 2*sqr...
```

Mupad [B] (verification not implemented)

Time = 19.99 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{1-x^3+x^6}\,dx$$
$$= \frac{\ln\left(x+\left(162\,x+\frac{27\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+\frac{\ln\left(x-\left(162\,x+\frac{27\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{2^{2/3}\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(x^4/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (162*x + (27*(3^(1/2)*12i - 36)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/1
62))*(3^(1/2)*12i - 36)^(1/3))/18 + (log(x - (162*x + (27*(- 3^(1/2)*12i -
 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(- 3^(1/2)*12i - 36)^(1/3))/18
- (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(- 3^(1/2)*1i - 3)^(2/3))/12 + (2^(1/3
)*3^(1/6)*(- 3^(1/2)*1i - 3)^(2/3)*1i)/4)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3
) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(
2/3))/12 - (2^(1/3)*3^(1/6)*(3^(1/2)*1i - 3)^(2/3)*1i)/4)*(3^(1/2)*1i - 3)
^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(- 3
^(1/2)*1i - 3)^(2/3))/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/
36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)
*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

Reduce [F]

$$\int \frac{x^4}{1-x^3+x^6}\,dx = \int \frac{x^4}{x^6-x^3+1}dx$$

input
```
int(x^4/(x^6-x^3+1),x)
```

output
```
int(x**4/(x**6 - x**3 + 1),x)
```

3.158 $\int \frac{x^3}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 411

$$\int \frac{x^3}{1-x^3+x^6}\,dx = -\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

output

```
-1/6*(3^(1/2)+I)*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*3^(1/2))*2^(
2/3)/(1-I*3^(1/2))^(2/3)+1/6*(I-3^(1/2))*arctan(1/3*(1+2*x/(1/2+1/2*I*3^(1
/2))^(1/3))*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/18*(3+I*3^(1/2))*ln((1-
I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/18*(3-I*3^(1/2))
*ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/36*(3+I*3
^(1/2))*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(2/3
)/(1-I*3^(1/2))^(2/3)-1/36*(3-I*3^(1/2))*ln((1+I*3^(1/2))^(2/3)+(2+2*I*3^(
1/2))^(1/3)*x+2^(2/3)*x^2)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.09

$$\int \frac{x^3}{1-x^3+x^6}\,dx=\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{-1+2\#1^3}\&\right]$$

input
```
Integrate[x^3/(1 - x^3 + x^6),x]
```

output
```
RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/3
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1710, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^6-x^3+1}\,dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)$$

$\downarrow$ 1142

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int \cdots}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int \cdots}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$\downarrow$ 1082

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

↓ 217

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{\cdots}{\sqrt[3]{\frac{1}{2}\left(1\cdots\right.}}}{\sqrt{\cdots}}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{\cdots}{\sqrt[3]{\frac{1}{2}\left(1\cdots\right.}}}{\sqrt{\cdots}}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\downarrow 1103$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\cdots\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\cdots\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

input
```
Int[x^3/(1 - x^3 + x^6),x]
```

output
```
((3 + I*Sqrt[3])*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 - I*Sqrt[3
])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]] + Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^
2]/2)/(3*((1 - I*Sqrt[3])/2)^(2/3))))/6 + ((3 - I*Sqrt[3])*(Log[(1 + I*Sqr
t[3])^(1/3) - 2^(1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(
1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sqrt[3])^(2/3)
+ (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[3])/2)^(2/
3))))/6
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	40

input
```
int(x^3/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{1-x^3+x^6}\,dx = -\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(3\sqrt{-\frac{1}{3}}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)+2\,x\right) - \frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(-3\sqrt{-\frac{1}{3}}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)+2\,x\right) + \frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(-3\sqrt{-\frac{1}{3}}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)+2\,x\right) + \frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(3\sqrt{-\frac{1}{3}}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)+2\,x\right) + \frac{1}{3}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\log\left(x-3\sqrt{-\frac{1}{3}}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\right) + \frac{1}{3}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\log\left(x+3\sqrt{-\frac{1}{3}}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^3/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/6*(1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) + 1)*log(3*sqrt(-1/3)*(1/6*sqr
t(-1/3) - 1/6)^(1/3)*(sqrt(-3) + 1) + 2*x) - 1/6*(-1/6*sqrt(-1/3) - 1/6)^(
1/3)*(sqrt(-3) + 1)*log(-3*sqrt(-1/3)*(-1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(
-3) + 1) + 2*x) + 1/6*(1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) - 1)*log(-3*s
qrt(-1/3)*(1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) - 1) + 2*x) + 1/6*(-1/6*s
qrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) - 1)*log(3*sqrt(-1/3)*(-1/6*sqrt(-1/3) -
1/6)^(1/3)*(sqrt(-3) - 1) + 2*x) + 1/3*(1/6*sqrt(-1/3) - 1/6)^(1/3)*log(x
- 3*sqrt(-1/3)*(1/6*sqrt(-1/3) - 1/6)^(1/3)) + 1/3*(-1/6*sqrt(-1/3) - 1/6)
^(1/3)*log(x + 3*sqrt(-1/3)*(-1/6*sqrt(-1/3) - 1/6)^(1/3))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{x^3}{1-x^3+x^6}\,dx = \mathrm{RootSum}\left(19683t^6+243t^3+1, \left(t\mapsto t\log\left(-1458t^4-9t+x\right)\right)\right)$$

input
```
integrate(x**3/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t
+ x)))
```

Maxima [F]

$$\int \frac{x^3}{1-x^3+x^6}\,dx = \int \frac{x^3}{x^6-x^3+1}\,dx$$

input
```
integrate(x^3/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
integrate(x^3/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(255) = 510.

Time = 0.14 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.56

$$\int \frac{x^3}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2
*sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4
/9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)
*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*
cos(2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9
*pi)^4 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(
3)*cos(2/9*pi) + sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2
*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4 -
12*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1
/9*pi)^3*sin(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) +
 sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqr
t(3) + 1/2)*sin(1/9*pi))) - 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 8*
sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 + 12*cos(4/9*pi)^2*sin
(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9*pi) - cos(4/9*pi))*log((-I*
sqrt(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3)*cos(2/9*
pi)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4
+ 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) -
 cos(2/9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/
18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*sin(1/9*pi
)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sin(1/9*pi)^...
```

Mupad [B] (verification not implemented)

Time = 20.15 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(x^3/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*12i -
 36)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/6)*(
3^(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(- 3^(1/2)
*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(4/3))/12)*(- 3^(1
/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3
^(1/3)*(3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(4/3)
)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x -
 (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/4 - (2^(2/3)*3^(5/6)*(- 3^(1/2
)*1i - 3)^(1/3)*1i)/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/3
6 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/4 + (2^(2/3)
*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) +
3^(5/6)*1i))/36
```

Reduce [F]

$$\int \frac{x^3}{1 - x^3 + x^6} \, dx = \int \frac{x^3}{x^6 - x^3 + 1} dx$$

input
```
int(x^3/(x^6-x^3+1),x)
```

output
```
int(x**3/(x**6 - x**3 + 1),x)
```

3.159 $\int \frac{x^2}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^2}{1-x^3+x^6} \, dx = -\frac{2\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
-2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1-x^3+x^6} \, dx = \frac{2\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Integrate[x^2/(1 - x^3 + x^6),x]
```

output
```
(2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1690, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^6-x^3+1} \, dx$$
$$\Big\downarrow \text{ 1690}$$
$$\frac{1}{3}\int \frac{1}{x^6-x^3+1}dx^3$$
$$\Big\downarrow \text{ 1083}$$
$$-\frac{2}{3}\int \frac{1}{-x^6-3}d\left(2x^3-1\right)$$
$$\Big\downarrow \text{ 217}$$
$$\frac{2\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Int[x^2/(1 - x^3 + x^6),x]
```

output
```
(2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}$	19
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}$	19

input
```
int(x^2/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

input
```
integrate(x^2/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**2/(x**6-x**3+1),x)
```

output
```
2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)$$

input
```
integrate(x^2/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)$$

input
```
integrate(x^2/(x^6-x^3+1),x, algorithm="giac")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Mupad [B] (verification not implemented)

Time = 19.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{1-x^3+x^6}\,dx=-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

input
```
int(x^2/(x^6 - x^3 + 1),x)
```

output
```
-(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

Reduce [F]

$$\int \frac{x^2}{1-x^3+x^6}\,dx=\int \frac{x^2}{x^6-x^3+1}dx$$

input
```
int(x^2/(x^6-x^3+1),x)
```

output
```
int(x**2/(x**6 - x**3 + 1),x)
```

3.160 $\int \frac{x}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 375

$$\int \frac{x}{1-x^3+x^6}\,dx = \frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}-\frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}$$

output
```
1/3*I*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*3^(1/2))/(1/2-1/2*I*3^(
1/2))^(1/3)-1/3*I*arctan(1/3*(1+2*x/(1/2+1/2*I*3^(1/2))^(1/3))*3^(1/2))/(1
/2+1/2*I*3^(1/2))^(1/3)+1/9*I*ln((1-I*3^(1/2))^(1/3)-2^(1/3)*x)*3^(1/2)/(1
/2-1/2*I*3^(1/2))^(1/3)-1/9*I*ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*3^(1/2)/(1
/2+1/2*I*3^(1/2))^(1/3)-1/18*I*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1/2))^(1/3
)*x+2^(2/3)*x^2)*2^(1/3)*3^(1/2)/(1-I*3^(1/2))^(1/3)+1/18*I*ln((1+I*3^(1/2
))^(2/3)+(2+2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(1/3)*3^(1/2)/(1+I*3^(1/2)
)^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.11

$$\int \frac{x}{1-x^3+x^6}\,dx = \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-\#1+2\#1^4}\&\right]$$

input
```
Integrate[x/(1 - x^3 + x^6),x]
```

output
```
RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1711, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{i\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{i\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx}{\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2i\int \frac{x}{-2x^3+i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{2i\int \frac{x}{-2x^3-i\sqrt{3}+1}dx}{\sqrt{3}}$$

$$\Big\downarrow \text{821}$$

$$\frac{2i\left(\frac{\int \frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{3}}-\frac{2i\left(\frac{\int \frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{16}$$

$$\frac{2i\left(-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)}{\sqrt{3}}-\frac{2i\left(-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{2i\left(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+\cdots}\right.}{3\,2^{2/3}\sqrt[3]{1+\cdots}}\right.}{\sqrt{3}}$$

$$\frac{2i\left(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\cdots}\right.}{3\,2^{2/3}\sqrt[3]{1-\cdots}}\right.}{\sqrt{3}}$$

$\downarrow$ 1082

$$\frac{2i\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1\cdots}}\right.}{\sqrt{3}}$$

$$\frac{2i\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1\cdots}}\right.}{\sqrt{3}}$$

$$\Big\downarrow \; 217$$

$$\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)}{\sqrt{3}}-\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)}{\sqrt{3}}$$

$\downarrow$ 1103

$$\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)}{\sqrt{3}}-\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)}{\sqrt{3}}$$

input
```
Int[x/(1 - x^3 + x^6),x]
```

output
```
((-2*I)*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1 - I*Sqrt[
3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3
]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^
(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3]))^(1/3))))/Sqrt[3] + ((2*I)*(
-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1 + I*Sqrt[3])^(1/3)
) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/2^(1/
3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2
]/(2*2^(1/3)))/(3*(2*(1 + I*Sqrt[3]))^(1/3))))/Sqrt[3]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	38
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	38

input
```
int(x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.70

$$\int \frac{x}{1-x^3+x^6}\,dx$$
$$=\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(-3\left(3\sqrt{-\frac{1}{3}}\left(\sqrt{-3}+1\right)+\sqrt{-3}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+4\,x\right)$$
$$-\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(3\left(3\sqrt{-\frac{1}{3}}\left(\sqrt{-3}-1\right)+\sqrt{-3}-1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+4\,x\right)$$
$$+\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(3\left(3\sqrt{-\frac{1}{3}}\left(\sqrt{-3}+1\right)-\sqrt{-3}-1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+4\,x\right)$$
$$-\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(-3\left(3\sqrt{-\frac{1}{3}}\left(\sqrt{-3}-1\right)-\sqrt{-3}+1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+4\,x\right)+\frac{1}{3}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(3\left(3\sqrt{-\frac{1}{3}}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+2\,x\right)$$
$$+\frac{1}{3}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(-3\left(3\sqrt{-\frac{1}{3}}-1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+2\,x\right)$$

input
```
integrate(x/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/6*(1/6*sqrt(-1/3) + 1/6)^(1/3)*(sqrt(-3) - 1)*log(-3*(3*sqrt(-1/3)*(sqrt
(-3) + 1) + sqrt(-3) + 1)*(1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) - 1/6*(1/6*s
qrt(-1/3) + 1/6)^(1/3)*(sqrt(-3) + 1)*log(3*(3*sqrt(-1/3)*(sqrt(-3) - 1) +
 sqrt(-3) - 1)*(1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) + 1/6*(-1/6*sqrt(-1/3)
+ 1/6)^(1/3)*(sqrt(-3) - 1)*log(3*(3*sqrt(-1/3)*(sqrt(-3) + 1) - sqrt(-3)
- 1)*(-1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) - 1/6*(-1/6*sqrt(-1/3) + 1/6)^(1
/3)*(sqrt(-3) + 1)*log(-3*(3*sqrt(-1/3)*(sqrt(-3) - 1) - sqrt(-3) + 1)*(-1
/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) + 1/3*(1/6*sqrt(-1/3) + 1/6)^(1/3)*log(3
*(3*sqrt(-1/3) + 1)*(1/6*sqrt(-1/3) + 1/6)^(2/3) + 2*x) + 1/3*(-1/6*sqrt(-
1/3) + 1/6)^(1/3)*log(-3*(3*sqrt(-1/3) - 1)*(-1/6*sqrt(-1/3) + 1/6)^(2/3)
+ 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.07

$$\int \frac{x}{1-x^3+x^6}\,dx = \text{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(6561t^5-27t^2+x\right)\right)\right)$$

input
```
integrate(x/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 - 27*_t*
*2 + x)))
```

Maxima [F]

$$\int \frac{x}{1-x^3+x^6}\,dx = \int \frac{x}{x^6-x^3+1}\,dx$$

input
```
integrate(x/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
integrate(x/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 815 vs. 2(217) = 434.

Time = 0.15 (sec) , antiderivative size = 815, normalized size of antiderivative = 2.17

$$\int \frac{x}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*s
qrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/
9*pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 + sqrt(3)*si
n(4/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(
4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9
*pi)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*si
n(2/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3
 - sin(2/9*pi)^5 - sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2 + 2*cos(2
/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*
I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*c
os(1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1
/9*pi)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 + sq
rt(3)*cos(1/9*pi)^2 - sqrt(3)*sin(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*pi))*a
rctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin
(1/9*pi))) - 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/
9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9
*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 - 2*sqrt(3)*cos(4/9*pi)
*sin(4/9*pi) - cos(4/9*pi)^2 + sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9*pi)
 - cos(4/9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) -
10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/...
```

Mupad [B] (verification not implemented)

Time = 19.95 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.81

$$\int \frac{x}{1-x^3+x^6}\,dx$$
$$= \frac{\ln\left(x+\left(81\,x-\frac{27\left(36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+\frac{\ln\left(x-\left(81\,x-\frac{27\left(36+\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(x/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/16
2))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i + 36)
^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(
2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 + (2^(1/3)*3^(1/6
)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*
1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12 - (
2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3^(1
/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)
^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*lo
g(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(
3^(1/3) + 3^(5/6)*1i))/36
```

Reduce [F]

$$\int \frac{x}{1-x^3+x^6}\,dx=\int \frac{x}{x^6-x^3+1}dx$$

input
```
int(x/(x^6-x^3+1),x)
```

output
```
int(x/(x**6 - x**3 + 1),x)
```

3.161 $\int \frac{1}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 186

$$\int \frac{1}{1-x^3+x^6}\,dx = -\frac{1}{3}(-1)^{13/18}\arctan\left(\frac{1+2\sqrt[9]{-1}x}{\sqrt{3}}\right) + \frac{1}{3}(-1)^{5/18}\arctan\left(\frac{1-2(-1)^{8/9}x}{\sqrt{3}}\right) - \frac{(-1)^{5/18}\left(\log(2)+3\log\left(\sqrt[9]{-1}-x\right)\right)}{9\sqrt{3}} + \frac{(-1)^{13/18}\log\left(-\sqrt[3]{2}((-1\right.}{3\sqrt{3}}$$

output
```
-1/3*(-1)^(13/18)*arctan(1/3*(1+2*(-1)^(1/9)*x)*3^(1/2))+1/3*(-1)^(5/18)*a
rctan(1/3*(1-2*(-1)^(8/9)*x)*3^(1/2))-1/27*(-1)^(5/18)*(ln(2)+3*ln((-1)^(1
/9)-x))*3^(1/2)+1/9*(-1)^(13/18)*ln(-2^(1/3)*((-1)^(8/9)+x))*3^(1/2)-1/18*
(-1)^(13/18)*ln(-2^(2/3)*((-1)^(7/9)+((-1)^(8/9)-x)*x))*3^(1/2)+1/18*(-1)^
(5/18)*ln(2^(2/3)*((-1)^(2/9)+x*((-1)^(1/9)+x)))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.23

$$\int \frac{1}{1-x^3+x^6}\,dx = \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-\#1^2+2\#1^5}\&\right]$$

input
```
Integrate[(1 - x^3 + x^6)^(-1),x]
```

output
```
RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.81, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1685, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{1685}$$

$$\frac{i\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx}{\sqrt{3}}-\frac{i\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx}{\sqrt{3}}$$

$$\Big\downarrow \text{750}$$

$$\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \; 16$$

$$\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \; 25$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int \frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int \frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \; 1142$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{\right.}\right.}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.}{\sqrt{3}}$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{\right.}\right.}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.}{\sqrt{3}}$$

$\downarrow$ 1082

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\right.}}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.}{\sqrt{3}}$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\right.}}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.}{\sqrt{3}}$$

$$\Big\downarrow \ 217$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}$$

$\downarrow$ 1103

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}$$

input
```
Int[(1 - x^3 + x^6)^(-1),x]
```

output
```
(I*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 - I*Sqrt[3])/2)^(2/3)) -
 (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 -
 I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 -
 I*Sqrt[3])/2)^(2/3))))/Sqrt[3] - (I*(Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*
x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt
[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^
(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[3])/2)^(2/3))))/Sqrt[3]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.20

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	37
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	37

input
```
int(1/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(1/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.36

$$\int \frac{1}{1-x^3+x^6}\,dx = -\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(-3\left(\sqrt{-\frac{1}{3}}\left(\sqrt{-3}+1\right)+\sqrt{-3}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+4\,x\right)$$
$$+\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(3\left(\sqrt{-\frac{1}{3}}\left(\sqrt{-3}-1\right)+\sqrt{-3}-1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+4\,x\right)$$
$$-\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(3\left(\sqrt{-\frac{1}{3}}\left(\sqrt{-3}+1\right)-\sqrt{-3}-1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+4\,x\right)$$
$$+\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(-3\left(\sqrt{-\frac{1}{3}}\left(\sqrt{-3}-1\right)-\sqrt{-3}+1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+4\,x\right)+\frac{1}{3}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(3\left(\sqrt{-\frac{1}{3}}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+2\,x\right)$$
$$+\frac{1}{3}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(-3\left(\sqrt{-\frac{1}{3}}-1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+2\,x\right)$$

input
```
integrate(1/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/6*(1/6*sqrt(-1/3) + 1/6)^(1/3)*(sqrt(-3) + 1)*log(-3*(sqrt(-1/3)*(sqrt(
-3) + 1) + sqrt(-3) + 1)*(1/6*sqrt(-1/3) + 1/6)^(1/3) + 4*x) + 1/6*(1/6*sq
rt(-1/3) + 1/6)^(1/3)*(sqrt(-3) - 1)*log(3*(sqrt(-1/3)*(sqrt(-3) - 1) + sq
rt(-3) - 1)*(1/6*sqrt(-1/3) + 1/6)^(1/3) + 4*x) - 1/6*(-1/6*sqrt(-1/3) + 1
/6)^(1/3)*(sqrt(-3) + 1)*log(3*(sqrt(-1/3)*(sqrt(-3) + 1) - sqrt(-3) - 1)*
(-1/6*sqrt(-1/3) + 1/6)^(1/3) + 4*x) + 1/6*(-1/6*sqrt(-1/3) + 1/6)^(1/3)*(
sqrt(-3) - 1)*log(-3*(sqrt(-1/3)*(sqrt(-3) - 1) - sqrt(-3) + 1)*(-1/6*sqrt
(-1/3) + 1/6)^(1/3) + 4*x) + 1/3*(1/6*sqrt(-1/3) + 1/6)^(1/3)*log(3*(sqrt(
-1/3) + 1)*(1/6*sqrt(-1/3) + 1/6)^(1/3) + 2*x) + 1/3*(-1/6*sqrt(-1/3) + 1/
6)^(1/3)*log(-3*(sqrt(-1/3) - 1)*(-1/6*sqrt(-1/3) + 1/6)^(1/3) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.11

$$\int \frac{1}{1-x^3+x^6}\,dx = \operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(729t^4+x\right)\right)\right)$$

input
```
integrate(1/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x)))
```

Maxima [F]

$$\int \frac{1}{1-x^3+x^6}\,dx = \int \frac{1}{x^6-x^3+1}\,dx$$

input
```
integrate(1/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^6 - x^3 + 1), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 632, normalized size of antiderivative = 3.40

$$\int \frac{1}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(1/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt
(3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi
)^3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(
4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9
*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4
*cos(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9
*pi) - sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*
I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*co
s(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/
9*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*a
rctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin
(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9
*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4
/9*pi)^4 - sqrt(3)*sin(4/9*pi) + cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi)
- cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) -
4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(
2/9*pi)^2 - sin(2/9*pi)^4 - sqrt(3)*sin(2/9*pi) + cos(2/9*pi))*log((-I*sqr
t(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*pi)
^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*c
os(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 + sqrt(3)*sin(1/9*pi) + cos(...
```

Mupad [B] (verification not implemented)

Time = 19.99 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.76

$$\int \frac{1}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}$$
$$+\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

input
```
int(1/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3
- 3^(1/2)*1i)^(1/3)*1i)/12)*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x + (2^(2/
3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(
1/3)*1i)/12)*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*3^(1
/3)*(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/1
2)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2
^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i +
3)^(4/3))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)
*log(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(
1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/
2)*1i + 3)^(1/3)*1i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

Reduce [F]

$$\int \frac{1}{1-x^3+x^6}\,dx = \int \frac{1}{x^6-x^3+1}dx$$

input
```
int(1/(x^6-x^3+1),x)
```

output
```
int(1/(x**6 - x**3 + 1),x)
```

3.162 $\int \frac{1}{x(1-x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x\left(1-x^3+x^6\right)}\,dx=-\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)+ln(x)-1/6*ln(x^6-x^3+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(1-x^3+x^6\right)}\,dx=\log(x)-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

input
```
Integrate[1/(x*(1 - x^3 + x^6)),x]
```

output
```
Log[x] - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-1
 + 2*#1^3) & ]/3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1693, 1144, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^6-x^3+1)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^3(x^6-x^3+1)}dx^3$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{3}\left(\int \frac{1-x^3}{x^6-x^3+1}dx^3+\log\left(x^3\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^6-x^3+1}dx^3-\frac{1}{2}\int -\frac{1-2x^3}{x^6-x^3+1}dx^3+\log\left(x^3\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^6-x^3+1}dx^3+\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3+\log\left(x^3\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(-\int \frac{1}{-x^6-3}d(2x^3-1)+\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3+\log\left(x^3\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3+\frac{\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^3\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\frac{\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^3\right)-\frac{1}{2}\log\left(x^6-x^3+1\right)\right)$$

input
```
Int[1/(x*(1 - x^3 + x^6)),x]
```

output
```
(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[x^3] - Log[1 - x^3 + x^6]/2)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln(x)+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}-\frac{\ln(x^6-x^3+1)}{6}$	33
default	$-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}+\ln(x)$	35

input
```
int(1/x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/9*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))-1/6*ln(x^6-x^3+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(1-x^3+x^6)}\,dx=\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

input
```
integrate(1/x/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-x^3+x^6\right)}\,dx = \log\left(x\right) - \frac{\log\left(x^6-x^3+1\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(1/x/(x**6-x**3+1),x)
```

output
```
log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(1-x^3+x^6\right)}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \frac{1}{3}\log\left(x^3\right)$$

input
```
integrate(1/x/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3
*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(1-x^3+x^6\right)}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log\left(|x|\right)$$

input
```
integrate(1/x/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(1-x^3+x^6\right)}\,dx = \ln(x) - \frac{\ln\left(x^6-x^3+1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

input
```
int(1/(x*(x^6 - x^3 + 1)),x)
```

output
```
log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/
3))/9
```

Reduce [F]

$$\int \frac{1}{x\left(1-x^3+x^6\right)}\,dx = \int \frac{1}{x^7-x^4+x}dx$$

input
```
int(1/x/(x^6-x^3+1),x)
```

output
```
int(1/(x**7 - x**4 + x),x)
```

3.163 $\int \frac{1}{x^2(1-x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 416

$$\int \frac{1}{x^2(1-x^3+x^6)}\,dx = -\frac{1}{x} + \frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{(3-i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{(3+i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

output
```
-1/x+1/6*(I-3^(1/2))*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*3^(1/2))
*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/6*(3^(1/2)+I)*arctan(1/3*(1+2*x/(1/2+1/2*I*
3^(1/2))^(1/3))*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/18*(3-I*3^(1/2))*ln
((1-I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/18*(3+I*3^(1
/2))*ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/36*(3
-I*3^(1/2))*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^
(1/3)/(1-I*3^(1/2))^(1/3)+1/36*(3+I*3^(1/2))*ln((1+I*3^(1/2))^(2/3)+(2+2*I
*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(1/3)/(1+I*3^(1/2))^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2 (1-x^3+x^6)} \, dx = -\frac{1}{x} - \frac{1}{3}\text{RootSum}\left[1-\#1^3 + \#1^6\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1+2\#1^4}\&\right]$$

input
```
Integrate[1/(x^2*(1 - x^3 + x^6)),x]
```

output
```
-x^(-1) - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-
#1 + 2*#1^4) & ]/3
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1704, 1834, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^6-x^3+1)} \, dx$$

$$\Big\downarrow \; 1704$$

$$\int \frac{x(1-x^3)}{x^6-x^3+1}dx-\frac{1}{x}$$

$$\Big\downarrow \; 1834$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx-\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\int \frac{x}{-2x^3-i\sqrt{3}+1}dx+\frac{1}{3}\left(3+i\sqrt{3}\right)\int \frac{x}{-2x^3+i\sqrt{3}+1}dx-\frac{1}{x}$$

$$\Big\downarrow \; 821$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)+$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)-\frac{1}{x}$$

$$\Big\downarrow \; 16$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)+$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)-\frac{1}{x}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int \frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int \frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{x}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{x}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\right.}}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\right.}}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right.$$

$$\frac{1}{x}$$

$$\downarrow 1103$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\ldots}\right.}{3\ 2^{2/3}\sqrt[3]{1-i\ldots}}\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\ldots}\right.}{3\ 2^{2/3}\sqrt[3]{1+i\ldots}}\right.$$

$$\frac{1}{x}$$

input
```
Int[1/(x^2*(1 - x^3 + x^6)),x]
```

output
```
-x^(-1) + ((3 - I*Sqrt[3])*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2
^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3
])/2)^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqr
t[3]))^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3]))^(1/3))))
/3 + ((3 + I*Sqrt[3])*(-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3
)*(1 + I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)
^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 + I*Sqrt[3]))^(1/3))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.08

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^{6}+9\_Z^{3}+1\right)}\_R\ln\left(-3\_R^{2}+x\right)\right)}{3}$	35
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(\_R^{4}-\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}-\frac{1}{x}$	50

input
```
int(1/x^2/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/3*sum(_R*ln(-3*_R^2+x),_R=RootOf(27*_Z^6+9*_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^2(1-x^3+x^6)}\,dx = \frac{(\sqrt{-3}x-x)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\log\left(3\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{2}{3}}(\sqrt{-3}+1)+2x\right)+(\sqrt{-3}x-x)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\cdots}{}$$

input
```
integrate(1/x^2/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/6*((sqrt(-3)*x - x)*(1/6*sqrt(-1/3) - 1/6)^(1/3)*log(3*(1/6*sqrt(-1/3) -
 1/6)^(2/3)*(sqrt(-3) + 1) + 2*x) + (sqrt(-3)*x - x)*(-1/6*sqrt(-1/3) - 1/
6)^(1/3)*log(3*(-1/6*sqrt(-1/3) - 1/6)^(2/3)*(sqrt(-3) + 1) + 2*x) - (sqrt
(-3)*x + x)*(1/6*sqrt(-1/3) - 1/6)^(1/3)*log(-3*(1/6*sqrt(-1/3) - 1/6)^(2/
3)*(sqrt(-3) - 1) + 2*x) - (sqrt(-3)*x + x)*(-1/6*sqrt(-1/3) - 1/6)^(1/3)*
log(-3*(-1/6*sqrt(-1/3) - 1/6)^(2/3)*(sqrt(-3) - 1) + 2*x) + 2*x*(1/6*sqrt
(-1/3) - 1/6)^(1/3)*log(x - 3*(1/6*sqrt(-1/3) - 1/6)^(2/3)) + 2*x*(-1/6*sq
rt(-1/3) - 1/6)^(1/3)*log(x - 3*(-1/6*sqrt(-1/3) - 1/6)^(2/3)) - 6)/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^2(1-x^3+x^6)}\,dx = \operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(-27t^2+x\right)\right)\right)-\frac{1}{x}$$

input
```
integrate(1/x**2/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x))) -
1/x
```

Maxima [F]

$$\int \frac{1}{x^2\left(1-x^3+x^6\right)}\,dx=\int \frac{1}{(x^6-x^3+1)x^2}\,dx$$

input
```
integrate(1/x^2/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/x - integrate((x^4 - x)/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 829 vs. $2(260) = 520$.

Time = 0.13 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.99

$$\int \frac{1}{x^2\left(1-x^3+x^6\right)}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^2/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sq
rt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9
*pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)
*sin(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*c
os(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(
2/9*pi)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)
*sin(2/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi
)^3 - sin(2/9*pi)^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 -
4*cos(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)
/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sq
rt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 +
5*cos(1/9*pi)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)
^5 - 2*sqrt(3)*cos(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin
(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3)
 + 1/2)*sin(1/9*pi))) + 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqr
t(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 -
 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 + 4*sqrt(3)*
cos(4/9*pi)*sin(4/9*pi) + 2*cos(4/9*pi)^2 - 2*sin(4/9*pi)^2)*log((-I*sqrt(
3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(5*sqrt(3)*cos(2/9*pi)^4
*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9...
```

Mupad [B] (verification not implemented)

Time = 20.04 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2(1-x^3+x^6)}\,dx$$
$$= \frac{\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{1}{x}+\frac{\ln\left(x-\frac{\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{12}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(1/(x^2*(x^6 - x^3 + 1)),x)
```

output
```
(log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1
/3))/18 - 1/x + (log(x - (- 3^(1/2)*12i - 36)^(2/3)/12)*(- 3^(1/2)*12i - 3
6)^(1/3))/18 - (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3)
 - 3^(5/6)*1i)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
- (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i
)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*lo
g(x - (2^(1/3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/
2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3
^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*
(3^(1/3) + 3^(5/6)*1i))/36
```

Reduce [F]

$$\int \frac{1}{x^2\left(1-x^3+x^6\right)}\,dx=\frac{-\left(\int \frac{x^4}{x^6-x^3+1}dx\right)x+\left(\int \frac{x}{x^6-x^3+1}dx\right)x-1}{x}$$

input
```
int(1/x^2/(x^6-x^3+1),x)
```

output
```
( - int(x**4/(x**6 - x**3 + 1),x)*x + int(x/(x**6 - x**3 + 1),x)*x - 1)/x
```

3.164 $\int \frac{1}{x^3(1-x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 418

$$\int \frac{1}{x^3(1-x^3+x^6)}\,dx = -\frac{1}{2x^2} - \frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{(3-i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(3+i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

output

```
-1/2/x^2-1/6*(I-3^(1/2))*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*3^(1
/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/6*(3^(1/2)+I)*arctan(1/3*(1+2*x/(1/2+1/
2*I*3^(1/2))^(1/3))*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/18*(3-I*3^(1/2)
)*ln((1-I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/18*(3+I*
3^(1/2))*ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/3
6*(3-I*3^(1/2))*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2
)*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/36*(3+I*3^(1/2))*ln((1+I*3^(1/2))^(2/3)+(2
+2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^3 (1 - x^3 + x^6)} dx = -\frac{1}{2x^2} - \frac{1}{3}\text{RootSum}\left[1 - \#1^3 + \#1^6\&, \frac{-\log(x - \#1) + \log(x - \#1)\#1^3}{-\#1^2 + 2\#1^5}\&\right]$$

input
```
Integrate[1/(x^3*(1 - x^3 + x^6)),x]
```

output
```
-1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)
/(-#1^2 + 2*#1^5) & ]/3
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1704, 27, 1752, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^6 - x^3 + 1)} dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{2} \int \frac{2(1 - x^3)}{x^6 - x^3 + 1} dx - \frac{1}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1 - x^3}{x^6 - x^3 + 1} dx - \frac{1}{2x^2}$$

$$\Big\downarrow \text{1752}$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx-\frac{1}{6}\left(3-i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx-\frac{1}{2x^2}$$

$$\downarrow 750$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}}\left(1-i\sqrt{3}\right)x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}}\left(1-i\sqrt{3}\right)}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}}\left(1+i\sqrt{3}\right)x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}}\left(1+i\sqrt{3}\right)}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{2x^2}$$

$$\downarrow 16$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}}\left(1-i\sqrt{3}\right)x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}}\left(1+i\sqrt{3}\right)x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{2x^2}$$

$$\downarrow 25$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{2x^2}$$

$\big\downarrow$ 1142

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int -\ldots}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\ldots}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

$\big\downarrow$ 1082

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right.}}\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{\cdots}{\sqrt[3]{\frac{1}{2}\cdots}}}{\cdots}\right)\cdots}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{\cdots}{\sqrt[3]{\frac{1}{2}\left(1\cdots\right.}}}{\sqrt{\cdots}}\right)\cdots}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

$$\Big\downarrow \text{ 1103}$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{\right.}\right.}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

input
```
Int[1/(x^3*(1 - x^3 + x^6)),x]
```

output
```
-1/2*1/x^2 - ((3 - I*Sqrt[3])*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*(
(1 - I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)
^(1/3))/Sqrt[3]] + Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x
 + 2^(2/3)*x^2]/2)/(3*((1 - I*Sqrt[3])/2)^(2/3))))/6 - ((3 + I*Sqrt[3])*(L
og[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqr
t[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sq
rt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sq
rt[3])/2)^(2/3))))/6
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.09

method	result	size
risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(27\_Z^6+9\_Z^3+1\right)}\_R\ln\left(9\_R^4+3\_R+x\right)\right)}{3}$	38
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(-\_R^3+1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}-\frac{1}{2x^2}$	50

input
```
int(1/x^3/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+1/3*sum(_R*ln(9*_R^4+3*_R+x),_R=RootOf(27*_Z^6+9*_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^3\left(1-x^3+x^6\right)}\,dx = \frac{2\,x^2\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\log\left(3\left(\sqrt{-\frac{1}{3}}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}+2\,x\right)+2\,x^2\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\log\left(-3\left(\sqrt{-\frac{1}{3}}\right.\right.}{}$$

input
```
integrate(1/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/6*(2*x^2*(1/6*sqrt(-1/3) - 1/6)^(1/3)*log(3*(sqrt(-1/3) + 1)*(1/6*sqrt(-
1/3) - 1/6)^(1/3) + 2*x) + 2*x^2*(-1/6*sqrt(-1/3) - 1/6)^(1/3)*log(-3*(sqr
t(-1/3) - 1)*(-1/6*sqrt(-1/3) - 1/6)^(1/3) + 2*x) - (sqrt(-3)*x^2 + x^2)*(
1/6*sqrt(-1/3) - 1/6)^(1/3)*log(-3*(sqrt(-1/3)*(sqrt(-3) + 1) + sqrt(-3) +
 1)*(1/6*sqrt(-1/3) - 1/6)^(1/3) + 4*x) + (sqrt(-3)*x^2 - x^2)*(1/6*sqrt(-
1/3) - 1/6)^(1/3)*log(3*(sqrt(-1/3)*(sqrt(-3) - 1) + sqrt(-3) - 1)*(1/6*sq
rt(-1/3) - 1/6)^(1/3) + 4*x) - (sqrt(-3)*x^2 + x^2)*(-1/6*sqrt(-1/3) - 1/6
)^(1/3)*log(3*(sqrt(-1/3)*(sqrt(-3) + 1) - sqrt(-3) - 1)*(-1/6*sqrt(-1/3)
- 1/6)^(1/3) + 4*x) + (sqrt(-3)*x^2 - x^2)*(-1/6*sqrt(-1/3) - 1/6)^(1/3)*l
og(-3*(sqrt(-1/3)*(sqrt(-3) - 1) - sqrt(-3) + 1)*(-1/6*sqrt(-1/3) - 1/6)^(
1/3) + 4*x) - 3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^3\left(1-x^3+x^6\right)}\,dx = \operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(729t^4+9t+x\right)\right)\right) - \frac{1}{2x^2}$$

input
```
integrate(1/x**3/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + 9*_t +
x))) - 1/(2*x**2)
```

Maxima [F]

$$\int \frac{1}{x^3(1-x^3+x^6)}\,dx = \int \frac{1}{(x^6-x^3+1)x^3}\,dx$$

input
```
integrate(1/x^3/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/2/x^2 - integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(260) = 520$.

Time = 0.15 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^3(1-x^3+x^6)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(x^6-x^3+1),x, algorithm="giac")
```

output

```
1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)
^3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*c
os(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(
2/9*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4
 + 4*cos(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*co
s(2/9*pi) + 2*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)
/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqr
t(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3
*sin(1/9*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin
(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3)
 + 1/2)*sin(1/9*pi))) + 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt
(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi
)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) - 2*cos(4/9*pi))*log((-I*sqrt(
3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(2/9*pi)^3
*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos
(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(2
/9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(4*
sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 +
cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 - 2*sqrt(...
```

Mupad [B] (verification not implemented)

Time = 19.76 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3\left(1-x^3+x^6\right)}\,dx=\frac{\ln\left(x-\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$

$$+\frac{\ln\left(x+\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{1}{2\,x^2}$$

$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^3}{16}+27\right)}{108}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^3}{16}-27\right)}{108}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(1/(x^3*(x^6 - x^3 + 1)),x)
```

output
```
(log(x - (((3^(1/2)*9i)/2 - 27/2)*(- 3^(1/2)*12i - 36)^(1/3))/54)*(- 3^(1/
2)*12i - 36)^(1/3))/18 + (log(x + (((3^(1/2)*9i)/2 + 27/2)*(3^(1/2)*12i -
36)^(1/3))/54)*(3^(1/2)*12i - 36)^(1/3))/18 - 1/(2*x^2) - (2^(2/3)*log(x -
 (2^(2/3)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)*((3*(3^(1/2)*1i
+ 3)*(3^(1/3) + 3^(5/6)*1i)^3)/16 + 27))/108)*(- 3^(1/2)*1i - 3)^(1/3)*(3^
(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i - 3)^(1/3)
*(3^(1/3) - 3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^(5/6)*1i)^3)/16
- 27))/108)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*l
og(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*1i - 3)
^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(
1/2)*1i - 3)^(1/3)*1i)/6)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/3
6
```

Reduce [F]

$$\int \frac{1}{x^3\left(1-x^3+x^6\right)}\,dx=\frac{-2\left(\int \frac{x^3}{x^6-x^3+1}dx\right)x^2+2\left(\int \frac{1}{x^6-x^3+1}dx\right)x^2-1}{2x^2}$$

input
```
int(1/x^3/(x^6-x^3+1),x)
```

output
```
( - 2*int(x**3/(x**6 - x**3 + 1),x)*x**2 + 2*int(1/(x**6 - x**3 + 1),x)*x*
*2 - 1)/(2*x**2)
```

3.165 $\int \frac{1}{x^4(1-x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{x^4(1-x^3+x^6)}\,dx = -\frac{1}{3x^3}+\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
-1/3/x^3+1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)+ln(x)-1/6*ln(x^6-x^3+1
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4(1-x^3+x^6)}\,dx = -\frac{1}{3x^3}+\log(x) - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

input
```
Integrate[1/(x^4*(1 - x^3 + x^6)),x]
```

output
```
-1/3*1/x^3 + Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 +
 2*#1^3) & ]/3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1693, 1145, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^6-x^3+1)} \, dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{1}{3} \int \frac{1}{x^6 (x^6-x^3+1)}dx^3$$
$$\Big\downarrow \text{1145}$$
$$\frac{1}{3} \left(\int \frac{1-x^3}{x^3 (x^6-x^3+1)}dx^3-\frac{1}{x^3}\right)$$
$$\Big\downarrow \text{1200}$$
$$\frac{1}{3} \left(\int \left(\frac{1}{x^3}-\frac{x^3}{x^6-x^3+1}\right)dx^3-\frac{1}{x^3}\right)$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3} \left(\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{x^3}+\log\left(x^3\right)-\frac{1}{2} \log\left(x^6-x^3+1\right)\right)$$

input
```
Int[1/(x^4*(1 - x^3 + x^6)),x]
```

output
```
(-x^(-3) + ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[x^3] - Log[1 - x^3 +
x^6]/2)/3
```

Defintions of rubi rules used

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{1}{3x^3}+\ln\left(x\right)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}-\frac{\ln\left(x^6-x^3+1\right)}{6}$	38
default	$-\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}+\ln\left(x\right)$	40

input
```
int(1/x^4/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+ln(x)-1/9*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))-1/6*ln(x^6-x^3+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4 (1 - x^3 + x^6)} \, dx = -\frac{2 \sqrt{3} x^3 \arctan\left(\frac{1}{3} \sqrt{3} (2 x^3 - 1)\right) + 3 x^3 \log\left(x^6 - x^3 + 1\right) - 18 x^3 \log\left(x\right) + 6}{18 x^3}$$

input
```
integrate(1/x^4/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/18*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3*x^3*log(x^6 - x^3
 + 1) - 18*x^3*log(x) + 6)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (1 - x^3 + x^6)} \, dx = \log\left(x\right) - \frac{\log\left(x^6 - x^3 + 1\right)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9} - \frac{1}{3x^3}$$

input
```
integrate(1/x**4/(x**6-x**3+1),x)
```

output
```
log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/9 - 1/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4 (1 - x^3 + x^6)} \, dx = -\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x^3 - 1)\right) - \frac{1}{3 x^3} - \frac{1}{6} \log\left(x^6 - x^3 + 1\right) + \frac{1}{3} \log\left(x^3\right)$$

input
```
integrate(1/x^4/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3 - 1/6*log(x^6 - x^3
 + 1) + 1/3*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4 (1-x^3+x^6)} \, dx = -\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2 x^3-1)\right) - \frac{x^3+1}{3 x^3} - \frac{1}{6} \log\left(x^6-x^3+1\right) + \log\left(|x|\right)$$

input
```
integrate(1/x^4/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3*(x^3 + 1)/x^3 - 1/6*log
(x^6 - x^3 + 1) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4 (1-x^3+x^6)} \, dx = \ln(x) - \frac{\ln\left(x^6-x^3+1\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9} - \frac{1}{3 x^3}$$

input
```
int(1/(x^4*(x^6 - x^3 + 1)),x)
```

output
```
log(x) - log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/
3))/9 - 1/(3*x^3)
```

Reduce [F]

$$\int \frac{1}{x^4 (1 - x^3 + x^6)} \, dx = \int \frac{1}{x^{10} - x^7 + x^4} dx$$

input
```
int(1/x^4/(x^6-x^3+1),x)
```

output
```
int(1/(x**10 - x**7 + x**4),x)
```

3.166 $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 423

$$\int \frac{1}{x^5(1-x^3+x^6)}\,dx = -\frac{1}{4x^4}-\frac{1}{x}-\frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{(3+i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{(3-i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

output
```
-1/4/x^4-1/x-1/6*(3^(1/2)+I)*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*
3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/6*(I-3^(1/2))*arctan(1/3*(1+2*x/(1/
2+1/2*I*3^(1/2))^(1/3))*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/18*(3+I*3^(
1/2))*ln((1-I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/18*(
3-I*3^(1/2))*ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(1/3)/(1+I*3^(1/2))^(1/3)
+1/36*(3+I*3^(1/2))*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1/2))^(1/3)*x+2^(2/3)
*x^2)*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/36*(3-I*3^(1/2))*ln((1+I*3^(1/2))^(2/3
)+(2+2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(1/3)/(1+I*3^(1/2))^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.13

$$\int \frac{1}{x^5(1-x^3+x^6)}\,dx = -\frac{1}{4x^4}-\frac{1}{x}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^3}\&\right]$$

input
```
Integrate[1/(x^5*(1 - x^3 + x^6)),x]
```

output
```
-1/4*1/x^4 - x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 +
 2*#1^3) & ]/3
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1704, 27, 1828, 1710, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5(x^6-x^3+1)}dx$$

$$\Big\downarrow \text{ 1704}$$

$$\frac{1}{4}\int \frac{4(1-x^3)}{x^2(x^6-x^3+1)}dx-\frac{1}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1-x^3}{x^2(x^6-x^3+1)}dx-\frac{1}{4x^4}$$

$$\Big\downarrow \text{1828}$$

$$-\int \frac{x^4}{x^6-x^3+1}dx-\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow \text{1710}$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx-\frac{1}{6}\left(3-i\sqrt{3}\right)\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx-\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\int \frac{x}{-2x^3-i\sqrt{3}+1}dx+\frac{1}{3}\left(3-i\sqrt{3}\right)\int \frac{x}{-2x^3+i\sqrt{3}+1}dx-\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)+$$
$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)-\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)+$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)-$$

$$\frac{1}{4x^4}-\frac{1}{x}$$

$\downarrow$ 1142

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int \frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int \frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\right.}}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\right.}}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right.$$

$$\frac{1}{4x^4}-\frac{1}{x}$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\right.}}{3\,2^{2/3}\sqrt[3]{1-i}}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\right.}}{3\,2^{2/3}\sqrt[3]{1+i}}\right.$$

$$\frac{1}{4x^4}-\frac{1}{x}$$

input
```
Int[1/(x^5*(1 - x^3 + x^6)),x]
```

output
```
-1/4*1/x^4 - x^(-1) + ((3 + I*Sqrt[3])*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2
^(1/3)*x]/(2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((
1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (
2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3
]))^(1/3))))/3 + ((3 - I*Sqrt[3])*(-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3
)*x]/(2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I
*Sqrt[3])/2)^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1
+ I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 + I*Sqrt[3]))^(
1/3))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.11

method	result	size
risch	$\frac{-x^3-\frac{1}{4}}{x^4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^6-9\_Z^3+1\right)}\_R\ln\left(-27\_R^5+6\_R^2+x\right)\right)}{3}$	46
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}-\frac{1}{4x^4}-\frac{1}{x}$	51

input
```
int(1/x^5/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
(-x^3-1/4)/x^4+1/3*sum(_R*ln(-27*_R^5+6*_R^2+x),_R=RootOf(27*_Z^6-9*_Z^3+1
))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^5 (1-x^3+x^6)} dx$$
$$= \frac{4\,x^4 \left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}} \log\left(-3\left(3\sqrt{-\frac{1}{3}}-1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+2\,x\right)+4\,x^4\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(3\left(3\sqrt{\right.\right.}{}$$

input
```
integrate(1/x^5/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/12*(4*x^4*(1/6*sqrt(-1/3) + 1/6)^(1/3)*log(-3*(3*sqrt(-1/3) - 1)*(1/6*sq
rt(-1/3) + 1/6)^(2/3) + 2*x) + 4*x^4*(-1/6*sqrt(-1/3) + 1/6)^(1/3)*log(3*(
3*sqrt(-1/3) + 1)*(-1/6*sqrt(-1/3) + 1/6)^(2/3) + 2*x) - 12*x^3 + 2*(sqrt(
-3)*x^4 - x^4)*(1/6*sqrt(-1/3) + 1/6)^(1/3)*log(3*(3*sqrt(-1/3)*(sqrt(-3)
+ 1) - sqrt(-3) - 1)*(1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) - 2*(sqrt(-3)*x^4
 + x^4)*(1/6*sqrt(-1/3) + 1/6)^(1/3)*log(-3*(3*sqrt(-1/3)*(sqrt(-3) - 1) -
 sqrt(-3) + 1)*(1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) + 2*(sqrt(-3)*x^4 - x^4
)*(-1/6*sqrt(-1/3) + 1/6)^(1/3)*log(-3*(3*sqrt(-1/3)*(sqrt(-3) + 1) + sqrt
(-3) + 1)*(-1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) - 2*(sqrt(-3)*x^4 + x^4)*(-
1/6*sqrt(-1/3) + 1/6)^(1/3)*log(3*(3*sqrt(-1/3)*(sqrt(-3) - 1) + sqrt(-3)
- 1)*(-1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) - 3)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^5 (1-x^3+x^6)} dx$$
$$= \operatorname{RootSum}\left(19683t^6-243t^3+1, \left(t\mapsto t\log\left(-6561t^5+54t^2+x\right)\right)\right)+\frac{-4x^3-1}{4x^4}$$

input
```
integrate(1/x**5/(x**6-x**3+1),x)
```

output
```
RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**5 + 54*_t
**2 + x))) + (-4*x**3 - 1)/(4*x**4)
```

Maxima [F]

$$\int \frac{1}{x^5 (1 - x^3 + x^6)} \, dx = \int \frac{1}{(x^6 - x^3 + 1)x^5} \, dx$$

input
```
integrate(1/x^5/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/4*(4*x^3 + 1)/x^4 - integrate(x^4/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 839 vs. $2(265) = 530$.

Time $= 0.14$ (sec) , antiderivative size $= 839$, normalized size of antiderivative $= 1.98$

$$\int \frac{1}{x^5 (1 - x^3 + x^6)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10
*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos
(4/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(
3)*sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)
*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*
cos(2/9*pi)^5 - 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/
9*pi)*sin(2/9*pi)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(
2/9*pi)^3 - 2*sin(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^
2 - 2*cos(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) +
2*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 -
 20*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*p
i)^4 + 10*cos(1/9*pi)^4*sin(1/9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*s
in(1/9*pi)^5 - sqrt(3)*cos(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*p
i)*sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*s
qrt(3) + 1/2)*sin(1/9*pi))) + 1/18*(10*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) -
 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqrt(3)*sin(4/9*pi)^5 + 2*cos(
4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*cos(4/9*pi)*sin(4/9*pi)^4
+ 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi)^2 - sin(4/9*pi)^2)*log((
-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(
2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + 2*sqrt...
```

Mupad [B] (verification not implemented)

Time = 19.85 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^5\left(1-x^3+x^6\right)}\,dx$$
$$= \frac{\ln\left(-x+\left(162\,x+\frac{27\,\left(36+\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+\frac{\ln\left(-x-\left(162\,x+\frac{27\,\left(36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{x^3+\frac{1}{4}}{x^4}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(1/(x^5*(x^6 - x^3 + 1)),x)
```

output
```
(log((162*x + (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162)
- x)*(3^(1/2)*12i + 36)^(1/3))/18 + (log(- x - (162*x + (27*(36 - 3^(1/2)*
12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 -
(x^3 + 1/4)/x^4 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3)
)/12 - (2^(1/3)*3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/
3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)
*1i + 3)^(2/3))/12 + (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/
2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^
(2/3)*(3 - 3^(1/2)*1i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)
*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3
^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

