Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.3-General-trinomial/125-1.2.3.2-b

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**143**]. This is test number [125].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (143)	0.00 (0)
Mathematica	100.00 (143)	0.00 (0)
Maple	94.41 (135)	5.59 (8)
Fricas	94.41 (135)	5.59 (8)
Mupad	94.41 (135)	5.59 (8)
Sympy	88.81 (127)	11.19 (16)
Giac	88.11 (126)	11.89 (17)
Reduce	56.64 (81)	43.36 (62)
Maxima	51.05 (73)	48.95 (70)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

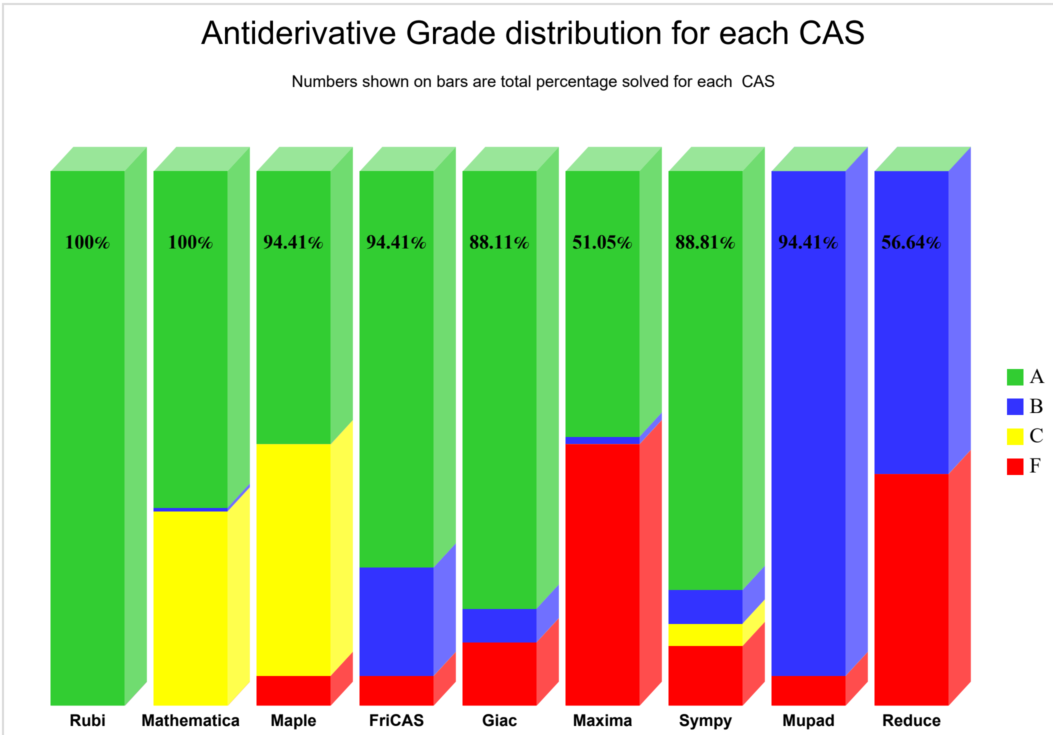
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

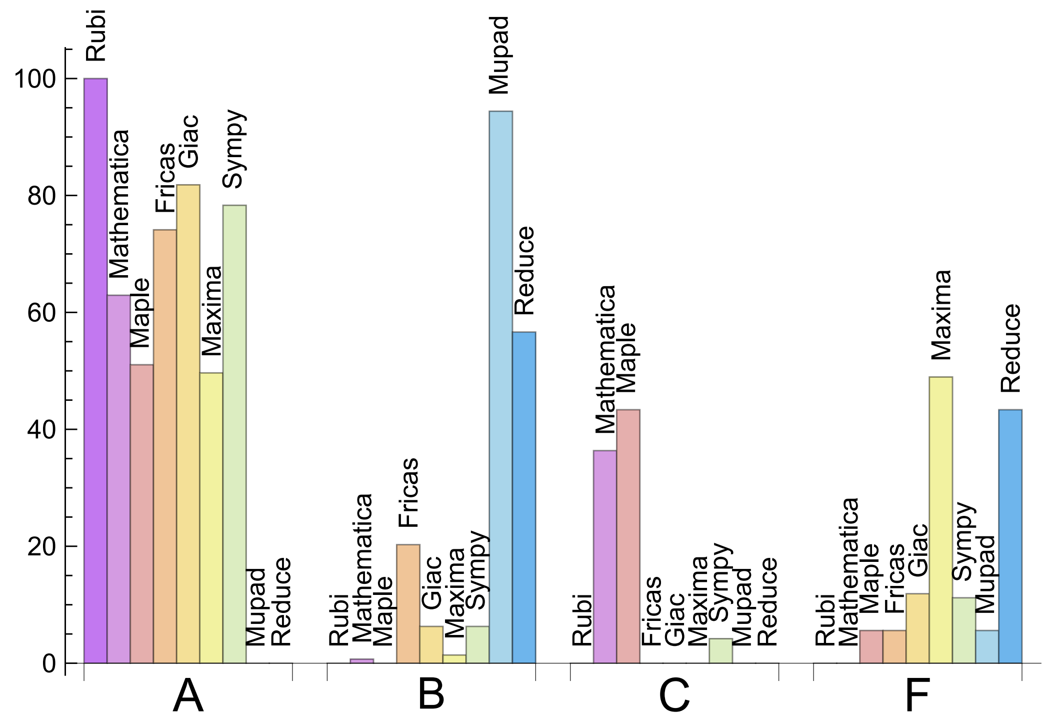
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Giac	81.818	6.294	0.000	11.888
Sympy	78.322	6.294	4.196	11.189
Fricas	74.126	20.280	0.000	5.594
Mathematica	62.937	0.699	36.364	0.000
Maple	51.049	0.000	43.357	5.594
Maxima	49.650	1.399	0.000	48.951
Mupad	0.000	94.406	0.000	5.594
Reduce	0.000	56.643	0.000	43.357

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	8	100.00	0.00	0.00
Maple	8	100.00	0.00	0.00
Mupad	8	0.00	100.00	0.00
Sympy	16	50.00	50.00	0.00
Giac	17	100.00	0.00	0.00
Reduce	62	100.00	0.00	0.00
Maxima	70	92.86	0.00	7.14

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maple	0.05
Mathematica	0.08
Maxima	0.08
Reduce	0.18
Giac	0.21
Fricas	0.26
Rubi	0.30
Sympy	1.98
Mupad	10.61

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	42.56	0.89	35.00	0.90
Maple	47.38	0.65	40.00	0.75
Sympy	75.37	0.97	53.00	0.94
Mathematica	79.91	1.02	59.00	1.00
Giac	131.37	1.20	63.50	0.94
Rubi	135.74	1.16	92.00	1.10
Reduce	138.91	2.40	92.00	1.99
Fricas	499.08	2.16	74.00	1.15
Mupad	1152.76	5.15	53.00	0.87

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

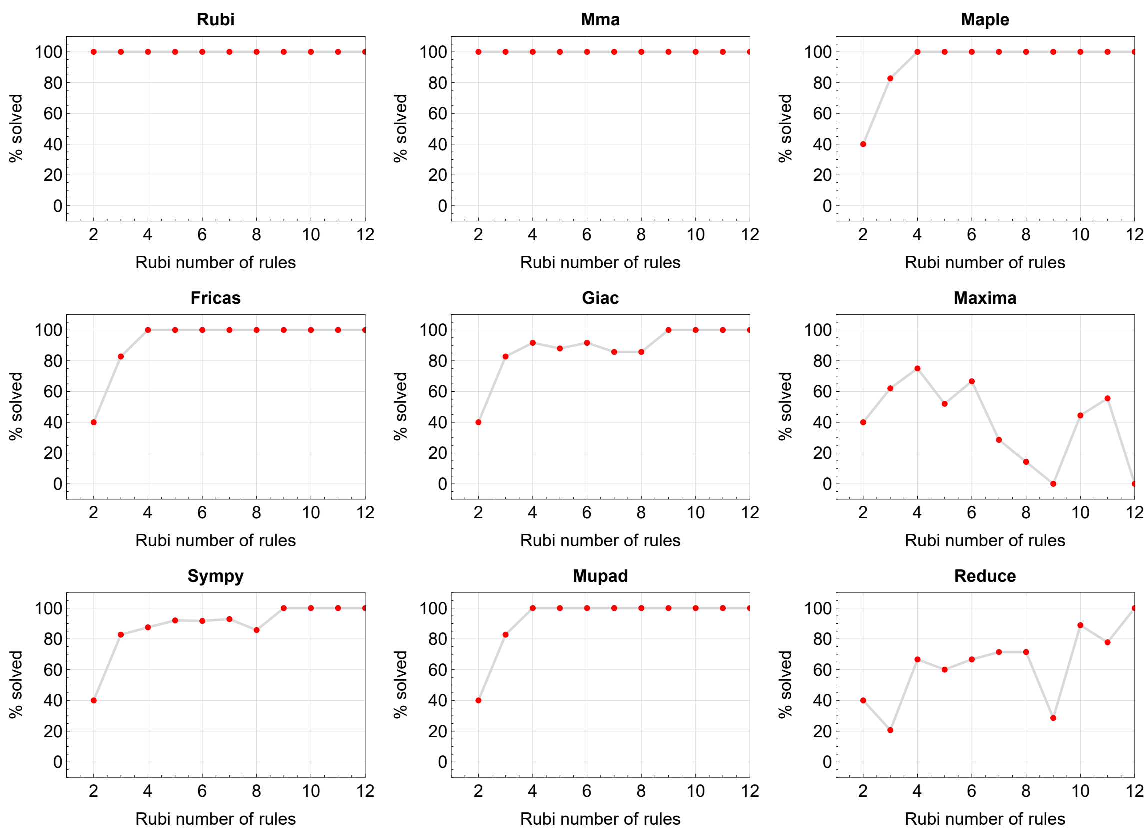


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

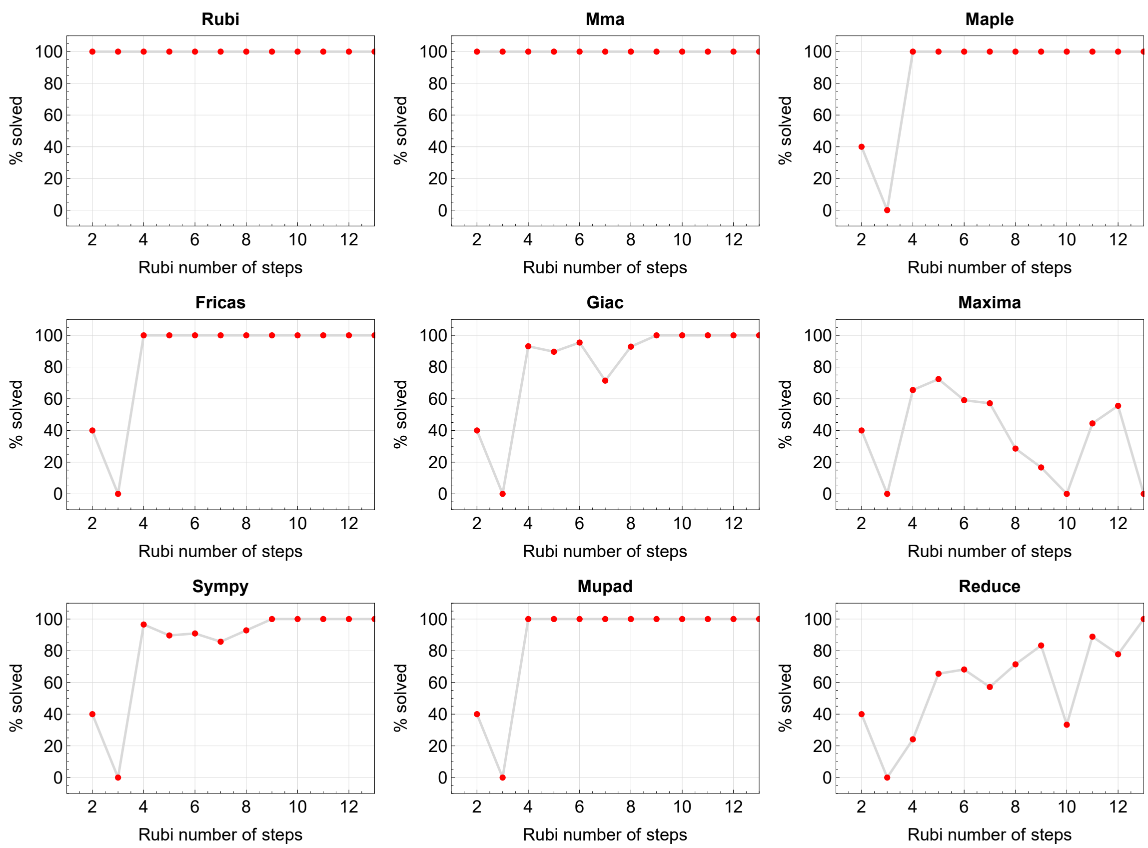


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

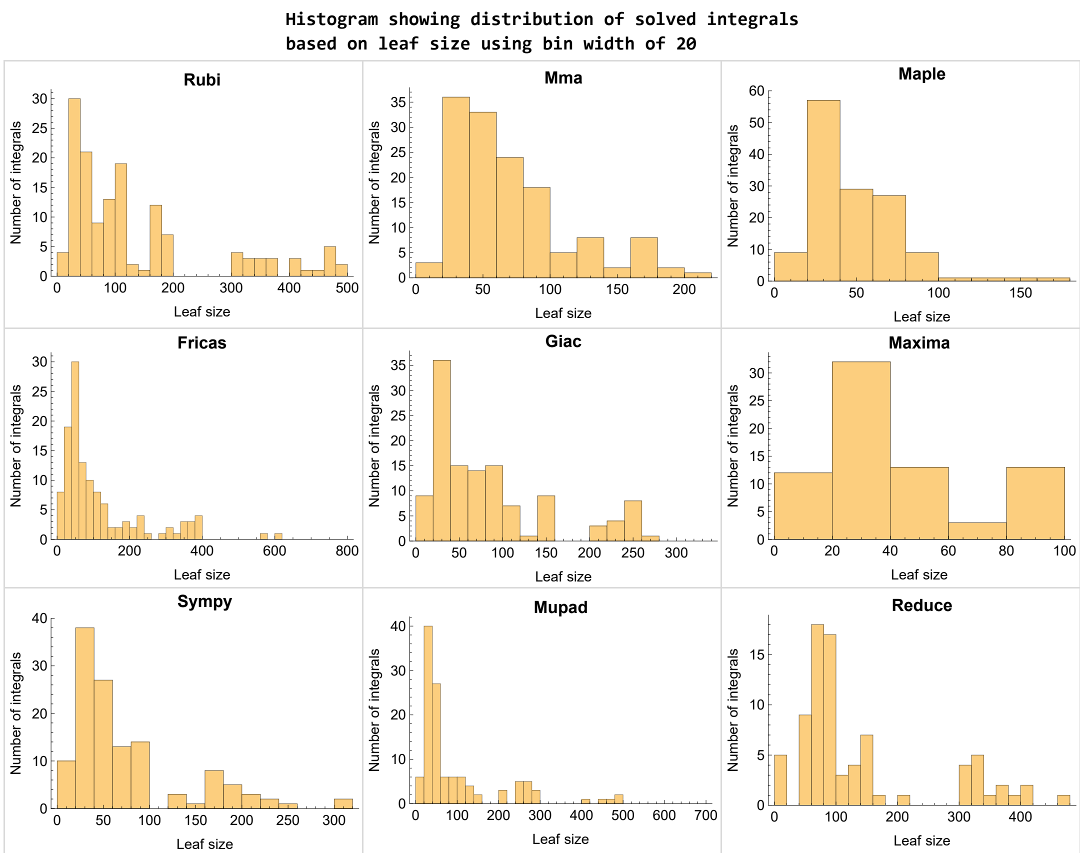


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

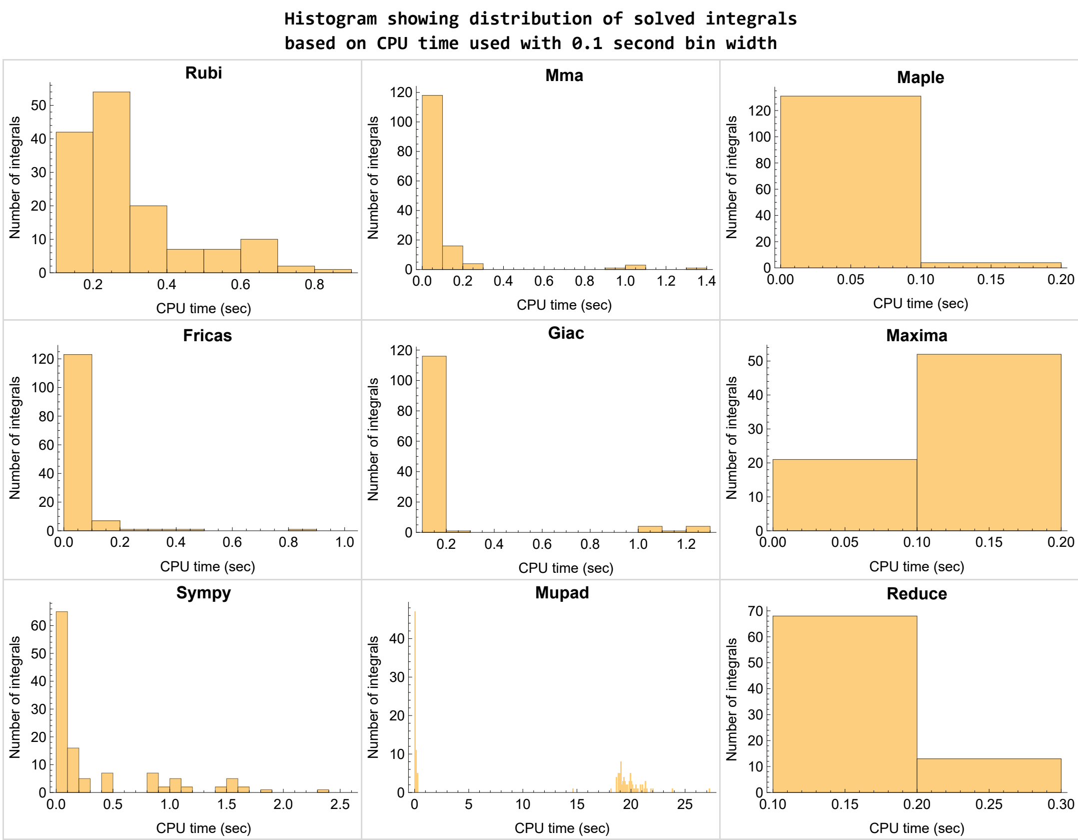


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

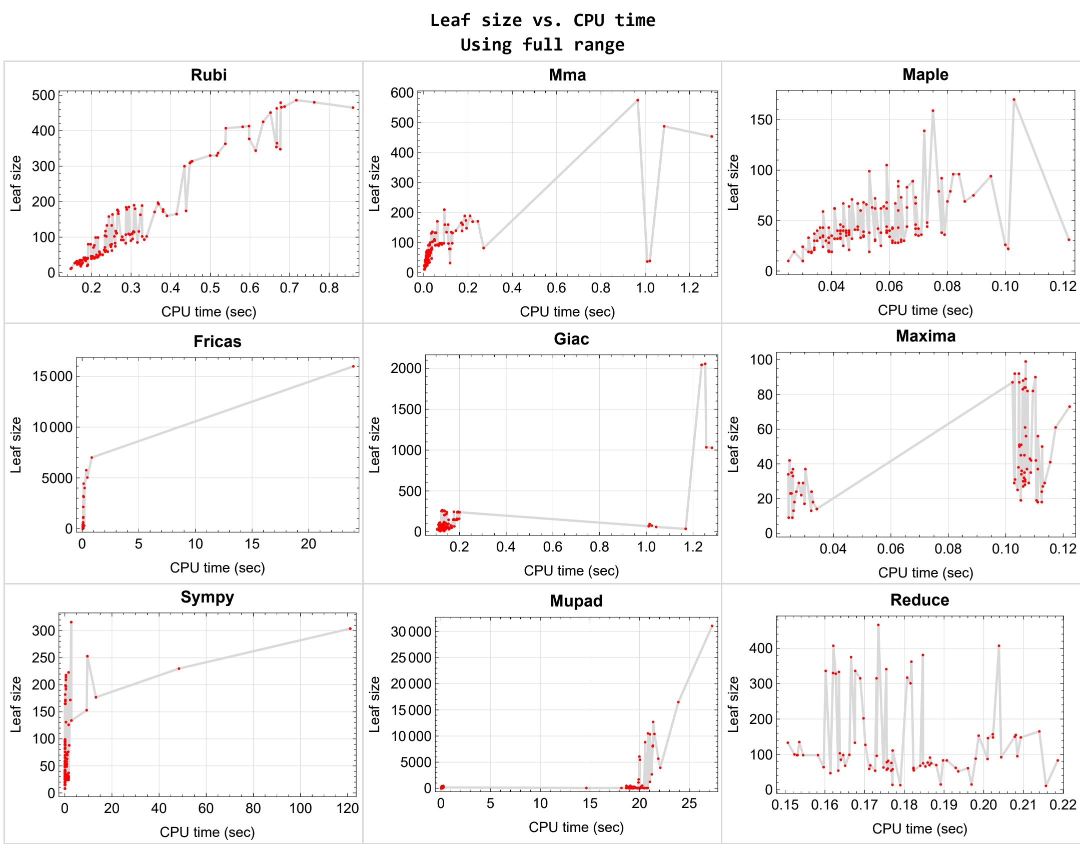


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{139, 140, 141, 142, 143\}$

Maple $\{39, 40, 41\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

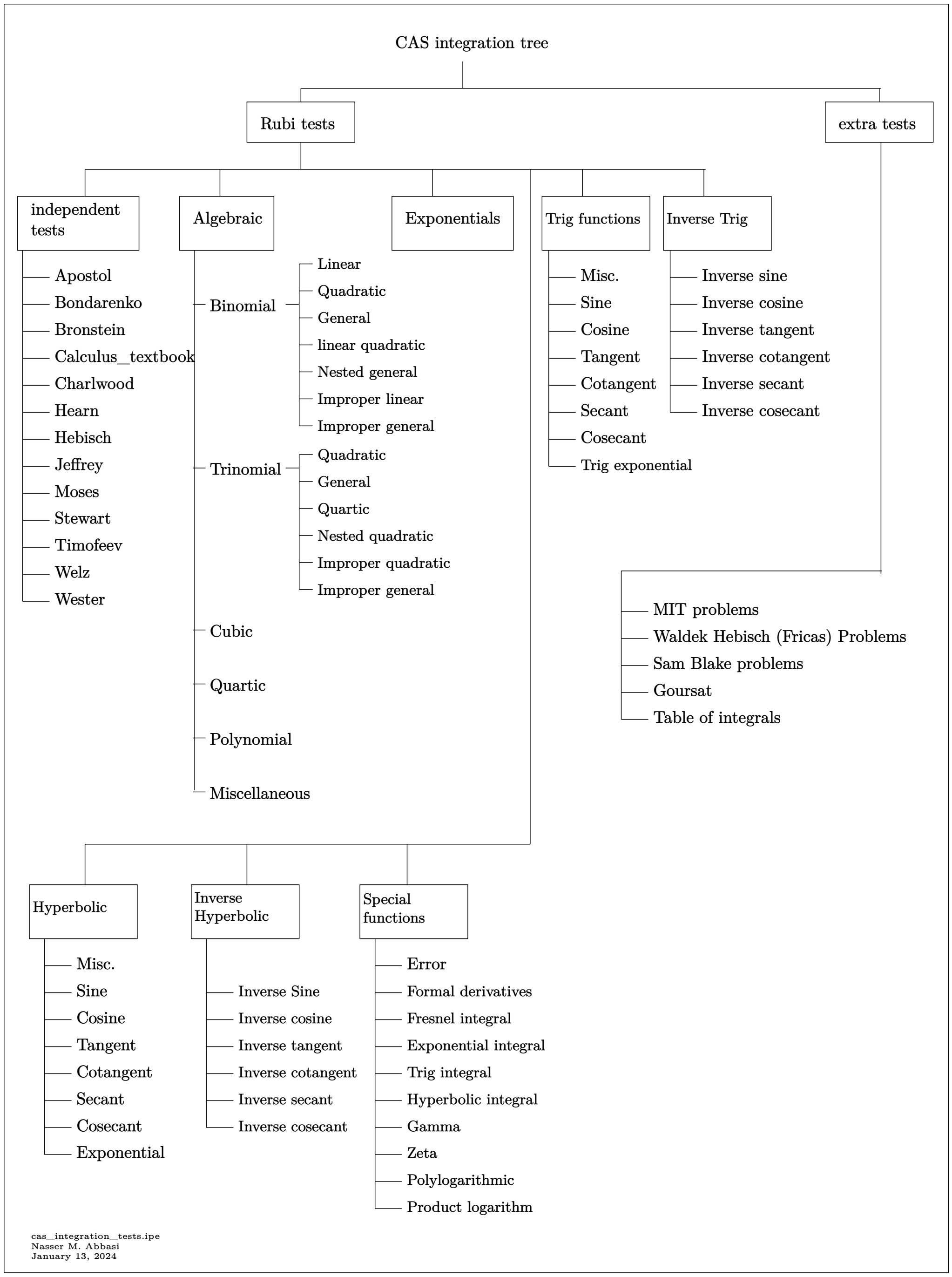
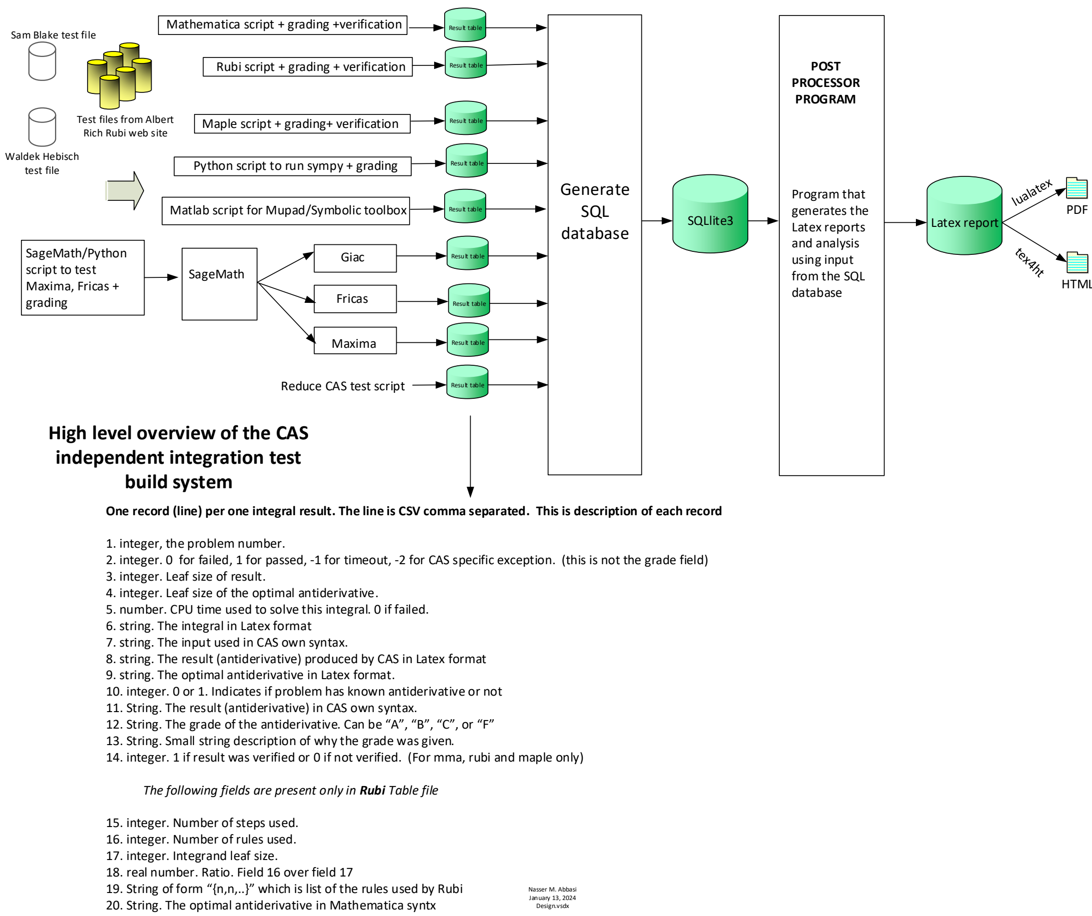


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

Rubi . . . 27
Mma . . . 28
Maple . . . 28
Fricas . . . 29
Maxima . . . 29
Giac . . . 30
Mupad . . . 30
Sympy . . . 31
Reduce . . . 31

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 63, 65, 67, 72, 74, 77, 80, 82, 83, 84, 86, 89, 101, 102, 103, 104, 105, 106, 107, 109, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138 }

B grade { 40 }

C grade { 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 69, 70, 71, 73, 75, 76, 78, 79, 81, 85, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 108, 110, 111, 112, 113, 114, 115, 116, 117, 139, 140, 141, 142, 143 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 45, 47, 49, 51, 52, 53, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 77, 80, 82, 83, 84, 86, 88, 89, 90, 101, 103, 105, 107, 109, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 137, 138 }

B grade { }

C grade { 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 46, 48, 50, 54, 55, 56, 57, 58, 59, 60, 61, 62, 73, 75, 76, 78, 79, 81, 85, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 104, 106, 108, 110, 111, 112, 113, 114, 115, 116, 117, 128, 129, 130, 131, 132, 133, 134, 135, 136 }

F normal fail { 42, 43, 44, 139, 140, 141, 142, 143 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 6, 8, 10, 12, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 45, 47, 49, 51, 53, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 122, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138 }

B grade { 5, 7, 9, 11, 13, 14, 15, 16, 17, 18, 19, 46, 48, 50, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 119, 121, 123, 125, 127 }

C grade { }

F normal fail { 42, 43, 44, 139, 140, 141, 142, 143 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 82, 84, 86, 88, 90, 101, 103, 105, 107, 109, 118, 119, 120, 122, 124, 125, 126, 127, 137, 138 }

B grade { 121, 123 }

C grade { }

F normal fail { 42, 43, 44, 46, 48, 50, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 85, 87, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 104, 106, 108, 110, 111, 112, 113, 114, 115, 116, 117, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 142, 143 }

F(-1) timedout fail { }

F(-2) exception fail { 45, 47, 49, 51, 53 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 45, 47, 49, 51, 53, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138 }

B grade { 46, 48, 50, 52, 83, 89, 121, 123, 127 }

C grade { }

F normal fail { 42, 43, 44, 54, 55, 56, 57, 58, 59, 60, 61, 62, 139, 140, 141, 142, 143 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138 }

C grade { }

F normal fail { }

F(-1) timedout fail { 42, 43, 44, 139, 140, 141, 142, 143 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 46, 48, 50, 52, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 77, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 122, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138 }

B grade { 45, 47, 49, 51, 119, 121, 123, 125, 127 }

C grade { 73, 75, 76, 78, 79, 81 }

F normal fail { 40, 42, 43, 44, 140, 141, 142, 143 }

F(-1) timedout fail { 39, 41, 53, 54, 55, 61, 62, 139 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 137, 138 }

C grade { }

F normal fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 142, 143 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	42	47	38	37	51	37	38	75	33
N.S.	1	1.20	1.34	1.09	1.06	1.46	1.06	1.09	2.14	0.94
time (sec)	N/A	0.185	0.036	0.046	0.026	0.061	0.060	0.123	0.185	0.059

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	27	25	24	23	32	20	31	68	25
N.S.	1	0.82	0.76	0.73	0.70	0.97	0.61	0.94	2.06	0.76
time (sec)	N/A	0.169	0.014	0.037	0.025	0.061	0.047	0.127	0.184	0.042

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	35	39	35	34	46	34	35	70	26
N.S.	1	1.09	1.22	1.09	1.06	1.44	1.06	1.09	2.19	0.81
time (sec)	N/A	0.170	0.024	0.045	0.024	0.067	0.045	0.119	0.188	0.034

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	22	22	19	18	23	15	19	62	20
N.S.	1	0.85	0.85	0.73	0.69	0.88	0.58	0.73	2.38	0.77
time (sec)	N/A	0.174	0.010	0.033	0.026	0.062	0.039	0.116	0.193	0.051

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	29	33	30	29	40	26	30	61	21
N.S.	1	1.16	1.32	1.20	1.16	1.60	1.04	1.20	2.44	0.84
time (sec)	N/A	0.164	0.014	0.035	0.029	0.061	0.046	0.129	0.196	0.035

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	9	8	9	14	11
N.S.	1	1.00	0.85	0.77	0.69	0.69	0.62	0.69	1.08	0.85
time (sec)	N/A	0.150	0.003	0.030	0.026	0.058	0.036	0.119	0.177	0.023

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	29	33	30	29	40	26	30	60	21
N.S.	1	1.16	1.32	1.20	1.16	1.60	1.04	1.20	2.40	0.84
time (sec)	N/A	0.163	0.010	0.034	0.028	0.061	0.054	0.110	0.176	0.035

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	24	32	19	30	70	22
N.S.	1	1.00	0.93	0.75	0.86	1.14	0.68	1.07	2.50	0.79
time (sec)	N/A	0.178	0.011	0.046	0.032	0.074	0.053	0.114	0.186	0.067

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	35	41	36	37	54	36	38	77	26
N.S.	1	1.09	1.28	1.12	1.16	1.69	1.12	1.19	2.41	0.81
time (sec)	N/A	0.173	0.024	0.046	0.030	0.062	0.079	0.116	0.186	0.048

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	35	35	32	35	50	29	36	91	32
N.S.	1	0.95	0.95	0.86	0.95	1.35	0.78	0.97	2.46	0.86
time (sec)	N/A	0.184	0.016	0.047	0.025	0.060	0.085	0.126	0.186	19.419

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	42	49	41	42	59	41	42	82	32
N.S.	1	1.14	1.32	1.11	1.14	1.59	1.11	1.14	2.22	0.86
time (sec)	N/A	0.181	0.017	0.049	0.025	0.064	0.104	0.111	0.176	0.053

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	41	40	35	34	51	37	36	62	32
N.S.	1	1.14	1.11	0.97	0.94	1.42	1.03	1.00	1.72	0.89
time (sec)	N/A	0.191	0.021	0.070	0.105	0.070	0.078	0.126	0.182	20.669

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	37	38	29	28	49	32	30	60	26
N.S.	1	1.32	1.36	1.04	1.00	1.75	1.14	1.07	2.14	0.93
time (sec)	N/A	0.185	0.018	0.064	0.107	0.075	0.078	0.126	0.182	0.040

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	34	35	30	29	46	32	31	57	23
N.S.	1	1.17	1.21	1.03	1.00	1.59	1.10	1.07	1.97	0.79
time (sec)	N/A	0.176	0.017	0.065	0.108	0.070	0.076	0.118	0.176	0.033

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	32	31	28	27	43	26	29	52	21
N.S.	1	1.19	1.15	1.04	1.00	1.59	0.96	1.07	1.93	0.78
time (sec)	N/A	0.170	0.016	0.061	0.106	0.072	0.066	0.118	0.194	0.033

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	34	33	30	29	45	27	31	54	23
N.S.	1	1.17	1.14	1.03	1.00	1.55	0.93	1.07	1.86	0.79
time (sec)	N/A	0.175	0.014	0.061	0.114	0.068	0.079	0.127	0.173	0.031

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	32	33	28	27	44	31	29	55	21
N.S.	1	1.19	1.22	1.04	1.00	1.63	1.15	1.07	2.04	0.78
time (sec)	N/A	0.163	0.011	0.062	0.113	0.068	0.086	0.122	0.182	0.030

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	39	40	36	35	55	37	37	63	26
N.S.	1	1.15	1.18	1.06	1.03	1.62	1.09	1.09	1.85	0.76
time (sec)	N/A	0.190	0.019	0.071	0.109	0.075	0.091	0.130	0.175	0.044

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	41	38	36	37	63	39	34	69	28
N.S.	1	1.21	1.12	1.06	1.09	1.85	1.15	1.00	2.03	0.82
time (sec)	N/A	0.185	0.021	0.079	0.107	0.064	0.090	0.112	0.171	0.045

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	40	32	30	29	36	26	29	88	31
N.S.	1	1.14	0.91	0.86	0.83	1.03	0.74	0.83	2.51	0.89
time (sec)	N/A	0.188	0.020	0.055	0.103	0.061	0.047	0.104	0.198	20.432