Reduce [F]

$$\int \frac{1}{x^5 (1 - x^3 + x^6)} \, dx = \frac{-4\left(\int \frac{x^4}{x^6 - x^3 + 1} dx\right) x^4 - 4x^3 - 1}{4x^4}$$

input
```
int(1/x^5/(x^6-x^3+1),x)
```

output
```
( - 4*int(x**4/(x**6 - x**3 + 1),x)*x**4 - 4*x**3 - 1)/(4*x**4)
```

3.167 $\int \frac{1}{2+x^3+x^6}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 381

$$\int \frac{1}{2+x^3+x^6}\,dx = \frac{i\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\left(1-i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}} - \frac{i\log\left(\left(1+i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}}$$

output
```
1/21*I*arctan(1/3*(1-2*x/(1/2-1/2*I*7^(1/2))^(1/3))*3^(1/2))*21^(1/2)/(1/2
-1/2*I*7^(1/2))^(2/3)-1/21*I*arctan(1/3*(1-2*x/(1/2+1/2*I*7^(1/2))^(1/3))*
3^(1/2))*21^(1/2)/(1/2+1/2*I*7^(1/2))^(2/3)-1/21*I*ln((1-I*7^(1/2))^(1/3)+
2^(1/3)*x)*7^(1/2)/(1/2-1/2*I*7^(1/2))^(2/3)+1/21*I*ln((1+I*7^(1/2))^(1/3)
+2^(1/3)*x)*7^(1/2)/(1/2+1/2*I*7^(1/2))^(2/3)+1/42*I*ln((1-I*7^(1/2))^(2/3
)-(2-2*I*7^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(2/3)*7^(1/2)/(1-I*7^(1/2))^(2/3)
-1/42*I*ln((1+I*7^(1/2))^(2/3)-(2+2*I*7^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(2/3
)*7^(1/2)/(1+I*7^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.10

$$\int \frac{1}{2+x^3+x^6}\,dx = \frac{1}{3}\text{RootSum}\left[2+\#1^3+\#1^6\&,\frac{\log(x-\#1)}{\#1^2+2\#1^5}\&\right]$$

input
```
Integrate[(2 + x^3 + x^6)^(-1),x]
```

output
```
RootSum[2 + #1^3 + #1^6 & , Log[x - #1]/(#1^2 + 2*#1^5) & ]/3
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {1685, 750, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6+x^3+2}\,dx$$

$$\Big\downarrow \; 1685$$

$$\frac{i\int \frac{1}{x^3+\frac{1}{2}\left(1+i\sqrt{7}\right)}dx}{\sqrt{7}}-\frac{i\int \frac{1}{x^3+\frac{1}{2}\left(1-i\sqrt{7}\right)}dx}{\sqrt{7}}$$

$$\Big\downarrow \; 750$$

$$\frac{i\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}-\frac{i\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}$$

$$\Big\downarrow \; 16$$

$$\frac{i\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}-\frac{i\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}$$

$$\Big\downarrow \; 1142$$

$$\frac{i\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\int\frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx-\frac{1}{2}\int-\frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\right.}{3\left(\frac{1}{2}\left(1+\right.\right.}\right.}{\sqrt{7}}$$

$$\frac{i\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\int\frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx-\frac{1}{2}\int-\frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\right.}{3\left(\frac{1}{2}\left(1-\right.\right.}\right.}{\sqrt{7}}$$

$$\Big\downarrow \; 25$$

$$\frac{i\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\int\frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{\right.}}{3\left(\frac{1}{2}\left(1+i\right.\right.}\right.}{\sqrt{7}}$$

$$\frac{i\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\int\frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{\right.}}{3\left(\frac{1}{2}\left(1-i\right.\right.}\right.}{\sqrt{7}}$$

$$\Big\downarrow \; 1082$$

$$\frac{i\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1-\cdots}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\cdots\right.}\cdots\right.}{\sqrt{7}}$$

$$\frac{i\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1-\cdots}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\cdots\right.}\cdots\right.}{\sqrt{7}}$$

$$\Big\downarrow \ 217$$

$$\frac{i\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}-\frac{i\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}$$

$$\Big\downarrow \ 1103$$

$$\frac{i\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}-\frac{i\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right)}{\sqrt{7}}$$

input
```
Int[(2 + x^3 + x^6)^(-1),x]
```

output
```
((-I)*(Log[(1 - I*Sqrt[7])^(1/3) + 2^(1/3)*x]/(3*((1 - I*Sqrt[7])/2)^(2/3)
) + (-(Sqrt[3]*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3))/Sqrt[3]]) - Lo
g[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3
*((1 - I*Sqrt[7])/2)^(2/3))))/Sqrt[7] + (I*(Log[(1 + I*Sqrt[7])^(1/3) + 2^
(1/3)*x]/(3*((1 + I*Sqrt[7])/2)^(2/3)) + (-(Sqrt[3]*ArcTan[(1 - (2*x)/((1
+ I*Sqrt[7])/2)^(1/3))/Sqrt[3]]) - Log[(1 + I*Sqrt[7])^(2/3) - (2*(1 + I*S
qrt[7]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[7])/2)^(2/3))))/Sqrt[7]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}+\_Z^{3}+2\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}+\_R^{2}}\right)}{3}$	33
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}+\_Z^{3}+2\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}+\_R^{2}}\right)}{3}$	33

input
```
int(1/(x^6+x^3+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(1/(2*_R^5+_R^2)*ln(x-_R),_R=RootOf(_Z^6+_Z^3+2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.66

$$\int \frac{1}{2+x^3+x^6}\,dx = -\frac{1}{12}\left(\frac{3}{7}\sqrt{-\frac{1}{7}}-\frac{1}{7}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(7\left(\sqrt{-\frac{1}{7}}\left(\sqrt{-3}+1\right)-\sqrt{-3}-1\right)\left(\frac{3}{7}\sqrt{-\frac{1}{7}}-\frac{1}{7}\right)^{\frac{1}{3}}+8\,x\right) + \frac{1}{12}\left(\frac{3}{7}\sqrt{-\frac{1}{7}}-\frac{1}{7}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(-7\left(\sqrt{-\frac{1}{7}}\left(\sqrt{-3}-1\right)-\sqrt{-3}+1\right)\left(\frac{3}{7}\sqrt{-\frac{1}{7}}-\frac{1}{7}\right)^{\frac{1}{3}}+8\,x\right) - \frac{1}{12}\left(-\frac{3}{7}\sqrt{-\frac{1}{7}}-\frac{1}{7}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(-7\left(\sqrt{-\frac{1}{7}}\left(\sqrt{-3}+1\right)+\sqrt{-3}+1\right)\left(-\frac{3}{7}\sqrt{-\frac{1}{7}}-\frac{1}{7}\right)^{\frac{1}{3}}+8\,x\right) + \frac{1}{12}\left(-\frac{3}{7}\sqrt{-\frac{1}{7}}-\frac{1}{7}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(7\left(\sqrt{-\frac{1}{7}}\left(\sqrt{-3}-1\right)+\sqrt{-3}-1\right)\left(-\frac{3}{7}\sqrt{-\frac{1}{7}}-\frac{1}{7}\right)^{\frac{1}{3}}+8\,x\right) + \frac{1}{6}\left(\frac{3}{7}\sqrt{-\frac{1}{7}}-\frac{1}{7}\right)^{\frac{1}{3}}\log\left(-7\left(\sqrt{-\frac{1}{7}}-1\right)\left(\frac{3}{7}\sqrt{-\frac{1}{7}}-\frac{1}{7}\right)^{\frac{1}{3}}+4\,x\right) + \frac{1}{6}\left(-\frac{3}{7}\sqrt{-\frac{1}{7}}-\frac{1}{7}\right)^{\frac{1}{3}}\log\left(7\left(\sqrt{-\frac{1}{7}}+1\right)\left(-\frac{3}{7}\sqrt{-\frac{1}{7}}-\frac{1}{7}\right)^{\frac{1}{3}}+4\,x\right)$$

input
```
integrate(1/(x^6+x^3+2),x, algorithm="fricas")
```

output
```
-1/12*(3/7*sqrt(-1/7) - 1/7)^(1/3)*(sqrt(-3) + 1)*log(7*(sqrt(-1/7)*(sqrt(
-3) + 1) - sqrt(-3) - 1)*(3/7*sqrt(-1/7) - 1/7)^(1/3) + 8*x) + 1/12*(3/7*s
qrt(-1/7) - 1/7)^(1/3)*(sqrt(-3) - 1)*log(-7*(sqrt(-1/7)*(sqrt(-3) - 1) -
sqrt(-3) + 1)*(3/7*sqrt(-1/7) - 1/7)^(1/3) + 8*x) - 1/12*(-3/7*sqrt(-1/7)
- 1/7)^(1/3)*(sqrt(-3) + 1)*log(-7*(sqrt(-1/7)*(sqrt(-3) + 1) + sqrt(-3) +
 1)*(-3/7*sqrt(-1/7) - 1/7)^(1/3) + 8*x) + 1/12*(-3/7*sqrt(-1/7) - 1/7)^(1
/3)*(sqrt(-3) - 1)*log(7*(sqrt(-1/7)*(sqrt(-3) - 1) + sqrt(-3) - 1)*(-3/7*
sqrt(-1/7) - 1/7)^(1/3) + 8*x) + 1/6*(3/7*sqrt(-1/7) - 1/7)^(1/3)*log(-7*(
sqrt(-1/7) - 1)*(3/7*sqrt(-1/7) - 1/7)^(1/3) + 4*x) + 1/6*(-3/7*sqrt(-1/7)
 - 1/7)^(1/3)*log(7*(sqrt(-1/7) + 1)*(-3/7*sqrt(-1/7) - 1/7)^(1/3) + 4*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{1}{2+x^3+x^6}\,dx = \operatorname{RootSum}\left(1000188t^6+1323t^3+1,\left(t\mapsto t\log\left(-5292t^4+7t+x\right)\right)\right)$$

input
```
integrate(1/(x**6+x**3+2),x)
```

output
```
RootSum(1000188*_t**6 + 1323*_t**3 + 1, Lambda(_t, _t*log(-5292*_t**4 + 7*
_t + x)))
```

Maxima [F]

$$\int \frac{1}{2+x^3+x^6}\,dx = \int \frac{1}{x^6+x^3+2}\,dx$$

input
```
integrate(1/(x^6+x^3+2),x, algorithm="maxima")
```

output
```
integrate(1/(x^6 + x^3 + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{2+x^3+x^6}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(x^6+x^3+2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Invalid _EXT in replace_ext Error: Bad Argument Valuein
tegrate(1/(sageVARx^6+sageVARx^3+2),sageVARx)
```

Mupad [B] (verification not implemented)

Time = 20.86 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.35

$$\int \frac{1}{2+x^3+x^6}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3 + x^6 + 2),x)
```

output
```
(log(x + (7^(1/3)*(- 7^(1/2)*3i - 7)^(1/3))/4 + (7^(5/6)*(- 7^(1/2)*3i - 7
)^(1/3)*1i)/28)*(- 7^(1/2)*21i - 49)^(1/3))/42 + (log(x + (7^(1/3)*(7^(1/2
)*3i - 7)^(1/3))/4 - (7^(5/6)*(7^(1/2)*3i - 7)^(1/3)*1i)/28)*(7^(1/2)*21i
- 49)^(1/3))/42 + (7^(1/3)*log(6*x + (7^(1/3)*(3^(1/2)*1i - 1)*(- 7^(1/2)*
3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i - 1)^2*(- 7^(1/2)*3i - 7)^(2/3)*(3969*
x + (567*7^(1/3)*(3^(1/2)*1i - 1)*(- 7^(1/2)*3i - 7)^(1/3))/2))/7056 + 63)
)/84)*(3^(1/2)*1i - 1)*(- 7^(1/2)*3i - 7)^(1/3))/84 + (7^(1/3)*log(6*x + (
7^(1/3)*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i - 1)
^2*(7^(1/2)*3i - 7)^(2/3)*(3969*x + (567*7^(1/3)*(3^(1/2)*1i - 1)*(7^(1/2)
*3i - 7)^(1/3))/2))/7056 + 63))/84)*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^(1/3
))/84 - (7^(1/3)*log(6*x - (7^(1/3)*(3^(1/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1
/3)*((7^(2/3)*(3^(1/2)*1i + 1)^2*(- 7^(1/2)*3i - 7)^(2/3)*(3969*x - (567*7
^(1/3)*(3^(1/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1/3))/2))/7056 + 63))/84)*(3^(
1/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1/3))/84 - (7^(1/3)*log(6*x - (7^(1/3)*(3
^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i + 1)^2*(7^(1/2
)*3i - 7)^(2/3)*(3969*x - (567*7^(1/3)*(3^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(
1/3))/2))/7056 + 63))/84)*(3^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(1/3))/84
```

Reduce [F]

$$\int \frac{1}{2+x^3+x^6}\,dx = \int \frac{1}{x^6+x^3+2}dx$$

input
```
int(1/(x^6+x^3+2),x)
```

output
```
int(1/(x**6 + x**3 + 2),x)
```

3.168 $\int \frac{x^2}{2+x^3+x^6} dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x^2}{2+x^3+x^6}\,dx=\frac{2\arctan\left(\frac{1+2x^3}{\sqrt{7}}\right)}{3\sqrt{7}}$$

output
```
2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{2+x^3+x^6}\,dx=\frac{2\arctan\left(\frac{1+2x^3}{\sqrt{7}}\right)}{3\sqrt{7}}$$

input
```
Integrate[x^2/(2 + x^3 + x^6),x]
```

output
```
(2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1690, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^6+x^3+2}dx$$
$$\Big\downarrow \text{ 1690}$$
$$\frac{1}{3}\int \frac{1}{x^6+x^3+2}dx^3$$
$$\Big\downarrow \text{ 1083}$$
$$-\frac{2}{3}\int \frac{1}{-x^6-7}d\left(2x^3+1\right)$$
$$\Big\downarrow \text{ 217}$$
$$\frac{2\arctan\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

input
```
Int[x^2/(2 + x^3 + x^6),x]
```

output
```
(2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2 \arctan\left(\frac{\left(2x^3+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{21}$	19
risch	$\frac{2 \arctan\left(\frac{\left(2x^3+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{21}$	19

input
```
int(x^2/(x^6+x^3+2),x,method=_RETURNVERBOSE)
```

output
```
2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{2}{21}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^3+1)\right)$$

input
```
integrate(x^2/(x^6+x^3+2),x, algorithm="fricas")
```

output
```
2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{2\sqrt{7}\,\text{atan}\left(\frac{2\sqrt{7}x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

input
```
integrate(x**2/(x**6+x**3+2),x)
```

output
```
2*sqrt(7)*atan(2*sqrt(7)*x**3/7 + sqrt(7)/7)/21
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{2}{21}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^3+1)\right)$$

input
```
integrate(x^2/(x^6+x^3+2),x, algorithm="maxima")
```

output
```
2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{2}{21}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^3+1)\right)$$

input
```
integrate(x^2/(x^6+x^3+2),x, algorithm="giac")
```

output
```
2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}\,x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

input
```
int(x^2/(x^3 + x^6 + 2),x)
```

output
```
(2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^3)/7))/21
```

Reduce [F]

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \int \frac{x^2}{x^6+x^3+2}dx$$

input
```
int(x^2/(x^6+x^3+2),x)
```

output
```
int(x**2/(x**6 + x**3 + 2),x)
```

3.169 $\int \frac{x^3}{2+x^3+x^6} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 399

$$\int \frac{x^3}{2+x^3+x^6}\,dx = -\frac{i\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}+\frac{i\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}+\frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}+\frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}-\frac{\left(7+i\sqrt{7}\right)\log\left(\left(1-i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{\left(7-i\sqrt{7}\right)\log\left(\left(1+i\sqrt{7}\right)^{2/3}-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+2^{2/3}x^2\right)}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

output
```
-1/21*I*(1/2-1/2*I*7^(1/2))^(1/3)*arctan(1/3*(1-2*x/(1/2-1/2*I*7^(1/2))^(1
/3))*3^(1/2))*21^(1/2)+1/21*I*(1/2+1/2*I*7^(1/2))^(1/3)*arctan(1/3*(1-2*x/
(1/2+1/2*I*7^(1/2))^(1/3))*3^(1/2))*21^(1/2)+1/42*(7+I*7^(1/2))*ln((1-I*7^
(1/2))^(1/3)+2^(1/3)*x)*2^(2/3)/(1-I*7^(1/2))^(2/3)+1/42*(7-I*7^(1/2))*ln(
(1+I*7^(1/2))^(1/3)+2^(1/3)*x)*2^(2/3)/(1+I*7^(1/2))^(2/3)-1/84*(7+I*7^(1/
2))*ln((1-I*7^(1/2))^(2/3)-(2-2*I*7^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(2/3)/(1
-I*7^(1/2))^(2/3)-1/84*(7-I*7^(1/2))*ln((1+I*7^(1/2))^(2/3)-(2+2*I*7^(1/2)
)^(1/3)*x+2^(2/3)*x^2)*2^(2/3)/(1+I*7^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.09

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \frac{1}{3}\text{RootSum}\left[2+\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{1+2\#1^3}\&\right]$$

input
```
Integrate[x^3/(2 + x^3 + x^6),x]
```

output
```
RootSum[2 + #1^3 + #1^6 & , (Log[x - #1]*#1)/(1 + 2*#1^3) & ]/3
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1710, 750, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^6+x^3+2}\,dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\int \frac{1}{x^3+\frac{1}{2}\left(1-i\sqrt{7}\right)}dx+\frac{1}{14}\left(7-i\sqrt{7}\right)\int \frac{1}{x^3+\frac{1}{2}\left(1+i\sqrt{7}\right)}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right)+\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\int \frac{1}{x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right)$$

$$\downarrow 16$$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right)+\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{7}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right)$$

$$\downarrow 1142$$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\int\frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx-\frac{1}{2}\int-\frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right.\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right.$$

$$\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\int\frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx-\frac{1}{2}\int-\frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right.\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right.$$

$$\Big\downarrow 25$$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\int\frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right.$$

$$\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\int\frac{1}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right.$$

$$\Big\downarrow 1082$$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{\right.}}}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}\right.$$

$$\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{\right.}}}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{\cdots}\right.}{3\left(\frac{1}{2}\left(1-i\cdots\right.\right.}\right.$$

$$\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}-2x}{x^2-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\left(\sqrt[3]{2}x+\sqrt[3]{\cdots}\right.}{3\left(\frac{1}{2}\left(1+i\cdots\right.\right.}\right.$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{14}\left(7+i\sqrt{7}\right)\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\cdots}{3\cdots}\right.$$

$$\frac{1}{14}\left(7-i\sqrt{7}\right)\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{\log\cdots}{3\cdots}\right.$$

input
```
Int[x^3/(2 + x^3 + x^6),x]
```

output
```
((7 + I*Sqrt[7])*(Log[(1 - I*Sqrt[7])^(1/3) + 2^(1/3)*x]/(3*((1 - I*Sqrt[7
])/2)^(2/3)) + (-(Sqrt[3]*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3))/Sqr
t[3]]) - Log[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)
*x^2]/2)/(3*((1 - I*Sqrt[7])/2)^(2/3))))/14 + ((7 - I*Sqrt[7])*(Log[(1 + I
*Sqrt[7])^(1/3) + 2^(1/3)*x]/(3*((1 + I*Sqrt[7])/2)^(2/3)) + (-(Sqrt[3]*Ar
cTan[(1 - (2*x)/((1 + I*Sqrt[7])/2)^(1/3))/Sqrt[3]]) - Log[(1 + I*Sqrt[7])
^(2/3) - (2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[7])
/2)^(2/3))))/14
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}+\_Z^{3}+2\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}+\_R^{2}}\right)}{3}$	36
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}+\_Z^{3}+2\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}+\_R^{2}}\right)}{3}$	36

input
```
int(x^3/(x^6+x^3+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R^3/(2*_R^5+_R^2)*ln(x-_R),_R=RootOf(_Z^6+_Z^3+2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.54

$$\int \frac{x^3}{2+x^3+x^6}\,dx$$
$$= -\frac{1}{6}\left(\frac{1}{14}\sqrt{-\frac{1}{7}}-\frac{1}{14}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(-7\sqrt{-\frac{1}{7}}\left(\frac{1}{14}\sqrt{-\frac{1}{7}}-\frac{1}{14}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)+2x\right)$$
$$-\frac{1}{6}\left(-\frac{1}{14}\sqrt{-\frac{1}{7}}-\frac{1}{14}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(7\sqrt{-\frac{1}{7}}\left(-\frac{1}{14}\sqrt{-\frac{1}{7}}-\frac{1}{14}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)+2x\right)$$
$$+\frac{1}{6}\left(\frac{1}{14}\sqrt{-\frac{1}{7}}-\frac{1}{14}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(7\sqrt{-\frac{1}{7}}\left(\frac{1}{14}\sqrt{-\frac{1}{7}}-\frac{1}{14}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)+2x\right)$$
$$+\frac{1}{6}\left(-\frac{1}{14}\sqrt{-\frac{1}{7}}-\frac{1}{14}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(-7\sqrt{-\frac{1}{7}}\left(-\frac{1}{14}\sqrt{-\frac{1}{7}}-\frac{1}{14}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)+2x\right)+\frac{1}{3}\left(\frac{1}{14}\sqrt{-\frac{1}{7}}-\frac{1}{14}\right)^{\frac{1}{3}}\log\left(x+7\sqrt{-\frac{1}{7}}\left(\frac{1}{14}\sqrt{-\frac{1}{7}}-\frac{1}{14}\right)^{\frac{1}{3}}\right)$$
$$+\frac{1}{3}\left(-\frac{1}{14}\sqrt{-\frac{1}{7}}-\frac{1}{14}\right)^{\frac{1}{3}}\log\left(x-7\sqrt{-\frac{1}{7}}\left(-\frac{1}{14}\sqrt{-\frac{1}{7}}-\frac{1}{14}\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^3/(x^6+x^3+2),x, algorithm="fricas")
```

output
```
-1/6*(1/14*sqrt(-1/7) - 1/14)^(1/3)*(sqrt(-3) + 1)*log(-7*sqrt(-1/7)*(1/14
*sqrt(-1/7) - 1/14)^(1/3)*(sqrt(-3) + 1) + 2*x) - 1/6*(-1/14*sqrt(-1/7) -
1/14)^(1/3)*(sqrt(-3) + 1)*log(7*sqrt(-1/7)*(-1/14*sqrt(-1/7) - 1/14)^(1/3
)*(sqrt(-3) + 1) + 2*x) + 1/6*(1/14*sqrt(-1/7) - 1/14)^(1/3)*(sqrt(-3) - 1
)*log(7*sqrt(-1/7)*(1/14*sqrt(-1/7) - 1/14)^(1/3)*(sqrt(-3) - 1) + 2*x) +
1/6*(-1/14*sqrt(-1/7) - 1/14)^(1/3)*(sqrt(-3) - 1)*log(-7*sqrt(-1/7)*(-1/1
4*sqrt(-1/7) - 1/14)^(1/3)*(sqrt(-3) - 1) + 2*x) + 1/3*(1/14*sqrt(-1/7) -
1/14)^(1/3)*log(x + 7*sqrt(-1/7)*(1/14*sqrt(-1/7) - 1/14)^(1/3)) + 1/3*(-1
/14*sqrt(-1/7) - 1/14)^(1/3)*log(x - 7*sqrt(-1/7)*(-1/14*sqrt(-1/7) - 1/14
)^(1/3))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \operatorname{RootSum}\left(250047t^6+1323t^3+2,\left(t\mapsto t\log\left(7938t^4+21t+x\right)\right)\right)$$

input
```
integrate(x**3/(x**6+x**3+2),x)
```

output
```
RootSum(250047*_t**6 + 1323*_t**3 + 2, Lambda(_t, _t*log(7938*_t**4 + 21*_
t + x)))
```

Maxima [F]

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \int \frac{x^3}{x^6+x^3+2}\,dx$$

input
```
integrate(x^3/(x^6+x^3+2),x, algorithm="maxima")
```

output
```
integrate(x^3/(x^6 + x^3 + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(x^6+x^3+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Invalid _EXT in replace_ext Error:
Bad Argument ValueDone
```

Mupad [B] (verification not implemented)

Time = 20.80 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.88

$$\begin{aligned}
\int \frac{x^3}{2+x^3+x^6}\,dx &= \frac{\ln\left(x-\frac{2^{2/3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{14}\right)\,\left(-196-\sqrt{7}\,28\mathrm{i}\right)^{1/3}}{42} \\
&+ \frac{2^{2/3}\,7^{1/3}\,\ln\left(x+\frac{2^{2/3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{14}\right)\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{42} \\
&- \frac{2^{2/3}\,7^{1/3}\,\ln\left(x+\frac{2^{2/3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}-\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{28}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{84} \\
&+ \frac{2^{2/3}\,7^{1/3}\,\ln\left(x+\frac{2^{2/3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}+\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{28}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7-\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{84} \\
&+ \frac{2^{2/3}\,7^{1/3}\,\ln\left(x-\frac{2^{2/3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}-\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{28}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{84} \\
&- \frac{2^{2/3}\,7^{1/3}\,\ln\left(x-\frac{2^{2/3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{28}+\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{28}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-7+\sqrt{7}\,1\mathrm{i}\right)^{1/3}}{84}
\end{aligned}$$

input
```
int(x^3/(x^3 + x^6 + 2),x)
```

output
```
(log(x - (2^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/14)*(- 7^(1/2)*28i
- 196)^(1/3))/42 + (2^(2/3)*7^(1/3)*log(x + (2^(2/3)*7^(5/6)*(7^(1/2)*1i -
 7)^(1/3)*1i)/14)*(7^(1/2)*1i - 7)^(1/3))/42 - (2^(2/3)*7^(1/3)*log(x + (2
^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/28 - (2^(2/3)*3^(1/2)*7^(5/6)*
(- 7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i + 1)*(- 7^(1/2)*1i - 7)^(1/3))/8
4 + (2^(2/3)*7^(1/3)*log(x + (2^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)
/28 + (2^(2/3)*3^(1/2)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i -
 1)*(- 7^(1/2)*1i - 7)^(1/3))/84 + (2^(2/3)*7^(1/3)*log(x - (2^(2/3)*7^(5/
6)*(7^(1/2)*1i - 7)^(1/3)*1i)/28 - (2^(2/3)*3^(1/2)*7^(5/6)*(7^(1/2)*1i -
7)^(1/3))/28)*(3^(1/2)*1i - 1)*(7^(1/2)*1i - 7)^(1/3))/84 - (2^(2/3)*7^(1/
3)*log(x - (2^(2/3)*7^(5/6)*(7^(1/2)*1i - 7)^(1/3)*1i)/28 + (2^(2/3)*3^(1/
2)*7^(5/6)*(7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i + 1)*(7^(1/2)*1i - 7)^(
1/3))/84
```

Reduce [F]

$$\int \frac{x^3}{2+x^3+x^6}\,dx=\int \frac{x^3}{x^6+x^3+2}dx$$

input
```
int(x^3/(x^6+x^3+2),x)
```

output
```
int(x**3/(x**6 + x**3 + 2),x)
```

3.170 $\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 231

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = \frac{(21b^4-56ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2} + \frac{x^9(a+bx^3+cx^6)^{3/2}}{18c} - \frac{(7b(15b^2-28ac)-6c(21b^2-20ac)x^3)(a+bx^3+cx^6)^{3/2}}{2880c^4} - \frac{(b^2-4ac)(21b^4-56ab^2c+16a^2c^2)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}}$$

output

```
1/1536*(16*a^2*c^2-56*a*b^2*c+21*b^4)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^
5-1/20*b*x^6*(c*x^6+b*x^3+a)^(3/2)/c^2+1/18*x^9*(c*x^6+b*x^3+a)^(3/2)/c-1/
2880*(7*b*(-28*a*c+15*b^2)-6*c*(-20*a*c+21*b^2)*x^3)*(c*x^6+b*x^3+a)^(3/2)
/c^4-1/3072*(-4*a*c+b^2)*(16*a^2*c^2-56*a*b^2*c+21*b^4)*arctanh(1/2*(2*c*x
^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.89

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}(315b^5-210b^4cx^3+16b^2c^2x^3(56a-9cx^6)+168b^3c(-10a+cx^6)+16bc^2(113a^2-3$$

input
```
Integrate[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(315*b^5 - 210*b^4*c*x^3 + 16*b^2*c^2*x
^3*(56*a - 9*c*x^6) + 168*b^3*c*(-10*a + c*x^6) + 16*b*c^2*(113*a^2 - 34*a
*c*x^6 + 8*c^2*x^12) + 160*c^3*x^3*(-3*a^2 + 2*a*c*x^6 + 8*c^2*x^12)) + 15
*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*Log[b + 2*c*x^3 - 2
*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(46080*c^(11/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1693, 1166, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^{12}\sqrt{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{3}{2}x^6\left(3bx^3+2a\right)\sqrt{cx^6+bx^3+a}dx^3}{6c}+\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\int x^6\left(3bx^3+2a\right)\sqrt{cx^6+bx^3+a}dx^3}{4c}\right)$$

$\big\downarrow$ 1236

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\frac{\int -\frac{1}{2}x^3\left(\left(21b^2-20ac\right)x^3+12ab\right)\sqrt{cx^6+bx^3+a}dx^3}{5c}+\frac{3bx^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}}{4c}\right)$$

$\big\downarrow$ 27

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\frac{3bx^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\int x^3\left(\left(21b^2-20ac\right)x^3+12ab\right)\sqrt{cx^6+bx^3+a}dx^3}{10c}}{4c}\right)$$

$\big\downarrow$ 1225

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\frac{3bx^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5\left(16a^2c^2-56ab^2c+21b^4\right)\int\sqrt{cx^6+bx^3+a}dx^3}{16c^2}-\frac{\left(7b\left(15b^2-28ac\right)-6cx^3\left(21b^2-20ac\right)\right)\left(a-\right.}{24c^2}}{10c}}{4c}\right.$$

$\big\downarrow$ 1087

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\frac{3bx^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5\left(16a^2c^2-56ab^2c+21b^4\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{16c^2}-}{10c}}{4c}\right.$$

$\big\downarrow$ 1092

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\frac{3bx^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5\left(16a^2c^2-56ab^2c+21b^4\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right.}{16c^2}}{10c}}{4c}\right.$$

$\big\downarrow$ 219

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{3/2}}{6c}-\frac{\frac{3bx^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5\left(16a^2c^2-56ab^2c+21b^4\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3}}\right.}{8c^{3/2}}\right.}{16c^2}}{10c}}{4c}\right.$$

input
```
Int[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((x^9*(a + b*x^3 + c*x^6)^(3/2))/(6*c) - ((3*b*x^6*(a + b*x^3 + c*x^6)^(3/
2))/(5*c) - (-1/24*((7*b*(15*b^2 - 28*a*c) - 6*c*(21*b^2 - 20*a*c)*x^3)*(a
 + b*x^3 + c*x^6)^(3/2))/c^2 + (5*(21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*(((b
+ 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*
c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2))))/(16*c^2))/(10*c
))/(4*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{14}\sqrt{c\,x^{6}+b\,x^{3}+a}dx$$

input
```
int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.95

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$

$$= \left[-\frac{15\,(21\,b^6-140\,ab^4c+240\,a^2b^2c^2-64\,a^3c^3)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3\right.}{}\right.$$

input
```
integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/92160*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c
)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 +
b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*(9*b^2*c^4 -
20*a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c - 1680*a*b^3*c
^2 + 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*x^3)*s
qrt(c*x^6 + b*x^3 + a))/c^6, 1/46080*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b
^2*c^2 - 64*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
+ b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(1280*c^6*x^15 + 128*b*c^5*x^
12 - 16*(9*b^2*c^4 - 20*a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315
*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3
+ 240*a^2*c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

Sympy [F]

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = \int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x**14*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**14*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}\,x^{14}\,dx$$

input
```
integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x^14, x)
```

Mupad [B] (verification not implemented)

Time = 21.10 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.35

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = \frac{x^9\,(c\,x^6+b\,x^3+a)^{3/2}}{18\,c}$$

$$-\frac{b\left(\frac{x^6\,(c\,x^6+b\,x^3+a)^{3/2}}{5\,c}+\frac{7\,b\left(\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^3\,(c\,x^6+b\,x^3+a)^{3/2}}{4\,c}+\frac{5\,b\left(\frac{\left(8\,c\left(c\right.\right.}{}\right.}{}\right.}{10\,c}\right.}{}$$

$$+\frac{a\left(\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^3\,(c\,x^6+b\,x^3+a)^{3/2}}{4\,c}+\frac{5\,b\left(\frac{\left(8\,c\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\sqrt{}}{24\,c^2}\right.}{}\right.}{6\,c}$$

input
```
int(x^14*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
(x^9*(a + b*x^3 + c*x^6)^(3/2))/(18*c) - (b*((x^6*(a + b*x^3 + c*x^6)^(3/2
))/(5*c) + (7*b*((a*((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a
 + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2)
)))/(4*c) - (x^3*(a + b*x^3 + c*x^6)^(3/2))/(4*c) + (5*b*(((8*c*(a + c*x^6
) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b
*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
))/(8*c)))/(10*c) - (2*a*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^
3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x
^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(5*c)))/(4*c) + (a*((a*((b/(4
*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) +
(b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^3*(a + b*x^
3 + c*x^6)^(3/2))/(4*c) + (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a
+ b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b +
 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(6*c)
```

Reduce [F]

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
( - 1920*sqrt(a + b*x**3 + c*x**6)*a**3*c**4 + 10816*sqrt(a + b*x**3 + c*x
**6)*a**2*b**2*c**3 - 960*sqrt(a + b*x**3 + c*x**6)*a**2*b*c**4*x**3 - 756
0*sqrt(a + b*x**3 + c*x**6)*a*b**4*c**2 + 1792*sqrt(a + b*x**3 + c*x**6)*a
*b**3*c**3*x**3 - 1088*sqrt(a + b*x**3 + c*x**6)*a*b**2*c**4*x**6 + 640*sq
rt(a + b*x**3 + c*x**6)*a*b*c**5*x**9 + 1260*sqrt(a + b*x**3 + c*x**6)*b**
6*c - 420*sqrt(a + b*x**3 + c*x**6)*b**5*c**2*x**3 + 336*sqrt(a + b*x**3 +
 c*x**6)*b**4*c**3*x**6 - 288*sqrt(a + b*x**3 + c*x**6)*b**3*c**4*x**9 + 2
56*sqrt(a + b*x**3 + c*x**6)*b**2*c**5*x**12 + 2560*sqrt(a + b*x**3 + c*x*
*6)*b*c**6*x**15 - 960*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**
3)*a**3*b*c**3 + 3600*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3
)*a**2*b**3*c**2 - 2100*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x*
*3)*a*b**5*c + 315*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*b
**7 + 960*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a**3*b*c**
3 - 3600*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a**2*b**3*c
**2 + 2100*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*b**5*c
- 315*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**7 + 5760*in
t((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x*
*6 + b*c*x**9),x)*a**3*b*c**5 - 21600*int((sqrt(a + b*x**3 + c*x**6)*x**8)
/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a**2*b**3*c**4 +
 12600*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x*...
```

3.171 $\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 171

$$\begin{aligned}\int x^{11}\sqrt{a+bx^3+cx^6}\,dx = &-\frac{b(7b^2-12ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4} \\ &+\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c} \\ &+\frac{\left(35b^2-32ac-42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3} \\ &+\frac{b(7b^2-12ac)\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}}\end{aligned}$$

output
```
-1/384*b*(-12*a*c+7*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^4+1/15*x^6*(c
*x^6+b*x^3+a)^(3/2)/c+1/720*(-42*b*c*x^3-32*a*c+35*b^2)*(c*x^6+b*x^3+a)^(3
/2)/c^3+1/768*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/
2)/(c*x^6+b*x^3+a)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$
$$=\frac{\sqrt{a+bx^3+cx^6}(-105b^4+70b^3cx^3+4b^2c(115a-14cx^6)+8bc^2x^3(-29a+6cx^6)+128c^2(-2a^2+acx^6}{5760c^4}$$
$$-\frac{(7b^5-40ab^3c+48a^2bc^2)\log\left(c^4\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)}{768c^{9/2}}$$

input
```
Integrate[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-105*b^4 + 70*b^3*c*x^3 + 4*b^2*c*(115*a - 14*c*
x^6) + 8*b*c^2*x^3*(-29*a + 6*c*x^6) + 128*c^2*(-2*a^2 + a*c*x^6 + 3*c^2*x
^12)))/(5760*c^4) - ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*Log[c^4*(b + 2*c*
x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(768*c^(9/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1693, 1166, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{1}{3}\int x^9\sqrt{cx^6+bx^3+a}dx^3$$
$$\Big\downarrow \text{1166}$$
$$\frac{1}{3}\left(\frac{\int -\frac{1}{2}x^3\left(7bx^3+4a\right)\sqrt{cx^6+bx^3+a}dx^3}{5c}+\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\int x^3\left(7bx^3+4a\right)\sqrt{cx^6+bx^3+a}dx^3}{10c}\right)$$

$$\Big\downarrow \ 1225$$

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5b(7b^2-12ac)\int\sqrt{cx^6+bx^3+a}dx^3}{16c^2}-\frac{\left(-32ac+35b^2-42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c^2}}{10c}\right)$$

$$\Big\downarrow \ 1087$$

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5b(7b^2-12ac)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{16c^2}-\frac{\left(-32ac+35b^2-42bcx^3\right)\left(a+bx^3+\right.}{24c^2}}{10c}\right.$$

$$\Big\downarrow \ 1092$$

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5b(7b^2-12ac)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right)}{16c^2}-\frac{\left(-32ac+35b^2-42bcx^3\right)\left(a+\right.}{24c^2}}{10c}\right.$$

$$\Big\downarrow \ 219$$

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{5c}-\frac{\frac{5b(7b^2-12ac)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)}{16c^2}-\frac{\left(-32ac+35b^2-42bcx\right.}{24c}}{10c}\right.$$

input
```
Int[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((x^6*(a + b*x^3 + c*x^6)^(3/2))/(5*c) - (-1/24*((35*b^2 - 32*a*c - 42*b*c
*x^3)*(a + b*x^3 + c*x^6)^(3/2))/c^2 + (5*b*(7*b^2 - 12*a*c)*(((b + 2*c*x^
3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(
2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2))))/(16*c^2))/(10*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{11}\sqrt{c\,x^{6}+b\,x^{3}+a}dx$$

input
```
int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.15

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$

$$= \left[\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+\right.$$

$$-\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(384\,c^5x^{12}+48\,bc^4x^9-8\,(7\,b^2c}{11520\,c^5}$$

input
```
integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 -
8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c)
 + 4*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a*c^4)*x^6 - 105*b^4
*c + 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^3)*sqrt(
c*x^6 + b*x^3 + a))/c^5, -1/11520*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*
sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^
6 + b*c*x^3 + a*c)) - 2*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a
*c^4)*x^6 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*
a*b*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

Sympy [F]

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx = \int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x**11*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**11*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}\,x^{11}\,dx$$

input
```
integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x^11, x)
```

Mupad [B] (verification not implemented)

Time = 19.84 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.84

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx=\frac{x^6\,(c\,x^6+b\,x^3+a)^{3/2}}{15\,c}+\frac{7\,b\left(\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^3\,(c\,x^6+b\,x^3+a)^{3/2}}{4\,c}+\frac{5\,b\left(\frac{(8\,c\,(c\,x^6+a)-3\,b^2+2\,b\,c\,x^3)\ldots}{24\,c^2}\ldots\right.}{\ldots}\right.}{30\,c\ldots}-\frac{2\,a\left(\frac{(8\,c\,(c\,x^6+a)-3\,b^2+2\,b\,c\,x^3)\,\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,(b^3-4\,a\,b\,c)}{16\,c^{5/2}}\right)}{15\,c}$$

input
```
int(x^11*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
(x^6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + (7*b*((a*((b/(4*c) + x^3/2)*(a +
b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1
/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^3*(a + b*x^3 + c*x^6)^(3/2))/
(4*c) + (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(
1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))
*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(30*c) - (2*a*(((8*c*(a + c*x^6)
- 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x
^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))
/(15*c)
```

Reduce [F]

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
( - 1952*sqrt(a + b*x**3 + c*x**6)*a**2*c**3 + 2120*sqrt(a + b*x**3 + c*x*
*6)*a*b**2*c**2 - 464*sqrt(a + b*x**3 + c*x**6)*a*b*c**3*x**3 + 256*sqrt(a
 + b*x**3 + c*x**6)*a*c**4*x**6 - 420*sqrt(a + b*x**3 + c*x**6)*b**4*c + 1
40*sqrt(a + b*x**3 + c*x**6)*b**3*c**2*x**3 - 112*sqrt(a + b*x**3 + c*x**6
)*b**2*c**3*x**6 + 96*sqrt(a + b*x**3 + c*x**6)*b*c**4*x**9 + 768*sqrt(a +
 b*x**3 + c*x**6)*c**5*x**12 - 720*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) -
 sqrt(c)*x**3)*a**2*b*c**2 + 600*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - s
qrt(c)*x**3)*a*b**3*c - 105*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c
)*x**3)*b**5 + 720*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a
**2*b*c**2 - 600*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*b
**3*c + 105*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**5 + 4
320*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b
**2*x**6 + b*c*x**9),x)*a**2*b*c**4 - 3600*int((sqrt(a + b*x**3 + c*x**6)*
x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a*b**3*c**3
 + 630*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6
+ b**2*x**6 + b*c*x**9),x)*b**5*c**2 + 4320*int((sqrt(a + b*x**3 + c*x**6)
*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a**3*c**4
- 1440*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6
+ b**2*x**6 + b*c*x**9),x)*a**2*b**2*c**3 - 1170*int((sqrt(a + b*x**3 + c*
x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a*...
```

3.172 $\int x^8\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 153

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx=\frac{(5b^2-4ac)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b(a+bx^3+cx^6)^{3/2}}{72c^2}+\frac{x^3(a+bx^3+cx^6)^{3/2}}{12c}-\frac{(b^2-4ac)(5b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}$$

output
```
1/192*(-4*a*c+5*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^3-5/72*b*(c*x^6+b
*x^3+a)^(3/2)/c^2+1/12*x^3*(c*x^6+b*x^3+a)^(3/2)/c-1/384*(-4*a*c+b^2)*(-4*
a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.86

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx=\frac{\sqrt{a+bx^3+cx^6}(15b^3-52abc-10b^2cx^3+24ac^2x^3+8bc^2x^6+48c^3x^9)}{576c^3}+\frac{(5b^4-24ab^2c+16a^2c^2)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{384c^{7/2}}$$

input
```
Integrate[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(15*b^3 - 52*a*b*c - 10*b^2*c*x^3 + 24*a*c^2*x^3
+ 8*b*c^2*x^6 + 48*c^3*x^9))/(576*c^3) + ((5*b^4 - 24*a*b^2*c + 16*a^2*c^2
)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(384*c^(7/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1693, 1166, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^6\sqrt{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{1}{2}\left(5bx^3+2a\right)\sqrt{cx^6+bx^3+a}dx^3}{4c}+\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{4c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{4c}-\frac{\int \left(5bx^3+2a\right)\sqrt{cx^6+bx^3+a}dx^3}{8c}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{\left(5b^2-4ac\right)\int \sqrt{cx^6+bx^3+a}dx^3}{2c}}{8c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{\left(5b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{2c}}{8c}\right)$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{\left(5b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right)}{2c}}{8c}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{\left(5b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)}{2c}}{8c}\right)$$

input
```
Int[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((x^3*(a + b*x^3 + c*x^6)^(3/2))/(4*c) - ((5*b*(a + b*x^3 + c*x^6)^(3/2))/
(3*c) - ((5*b^2 - 4*a*c)*(((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) -
((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])
/(8*c^(3/2))))/(2*c))/(8*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^8\sqrt{c\,x^6+b\,x^3+a}dx$$

input
```
int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.98

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx$$
$$= \left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+4}{2304\,c^4}\right.$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*
c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4
*(48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c
^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/1152*(3*(5*b^4 - 24*a*b^2*c + 16*
a^2*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c
)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52
*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

Sympy [F]

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx=\int x^8\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x**8*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**8*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx=\int\sqrt{cx^6+bx^3+a}x^8\,dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x^8, x)
```

Mupad [B] (verification not implemented)

Time = 19.63 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.26

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx$$
$$= \frac{x^3\,(c\,x^6+b\,x^3+a)^{3/2}}{12\,c} - \frac{a\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{12\,c} - \frac{5\,b\left(\frac{(8\,c\,(c\,x^6+a)-3\,b^2+2\,b\,c\,x^3)\,\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)(b^3-4\,a\,b\,c)}{16\,c^{5/2}}\right)}{24\,c}$$

input
```
int(x^8*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
(x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c) - (a*((b/(4*c) + x^3/2)*(a + b*x^3
+ c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(
a*c - b^2/4))/(2*c^(3/2))))/(12*c) - (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b
*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(
1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(24*c)
```

Reduce [F]

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx$$
$$= \frac{96\sqrt{c\,x^6+b\,x^3+a}\,a^2c^3 - 248\sqrt{c\,x^6+b\,x^3+a}\,a\,b^2c^2 + 48\sqrt{c\,x^6+b\,x^3+a}\,ab\,c^3x^3 + 60\sqrt{c\,x^6+b\,x^3+a}}{}$$

input
```
int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
(96*sqrt(a + b*x**3 + c*x**6)*a**2*c**3 - 248*sqrt(a + b*x**3 + c*x**6)*a*
b**2*c**2 + 48*sqrt(a + b*x**3 + c*x**6)*a*b*c**3*x**3 + 60*sqrt(a + b*x**
3 + c*x**6)*b**4*c - 20*sqrt(a + b*x**3 + c*x**6)*b**3*c**2*x**3 + 16*sqrt
(a + b*x**3 + c*x**6)*b**2*c**3*x**6 + 96*sqrt(a + b*x**3 + c*x**6)*b*c**4
*x**9 + 48*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a**2*b*c*
*2 - 72*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a*b**3*c + 1
5*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*b**5 - 48*sqrt(c)*
log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a**2*b*c**2 + 72*sqrt(c)*log
(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*b**3*c - 15*sqrt(c)*log(sqrt(
a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**5 - 288*int((sqrt(a + b*x**3 + c*x
**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a**2*b
*c**4 + 432*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*
x**6 + b**2*x**6 + b*c*x**9),x)*a*b**3*c**3 - 90*int((sqrt(a + b*x**3 + c*
x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*b**5*
c**2 - 288*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x
**6 + b**2*x**6 + b*c*x**9),x)*a**3*c**4 + 288*int((sqrt(a + b*x**3 + c*x*
*6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a**2*b*
*2*c**3 + 126*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*
c*x**6 + b**2*x**6 + b*c*x**9),x)*a*b**4*c**2 - 45*int((sqrt(a + b*x**3 +
c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*...
```

3.173 $\int x^5\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=-\frac{b(b+2cx^3)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{(a+bx^3+cx^6)^{3/2}}{9c}+\frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}$$

output
```
-1/24*b*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^2+1/9*(c*x^6+b*x^3+a)^(3/2)/c+
1/48*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))
/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\frac{\sqrt{a+bx^3+cx^6}(-3b^2+2bcx^3+8c(a+cx^6))}{72c^2}-\frac{(b^3-4abc)\log\left(c^2\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)}{48c^{5/2}}$$

input
```
Integrate[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 2*b*c*x^3 + 8*c*(a + c*x^6)))/(72*c^2)
- ((b^3 - 4*a*b*c)*Log[c^2*(b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6
])])/(48*c^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1693, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \ 1693$$

$$\frac{1}{3}\int x^3 \sqrt{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \ 1160$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{b\int \sqrt{cx^6+bx^3+a}dx^3}{2c}\right)$$

$$\Big\downarrow \ 1087$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{2c}\right)$$

$$\Big\downarrow \ 1092$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4ac)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right)}{2c}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{3}\left(\frac{(a+bx^3+cx^6)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)}{2c}\right)$$

input
```
Int[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((a + b*x^3 + c*x^6)^(3/2)/(3*c) - (b*(((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x
^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^
3 + c*x^6])])/(8*c^(3/2))))/(2*c))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^5\sqrt{c\,x^6+b\,x^3+a}dx$$

input
```
int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.19

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx$$
$$= \left[-\frac{3\,(b^3-4\,abc)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,(8\,c^3x^6+2}{288\,c^3}\right.$$
$$-\frac{3\,(b^3-4\,abc)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(8\,c^3x^6+2\,bc^2x^3-3\,b^2c+8\,ac^2)\sqrt{cx^6+bx^3+}}{144\,c^3}$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/288*(3*(b^3 - 4*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sq
rt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 + 2*b*
c^2*x^3 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^3, -1/144*(3*(b^3
- 4*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(
-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(8*c^3*x^6 + 2*b*c^2*x^3 - 3*b^2*c + 8*
a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

Sympy [F]

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\int x^5\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x**5*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**5*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\frac{1}{72}\sqrt{cx^6+bx^3+a}\left(2\left(4x^3+\frac{b}{c}\right)x^3-\frac{3b^2-8ac}{c^2}\right)-\frac{(b^3-4abc)\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{48c^{\frac{5}{2}}}$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/72*sqrt(c*x^6 + b*x^3 + a)*(2*(4*x^3 + b/c)*x^3 - (3*b^2 - 8*a*c)/c^2) -
 1/48*(b^3 - 4*a*b*c)*log(abs(2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sq
rt(c) + b))/c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 19.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx=\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{c\,x^6+b\,x^3+a}}{72\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{48\,c^{5/2}}$$

input
```
int(x^5*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(72*c^2)
 + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*
c))/(48*c^(5/2))
```

Reduce [F]

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx$$

$$=\frac{40\sqrt{c\,x^6+b\,x^3+a}\,a\,c^2-12\sqrt{c\,x^6+b\,x^3+a}\,b^2c+4\sqrt{c\,x^6+b\,x^3+a}\,b\,c^2x^3+16\sqrt{c\,x^6+b\,x^3+a}\,c^3x^6+}{}$$

input
```
int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
(40*sqrt(a + b*x**3 + c*x**6)*a*c**2 - 12*sqrt(a + b*x**3 + c*x**6)*b**2*c
 + 4*sqrt(a + b*x**3 + c*x**6)*b*c**2*x**3 + 16*sqrt(a + b*x**3 + c*x**6)*
c**3*x**6 + 12*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a*b*c
 - 3*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*b**3 - 12*sqrt(
c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*b*c + 3*sqrt(c)*log(sqr
t(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**3 - 72*int((sqrt(a + b*x**3 + c*
x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a*b*c
**3 + 18*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**
6 + b**2*x**6 + b*c*x**9),x)*b**3*c**2 - 72*int((sqrt(a + b*x**3 + c*x**6)
*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a**2*c**3
- 18*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 +
b**2*x**6 + b*c*x**9),x)*a*b**2*c**2 + 9*int((sqrt(a + b*x**3 + c*x**6)*x*
*5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*b**4*c)/(144*
c**3)
```

3.174 $\int x^2\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c} - \frac{(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

output
```
1/12*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c-1/24*(-4*a*c+b^2)*arctanh(1/2*(2*
c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c} + \frac{(-b^2+4ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^3}{-\sqrt{a}+\sqrt{a+bx^3+cx^6}}\right)}{12c^{3/2}}$$

input
```
Integrate[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) + ((-b^2 + 4*a*c)*ArcTanh[(
Sqrt[c]*x^3)/(-Sqrt[a] + Sqrt[a + b*x^3 + c*x^6])])/(12*c^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1690, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \sqrt{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3}\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4ac)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)$$

input
```
Int[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b
 + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2)))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [F]

$$\int x^2\sqrt{c\,x^6+b\,x^3+a}dx$$

input
```
int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.37

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx$$

$$= \left[-\frac{(b^2-4\,ac)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^6+bx^3+}}{48\,c^2}\right.$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*
x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a
)*(2*c^2*x^3 + b*c))/c^2, 1/24*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x
^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt
(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c))/c^2]
```

Sympy [F]

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx=\int x^2\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x**2*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{12}\sqrt{cx^6+bx^3+a}\left(2\,x^3+\frac{b}{c}\right) + \frac{(b^2-4\,ac)\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{24\,c^{\frac{3}{2}}}$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(c*x^6 + b*x^3 + a)*(2*x^3 + b/c) + 1/24*(b^2 - 4*a*c)*log(abs(2*
(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 19.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \frac{\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}}{3} + \frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{6\,c^{3/2}}$$

input
```
int(x^2*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2))/3 + (log((a + b*x^3 + c*x^6)
^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(6*c^(3/2))
```

Reduce [F]

$$\int x^2\sqrt{a+bx^3+cx^6}\,dx = \frac{-8\sqrt{c\,x^6+b\,x^3+a}\,a\,c^2+4\sqrt{c\,x^6+b\,x^3+a}\,b^2c+4\sqrt{c\,x^6+b\,x^3+a}\,b\,c^2x^3-4\sqrt{c}\log\left(\sqrt{c\,x^6+b\,x^3+a}\right.}{}$$

input
```
int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

output

```
( - 8*sqrt(a + b*x**3 + c*x**6)*a*c**2 + 4*sqrt(a + b*x**3 + c*x**6)*b**2*
c + 4*sqrt(a + b*x**3 + c*x**6)*b*c**2*x**3 - 4*sqrt(c)*log(sqrt(a + b*x**
3 + c*x**6) - sqrt(c)*x**3)*a*b*c + sqrt(c)*log(sqrt(a + b*x**3 + c*x**6)
- sqrt(c)*x**3)*b**3 + 4*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x
**3)*a*b*c - sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**3 +
24*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b*
*2*x**6 + b*c*x**9),x)*a*b*c**3 - 6*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(
a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*b**3*c**2 + 24*int
((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**
6 + b*c*x**9),x)*a**2*c**3 + 6*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2
+ 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a*b**2*c**2 - 3*int((sq
rt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 +
b*c*x**9),x)*b**4*c)/(24*b*c**2)
```

3.175 $\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = \frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}\sqrt{a}\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{b\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

output
```
1/3*(c*x^6+b*x^3+a)^(1/2)-1/3*a^(1/2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x
^6+b*x^3+a)^(1/2))+1/6*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(
1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = \frac{1}{6}\left(2\sqrt{a+bx^3+cx^6}+4\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)-\frac{b\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

output
```
(2*Sqrt[a + b*x^3 + c*x^6] + 4*Sqrt[a]*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x
^3 + c*x^6])/Sqrt[a]] - (b*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*
x^6]])/Sqrt[c])/6
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1693, 1162, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{\sqrt{cx^6+bx^3+a}}{x^3}dx^3$$

$$\Big\downarrow \text{1162}$$

$$\frac{1}{3}\left(\sqrt{a+bx^3+cx^6}-\frac{1}{2}\int -\frac{bx^3+2a}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{bx^3+2a}{x^3\sqrt{cx^6+bx^3+a}}dx^3+\sqrt{a+bx^3+cx^6}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(b\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3+2a\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)+\sqrt{a+bx^3+cx^6}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2b\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}+2a\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)+\sqrt{a+bx^3+cx^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2a\int\frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3+\frac{b\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}\right)+\sqrt{a+bx^3+cx^6}\right)$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}-4a\int\frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}\right)+\sqrt{a+bx^3+cx^6}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}-2\sqrt{a}\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)\right)+\sqrt{a+bx^3+cx^6}\right)$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6] + (-2*Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sq
rt[a + b*x^3 + c*x^6])] + (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x
^3 + c*x^6])])/Sqrt[c])/2)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 566, normalized size of antiderivative = 5.19

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$$
$$= \left[\frac{b\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+2\,\sqrt{a}c\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3\ldots}{}\right.}{12\,c}\right.$$
$$-\frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-\sqrt{a}c\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-2\,\sqrt{\ldots}}{6\,c}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 +
 a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 2*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^6 +
 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6)
 + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c, -1/6*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^6
+ b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - sqrt(a)*c
*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(a) + 8*a^2)/x^6) - 2*sqrt(c*x^6 + b*x^3 + a)*c)/c, 1/12*(4*sqrt(
-a)*c*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 +
 a*b*x^3 + a^2)) + b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x
^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)
*c)/c, 1/6*(2*sqrt(-a)*c*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*
sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b
*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6
 + b*x^3 + a)*c)/c]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx=\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 19.80 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = \frac{\sqrt{c\,x^6+b\,x^3+a}}{3} - \frac{\sqrt{a}\,\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{3} + \frac{b\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{6\,\sqrt{c}}$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x,x)
```

output
```
(a + b*x^3 + c*x^6)^(1/2)/3 - (a^(1/2)*log(b/2 + a/x^3 + (a^(1/2)*(a + b*x
^3 + c*x^6)^(1/2))/x^3))/3 + (b*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x
^3)/c^(1/2)))/(6*c^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx = \text{too large to display}$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

output
```
(2*sqrt(a + b*x**3 + c*x**6)*b**2*c + 6*sqrt(a)*log(sqrt(a + b*x**3 + c*x*
*6) - sqrt(a))*a*c**2 + 2*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))
*b**2*c - 6*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*a*c**2 - 2*sq
rt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*b**2*c - 3*sqrt(c)*log(sqrt
(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a*b*c - sqrt(c)*log(sqrt(a + b*x**3
+ c*x**6) - sqrt(c)*x**3)*b**3 + 3*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) +
 sqrt(c)*x**3)*a*b*c + sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**
3)*b**3 + 54*int((sqrt(a + b*x**3 + c*x**6)*x**11)/(3*a**3*b*c + 3*a**3*c*
*2*x**3 + a**2*b**3 + 7*a**2*b**2*c*x**3 + 9*a**2*b*c**2*x**6 + 3*a**2*c**
3*x**9 + 2*a*b**4*x**3 + 6*a*b**3*c*x**6 + 7*a*b**2*c**2*x**9 + 3*a*b*c**3
*x**12 + b**5*x**6 + 2*b**4*c*x**9 + b**3*c**2*x**12),x)*a**2*b*c**5 + 18*
int((sqrt(a + b*x**3 + c*x**6)*x**11)/(3*a**3*b*c + 3*a**3*c**2*x**3 + a**
2*b**3 + 7*a**2*b**2*c*x**3 + 9*a**2*b*c**2*x**6 + 3*a**2*c**3*x**9 + 2*a*
b**4*x**3 + 6*a*b**3*c*x**6 + 7*a*b**2*c**2*x**9 + 3*a*b*c**3*x**12 + b**5
*x**6 + 2*b**4*c*x**9 + b**3*c**2*x**12),x)*a*b**3*c**4 + 54*int((sqrt(a +
 b*x**3 + c*x**6)*x**8)/(3*a**3*b*c + 3*a**3*c**2*x**3 + a**2*b**3 + 7*a**
2*b**2*c*x**3 + 9*a**2*b*c**2*x**6 + 3*a**2*c**3*x**9 + 2*a*b**4*x**3 + 6*
a*b**3*c*x**6 + 7*a*b**2*c**2*x**9 + 3*a*b*c**3*x**12 + b**5*x**6 + 2*b**4
*c*x**9 + b**3*c**2*x**12),x)*a**3*c**5 + 99*int((sqrt(a + b*x**3 + c*x**6
)*x**8)/(3*a**3*b*c + 3*a**3*c**2*x**3 + a**2*b**3 + 7*a**2*b**2*c*x**3...
```

3.176 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4} dx = -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{b \operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}} + \frac{1}{3}\sqrt{c} \operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

output
```
-1/3*(c*x^6+b*x^3+a)^(1/2)/x^3-1/6*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^
6+b*x^3+a)^(1/2))/a^(1/2)+1/3*c^(1/2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x
^6+b*x^3+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4} dx = \frac{1}{3}\left(-\frac{\sqrt{a+bx^3+cx^6}}{x^3} + \frac{b \operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{\sqrt{a}} - \sqrt{c}\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

output
```
(-(Sqrt[a + b*x^3 + c*x^6]/x^3) + (b*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3
 + c*x^6])/Sqrt[a]])/Sqrt[a] - Sqrt[c]*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a
+ b*x^3 + c*x^6]])/3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1693, 1161, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3}\int \frac{\sqrt{cx^6+bx^3+a}}{x^6}dx^3$$

$$\Big\downarrow \text{ 1161}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{2cx^3+b}{x^3\sqrt{cx^6+bx^3+a}}dx^3-\frac{\sqrt{a+bx^3+cx^6}}{x^3}\right)$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2c\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3+b\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)-\frac{\sqrt{a+bx^3+cx^6}}{x^3}\right)$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(4c\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}+b\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)-\frac{\sqrt{a+bx^3+cx^6}}{x^3}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(b\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3+2\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)-\frac{\sqrt{a+bx^3+cx^6}}{x^3}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-2b\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}\right)-\frac{\sqrt{a+bx^3+cx^6}}{x^3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(2\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-\frac{b\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{a}}\right)-\frac{\sqrt{a+bx^3+cx^6}}{x^3}\right)$$

input

```
Int[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

output

```
(-(Sqrt[a + b*x^3 + c*x^6]/x^3) + (-((b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*S
qrt[a + b*x^3 + c*x^6])])/Sqrt[a]) + 2*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sq
rt[c]*Sqrt[a + b*x^3 + c*x^6])])/2)/3
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{4}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 601, normalized size of antiderivative = 5.37

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx$$

$$= \left[\frac{2\,a\sqrt{c}x^3\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+\sqrt{a}bx^3\log\left(-\frac{(b^2+4\,ac)x^6}{\ldots}\right.}{12\,ax^3}\right.$$

$$-\frac{4\,a\sqrt{-c}x^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-\sqrt{a}bx^3\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right.}{12\,ax^3}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/12*(2*a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b
*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + sqrt(a)*b*x^3*log(-((b^2 + 4*a*
c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a
^2)/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), -1/12*(4*a*sqrt(-c)*x^3*a
rctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^
3 + a*c)) - sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x
^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3
 + a)*a)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(
b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + a*sqrt(c)*x^3*log(-8*c^
2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c)
- 4*a*c) - 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arcta
n(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 +
a^2)) - 2*a*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*
sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3)
]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx = \int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 20.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx = \frac{\sqrt{c}\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3} - \frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,x^3} - \frac{b\,\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{6\,\sqrt{a}}$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^4,x)
```

output
```
(c^(1/2)*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2)))/3 - (a +
b*x^3 + c*x^6)^(1/2)/(3*x^3) - (b*log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^3 +
c*x^6)^(1/2))/x^3))/(6*a^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx = \text{Too large to display}$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

output
```
( - 2*sqrt(a + b*x**3 + c*x**6)*a + sqrt(a)*log(sqrt(a + b*x**3 + c*x**6)
- sqrt(a))*b*x**3 - sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*b*x**
3 - 2*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a*x**3 + 2*sqr
t(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*x**3 - 18*int((sqrt(a
 + b*x**3 + c*x**6)*x**5)/(3*a**3*b*c + 3*a**3*c**2*x**3 + a**2*b**3 + 7*a
**2*b**2*c*x**3 + 9*a**2*b*c**2*x**6 + 3*a**2*c**3*x**9 + 2*a*b**4*x**3 +
6*a*b**3*c*x**6 + 7*a*b**2*c**2*x**9 + 3*a*b*c**3*x**12 + b**5*x**6 + 2*b*
*4*c*x**9 + b**3*c**2*x**12),x)*a**3*c**3*x**3 - 15*int((sqrt(a + b*x**3 +
 c*x**6)*x**5)/(3*a**3*b*c + 3*a**3*c**2*x**3 + a**2*b**3 + 7*a**2*b**2*c*
x**3 + 9*a**2*b*c**2*x**6 + 3*a**2*c**3*x**9 + 2*a*b**4*x**3 + 6*a*b**3*c*
x**6 + 7*a*b**2*c**2*x**9 + 3*a*b*c**3*x**12 + b**5*x**6 + 2*b**4*c*x**9 +
 b**3*c**2*x**12),x)*a**2*b**2*c**2*x**3 - 3*int((sqrt(a + b*x**3 + c*x**6
)*x**5)/(3*a**3*b*c + 3*a**3*c**2*x**3 + a**2*b**3 + 7*a**2*b**2*c*x**3 +
9*a**2*b*c**2*x**6 + 3*a**2*c**3*x**9 + 2*a*b**4*x**3 + 6*a*b**3*c*x**6 +
7*a*b**2*c**2*x**9 + 3*a*b*c**3*x**12 + b**5*x**6 + 2*b**4*c*x**9 + b**3*c
**2*x**12),x)*a*b**4*c*x**3 - 27*int((sqrt(a + b*x**3 + c*x**6)*x**2)/(3*a
**3*b*c + 3*a**3*c**2*x**3 + a**2*b**3 + 7*a**2*b**2*c*x**3 + 9*a**2*b*c**
2*x**6 + 3*a**2*c**3*x**9 + 2*a*b**4*x**3 + 6*a*b**3*c*x**6 + 7*a*b**2*c**
2*x**9 + 3*a*b*c**3*x**12 + b**5*x**6 + 2*b**4*c*x**9 + b**3*c**2*x**12),x
)*a**3*b*c**2*x**3 - 9*int((sqrt(a + b*x**3 + c*x**6)*x**2)/(3*a**3*b*c...
```

3.177 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = -\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}+\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}$$

output
```
-1/12*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a/x^6+1/24*(-4*a*c+b^2)*arctanh(1/
2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \frac{(-2a-bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6} + \frac{(-b^2+4ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{12a^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

output
```
((-2*a - b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a*x^6) + ((-b^2 + 4*a*c)*ArcT
anh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(12*a^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1693, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{3}\left(-\frac{(b^2-4ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{(b^2-4ac)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

output
```
(-1/4*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) + ((b^2 - 4*a*c)*Arc
Tanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(8*a^(3/2)))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{7}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \left[-\frac{(b^2-4\,ac)\sqrt{a}x^6\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,\sqrt{cx^6+bx^3+a}(abx^3+2\,a^2)}{48\,a^2x^6}, -\frac{(b^2-4\,ac)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,\sqrt{cx^6+bx^3+a}(abx^3+2\,a^2)}{24\,a^2x^6}\right]$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
[-1/48*((b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*
sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6
 + b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6), -1/24*((b^2 - 4*a*c)*sqrt(-a)*
x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a
*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**7,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^7}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^7}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^7,x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2)/x^7, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx = \text{too large to display}$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

output
```
(64*sqrt(a + b*x**3 + c*x**6)*a**4*b*c**3*x**3 + 64*sqrt(a + b*x**3 + c*x*
*6)*a**4*c**4*x**6 - 24*sqrt(a + b*x**3 + c*x**6)*a**3*b**4*c - 16*sqrt(a
+ b*x**3 + c*x**6)*a**3*b**3*c**2*x**3 - 64*sqrt(a + b*x**3 + c*x**6)*a**3
*b**2*c**3*x**6 - 12*sqrt(a + b*x**3 + c*x**6)*a**2*b**6 + 12*sqrt(a + b*x
**3 + c*x**6)*a**2*b**5*c*x**3 + 12*sqrt(a + b*x**3 + c*x**6)*a**2*b**4*c*
*2*x**6 - 12*sqrt(a + b*x**3 + c*x**6)*a*b**7*x**3 + 24*sqrt(a)*log(sqrt(a
 + b*x**3 + c*x**6) - sqrt(a))*a**2*b**4*c**2*x**6 + 6*sqrt(a)*log(sqrt(a
+ b*x**3 + c*x**6) - sqrt(a))*a*b**6*c*x**6 - 3*sqrt(a)*log(sqrt(a + b*x**
3 + c*x**6) - sqrt(a))*b**8*x**6 - 24*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6
) + sqrt(a))*a**2*b**4*c**2*x**6 - 6*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6)
 + sqrt(a))*a*b**6*c*x**6 + 3*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt
(a))*b**8*x**6 + 192*int(sqrt(a + b*x**3 + c*x**6)/(a*b*x + a*c*x**4 + b**
2*x**4 + 2*b*c*x**7 + c**2*x**10),x)*a**5*b*c**4*x**6 + 48*int(sqrt(a + b*
x**3 + c*x**6)/(a*b*x + a*c*x**4 + b**2*x**4 + 2*b*c*x**7 + c**2*x**10),x)
*a**4*b**3*c**3*x**6 + 48*int(sqrt(a + b*x**3 + c*x**6)/(a*b*x + a*c*x**4
+ b**2*x**4 + 2*b*c*x**7 + c**2*x**10),x)*a**3*b**5*c**2*x**6 + 18*int(sqr
t(a + b*x**3 + c*x**6)/(a*b*x + a*c*x**4 + b**2*x**4 + 2*b*c*x**7 + c**2*x
**10),x)*a**2*b**7*c*x**6 - 9*int(sqrt(a + b*x**3 + c*x**6)/(a*b*x + a*c*x
**4 + b**2*x**4 + 2*b*c*x**7 + c**2*x**10),x)*a*b**9*x**6 + 384*int(sqrt(a
 + b*x**3 + c*x**6)/(2*a**2*b*c*x**4 + 2*a**2*c**2*x**7 + a*b**3*x**4 +...
```

3.178 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}} dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}} dx = \frac{b(2a+bx^3)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9ax^9} - \frac{b(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}$$

output
```
1/24*b*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^2/x^6-1/9*(c*x^6+b*x^3+a)^(3/2)
/a/x^9-1/48*b*(-4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)
^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}} dx = \frac{\sqrt{a+bx^3+cx^6}(-8a^2-2abx^3+3b^2x^6-8acx^6)}{72a^2x^9} + \frac{(b^3-4abc)\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{5/2}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 2*a*b*x^3 + 3*b^2*x^6 - 8*a*c*x^6))/(72
*a^2*x^9) + ((b^3 - 4*a*b*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6
])/Sqrt[a]])/(24*a^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1693, 1157, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{\sqrt{cx^6+bx^3+a}}{x^{12}}dx^3$$

$$\Big\downarrow \text{1157}$$

$$\frac{1}{3}\left(-\frac{b\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{2a}-\frac{(a+bx^3+cx^6)^{3/2}}{3ax^9}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{3}\left(-\frac{b\left(-\frac{(b^2-4ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{(a+bx^3+cx^6)^{3/2}}{3ax^9}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(-\frac{b\left(\frac{(b^2-4ac)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{(a+bx^3+cx^6)^{3/2}}{3ax^9}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(-\frac{b\left(\frac{(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3ax^9}\right)$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

output
```
(-1/3*(a + b*x^3 + c*x^6)^(3/2)/(a*x^9) - (b*(-1/4*((2*a + b*x^3)*Sqrt[a +
 b*x^3 + c*x^6])/(a*x^6) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]
*Sqrt[a + b*x^3 + c*x^6])])/(8*a^(3/2))))/(2*a))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{10}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \left[-\frac{3\,(b^3-4\,abc)\sqrt{a}x^9\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,((3\,ab^2-8\,a^2c)x^6-2\,a^2bx}{288\,a^3x^9}\right.$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="fricas")
```

output
```
[-1/288*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3
 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a
*b^2 - 8*a^2*c)*x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x
^9), 1/144*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 +
 a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 - 8*a^
2*c)*x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^9)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**10,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**10, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^{10}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{10}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^10,x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2)/x^10, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \text{too large to display}$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

output
```
( - 256*sqrt(a + b*x**3 + c*x**6)*a**5*c**3 - 192*sqrt(a + b*x**3 + c*x**6
)*a**4*b**2*c**2 + 128*sqrt(a + b*x**3 + c*x**6)*a**4*b*c**3*x**3 - 640*sq
rt(a + b*x**3 + c*x**6)*a**4*c**4*x**6 + 160*sqrt(a + b*x**3 + c*x**6)*a**
3*b**4*c - 160*sqrt(a + b*x**3 + c*x**6)*a**3*b**3*c**2*x**3 - 160*sqrt(a
 + b*x**3 + c*x**6)*a**3*b**2*c**3*x**6 - 24*sqrt(a + b*x**3 + c*x**6)*a**2
*b**5*c*x**3 + 528*sqrt(a + b*x**3 + c*x**6)*a**2*b**4*c**2*x**6 + 20*sqrt
(a + b*x**3 + c*x**6)*a*b**7*x**3 - 4*sqrt(a + b*x**3 + c*x**6)*a*b**6*c*x
**6 - 30*sqrt(a + b*x**3 + c*x**6)*b**8*x**6 - 192*sqrt(a)*log(sqrt(a + b*
x**3 + c*x**6) - sqrt(a))*a**3*b*c**4*x**9 - 96*sqrt(a)*log(sqrt(a + b*x**
3 + c*x**6) - sqrt(a))*a**2*b**3*c**3*x**9 + 156*sqrt(a)*log(sqrt(a + b*x*
*3 + c*x**6) - sqrt(a))*a*b**5*c**2*x**9 - 30*sqrt(a)*log(sqrt(a + b*x**3
 + c*x**6) - sqrt(a))*b**7*c*x**9 + 192*sqrt(a)*log(sqrt(a + b*x**3 + c*x**
6) + sqrt(a))*a**3*b*c**4*x**9 + 96*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6)
 + sqrt(a))*a**2*b**3*c**3*x**9 - 156*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6)
 + sqrt(a))*a*b**5*c**2*x**9 + 30*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) +
sqrt(a))*b**7*c*x**9 + 9216*int(sqrt(a + b*x**3 + c*x**6)/(8*a**3*b*c**2*x
**7 + 8*a**3*c**3*x**10 + 6*a**2*b**3*c*x**7 + 14*a**2*b**2*c**2*x**10 + 1
6*a**2*b*c**3*x**13 + 8*a**2*c**4*x**16 - 5*a*b**5*x**7 + a*b**4*c*x**10 +
 12*a*b**3*c**2*x**13 + 6*a*b**2*c**3*x**16 - 5*b**6*x**10 - 10*b**5*c*x**
13 - 5*b**4*c**2*x**16),x)*a**7*b**2*c**5*x**9 + 1536*int(sqrt(a + b*x*...
```

3.179 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 161

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = -\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9} + \frac{(b^2-4ac)(5b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}$$

output
```
-1/192*(-4*a*c+5*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^3/x^6-1/12*(c*x^
6+b*x^3+a)^(3/2)/a/x^12+5/72*b*(c*x^6+b*x^3+a)^(3/2)/a^2/x^9+1/384*(-4*a*c
+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2)
)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-48a^3-8a^2bx^3+10ab^2x^6-24a^2cx^6-15b^3x^9+52abcx^9)}{576a^3x^{12}} + \frac{(-5b^4+24ab^2c-16a^2c^2)\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{192a^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 - 8*a^2*b*x^3 + 10*a*b^2*x^6 - 24*a^2*c*
x^6 - 15*b^3*x^9 + 52*a*b*c*x^9))/(576*a^3*x^12) + ((-5*b^4 + 24*a*b^2*c -
 16*a^2*c^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(19
2*a^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1693, 1167, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{\sqrt{cx^6+bx^3+a}}{x^{15}}dx^3$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{3}\left(-\frac{\int \frac{(2cx^3+5b)\sqrt{cx^6+bx^3+a}}{2x^{12}}dx^3}{4a}-\frac{(a+bx^3+cx^6)^{3/2}}{4ax^{12}}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(-\frac{\int \frac{(2cx^3+5b)\sqrt{cx^6+bx^3+a}}{x^{12}}dx^3}{8a}-\frac{(a+bx^3+cx^6)^{3/2}}{4ax^{12}}\right)$$

$$\Big\downarrow 1228$$

$$\frac{1}{3}\left(-\frac{-\frac{(5b^2-4ac)\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{2a}-\frac{5b(a+bx^3+cx^6)^{3/2}}{3ax^9}}{8a}-\frac{(a+bx^3+cx^6)^{3/2}}{4ax^{12}}\right)$$

$$\Big\downarrow 1152$$

$$\frac{1}{3}\left(-\frac{-\frac{(5b^2-4ac)\left(-\frac{(b^2-4ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{5b(a+bx^3+cx^6)^{3/2}}{3ax^9}}{8a}-\frac{(a+bx^3+cx^6)^{3/2}}{4ax^{12}}\right)$$

$$\Big\downarrow 1154$$

$$\frac{1}{3}\left(-\frac{-\frac{(5b^2-4ac)\left(\frac{(b^2-4ac)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{5b(a+bx^3+cx^6)^{3/2}}{3ax^9}}{8a}-\frac{(a+bx^3+cx^6)^{3/2}}{4ax^{12}}\right)$$

$$\Big\downarrow 219$$

$$\frac{1}{3}\left(-\frac{-\frac{(5b^2-4ac)\left(\frac{(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{5b(a+bx^3+cx^6)^{3/2}}{3ax^9}}{8a}-\frac{(a+bx^3+cx^6)^{3/2}}{4ax^{12}}\right)$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

output
```
(-1/4*(a + b*x^3 + c*x^6)^(3/2)/(a*x^12) - ((-5*b*(a + b*x^3 + c*x^6)^(3/2
))/(3*a*x^9) - ((5*b^2 - 4*a*c)*(-1/4*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^
6])/(a*x^6) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x
^3 + c*x^6])])/(8*a^(3/2))))/(2*a))/(8*a))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{13}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = \left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{a}x^{12}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,((15\,ab^3-52\,a\right.}{2304\,a^4x^{12}}$$

$$-\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{-a}x^{12}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((15\,ab^3-52\,a^2bc)x^9+8\,a^3bx}{1152\,a^4x^{12}}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="fricas")
```

output
```
[1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a
*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*
a^2)/x^6) - 4*((15*a*b^3 - 52*a^2*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 -
12*a^3*c)*x^6 + 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^12), -1/1152*(3*(5
*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^
3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^3 -
52*a^2*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^6 + 48*a^4)*sqr
t(c*x^6 + b*x^3 + a))/(a^4*x^12)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = \int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**13,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**13, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x^{13}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^13, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx=\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{13}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^13,x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2)/x^13, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx=\text{too large to display}$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

output
```
( - 294912*sqrt(a + b*x**3 + c*x**6)*a**9*b**4*c**7 + 131072*sqrt(a + b*x*
*3 + c*x**6)*a**9*b**3*c**8*x**3 - 196608*sqrt(a + b*x**3 + c*x**6)*a**9*b
**2*c**9*x**6 - 393216*sqrt(a + b*x**3 + c*x**6)*a**9*c**11*x**12 - 860160
*sqrt(a + b*x**3 + c*x**6)*a**8*b**6*c**6 - 180224*sqrt(a + b*x**3 + c*x**
6)*a**8*b**5*c**7*x**3 - 114688*sqrt(a + b*x**3 + c*x**6)*a**8*b**4*c**8*x
**6 + 229376*sqrt(a + b*x**3 + c*x**6)*a**8*b**3*c**9*x**9 + 589824*sqrt(a
 + b*x**3 + c*x**6)*a**8*b**2*c**10*x**12 - 325632*sqrt(a + b*x**3 + c*x**
6)*a**7*b**8*c**5 + 12288*sqrt(a + b*x**3 + c*x**6)*a**7*b**7*c**6*x**3 -
438272*sqrt(a + b*x**3 + c*x**6)*a**7*b**6*c**7*x**6 + 385024*sqrt(a + b*x
**3 + c*x**6)*a**7*b**5*c**8*x**9 + 344064*sqrt(a + b*x**3 + c*x**6)*a**7*
b**4*c**9*x**12 + 483840*sqrt(a + b*x**3 + c*x**6)*a**6*b**10*c**4 - 31846
4*sqrt(a + b*x**3 + c*x**6)*a**6*b**9*c**5*x**3 + 218112*sqrt(a + b*x**3 +
 c*x**6)*a**6*b**8*c**6*x**6 + 669696*sqrt(a + b*x**3 + c*x**6)*a**6*b**7*
c**7*x**9 - 638976*sqrt(a + b*x**3 + c*x**6)*a**6*b**6*c**8*x**12 + 99840*
sqrt(a + b*x**3 + c*x**6)*a**5*b**12*c**3 + 120576*sqrt(a + b*x**3 + c*x**
6)*a**5*b**11*c**4*x**3 + 580096*sqrt(a + b*x**3 + c*x**6)*a**5*b**10*c**5
*x**6 + 193024*sqrt(a + b*x**3 + c*x**6)*a**5*b**9*c**6*x**9 - 53760*sqrt(
a + b*x**3 + c*x**6)*a**5*b**8*c**7*x**12 - 67200*sqrt(a + b*x**3 + c*x**6
)*a**4*b**14*c**2 - 7040*sqrt(a + b*x**3 + c*x**6)*a**4*b**13*c**3*x**3 -
65280*sqrt(a + b*x**3 + c*x**6)*a**4*b**12*c**4*x**6 - 1436160*sqrt(a +...
```

3.180 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 199

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \frac{b(7b^2-12ac)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} - \frac{(35b^2-32ac)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9} - \frac{b(7b^2-12ac)\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}}$$

output

```
1/384*b*(-12*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^4/x^6-1/15*(c*
x^6+b*x^3+a)^(3/2)/a/x^15+7/120*b*(c*x^6+b*x^3+a)^(3/2)/a^2/x^12-1/720*(-3
2*a*c+35*b^2)*(c*x^6+b*x^3+a)^(3/2)/a^3/x^9-1/768*b*(-12*a*c+7*b^2)*(-4*a*
c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$$
$$=\frac{\sqrt{a+bx^3+cx^6}(-384a^4-48a^3bx^3+56a^2b^2x^6-128a^3cx^6-70ab^3x^9+232a^2bcx^9+105b^4x^{12}-460ab^2}{5760a^4x^{15}}$$
$$+\frac{(7b^5-40ab^3c+48a^2bc^2)\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{384a^{9/2}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-384*a^4 - 48*a^3*b*x^3 + 56*a^2*b^2*x^6 - 128*a
^3*c*x^6 - 70*a*b^3*x^9 + 232*a^2*b*c*x^9 + 105*b^4*x^12 - 460*a*b^2*c*x^1
2 + 256*a^2*c^2*x^12))/(5760*a^4*x^15) + ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c
^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(384*a^(9/2)
)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1693, 1167, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$$

$$\Big\downarrow \; 1693$$

$$\frac{1}{3}\int \frac{\sqrt{cx^6+bx^3+a}}{x^{18}}dx^3$$

$$\Big\downarrow \; 1167$$

$$\frac{1}{3}\left(-\frac{\int \frac{(4cx^3+7b)\sqrt{cx^6+bx^3+a}}{2x^{15}}dx^3}{5a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{5ax^{15}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{(4cx^3+7b)\sqrt{cx^6+bx^3+a}}{x^{15}}dx^3}{10a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{5ax^{15}}\right)$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{\left(14bcx^3+35b^2-32ac\right)\sqrt{cx^6+bx^3+a}}{2x^{12}}dx^3}{4a}-\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{4ax^{12}}}{10a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{5ax^{15}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{\left(14bcx^3+35b^2-32ac\right)\sqrt{cx^6+bx^3+a}}{x^{12}}dx^3}{8a}-\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{4ax^{12}}}{10a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{5ax^{15}}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{5b\left(7b^2-12ac\right)\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{2a}-\frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{3ax^9}}{8a}-\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{4ax^{12}}}{10a}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{5ax^{15}}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{5b\left(7b^2-12ac\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{\left(35b^2-32ac\right)\left(a+bx^3+cx^6\right)^{3/2}}{3ax^9}}{8a}-\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{4ax^{12}}}{10a}-\right.$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{5b(7b^2-12ac)\left(\frac{(b^2-4ac)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{(35b^2-32ac)(a+bx^3+cx^6)^{3/2}}{3ax^9}}{8a}-\frac{7b(a+bx^3+cx^6)^{3/2}}{4ax^{12}}}{10a}-\right.$$

$\downarrow$ 219

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{5b(7b^2-12ac)\left(\frac{(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{2a}-\frac{(35b^2-32ac)(a+bx^3+cx^6)^{3/2}}{3ax^9}}{8a}-\frac{7b(a+bx^3+cx^6)}{4ax^{12}}\right.$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

output
```
(-1/5*(a + b*x^3 + c*x^6)^(3/2)/(a*x^15) - ((-7*b*(a + b*x^3 + c*x^6)^(3/2
))/(4*a*x^12) - (-1/3*((35*b^2 - 32*a*c)*(a + b*x^3 + c*x^6)^(3/2))/(a*x^9
) - (5*b*(7*b^2 - 12*a*c)*(-1/4*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a
*x^6) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c
*x^6])])/(8*a^(3/2))))/(2*a))/(8*a))/(10*a))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{16}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \left[\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{a}x^{15}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,((105\,ab^4-}{2}\right.$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="fricas")
```

output
```
[1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 +
 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a)
+ 8*a^2)/x^6) + 4*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^12 - 2*(35*
a^2*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a^3*b^2 - 16*a^4*c)*x^6 -
 384*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15), 1/11520*(15*(7*b^5 - 40*a*b
^3*c + 48*a^2*b*c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x
^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^4 - 460*a^2*b^
2*c + 256*a^3*c^2)*x^12 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3
+ 8*(7*a^3*b^2 - 16*a^4*c)*x^6 - 384*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^
15)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**16,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**16, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^{16}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^16, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{16}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^16,x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2)/x^16, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \text{too large to display}$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

output
```
( - 7077888*sqrt(a + b*x**3 + c*x**6)*a**10*b**2*c**8 - 11796480*sqrt(a +
b*x**3 + c*x**6)*a**9*b**4*c**7 - 5308416*sqrt(a + b*x**3 + c*x**6)*a**9*b
**3*c**8*x**3 + 3538944*sqrt(a + b*x**3 + c*x**6)*a**9*b**2*c**9*x**6 - 17
694720*sqrt(a + b*x**3 + c*x**6)*a**9*b*c**10*x**9 + 35389440*sqrt(a + b*x
**3 + c*x**6)*a**9*c**11*x**12 + 20643840*sqrt(a + b*x**3 + c*x**6)*a**8*b
**6*c**6 + 5898240*sqrt(a + b*x**3 + c*x**6)*a**8*b**5*c**7*x**3 - 7569408
*sqrt(a + b*x**3 + c*x**6)*a**8*b**4*c**8*x**6 - 3833856*sqrt(a + b*x**3 +
 c*x**6)*a**8*b**3*c**9*x**9 + 34209792*sqrt(a + b*x**3 + c*x**6)*a**8*b**
2*c**10*x**12 + 21159936*sqrt(a + b*x**3 + c*x**6)*a**7*b**8*c**5 - 915456
0*sqrt(a + b*x**3 + c*x**6)*a**7*b**7*c**6*x**3 + 15646720*sqrt(a + b*x**3
 + c*x**6)*a**7*b**6*c**7*x**6 + 44974080*sqrt(a + b*x**3 + c*x**6)*a**7*b
**5*c**8*x**9 - 67829760*sqrt(a + b*x**3 + c*x**6)*a**7*b**4*c**9*x**12 -
19031040*sqrt(a + b*x**3 + c*x**6)*a**6*b**10*c**4 + 10002432*sqrt(a + b*x
**3 + c*x**6)*a**6*b**9*c**5*x**3 + 7923712*sqrt(a + b*x**3 + c*x**6)*a**6
*b**8*c**6*x**6 - 38184960*sqrt(a + b*x**3 + c*x**6)*a**6*b**7*c**7*x**9 -
 34037760*sqrt(a + b*x**3 + c*x**6)*a**6*b**6*c**8*x**12 + 3386880*sqrt(a
+ b*x**3 + c*x**6)*a**5*b**12*c**3 + 1117440*sqrt(a + b*x**3 + c*x**6)*a**
5*b**11*c**4*x**3 - 22674944*sqrt(a + b*x**3 + c*x**6)*a**5*b**10*c**5*x**
6 - 18344448*sqrt(a + b*x**3 + c*x**6)*a**5*b**9*c**6*x**9 + 28302336*sqrt
(a + b*x**3 + c*x**6)*a**5*b**8*c**7*x**12 - 7653120*sqrt(a + b*x**3 + ...
```

3.181 $\int x^3\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx = \frac{x^4\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{2},-\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/4*x^4*(c*x^6+b*x^3+a)^(1/2)*AppellF1(4/3,-1/2,-1/2,7/3,-2*c*x^3/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 358 vs. 2(140) = 280.

Time = 10.61 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.56

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx = \frac{x\left(8(3b+8cx^3)\left(a+bx^3+cx^6\right)-24ab\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4a}}\right.}{448c}$$

input
```
Integrate[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*(8*(3*b + 8*c*x^3)*(a + b*x^3 + c*x^6) - 24*a*b*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*(-5*b^2 + 16
*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4
/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqr
t[b^2 - 4*a*c])]))/(448*c*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{a+bx^3+cx^6}\int x^3\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{2},-\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -1/2, -1/2, 7/3, (-2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^3\sqrt{c\,x^6+b\,x^3+a}dx$$

input
```
int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

Sympy [F]

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int x^3\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x**3*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**3*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx=\int x^3\,\sqrt{c\,x^6+b\,x^3+a}\,dx$$

input
```
int(x^3*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x^3*(a + b*x^3 + c*x^6)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx$$
$$=\frac{6\sqrt{c\,x^6+b\,x^3+a}\,bx+16\sqrt{c\,x^6+b\,x^3+a}\,c\,x^4-6\left(\int\frac{\sqrt{c\,x^6+b\,x^3+a}}{c\,x^6+b\,x^3+a}dx\right)ab+48\left(\int\frac{\sqrt{c\,x^6+b\,x^3+a}\,x^3}{c\,x^6+b\,x^3+a}dx\right)ac-}{112c}$$

input
```
int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
(6*sqrt(a + b*x**3 + c*x**6)*b*x + 16*sqrt(a + b*x**3 + c*x**6)*c*x**4 - 6
*int(sqrt(a + b*x**3 + c*x**6)/(a + b*x**3 + c*x**6),x)*a*b + 48*int((sqrt
(a + b*x**3 + c*x**6)*x**3)/(a + b*x**3 + c*x**6),x)*a*c - 15*int((sqrt(a
+ b*x**3 + c*x**6)*x**3)/(a + b*x**3 + c*x**6),x)*b**2)/(112*c)
```

3.182 $\int x\sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 140

$$\int x\sqrt{a+bx^3+cx^6}\,dx = \frac{x^2\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output

```
1/2*x^2*(c*x^6+b*x^3+a)^(1/2)*AppellF1(2/3,-1/2,-1/2,5/3,-2*c*x^3/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 337 vs. 2(140) = 280.

Time = 10.45 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.41

$$\int x\sqrt{a+bx^3+cx^6}\,dx = \frac{x^2\left(10(a+bx^3+cx^6)+15a\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4 \cdots}}\right.\right.}{50\sqrt{a+bx^3+c \cdots}}$$

input
```
Integrate[x*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^2*(10*(a + b*x^3 + c*x^6) + 15*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*x^3*Sqrt[(b - Sqrt[b^2 -
4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(50*Sqrt[a +
 b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{a+bx^3+cx^6}\int x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{a+bx^3+cx^6}\operatorname{AppellF1}\left(\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -1/2, -1/2, 5/3, (-2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x\sqrt{c\,x^{6}+b\,x^{3}+a}dx$$

input
```
int(x*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int x\sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

Sympy [F]

$$\int x\sqrt{a+bx^3+cx^6}\,dx = \int x\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate(x*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int x\sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^3+cx^6}\,dx=\int x\sqrt{c\,x^6+b\,x^3+a}\,dx$$

input
```
int(x*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x*(a + b*x^3 + c*x^6)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{a+bx^3+cx^6}\,dx=\frac{\sqrt{c\,x^6+b\,x^3+a}\,x^2}{5}+\frac{3\left(\int \frac{\sqrt{c\,x^6+b\,x^3+a}\,x^4}{c\,x^6+b\,x^3+a}dx\right)b}{10}+\frac{3\left(\int \frac{\sqrt{c\,x^6+b\,x^3+a}\,x}{c\,x^6+b\,x^3+a}dx\right)a}{5}$$

input
```
int(x*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3 + c*x**6)*x**2 + 3*int((sqrt(a + b*x**3 + c*x**6)*x**4)
/(a + b*x**3 + c*x**6),x)*b + 6*int((sqrt(a + b*x**3 + c*x**6)*x)/(a + b*x
**3 + c*x**6),x)*a)/10
```

3.183 $\int \sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \sqrt{a+bx^3+cx^6}\,dx = \frac{x\sqrt{a+bx^3+cx^6}\,\mathrm{AppellF1}\left(\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
x*(c*x^6+b*x^3+a)^(1/2)*AppellF1(1/3,-1/2,-1/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2
)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 335 vs. 2(135) = 270.

Time = 10.42 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.48

$$\int \sqrt{a+bx^3+cx^6}\,dx = \frac{x\left(8(a+bx^3+cx^6)+24a\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right.\right.}{32\sqrt{a+bx^3+cx}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*(8*(a + b*x^3 + c*x^6) + 24*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(32*Sqrt[a + b*
x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1686}$$

$$\frac{\sqrt{a+bx^3+cx^6}\int \sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -1/2, -1/2, 4/3, (-2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sqrt{c\,x^{6}+b\,x^{3}+a}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]

$$\int \sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int \sqrt{a+bx^3+cx^6}\,dx = \int \sqrt{cx^6+bx^3+a}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{c\,x^6+b\,x^3+a}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^3+cx^6}\,dx=\frac{\sqrt{c\,x^6+b\,x^3+a}\,x}{4}+\frac{3\left(\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{c\,x^6+b\,x^3+a}dx\right)a}{4}+\frac{3\left(\int \frac{\sqrt{c\,x^6+b\,x^3+a}\,x^3}{c\,x^6+b\,x^3+a}dx\right)b}{8}$$

input
```
int((c*x^6+b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3 + c*x**6)*x + 6*int(sqrt(a + b*x**3 + c*x**6)/(a + b*x**
*3 + c*x**6),x)*a + 3*int((sqrt(a + b*x**3 + c*x**6)*x**3)/(a + b*x**3 + c
*x**6),x)*b)/8
```

3.184 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = -\frac{\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output

```
-(c*x^6+b*x^3+a)^(1/2)*AppellF1(-1/3,-1/2,-1/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2
)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 340 vs. 2(138) = 276.

Time = 10.41 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.46

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \frac{-20(a+bx^3+cx^6)+15bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4}}\right.}{20x\sqrt{a+bx^3+}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

output
```
(-20*(a + b*x^3 + c*x^6) + 15*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 12*c*x^6*Sqrt[(b - Sqrt[b^2 -
 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*x*Sqrt[a
 + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx$$
$$\Big\downarrow \ 1721$$
$$\frac{\sqrt{a+bx^3+cx^6}\int \frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}{x^2}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$
$$\Big\downarrow \ 1012$$
$$-\frac{\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{1}{3},-\frac{1}{2},-\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

output
```
-((Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -1/2, -1/2, 2/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{2}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx=\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx=\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^2}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^2,x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx=\frac{2\sqrt{c\,x^6+b\,x^3+a}+3\left(\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{c\,x^8+b\,x^5+a\,x^2}dx\right)ax-3\left(\int \frac{\sqrt{c\,x^6+b\,x^3+a}\,x^4}{c\,x^6+b\,x^3+a}dx\right)cx}{x}$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

output
```
(2*sqrt(a + b*x**3 + c*x**6) + 3*int(sqrt(a + b*x**3 + c*x**6)/(a*x**2 + b
*x**5 + c*x**8),x)*a*x - 3*int((sqrt(a + b*x**3 + c*x**6)*x**4)/(a + b*x**
3 + c*x**6),x)*c*x)/x
```

3.185 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3} \, dx = -\frac{\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
-1/2*(c*x^6+b*x^3+a)^(1/2)*AppellF1(-2/3,-1/2,-1/2,1/3,-2*c*x^3/(b-(-4*a*c
+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^2/(1+2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 340 vs. 2(140) = 280.

Time = 10.38 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.43

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3} \, dx = \frac{-4(a+bx^3+cx^6)+6bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right.}{8x^2\sqrt{a+bx^3+cx}}$$

input
```
Integrate[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

output
```
(-4*(a + b*x^3 + c*x^6) + 6*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(8*x^2*Sqrt[a +
 b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx$$

$$\Big\downarrow \ 1721$$

$$\frac{\sqrt{a+bx^3+cx^6}\int \frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}{x^3}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \ 1012$$

$$-\frac{\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{1}{2},-\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

output
```
-1/2*(Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -1/2, -1/2, 1/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[1 +
 (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{6}+b\,x^{3}+a}}{x^{3}}dx$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

output
```
int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a + b*x**3 + c*x**6)/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \int \frac{\sqrt{cx^6+bx^3+a}}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx=\int \frac{\sqrt{cx^6+bx^3+a}}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx=\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^3}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(1/2)/x^3,x)
```

output
```
int((a + b*x^3 + c*x^6)^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx = \frac{-2\sqrt{c\,x^6+b\,x^3+a}-3\left(\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{c\,x^9+b\,x^6+a\,x^3}dx\right)a\,x^2+3\left(\int \frac{\sqrt{c\,x^6+b\,x^3+a}\,x^3}{c\,x^6+b\,x^3+a}dx\right)c\,x^2}{x^2}$$

input
```
int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

output
```
( - 2*sqrt(a + b*x**3 + c*x**6) - 3*int(sqrt(a + b*x**3 + c*x**6)/(a*x**3
+ b*x**6 + c*x**9),x)*a*x**2 + 3*int((sqrt(a + b*x**3 + c*x**6)*x**3)/(a +
 b*x**3 + c*x**6),x)*c*x**2)/x**2
```

3.186 $\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 293

$$\int x^{14}\left(a+bx^3+cx^6\right)^{3/2}\,dx = -\frac{(b^2-4ac)\,(33b^4-72ab^2c+16a^2c^2)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{16384c^6} + \frac{(33b^4-72ab^2c+16a^2c^2)\,(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{6144c^5} - \frac{11bx^6(a+bx^3+cx^6)^{5/2}}{336c^2} + \frac{x^9(a+bx^3+cx^6)^{5/2}}{24c} - \frac{(3b(77b^2-124ac)-10c(33b^2-28ac)\,x^3)\,(a+bx^3+cx^6)^{5/2}}{13440c^4} + \frac{(b^2-4ac)^2\,(33b^4-72ab^2c+16a^2c^2)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{32768c^{13/2}}$$

output
```
-1/16384*(-4*a*c+b^2)*(16*a^2*c^2-72*a*b^2*c+33*b^4)*(2*c*x^3+b)*(c*x^6+b*
x^3+a)^(1/2)/c^6+1/6144*(16*a^2*c^2-72*a*b^2*c+33*b^4)*(2*c*x^3+b)*(c*x^6+
b*x^3+a)^(3/2)/c^5-11/336*b*x^6*(c*x^6+b*x^3+a)^(5/2)/c^2+1/24*x^9*(c*x^6+
b*x^3+a)^(5/2)/c-1/13440*(3*b*(-124*a*c+77*b^2)-10*c*(-28*a*c+33*b^2)*x^3)
*(c*x^6+b*x^3+a)^(5/2)/c^4+1/32768*(-4*a*c+b^2)^2*(16*a^2*c^2-72*a*b^2*c+3
3*b^4)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(13/2)
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.99

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx = \frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}(-3465b^7+2310b^6cx^3+84b^5c(365a-22cx^6)+24b^4c^2x^3(-749a+66c$$

input
```
Integrate[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(-3465*b^7 + 2310*b^6*c*x^3 + 84*b^5*c*
(365*a - 22*c*x^6) + 24*b^4*c^2*x^3*(-749*a + 66*c*x^6) + 32*b^2*c^3*x^3*(
1181*a^2 - 284*a*c*x^6 + 40*c^2*x^12) - 16*b^3*c^2*(5103*a^2 - 780*a*c*x^6
 + 88*c^2*x^12) + 4480*c^4*x^3*(-3*a^3 + 2*a^2*c*x^6 + 24*a*c^2*x^12 + 16*
c^3*x^18) + 64*b*c^3*(919*a^3 - 302*a^2*c*x^6 + 104*a*c^2*x^12 + 1360*c^3*
x^18)) - 105*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*Log[b + 2*
c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(3440640*c^(13/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1693, 1166, 27, 1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^{12}(cx^6+bx^3+a)^{3/2}\,dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{1}{2}x^6(11bx^3+6a)\,(cx^6+bx^3+a)^{3/2}\,dx^3}{8c}+\frac{x^9(a+bx^3+cx^6)^{5/2}}{8c}\right)$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\int x^6\left(11bx^3+6a\right)\left(cx^6+bx^3+a\right)^{3/2}dx^3}{16c}\right)$$

$\downarrow$ 1236

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{\int -\frac{1}{2}x^3\left(3\left(33b^2-28ac\right)x^3+44ab\right)\left(cx^6+bx^3+a\right)^{3/2}dx^3}{7c}+\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}}{16c}\right)$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\int x^3\left(3\left(33b^2-28ac\right)x^3+44ab\right)\left(cx^6+bx^3+a\right)^{3/2}dx^3}{14c}}{16c}\right)$$

$\downarrow$ 1225

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7\left(16a^2c^2-72ab^2c+33b^4\right)\int\left(cx^6+bx^3+a\right)^{3/2}dx^3}{8c^2}-\frac{\left(3b\left(77b^2-124ac\right)-10cx^3\left(33b^2-28a\right.\right.}{20c^2}}{14c}}{16c}\right.$$

$\downarrow$ 1087

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7\left(16a^2c^2-72ab^2c+33b^4\right)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int\sqrt{cx^6+bx^3+a}dx}{16c}\right.}{8c^2}}{14c}}{16c}\right.$$

$\downarrow$ 1087

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{7\left(16a^2c^2-72ab^2c+33b^4\right)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+b}}{4c}\right.}{}\right.}{8c^2}}{16c}\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7\left(16a^2c^2-72ab^2c+33b^4\right)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3\cdots}}{4c}\cdots\right)}{\cdots}\right)}{8c^2}\cdots}{\cdots}}{16c}\cdots\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{8c}-\frac{\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7\left(16a^2c^2-72ab^2c+33b^4\right)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3\cdots}}{4c}\cdots\right)}{\cdots}\right)}{8c^2}\cdots}{\cdots}}{16\cdots}\cdots\right)$$

input
```
Int[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((x^9*(a + b*x^3 + c*x^6)^(5/2))/(8*c) - ((11*b*x^6*(a + b*x^3 + c*x^6)^(5
/2))/(7*c) - (-1/20*((3*b*(77*b^2 - 124*a*c) - 10*c*(33*b^2 - 28*a*c)*x^3)
*(a + b*x^3 + c*x^6)^(5/2))/c^2 + (7*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*((
(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2
*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x
^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2))))/(16*c)))/(8*c^2))/
(14*c))/(16*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{14}\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}dx$$

input
```
int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.19

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx = \left[\frac{105\,(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b\right.}{}\right.$$

$$-\frac{105\,(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(7}{}$$

input
```
integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/6881280*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^
3 + 256*a^4*c^4)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 +
 b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(71680*c^8*x^21 + 87040*b*c
^7*x^18 + 1280*(b^2*c^6 + 84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x
^12 + 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 306
60*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560
*a*b^3*c^4 + 2416*a^2*b*c^5)*x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 1889
6*a^2*b^2*c^4 - 6720*a^3*c^5)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7, -1/344064
0*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a
^4*c^4)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)
/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(71680*c^8*x^21 + 87040*b*c^7*x^18 + 1280*
(b^2*c^6 + 84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12 + 16*(99*b^
4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*b^5*c^2 -
81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 + 24
16*a^2*b*c^5)*x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 -
 6720*a^3*c^5)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7]
```

Sympy [F]

$$\int x^{14}\left(a+bx^3+cx^6\right)^{3/2}\,dx=\int x^{14}\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**14*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**14*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{14}\left(a+bx^3+cx^6\right)^{3/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx=\int (cx^6+bx^3+a)^{\frac{3}{2}}x^{14}\,dx$$

input
```
integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x^14, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx=\int x^{14}\,(c\,x^6+b\,x^3+a)^{3/2}\,dx$$

input
```
int(x^14*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x^14*(a + b*x^3 + c*x^6)^(3/2), x)
```

Reduce [F]

$$\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx=\text{too large to display}$$

input
```
int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
( - 53760*sqrt(a + b*x**3 + c*x**6)*a**4*c**5 + 386432*sqrt(a + b*x**3 + c
*x**6)*a**3*b**2*c**4 - 26880*sqrt(a + b*x**3 + c*x**6)*a**3*b*c**5*x**3 -
 398496*sqrt(a + b*x**3 + c*x**6)*a**2*b**4*c**3 + 75584*sqrt(a + b*x**3 +
 c*x**6)*a**2*b**3*c**4*x**3 - 38656*sqrt(a + b*x**3 + c*x**6)*a**2*b**2*c
**5*x**6 + 17920*sqrt(a + b*x**3 + c*x**6)*a**2*b*c**6*x**9 + 131880*sqrt(
a + b*x**3 + c*x**6)*a*b**6*c**2 - 35952*sqrt(a + b*x**3 + c*x**6)*a*b**5*
c**3*x**3 + 24960*sqrt(a + b*x**3 + c*x**6)*a*b**4*c**4*x**6 - 18176*sqrt(
a + b*x**3 + c*x**6)*a*b**3*c**5*x**9 + 13312*sqrt(a + b*x**3 + c*x**6)*a*
b**2*c**6*x**12 + 215040*sqrt(a + b*x**3 + c*x**6)*a*b*c**7*x**15 - 13860*
sqrt(a + b*x**3 + c*x**6)*b**8*c + 4620*sqrt(a + b*x**3 + c*x**6)*b**7*c**
2*x**3 - 3696*sqrt(a + b*x**3 + c*x**6)*b**6*c**3*x**6 + 3168*sqrt(a + b*x
**3 + c*x**6)*b**5*c**4*x**9 - 2816*sqrt(a + b*x**3 + c*x**6)*b**4*c**5*x*
*12 + 2560*sqrt(a + b*x**3 + c*x**6)*b**3*c**6*x**15 + 174080*sqrt(a + b*x
**3 + c*x**6)*b**2*c**7*x**18 + 143360*sqrt(a + b*x**3 + c*x**6)*b*c**8*x*
*21 - 26880*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a**4*b*c
**4 + 134400*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a**3*b*
*3*c**3 - 117600*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a**
2*b**5*c**2 + 35280*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*
a*b**7*c - 3465*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*b**9
 + 26880*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a**4*b*c...
```

3.187 $\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 223

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx = \frac{b(b^2-4ac)(3b^2-4ac)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1024c^5} - \frac{b(3b^2-4ac)(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{384c^4} + \frac{x^6(a+bx^3+cx^6)^{5/2}}{21c} + \frac{(21b^2-16ac-30bcx^3)(a+bx^3+cx^6)^{5/2}}{840c^3} - \frac{b(b^2-4ac)^2(3b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2048c^{11/2}}$$

output
```
1/1024*b*(-4*a*c+b^2)*(-4*a*c+3*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^5
-1/384*b*(-4*a*c+3*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^4+1/21*x^6*(c*
x^6+b*x^3+a)^(5/2)/c+1/840*(-30*b*c*x^3-16*a*c+21*b^2)*(c*x^6+b*x^3+a)^(5/
2)/c^3-1/2048*b*(-4*a*c+b^2)^2*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1
/2)/(c*x^6+b*x^3+a)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.99

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx = \frac{\sqrt{a+bx^3+cx^6}\left(315b^6-210b^5cx^3+16b^3c^2x^3(91a-9cx^6)+168b^4c(-15a+cx^6)+1024c^3\right.}{}$$
$$+\frac{b(b^2-4ac)^2\,(3b^2-4ac)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{2048c^{11/2}}$$

input
```
Integrate[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(315*b^6 - 210*b^5*c*x^3 + 16*b^3*c^2*x^3*(91*a -
 9*c*x^6) + 168*b^4*c*(-15*a + c*x^6) + 1024*c^3*(a + c*x^6)^2*(-2*a + 5*c
*x^6) + 16*b^2*c^2*(343*a^2 - 62*a*c*x^6 + 8*c^2*x^12) + 32*b*c^3*x^3*(-73
*a^2 + 22*a*c*x^6 + 200*c^2*x^12)))/(107520*c^5) + (b*(b^2 - 4*a*c)^2*(3*b
^2 - 4*a*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(2048*c^
(11/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1693, 1166, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3}\int x^9\left(cx^6+bx^3+a\right)^{3/2}dx^3$$

$$\Big\downarrow \text{ 1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{1}{2}x^3\left(9bx^3+4a\right)\left(cx^6+bx^3+a\right)^{3/2}dx^3}{7c}+\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\int x^3\left(9bx^3+4a\right)\left(cx^6+bx^3+a\right)^{3/2}dx^3}{14c}\right)$$

$\Big\downarrow$ 1225

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\int\left(cx^6+bx^3+a\right)^{3/2}dx^3}{8c^2}-\frac{\left(-16ac+21b^2-30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{20c^2}}{14c}\right)$$

$\Big\downarrow$ 1087

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int\sqrt{cx^6+bx^3+a}dx^3}{16c}\right)}{8c^2}-\frac{\left(-16ac+21b^2-30bcx^3\right)\left(a+bx^3+cx^6\right)^{5/2}}{20c^2}}{14c}\right)$$

$\Big\downarrow$ 1087

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{16c}\right)}{8c^2}}{14c}\right)$$

$\Big\downarrow$ 1092

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3}}}{4c}\right.}{16c}\right.}{8c^2}}{14c}\right.$$

$\downarrow$ 219

$$\frac{1}{3}\left(\frac{x^6\left(a+bx^3+cx^6\right)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+}{2\sqrt{c}\sqrt{a+}}\right.}{8c^{3/2}}\right.}{16c}\right.}{8c^2}}{14c}\right.$$

input
```
Int[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((x^6*(a + b*x^3 + c*x^6)^(5/2))/(7*c) - (-1/20*((21*b^2 - 16*a*c - 30*b*c
*x^3)*(a + b*x^3 + c*x^6)^(5/2))/c^2 + (7*b*(3*b^2 - 4*a*c)*(((b + 2*c*x^3
)*(a + b*x^3 + c*x^6)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x^3)*Sqrt
[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[
c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2))))/(16*c)))/(8*c^2))/(14*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{11}\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}dx$$

input
```
int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.40

$$\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}\,dx = \left[-\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3}\right.}{}\right.$$

input
```
integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(
c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 +
 b)*sqrt(c) - 4*a*c) - 4*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128*(b^2*c^5 +
 64*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c - 2520*a*b^4
*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 1
28*a^2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x^3)*sq
rt(c*x^6 + b*x^3 + a))/c^6, 1/215040*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3
*c^2 - 64*a^3*b*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
+ b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(5120*c^7*x^18 + 6400*b*c^6*x
^15 + 128*(b^2*c^5 + 64*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 31
5*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3
 - 124*a*b^2*c^4 + 128*a^2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 116
8*a^2*b*c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

Sympy [F]

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx = \int x^{11}\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**11*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**11*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}\,dx=\int\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^{11}\,dx$$

input
```
integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}\,dx=\int x^{11}\,\left(c\,x^6+b\,x^3+a\right)^{3/2}\,dx$$

input
```
int(x^11*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x^11*(a + b*x^3 + c*x^6)^(3/2), x)
```