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	25	25	24	23	30	20	30	75	25
N.S.	1	0.86	0.86	0.83	0.79	1.03	0.69	1.03	2.59	0.86
time (sec)	N/A	0.176	0.013	0.030	0.025	0.061	0.045	0.126	0.187	0.038

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	33	24	25	24	31	22	24	83	25
N.S.	1	1.10	0.80	0.83	0.80	1.03	0.73	0.80	2.77	0.83
time (sec)	N/A	0.172	0.016	0.044	0.113	0.061	0.059	0.111	0.191	19.930

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	18	18	19	18	23	15	18	66	18
N.S.	1	0.82	0.82	0.86	0.82	1.05	0.68	0.82	3.00	0.82
time (sec)	N/A	0.173	0.007	0.027	0.033	0.065	0.039	0.122	0.185	0.031

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	27	23	20	19	24	15	19	78	21
N.S.	1	1.17	1.00	0.87	0.83	1.04	0.65	0.83	3.39	0.91
time (sec)	N/A	0.163	0.011	0.039	0.105	0.061	0.050	0.113	0.176	0.029

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	13	11
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	1.18	1.00
time (sec)	N/A	0.148	0.003	0.025	0.024	0.061	0.041	0.135	0.179	0.019

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	27	20	20	19	23	15	19	76	20
N.S.	1	1.17	0.87	0.87	0.83	1.00	0.65	0.83	3.30	0.87
time (sec)	N/A	0.158	0.008	0.039	0.111	0.059	0.045	0.130	0.177	0.028

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	24	32	19	29	83	20
N.S.	1	1.00	1.00	0.88	1.00	1.33	0.79	1.21	3.46	0.83
time (sec)	N/A	0.172	0.011	0.034	0.027	0.058	0.055	0.131	0.219	20.073

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	35	30	25	25	31	26	25	87	25
N.S.	1	1.17	1.00	0.83	0.83	1.03	0.87	0.83	2.90	0.83
time (sec)	N/A	0.172	0.013	0.057	0.104	0.058	0.072	0.120	0.201	19.808

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	33	44	31	33	95	31
N.S.	1	1.00	1.00	0.85	1.00	1.33	0.94	1.00	2.88	0.94
time (sec)	N/A	0.185	0.013	0.039	0.026	0.056	0.071	0.111	0.208	19.999

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	38	33	28	30	36	29	31	92	30
N.S.	1	1.15	1.00	0.85	0.91	1.09	0.88	0.94	2.79	0.91
time (sec)	N/A	0.181	0.013	0.063	0.106	0.062	0.072	0.127	0.204	19.987

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	116	98	40	89	99	95	89	155	51
N.S.	1	1.36	1.15	0.47	1.05	1.16	1.12	1.05	1.82	0.60
time (sec)	N/A	0.320	0.081	0.045	0.107	0.067	0.073	0.125	0.208	20.816

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	110	94	34	83	97	90	83	153	45
N.S.	1	1.43	1.22	0.44	1.08	1.26	1.17	1.08	1.99	0.58
time (sec)	N/A	0.302	0.066	0.037	0.106	0.063	0.070	0.124	0.199	14.643

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	109	93	35	84	94	90	84	150	47
N.S.	1	1.40	1.19	0.45	1.08	1.21	1.15	1.08	1.92	0.60
time (sec)	N/A	0.294	0.064	0.040	0.107	0.069	0.077	0.122	0.208	0.051

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	107	90	33	82	91	82	82	146	45
N.S.	1	1.41	1.18	0.43	1.08	1.20	1.08	1.08	1.92	0.59
time (sec)	N/A	0.297	0.067	0.039	0.108	0.063	0.069	0.120	0.201	0.050

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	109	92	35	84	93	83	84	148	46
N.S.	1	1.40	1.18	0.45	1.08	1.19	1.06	1.08	1.90	0.59
time (sec)	N/A	0.302	0.057	0.037	0.107	0.066	0.072	0.137	0.209	0.026

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	107	91	33	82	92	88	82	148	44
N.S.	1	1.41	1.20	0.43	1.08	1.21	1.16	1.08	1.95	0.58
time (sec)	N/A	0.287	0.052	0.036	0.109	0.070	0.093	0.126	0.202	18.170

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	114	98	41	88	95	97	88	157	49
N.S.	1	1.37	1.18	0.49	1.06	1.14	1.17	1.06	1.89	0.59
time (sec)	N/A	0.308	0.073	0.048	0.106	0.066	0.088	0.128	0.202	19.849

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	116	96	39	90	105	99	87	165	51
N.S.	1	1.40	1.16	0.47	1.08	1.27	1.19	1.05	1.99	0.61
time (sec)	N/A	0.306	0.075	0.051	0.110	0.070	0.096	0.110	0.214	19.674

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	39	31	13	13	0	23	15	59
N.S.	1	1.06	0.58	0.46	0.19	0.19	0.00	0.34	0.22	0.88
time (sec)	N/A	0.205	1.021	0.122	0.032	0.062	0.000	0.121	0.197	20.045

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	45	454	26	14	11	0	30	11	109
N.S.	1	0.60	6.05	0.35	0.19	0.15	0.00	0.40	0.15	1.45
time (sec)	N/A	0.200	1.300	0.100	0.034	0.069	0.000	0.117	0.216	20.141

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	22	13	13	0	30	15	33
N.S.	1	1.00	0.95	0.56	0.33	0.33	0.00	0.77	0.38	0.85
time (sec)	N/A	0.187	1.009	0.101	0.026	0.059	0.000	0.129	0.189	19.353

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	0	0	0	0	0	18	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.163	0.026	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	0	0	0	0	0	18	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.161	0.119	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	0	0	0	0	0	218	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	2.83	0.00
time (sec)	N/A	0.207	0.032	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	80	78	83	0	254	316	75	20	3916
N.S.	1	0.99	0.96	1.02	0.00	3.14	3.90	0.93	0.25	48.35
time (sec)	N/A	0.251	0.038	0.066	0.000	0.113	2.721	1.023	200.023	22.076

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	193	210	139	0	1071	134	2043	20	5659
N.S.	1	1.01	1.09	0.72	0.00	5.58	0.70	10.64	0.10	29.47
time (sec)	N/A	0.370	0.093	0.072	0.000	0.091	2.790	1.237	200.024	21.882

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	64	62	60	0	197	223	59	20	2654
N.S.	1	1.02	0.98	0.95	0.00	3.13	3.54	0.94	0.32	42.13
time (sec)	N/A	0.232	0.016	0.050	0.000	0.083	1.584	1.042	200.019	21.223

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	166	171	105	0	567	76	1034	20	1220
N.S.	1	1.04	1.08	0.66	0.00	3.57	0.48	6.50	0.13	7.67
time (sec)	N/A	0.268	0.062	0.059	0.000	0.085	1.438	1.257	200.023	21.031

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	129	131	36	20	260
N.S.	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	0.53	6.84
time (sec)	N/A	0.183	0.007	0.034	0.000	0.070	0.464	1.168	200.022	18.963

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	158	133	99	0	619	88	1028	18	1105
N.S.	1	1.03	0.86	0.64	0.00	4.02	0.57	6.68	0.12	7.18
time (sec)	N/A	0.244	0.056	0.053	0.000	0.086	1.845	1.281	200.022	19.886

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	66	66	0	223	253	68	20	1690
N.S.	1	1.06	0.96	0.96	0.00	3.23	3.67	0.99	0.29	24.49
time (sec)	N/A	0.249	0.020	0.052	0.000	0.103	9.570	1.010	200.021	19.905

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	180	75	159	0	1134	153	2055	20	5451
N.S.	1	0.98	0.41	0.86	0.00	6.16	0.83	11.17	0.11	29.62
time (sec)	N/A	0.310	0.022	0.075	0.000	0.106	9.263	1.252	200.016	20.036

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	95	92	84	0	293	0	94	20	8817
N.S.	1	1.07	1.03	0.94	0.00	3.29	0.00	1.06	0.22	99.07
time (sec)	N/A	0.304	0.022	0.063	0.000	0.179	0.000	1.014	200.020	20.554

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	344	70	63	0	7003	0	0	20	12709
N.S.	1	0.90	0.18	0.17	0.00	18.38	0.00	0.00	0.05	33.36
time (sec)	N/A	0.614	0.030	0.066	0.000	0.845	0.000	0.000	200.021	21.358

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	337	70	59	0	4001	0	0	20	10382
N.S.	1	0.90	0.19	0.16	0.00	10.64	0.00	0.00	0.05	27.61
time (sec)	N/A	0.519	0.030	0.037	0.000	0.216	0.000	0.000	200.023	21.491

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	314	44	43	0	4433	230	0	20	8033
N.S.	1	0.97	0.14	0.13	0.00	13.64	0.71	0.00	0.06	24.72
time (sec)	N/A	0.454	0.019	0.036	0.000	0.129	48.430	0.000	200.017	21.305

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	312	42	43	0	2141	126	0	20	8169
N.S.	1	0.96	0.13	0.13	0.00	6.59	0.39	0.00	0.06	25.14
time (sec)	N/A	0.451	0.017	0.037	0.000	0.098	1.598	0.000	200.022	21.344

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	309	43	43	0	3193	172	0	20	6067
N.S.	1	0.98	0.14	0.14	0.00	10.14	0.55	0.00	0.06	19.26
time (sec)	N/A	0.448	0.017	0.039	0.000	0.106	2.325	0.000	200.019	19.976

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	300	45	40	0	3125	177	0	16	10337
N.S.	1	0.95	0.14	0.13	0.00	9.92	0.56	0.00	0.05	32.82
time (sec)	N/A	0.435	0.020	0.035	0.000	0.140	13.197	0.000	200.020	21.040

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	330	71	63	0	5758	304	0	20	10509
N.S.	1	0.91	0.20	0.17	0.00	15.86	0.84	0.00	0.06	28.95
time (sec)	N/A	0.516	0.029	0.054	0.000	0.365	121.143	0.000	200.013	20.830

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	330	75	62	0	5030	0	0	20	16497
N.S.	1	0.90	0.21	0.17	0.00	13.78	0.00	0.00	0.05	45.20
time (sec)	N/A	0.500	0.031	0.059	0.000	0.459	0.000	0.000	200.020	23.898

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	465	170	170	0	15984	0	0	20	31085
N.S.	1	0.84	0.31	0.31	0.00	28.90	0.00	0.00	0.04	56.21
time (sec)	N/A	0.860	0.221	0.103	0.000	23.941	0.000	0.000	200.019	27.298

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	42	44	36	35	35	42	35	103	37
N.S.	1	0.95	1.00	0.82	0.80	0.80	0.95	0.80	2.34	0.84
time (sec)	N/A	0.202	0.010	0.069	0.107	0.067	0.089	0.126	0.164	19.669

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	50	98	43	42	40	51	42	58	43
N.S.	1	0.93	1.81	0.80	0.78	0.74	0.94	0.78	1.07	0.80
time (sec)	N/A	0.213	0.129	0.063	0.110	0.061	0.076	0.132	0.177	0.047

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	39	37	31	30	30	37	30	98	32
N.S.	1	1.05	1.00	0.84	0.81	0.81	1.00	0.81	2.65	0.86
time (sec)	N/A	0.200	0.006	0.055	0.107	0.062	0.069	0.116	0.153	0.045

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	83	94	62	61	61	76	61	98	51
N.S.	1	1.32	1.49	0.98	0.97	0.97	1.21	0.97	1.56	0.81
time (sec)	N/A	0.287	0.082	0.057	0.107	0.070	0.116	0.132	0.153	19.022

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	26	18	47	17
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	2.04	0.74
time (sec)	N/A	0.171	0.005	0.053	0.113	0.061	0.061	0.112	0.161	0.070

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	83	79	62	61	61	76	61	98	51
N.S.	1	1.32	1.25	0.98	0.97	0.97	1.21	0.97	1.56	0.81
time (sec)	N/A	0.260	0.034	0.055	0.117	0.066	0.128	0.116	0.155	19.217

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	133	31	36	32	41	36	100	34
N.S.	1	1.10	3.41	0.79	0.92	0.82	1.05	0.92	2.56	0.87
time (sec)	N/A	0.214	0.052	0.064	0.106	0.060	0.086	0.133	0.152	18.936

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	52	100	44	42	45	53	42	68	43
N.S.	1	0.96	1.85	0.81	0.78	0.83	0.98	0.78	1.26	0.80
time (sec)	N/A	0.222	0.029	0.073	0.109	0.064	0.093	0.132	0.165	0.038

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	50	136	38	41	49	48	46	135	41
N.S.	1	1.04	2.83	0.79	0.85	1.02	1.00	0.96	2.81	0.85
time (sec)	N/A	0.228	0.039	0.078	0.115	0.064	0.097	0.134	0.154	0.066

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	93	107	69	73	84	88	73	133	62
N.S.	1	1.21	1.39	0.90	0.95	1.09	1.14	0.95	1.73	0.81
time (sec)	N/A	0.333	0.050	0.086	0.122	0.064	0.141	0.114	0.151	19.040

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	172	139	92	0	107	192	109	99	100
N.S.	1	1.56	1.26	0.84	0.00	0.97	1.75	0.99	0.90	0.91
time (sec)	N/A	0.381	0.173	0.078	0.000	0.069	0.440	0.139	0.166	0.126

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	92	68	67	0	70	82	66	54	38
N.S.	1	1.37	1.01	1.00	0.00	1.04	1.22	0.99	0.81	0.57
time (sec)	N/A	0.280	0.018	0.069	0.000	0.068	0.098	0.114	0.163	0.084

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	160	135	89	0	106	197	108	98	99
N.S.	1	1.47	1.24	0.82	0.00	0.97	1.81	0.99	0.90	0.91
time (sec)	N/A	0.391	0.125	0.068	0.000	0.074	0.407	0.141	0.158	18.917

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	171	135	89	0	106	214	108	98	97
N.S.	1	1.57	1.24	0.82	0.00	0.97	1.96	0.99	0.90	0.89
time (sec)	N/A	0.359	0.103	0.063	0.000	0.067	0.449	0.114	0.165	0.069

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	92	68	67	0	70	82	66	54	40
N.S.	1	1.37	1.01	1.00	0.00	1.04	1.22	0.99	0.81	0.60
time (sec)	N/A	0.275	0.012	0.062	0.000	0.083	0.098	0.130	0.177	0.040

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	165	140	96	0	119	218	113	111	102
N.S.	1	1.45	1.23	0.84	0.00	1.04	1.91	0.99	0.97	0.89
time (sec)	N/A	0.415	0.145	0.082	0.000	0.068	0.477	0.117	0.177	0.052

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	178	148	96	0	135	197	113	127	104
N.S.	1	1.53	1.28	0.83	0.00	1.16	1.70	0.97	1.09	0.90
time (sec)	N/A	0.381	0.193	0.084	0.000	0.072	0.442	0.133	0.170	0.031

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	102	95	75	0	90	94	100	83	52
N.S.	1	1.32	1.23	0.97	0.00	1.17	1.22	1.30	1.08	0.68
time (sec)	N/A	0.328	0.027	0.089	0.000	0.067	0.112	0.116	0.190	0.040

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	174	171	94	0	141	209	120	133	110
N.S.	1	1.41	1.39	0.76	0.00	1.15	1.70	0.98	1.08	0.89
time (sec)	N/A	0.439	0.245	0.095	0.000	0.069	0.434	0.134	0.168	18.957

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	43	46	38	37	37	42	37	333	39
N.S.	1	0.93	1.00	0.83	0.80	0.80	0.91	0.80	7.24	0.85
time (sec)	N/A	0.209	0.010	0.053	0.111	0.065	0.075	0.121	0.164	19.086

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	57	55	44	0	47	48	99	85	29
N.S.	1	1.68	1.62	1.29	0.00	1.38	1.41	2.91	2.50	0.85
time (sec)	N/A	0.220	0.011	0.044	0.000	0.062	0.062	0.138	0.164	0.047

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	40	39	33	32	32	37	32	328	34
N.S.	1	1.03	1.00	0.85	0.82	0.82	0.95	0.82	8.41	0.87
time (sec)	N/A	0.207	0.006	0.039	0.106	0.065	0.069	0.134	0.163	18.649

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	90	98	32	0	62	70	76	336	53
N.S.	1	1.53	1.66	0.54	0.00	1.05	1.19	1.29	5.69	0.90
time (sec)	N/A	0.300	0.091	0.043	0.000	0.069	0.108	0.120	0.168	18.680

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	26	18	202	17
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	8.78	0.74
time (sec)	N/A	0.164	0.004	0.039	0.111	0.063	0.068	0.116	0.170	0.045

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	101	83	32	0	62	70	64	336	53
N.S.	1	1.71	1.41	0.54	0.00	1.05	1.19	1.08	5.69	0.90
time (sec)	N/A	0.278	0.037	0.042	0.000	0.064	0.109	0.141	0.160	19.100

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	44	55	33	38	34	41	38	330	36
N.S.	1	1.07	1.34	0.80	0.93	0.83	1.00	0.93	8.05	0.88
time (sec)	N/A	0.217	0.011	0.050	0.104	0.065	0.093	0.115	0.162	0.060

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	59	55	44	0	50	49	99	96	29
N.S.	1	1.74	1.62	1.29	0.00	1.47	1.44	2.91	2.82	0.85
time (sec)	N/A	0.220	0.013	0.059	0.000	0.060	0.082	0.134	0.173	19.077

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	49	51	38	43	51	48	48	375	41
N.S.	1	1.02	1.06	0.79	0.90	1.06	1.00	1.00	7.81	0.85
time (sec)	N/A	0.220	0.011	0.059	0.108	0.066	0.098	0.116	0.167	0.070

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	102	56	46	0	84	83	56	381	63
N.S.	1	1.40	0.77	0.63	0.00	1.15	1.14	0.77	5.22	0.86
time (sec)	N/A	0.339	0.015	0.069	0.000	0.068	0.134	0.134	0.185	19.498

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	348	59	44	0	338	26	254	301	209
N.S.	1	1.22	0.21	0.15	0.00	1.18	0.09	0.89	1.05	0.73
time (sec)	N/A	0.677	0.014	0.049	0.000	0.070	1.510	0.123	0.182	19.009

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	413	41	32	0	129	165	205	315	53
N.S.	1	1.89	0.19	0.15	0.00	0.59	0.75	0.94	1.44	0.24
time (sec)	N/A	0.598	0.011	0.041	0.000	0.068	0.132	0.136	0.169	0.102

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	363	39	40	0	305	24	253	407	474
N.S.	1	1.33	0.14	0.15	0.00	1.12	0.09	0.93	1.49	1.74
time (sec)	N/A	0.538	0.011	0.044	0.000	0.070	1.584	0.137	0.204	18.668

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	407	40	40	0	373	26	253	407	286
N.S.	1	1.49	0.15	0.15	0.00	1.37	0.10	0.93	1.49	1.05
time (sec)	N/A	0.539	0.012	0.046	0.000	0.077	1.499	0.128	0.162	19.105

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	411	42	30	0	129	165	205	315	53
N.S.	1	1.88	0.19	0.14	0.00	0.59	0.75	0.94	1.44	0.24
time (sec)	N/A	0.582	0.010	0.038	0.000	0.071	0.121	0.135	0.173	0.039

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	365	61	40	0	383	29	258	317	253
N.S.	1	1.26	0.21	0.14	0.00	1.32	0.10	0.89	1.09	0.87
time (sec)	N/A	0.668	0.014	0.060	0.000	0.073	1.658	0.135	0.181	19.364

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	354	65	38	0	367	31	258	341	213
N.S.	1	1.21	0.22	0.13	0.00	1.26	0.11	0.88	1.17	0.73
time (sec)	N/A	0.667	0.015	0.059	0.000	0.077	1.572	0.126	0.175	0.061

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	425	54	43	0	158	182	217	362	63
N.S.	1	1.84	0.23	0.19	0.00	0.68	0.79	0.94	1.57	0.27
time (sec)	N/A	0.633	0.016	0.061	0.000	0.071	0.144	0.144	0.182	19.153

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	377	54	44	0	340	37	265	466	486
N.S.	1	1.31	0.19	0.15	0.00	1.18	0.13	0.92	1.62	1.69
time (sec)	N/A	0.598	0.014	0.066	0.000	0.075	1.605	0.127	0.173	19.070

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	38	50	62	60	50	39	64
N.S.	1	1.00	0.92	0.61	0.81	1.00	0.97	0.81	0.63	1.03
time (sec)	N/A	0.252	0.027	0.057	0.105	0.065	0.086	0.163	0.172	18.667

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	102	97	42	0	54	54	66	44	130
N.S.	1	1.16	1.10	0.48	0.00	0.61	0.61	0.75	0.50	1.48
time (sec)	N/A	0.258	0.112	0.070	0.000	0.074	0.123	0.178	0.165	0.127

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	59	53	33	45	56	53	45	34	59
N.S.	1	1.07	0.96	0.60	0.82	1.02	0.96	0.82	0.62	1.07
time (sec)	N/A	0.241	0.016	0.043	0.105	0.062	0.066	0.161	0.170	0.133

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	99	75	34	0	45	49	47	18	117
N.S.	1	1.29	0.97	0.44	0.00	0.58	0.64	0.61	0.23	1.52
time (sec)	N/A	0.211	0.032	0.060	0.000	0.071	0.109	0.146	0.163	18.898

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	46	38	19	31	43	42	31	18	30
N.S.	1	0.98	0.81	0.40	0.66	0.91	0.89	0.66	0.38	0.64
time (sec)	N/A	0.199	0.009	0.040	0.107	0.061	0.060	0.155	0.161	18.876

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	80	74	34	0	45	49	41	16	125
N.S.	1	1.07	0.99	0.45	0.00	0.60	0.65	0.55	0.21	1.67
time (sec)	N/A	0.192	0.027	0.051	0.000	0.076	0.109	0.138	0.158	0.059

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	74	55	35	51	58	58	51	14	42
N.S.	1	1.30	0.96	0.61	0.89	1.02	1.02	0.89	0.25	0.74
time (sec)	N/A	0.260	0.025	0.051	0.105	0.063	0.084	0.167	0.171	0.193

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	104	65	42	0	67	56	68	51	130
N.S.	1	1.18	0.74	0.48	0.00	0.76	0.64	0.77	0.58	1.48
time (sec)	N/A	0.234	0.016	0.069	0.000	0.075	0.137	0.167	0.159	18.859

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	85	60	42	56	76	65	63	16	49
N.S.	1	1.02	0.72	0.51	0.67	0.92	0.78	0.76	0.19	0.59
time (sec)	N/A	0.316	0.023	0.056	0.111	0.063	0.095	0.149	0.164	0.111

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	111	73	48	0	74	65	77	56	136
N.S.	1	1.12	0.74	0.48	0.00	0.75	0.66	0.78	0.57	1.37
time (sec)	N/A	0.263	0.015	0.073	0.000	0.074	0.148	0.163	0.168	18.856

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	480	58	46	0	350	29	240	38	216
N.S.	1	1.40	0.17	0.13	0.00	1.02	0.08	0.70	0.11	0.63
time (sec)	N/A	0.762	0.015	0.043	0.000	0.074	1.152	0.198	0.160	18.969

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	463	41	40	0	385	26	239	18	149
N.S.	1	1.45	0.13	0.12	0.00	1.20	0.08	0.75	0.06	0.47
time (sec)	N/A	0.668	0.012	0.042	0.000	0.074	1.036	0.198	0.158	0.251

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	479	39	40	0	301	24	239	18	454
N.S.	1	1.43	0.12	0.12	0.00	0.90	0.07	0.71	0.05	1.36
time (sec)	N/A	0.677	0.011	0.043	0.000	0.075	0.988	0.190	0.176	0.238

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	451	40	40	0	385	26	239	18	275
N.S.	1	1.41	0.12	0.12	0.00	1.20	0.08	0.75	0.06	0.86
time (sec)	N/A	0.652	0.010	0.043	0.000	0.077	1.033	0.201	0.160	18.829

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	466	42	37	0	349	26	239	14	403
N.S.	1	1.53	0.14	0.12	0.00	1.15	0.09	0.79	0.05	1.33
time (sec)	N/A	0.679	0.010	0.043	0.000	0.073	1.101	0.146	0.162	0.083

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	468	61	42	0	395	32	244	48	292
N.S.	1	1.38	0.18	0.12	0.00	1.16	0.09	0.72	0.14	0.86
time (sec)	N/A	0.687	0.014	0.061	0.000	0.072	1.064	0.194	0.159	20.142

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	486	65	40	0	379	34	244	49	492
N.S.	1	1.40	0.19	0.11	0.00	1.09	0.10	0.70	0.14	1.41
time (sec)	N/A	0.717	0.015	0.059	0.000	0.073	1.063	0.176	0.157	20.264

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	38	50	62	58	53	123	64
N.S.	1	1.00	0.90	0.61	0.81	1.00	0.94	0.85	1.98	1.03
time (sec)	N/A	0.236	0.019	0.045	0.113	0.061	0.077	0.143	0.171	0.132

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	102	103	67	92	114	170	97	77	90
N.S.	1	1.16	1.17	0.76	1.05	1.30	1.93	1.10	0.88	1.02
time (sec)	N/A	0.238	0.040	0.044	0.105	0.068	0.235	0.153	0.169	0.108

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	59	53	33	45	57	53	48	118	59
N.S.	1	1.07	0.96	0.60	0.82	1.04	0.96	0.87	2.15	1.07
time (sec)	N/A	0.235	0.014	0.032	0.107	0.060	0.075	0.156	0.163	0.112

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	99	91	62	87	109	165	92	72	77
N.S.	1	1.30	1.20	0.82	1.14	1.43	2.17	1.21	0.95	1.01
time (sec)	N/A	0.216	0.026	0.041	0.105	0.070	0.228	0.148	0.177	0.112

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	46	38	19	31	43	42	33	118	30
N.S.	1	0.98	0.81	0.40	0.66	0.91	0.89	0.70	2.51	0.64
time (sec)	N/A	0.203	0.007	0.032	0.103	0.061	0.060	0.142	0.164	19.085

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	80	91	62	87	107	165	92	16	83
N.S.	1	1.07	1.21	0.83	1.16	1.43	2.20	1.23	0.21	1.11
time (sec)	N/A	0.198	0.022	0.041	0.102	0.066	0.211	0.130	0.158	20.301

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	71	55	64	51	59	58	54	14	42
N.S.	1	1.25	0.96	1.12	0.89	1.04	1.02	0.95	0.25	0.74
time (sec)	N/A	0.234	0.019	0.046	0.105	0.064	0.089	0.122	0.167	20.466

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	104	103	67	92	125	172	97	16	88
N.S.	1	1.18	1.17	0.76	1.05	1.42	1.95	1.10	0.18	1.00
time (sec)	N/A	0.234	0.040	0.049	0.103	0.066	0.240	0.129	0.172	19.405

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	78	61	71	56	76	66	66	16	49
N.S.	1	1.07	0.84	0.97	0.77	1.04	0.90	0.90	0.22	0.67
time (sec)	N/A	0.248	0.027	0.047	0.107	0.065	0.095	0.136	0.164	19.324

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	111	111	72	99	130	199	104	16	95
N.S.	1	1.12	1.12	0.73	1.00	1.31	2.01	1.05	0.16	0.96
time (sec)	N/A	0.264	0.049	0.055	0.107	0.082	0.265	0.134	0.160	0.127

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	160	69	0	200	58	148	170	246
N.S.	1	1.00	0.90	0.39	0.00	1.12	0.33	0.83	0.96	1.38
time (sec)	N/A	0.289	0.176	0.080	0.000	0.081	0.823	0.180	0.163	0.220

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	185	160	68	0	193	53	147	167	147
N.S.	1	1.11	0.96	0.41	0.00	1.16	0.32	0.88	1.00	0.88
time (sec)	N/A	0.295	0.110	0.059	0.000	0.074	0.822	0.173	0.173	19.529

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	177	132	60	0	175	49	147	165	269
N.S.	1	1.02	0.76	0.35	0.00	1.01	0.28	0.85	0.95	1.55
time (sec)	N/A	0.265	0.130	0.063	0.000	0.076	0.844	0.190	0.162	19.505

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	173	131	64	0	179	53	147	18	269
N.S.	1	1.19	0.90	0.44	0.00	1.23	0.37	1.01	0.12	1.86
time (sec)	N/A	0.267	0.034	0.058	0.000	0.078	0.850	0.186	0.159	0.082

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	164	160	68	0	199	53	147	14	245
N.S.	1	0.97	0.95	0.40	0.00	1.18	0.31	0.87	0.08	1.45
time (sec)	N/A	0.253	0.095	0.051	0.000	0.080	0.819	0.153	0.168	0.086

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	190	174	73	0	203	63	152	16	250
N.S.	1	1.06	0.97	0.41	0.00	1.13	0.35	0.84	0.09	1.39
time (sec)	N/A	0.308	0.191	0.064	0.000	0.080	0.846	0.190	0.159	19.272

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	184	166	73	0	223	63	152	16	268
N.S.	1	1.01	0.91	0.40	0.00	1.23	0.35	0.84	0.09	1.47
time (sec)	N/A	0.288	0.170	0.069	0.000	0.088	0.942	0.175	0.169	0.233

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	197	189	79	0	230	73	159	16	257
N.S.	1	1.14	1.09	0.46	0.00	1.33	0.42	0.92	0.09	1.49
time (sec)	N/A	0.368	0.185	0.077	0.000	0.090	0.842	0.189	0.168	19.364

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	189	79	0	222	70	159	16	291
N.S.	1	1.00	1.00	0.42	0.00	1.17	0.37	0.84	0.08	1.54
time (sec)	N/A	0.328	0.209	0.081	0.000	0.075	1.070	0.199	0.167	0.241

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	19	21	18	17	17	15	17	59	16
N.S.	1	0.90	1.00	0.86	0.81	0.81	0.71	0.81	2.81	0.76
time (sec)	N/A	0.189	0.004	0.033	0.030	0.060	0.067	0.136	0.171	19.279

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	22	26	23	22	22	19	22	64	22
N.S.	1	0.85	1.00	0.88	0.85	0.85	0.73	0.85	2.46	0.85
time (sec)	N/A	0.191	0.004	0.034	0.029	0.062	0.082	0.115	0.160	0.046

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	82	0	0	0	0	0	20	0
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.324	0.271	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	133	488	0	0	0	0	0	16	0
N.S.	1	1.05	3.84	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.251	1.085	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	133	79	0	0	0	0	0	18	0
N.S.	1	1.05	0.62	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.239	0.116	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	79	0	0	0	0	0	18	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.261	0.120	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	575	0	0	0	0	0	18	0
N.S.	1	1.00	4.91	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.235	0.966	0.000	0.000	0.000	0.000	0.000	0.157	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [81] had the largest ratio of [.8571429999999999988]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.20	16	0.312
2	A	5	4	0.82	16	0.250
3	A	6	5	1.09	16	0.312
4	A	5	4	0.85	16	0.250
5	A	5	4	1.16	16	0.250
6	A	2	2	1.00	16	0.125
7	A	5	4	1.16	14	0.286
8	A	5	4	1.00	16	0.250
9	A	6	5	1.09	16	0.312
10	A	5	4	0.95	16	0.250
11	A	7	6	1.14	16	0.375
12	A	6	6	1.14	16	0.375
13	A	6	6	1.32	16	0.375
14	A	5	5	1.17	16	0.312
15	A	5	5	1.19	16	0.312
16	A	5	5	1.17	16	0.312
17	A	5	5	1.19	12	0.417
18	A	6	6	1.15	16	0.375
19	A	6	6	1.21	16	0.375
20	A	6	5	1.14	16	0.312
21	A	5	4	0.86	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	6	5	1.10	16	0.312
23	A	5	4	0.82	16	0.250
24	A	5	4	1.17	16	0.250
25	A	2	2	1.00	16	0.125
26	A	5	4	1.17	14	0.286
27	A	5	4	1.00	16	0.250
28	A	6	5	1.17	16	0.312
29	A	5	4	1.00	16	0.250
30	A	7	6	1.15	16	0.375
31	A	12	11	1.36	16	0.688
32	A	12	11	1.43	16	0.688
33	A	11	10	1.40	16	0.625
34	A	11	10	1.41	16	0.625
35	A	11	10	1.40	16	0.625
36	A	11	10	1.41	12	0.833
37	A	12	11	1.37	16	0.688
38	A	12	11	1.40	16	0.688
39	A	5	4	1.06	26	0.154
40	A	4	4	0.60	26	0.154
41	A	5	4	1.00	26	0.154
42	A	2	2	1.00	16	0.125
43	A	2	2	1.00	16	0.125
44	A	2	2	1.00	26	0.077
45	A	4	3	0.99	18	0.167
46	A	5	4	1.01	18	0.222
47	A	6	5	1.02	18	0.278
48	A	4	3	1.04	18	0.167
49	A	4	3	1.00	18	0.167
50	A	4	3	1.03	16	0.188
51	A	8	7	1.06	18	0.389
52	A	6	5	0.98	18	0.278
53	A	6	5	1.07	18	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	7	7	0.90	18	0.389
55	A	5	5	0.90	18	0.278
56	A	5	5	0.97	18	0.278
57	A	4	4	0.96	18	0.222
58	A	5	5	0.98	18	0.278
59	A	4	4	0.95	14	0.286
60	A	7	7	0.91	18	0.389
61	A	6	6	0.90	18	0.333
62	A	8	8	0.84	18	0.444
63	A	4	3	0.95	14	0.214
64	A	6	5	0.93	14	0.357
65	A	6	5	1.05	14	0.357
66	A	9	8	1.32	14	0.571
67	A	4	3	1.00	14	0.214
68	A	8	7	1.32	12	0.583
69	A	8	7	1.10	14	0.500
70	A	7	6	0.96	14	0.429
71	A	6	5	1.04	14	0.357
72	A	12	11	1.21	14	0.786
73	A	9	8	1.56	14	0.571
74	A	8	7	1.37	14	0.500
75	A	9	8	1.47	14	0.571
76	A	8	7	1.57	14	0.500
77	A	8	7	1.37	10	0.700
78	A	11	10	1.45	14	0.714
79	A	10	9	1.53	14	0.643
80	A	11	10	1.32	14	0.714
81	A	13	12	1.41	14	0.857
82	A	4	3	0.93	16	0.188
83	A	6	5	1.68	16	0.312
84	A	7	6	1.03	16	0.375
85	A	9	8	1.53	16	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	3	1.00	16	0.188
87	A	8	7	1.71	14	0.500
88	A	8	7	1.07	16	0.438
89	A	6	5	1.74	16	0.312
90	A	5	4	1.02	16	0.250
91	A	12	11	1.40	16	0.688
92	A	10	9	1.22	16	0.562
93	A	8	7	1.89	16	0.438
94	A	9	8	1.33	16	0.500
95	A	8	7	1.49	16	0.438
96	A	8	7	1.88	12	0.583
97	A	12	11	1.26	16	0.688
98	A	11	10	1.21	16	0.625
99	A	11	10	1.84	16	0.625
100	A	13	12	1.31	16	0.750
101	A	4	3	1.00	16	0.188
102	A	5	4	1.16	16	0.250
103	A	4	3	1.07	16	0.188
104	A	4	3	1.29	16	0.188
105	A	4	3	0.98	16	0.188
106	A	4	3	1.07	14	0.214
107	A	4	3	1.30	16	0.188
108	A	6	5	1.18	16	0.312
109	A	4	3	1.02	16	0.188
110	A	7	6	1.12	16	0.375
111	A	11	10	1.40	16	0.625
112	A	10	9	1.45	16	0.562
113	A	10	9	1.43	16	0.562
114	A	10	9	1.41	16	0.562
115	A	10	9	1.53	12	0.750
116	A	12	11	1.38	16	0.688
117	A	12	11	1.40	16	0.688

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	4	3	1.00	16	0.188
119	A	5	4	1.16	16	0.250
120	A	4	3	1.07	16	0.188
121	A	4	3	1.30	16	0.188
122	A	4	3	0.98	16	0.188
123	A	4	3	1.07	14	0.214
124	A	4	3	1.25	16	0.188
125	A	5	4	1.18	16	0.250
126	A	4	3	1.07	16	0.188
127	A	8	7	1.12	16	0.438
128	A	5	5	1.00	16	0.312
129	A	5	5	1.11	16	0.312
130	A	4	4	1.02	16	0.250
131	A	5	5	1.19	16	0.312
132	A	4	4	0.97	12	0.333
133	A	6	6	1.06	16	0.375
134	A	6	6	1.01	16	0.375
135	A	9	9	1.14	16	0.562
136	A	8	8	1.00	16	0.500
137	A	4	3	0.90	16	0.188
138	A	4	3	0.85	16	0.188
139	A	3	3	1.00	18	0.167
140	A	3	3	1.05	14	0.214
141	A	3	3	1.05	16	0.188
142	A	3	3	1.00	16	0.188
143	A	3	3	1.00	16	0.188