Reduce [F]

$$\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
( - 17536*sqrt(a + b*x**3 + c*x**6)*a**3*c**4 + 27776*sqrt(a + b*x**3 + c*
x**6)*a**2*b**2*c**3 - 4672*sqrt(a + b*x**3 + c*x**6)*a**2*b*c**4*x**3 + 2
048*sqrt(a + b*x**3 + c*x**6)*a**2*c**5*x**6 - 10920*sqrt(a + b*x**3 + c*x
**6)*a*b**4*c**2 + 2912*sqrt(a + b*x**3 + c*x**6)*a*b**3*c**3*x**3 - 1984*
sqrt(a + b*x**3 + c*x**6)*a*b**2*c**4*x**6 + 1408*sqrt(a + b*x**3 + c*x**6
)*a*b*c**5*x**9 + 16384*sqrt(a + b*x**3 + c*x**6)*a*c**6*x**12 + 1260*sqrt
(a + b*x**3 + c*x**6)*b**6*c - 420*sqrt(a + b*x**3 + c*x**6)*b**5*c**2*x**
3 + 336*sqrt(a + b*x**3 + c*x**6)*b**4*c**3*x**6 - 288*sqrt(a + b*x**3 + c
*x**6)*b**3*c**4*x**9 + 256*sqrt(a + b*x**3 + c*x**6)*b**2*c**5*x**12 + 12
800*sqrt(a + b*x**3 + c*x**6)*b*c**6*x**15 + 10240*sqrt(a + b*x**3 + c*x**
6)*c**7*x**18 - 6720*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)
*a**3*b*c**3 + 8400*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*
a**2*b**3*c**2 - 2940*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3
)*a*b**5*c + 315*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*b**
7 + 6720*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a**3*b*c**3
 - 8400*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a**2*b**3*c*
*2 + 2940*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*b**5*c -
 315*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**7 + 40320*in
t((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x*
*6 + b*c*x**9),x)*a**3*b*c**5 - 50400*int((sqrt(a + b*x**3 + c*x**6)*x*...
```

3.188 $\int x^8(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 204

$$\int x^8\left(a+bx^3+cx^6\right)^{3/2}\,dx=-\frac{(b^2-4ac)\,(7b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{(7b^2-4ac)\,(b+2cx^3)\,\left(a+bx^3+cx^6\right)^{3/2}}{576c^3}-\frac{7b(a+bx^3+cx^6)^{5/2}}{180c^2}+\frac{x^3(a+bx^3+cx^6)^{5/2}}{18c}+\frac{(b^2-4ac)^2\,(7b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{9/2}}$$

output

```
-1/1536*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^4+
1/576*(-4*a*c+7*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^3-7/180*b*(c*x^6+
b*x^3+a)^(5/2)/c^2+1/18*x^3*(c*x^6+b*x^3+a)^(5/2)/c+1/3072*(-4*a*c+b^2)^2*
(-4*a*c+7*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(9
/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95

$$\int x^8 \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}(-105b^5+70b^4cx^3+8b^3c(95a-7cx^6)+48b^2c^2x^3(-9a+cx^6)+160c^3x^3 \ldots}{}$$

input
```
Integrate[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(-105*b^5 + 70*b^4*c*x^3 + 8*b^3*c*(95*
a - 7*c*x^6) + 48*b^2*c^2*x^3*(-9*a + c*x^6) + 160*c^3*x^3*(3*a^2 + 14*a*c
*x^6 + 8*c^2*x^12) + 16*b*c^2*(-81*a^2 + 18*a*c*x^6 + 104*c^2*x^12)) - 15*
(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3
 + c*x^6]])/(46080*c^(9/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1693, 1166, 27, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3+cx^6\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3}\int x^6 \left(cx^6+bx^3+a\right)^{3/2} dx^3$$

$$\Big\downarrow \text{ 1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{1}{2}\left(7bx^3+2a\right)\left(cx^6+bx^3+a\right)^{3/2} dx^3}{6c}+\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}-\frac{\int \left(7bx^3+2a\right)\left(cx^6+bx^3+a\right)^{3/2}dx^3}{12c}\right)$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{\left(7b^2-4ac\right)\int \left(cx^6+bx^3+a\right)^{3/2}dx^3}{2c}}{12c}\right)$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{\left(7b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^6+bx^3+a}\,dx^3}{16c}\right)}{2c}}{12c}\right)$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{\left(7b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4a\right.}{}\right.}{16c}\right.}{2c}}{12c}\right.$$

$$\Big\downarrow \; 1092$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{\left(7b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4a\right.}{}\right.}{16c}\right.}{2c}}{12c}\right.$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{(7b^2-4ac)\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4a}{}\right)}{16c}\right)}{2c}}{12c}\right)$$

input
```
Int[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((x^3*(a + b*x^3 + c*x^6)^(5/2))/(6*c) - ((7*b*(a + b*x^3 + c*x^6)^(5/2))/
(5*c) - ((7*b^2 - 4*a*c)*(((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(8*c)
- (3*(b^2 - 4*a*c)*(((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2
- 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^
(3/2))))/(16*c)))/(2*c))/(12*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{8} \left(c\,x^{6}+b\,x^{3}+a \right)^{\frac{3}{2}}d x$$

input
```
int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.21

$$\int x^8(a+bx^3+cx^6)^{3/2}\,dx = \Bigg[-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3}\right.}{}$$

$$-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(1280\,c^6x^{15}+1664\,bc^5}{}$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*
log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)
*sqrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*b^2*c^4 + 1
40*a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760*a*b^3*c^2
 - 1296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt
(c*x^6 + b*x^3 + a))/c^5, -1/46080*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c
^2 - 64*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)
*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(1280*c^6*x^15 + 1664*b*c^5*x^12
+ 16*(3*b^2*c^4 + 140*a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^
5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240
*a^2*c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

Sympy [F]

$$\int x^8(a+bx^3+cx^6)^{3/2}\,dx = \int x^8(a+bx^3+cx^6)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**8*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**8*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^8 \left(a + bx^3 + cx^6\right)^{3/2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int x^8 \left(a + bx^3 + cx^6\right)^{3/2} \, dx = \int \left(cx^6 + bx^3 + a\right)^{\frac{3}{2}} x^8 \, dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \left(a + bx^3 + cx^6\right)^{3/2} \, dx = \int x^8 \left(c\,x^6 + b\,x^3 + a\right)^{3/2} \,dx$$

input
```
int(x^8*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x^8*(a + b*x^3 + c*x^6)^(3/2), x)
```

Reduce [F]

$$\int x^8 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \text{Too large to display}$$

input
```
int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
(1920*sqrt(a + b*x**3 + c*x**6)*a**3*c**4 - 6912*sqrt(a + b*x**3 + c*x**6)
*a**2*b**2*c**3 + 960*sqrt(a + b*x**3 + c*x**6)*a**2*b*c**4*x**3 + 3320*sq
rt(a + b*x**3 + c*x**6)*a*b**4*c**2 - 864*sqrt(a + b*x**3 + c*x**6)*a*b**3
*c**3*x**3 + 576*sqrt(a + b*x**3 + c*x**6)*a*b**2*c**4*x**6 + 4480*sqrt(a
+ b*x**3 + c*x**6)*a*b*c**5*x**9 - 420*sqrt(a + b*x**3 + c*x**6)*b**6*c +
140*sqrt(a + b*x**3 + c*x**6)*b**5*c**2*x**3 - 112*sqrt(a + b*x**3 + c*x**
6)*b**4*c**3*x**6 + 96*sqrt(a + b*x**3 + c*x**6)*b**3*c**4*x**9 + 3328*sqr
t(a + b*x**3 + c*x**6)*b**2*c**5*x**12 + 2560*sqrt(a + b*x**3 + c*x**6)*b*
c**6*x**15 + 960*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a**
3*b*c**3 - 2160*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a**2
*b**3*c**2 + 900*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a*b
**5*c - 105*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*b**7 - 9
60*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a**3*b*c**3 + 216
0*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a**2*b**3*c**2 - 9
00*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*b**5*c + 105*sq
rt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**7 - 5760*int((sqrt(
a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c
*x**9),x)*a**3*b*c**5 + 12960*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 +
 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a**2*b**3*c**4 - 5400*in
t((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2...
```

3.189 $\int x^5(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\int x^5\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{b(b^2-4ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c}-\frac{b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}$$

output
```
1/128*b*(-4*a*c+b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^3-1/48*b*(2*c*x^3
+b)*(c*x^6+b*x^3+a)^(3/2)/c^2+1/15*(c*x^6+b*x^3+a)^(5/2)/c-1/256*b*(-4*a*c
+b^2)^2*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int x^5\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{\sqrt{a+bx^3+cx^6}\left(15b^4-10b^3cx^3+128c^2\left(a+cx^6\right)^2+4b^2c\left(-25a+2cx^6\right)+8bc^2x^3(7a+22c\right.}{1920c^3}+\frac{b(b^2-4ac)^2\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{256c^{7/2}}$$

input
```
Integrate[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(15*b^4 - 10*b^3*c*x^3 + 128*c^2*(a + c*x^6)^2 +
4*b^2*c*(-25*a + 2*c*x^6) + 8*b*c^2*x^3*(7*a + 22*c*x^6)))/(1920*c^3) + (b
*(b^2 - 4*a*c)^2*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(25
6*c^(7/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1693, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3+cx^6\right)^{3/2} dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^3 \left(cx^6+bx^3+a\right)^{3/2} dx^3$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{b\int \left(cx^6+bx^3+a\right)^{3/2} dx^3}{2c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{b\left(\frac{(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3(b^2-4ac)\int \sqrt{cx^6+bx^3+a}dx^3}{16c}\right)}{2c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{b\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{16c}\right)}{2c}\right)$$

$\big\downarrow$ 1092

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{b\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right)}{16c}\right)}{2c}\right)$$

$\big\downarrow$ 219

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{5/2}}{5c}-\frac{b\left(\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)}{16c}\right)}{2c}\right)$$

input
```
Int[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((a + b*x^3 + c*x^6)^(5/2)/(5*c) - (b*(((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^
(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(
4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*
x^6])])/(8*c^(3/2))))/(16*c)))/(2*c))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^5 (c x^6 + b x^3 + a)^{\frac{3}{2}} dx$$

input
```
int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.41

$$\int x^5 (a + bx^3 + cx^6)^{3/2} dx = \left[\frac{15 (b^5 - 8 ab^3 c + 16 a^2 bc^2) \sqrt{c} \log \left(-8 c^2 x^6 - 8 bcx^3 - b^2 + 4 \sqrt{cx^6 + bx^3 + a}(2 cx^3 + b)\sqrt{c}\right.}{}\right.$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*
c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4
*(128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c - 1
00*a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6 +
b*x^3 + a))/c^4, 1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arct
an(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 +
 a*c)) + 2*(128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15
*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqr
t(c*x^6 + b*x^3 + a))/c^4]
```

Sympy [F]

$$\int x^5 \left(a+bx^3+cx^6\right)^{3/2}\,dx=\int x^5 \left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**5*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**5*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^5 \left(a+bx^3+cx^6\right)^{3/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.13

$$\int x^5 \left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{1}{1920}\,\sqrt{cx^6+bx^3+a}\left(2\left(4\left(2\,(8\,cx^3+11\,b)x^3+\frac{b^2c^3+32\,ac^4}{c^4}\right)x^3-\frac{5\,b^3c^2-28\,abc^3}{c^4}\right)x^3\right.$$
$$+\frac{(b^5-8\,ab^3c+16\,a^2bc^2)\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{7}{2}}}$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*(8*c*x^3 + 11*b)*x^3 + (b^2*c^3 +
32*a*c^4)/c^4)*x^3 - (5*b^3*c^2 - 28*a*b*c^3)/c^4)*x^3 + (15*b^4*c - 100*a
*b^2*c^2 + 128*a^2*c^3)/c^4) + 1/256*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*log(
abs(2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 20.74 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.49

$$\int x^5 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \frac{\left(c\,x^6+b\,x^3+a\right)^{5/2}}{15\,c} - \frac{b\left(\frac{3\,a\left(\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\left(\frac{a}{2\,\sqrt{c}}-\frac{b^2}{8\,c^{3/2}}\right)+\frac{\left(2\,c\,x^3+b\right)\,\sqrt{c\,x^6+b\,x^3+a}}{4\,c}\right)}{4} + \frac{x^3\,\left(c\,x^6+b\,x^3+a\right)^{3/2}}{4} - \frac{3\,b^2\left(\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3}{\sqrt{}}\right.\right.}{}\right.}{6\,c}$$

input
```
int(x^5*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
(a + b*x^3 + c*x^6)^(5/2)/(15*c) - (b*((3*a*(log((a + b*x^3 + c*x^6)^(1/2)
 + (b/2 + c*x^3)/c^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^
3)*(a + b*x^3 + c*x^6)^(1/2))/(4*c)))/4 + (x^3*(a + b*x^3 + c*x^6)^(3/2))/
4 - (3*b^2*(log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a/(2*c
^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(4*
c)))/(16*c) + (b*(a + b*x^3 + c*x^6)^(3/2))/(8*c)))/(6*c)
```

Reduce [F]

$$\int x^5 \left(a+bx^3 +cx^6\right)^{3/2} \, dx = \frac{736\sqrt{c\,x^6+b\,x^3+a}\,a^2c^3-440\sqrt{c\,x^6+b\,x^3+a}\,a\,b^2c^2+112\sqrt{c\,x^6+b\,x^3+a}\,ab\,c^3x^3+512\sqrt{}}{}$$

input

```
int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

output

```
(736*sqrt(a + b*x**3 + c*x**6)*a**2*c**3 - 440*sqrt(a + b*x**3 + c*x**6)*a
*b**2*c**2 + 112*sqrt(a + b*x**3 + c*x**6)*a*b*c**3*x**3 + 512*sqrt(a + b*
x**3 + c*x**6)*a*c**4*x**6 + 60*sqrt(a + b*x**3 + c*x**6)*b**4*c - 20*sqrt
(a + b*x**3 + c*x**6)*b**3*c**2*x**3 + 16*sqrt(a + b*x**3 + c*x**6)*b**2*c
**3*x**6 + 352*sqrt(a + b*x**3 + c*x**6)*b*c**4*x**9 + 256*sqrt(a + b*x**3
 + c*x**6)*c**5*x**12 + 240*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c
)*x**3)*a**2*b*c**2 - 120*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*
x**3)*a*b**3*c + 15*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*
b**5 - 240*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a**2*b*c*
*2 + 120*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*b**3*c -
15*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**5 - 1440*int((
sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6
+ b*c*x**9),x)*a**2*b*c**4 + 720*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**
2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a*b**3*c**3 - 90*int(
(sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6
 + b*c*x**9),x)*b**5*c**2 - 1440*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**
2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a**3*c**4 + 270*int((
sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6
+ b*c*x**9),x)*a*b**4*c**2 - 45*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2
 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*b**6*c)/(3840*c**4)
```

3.190 $\int x^2(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 124

$$\int x^2\left(a+bx^3+cx^6\right)^{3/2}\,dx=-\frac{(b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{24c}+\frac{(b^2-4ac)^2\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}$$

output
```
-1/64*(-4*a*c+b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^2+1/24*(2*c*x^3+b)*
(c*x^6+b*x^3+a)^(3/2)/c+1/128*(-4*a*c+b^2)^2*arctanh(1/2*(2*c*x^3+b)/c^(1/
2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int x^2\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{(b+2cx^3)\,\sqrt{a+bx^3+cx^6}(-3b^2+20ac+8bcx^3+8c^2x^6)}{192c^2}+\frac{(-b^2+4ac)^2\,\text{arctanh}\left(\frac{\sqrt{c}x^3}{-\sqrt{a}+\sqrt{a+bx^3+cx^6}}\right)}{64c^{5/2}}$$

input
```
Integrate[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 20*a*c + 8*b*c*x^3 + 8*c^
2*x^6))/(192*c^2) + ((-b^2 + 4*a*c)^2*ArcTanh[(Sqrt[c]*x^3)/(-Sqrt[a] + Sq
rt[a + b*x^3 + c*x^6])])/(64*c^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1690, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3+cx^6\right)^{3/2} dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \left(cx^6+bx^3+a\right)^{3/2} dx^3$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3}\left(\frac{(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3(b^2-4ac)\int \sqrt{cx^6+bx^3+a}dx^3}{16c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{3}\left(\frac{(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)}{16c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4ac)\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c}\right)}{16c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{3/2}}\right)}{16c}\right)$$

input
```
Int[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b +
 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c
*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2))))/(16*c))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [F]

$$\int x^2 \left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}} dx$$

input
```
int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.40

$$\int x^2 \left(a+bx^3+cx^6\right)^{3/2} dx = \left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-\right.}{768\,c^3}\right.$$

$$-\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(16\,c^4x^9+24\,bc^3x^6-3\,b^3c+20\,abc^2+}{384\,c^3}$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^
3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(16
*c^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^
3)*sqrt(c*x^6 + b*x^3 + a))/c^3, -1/384*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*
sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^
6 + b*c*x^3 + a*c)) - 2*(16*c^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2
+ 2*(b^2*c^2 + 20*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

Sympy [F]

$$\int x^2 \left(a+bx^3+cx^6\right)^{3/2} dx = \int x^2 \left(a+bx^3+cx^6\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**2*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**2*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(a+bx^3+cx^6\right)^{3/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int x^2 \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{1}{192} \sqrt{cx^6+bx^3+a} \left(2 \left(4 \left(2 cx^3+3 b\right) x^3 + \frac{b^2c^2+20 ac^3}{c^3}\right) x^3 - \frac{3 b^3 c - 20 abc^2}{c^3}\right) - \frac{\left(b^4-8 ab^2 c+16 a^2 c^2\right) \log \left(\left|2 \left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{128 c^{\frac{5}{2}}}$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*c*x^3 + 3*b)*x^3 + (b^2*c^2 + 20*a*
c^3)/c^3)*x^3 - (3*b^3*c - 20*a*b*c^2)/c^3) - 1/128*(b^4 - 8*a*b^2*c + 16*
a^2*c^2)*log(abs(2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c
^(5/2)
```

Mupad [B] (verification not implemented)

Time = 21.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int x^2\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{\left(c\,x^3+\frac{b}{2}\right)\,\left(c\,x^6+b\,x^3+a\right)^{3/2}}{12\,c}+\frac{\left(3\,a\,c-\frac{3\,b^2}{4}\right)\,\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{12\,c}$$

input
```
int(x^2*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
((b/2 + c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(12*c) + ((3*a*c - (3*b^2)/4)*((
b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2
) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(12*c)
```

Reduce [F]

$$\int x^2\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{-96\sqrt{c\,x^6+b\,x^3+a}\,a^2c^3+88\sqrt{c\,x^6+b\,x^3+a}\,a\,b^2c^2+80\sqrt{c\,x^6+b\,x^3+a}\,ab\,c^3x^3-12\sqrt{c}}{}$$

input
```
int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

output

```
( - 96*sqrt(a + b*x**3 + c*x**6)*a**2*c**3 + 88*sqrt(a + b*x**3 + c*x**6)*
a*b**2*c**2 + 80*sqrt(a + b*x**3 + c*x**6)*a*b*c**3*x**3 - 12*sqrt(a + b*x
**3 + c*x**6)*b**4*c + 4*sqrt(a + b*x**3 + c*x**6)*b**3*c**2*x**3 + 48*sqr
t(a + b*x**3 + c*x**6)*b**2*c**3*x**6 + 32*sqrt(a + b*x**3 + c*x**6)*b*c**
4*x**9 - 48*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a**2*b*c
**2 + 24*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a*b**3*c -
3*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*b**5 + 48*sqrt(c)*
log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a**2*b*c**2 - 24*sqrt(c)*log
(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*b**3*c + 3*sqrt(c)*log(sqrt(a
 + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**5 + 288*int((sqrt(a + b*x**3 + c*x*
*6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a**2*b*
c**4 - 144*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x
**6 + b**2*x**6 + b*c*x**9),x)*a*b**3*c**3 + 18*int((sqrt(a + b*x**3 + c*x
**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*b**5*c
**2 + 288*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x*
*6 + b**2*x**6 + b*c*x**9),x)*a**3*c**4 - 54*int((sqrt(a + b*x**3 + c*x**6
)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a*b**4*c*
*2 + 9*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6
+ b**2*x**6 + b*c*x**9),x)*b**6*c)/(384*b*c**3)
```

3.191 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 155

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \frac{(b^2+8ac+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}(a+bx^3+cx^6)^{3/2} - \frac{1}{3}a^{3/2}\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) - \frac{b(b^2-12ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}}$$

output
```
1/24*(2*b*c*x^3+8*a*c+b^2)*(c*x^6+b*x^3+a)^(1/2)/c+1/9*(c*x^6+b*x^3+a)^(3/
2)-1/3*a^(3/2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))-1/48
*b*(-12*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^
(3/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \frac{1}{144}\left(\frac{2\sqrt{a+bx^3+cx^6}(3b^2+14bcx^3+8c(4a+cx^6))}{c} - \frac{3(b^3-12abc)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{c^{3/2}} + 96a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)\right)$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

output
```
((2*Sqrt[a + b*x^3 + c*x^6]*(3*b^2 + 14*b*c*x^3 + 8*c*(4*a + c*x^6)))/c -
(3*(b^3 - 12*a*b*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^
6])])/c^(3/2) + 96*a^(3/2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])
/Sqrt[a]])/144
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1693, 1162, 25, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^{3/2}}{x^3}dx^3$$

$$\Big\downarrow \text{1162}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(a+bx^3+cx^6\right)^{3/2}-\frac{1}{2}\int -\frac{\left(bx^3+2a\right)\sqrt{cx^6+bx^3+a}}{x^3}dx^3\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{\left(bx^3+2a\right)\sqrt{cx^6+bx^3+a}}{x^3}dx^3+\frac{1}{3}\left(a+bx^3+cx^6\right)^{3/2}\right)$$

$$\Big\downarrow \; 1231$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\left(8ac+b^2+2bcx^3\right)\sqrt{a+bx^3+cx^6}}{4c}-\frac{\int -\frac{16a^2c-b\left(b^2-12ac\right)x^3}{2x^3\sqrt{cx^6+bx^3+a}}dx^3}{4c}\right)+\frac{1}{3}\left(a+bx^3+cx^6\right)^{3/2}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{\int \frac{16a^2c-b\left(b^2-12ac\right)x^3}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8c}+\frac{\sqrt{a+bx^3+cx^6}\left(8ac+b^2+2bcx^3\right)}{4c}\right)+\frac{1}{3}\left(a+bx^3+cx^6\right)^{3/2}\right)$$

$$\Big\downarrow \; 1269$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{16a^2c\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3-b\left(b^2-12ac\right)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}+\frac{\sqrt{a+bx^3+cx^6}\left(8ac+b^2+2bcx^3\right)}{4c}\right)+\frac{1}{3}(a\right.$$

$$\Big\downarrow \; 1092$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{16a^2c\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3-2b\left(b^2-12ac\right)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{8c}+\frac{\sqrt{a+bx^3+cx^6}\left(8ac+b^2+2bcx^3\right)}{4c}\right)+\right.$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{16a^2c\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3-\frac{b\left(b^2-12ac\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^3+cx^6}\left(8ac+b^2+2bcx^3\right)}{4c}\right)+\right.$$

$$\Big\downarrow \; 1154$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{-32a^2c\int\frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^3+cx^6}\,(8ac+b^2+2bcx^3)}{4c}\right)\right.$$

$\downarrow$ 219

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{-16a^{3/2}c\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^3+cx^6}\,(8ac+b^2+2b}{4c}\right.\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

output
```
((a + b*x^3 + c*x^6)^(3/2)/3 + (((b^2 + 8*a*c + 2*b*c*x^3)*Sqrt[a + b*x^3
+ c*x^6])/(4*c) + (-16*a^(3/2)*c*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a +
 b*x^3 + c*x^6])] - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqr
t[a + b*x^3 + c*x^6])])/Sqrt[c])/(8*c))/2)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 727, normalized size of antiderivative = 4.69

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/288*(48*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6
+ b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 3*(b^3 - 12*a*b*c)*sqrt
(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
+ b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*
sqrt(c*x^6 + b*x^3 + a))/c^2, 1/144*(24*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^
6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x
^6) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*
x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 + 14*b*c^2*x^3
 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/288*(96*sqrt(-a)*a*
c^2*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a
*b*x^3 + a^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b
^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x
^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/14
4*(48*sqrt(-a)*a*c^2*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt
(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*s
qrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) +
 2*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a)
)/c^2]
```

Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx=\text{too large to display}$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

output
```
(24*sqrt(a + b*x**3 + c*x**6)*a**2*c**3 + 88*sqrt(a + b*x**3 + c*x**6)*a*b
**2*c**2 + 84*sqrt(a + b*x**3 + c*x**6)*a*b*c**3*x**3 + 48*sqrt(a + b*x**3
 + c*x**6)*a*c**4*x**6 + 6*sqrt(a + b*x**3 + c*x**6)*b**4*c + 28*sqrt(a +
b*x**3 + c*x**6)*b**3*c**2*x**3 + 16*sqrt(a + b*x**3 + c*x**6)*b**2*c**3*x
**6 + 144*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*a**2*c**3 + 48*
sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*a*b**2*c**2 - 144*sqrt(a)
*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*a**2*c**3 - 48*sqrt(a)*log(sqrt(
a + b*x**3 + c*x**6) + sqrt(a))*a*b**2*c**2 - 108*sqrt(c)*log(sqrt(a + b*x
**3 + c*x**6) - sqrt(c)*x**3)*a**2*b*c**2 - 27*sqrt(c)*log(sqrt(a + b*x**3
 + c*x**6) - sqrt(c)*x**3)*a*b**3*c + 3*sqrt(c)*log(sqrt(a + b*x**3 + c*x*
*6) - sqrt(c)*x**3)*b**5 + 108*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqr
t(c)*x**3)*a**2*b*c**2 + 27*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c
)*x**3)*a*b**3*c - 3*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)
*b**5 + 1512*int((sqrt(a + b*x**3 + c*x**6)*x**11)/(3*a**3*b*c + 3*a**3*c*
*2*x**3 + a**2*b**3 + 7*a**2*b**2*c*x**3 + 9*a**2*b*c**2*x**6 + 3*a**2*c**
3*x**9 + 2*a*b**4*x**3 + 6*a*b**3*c*x**6 + 7*a*b**2*c**2*x**9 + 3*a*b*c**3
*x**12 + b**5*x**6 + 2*b**4*c*x**9 + b**3*c**2*x**12),x)*a**3*b*c**6 + 450
*int((sqrt(a + b*x**3 + c*x**6)*x**11)/(3*a**3*b*c + 3*a**3*c**2*x**3 + a*
*2*b**3 + 7*a**2*b**2*c*x**3 + 9*a**2*b*c**2*x**6 + 3*a**2*c**3*x**9 + 2*a
*b**4*x**3 + 6*a*b**3*c*x**6 + 7*a*b**2*c**2*x**9 + 3*a*b*c**3*x**12 + ...
```

3.192 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6} - \frac{(a+bx^3+cx^6)^{3/2}}{3x^3} - \frac{1}{2}\sqrt{a}b\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{(b^2+4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}$$

output
```
1/4*(2*c*x^3+3*b)*(c*x^6+b*x^3+a)^(1/2)-1/3*(c*x^6+b*x^3+a)^(3/2)/x^3-1/2*
a^(1/2)*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))+1/8*(4*a*
c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-4a+5bx^3+2cx^6)}{12x^3} + \sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right) - \frac{(b^2+4ac)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{8\sqrt{c}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-4*a + 5*b*x^3 + 2*c*x^6))/(12*x^3) + Sqrt[a]*b*
ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]] - ((b^2 + 4*a*c)*
Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(8*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1693, 1161, 1231, 25, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^{3/2}}{x^6}dx^3$$

$$\Big\downarrow \text{ 1161}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{(2cx^3+b)\sqrt{cx^6+bx^3+a}}{x^3}dx^3-\frac{(a+bx^3+cx^6)^{3/2}}{x^3}\right)$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{2}(3b+2cx^3)\sqrt{a+bx^3+cx^6}-\frac{\int -\frac{c((b^2+4ac)x^3+4ab)}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{4c}\right)-\frac{(a+bx^3+cx^6)^{3/2}}{x^3}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{\int \frac{c((b^2+4ac)x^3+4ab)}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{4c}+\frac{1}{2}(3b+2cx^3)\sqrt{a+bx^3+cx^6}\right)-\frac{(a+bx^3+cx^6)^{3/2}}{x^3}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{4}\int\frac{(b^2+4ac)\,x^3+4ab}{x^3\sqrt{cx^6+bx^3+a}}dx^3+\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}\right)-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^3}\right)$$

$\big\downarrow$ 1269

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{4}\left(\left(4ac+b^2\right)\int\frac{1}{\sqrt{cx^6+bx^3+a}}dx^3+4ab\int\frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)+\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+bx^3+cx^6}\right)\right.$$

$\big\downarrow$ 1092

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{4}\left(2\left(4ac+b^2\right)\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}+4ab\int\frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3\right)+\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+bx^3+\right.\right.\right.$$

$\big\downarrow$ 219

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{4}\left(4ab\int\frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3+\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}\right)+\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+bx^3+c\right.\right.$$

$\big\downarrow$ 1154

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{4}\left(\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}-8ab\int\frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}\right)+\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+bx^3\right.\right.$$

$\big\downarrow$ 219

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{1}{4}\left(\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{c}}-4\sqrt{a}b\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)\right)+\frac{1}{2}\left(3b+2cx^3\right)\sqrt{a+\right.\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

output
```
(-((a + b*x^3 + c*x^6)^(3/2)/x^3) + (3*(((3*b + 2*c*x^3)*Sqrt[a + b*x^3 +
c*x^6])/2 + (-4*Sqrt[a]*b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3
+ c*x^6])] + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^
3 + c*x^6])])/Sqrt[c])/4))/2)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{4}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.75

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(12*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x
^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 3*(b^2 + 4*a*c)*sqrt
(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*
x^3 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c*x^6 +
 b*x^3 + a))/(c*x^3), 1/24*(6*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*
a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) -
3*(b^2 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 +
 b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c
)*sqrt(c*x^6 + b*x^3 + a))/(c*x^3), 1/48*(24*sqrt(-a)*b*c*x^3*arctan(1/2*s
qrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) +
 3*(b^2 + 4*a*c)*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x
^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3
- 4*a*c)*sqrt(c*x^6 + b*x^3 + a))/(c*x^3), 1/24*(12*sqrt(-a)*b*c*x^3*arcta
n(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 +
a^2)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2
*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3
 - 4*a*c)*sqrt(c*x^6 + b*x^3 + a))/(c*x^3)]
```

Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**4,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^4}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^4,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \text{Too large to display}$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

output
```
( - 8*sqrt(a + b*x**3 + c*x**6)*a*c + 10*sqrt(a + b*x**3 + c*x**6)*b*c*x**
3 + 4*sqrt(a + b*x**3 + c*x**6)*c**2*x**6 + 12*sqrt(a)*log(sqrt(a + b*x**3
 + c*x**6) - sqrt(a))*b*c*x**3 - 12*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6)
+ sqrt(a))*b*c*x**3 - 12*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x
**3)*a*c*x**3 - 3*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*b*
*2*x**3 + 12*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*c*x**
3 + 3*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**2*x**3 - 10
8*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(3*a**3*b*c + 3*a**3*c**2*x**3 + a*
*2*b**3 + 7*a**2*b**2*c*x**3 + 9*a**2*b*c**2*x**6 + 3*a**2*c**3*x**9 + 2*a
*b**4*x**3 + 6*a*b**3*c*x**6 + 7*a*b**2*c**2*x**9 + 3*a*b*c**3*x**12 + b**
5*x**6 + 2*b**4*c*x**9 + b**3*c**2*x**12),x)*a**3*c**4*x**3 - 171*int((sqr
t(a + b*x**3 + c*x**6)*x**5)/(3*a**3*b*c + 3*a**3*c**2*x**3 + a**2*b**3 +
7*a**2*b**2*c*x**3 + 9*a**2*b*c**2*x**6 + 3*a**2*c**3*x**9 + 2*a*b**4*x**3
 + 6*a*b**3*c*x**6 + 7*a*b**2*c**2*x**9 + 3*a*b*c**3*x**12 + b**5*x**6 + 2
*b**4*c*x**9 + b**3*c**2*x**12),x)*a**2*b**2*c**3*x**3 - 45*int((sqrt(a +
b*x**3 + c*x**6)*x**5)/(3*a**3*b*c + 3*a**3*c**2*x**3 + a**2*b**3 + 7*a**2
*b**2*c*x**3 + 9*a**2*b*c**2*x**6 + 3*a**2*c**3*x**9 + 2*a*b**4*x**3 + 6*a
*b**3*c*x**6 + 7*a*b**2*c**2*x**9 + 3*a*b*c**3*x**12 + b**5*x**6 + 2*b**4*
c*x**9 + b**3*c**2*x**12),x)*a*b**4*c**2*x**3 - 216*int((sqrt(a + b*x**3 +
 c*x**6)*x**2)/(3*a**3*b*c + 3*a**3*c**2*x**3 + a**2*b**3 + 7*a**2*b**2...
```

3.193 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 151

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = -\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{(a+bx^3+cx^6)^{3/2}}{6x^6} - \frac{(b^2+4ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}} + \frac{1}{2}b\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

output
```
-1/4*(-2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/x^3-1/6*(c*x^6+b*x^3+a)^(3/2)/x^6-
1/8*(4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(
1/2)+1/2*b*c^(1/2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \frac{1}{12}\left(\frac{\sqrt{a+bx^3+cx^6}(-2a-5bx^3+4cx^6)}{x^6} + \frac{3(b^2+4ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{\sqrt{a}} - 6b\sqrt{c}\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

output
```
((Sqrt[a + b*x^3 + c*x^6]*(-2*a - 5*b*x^3 + 4*c*x^6))/x^6 + (3*(b^2 + 4*a*
c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/Sqrt[a] - 6*b
*Sqrt[c]*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/12
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1693, 1161, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^{3/2}}{x^9}dx^3$$

$$\Big\downarrow \text{1161}$$

$$\frac{1}{3}\left(\frac{3}{4}\int \frac{\left(2cx^3+b\right)\sqrt{cx^6+bx^3+a}}{x^6}dx^3-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{2x^6}\right)$$

$$\Big\downarrow \text{ 1230}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(-\frac{1}{2}\int -\frac{4bcx^3+b^2+4ac}{x^3\sqrt{cx^6+bx^3+a}}dx^3-\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{x^3}\right)-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{2x^6}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{4bcx^3+b^2+4ac}{x^3\sqrt{cx^6+bx^3+a}}dx^3-\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{x^3}\right)-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{2x^6}\right)$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\left(\left(4ac+b^2\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3+4bc\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3\right)-\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{x^3}\right)-\right.$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\left(\left(4ac+b^2\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3+8bc\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}\right)-\frac{\left(b-2cx^3\right)\sqrt{a+bx^3+cx^6}}{x^3}\right.\right.$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\left(\left(4ac+b^2\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3+4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)\right)\right)-\frac{\left(b-2cx^3\right)\sqrt{a+bx^3}}{x^3}\right.$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\left(4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-2\left(4ac+b^2\right)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}\right)-\frac{\left(b-2cx^3\right)\sqrt{a+}}{x^3}\right.\right.$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\left(4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{a}}\right)\right)-\frac{\left(b-2cx^3\right)\sqrt{a+bx}}{x^3}\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

output
```
(-1/2*(a + b*x^3 + c*x^6)^(3/2)/x^6 + (3*(-(((b - 2*c*x^3)*Sqrt[a + b*x^3
+ c*x^6])/x^3) + (-(((b^2 + 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a
 + b*x^3 + c*x^6])])/Sqrt[a]) + 4*b*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[
c]*Sqrt[a + b*x^3 + c*x^6])])/2))/4)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{7}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.72

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^7} \, dx = \text{Too large to display}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/48*(12*a*b*sqrt(c)*x^6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6
+ b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 3*(b^2 + 4*a*c)*sqrt(a)*x^6*
log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2
*a)*sqrt(a) + 8*a^2)/x^6) + 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt(c*x^6 +
 b*x^3 + a))/(a*x^6), -1/48*(24*a*b*sqrt(-c)*x^6*arctan(1/2*sqrt(c*x^6 + b
*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 3*(b^2 + 4*a
*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3
 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a
^2)*sqrt(c*x^6 + b*x^3 + a))/(a*x^6), 1/24*(6*a*b*sqrt(c)*x^6*log(-8*c^2*x
^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4
*a*c) + 3*(b^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b
*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(4*a*c*x^6 - 5*a*b*x^3
 - 2*a^2)*sqrt(c*x^6 + b*x^3 + a))/(a*x^6), -1/24*(12*a*b*sqrt(-c)*x^6*arc
tan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3
+ a*c)) - 3*(b^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*
(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*(4*a*c*x^6 - 5*a*b*x
^3 - 2*a^2)*sqrt(c*x^6 + b*x^3 + a))/(a*x^6)]
```

Sympy [F]

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^7} \, dx = \int \frac{(a + bx^3 + cx^6)^{\frac{3}{2}}}{x^7} \, dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**7,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^7}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^7,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^7, x)
```

Reduce [F]

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^7} \, dx = \text{too large to display}$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

output
```
( - 96*sqrt(a + b*x**3 + c*x**6)*a**5*b*c**3 - 768*sqrt(a + b*x**3 + c*x**
6)*a**5*c**4*x**3 + 40*sqrt(a + b*x**3 + c*x**6)*a**4*b**3*c**2 - 2928*sqr
t(a + b*x**3 + c*x**6)*a**4*b**2*c**3*x**3 + 192*sqrt(a + b*x**3 + c*x**6)
*a**4*b*c**4*x**6 + 96*sqrt(a + b*x**3 + c*x**6)*a**3*b**5*c - 3056*sqrt(a
 + b*x**3 + c*x**6)*a**3*b**4*c**2*x**3 - 80*sqrt(a + b*x**3 + c*x**6)*a**
3*b**3*c**3*x**6 + 24*sqrt(a + b*x**3 + c*x**6)*a**2*b**7 - 1122*sqrt(a +
b*x**3 + c*x**6)*a**2*b**6*c*x**3 - 192*sqrt(a + b*x**3 + c*x**6)*a**2*b**
5*c**2*x**6 - 66*sqrt(a + b*x**3 + c*x**6)*a*b**8*x**3 - 48*sqrt(a + b*x**
3 + c*x**6)*a*b**7*c*x**6 + 288*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sq
rt(a))*a**4*b*c**4*x**6 - 48*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(
a))*a**3*b**3*c**3*x**6 - 318*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt
(a))*a**2*b**5*c**2*x**6 - 144*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqr
t(a))*a*b**7*c*x**6 - 18*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*
b**9*x**6 - 288*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*a**4*b*c*
*4*x**6 + 48*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*a**3*b**3*c*
*3*x**6 + 318*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*a**2*b**5*c
**2*x**6 + 144*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*a*b**7*c*x
**6 + 18*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*b**9*x**6 - 288*
sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a**4*b**2*c**3*x**6
+ 120*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a**3*b**4*c...
```

3.194 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 163

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = -\frac{(2ab+(b^2+8ac)x^3)\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9x^9} + \frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}} + \frac{1}{3}c^{3/2}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

output
```
-1/24*(2*a*b+(8*a*c+b^2)*x^3)*(c*x^6+b*x^3+a)^(1/2)/a/x^6-1/9*(c*x^6+b*x^3
+a)^(3/2)/x^9+1/48*b*(-12*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+
b*x^3+a)^(1/2))/a^(3/2)+1/3*c^(3/2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6
+b*x^3+a)^(1/2))
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-8a^2-14abx^3-3b^2x^6-32acx^6)}{72ax^9} + \frac{(b^3-12abc)\operatorname{arctanh}\left(\frac{-\sqrt{c}x^3+\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{3/2}} - \frac{1}{3}c^{3/2}\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 14*a*b*x^3 - 3*b^2*x^6 - 32*a*c*x^6))/(
72*a*x^9) + ((b^3 - 12*a*b*c)*ArcTanh[(-(Sqrt[c]*x^3) + Sqrt[a + b*x^3 + c
*x^6])/Sqrt[a]])/(24*a^(3/2)) - (c^(3/2)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[
a + b*x^3 + c*x^6]])/3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1693, 1161, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^{3/2}}{x^{12}}dx^3$$

$$\Big\downarrow \text{1161}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{(2cx^3+b)\sqrt{cx^6+bx^3+a}}{x^9}dx^3 - \frac{(a+bx^3+cx^6)^{3/2}}{3x^9}\right)$$

$$\Big\downarrow \text{1229}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{\int \frac{b(b^2-12ac)-16ac^2x^3}{2x^3\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{\sqrt{a+bx^3+cx^6}\left(x^3\left(8ac+b^2\right)+2ab\right)}{4ax^6}\right)-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^9}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{\int \frac{b(b^2-12ac)-16ac^2x^3}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\sqrt{a+bx^3+cx^6}\left(x^3\left(8ac+b^2\right)+2ab\right)}{4ax^6}\right)-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{3x^9}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{b\left(b^2-12ac\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3-16ac^2\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\sqrt{a+bx^3+cx^6}\left(x^3\left(8ac+b^2\right)+2ab\right)}{4ax^6}\right)-\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{b\left(b^2-12ac\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3-32ac^2\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{8a}-\frac{\sqrt{a+bx^3+cx^6}\left(x^3\left(8ac+b^2\right)+2ab\right)}{4ax^6}\right)\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{b\left(b^2-12ac\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8a}-\frac{\sqrt{a+bx^3+cx^6}\left(x^3\left(8ac+b^2\right)+\right.}{4ax^6}\right.\right.$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{-2b\left(b^2-12ac\right)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8a}-\frac{\sqrt{a+bx^3+cx^6}\left(x^3\left(8ac+b\right.\right.}{4ax^6}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{\sqrt{a}}-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8a}-\frac{\sqrt{a+bx^3+cx^6}\left(x^3\left(8ac+b^2\right.\right.}{4ax^6}\right.\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

output
```
(-1/3*(a + b*x^3 + c*x^6)^(3/2)/x^9 + (-1/4*((2*a*b + (b^2 + 8*a*c)*x^3)*S
qrt[a + b*x^3 + c*x^6])/(a*x^6) - (-((b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^
3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/Sqrt[a]) - 16*a*c^(3/2)*ArcTanh[(
b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*a))/2)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{10}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 771, normalized size of antiderivative = 4.73

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
[1/288*(48*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6
 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(a)*
x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3
 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/288*(96*a^2*sqrt(-c)*c*x^
9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c
*x^3 + a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*
a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) +
4*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a
))/(a^2*x^9), 1/144*(24*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 -
 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*
b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a
)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3
+ 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/144*(48*a^2*sqrt(-c)*c*x^9
*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*
x^3 + a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^
3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 + 3
2*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9)]
```

Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**10,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**10, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^{10}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^10,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^10, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^{10}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

3.195 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 133

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2-4ac)^2\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}$$

output
```
1/64*(-4*a*c+b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^2/x^6-1/24*(b*x^3+2*
a)*(c*x^6+b*x^3+a)^(3/2)/a/x^12-1/128*(-4*a*c+b^2)^2*arctanh(1/2*(b*x^3+2*
a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = -\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}(8a^2+8abx^3-3b^2x^6+20acx^6)}{192a^2x^{12}} + \frac{(b^2-4ac)^2\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{64a^{5/2}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

output
```
-1/192*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6]*(8*a^2 + 8*a*b*x^3 - 3*b^2*x
^6 + 20*a*c*x^6))/(a^2*x^12) + ((b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]*x^3 - Sqr
t[a + b*x^3 + c*x^6])/Sqrt[a]])/(64*a^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1693, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{13}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{\left(cx^6+bx^3+a\right)^{3/2}}{x^{15}}dx^3$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{3}\left(-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{3}\left(-\frac{3\left(b^2-4ac\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

output
```
(-1/8*((2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(a*x^12) - (3*(b^2 - 4*a*c
)*(-1/4*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) + ((b^2 - 4*a*c)*A
rcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(8*a^(3/2))))/(
16*a))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{13}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.40

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{a}x^{12}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8}{x^6}\right.}{7}\right.$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="fricas")
```

output
```
[1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a*c)*
x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)
/x^6) + 4*((3*a*b^3 - 20*a^2*b*c)*x^9 - 24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3
*c)*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^12), 1/384*(3*(b^4 - 8*a
*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x
^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^3 - 20*a^2*b*c)*
x^9 - 24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)*x^6 - 16*a^4)*sqrt(c*x^6 + b*x
^3 + a))/(a^3*x^12)]
```

Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^{13}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**13,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**13, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{13}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^13, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^{13}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^13,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^13, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \text{too large to display}$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

output
```
( - 98304*sqrt(a + b*x**3 + c*x**6)*a**9*b**4*c**7 - 131072*sqrt(a + b*x**
3 + c*x**6)*a**9*b**3*c**8*x**3 + 196608*sqrt(a + b*x**3 + c*x**6)*a**9*b*
*2*c**9*x**6 + 393216*sqrt(a + b*x**3 + c*x**6)*a**9*c**11*x**12 - 286720*
sqrt(a + b*x**3 + c*x**6)*a**8*b**6*c**6 - 147456*sqrt(a + b*x**3 + c*x**6
)*a**8*b**5*c**7*x**3 - 81920*sqrt(a + b*x**3 + c*x**6)*a**8*b**4*c**8*x**
6 - 229376*sqrt(a + b*x**3 + c*x**6)*a**8*b**3*c**9*x**9 - 196608*sqrt(a +
 b*x**3 + c*x**6)*a**8*b**2*c**10*x**12 - 108544*sqrt(a + b*x**3 + c*x**6)
*a**7*b**8*c**5 - 618496*sqrt(a + b*x**3 + c*x**6)*a**7*b**7*c**6*x**3 - 4
46464*sqrt(a + b*x**3 + c*x**6)*a**7*b**6*c**7*x**6 - 417792*sqrt(a + b*x*
*3 + c*x**6)*a**7*b**5*c**8*x**9 - 442368*sqrt(a + b*x**3 + c*x**6)*a**7*b
**4*c**9*x**12 + 161280*sqrt(a + b*x**3 + c*x**6)*a**6*b**10*c**4 - 95232*
sqrt(a + b*x**3 + c*x**6)*a**6*b**9*c**5*x**3 - 173056*sqrt(a + b*x**3 + c
*x**6)*a**6*b**8*c**6*x**6 - 522240*sqrt(a + b*x**3 + c*x**6)*a**6*b**7*c*
*7*x**9 + 172032*sqrt(a + b*x**3 + c*x**6)*a**6*b**6*c**8*x**12 + 33280*sq
rt(a + b*x**3 + c*x**6)*a**5*b**12*c**3 + 220416*sqrt(a + b*x**3 + c*x**6)
*a**5*b**11*c**4*x**3 + 337408*sqrt(a + b*x**3 + c*x**6)*a**5*b**10*c**5*x
**6 - 461312*sqrt(a + b*x**3 + c*x**6)*a**5*b**9*c**6*x**9 + 109056*sqrt(a
 + b*x**3 + c*x**6)*a**5*b**8*c**7*x**12 - 22400*sqrt(a + b*x**3 + c*x**6)
*a**4*b**14*c**2 + 56704*sqrt(a + b*x**3 + c*x**6)*a**4*b**13*c**3*x**3 +
120064*sqrt(a + b*x**3 + c*x**6)*a**4*b**12*c**4*x**6 + 585216*sqrt(a +...
```

3.196 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 162

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}} \, dx = -\frac{b(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{15ax^{15}} + \frac{b(b^2-4ac)^2 \operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}$$

output
```
-1/128*b*(-4*a*c+b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^3/x^6+1/48*b*(b*
x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^2/x^12-1/15*(c*x^6+b*x^3+a)^(5/2)/a/x^15+
1/256*b*(-4*a*c+b^2)^2*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/
2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+bx^3+cx^6}\left(128a^4+15b^4x^{12}-10ab^2x^9(b+10cx^3)+16a^3(11bx^3+16cx^6)+8a^2x^6(b^2+7bcx^3+16c^2x^6)\right)}{x^{15}}}{1920a^{7/2}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]*(128*a^4 + 15*b^4*x^12 - 10*a*b^2*x^9*
(b + 10*c*x^3) + 16*a^3*(11*b*x^3 + 16*c*x^6) + 8*a^2*x^6*(b^2 + 7*b*c*x^3
 + 16*c^2*x^6)))/x^15) - 15*b*(b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]*x^3 - Sqrt[
a + b*x^3 + c*x^6])/Sqrt[a]])/(1920*a^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1693, 1157, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^{3/2}}{x^{18}}dx^3$$

$$\Big\downarrow \text{1157}$$

$$\frac{1}{3}\left(-\frac{b\int \frac{(cx^6+bx^3+a)^{3/2}}{x^{15}}dx^3}{2a}-\frac{(a+bx^3+cx^6)^{5/2}}{5ax^{15}}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{3}\left(-\frac{b\left(-\frac{3(b^2-4ac)\int\frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{16a}-\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{(a+bx^3+cx^6)^{5/2}}{5ax^{15}}\right)$$

$\Big\downarrow$ 1152

$$\frac{1}{3}\left(-\frac{b\left(-\frac{3(b^2-4ac)\left(-\frac{(b^2-4ac)\int\frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{(a+bx^3+cx^6)^{5/2}}{5ax^{15}}\right)$$

$\Big\downarrow$ 1154

$$\frac{1}{3}\left(-\frac{b\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\int\frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{(a+bx^3+cx^6)^{5/2}}{5ax^{15}}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{3}\left(-\frac{b\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{(a+bx^3+cx^6)^{5/2}}{5ax^{15}}\right)$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

output
```
(-1/5*(a + b*x^3 + c*x^6)^(5/2)/(a*x^15) - (b*(-1/8*((2*a + b*x^3)*(a + b*
x^3 + c*x^6)^(3/2))/(a*x^12) - (3*(b^2 - 4*a*c)*(-1/4*((2*a + b*x^3)*Sqrt[
a + b*x^3 + c*x^6])/(a*x^6) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt
[a]*Sqrt[a + b*x^3 + c*x^6])])/(8*a^(3/2))))/(16*a)))/(2*a))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{16}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{a}x^{15}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}}{x^6}\right.}{}\right.$$

$$-\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{-a}x^{15}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((15\,ab^4-100\,a^2b^2c+128\,a^3c^2)x}{3840\,a^4x^{15}}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="fricas")
```

output
```
[1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 + 4*a
*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*
a^2)/x^6) - 4*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^12 - 2*(5*a^2*b^
3 - 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^6 + 128*a^5
)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^15), -1/3840*(15*(b^5 - 8*a*b^3*c + 16*a
^2*b*c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*s
qrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^4 - 100*a^2*b^2*c + 128*a^
3*c^2)*x^12 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2
+ 32*a^4*c)*x^6 + 128*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^15)]
```

Sympy [F]

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^{16}}\, dx = \int \frac{(a + bx^3 + cx^6)^{\frac{3}{2}}}{x^{16}}\, dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**16,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**16, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^{16}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{16}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^16, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^{16}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^16,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^16, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx = \text{too large to display}$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

output
```
( - 2359296*sqrt(a + b*x**3 + c*x**6)*a**10*b**2*c**8 - 3932160*sqrt(a + b
*x**3 + c*x**6)*a**9*b**4*c**7 - 1769472*sqrt(a + b*x**3 + c*x**6)*a**9*b*
*3*c**8*x**3 - 6684672*sqrt(a + b*x**3 + c*x**6)*a**9*b**2*c**9*x**6 + 589
8240*sqrt(a + b*x**3 + c*x**6)*a**9*b*c**10*x**9 - 11796480*sqrt(a + b*x**
3 + c*x**6)*a**9*c**11*x**12 + 6881280*sqrt(a + b*x**3 + c*x**6)*a**8*b**6
*c**6 - 7372800*sqrt(a + b*x**3 + c*x**6)*a**8*b**5*c**7*x**3 - 7110656*sq
rt(a + b*x**3 + c*x**6)*a**8*b**4*c**8*x**6 + 3637248*sqrt(a + b*x**3 + c*
x**6)*a**8*b**3*c**9*x**9 - 16121856*sqrt(a + b*x**3 + c*x**6)*a**8*b**2*c
**10*x**12 + 7053312*sqrt(a + b*x**3 + c*x**6)*a**7*b**8*c**5 + 12840960*s
qrt(a + b*x**3 + c*x**6)*a**7*b**7*c**6*x**3 + 11304960*sqrt(a + b*x**3 +
c*x**6)*a**7*b**6*c**7*x**6 - 12533760*sqrt(a + b*x**3 + c*x**6)*a**7*b**5
*c**8*x**9 + 14745600*sqrt(a + b*x**3 + c*x**6)*a**7*b**4*c**9*x**12 - 634
3680*sqrt(a + b*x**3 + c*x**6)*a**6*b**10*c**4 + 8239104*sqrt(a + b*x**3 +
 c*x**6)*a**6*b**9*c**5*x**3 + 12539904*sqrt(a + b*x**3 + c*x**6)*a**6*b**
8*c**6*x**6 + 8017920*sqrt(a + b*x**3 + c*x**6)*a**6*b**7*c**7*x**9 + 1658
8800*sqrt(a + b*x**3 + c*x**6)*a**6*b**6*c**8*x**12 + 1128960*sqrt(a + b*x
**3 + c*x**6)*a**5*b**12*c**3 - 10126080*sqrt(a + b*x**3 + c*x**6)*a**5*b*
*11*c**4*x**3 - 8672768*sqrt(a + b*x**3 + c*x**6)*a**5*b**10*c**5*x**6 + 6
434304*sqrt(a + b*x**3 + c*x**6)*a**5*b**9*c**6*x**9 - 242688*sqrt(a + b*x
**3 + c*x**6)*a**5*b**8*c**7*x**12 + 3717120*sqrt(a + b*x**3 + c*x**6)*...
```

3.197 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 216

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}} + \frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}} - \frac{(b^2-4ac)^2(7b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}}$$

output

```
1/1536*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^4/x
^6-1/576*(-4*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^3/x^12-1/18*(c
*x^6+b*x^3+a)^(5/2)/a/x^18+7/180*b*(c*x^6+b*x^3+a)^(5/2)/a^2/x^15-1/3072*(
-4*a*c+b^2)^2*(-4*a*c+7*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+
a)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+bx^3+cx^6}(1280a^5-105b^5x^{15}+10ab^3x^{12}(7b+76cx^3)+64a^4(26bx^3+35cx^6)+48a^3x^6(b^2+6bcx^3+10}{x^{18}}}{23}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]*(1280*a^5 - 105*b^5*x^15 + 10*a*b^3*x^
12*(7*b + 76*c*x^3) + 64*a^4*(26*b*x^3 + 35*c*x^6) + 48*a^3*x^6*(b^2 + 6*b
*c*x^3 + 10*c^2*x^6) - 8*a^2*b*x^9*(7*b^2 + 54*b*c*x^3 + 162*c^2*x^6)))/x^
18) + 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b
*x^3 + c*x^6])/Sqrt[a]])/(23040*a^(9/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1693, 1167, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^{3/2}}{x^{21}}dx^3$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{3}\left(-\frac{\int \frac{(2cx^3+7b)(cx^6+bx^3+a)^{3/2}}{2x^{18}}dx^3}{6a}-\frac{(a+bx^3+cx^6)^{5/2}}{6ax^{18}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{(2cx^3+7b)\left(cx^6+bx^3+a\right)^{3/2}}{x^{18}}dx^3}{12a}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{6ax^{18}}\right)$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{1}{3}\left(-\frac{-\frac{(7b^2-4ac)\int \frac{\left(cx^6+bx^3+a\right)^{3/2}}{x^{15}}dx^3}{2a}-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5ax^{15}}}{12a}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{6ax^{18}}\right)$$

$$\Big\downarrow \text{ 1152}$$

$$\frac{1}{3}\left(-\frac{-\frac{(7b^2-4ac)\left(-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{7b\left(a+bx^3+cx^6\right)^{5/2}}{5ax^{15}}}{12a}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{6ax^{18}}\right)$$

$$\Big\downarrow \text{ 1152}$$

$$\frac{1}{3}\left(-\frac{-\frac{(7b^2-4ac)\left(-\frac{3\left(b^2-4ac\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{7b\left(a+bx^3+cx^6\right.}{5ax^{15}}}{12a}\right.$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{3}\left(-\frac{-\frac{(7b^2-4ac)\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{7b\left(a+bx^3+cx^6\right.}{5ax^{15}}}{12a}\right.$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(-\frac{-\frac{(7b^2-4ac)\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{7b(a+b}{5}}{12a}\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

output
```
(-1/6*(a + b*x^3 + c*x^6)^(5/2)/(a*x^18) - ((-7*b*(a + b*x^3 + c*x^6)^(5/2
))/(5*a*x^15) - ((7*b^2 - 4*a*c)*(-1/8*((2*a + b*x^3)*(a + b*x^3 + c*x^6)^
(3/2))/(a*x^12) - (3*(b^2 - 4*a*c)*(-1/4*((2*a + b*x^3)*Sqrt[a + b*x^3 + c
*x^6])/(a*x^6) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a +
b*x^3 + c*x^6])])/(8*a^(3/2))))/(16*a)))/(2*a))/(12*a))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^{19}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \left[-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{a}x^{18}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx}}{x}\right.}{}\right.$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="fricas")
```

output
```
[-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*
x^18*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^
3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*
b*c^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^12 + 8*(7*a^3
*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*b^2 + 140*a^5*c)*x^6 -
 1280*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18), 1/46080*(15*(7*b^6 - 60*a*
b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^18*arctan(1/2*sqrt(c*x^6
+ b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a
*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*
c + 240*a^4*c^2)*x^12 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 -
16*(3*a^4*b^2 + 140*a^5*c)*x^6 - 1280*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x
^18)]
```

Sympy [F]

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{19}}\,dx = \int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^{19}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**19,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**19, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{19}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{19}}\,dx = \int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^{19}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^19, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^{19}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^19,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^19, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^{19}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

3.198 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 255

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = -\frac{b(b^2-4ac)(3b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b(3b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{384a^4x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}} + \frac{b(a+bx^3+cx^6)^{5/2}}{28a^2x^{18}} - \frac{(21b^2-16ac)(a+bx^3+cx^6)^{5/2}}{840a^3x^{15}} + \frac{b(b^2-4ac)^2(3b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{11/2}}$$

output

```
-1/1024*b*(-4*a*c+b^2)*(-4*a*c+3*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^
5/x^6+1/384*b*(-4*a*c+3*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^4/x^12-1/
21*(c*x^6+b*x^3+a)^(5/2)/a/x^21+1/28*b*(c*x^6+b*x^3+a)^(5/2)/a^2/x^18-1/84
0*(-16*a*c+21*b^2)*(c*x^6+b*x^3+a)^(5/2)/a^3/x^15+1/2048*b*(-4*a*c+b^2)^2*
(-4*a*c+3*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1
1/2)
```

Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.96

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^{22}} dx = -\frac{\frac{\sqrt{a}\sqrt{a+bx^3+cx^6}(5120a^6+315b^6x^{18}-210ab^4x^{15}(b+12cx^3)+256a^5(25bx^3+32cx^6)+64a^4x^6(2b^2+11bcx^3\cdots}{x^{21}}}{\cdots}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]*(5120*a^6 + 315*b^6*x^18 - 210*a*b^4*x
^15*(b + 12*c*x^3) + 256*a^5*(25*b*x^3 + 32*c*x^6) + 64*a^4*x^6*(2*b^2 + 1
1*b*c*x^3 + 16*c^2*x^6) + 56*a^2*b^2*x^12*(3*b^2 + 26*b*c*x^3 + 98*c^2*x^6
) - 16*a^3*x^9*(9*b^3 + 62*b^2*c*x^3 + 146*b*c^2*x^6 + 128*c^3*x^9)))/x^21
) - 105*b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a +
b*x^3 + c*x^6])/Sqrt[a]])/(107520*a^(11/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1693, 1167, 27, 1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^{22}} dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3}\int \frac{(cx^6 + bx^3 + a)^{3/2}}{x^{24}}dx^3$$

$$\Big\downarrow \text{ 1167}$$

$$\frac{1}{3}\left(-\frac{\int \frac{(4cx^3+9b)(cx^6+bx^3+a)^{3/2}}{2x^{21}}dx^3}{7a} - \frac{(a + bx^3 + cx^6)^{5/2}}{7ax^{21}}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(-\frac{\int \frac{(4cx^3+9b)(cx^6+bx^3+a)^{3/2}}{x^{21}}dx^3}{14a}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{7ax^{21}}\right)$$

$$\Big\downarrow \ 1237$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{3\left(6bcx^3+21b^2-16ac\right)\left(cx^6+bx^3+a\right)^{3/2}}{2x^{18}}dx^3}{6a}-\frac{3b(a+bx^3+cx^6)^{5/2}}{2ax^{18}}}{14a}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{7ax^{21}}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{\left(6bcx^3+21b^2-16ac\right)\left(cx^6+bx^3+a\right)^{3/2}}{x^{18}}dx^3}{4a}-\frac{3b(a+bx^3+cx^6)^{5/2}}{2ax^{18}}}{14a}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{7ax^{21}}\right)$$

$$\Big\downarrow \ 1228$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{7b\left(3b^2-4ac\right)\int \frac{\left(cx^6+bx^3+a\right)^{3/2}}{x^{15}}dx^3}{2a}-\frac{\left(21b^2-16ac\right)\left(a+bx^3+cx^6\right)^{5/2}}{5ax^{15}}}{4a}-\frac{3b(a+bx^3+cx^6)^{5/2}}{2ax^{18}}}{14a}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{7ax^{21}}\right)$$

$$\Big\downarrow \ 1152$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{7b\left(3b^2-4ac\right)\left(-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^6+bx^3+a}}{x^9}dx^3}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{\left(21b^2-16ac\right)\left(a+bx^3+cx^6\right)^{5/2}}{5ax^{15}}}{4a}-\frac{3b(a+bx^3+cx^6)^{5/2}}{2ax^{18}}}{14a}-\right.$$

$$\Big\downarrow \ 1152$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{7b\left(3b^2-4ac\right)\left(-\frac{3\left(b^2-4ac\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{\left(21b^2-16ac\right)\left(a\cdots\right.}{5ax\cdots}}{4a}}{14a}\right.$$

$\Big\downarrow$ 1154

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{7b\left(3b^2-4ac\right)\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{4a}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{\left(21b^2-16ac\right)\left(a\cdots\right.}{5ax\cdots}}{4a}}{14a}\right.$$

$\Big\downarrow$ 219

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{7b\left(3b^2-4ac\right)\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{3/2}}-\frac{\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{4ax^6}\right)}{16a}-\frac{\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{8ax^{12}}\right)}{2a}-\frac{\left(21b^2-\cdots\right.}{\cdots}}{4a}}{14a}\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

output
```
(-1/7*(a + b*x^3 + c*x^6)^(5/2)/(a*x^21) - ((-3*b*(a + b*x^3 + c*x^6)^(5/2
))/(2*a*x^18) - (-1/5*((21*b^2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))/(a*x^1
5) - (7*b*(3*b^2 - 4*a*c)*(-1/8*((2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/
(a*x^12) - (3*(b^2 - 4*a*c)*(-1/4*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/
(a*x^6) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
 c*x^6])])/(8*a^(3/2))))/(16*a)))/(2*a))/(4*a))/(14*a))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{22}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.18

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \left[-\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{a}x^{21}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{c}\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{-a}x^{21}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((315\,ab^6-2520\,\cdots}{\cdots}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="fricas")
```

output
```
[-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(
a)*x^21*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b
*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((315*a*b^6 - 2520*a^2*b^4*c + 5488*
a^3*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5 - 728*a^3*b^3*c + 1168*a
^4*b*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 128*a^5*c^2)*x^12 + 6400*
a^6*b*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*a^7 + 128*(a^5*b^2 + 64
*a^6*c)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(a^6*x^21), -1/215040*(105*(3*b^7 -
28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-a)*x^21*arctan(1/2*sqrt(
c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(
(315*a*b^6 - 2520*a^2*b^4*c + 5488*a^3*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(1
05*a^2*b^5 - 728*a^3*b^3*c + 1168*a^4*b*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^
4*b^2*c + 128*a^5*c^2)*x^12 + 6400*a^6*b*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)
*x^9 + 5120*a^7 + 128*(a^5*b^2 + 64*a^6*c)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(
a^6*x^21)]
```

Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx=\int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^{22}}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**22,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**22, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx=\int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^{22}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^22, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx=\int \frac{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{x^{22}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^22,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^22, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx=\int \frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}{x^{22}}d x$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

3.199 $\int x^3(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int x^3\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{ax^4\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{4}{3},-\frac{3}{2},-\frac{3}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/4*a*x^4*(c*x^6+b*x^3+a)^(1/2)*AppellF1(4/3,-3/2,-3/2,7/3,-2*c*x^3/(b-(-4
*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 453 vs. 2(141) = 282.

Time = 10.98 (sec) , antiderivative size = 453, normalized size of antiderivative = 3.21

$$\int x^3\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{x\left(8(-297b^4x^3-81b^3cx^6+3464b^2c^2x^9+5488bc^3x^{12}+2240c^4x^{15}+4a^2c(459b+1280cx^3)\right.}{}$$

input
```
Integrate[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x*(8*(-297*b^4*x^3 - 81*b^3*c*x^6 + 3464*b^2*c^2*x^9 + 5488*b*c^3*x^12 +
2240*c^4*x^15 + 4*a^2*c*(459*b + 1280*c*x^3) + a*(-297*b^3 + 2052*b^2*c*x^
3 + 10204*b*c^2*x^6 + 7360*c^3*x^9)) + 216*a*b*(11*b^2 - 68*a*c)*Sqrt[(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3,
(-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] +
27*(55*b^4 - 404*a*b^2*c + 640*a^2*c^2)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(232960*c^2*Sqrt[a +
b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^3+cx^6}\int x^3\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{ax^4\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{4}{3},-\frac{3}{2},-\frac{3}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(a*x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -3/2, -3/2, 7/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (
2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^3 \left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}dx$$

input
```
int(x^3*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^3*(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} x^3 \, dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^9 + b*x^6 + a*x^3)*sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \int x^3 \left(a+bx^3+cx^6\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**3*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**3*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} x^3 \, dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

Giac [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} x^3 dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+bx^3+cx^6\right)^{3/2} dx = \int x^3 \left(c x^6+b x^3+a\right)^{3/2} dx$$

Reduce [F]

$$\int x^3 \left(a+bx^3 +cx^6\right)^{3/2} dx = \frac{3672\sqrt{c x^6+b x^3+a}\, abcx+10240\sqrt{c x^6+b x^3+a}\, a c^2x^4-594\sqrt{c x^6+b x^3+a}\, b^3x+432}{}$$

input
```
int(x^3*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^3*(a + b*x^3 + c*x^6)^(3/2), x)
```

output
```
(3672*sqrt(a + b*x**3 + c*x**6)*a*b*c*x + 10240*sqrt(a + b*x**3 + c*x**6)*
a*c**2*x**4 - 594*sqrt(a + b*x**3 + c*x**6)*b**3*x + 432*sqrt(a + b*x**3 +
 c*x**6)*b**2*c*x**4 + 6496*sqrt(a + b*x**3 + c*x**6)*b*c**2*x**7 + 4480*s
qrt(a + b*x**3 + c*x**6)*c**3*x**10 - 3672*int(sqrt(a + b*x**3 + c*x**6)/(
a + b*x**3 + c*x**6),x)*a**2*b*c + 594*int(sqrt(a + b*x**3 + c*x**6)/(a +
b*x**3 + c*x**6),x)*a*b**3 + 17280*int((sqrt(a + b*x**3 + c*x**6)*x**3)/(a
 + b*x**3 + c*x**6),x)*a**2*c**2 - 10908*int((sqrt(a + b*x**3 + c*x**6)*x*
*3)/(a + b*x**3 + c*x**6),x)*a*b**2*c + 1485*int((sqrt(a + b*x**3 + c*x**6
)*x**3)/(a + b*x**3 + c*x**6),x)*b**4)/(58240*c**2)
```

3.200 $\int x(a+bx^3+cx^6)^{3/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 141

$$\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{ax^2\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output

```
1/2*a*x^2*(c*x^6+b*x^3+a)^(1/2)*AppellF1(2/3,-3/2,-3/2,5/3,-2*c*x^3/(b-(-4
*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 410 vs. 2(141) = 282.

Time = 10.84 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.91

$$\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{x^2\Big(10(27ab^2+448a^2c+27b^3x^3+698abcx^3+277b^2cx^6+608ac^2x^6+410bc^2x^9+160c^3x^{12})}{}$$

input
```
Integrate[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x^2*(10*(27*a*b^2 + 448*a^2*c + 27*b^3*x^3 + 698*a*b*c*x^3 + 277*b^2*c*x^
6 + 608*a*c^2*x^6 + 410*b*c^2*x^9 + 160*c^3*x^12) - 270*a*(b^2 - 16*a*c)*S
qrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/
2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a
*c])] - 27*b*(7*b^2 - 52*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(17600*c*Sqrt[a + b*x^3 + c*x^6
])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \left(a+bx^3+cx^6\right)^{3/2} dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^3+cx^6}\int x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{ax^2\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(a*x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -3/2, -3/2, 5/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (
2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}\,dx$$

input
```
int(x*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x*(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int x(a+bx^3+cx^6)^{3/2}\,dx=\int (cx^6+bx^3+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^7 + b*x^4 + a*x)*sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]

$$\int x(a+bx^3+cx^6)^{3/2}\,dx=\int x(a+bx^3+cx^6)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F]

$$\int x(a+bx^3+cx^6)^{3/2}\,dx=\int (cx^6+bx^3+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

Giac [F]

$$\int x(a+bx^3+cx^6)^{3/2}\,dx=\int (cx^6+bx^3+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^3+cx^6)^{3/2}\,dx=\int x\,(c\,x^6+b\,x^3+a)^{3/2}\,dx$$

input
```
int(x*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x*(a + b*x^3 + c*x^6)^(3/2), x)
```

Reduce [F]

$$\int x(a+bx^3+cx^6)^{3/2}\,dx=\frac{896\sqrt{c\,x^6+b\,x^3+a}\,ac\,x^2+54\sqrt{c\,x^6+b\,x^3+a}\,b^2x^2+500\sqrt{c\,x^6+b\,x^3+a}\,bc\,x^5+320\sqrt{c\,x^6}}{}$$

input
```
int(x*(c*x^6+b*x^3+a)^(3/2),x)
```

output

```
(896*sqrt(a + b*x**3 + c*x**6)*a*c*x**2 + 54*sqrt(a + b*x**3 + c*x**6)*b**
2*x**2 + 500*sqrt(a + b*x**3 + c*x**6)*b*c*x**5 + 320*sqrt(a + b*x**3 + c*
x**6)*c**2*x**8 + 1404*int((sqrt(a + b*x**3 + c*x**6)*x**4)/(a + b*x**3 +
c*x**6),x)*a*b*c - 189*int((sqrt(a + b*x**3 + c*x**6)*x**4)/(a + b*x**3 +
c*x**6),x)*b**3 + 1728*int((sqrt(a + b*x**3 + c*x**6)*x)/(a + b*x**3 + c*x
**6),x)*a**2*c - 108*int((sqrt(a + b*x**3 + c*x**6)*x)/(a + b*x**3 + c*x**
6),x)*a*b**2)/(3520*c)
```

3.201 $\int \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal result

Integrand size = 16, antiderivative size = 136

$$\int \left(a+bx^3+cx^6\right)^{3/2} dx=\frac{ax\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output

```
a*x*(c*x^6+b*x^3+a)^(1/2)*AppellF1(1/3,-3/2,-3/2,4/3,-2*c*x^3/(b-(-4*a*c+b
^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 408 vs. 2(136) = 272.

Time = 10.77 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.00

$$\int \left(a+bx^3+cx^6\right)^{3/2} dx=\frac{x\left(8(27ab^2+364a^2c+27b^3x^3+548abcx^3+211b^2cx^6+476ac^2x^6+296bc^2x^9+112c^3x^{12})-\right.}{}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x*(8*(27*a*b^2 + 364*a^2*c + 27*b^3*x^3 + 548*a*b*c*x^3 + 211*b^2*c*x^6 +
 476*a*c^2*x^6 + 296*b*c^2*x^9 + 112*c^3*x^12) - 216*a*(b^2 - 28*a*c)*Sqrt
[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2,
4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c]
)] - 27*b*(5*b^2 - 44*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(8960*c*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3+cx^6\right)^{3/2} dx$$

$$\Big\downarrow \text{1686}$$

$$\frac{a\sqrt{a+bx^3+cx^6}\int \left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{ax\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(a*x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -3/2, -3/2, 4/3, (-2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}d x$$

input
```
int((c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \left(a+bx^3+cx^6\right)^{3/2} dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^(3/2), x)
```

Sympy [F]

$$\int \left(a+bx^3+cx^6\right)^{3/2} dx = \int \left(a+bx^3+cx^6\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F]

$$\int \left(a+bx^3+cx^6\right)^{3/2} dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^3+cx^6\right)^{3/2} dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3+cx^6\right)^{3/2} dx = \int \left(c\,x^6+b\,x^3+a\right)^{3/2} dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2), x)
```

Reduce [F]

$$\int \left(a+bx^3 +cx^6\right)^{3/2} dx = \frac{728\sqrt{c\,x^6+b\,x^3+a}\,acx+54\sqrt{c\,x^6+b\,x^3+a}\,b^2x+368\sqrt{c\,x^6+b\,x^3+a}\,bc\,x^4+224\sqrt{c\,x^6+}}{}$$

input
```
int((c*x^6+b*x^3+a)^(3/2),x)
```

output
```
(728*sqrt(a + b*x**3 + c*x**6)*a*c*x + 54*sqrt(a + b*x**3 + c*x**6)*b**2*x
 + 368*sqrt(a + b*x**3 + c*x**6)*b*c*x**4 + 224*sqrt(a + b*x**3 + c*x**6)*
c**2*x**7 + 1512*int(sqrt(a + b*x**3 + c*x**6)/(a + b*x**3 + c*x**6),x)*a*
*2*c - 54*int(sqrt(a + b*x**3 + c*x**6)/(a + b*x**3 + c*x**6),x)*a*b**2 +
1188*int((sqrt(a + b*x**3 + c*x**6)*x**3)/(a + b*x**3 + c*x**6),x)*a*b*c -
 135*int((sqrt(a + b*x**3 + c*x**6)*x**3)/(a + b*x**3 + c*x**6),x)*b**3)/(
2240*c)
```

3.202 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 139

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx = -\frac{a\sqrt{a+bx^3+cx^6}\,\mathrm{AppellF1}\left(-\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
-a*(c*x^6+b*x^3+a)^(1/2)*AppellF1(-1/3,-3/2,-3/2,2/3,-2*c*x^3/(b-(-4*a*c+b
^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x/(1+2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 379 vs. 2(139) = 278.

Time = 10.59 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.73

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx = \frac{10(-80a^2-61abx^3+19b^2x^6-70acx^6+29bcx^9+10c^2x^{12})+810abx^3\sqrt{\frac{b-\sqrt{b^2\cdots}}{b-\cdots}}\cdots}{\cdots}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

output
```
(10*(-80*a^2 - 61*a*b*x^3 + 19*b^2*x^6 - 70*a*c*x^6 + 29*b*c*x^9 + 10*c^2*
x^12) + 810*a*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*
AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)
/(-b + Sqrt[b^2 - 4*a*c])] + 27*(b^2 + 20*a*c)*x^6*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(800*x*Sqrt[a +
 b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^2} \, dx$$

$$\Big\downarrow \ 1721$$

$$\frac{a\sqrt{a+bx^3+cx^6}\int \frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}{x^2}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \ 1012$$

$$-\frac{a\sqrt{a+bx^3+cx^6}\operatorname{AppellF1}\left(-\frac{1}{3},-\frac{3}{2},-\frac{3}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

output
```
-((a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -3/2, -3/2, 2/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2
*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
)]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{2}}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^2}\,dx=\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**2,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^2}\,dx=\int \frac{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx=\int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^2}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^2,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx=\frac{488\sqrt{c\,x^6+b\,x^3+a}\,a+38\sqrt{c\,x^6+b\,x^3+a}\,b\,x^3+20\sqrt{c\,x^6+b\,x^3+a}\,c\,x^6+64}{}$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
```

output
```
(488*sqrt(a + b*x**3 + c*x**6)*a + 38*sqrt(a + b*x**3 + c*x**6)*b*x**3 + 2
0*sqrt(a + b*x**3 + c*x**6)*c*x**6 + 648*int(sqrt(a + b*x**3 + c*x**6)/(a*
x**2 + b*x**5 + c*x**8),x)*a**2*x - 756*int((sqrt(a + b*x**3 + c*x**6)*x**
4)/(a + b*x**3 + c*x**6),x)*a*c*x + 27*int((sqrt(a + b*x**3 + c*x**6)*x**4
)/(a + b*x**3 + c*x**6),x)*b**2*x)/(160*x)
```

3.203 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = -\frac{a\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
-1/2*a*(c*x^6+b*x^3+a)^(1/2)*AppellF1(-2/3,-3/2,-3/2,1/3,-2*c*x^3/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^2/(1+2*c*x^3/(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 379 vs. 2(141) = 282.

Time = 10.68 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.69

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \frac{8(-28a^2-11abx^3+17b^2x^6-20acx^6+25bcx^9+8c^2x^{12})+648abx^3\sqrt{\frac{b-\sqrt{b^2-4a}}{b-\sqrt{b^2}}}}{}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

output
```
(8*(-28*a^2 - 11*a*b*x^3 + 17*b^2*x^6 - 20*a*c*x^6 + 25*b*c*x^9 + 8*c^2*x^
12) + 648*a*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(
-b + Sqrt[b^2 - 4*a*c])] + 27*(b^2 + 8*a*c)*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(448*x^2*Sqrt[a +
b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^3+cx^6}\int \frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}{x^3}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{a\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{3}{2},-\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

output
```
-1/2*(a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -3/2, -3/2, 1/3, (-2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[1
 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4
*a*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(c\,x^6 + b\,x^3 + a)^{\frac{3}{2}}}{x^3}dx$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
```

output
```
int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

Sympy [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x**6+b*x**3+a)**(3/2)/x**3,x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx=\int \frac{(cx^6+bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^{3/2}}{x^3}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^(3/2)/x^3,x)
```

output
```
int((a + b*x^3 + c*x^6)^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx=\frac{-380\sqrt{c\,x^6+b\,x^3+a}\,a+34\sqrt{c\,x^6+b\,x^3+a}\,b\,x^3+16\sqrt{c\,x^6+b\,x^3+a}\,c\,x^6-6}{}$$

input
```
int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
```

output
```
( - 380*sqrt(a + b*x**3 + c*x**6)*a + 34*sqrt(a + b*x**3 + c*x**6)*b*x**3
+ 16*sqrt(a + b*x**3 + c*x**6)*c*x**6 - 648*int(sqrt(a + b*x**3 + c*x**6)/
(a*x**3 + b*x**6 + c*x**9),x)*a**2*x**2 + 540*int((sqrt(a + b*x**3 + c*x**
6)*x**3)/(a + b*x**3 + c*x**6),x)*a*c*x**2 + 27*int((sqrt(a + b*x**3 + c*x
**6)*x**3)/(a + b*x**3 + c*x**6),x)*b**2*x**2)/(112*x**2)
```

3.204 $\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 171

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2}+\frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{(5b(21b^2-44ac)-2c(35b^2-36ac)\,x^3)\sqrt{a+bx^3+cx^6}}{576c^4} + \frac{(35b^4-120ab^2c+48a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}}$$

output
```
-7/72*b*x^6*(c*x^6+b*x^3+a)^(1/2)/c^2+1/12*x^9*(c*x^6+b*x^3+a)^(1/2)/c-1/5
76*(5*b*(-44*a*c+21*b^2)-2*c*(-36*a*c+35*b^2)*x^3)*(c*x^6+b*x^3+a)^(1/2)/c
^4+1/384*(48*a^2*c^2-120*a*b^2*c+35*b^4)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(
c*x^6+b*x^3+a)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.81

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-105b^3+220abc+70b^2cx^3-72ac^2x^3-56bc^2x^6+48c^3x^9)}{576c^4} + \frac{(-35b^4+120ab^2c-48a^2c^2)\log\left(bc^4+2c^5x^3-2c^{9/2}\sqrt{a+bx^3+cx^6}\right)}{384c^{9/2}}$$

input
```
Integrate[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-105*b^3 + 220*a*b*c + 70*b^2*c*x^3 - 72*a*c^2*x
^3 - 56*b*c^2*x^6 + 48*c^3*x^9))/(576*c^4) + ((-35*b^4 + 120*a*b^2*c - 48*
a^2*c^2)*Log[b*c^4 + 2*c^5*x^3 - 2*c^(9/2)*Sqrt[a + b*x^3 + c*x^6]])/(384*
c^(9/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1693, 1166, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^{12}}{\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{x^6(7bx^3+6a)}{2\sqrt{cx^6+bx^3+a}}dx^3}{4c}+\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}-\frac{\int \frac{x^6(7bx^3+6a)}{\sqrt{cx^6+bx^3+a}}dx^3}{8c}\right)$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}-\frac{\frac{\int -\frac{x^3\left(\left(35b^2-36ac\right)x^3+28ab\right)}{2\sqrt{cx^6+bx^3+a}}dx^3}{3c}+\frac{7bx^6\sqrt{a+bx^3+cx^6}}{3c}}{8c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}-\frac{\frac{7bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\int \frac{x^3\left(\left(35b^2-36ac\right)x^3+28ab\right)}{\sqrt{cx^6+bx^3+a}}dx^3}{6c}}{8c}\right)$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}-\frac{\frac{7bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3\left(48a^2c^2-120ab^2c+35b^4\right)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c^2}-\frac{\left(5b\left(21b^2-44ac\right)-2cx^3\left(35b^2-36ac\right)\right)\sqrt{a+bx^3+}}{4c^2}}{6c}}{8c}\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}-\frac{\frac{7bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3\left(48a^2c^2-120ab^2c+35b^4\right)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c^2}-\frac{\left(5b\left(21b^2-44ac\right)-2cx^3\left(35b^2-36ac\right)\right)\sqrt{a+}}{4c^2}}{6c}}{8c}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{x^9\sqrt{a+bx^3+cx^6}}{4c}-\frac{\frac{7bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3\left(48a^2c^2-120ab^2c+35b^4\right)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{5/2}}-\frac{\left(5b\left(21b^2-44ac\right)-2cx^3\left(35b^2-36a\right.\right.}{4c^2}}{6c}}{8c}\right.$$

input
```
Int[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((x^9*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((7*b*x^6*Sqrt[a + b*x^3 + c*x^6])/
(3*c) - (-1/4*((5*b*(21*b^2 - 44*a*c) - 2*c*(35*b^2 - 36*a*c)*x^3)*Sqrt[a
+ b*x^3 + c*x^6])/c^2 + (3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(b
+ 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(5/2)))/(6*c))/(8*c)
)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{14}}{\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.77

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \left[\frac{3\,(35\,b^4-120\,ab^2c+48\,a^2c^2)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+}{2304\,c^5}\right.$$

$$-\frac{3\,(35\,b^4-120\,ab^2c+48\,a^2c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(48\,c^4x^9-56\,bc^3x^6-105\,b^3c+}{1152\,c^5}$$

input
```
integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*
b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) +
 4*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c + 220*a*b*c^2 + 2*(35*b^2*c^2 -
36*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5, -1/1152*(3*(35*b^4 - 120*a*b^
2*c + 48*a^2*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(48*c^4*x^9 - 56*b*c^3*x^6 - 105
*b^3*c + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 +
 a))/c^5]
```

Sympy [F]

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x**14/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**14/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{14}}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^14/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{14}}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(x^14/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x^14/(a + b*x^3 + c*x^6)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{-288\sqrt{c\,x^6+b\,x^3+a}\,a^2c^3+1160\sqrt{c\,x^6+b\,x^3+a}\,a\,b^2c^2-144\sqrt{c\,x^6+b\,x^3+a}\,ab\,c^3x^3-420\sqrt{c\,x^6+b}}{}$$

input
```
int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
( - 288*sqrt(a + b*x**3 + c*x**6)*a**2*c**3 + 1160*sqrt(a + b*x**3 + c*x**
6)*a*b**2*c**2 - 144*sqrt(a + b*x**3 + c*x**6)*a*b*c**3*x**3 - 420*sqrt(a
+ b*x**3 + c*x**6)*b**4*c + 140*sqrt(a + b*x**3 + c*x**6)*b**3*c**2*x**3 -
 112*sqrt(a + b*x**3 + c*x**6)*b**2*c**3*x**6 + 96*sqrt(a + b*x**3 + c*x**
6)*b*c**4*x**9 - 144*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)
*a**2*b*c**2 + 360*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a
*b**3*c - 105*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*b**5 +
 144*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a**2*b*c**2 - 3
60*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*b**3*c + 105*sq
rt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**5 + 864*int((sqrt(a
 + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*
x**9),x)*a**2*b*c**4 - 2160*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2
*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a*b**3*c**3 + 630*int((sqr
t(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b
*c*x**9),x)*b**5*c**2 + 864*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2
*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a**3*c**4 - 1728*int((sqrt
(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*
c*x**9),x)*a**2*b**2*c**3 - 450*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2
 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a*b**4*c**2 + 315*int(
(sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x...
```

3.205 $\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 121

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{(15b^2-16ac-10bcx^3)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{b(5b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}$$

output
```
1/9*x^6*(c*x^6+b*x^3+a)^(1/2)/c+1/72*(-10*b*c*x^3-16*a*c+15*b^2)*(c*x^6+b*
x^3+a)^(1/2)/c^3-1/48*b*(-12*a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c
*x^6+b*x^3+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(15b^2-16ac-10bcx^3+8c^2x^6)}{72c^3} + \frac{(5b^3-12abc)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{48c^{7/2}}$$

input
```
Integrate[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(15*b^2 - 16*a*c - 10*b*c*x^3 + 8*c^2*x^6))/(72*c
^3) + ((5*b^3 - 12*a*b*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x
^6]])/(48*c^(7/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1693, 1166, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^9}{\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{x^3(5bx^3+4a)}{2\sqrt{cx^6+bx^3+a}}dx^3}{3c}+\frac{x^6\sqrt{a+bx^3+cx^6}}{3c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\int \frac{x^3(5bx^3+4a)}{\sqrt{cx^6+bx^3+a}}dx^3}{6c}\right)$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{3}\left(\frac{x^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3b(5b^2-12ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c^2}-\frac{(-16ac+15b^2-10bcx^3)\sqrt{a+bx^3+cx^6}}{4c^2}}{6c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{x^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3b(5b^2-12ac)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c^2}-\frac{(-16ac+15b^2-10bcx^3)\sqrt{a+bx^3+cx^6}}{4c^2}}{6c}\right)$$

$$\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{x^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3b(5b^2-12ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{5/2}}-\frac{(-16ac+15b^2-10bcx^3)\sqrt{a+bx^3+cx^6}}{4c^2}}{6c}\right)$$

input
```
Int[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((x^6*Sqrt[a + b*x^3 + c*x^6])/(3*c) - (-1/4*((15*b^2 - 16*a*c - 10*b*c*x^
3)*Sqrt[a + b*x^3 + c*x^6])/c^2 + (3*b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x
^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(5/2)))/(6*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{11}}{\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.99

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \left[-\frac{3\,(5\,b^3-12\,abc)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,(8\,c^3x^6-}{288\,c^4}\right.$$

input
```
integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4
*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 - 1
0*b*c^2*x^3 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/144*(3*
(5*b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 +
b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 - 10*b*c^2*x^3 + 15*
b^2*c - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

Sympy [F]

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x**11/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**11/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{11}}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^11/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^{11}}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(x^11/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x^11/(a + b*x^3 + c*x^6)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$$
$$= \frac{-104\sqrt{c\,x^6+b\,x^3+a}\,a\,c^2+60\sqrt{c\,x^6+b\,x^3+a}\,b^2c-20\sqrt{c\,x^6+b\,x^3+a}\,b\,c^2x^3+16\sqrt{c\,x^6+b\,x^3+a}\,c^3x}{}$$

input
```
int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
( - 104*sqrt(a + b*x**3 + c*x**6)*a*c**2 + 60*sqrt(a + b*x**3 + c*x**6)*b*
*2*c - 20*sqrt(a + b*x**3 + c*x**6)*b*c**2*x**3 + 16*sqrt(a + b*x**3 + c*x
**6)*c**3*x**6 - 36*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*
a*b*c + 15*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*b**3 + 36
*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*b*c - 15*sqrt(c)*
log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**3 + 216*int((sqrt(a + b*x
**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),
x)*a*b*c**3 - 90*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 +
 a*c*x**6 + b**2*x**6 + b*c*x**9),x)*b**3*c**2 + 216*int((sqrt(a + b*x**3
+ c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a
**2*c**3 + 18*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*
c*x**6 + b**2*x**6 + b*c*x**9),x)*a*b**2*c**2 - 45*int((sqrt(a + b*x**3 +
c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*b**
4*c)/(144*c**4)
```

3.206 $\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}+\frac{(3b^2-4ac)\,\mathrm{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}$$

output
```
-1/4*b*(c*x^6+b*x^3+a)^(1/2)/c^2+1/6*x^3*(c*x^6+b*x^3+a)^(1/2)/c+1/24*(-4*
a*c+3*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{(-3b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c^2}+\frac{(-3b^2+4ac)\log\left(bc^2+2c^3x^3-2c^{5/2}\sqrt{a+bx^3+cx^6}\right)}{24c^{5/2}}$$

input
```
Integrate[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((-3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c^2) + ((-3*b^2 + 4*a*c)*Lo
g[b*c^2 + 2*c^3*x^3 - 2*c^(5/2)*Sqrt[a + b*x^3 + c*x^6]])/(24*c^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1693, 1166, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^6}{\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\frac{3bx^3+2a}{2\sqrt{cx^6+bx^3+a}}dx^3}{2c}+\frac{x^3\sqrt{a+bx^3+cx^6}}{2c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{a+bx^3+cx^6}}{2c}-\frac{\int \frac{3bx^3+2a}{\sqrt{cx^6+bx^3+a}}dx^3}{4c}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{a+bx^3+cx^6}}{2c}-\frac{\frac{3b\sqrt{a+bx^3+cx^6}}{c}-\frac{(3b^2-4ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{2c}}{4c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{a+bx^3+cx^6}}{2c}-\frac{\frac{3b\sqrt{a+bx^3+cx^6}}{c}-\frac{(3b^2-4ac)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{c}}{4c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{a+bx^3+cx^6}}{2c}-\frac{\frac{3b\sqrt{a+bx^3+cx^6}}{c}-\frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{3/2}}}{4c}\right)$$

input
```
Int[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((x^3*Sqrt[a + b*x^3 + c*x^6])/(2*c) - ((3*b*Sqrt[a + b*x^3 + c*x^6])/c -
((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])
])/(2*c^(3/2)))/(4*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{8}}{\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.95

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \left[-\frac{(3\,b^2-4\,ac)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^6+bx^3}}{48\,c^3}\right.$$
$$\left.-\frac{(3\,b^2-4\,ac)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,\sqrt{cx^6+bx^3+a}(2\,c^2x^3-3\,bc)}{24\,c^3}\right]$$

input
```
integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*((3*b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(
c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 +
 a)*(2*c^2*x^3 - 3*b*c))/c^3, -1/24*((3*b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*s
qrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) -
 2*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 - 3*b*c))/c^3]
```

Sympy [F]

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x**8/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**8/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x^8}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^8/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x^8}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(x^8/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x^8/(a + b*x^3 + c*x^6)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$$
$$=\frac{8\sqrt{c\,x^6+b\,x^3+a}\,a\,c^2-12\sqrt{c\,x^6+b\,x^3+a}\,b^2c+4\sqrt{c\,x^6+b\,x^3+a}\,b\,c^2x^3+4\sqrt{c}\log\left(\sqrt{c\,x^6+b\,x^3+a}-\right.}{}$$

input
```
int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

output

```
(8*sqrt(a + b*x**3 + c*x**6)*a*c**2 - 12*sqrt(a + b*x**3 + c*x**6)*b**2*c
+ 4*sqrt(a + b*x**3 + c*x**6)*b*c**2*x**3 + 4*sqrt(c)*log(sqrt(a + b*x**3
+ c*x**6) - sqrt(c)*x**3)*a*b*c - 3*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6)
- sqrt(c)*x**3)*b**3 - 4*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x
**3)*a*b*c + 3*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**3
- 24*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 +
b**2*x**6 + b*c*x**9),x)*a*b*c**3 + 18*int((sqrt(a + b*x**3 + c*x**6)*x**8
)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*b**3*c**2 - 24*
int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*
x**6 + b*c*x**9),x)*a**2*c**3 + 6*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a*
*2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a*b**2*c**2 + 9*int(
(sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6
 + b*c*x**9),x)*b**4*c)/(24*b*c**3)
```

3.207 $\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

output
```
1/3*(c*x^6+b*x^3+a)^(1/2)/c-1/6*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b
*x^3+a)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

input
```
Integrate[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a
 + b*x^3 + c*x^6])])/(6*c^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1693, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3+cx^6}}{c}-\frac{b\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{2c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3+cx^6}}{c}-\frac{b\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3+cx^6}}{c}-\frac{b\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{3/2}}\right)$$

input
```
Int[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]/c - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a +
b*x^3 + c*x^6])])/(2*c^(3/2)))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^5}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.37

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$$
$$= \left[\frac{b\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)+4\,\sqrt{cx^6+bx^3+a}\,c}{12\,c^2}, \frac{b\sqrt{-c}\,a}{}\right.$$

input
```
integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 +
 a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c^2, 1/6
*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^
2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*c)/c^2]
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x**5/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**5/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{b\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{6\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^6+bx^3+a}}{3\,c}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*b*log(abs(2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c^(3
/2) + 1/3*sqrt(c*x^6 + b*x^3 + a)/c
```

Mupad [B] (verification not implemented)

Time = 19.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,c}-\frac{b\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{6\,c^{3/2}}$$

input
```
int(x^5/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
(a + b*x^3 + c*x^6)^(1/2)/(3*c) - (b*log((a + b*x^3 + c*x^6)^(1/2) + (b/2
+ c*x^3)/c^(1/2)))/(6*c^(3/2))
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$$
$$= \frac{4\sqrt{c\,x^6+b\,x^3+a}\,c+\sqrt{c}\log\left(\sqrt{c\,x^6+b\,x^3+a}-\sqrt{c}\,x^3\right)b-\sqrt{c}\log\left(\sqrt{c\,x^6+b\,x^3+a}+\sqrt{c}\,x^3\right)b-6\left(\int \cdots\right.}{6\cdots}$$

input
```
int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
(4*sqrt(a + b*x**3 + c*x**6)*c + sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - s
qrt(c)*x**3)*b - sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b -
 6*int((sqrt(a + b*x**3 + c*x**6)*x**8)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b*
*2*x**6 + b*c*x**9),x)*b*c**2 - 6*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a*
*2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a*c**2 - 3*int((sqrt
(a + b*x**3 + c*x**6)*x**5)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*
c*x**9),x)*b**2*c)/(6*c**2)
```

3.208 $\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx=\frac{\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

output
```
1/3*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx=-\frac{\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{3\sqrt{c}}$$

input
```
Integrate[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
-1/3*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]]/Sqrt[c]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1690, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}dx$$
$$\Big\downarrow \text{ 1690}$$
$$\frac{1}{3}\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3$$
$$\Big\downarrow \text{ 1092}$$
$$\frac{2}{3}\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

input
```
Int[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[c])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.74

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \left[\frac{\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}(2\,cx^3+b)\sqrt{c}-4\,ac\right)}{6\,\sqrt{c}},\ -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)}{3\,c}\right]$$

input
```
integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
 + b)*sqrt(c) - 4*a*c)/sqrt(c), -1/3*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^
3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c))/c]
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(33) = 66$.

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{12}\sqrt{cx^6+bx^3+a}\left(2\,x^3+\frac{b}{c}\right) + \frac{(b^2-4\,ac)\log\left(\left|2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}+b\right|\right)}{24\,c^{\frac{3}{2}}}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(c*x^6 + b*x^3 + a)*(2*x^3 + b/c) + 1/24*(b^2 - 4*a*c)*log(abs(2*
(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 19.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3\,\sqrt{c}}$$

input
```
int(x^2/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))/(3*c^(1/2))
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$$
$$= \frac{-\sqrt{c}\log\left(\sqrt{c\,x^6+b\,x^3+a}-\sqrt{c}\,x^3\right)+\sqrt{c}\log\left(\sqrt{c\,x^6+b\,x^3+a}+\sqrt{c}\,x^3\right)-3\left(\int \frac{\sqrt{c\,x^6+b\,x^3+a}\,x^2}{bc\,x^9+ac\,x^6+b^2x^6+2ab\,x^3+a^2}\,d\right.}{3c}$$

input
```
int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
( - sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3) + sqrt(c)*log(sq
rt(a + b*x**3 + c*x**6) + sqrt(c)*x**3) - 3*int((sqrt(a + b*x**3 + c*x**6)
*x**2)/(a**2 + 2*a*b*x**3 + a*c*x**6 + b**2*x**6 + b*c*x**9),x)*a*c)/(3*c)
```

3.209 $\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

output
```
-1/3*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(2*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(3*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1693, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1154}$$

$$-\frac{2}{3}\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
-1/3*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/Sqrt[a]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{x\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.82

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = \left[\frac{\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)}{6\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)}{3\,a}\right]$$

input
```
integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^
3 + 2*a)*sqrt(a) + 8*a^2)/x^6)/sqrt(a), 1/3*sqrt(-a)*arctan(1/2*sqrt(c*x^6
 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2))/a]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*x**3 + c*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}\,x}\,dx$$

input
```
integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 19.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{3\,\sqrt{a}}$$

input
```
int(1/(x*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
-log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^3 + c*x^6)^(1/2))/x^3)/(3*a^(1/2))
```

Reduce [F]

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a}\log\left(\sqrt{c\,x^6+b\,x^3+a}-\sqrt{a}\right)-\sqrt{a}\log\left(\sqrt{c\,x^6+b\,x^3+a}+\sqrt{a}\right)-3\left(\int \frac{\sqrt{c\,x^6+b\,x^3+a}\,x^2}{c^2x^9+2bc\,x^6+ac\,x^3+b^2x^3+ab}dx\right)ac}{3a}$$

input
```
int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
(sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a)) - sqrt(a)*log(sqrt(a + b
*x**3 + c*x**6) + sqrt(a)) - 3*int((sqrt(a + b*x**3 + c*x**6)*x**2)/(a*b +
 a*c*x**3 + b**2*x**3 + 2*b*c*x**6 + c**2*x**9),x)*a*c)/(3*a)
```

3.210 $\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}+\frac{b\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}$$

output
```
-1/3*(c*x^6+b*x^3+a)^(1/2)/a/x^3+1/6*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*
x^6+b*x^3+a)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3a^{3/2}}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
-1/3*Sqrt[a + b*x^3 + c*x^6]/(a*x^3) - (b*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a +
b*x^3 + c*x^6])/Sqrt[a]])/(3*a^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1693, 1157, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1157}$$

$$\frac{1}{3}\left(-\frac{b\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{\sqrt{a+bx^3+cx^6}}{ax^3}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{b\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{\sqrt{a+bx^3+cx^6}}{ax^3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{b\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^3+cx^6}}{ax^3}\right)$$

input
```
Int[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(-(Sqrt[a + b*x^3 + c*x^6]/(a*x^3)) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*
Sqrt[a + b*x^3 + c*x^6])])/(2*a^(3/2)))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{x^4\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.49

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \left[\frac{\sqrt{a}bx^3\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,\sqrt{cx^6+bx^3+a}a}{12\,a^2x^3}, -\frac{\sqrt{-a}bx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,\sqrt{cx^6+bx^3+a}a}{6\,a^2x^3}\right]$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 +
b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)
*a)/(a^2*x^3), -1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*
x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)
*a)/(a^2*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x**4/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a + b*x**3 + c*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^4}\,dx$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 19.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\frac{b\,x^3}{2}+a}{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}\right)}{6\,a^{3/2}} - \frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,a\,x^3}$$

input
```
int(1/(x^4*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
(b*atanh((a + (b*x^3)/2)/(a^(1/2)*(a + b*x^3 + c*x^6)^(1/2))))/(6*a^(3/2))
 - (a + b*x^3 + c*x^6)^(1/2)/(3*a*x^3)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$$
$$= \frac{-2\sqrt{c\,x^6+b\,x^3+a}\,ab+2\sqrt{c\,x^6+b\,x^3+a}\,ac\,x^3-\sqrt{a}\log\left(\sqrt{c\,x^6+b\,x^3+a}-\sqrt{a}\right)b^2x^3+\sqrt{a}\log\left(\sqrt{c\,x^6\right.}{6a^2}$$

input
```
int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**3 + c*x**6)*a*b + 2*sqrt(a + b*x**3 + c*x**6)*a*c*x**3
 - sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*b**2*x**3 + sqrt(a)*lo
g(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*b**2*x**3 - 6*int((sqrt(a + b*x**3
+ c*x**6)*x**8)/(a*b + a*c*x**3 + b**2*x**3 + 2*b*c*x**6 + c**2*x**9),x)*a
*c**3*x**3 - 9*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a*b + a*c*x**3 + b**2
*x**3 + 2*b*c*x**6 + c**2*x**9),x)*a*b*c**2*x**3)/(6*a**2*b*x**3)
```

3.211 $\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} - \frac{(3b^2-4ac)\,\mathrm{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}$$

output
```
-1/6*(c*x^6+b*x^3+a)^(1/2)/a/x^6+1/4*b*(c*x^6+b*x^3+a)^(1/2)/a^2/x^3-1/24*
(-4*a*c+3*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5
/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \frac{(-2a+3bx^3)\sqrt{a+bx^3+cx^6}}{12a^2x^6} + \frac{(3b^2-4ac)\,\mathrm{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{12a^{5/2}}$$

input
```
Integrate[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
((-2*a + 3*b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a^2*x^6) + ((3*b^2 - 4*a*c)
*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(12*a^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1693, 1167, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^9\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{3}\left(-\frac{\int \frac{2cx^3+3b}{2x^6\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{\sqrt{a+bx^3+cx^6}}{2ax^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{2cx^3+3b}{x^6\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{\sqrt{a+bx^3+cx^6}}{2ax^6}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{3}\left(-\frac{-\frac{(3b^2-4ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{3b\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{\sqrt{a+bx^3+cx^6}}{2ax^6}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(-\frac{\frac{(3b^2-4ac)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{3b\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{\sqrt{a+bx^3+cx^6}}{2ax^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(-\frac{\frac{(3b^2-4ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{3b\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{\sqrt{a+bx^3+cx^6}}{2ax^6}\right)$$

input
```
Int[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(-1/2*Sqrt[a + b*x^3 + c*x^6]/(a*x^6) - ((-3*b*Sqrt[a + b*x^3 + c*x^6])/(a
*x^3) + ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
 c*x^6])])/(2*a^(3/2)))/(4*a))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{x^{7}\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.05

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \left[-\frac{(3\,b^2-4\,ac)\sqrt{a}x^6\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,\sqrt{cx^6+bx^3+a}(3\,abx^3-2}{48\,a^3x^6}\right.$$

input
```
integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*((3*b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 +
4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x
^6 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6), 1/24*((3*b^2 - 4*a*c)*sqrt
(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^
6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*
x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x**7/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x**7*sqrt(a + b*x**3 + c*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}\,x^7}\,dx$$

input
```
integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^7\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx = \text{too large to display}$$

input
```
int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

output

```
( - 64*sqrt(a + b*x**3 + c*x**6)*a**4*b*c**3*x**3 - 64*sqrt(a + b*x**3 + c
*x**6)*a**4*c**4*x**6 - 24*sqrt(a + b*x**3 + c*x**6)*a**3*b**4*c + 48*sqrt
(a + b*x**3 + c*x**6)*a**3*b**3*c**2*x**3 + 96*sqrt(a + b*x**3 + c*x**6)*a
**3*b**2*c**3*x**6 - 12*sqrt(a + b*x**3 + c*x**6)*a**2*b**6 + 12*sqrt(a +
b*x**3 + c*x**6)*a**2*b**5*c*x**3 - 36*sqrt(a + b*x**3 + c*x**6)*a**2*b**4
*c**2*x**6 + 36*sqrt(a + b*x**3 + c*x**6)*a*b**7*x**3 - 24*sqrt(a)*log(sqr
t(a + b*x**3 + c*x**6) - sqrt(a))*a**2*b**4*c**2*x**6 + 6*sqrt(a)*log(sqrt
(a + b*x**3 + c*x**6) - sqrt(a))*a*b**6*c*x**6 + 9*sqrt(a)*log(sqrt(a + b*
x**3 + c*x**6) - sqrt(a))*b**8*x**6 + 24*sqrt(a)*log(sqrt(a + b*x**3 + c*x
**6) + sqrt(a))*a**2*b**4*c**2*x**6 - 6*sqrt(a)*log(sqrt(a + b*x**3 + c*x*
*6) + sqrt(a))*a*b**6*c*x**6 - 9*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + s
qrt(a))*b**8*x**6 - 192*int(sqrt(a + b*x**3 + c*x**6)/(a*b*x + a*c*x**4 +
b**2*x**4 + 2*b*c*x**7 + c**2*x**10),x)*a**5*b*c**4*x**6 + 48*int(sqrt(a +
 b*x**3 + c*x**6)/(a*b*x + a*c*x**4 + b**2*x**4 + 2*b*c*x**7 + c**2*x**10)
,x)*a**4*b**3*c**3*x**6 + 18*int(sqrt(a + b*x**3 + c*x**6)/(a*b*x + a*c*x*
*4 + b**2*x**4 + 2*b*c*x**7 + c**2*x**10),x)*a**2*b**7*c*x**6 + 27*int(sqr
t(a + b*x**3 + c*x**6)/(a*b*x + a*c*x**4 + b**2*x**4 + 2*b*c*x**7 + c**2*x
**10),x)*a*b**9*x**6 - 384*int(sqrt(a + b*x**3 + c*x**6)/(2*a**2*b*c*x**4
+ 2*a**2*c**2*x**7 + a*b**3*x**4 + 3*a*b**2*c*x**7 + 4*a*b*c**2*x**10 + 2*
a*c**3*x**13 + b**4*x**7 + 2*b**3*c*x**10 + b**2*c**2*x**13),x)*a**6*b*...
```

3.212 $\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 145

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3} + \frac{b(5b^2-12ac)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}$$

output
```
-1/9*(c*x^6+b*x^3+a)^(1/2)/a/x^9+5/36*b*(c*x^6+b*x^3+a)^(1/2)/a^2/x^6-1/72
*(-16*a*c+15*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/x^3+1/48*b*(-12*a*c+5*b^2)*arc
tanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \frac{\sqrt{a+bx^3+cx^6}(-8a^2+10abx^3-15b^2x^6+16acx^6)}{72a^3x^9} + \frac{(-5b^3+12abc)\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{7/2}}$$

input
```
Integrate[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 + 10*a*b*x^3 - 15*b^2*x^6 + 16*a*c*x^6))/
(72*a^3*x^9) + ((-5*b^3 + 12*a*b*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3
+ c*x^6])/Sqrt[a]])/(24*a^(7/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1693, 1167, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^{12}\sqrt{cx^6+bx^3+a}}dx^3$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{3}\left(-\frac{\int \frac{4cx^3+5b}{2x^9\sqrt{cx^6+bx^3+a}}dx^3}{3a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{4cx^3+5b}{x^9\sqrt{cx^6+bx^3+a}}dx^3}{6a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{10bcx^3+15b^2-16ac}{2x^6\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{5b\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{10bcx^3+15b^2-16ac}{x^6\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{5b\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{3b(5b^2-12ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{5b\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{3}\left(-\frac{-\frac{\frac{3b(5b^2-12ac)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{5b\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(-\frac{-\frac{\frac{3b(5b^2-12ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{5b\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^9}\right)$$

input
```
Int[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(-1/3*Sqrt[a + b*x^3 + c*x^6]/(a*x^9) - ((-5*b*Sqrt[a + b*x^3 + c*x^6])/(2
*a*x^6) - (-(((15*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^3)) + (3*b*(
5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]
)/(2*a^(3/2)))/(4*a))/(6*a))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{x^{10}\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \left[-\frac{3\,(5\,b^3-12\,abc)\sqrt{a}x^9\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,((15\,ab^2-16\,a^2c)x^6-1}{288\,a^4x^9}\right.$$

$$-\frac{3\,(5\,b^3-12\,abc)\sqrt{-a}x^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((15\,ab^2-16\,a^2c)x^6-10\,a^2bx^3+8\,a^3)\sqrt{}}{144\,a^4x^9}$$

input
```
integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*
x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((
15*a*b^2 - 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/
(a^4*x^9), -1/144*(3*(5*b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6
 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a
*b^2 - 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^4
*x^9)]
```

Sympy [F]

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x**10/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x**10*sqrt(a + b*x**3 + c*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^{10}}\,dx$$

input
```
integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^{10}\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \text{too large to display}$$

input
```
int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
( - 256*sqrt(a + b*x**3 + c*x**6)*a**5*c**3 - 192*sqrt(a + b*x**3 + c*x**6
)*a**4*b**2*c**2 - 256*sqrt(a + b*x**3 + c*x**6)*a**4*b*c**3*x**3 + 1664*s
qrt(a + b*x**3 + c*x**6)*a**4*c**4*x**6 + 160*sqrt(a + b*x**3 + c*x**6)*a*
*3*b**4*c + 672*sqrt(a + b*x**3 + c*x**6)*a**3*b**3*c**2*x**3 - 96*sqrt(a
+ b*x**3 + c*x**6)*a**3*b**2*c**3*x**6 - 40*sqrt(a + b*x**3 + c*x**6)*a**2
*b**5*c*x**3 - 1648*sqrt(a + b*x**3 + c*x**6)*a**2*b**4*c**2*x**6 - 100*sq
rt(a + b*x**3 + c*x**6)*a*b**7*x**3 + 260*sqrt(a + b*x**3 + c*x**6)*a*b**6
*c*x**6 + 150*sqrt(a + b*x**3 + c*x**6)*b**8*x**6 + 576*sqrt(a)*log(sqrt(a
 + b*x**3 + c*x**6) - sqrt(a))*a**3*b*c**4*x**9 + 192*sqrt(a)*log(sqrt(a +
 b*x**3 + c*x**6) - sqrt(a))*a**2*b**3*c**3*x**9 - 540*sqrt(a)*log(sqrt(a
+ b*x**3 + c*x**6) - sqrt(a))*a*b**5*c**2*x**9 + 150*sqrt(a)*log(sqrt(a +
b*x**3 + c*x**6) - sqrt(a))*b**7*c*x**9 - 576*sqrt(a)*log(sqrt(a + b*x**3
+ c*x**6) + sqrt(a))*a**3*b*c**4*x**9 - 192*sqrt(a)*log(sqrt(a + b*x**3 +
c*x**6) + sqrt(a))*a**2*b**3*c**3*x**9 + 540*sqrt(a)*log(sqrt(a + b*x**3 +
 c*x**6) + sqrt(a))*a*b**5*c**2*x**9 - 150*sqrt(a)*log(sqrt(a + b*x**3 + c
*x**6) + sqrt(a))*b**7*c*x**9 - 27648*int(sqrt(a + b*x**3 + c*x**6)/(8*a**
3*b*c**2*x**7 + 8*a**3*c**3*x**10 + 6*a**2*b**3*c*x**7 + 14*a**2*b**2*c**2
*x**10 + 16*a**2*b*c**3*x**13 + 8*a**2*c**4*x**16 - 5*a*b**5*x**7 + a*b**4
*c*x**10 + 12*a*b**3*c**2*x**13 + 6*a*b**2*c**3*x**16 - 5*b**6*x**10 - 10*
b**5*c*x**13 - 5*b**4*c**2*x**16),x)*a**7*b**2*c**5*x**9 + 40512*int(sq...
```