CHAPTER 3

Listing of integrals

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3.1 $\int \frac{x^{13}}{1-2x^4+x^8}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{x^{13}}{1-2x^4+x^8}\,dx = x^2+\frac{x^6}{6}+\frac{x^2}{4(1-x^4)}-\frac{5\operatorname{arctanh}(x^2)}{4}$$

output
```
x^2+1/6*x^6+x^2/(-4*x^4+4)-5/4*arctanh(x^2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{x^{13}}{1-2x^4+x^8}\,dx = x^2+\frac{x^6}{6}-\frac{x^2}{4(-1+x^4)}+\frac{5}{8}\log\left(1-x^2\right)-\frac{5}{8}\log\left(1+x^2\right)$$

input
```
Integrate[x^13/(1 - 2*x^4 + x^8),x]
```

output
```
x^2 + x^6/6 - x^2/(4*(-1 + x^4)) + (5*Log[1 - x^2])/8 - (5*Log[1 + x^2])/8
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1380, 807, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^{13}}{(1-x^4)^2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^{12}}{(1-x^4)^2}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{x^{10}}{2(1-x^4)}-\frac{5}{2}\int \frac{x^8}{1-x^4}dx^2\right)$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{2}\left(\frac{x^{10}}{2(1-x^4)}-\frac{5}{2}\int \left(-x^4+\frac{1}{1-x^4}-1\right)dx^2\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{x^{10}}{2(1-x^4)}-\frac{5}{2}\left(\operatorname{arctanh}(x^2)-\frac{x^6}{3}-x^2\right)\right)$$

input
```
Int[x^13/(1 - 2*x^4 + x^8),x]
```

output
```
(x^10/(2*(1 - x^4)) - (5*(-x^2 - x^6/3 + ArcTanh[x^2]))/2)/2
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

method	result	size
risch	$\frac{x^6}{6}+x^2-\frac{x^2}{4(x^4-1)}-\frac{5\ln(x^2+1)}{8}+\frac{5\ln(x^2-1)}{8}$	38
default	$\frac{x^6}{6}+x^2-\frac{1}{8(x^2-1)}+\frac{5\ln(x^2-1)}{8}-\frac{1}{8(x^2+1)}-\frac{5\ln(x^2+1)}{8}$	44
norman	$\frac{-\frac{5}{4}x^2+\frac{5}{6}x^6+\frac{1}{6}x^{10}}{x^4-1}+\frac{5\ln(x-1)}{8}+\frac{5\ln(x+1)}{8}-\frac{5\ln(x^2+1)}{8}$	46
parallelrisch	$\frac{4x^{10}+20x^6+15\ln(x-1)x^4+15\ln(x+1)x^4-15\ln(x^2+1)x^4-30x^2-15\ln(x-1)-15\ln(x+1)+15\ln(x^2+1)}{24x^4-24}$	75

input
```
int(x^13/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6+x^2-1/4*x^2/(x^4-1)-5/8*ln(x^2+1)+5/8*ln(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int \frac{x^{13}}{1-2x^4+x^8}\,dx = \frac{4\,x^{10}+20\,x^6-30\,x^2-15\,(x^4-1)\log(x^2+1)+15\,(x^4-1)\log(x^2-1)}{24\,(x^4-1)}$$

input
```
integrate(x^13/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
1/24*(4*x^10 + 20*x^6 - 30*x^2 - 15*(x^4 - 1)*log(x^2 + 1) + 15*(x^4 - 1)*
log(x^2 - 1))/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{x^{13}}{1-2x^4+x^8}\,dx = \frac{x^6}{6}+x^2-\frac{x^2}{4x^4-4}+\frac{5\log(x^2-1)}{8}-\frac{5\log(x^2+1)}{8}$$

input
```
integrate(x**13/(x**8-2*x**4+1),x)
```

output
```
x**6/6 + x**2 - x**2/(4*x**4 - 4) + 5*log(x**2 - 1)/8 - 5*log(x**2 + 1)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{x^{13}}{1-2x^4+x^8}\,dx = \frac{1}{6}\,x^6 + x^2 - \frac{x^2}{4\,(x^4-1)} - \frac{5}{8}\,\log\left(x^2+1\right) + \frac{5}{8}\,\log\left(x^2-1\right)$$

input
```
integrate(x^13/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
1/6*x^6 + x^2 - 1/4*x^2/(x^4 - 1) - 5/8*log(x^2 + 1) + 5/8*log(x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{x^{13}}{1-2x^4+x^8}\,dx = \frac{1}{6}\,x^6 + x^2 - \frac{x^2}{4\,(x^4-1)} - \frac{5}{8}\,\log\left(x^2+1\right) + \frac{5}{8}\,\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(x^13/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
1/6*x^6 + x^2 - 1/4*x^2/(x^4 - 1) - 5/8*log(x^2 + 1) + 5/8*log(abs(x^2 - 1
))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{x^{13}}{1-2x^4+x^8}\,dx = x^2 - \frac{x^2}{4\,(x^4-1)} + \frac{x^6}{6} + \frac{\mathrm{atan}(x^2\,1\mathrm{i})\;5\mathrm{i}}{4}$$

input
```
int(x^13/(x^8 - 2*x^4 + 1),x)
```

output
```
(atan(x^2*1i)*5i)/4 - x^2/(4*(x^4 - 1)) + x^2 + x^6/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.14

$$\int \frac{x^{13}}{1-2x^4+x^8}\,dx$$
$$= \frac{-15\log(x^2+1)\,x^4+15\log(x^2+1)+15\log(x-1)\,x^4-15\log(x-1)+15\log(x+1)\,x^4-15\log(x+1}{24x^4-24}$$

input
```
int(x^13/(x^8-2*x^4+1),x)
```

output
```
( - 15*log(x**2 + 1)*x***4 + 15*log(x**2 + 1) + 15*log(x - 1)*x***4 - 15*log
(x - 1) + 15*log(x + 1)*x***4 - 15*log(x + 1) + 4*x**10 + 20*x***6 - 30*x**2
)/(24*(x**4 - 1))
```

3.2 $\int \frac{x^{11}}{1-2x^4+x^8} \, dx$

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Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{x^{11}}{1-2x^4+x^8} \, dx = \frac{x^4}{4}+\frac{1}{4(1-x^4)}+\frac{1}{2}\log\left(1-x^4\right)$$

output
```
1/4*x^4+1/(-4*x^4+4)+1/2*ln(-x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^{11}}{1-2x^4+x^8} \, dx = \frac{1}{4}\left(x^4+\frac{1}{1-x^4}+2\log\left(-1+x^4\right)\right)$$

input
```
Integrate[x^11/(1 - 2*x^4 + x^8),x]
```

output
```
(x^4 + (1 - x^4)^(-1) + 2*Log[-1 + x^4])/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^8-2x^4+1}dx$$

$$\Big\downarrow \ 1380$$

$$\int \frac{x^{11}}{\left(1-x^4\right)^2}dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4}\int \frac{x^8}{\left(1-x^4\right)^2}dx^4$$

$$\Big\downarrow \ 49$$

$$\frac{1}{4}\int \left(1+\frac{2}{x^4-1}+\frac{1}{\left(x^4-1\right)^2}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(x^4+\frac{1}{1-x^4}+2\log\left(1-x^4\right)\right)$$

input
```
Int[x^11/(1 - 2*x^4 + x^8),x]
```

output
```
(x^4 + (1 - x^4)^(-1) + 2*Log[1 - x^4])/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{x^4}{4}+\frac{\ln(x^4-1)}{2}-\frac{1}{4(x^4-1)}$	24
risch	$\frac{x^4}{4}+\frac{\ln(x^4-1)}{2}-\frac{1}{4(x^4-1)}$	24
norman	$\frac{\frac{x^8}{4}-\frac{1}{2}}{x^4-1}+\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}+\frac{\ln(x^2+1)}{2}$	37
parallelrisch	$\frac{x^8+2\ln(x-1)x^4+2\ln(x+1)x^4+2\ln(x^2+1)x^4-2-2\ln(x-1)-2\ln(x+1)-2\ln(x^2+1)}{4x^4-4}$	64

input
```
int(x^11/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+1/2*ln(x^4-1)-1/4/(x^4-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{1-2x^4+x^8}\,dx = \frac{x^8 - x^4 + 2\,(x^4-1)\log\left(x^4-1\right) - 1}{4\,(x^4-1)}$$

input
```
integrate(x^11/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
1/4*(x^8 - x^4 + 2*(x^4 - 1)*log(x^4 - 1) - 1)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

$$\int \frac{x^{11}}{1-2x^4+x^8}\,dx = \frac{x^4}{4} + \frac{\log\left(x^4-1\right)}{2} - \frac{1}{4x^4-4}$$

input
```
integrate(x**11/(x**8-2*x**4+1),x)
```

output
```
x**4/4 + log(x**4 - 1)/2 - 1/(4*x**4 - 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{x^{11}}{1-2x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{4\,(x^4-1)} + \frac{1}{2}\,\log\left(x^4-1\right)$$

input
```
integrate(x^11/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^4 - 1/4/(x^4 - 1) + 1/2*log(x^4 - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{x^{11}}{1-2x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{2\,x^4-1}{4\,(x^4-1)} + \frac{1}{2}\,\log\left(|x^4-1|\right)$$

input
```
integrate(x^11/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
1/4*x^4 - 1/4*(2*x^4 - 1)/(x^4 - 1) + 1/2*log(abs(x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^{11}}{1-2x^4+x^8}\,dx = \frac{\ln\left(x^4-1\right)}{2} - \frac{1}{4\,\left(x^4-1\right)} + \frac{x^4}{4}$$

input
```
int(x^11/(x^8 - 2*x^4 + 1),x)
```

output
```
log(x^4 - 1)/2 - 1/(4*(x^4 - 1)) + x^4/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.06

$$\int \frac{x^{11}}{1-2x^4+x^8}\,dx = \frac{2\log(x^2+1)\,x^4 - 2\log(x^2+1) + 2\log(x-1)\,x^4 - 2\log(x-1) + 2\log(x+1)\,x^4 - 2\log(x+1) + x^8 -}{4x^4-4}$$

input
```
int(x^11/(x^8-2*x^4+1),x)
```

output
```
(2*log(x**2 + 1)*x**4 - 2*log(x**2 + 1) + 2*log(x - 1)*x**4 - 2*log(x - 1)
 + 2*log(x + 1)*x**4 - 2*log(x + 1) + x**8 - 2*x**4)/(4*(x**4 - 1))
```

3.3 $\int \frac{x^9}{1-2x^4+x^8}\,dx$

Optimal result 89
Mathematica [A] (verified) 89
Rubi [A] (verified) 90
Maple [A] (verified) 92
Fricas [A] (verification not implemented) 92
Sympy [A] (verification not implemented) 93
Maxima [A] (verification not implemented) 93
Giac [A] (verification not implemented) 93
Mupad [B] (verification not implemented) 94
Reduce [B] (verification not implemented) 94

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{x^9}{1-2x^4+x^8}\,dx=\frac{x^2}{2}+\frac{x^2}{4(1-x^4)}-\frac{3\text{arctanh}(x^2)}{4}$$

output
```
1/2*x^2+x^2/(-4*x^4+4)-3/4*arctanh(x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{x^9}{1-2x^4+x^8}\,dx=\frac{1}{8}\left(2x^2\left(2+\frac{1}{1-x^4}\right)+3\log\left(1-x^2\right)-3\log\left(1+x^2\right)\right)$$

input
```
Integrate[x^9/(1 - 2*x^4 + x^8),x]
```

output
```
(2*x^2*(2 + (1 - x^4)^(-1)) + 3*Log[1 - x^2] - 3*Log[1 + x^2])/8
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1380, 807, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^9}{(1-x^4)^2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^8}{(1-x^4)^2}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{x^6}{2(1-x^4)}-\frac{3}{2}\int \frac{x^4}{1-x^4}dx^2\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{x^6}{2(1-x^4)}-\frac{3}{2}\left(\int \frac{1}{1-x^4}dx^2-x^2\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x^6}{2(1-x^4)}-\frac{3}{2}(\text{arctanh}(x^2)-x^2)\right)$$

input
```
Int[x^9/(1 - 2*x^4 + x^8),x]
```

output
```
(x^6/(2*(1 - x^4)) - (3*(-x^2 + ArcTanh[x^2]))/2)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

method	result	size
risch	$\frac{x^2}{2}-\frac{x^2}{4(x^4-1)}+\frac{3\ln(x^2-1)}{8}-\frac{3\ln(x^2+1)}{8}$	35
default	$\frac{x^2}{2}-\frac{1}{8(x^2-1)}+\frac{3\ln(x^2-1)}{8}-\frac{1}{8(x^2+1)}-\frac{3\ln(x^2+1)}{8}$	41
norman	$\frac{-\frac{3}{4}x^2+\frac{1}{2}x^6}{x^4-1}+\frac{3\ln(x-1)}{8}+\frac{3\ln(x+1)}{8}-\frac{3\ln(x^2+1)}{8}$	41
parallelrisch	$\frac{4x^6+3\ln(x-1)x^4+3\ln(x+1)x^4-3\ln(x^2+1)x^4-6x^2-3\ln(x-1)-3\ln(x+1)+3\ln(x^2+1)}{8x^4-8}$	70

input
```
int(x^9/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-1/4*x^2/(x^4-1)+3/8*ln(x^2-1)-3/8*ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{4\,x^6-6\,x^2-3\,(x^4-1)\log(x^2+1)+3\,(x^4-1)\log(x^2-1)}{8\,(x^4-1)}$$

input
```
integrate(x^9/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
1/8*(4*x^6 - 6*x^2 - 3*(x^4 - 1)*log(x^2 + 1) + 3*(x^4 - 1)*log(x^2 - 1))/
(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{x^2}{2} - \frac{x^2}{4x^4-4} + \frac{3\log(x^2-1)}{8} - \frac{3\log(x^2+1)}{8}$$

input
```
integrate(x**9/(x**8-2*x**4+1),x)
```

output
```
x**2/2 - x**2/(4*x**4 - 4) + 3*log(x**2 - 1)/8 - 3*log(x**2 + 1)/8
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{1}{2}x^2 - \frac{x^2}{4(x^4-1)} - \frac{3}{8}\log(x^2+1) + \frac{3}{8}\log(x^2-1)$$

input
```
integrate(x^9/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{1}{2}x^2 - \frac{x^2}{4(x^4-1)} - \frac{3}{8}\log(x^2+1) + \frac{3}{8}\log(|x^2-1|)$$

input
```
integrate(x^9/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{x^2}{2} - \frac{x^2}{4\,(x^4-1)} - \frac{3\,\mathrm{atanh}(x^2)}{4}$$

input
```
int(x^9/(x^8 - 2*x^4 + 1),x)
```

output
```
x^2/2 - x^2/(4*(x^4 - 1)) - (3*atanh(x^2))/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

$$\int \frac{x^9}{1-2x^4+x^8}\,dx = \frac{-3\log(x^2+1)\,x^4+3\log(x^2+1)+3\log(x-1)\,x^4-3\log(x-1)+3\log(x+1)\,x^4-3\log(x+1)+4x}{8x^4-8}$$

input
```
int(x^9/(x^8-2*x^4+1),x)
```

output
```
( - 3*log(x**2 + 1)*x**4 + 3*log(x**2 + 1) + 3*log(x - 1)*x**4 - 3*log(x -
 1) + 3*log(x + 1)*x**4 - 3*log(x + 1) + 4*x**6 - 6*x**2)/(8*(x**4 - 1))
```

3.4 $\int \frac{x^7}{1-2x^4+x^8} \, dx$

Optimal result 95
Mathematica [A] (verified) 95
Rubi [A] (verified) 96
Maple [A] (verified) 97
Fricas [A] (verification not implemented) 98
Sympy [A] (verification not implemented) 98
Maxima [A] (verification not implemented) 98
Giac [A] (verification not implemented) 99
Mupad [B] (verification not implemented) 99
Reduce [B] (verification not implemented) 99

Optimal result

Integrand size = 16, antiderivative size = 26

$$\int \frac{x^7}{1-2x^4+x^8} \, dx = \frac{1}{4(1-x^4)} + \frac{1}{4}\log\left(1-x^4\right)$$

output
```
1/(-4*x^4+4)+1/4*ln(-x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{1-2x^4+x^8} \, dx = -\frac{1}{4(-1+x^4)} + \frac{1}{4}\log\left(-1+x^4\right)$$

input
```
Integrate[x^7/(1 - 2*x^4 + x^8),x]
```

output
```
-1/4*1/(-1 + x^4) + Log[-1 + x^4]/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^7}{(1-x^4)^2}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^4}{(1-x^4)^2}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\int \left(\frac{1}{x^4-1}+\frac{1}{(x^4-1)^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{1-x^4}+\log\left(1-x^4\right)\right)$$

input
```
Int[x^7/(1 - 2*x^4 + x^8),x]
```

output
```
((1 - x^4)^(-1) + Log[1 - x^4])/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\ln(x^4-1)}{4}-\frac{1}{4(x^4-1)}$	19
risch	$\frac{\ln(x^4-1)}{4}-\frac{1}{4(x^4-1)}$	19
norman	$-\frac{1}{4(x^4-1)}+\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}+\frac{\ln(x^2+1)}{4}$	31
parallelrisch	$\frac{\ln(x-1)x^4+\ln(x+1)x^4+\ln(x^2+1)x^4-1-\ln(x-1)-\ln(x+1)-\ln(x^2+1)}{4x^4-4}$	58

input
```
int(x^7/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^4-1)-1/4/(x^4-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = \frac{(x^4-1)\log(x^4-1)-1}{4\,(x^4-1)}$$

input
```
integrate(x^7/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
1/4*((x^4 - 1)*log(x^4 - 1) - 1)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = \frac{\log(x^4-1)}{4} - \frac{1}{4x^4-4}$$

input
```
integrate(x**7/(x**8-2*x**4+1),x)
```

output
```
log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = -\frac{1}{4\,(x^4-1)} + \frac{1}{4}\log(x^4-1)$$

input
```
integrate(x^7/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4/(x^4 - 1) + 1/4*log(x^4 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = -\frac{1}{4\left(x^4-1\right)}+\frac{1}{4}\log\left(\left|x^4-1\right|\right)$$

input
```
integrate(x^7/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4/(x^4 - 1) + 1/4*log(abs(x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = \frac{\ln\left(x^4-1\right)}{4}-\frac{1}{4\left(x^4-1\right)}$$

input
```
int(x^7/(x^8 - 2*x^4 + 1),x)
```

output
```
log(x^4 - 1)/4 - 1/(4*(x^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{x^7}{1-2x^4+x^8}\,dx$$
$$= \frac{\log(x^2+1)\,x^4-\log(x^2+1)+\log(x-1)\,x^4-\log(x-1)+\log(x+1)\,x^4-\log(x+1)-x^4}{4x^4-4}$$

input
```
int(x^7/(x^8-2*x^4+1),x)
```

output
```
(log(x**2 + 1)*x**4 - log(x**2 + 1) + log(x - 1)*x**4 - log(x - 1) + log(x
 + 1)*x**4 - log(x + 1) - x**4)/(4*(x**4 - 1))
```

3.5 $\int \frac{x^5}{1-2x^4+x^8} \, dx$

Optimal result 100
Mathematica [A] (verified) 100
Rubi [A] (verified) 101
Maple [A] (verified) 102
Fricas [B] (verification not implemented) 103
Sympy [A] (verification not implemented) 103
Maxima [A] (verification not implemented) 103
Giac [A] (verification not implemented) 104
Mupad [B] (verification not implemented) 104
Reduce [B] (verification not implemented) 104

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{x^5}{1-2x^4+x^8} \, dx = \frac{x^2}{4(1-x^4)} - \frac{\operatorname{arctanh}(x^2)}{4}$$

output
```
x^2/(-4*x^4+4)-1/4*arctanh(x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{x^5}{1-2x^4+x^8} \, dx = \frac{1}{8}\left(-\frac{2x^2}{-1+x^4} + \log\left(1-x^2\right) - \log\left(1+x^2\right)\right)$$

input
```
Integrate[x^5/(1 - 2*x^4 + x^8),x]
```

output
```
((-2*x^2)/(-1 + x^4) + Log[1 - x^2] - Log[1 + x^2])/8
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 807, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^8-2x^4+1} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^5}{\left(1-x^4\right)^2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{x^4}{\left(1-x^4\right)^2}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2} \left(\frac{x^2}{2 \left(1-x^4\right)}-\frac{1}{2} \int \frac{1}{1-x^4}dx^2\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2} \left(\frac{x^2}{2 \left(1-x^4\right)}-\frac{\text{arctanh}\left(x^2\right)}{2}\right)$$

input
```
Int[x^5/(1 - 2*x^4 + x^8),x]
```

output
```
(x^2/(2*(1 - x^4)) - ArcTanh[x^2]/2)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
risch	$-\frac{x^2}{4(x^4-1)}+\frac{\ln(x^2-1)}{8}-\frac{\ln(x^2+1)}{8}$	30
norman	$-\frac{x^2}{4(x^4-1)}+\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}-\frac{\ln(x^2+1)}{8}$	34
default	$-\frac{1}{8(x^2-1)}+\frac{\ln(x^2-1)}{8}-\frac{1}{8(x^2+1)}-\frac{\ln(x^2+1)}{8}$	36
parallelrisch	$\frac{\ln(x-1)x^4+\ln(x+1)x^4-\ln(x^2+1)x^4-2x^2-\ln(x-1)-\ln(x+1)+\ln(x^2+1)}{8x^4-8}$	61

input
```
int(x^5/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2/(x^4-1)+1/8*ln(x^2-1)-1/8*ln(x^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = -\frac{2\,x^2+(x^4-1)\log(x^2+1)-(x^4-1)\log(x^2-1)}{8\,(x^4-1)}$$

input
```
integrate(x^5/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/8*(2*x^2 + (x^4 - 1)*log(x^2 + 1) - (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = -\frac{x^2}{4x^4-4}+\frac{\log(x^2-1)}{8}-\frac{\log(x^2+1)}{8}$$

input
```
integrate(x**5/(x**8-2*x**4+1),x)
```

output
```
-x**2/(4*x**4 - 4) + log(x**2 - 1)/8 - log(x**2 + 1)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = -\frac{x^2}{4\,(x^4-1)}-\frac{1}{8}\log(x^2+1)+\frac{1}{8}\log(x^2-1)$$

input
```
integrate(x^5/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = -\frac{x^2}{4\,(x^4-1)} - \frac{1}{8}\log\left(x^2+1\right) + \frac{1}{8}\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(x^5/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = -\frac{\operatorname{atanh}(x^2)}{4} - \frac{x^2}{4\,(x^4-1)}$$

input
```
int(x^5/(x^8 - 2*x^4 + 1),x)
```

output
```
- atanh(x^2)/4 - x^2/(4*(x^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.44

$$\int \frac{x^5}{1-2x^4+x^8}\,dx = \frac{-\log(x^2+1)\,x^4+\log(x^2+1)+\log(x-1)\,x^4-\log(x-1)+\log(x+1)\,x^4-\log(x+1)-2x^2}{8x^4-8}$$

input
```
int(x^5/(x^8-2*x^4+1),x)
```

output
```
( - log(x**2 + 1)*x**4 + log(x**2 + 1) + log(x - 1)*x**4 - log(x - 1) + lo
g(x + 1)*x**4 - log(x + 1) - 2*x**2)/(8*(x**4 - 1))
```

3.6 $\int \frac{x^3}{1-2x^4+x^8}\,dx$

Optimal result 106
Mathematica [A] (verified) 106
Rubi [A] (verified) 107
Maple [A] (verified) 108
Fricas [A] (verification not implemented) 108
Sympy [A] (verification not implemented) 109
Maxima [A] (verification not implemented) 109
Giac [A] (verification not implemented) 109
Mupad [B] (verification not implemented) 110
Reduce [B] (verification not implemented) 110

Optimal result

Integrand size = 16, antiderivative size = 13

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = \frac{1}{4(1-x^4)}$$

output
```
1/(-4*x^4+4)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4(-1+x^4)}$$

input
```
Integrate[x^3/(1 - 2*x^4 + x^8),x]
```

output
```
-1/4*1/(-1 + x^4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1380, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{ 1380}$$

$$\int \frac{x^3}{(1-x^4)^2}dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{1}{4(1-x^4)}$$

input
```
Int[x^3/(1 - 2*x^4 + x^8),x]
```

output
```
1/(4*(1 - x^4))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{1}{4(x^4-1)}$	10
default	$-\frac{1}{4(x^4-1)}$	10
norman	$-\frac{1}{4(x^4-1)}$	10
risch	$-\frac{1}{4(x^4-1)}$	10
parallelrisch	$-\frac{1}{4(x^4-1)}$	10
orering	$-\frac{(x-1)(x+1)(x^2+1)}{4(x^8-2x^4+1)}$	26

input
```
int(x^3/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(x^4-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4\,(x^4-1)}$$

input
```
integrate(x^3/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4x^4-4}$$

input
```
integrate(x**3/(x**8-2*x**4+1),x)
```

output
```
-1/(4*x**4 - 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4\,(x^4-1)}$$

input
```
integrate(x^3/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4/(x^4 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4\,(x^4-1)}$$

input
```
integrate(x^3/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4/(x^4 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{1}{4\left(x^4-1\right)}$$

input
```
int(x^3/(x^8 - 2*x^4 + 1),x)
```

output
```
-1/(4*(x^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{1-2x^4+x^8}\,dx = -\frac{x^4}{4x^4-4}$$

input
```
int(x^3/(x^8-2*x^4+1),x)
```

output
```
( - x**4)/(4*(x**4 - 1))
```

3.7 $\int \frac{x}{1-2x^4+x^8} \, dx$

Optimal result 111
Mathematica [A] (verified) 111
Rubi [A] (verified) 112
Maple [A] (verified) 113
Fricas [B] (verification not implemented) 114
Sympy [A] (verification not implemented) 114
Maxima [A] (verification not implemented) 114
Giac [A] (verification not implemented) 115
Mupad [B] (verification not implemented) 115
Reduce [B] (verification not implemented) 115

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{x}{1-2x^4+x^8} \, dx = \frac{x^2}{4(1-x^4)} + \frac{\operatorname{arctanh}(x^2)}{4}$$

output
```
x^2/(-4*x^4+4)+1/4*arctanh(x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{x}{1-2x^4+x^8} \, dx = \frac{1}{8}\left(-\frac{2x^2}{-1+x^4} - \log\left(1-x^2\right) + \log\left(1+x^2\right)\right)$$

input
```
Integrate[x/(1 - 2*x^4 + x^8),x]
```

output
```
((-2*x^2)/(-1 + x^4) - Log[1 - x^2] + Log[1 + x^2])/8
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1380, 807, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x}{(1-x^4)^2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{(1-x^4)^2}dx^2$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-x^4}dx^2+\frac{x^2}{2(1-x^4)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\text{arctanh}(x^2)}{2}+\frac{x^2}{2(1-x^4)}\right)$$

input
```
Int[x/(1 - 2*x^4 + x^8),x]
```

output
```
(x^2/(2*(1 - x^4)) + ArcTanh[x^2]/2)/2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
risch	$-\frac{x^2}{4(x^4-1)}+\frac{\ln(x^2+1)}{8}-\frac{\ln(x^2-1)}{8}$	30
norman	$-\frac{x^2}{4(x^4-1)}-\frac{\ln(x-1)}{8}-\frac{\ln(x+1)}{8}+\frac{\ln(x^2+1)}{8}$	34
default	$-\frac{1}{8(x^2-1)}-\frac{\ln(x^2-1)}{8}-\frac{1}{8(x^2+1)}+\frac{\ln(x^2+1)}{8}$	36
parallelrisch	$-\frac{\ln(x-1)x^4+\ln(x+1)x^4-\ln(x^2+1)x^4+2x^2-\ln(x-1)-\ln(x+1)+\ln(x^2+1)}{8(x^4-1)}$	61

input
```
int(x/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2/(x^4-1)+1/8*ln(x^2+1)-1/8*ln(x^2-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(19) = 38$.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{x}{1-2x^4+x^8}\,dx = -\frac{2\,x^2-(x^4-1)\log(x^2+1)+(x^4-1)\log(x^2-1)}{8\,(x^4-1)}$$

input
```
integrate(x/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/8*(2*x^2 - (x^4 - 1)*log(x^2 + 1) + (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{x}{1-2x^4+x^8}\,dx = -\frac{x^2}{4x^4-4} - \frac{\log(x^2-1)}{8} + \frac{\log(x^2+1)}{8}$$

input
```
integrate(x/(x**8-2*x**4+1),x)
```

output
```
-x**2/(4*x**4 - 4) - log(x**2 - 1)/8 + log(x**2 + 1)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x}{1-2x^4+x^8}\,dx = -\frac{x^2}{4\,(x^4-1)} + \frac{1}{8}\log(x^2+1) - \frac{1}{8}\log(x^2-1)$$

input
```
integrate(x/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{x}{1-2x^4+x^8}\,dx = -\frac{x^2}{4\,(x^4-1)} + \frac{1}{8}\log\left(x^2+1\right) - \frac{1}{8}\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(x/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x}{1-2x^4+x^8}\,dx = \frac{\operatorname{atanh}(x^2)}{4} - \frac{x^2}{4\,(x^4-1)}$$

input
```
int(x/(x^8 - 2*x^4 + 1),x)
```

output
```
atanh(x^2)/4 - x^2/(4*(x^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{x}{1-2x^4+x^8}\,dx = \frac{\log(x^2+1)\,x^4 - \log(x^2+1) - \log(x-1)\,x^4 + \log(x-1) - \log(x+1)\,x^4 + \log(x+1) - 2x^2}{8x^4-8}$$

input
```
int(x/(x^8-2*x^4+1),x)
```

output
```
(log(x**2 + 1)*x**4 - log(x**2 + 1) - log(x - 1)*x**4 + log(x - 1) - log(x
 + 1)*x**4 + log(x + 1) - 2*x**2)/(8*(x**4 - 1))
```

3.8 $\int \frac{1}{x(1-2x^4+x^8)}\,dx$

Optimal result 116
Mathematica [A] (verified) 116
Rubi [A] (verified) 117
Maple [A] (verified) 118
Fricas [A] (verification not implemented) 119
Sympy [A] (verification not implemented) 119
Maxima [A] (verification not implemented) 119
Giac [A] (verification not implemented) 120
Mupad [B] (verification not implemented) 120
Reduce [B] (verification not implemented) 120

Optimal result

Integrand size = 16, antiderivative size = 28

$$\int \frac{1}{x(1-2x^4+x^8)}\,dx = \frac{1}{4(1-x^4)} + \log(x) - \frac{1}{4}\log\left(1-x^4\right)$$

output
```
1/(-4*x^4+4)+ln(x)-1/4*ln(-x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x(1-2x^4+x^8)}\,dx = -\frac{1}{4(-1+x^4)} + \log(x) - \frac{1}{4}\log\left(1-x^4\right)$$

input
```
Integrate[1/(x*(1 - 2*x^4 + x^8)),x]
```

output
```
-1/4*1/(-1 + x^4) + Log[x] - Log[1 - x^4]/4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1380, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^8-2x^4+1)}dx$$
$$\Big\downarrow \ 1380$$
$$\int \frac{1}{x(1-x^4)^2}dx$$
$$\Big\downarrow \ 798$$
$$\frac{1}{4}\int \frac{1}{x^4(1-x^4)^2}dx^4$$
$$\Big\downarrow \ 54$$
$$\frac{1}{4}\int \left(\frac{1}{(x^4-1)^2}+\frac{1}{x^4}+\frac{1}{1-x^4}\right)dx^4$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{4}\left(\frac{1}{1-x^4}+\log\left(x^4\right)-\log\left(1-x^4\right)\right)$$

input
```
Int[1/(x*(1 - 2*x^4 + x^8)),x]
```

output
```
((1 - x^4)^(-1) + Log[x^4] - Log[1 - x^4])/4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{1}{4(x^4-1)}+\ln\left(x\right)-\frac{\ln(x^4-1)}{4}$	21
norman	$-\frac{1}{4(x^4-1)}-\frac{\ln(x-1)}{4}-\frac{\ln(x+1)}{4}-\frac{\ln(x^2+1)}{4}+\ln\left(x\right)$	33
default	$-\frac{1}{16(x-1)}-\frac{\ln(x-1)}{4}-\frac{\ln(x^2+1)}{4}+\frac{1}{8x^2+8}+\frac{1}{16x+16}-\frac{\ln(x+1)}{4}+\ln\left(x\right)$	47
parallelrisch	$\frac{4\ln(x)x^4-\ln(x-1)x^4-\ln(x+1)x^4-\ln(x^2+1)x^4-1-4\ln(x)+\ln(x-1)+\ln(x+1)+\ln(x^2+1)}{4x^4-4}$	66

input
```
int(1/x/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(x^4-1)+ln(x)-1/4*ln(x^4-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx = -\frac{(x^4-1)\log\left(x^4-1\right)-4\,(x^4-1)\log\left(x\right)+1}{4\,(x^4-1)}$$

input
```
integrate(1/x/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*((x^4 - 1)*log(x^4 - 1) - 4*(x^4 - 1)*log(x) + 1)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx = \log\left(x\right)-\frac{\log\left(x^4-1\right)}{4}-\frac{1}{4x^4-4}$$

input
```
integrate(1/x/(x**8-2*x**4+1),x)
```

output
```
log(x) - log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx = -\frac{1}{4\,(x^4-1)}-\frac{1}{4}\log\left(x^4-1\right)+\frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4/(x^4 - 1) - 1/4*log(x^4 - 1) + 1/4*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx=\frac{x^4-2}{4\left(x^4-1\right)}+\frac{1}{4}\log\left(x^4\right)-\frac{1}{4}\log\left(\left|x^4-1\right|\right)$$

input
```
integrate(1/x/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
1/4*(x^4 - 2)/(x^4 - 1) + 1/4*log(x^4) - 1/4*log(abs(x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx=\ln\left(x\right)-\frac{\ln\left(x^4-1\right)}{4}-\frac{1}{4\left(x^4-1\right)}$$

input
```
int(1/(x*(x^8 - 2*x^4 + 1)),x)
```

output
```
log(x) - log(x^4 - 1)/4 - 1/(4*(x^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\left(1-2x^4+x^8\right)}\,dx$$
$$=\frac{-\log(x^2+1)\,x^4+\log(x^2+1)-\log(x-1)\,x^4+\log(x-1)-\log(x+1)\,x^4+\log(x+1)+4\log(x)\,x^4-}{4x^4-4}$$

input
```
int(1/x/(x^8-2*x^4+1),x)
```

output
```
( - log(x**2 + 1)*x**4 + log(x**2 + 1) - log(x - 1)*x**4 + log(x - 1) - lo
g(x + 1)*x**4 + log(x + 1) + 4*log(x)*x**4 - 4*log(x) - x**4)/(4*(x**4 - 1
))
```

3.9 $\int \frac{1}{x^3(1-2x^4+x^8)}\,dx$

Optimal result 121
Mathematica [A] (verified) 121
Rubi [A] (verified) 122
Maple [A] (verified) 124
Fricas [B] (verification not implemented) 124
Sympy [A] (verification not implemented) 125
Maxima [A] (verification not implemented) 125
Giac [A] (verification not implemented) 125
Mupad [B] (verification not implemented) 126
Reduce [B] (verification not implemented) 126