3.213 $\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 192

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}}+\frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9}-\frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6}+\frac{5b(21b^2-44ac)\sqrt{a+bx^3+cx^6}}{576a^4x^3}-\frac{(35b^4-120ab^2c+48a^2c^2)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}}$$

output
```
-1/12*(c*x^6+b*x^3+a)^(1/2)/a/x^12+7/72*b*(c*x^6+b*x^3+a)^(1/2)/a^2/x^9-1/
288*(-36*a*c+35*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/x^6+5/576*b*(-44*a*c+21*b^2
)*(c*x^6+b*x^3+a)^(1/2)/a^4/x^3-1/384*(48*a^2*c^2-120*a*b^2*c+35*b^4)*arct
anh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$$
$$=\frac{\sqrt{a+bx^3+cx^6}(-48a^3+56a^2bx^3-70ab^2x^6+72a^2cx^6+105b^3x^9-220abcx^9)}{576a^4x^{12}}$$
$$+\frac{(35b^4-120ab^2c+48a^2c^2)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{192a^{9/2}}$$

input
```
Integrate[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 + 56*a^2*b*x^3 - 70*a*b^2*x^6 + 72*a^2*c
*x^6 + 105*b^3*x^9 - 220*a*b*c*x^9))/(576*a^4*x^12) + ((35*b^4 - 120*a*b^2
*c + 48*a^2*c^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])
/(192*a^(9/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1693, 1167, 27, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{1}{3}\int \frac{1}{x^{15}\sqrt{cx^6+bx^3+a}}dx^3$$
$$\Big\downarrow \text{1167}$$
$$\frac{1}{3}\left(-\frac{\int \frac{6cx^3+7b}{2x^{12}\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{\sqrt{a+bx^3+cx^6}}{4ax^{12}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(-\frac{\int \frac{6cx^3+7b}{x^{12}\sqrt{cx^6+bx^3+a}}dx^3}{8a}-\frac{\sqrt{a+bx^3+cx^6}}{4ax^{12}}\right)$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{28bcx^3+35b^2-36ac}{2x^9\sqrt{cx^6+bx^3+a}}dx^3}{3a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a}-\frac{\sqrt{a+bx^3+cx^6}}{4ax^{12}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(-\frac{-\frac{\int \frac{28bcx^3+35b^2-36ac}{x^9\sqrt{cx^6+bx^3+a}}dx^3}{6a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a}-\frac{\sqrt{a+bx^3+cx^6}}{4ax^{12}}\right)$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{\int \frac{2c(35b^2-36ac)x^3+5b(21b^2-44ac)}{2x^6\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a}-\frac{\sqrt{a+bx^3+cx^6}}{4ax^{12}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{\int \frac{2c(35b^2-36ac)x^3+5b(21b^2-44ac)}{x^6\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a}-\frac{\sqrt{a+bx^3+cx^6}}{4ax^{12}}\right)$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{-\frac{3(48a^2c^2-120ab^2c+35b^4)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{5b(21b^2-44ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a}-\sqrt{a\right.$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{\frac{3(48a^2c^2-120ab^2c+35b^4)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{5b(21b^2-44ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a}-\frac{\sqrt{\vphantom{a}}}{}\right.$$

$\downarrow$ 219

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{\frac{3(48a^2c^2-120ab^2c+35b^4)\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{5b(21b^2-44ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{7b\sqrt{a+bx^3+cx^6}}{3ax^9}}{8a}\right.$$

input
```
Int[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(-1/4*Sqrt[a + b*x^3 + c*x^6]/(a*x^12) - ((-7*b*Sqrt[a + b*x^3 + c*x^6])/(
3*a*x^9) - (-1/2*((35*b^2 - 36*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) - ((-
5*b*(21*b^2 - 44*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^3) + (3*(35*b^4 - 120*
a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*
x^6])])/(2*a^(3/2)))/(4*a))/(6*a))/(8*a))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{x^{13}\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \left[\frac{3\,(35\,b^4-120\,ab^2c+48\,a^2c^2)\sqrt{a}x^{12}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,(5\,(21\,ab^3-\right.}{2304\,a^5x^{12}}$$

input
```
integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4
*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) +
8*a^2)/x^6) + 4*(5*(21*a*b^3 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*
b^2 - 36*a^3*c)*x^6 - 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12), 1/1152*
(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6
 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(5*(21
*a*b^3 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2 - 36*a^3*c)*x^6 -
48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12)]
```

Sympy [F]

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x**13/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x**13*sqrt(a + b*x**3 + c*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}\,x^{13}}\,dx$$

input
```
integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^13), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^{13}\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \text{too large to display}$$

input
```
int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

output

```
( - 294912*sqrt(a + b*x**3 + c*x**6)*a**9*b**4*c**7 - 393216*sqrt(a + b*x*
*3 + c*x**6)*a**9*b**3*c**8*x**3 + 589824*sqrt(a + b*x**3 + c*x**6)*a**9*b
**2*c**9*x**6 + 1179648*sqrt(a + b*x**3 + c*x**6)*a**9*c**11*x**12 - 86016
0*sqrt(a + b*x**3 + c*x**6)*a**8*b**6*c**6 + 1130496*sqrt(a + b*x**3 + c*x
**6)*a**8*b**5*c**7*x**3 - 245760*sqrt(a + b*x**3 + c*x**6)*a**8*b**4*c**8
*x**6 - 688128*sqrt(a + b*x**3 + c*x**6)*a**8*b**3*c**9*x**9 - 2949120*sqr
t(a + b*x**3 + c*x**6)*a**8*b**2*c**10*x**12 - 325632*sqrt(a + b*x**3 + c*
x**6)*a**7*b**8*c**5 + 569344*sqrt(a + b*x**3 + c*x**6)*a**7*b**7*c**6*x**
3 + 528384*sqrt(a + b*x**3 + c*x**6)*a**7*b**6*c**7*x**6 - 860160*sqrt(a +
 b*x**3 + c*x**6)*a**7*b**5*c**8*x**9 - 540672*sqrt(a + b*x**3 + c*x**6)*a
**7*b**4*c**9*x**12 + 483840*sqrt(a + b*x**3 + c*x**6)*a**6*b**10*c**4 + 1
729536*sqrt(a + b*x**3 + c*x**6)*a**6*b**9*c**5*x**3 - 2247680*sqrt(a + b*
x**3 + c*x**6)*a**6*b**8*c**6*x**6 - 1845248*sqrt(a + b*x**3 + c*x**6)*a**
6*b**7*c**7*x**9 + 3317760*sqrt(a + b*x**3 + c*x**6)*a**6*b**6*c**8*x**12
+ 99840*sqrt(a + b*x**3 + c*x**6)*a**5*b**12*c**3 - 846080*sqrt(a + b*x**3
 + c*x**6)*a**5*b**11*c**4*x**3 - 2465280*sqrt(a + b*x**3 + c*x**6)*a**5*b
**10*c**5*x**6 + 668160*sqrt(a + b*x**3 + c*x**6)*a**5*b**9*c**6*x**9 - 23
8080*sqrt(a + b*x**3 + c*x**6)*a**5*b**8*c**7*x**12 - 67200*sqrt(a + b*x**
3 + c*x**6)*a**4*b**14*c**2 - 12160*sqrt(a + b*x**3 + c*x**6)*a**4*b**13*c
**3*x**3 + 751360*sqrt(a + b*x**3 + c*x**6)*a**4*b**12*c**4*x**6 + 6515...
```

3.214 $\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

output
```
1/4*x^4*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)*AppellF1(4/3,1/2,1/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-
2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/(c*x^6+b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 10.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[x^3/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1
/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
 - 4*a*c])])/(4*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{x^3}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

input
```
Int[x^3/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^3 + c*x^6])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x^3}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x**3/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x**3/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^3}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{x^3}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(x^3/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x^3/(a + b*x^3 + c*x^6)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{\sqrt{c\,x^6+b\,x^3+a}\,x^3}{c\,x^6+b\,x^3+a}dx$$

input
```
int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int((sqrt(a + b*x**3 + c*x**6)*x**3)/(a + b*x**3 + c*x**6),x)
```

3.215 $\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 140

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

output
```
1/2*x^2*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)*AppellF1(2/3,1/2,1/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-
2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/(c*x^6+b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 10.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.20

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[x/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1
/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
 - 4*a*c])])/(2*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{x}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

input
```
Int[x/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^3 + c*x^6])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x}{\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(x/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(x/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(x/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{x}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(x/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(x/(a + b*x^3 + c*x^6)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{\sqrt{c\,x^6+b\,x^3+a}\,x}{c\,x^6+b\,x^3+a}dx$$

input
```
int(x/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int((sqrt(a + b*x**3 + c*x**6)*x)/(a + b*x**3 + c*x**6),x)
```

3.216 $\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

output
```
x*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)*AppellF1(1/3,1/2,1/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
3/(b+(-4*a*c+b^2)^(1/2)))/(c*x^6+b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 10.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[1/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2
, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
 4*a*c])])/Sqrt[a + b*x^3 + c*x^6]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx$$

$$\Big\downarrow \text{ 1686}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

input
```
Int[1/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^3 + c*x^6]
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sqrt{c\,x^{6}+b\,x^{3}+a}}dx$$

input
```
int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(1/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int(1/(a + b*x^3 + c*x^6)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{c\,x^6+b\,x^3+a}dx$$

input
```
int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3 + c*x**6)/(a + b*x**3 + c*x**6),x)
```

3.217 $\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

output
```
-(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)
))^(1/2)*AppellF1(-1/3,1/2,1/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
3/(b+(-4*a*c+b^2)^(1/2)))/x/(c*x^6+b*x^3+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(138) = 276.

Time = 10.41 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.49

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \frac{-20(a+bx^3+cx^6)+5bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4a}}\right.}{20ax\sqrt{a+bx^3+c}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(-20*(a + b*x^3 + c*x^6) + 5*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 8*c*x^6*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*a*x*Sqrt[a
 + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
-((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-1/3, 1/2, 1/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 -
4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^3 + c*x^6]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{1}{x^2\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+ax^2}}\,dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/(c*x^8 + b*x^5 + a*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**3 + c*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+ax^2}}\,dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+a}x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^2\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(1/(x^2*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3 + c*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{\sqrt{c\,x^6+b\,x^3+a}}{c\,x^8+b\,x^5+a\,x^2}dx$$

input
```
int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3 + c*x**6)/(a*x**2 + b*x**5 + c*x**8),x)
```

3.218 $\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

output
```
-1/2*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(
1/2)))^(1/2)*AppellF1(-2/3,1/2,1/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*
c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^2/(c*x^6+b*x^3+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 342 vs. 2(140) = 280.

Time = 10.39 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = \frac{-4(a+bx^3+cx^6)-2bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{8ax^2\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
(-4*(a + b*x^3 + c*x^6) - 2*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(8*a*x^2*Sqrt[a +
 b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^3\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

input
```
Int[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

output
```
-1/2*(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[-2/3, 1/2, 1/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2
 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]])/(x^2*Sqrt[a + b*x^3 + c*x
^6])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{1}{x^3\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+ax^3}}\,dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/(c*x^9 + b*x^6 + a*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate(1/x**3/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a + b*x**3 + c*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{1}{\sqrt{cx^6+bx^3+ax^3}}\,dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{\sqrt{cx^6+bx^3+a}x^3}\,dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{1}{x^3\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int(1/(x^3*(a + b*x^3 + c*x^6)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3 + c*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{c\,x^9+b\,x^6+a\,x^3}dx$$

input
```
int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3 + c*x**6)/(a*x**3 + b*x**6 + c*x**9),x)
```

3.219 $\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 195

$$\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{2x^9(2a+bx^3)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)} - \frac{\left(b(15b^2-52ac)-2c(5b^2-12ac)\,x^3\right)\sqrt{a+bx^3+cx^6}}{12c^3\left(b^2-4ac\right)} + \frac{(5b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}}$$

output
```
2/3*x^9*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)-2/3*b*x^6*(c*x^6+b*
x^3+a)^(1/2)/c/(-4*a*c+b^2)-1/12*(b*(-52*a*c+15*b^2)-2*c*(-12*a*c+5*b^2)*x
^3)*(c*x^6+b*x^3+a)^(1/2)/c^3/(-4*a*c+b^2)+1/8*(-4*a*c+5*b^2)*arctanh(1/2*
(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.87

$$\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{4a^2c(-13b+6cx^3)+b^2x^3(15b^2+5bcx^3-2c^2x^6)+a(15b^3-62b^2cx^3-20bc^2x}{12c^3\left(-b^2+4ac\right)\sqrt{a+bx^3+cx^6}}$$
$$+\frac{(-5b^2+4ac)\log\left(c^3\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)}{8c^{7/2}}$$

input
```
Integrate[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(4*a^2*c*(-13*b + 6*c*x^3) + b^2*x^3*(15*b^2 + 5*b*c*x^3 - 2*c^2*x^6) + a*
(15*b^3 - 62*b^2*c*x^3 - 20*b*c^2*x^6 + 8*c^3*x^9))/(12*c^3*(-b^2 + 4*a*c)
*Sqrt[a + b*x^3 + c*x^6]) + ((-5*b^2 + 4*a*c)*Log[c^3*(b + 2*c*x^3 - 2*Sqr
t[c]*Sqrt[a + b*x^3 + c*x^6]])])/(8*c^(7/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1693, 1164, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^{12}}{\left(cx^6+bx^3+a\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{1164}$$

$$\frac{1}{3}\left(\frac{2x^9\left(2a+bx^3\right)}{\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2\int \frac{3x^6\left(bx^3+2a\right)}{\sqrt{cx^6+bx^3+a}}dx^3}{b^2-4ac}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{2x^9\left(2a+bx^3\right)}{\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{6\int \frac{x^6\left(bx^3+2a\right)}{\sqrt{cx^6+bx^3+a}}dx^3}{b^2-4ac}\right)$$

$\big\downarrow$ 1236

$$\frac{1}{3}\left(\frac{2x^9\left(2a+bx^3\right)}{\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{6\left(\frac{\int -\frac{x^3\left(\left(5b^2-12ac\right)x^3+4ab\right)}{2\sqrt{cx^6+bx^3+a}}dx^3}{3c}+\frac{bx^6\sqrt{a+bx^3+cx^6}}{3c}\right)}{b^2-4ac}\right)$$

$\big\downarrow$ 27

$$\frac{1}{3}\left(\frac{2x^9\left(2a+bx^3\right)}{\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{6\left(\frac{bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\int \frac{x^3\left(\left(5b^2-12ac\right)x^3+4ab\right)}{\sqrt{cx^6+bx^3+a}}dx^3}{6c}\right)}{b^2-4ac}\right)$$

$\big\downarrow$ 1225

$$\frac{1}{3}\left(\frac{2x^9\left(2a+bx^3\right)}{\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{6\left(\frac{bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c^2}-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5b^2-12ac\right)\right)\sqrt{\cdots}}{4c^2}}{6c}\cdots\right.}{b^2-4ac}\cdots\right.$$

$\big\downarrow$ 1092

$$\frac{1}{3}\left(\frac{2x^9\left(2a+bx^3\right)}{\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{6\left(\frac{bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c^2}-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5b^2-12a\cdots\right.\right.}{4c^2}}{6c}\cdots\right.}{b^2-4ac}\cdots\right.$$

$\big\downarrow$ 219

$$\frac{1}{3}\left(\frac{2x^9(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{6\left(\frac{bx^6\sqrt{a+bx^3+cx^6}}{3c}-\frac{\frac{3(b^2-4ac)(5b^2-4ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{5/2}}-\frac{\left(b\left(15b^2-52ac\right)-2cx^3\left(5\right.\right.}{4}}{6c}\right.}{b^2-4ac}\right.$$

input
```
Int[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((2*x^9*(2*a + b*x^3))/((b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (6*((b*x^
6*Sqrt[a + b*x^3 + c*x^6])/(3*c) - (-1/4*((b*(15*b^2 - 52*a*c) - 2*c*(5*b^
2 - 12*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/c^2 + (3*(b^2 - 4*a*c)*(5*b^2 -
4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(5
/2)))/(6*c)))/(b^2 - 4*a*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{14}}{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 591, normalized size of antiderivative = 3.03

$$\int \frac{x^{14}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \Bigg[-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^6+5\,ab^4-24\,a^2b^2c+16\,a^3c^2+\left(5\,b^5-24\,ab\right.\right.}{}$$

$$-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^6+5\,ab^4-24\,a^2b^2c+16\,a^3c^2+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^3\right)\sqrt{-c}\,\mathrm{arctan}}{24\left(ab^2c^4-\right.}$$

input
```
integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6 + 5*a*b^4 - 24*a^2*b^
2*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^3)*sqrt(c)*log(-8
*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(
c) - 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*b*c^3)*x^6 -
 15*a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^3)*s
qrt(c*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^6 +
 (b^3*c^4 - 4*a*b*c^5)*x^3), -1/24*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^
3)*x^6 + 5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^
2*b*c^2)*x^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sq
rt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*
c^2 - 4*a*b*c^3)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^
2 + 24*a^2*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^
2*c^5 - 4*a*c^6)*x^6 + (b^3*c^4 - 4*a*b*c^5)*x^3)]
```

Sympy [F]

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{14}}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**14/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**14/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{14}}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^14/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{14}}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

input
```
int(x^14/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x^14/(a + b*x^3 + c*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{too large to display}$$

input
```
int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
( - 48*sqrt(a + b*x**3 + c*x**6)*a**3*c**3 - 20*sqrt(a + b*x**3 + c*x**6)*
a**2*b**2*c**2 + 72*sqrt(a + b*x**3 + c*x**6)*a**2*b*c**3*x**3 - 154*sqrt(
a + b*x**3 + c*x**6)*a*b**3*c**2*x**3 - 40*sqrt(a + b*x**3 + c*x**6)*a*b**
2*c**3*x**6 + 16*sqrt(a + b*x**3 + c*x**6)*a*b*c**4*x**9 + 30*sqrt(a + b*x
**3 + c*x**6)*b**5*c*x**3 + 10*sqrt(a + b*x**3 + c*x**6)*b**4*c**2*x**6 -
4*sqrt(a + b*x**3 + c*x**6)*b**3*c**3*x**9 + 48*sqrt(c)*log(sqrt(a + b*x**
3 + c*x**6) - sqrt(c)*x**3)*a**3*b*c**2 - 72*sqrt(c)*log(sqrt(a + b*x**3 +
 c*x**6) - sqrt(c)*x**3)*a**2*b**3*c + 48*sqrt(c)*log(sqrt(a + b*x**3 + c*
x**6) - sqrt(c)*x**3)*a**2*b**2*c**2*x**3 + 48*sqrt(c)*log(sqrt(a + b*x**3
 + c*x**6) - sqrt(c)*x**3)*a**2*b*c**3*x**6 + 15*sqrt(c)*log(sqrt(a + b*x*
*3 + c*x**6) - sqrt(c)*x**3)*a*b**5 - 72*sqrt(c)*log(sqrt(a + b*x**3 + c*x
**6) - sqrt(c)*x**3)*a*b**4*c*x**3 - 72*sqrt(c)*log(sqrt(a + b*x**3 + c*x*
*6) - sqrt(c)*x**3)*a*b**3*c**2*x**6 + 15*sqrt(c)*log(sqrt(a + b*x**3 + c*
x**6) - sqrt(c)*x**3)*b**6*x**3 + 15*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6)
 - sqrt(c)*x**3)*b**5*c*x**6 - 48*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) +
sqrt(c)*x**3)*a**3*b*c**2 + 72*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqr
t(c)*x**3)*a**2*b**3*c - 48*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c
)*x**3)*a**2*b**2*c**2*x**3 - 48*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + s
qrt(c)*x**3)*a**2*b*c**3*x**6 - 15*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) +
 sqrt(c)*x**3)*a*b**5 + 72*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt...
```

3.220 $\int \frac{x^{11}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int \frac{x^{11}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{2x^6(2a+bx^3)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}+\frac{\left(3b^2-8ac-2bcx^3\right)\sqrt{a+bx^3+cx^6}}{3c^2\left(b^2-4ac\right)}-\frac{b\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

output
```
2/3*x^6*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)+1/3*(-2*b*c*x^3-8*a
*c+3*b^2)*(c*x^6+b*x^3+a)^(1/2)/c^2/(-4*a*c+b^2)-1/2*b*arctanh(1/2*(2*c*x^
3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{-3ab^2+8a^2c-3b^3x^3+10abcx^3-b^2cx^6+4ac^2x^6}{3c^2\left(-b^2+4ac\right)\sqrt{a+bx^3+cx^6}}+\frac{b\log\left(bc^2+2c^3x^3-2c^{5/2}\sqrt{a+bx^3+cx^6}\right)}{2c^{5/2}}$$

input
```
Integrate[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(-3*a*b^2 + 8*a^2*c - 3*b^3*x^3 + 10*a*b*c*x^3 - b^2*c*x^6 + 4*a*c^2*x^6)/
(3*c^2*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) + (b*Log[b*c^2 + 2*c^3*x^3
- 2*c^(5/2)*Sqrt[a + b*x^3 + c*x^6]])/(2*c^(5/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1693, 1164, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{x^9}{(cx^6+bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{1164}$$

$$\frac{1}{3}\left(\frac{2x^6(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\int \frac{2x^3(bx^3+2a)}{\sqrt{cx^6+bx^3+a}}dx^3}{b^2-4ac}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{2x^6(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{4\int \frac{x^3(bx^3+2a)}{\sqrt{cx^6+bx^3+a}}dx^3}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{3}\left(\frac{2x^6(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{4\left(\frac{3b(b^2-4ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{8c^2}-\frac{(-8ac+3b^2-2bcx^3)\sqrt{a+bx^3+cx^6}}{4c^2}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{3}\left(\frac{2x^6(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{4\left(\frac{3b(b^2-4ac)\int\frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{4c^2}-\frac{(-8ac+3b^2-2bcx^3)\sqrt{a+bx^3+cx^6}}{4c^2}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{2x^6(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{4\left(\frac{3b(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{5/2}}-\frac{(-8ac+3b^2-2bcx^3)\sqrt{a+bx^3+cx^6}}{4c^2}\right)}{b^2-4ac}\right)$$

input
```
Int[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((2*x^6*(2*a + b*x^3))/((b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (4*(-1/4*
((3*b^2 - 8*a*c - 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/c^2 + (3*b*(b^2 - 4*
a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(5/2
))))/(b^2 - 4*a*c))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{11}}{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}d x$$

input
```
int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.35

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \left[\frac{3\left((b^3c-4\,abc^2)x^6+ab^3-4\,a^2bc+(b^4-4\,ab^2c)x^3\right)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3\right.}{12\left((b\right.}\right.$$

input
```
integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*
x^3)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*
(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c -
8*a^2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4
 - 4*a*c^5)*x^6 + a*b^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3), 1/6*
(3*((b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^3)*s
qrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6
 + b*c*x^3 + a*c)) + 2*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a^2*c^2 +
(3*b^3*c - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4*a*c^5)*
x^6 + a*b^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3)]
```

Sympy [F]

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{11}}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**11/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**11/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{11}}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^11/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^{11}}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

input
```
int(x^11/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x^11/(a + b*x^3 + c*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
(4*sqrt(a + b*x**3 + c*x**6)*a**2*c**2 + 26*sqrt(a + b*x**3 + c*x**6)*a*b*
c**2*x**3 + 8*sqrt(a + b*x**3 + c*x**6)*a*c**3*x**6 - 6*sqrt(a + b*x**3 +
c*x**6)*b**3*c*x**3 - 2*sqrt(a + b*x**3 + c*x**6)*b**2*c**2*x**6 + 12*sqrt
(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a**2*b*c - 3*sqrt(c)*log
(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**3)*a*b**3 + 12*sqrt(c)*log(sqrt(a
+ b*x**3 + c*x**6) - sqrt(c)*x**3)*a*b**2*c*x**3 + 12*sqrt(c)*log(sqrt(a +
 b*x**3 + c*x**6) - sqrt(c)*x**3)*a*b*c**2*x**6 - 3*sqrt(c)*log(sqrt(a + b
*x**3 + c*x**6) - sqrt(c)*x**3)*b**4*x**3 - 3*sqrt(c)*log(sqrt(a + b*x**3
+ c*x**6) - sqrt(c)*x**3)*b**3*c*x**6 - 12*sqrt(c)*log(sqrt(a + b*x**3 + c
*x**6) + sqrt(c)*x**3)*a**2*b*c + 3*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6)
+ sqrt(c)*x**3)*a*b**3 - 12*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c
)*x**3)*a*b**2*c*x**3 - 12*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)
*x**3)*a*b*c**2*x**6 + 3*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x
**3)*b**4*x**3 + 3*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b
**3*c*x**6 - 36*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**3 + 3*a**2*b*x**3
 + 2*a**2*c*x**6 + 3*a*b**2*x**6 + 4*a*b*c*x**9 + a*c**2*x**12 + b**3*x**9
 + 2*b**2*c*x**12 + b*c**2*x**15),x)*a**4*c**3 + 9*int((sqrt(a + b*x**3 +
c*x**6)*x**5)/(a**3 + 3*a**2*b*x**3 + 2*a**2*c*x**6 + 3*a*b**2*x**6 + 4*a*
b*c*x**9 + a*c**2*x**12 + b**3*x**9 + 2*b**2*c*x**12 + b*c**2*x**15),x)*a*
*3*b**2*c**2 - 36*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**3 + 3*a**2*b...
```

3.221 $\int \frac{x^8}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 120

$$\int \frac{x^8}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{2x^3(2a+bx^3)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

output
```
2/3*x^3*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)-2/3*b*(c*x^6+b*x^3+
a)^(1/2)/c/(-4*a*c+b^2)+1/3*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a
)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{-\frac{2\sqrt{c}\left(b^2x^3+a\left(b-2cx^3\right)\right)}{\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}+\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

input
```
Integrate[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((-2*Sqrt[c]*(b^2*x^3 + a*(b - 2*c*x^3)))/((b^2 - 4*a*c)*Sqrt[a + b*x^3 +
c*x^6]) + ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(3*c
^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1693, 1164, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3}\int \frac{x^6}{(cx^6+bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{ 1164}$$

$$\frac{1}{3}\left(\frac{2x^3(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\int \frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}dx^3}{b^2-4ac}\right)$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{3}\left(\frac{2x^3(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\left(\frac{b\sqrt{a+bx^3+cx^6}}{c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^6+bx^3+a}}dx^3}{2c}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{3}\left(\frac{2x^3(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\left(\frac{b\sqrt{a+bx^3+cx^6}}{c}-\frac{(b^2-4ac)\int \frac{1}{4c-x^6}d\frac{2cx^3+b}{\sqrt{cx^6+bx^3+a}}}{c}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{2x^3(2a+bx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\left(\frac{b\sqrt{a+bx^3+cx^6}}{c}-\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{3/2}}\right)}{b^2-4ac}\right)$$

input
```
Int[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((2*x^3*(2*a + b*x^3))/((b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*((b*Sq
rt[a + b*x^3 + c*x^6])/c - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]
*Sqrt[a + b*x^3 + c*x^6])])/(2*c^(3/2))))/(b^2 - 4*a*c))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^8}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}dx$$

input
```
int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.22

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2\right.}{6\,((b^2c^3-4\,ac^4)x^6+ab^2\right.$$

$$-\frac{((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)+2\,\sqrt{cx^6+bx^3+a}}{3\,((b^2c^3-4\,ac^4)x^6+ab^2c^2-4\,a^2c^3+(b^3c^2-4\,abc^3)x^3)}$$

input
```
integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt
(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
+ b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3 +
 a*b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a
*b*c^3)*x^3), -1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 -
 4*a^2*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-
c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^
2)*x^3 + a*b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c
^2 - 4*a*b*c^3)*x^3)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^8}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**8/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^8}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^8/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 19.98 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.70

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3\,c^{3/2}} + \frac{\frac{a\,b}{2}-x^3\,\left(a\,c-\frac{b^2}{2}\right)}{3\,c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^6+b\,x^3+a}}$$

input
```
int(x^8/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))/(3*c^(3/2)) + ((a*b
)/2 - x^3*(a*c - b^2/2))/(3*c*(a*c - b^2/4)*(a + b*x^3 + c*x^6)^(1/2))
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

output

```
(4*sqrt(a + b*x**3 + c*x**6)*a**2*c**2 - 6*sqrt(a + b*x**3 + c*x**6)*a*b*c
**2*x**3 + 2*sqrt(a + b*x**3 + c*x**6)*b**3*c*x**3 - 4*sqrt(c)*log(sqrt(a
+ b*x**3 + c*x**6) - sqrt(c)*x**3)*a**2*b*c + sqrt(c)*log(sqrt(a + b*x**3
+ c*x**6) - sqrt(c)*x**3)*a*b**3 - 4*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6)
 - sqrt(c)*x**3)*a*b**2*c*x**3 - 4*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) -
 sqrt(c)*x**3)*a*b*c**2*x**6 + sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqr
t(c)*x**3)*b**4*x**3 + sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(c)*x**
3)*b**3*c*x**6 + 4*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a
**2*b*c - sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*b**3 + 4
*sqrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*b**2*c*x**3 + 4*s
qrt(c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*a*b*c**2*x**6 - sqrt(
c)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**4*x**3 - sqrt(c)*log(s
qrt(a + b*x**3 + c*x**6) + sqrt(c)*x**3)*b**3*c*x**6 + 12*int((sqrt(a + b*
x**3 + c*x**6)*x**5)/(a**3 + 3*a**2*b*x**3 + 2*a**2*c*x**6 + 3*a*b**2*x**6
 + 4*a*b*c*x**9 + a*c**2*x**12 + b**3*x**9 + 2*b**2*c*x**12 + b*c**2*x**15
),x)*a**4*c**3 - 3*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**3 + 3*a**2*b*x
**3 + 2*a**2*c*x**6 + 3*a*b**2*x**6 + 4*a*b*c*x**9 + a*c**2*x**12 + b**3*x
**9 + 2*b**2*c*x**12 + b*c**2*x**15),x)*a**3*b**2*c**2 + 12*int((sqrt(a +
b*x**3 + c*x**6)*x**5)/(a**3 + 3*a**2*b*x**3 + 2*a**2*c*x**6 + 3*a*b**2*x*
*6 + 4*a*b*c*x**9 + a*c**2*x**12 + b**3*x**9 + 2*b**2*c*x**12 + b*c**2*...
```

3.222 $\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx=\frac{2(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

output
```
2/3*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx=\frac{2(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1693, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 1693}$$
$$\frac{1}{3}\int \frac{x^3}{(cx^6+bx^3+a)^{3/2}}dx^3$$
$$\Big\downarrow \text{ 1158}$$
$$\frac{2(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

input
```
Int[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
gosper	$-\frac{2(b\,x^{3}+2a)}{3\sqrt{c\,x^{6}+b\,x^{3}+a}\,(4ac-b^{2})}$	38
trager	$-\frac{2(b\,x^{3}+2a)}{3\sqrt{c\,x^{6}+b\,x^{3}+a}\,(4ac-b^{2})}$	38
orering	$-\frac{2(b\,x^{3}+2a)}{3\sqrt{c\,x^{6}+b\,x^{3}+a}\,(4ac-b^{2})}$	38

input
```
int(x^5/(c*x^6+b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(c*x^6+b*x^3+a)^(1/2)*(b*x^3+2*a)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.74

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2\sqrt{cx^6+bx^3+a}(bx^3+2a)}{3\left((b^2c-4ac^2)x^6+(b^3-4abc)x^3+ab^2-4a^2c\right)}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)/((b^2*c - 4*a*c^2)*x^6 + (b^3 -
4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^5}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**5/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2\left(\frac{bx^3}{b^2-4\,ac}+\frac{2\,a}{b^2-4\,ac}\right)}{3\sqrt{cx^6+bx^3+a}}$$

input
```
integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/3*(b*x^3/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 19.71 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2\,b\,x^3+4\,a}{(12\,a\,c-3\,b^2)\,\sqrt{c\,x^6+b\,x^3+a}}$$

input
```
int(x^5/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
-(4*a + 2*b*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.90

$$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2\sqrt{c\,x^6+b\,x^3+a}\,(-b\,x^3-2a)}{12a\,c^2x^6-3b^2c\,x^6+12abc\,x^3-3b^3x^3+12a^2c-3a\,b^2}$$

input
```
int(x^5/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**3 + c*x**6)*( - 2*a - b*x**3))/(3*(4*a**2*c - a*b**2 + 4*
a*b*c*x**3 + 4*a*c**2*x**6 - b**3*x**3 - b**2*c*x**6))
```

3.223 $\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

output
```
1/3*(-4*c*x^3-2*b)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1690, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx$$
$$\Big\downarrow \text{1690}$$
$$\frac{1}{3}\int \frac{1}{(cx^6+bx^3+a)^{3/2}}dx^3$$
$$\Big\downarrow \text{1088}$$
$$-\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

input
```
Int[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
gosper	$\frac{\frac{4c\,x^3}{3}+\frac{2b}{3}}{\sqrt{c\,x^6+b\,x^3+a}\,(4ac-b^2)}$	37
trager	$\frac{\frac{4c\,x^3}{3}+\frac{2b}{3}}{\sqrt{c\,x^6+b\,x^3+a}\,(4ac-b^2)}$	37
orering	$\frac{\frac{4c\,x^3}{3}+\frac{2b}{3}}{\sqrt{c\,x^6+b\,x^3+a}\,(4ac-b^2)}$	37

input
```
int(x^2/(c*x^6+b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(c*x^6+b*x^3+a)^(1/2)*(2*c*x^3+b)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2\sqrt{cx^6+bx^3+a}(2\,cx^3+b)}{3\left((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c\right)}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)/((b^2*c - 4*a*c^2)*x^6 + (b^3 -
 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^2}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**2/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{2\left(\frac{2\,cx^3}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}\right)}{3\,\sqrt{cx^6+bx^3+a}}$$

input
```
integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(2*c*x^3/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 19.75 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{4\,c\,x^3+2\,b}{(12\,a\,c-3\,b^2)\,\sqrt{c\,x^6+b\,x^3+a}}$$

input
```
int(x^2/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
(2*b + 4*c*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{2\sqrt{c\,x^6+b\,x^3+a}\,(2c\,x^3+b)}{12a\,c^2x^6-3b^2c\,x^6+12abc\,x^3-3b^3x^3+12a^2c-3a\,b^2}$$

input
```
int(x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**3 + c*x**6)*(b + 2*c*x**3))/(3*(4*a**2*c - a*b**2 + 4*a*b
*c*x**3 + 4*a*c**2*x**6 - b**3*x**3 - b**2*c*x**6))
```

3.224 $\int \frac{1}{x(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

output
```
2/3*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)-1/3*arctanh(1
/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{2\left(\frac{\sqrt{a}(b^2-2ac+bcx^3)}{(b^2-4ac)\sqrt{a+bx^3+cx^6}}+\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)\right)}{3a^{3/2}}$$

input
```
Integrate[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
(2*((Sqrt[a]*(b^2 - 2*a*c + b*c*x^3))/((b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^
6]) + ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]]))/(3*a^(3/2
))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1693, 1165, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(cx^6+bx^3+a\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{1165}$$

$$\frac{1}{3}\left(\frac{2\left(-2ac+b^2+bcx^3\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2\int -\frac{b^2-4ac}{2x^3\sqrt{cx^6+bx^3+a}}dx^3}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{a}+\frac{2\left(-2ac+b^2+bcx^3\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{2\left(-2ac+b^2+bcx^3\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{2\left(-2ac+b^2+bcx^3\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{a^{3/2}}\right)$$

input
```
Int[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^3))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - A
rcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/a^(3/2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{x\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(78) = 156$.

Time = 0.12 (sec) , antiderivative size = 389, normalized size of antiderivative = 4.23

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)\sqrt{a}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4}{}\right.}{6\,((a^2b^2c-4\,a^3c^2)x^6+a^3b^2-4}\right.$$

input
```
integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt
(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b
^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3
 - 4*a^3*b*c)*x^3), 1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*
b^2 - 4*a^2*c)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*s
qrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3
+ a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^
2*b^3 - 4*a^3*b*c)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{x\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(1/(x*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{x\left(c\,x^6+b\,x^3+a\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*x^3 + c*x^6)^(3/2)),x)
```

output
```
int(1/(x*(a + b*x^3 + c*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
(6*sqrt(a + b*x**3 + c*x**6)*a**2*b*c - 4*sqrt(a + b*x**3 + c*x**6)*a**2*c
**2*x**3 - 2*sqrt(a + b*x**3 + c*x**6)*a*b**3 + 4*sqrt(a)*log(sqrt(a + b*x
**3 + c*x**6) - sqrt(a))*a**2*b*c - sqrt(a)*log(sqrt(a + b*x**3 + c*x**6)
- sqrt(a))*a*b**3 + 4*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*a*b
**2*c*x**3 + 4*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*a*b*c**2*x
**6 - sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*b**4*x**3 - sqrt(a)
*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*b**3*c*x**6 - 4*sqrt(a)*log(sqrt
(a + b*x**3 + c*x**6) + sqrt(a))*a**2*b*c + sqrt(a)*log(sqrt(a + b*x**3 +
c*x**6) + sqrt(a))*a*b**3 - 4*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt
(a))*a*b**2*c*x**3 - 4*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*a*
b*c**2*x**6 + sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*b**4*x**3 +
 sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*b**3*c*x**6 + 12*int((sq
rt(a + b*x**3 + c*x**6)*x**5)/(a**2*b + a**2*c*x**3 + 2*a*b**2*x**3 + 4*a*
b*c*x**6 + 2*a*c**2*x**9 + b**3*x**6 + 3*b**2*c*x**9 + 3*b*c**2*x**12 + c*
*3*x**15),x)*a**4*c**3 - 3*int((sqrt(a + b*x**3 + c*x**6)*x**5)/(a**2*b +
a**2*c*x**3 + 2*a*b**2*x**3 + 4*a*b*c*x**6 + 2*a*c**2*x**9 + b**3*x**6 + 3
*b**2*c*x**9 + 3*b*c**2*x**12 + c**3*x**15),x)*a**3*b**2*c**2 + 12*int((sq
rt(a + b*x**3 + c*x**6)*x**5)/(a**2*b + a**2*c*x**3 + 2*a*b**2*x**3 + 4*a*
b*c*x**6 + 2*a*c**2*x**9 + b**3*x**6 + 3*b**2*c*x**9 + 3*b*c**2*x**12 + c*
*3*x**15),x)*a**3*b*c**3*x**3 + 12*int((sqrt(a + b*x**3 + c*x**6)*x**5)...
```

3.225 $\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\operatorname{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}$$

output
```
2/3*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c*x^6+b*x^3+a)^(1/2)-1/3*(-8*a
*c+3*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^3+1/2*b*arctanh(1/2*(b*
x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{-4a^2c+3b^2x^3(b+cx^3)+a(b^2-10bcx^3-8c^2x^6)}{3a^2\left(-b^2+4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
(-4*a^2*c + 3*b^2*x^3*(b + c*x^3) + a*(b^2 - 10*b*c*x^3 - 8*c^2*x^6))/(3*a
^2*(-b^2 + 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6]) - (b*ArcTanh[(Sqrt[c]*x^3 -
 Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1693, 1165, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3+cx^6)^{3/2}} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3} \int \frac{1}{x^6 (cx^6+bx^3+a)^{3/2}} dx^3$$

$$\Big\downarrow \text{1165}$$

$$\frac{1}{3} \left(\frac{2(-2ac+b^2+bcx^3)}{ax^3 (b^2-4ac) \sqrt{a+bx^3+cx^6}} - \frac{2\int -\frac{2bcx^3+3b^2-8ac}{2x^6\sqrt{cx^6+bx^3+a}}dx^3}{a(b^2-4ac)} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(\frac{\int \frac{2bcx^3+3b^2-8ac}{x^6\sqrt{cx^6+bx^3+a}}dx^3}{a(b^2-4ac)} + \frac{2(-2ac+b^2+bcx^3)}{ax^3 (b^2-4ac) \sqrt{a+bx^3+cx^6}} \right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{3} \left(\frac{-\frac{3b(b^2-4ac)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a} - \frac{(3b^2-8ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{a(b^2-4ac)} + \frac{2(-2ac+b^2+bcx^3)}{ax^3 (b^2-4ac) \sqrt{a+bx^3+cx^6}} \right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{\frac{3b(b^2-4ac)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{(3b^2-8ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx^3)}{ax^3(b^2-4ac)\sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{\frac{3b(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{(3b^2-8ac)\sqrt{a+bx^3+cx^6}}{ax^3}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx^3)}{ax^3(b^2-4ac)\sqrt{a+bx^3+cx^6}}\right)$$

input
```
Int[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^3))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6])
 + (-(((3*b^2 - 8*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^3)) + (3*b*(b^2 - 4*a
*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2*a^(3/2)
))/(a*(b^2 - 4*a*c)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{x^{4} \left(c\,x^{6}+b\,x^{3}+a \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 485, normalized size of antiderivative = 3.42

$$\int \frac{1}{x^4 (a+bx^3+cx^6)^{3/2}}\,dx = \left[\frac{3\left((b^3c-4\,abc^2)x^9+(b^4-4\,ab^2c)x^6+(ab^3-4\,a^2bc)x^3\right)\sqrt{a}\log\left(-\frac{(b^2+4\,ac)x}{}\right.}{12\left((a^3b^2c-\right.}\right.$$

$$-\frac{3\left((b^3c-4\,abc^2)x^9+(b^4-4\,ab^2c)x^6+(ab^3-4\,a^2bc)x^3\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\left((3\,ab^2\right.}{6\left((a^3b^2c-4\,a^4c^2)x^9+(a^3b^3-4\,a^4bc)x^6+(a^4b^2\right.}$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^3 - 4*a^2
*b*c)*x^3)*sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*
x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2*c - 8*a^2*c^2)*
x^6 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3 +
 a))/((a^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 -
 4*a^5*c)*x^3), -1/6*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 +
 (a*b^3 - 4*a^2*b*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x
^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2*c - 8*a^2*c^2)
*x^6 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3
+ a))/((a^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2
- 4*a^5*c)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{x^4 (a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)^{3/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)^{3/2}}\, dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^4}\, dx$$

input
```
integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)^{3/2}}\, dx = \int \frac{1}{x^4 \left(c\, x^6 + b\, x^3 + a\right)^{3/2}}\, dx$$

input
```
int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x)
```

output
```
int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
( - 8*sqrt(a + b*x**3 + c*x**6)*a**3*c + 2*sqrt(a + b*x**3 + c*x**6)*a**2*
b**2 - 32*sqrt(a + b*x**3 + c*x**6)*a**2*b*c*x**3 - 16*sqrt(a + b*x**3 + c
*x**6)*a**2*c**2*x**6 + 6*sqrt(a + b*x**3 + c*x**6)*a*b**3*x**3 - 12*sqrt(
a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*a**2*b*c*x**3 + 3*sqrt(a)*log(
sqrt(a + b*x**3 + c*x**6) - sqrt(a))*a*b**3*x**3 - 12*sqrt(a)*log(sqrt(a +
 b*x**3 + c*x**6) - sqrt(a))*a*b**2*c*x**6 - 12*sqrt(a)*log(sqrt(a + b*x**
3 + c*x**6) - sqrt(a))*a*b*c**2*x**9 + 3*sqrt(a)*log(sqrt(a + b*x**3 + c*x
**6) - sqrt(a))*b**4*x**6 + 3*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt
(a))*b**3*c*x**9 + 12*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*a**
2*b*c*x**3 - 3*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*a*b**3*x**
3 + 12*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*a*b**2*c*x**6 + 12
*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*a*b*c**2*x**9 - 3*sqrt(a
)*log(sqrt(a + b*x**3 + c*x**6) + sqrt(a))*b**4*x**6 - 3*sqrt(a)*log(sqrt(
a + b*x**3 + c*x**6) + sqrt(a))*b**3*c*x**9 + 36*int((sqrt(a + b*x**3 + c*
x**6)*x**2)/(a**2*b + a**2*c*x**3 + 2*a*b**2*x**3 + 4*a*b*c*x**6 + 2*a*c**
2*x**9 + b**3*x**6 + 3*b**2*c*x**9 + 3*b*c**2*x**12 + c**3*x**15),x)*a**4*
b*c**2*x**3 - 9*int((sqrt(a + b*x**3 + c*x**6)*x**2)/(a**2*b + a**2*c*x**3
 + 2*a*b**2*x**3 + 4*a*b*c*x**6 + 2*a*c**2*x**9 + b**3*x**6 + 3*b**2*c*x**
9 + 3*b*c**2*x**12 + c**3*x**15),x)*a**3*b**3*c*x**3 + 36*int((sqrt(a + b*
x**3 + c*x**6)*x**2)/(a**2*b + a**2*c*x**3 + 2*a*b**2*x**3 + 4*a*b*c*x*...
```

3.226 $\int \frac{1}{x^7(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 198

$$\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{(5b^2-12ac)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b(15b^2-52ac)\sqrt{a+bx^3+cx^6}}{12a^3\left(b^2-4ac\right)x^3} - \frac{(5b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}}$$

output
```
2/3*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^6/(c*x^6+b*x^3+a)^(1/2)-1/6*(-12*
a*c+5*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^6+1/12*b*(-52*a*c+15*b
^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/(-4*a*c+b^2)/x^3-1/8*(-4*a*c+5*b^2)*arctanh(
1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^7 (a+bx^3+cx^6)^{3/2}}\,dx = \frac{-8a^3c-15b^3x^6(b+cx^3)+2a^2(b^2+10bcx^3-12c^2x^6)+abx^3(-5b^2+62bcx}{12a^3(-b^2+4ac)\,x^6\sqrt{a+bx^3+cx^6}}$$
$$+\frac{(5b^2-4ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{4a^{7/2}}$$

input
```
Integrate[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
(-8*a^3*c - 15*b^3*x^6*(b + c*x^3) + 2*a^2*(b^2 + 10*b*c*x^3 - 12*c^2*x^6)
 + a*b*x^3*(-5*b^2 + 62*b*c*x^3 + 52*c^2*x^6))/(12*a^3*(-b^2 + 4*a*c)*x^6*
Sqrt[a + b*x^3 + c*x^6]) + ((5*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a
+ b*x^3 + c*x^6])/Sqrt[a]])/(4*a^(7/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1693, 1165, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^3+cx^6)^{3/2}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{1}{x^9 (cx^6+bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{1165}$$

$$\frac{1}{3}\left(\frac{2(-2ac+b^2+bcx^3)}{ax^6(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2\int -\frac{4bcx^3+5b^2-12ac}{2x^9\sqrt{cx^6+bx^3+a}}dx^3}{a(b^2-4ac)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{4bcx^3+5b^2-12ac}{x^9\sqrt{cx^6+bx^3+a}}dx^3}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^6\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

$\big\downarrow$ 1237

$$\frac{1}{3}\left(\frac{-\frac{\int \frac{2c\left(5b^2-12ac\right)x^3+b\left(15b^2-52ac\right)}{2x^6\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^6\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

$\big\downarrow$ 27

$$\frac{1}{3}\left(\frac{-\frac{\int \frac{2c\left(5b^2-12ac\right)x^3+b\left(15b^2-52ac\right)}{x^6\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^6\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

$\big\downarrow$ 1228

$$\frac{1}{3}\left(\frac{-\frac{-\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right.}{ax^6\left(b^2-4ac\right)\sqrt{a+bx}}$$

$\big\downarrow$ 1154

$$\frac{1}{3}\left(\frac{-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right.}{ax^6\left(b^2-4ac\right)\sqrt{a+bx}}$$

$\big\downarrow$ 219

$$\frac{1}{3}\left(\frac{-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2\right.}{ax^6\left(b^2-4ac\right)\sqrt{a}}$$

input
```
Int[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^3))/(a*(b^2 - 4*a*c)*x^6*Sqrt[a + b*x^3 + c*x^6])
 + (-1/2*((5*b^2 - 12*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) - (-((b*(15*b^
2 - 52*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^3)) + (3*(b^2 - 4*a*c)*(5*b^2 -
4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2*a^(3
/2)))/(4*a))/(a*(b^2 - 4*a*c)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{x^{7} \left(c \,x^{6}+b \,x^{3}+a \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 615, normalized size of antiderivative = 3.11

$$\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \left[-\frac{3\left((5b^4c-24ab^2c^2+16a^2c^3)x^{12}+(5b^5-24ab^3c+16a^2bc^2)x^9+(5ab^4\right.}{}\right.$$

input
```
integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*
c + 16*a^2*b*c^2)*x^9 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(a)
*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((15*a*b^3*c - 52*a^2*b*c^2)*x^9 + (15*a*b^
4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*a^3*b^2 + 8*a^4*c + 5*(a^2*b^3 - 4*
a^3*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^4*b^2*c - 4*a^5*c^2)*x^12 + (a^
4*b^3 - 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a^6*c)*x^6), 1/24*(3*((5*b^4*c - 24*
a*b^2*c^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^9 + (
5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 +
 b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b
^3*c - 52*a^2*b*c^2)*x^9 + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*
a^3*b^2 + 8*a^4*c + 5*(a^2*b^3 - 4*a^3*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/
((a^4*b^2*c - 4*a^5*c^2)*x^12 + (a^4*b^3 - 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a
^6*c)*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \int \frac{1}{x^7\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**7/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^7 (a + bx^3 + cx^6)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{1}{x^7 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^7} \, dx$$

input
```
integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{x^7 (c x^6 + b x^3 + a)^{3/2}} \, dx$$

input
```
int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x)
```

output
```
int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^7 (a + bx^3 + cx^6)^{3/2}} \, dx = \text{too large to display}$$

input
```
int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
( - 256*sqrt(a + b*x**3 + c*x**6)*a**6*c**4*x**3 - 160*sqrt(a + b*x**3 + c
*x**6)*a**5*b**3*c**2 + 384*sqrt(a + b*x**3 + c*x**6)*a**5*b**2*c**3*x**3
- 256*sqrt(a + b*x**3 + c*x**6)*a**5*b*c**4*x**6 - 512*sqrt(a + b*x**3 + c
*x**6)*a**5*c**5*x**9 - 200*sqrt(a + b*x**3 + c*x**6)*a**4*b**5*c - 160*sq
rt(a + b*x**3 + c*x**6)*a**4*b**4*c**2*x**3 - 640*sqrt(a + b*x**3 + c*x**6
)*a**4*b**3*c**3*x**6 + 640*sqrt(a + b*x**3 + c*x**6)*a**4*b**2*c**4*x**9
+ 60*sqrt(a + b*x**3 + c*x**6)*a**3*b**7 + 1220*sqrt(a + b*x**3 + c*x**6)*
a**3*b**6*c*x**3 - 80*sqrt(a + b*x**3 + c*x**6)*a**3*b**5*c**2*x**6 - 160*
sqrt(a + b*x**3 + c*x**6)*a**3*b**4*c**3*x**9 - 300*sqrt(a + b*x**3 + c*x*
*6)*a**2*b**8*x**3 + 3060*sqrt(a + b*x**3 + c*x**6)*a**2*b**7*c*x**6 + 240
0*sqrt(a + b*x**3 + c*x**6)*a**2*b**6*c**2*x**9 - 450*sqrt(a + b*x**3 + c*
x**6)*a*b**9*x**6 - 480*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*a
**4*b**3*c**3*x**6 - 480*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*
a**3*b**4*c**3*x**9 - 480*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))
*a**3*b**3*c**4*x**12 + 930*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a
))*a**2*b**7*c*x**6 - 225*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))
*a*b**9*x**6 + 930*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*a*b**8
*c*x**9 + 930*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*a*b**7*c**2
*x**12 - 225*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*b**10*x**9 -
 225*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sqrt(a))*b**9*c*x**12 + 48...
```

3.227 $\int \frac{1}{x^{10}(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 256