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx = -\frac{1}{2x^2}+\frac{x^2}{4(1-x^4)}+\frac{3\text{arctanh}(x^2)}{4}$$

output
```
-1/2/x^2+x^2/(-4*x^4+4)+3/4*arctanh(x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx = \frac{1}{8}\left(\frac{4-6x^4}{x^2(-1+x^4)}-3\log\left(1-x^2\right)+3\log\left(1+x^2\right)\right)$$

input
```
Integrate[1/(x^3*(1 - 2*x^4 + x^8)),x]
```

output
```
((4 - 6*x^4)/(x^2*(-1 + x^4)) - 3*Log[1 - x^2] + 3*Log[1 + x^2])/8
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1380, 807, 253, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^8 - 2x^4 + 1)} \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\int \frac{1}{x^3 (1 - x^4)^2} dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2} \int \frac{1}{x^4 (1 - x^4)^2} dx^2$$

$$\Big\downarrow \text{ 253}$$

$$\frac{1}{2} \left(\frac{3}{2} \int \frac{1}{x^4 (1 - x^4)} dx^2 + \frac{1}{2x^2 (1 - x^4)} \right)$$

$$\Big\downarrow \text{ 264}$$

$$\frac{1}{2} \left(\frac{3}{2} \left(\int \frac{1}{1 - x^4} dx^2 - \frac{1}{x^2} \right) + \frac{1}{2x^2 (1 - x^4)} \right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2} \left(\frac{3}{2} \left(\text{arctanh}(x^2) - \frac{1}{x^2} \right) + \frac{1}{2x^2 (1 - x^4)} \right)$$

input
```
Int[1/(x^3*(1 - 2*x^4 + x^8)),x]
```

output
```
(1/(2*x^2*(1 - x^4)) + (3*(-x^(-2) + ArcTanh[x^2]))/2)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

method	result	size
risch	$\frac{\frac{1}{2}-\frac{3x^4}{4}}{x^2(x^4-1)}+\frac{3\ln(x^2+1)}{8}-\frac{3\ln(x^2-1)}{8}$	36
norman	$\frac{\frac{1}{2}-\frac{3x^4}{4}}{x^2(x^4-1)}-\frac{3\ln(x-1)}{8}-\frac{3\ln(x+1)}{8}+\frac{3\ln(x^2+1)}{8}$	40
default	$-\frac{1}{16(x-1)}-\frac{3\ln(x-1)}{8}+\frac{3\ln(x^2+1)}{8}-\frac{1}{8(x^2+1)}+\frac{1}{16x+16}-\frac{3\ln(x+1)}{8}-\frac{1}{2x^2}$	50
parallelrisch	$-\frac{3\ln(x-1)x^6+3\ln(x+1)x^6-3\ln(x^2+1)x^6-4+6x^4-3\ln(x-1)x^2-3\ln(x+1)x^2+3\ln(x^2+1)x^2}{8x^2(x^4-1)}$	78

input
```
int(1/x^3/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(1/2-3/4*x^4)/x^2/(x^4-1)+3/8*ln(x^2+1)-3/8*ln(x^2-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^3(1-2x^4+x^8)}\,dx=-\frac{6\,x^4-3\,(x^6-x^2)\log\left(x^2+1\right)+3\,(x^6-x^2)\log\left(x^2-1\right)-4}{8\,(x^6-x^2)}$$

input
```
integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/8*(6*x^4 - 3*(x^6 - x^2)*log(x^2 + 1) + 3*(x^6 - x^2)*log(x^2 - 1) - 4)
/(x^6 - x^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^3\left(1-2x^4+x^8\right)}\,dx = \frac{2-3x^4}{4x^6-4x^2} - \frac{3\log\left(x^2-1\right)}{8} + \frac{3\log\left(x^2+1\right)}{8}$$

input
```
integrate(1/x**3/(x**8-2*x**4+1),x)
```

output
```
(2 - 3*x**4)/(4*x**6 - 4*x**2) - 3*log(x**2 - 1)/8 + 3*log(x**2 + 1)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3\left(1-2x^4+x^8\right)}\,dx = -\frac{3\,x^4-2}{4\left(x^6-x^2\right)} + \frac{3}{8}\log\left(x^2+1\right) - \frac{3}{8}\log\left(x^2-1\right)$$

input
```
integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^3\left(1-2x^4+x^8\right)}\,dx = -\frac{3\,x^4-2}{4\left(x^6-x^2\right)} + \frac{3}{8}\log\left(x^2+1\right) - \frac{3}{8}\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\left(1-2x^4+x^8\right)}\,dx=\frac{3\,\mathrm{atanh}\left(x^2\right)}{4}+\frac{\frac{3\,x^4}{4}-\frac{1}{2}}{x^2-x^6}$$

input
```
int(1/(x^3*(x^8 - 2*x^4 + 1)),x)
```

output
```
(3*atanh(x^2))/4 + ((3*x^4)/4 - 1/2)/(x^2 - x^6)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.41

$$\int \frac{1}{x^3\left(1-2x^4+x^8\right)}\,dx$$
$$=\frac{3\log\left(x^2+1\right)x^6-3\log\left(x^2+1\right)x^2-3\log\left(x-1\right)x^6+3\log\left(x-1\right)x^2-3\log\left(x+1\right)x^6+3\log\left(x+1\right)x^2}{8x^2\left(x^4-1\right)}$$

input
```
int(1/x^3/(x^8-2*x^4+1),x)
```

output
```
(3*log(x**2 + 1)*x**6 - 3*log(x**2 + 1)*x**2 - 3*log(x - 1)*x**6 + 3*log(x
 - 1)*x**2 - 3*log(x + 1)*x**6 + 3*log(x + 1)*x**2 - 6*x**4 + 4)/(8*x**2*(
x**4 - 1))
```

3.10 $\int \frac{1}{x^5(1-2x^4+x^8)}\,dx$

Optimal result 127
Mathematica [A] (verified) 127
Rubi [A] (verified) 128
Maple [A] (verified) 129
Fricas [A] (verification not implemented) 130
Sympy [A] (verification not implemented) 130
Maxima [A] (verification not implemented) 130
Giac [A] (verification not implemented) 131
Mupad [B] (verification not implemented) 131
Reduce [B] (verification not implemented) 131

Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \frac{1}{x^5(1-2x^4+x^8)}\,dx = -\frac{1}{4x^4}+\frac{1}{4(1-x^4)}+2\log(x)-\frac{1}{2}\log\left(1-x^4\right)$$

output
```
-1/4/x^4+1/(-4*x^4+4)+2*ln(x)-1/2*ln(-x^4+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5(1-2x^4+x^8)}\,dx = -\frac{1}{4x^4}-\frac{1}{4(-1+x^4)}+2\log(x)-\frac{1}{2}\log\left(1-x^4\right)$$

input
```
Integrate[1/(x^5*(1 - 2*x^4 + x^8)),x]
```

output
```
-1/4*1/x^4 - 1/(4*(-1 + x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1380, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^8-2x^4+1)} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^5 (1-x^4)^2}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^8 (1-x^4)^2}dx^4$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{4}\int \left(\frac{2}{x^4}+\frac{1}{x^8}-\frac{2}{x^4-1}+\frac{1}{(x^4-1)^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{1-x^4}-\frac{1}{x^4}+2\log\left(x^4\right)-2\log\left(1-x^4\right)\right)$$

input
```
Int[1/(x^5*(1 - 2*x^4 + x^8)),x]
```

output
```
(-x^(-4) + (1 - x^4)^(-1) + 2*Log[x^4] - 2*Log[1 - x^4])/4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4-1)}+2\ln(x)-\frac{\ln(x^4-1)}{2}$	32
norman	$\frac{\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4-1)}+2\ln(x)-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{2}$	44
default	$-\frac{1}{16(x-1)}-\frac{\ln(x-1)}{2}-\frac{\ln(x^2+1)}{2}+\frac{1}{8x^2+8}+\frac{1}{16x+16}-\frac{\ln(x+1)}{2}-\frac{1}{4x^4}+2\ln(x)$	54
parallelrisch	$\frac{8\ln(x)x^8-2\ln(x-1)x^8-2\ln(x+1)x^8-2\ln(x^2+1)x^8+1-8\ln(x)x^4+2\ln(x-1)x^4+2\ln(x+1)x^4+2\ln(x^2+1)x^4-2x^4}{4x^4(x^4-1)}$	92

input
```
int(1/x^5/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(1/4-1/2*x^4)/x^4/(x^4-1)+2*ln(x)-1/2*ln(x^4-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^5 (1-2x^4+x^8)} \, dx = -\frac{2 x^4 + 2 (x^8-x^4) \log (x^4-1) - 8 (x^8-x^4) \log (x) - 1}{4 (x^8-x^4)}$$

input
```
integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*(2*x^4 + 2*(x^8 - x^4)*log(x^4 - 1) - 8*(x^8 - x^4)*log(x) - 1)/(x^8
- x^4)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^5 (1-2x^4+x^8)} \, dx = \frac{1-2x^4}{4x^8-4x^4} + 2 \log (x) - \frac{\log (x^4-1)}{2}$$

input
```
integrate(1/x**5/(x**8-2*x**4+1),x)
```

output
```
(1 - 2*x**4)/(4*x**8 - 4*x**4) + 2*log(x) - log(x**4 - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5 (1-2x^4+x^8)} \, dx = -\frac{2 x^4-1}{4 (x^8-x^4)} - \frac{1}{2} \log \left(x^4-1\right) + \frac{1}{2} \log \left(x^4\right)$$

input
```
integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*(2*x^4 - 1)/(x^8 - x^4) - 1/2*log(x^4 - 1) + 1/2*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^5 (1-2x^4+x^8)}\,dx = -\frac{2\,x^4-1}{4\,(x^8-x^4)}+\frac{1}{2}\,\log\left(x^4\right)-\frac{1}{2}\,\log\left(\left|x^4-1\right|\right)$$

input
```
integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*(2*x^4 - 1)/(x^8 - x^4) + 1/2*log(x^4) - 1/2*log(abs(x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5 (1-2x^4+x^8)}\,dx = 2\,\ln\left(x\right)-\frac{\ln\left(x^4-1\right)}{2}+\frac{\frac{x^4}{2}-\frac{1}{4}}{x^4-x^8}$$

input
```
int(1/(x^5*(x^8 - 2*x^4 + 1)),x)
```

output
```
2*log(x) - log(x^4 - 1)/2 + (x^4/2 - 1/4)/(x^4 - x^8)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^5 (1-2x^4+x^8)}\,dx = \frac{-2\log(x^2+1)\,x^8+2\log(x^2+1)\,x^4-2\log(x-1)\,x^8+2\log(x-1)\,x^4-2\log(x+1)\,x^8+2\log(x+1)}{4x^4\,(x^4-1)}$$

input
```
int(1/x^5/(x^8-2*x^4+1),x)
```

output
```
( - 2*log(x**2 + 1)*x***8 + 2*log(x**2 + 1)*x***4 - 2*log(x - 1)*x***8 + 2*lo
g(x - 1)*x***4 - 2*log(x + 1)*x***8 + 2*log(x + 1)*x***4 + 8*log(x)*x***8 - 8*
log(x)*x***4 - 2*x***8 + 1)/(4*x***4*(x***4 - 1))
```

3.11 $\int \frac{1}{x^7(1-2x^4+x^8)}\,dx$

Optimal result 133
Mathematica [A] (verified) 133
Rubi [A] (verified) 134
Maple [A] (verified) 135
Fricas [B] (verification not implemented) 136
Sympy [A] (verification not implemented) 136
Maxima [A] (verification not implemented) 137
Giac [A] (verification not implemented) 137
Mupad [B] (verification not implemented) 137
Reduce [B] (verification not implemented) 138

Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \frac{1}{x^7(1-2x^4+x^8)}\,dx = -\frac{1}{6x^6} - \frac{1}{x^2} + \frac{x^2}{4(1-x^4)} + \frac{5\operatorname{arctanh}(x^2)}{4}$$

output
```
-1/6/x^6-1/x^2+x^2/(-4*x^4+4)+5/4*arctanh(x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^7(1-2x^4+x^8)}\,dx = -\frac{1}{6x^6} - \frac{1}{x^2} - \frac{x^2}{4(-1+x^4)} - \frac{5}{8}\log\left(1-x^2\right) + \frac{5}{8}\log\left(1+x^2\right)$$

input
```
Integrate[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

output
```
-1/6*1/x^6 - x^(-2) - x^2/(4*(-1 + x^4)) - (5*Log[1 - x^2])/8 + (5*Log[1 +
 x^2])/8
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1380, 807, 253, 264, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (x^8-2x^4+1)}dx$$
$$\Big\downarrow \text{1380}$$
$$\int \frac{1}{x^7 (1-x^4)^2}dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}\int \frac{1}{x^8 (1-x^4)^2}dx^2$$
$$\Big\downarrow \text{253}$$
$$\frac{1}{2}\left(\frac{5}{2}\int \frac{1}{x^8 (1-x^4)}dx^2+\frac{1}{2x^6 (1-x^4)}\right)$$
$$\Big\downarrow \text{264}$$
$$\frac{1}{2}\left(\frac{5}{2}\left(\int \frac{1}{x^4 (1-x^4)}dx^2-\frac{1}{3x^6}\right)+\frac{1}{2x^6 (1-x^4)}\right)$$
$$\Big\downarrow \text{264}$$
$$\frac{1}{2}\left(\frac{5}{2}\left(\int \frac{1}{1-x^4}dx^2-\frac{1}{3x^6}-\frac{1}{x^2}\right)+\frac{1}{2x^6 (1-x^4)}\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{2}\left(\frac{5}{2}\left(\text{arctanh}(x^2)-\frac{1}{3x^6}-\frac{1}{x^2}\right)+\frac{1}{2x^6 (1-x^4)}\right)$$

input
```
Int[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

output
```
(1/(2*x^6*(1 - x^4)) + (5*(-1/3*1/x^6 - x^(-2) + ArcTanh[x^2]))/2)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{\frac{1}{6}+\frac{5}{6}x^4-\frac{5}{4}x^8}{x^6(x^4-1)}+\frac{5\ln(x^2+1)}{8}-\frac{5\ln(x^2-1)}{8}$	41
norman	$\frac{\frac{1}{6}+\frac{5}{6}x^4-\frac{5}{4}x^8}{x^6(x^4-1)}-\frac{5\ln(x-1)}{8}-\frac{5\ln(x+1)}{8}+\frac{5\ln(x^2+1)}{8}$	45
default	$-\frac{1}{16(x-1)}-\frac{5\ln(x-1)}{8}+\frac{5\ln(x^2+1)}{8}-\frac{1}{8(x^2+1)}+\frac{1}{16x+16}-\frac{5\ln(x+1)}{8}-\frac{1}{6x^6}-\frac{1}{x^2}$	55
parallelrisch	$-\frac{15\ln(x-1)x^{10}+15\ln(x+1)x^{10}-15\ln(x^2+1)x^{10}-4+30x^8-15\ln(x-1)x^6-15\ln(x+1)x^6+15\ln(x^2+1)x^6-20x^4}{24x^6(x^4-1)}$	83

input
```
int(1/x^7/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(1/6+5/6*x^4-5/4*x^8)/x^6/(x^4-1)+5/8*ln(x^2+1)-5/8*ln(x^2-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(29) = 58$.

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^7 (1 - 2x^4 + x^8)} \, dx = -\frac{30\,x^8 - 20\,x^4 - 15\,(x^{10} - x^6)\log(x^2 + 1) + 15\,(x^{10} - x^6)\log(x^2 - 1) - 4}{24\,(x^{10} - x^6)}$$

input
```
integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/24*(30*x^8 - 20*x^4 - 15*(x^10 - x^6)*log(x^2 + 1) + 15*(x^10 - x^6)*lo
g(x^2 - 1) - 4)/(x^10 - x^6)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^7 (1 - 2x^4 + x^8)} \, dx = -\frac{5\log(x^2 - 1)}{8} + \frac{5\log(x^2 + 1)}{8} + \frac{-15x^8 + 10x^4 + 2}{12x^{10} - 12x^6}$$

input
```
integrate(1/x**7/(x**8-2*x**4+1),x)
```

output
```
-5*log(x**2 - 1)/8 + 5*log(x**2 + 1)/8 + (-15*x**8 + 10*x**4 + 2)/(12*x**1
0 - 12*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^7 (1-2x^4+x^8)}\,dx = -\frac{15\,x^8-10\,x^4-2}{12\,(x^{10}-x^6)} + \frac{5}{8}\,\log\left(x^2+1\right) - \frac{5}{8}\,\log\left(x^2-1\right)$$

input
```
integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/12*(15*x^8 - 10*x^4 - 2)/(x^10 - x^6) + 5/8*log(x^2 + 1) - 5/8*log(x^2
- 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^7 (1-2x^4+x^8)}\,dx = -\frac{x^2}{4\,(x^4-1)} - \frac{6\,x^4+1}{6\,x^6} + \frac{5}{8}\,\log\left(x^2+1\right) - \frac{5}{8}\,\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^2/(x^4 - 1) - 1/6*(6*x^4 + 1)/x^6 + 5/8*log(x^2 + 1) - 5/8*log(abs(
x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^7 (1-2x^4+x^8)}\,dx = \frac{5\,\mathrm{atanh}(x^2)}{4} - \frac{-\frac{5\,x^8}{4}+\frac{5\,x^4}{6}+\frac{1}{6}}{x^6-x^{10}}$$

input
```
int(1/(x^7*(x^8 - 2*x^4 + 1)),x)
```

output
```
(5*atanh(x^2))/4 - ((5*x^4)/6 - (5*x^8)/4 + 1/6)/(x^6 - x^10)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.22

$$\int \frac{1}{x^7 (1 - 2x^4 + x^8)} \, dx = \frac{15 \log(x^2+1) \, x^{10} - 15 \log(x^2+1) \, x^6 - 15 \log(x-1) \, x^{10} + 15 \log(x-1) \, x^6 - 15 \log(x+1) \, x^{10} + 15 \log}{24x^6 (x^4 - 1)}$$

input
```
int(1/x^7/(x^8-2*x^4+1),x)
```

output
```
(15*log(x**2 + 1)*x**10 - 15*log(x**2 + 1)*x**6 - 15*log(x - 1)*x**10 + 15
*log(x - 1)*x**6 - 15*log(x + 1)*x**10 + 15*log(x + 1)*x**6 - 30*x**8 + 20
*x**4 + 4)/(24*x**6*(x**4 - 1))
```

3.12 $\int \frac{x^{10}}{1-2x^4+x^8}\,dx$

Optimal result 139
Mathematica [A] (verified) 139
Rubi [A] (verified) 140
Maple [A] (verified) 142
Fricas [A] (verification not implemented) 142
Sympy [A] (verification not implemented) 143
Maxima [A] (verification not implemented) 143
Giac [A] (verification not implemented) 143
Mupad [B] (verification not implemented) 144
Reduce [B] (verification not implemented) 144

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \frac{x^{10}}{1-2x^4+x^8}\,dx = \frac{x^3}{3}+\frac{x^3}{4\left(1-x^4\right)}+\frac{7\arctan(x)}{8}-\frac{7\text{arctanh}(x)}{8}$$

output
```
1/3*x^3+x^3/(-4*x^4+4)+7/8*arctan(x)-7/8*arctanh(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{x^{10}}{1-2x^4+x^8}\,dx = \frac{1}{48}\left(16x^3-\frac{12x^3}{-1+x^4}+42\arctan(x)+21\log(1-x)-21\log(1+x)\right)$$

input
```
Integrate[x^10/(1 - 2*x^4 + x^8),x]
```

output
```
(16*x^3 - (12*x^3)/(-1 + x^4) + 42*ArcTan[x] + 21*Log[1 - x] - 21*Log[1 +
x])/48
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1380, 817, 843, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^{10}}{(1-x^4)^2}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{x^7}{4(1-x^4)}-\frac{7}{4}\int \frac{x^6}{1-x^4}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x^7}{4(1-x^4)}-\frac{7}{4}\left(\int \frac{x^2}{1-x^4}dx-\frac{x^3}{3}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{x^7}{4(1-x^4)}-\frac{7}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx-\frac{x^3}{3}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{x^7}{4(1-x^4)}-\frac{7}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}-\frac{x^3}{3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{x^7}{4(1-x^4)}-\frac{7}{4}\left(-\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}-\frac{x^3}{3}\right)$$

input
```
Int[x^10/(1 - 2*x^4 + x^8),x]
```

output
```
x^7/(4*(1 - x^4)) - (7*(-1/3*x^3 - ArcTan[x]/2 + ArcTanh[x]/2))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result
risch	$\frac{x^3}{3}-\frac{x^3}{4(x^4-1)}+\frac{7\ln(x-1)}{16}-\frac{7\ln(x+1)}{16}+\frac{7\arctan(x)}{8}$
default	$\frac{x^3}{3}-\frac{1}{16(x-1)}+\frac{7\ln(x-1)}{16}-\frac{x}{8(x^2+1)}+\frac{7\arctan(x)}{8}-\frac{1}{16(x+1)}-\frac{7\ln(x+1)}{16}$
parallelrisch	$\frac{16x^7-21i\ln(x-i)x^4+21i\ln(x+i)x^4+21\ln(x-1)x^4-21\ln(x+1)x^4-28x^3+21i\ln(x-i)-21i\ln(x+i)-21\ln(x-1)+21\ln(x+1}{48x^4-48}$

input
```
int(x^10/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3-1/4*x^3/(x^4-1)+7/16*ln(x-1)-7/16*ln(x+1)+7/8*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{x^{10}}{1-2x^4+x^8}\,dx = \frac{16\,x^7-28\,x^3+42\,(x^4-1)\arctan(x)-21\,(x^4-1)\log(x+1)+21\,(x^4-1)\log(x-1)}{48\,(x^4-1)}$$

input
```
integrate(x^10/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
1/48*(16*x^7 - 28*x^3 + 42*(x^4 - 1)*arctan(x) - 21*(x^4 - 1)*log(x + 1) +
 21*(x^4 - 1)*log(x - 1))/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{x^{10}}{1-2x^4+x^8}\,dx = \frac{x^3}{3} - \frac{x^3}{4x^4-4} + \frac{7\log(x-1)}{16} - \frac{7\log(x+1)}{16} + \frac{7\operatorname{atan}(x)}{8}$$

input
```
integrate(x**10/(x**8-2*x**4+1),x)
```

output
```
x**3/3 - x**3/(4*x**4 - 4) + 7*log(x - 1)/16 - 7*log(x + 1)/16 + 7*atan(x)
/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{x^{10}}{1-2x^4+x^8}\,dx = \frac{1}{3}x^3 - \frac{x^3}{4(x^4-1)} + \frac{7}{8}\arctan(x) - \frac{7}{16}\log(x+1) + \frac{7}{16}\log(x-1)$$

input
```
integrate(x^10/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
1/3*x^3 - 1/4*x^3/(x^4 - 1) + 7/8*arctan(x) - 7/16*log(x + 1) + 7/16*log(x
 - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{x^{10}}{1-2x^4+x^8}\,dx = \frac{1}{3}x^3 - \frac{x^3}{4(x^4-1)} + \frac{7}{8}\arctan(x) - \frac{7}{16}\log(|x+1|) + \frac{7}{16}\log(|x-1|)$$

input
```
integrate(x^10/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
1/3*x^3 - 1/4*x^3/(x^4 - 1) + 7/8*arctan(x) - 7/16*log(abs(x + 1)) + 7/16*
log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 20.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^{10}}{1-2x^4+x^8}\,dx = \frac{7\,\mathrm{atan}(x)}{8} - \frac{x^3}{4\,(x^4-1)} + \frac{x^3}{3} + \frac{\mathrm{atan}(x\,1\mathrm{i})\;7\mathrm{i}}{8}$$

input
```
int(x^10/(x^8 - 2*x^4 + 1),x)
```

output
```
(atan(x*1i)*7i)/8 + (7*atan(x))/8 - x^3/(4*(x^4 - 1)) + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.72

$$\int \frac{x^{10}}{1-2x^4+x^8}\,dx$$
$$= \frac{42 atan(x)\,x^4 - 42 atan(x) + 21\log(x-1)\,x^4 - 21\log(x-1) - 21\log(x+1)\,x^4 + 21\log(x+1) + 16x^7}{48x^4-48}$$

input
```
int(x^10/(x^8-2*x^4+1),x)
```

output
```
(42*atan(x)*x**4 - 42*atan(x) + 21*log(x - 1)*x**4 - 21*log(x - 1) - 21*lo
g(x + 1)*x**4 + 21*log(x + 1) + 16*x**7 - 28*x**3)/(48*(x**4 - 1))
```

3.13 $\int \frac{x^8}{1-2x^4+x^8} \, dx$

Optimal result 145
Mathematica [A] (verified) 145
Rubi [A] (verified) 146
Maple [A] (verified) 148
Fricas [B] (verification not implemented) 148
Sympy [A] (verification not implemented) 149
Maxima [A] (verification not implemented) 149
Giac [A] (verification not implemented) 149
Mupad [B] (verification not implemented) 150
Reduce [B] (verification not implemented) 150

Optimal result

Integrand size = 16, antiderivative size = 28

$$\int \frac{x^8}{1-2x^4+x^8} \, dx = x+\frac{x}{4(1-x^4)}-\frac{5\arctan(x)}{8}-\frac{5\text{arctanh}(x)}{8}$$

output
```
x+x/(-4*x^4+4)-5/8*arctan(x)-5/8*arctanh(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x^8}{1-2x^4+x^8} \, dx = x-\frac{x}{4(-1+x^4)}-\frac{5\arctan(x)}{8}+\frac{5}{16}\log(1-x)-\frac{5}{16}\log(1+x)$$

input
```
Integrate[x^8/(1 - 2*x^4 + x^8),x]
```

output
```
x - x/(4*(-1 + x^4)) - (5*ArcTan[x])/8 + (5*Log[1 - x])/16 - (5*Log[1 + x]
)/16
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1380, 817, 843, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^8}{\left(1-x^4\right)^2}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{x^5}{4\left(1-x^4\right)}-\frac{5}{4}\int \frac{x^4}{1-x^4}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x^5}{4\left(1-x^4\right)}-\frac{5}{4}\left(\int \frac{1}{1-x^4}dx-x\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{x^5}{4\left(1-x^4\right)}-\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{1}{2}\int \frac{1}{x^2+1}dx-x\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{x^5}{4\left(1-x^4\right)}-\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{\arctan(x)}{2}-x\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{x^5}{4\left(1-x^4\right)}-\frac{5}{4}\left(\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}-x\right)$$

input
```
Int[x^8/(1 - 2*x^4 + x^8),x]
```

output
```
x^5/(4*(1 - x^4)) - (5*(-x + ArcTan[x]/2 + ArcTanh[x]/2))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
risch	$x-\frac{x}{4(x^4-1)}+\frac{5\ln(x-1)}{16}-\frac{5\arctan(x)}{8}-\frac{5\ln(x+1)}{16}$	29
default	$x-\frac{1}{16(x-1)}+\frac{5\ln(x-1)}{16}+\frac{x}{8x^2+8}-\frac{5\arctan(x)}{8}-\frac{1}{16(x+1)}-\frac{5\ln(x+1)}{16}$	43
parallelrisch	$\frac{5i\ln(x-i)x^4-5i\ln(x+i)x^4+5\ln(x-1)x^4-5\ln(x+1)x^4+16x^5-5i\ln(x-i)+5i\ln(x+i)-5\ln(x-1)+5\ln(x+1)-20x}{16x^4-16}$	87

input
```
int(x^8/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
x-1/4*x/(x^4-1)+5/16*ln(x-1)-5/8*arctan(x)-5/16*ln(x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(20) = 40.

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = \frac{16\,x^5-10\,(x^4-1)\arctan(x)-5\,(x^4-1)\log(x+1)+5\,(x^4-1)\log(x-1)-20\,x}{16\,(x^4-1)}$$

input
```
integrate(x^8/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
1/16*(16*x^5 - 10*(x^4 - 1)*arctan(x) - 5*(x^4 - 1)*log(x + 1) + 5*(x^4 -
1)*log(x - 1) - 20*x)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = x - \frac{x}{4x^4-4} + \frac{5\log(x-1)}{16} - \frac{5\log(x+1)}{16} - \frac{5\,\mathrm{atan}(x)}{8}$$

input
```
integrate(x**8/(x**8-2*x**4+1),x)
```

output
```
x - x/(4*x**4 - 4) + 5*log(x - 1)/16 - 5*log(x + 1)/16 - 5*atan(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = x - \frac{x}{4(x^4-1)} - \frac{5}{8}\arctan(x) - \frac{5}{16}\log(x+1) + \frac{5}{16}\log(x-1)$$

input
```
integrate(x^8/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(x + 1) + 5/16*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = x - \frac{x}{4(x^4-1)} - \frac{5}{8}\arctan(x) - \frac{5}{16}\log(|x+1|) + \frac{5}{16}\log(|x-1|)$$

input
```
integrate(x^8/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(abs(x + 1)) + 5/16*log(abs(
x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{1-2x^4+x^8}\,dx = x - \frac{5\,\mathrm{atan}(x)}{8} - \frac{x}{4\,(x^4-1)} + \frac{\mathrm{atan}(x\,1\mathrm{i})\;5\mathrm{i}}{8}$$

input
```
int(x^8/(x^8 - 2*x^4 + 1),x)
```

output
```
x + (atan(x*1i)*5i)/8 - (5*atan(x))/8 - x/(4*(x^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \frac{x^8}{1-2x^4+x^8}\,dx$$
$$= \frac{-10atan(x)\,x^4 + 10atan(x) + 5\log(x-1)\,x^4 - 5\log(x-1) - 5\log(x+1)\,x^4 + 5\log(x+1) + 16x^5 - 2}{16x^4-16}$$

input
```
int(x^8/(x^8-2*x^4+1),x)
```

output
```
( - 10*atan(x)*x**4 + 10*atan(x) + 5*log(x - 1)*x**4 - 5*log(x - 1) - 5*lo
g(x + 1)*x**4 + 5*log(x + 1) + 16*x**5 - 20*x)/(16*(x**4 - 1))
```

3.14 $\int \frac{x^6}{1-2x^4+x^8} \, dx$

Optimal result 151
Mathematica [A] (verified) 151
Rubi [A] (verified) 152
Maple [A] (verified) 153
Fricas [B] (verification not implemented) 154
Sympy [A] (verification not implemented) 154
Maxima [A] (verification not implemented) 155
Giac [A] (verification not implemented) 155
Mupad [B] (verification not implemented) 155
Reduce [B] (verification not implemented) 156

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{x^6}{1-2x^4+x^8} \, dx = \frac{x^3}{4(1-x^4)} + \frac{3\arctan(x)}{8} - \frac{3\text{arctanh}(x)}{8}$$

output
```
x^3/(-4*x^4+4)+3/8*arctan(x)-3/8*arctanh(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{x^6}{1-2x^4+x^8} \, dx = \frac{1}{16}\left(-\frac{4x^3}{-1+x^4} + 6\arctan(x) + 3\log(1-x) - 3\log(1+x)\right)$$

input
```
Integrate[x^6/(1 - 2*x^4 + x^8),x]
```

output
```
((-4*x^3)/(-1 + x^4) + 6*ArcTan[x] + 3*Log[1 - x] - 3*Log[1 + x])/16
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1380, 817, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{ 1380}$$

$$\int \frac{x^6}{(1-x^4)^2}dx$$

$$\Big\downarrow \text{ 817}$$

$$\frac{x^3}{4(1-x^4)}-\frac{3}{4}\int \frac{x^2}{1-x^4}dx$$

$$\Big\downarrow \text{ 827}$$

$$\frac{x^3}{4(1-x^4)}-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{x^3}{4(1-x^4)}-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x^3}{4(1-x^4)}-\frac{3}{4}\left(\frac{\text{arctanh}(x)}{2}-\frac{\arctan(x)}{2}\right)$$

input
```
Int[x^6/(1 - 2*x^4 + x^8),x]
```

output
```
x^3/(4*(1 - x^4)) - (3*(-1/2*ArcTan[x] + ArcTanh[x]/2))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{x^3}{4(x^4-1)}-\frac{3\ln(x+1)}{16}+\frac{3\arctan(x)}{8}+\frac{3\ln(x-1)}{16}$	30
default	$-\frac{1}{16(x-1)}+\frac{3\ln(x-1)}{16}-\frac{x}{8(x^2+1)}+\frac{3\arctan(x)}{8}-\frac{1}{16(x+1)}-\frac{3\ln(x+1)}{16}$	42
parallelrisch	$\frac{-3i\ln(x-i)x^4+3i\ln(x+i)x^4+3\ln(x-1)x^4-3\ln(x+1)x^4-4x^3+3i\ln(x-i)-3i\ln(x+i)-3\ln(x-1)+3\ln(x+1)}{16x^4-16}$	84

input
```
int(x^6/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^3/(x^4-1)-3/16*ln(x+1)+3/8*arctan(x)+3/16*ln(x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = -\frac{4\,x^3-6\,(x^4-1)\arctan(x)+3\,(x^4-1)\log(x+1)-3\,(x^4-1)\log(x-1)}{16\,(x^4-1)}$$

input
```
integrate(x^6/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/16*(4*x^3 - 6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1
)*log(x - 1))/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = -\frac{x^3}{4x^4-4}+\frac{3\log(x-1)}{16}-\frac{3\log(x+1)}{16}+\frac{3\,\mathrm{atan}\,(x)}{8}$$

input
```
integrate(x**6/(x**8-2*x**4+1),x)
```

output
```
-x**3/(4*x**4 - 4) + 3*log(x - 1)/16 - 3*log(x + 1)/16 + 3*atan(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4-1)} + \frac{3}{8}\arctan(x) - \frac{3}{16}\log(x+1) + \frac{3}{16}\log(x-1)$$

input
```
integrate(x^6/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(x + 1) + 3/16*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4-1)} + \frac{3}{8}\arctan(x) - \frac{3}{16}\log(|x+1|) + \frac{3}{16}\log(|x-1|)$$

input
```
integrate(x^6/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(abs(x + 1)) + 3/16*log(abs(x
 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x^6}{1-2x^4+x^8}\,dx = \frac{3\,\mathrm{atan}(x)}{8} - \frac{3\,\mathrm{atanh}(x)}{8} - \frac{x^3}{4\,(x^4-1)}$$

input
```
int(x^6/(x^8 - 2*x^4 + 1),x)
```

output
```
(3*atan(x))/8 - (3*atanh(x))/8 - x^3/(4*(x^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

$$\int \frac{x^6}{1-2x^4+x^8} \, dx$$
$$= \frac{6atan(x)\,x^4 - 6atan(x) + 3\log(x-1)\,x^4 - 3\log(x-1) - 3\log(x+1)\,x^4 + 3\log(x+1) - 4x^3}{16x^4 - 16}$$

input
```
int(x^6/(x^8-2*x^4+1),x)
```

output
```
(6*atan(x)*x**4 - 6*atan(x) + 3*log(x - 1)*x**4 - 3*log(x - 1) - 3*log(x +
 1)*x**4 + 3*log(x + 1) - 4*x**3)/(16*(x**4 - 1))
```

3.15 $\int \frac{x^4}{1-2x^4+x^8}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{x^4}{1-2x^4+x^8}\,dx=\frac{x}{4(1-x^4)}-\frac{\arctan(x)}{8}-\frac{\operatorname{arctanh}(x)}{8}$$

output
```
x/(-4*x^4+4)-1/8*arctan(x)-1/8*arctanh(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{x^4}{1-2x^4+x^8}\,dx=\frac{1}{16}\left(-\frac{4x}{-1+x^4}-2\arctan(x)+\log(1-x)-\log(1+x)\right)$$

input
```
Integrate[x^4/(1 - 2*x^4 + x^8),x]
```

output
```
((-4*x)/(-1 + x^4) - 2*ArcTan[x] + Log[1 - x] - Log[1 + x])/16
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1380, 817, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^4}{(1-x^4)^2}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{x}{4(1-x^4)}-\frac{1}{4}\int \frac{1}{1-x^4}dx$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx\right)+\frac{x}{4(1-x^4)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}\right)+\frac{x}{4(1-x^4)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(-\frac{\arctan(x)}{2}-\frac{\text{arctanh}(x)}{2}\right)+\frac{x}{4(1-x^4)}$$

input
```
Int[x^4/(1 - 2*x^4 + x^8),x]
```

output
```
x/(4*(1 - x^4)) + (-1/2*ArcTan[x] - ArcTanh[x]/2)/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{x}{4(x^4-1)}-\frac{\arctan(x)}{8}-\frac{\ln(x+1)}{16}+\frac{\ln(x-1)}{16}$	28
default	$-\frac{1}{16(x-1)}+\frac{\ln(x-1)}{16}+\frac{x}{8x^2+8}-\frac{\arctan(x)}{8}-\frac{1}{16(x+1)}-\frac{\ln(x+1)}{16}$	42
parallelrisch	$\frac{i\ln(x-i)x^4-i\ln(x+i)x^4+\ln(x-1)x^4-\ln(x+1)x^4-i\ln(x-i)+i\ln(x+i)-\ln(x-1)+\ln(x+1)-4x}{16x^4-16}$	79

input
```
int(x^4/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x/(x^4-1)-1/8*arctan(x)-1/16*ln(x+1)+1/16*ln(x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(19) = 38$.