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{(7b^2-16ac)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b(35b^2-116ac)\sqrt{a+bx^3+cx^6}}{36a^3\left(b^2-4ac\right)x^6} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{a+bx^3+cx^6}}{72a^4\left(b^2-4ac\right)x^3} + \frac{5b(7b^2-12ac)\,\mathrm{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{9/2}}$$

output

```
2/3*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^9/(c*x^6+b*x^3+a)^(1/2)-1/9*(-16*
a*c+7*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^9+1/36*b*(-116*a*c+35*
b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/(-4*a*c+b^2)/x^6-1/72*(256*a^2*c^2-460*a*b^
2*c+105*b^4)*(c*x^6+b*x^3+a)^(1/2)/a^4/(-4*a*c+b^2)/x^3+5/48*b*(-12*a*c+7*
b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{10} (a + bx^3 + cx^6)^{3/2}} dx = \frac{-32a^4c + 105b^4x^9(b + cx^3) + 5ab^2x^6(7b^2 - 106bcx^3 - 92c^2x^6) + 8a^3(b^2 + 7}{72a^4 (-b^2 + 4ac) x^9 \sqrt{a}}$$
$$+ \frac{5b(-7b^2 + 12ac) \operatorname{arctanh}\left(\frac{\sqrt{c}x^3 - \sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{9/2}}$$

input
```
Integrate[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
(-32*a^4*c + 105*b^4*x^9*(b + c*x^3) + 5*a*b^2*x^6*(7*b^2 - 106*b*c*x^3 -
92*c^2*x^6) + 8*a^3*(b^2 + 7*b*c*x^3 + 16*c^2*x^6) + 2*a^2*x^3*(-7*b^3 - 8
6*b^2*c*x^3 + 244*b*c^2*x^6 + 128*c^3*x^9))/(72*a^4*(-b^2 + 4*a*c)*x^9*Sqr
t[a + b*x^3 + c*x^6]) + (5*b*(-7*b^2 + 12*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt
[a + b*x^3 + c*x^6])/Sqrt[a]])/(24*a^(9/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1693, 1165, 27, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10} (a + bx^3 + cx^6)^{3/2}} dx$$
$$\Big\downarrow \text{ 1693}$$
$$\frac{1}{3} \int \frac{1}{x^{12} (cx^6 + bx^3 + a)^{3/2}} dx^3$$
$$\Big\downarrow \text{ 1165}$$
$$\frac{1}{3} \left(\frac{2(-2ac + b^2 + bcx^3)}{ax^9 (b^2 - 4ac) \sqrt{a + bx^3 + cx^6}} - \frac{2 \int -\frac{6bcx^3 + 7b^2 - 16ac}{2x^{12}\sqrt{cx^6 + bx^3 + a}} dx^3}{a (b^2 - 4ac)} \right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{\int \frac{6bcx^3+7b^2-16ac}{x^{12}\sqrt{cx^6+bx^3+a}}dx^3}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^9\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{1}{3}\left(\frac{-\frac{\int \frac{4c\left(7b^2-16ac\right)x^3+b\left(35b^2-116ac\right)}{2x^9\sqrt{cx^6+bx^3+a}}dx^3}{3a}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{3ax^9}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^9\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{-\frac{\int \frac{4c\left(7b^2-16ac\right)x^3+b\left(35b^2-116ac\right)}{x^9\sqrt{cx^6+bx^3+a}}dx^3}{6a}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{3ax^9}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^3\right)}{ax^9\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}\right)$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{\int \frac{105b^4-460acb^2+2c\left(35b^2-116ac\right)x^3b+256a^2c^2}{2x^6\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{3ax^9}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2\right.}{ax^9\left(b^2-4ac\right)\sqrt{}}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{\int \frac{105b^4-460acb^2+2c\left(35b^2-116ac\right)x^3b+256a^2c^2}{x^6\sqrt{cx^6+bx^3+a}}dx^3}{4a}-\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{3ax^9}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2\right.}{ax^9\left(b^2-4ac\right)\sqrt{}}\right.$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{-\frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)\int \frac{1}{x^3\sqrt{cx^6+bx^3+a}}dx^3}{2a}-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{b\left(35b^2-116ac\right)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{\left(7b^2-16ac\right)\sqrt{}}{3ax}}{a\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{\frac{15b(7b^2-12ac)(b^2-4ac)\int \frac{1}{4a-x^6}d\frac{bx^3+2a}{\sqrt{cx^6+bx^3+a}}}{a}-\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{b(35b^2-116ac)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{(7b^2-16ac)\sqrt{}}{3ax}}{a(b^2-4ac)}\right.$$

$\downarrow$ 219

$$\frac{1}{3}\left(-\frac{-\frac{-\frac{\frac{15b(7b^2-12ac)(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{3/2}}-\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{a+bx^3+cx^6}}{ax^3}}{4a}-\frac{b(35b^2-116ac)\sqrt{a+bx^3+cx^6}}{2ax^6}}{6a}-\frac{(7b^2-1}{}}{a(b^2-4ac)}\right.$$

input
```
Int[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^3))/(a*(b^2 - 4*a*c)*x^9*Sqrt[a + b*x^3 + c*x^6])
 + (-1/3*((7*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^9) - (-1/2*(b*(35
*b^2 - 116*a*c)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) - (-(((105*b^4 - 460*a*b^
2*c + 256*a^2*c^2)*Sqrt[a + b*x^3 + c*x^6])/(a*x^3)) + (15*b*(7*b^2 - 12*a
*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]
)])/(2*a^(3/2)))/(4*a))/(6*a))/(a*(b^2 - 4*a*c)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{x^{10} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^{10} \left(a+bx^3+cx^6\right)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/288*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*
b^4*c + 48*a^2*b^2*c^2)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9
)*sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*
(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((105*a*b^4*c - 460*a^2*b^2*c^2 +
256*a^3*c^3)*x^12 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*
a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^
3*b^3 - 4*a^4*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^5*b^2*c - 4*a^6*c^2)*
x^15 + (a^5*b^3 - 4*a^6*b*c)*x^12 + (a^6*b^2 - 4*a^7*c)*x^9), -1/144*(15*(
(7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b^4*c + 48*a^
2*b^2*c^2)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*sqrt(-a)*ar
ctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3
 + a^2)) + 2*((105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^12 + (105*a*
b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*a^2*b^4 - 172*a^3*b^2*c + 1
28*a^4*c^2)*x^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^3*b^3 - 4*a^4*b*c)*x^3)*sqr
t(c*x^6 + b*x^3 + a))/((a^5*b^2*c - 4*a^6*c^2)*x^15 + (a^5*b^3 - 4*a^6*b*c
)*x^12 + (a^6*b^2 - 4*a^7*c)*x^9)]
```

Sympy [F]

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**10/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(1/(x**10*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{x^{10}\,\left(c\,x^6+b\,x^3+a\right)^{3/2}} \,dx$$

input
```
int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x)
```

output
```
int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\text{too large to display}$$

input
```
int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
( - 3072*sqrt(a + b*x**3 + c*x**6)*a**7*c**4 - 3072*sqrt(a + b*x**3 + c*x*
*6)*a**6*b**2*c**3 - 6144*sqrt(a + b*x**3 + c*x**6)*a**6*b*c**4*x**3 + 353
28*sqrt(a + b*x**3 + c*x**6)*a**6*c**5*x**6 + 5440*sqrt(a + b*x**3 + c*x**
6)*a**5*b**4*c**2 + 11136*sqrt(a + b*x**3 + c*x**6)*a**5*b**3*c**3*x**3 +
16128*sqrt(a + b*x**3 + c*x**6)*a**5*b**2*c**4*x**6 + 87168*sqrt(a + b*x**
3 + c*x**6)*a**5*b*c**5*x**9 + 36096*sqrt(a + b*x**3 + c*x**6)*a**5*c**6*x
**12 - 1120*sqrt(a + b*x**3 + c*x**6)*a**4*b**6*c + 12160*sqrt(a + b*x**3
+ c*x**6)*a**4*b**5*c**2*x**3 - 92960*sqrt(a + b*x**3 + c*x**6)*a**4*b**4*
c**3*x**6 + 63840*sqrt(a + b*x**3 + c*x**6)*a**4*b**3*c**4*x**9 + 70080*sq
rt(a + b*x**3 + c*x**6)*a**4*b**2*c**5*x**12 - 15400*sqrt(a + b*x**3 + c*x
**6)*a**3*b**7*c*x**3 + 8800*sqrt(a + b*x**3 + c*x**6)*a**3*b**6*c**2*x**6
 - 217040*sqrt(a + b*x**3 + c*x**6)*a**3*b**5*c**3*x**9 - 108640*sqrt(a +
b*x**3 + c*x**6)*a**3*b**4*c**4*x**12 + 2940*sqrt(a + b*x**3 + c*x**6)*a**
2*b**9*x**3 + 32620*sqrt(a + b*x**3 + c*x**6)*a**2*b**8*c*x**6 + 27020*sqr
t(a + b*x**3 + c*x**6)*a**2*b**7*c**2*x**9 - 139160*sqrt(a + b*x**3 + c*x*
*6)*a**2*b**6*c**3*x**12 - 7350*sqrt(a + b*x**3 + c*x**6)*a*b**10*x**6 + 3
6750*sqrt(a + b*x**3 + c*x**6)*a*b**9*c*x**9 + 102900*sqrt(a + b*x**3 + c*
x**6)*a*b**8*c**2*x**12 + 34560*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sq
rt(a))*a**5*b*c**5*x**9 + 14400*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6) - sq
rt(a))*a**4*b**3*c**4*x**9 + 34560*sqrt(a)*log(sqrt(a + b*x**3 + c*x**6...
```

3.228 $\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(\frac{4}{3},\frac{3}{2},\frac{3}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.}{4a\sqrt{a+bx^3+cx^6}}$$

output
```
1/4*x^4*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)*AppellF1(4/3,3/2,3/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-
2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/a/(c*x^6+b*x^3+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 340 vs. 2(143) = 286.

Time = 10.40 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.38

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{x\left(-2(b+2cx^3)+2b\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{}{b}\right.\right.}{}$$

input
```
Integrate[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x*(-2*(b + 2*c*x^3) + 2*b*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(3*(b^2 - 4*a*c)*Sqrt[a
 + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}} \, dx$$
$$\Big\downarrow \text{1721}$$
$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{x^3}{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3+cx^6}}$$
$$\Big\downarrow \text{1012}$$
$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{4}{3},\frac{3}{2},\frac{3}{2},\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

input
```
Int[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[4/3, 3/2, 3/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^3 + c*x^6
])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x^3}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}dx$$

input
```
int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx=\int \frac{x^3}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)*x^3/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)
*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^3}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x**3/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^3}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x^3}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx=\int \frac{x^3}{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}\,dx$$

input
```
int(x^3/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x^3/(a + b*x^3 + c*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^3+cx^6)^{3/2}}\,dx=\int \frac{\sqrt{c\,x^6+b\,x^3+a}\,x^3}{c^2x^{12}+2bc\,x^9+2ac\,x^6+b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int((sqrt(a + b*x**3 + c*x**6)*x**3)/(a**2 + 2*a*b*x**3 + 2*a*c*x**6 + b**
2*x**6 + 2*b*c*x**9 + c**2*x**12),x)
```

3.229 $\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 143

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx=\frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.}{2a\sqrt{a+bx^3+cx^6}}$$

output
```
1/2*x^2*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)*AppellF1(2/3,3/2,3/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-
2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/a/(c*x^6+b*x^3+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 362 vs. 2(143) = 286.

Time = 10.69 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.53

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx=\frac{x^2\left(-20(b^2-2ac+bcx^3)+5(b^2+4ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{Appe}\right.}{}$$

input
```
Integrate[x/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x^2*(-20*(b^2 - 2*a*c + b*c*x^3) + 5*(b^2 + 4*a*c)*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 8*b*c*x^3*Sqr
t[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2,
 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c
])]))/(30*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+bx^3+cx^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{x}{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

input
```
Int[x/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[2/3, 3/2, 3/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^3 + c*x^6
])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x}{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(x/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{x}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{x}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)*x/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x
^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(x/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{x}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

input
```
int(x/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(x/(a + b*x^3 + c*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^6+b\,x^3+a}\,x}{c^2x^{12}+2bc\,x^9+2ac\,x^6+b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(x/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int((sqrt(a + b*x**3 + c*x**6)*x)/(a**2 + 2*a*b*x**3 + 2*a*c*x**6 + b**2*x
**6 + 2*b*c*x**9 + c**2*x**12),x)
```

3.230 $\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 138

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx=\frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

output
```
x*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)*AppellF1(1/3,3/2,3/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
3/(b+(-4*a*c+b^2)^(1/2)))/a/(c*x^6+b*x^3+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 359 vs. 2(138) = 276.

Time = 10.57 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.60

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx=\frac{x\Big(-4(b^2-2ac+bcx^3)-2(b^2-8ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF}}{}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^(-3/2),x]
```

output
```
(x*(-4*(b^2 - 2*a*c + b*c*x^3) - 2*(b^2 - 8*a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + b*c*x^3*Sqrt[(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))
/(6*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1686}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

input
```
Int[(a + b*x^3 + c*x^6)^(-3/2),x]
```

output
```
(x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[1/3, 3/2, 3/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^3 + c*x^6])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^{6}+bx^{3}+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6
 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral((a + b*x**3 + c*x**6)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx=\int \frac{1}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

input
```
int(1/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int(1/(a + b*x^3 + c*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx=\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{c^2x^{12}+2bc\,x^9+2ac\,x^6+b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(1/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**3 + c*x**6)/(a**2 + 2*a*b*x**3 + 2*a*c*x**6 + b**2*x**6
+ 2*b*c*x**9 + c**2*x**12),x)
```

3.231 $\int \frac{1}{x^2(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int \frac{1}{x^2(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

output
```
-(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)
))^(1/2)*AppellF1(-1/3,3/2,3/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
3/(b+(-4*a*c+b^2)^(1/2)))/a/x/(c*x^6+b*x^3+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 407 vs. 2(141) = 282.

Time = 10.91 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.89

$$\int \frac{1}{x^2(a+bx^3+cx^6)^{3/2}}\,dx = -\frac{5b(-5b^2+12ac)\,x^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)-4(\cdots}{\cdots}$$

input
```
Integrate[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
-1/60*(5*b*(-5*b^2 + 12*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 4*(60*a^2*c - 25*b^2*x^3*(b + c*
x^3) + 5*a*(-3*b^2 + 18*b*c*x^3 + 16*c^2*x^6) + 2*c*(5*b^2 - 16*a*c)*x^6*S
qrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/
2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a
*c])]))/(a^2*(b^2 - 4*a*c)*x*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3+cx^6)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^2\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

input
```
Int[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
-((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-1/3, 3/2, 3/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 -
4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*x^3 + c*x^6]
))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^14 + 2*b*c*x^11 + (b^2 + 2*a*c)*x^
8 + 2*a*b*x^5 + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{x^2 (a + bx^3 + cx^6)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^3 + cx^6)^{3/2}} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*x^3 + c*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 \left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{-2\sqrt{c\,x^6+b\,x^3+a}-8\left(\int \frac{\sqrt{c\,x^6+b\,x^3+a}\,x^4}{c^2x^{12}+2bc\,x^9+2ac\,x^6+b^2x^6+2ab\,x^3+a^2}dx\right)acx-8\left(\int \frac{}{c^2x^1}\right.}{}$$

input
```
int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**3 + c*x**6) - 8*int((sqrt(a + b*x**3 + c*x**6)*x**4)/(
a**2 + 2*a*b*x**3 + 2*a*c*x**6 + b**2*x**6 + 2*b*c*x**9 + c**2*x**12),x)*a
*c*x - 8*int((sqrt(a + b*x**3 + c*x**6)*x**4)/(a**2 + 2*a*b*x**3 + 2*a*c*x
**6 + b**2*x**6 + 2*b*c*x**9 + c**2*x**12),x)*b*c*x**4 - 8*int((sqrt(a + b
*x**3 + c*x**6)*x**4)/(a**2 + 2*a*b*x**3 + 2*a*c*x**6 + b**2*x**6 + 2*b*c*
x**9 + c**2*x**12),x)*c**2*x**7 - 5*int((sqrt(a + b*x**3 + c*x**6)*x)/(a**
2 + 2*a*b*x**3 + 2*a*c*x**6 + b**2*x**6 + 2*b*c*x**9 + c**2*x**12),x)*a*b*
x - 5*int((sqrt(a + b*x**3 + c*x**6)*x)/(a**2 + 2*a*b*x**3 + 2*a*c*x**6 +
b**2*x**6 + 2*b*c*x**9 + c**2*x**12),x)*b**2*x**4 - 5*int((sqrt(a + b*x**3
 + c*x**6)*x)/(a**2 + 2*a*b*x**3 + 2*a*c*x**6 + b**2*x**6 + 2*b*c*x**9 + c
**2*x**12),x)*b*c*x**7)/(2*a*x*(a + b*x**3 + c*x**6))
```

3.232 $\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

output
```
-1/2*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(
1/2)))^(1/2)*AppellF1(-2/3,3/2,3/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*
c*x^3/(b+(-4*a*c+b^2)^(1/2)))/a/x^2/(c*x^6+b*x^3+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 405 vs. 2(143) = 286.

Time = 10.74 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{-48a^2c+28b^2x^3(b+cx^3)+4a(3b^2-24bcx^3-20c^2x^6)+2b(7b^2-36ac)\,x^3}{}$$

input
```
Integrate[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
(-48*a^2*c + 28*b^2*x^3*(b + c*x^3) + 4*a*(3*b^2 - 24*b*c*x^3 - 20*c^2*x^6
) + 2*b*(7*b^2 - 36*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 -
4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*(-7*b^2 + 20*a*c)*x^6*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2
*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(24*
a^2*(-b^2 + 4*a*c)*x^2*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^3\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{2}{3},\frac{3}{2},\frac{3}{2},\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

input
```
Int[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[-2/3, 3/2, 3/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2
 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*x^2*Sqrt[a + b*x^3 + c
*x^6])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{1}{x^{3} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^15 + 2*b*c*x^12 + (b^2 + 2*a*c)*x^
9 + 2*a*b*x^6 + a^2*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^6+bx^3+a\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{(cx^6+bx^3+a)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{1}{x^3\,(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3 + c*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^3+cx^6)^{3/2}}\,dx = \frac{-2\sqrt{c\,x^6+b\,x^3+a}-7\left(\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{c^2x^{12}+2bc\,x^9+2ac\,x^6+b^2x^6+2ab\,x^3+a^2}dx\right)ab\,x^2-7\left(\int \frac{}{c^2}\right.}{}$$

input
```
int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**3 + c*x**6) - 7*int(sqrt(a + b*x**3 + c*x**6)/(a**2 +
2*a*b*x**3 + 2*a*c*x**6 + b**2*x**6 + 2*b*c*x**9 + c**2*x**12),x)*a*b*x**2
 - 7*int(sqrt(a + b*x**3 + c*x**6)/(a**2 + 2*a*b*x**3 + 2*a*c*x**6 + b**2*
x**6 + 2*b*c*x**9 + c**2*x**12),x)*b**2*x**5 - 7*int(sqrt(a + b*x**3 + c*x
**6)/(a**2 + 2*a*b*x**3 + 2*a*c*x**6 + b**2*x**6 + 2*b*c*x**9 + c**2*x**12
),x)*b*c*x**8 - 10*int((sqrt(a + b*x**3 + c*x**6)*x**3)/(a**2 + 2*a*b*x**3
 + 2*a*c*x**6 + b**2*x**6 + 2*b*c*x**9 + c**2*x**12),x)*a*c*x**2 - 10*int(
(sqrt(a + b*x**3 + c*x**6)*x**3)/(a**2 + 2*a*b*x**3 + 2*a*c*x**6 + b**2*x*
*6 + 2*b*c*x**9 + c**2*x**12),x)*b*c*x**5 - 10*int((sqrt(a + b*x**3 + c*x*
*6)*x**3)/(a**2 + 2*a*b*x**3 + 2*a*c*x**6 + b**2*x**6 + 2*b*c*x**9 + c**2*
x**12),x)*c**2*x**8)/(4*a*x**2*(a + b*x**3 + c*x**6))
```

3.233 $\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{4+m}}{d^4(4+m)}+\frac{\left(b^2+2ac\right)(dx)^{7+m}}{d^7(7+m)} +\frac{2bc(dx)^{10+m}}{d^{10}(10+m)}+\frac{c^2(dx)^{13+m}}{d^{13}(13+m)}$$

output
```
a^2*(d*x)^(1+m)/d/(1+m)+2*a*b*(d*x)^(4+m)/d^4/(4+m)+(2*a*c+b^2)*(d*x)^(7+m
)/d^7/(7+m)+2*b*c*(d*x)^(10+m)/d^10/(10+m)+c^2*(d*x)^(13+m)/d^13/(13+m)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = x(dx)^m \left(\frac{a^2}{1+m}+\frac{2abx^3}{4+m}+\frac{\left(b^2+2ac\right)x^6}{7+m}+\frac{2bcx^9}{10+m} +\frac{c^2x^{12}}{13+m}\right)$$

input
```
Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

output
```
x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^3)/(4 + m) + ((b^2 + 2*a*c)*x^6)/(7 + m)
 + (2*b*c*x^9)/(10 + m) + (c^2*x^12)/(13 + m))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx$$

$$\Big\downarrow \text{1691}$$

$$\int \left(a^2(dx)^m+\frac{\left(2ac+b^2\right)(dx)^{m+6}}{d^6}+\frac{2ab(dx)^{m+3}}{d^3}+\frac{2bc(dx)^{m+9}}{d^9}+\frac{c^2(dx)^{m+12}}{d^{12}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{2ab(dx)^{m+4}}{d^4(m+4)}+\frac{2bc(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

input
```
Int[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

output
```
(a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(4 + m))/(d^4*(4 + m)) + ((
b^2 + 2*a*c)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*c*(d*x)^(10 + m))/(d^10*(
10 + m)) + (c^2*(d*x)^(13 + m))/(d^13*(13 + m))
```

Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(101) = 202$.

Time = 0.12 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.98

method	result
gosper	$\frac{x(c^2m^4x^{12}+22c^2m^3x^{12}+159c^2m^2x^{12}+2bc\,m^4x^9+418m\,x^{12}c^2+50bc\,m^3x^9+280c^2x^{12}+390bc\,m^2x^9+2ac\,m^4x^6+b^2m^4x^6+}{}$
risch	$\frac{x(c^2m^4x^{12}+22c^2m^3x^{12}+159c^2m^2x^{12}+2bc\,m^4x^9+418m\,x^{12}c^2+50bc\,m^3x^9+280c^2x^{12}+390bc\,m^2x^9+2ac\,m^4x^6+b^2m^4x^6+}{}$
orering	$\frac{x(c^2m^4x^{12}+22c^2m^3x^{12}+159c^2m^2x^{12}+2bc\,m^4x^9+418m\,x^{12}c^2+50bc\,m^3x^9+280c^2x^{12}+390bc\,m^2x^9+2ac\,m^4x^6+b^2m^4x^6+}{}$
parallelrisch	$\frac{2x^{10}(dx)^m bc\,m^4+50x^{10}(dx)^m bc\,m^3+390x^{10}(dx)^m bc\,m^2+1070x^{10}(dx)^m bcm+2x^7(dx)^m ac\,m^4+56x^7(dx)^m ac\,m^3+498x^7(}{}$

input
```
int((d*x)^m*(c*x^6+b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(c^2*m^4*x^12+22*c^2*m^3*x^12+159*c^2*m^2*x^12+2*b*c*m^4*x^9+418*c^2*m*x
^12+50*b*c*m^3*x^9+280*c^2*x^12+390*b*c*m^2*x^9+2*a*c*m^4*x^6+b^2*m^4*x^6+
1070*b*c*m*x^9+56*a*c*m^3*x^6+28*b^2*m^3*x^6+728*b*c*x^9+498*a*c*m^2*x^6+2
49*b^2*m^2*x^6+2*a*b*m^4*x^3+1484*a*c*m*x^6+742*b^2*m*x^6+62*a*b*m^3*x^3+1
040*a*c*x^6+520*b^2*x^6+642*a*b*m^2*x^3+a^2*m^4+2402*a*b*m*x^3+34*a^2*m^3+
1820*a*b*x^3+411*a^2*m^2+2074*a^2*m+3640*a^2)*(d*x)^m/(13+m)/(10+m)/(7+m)/
(4+m)/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(101) = 202$.

Time = 0.08 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.39

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = \frac{((c^2m^4+22\,c^2m^3+159\,c^2m^2+418\,c^2m+280\,c^2)x^{13}+2\,(bcm^4+25\,bcm^3+195\,bcm^2+535\,bcm+364}{}$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="fricas")
```

output
```
((c^2*m^4 + 22*c^2*m^3 + 159*c^2*m^2 + 418*c^2*m + 280*c^2)*x^13 + 2*(b*c*
m^4 + 25*b*c*m^3 + 195*b*c*m^2 + 535*b*c*m + 364*b*c)*x^10 + ((b^2 + 2*a*c
)*m^4 + 28*(b^2 + 2*a*c)*m^3 + 249*(b^2 + 2*a*c)*m^2 + 520*b^2 + 1040*a*c
+ 742*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 31*a*b*m^3 + 321*a*b*m^2 + 1201*
a*b*m + 910*a*b)*x^4 + (a^2*m^4 + 34*a^2*m^3 + 411*a^2*m^2 + 2074*a^2*m +
3640*a^2)*x)*(d*x)^m/(m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1459 vs. $2(90) = 180$.

Time = 0.97 (sec) , antiderivative size = 1459, normalized size of antiderivative = 14.45

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(c*x**6+b*x**3+a)**2,x)
```

output
```
Piecewise(((-a**2/(12*x**12) - 2*a*b/(9*x**9) - a*c/(3*x**6) - b**2/(6*x**
6) - 2*b*c/(3*x**3) + c**2*log(x))/d**13, Eq(m, -13)), ((-a**2/(9*x**9) -
a*b/(3*x**6) - 2*a*c/(3*x**3) - b**2/(3*x**3) + 2*b*c*log(x) + c**2*x**3/3
)/d**10, Eq(m, -10)), ((-a**2/(6*x**6) - 2*a*b/(3*x**3) + 2*a*c*log(x) + b
**2*log(x) + 2*b*c*x**3/3 + c**2*x**6/6)/d**7, Eq(m, -7)), ((-a**2/(3*x**3
) + 2*a*b*log(x) + 2*a*c*x**3/3 + b**2*x**3/3 + b*c*x**6/3 + c**2*x**9/9)/
d**4, Eq(m, -4)), ((a**2*log(x) + 2*a*b*x**3/3 + a*c*x**6/3 + b**2*x**6/6
+ 2*b*c*x**9/9 + c**2*x**12/12)/d, Eq(m, -1)), (a**2*m**4*x*(d*x)**m/(m**5
 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 34*a**2*m**3*x*(d*x)*
*m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 411*a**2*m**2
*x*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2074
*a**2*m*x*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640)
 + 3640*a**2*x*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
3640) + 2*a*b*m**4*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 +
5714*m + 3640) + 62*a*b*m**3*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 24
85*m**2 + 5714*m + 3640) + 642*a*b*m**2*x**4*(d*x)**m/(m**5 + 35*m**4 + 44
5*m**3 + 2485*m**2 + 5714*m + 3640) + 2402*a*b*m*x**4*(d*x)**m/(m**5 + 35*
m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 1820*a*b*x**4*(d*x)**m/(m**
5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2*a*c*m**4*x**7*(d*x
)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 56*a*c*m...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int (dx)^m \left(a + bx^3 + cx^6\right)^2 \, dx = \frac{c^2 d^m x^{13} x^m}{m+13} + \frac{2\, bcd^m x^{10} x^m}{m+10} + \frac{b^2 d^m x^7 x^m}{m+7} + \frac{2\, acd^m x^7 x^m}{m+7} + \frac{2\, abd^m x^4 x^m}{m+4} + \frac{(dx)^{m+1} a^2}{d(m+1)}$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="maxima")
```

output
```
c^2*d^m*x^13*x^m/(m + 13) + 2*b*c*d^m*x^10*x^m/(m + 10) + b^2*d^m*x^7*x^m/
(m + 7) + 2*a*c*d^m*x^7*x^m/(m + 7) + 2*a*b*d^m*x^4*x^m/(m + 4) + (d*x)^(m
 + 1)*a^2/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(101) = 202$.

Time = 0.12 (sec) , antiderivative size = 449, normalized size of antiderivative = 4.45

$$\int (dx)^m \left(a + bx^3 + cx^6\right)^2 \, dx$$
$$= \frac{(dx)^m c^2 m^4 x^{13} + 22 (dx)^m c^2 m^3 x^{13} + 159 (dx)^m c^2 m^2 x^{13} + 2 (dx)^m bcm^4 x^{10} + 418 (dx)^m c^2 m x^{13} + 50 (d}{}$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="giac")
```

output
```
((d*x)^m*c^2*m^4*x^13 + 22*(d*x)^m*c^2*m^3*x^13 + 159*(d*x)^m*c^2*m^2*x^13
 + 2*(d*x)^m*b*c*m^4*x^10 + 418*(d*x)^m*c^2*m*x^13 + 50*(d*x)^m*b*c*m^3*x^
10 + 280*(d*x)^m*c^2*x^13 + 390*(d*x)^m*b*c*m^2*x^10 + (d*x)^m*b^2*m^4*x^7
 + 2*(d*x)^m*a*c*m^4*x^7 + 1070*(d*x)^m*b*c*m*x^10 + 28*(d*x)^m*b^2*m^3*x^
7 + 56*(d*x)^m*a*c*m^3*x^7 + 728*(d*x)^m*b*c*x^10 + 249*(d*x)^m*b^2*m^2*x^
7 + 498*(d*x)^m*a*c*m^2*x^7 + 2*(d*x)^m*a*b*m^4*x^4 + 742*(d*x)^m*b^2*m*x^
7 + 1484*(d*x)^m*a*c*m*x^7 + 62*(d*x)^m*a*b*m^3*x^4 + 520*(d*x)^m*b^2*x^7
+ 1040*(d*x)^m*a*c*x^7 + 642*(d*x)^m*a*b*m^2*x^4 + (d*x)^m*a^2*m^4*x + 240
2*(d*x)^m*a*b*m*x^4 + 34*(d*x)^m*a^2*m^3*x + 1820*(d*x)^m*a*b*x^4 + 411*(d
*x)^m*a^2*m^2*x + 2074*(d*x)^m*a^2*m*x + 3640*(d*x)^m*a^2*x)/(m^5 + 35*m^4
 + 445*m^3 + 2485*m^2 + 5714*m + 3640)
```

Mupad [B] (verification not implemented)

Time = 20.66 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.57

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = (d\,x)^m \left(\frac{c^2\,x^{13}\,\left(m^4+22\,m^3+159\,m^2+418\,m+280\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640} + \frac{x^7\,\left(b^2+2\,a\,c\right)\,\left(m^4+28\,m^3+249\,m^2+742\,m+520\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640} + \frac{a^2\,x\,\left(m^4+34\,m^3+411\,m^2+2074\,m+3640\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640} + \frac{2\,a\,b\,x^4\,\left(m^4+31\,m^3+321\,m^2+1201\,m+910\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640} + \frac{2\,b\,c\,x^{10}\,\left(m^4+25\,m^3+195\,m^2+535\,m+364\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}\right)$$

input
```
int((d*x)^m*(a + b*x^3 + c*x^6)^2,x)
```

output
```
(d*x)^m*((c^2*x^13*(418*m + 159*m^2 + 22*m^3 + m^4 + 280))/(5714*m + 2485*
m^2 + 445*m^3 + 35*m^4 + m^5 + 3640) + (x^7*(2*a*c + b^2)*(742*m + 249*m^2
 + 28*m^3 + m^4 + 520))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640
) + (a^2*x*(2074*m + 411*m^2 + 34*m^3 + m^4 + 3640))/(5714*m + 2485*m^2 +
445*m^3 + 35*m^4 + m^5 + 3640) + (2*a*b*x^4*(1201*m + 321*m^2 + 31*m^3 + m
^4 + 910))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640) + (2*b*c*x^
10*(535*m + 195*m^2 + 25*m^3 + m^4 + 364))/(5714*m + 2485*m^2 + 445*m^3 +
35*m^4 + m^5 + 3640))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.98

$$\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx = \frac{x^m d^m x(c^2m^4x^{12} + 22c^2m^3x^{12} + 159c^2m^2x^{12} + 2bc\,m^4x^9 + 418c^2m\,x^{12} + 50bc\,m^3x^9 + 280c^2x^{12} + 390bc}{}$$

input
```
int((d*x)^m*(c*x^6+b*x^3+a)^2,x)
```

output
```
(x**m*d**m*x*(a**2*m**4 + 34*a**2*m**3 + 411*a**2*m**2 + 2074*a**2*m + 364
0*a**2 + 2*a*b*m**4*x**3 + 62*a*b*m**3*x**3 + 642*a*b*m**2*x**3 + 2402*a*b
*m*x**3 + 1820*a*b*x**3 + 2*a*c*m**4*x**6 + 56*a*c*m**3*x**6 + 498*a*c*m**
2*x**6 + 1484*a*c*m*x**6 + 1040*a*c*x**6 + b**2*m**4*x**6 + 28*b**2*m**3*x
**6 + 249*b**2*m**2*x**6 + 742*b**2*m*x**6 + 520*b**2*x**6 + 2*b*c*m**4*x*
*9 + 50*b*c*m**3*x**9 + 390*b*c*m**2*x**9 + 1070*b*c*m*x**9 + 728*b*c*x**9
 + c**2*m**4*x**12 + 22*c**2*m**3*x**12 + 159*c**2*m**2*x**12 + 418*c**2*m
*x**12 + 280*c**2*x**12))/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m
+ 3640)
```

3.234 $\int (dx)^m \left(a+bx^3+cx^6\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int (dx)^m \left(a+bx^3+cx^6\right) \, dx=\frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{4+m}}{d^4(4+m)}+\frac{c(dx)^{7+m}}{d^7(7+m)}$$

output
```
a*(d*x)^(1+m)/d/(1+m)+b*(d*x)^(4+m)/d^4/(4+m)+c*(d*x)^(7+m)/d^7/(7+m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int (dx)^m \left(a+bx^3+cx^6\right) \, dx=x(dx)^m \left(\frac{a}{1+m}+\frac{bx^3}{4+m}+\frac{cx^6}{7+m}\right)$$

input
```
Integrate[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

output
```
x*(d*x)^m*(a/(1 + m) + (b*x^3)/(4 + m) + (c*x^6)/(7 + m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^3+cx^6\right) \, dx$$

$$\Big\downarrow \ 1691$$

$$\int \left(a(dx)^m+\frac{b(dx)^{m+3}}{d^3}+\frac{c(dx)^{m+6}}{d^6}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+7}}{d^7(m+7)}$$

input
```
Int[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

output
```
(a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(4 + m))/(d^4*(4 + m)) + (c*(d*x)
^(7 + m))/(d^7*(7 + m))
```

Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

method	result
norman	$\frac{a x\,\mathrm{e}^{m \ln(dx)}}{1+m}+\frac{b\,x^{4} \mathrm{e}^{m \ln(dx)}}{4+m}+\frac{c\,x^{7} \mathrm{e}^{m \ln(dx)}}{7+m}$
gosper	$\frac{x\left(c\,m^{2} x^{6}+5 c m\,x^{6}+4 c\,x^{6}+b\,m^{2} x^{3}+8 b m\,x^{3}+7 b\,x^{3}+a\,m^{2}+11 a m+28 a\right)\left(dx\right)^{m}}{(7+m)(4+m)(1+m)}$
risch	$\frac{x\left(c\,m^{2} x^{6}+5 c m\,x^{6}+4 c\,x^{6}+b\,m^{2} x^{3}+8 b m\,x^{3}+7 b\,x^{3}+a\,m^{2}+11 a m+28 a\right)\left(dx\right)^{m}}{(7+m)(4+m)(1+m)}$
orering	$\frac{x\left(c\,m^{2} x^{6}+5 c m\,x^{6}+4 c\,x^{6}+b\,m^{2} x^{3}+8 b m\,x^{3}+7 b\,x^{3}+a\,m^{2}+11 a m+28 a\right)\left(dx\right)^{m}}{(7+m)(4+m)(1+m)}$
parallelrisch	$\frac{x^{7} \left(dx\right)^{m} c\,m^{2}+5 x^{7} \left(dx\right)^{m} c m+4 x^{7} \left(dx\right)^{m} c+x^{4} \left(dx\right)^{m} b\,m^{2}+8 x^{4} \left(dx\right)^{m} b m+7 x^{4} \left(dx\right)^{m} b+x \left(dx\right)^{m} a\,m^{2}+11 x \left(dx\right)^{m} a m+28 x \left(dx\right)}{(7+m)(4+m)(1+m)}$

input
```
int((d*x)^m*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
a/(1+m)*x*exp(m*ln(d*x))+b/(4+m)*x^4*exp(m*ln(d*x))+c/(7+m)*x^7*exp(m*ln(d
*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int (dx)^m \left(a+bx^3+cx^6\right)\,dx = \frac{((cm^2+5\,cm+4\,c)x^7+(bm^2+8\,bm+7\,b)x^4+(am^2+11\,am+28\,a)x)(dx)^m}{m^3+12\,m^2+39\,m+28}$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
((c*m^2 + 5*c*m + 4*c)*x^7 + (b*m^2 + 8*b*m + 7*b)*x^4 + (a*m^2 + 11*a*m +
 28*a)*x)*(d*x)^m/(m^3 + 12*m^2 + 39*m + 28)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(42) = 84$.

Time = 0.37 (sec) , antiderivative size = 299, normalized size of antiderivative = 5.75

$$\int (dx)^m \left(a + bx^3 + cx^6\right) dx$$
$$= \begin{cases} \frac{-\frac{a}{6x^6} - \frac{b}{3x^3} + c\log(x)}{d^7} \\ \frac{-\frac{a}{3x^3} + b\log(x) + \frac{cx^3}{3}}{d^4} \\ \frac{a\log(x) + \frac{bx^3}{3} + \frac{cx^6}{6}}{d} \\ \frac{am^2x(dx)^m}{m^3+12m^2+39m+28} + \frac{11amx(dx)^m}{m^3+12m^2+39m+28} + \frac{28ax(dx)^m}{m^3+12m^2+39m+28} + \frac{bm^2x^4(dx)^m}{m^3+12m^2+39m+28} + \frac{8bmx^4(dx)^m}{m^3+12m^2+39m+28} + \frac{7bx^4(}{m^3+12m^2} \end{cases}$$

input
```
integrate((d*x)**m*(c*x**6+b*x**3+a),x)
```

output
```
Piecewise(((-a/(6*x**6) - b/(3*x**3) + c*log(x))/d**7, Eq(m, -7)), ((-a/(3
*x**3) + b*log(x) + c*x**3/3)/d**4, Eq(m, -4)), ((a*log(x) + b*x**3/3 + c*
x**6/6)/d, Eq(m, -1)), (a*m**2*x*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 1
1*a*m*x*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 28*a*x*(d*x)**m/(m**3 + 12
*m**2 + 39*m + 28) + b*m**2*x**4*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 8
*b*m*x**4*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 7*b*x**4*(d*x)**m/(m**3
+ 12*m**2 + 39*m + 28) + c*m**2*x**7*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28)
 + 5*c*m*x**7*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 4*c*x**7*(d*x)**m/(m
**3 + 12*m**2 + 39*m + 28), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int (dx)^m \left(a + bx^3 + cx^6\right) dx = \frac{cd^m x^7 x^m}{m+7} + \frac{bd^m x^4 x^m}{m+4} + \frac{(dx)^{m+1} a}{d(m+1)}$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
c*d^m*x^7*x^m/(m + 7) + b*d^m*x^4*x^m/(m + 4) + (d*x)^(m + 1)*a/(d*(m + 1)
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.29

$$\int (dx)^m \left(a+bx^3+cx^6\right)\,dx = \frac{(dx)^m cm^2x^7 + 5\,(dx)^m cmx^7 + 4\,(dx)^m cx^7 + (dx)^m bm^2x^4 + 8\,(dx)^m bmx^4 + 7\,(dx)^m bx^4 + (dx)^m am^2x}{m^3 + 12\,m^2 + 39\,m + 28}$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
((d*x)^m*c*m^2*x^7 + 5*(d*x)^m*c*m*x^7 + 4*(d*x)^m*c*x^7 + (d*x)^m*b*m^2*x
^4 + 8*(d*x)^m*b*m*x^4 + 7*(d*x)^m*b*x^4 + (d*x)^m*a*m^2*x + 11*(d*x)^m*a*
m*x + 28*(d*x)^m*a*x)/(m^3 + 12*m^2 + 39*m + 28)
```

Mupad [B] (verification not implemented)

Time = 20.64 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.71

$$\int (dx)^m \left(a+bx^3+cx^6\right)\,dx = (d\,x)^m \left(\frac{b\,x^4\,(m^2+8\,m+7)}{m^3+12\,m^2+39\,m+28} + \frac{c\,x^7\,(m^2+5\,m+4)}{m^3+12\,m^2+39\,m+28} + \frac{a\,x\,(m^2+11\,m+28)}{m^3+12\,m^2+39\,m+28}\right)$$

input
```
int((d*x)^m*(a + b*x^3 + c*x^6),x)
```

output
```
(d*x)^m*((b*x^4*(8*m + m^2 + 7))/(39*m + 12*m^2 + m^3 + 28) + (c*x^7*(5*m
+ m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (a*x*(11*m + m^2 + 28))/(39*m + 1
2*m^2 + m^3 + 28))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

$$\int (dx)^m \left(a+bx^3+cx^6\right) dx$$
$$= \frac{x^m d^m x(c\,m^2x^6+5cm\,x^6+4c\,x^6+b\,m^2x^3+8bm\,x^3+7b\,x^3+a\,m^2+11am+28a)}{m^3+12m^2+39m+28}$$

input
```
int((d*x)^m*(c*x^6+b*x^3+a),x)
```

output
```
(x**m*d**m*x*(a*m**2 + 11*a*m + 28*a + b*m**2*x**3 + 8*b*m*x**3 + 7*b*x**3
 + c*m**2*x**6 + 5*c*m*x**6 + 4*c*x**6))/(m**3 + 12*m**2 + 39*m + 28)
```

3.235 $\int \frac{(dx)^m}{a+bx^3+cx^6} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 173

$$\int \frac{(dx)^m}{a+bx^3+cx^6} \, dx = \frac{2c(dx)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

output
```
2*c*(d*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b-(-4*a*c+b
^2)^(1/2)))/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))/d/(1+m)-2*c*(d*x)^(1
+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/
(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/d/(1+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.49

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \frac{(dx)^m\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1^2+2c\#1^5}\&\right]}{3m}$$

input
```
Integrate[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

output
```
((d*x)^m*RootSum[a + b*#1^3 + c*#1^6 & , Hypergeometric2F1[-m, -m, 1 - m,
-(#1/(x - #1))]/((x/(x - #1))^m*(b*#1^2 + 2*c*#1^5)) & ])/(3*m)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1711, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{ 1711}$$

$$\frac{c\int \frac{2(dx)^m}{2cx^3+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{2(dx)^m}{2cx^3+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2c\int \frac{(dx)^m}{2cx^3+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{(dx)^m}{2cx^3+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2c(dx)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

output
```
(2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 +
 m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])
*d*(1 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{(dx)^m}{c\,x^6+b\,x^3+a}dx$$

input
```
int((d*x)^m/(c*x^6+b*x^3+a),x)
```

output
```
int((d*x)^m/(c*x^6+b*x^3+a),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx=\int \frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx=\text{Timed out}$$

input
```
integrate((d*x)**m/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \int \frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \int \frac{(dx)^m}{cx^6+bx^3+a}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \int \frac{(d\,x)^m}{c\,x^6+b\,x^3+a}\,dx$$

input
```
int((d*x)^m/(a + b*x^3 + c*x^6),x)
```

output
```
int((d*x)^m/(a + b*x^3 + c*x^6), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = d^m\left(\int \frac{x^m}{c\,x^6+b\,x^3+a}dx\right)$$

input
```
int((d*x)^m/(c*x^6+b*x^3+a),x)
```

output
```
d**m*int(x**m/(a + b*x**3 + c*x**6),x)
```

3.236 $\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 315

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \frac{(dx)^{1+m}(b^2-2ac+bcx^3)}{3a(b^2-4ac)\,d\,(a+bx^3+cx^6)} + \frac{c\left(b^2(2-m)+b\sqrt{b^2-4ac}(2-m)-4ac(5-m)\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{3a(b^2-4ac)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{c\left(b^2(2-m)-b\sqrt{b^2-4ac}(2-m)-4ac(5-m)\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{3a(b^2-4ac)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

output
```
1/3*(d*x)^(1+m)*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^6+b*x^3+a)+1/3*c
*(b^2*(2-m)+b*(-4*a*c+b^2)^(1/2)*(2-m)-4*a*c*(5-m))*(d*x)^(1+m)*hypergeom(
[1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)
^(3/2)/(b-(-4*a*c+b^2)^(1/2))/d/(1+m)-1/3*c*(b^2*(2-m)-b*(-4*a*c+b^2)^(1/2
)*(2-m)-4*a*c*(5-m))*(d*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c
*x^3/(b+(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))/d
/(1+m)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.61 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.25

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \frac{x(dx)^m \operatorname{AppellF1}\left(\frac{1+m}{3},2,2,\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{a^2(1+m)}$$

input
```
Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
```

output
```
(x*(d*x)^m*AppellF1[(1 + m)/3, 2, 2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(a^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1702, 25, 1834, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx$$

$$\Big\downarrow \text{1702}$$

$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}-\frac{\int -\frac{(dx)^m\left(bc(2-m)x^3+b^2(2-m)-2ac(5-m)\right)}{cx^6+bx^3+a}dx}{3a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx)^m\left(bc(2-m)x^3+b^2(2-m)-2ac(5-m)\right)}{cx^6+bx^3+a}dx}{3a\left(b^2-4ac\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}$$

$$\Big\downarrow \text{1834}$$

$$\frac{\frac{c\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\int \frac{2(dx)^m}{2cx^3+b-\sqrt{b^2-4ac}}dx}{2\sqrt{b^2-4ac}}-\frac{c\left(-b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\int \frac{2(dx)^m}{2cx^3+b+\sqrt{b^2-4ac}}dx}{2\sqrt{b^2-4ac}}}{3a\left(b^2-4ac\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{c\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\int \frac{(dx)^m}{2cx^3+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\left(-b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\int \frac{(dx)^m}{2cx^3+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}}{3a\left(b^2-4ac\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}$$

$$\Big\downarrow \text{888}$$

$$\frac{\frac{c(dx)^{m+1}\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(2-m)\sqrt{b^2-4ac}-4ac\right.}{}}{3a\left(b^2-4ac\right)}$$

$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}$$

input
```
Int[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
```

output
```
((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*d*(a + b*x^3 +
c*x^6)) + ((c*(b^2*(2 - m) + b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*
(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m))
- (c*(b^2*(2 - m) - b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*x)^(1
+ m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m)))/(3*a*(b^
2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1702
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*
(p + 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && IGtQ[n, 0] && ILtQ[p, -1]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [F]

$$\int \frac{(dx)^m}{(c\,x^6+b\,x^3+a)^2}dx$$

input
```
int((d*x)^m/(c*x^6+b*x^3+a)^2,x)
```

output
```
int((d*x)^m/(c*x^6+b*x^3+a)^2,x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^2}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6 + 2*a*b*x^3 + a
^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(c*x**6+b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^2}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^2}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \int \frac{(d\,x)^m}{(c\,x^6+b\,x^3+a)^2}\,dx$$

input
```
int((d*x)^m/(a + b*x^3 + c*x^6)^2,x)
```

output
```
int((d*x)^m/(a + b*x^3 + c*x^6)^2, x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = d^m\left(\int \frac{x^m}{c^2x^{12}+2bc\,x^9+2ac\,x^6+b^2x^6+2ab\,x^3+a^2}dx\right)$$

input
```
int((d*x)^m/(c*x^6+b*x^3+a)^2,x)
```

output
```
d**m*int(x**m/(a**2 + 2*a*b*x**3 + 2*a*c*x**6 + b**2*x**6 + 2*b*c*x**9 + c
**2*x**12),x)
```

3.237 $\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal result

Integrand size = 22, antiderivative size = 158

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{a(dx)^{1+m}\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1+m}{3},-\frac{3}{2},-\frac{3}{2},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
a*(d*x)^(1+m)*(c*x^6+b*x^3+a)^(1/2)*AppellF1(1/3+1/3*m,-3/2,-3/2,4/3+1/3*m
,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/
(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2))
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 357 vs. 2(158) = 316.

Time = 2.35 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.26

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{x(dx)^m\sqrt{a+bx^3+cx^6}\left(a(28+11m+m^2)\,\text{AppellF1}\left(\frac{1+m}{3},-\frac{1}{2},-\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2c}{-b+\sqrt{\ldots}}\right.\right.}{\ldots}$$

input
```
Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x*(d*x)^m*Sqrt[a + b*x^3 + c*x^6]*(a*(28 + 11*m + m^2)*AppellF1[(1 + m)/3
, -1/2, -1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
 + Sqrt[b^2 - 4*a*c])] + (1 + m)*x^3*(b*(7 + m)*AppellF1[(4 + m)/3, -1/2,
-1/2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[
b^2 - 4*a*c])] + c*(4 + m)*x^3*AppellF1[(7 + m)/3, -1/2, -1/2, (10 + m)/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))
)/((1 + m)*(4 + m)*(7 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c])])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^3+cx^6} \int (dx)^m \left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2} \left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2} dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{a(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{m+1}{3},-\frac{3}{2},-\frac{3}{2},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(a*(d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -3/2, -3/2, (
4 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4
*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (
2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int (dx)^m \left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}dx$$

input
```
int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \int (dx)^m \left(a+bx^3+cx^6\right)^{\frac{3}{2}} dx$$

input
```
integrate((d*x)**m*(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral((d*x)**m*(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \int \left(cx^6+bx^3+a\right)^{\frac{3}{2}} (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \int (d\,x)^m \left(c\,x^6+b\,x^3+a\right)^{3/2} dx$$

input
```
int((d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int((d*x)^m*(a + b*x^3 + c*x^6)^(3/2), x)
```

Reduce [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \text{too large to display}$$

input
```
int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
(d**m*(8*x**m*sqrt(a + b*x**3 + c*x**6)*a*c*m**2*x + 160*x**m*sqrt(a + b*x
**3 + c*x**6)*a*c*m*x + 728*x**m*sqrt(a + b*x**3 + c*x**6)*a*c*x + 54*x**m
*sqrt(a + b*x**3 + c*x**6)*b**2*x + 8*x**m*sqrt(a + b*x**3 + c*x**6)*b*c*m
**2*x**4 + 124*x**m*sqrt(a + b*x**3 + c*x**6)*b*c*m*x**4 + 368*x**m*sqrt(a
 + b*x**3 + c*x**6)*b*c*x**4 + 8*x**m*sqrt(a + b*x**3 + c*x**6)*c**2*m**2*
x**7 + 88*x**m*sqrt(a + b*x**3 + c*x**6)*c**2*m*x**7 + 224*x**m*sqrt(a + b
*x**3 + c*x**6)*c**2*x**7 + 216*int((x**m*sqrt(a + b*x**3 + c*x**6)*x**3)/
(a*m**3 + 21*a*m**2 + 138*a*m + 280*a + b*m**3*x**3 + 21*b*m**2*x**3 + 138
*b*m*x**3 + 280*b*x**3 + c*m**3*x**6 + 21*c*m**2*x**6 + 138*c*m*x**6 + 280
*c*x**6),x)*a*b*c*m**4 + 5724*int((x**m*sqrt(a + b*x**3 + c*x**6)*x**3)/(a
*m**3 + 21*a*m**2 + 138*a*m + 280*a + b*m**3*x**3 + 21*b*m**2*x**3 + 138*b
*m*x**3 + 280*b*x**3 + c*m**3*x**6 + 21*c*m**2*x**6 + 138*c*m*x**6 + 280*c
*x**6),x)*a*b*c*m**3 + 54756*int((x**m*sqrt(a + b*x**3 + c*x**6)*x**3)/(a*
m**3 + 21*a*m**2 + 138*a*m + 280*a + b*m**3*x**3 + 21*b*m**2*x**3 + 138*b*
m*x**3 + 280*b*x**3 + c*m**3*x**6 + 21*c*m**2*x**6 + 138*c*m*x**6 + 280*c*
x**6),x)*a*b*c*m**2 + 224424*int((x**m*sqrt(a + b*x**3 + c*x**6)*x**3)/(a*
m**3 + 21*a*m**2 + 138*a*m + 280*a + b*m**3*x**3 + 21*b*m**2*x**3 + 138*b*
m*x**3 + 280*b*x**3 + c*m**3*x**6 + 21*c*m**2*x**6 + 138*c*m*x**6 + 280*c*
x**6),x)*a*b*c*m + 332640*int((x**m*sqrt(a + b*x**3 + c*x**6)*x**3)/(a*m**
3 + 21*a*m**2 + 138*a*m + 280*a + b*m**3*x**3 + 21*b*m**2*x**3 + 138*b*...
```

3.238 $\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx = \frac{(dx)^{1+m}\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1+m}{3},-\frac{1}{2},-\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

output
```
(d*x)^(1+m)*(c*x^6+b*x^3+a)^(1/2)*AppellF1(1/3+1/3*m,-1/2,-1/2,4/3+1/3*m,-
2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/(1
+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^
(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.91 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx = \frac{x(dx)^m\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{1+m}{3},-\frac{1}{2},-\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}}$$

input
```
Integrate[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*(d*x)^m*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2, -1/2, (4 + m)
/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])
])/((1 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{a+bx^3+cx^6}\int (dx)^m\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,\text{AppellF1}\left(\frac{m+1}{3},-\frac{1}{2},-\frac{1}{2},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2, -1/2, (4
+ m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int (dx)^m \sqrt{c\,x^6+b\,x^3+a}dx$$

input
```
int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx=\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx$$

input
```
integrate((d*x)**m*(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx=\int \sqrt{cx^6+bx^3+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx=\int (d\,x)^m\,\sqrt{c\,x^6+b\,x^3+a}\,dx$$

input
```
int((d*x)^m*(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int((d*x)^m*(a + b*x^3 + c*x^6)^(1/2), x)
```

Reduce [F]

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx$$
$$=\frac{d^m\left(2x^m\sqrt{c\,x^6+b\,x^3+a}\,x+3\left(\int \frac{x^m\sqrt{c\,x^6+b\,x^3+a}\,x^3}{cm\,x^6+4c\,x^6+bm\,x^3+4b\,x^3+am+4a}dx\right)bm+12\left(\int \frac{x^m\sqrt{c\,x^6+b\,x^3+a}\,x^3}{cm\,x^6+4c\,x^6+bm\,x^3+4b\,x^3+am+4}\right.\right.}{2m+8}$$

input
```
int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
(d**m*(2*x**m*sqrt(a + b*x**3 + c*x**6)*x + 3*int((x**m*sqrt(a + b*x**3 +
c*x**6)*x**3)/(a*m + 4*a + b*m*x**3 + 4*b*x**3 + c*m*x**6 + 4*c*x**6),x)*b
*m + 12*int((x**m*sqrt(a + b*x**3 + c*x**6)*x**3)/(a*m + 4*a + b*m*x**3 +
4*b*x**3 + c*m*x**6 + 4*c*x**6),x)*b + 6*int((x**m*sqrt(a + b*x**3 + c*x**
6))/(a*m + 4*a + b*m*x**3 + 4*b*x**3 + c*m*x**6 + 4*c*x**6),x)*a*m + 24*in
t((x**m*sqrt(a + b*x**3 + c*x**6))/(a*m + 4*a + b*m*x**3 + 4*b*x**3 + c*m*
x**6 + 4*c*x**6),x)*a))/(2*(m + 4))
```

3.239 $\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{3},\frac{1}{2},\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^3+cx^6}}$$

output
```
(d*x)^(1+m)*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4*a*c
+b^2)^(1/2)))^(1/2)*AppellF1(1/3+1/3*m,1/2,1/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/(c*x^6+b*x^3+a)^(1
/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.36 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{x(dx)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{3},\frac{1}{2},\frac{1}{2},\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{a+bx^3+cx^6}}$$

input
```
Integrate[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
(x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[
(1 + m)/3, 1/2, 1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^3)/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + b*x^3 + c*x^6])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{(dx)^m}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^3+cx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{m+1}{3},\frac{1}{2},\frac{1}{2},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^3+cx^6}}$$

input
```
Int[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

output
```
((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 1/2, 1/2, (4 + m)/3, (-2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1
 + m)*Sqrt[a + b*x^3 + c*x^6])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(dx)^m}{\sqrt{c\,x^6+b\,x^3+a}}dx$$

input
```
int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{(dx)^m}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx$$

input
```
integrate((d*x)**m/(c*x**6+b*x**3+a)**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt(a + b*x**3 + c*x**6), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{(dx)^m}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx=\int \frac{(dx)^m}{\sqrt{cx^6+bx^3+a}}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \int \frac{(d\,x)^m}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

input
```
int((d*x)^m/(a + b*x^3 + c*x^6)^(1/2),x)
```

output
```
int((d*x)^m/(a + b*x^3 + c*x^6)^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = d^m\left(\int \frac{x^m\sqrt{c\,x^6+b\,x^3+a}}{c\,x^6+b\,x^3+a}dx\right)$$

input
```
int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

output
```
d**m*int((x**m*sqrt(a + b*x**3 + c*x**6))/(a + b*x**3 + c*x**6),x)
```

3.240 $\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{3},\frac{3}{2},\frac{3}{2},\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},\right.}{ad(1+m)\sqrt{a+bx^3+cx^6}}$$

output
```
(d*x)^(1+m)*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4*a*c
+b^2)^(1/2)))^(1/2)*AppellF1(1/3+1/3*m,3/2,3/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/a/d/(1+m)/(c*x^6+b*x^3+a)^
(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 10.10 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx=\frac{x(dx)^m\left(-b+\sqrt{b^2-4ac}-2cx^3\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}\text{AppellF}}{\left(-b+\sqrt{b^2-4ac}\right)(1+m)\left(a+bx^3+cx\right.}$$

input
```
Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
(x*(d*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c]))^(3/2)*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((-b +
 Sqrt[b^2 - 4*a*c])*(1 + m)*(a + b*x^3 + c*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}} \, dx$$
$$\Big\downarrow \text{1721}$$
$$\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\int \frac{(dx)^m}{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3+cx^6}}$$
$$\Big\downarrow \text{1012}$$
$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{m+1}{3},\frac{3}{2},\frac{3}{2},\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^3+cx^6}}$$

input
```
Int[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
```

output
```
((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)/3, (-2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*
(1 + m)*Sqrt[a + b*x^3 + c*x^6])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(dx)^m}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*
a*c)*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**m/(c*x**6+b*x**3+a)**(3/2),x)
```

output
```
Integral((d*x)**m/(a + b*x**3 + c*x**6)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^6+bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx=\int \frac{(d\,x)^m}{(c\,x^6+b\,x^3+a)^{3/2}}\,dx$$

input
```
int((d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x)
```

output
```
int((d*x)^m/(a + b*x^3 + c*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx=d^m\left(\int \frac{x^m\sqrt{c\,x^6+b\,x^3+a}}{c^2x^{12}+2bc\,x^9+2ac\,x^6+b^2x^6+2ab\,x^3+a^2}dx\right)$$

input
```
int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)
```

output
```
d**m*int((x**m*sqrt(a + b*x**3 + c*x**6))/(a**2 + 2*a*b*x**3 + 2*a*c*x**6
+ b**2*x**6 + 2*b*c*x**9 + c**2*x**12),x)
```

3.241 $\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 155

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx$$
$$= \frac{(dx)^{1+m}\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p \text{AppellF1}\left(\frac{1+m}{3},-p,-p,\frac{4+m}{3},-\frac{2cx^3}{b-\sqrt{b^2-4a}}\right.}{d(1+m)}$$

output
```
(d*x)^(1+m)*(c*x^6+b*x^3+a)^p*AppellF1(1/3+1/3*m,-p,-p,4/3+1/3*m,-2*c*x^3/
(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*x^
3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.15

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx$$
$$= \frac{x(dx)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p \text{AppellF1}\left(\frac{1+m}{3},-p,-p,\frac{4+m}{3},-\frac{2cx^3}{b+\sqrt{b^2-4a}}\right.}{1+m}$$

input
```
Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x*(d*x)^m*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3, (-
2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((1
 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b +
Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\int (dx)^m\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-}}\right.$$

$$\Big\downarrow \text{1012}$$

$$\frac{(dx)^{m+1}\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{m+1}{3},-p,-p,\frac{m+4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{}{b}\right.}{d(m+1)}$$

input
```
Int[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

output
```
((d*x)^(1 + m)*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3
, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
/(d*(1 + m)*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int (dx)^m \left(c\,x^6+b\,x^3+a\right)^p dx$$

input
```
int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

output
```
int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \int \left(cx^6+bx^3+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \int \left(cx^6+bx^3+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx = \int \left(cx^6+bx^3+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p\,dx=\int (d\,x)^m\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

input
```
int((d*x)^m*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int((d*x)^m*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p\,dx$$
$$=\frac{d^m\left(x^m(c\,x^6+b\,x^3+a)^p\,x+3\left(\int\frac{x^m\,(c\,x^6+b\,x^3+a)^p\,x^3}{cm\,x^6+6cp\,x^6+c\,x^6+bm\,x^3+6bp\,x^3+b\,x^3+am+6ap+a}dx\right)bmp+18\left(\int\frac{}{cm\,x^6+6cp\,x^6+}\right.\right.}{}$$

input
```
int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

output
```
(d**m*(x**m*(a + b*x**3 + c*x**6)**p*x + 3*int((x**m*(a + b*x**3 + c*x**6)
**p*x**3)/(a*m + 6*a*p + a + b*m*x**3 + 6*b*p*x**3 + b*x**3 + c*m*x**6 + 6
*c*p*x**6 + c*x**6),x)*b*m*p + 18*int((x**m*(a + b*x**3 + c*x**6)**p*x**3)
/(a*m + 6*a*p + a + b*m*x**3 + 6*b*p*x**3 + b*x**3 + c*m*x**6 + 6*c*p*x**6
 + c*x**6),x)*b*p**2 + 3*int((x**m*(a + b*x**3 + c*x**6)**p*x**3)/(a*m + 6
*a*p + a + b*m*x**3 + 6*b*p*x**3 + b*x**3 + c*m*x**6 + 6*c*p*x**6 + c*x**6
),x)*b*p + 6*int((x**m*(a + b*x**3 + c*x**6)**p)/(a*m + 6*a*p + a + b*m*x*
*3 + 6*b*p*x**3 + b*x**3 + c*m*x**6 + 6*c*p*x**6 + c*x**6),x)*a*m*p + 36*i
nt((x**m*(a + b*x**3 + c*x**6)**p)/(a*m + 6*a*p + a + b*m*x**3 + 6*b*p*x**
3 + b*x**3 + c*m*x**6 + 6*c*p*x**6 + c*x**6),x)*a*p**2 + 6*int((x**m*(a +
b*x**3 + c*x**6)**p)/(a*m + 6*a*p + a + b*m*x**3 + 6*b*p*x**3 + b*x**3 + c
*m*x**6 + 6*c*p*x**6 + c*x**6),x)*a*p))/(m + 6*p + 1)
```

3.242 $\int x^8(a+bx^3+cx^6)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 224

$$\int x^8\left(a+bx^3+cx^6\right)^p\,dx=-\frac{b(2+p)\left(a+bx^3+cx^6\right)^{1+p}}{6c^2(1+p)(3+2p)}+\frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)}+\frac{2^p(2ac-b^2(2+p))\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2+p,\right.}{3c^2\sqrt{b^2-4ac}(1+p)(3+2p)}$$

output
```
-1/6*b*(2+p)*(c*x^6+b*x^3+a)^(p+1)/c^2/(p+1)/(3+2*p)+1/3*x^3*(c*x^6+b*x^3+
a)^(p+1)/c/(3+2*p)+1/3*2^p*(2*a*c-b^2*(2+p))*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x
^3)/(-4*a*c+b^2)^(1/2))^(-1-p)*(c*x^6+b*x^3+a)^(p+1)*hypergeom([-p, p+1],[
2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^
2)^(1/2)/(p+1)/(3+2*p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.52 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.72

$$\int x^8\left(a+bx^3+cx^6\right)^p\,dx$$
$$=\frac{1}{9}x^9\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(3,-p,-p,4,-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^8*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^9*(a + b*x^3 + c*x^6)^p*AppellF1[3, -p, -p, 4, (-2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(9*((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1693, 1166, 25, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8\left(a+bx^3+cx^6\right)^p\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int x^6\left(cx^6+bx^3+a\right)^p\,dx^3$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\left(\frac{\int -\left(\left(b(p+2)x^3+a\right)\left(cx^6+bx^3+a\right)^p\right)dx^3}{c(2p+3)}+\frac{x^3\left(a+bx^3+cx^6\right)^{p+1}}{c(2p+3)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{p+1}}{c(2p+3)}-\frac{\int \left(b(p+2)x^3+a\right)\left(cx^6+bx^3+a\right)^p dx^3}{c(2p+3)}\right)$$

$\Big\downarrow$ 1160

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{p+1}}{c(2p+3)}-\frac{\frac{\left(2ac-b^2(p+2)\right)\int \left(cx^6+bx^3+a\right)^p dx^3}{2c}+\frac{b(p+2)\left(a+bx^3+cx^6\right)^{p+1}}{2c(p+1)}}{c(2p+3)}\right)$$

$\Big\downarrow$ 1096

$$\frac{1}{3}\left(\frac{x^3\left(a+bx^3+cx^6\right)^{p+1}}{c(2p+3)}-\frac{\frac{b(p+2)\left(a+bx^3+cx^6\right)^{p+1}}{2c(p+1)}-\frac{2^p\left(2ac-b^2(p+2)\right)\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^3+cx^6\right)^{p+1}\text{Hypergeom}}{c(p+1)\sqrt{b^2-4ac}}}{c(2p+3)}\right.$$

input
```
Int[x^8*(a + b*x^3 + c*x^6)^p,x]
```

output
```
((x^3*(a + b*x^3 + c*x^6)^(1 + p))/(c*(3 + 2*p)) - ((b*(2 + p)*(a + b*x^3
+ c*x^6)^(1 + p))/(2*c*(1 + p)) - (2^p*(2*a*c - b^2*(2 + p))*(-((b - Sqrt[
b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(
1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^
3)/(2*Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p)))/(c*(3 + 2*p)))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{8} \left(c\,x^{6}+b\,x^{3}+a\right)^{p}dx$$

input
```
int(x^8*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^8*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^8 \left(a+bx^3+cx^6\right)^p\,dx=\int \left(cx^{6}+bx^{3}+a\right)^{p}x^{8}\,dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^8, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^8 \left(a + bx^3 + cx^6\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x**8*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^8 \left(a + bx^3 + cx^6\right)^p \, dx = \int \left(cx^6 + bx^3 + a\right)^p x^8 \, dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

Giac [F]

$$\int x^8 \left(a + bx^3 + cx^6\right)^p \, dx = \int \left(cx^6 + bx^3 + a\right)^p x^8 \, dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8(a+bx^3+cx^6)^p\,dx=\int x^8\,(c\,x^6+b\,x^3+a)^p\,dx$$

input
```
int(x^8*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^8*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int x^8(a+bx^3+cx^6)^p\,dx=\text{Too large to display}$$

input
```
int(x^8*(c*x^6+b*x^3+a)^p,x)
```

output

```
( - 4*(a + b*x**3 + c*x**6)**p*a**2*c*p - 4*(a + b*x**3 + c*x**6)**p*a**2*
c + (a + b*x**3 + c*x**6)**p*a*b**2*p + 2*(a + b*x**3 + c*x**6)**p*a*b**2
+ 4*(a + b*x**3 + c*x**6)**p*a*b*c*p**2*x**3 + 4*(a + b*x**3 + c*x**6)**p*
a*b*c*p*x**3 - (a + b*x**3 + c*x**6)**p*b**3*p**2*x**3 - 2*(a + b*x**3 + c
*x**6)**p*b**3*p*x**3 + 2*(a + b*x**3 + c*x**6)**p*b**2*c*p**2*x**6 + (a +
 b*x**3 + c*x**6)**p*b**2*c*p*x**6 + 4*(a + b*x**3 + c*x**6)**p*b*c**2*p**
2*x**9 + 6*(a + b*x**3 + c*x**6)**p*b*c**2*p*x**9 + 2*(a + b*x**3 + c*x**6
)**p*b*c**2*x**9 + 96*int(((a + b*x**3 + c*x**6)**p*x**5)/(4*a*p**2 + 8*a*
p + 3*a + 4*b*p**2*x**3 + 8*b*p*x**3 + 3*b*x**3 + 4*c*p**2*x**6 + 8*c*p*x*
*6 + 3*c*x**6),x)*a**2*c**2*p**4 + 288*int(((a + b*x**3 + c*x**6)**p*x**5)
/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**3 + 8*b*p*x**3 + 3*b*x**3 + 4*c*p**
2*x**6 + 8*c*p*x**6 + 3*c*x**6),x)*a**2*c**2*p**3 + 264*int(((a + b*x**3 +
 c*x**6)**p*x**5)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**3 + 8*b*p*x**3 + 3
*b*x**3 + 4*c*p**2*x**6 + 8*c*p*x**6 + 3*c*x**6),x)*a**2*c**2*p**2 + 72*in
t(((a + b*x**3 + c*x**6)**p*x**5)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**3
+ 8*b*p*x**3 + 3*b*x**3 + 4*c*p**2*x**6 + 8*c*p*x**6 + 3*c*x**6),x)*a**2*c
**2*p - 48*int(((a + b*x**3 + c*x**6)**p*x**5)/(4*a*p**2 + 8*a*p + 3*a + 4
*b*p**2*x**3 + 8*b*p*x**3 + 3*b*x**3 + 4*c*p**2*x**6 + 8*c*p*x**6 + 3*c*x*
*6),x)*a*b**2*c*p**5 - 264*int(((a + b*x**3 + c*x**6)**p*x**5)/(4*a*p**2 +
 8*a*p + 3*a + 4*b*p**2*x**3 + 8*b*p*x**3 + 3*b*x**3 + 4*c*p**2*x**6 + ...
```

3.243 $\int x^5(a+bx^3+cx^6)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 161

$$\int x^5\left(a+bx^3+cx^6\right)^p\,dx=\frac{\left(a+bx^3+cx^6\right)^{1+p}}{6c(1+p)}+\frac{2^pb\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-4ac}+2cx^3}{2\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}(1+p)}$$

output
```
1/6*(c*x^6+b*x^3+a)^(p+1)/c/(p+1)+1/3*2^p*b*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x^
3)/(-4*a*c+b^2)^(1/2))^(-1-p)*(c*x^6+b*x^3+a)^(p+1)*hypergeom([-p, p+1],[2
+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^
(1/2)/(p+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01

$$\int x^5 \left(a+bx^3+cx^6\right)^p \, dx = \frac{1}{6}x^6 \left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p} \left(a+bx^3+cx^6\right)^p \text{AppellF1}\left(2,-p,-p,3,-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^5*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^6*(a + b*x^3 + c*x^6)^p*AppellF1[2, -p, -p, 3, (-2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(6*((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3+cx^6\right)^p \, dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{3}\int x^3 \left(cx^6+bx^3+a\right)^p dx^3$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{p+1}}{2c(p+1)}-\frac{b\int \left(cx^6+bx^3+a\right)^p dx^3}{2c}\right)$$

$$\Big\downarrow \text{ 1096}$$

$$\frac{1}{3}\left(\frac{b2^p\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}+\right.$$

input
```
Int[x^5*(a + b*x^3 + c*x^6)^p,x]
```

output
```
((a + b*x^3 + c*x^6)^(1 + p)/(2*c*(1 + p)) + (2^p*b*(-((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hy
pergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqr
t[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p)))/3
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^5(c\,x^6+b\,x^3+a)^p\,dx$$

input
```
int(x^5*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^5*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^5\left(a+bx^3+cx^6\right)^p\,dx=\int \left(cx^6+bx^3+a\right)^p x^5\,dx$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^5\left(a+bx^3+cx^6\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**5*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^5 \left(a+bx^3+cx^6\right)^p \, dx = \int \left(cx^6+bx^3+a\right)^p x^5 \, dx$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

Giac [F]

$$\int x^5 \left(a+bx^3+cx^6\right)^p \, dx = \int \left(cx^6+bx^3+a\right)^p x^5 \, dx$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a+bx^3+cx^6\right)^p \, dx = \int x^5 \left(c\,x^6+b\,x^3+a\right)^p \, dx$$

input
```
int(x^5*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^5*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int x^5(a+bx^3+cx^6)^p\,dx$$
$$=\frac{-(c\,x^6+b\,x^3+a)^p\,a+(c\,x^6+b\,x^3+a)^p\,bp\,x^3+2(c\,x^6+b\,x^3+a)^p\,cp\,x^6+(c\,x^6+b\,x^3+a)^p\,c\,x^6+24\Big(}{}$$

input
```
int(x^5*(c*x^6+b*x^3+a)^p,x)
```

output
```
( - (a + b*x**3 + c*x**6)**p*a + (a + b*x**3 + c*x**6)**p*b*p*x**3 + 2*(a
+ b*x**3 + c*x**6)**p*c*p*x**6 + (a + b*x**3 + c*x**6)**p*c*x**6 + 24*int(
((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x**3 + b*x**3 + 2*c*p*x
**6 + c*x**6),x)*a*c*p**3 + 36*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p
+ a + 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*a*c*p**2 + 12*int(((a
+ b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x**3 + b*x**3 + 2*c*p*x**6
+ c*x**6),x)*a*c*p - 6*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*
b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*b**2*p**3 - 9*int(((a + b*x**3
 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**6
),x)*b**2*p**2 - 3*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*
x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*b**2*p)/(6*c*(2*p**2 + 3*p + 1))
```

3.244 $\int x^2(a+bx^3+cx^6)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 130

$$\int x^2\left(a+bx^3+cx^6\right)^p\,dx = -\frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-4ac}+2cx^3}{2\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}(1+p)}$$

output
```
-1/3*2^(p+1)*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))^(-1-p)*(
c*x^6+b*x^3+a)^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2
*c*x^3)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/(p+1)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06

$$\int x^2\left(a+bx^3+cx^6\right)^p\,dx = \frac{2^{-1+p}\left(b-\sqrt{b^2-4ac}+2cx^3\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\text{Hypergeometric2F1}\Big(-p,1+p,2+p}{3c(1+p)}$$

input
```
Integrate[x^2*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(a + b*x^3 + c*x^6)^p*Hyperg
eometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)/(2*Sqrt[b
^2 - 4*a*c])])/(3*c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 -
4*a*c])^p)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1690, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a + bx^3 + cx^6)^p \, dx$$

$$\Big\downarrow \text{ 1690}$$

$$\frac{1}{3} \int (cx^6 + bx^3 + a)^p \, dx^3$$

$$\Big\downarrow \text{ 1096}$$

$$-\frac{2^{p+1} \left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1} (a + bx^3 + cx^6)^{p+1} \text{Hypergeometric2F1}\left(-p, p+1, p+2, \frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3(p+1)\sqrt{b^2-4ac}}$$

input
```
Int[x^2*(a + b*x^3 + c*x^6)^p,x]
```

output
```
-1/3*(2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(
-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]
*(1 + p))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [F]

$$\int x^{2} \left(c\,x^{6}+b\,x^{3}+a \right)^{p}d x$$

input
```
int(x^2*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^2*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^2 \left(a+bx^3+cx^6\right)^p \,dx = \int (cx^6+bx^3+a)^p x^2 \,dx$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+bx^3+cx^6)^p\,dx = \text{Timed out}$$

input
```
integrate(x**2*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(a+bx^3+cx^6)^p\,dx = \int (cx^6+bx^3+a)^p x^2\,dx$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(a+bx^3+cx^6)^p\,dx = \int (cx^6+bx^3+a)^p x^2\,dx$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a + bx^3 + cx^6)^p \, dx = \int x^2 \left(c\,x^6 + b\,x^3 + a\right)^p \, dx$$

input
```
int(x^2*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^2*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int x^2 (a + bx^3 + cx^6)^p \, dx$$
$$= \frac{2(c\,x^6 + b\,x^3 + a)^p\,a + (c\,x^6 + b\,x^3 + a)^p\,b\,x^3 - 24\left(\int \frac{(c\,x^6+b\,x^3+a)^p x^5}{2cp\,x^6+c\,x^6+2bp\,x^3+b\,x^3+2ap+a}dx\right)ac\,p^2 - 12\left(\int \frac{(c}{2cp\,x^6+c}\right.}{3b\,(}$$

input
```
int(x^2*(c*x^6+b*x^3+a)^p,x)
```

output
```
(2*(a + b*x**3 + c*x**6)**p*a + (a + b*x**3 + c*x**6)**p*b*x**3 - 24*int((
(a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x**3 + b*x**3 + 2*c*p*x*
*6 + c*x**6),x)*a*c*p**2 - 12*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p +
 a + 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*a*c*p + 6*int(((a + b*x
**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x
**6),x)*b**2*p**2 + 3*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b
*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*b**2*p)/(3*b*(2*p + 1))
```

3.245 $\int x^4(a+bx^3+cx^6)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int x^4(a+bx^3+cx^6)^p\,dx=\frac{1}{5}x^5\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/5*x^5*(c*x^6+b*x^3+a)^p*AppellF1(5/3,-p,-p,8/3,-2*c*x^3/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2))
)^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int x^4(a+bx^3+cx^6)^p\,dx=\frac{1}{5}x^5\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^4*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(5*((b - Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^3+cx^6\right)^p \, dx$$

$$\Big\downarrow \text{ 1721}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\int x^4\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{1}{5}x^5\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b-\sqrt{b^2-4a}}\right.$$

input
```
Int[x^4*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^{4} \left(c\,x^{6}+b\,x^{3}+a \right)^{p}d x$$

input
```
int(x^4*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^4*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^4 \left(a+bx^3+cx^6\right)^p \, dx = \int { {\left(c x^{6} + b x^{3} + a\right)}^{p} x^{4} \,d x }$$

input
```
integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4 (a + bx^3 + cx^6)^p \, dx = \text{Timed out}$$

input
```
integrate(x**4*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^4 (a + bx^3 + cx^6)^p \, dx = \int (cx^6 + bx^3 + a)^p x^4 \, dx$$

input
```
integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^4, x)
```

Giac [F]

$$\int x^4 (a + bx^3 + cx^6)^p \, dx = \int (cx^6 + bx^3 + a)^p x^4 \, dx$$

input
```
integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a+bx^3+cx^6\right)^p \, dx = \int x^4 \left(c\,x^6+b\,x^3+a\right)^p \,dx$$

input
```
int(x^4*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^4*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int x^4 \left(a+bx^3+cx^6\right)^p \, dx = \text{Too large to display}$$

input
```
int(x^4*(c*x^6+b*x^3+a)^p,x)
```

output
```
(3*(a + b*x**3 + c*x**6)**p*b*p*x**2 + 6*(a + b*x**3 + c*x**6)**p*c*p*x**5
 + 2*(a + b*x**3 + c*x**6)**p*c*x**5 + 648*int(((a + b*x**3 + c*x**6)**p*x
**4)/(18*a*p**2 + 21*a*p + 5*a + 18*b*p**2*x**3 + 21*b*p*x**3 + 5*b*x**3 +
 18*c*p**2*x**6 + 21*c*p*x**6 + 5*c*x**6),x)*a*c*p**4 + 972*int(((a + b*x*
*3 + c*x**6)**p*x**4)/(18*a*p**2 + 21*a*p + 5*a + 18*b*p**2*x**3 + 21*b*p*
x**3 + 5*b*x**3 + 18*c*p**2*x**6 + 21*c*p*x**6 + 5*c*x**6),x)*a*c*p**3 + 4
32*int(((a + b*x**3 + c*x**6)**p*x**4)/(18*a*p**2 + 21*a*p + 5*a + 18*b*p*
*2*x**3 + 21*b*p*x**3 + 5*b*x**3 + 18*c*p**2*x**6 + 21*c*p*x**6 + 5*c*x**6
),x)*a*c*p**2 + 60*int(((a + b*x**3 + c*x**6)**p*x**4)/(18*a*p**2 + 21*a*p
 + 5*a + 18*b*p**2*x**3 + 21*b*p*x**3 + 5*b*x**3 + 18*c*p**2*x**6 + 21*c*p
*x**6 + 5*c*x**6),x)*a*c*p - 162*int(((a + b*x**3 + c*x**6)**p*x**4)/(18*a
*p**2 + 21*a*p + 5*a + 18*b*p**2*x**3 + 21*b*p*x**3 + 5*b*x**3 + 18*c*p**2
*x**6 + 21*c*p*x**6 + 5*c*x**6),x)*b**2*p**4 - 297*int(((a + b*x**3 + c*x*
*6)**p*x**4)/(18*a*p**2 + 21*a*p + 5*a + 18*b*p**2*x**3 + 21*b*p*x**3 + 5*
b*x**3 + 18*c*p**2*x**6 + 21*c*p*x**6 + 5*c*x**6),x)*b**2*p**3 - 171*int((
(a + b*x**3 + c*x**6)**p*x**4)/(18*a*p**2 + 21*a*p + 5*a + 18*b*p**2*x**3
+ 21*b*p*x**3 + 5*b*x**3 + 18*c*p**2*x**6 + 21*c*p*x**6 + 5*c*x**6),x)*b**
2*p**2 - 30*int(((a + b*x**3 + c*x**6)**p*x**4)/(18*a*p**2 + 21*a*p + 5*a
+ 18*b*p**2*x**3 + 21*b*p*x**3 + 5*b*x**3 + 18*c*p**2*x**6 + 21*c*p*x**6 +
 5*c*x**6),x)*b**2*p - 108*int(((a + b*x**3 + c*x**6)**p*x)/(18*a*p**2 ...
```

3.246 $\int x^3(a+bx^3+cx^6)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int x^3\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{4}x^4\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\text{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/4*x^4*(c*x^6+b*x^3+a)^p*AppellF1(4/3,-p,-p,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2))
)^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int x^3\left(a+bx^3+cx^6\right)^p\,dx$$
$$=\frac{1}{4}x^4\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\text{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^3*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4*((b - Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^3+cx^6\right)^p \, dx$$

$$\Big\downarrow \ 1721$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^3+cx^6\right)^p \int x^3 \left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p \left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.$$

$$\Big\downarrow \ 1012$$

$$\frac{1}{4}x^4 \left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^3+cx^6\right)^p \text{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4a}}\right.$$

input
```
Int[x^3*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*(1 + (2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^{3} \left(c\,x^{6}+b\,x^{3}+a\right)^{p}dx$$

input
```
int(x^3*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^3*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^3 \left(a+bx^3+cx^6\right)^p \, dx = \int \left(cx^{6}+bx^{3}+a\right)^{p} x^{3} \, dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(a+bx^3+cx^6)^p\,dx = \text{Timed out}$$

input
```
integrate(x**3*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3(a+bx^3+cx^6)^p\,dx = \int (cx^6+bx^3+a)^p x^3\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

Giac [F]

$$\int x^3(a+bx^3+cx^6)^p\,dx = \int (cx^6+bx^3+a)^p x^3\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (a + bx^3 + cx^6)^p \, dx = \int x^3 \left(c\,x^6 + b\,x^3 + a\right)^p \, dx$$

input
```
int(x^3*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^3*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int x^3 (a + bx^3 + cx^6)^p \, dx = \text{Too large to display}$$

input
```
int(x^3*(c*x^6+b*x^3+a)^p,x)
```

output
```
(3*(a + b*x**3 + c*x**6)**p*b*p*x + 6*(a + b*x**3 + c*x**6)**p*c*p*x**4 +
(a + b*x**3 + c*x**6)**p*c*x**4 - 54*int((a + b*x**3 + c*x**6)**p/(18*a*p*
*2 + 15*a*p + 2*a + 18*b*p**2*x**3 + 15*b*p*x**3 + 2*b*x**3 + 18*c*p**2*x*
*6 + 15*c*p*x**6 + 2*c*x**6),x)*a*b*p**3 - 45*int((a + b*x**3 + c*x**6)**p
/(18*a*p**2 + 15*a*p + 2*a + 18*b*p**2*x**3 + 15*b*p*x**3 + 2*b*x**3 + 18*
c*p**2*x**6 + 15*c*p*x**6 + 2*c*x**6),x)*a*b*p**2 - 6*int((a + b*x**3 + c*
x**6)**p/(18*a*p**2 + 15*a*p + 2*a + 18*b*p**2*x**3 + 15*b*p*x**3 + 2*b*x*
*3 + 18*c*p**2*x**6 + 15*c*p*x**6 + 2*c*x**6),x)*a*b*p + 648*int(((a + b*x
**3 + c*x**6)**p*x**3)/(18*a*p**2 + 15*a*p + 2*a + 18*b*p**2*x**3 + 15*b*p
*x**3 + 2*b*x**3 + 18*c*p**2*x**6 + 15*c*p*x**6 + 2*c*x**6),x)*a*c*p**4 +
648*int(((a + b*x**3 + c*x**6)**p*x**3)/(18*a*p**2 + 15*a*p + 2*a + 18*b*p
**2*x**3 + 15*b*p*x**3 + 2*b*x**3 + 18*c*p**2*x**6 + 15*c*p*x**6 + 2*c*x**
6),x)*a*c*p**3 + 162*int(((a + b*x**3 + c*x**6)**p*x**3)/(18*a*p**2 + 15*a
*p + 2*a + 18*b*p**2*x**3 + 15*b*p*x**3 + 2*b*x**3 + 18*c*p**2*x**6 + 15*c
*p*x**6 + 2*c*x**6),x)*a*c*p**2 + 12*int(((a + b*x**3 + c*x**6)**p*x**3)/(
18*a*p**2 + 15*a*p + 2*a + 18*b*p**2*x**3 + 15*b*p*x**3 + 2*b*x**3 + 18*c*
p**2*x**6 + 15*c*p*x**6 + 2*c*x**6),x)*a*c*p - 162*int(((a + b*x**3 + c*x*
*6)**p*x**3)/(18*a*p**2 + 15*a*p + 2*a + 18*b*p**2*x**3 + 15*b*p*x**3 + 2*
b*x**3 + 18*c*p**2*x**6 + 15*c*p*x**6 + 2*c*x**6),x)*b**2*p**4 - 189*int((
(a + b*x**3 + c*x**6)**p*x**3)/(18*a*p**2 + 15*a*p + 2*a + 18*b*p**2*x*...
```

3.247 $\int x(a+bx^3+cx^6)^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 138

$$\int x\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{2}x^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/2*x^2*(c*x^6+b*x^3+a)^p*AppellF1(2/3,-p,-p,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2))
)^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int x(a+bx^3+cx^6)^p\,dx$$
$$=\frac{1}{2}x^2\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(2*((b - Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^3+cx^6)^p\,dx$$
$$\Big\downarrow \text{1721}$$
$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\int x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.$$
$$\Big\downarrow \text{1012}$$

$$\frac{1}{2}x^2\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p \text{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4a}}\right.$$

input
```
Int[x*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*(1 + (2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x\left(c\,x^{6}+b\,x^{3}+a\right)^{p}dx$$

input
```
int(x*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x\left(a+bx^3+cx^6\right)^p\,dx=\int \left(cx^6+bx^3+a\right)^p x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+bx^3+cx^6\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+bx^3+cx^6\right)^p\,dx=\int \left(cx^6+bx^3+a\right)^p x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+bx^3+cx^6\right)^p\,dx=\int \left(cx^6+bx^3+a\right)^p x\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+bx^3+cx^6\right)^p\,dx=\int x\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

input
```
int(x*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int x\left(a+bx^3+cx^6\right)^p\,dx$$
$$=\frac{\left(c\,x^6+b\,x^3+a\right)^p x^2+9\left(\int \frac{\left(c\,x^6+b\,x^3+a\right)^p x^4}{3cp\,x^6+c\,x^6+3bp\,x^3+b\,x^3+3ap+a}dx\right)b\,p^2+3\left(\int \frac{\left(c\,x^6+b\,x^3+a\right)^p x^4}{3cp\,x^6+c\,x^6+3bp\,x^3+b\,x^3+3ap+a}dx\right)bp+}{6p+2}$$

input
```
int(x*(c*x^6+b*x^3+a)^p,x)
```

output
```
((a + b*x**3 + c*x**6)**p*x**2 + 9*int(((a + b*x**3 + c*x**6)**p*x**4)/(3*
a*p + a + 3*b*p*x**3 + b*x**3 + 3*c*p*x**6 + c*x**6),x)*b*p**2 + 3*int(((a
 + b*x**3 + c*x**6)**p*x**4)/(3*a*p + a + 3*b*p*x**3 + b*x**3 + 3*c*p*x**6
 + c*x**6),x)*b*p + 18*int(((a + b*x**3 + c*x**6)**p*x)/(3*a*p + a + 3*b*p
*x**3 + b*x**3 + 3*c*p*x**6 + c*x**6),x)*a*p**2 + 6*int(((a + b*x**3 + c*x
**6)**p*x)/(3*a*p + a + 3*b*p*x**3 + b*x**3 + 3*c*p*x**6 + c*x**6),x)*a*p)
/(2*(3*p + 1))
```

3.248 $\int (a+bx^3+cx^6)^p\,dx$

Optimal result

Integrand size = 14, antiderivative size = 133

$$\int (a+bx^3+cx^6)^p\,dx = x\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\mathrm{AppellF1}\left(\frac{1}{3},-p,-p,\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
x*(c*x^6+b*x^3+a)^p*AppellF1(1/3,-p,-p,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/(
(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.21

$$\int (a+bx^3+cx^6)^p\,dx = x\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\mathrm{AppellF1}\left(\frac{1}{3},-p,-p,\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3+cx^6\right)^p \, dx$$

$$\Big\downarrow \text{1686}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\int \left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+\right.$$

$$\Big\downarrow \text{936}$$

$$x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p \text{AppellF1}\left(\frac{1}{3},-p,-p,\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \left(c\,x^{6}+b\,x^{3}+a\right)^{p}dx$$

input
```
int((c*x^6+b*x^3+a)^p,x)
```

output
```
int((c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx^3+cx^6\right)^p\,dx=\int \left(cx^6+bx^3+a\right)^p\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^3+cx^6\right)^p\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx^3+cx^6\right)^p\,dx = \int \left(cx^6+bx^3+a\right)^p\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^3+cx^6\right)^p\,dx = \int \left(cx^6+bx^3+a\right)^p\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3+cx^6\right)^p\,dx=\int \left(c\,x^6+b\,x^3+a\right)^p\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p,x)
```

output
```
int((a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int \left(a+bx^3+cx^6\right)^p\,dx = \frac{\left(c\,x^6+b\,x^3+a\right)^p x+36\left(\int \frac{\left(c\,x^6+b\,x^3+a\right)^p}{6cp\,x^6+c\,x^6+6bp\,x^3+b\,x^3+6ap+a}dx\right) a\,p^2+6\left(\int \frac{\left(c\,x^6+b\,x^3+a\right)^p}{6cp\,x^6+c\,x^6+6bp\,x^3+b\,x^3+6ap+a}dx\right) ap+}{6p+1}$$

input
```
int((c*x^6+b*x^3+a)^p,x)
```

output
```
((a + b*x**3 + c*x**6)**p*x + 36*int((a + b*x**3 + c*x**6)**p/(6*a*p + a +
 6*b*p*x**3 + b*x**3 + 6*c*p*x**6 + c*x**6),x)*a*p**2 + 6*int((a + b*x**3
+ c*x**6)**p/(6*a*p + a + 6*b*p*x**3 + b*x**3 + 6*c*p*x**6 + c*x**6),x)*a*
p + 18*int(((a + b*x**3 + c*x**6)**p*x**3)/(6*a*p + a + 6*b*p*x**3 + b*x**
3 + 6*c*p*x**6 + c*x**6),x)*b*p**2 + 3*int(((a + b*x**3 + c*x**6)**p*x**3)
/(6*a*p + a + 6*b*p*x**3 + b*x**3 + 6*c*p*x**6 + c*x**6),x)*b*p)/(6*p + 1)
```

3.249 $\int \frac{(a+bx^3+cx^6)^p}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 157

$$\int \frac{(a+bx^3+cx^6)^p}{x} dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-}}{2cx^3}\right.}{3p}$$

output
```
1/3*2^(-1+2*p)*(c*x^6+b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*a*c
+b^2)^(1/2))/c/x^3,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^3)/p/(((b-(-4*a*c+b^2)^
(1/2)+2*c*x^3)/c/x^3)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3+cx^6)^p}{x} dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b+\sqrt{b^2-}}{2cx^3}\right.}{3p}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x,x]
```

output
```
(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(
b + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*p*
((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(c*x^3))^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^p}{x^3}dx^3$$

$$\Big\downarrow \text{1178}$$

$$-\frac{1}{3}4^p\left(\frac{1}{x^3}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\int\left(\frac{b-\sqrt{b^2-4ac}}{2cx^3}+\right.$$

$$\Big\downarrow \text{150}$$

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\right.}{3p}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x,x]
```

output
```
(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(
b - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/(3*
p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] +
2*c*x^3)/(c*x^3))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x} \, dx = \int \frac{(cx^6+bx^3+a)^p}{x} \, dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x} \, dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x} \, dx = \int \frac{(cx^6+bx^3+a)^p}{x} \, dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \frac{(c\,x^6+b\,x^3+a)^p+3\left(\int \frac{(c\,x^6+b\,x^3+a)^p}{c\,x^7+b\,x^4+ax}dx\right)ap-3\left(\int \frac{(c\,x^6+b\,x^3+a)^p x^5}{c\,x^6+b\,x^3+a}dx\right)cp}{3p}$$

input
```
int((c*x^6+b*x^3+a)^p/x,x)
```

output
```
((a + b*x**3 + c*x**6)**p + 3*int((a + b*x**3 + c*x**6)**p/(a*x + b*x**4 +
 c*x**7),x)*a*p - 3*int(((a + b*x**3 + c*x**6)**p*x**5)/(a + b*x**3 + c*x*
*6),x)*c*p)/(3*p)
```

3.250 $\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 136

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-}\right.}{x}$$

output
```
-(c*x^6+b*x^3+a)^p*AppellF1(-1/3,-p,-p,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)
/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2}}\right.}{x}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x^2,x]
```

output
```
-(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x*((b - Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^2} \, dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\int \frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^p}{x^2}dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p \text{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x^2,x]
```

output
```
-(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x^{2}}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^2,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^2}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x^2,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx$$
$$= \frac{(c\,x^6+b\,x^3+a)^p+9\left(\int \frac{(c\,x^6+b\,x^3+a)^p}{3cp\,x^8-c\,x^8+3bp\,x^5-b\,x^5+3ap\,x^2-a\,x^2}dx\right)a\,p^2x-3\left(\int \frac{(c\,x^6+b\,x^3+a)^p}{3cp\,x^8-c\,x^8+3bp\,x^5-b\,x^5+3ap\,x^2-a\,x^2}dx\right.}{x\,(3p-1)}$$

input
```
int((c*x^6+b*x^3+a)^p/x^2,x)
```

output
```
((a + b*x**3 + c*x**6)**p + 9*int((a + b*x**3 + c*x**6)**p/(3*a*p*x**2 - a
*x**2 + 3*b*p*x**5 - b*x**5 + 3*c*p*x**8 - c*x**8),x)*a*p**2*x - 3*int((a
+ b*x**3 + c*x**6)**p/(3*a*p*x**2 - a*x**2 + 3*b*p*x**5 - b*x**5 + 3*c*p*x
**8 - c*x**8),x)*a*p*x - 9*int(((a + b*x**3 + c*x**6)**p*x**4)/(3*a*p - a
+ 3*b*p*x**3 - b*x**3 + 3*c*p*x**6 - c*x**6),x)*c*p**2*x + 3*int(((a + b*x
**3 + c*x**6)**p*x**4)/(3*a*p - a + 3*b*p*x**3 - b*x**3 + 3*c*p*x**6 - c*x
**6),x)*c*p*x)/(x*(3*p - 1))
```

3.251 $\int \frac{(a+bx^3+cx^6)^p}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int \frac{(a+bx^3+cx^6)^p}{x^3} dx = -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-}}\right.}{2x^2}$$

output
```
-1/2*(c*x^6+b*x^3+a)^p*AppellF1(-2/3,-p,-p,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^2/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2
)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3+cx^6)^p}{x^3} dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2}}\right.}{2x^2}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x^3,x]
```

output
```
-1/2*((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^2*((b - Sqrt[b^2
 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] +
2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\int \frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^p}{x^3}dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p \text{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x^3,x]
```

output
```
-1/2*((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*(1 + (2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x^{3}}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^3,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^3}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x^3,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx$$
$$= \frac{(c\,x^6+b\,x^3+a)^p+9\left(\int \frac{(c\,x^6+b\,x^3+a)^p}{3cp\,x^9-2c\,x^9+3bp\,x^6-2b\,x^6+3ap\,x^3-2a\,x^3}dx\right)a\,p^2x^2-6\left(\int \frac{(c\,x^6+b\,x^3+a)^p}{3cp\,x^9-2c\,x^9+3bp\,x^6-2b\,x^6+3ap\,x^3-2}\right.}{x^2\,(3p}$$

input
```
int((c*x^6+b*x^3+a)^p/x^3,x)
```

output
```
((a + b*x**3 + c*x**6)**p + 9*int((a + b*x**3 + c*x**6)**p/(3*a*p*x**3 - 2
*a*x**3 + 3*b*p*x**6 - 2*b*x**6 + 3*c*p*x**9 - 2*c*x**9),x)*a*p**2*x**2 -
6*int((a + b*x**3 + c*x**6)**p/(3*a*p*x**3 - 2*a*x**3 + 3*b*p*x**6 - 2*b*x
**6 + 3*c*p*x**9 - 2*c*x**9),x)*a*p*x**2 - 9*int(((a + b*x**3 + c*x**6)**p
*x**3)/(3*a*p - 2*a + 3*b*p*x**3 - 2*b*x**3 + 3*c*p*x**6 - 2*c*x**6),x)*c*
p**2*x**2 + 6*int(((a + b*x**3 + c*x**6)**p*x**3)/(3*a*p - 2*a + 3*b*p*x**
3 - 2*b*x**3 + 3*c*p*x**6 - 2*c*x**6),x)*c*p*x**2)/(x**2*(3*p - 2))
```

3.252 $\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 164

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = -\frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{b-\sqrt{b^2-4ac}}{2c\ldots}\right.}{3(1-2p)x^3}$$

output
```
-1/3*4^p*(c*x^6+b*x^3+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,-1/2*(b-(-4*a*c+b^2)
^(1/2))/c/x^3,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^3)/(1-2*p)/x^3/(((b-(-4*a*c+
b^2)^(1/2)+2*c*x^3)/c/x^3)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = \frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\ldots\right.}{3(-1+2p)x^3}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x^4,x]
```

output
```
(4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -1/2*(b + Sq
rt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*(-1 + 2*
p)*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(c*x^3))^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^4} \, dx$$

$$\Big\downarrow \; 1693$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^p}{x^6}dx^3$$

$$\Big\downarrow \; 1178$$

$$-\frac{1}{3}4^p\left(\frac{1}{x^3}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\int \left(\frac{b-\sqrt{b^2-4ac}}{2cx^3}+\right.$$

$$\Big\downarrow \; 150$$

$$-\frac{4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx^3},\right.}{3(1-2p)x^3}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x^4,x]
```

output
```
-1/3*(4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -1/2*(b
 - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/((1
- 2*p)*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 -
4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(c\,x^6 + b\,x^3 + a)^p}{x^4}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^4,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{x^4}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x^4,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx$$
$$=\frac{-(c\,x^6+b\,x^3+a)^p\,bp+(c\,x^6+b\,x^3+a)^p\,b-(c\,x^6+b\,x^3+a)^p\,c\,x^3+3\left(\int \frac{(c\,x^6+b\,x^3+a)^p}{cp\,x^7-c\,x^7+bp\,x^4-b\,x^4+apx-ax}dx\right)}{}$$

input
```
int((c*x^6+b*x^3+a)^p/x^4,x)
```

output
```
( - (a + b*x**3 + c*x**6)**p*b*p + (a + b*x**3 + c*x**6)**p*b - (a + b*x**
3 + c*x**6)**p*c*x**3 + 3*int((a + b*x**3 + c*x**6)**p/(a*p*x - a*x + b*p*
x**4 - b*x**4 + c*p*x**7 - c*x**7),x)*b**2*p**3*x**3 - 6*int((a + b*x**3 +
 c*x**6)**p/(a*p*x - a*x + b*p*x**4 - b*x**4 + c*p*x**7 - c*x**7),x)*b**2*
p**2*x**3 + 3*int((a + b*x**3 + c*x**6)**p/(a*p*x - a*x + b*p*x**4 - b*x**
4 + c*p*x**7 - c*x**7),x)*b**2*p*x**3 + 6*int(((a + b*x**3 + c*x**6)**p*x*
*5)/(a*p - a + b*p*x**3 - b*x**3 + c*p*x**6 - c*x**6),x)*c**2*p**2*x**3 -
6*int(((a + b*x**3 + c*x**6)**p*x**5)/(a*p - a + b*p*x**3 - b*x**3 + c*p*x
**6 - c*x**6),x)*c**2*p*x**3 + 6*int(((a + b*x**3 + c*x**6)**p*x**2)/(a*p
- a + b*p*x**3 - b*x**3 + c*p*x**6 - c*x**6),x)*b*c*p**3*x**3 - 9*int(((a
+ b*x**3 + c*x**6)**p*x**2)/(a*p - a + b*p*x**3 - b*x**3 + c*p*x**6 - c*x*
*6),x)*b*c*p**2*x**3 + 3*int(((a + b*x**3 + c*x**6)**p*x**2)/(a*p - a + b*
p*x**3 - b*x**3 + c*p*x**6 - c*x**6),x)*b*c*p*x**3)/(3*b*x**3*(p - 1))
```

3.253 $\int \frac{(a+bx^3+cx^6)^p}{x^5} dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int \frac{(a+bx^3+cx^6)^p}{x^5} dx = -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{4}{3},-p,-p,-\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2}{b+\sqrt{\ }}\right.}{4x^4}$$

output
```
-1/4*(c*x^6+b*x^3+a)^p*AppellF1(-4/3,-p,-p,-1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^4/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3+cx^6)^p}{x^5} dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{4}{3},-p,-p,-\frac{1}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2}{-b+\sqrt{\ }}\right.}{4x^4}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x^5,x]
```

output
```
-1/4*((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^4*((b - Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}dx$$

$$\Big\downarrow \text{ 1721}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\int \frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^p}{x^5}dx$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p \text{AppellF1}\left(-\frac{4}{3},-p,-p,-\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4x^4}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x^5,x]
```

output
```
-1/4*((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^4*(1 + (2*c*x^3
)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x^{5}}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^5,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^5,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^5}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^5}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^5, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^5}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^5}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x^5,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \frac{-(c\,x^6+b\,x^3+a)^p + 9\left(\int \frac{(c\,x^6+b\,x^3+a)^p}{3cp\,x^8-4c\,x^8+3bp\,x^5-4b\,x^5+3ap\,x^2-4a\,x^2}dx\right) b\,p^2 x^4 - 12\left(\int \frac{(c\,x^6+b\,x^3+a)^p}{3cp\,x^8-4c\,x^8+3bp\,x^5-4b\,x^5+3ap\,x}\right)}{}$$

input
```
int((c*x^6+b*x^3+a)^p/x^5,x)
```

output
```
( - (a + b*x**3 + c*x**6)**p + 9*int((a + b*x**3 + c*x**6)**p/(3*a*p*x**2
- 4*a*x**2 + 3*b*p*x**5 - 4*b*x**5 + 3*c*p*x**8 - 4*c*x**8),x)*b*p**2*x**4
 - 12*int((a + b*x**3 + c*x**6)**p/(3*a*p*x**2 - 4*a*x**2 + 3*b*p*x**5 - 4
*b*x**5 + 3*c*p*x**8 - 4*c*x**8),x)*b*p*x**4 + 18*int(((a + b*x**3 + c*x**
6)**p*x)/(3*a*p - 4*a + 3*b*p*x**3 - 4*b*x**3 + 3*c*p*x**6 - 4*c*x**6),x)*
c*p**2*x**4 - 24*int(((a + b*x**3 + c*x**6)**p*x)/(3*a*p - 4*a + 3*b*p*x**
3 - 4*b*x**3 + 3*c*p*x**6 - 4*c*x**6),x)*c*p*x**4)/(4*x**4)
```

3.254 $\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-\frac{5}{3},-p,-p,-\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

output
```
-1/5*(c*x^6+b*x^3+a)^p*AppellF1(-5/3,-p,-p,-2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^5/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-\frac{5}{3},-p,-p,-\frac{2}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2 \cdots}{-b+\sqrt{\cdots}}\right)}{5x^5}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x^6,x]
```

output
```
-1/5*((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^5*((b - Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}dx$$

$$\Big\downarrow \textbf{1721}$$

$$\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\int \frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^3}{b+\sqrt{b^2-4ac}}+1\right)^p}{x^6}dx$$

$$\Big\downarrow \textbf{1012}$$

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(-\frac{5}{3},-p,-p,-\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x^6,x]
```

output
```
-1/5*((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^5*(1 + (2*c*x^3
)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x^{6}}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^6,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^6,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^6}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^6}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = \int \frac{(cx^6+bx^3+a)^p}{x^6}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^6}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x^6,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx$$
$$= \frac{-(c\,x^6+b\,x^3+a)^p+9\left(\int \frac{(c\,x^6+b\,x^3+a)^p}{3cp\,x^9-5c\,x^9+3bp\,x^6-5b\,x^6+3ap\,x^3-5a\,x^3}dx\right)b\,p^2x^5-15\left(\int \frac{(c\,x^6+b\,x^3+a)^p}{3cp\,x^9-5c\,x^9+3bp\,x^6-5b\,x^6+3ap\,x}\right.}{}$$

input
```
int((c*x^6+b*x^3+a)^p/x^6,x)
```

output
```
( - (a + b*x**3 + c*x**6)**p + 9*int((a + b*x**3 + c*x**6)**p/(3*a*p*x**3
- 5*a*x**3 + 3*b*p*x**6 - 5*b*x**6 + 3*c*p*x**9 - 5*c*x**9),x)*b*p**2*x**5
 - 15*int((a + b*x**3 + c*x**6)**p/(3*a*p*x**3 - 5*a*x**3 + 3*b*p*x**6 - 5
*b*x**6 + 3*c*p*x**9 - 5*c*x**9),x)*b*p*x**5 + 18*int((a + b*x**3 + c*x**6
)**p/(3*a*p - 5*a + 3*b*p*x**3 - 5*b*x**3 + 3*c*p*x**6 - 5*c*x**6),x)*c*p*
*2*x**5 - 30*int((a + b*x**3 + c*x**6)**p/(3*a*p - 5*a + 3*b*p*x**3 - 5*b*
x**3 + 3*c*p*x**6 - 5*c*x**6),x)*c*p*x**5)/(5*x**5)
```

3.255 $\int \frac{(a+bx^3+cx^6)^p}{x^7} dx$

Optimal result

Integrand size = 18, antiderivative size = 168

$$\int \frac{(a+bx^3+cx^6)^p}{x^7} dx = -\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(2(1-p),-p,-p,3-2p,-\right.}{3(1-p)x^6}$$

output
```
-1/3*2^(-1+2*p)*(c*x^6+b*x^3+a)^p*AppellF1(2-2*p,-p,-p,3-2*p,-1/2*(b-(-4*a
*c+b^2)^(1/2))/c/x^3,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^3)/(1-p)/x^6/(((b-(-4
*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)
^p)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3+cx^6)^p}{x^7} dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(2-2p,-p,-p,3-2p,-\frac{b+\sqrt{b}}{2c}\right.}{3(-1+p)x^6}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/x^7,x]
```

output
```
(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -1/
2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3
*(-1 + p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2
 - 4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^p}{x^9}dx^3$$

$$\Big\downarrow \text{1178}$$

$$-\frac{1}{3}4^p\left(\frac{1}{x^3}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\int\left(\frac{b-\sqrt{b^2-4ac}}{2cx^3}+\right.$$

$$\Big\downarrow \text{150}$$

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(2-2p,-p,-p,3-2p,-\frac{b-\sqrt{b^2-4a}}{2cx^3}\right.}{3(1-p)x^6}$$

input
```
Int[(a + b*x^3 + c*x^6)^p/x^7,x]
```

output
```
-1/3*(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p
, -1/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^
3)])/((1 - p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt
[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^7}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^7,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^7,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^7}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/x^7, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx=\text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**7,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^7}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^7, x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx=\int \frac{(cx^6+bx^3+a)^p}{x^7}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{x^7}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/x^7,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/x^7, x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx$$
$$=\frac{-(c\,x^6+b\,x^3+a)^p+3\left(\int \frac{(c\,x^6+b\,x^3+a)^p}{cp\,x^{10}-2c\,x^{10}+bp\,x^7-2b\,x^7+ap\,x^4-2a\,x^4}dx\right)b\,p^2x^6-6\left(\int \frac{(c\,x^6+b\,x^3+a)^p}{cp\,x^{10}-2c\,x^{10}+bp\,x^7-2b\,x^7+ap\,x^4-2c}\right.}{6x}$$

input
```
int((c*x^6+b*x^3+a)^p/x^7,x)
```

output

```
( - (a + b*x**3 + c*x**6)**p + 3*int((a + b*x**3 + c*x**6)**p/(a*p*x**4 -
2*a*x**4 + b*p*x**7 - 2*b*x**7 + c*p*x**10 - 2*c*x**10),x)*b*p**2*x**6 - 6
*int((a + b*x**3 + c*x**6)**p/(a*p*x**4 - 2*a*x**4 + b*p*x**7 - 2*b*x**7 +
 c*p*x**10 - 2*c*x**10),x)*b*p*x**6 + 6*int((a + b*x**3 + c*x**6)**p/(a*p*
x - 2*a*x + b*p*x**4 - 2*b*x**4 + c*p*x**7 - 2*c*x**7),x)*c*p**2*x**6 - 12
*int((a + b*x**3 + c*x**6)**p/(a*p*x - 2*a*x + b*p*x**4 - 2*b*x**4 + c*p*x
**7 - 2*c*x**7),x)*c*p*x**6)/(6*x**6)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:  #isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:  #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file