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{2\,(x^4-1)\arctan(x)+(x^4-1)\log(x+1)-(x^4-1)\log(x-1)+4\,x}{16\,(x^4-1)}$$

input
```
integrate(x^4/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/16*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*log(x - 1)
 + 4*x)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{x}{4x^4-4}+\frac{\log(x-1)}{16}-\frac{\log(x+1)}{16}-\frac{\operatorname{atan}(x)}{8}$$

input
```
integrate(x**4/(x**8-2*x**4+1),x)
```

output
```
-x/(4*x**4 - 4) + log(x - 1)/16 - log(x + 1)/16 - atan(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{x}{4\,(x^4-1)} - \frac{1}{8}\,\arctan\left(x\right) - \frac{1}{16}\,\log\left(x+1\right) + \frac{1}{16}\,\log\left(x-1\right)$$

input
```
integrate(x^4/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(x + 1) + 1/16*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{x}{4\,(x^4-1)} - \frac{1}{8}\,\arctan\left(x\right) - \frac{1}{16}\,\log\left(|x+1|\right) + \frac{1}{16}\,\log\left(|x-1|\right)$$

input
```
integrate(x^4/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(abs(x + 1)) + 1/16*log(abs(x -
1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{1-2x^4+x^8}\,dx = -\frac{\operatorname{atan}(x)}{8} - \frac{\operatorname{atanh}(x)}{8} - \frac{x}{4\,\left(x^4-1\right)}$$

input
```
int(x^4/(x^8 - 2*x^4 + 1),x)
```

output
```
- atan(x)/8 - atanh(x)/8 - x/(4*(x^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int \frac{x^4}{1-2x^4+x^8}\,dx$$
$$= \frac{-2atan(x)\,x^4+2atan(x)+\log(x-1)\,x^4-\log(x-1)-\log(x+1)\,x^4+\log(x+1)-4x}{16x^4-16}$$

input
```
int(x^4/(x^8-2*x^4+1),x)
```

output
```
( - 2*atan(x)*x***4 + 2*atan(x) + log(x - 1)*x***4 - log(x - 1) - log(x + 1)
*x***4 + log(x + 1) - 4*x)/(16*(x***4 - 1))
```

3.16 $\int \frac{x^2}{1-2x^4+x^8} \, dx$

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Mathematica [A] (verified) 163
Rubi [A] (verified) 164
Maple [A] (verified) 165
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Sympy [A] (verification not implemented) 166
Maxima [A] (verification not implemented) 167
Giac [A] (verification not implemented) 167
Mupad [B] (verification not implemented) 167
Reduce [B] (verification not implemented) 168

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{x^2}{1-2x^4+x^8} \, dx = \frac{x^3}{4(1-x^4)} - \frac{\arctan(x)}{8} + \frac{\arctanh(x)}{8}$$

output
```
x^3/(-4*x^4+4)-1/8*arctan(x)+1/8*arctanh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{1-2x^4+x^8} \, dx = \frac{1}{16}\left(-\frac{4x^3}{-1+x^4} - 2\arctan(x) - \log(1-x) + \log(1+x)\right)$$

input
```
Integrate[x^2/(1 - 2*x^4 + x^8),x]
```

output
```
((-4*x^3)/(-1 + x^4) - 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/16
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1380, 819, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^8-2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^2}{(1-x^4)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{1}{4}\int \frac{x^2}{1-x^4}dx+\frac{x^3}{4(1-x^4)}$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx\right)+\frac{x^3}{4(1-x^4)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}\right)+\frac{x^3}{4(1-x^4)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{\text{arctanh}(x)}{2}-\frac{\arctan(x)}{2}\right)+\frac{x^3}{4(1-x^4)}$$

input
```
Int[x^2/(1 - 2*x^4 + x^8),x]
```

output
```
x^3/(4*(1 - x^4)) + (-1/2*ArcTan[x] + ArcTanh[x]/2)/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{x^{3}}{4(x^{4}-1)}-\frac{\arctan(x)}{8}+\frac{\ln(x+1)}{16}-\frac{\ln(x-1)}{16}$	30
default	$-\frac{1}{16(x-1)}-\frac{\ln(x-1)}{16}-\frac{x}{8(x^{2}+1)}-\frac{\arctan(x)}{8}-\frac{1}{16(x+1)}+\frac{\ln(x+1)}{16}$	42
parallelrisch	$-\frac{-i\ln(x-i)x^{4}+i\ln(x+i)x^{4}+\ln(x-1)x^{4}-\ln(x+1)x^{4}+4x^{3}+i\ln(x-i)-i\ln(x+i)-\ln(x-1)+\ln(x+1)}{16(x^{4}-1)}$	81

input
```
int(x^2/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^3/(x^4-1)-1/8*arctan(x)+1/16*ln(x+1)-1/16*ln(x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(21) = 42$.

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = -\frac{4\,x^3+2\,(x^4-1)\arctan(x)-(x^4-1)\log(x+1)+(x^4-1)\log(x-1)}{16\,(x^4-1)}$$

input
```
integrate(x^2/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/16*(4*x^3 + 2*(x^4 - 1)*arctan(x) - (x^4 - 1)*log(x + 1) + (x^4 - 1)*lo
g(x - 1))/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = -\frac{x^3}{4x^4-4} - \frac{\log(x-1)}{16} + \frac{\log(x+1)}{16} - \frac{\operatorname{atan}(x)}{8}$$

input
```
integrate(x**2/(x**8-2*x**4+1),x)
```

output
```
-x**3/(4*x**4 - 4) - log(x - 1)/16 + log(x + 1)/16 - atan(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4-1)} - \frac{1}{8}\,\arctan\left(x\right) + \frac{1}{16}\,\log\left(x+1\right) - \frac{1}{16}\,\log\left(x-1\right)$$

input
```
integrate(x^2/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(x + 1) - 1/16*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4-1)} - \frac{1}{8}\,\arctan\left(x\right) + \frac{1}{16}\,\log\left(|x+1|\right) - \frac{1}{16}\,\log\left(|x-1|\right)$$

input
```
integrate(x^2/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(abs(x + 1)) - 1/16*log(abs(x
 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{1-2x^4+x^8}\,dx = \frac{\mathrm{atanh}(x)}{8} - \frac{\mathrm{atan}(x)}{8} - \frac{x^3}{4\,(x^4-1)}$$

input
```
int(x^2/(x^8 - 2*x^4 + 1),x)
```

output
```
atanh(x)/8 - atan(x)/8 - x^3/(4*(x^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{x^2}{1-2x^4+x^8}\,dx$$
$$=\frac{-2atan(x)\,x^4+2atan(x)-\log(x-1)\,x^4+\log(x-1)+\log(x+1)\,x^4-\log(x+1)-4x^3}{16x^4-16}$$

input
```
int(x^2/(x^8-2*x^4+1),x)
```

output
```
( - 2*atan(x)*x**4 + 2*atan(x) - log(x - 1)*x**4 + log(x - 1) + log(x + 1)
*x**4 - log(x + 1) - 4*x**3)/(16*(x**4 - 1))
```

3.17 $\int \frac{1}{1-2x^4+x^8}\,dx$

Optimal result 169
Mathematica [A] (verified) 169
Rubi [A] (verified) 170
Maple [A] (verified) 171
Fricas [B] (verification not implemented) 172
Sympy [A] (verification not implemented) 172
Maxima [A] (verification not implemented) 173
Giac [A] (verification not implemented) 173
Mupad [B] (verification not implemented) 173
Reduce [B] (verification not implemented) 174

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{1}{1-2x^4+x^8}\,dx = \frac{x}{4(1-x^4)} + \frac{3\arctan(x)}{8} + \frac{3\text{arctanh}(x)}{8}$$

output
```
x/(-4*x^4+4)+3/8*arctan(x)+3/8*arctanh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1}{1-2x^4+x^8}\,dx = \frac{1}{16}\left(-\frac{4x}{-1+x^4} + 6\arctan(x) - 3\log(1-x) + 3\log(1+x)\right)$$

input
```
Integrate[(1 - 2*x^4 + x^8)^(-1),x]
```

output
```
((-4*x)/(-1 + x^4) + 6*ArcTan[x] - 3*Log[1 - x] + 3*Log[1 + x])/16
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1379, 749, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8-2x^4+1}dx$$
$$\Big\downarrow \text{1379}$$
$$\int \frac{1}{(x^4-1)^2}dx$$
$$\Big\downarrow \text{749}$$
$$\frac{x}{4(1-x^4)}-\frac{3}{4}\int \frac{1}{x^4-1}dx$$
$$\Big\downarrow \text{756}$$
$$\frac{x}{4(1-x^4)}-\frac{3}{4}\left(-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx\right)$$
$$\Big\downarrow \text{216}$$
$$\frac{x}{4(1-x^4)}-\frac{3}{4}\left(-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{x}{4(1-x^4)}-\frac{3}{4}\left(-\frac{\arctan(x)}{2}-\frac{\text{arctanh}(x)}{2}\right)$$

input
```
Int[(1 - 2*x^4 + x^8)^(-1),x]
```

output
```
x/(4*(1 - x^4)) - (3*(-1/2*ArcTan[x] - ArcTanh[x]/2))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{x}{4(x^4-1)}-\frac{3\ln(x-1)}{16}+\frac{3\ln(x+1)}{16}+\frac{3\arctan(x)}{8}$	28
default	$-\frac{1}{16(x-1)}-\frac{3\ln(x-1)}{16}+\frac{x}{8x^2+8}+\frac{3\arctan(x)}{8}-\frac{1}{16(x+1)}+\frac{3\ln(x+1)}{16}$	42
parallelrisch	$-\frac{3i\ln(x-i)x^4-3i\ln(x+i)x^4+3\ln(x-1)x^4-3\ln(x+1)x^4-3i\ln(x-i)+3i\ln(x+i)-3\ln(x-1)+3\ln(x+1)+4x}{16(x^4-1)}$	82

input
```
int(1/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x/(x^4-1)-3/16*ln(x-1)+3/16*ln(x+1)+3/8*arctan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(19) = 38$.

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \frac{1}{1-2x^4+x^8}\,dx = \frac{6\,(x^4-1)\arctan(x)+3\,(x^4-1)\log(x+1)-3\,(x^4-1)\log(x-1)-4\,x}{16\,(x^4-1)}$$

input
```
integrate(1/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
1/16*(6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)*log(x -
 1) - 4*x)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1}{1-2x^4+x^8}\,dx = -\frac{x}{4x^4-4} - \frac{3\log(x-1)}{16} + \frac{3\log(x+1)}{16} + \frac{3\,\mathrm{atan}\,(x)}{8}$$

input
```
integrate(1/(x**8-2*x**4+1),x)
```

output
```
-x/(4*x**4 - 4) - 3*log(x - 1)/16 + 3*log(x + 1)/16 + 3*atan(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-2x^4+x^8}\,dx = -\frac{x}{4\,(x^4-1)} + \frac{3}{8}\arctan(x) + \frac{3}{16}\log(x+1) - \frac{3}{16}\log(x-1)$$

input
```
integrate(1/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(x + 1) - 3/16*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{1}{1-2x^4+x^8}\,dx = -\frac{x}{4\,(x^4-1)} + \frac{3}{8}\arctan(x) + \frac{3}{16}\log(|x+1|) - \frac{3}{16}\log(|x-1|)$$

input
```
integrate(1/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(abs(x + 1)) - 3/16*log(abs(x -
 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{1-2x^4+x^8}\,dx = \frac{3\,\mathrm{atan}(x)}{8} + \frac{3\,\mathrm{atanh}(x)}{8} - \frac{x}{4\,(x^4-1)}$$

input
```
int(1/(x^8 - 2*x^4 + 1),x)
```

output
```
(3*atan(x))/8 + (3*atanh(x))/8 - x/(4*(x^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{1}{1-2x^4+x^8}\,dx = \frac{6atan(x)\,x^4-6atan(x)-3\log(x-1)\,x^4+3\log(x-1)+3\log(x+1)\,x^4-3\log(x+1)-4x}{16x^4-16}$$

input
```
int(1/(x^8-2*x^4+1),x)
```

output
```
(6*atan(x)*x**4 - 6*atan(x) - 3*log(x - 1)*x**4 + 3*log(x - 1) + 3*log(x +
 1)*x**4 - 3*log(x + 1) - 4*x)/(16*(x**4 - 1))
```

3.18 $\int \frac{1}{x^2(1-2x^4+x^8)}\,dx$

Optimal result 175
Mathematica [A] (verified) 175
Rubi [A] (verified) 176
Maple [A] (verified) 178
Fricas [B] (verification not implemented) 178
Sympy [A] (verification not implemented) 179
Maxima [A] (verification not implemented) 179
Giac [A] (verification not implemented) 179
Mupad [B] (verification not implemented) 180
Reduce [B] (verification not implemented) 180

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx=-\frac{1}{x}+\frac{x^3}{4\left(1-x^4\right)}-\frac{5\arctan(x)}{8}+\frac{5\operatorname{arctanh}(x)}{8}$$

output
```
-1/x+x^3/(-4*x^4+4)-5/8*arctan(x)+5/8*arctanh(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx=\frac{1}{16}\left(-\frac{16}{x}-\frac{4x^3}{-1+x^4}-10\arctan(x)-5\log(1-x)+5\log(1+x)\right)$$

input
```
Integrate[1/(x^2*(1 - 2*x^4 + x^8)),x]
```

output
```
(-16/x - (4*x^3)/(-1 + x^4) - 10*ArcTan[x] - 5*Log[1 - x] + 5*Log[1 + x])/
16
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1380, 819, 847, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^8-2x^4+1)}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^2 (1-x^4)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{5}{4} \int \frac{1}{x^2 (1-x^4)}dx+\frac{1}{4x (1-x^4)}$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{4} \left(\int \frac{x^2}{1-x^4}dx-\frac{1}{x}\right)+\frac{1}{4x (1-x^4)}$$

$$\Big\downarrow \text{827}$$

$$\frac{5}{4} \left(\frac{1}{2} \int \frac{1}{1-x^2}dx-\frac{1}{2} \int \frac{1}{x^2+1}dx-\frac{1}{x}\right)+\frac{1}{4x (1-x^4)}$$

$$\Big\downarrow \text{216}$$

$$\frac{5}{4} \left(\frac{1}{2} \int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}-\frac{1}{x}\right)+\frac{1}{4x (1-x^4)}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{4} \left(-\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}-\frac{1}{x}\right)+\frac{1}{4x (1-x^4)}$$

input
```
Int[1/(x^2*(1 - 2*x^4 + x^8)),x]
```

output
```
1/(4*x*(1 - x^4)) + (5*(-x^(-1) - ArcTan[x]/2 + ArcTanh[x]/2))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result
risch	$\frac{-\frac{5x^4}{4}+1}{x(x^4-1)}-\frac{5\arctan(x)}{8}-\frac{5\ln(x-1)}{16}+\frac{5\ln(x+1)}{16}$
default	$-\frac{1}{16(x-1)}-\frac{5\ln(x-1)}{16}-\frac{x}{8(x^2+1)}-\frac{5\arctan(x)}{8}-\frac{1}{16(x+1)}+\frac{5\ln(x+1)}{16}-\frac{1}{x}$
parallelrisch	$-\frac{-5i\ln(x-i)x^5+5i\ln(x+i)x^5+5\ln(x-1)x^5-5\ln(x+1)x^5-16+20x^4+5i\ln(x-i)x-5i\ln(x+i)x-5\ln(x-1)x+5\ln(x+1)x}{16x(x^4-1)}$

input
```
int(1/x^2/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-5/4*x^4+1)/x/(x^4-1)-5/8*arctan(x)-5/16*ln(x-1)+5/16*ln(x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(26) = 52.

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{1}{x^2(1-2x^4+x^8)}\,dx = -\frac{20\,x^4+10\,(x^5-x)\arctan(x)-5\,(x^5-x)\log(x+1)+5\,(x^5-x)\log(x-1)-16}{16\,(x^5-x)}$$

input
```
integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/16*(20*x^4 + 10*(x^5 - x)*arctan(x) - 5*(x^5 - x)*log(x + 1) + 5*(x^5 -
 x)*log(x - 1) - 16)/(x^5 - x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx = \frac{4-5x^4}{4x^5-4x} - \frac{5\log\left(x-1\right)}{16} + \frac{5\log\left(x+1\right)}{16} - \frac{5\,\mathrm{atan}\left(x\right)}{8}$$

input
```
integrate(1/x**2/(x**8-2*x**4+1),x)
```

output
```
(4 - 5*x**4)/(4*x**5 - 4*x) - 5*log(x - 1)/16 + 5*log(x + 1)/16 - 5*atan(x
)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx = -\frac{5\,x^4-4}{4\left(x^5-x\right)} - \frac{5}{8}\,\arctan\left(x\right) + \frac{5}{16}\,\log\left(x+1\right) - \frac{5}{16}\,\log\left(x-1\right)$$

input
```
integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(x + 1) - 5/16*log(x
- 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx = -\frac{5\,x^4-4}{4\left(x^5-x\right)} - \frac{5}{8}\,\arctan\left(x\right) + \frac{5}{16}\,\log\left(|x+1|\right) - \frac{5}{16}\,\log\left(|x-1|\right)$$

input
```
integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(abs(x + 1)) - 5/16*l
og(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx = \frac{5\,\mathrm{atanh}(x)}{8} - \frac{5\,\mathrm{atan}(x)}{8} + \frac{\frac{5\,x^4}{4}-1}{x-x^5}$$

input
```
int(1/(x^2*(x^8 - 2*x^4 + 1)),x)
```

output
```
(5*atanh(x))/8 - (5*atan(x))/8 + ((5*x^4)/4 - 1)/(x - x^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

$$\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx$$
$$= \frac{-10atan(x)\,x^5+10atan(x)\,x-5\log(x-1)\,x^5+5\log(x-1)\,x+5\log(x+1)\,x^5-5\log(x+1)\,x-20x}{16x\left(x^4-1\right)}$$

input
```
int(1/x^2/(x^8-2*x^4+1),x)
```

output
```
( - 10*atan(x)*x**5 + 10*atan(x)*x - 5*log(x - 1)*x**5 + 5*log(x - 1)*x +
5*log(x + 1)*x**5 - 5*log(x + 1)*x - 20*x**4 + 16)/(16*x*(x**4 - 1))
```

3.19 $\int \frac{1}{x^4(1-2x^4+x^8)}\,dx$

Optimal result 181
Mathematica [A] (verified) 181
Rubi [A] (verified) 182
Maple [A] (verified) 184
Fricas [B] (verification not implemented) 184
Sympy [A] (verification not implemented) 185
Maxima [A] (verification not implemented) 185
Giac [A] (verification not implemented) 185
Mupad [B] (verification not implemented) 186
Reduce [B] (verification not implemented) 186

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{1}{x^4\left(1-2x^4+x^8\right)}\,dx=-\frac{1}{3x^3}+\frac{x}{4\left(1-x^4\right)}+\frac{7\arctan(x)}{8}+\frac{7\text{arctanh}(x)}{8}$$

output
```
-1/3/x^3+x/(-4*x^4+4)+7/8*arctan(x)+7/8*arctanh(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^4\left(1-2x^4+x^8\right)}\,dx=\frac{1}{48}\left(-\frac{16}{x^3}-\frac{12x}{-1+x^4}+42\arctan(x)-21\log(1-x)+21\log(1+x)\right)$$

input
```
Integrate[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

output
```
(-16/x^3 - (12*x)/(-1 + x^4) + 42*ArcTan[x] - 21*Log[1 - x] + 21*Log[1 + x
])/48
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1380, 819, 847, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^8-2x^4+1)} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^4 (1-x^4)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{7}{4} \int \frac{1}{x^4 (1-x^4)}dx+\frac{1}{4 x^3 (1-x^4)}$$

$$\Big\downarrow \text{847}$$

$$\frac{7}{4} \left(\int \frac{1}{1-x^4}dx-\frac{1}{3 x^3}\right)+\frac{1}{4 x^3 (1-x^4)}$$

$$\Big\downarrow \text{756}$$

$$\frac{7}{4} \left(\frac{1}{2} \int \frac{1}{1-x^2}dx+\frac{1}{2} \int \frac{1}{x^2+1}dx-\frac{1}{3 x^3}\right)+\frac{1}{4 x^3 (1-x^4)}$$

$$\Big\downarrow \text{216}$$

$$\frac{7}{4} \left(\frac{1}{2} \int \frac{1}{1-x^2}dx+\frac{\arctan (x)}{2}-\frac{1}{3 x^3}\right)+\frac{1}{4 x^3 (1-x^4)}$$

$$\Big\downarrow \text{219}$$

$$\frac{7}{4} \left(\frac{\arctan (x)}{2}+\frac{\text{arctanh}(x)}{2}-\frac{1}{3 x^3}\right)+\frac{1}{4 x^3 (1-x^4)}$$

input
```
Int[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

output
```
1/(4*x^3*(1 - x^4)) + (7*(-1/3*1/x^3 + ArcTan[x]/2 + ArcTanh[x]/2))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result
risch	$\frac{-\frac{7x^4}{12}+\frac{1}{3}}{x^3(x^4-1)}+\frac{7\ln(x+1)}{16}-\frac{7\ln(x-1)}{16}+\frac{7\arctan(x)}{8}$
default	$-\frac{1}{16(x-1)}-\frac{7\ln(x-1)}{16}+\frac{x}{8x^2+8}+\frac{7\arctan(x)}{8}-\frac{1}{16(x+1)}+\frac{7\ln(x+1)}{16}-\frac{1}{3x^3}$
parallelrisch	$-\frac{21i\ln(x-i)x^7-21i\ln(x+i)x^7+21\ln(x-1)x^7-21\ln(x+1)x^7-16-21i\ln(x-i)x^3+21i\ln(x+i)x^3-21\ln(x-1)x^3+21\ln(x+}{48x^3(x^4-1)}$

input
```
int(1/x^4/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-7/12*x^4+1/3)/x^3/(x^4-1)+7/16*ln(x+1)-7/16*ln(x-1)+7/8*arctan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(24) = 48.

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

$$\int \frac{1}{x^4(1-2x^4+x^8)}\,dx = -\frac{28\,x^4-42\,(x^7-x^3)\arctan(x)-21\,(x^7-x^3)\log(x+1)+21\,(x^7-x^3)\log(x-1)-16}{48\,(x^7-x^3)}$$

input
```
integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
-1/48*(28*x^4 - 42*(x^7 - x^3)*arctan(x) - 21*(x^7 - x^3)*log(x + 1) + 21*
(x^7 - x^3)*log(x - 1) - 16)/(x^7 - x^3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^4\left(1-2x^4+x^8\right)}\,dx=\frac{4-7x^4}{12x^7-12x^3}-\frac{7\log\left(x-1\right)}{16}+\frac{7\log\left(x+1\right)}{16}+\frac{7\,\mathrm{atan}\left(x\right)}{8}$$

input
```
integrate(1/x**4/(x**8-2*x**4+1),x)
```

output
```
(4 - 7*x**4)/(12*x**7 - 12*x**3) - 7*log(x - 1)/16 + 7*log(x + 1)/16 + 7*a
tan(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4\left(1-2x^4+x^8\right)}\,dx=-\frac{7\,x^4-4}{12\left(x^7-x^3\right)}+\frac{7}{8}\,\arctan\left(x\right)+\frac{7}{16}\,\log\left(x+1\right)-\frac{7}{16}\,\log\left(x-1\right)$$

input
```
integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
-1/12*(7*x^4 - 4)/(x^7 - x^3) + 7/8*arctan(x) + 7/16*log(x + 1) - 7/16*log
(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(1-2x^4+x^8\right)}\,dx=-\frac{x}{4\left(x^4-1\right)}-\frac{1}{3\,x^3}+\frac{7}{8}\,\arctan\left(x\right)+\frac{7}{16}\,\log\left(|x+1|\right)-\frac{7}{16}\,\log\left(|x-1|\right)$$

input
```
integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x/(x^4 - 1) - 1/3/x^3 + 7/8*arctan(x) + 7/16*log(abs(x + 1)) - 7/16*l
og(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4\left(1-2x^4+x^8\right)}\,dx=\frac{7\,\mathrm{atan}(x)}{8}+\frac{7\,\mathrm{atanh}(x)}{8}+\frac{\frac{7\,x^4}{12}-\frac{1}{3}}{x^3-x^7}$$

input
```
int(1/(x^4*(x^8 - 2*x^4 + 1)),x)
```

output
```
(7*atan(x))/8 + (7*atanh(x))/8 + ((7*x^4)/12 - 1/3)/(x^3 - x^7)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int \frac{1}{x^4\left(1-2x^4+x^8\right)}\,dx$$
$$=\frac{42atan(x)\,x^7-42atan(x)\,x^3-21\log(x-1)\,x^7+21\log(x-1)\,x^3+21\log(x+1)\,x^7-21\log(x+1)\,x^3}{48x^3\left(x^4-1\right)}$$

input
```
int(1/x^4/(x^8-2*x^4+1),x)
```

output
```
(42*atan(x)*x**7 - 42*atan(x)*x**3 - 21*log(x - 1)*x**7 + 21*log(x - 1)*x*
*3 + 21*log(x + 1)*x**7 - 21*log(x + 1)*x**3 - 28*x**4 + 16)/(48*x**3*(x**
4 - 1))
```

3.20 $\int \frac{x^{13}}{1+2x^4+x^8}\,dx$

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Rubi [A] (verified) 188
Maple [A] (verified) 189
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Maxima [A] (verification not implemented) 190
Giac [A] (verification not implemented) 191
Mupad [B] (verification not implemented) 191
Reduce [B] (verification not implemented) 191

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{x^{13}}{1+2x^4+x^8}\,dx = -x^2+\frac{x^6}{6}-\frac{x^2}{4(1+x^4)}+\frac{5\arctan(x^2)}{4}$$

output
```
-x^2+1/6*x^6-x^2/(4*x^4+4)+5/4*arctan(x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x^{13}}{1+2x^4+x^8}\,dx = \frac{1}{12}x^2\left(-12+2x^4-\frac{3}{1+x^4}\right)+\frac{5\arctan(x^2)}{4}$$

input
```
Integrate[x^13/(1 + 2*x^4 + x^8),x]
```

output
```
(x^2*(-12 + 2*x^4 - 3/(1 + x^4)))/12 + (5*ArcTan[x^2])/4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1380, 807, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{x^8+2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^{13}}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^{12}}{(x^4+1)^2}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{5}{2}\int \frac{x^8}{x^4+1}dx^2-\frac{x^{10}}{2(x^4+1)}\right)$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{2}\left(\frac{5}{2}\int \left(x^4+\frac{1}{x^4+1}-1\right)dx^2-\frac{x^{10}}{2(x^4+1)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{5}{2}\left(\arctan\left(x^2\right)+\frac{x^6}{3}-x^2\right)-\frac{x^{10}}{2(x^4+1)}\right)$$

input
```
Int[x^13/(1 + 2*x^4 + x^8),x]
```

output
```
(-1/2*x^10/(1 + x^4) + (5*(-x^2 + x^6/3 + ArcTan[x^2]))/2)/2
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^6}{6}-x^2-\frac{x^2}{4(x^4+1)}+\frac{5\arctan(x^2)}{4}$	30
risch	$\frac{x^6}{6}-x^2-\frac{x^2}{4(x^4+1)}+\frac{5\arctan(x^2)}{4}$	30
parallelrisch	$-\frac{-4x^{10}+15i\ln(x^2-i)x^4-15i\ln(x^2+i)x^4+20x^6+15i\ln(x^2-i)-15i\ln(x^2+i)+30x^2}{24(x^4+1)}$	72

input
```
int(x^13/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6-x^2-1/4*x^2/(x^4+1)+5/4*arctan(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{x^{13}}{1+2x^4+x^8}\,dx = \frac{2\,x^{10}-10\,x^6-15\,x^2+15\,(x^4+1)\arctan\left(x^2\right)}{12\,(x^4+1)}$$

input
```
integrate(x^13/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/12*(2*x^10 - 10*x^6 - 15*x^2 + 15*(x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{x^{13}}{1+2x^4+x^8}\,dx = \frac{x^6}{6} - x^2 - \frac{x^2}{4x^4+4} + \frac{5\,\mathrm{atan}\left(x^2\right)}{4}$$

input
```
integrate(x**13/(x**8+2*x**4+1),x)
```

output
```
x**6/6 - x**2 - x**2/(4*x**4 + 4) + 5*atan(x**2)/4
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^{13}}{1+2x^4+x^8}\,dx = \frac{1}{6}\,x^6 - x^2 - \frac{x^2}{4\,(x^4+1)} + \frac{5}{4}\,\arctan\left(x^2\right)$$

input
```
integrate(x^13/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/6*x^6 - x^2 - 1/4*x^2/(x^4 + 1) + 5/4*arctan(x^2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^{13}}{1+2x^4+x^8}\,dx = \frac{1}{6}\,x^6 - x^2 - \frac{x^2}{4\,(x^4+1)} + \frac{5}{4}\,\arctan\left(x^2\right)$$

input
```
integrate(x^13/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/6*x^6 - x^2 - 1/4*x^2/(x^4 + 1) + 5/4*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 20.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^{13}}{1+2x^4+x^8}\,dx = \frac{5\,\mathrm{atan}(x^2)}{4} - \frac{x^2}{4\,(x^4+1)} - x^2 + \frac{x^6}{6}$$

input
```
int(x^13/(2*x^4 + x^8 + 1),x)
```

output
```
(5*atan(x^2))/4 - x^2/(4*(x^4 + 1)) - x^2 + x^6/6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.51

$$\int \frac{x^{13}}{1+2x^4+x^8}\,dx = \frac{-15atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^4 - 15atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) - 15atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^4 - 15atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) + 2x^{10} - 10x^6 - 15x^2}{12x^4+12}$$

input
```
int(x^13/(x^8+2*x^4+1),x)
```

output
```
( - 15*atan((sqrt(2) - 2*x)/sqrt(2))*x**4 - 15*atan((sqrt(2) - 2*x)/sqrt(2
)) - 15*atan((sqrt(2) + 2*x)/sqrt(2))*x**4 - 15*atan((sqrt(2) + 2*x)/sqrt(
2)) + 2*x**10 - 10*x**6 - 15*x**2)/(12*(x**4 + 1))
```

3.21 $\int \frac{x^{11}}{1+2x^4+x^8} dx$

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Rubi [A] (verified) 194
Maple [A] (verified) 195
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Maxima [A] (verification not implemented) 196
Giac [A] (verification not implemented) 197
Mupad [B] (verification not implemented) 197
Reduce [B] (verification not implemented) 197

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{x^{11}}{1+2x^4+x^8} dx = \frac{x^4}{4} - \frac{1}{4(1+x^4)} - \frac{1}{2}\log(1+x^4)$$

output
```
1/4*x^4-1/(4*x^4+4)-1/2*ln(x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}}{1+2x^4+x^8} dx = \frac{1}{4}\left(x^4 - \frac{1}{1+x^4} - 2\log(1+x^4)\right)$$

input
```
Integrate[x^11/(1 + 2*x^4 + x^8),x]
```

output
```
(x^4 - (1 + x^4)^(-1) - 2*Log[1 + x^4])/4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^8+2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^{11}}{\left(x^4+1\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^8}{\left(x^4+1\right)^2}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\int \left(1-\frac{2}{x^4+1}+\frac{1}{\left(x^4+1\right)^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(x^4-\frac{1}{x^4+1}-2\log\left(x^4+1\right)\right)$$

input
```
Int[x^11/(1 + 2*x^4 + x^8),x]
```

output
```
(x^4 - (1 + x^4)^(-1) - 2*Log[1 + x^4])/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^{4}}{4}-\frac{\ln\left(x^{4}+1\right)}{2}-\frac{1}{4\left(x^{4}+1\right)}$	24
risch	$\frac{x^{4}}{4}-\frac{\ln\left(x^{4}+1\right)}{2}-\frac{1}{4\left(x^{4}+1\right)}$	24
norman	$\frac{\frac{x^{8}}{4}-\frac{1}{2}}{x^{4}+1}-\frac{\ln\left(x^{4}+1\right)}{2}$	25
parallelrisch	$-\frac{-x^{8}+2\ln\left(x^{4}+1\right)x^{4}+2+2\ln\left(x^{4}+1\right)}{4\left(x^{4}+1\right)}$	36

input
```
int(x^11/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4-1/2*ln(x^4+1)-1/4/(x^4+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{1+2x^4+x^8}\,dx = \frac{x^8+x^4-2\,(x^4+1)\log\left(x^4+1\right)-1}{4\,(x^4+1)}$$

input
```
integrate(x^11/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/4*(x^8 + x^4 - 2*(x^4 + 1)*log(x^4 + 1) - 1)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{x^{11}}{1+2x^4+x^8}\,dx = \frac{x^4}{4} - \frac{\log\left(x^4+1\right)}{2} - \frac{1}{4x^4+4}$$

input
```
integrate(x**11/(x**8+2*x**4+1),x)
```

output
```
x**4/4 - log(x**4 + 1)/2 - 1/(4*x**4 + 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x^{11}}{1+2x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{4\,(x^4+1)} - \frac{1}{2}\,\log\left(x^4+1\right)$$

input
```
integrate(x^11/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^4 - 1/4/(x^4 + 1) - 1/2*log(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{1+2x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{2\,x^4+1}{4\,(x^4+1)} - \frac{1}{2}\,\log\left(x^4+1\right)$$

input
```
integrate(x^11/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/4*x^4 + 1/4*(2*x^4 + 1)/(x^4 + 1) - 1/2*log(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}}{1+2x^4+x^8}\,dx = \frac{x^4}{4} - \frac{1}{4\,\left(x^4+1\right)} - \frac{\ln\left(x^4+1\right)}{2}$$

input
```
int(x^11/(2*x^4 + x^8 + 1),x)
```

output
```
x^4/4 - 1/(4*(x^4 + 1)) - log(x^4 + 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.59

$$\int \frac{x^{11}}{1+2x^4+x^8}\,dx$$
$$= \frac{-2\log\left(-\sqrt{2}\,x+x^2+1\right)x^4 - 2\log\left(-\sqrt{2}\,x+x^2+1\right) - 2\log\left(\sqrt{2}\,x+x^2+1\right)x^4 - 2\log\left(\sqrt{2}\,x+x^2+1\right)}{4x^4+4}$$

input
```
int(x^11/(x^8+2*x^4+1),x)
```

output
```
( - 2*log( - sqrt(2)*x + x**2 + 1)*x***4 - 2*log( - sqrt(2)*x + x**2 + 1) -
 2*log(sqrt(2)*x + x**2 + 1)*x***4 - 2*log(sqrt(2)*x + x**2 + 1) + x**8 + 2
*x***4)/(4*(x***4 + 1))
```

3.22 $\int \frac{x^9}{1+2x^4+x^8} dx$

Optimal result 199
Mathematica [A] (verified) 199
Rubi [A] (verified) 200
Maple [A] (verified) 201
Fricas [A] (verification not implemented) 202
Sympy [A] (verification not implemented) 202
Maxima [A] (verification not implemented) 203
Giac [A] (verification not implemented) 203
Mupad [B] (verification not implemented) 203
Reduce [B] (verification not implemented) 204

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{x^9}{1+2x^4+x^8} dx = \frac{x^2}{2} + \frac{x^2}{4(1+x^4)} - \frac{3\arctan(x^2)}{4}$$

output
```
1/2*x^2+x^2/(4*x^4+4)-3/4*arctan(x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^9}{1+2x^4+x^8} dx = \frac{1}{4}\left(x^2\left(2+\frac{1}{1+x^4}\right) - 3\arctan(x^2)\right)$$

input
```
Integrate[x^9/(1 + 2*x^4 + x^8),x]
```

output
```
(x^2*(2 + (1 + x^4)^(-1)) - 3*ArcTan[x^2])/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1380, 807, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^8+2x^4+1} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^9}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^8}{(x^4+1)^2}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{x^4}{x^4+1}dx^2-\frac{x^6}{2(x^4+1)}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(x^2-\int \frac{1}{x^4+1}dx^2\right)-\frac{x^6}{2(x^4+1)}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(x^2-\arctan\left(x^2\right)\right)-\frac{x^6}{2(x^4+1)}\right)$$

input
```
Int[x^9/(1 + 2*x^4 + x^8),x]
```

output
```
(-1/2*x^6/(1 + x^4) + (3*(x^2 - ArcTan[x^2]))/2)/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^2}{2}+\frac{x^2}{4x^4+4}-\frac{3\arctan\left(x^2\right)}{4}$	25
risch	$\frac{x^2}{2}+\frac{x^2}{4x^4+4}-\frac{3\arctan\left(x^2\right)}{4}$	25
parallelrisch	$\frac{3i\ln\left(x^2-i\right)x^4-3i\ln\left(x^2+i\right)x^4+4x^6+3i\ln\left(x^2-i\right)-3i\ln\left(x^2+i\right)+6x^2}{8x^4+8}$	67

input
```
int(x^9/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/4*x^2/(x^4+1)-3/4*arctan(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{2\,x^6+3\,x^2-3\,(x^4+1)\arctan\left(x^2\right)}{4\,(x^4+1)}$$

input
```
integrate(x^9/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/4*(2*x^6 + 3*x^2 - 3*(x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{x^2}{2}+\frac{x^2}{4x^4+4}-\frac{3\operatorname{atan}\left(x^2\right)}{4}$$

input
```
integrate(x**9/(x**8+2*x**4+1),x)
```

output
```
x**2/2 + x**2/(4*x**4 + 4) - 3*atan(x**2)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{x^2}{4\,(x^4+1)} - \frac{3}{4}\,\arctan\left(x^2\right)$$

input
```
integrate(x^9/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{x^2}{4\,(x^4+1)} - \frac{3}{4}\,\arctan\left(x^2\right)$$

input
```
integrate(x^9/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 19.93 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{x^9}{1+2x^4+x^8}\,dx = \frac{x^2}{4\,(x^4+1)} - \frac{3\,\mathrm{atan}(x^2)}{4} + \frac{x^2}{2}$$

input
```
int(x^9/(2*x^4 + x^8 + 1),x)
```

output
```
x^2/(4*(x^4 + 1)) - (3*atan(x^2))/4 + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.77

$$\int \frac{x^9}{1+2x^4+x^8}\,dx$$
$$=\frac{3atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^4+3atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)+3atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^4+3atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)+2x^6+3x^2}{4x^4+4}$$

input
```
int(x^9/(x^8+2*x^4+1),x)
```

output
```
(3*atan((sqrt(2) - 2*x)/sqrt(2))*x**4 + 3*atan((sqrt(2) - 2*x)/sqrt(2)) +
3*atan((sqrt(2) + 2*x)/sqrt(2))*x**4 + 3*atan((sqrt(2) + 2*x)/sqrt(2)) + 2
*x**6 + 3*x**2)/(4*(x**4 + 1))
```

3.23 $\int \frac{x^7}{1+2x^4+x^8}\,dx$

Optimal result 205
Mathematica [A] (verified) 205
Rubi [A] (verified) 206
Maple [A] (verified) 207
Fricas [A] (verification not implemented) 208
Sympy [A] (verification not implemented) 208
Maxima [A] (verification not implemented) 208
Giac [A] (verification not implemented) 209
Mupad [B] (verification not implemented) 209
Reduce [B] (verification not implemented) 209

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{x^7}{1+2x^4+x^8}\,dx=\frac{1}{4\left(1+x^4\right)}+\frac{1}{4}\log\left(1+x^4\right)$$

output
```
1/(4*x^4+4)+1/4*ln(x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+2x^4+x^8}\,dx=\frac{1}{4}\left(\frac{1}{1+x^4}+\log\left(1+x^4\right)\right)$$

input
```
Integrate[x^7/(1 + 2*x^4 + x^8),x]
```

output
```
((1 + x^4)^(-1) + Log[1 + x^4])/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^8+2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^7}{\left(x^4+1\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^4}{\left(x^4+1\right)^2}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\int \left(\frac{1}{x^4+1}-\frac{1}{\left(x^4+1\right)^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{x^4+1}+\log\left(x^4+1\right)\right)$$

input
```
Int[x^7/(1 + 2*x^4 + x^8),x]
```

output
```
((1 + x^4)^(-1) + Log[1 + x^4])/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{1}{4x^4+4}+\frac{\ln(x^4+1)}{4}$	19
norman	$\frac{1}{4x^4+4}+\frac{\ln(x^4+1)}{4}$	19
risch	$\frac{1}{4x^4+4}+\frac{\ln(x^4+1)}{4}$	19
parallelrisch	$\frac{\ln(x^4+1)x^4+1+\ln(x^4+1)}{4x^4+4}$	28

input
```
int(x^7/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^4+1)+1/4/(x^4+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{(x^4+1)\log(x^4+1)+1}{4\,(x^4+1)}$$

input
```
integrate(x^7/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/4*((x^4 + 1)*log(x^4 + 1) + 1)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{\log(x^4+1)}{4} + \frac{1}{4x^4+4}$$

input
```
integrate(x**7/(x**8+2*x**4+1),x)
```

output
```
log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{1}{4\,(x^4+1)} + \frac{1}{4}\log(x^4+1)$$

input
```
integrate(x^7/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{1}{4\,(x^4+1)} + \frac{1}{4}\log\left(x^4+1\right)$$

input
```
integrate(x^7/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{\ln\left(x^4+1\right)}{4} + \frac{1}{4\,\left(x^4+1\right)}$$

input
```
int(x^7/(2*x^4 + x^8 + 1),x)
```

output
```
log(x^4 + 1)/4 + 1/(4*(x^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.00

$$\int \frac{x^7}{1+2x^4+x^8}\,dx = \frac{\log\left(-\sqrt{2}\,x+x^2+1\right)x^4+\log\left(-\sqrt{2}\,x+x^2+1\right)+\log\left(\sqrt{2}\,x+x^2+1\right)x^4+\log\left(\sqrt{2}\,x+x^2+1\right)-x^4}{4x^4+4}$$

input
```
int(x^7/(x^8+2*x^4+1),x)
```

output
```
(log( - sqrt(2)*x + x**2 + 1)*x**4 + log( - sqrt(2)*x + x**2 + 1) + log(sq
rt(2)*x + x**2 + 1)*x**4 + log(sqrt(2)*x + x**2 + 1) - x**4)/(4*(x**4 + 1)
)
```

3.24 $\int \frac{x^5}{1+2x^4+x^8} \, dx$

Optimal result 211
Mathematica [A] (verified) 211
Rubi [A] (verified) 212
Maple [A] (verified) 213
Fricas [A] (verification not implemented) 214
Sympy [A] (verification not implemented) 214
Maxima [A] (verification not implemented) 214
Giac [A] (verification not implemented) 215
Mupad [B] (verification not implemented) 215
Reduce [B] (verification not implemented) 215

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^5}{1+2x^4+x^8} \, dx = -\frac{x^2}{4(1+x^4)} + \frac{\arctan(x^2)}{4}$$

output
```
-1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{1+2x^4+x^8} \, dx = -\frac{x^2}{4(1+x^4)} + \frac{\arctan(x^2)}{4}$$

input
```
Integrate[x^5/(1 + 2*x^4 + x^8),x]
```

output
```
-1/4*x^2/(1 + x^4) + ArcTan[x^2]/4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 807, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^8+2x^4+1} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^5}{\left(x^4+1\right)^2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{x^4}{\left(x^4+1\right)^2}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{x^4+1}dx^2-\frac{x^2}{2 \left(x^4+1\right)}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2} \left(\frac{\arctan \left(x^2\right)}{2}-\frac{x^2}{2 \left(x^4+1\right)}\right)$$

input
```
Int[x^5/(1 + 2*x^4 + x^8),x]
```

output
```
(-1/2*x^2/(1 + x^4) + ArcTan[x^2]/2)/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{x^{2}}{4(x^{4}+1)}+\frac{\arctan\left(x^{2}\right)}{4}$	20
risch	$-\frac{x^{2}}{4(x^{4}+1)}+\frac{\arctan\left(x^{2}\right)}{4}$	20
parallelrisch	$-\frac{i\ln\left(x^{2}-i\right)x^{4}-i\ln\left(x^{2}+i\right)x^{4}+i\ln\left(x^{2}-i\right)-i\ln\left(x^{2}+i\right)+2x^{2}}{8(x^{4}+1)}$	62

input
```
int(x^5/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{1+2x^4+x^8}\,dx = -\frac{x^2-(x^4+1)\arctan(x^2)}{4(x^4+1)}$$

input
```
integrate(x^5/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*(x^2 - (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{1+2x^4+x^8}\,dx = -\frac{x^2}{4x^4+4}+\frac{\operatorname{atan}(x^2)}{4}$$

input
```
integrate(x**5/(x**8+2*x**4+1),x)
```

output
```
-x**2/(4*x**4 + 4) + atan(x**2)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{1+2x^4+x^8}\,dx = -\frac{x^2}{4(x^4+1)}+\frac{1}{4}\arctan\left(x^2\right)$$

input
```
integrate(x^5/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{1+2x^4+x^8}\,dx = -\frac{x^2}{4(x^4+1)} + \frac{1}{4}\arctan\left(x^2\right)$$

input
```
integrate(x^5/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{1+2x^4+x^8}\,dx = \frac{\mathrm{atan}(x^2)}{4} - \frac{x^2}{4\,(x^4+1)}$$

input
```
int(x^5/(2*x^4 + x^8 + 1),x)
```

output
```
atan(x^2)/4 - x^2/(4*(x^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.39

$$\int \frac{x^5}{1+2x^4+x^8}\,dx = \frac{-atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^4 - atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) - atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^4 - atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) - x^2}{4x^4+4}$$

input
```
int(x^5/(x^8+2*x^4+1),x)
```

output
```
( - (atan((sqrt(2) - 2*x)/sqrt(2))*x**4 + atan((sqrt(2) - 2*x)/sqrt(2)) +
atan((sqrt(2) + 2*x)/sqrt(2))*x**4 + atan((sqrt(2) + 2*x)/sqrt(2)) + x**2)
)/(4*(x**4 + 1))
```

3.25 $\int \frac{x^3}{1+2x^4+x^8}\,dx$

Optimal result 217
Mathematica [A] (verified) 217
Rubi [A] (verified) 218
Maple [A] (verified) 219
Fricas [A] (verification not implemented) 219
Sympy [A] (verification not implemented) 220
Maxima [A] (verification not implemented) 220
Giac [A] (verification not implemented) 220
Mupad [B] (verification not implemented) 221
Reduce [B] (verification not implemented) 221

Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{x^3}{1+2x^4+x^8}\,dx=-\frac{1}{4\,(1+x^4)}$$

output
```
-1/4/(x^4+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+2x^4+x^8}\,dx=-\frac{1}{4\,(1+x^4)}$$

input
```
Integrate[x^3/(1 + 2*x^4 + x^8),x]
```

output
```
-1/4*1/(1 + x^4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1380, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8+2x^4+1}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^3}{\left(x^4+1\right)^2}dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{1}{4\left(x^4+1\right)}$$

input
```
Int[x^3/(1 + 2*x^4 + x^8),x]
```

output
```
-1/4*1/(1 + x^4)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gosper	$-\frac{1}{4(x^4+1)}$	10
default	$-\frac{1}{4(x^4+1)}$	10
norman	$-\frac{1}{4(x^4+1)}$	10
risch	$-\frac{1}{4(x^4+1)}$	10
parallelrisch	$-\frac{1}{4(x^4+1)}$	10
orering	$-\frac{x^4+1}{4(x^8+2x^4+1)}$	20

input
```
int(x^3/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(x^4+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+2x^4+x^8}\,dx = -\frac{1}{4\,(x^4+1)}$$

input
```
integrate(x^3/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{1+2x^4+x^8}\,dx=-\frac{1}{4x^4+4}$$

input
```
integrate(x**3/(x**8+2*x**4+1),x)
```

output
```
-1/(4*x**4 + 4)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+2x^4+x^8}\,dx=-\frac{1}{4\,(x^4+1)}$$

input
```
integrate(x^3/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4/(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+2x^4+x^8}\,dx=-\frac{1}{4\,(x^4+1)}$$

input
```
integrate(x^3/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-1/4/(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+2x^4+x^8}\,dx = -\frac{1}{4\,(x^4+1)}$$

input
```
int(x^3/(2*x^4 + x^8 + 1),x)
```

output
```
-1/(4*(x^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{1+2x^4+x^8}\,dx = \frac{x^4}{4x^4+4}$$

input
```
int(x^3/(x^8+2*x^4+1),x)
```

output
```
x**4/(4*(x**4 + 1))
```

3.26 $\int \frac{x}{1+2x^4+x^8}\,dx$

Optimal result 222
Mathematica [A] (verified) 222
Rubi [A] (verified) 223
Maple [A] (verified) 224
Fricas [A] (verification not implemented) 225
Sympy [A] (verification not implemented) 225
Maxima [A] (verification not implemented) 225
Giac [A] (verification not implemented) 226
Mupad [B] (verification not implemented) 226
Reduce [B] (verification not implemented) 226

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x}{1+2x^4+x^8}\,dx=\frac{x^2}{4(1+x^4)}+\frac{\arctan(x^2)}{4}$$

output
```
x^2/(4*x^4+4)+1/4*arctan(x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{1+2x^4+x^8}\,dx=\frac{1}{4}\left(\frac{x^2}{1+x^4}+\arctan\left(x^2\right)\right)$$

input
```
Integrate[x/(1 + 2*x^4 + x^8),x]
```

output
```
(x^2/(1 + x^4) + ArcTan[x^2])/4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1380, 807, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^8+2x^4+1}dx$$

$$\Big\downarrow \text{ 1380}$$

$$\int \frac{x}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{1}{(x^4+1)^2}dx^2$$

$$\Big\downarrow \text{ 215}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4+1}dx^2+\frac{x^2}{2(x^4+1)}\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\left(\frac{\arctan(x^2)}{2}+\frac{x^2}{2(x^4+1)}\right)$$

input
```
Int[x/(1 + 2*x^4 + x^8),x]
```

output
```
(x^2/(2*(1 + x^4)) + ArcTan[x^2]/2)/2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x^2}{4x^4+4}+\frac{\arctan(x^2)}{4}$	20
risch	$\frac{x^2}{4x^4+4}+\frac{\arctan(x^2)}{4}$	20
parallelrisch	$-\frac{i\ln(x^2-i)x^4-i\ln(x^2+i)x^4+i\ln(x^2-i)-i\ln(x^2+i)-2x^2}{8(x^4+1)}$	62

input
```
int(x/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{x^2+(x^4+1)\arctan(x^2)}{4\,(x^4+1)}$$

input
```
integrate(x/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/4*(x^2 + (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{x^2}{4x^4+4} + \frac{\operatorname{atan}(x^2)}{4}$$

input
```
integrate(x/(x**8+2*x**4+1),x)
```

output
```
x**2/(4*x**4 + 4) + atan(x**2)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{x^2}{4\,(x^4+1)} + \frac{1}{4}\arctan(x^2)$$

input
```
integrate(x/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{x^2}{4(x^4+1)} + \frac{1}{4}\arctan\left(x^2\right)$$

input
```
integrate(x/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{\operatorname{atan}(x^2)}{4} + \frac{x^2}{4\,(x^4+1)}$$

input
```
int(x/(2*x^4 + x^8 + 1),x)
```

output
```
atan(x^2)/4 + x^2/(4*(x^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

$$\int \frac{x}{1+2x^4+x^8}\,dx = \frac{-atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^4 - atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) - atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^4 - atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) + x^2}{4x^4+4}$$

input
```
int(x/(x^8+2*x^4+1),x)
```

output
```
( - atan((sqrt(2) - 2*x)/sqrt(2))*x**4 - atan((sqrt(2) - 2*x)/sqrt(2)) - a
tan((sqrt(2) + 2*x)/sqrt(2))*x**4 - atan((sqrt(2) + 2*x)/sqrt(2)) + x**2)/
(4*(x**4 + 1))
```

3.27 $\int \frac{1}{x(1+2x^4+x^8)}\,dx$

Optimal result . . . 228
Mathematica [A] (verified) . . . 228
Rubi [A] (verified) . . . 229
Maple [A] (verified) . . . 230
Fricas [A] (verification not implemented) . . . 231
Sympy [A] (verification not implemented) . . . 231
Maxima [A] (verification not implemented) . . . 231
Giac [A] (verification not implemented) . . . 232
Mupad [B] (verification not implemented) . . . 232
Reduce [B] (verification not implemented) . . . 232

Optimal result

Integrand size = 16, antiderivative size = 24

$$\int \frac{1}{x(1+2x^4+x^8)}\,dx = \frac{1}{4(1+x^4)}+\log(x)-\frac{1}{4}\log\left(1+x^4\right)$$

output
```
1/(4*x^4+4)+ln(x)-1/4*ln(x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+2x^4+x^8)}\,dx = \frac{1}{4(1+x^4)}+\log(x)-\frac{1}{4}\log\left(1+x^4\right)$$

input
```
Integrate[1/(x*(1 + 2*x^4 + x^8)),x]
```

output
```
1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (x^8+2x^4+1)} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x (x^4+1)^2}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^4 (x^4+1)^2}dx^4$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{4}\int \left(-\frac{1}{(x^4+1)^2}+\frac{1}{x^4}+\frac{1}{-x^4-1}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{x^4+1}+\log\left(x^4\right)-\log\left(x^4+1\right)\right)$$

input
```
Int[1/(x*(1 + 2*x^4 + x^8)),x]
```

output
```
((1 + x^4)^(-1) + Log[x^4] - Log[1 + x^4])/4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{1}{4x^4+4}+\ln(x)-\frac{\ln(x^4+1)}{4}$	21
norman	$\frac{1}{4x^4+4}+\ln(x)-\frac{\ln(x^4+1)}{4}$	21
risch	$\frac{1}{4x^4+4}+\ln(x)-\frac{\ln(x^4+1)}{4}$	21
parallelrisch	$\frac{4\ln(x)x^4-\ln(x^4+1)x^4+1+4\ln(x)-\ln(x^4+1)}{4x^4+4}$	42

input
```
int(1/x/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(x^4+1)+1/4/(x^4+1)+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = -\frac{\left(x^4+1\right)\log\left(x^4+1\right)-4\left(x^4+1\right)\log\left(x\right)-1}{4\left(x^4+1\right)}$$

input
```
integrate(1/x/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*((x^4 + 1)*log(x^4 + 1) - 4*(x^4 + 1)*log(x) - 1)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = \log\left(x\right)-\frac{\log\left(x^4+1\right)}{4}+\frac{1}{4x^4+4}$$

input
```
integrate(1/x/(x**8+2*x**4+1),x)
```

output
```
log(x) - log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = \frac{1}{4\left(x^4+1\right)}-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/4/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = \frac{x^4+2}{4\left(x^4+1\right)} - \frac{1}{4}\,\log\left(x^4+1\right) + \frac{1}{4}\,\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/4*(x^4 + 2)/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 20.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx = \ln\left(x\right) - \frac{\ln\left(x^4+1\right)}{4} + \frac{1}{4\,\left(x^4+1\right)}$$

input
```
int(1/(x*(2*x^4 + x^8 + 1)),x)
```

output
```
log(x) - log(x^4 + 1)/4 + 1/(4*(x^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.46

$$\int \frac{1}{x\left(1+2x^4+x^8\right)}\,dx$$
$$= \frac{-\log\left(-\sqrt{2}\,x+x^2+1\right)x^4 - \log\left(-\sqrt{2}\,x+x^2+1\right) - \log\left(\sqrt{2}\,x+x^2+1\right)x^4 - \log\left(\sqrt{2}\,x+x^2+1\right) + 4\,\mathrm{lo}}{4x^4+4}$$

input
```
int(1/x/(x^8+2*x^4+1),x)
```

output
```
( - log( - sqrt(2)*x + x**2 + 1)*x**4 - log( - sqrt(2)*x + x**2 + 1) - log
(sqrt(2)*x + x**2 + 1)*x**4 - log(sqrt(2)*x + x**2 + 1) + 4*log(x)*x**4 +
4*log(x) - x**4)/(4*(x**4 + 1))
```

3.28 $\int \frac{1}{x^3(1+2x^4+x^8)} \, dx$

Optimal result 234
Mathematica [A] (verified) 234
Rubi [A] (verified) 235
Maple [A] (verified) 236
Fricas [A] (verification not implemented) 237
Sympy [A] (verification not implemented) 237
Maxima [A] (verification not implemented) 238
Giac [A] (verification not implemented) 238
Mupad [B] (verification not implemented) 238
Reduce [B] (verification not implemented) 239

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{1}{x^3 (1+2x^4+x^8)} \, dx = -\frac{1}{2x^2} - \frac{x^2}{4(1+x^4)} - \frac{3\arctan(x^2)}{4}$$

output
```
-1/2/x^2-x^2/(4*x^4+4)-3/4*arctan(x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (1+2x^4+x^8)} \, dx = -\frac{1}{2x^2} - \frac{x^2}{4(1+x^4)} + \frac{3}{4}\arctan\left(\frac{1}{x^2}\right)$$

input
```
Integrate[1/(x^3*(1 + 2*x^4 + x^8)),x]
```

output
```
-1/2*1/x^2 - x^2/(4*(1 + x^4)) + (3*ArcTan[x^(-2)])/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1380, 807, 253, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^8+2x^4+1)}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^3 (x^4+1)^2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{x^4 (x^4+1)^2}dx^2$$

$$\Big\downarrow \text{253}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^4 (x^4+1)}dx^2+\frac{1}{2x^2 (x^4+1)}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(-\int \frac{1}{x^4+1}dx^2-\frac{1}{x^2}\right)+\frac{1}{2x^2 (x^4+1)}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(-\arctan\left(x^2\right)-\frac{1}{x^2}\right)+\frac{1}{2x^2 (x^4+1)}\right)$$

input
```
Int[1/(x^3*(1 + 2*x^4 + x^8)),x]
```

output
```
(1/(2*x^2*(1 + x^4)) + (3*(-x^(-2) - ArcTan[x^2]))/2)/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{x^{2}}{4\left(x^{4}+1\right)}-\frac{3 \arctan\left(x^{2}\right)}{4}-\frac{1}{2 x^{2}}$	25
risch	$\frac{-\frac{3 x^{4}}{4}-\frac{1}{2}}{x^{2}\left(x^{4}+1\right)}-\frac{3 \arctan\left(x^{2}\right)}{4}$	26
parallelrisch	$\frac{3 i \ln\left(x^{2}-i\right) x^{6}-3 i \ln\left(x^{2}+i\right) x^{6}-4+3 i \ln\left(x^{2}-i\right) x^{2}-3 i \ln\left(x^{2}+i\right) x^{2}-6 x^{4}}{8 x^{2}\left(x^{4}+1\right)}$	72

input
```
int(1/x^3/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2/(x^4+1)-3/4*arctan(x^2)-1/2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = -\frac{3\, x^4 + 3\, (x^6 + x^2) \arctan{(x^2)} + 2}{4\, (x^6 + x^2)}$$

input
```
integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*(3*x^4 + 3*(x^6 + x^2)*arctan(x^2) + 2)/(x^6 + x^2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = \frac{-3x^4 - 2}{4x^6 + 4x^2} - \frac{3 \operatorname{atan}{(x^2)}}{4}$$

input
```
integrate(1/x**3/(x**8+2*x**4+1),x)
```

output
```
(-3*x**4 - 2)/(4*x**6 + 4*x**2) - 3*atan(x**2)/4
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = -\frac{3\,x^4 + 2}{4\,(x^6 + x^2)} - \frac{3}{4} \arctan\left(x^2\right)$$

input
```
integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = -\frac{3\,x^4 + 2}{4\,(x^6 + x^2)} - \frac{3}{4} \arctan\left(x^2\right)$$

input
```
integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 19.81 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)} \, dx = -\frac{3\,\mathrm{atan}(x^2)}{4} - \frac{\frac{3\,x^4}{4} + \frac{1}{2}}{x^6 + x^2}$$

input
```
int(1/(x^3*(2*x^4 + x^8 + 1)),x)
```

output
```
- (3*atan(x^2))/4 - ((3*x^4)/4 + 1/2)/(x^2 + x^6)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.90

$$\int \frac{1}{x^3 (1 + 2x^4 + x^8)}\,dx$$
$$= \frac{3atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^6 + 3atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^2 + 3atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^6 + 3atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^2 - 3x^4 - 2}{4x^2\,(x^4+1)}$$

input
```
int(1/x^3/(x^8+2*x^4+1),x)
```

output
```
(3*atan((sqrt(2) - 2*x)/sqrt(2))*x**6 + 3*atan((sqrt(2) - 2*x)/sqrt(2))*x*
*2 + 3*atan((sqrt(2) + 2*x)/sqrt(2))*x**6 + 3*atan((sqrt(2) + 2*x)/sqrt(2)
)*x**2 - 3*x**4 - 2)/(4*x**2*(x**4 + 1))
```

3.29 $\int \frac{1}{x^5(1+2x^4+x^8)}\,dx$

Optimal result 240
Mathematica [A] (verified) 240
Rubi [A] (verified) 241
Maple [A] (verified) 242
Fricas [A] (verification not implemented) 243
Sympy [A] (verification not implemented) 243
Maxima [A] (verification not implemented) 243
Giac [A] (verification not implemented) 244
Mupad [B] (verification not implemented) 244
Reduce [B] (verification not implemented) 244

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{x^5(1+2x^4+x^8)}\,dx=-\frac{1}{4x^4}-\frac{1}{4(1+x^4)}-2\log(x)+\frac{1}{2}\log\left(1+x^4\right)$$

output
```
-1/4/x^4-1/(4*x^4+4)-2*ln(x)+1/2*ln(x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(1+2x^4+x^8)}\,dx=-\frac{1}{4x^4}-\frac{1}{4(1+x^4)}-2\log(x)+\frac{1}{2}\log\left(1+x^4\right)$$

input
```
Integrate[1/(x^5*(1 + 2*x^4 + x^8)),x]
```

output
```
-1/4*1/x^4 - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^8+2x^4+1)}dx$$

$$\Big\downarrow \; 1380$$

$$\int \frac{1}{x^5 (x^4+1)^2}dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{4}\int \frac{1}{x^8 (x^4+1)^2}dx^4$$

$$\Big\downarrow \; 54$$

$$\frac{1}{4}\int \left(-\frac{2}{x^4}+\frac{1}{x^8}+\frac{2}{x^4+1}+\frac{1}{(x^4+1)^2}\right)dx^4$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(-\frac{1}{x^4+1}-\frac{1}{x^4}-2\log\left(x^4\right)+2\log\left(x^4+1\right)\right)$$

input
```
Int[1/(x^5*(1 + 2*x^4 + x^8)),x]
```

output
```
(-x^(-4) - (1 + x^4)^(-1) - 2*Log[x^4] + 2*Log[1 + x^4])/4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln(x^4+1)}{2} - \frac{1}{4(x^4+1)} - \frac{1}{4x^4} - 2\ln(x)$	28
norman	$\frac{-\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4+1)} - 2\ln(x) + \frac{\ln(x^4+1)}{2}$	32
risch	$\frac{-\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4+1)} - 2\ln(x) + \frac{\ln(x^4+1)}{2}$	32
parallelrisch	$-\frac{8\ln(x)x^8-2\ln(x^4+1)x^8+1+8\ln(x)x^4-2\ln(x^4+1)x^4+2x^4}{4x^4(x^4+1)}$	56

input
```
int(1/x^5/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^4+1)-1/4/(x^4+1)-1/4/x^4-2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^5 (1+2x^4+x^8)} \, dx = -\frac{2\,x^4 - 2\,(x^8 + x^4) \log{(x^4 + 1)} + 8\,(x^8 + x^4) \log{(x)} + 1}{4\,(x^8 + x^4)}$$

input
```
integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*(2*x^4 - 2*(x^8 + x^4)*log(x^4 + 1) + 8*(x^8 + x^4)*log(x) + 1)/(x^8
+ x^4)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5 (1+2x^4+x^8)} \, dx = \frac{-2x^4 - 1}{4x^8 + 4x^4} - 2\log{(x)} + \frac{\log{(x^4 + 1)}}{2}$$

input
```
integrate(1/x**5/(x**8+2*x**4+1),x)
```

output
```
(-2*x**4 - 1)/(4*x**8 + 4*x**4) - 2*log(x) + log(x**4 + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (1+2x^4+x^8)} \, dx = -\frac{2\,x^4 + 1}{4\,(x^8 + x^4)} + \frac{1}{2}\,\log\left(x^4 + 1\right) - \frac{1}{2}\,\log\left(x^4\right)$$

input
```
integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (1+2x^4+x^8)} \, dx = -\frac{2 x^4+1}{4 (x^8+x^4)} + \frac{1}{2} \log \left(x^4+1\right) - \frac{1}{2} \log \left(x^4\right)$$

input
```
integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 20.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5 (1+2x^4+x^8)} \, dx = \frac{\ln \left(x^4+1\right)}{2} - 2 \ln \left(x\right) - \frac{\frac{x^4}{2}+\frac{1}{4}}{x^8+x^4}$$

input
```
int(1/(x^5*(2*x^4 + x^8 + 1)),x)
```

output
```
log(x^4 + 1)/2 - 2*log(x) - (x^4/2 + 1/4)/(x^4 + x^8)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.88

$$\int \frac{1}{x^5 (1+2x^4+x^8)} \, dx = \frac{2 \log\left(-\sqrt{2}\, x+x^2+1\right) x^8 + 2 \log\left(-\sqrt{2}\, x+x^2+1\right) x^4 + 2 \log\left(\sqrt{2}\, x+x^2+1\right) x^8 + 2 \log\left(\sqrt{2}\, x+x^2+1\right.}{4x^4 \left(x^4+1\right)}$$

input
```
int(1/x^5/(x^8+2*x^4+1),x)
```

output
```
(2*log( - sqrt(2)*x + x**2 + 1)*x**8 + 2*log( - sqrt(2)*x + x**2 + 1)*x**4
 + 2*log(sqrt(2)*x + x**2 + 1)*x**8 + 2*log(sqrt(2)*x + x**2 + 1)*x**4 - 8
*log(x)*x**8 - 8*log(x)*x**4 + 2*x**8 - 1)/(4*x**4*(x**4 + 1))
```

3.30 $\int \frac{1}{x^7(1+2x^4+x^8)} dx$

Optimal result 246
Mathematica [A] (verified) 246
Rubi [A] (verified) 247
Maple [A] (verified) 248
Fricas [A] (verification not implemented) 249
Sympy [A] (verification not implemented) 249
Maxima [A] (verification not implemented) 250
Giac [A] (verification not implemented) 250
Mupad [B] (verification not implemented) 250
Reduce [B] (verification not implemented) 251

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{x^7(1+2x^4+x^8)} dx = -\frac{1}{6x^6} + \frac{1}{x^2} + \frac{x^2}{4(1+x^4)} + \frac{5\arctan(x^2)}{4}$$

output
```
-1/6/x^6+1/x^2+x^2/(4*x^4+4)+5/4*arctan(x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^7(1+2x^4+x^8)} dx = -\frac{1}{6x^6} + \frac{1}{x^2} + \frac{x^2}{4(1+x^4)} - \frac{5}{4}\arctan\left(\frac{1}{x^2}\right)$$

input
```
Integrate[1/(x^7*(1 + 2*x^4 + x^8)),x]
```

output
```
-1/6*1/x^6 + x^(-2) + x^2/(4*(1 + x^4)) - (5*ArcTan[x^(-2)])/4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1380, 807, 253, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (x^8+2x^4+1)}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^7 (x^4+1)^2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{x^8 (x^4+1)^2}dx^2$$

$$\Big\downarrow \text{253}$$

$$\frac{1}{2}\left(\frac{5}{2}\int \frac{1}{x^8 (x^4+1)}dx^2+\frac{1}{2x^6 (x^4+1)}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\frac{5}{2}\left(-\int \frac{1}{x^4 (x^4+1)}dx^2-\frac{1}{3x^6}\right)+\frac{1}{2x^6 (x^4+1)}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\frac{5}{2}\left(\int \frac{1}{x^4+1}dx^2-\frac{1}{3x^6}+\frac{1}{x^2}\right)+\frac{1}{2x^6 (x^4+1)}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{5}{2}\left(\arctan\left(x^2\right)-\frac{1}{3x^6}+\frac{1}{x^2}\right)+\frac{1}{2x^6 (x^4+1)}\right)$$

input
```
Int[1/(x^7*(1 + 2*x^4 + x^8)),x]
```

output
```
(1/(2*x^6*(1 + x^4)) + (5*(-1/3*1/x^6 + x^(-2) + ArcTan[x^2]))/2)/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{1}{6x^{6}}+\frac{1}{x^{2}}+\frac{x^{2}}{4x^{4}+4}+\frac{5\arctan\left(x^{2}\right)}{4}$	28
risch	$\frac{\frac{5}{4}x^{8}+\frac{5}{6}x^{4}-\frac{1}{6}}{x^{6}\left(x^{4}+1\right)}+\frac{5\arctan\left(x^{2}\right)}{4}$	31
parallelrisch	$-\frac{15i\ln\left(x^{2}-i\right)x^{10}-15i\ln\left(x^{2}+i\right)x^{10}+4+15i\ln\left(x^{2}-i\right)x^{6}-15i\ln\left(x^{2}+i\right)x^{6}-30x^{8}-20x^{4}}{24x^{6}\left(x^{4}+1\right)}$	77

input
```
int(1/x^7/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2/(x^4+1)+5/4*arctan(x^2)-1/6/x^6+1/x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^7 (1+2x^4+x^8)}\,dx = \frac{15\,x^8+10\,x^4+15\,(x^{10}+x^6)\arctan{(x^2)}-2}{12\,(x^{10}+x^6)}$$

input
```
integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/12*(15*x^8 + 10*x^4 + 15*(x^10 + x^6)*arctan(x^2) - 2)/(x^10 + x^6)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^7 (1+2x^4+x^8)}\,dx = \frac{5\,\mathrm{atan}\,(x^2)}{4} + \frac{15x^8+10x^4-2}{12x^{10}+12x^6}$$

input
```
integrate(1/x**7/(x**8+2*x**4+1),x)
```

output
```
5*atan(x**2)/4 + (15*x**8 + 10*x**4 - 2)/(12*x**10 + 12*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^7 (1+2x^4+x^8)} \, dx = \frac{15 \, x^8 + 10 \, x^4 - 2}{12 \, (x^{10} + x^6)} + \frac{5}{4} \arctan \left(x^2\right)$$

input
```
integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/12*(15*x^8 + 10*x^4 - 2)/(x^10 + x^6) + 5/4*arctan(x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^7 (1+2x^4+x^8)} \, dx = \frac{x^2}{4 \, (x^4+1)} + \frac{6 \, x^4 - 1}{6 \, x^6} + \frac{5}{4} \arctan \left(x^2\right)$$

input
```
integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/4*x^2/(x^4 + 1) + 1/6*(6*x^4 - 1)/x^6 + 5/4*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 19.99 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^7 (1+2x^4+x^8)} \, dx = \frac{5 \, \mathrm{atan}(x^2)}{4} + \frac{\frac{5 \, x^8}{4} + \frac{5 \, x^4}{6} - \frac{1}{6}}{x^6 \, (x^4+1)}$$

input
```
int(1/(x^7*(2*x^4 + x^8 + 1)),x)
```

output
```
(5*atan(x^2))/4 + ((5*x^4)/6 + (5*x^8)/4 - 1/6)/(x^6*(x^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.79

$$\int \frac{1}{x^7 (1+2x^4+x^8)}\,dx = \frac{-15atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^{10} - 15atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^6 - 15atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^{10} - 15atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^6 + 15x^8 + 10x^4 -}{12x^6(x^4+1)}$$

input
```
int(1/x^7/(x^8+2*x^4+1),x)
```

output
```
( - 15*atan((sqrt(2) - 2*x)/sqrt(2))*x***10 - 15*atan((sqrt(2) - 2*x)/sqrt(
2))*x***6 - 15*atan((sqrt(2) + 2*x)/sqrt(2))*x***10 - 15*atan((sqrt(2) + 2*x
)/sqrt(2))*x***6 + 15*x***8 + 10*x***4 - 2)/(12*x***6*(x***4 + 1))
```

3.31 $\int \frac{x^{10}}{1+2x^4+x^8}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{x^{10}}{1+2x^4+x^8}\,dx = \frac{x^3}{3}+\frac{x^3}{4(1+x^4)}+\frac{7\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}}-\frac{7\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}}+\frac{7\text{arctanh}\left(\frac{\sqrt{2}x}{1+x^2}\right)}{8\sqrt{2}}$$

output
```
1/3*x^3+x^3/(4*x^4+4)-7/16*arctan(-1+x*2^(1/2))*2^(1/2)-7/16*arctan(1+x*2^
(1/2))*2^(1/2)+7/16*arctanh(2^(1/2)*x/(x^2+1))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.15

$$\int \frac{x^{10}}{1+2x^4+x^8}\,dx = \frac{1}{96}\left(32x^3+\frac{24x^3}{1+x^4}+42\sqrt{2}\arctan\left(1-\sqrt{2}x\right)-42\sqrt{2}\arctan\left(1+\sqrt{2}x\right)-21\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right)+21\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[x^10/(1 + 2*x^4 + x^8),x]
```

output
```
(32*x^3 + (24*x^3)/(1 + x^4) + 42*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 42*Sqrt[
2]*ArcTan[1 + Sqrt[2]*x] - 21*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 21*Sqrt[2
]*Log[1 + Sqrt[2]*x + x^2])/96
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1380, 817, 843, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{x^8+2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^{10}}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{7}{4}\int \frac{x^6}{x^4+1}dx-\frac{x^7}{4(x^4+1)}$$

$$\Big\downarrow \text{843}$$

$$\frac{7}{4}\left(\frac{x^3}{3}-\int \frac{x^2}{x^4+1}dx\right)-\frac{x^7}{4(x^4+1)}$$

$$\Big\downarrow \text{826}$$

$$\frac{7}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx+\frac{x^3}{3}\right)-\frac{x^7}{4(x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{7}{4}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{x^3}{3}\right)-\frac{x^7}{4(x^4+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{7}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{x^3}{3}\right)-\frac{x^7}{4\left(x^4+1\right)}$$

$$\Big\downarrow \ 217$$

$$\frac{7}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+\frac{x^3}{3}\right)-\frac{x^7}{4\left(x^4+1\right)}$$

$$\Big\downarrow \ 1479$$

$$\frac{7}{4}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+\frac{x^3}{3}\right)-\frac{x^7}{4\left(x^4+1\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{7}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+\frac{x^3}{3}\right)-\frac{x^7}{4\left(x^4+1\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{7}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+\frac{x^3}{3}\right)-\frac{x^7}{4\left(x^4+1\right)}$$

$$\Big\downarrow \ 1103$$

$$\frac{7}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+\frac{x^3}{3}+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right)-\frac{x^7}{4\left(x^4+1\right)}$$

input
```
Int[x^10/(1 + 2*x^4 + x^8),x]
```

output
```
-1/4*x^7/(1 + x^4) + (7*(x^3/3 + (ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - ArcTan[1
 + Sqrt[2]*x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1
+ Sqrt[2]*x + x^2]/(2*Sqrt[2]))/2))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.47

method	result	size
risch	$\frac{x^3}{3}+\frac{x^3}{4x^4+4}-\frac{7\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{16}$	40
default	$\frac{x^3}{3}+\frac{x^3}{4x^4+4}-\frac{7\sqrt{2}\left(\ln\left(\frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{32}$	70

input
```
int(x^10/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+1/4*x^3/(x^4+1)-7/16*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \frac{x^{10}}{1+2x^4+x^8}\,dx$$
$$=\frac{32\,x^7+56\,x^3-42\,\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x+1\right)-42\,\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x-1\right)+21\,\sqrt{2}(x^4+1)\log}{96\,(x^4+1)}$$

input
```
integrate(x^10/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/96*(32*x^7 + 56*x^3 - 42*sqrt(2)*(x^4 + 1)*arctan(sqrt(2)*x + 1) - 42*sq
rt(2)*(x^4 + 1)*arctan(sqrt(2)*x - 1) + 21*sqrt(2)*(x^4 + 1)*log(x^2 + sqr
t(2)*x + 1) - 21*sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1))/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \frac{x^{10}}{1+2x^4+x^8}\,dx = \frac{x^3}{3}+\frac{x^3}{4x^4+4}-\frac{7\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{7\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} - \frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}-\frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(x**10/(x**8+2*x**4+1),x)
```

output
```
x**3/3 + x**3/(4*x**4 + 4) - 7*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 7*sq
rt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 7*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 7*
sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \frac{x^{10}}{1+2x^4+x^8}\,dx = \frac{1}{3}\,x^3+\frac{x^3}{4\,(x^4+1)}-\frac{7}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) - \frac{7}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{7}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{7}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(x^10/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/3*x^3 + 1/4*x^3/(x^4 + 1) - 7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2))) - 7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 7/32*sqrt(2)*log
(x^2 + sqrt(2)*x + 1) - 7/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \frac{x^{10}}{1+2x^4+x^8}\,dx = \frac{1}{3}\,x^3 + \frac{x^3}{4\,(x^4+1)} - \frac{7}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) - \frac{7}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{7}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{7}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(x^10/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/3*x^3 + 1/4*x^3/(x^4 + 1) - 7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2))) - 7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 7/32*sqrt(2)*log
(x^2 + sqrt(2)*x + 1) - 7/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 20.82 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\int \frac{x^{10}}{1+2x^4+x^8}\,dx = \frac{x^3}{4\,(x^4+1)} + \frac{x^3}{3} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{7}{16}+\frac{7}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{7}{16}-\frac{7}{16}\mathrm{i}\right)$$

input
```
int(x^10/(2*x^4 + x^8 + 1),x)
```

output
```
x^3/(4*(x^4 + 1)) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(7/16 + 7i/16) -
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(7/16 - 7i/16) + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.82

$$\int \frac{x^{10}}{1+2x^4+x^8}\,dx$$
$$= \frac{42\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^4+42\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)-42\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^4-42\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)-21\sqrt{2}\log}{}$$

input
```
int(x^10/(x^8+2*x^4+1),x)
```

output
```
(42*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2))*x**4 + 42*sqrt(2)*atan((sqrt(2)
- 2*x)/sqrt(2)) - 42*sqrt(2)*atan((sqrt(2) + 2*x)/sqrt(2))*x**4 - 42*sqrt(
2)*atan((sqrt(2) + 2*x)/sqrt(2)) - 21*sqrt(2)*log( - sqrt(2)*x + x**2 + 1)
*x**4 - 21*sqrt(2)*log( - sqrt(2)*x + x**2 + 1) + 21*sqrt(2)*log(sqrt(2)*x
 + x**2 + 1)*x**4 + 21*sqrt(2)*log(sqrt(2)*x + x**2 + 1) + 32*x**7 + 56*x*
*3)/(96*(x**4 + 1))
```

3.32 $\int \frac{x^8}{1+2x^4+x^8}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = x+\frac{x}{4\left(1+x^4\right)}+\frac{5\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{5\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}}-\frac{5\text{arctanh}\left(\frac{\sqrt{2}x}{1+x^2}\right)}{8\sqrt{2}}$$

output
```
x+x/(4*x^4+4)-5/16*arctan(-1+x*2^(1/2))*2^(1/2)-5/16*arctan(1+x*2^(1/2))*2
^(1/2)-5/16*arctanh(2^(1/2)*x/(x^2+1))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = \frac{1}{32}\left(32x+\frac{8x}{1+x^4}+10\sqrt{2}\arctan\left(1-\sqrt{2}x\right) - 10\sqrt{2}\arctan\left(1+\sqrt{2}x\right)+5\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) - 5\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[x^8/(1 + 2*x^4 + x^8),x]
```

output
```
(32*x + (8*x)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*Ar
cTan[1 + Sqrt[2]*x] + 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 5*Sqrt[2]*Log[1
 + Sqrt[2]*x + x^2])/32
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.43, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1380, 817, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^8+2x^4+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^8}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{5}{4}\int \frac{x^4}{x^4+1}dx-\frac{x^5}{4(x^4+1)}$$

$$\Big\downarrow \text{843}$$

$$\frac{5}{4}\left(x-\int \frac{1}{x^4+1}dx\right)-\frac{x^5}{4(x^4+1)}$$

$$\Big\downarrow \text{755}$$

$$\frac{5}{4}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx+x\right)-\frac{x^5}{4(x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+x\right)-\frac{x^5}{4(x^4+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{5}{4}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d(1-\sqrt{2}x)}{\sqrt{2}}\right)+x\right)-\frac{x^5}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{5}{4}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+x\right)-\frac{x^5}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{5}{4}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+x\right)-\frac{x^5}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{5}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+x\right)-\frac{x^5}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{5}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+x\right)-\frac{x^5}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{5}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}\right)+x\right)-\frac{x^5}{4\left(x^4+1\right)}$$

input
```
Int[x^8/(1 + 2*x^4 + x^8),x]
```

output
```
-1/4*x^5/(1 + x^4) + (5*(x + (ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - ArcTan[1 + S
qrt[2]*x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) - Log[1 + Sqr
t[2]*x + x^2]/(2*Sqrt[2]))/2))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.44

method	result	size
risch	$x+\frac{x}{4x^4+4}-\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16}$	34
default	$x+\frac{x}{4x^4+4}-\frac{5\sqrt{2}\left(\ln\left(\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{32}$	64

input
```
int(x^8/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
x+1/4*x/(x^4+1)-5/16*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26

$$\int \frac{x^8}{1+2x^4+x^8}\,dx$$
$$=\frac{32\,x^5-10\,\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x+1\right)-10\,\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x-1\right)-5\,\sqrt{2}(x^4+1)\log\left(x^2+\sqrt{\right.}}{32\,(x^4+1)}$$

input
```
integrate(x^8/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/32*(32*x^5 - 10*sqrt(2)*(x^4 + 1)*arctan(sqrt(2)*x + 1) - 10*sqrt(2)*(x^
4 + 1)*arctan(sqrt(2)*x - 1) - 5*sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1
) + 5*sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1) + 40*x)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = x+\frac{x}{4x^4+4}+\frac{5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} - \frac{5\sqrt{2}\,\text{atan}\left(\sqrt{2}x-1\right)}{16}-\frac{5\sqrt{2}\,\text{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(x**8/(x**8+2*x**4+1),x)
```

output
```
x + x/(4*x**4 + 4) + 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 5*sqrt(2)*lo
g(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 5*sqrt(2)*
atan(sqrt(2)*x + 1)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = -\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+x+\frac{x}{4\left(x^4+1\right)}$$

input
```
integrate(x^8/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = -\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + x + \frac{x}{4\,(x^4+1)}$$

input
```
integrate(x^8/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 14.64 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = x + \frac{x}{4\,(x^4+1)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{16}-\frac{5}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{16}+\frac{5}{16}\mathrm{i}\right)$$

input
```
int(x^8/(2*x^4 + x^8 + 1),x)
```

output
```
x - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(5/16 + 5i/16) - 2^(1/2)*atan(2^(
1/2)*x*(1/2 + 1i/2))*(5/16 - 5i/16) + x/(4*(x^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.99

$$\int \frac{x^8}{1+2x^4+x^8}\,dx$$
$$= \frac{10\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^4+10\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)-10\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^4-10\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)+5\sqrt{2}\log(}{}$$

input
```
int(x^8/(x^8+2*x^4+1),x)
```

output
```
(10*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2))*x**4 + 10*sqrt(2)*atan((sqrt(2)
- 2*x)/sqrt(2)) - 10*sqrt(2)*atan((sqrt(2) + 2*x)/sqrt(2))*x**4 - 10*sqrt(
2)*atan((sqrt(2) + 2*x)/sqrt(2)) + 5*sqrt(2)*log( - sqrt(2)*x + x**2 + 1)*
x**4 + 5*sqrt(2)*log( - sqrt(2)*x + x**2 + 1) - 5*sqrt(2)*log(sqrt(2)*x +
x**2 + 1)*x**4 - 5*sqrt(2)*log(sqrt(2)*x + x**2 + 1) + 32*x**5 + 40*x)/(32
*(x**4 + 1))
```

3.33 $\int \frac{x^6}{1+2x^4+x^8} dx$

Optimal result 270
Mathematica [A] (verified) 270
Rubi [A] (verified) 271
Maple [C] (verified) 274
Fricas [A] (verification not implemented) 275
Sympy [A] (verification not implemented) 275
Maxima [A] (verification not implemented) 276
Giac [A] (verification not implemented) 276
Mupad [B] (verification not implemented) 277
Reduce [B] (verification not implemented) 277

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{x^6}{1+2x^4+x^8} dx = -\frac{x^3}{4(1+x^4)} - \frac{3\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} - \frac{3\text{arctanh}\left(\frac{\sqrt{2}x}{1+x^2}\right)}{8\sqrt{2}}$$

output
```
-1/4*x^3/(x^4+1)+3/16*arctan(-1+x*2^(1/2))*2^(1/2)+3/16*arctan(1+x*2^(1/2)
)*2^(1/2)-3/16*arctanh(2^(1/2)*x/(x^2+1))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

$$\int \frac{x^6}{1+2x^4+x^8} dx = \frac{1}{32}\left(-\frac{8x^3}{1+x^4} - 6\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 6\sqrt{2}\arctan\left(1+\sqrt{2}x\right) + 3\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) - 3\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[x^6/(1 + 2*x^4 + x^8),x]
```

output
```
((-8*x^3)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1
 + Sqrt[2]*x] + 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 3*Sqrt[2]*Log[1 + Sqr
t[2]*x + x^2])/32
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1380, 817, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^8+2x^4+1}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^6}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{3}{4}\int \frac{x^2}{x^4+1}dx-\frac{x^3}{4(x^4+1)}$$

$$\Big\downarrow \text{826}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)-\frac{x^3}{4(x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)-\frac{x^3}{4(x^4+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)-\frac{x^3}{4(x^4+1)}$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-x^2}{x^4+1}dx\right)-\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right)-\frac{x^3}{4\left(x^4+1\right)}$$

input
```
Int[x^6/(1 + 2*x^4 + x^8),x]
```

output
```
-1/4*x^3/(1 + x^4) + (3*((-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sq
rt[2]*x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) - Log[1 + Sqrt
[2]*x + x^2]/(2*Sqrt[2]))/2))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.45

method	result	size
risch	$-\frac{x^{3}}{4(x^{4}+1)}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}+1\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{16}$	35
default	$-\frac{x^{3}}{4(x^{4}+1)}+\frac{3\sqrt{2}\,\left(\ln\left(\frac{x^{2}-x\sqrt{2}+1}{x^{2}+x\sqrt{2}+1}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{32}$	65

input
```
int(x^6/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^3/(x^4+1)+3/16*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{8\,x^3 - 6\,\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x+1\right) - 6\,\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x-1\right) + 3\,\sqrt{2}(x^4+1)\log\left(x^2+\sqrt{2}\right.}{32\,(x^4+1)}$$

input
```
integrate(x^6/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/32*(8*x^3 - 6*sqrt(2)*(x^4 + 1)*arctan(sqrt(2)*x + 1) - 6*sqrt(2)*(x^4
+ 1)*arctan(sqrt(2)*x - 1) + 3*sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1)
- 3*sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1))/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.15

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4x^4+4} + \frac{3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} - \frac{3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16} + \frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(x**6/(x**8+2*x**4+1),x)
```

output
```
-x**3/(4*x**4 + 4) + 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 3*sqrt(2)*lo
g(x**2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*
atan(sqrt(2)*x + 1)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4+1)} + \frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{3}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{3}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(x^6/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/
16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sq
rt(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4+1)} + \frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{3}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{3}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{3}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(x^6/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/
16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sq
rt(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = -\frac{x^3}{4\,(x^4+1)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)$$

input
```
int(x^6/(2*x^4 + x^8 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(3/16 - 3i/16) + 2^(1/2)*atan(2^(1/2)
*x*(1/2 + 1i/2))*(3/16 + 3i/16) - x^3/(4*(x^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.92

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = \frac{-6\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^4 - 6\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) + 6\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^4 + 6\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) + 3\sqrt{2}\log(-\sqrt{}}{32}$$

input
```
int(x^6/(x^8+2*x^4+1),x)
```

output
```
( - 6*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2))*x**4 - 6*sqrt(2)*atan((sqrt(2)
 - 2*x)/sqrt(2)) + 6*sqrt(2)*atan((sqrt(2) + 2*x)/sqrt(2))*x**4 + 6*sqrt(2
)*atan((sqrt(2) + 2*x)/sqrt(2)) + 3*sqrt(2)*log( - sqrt(2)*x + x**2 + 1)*x
**4 + 3*sqrt(2)*log( - sqrt(2)*x + x**2 + 1) - 3*sqrt(2)*log(sqrt(2)*x + x
**2 + 1)*x**4 - 3*sqrt(2)*log(sqrt(2)*x + x**2 + 1) - 8*x**3)/(32*(x**4 +
1))
```

3.34 $\int \frac{x^4}{1+2x^4+x^8} \, dx$

Optimal result 278
Mathematica [A] (verified) 278
Rubi [A] (verified) 279
Maple [C] (verified) 282
Fricas [A] (verification not implemented) 283
Sympy [A] (verification not implemented) 283
Maxima [A] (verification not implemented) 284
Giac [A] (verification not implemented) 284
Mupad [B] (verification not implemented) 285
Reduce [B] (verification not implemented) 285

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{x^4}{1+2x^4+x^8} \, dx = -\frac{x}{4(1+x^4)} - \frac{\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{1+x^2}\right)}{8\sqrt{2}}$$

output
```
-1/4*x/(x^4+1)+1/16*arctan(-1+x*2^(1/2))*2^(1/2)+1/16*arctan(1+x*2^(1/2))*
2^(1/2)+1/16*arctanh(2^(1/2)*x/(x^2+1))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18

$$\int \frac{x^4}{1+2x^4+x^8} \, dx = \frac{1}{32}\left(-\frac{8x}{1+x^4} - 2\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 2\sqrt{2}\arctan\left(1+\sqrt{2}x\right) - \sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) + \sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[x^4/(1 + 2*x^4 + x^8),x]
```

output
```
((-8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]*Log[1 + Sqrt[2]*x
 + x^2])/32
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.41, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1380, 817, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^8+2x^4+1}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^4}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{1}{4}\int \frac{1}{x^4+1}dx-\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx\right)-\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)-\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)\right)-\frac{x}{4(x^4+1)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)-\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right)-\frac{x}{4\left(x^4+1\right)}$$

input
```
Int[x^4/(1 + 2*x^4 + x^8),x]
```

output
```
-1/4*x/(1 + x^4) + ((-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]
*x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1 + Sqrt[2]*
x + x^2]/(2*Sqrt[2]))/2)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.43

method	result	size
risch	$-\frac{x}{4(x^4+1)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16}$	33
default	$-\frac{x}{4(x^4+1)}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{32}$	63

input
```
int(x^4/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x/(x^4+1)+1/16*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \frac{x^4}{1+2x^4+x^8}\,dx$$
$$= \frac{2\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x+1\right)+2\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x-1\right)+\sqrt{2}(x^4+1)\log\left(x^2+\sqrt{2}x+1\right)-}{32\,(x^4+1)}$$

input
```
integrate(x^4/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/32*(2*sqrt(2)*(x^4 + 1)*arctan(sqrt(2)*x + 1) + 2*sqrt(2)*(x^4 + 1)*arct
an(sqrt(2)*x - 1) + sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) - sqrt(2)*(
x^4 + 1)*log(x^2 - sqrt(2)*x + 1) - 8*x)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = -\frac{x}{4x^4+4}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(x**4/(x**8+2*x**4+1),x)
```

output
```
-x/(4*x**4 + 4) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + sqrt(2)*log(x**2
+ sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt(2
)*x + 1)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{x}{4\left(x^4+1\right)}$$

input
```
integrate(x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*s
qrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{x}{4\left(x^4+1\right)}$$

input
```
integrate(x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*s
qrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = -\frac{x}{4\,(x^4+1)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}+\frac{1}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}-\frac{1}{16}\mathrm{i}\right)$$

input
```
int(x^4/(2*x^4 + x^8 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/16 + 1i/16) + 2^(1/2)*atan(2^(1/2)
*x*(1/2 + 1i/2))*(1/16 - 1i/16) - x/(4*(x^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.92

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = \frac{-2\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^4 - 2\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) + 2\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^4 + 2\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) - \sqrt{2}\log(-\sqrt{}}{32x^4}$$

input
```
int(x^4/(x^8+2*x^4+1),x)
```

output
```
( - 2*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2))*x**4 - 2*sqrt(2)*atan((sqrt(2)
 - 2*x)/sqrt(2)) + 2*sqrt(2)*atan((sqrt(2) + 2*x)/sqrt(2))*x**4 + 2*sqrt(2
)*atan((sqrt(2) + 2*x)/sqrt(2)) - sqrt(2)*log( - sqrt(2)*x + x**2 + 1)*x**
4 - sqrt(2)*log( - sqrt(2)*x + x**2 + 1) + sqrt(2)*log(sqrt(2)*x + x**2 +
1)*x**4 + sqrt(2)*log(sqrt(2)*x + x**2 + 1) - 8*x)/(32*(x**4 + 1))
```

3.35 $\int \frac{x^2}{1+2x^4+x^8} \, dx$

Optimal result 286
Mathematica [A] (verified) 286
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Reduce [B] (verification not implemented) 293

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{x^2}{1+2x^4+x^8} \, dx = \frac{x^3}{4(1+x^4)} - \frac{\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{1+x^2}\right)}{8\sqrt{2}}$$

output
```
x^3/(4*x^4+4)+1/16*arctan(-1+x*2^(1/2))*2^(1/2)+1/16*arctan(1+x*2^(1/2))*2
^(1/2)-1/16*arctanh(2^(1/2)*x/(x^2+1))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{1+2x^4+x^8} \, dx = \frac{1}{32}\left(\frac{8x^3}{1+x^4} - 2\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 2\sqrt{2}\arctan\left(1+\sqrt{2}x\right) + \sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) - \sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[x^2/(1 + 2*x^4 + x^8),x]
```

output
```
((8*x^3)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1
+ Sqrt[2]*x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1 + Sqrt[2]*
x + x^2])/32
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1380, 819, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^8+2x^4+1}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^2}{(x^4+1)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{1}{4}\int \frac{x^2}{x^4+1}dx+\frac{x^3}{4(x^4+1)}$$

$$\Big\downarrow \text{826}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)+\frac{x^3}{4(x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)+\frac{x^3}{4(x^4+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)+$$
$$\frac{x^3}{4(x^4+1)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-x^2}{x^4+1}dx\right)+\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x^3}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right)+\frac{x^3}{4\left(x^4+1\right)}$$

input

```
Int[x^2/(1 + 2*x^4 + x^8),x]
```

output
```
x^3/(4*(1 + x^4)) + ((-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2
]*x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*
x + x^2]/(2*Sqrt[2]))/2)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.45

method	result	size
risch	$\frac{x^3}{4x^4+4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{16}$	35
default	$\frac{x^3}{4x^4+4}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{32}$	65

input
```
int(x^2/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^3/(x^4+1)+1/16*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{1+2x^4+x^8}\,dx$$
$$= \frac{8\,x^3+2\,\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x+1\right)+2\,\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x-1\right)-\sqrt{2}(x^4+1)\log\left(x^2+\sqrt{2}x+\right.}{32\,(x^4+1)}$$

input
```
integrate(x^2/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/32*(8*x^3 + 2*sqrt(2)*(x^4 + 1)*arctan(sqrt(2)*x + 1) + 2*sqrt(2)*(x^4 +
 1)*arctan(sqrt(2)*x - 1) - sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + s
qrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1))/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{x^3}{4x^4+4}+\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(x**2/(x**8+2*x**4+1),x)
```

output
```
x**3/(4*x**4 + 4) + sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - sqrt(2)*log(x**
2 + sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt
(2)*x + 1)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{x^3}{4\,(x^4+1)} + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqr
t(2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{x^3}{4\,(x^4+1)} + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{32}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{32}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate(x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqr
t(2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{x^3}{4\,(x^4+1)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{16}-\frac{1}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{16}+\frac{1}{16}\mathrm{i}\right)$$

input
```
int(x^2/(2*x^4 + x^8 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/16 - 1i/16) + 2^(1/2)*atan(2^(1/2)
*x*(1/2 + 1i/2))*(1/16 + 1i/16) + x^3/(4*(x^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \frac{-2\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^4 - 2\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) + 2\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^4 + 2\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) + \sqrt{2}\log(-\sqrt{}}{32x^4}$$

input
```
int(x^2/(x^8+2*x^4+1),x)
```

output
```
( - 2*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2))*x**4 - 2*sqrt(2)*atan((sqrt(2)
 - 2*x)/sqrt(2)) + 2*sqrt(2)*atan((sqrt(2) + 2*x)/sqrt(2))*x**4 + 2*sqrt(2
)*atan((sqrt(2) + 2*x)/sqrt(2)) + sqrt(2)*log( - sqrt(2)*x + x**2 + 1)*x**
4 + sqrt(2)*log( - sqrt(2)*x + x**2 + 1) - sqrt(2)*log(sqrt(2)*x + x**2 +
1)*x**4 - sqrt(2)*log(sqrt(2)*x + x**2 + 1) + 8*x**3)/(32*(x**4 + 1))
```

3.36 $\int \frac{1}{1+2x^4+x^8} dx$

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Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{1}{1+2x^4+x^8} dx = \frac{x}{4(1+x^4)} - \frac{3\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{3\text{arctanh}\left(\frac{\sqrt{2}x}{1+x^2}\right)}{8\sqrt{2}}$$

output
```
x/(4*x^4+4)+3/16*arctan(-1+x*2^(1/2))*2^(1/2)+3/16*arctan(1+x*2^(1/2))*2^(
1/2)+3/16*arctanh(2^(1/2)*x/(x^2+1))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \frac{1}{1+2x^4+x^8} dx = \frac{1}{32}\left(\frac{8x}{1+x^4} - 6\sqrt{2}\arctan\left(1-\sqrt{2}x\right) + 6\sqrt{2}\arctan\left(1+\sqrt{2}x\right) - 3\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) + 3\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[(1 + 2*x^4 + x^8)^(-1),x]
```

output
```
((8*x)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1 +
Sqrt[2]*x] - 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 3*Sqrt[2]*Log[1 + Sqrt[2
]*x + x^2])/32
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.41, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {1379, 749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8+2x^4+1}dx$$

$$\Big\downarrow \text{1379}$$

$$\int \frac{1}{\left(x^4+1\right)^2}dx$$

$$\Big\downarrow \text{749}$$

$$\frac{3}{4}\int \frac{1}{x^4+1}dx+\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx\right)+\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx\right)+\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)\right)+\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)\right)+\frac{x}{4\left(x^4+1\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right)+\frac{x}{4\left(x^4+1\right)}$$

input
```
Int[(1 + 2*x^4 + x^8)^(-1),x]
```

output
```
x/(4*(1 + x^4)) + (3*((-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[
2]*x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1 + Sqrt[2
]*x + x^2]/(2*Sqrt[2]))/2))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{x}{4x^4+4}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16}$	33
default	$\frac{x}{4x^4+4}+\frac{3\sqrt{2}\left(\ln\left(\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{32}$	63

input
```
int(1/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x/(x^4+1)+3/16*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{6\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x+1\right)+6\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x-1\right)+3\sqrt{2}(x^4+1)\log\left(x^2+\sqrt{2}x+1\right)-}{32\,(x^4+1)}$$

input
```
integrate(1/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
1/32*(6*sqrt(2)*(x^4 + 1)*arctan(sqrt(2)*x + 1) + 6*sqrt(2)*(x^4 + 1)*arct
an(sqrt(2)*x - 1) + 3*sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) - 3*sqrt(
2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1) + 8*x)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{x}{4x^4+4} - \frac{3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} + \frac{3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} + \frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16} + \frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(1/(x**8+2*x**4+1),x)
```

output
```
x/(4*x**4 + 4) - 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 3*sqrt(2)*log(x*
*2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*atan
(sqrt(2)*x + 1)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{3}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \frac{x}{4\,(x^4+1)}$$

input
```
integrate(1/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*s
qrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{3}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \frac{x}{4\,(x^4+1)}$$

input
```
integrate(1/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*s
qrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.58

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{x}{4\,(x^4+1)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)$$

input
```
int(1/(2*x^4 + x^8 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(3/16 + 3i/16) + 2^(1/2)*atan(2^(1/2)
*x*(1/2 + 1i/2))*(3/16 - 3i/16) + x/(4*(x^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.95

$$\int \frac{1}{1+2x^4+x^8}\,dx = \frac{-6\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^4 - 6\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) + 6\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^4 + 6\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) - 3\sqrt{2}\log(-\sqrt{}}{32}$$

input
```
int(1/(x^8+2*x^4+1),x)
```

output
```
( - 6*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2))*x**4 - 6*sqrt(2)*atan((sqrt(2)
 - 2*x)/sqrt(2)) + 6*sqrt(2)*atan((sqrt(2) + 2*x)/sqrt(2))*x**4 + 6*sqrt(2
)*atan((sqrt(2) + 2*x)/sqrt(2)) - 3*sqrt(2)*log( - sqrt(2)*x + x**2 + 1)*x
**4 - 3*sqrt(2)*log( - sqrt(2)*x + x**2 + 1) + 3*sqrt(2)*log(sqrt(2)*x + x
**2 + 1)*x**4 + 3*sqrt(2)*log(sqrt(2)*x + x**2 + 1) + 8*x)/(32*(x**4 + 1))
```

3.37 $\int \frac{1}{x^2(1+2x^4+x^8)}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 83

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = -\frac{1}{x} - \frac{x^3}{4(1+x^4)} + \frac{5\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{5\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}} + \frac{5\text{arctanh}\left(\frac{\sqrt{2}x}{1+x^2}\right)}{8\sqrt{2}}$$

output
```
-1/x-x^3/(4*x^4+4)-5/16*arctan(-1+x*2^(1/2))*2^(1/2)-5/16*arctan(1+x*2^(1/
2))*2^(1/2)+5/16*arctanh(2^(1/2)*x/(x^2+1))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = \frac{1}{32}\left(-\frac{32}{x} - \frac{8x^3}{1+x^4} + 10\sqrt{2}\arctan\left(1-\sqrt{2}x\right) - 10\sqrt{2}\arctan\left(1+\sqrt{2}x\right) - 5\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) + 5\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

output
```
(-32/x - (8*x^3)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]
*ArcTan[1 + Sqrt[2]*x] - 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 5*Sqrt[2]*Lo
g[1 + Sqrt[2]*x + x^2])/32
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.37, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1380, 819, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^8+2x^4+1)} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^2 (x^4+1)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{5}{4}\int \frac{1}{x^2 (x^4+1)}dx+\frac{1}{4x (x^4+1)}$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{4}\left(-\int \frac{x^2}{x^4+1}dx-\frac{1}{x}\right)+\frac{1}{4x (x^4+1)}$$

$$\Big\downarrow \text{826}$$

$$\frac{5}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx-\frac{1}{x}\right)+\frac{1}{4x (x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{x}\right)+\frac{1}{4x (x^4+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{5}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{x}\right)+\frac{1}{4x\left(x^4+1\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{5}{4}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{x}\right)+\frac{1}{4x\left(x^4+1\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{5}{4}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{x}\right)+\frac{1}{4x\left(x^4+1\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{5}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{x}\right)+\frac{1}{4x\left(x^4+1\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{5}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{x}\right)+\frac{1}{4x\left(x^4+1\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{5}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}\right)-\frac{1}{x}\right)+\frac{1}{4x\left(x^4+1\right)}$$

input
```
Int[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

output
```
1/(4*x*(1 + x^4)) + (5*(-x^(-1) + (ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - ArcTan[
1 + Sqrt[2]*x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1
 + Sqrt[2]*x + x^2]/(2*Sqrt[2]))/2))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{-\frac{5x^4}{4}-1}{x(x^4+1)}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}(\_Z^4+1)}\_R\ln\left(-\_R^3+x\right)\right)}{16}$	41
default	$-\frac{x^3}{4(x^4+1)}-\frac{5\sqrt{2}\left(\ln\left(\frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{32}-\frac{1}{x}$	70

input
```
int(1/x^2/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-5/4*x^4-1)/x/(x^4+1)+5/16*sum(_R*ln(-_R^3+x),_R=RootOf(_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = -\frac{40\,x^4+10\sqrt{2}(x^5+x)\arctan\left(\sqrt{2}x+1\right)+10\sqrt{2}(x^5+x)\arctan\left(\sqrt{2}x-1\right)-5\sqrt{2}(x^5+x)\log\left(x^2+\right.}{32\,(x^5+x)}$$

input
```
integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/32*(40*x^4 + 10*sqrt(2)*(x^5 + x)*arctan(sqrt(2)*x + 1) + 10*sqrt(2)*(x
^5 + x)*arctan(sqrt(2)*x - 1) - 5*sqrt(2)*(x^5 + x)*log(x^2 + sqrt(2)*x +
1) + 5*sqrt(2)*(x^5 + x)*log(x^2 - sqrt(2)*x + 1) + 32)/(x^5 + x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = \frac{-5x^4-4}{4x^5+4x} - \frac{5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} + \frac{5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} - \frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16} - \frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(1/x**2/(x**8+2*x**4+1),x)
```

output
```
(-5*x**4 - 4)/(4*x**5 + 4*x) - 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 5*
sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 -
5*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(1+2x^4+x^8)}\,dx = -\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{5x^4+4}{4(x^5+x)}$$

input
```
integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(1+2x^4+x^8)} dx = -\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{5x^4+4}{4(x^5+x)}$$

input
```
integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

Mupad [B] (verification not implemented)

Time = 19.85 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^2(1+2x^4+x^8)} dx = -\frac{\frac{5x^4}{4}+1}{x^5+x} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{16}+\frac{5}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{16}-\frac{5}{16}\mathrm{i}\right)$$

input
```
int(1/(x^2*(2*x^4 + x^8 + 1)),x)
```

output
```
- ((5*x^4)/4 + 1)/(x + x^5) - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(5/16 -
 5i/16) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(5/16 + 5i/16)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^2 (1 + 2x^4 + x^8)} \, dx$$
$$= \frac{10\sqrt{2}\, atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) x^5 + 10\sqrt{2}\, atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) x - 10\sqrt{2}\, atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) x^5 - 10\sqrt{2}\, atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) x - 5\sqrt{2}\, l}{}$$

input
```
int(1/x^2/(x^8+2*x^4+1),x)
```

output
```
(10*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2))*x**5 + 10*sqrt(2)*atan((sqrt(2)
- 2*x)/sqrt(2))*x - 10*sqrt(2)*atan((sqrt(2) + 2*x)/sqrt(2))*x**5 - 10*sqr
t(2)*atan((sqrt(2) + 2*x)/sqrt(2))*x - 5*sqrt(2)*log( - sqrt(2)*x + x**2 +
 1)*x**5 - 5*sqrt(2)*log( - sqrt(2)*x + x**2 + 1)*x + 5*sqrt(2)*log(sqrt(2
)*x + x**2 + 1)*x**5 + 5*sqrt(2)*log(sqrt(2)*x + x**2 + 1)*x - 40*x**4 - 3
2)/(32*x*(x**4 + 1))
```

3.38 $\int \frac{1}{x^4(1+2x^4+x^8)} dx$

Optimal result 311
Mathematica [A] (verified) 311
Rubi [A] (verified) 312
Maple [C] (verified) 316
Fricas [A] (verification not implemented) 316
Sympy [A] (verification not implemented) 317
Maxima [A] (verification not implemented) 317
Giac [A] (verification not implemented) 318
Mupad [B] (verification not implemented) 318
Reduce [B] (verification not implemented) 319

Optimal result

Integrand size = 16, antiderivative size = 83

$$\int \frac{1}{x^4\left(1+2x^4+x^8\right)}\,dx = -\frac{1}{3x^3}-\frac{x}{4\left(1+x^4\right)}+\frac{7\arctan\left(1-\sqrt{2}x\right)}{8\sqrt{2}} -\frac{7\arctan\left(1+\sqrt{2}x\right)}{8\sqrt{2}}-\frac{7\text{arctanh}\left(\frac{\sqrt{2}x}{1+x^2}\right)}{8\sqrt{2}}$$

output
```
-1/3/x^3-x/(4*x^4+4)-7/16*arctan(-1+x*2^(1/2))*2^(1/2)-7/16*arctan(1+x*2^(
1/2))*2^(1/2)-7/16*arctanh(2^(1/2)*x/(x^2+1))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^4\left(1+2x^4+x^8\right)}\,dx = \frac{1}{96}\left(-\frac{32}{x^3}-\frac{24x}{1+x^4}+42\sqrt{2}\arctan\left(1-\sqrt{2}x\right) -42\sqrt{2}\arctan\left(1+\sqrt{2}x\right)+21\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) -21\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

output
```
(-32/x^3 - (24*x)/(1 + x^4) + 42*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 42*Sqrt[2
]*ArcTan[1 + Sqrt[2]*x] + 21*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 21*Sqrt[2]
*Log[1 + Sqrt[2]*x + x^2])/96
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.40, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1380, 819, 847, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^8+2x^4+1)} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^4 (x^4+1)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{7}{4} \int \frac{1}{x^4 (x^4+1)}dx+\frac{1}{4x^3 (x^4+1)}$$

$$\Big\downarrow \text{847}$$

$$\frac{7}{4} \left(-\int \frac{1}{x^4+1}dx-\frac{1}{3x^3}\right)+\frac{1}{4x^3 (x^4+1)}$$

$$\Big\downarrow \text{755}$$

$$\frac{7}{4} \left(-\frac{1}{2} \int \frac{1-x^2}{x^4+1}dx-\frac{1}{2} \int \frac{x^2+1}{x^4+1}dx-\frac{1}{3x^3}\right)+\frac{1}{4x^3 (x^4+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{7}{4} \left(\frac{1}{2} \left(-\frac{1}{2} \int \frac{1}{x^2-\sqrt{2}x+1}dx-\frac{1}{2} \int \frac{1}{x^2+\sqrt{2}x+1}dx\right)-\frac{1}{2} \int \frac{1-x^2}{x^4+1}dx-\frac{1}{3x^3}\right)+\frac{1}{4x^3 (x^4+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{7}{4}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)-\frac{1}{3x^3}\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{7}{4}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{3x^3}\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{7}{4}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{3x^3}\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{7}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{3x^3}\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{4}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{3x^3}\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{7}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)-\frac{1}{3x^3}+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}\right)\right)+\frac{1}{4x^3\left(x^4+1\right)}$$

input
```
Int[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

output
```
1/(4*x^3*(1 + x^4)) + (7*(-1/3*1/x^3 + (ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - Ar
cTan[1 + Sqrt[2]*x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) - L
og[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]))/2))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.47

method	result	size
risch	$\frac{-\frac{7x^4}{12}-\frac{1}{3}}{x^3(x^4+1)}+\frac{7\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\_R\ln\left(x-\_R\right)\right)}{16}$	39
default	$-\frac{x}{4(x^4+1)}-\frac{7\sqrt{2}\left(\ln\left(\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{32}-\frac{1}{3x^3}$	68

input
```
int(1/x^4/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-7/12*x^4-1/3)/x^3/(x^4+1)+7/16*sum(_R*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^4\left(1+2x^4+x^8\right)}\,dx = -\frac{56\,x^4+42\,\sqrt{2}(x^7+x^3)\arctan\left(\sqrt{2}x+1\right)+42\,\sqrt{2}(x^7+x^3)\arctan\left(\sqrt{2}x-1\right)+21\,\sqrt{2}(x^7+x^3)\log\left(\right.}{96\left(x^7+x^3\right)}$$

input
```
integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
-1/96*(56*x^4 + 42*sqrt(2)*(x^7 + x^3)*arctan(sqrt(2)*x + 1) + 42*sqrt(2)*
(x^7 + x^3)*arctan(sqrt(2)*x - 1) + 21*sqrt(2)*(x^7 + x^3)*log(x^2 + sqrt(
2)*x + 1) - 21*sqrt(2)*(x^7 + x^3)*log(x^2 - sqrt(2)*x + 1) + 32)/(x^7 + x
^3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4 (1+2x^4+x^8)} \, dx = \frac{-7x^4-4}{12x^7+12x^3} + \frac{7\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32} - \frac{7\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32} - \frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{16} - \frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{16}$$

input
```
integrate(1/x**4/(x**8+2*x**4+1),x)
```

output
```
(-7*x**4 - 4)/(12*x**7 + 12*x**3) + 7*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32
 - 7*sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 7*sqrt(2)*atan(sqrt(2)*x - 1)/
16 - 7*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (1+2x^4+x^8)} \, dx = -\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{7}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{7}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{7x^4+4}{12(x^7+x^3)}$$

input
```
integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
-7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/12*(7*x^4 + 4)/(x^7 + x^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^4(1+2x^4+x^8)}\,dx = -\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{7}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{7}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{x}{4(x^4+1)} - \frac{1}{3x^3}$$

input
```
integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
-7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1) - 1/3/x^3
```

Mupad [B] (verification not implemented)

Time = 19.67 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^4(1+2x^4+x^8)}\,dx = -\frac{\frac{7x^4}{12}+\frac{1}{3}}{x^7+x^3} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{16}-\frac{7}{16}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{16}+\frac{7}{16}\mathrm{i}\right)$$

input
```
int(1/(x^4*(2*x^4 + x^8 + 1)),x)
```

output
```
- 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(7/16 + 7i/16) - 2^(1/2)*atan(2^(1/
2)*x*(1/2 + 1i/2))*(7/16 - 7i/16) - ((7*x^4)/12 + 1/3)/(x^3 + x^7)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.99

$$\int \frac{1}{x^4 (1+2x^4+x^8)} \, dx$$
$$= \frac{42\sqrt{2}\, atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) x^7 + 42\sqrt{2}\, atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) x^3 - 42\sqrt{2}\, atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) x^7 - 42\sqrt{2}\, atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) x^3 + 21\sqrt{}$$

input
```
int(1/x^4/(x^8+2*x^4+1),x)
```

output
```
(42*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2))*x**7 + 42*sqrt(2)*atan((sqrt(2)
- 2*x)/sqrt(2))*x**3 - 42*sqrt(2)*atan((sqrt(2) + 2*x)/sqrt(2))*x**7 - 42*
sqrt(2)*atan((sqrt(2) + 2*x)/sqrt(2))*x**3 + 21*sqrt(2)*log( - sqrt(2)*x +
 x**2 + 1)*x**7 + 21*sqrt(2)*log( - sqrt(2)*x + x**2 + 1)*x**3 - 21*sqrt(2
)*log(sqrt(2)*x + x**2 + 1)*x**7 - 21*sqrt(2)*log(sqrt(2)*x + x**2 + 1)*x*
*3 - 56*x**4 - 32)/(96*x**3*(x**4 + 1))
```

3.39 $\int x^7\sqrt{a^2+2abx^4+b^2x^8}\,dx$

Optimal result 320
Mathematica [A] (verified) 320
Rubi [A] (verified) 321
Maple [C] (warning: unable to verify) 322
Fricas [A] (verification not implemented) 323
Sympy [F(-1)] 323
Maxima [A] (verification not implemented) 324
Giac [A] (verification not implemented) 324
Mupad [B] (verification not implemented) 324
Reduce [B] (verification not implemented) 325

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int x^7\sqrt{a^2+2abx^4+b^2x^8}\,dx=-\frac{a(a+bx^4)\sqrt{a^2+2abx^4+b^2x^8}}{8b^2}+\frac{(a^2+2abx^4+b^2x^8)^{3/2}}{12b^2}$$

output
```
-1/8*a*(b*x^4+a)*((b*x^4+a)^2)^(1/2)/b^2+1/12*(b^2*x^8+2*a*b*x^4+a^2)^(3/2
)/b^2
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int x^7\sqrt{a^2+2abx^4+b^2x^8}\,dx=\frac{x^8\sqrt{(a+bx^4)^2}(3a+2bx^4)}{24\,(a+bx^4)}$$

input
```
Integrate[x^7*Sqrt[a^2 + 2*a*b*x^4 + b^2*x^8],x]
```

output
```
(x^8*Sqrt[(a + b*x^4)^2]*(3*a + 2*b*x^4))/(24*(a + b*x^4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \sqrt{a^2+2abx^4+b^2x^8}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int x^4\sqrt{b^2x^8+2abx^4+a^2}dx^4$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{4}\left(\frac{\left(a^2+2abx^4+b^2x^8\right)^{3/2}}{3b^2}-\frac{a\int \sqrt{b^2x^8+2abx^4+a^2}dx^4}{b}\right)$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{4}\left(\frac{\left(a^2+2abx^4+b^2x^8\right)^{3/2}}{3b^2}-\frac{a\sqrt{a^2+2abx^4+b^2x^8}\int \left(b^2x^4+ab\right)dx^4}{b^2\left(a+bx^4\right)}\right)$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{4}\left(\frac{\left(a^2+2abx^4+b^2x^8\right)^{3/2}}{3b^2}-\frac{a\left(a+bx^4\right)\sqrt{a^2+2abx^4+b^2x^8}}{2b^2}\right)$$

input
```
Int[x^7*Sqrt[a^2 + 2*a*b*x^4 + b^2*x^8],x]
```

output
```
(-1/2*(a*(a + b*x^4)*Sqrt[a^2 + 2*a*b*x^4 + b^2*x^8])/b^2 + (a^2 + 2*a*b*x
^4 + b^2*x^8)^(3/2)/(3*b^2))/4
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{4}+a\right)\left(b\,x^{4}+a\right)^{2}\left(-2b\,x^{4}+a\right)}{24b^{2}}$	31
gosper	$\frac{x^{8}\left(2b\,x^{4}+3a\right)\sqrt{\left(b\,x^{4}+a\right)^{2}}}{24b\,x^{4}+24a}$	36
default	$\frac{x^{8}\left(2b\,x^{4}+3a\right)\sqrt{\left(b\,x^{4}+a\right)^{2}}}{24b\,x^{4}+24a}$	36
orering	$\frac{x^{8}\left(2b\,x^{4}+3a\right)\sqrt{\left(b\,x^{4}+a\right)^{2}}}{24b\,x^{4}+24a}$	36
risch	$\frac{\sqrt{\left(b\,x^{4}+a\right)^{2}}\,b\,x^{12}}{12b\,x^{4}+12a}+\frac{\sqrt{\left(b\,x^{4}+a\right)^{2}}\,a\,x^{8}}{8b\,x^{4}+8a}$	54

input
```
int(x^7*((b*x^4+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*csgn(b*x^4+a)*(b*x^4+a)^2*(-2*b*x^4+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.19

$$\int x^7\sqrt{a^2+2abx^4+b^2x^8}\,dx=\frac{1}{12}\,bx^{12}+\frac{1}{8}\,ax^8$$

input
```
integrate(x^7*((b*x^4+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*b*x^12 + 1/8*a*x^8
```

Sympy [F(-1)]

Timed out.

$$\int x^7\sqrt{a^2+2abx^4+b^2x^8}\,dx=\text{Timed out}$$

input
```
integrate(x**7*((b*x**4+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.19

$$\int x^7\sqrt{a^2+2abx^4+b^2x^8}\,dx=\frac{1}{12}\,bx^{12}+\frac{1}{8}\,ax^8$$

input
```
integrate(x^7*((b*x^4+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/12*b*x^12 + 1/8*a*x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.34

$$\int x^7\sqrt{a^2+2abx^4+b^2x^8}\,dx=\frac{1}{24}\,\left(2\,bx^{12}+3\,ax^8\right)\mathrm{sgn}\left(bx^4+a\right)$$

input
```
integrate(x^7*((b*x^4+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/24*(2*b*x^12 + 3*a*x^8)*sgn(b*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 20.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int x^7\sqrt{a^2+2abx^4+b^2x^8}\,dx$$
$$=\frac{\sqrt{a^2+2\,a\,b\,x^4+b^2\,x^8}\,\left(8\,b^2\,\left(a^2+b^2\,x^8\right)-12\,a^2\,b^2+4\,a\,b^3\,x^4\right)}{96\,b^4}$$

input
```
int(x^7*((a + b*x^4)^2)^(1/2),x)
```

output
```
((a^2 + b^2*x^8 + 2*a*b*x^4)^(1/2)*(8*b^2*(a^2 + b^2*x^8) - 12*a^2*b^2 + 4
*a*b^3*x^4))/(96*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.22

$$\int x^7\sqrt{a^2+2abx^4+b^2x^8}\,dx=\frac{x^8(2b\,x^4+3a)}{24}$$

input
```
int(x^7*((b*x^4+a)^2)^(1/2),x)
```

output
```
(x**8*(3*a + 2*b*x**4))/24
```

3.40 $\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x} \, dx$

Optimal result 326
Mathematica [B] (verified) 326
Rubi [A] (verified) 327
Maple [C] (warning: unable to verify) 328
Fricas [A] (verification not implemented) 329
Sympy [F] 329
Maxima [A] (verification not implemented) 330
Giac [A] (verification not implemented) 330
Mupad [B] (verification not implemented) 330
Reduce [B] (verification not implemented) 331

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x} \, dx = \frac{bx^4\sqrt{a^2+2abx^4+b^2x^8}}{4(a+bx^4)} + \frac{a\sqrt{a^2+2abx^4+b^2x^8}\log(x)}{a+bx^4}$$

output
```
b*x^4*((b*x^4+a)^2)^(1/2)/(4*b*x^4+4*a)+a*((b*x^4+a)^2)^(1/2)*ln(x)/(b*x^4
+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 454 vs. 2(75) = 150.

Time = 1.30 (sec) , antiderivative size = 454, normalized size of antiderivative = 6.05

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x} \, dx$$
$$= \frac{-2a\sqrt{a^2}bx^4 - 2\sqrt{a^2}b^2x^8 + 2abx^4\sqrt{(a+bx^4)^2} - 2a\left(a^2+abx^4-\sqrt{a^2}\sqrt{(a+bx^4)^2}\right)\operatorname{arctanh}\left(\frac{bx^4}{\sqrt{a^2}-\sqrt{(a+\cdots}}\right.}{}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^4 + b^2*x^8]/x,x]
```

output
```
(-2*a*Sqrt[a^2]*b*x^4 - 2*Sqrt[a^2]*b^2*x^8 + 2*a*b*x^4*Sqrt[(a + b*x^4)^2
] - 2*a*(a^2 + a*b*x^4 - Sqrt[a^2]*Sqrt[(a + b*x^4)^2])*ArcTanh[(b*x^4)/(S
qrt[a^2] - Sqrt[(a + b*x^4)^2])] - 2*((a^2)^(3/2) + a*Sqrt[a^2]*b*x^4 - a^
2*Sqrt[(a + b*x^4)^2])*Log[x^4] + (a^2)^(3/2)*Log[Sqrt[a^2] - b*x^4 - Sqrt
[(a + b*x^4)^2]] + a*Sqrt[a^2]*b*x^4*Log[Sqrt[a^2] - b*x^4 - Sqrt[(a + b*x
^4)^2]] - a^2*Sqrt[(a + b*x^4)^2]*Log[Sqrt[a^2] - b*x^4 - Sqrt[(a + b*x^4)
^2]] + (a^2)^(3/2)*Log[Sqrt[a^2] + b*x^4 - Sqrt[(a + b*x^4)^2]] + a*Sqrt[a
^2]*b*x^4*Log[Sqrt[a^2] + b*x^4 - Sqrt[(a + b*x^4)^2]] - a^2*Sqrt[(a + b*x
^4)^2]*Log[Sqrt[a^2] + b*x^4 - Sqrt[(a + b*x^4)^2]])/(8*(a^2 + a*b*x^4 - S
qrt[a^2]*Sqrt[(a + b*x^4)^2]))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^4+b^2x^8}\int \frac{b(bx^4+a)}{x}dx}{b(a+bx^4)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^4+b^2x^8}\int \frac{bx^4+a}{x}dx}{a+bx^4}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^4+b^2x^8}\int \left(bx^3+\frac{a}{x}\right)dx}{a+bx^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^4+b^2x^8}\left(a\log(x)+\frac{bx^4}{4}\right)}{a+bx^4}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^4 + b^2*x^8]/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^4 + b^2*x^8]*((b*x^4)/4 + a*Log[x]))/(a + b*x^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{4}+a\right)\left(b\,x^{4}+a+a\ln\left(b\,x^{4}\right)\right)}{4}$	26
default	$\frac{\sqrt{\left(b\,x^{4}+a\right)^{2}}\,\left(b\,x^{4}+4 a\ln\left(x\right)\right)}{4 b\,x^{4}+4 a}$	34
risch	$\frac{b\,x^{4}\sqrt{\left(b\,x^{4}+a\right)^{2}}}{4 b\,x^{4}+4 a}+\frac{a\sqrt{\left(b\,x^{4}+a\right)^{2}}\,\ln\left(x\right)}{b\,x^{4}+a}$	52

input
```
int(((b*x^4+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*csgn(b*x^4+a)*(b*x^4+a+a*ln(b*x^4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x}\,dx = \frac{1}{4}\,bx^4 + a\log\left(x\right)$$

input
```
integrate(((b*x^4+a)^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/4*b*x^4 + a*log(x)
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x}\,dx = \int \frac{\sqrt{\left(a+bx^4\right)^2}}{x}\,dx$$

input
```
integrate(((b*x**4+a)**2)**(1/2)/x,x)
```

output
```
Integral(sqrt((a + b*x**4)**2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x}\,dx = \frac{1}{4}\,bx^4 + \frac{1}{4}\,a\log\left(x^4\right)$$

input
```
integrate(((b*x^4+a)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/4*b*x^4 + 1/4*a*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x}\,dx = \frac{1}{4}\,bx^4\mathrm{sgn}\left(bx^4+a\right) + \frac{1}{4}\,a\log\left(x^4\right)\mathrm{sgn}\left(bx^4+a\right)$$

input
```
integrate(((b*x^4+a)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/4*b*x^4*sgn(b*x^4 + a) + 1/4*a*log(x^4)*sgn(b*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 20.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^4+b^2\,x^8}}{4} - \frac{\ln\left(a\,b+\frac{a^2}{x^4}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^4+b^2\,x^8}}{x^4}\right)\sqrt{a^2}}{4} + \frac{a\,b\,\ln\left(a\,b+\sqrt{\left(b\,x^4+a\right)^2}\,\sqrt{b^2}+b^2\,x^4\right)}{4\,\sqrt{b^2}}$$

input
```
int(((a + b*x^4)^2)^(1/2)/x,x)
```

output
```
(a^2 + b^2*x^8 + 2*a*b*x^4)^(1/2)/4 - (log(a*b + a^2/x^4 + ((a^2)^(1/2)*(a
^2 + b^2*x^8 + 2*a*b*x^4)^(1/2))/x^4)*(a^2)^(1/2))/4 + (a*b*log(a*b + ((a
+ b*x^4)^2)^(1/2)*(b^2)^(1/2) + b^2*x^4))/(4*(b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x}\,dx = \log(x)\,a+\frac{b\,x^4}{4}$$

input
```
int(((b*x^4+a)^2)^(1/2)/x,x)
```

output
```
(4*log(x)*a + b*x**4)/4
```

3.41 $\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x^9}\,dx$

Optimal result 332
Mathematica [A] (verified) 332
Rubi [A] (verified) 333
Maple [C] (warning: unable to verify) 334
Fricas [A] (verification not implemented) 335
Sympy [F(-1)] 335
Maxima [A] (verification not implemented) 336
Giac [A] (verification not implemented) 336
Mupad [B] (verification not implemented) 336
Reduce [B] (verification not implemented) 337

Optimal result

Integrand size = 26, antiderivative size = 39

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x^9}\,dx=-\frac{(a+bx^4)\sqrt{a^2+2abx^4+b^2x^8}}{8ax^8}$$

output
```
-1/8*(b*x^4+a)*((b*x^4+a)^2)^(1/2)/a/x^8
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x^9}\,dx=-\frac{\sqrt{(a+bx^4)^2}(a+2bx^4)}{8x^8(a+bx^4)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^4 + b^2*x^8]/x^9,x]
```

output
```
-1/8*(Sqrt[(a + b*x^4)^2]*(a + 2*b*x^4))/(x^8*(a + b*x^4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1693, 1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x^9}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{4}\int \frac{\sqrt{b^2x^8+2abx^4+a^2}}{x^{12}}dx^4$$

$$\Big\downarrow \text{ 1102}$$

$$\frac{\sqrt{a^2+2abx^4+b^2x^8}\int \frac{b(bx^4+a)}{x^{12}}dx^4}{4b(a+bx^4)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^4+b^2x^8}\int \frac{bx^4+a}{x^{12}}dx^4}{4(a+bx^4)}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{(a+bx^4)\sqrt{a^2+2abx^4+b^2x^8}}{8ax^8}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^4 + b^2*x^8]/x^9,x]
```

output
```
-1/8*((a + b*x^4)*Sqrt[a^2 + 2*a*b*x^4 + b^2*x^8])/(a*x^8)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{4}+a\right)\left(2 b\,x^{4}+a\right)}{8 x^{8}}$	22
gosper	$-\frac{\left(2 b\,x^{4}+a\right)\sqrt{\left(b\,x^{4}+a\right)^{2}}}{8 x^{8}\left(b\,x^{4}+a\right)}$	34
default	$-\frac{\left(2 b\,x^{4}+a\right)\sqrt{\left(b\,x^{4}+a\right)^{2}}}{8 x^{8}\left(b\,x^{4}+a\right)}$	34
orering	$-\frac{\left(2 b\,x^{4}+a\right)\sqrt{\left(b\,x^{4}+a\right)^{2}}}{8 x^{8}\left(b\,x^{4}+a\right)}$	34
risch	$\frac{\left(-\frac{b\,x^{4}}{4}-\frac{a}{8}\right)\sqrt{\left(b\,x^{4}+a\right)^{2}}}{x^{8}\left(b\,x^{4}+a\right)}$	35

input
```
int(((b*x^4+a)^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*csgn(b*x^4+a)*(2*b*x^4+a)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x^9}\,dx=-\frac{2\,bx^4+a}{8\,x^8}$$

input
```
integrate(((b*x^4+a)^2)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
-1/8*(2*b*x^4 + a)/x^8
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^4+b^2x^8}}{x^9}\,dx=\text{Timed out}$$

input
```
integrate(((b*x**4+a)**2)**(1/2)/x**9,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{a^2 + 2abx^4 + b^2x^8}}{x^9}\,dx = -\frac{2\,bx^4 + a}{8\,x^8}$$

input
```
integrate(((b*x^4+a)^2)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
-1/8*(2*b*x^4 + a)/x^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a^2 + 2abx^4 + b^2x^8}}{x^9}\,dx = -\frac{2\,bx^4\mathrm{sgn}(bx^4 + a) + a\mathrm{sgn}(bx^4 + a)}{8\,x^8}$$

input
```
integrate(((b*x^4+a)^2)^(1/2)/x^9,x, algorithm="giac")
```

output
```
-1/8*(2*b*x^4*sgn(b*x^4 + a) + a*sgn(b*x^4 + a))/x^8
```

Mupad [B] (verification not implemented)

Time = 19.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a^2 + 2abx^4 + b^2x^8}}{x^9}\,dx = -\frac{(2\,b\,x^4 + a)\,\sqrt{(b\,x^4 + a)^2}}{8\,x^8\,(b\,x^4 + a)}$$

input
```
int(((a + b*x^4)^2)^(1/2)/x^9,x)
```

output
```
-((a + 2*b*x^4)*((a + b*x^4)^2)^(1/2))/(8*x^8*(a + b*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a^2 + 2abx^4 + b^2x^8}}{x^9}\,dx = \frac{-2b\,x^4 - a}{8x^8}$$

input
```
int(((b*x^4+a)^2)^(1/2)/x^9,x)
```

output
```
( - a - 2*b*x**4)/(8*x**8)
```

3.42 $\int \frac{x^m}{1+2x^4+x^8} \, dx$

Optimal result . . . 338
Mathematica [A] (verified) . . . 338
Rubi [A] (verified) . . . 339
Maple [F] . . . 340
Fricas [F] . . . 340
Sympy [F] . . . 340
Maxima [F] . . . 341
Giac [F] . . . 341
Mupad [F(-1)] . . . 341
Reduce [F] . . . 342

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{x^m}{1+2x^4+x^8} \, dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(2, \frac{1+m}{4}, \frac{5+m}{4}, -x^4\right)}{1+m}$$

output
```
x^(1+m)*hypergeom([2, 1/4+1/4*m],[5/4+1/4*m],-x^4)/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{x^m}{1+2x^4+x^8} \, dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(2, \frac{1+m}{4}, 1+\frac{1+m}{4}, -x^4\right)}{1+m}$$

input
```
Integrate[x^m/(1 + 2*x^4 + x^8),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, -x^4])/(1 + m)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1380, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{x^8+2x^4+1}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^m}{\left(x^4+1\right)^2}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{4},\frac{m+5}{4},-x^4\right)}{m+1}$$

input
```
Int[x^m/(1 + 2*x^4 + x^8),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, -x^4])/(1 + m)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x^m}{x^8 + 2x^4 + 1} dx$$

input
```
int(x^m/(x^8+2*x^4+1),x)
```

output
```
int(x^m/(x^8+2*x^4+1),x)
```

Fricas [F]

$$\int \frac{x^m}{1 + 2x^4 + x^8} \, dx = \int \frac{x^m}{x^8 + 2\,x^4 + 1} \, dx$$

input
```
integrate(x^m/(x^8+2*x^4+1),x, algorithm="fricas")
```

output
```
integral(x^m/(x^8 + 2*x^4 + 1), x)
```

Sympy [F]

$$\int \frac{x^m}{1 + 2x^4 + x^8} \, dx = \int \frac{x^m}{(x^4 + 1)^2} \, dx$$

input
```
integrate(x**m/(x**8+2*x**4+1),x)
```

output
```
Integral(x**m/(x**4 + 1)**2, x)
```

Maxima [F]

$$\int \frac{x^m}{1+2x^4+x^8}\,dx=\int \frac{x^m}{x^8+2\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+2*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^m}{1+2x^4+x^8}\,dx=\int \frac{x^m}{x^8+2\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+2*x^4+1),x, algorithm="giac")
```

output
```
integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1+2x^4+x^8}\,dx=\int \frac{x^m}{x^8+2\,x^4+1}\,dx$$

input
```
int(x^m/(2*x^4 + x^8 + 1),x)
```

output
```
int(x^m/(2*x^4 + x^8 + 1), x)
```

Reduce [F]

$$\int \frac{x^m}{1+2x^4+x^8}\,dx = \int \frac{x^m}{x^8+2x^4+1}dx$$

input
```
int(x^m/(x^8+2*x^4+1),x)
```

output
```
int(x**m/(x**8 + 2*x**4 + 1),x)
```

3.43 $\int \frac{x^m}{1-2x^4+x^8} \, dx$

Optimal result 343
Mathematica [A] (verified) 343
Rubi [A] (verified) 344
Maple [F] 345
Fricas [F] 345
Sympy [F] 345
Maxima [F] 346
Giac [F] 346
Mupad [F(-1)] 346
Reduce [F] 347

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{x^m}{1-2x^4+x^8} \, dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(2, \frac{1+m}{4}, \frac{5+m}{4}, x^4\right)}{1+m}$$

output
```
x^(1+m)*hypergeom([2, 1/4+1/4*m],[5/4+1/4*m],x^4)/(1+m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{1-2x^4+x^8} \, dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(2, \frac{1+m}{4}, 1+\frac{1+m}{4}, x^4\right)}{1+m}$$

input
```
Integrate[x^m/(1 - 2*x^4 + x^8),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, x^4])/(1 + m)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1380, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{x^8-2x^4+1}\,dx$$
$$\Big\downarrow \; 1380$$
$$\int \frac{x^m}{(1-x^4)^2}dx$$
$$\Big\downarrow \; 888$$
$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(2,\frac{m+1}{4},\frac{m+5}{4},x^4\right)}{m+1}$$

input
```
Int[x^m/(1 - 2*x^4 + x^8),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4])/(1 + m)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x^m}{x^8 - 2x^4 + 1} dx$$

input
```
int(x^m/(x^8-2*x^4+1),x)
```

output
```
int(x^m/(x^8-2*x^4+1),x)
```

Fricas [F]

$$\int \frac{x^m}{1 - 2x^4 + x^8} dx = \int \frac{x^m}{x^8 - 2\,x^4 + 1} dx$$

input
```
integrate(x^m/(x^8-2*x^4+1),x, algorithm="fricas")
```

output
```
integral(x^m/(x^8 - 2*x^4 + 1), x)
```

Sympy [F]

$$\int \frac{x^m}{1 - 2x^4 + x^8} dx = \int \frac{x^m}{(x-1)^2 (x+1)^2 (x^2+1)^2} dx$$

input
```
integrate(x**m/(x**8-2*x**4+1),x)
```

output
```
Integral(x**m/((x - 1)**2*(x + 1)**2*(x**2 + 1)**2), x)
```

Maxima [F]

$$\int \frac{x^m}{1-2x^4+x^8}\,dx = \int \frac{x^m}{x^8-2\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8-2*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^m}{1-2x^4+x^8}\,dx = \int \frac{x^m}{x^8-2\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8-2*x^4+1),x, algorithm="giac")
```

output
```
integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1-2x^4+x^8}\,dx = \int \frac{x^m}{x^8-2\,x^4+1}\,dx$$

input
```
int(x^m/(x^8 - 2*x^4 + 1),x)
```

output
```
int(x^m/(x^8 - 2*x^4 + 1), x)
```

Reduce [F]

$$\int \frac{x^m}{1-2x^4+x^8}\,dx = \int \frac{x^m}{x^8-2x^4+1}dx$$

input
```
int(x^m/(x^8-2*x^4+1),x)
```

output
```
int(x**m/(x**8 - 2*x**4 + 1),x)
```

3.44 $\int (dx)^m \left(a^2+2abx^4+b^2x^8\right)^p\, dx$

Optimal result 348
Mathematica [A] (verified) 348
Rubi [A] (verified) 349
Maple [F] 350
Fricas [F] 350
Sympy [F] 351
Maxima [F] 351
Giac [F] 351
Mupad [F(-1)] 352
Reduce [F] 352

Optimal result

Integrand size = 26, antiderivative size = 77

$$\int (dx)^m \left(a^2+2abx^4+b^2x^8\right)^p\, dx = \frac{(dx)^{1+m}\left(1+\frac{bx^4}{a}\right)^{-2p}\left(a^2+2abx^4+b^2x^8\right)^p \text{Hypergeometric2F1}\left(\frac{1+m}{4},-2p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{d(1+m)}$$

output
```
(d*x)^(1+m)*(b^2*x^8+2*a*b*x^4+a^2)^p*hypergeom([-2*p, 1/4+1/4*m],[5/4+1/4
*m],-b*x^4/a)/d/(1+m)/((1+b*x^4/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int (dx)^m \left(a^2+2abx^4+b^2x^8\right)^p\, dx = \frac{x(dx)^m\left(\left(a+bx^4\right)^2\right)^p\left(1+\frac{bx^4}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{1+m}{4},-2p,1+\frac{1+m}{4},-\frac{bx^4}{a}\right)}{1+m}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^4 + b^2*x^8)^p,x]
```

output
```
(x*(d*x)^m*((a + b*x^4)^2)^p*Hypergeometric2F1[(1 + m)/4, -2*p, 1 + (1 + m
)/4, -((b*x^4)/a)])/((1 + m)*(1 + (b*x^4)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1385, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a^2+2abx^4+b^2x^8\right)^p \, dx$$

$$\Big\downarrow \ 1385$$

$$\left(\frac{bx^4}{a}+1\right)^{-2p} \left(a^2+2abx^4+b^2x^8\right)^p \int (dx)^m \left(\frac{bx^4}{a}+1\right)^{2p} dx$$

$$\Big\downarrow \ 888$$

$$\frac{(dx)^{m+1} \left(\frac{bx^4}{a}+1\right)^{-2p} \left(a^2+2abx^4+b^2x^8\right)^p \text{Hypergeometric2F1}\left(\frac{m+1}{4},-2p,\frac{m+5}{4},-\frac{bx^4}{a}\right)}{d(m+1)}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^4 + b^2*x^8)^p,x]
```

output
```
((d*x)^(1 + m)*(a^2 + 2*a*b*x^4 + b^2*x^8)^p*Hypergeometric2F1[(1 + m)/4,
-2*p, (5 + m)/4, -((b*x^4)/a)])/(d*(1 + m)*(1 + (b*x^4)/a)^(2*p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int (dx)^m \left(b^2x^8 + 2ab\,x^4 + a^2\right)^p dx$$

input
```
int((d*x)^m*(b^2*x^8+2*a*b*x^4+a^2)^p,x)
```

output
```
int((d*x)^m*(b^2*x^8+2*a*b*x^4+a^2)^p,x)
```

Fricas [F]

$$\int (dx)^m \left(a^2 + 2abx^4 + b^2x^8\right)^p dx = \int \left(b^2x^8 + 2\,abx^4 + a^2\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(b^2*x^8+2*a*b*x^4+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^8 + 2*a*b*x^4 + a^2)^p*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a^2+2abx^4+b^2x^8\right)^p\,dx=\int (dx)^m \left(\left(a+bx^4\right)^2\right)^p\,dx$$

input
```
integrate((d*x)**m*(b**2*x**8+2*a*b*x**4+a**2)**p,x)
```

output
```
Integral((d*x)**m*((a + b*x**4)**2)**p, x)
```

Maxima [F]

$$\int (dx)^m \left(a^2+2abx^4+b^2x^8\right)^p\,dx=\int \left(b^2x^8+2\,abx^4+a^2\right)^p(dx)^m\,dx$$

input
```
integrate((d*x)^m*(b^2*x^8+2*a*b*x^4+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^8 + 2*a*b*x^4 + a^2)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a^2+2abx^4+b^2x^8\right)^p\,dx=\int \left(b^2x^8+2\,abx^4+a^2\right)^p(dx)^m\,dx$$

input
```
integrate((d*x)^m*(b^2*x^8+2*a*b*x^4+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^8 + 2*a*b*x^4 + a^2)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^4+b^2x^8\right)^p \, dx = \int (d\,x)^m \left(a^2+2\,a\,b\,x^4+b^2\,x^8\right)^p \, dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^8 + 2*a*b*x^4)^p,x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^8 + 2*a*b*x^4)^p, x)
```

Reduce [F]

$$\int (dx)^m \left(a^2+2abx^4+b^2x^8\right)^p \, dx = \frac{d^m\left(x^m(b^2x^8+2ab\,x^4+a^2)^p\,x+8\left(\int \frac{x^m(b^2x^8+2ab\,x^4+a^2)^p}{bm\,x^4+8bp\,x^4+b\,x^4+am+8ap+a}dx\right)amp+64\left(\int \frac{x^m(b^2x^8+2ab\,x^4+a^2)^p}{bm\,x^4+8bp\,x^4+b\,x^4+am+8ap+}\right.\right.}{m+8p+1}$$

input
```
int((d*x)^m*(b^2*x^8+2*a*b*x^4+a^2)^p,x)
```

output
```
(d**m*(x**m*(a**2 + 2*a*b*x**4 + b**2*x**8)**p*x + 8*int((x**m*(a**2 + 2*a
*b*x**4 + b**2*x**8)**p)/(a*m + 8*a*p + a + b*m*x**4 + 8*b*p*x**4 + b*x**4
),x)*a*m*p + 64*int((x**m*(a**2 + 2*a*b*x**4 + b**2*x**8)**p)/(a*m + 8*a*p
 + a + b*m*x**4 + 8*b*p*x**4 + b*x**4),x)*a*p**2 + 8*int((x**m*(a**2 + 2*a
*b*x**4 + b**2*x**8)**p)/(a*m + 8*a*p + a + b*m*x**4 + 8*b*p*x**4 + b*x**4
),x)*a*p))/(m + 8*p + 1)
```

3.45 $\int \frac{x^{11}}{a+bx^4+cx^8}\,dx$

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Mathematica [A] (verified) 353
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Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx=\frac{x^4}{4c}-\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}$$

output
```
1/4*x^4/c-1/4*(-2*a*c+b^2)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4
*a*c+b^2)^(1/2)-1/8*b*ln(c*x^8+b*x^4+a)/c^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx=\frac{2cx^4+\frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log\left(a+bx^4+cx^8\right)}{8c^2}$$

input
```
Integrate[x^11/(a + b*x^4 + c*x^8),x]
```

output
```
(2*c*x^4 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] - b*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^8}{cx^8+bx^4+a}dx^4$$

$$\Big\downarrow \text{1143}$$

$$\frac{1}{4}\int \left(\frac{1}{c}-\frac{bx^4+a}{c\left(cx^8+bx^4+a\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{\left(b^2-2ac\right)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{2c^2}+\frac{x^4}{c}\right)$$

input
```
Int[x^11/(a + b*x^4 + c*x^8),x]
```

output
```
(x^4/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqr
t[b^2 - 4*a*c]) - (b*Log[a + b*x^4 + c*x^8])/(2*c^2))/4
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

method	result
default	$\frac{x^4}{4c}+\frac{-\frac{b\ln\left(c\,x^8+b\,x^4+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$
risch	$\frac{x^4}{4c}-\frac{\ln\left(\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,b\right)x^4+2\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,a\right)ab}{2c(4ac-b^2)}+\frac{\ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{\cdots}\right.\right.}{\cdots}$

input
```
int(x^11/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4/c+1/4/c*(-1/2*b/c*ln(c*x^8+b*x^4+a)+2*(-a+1/2*b^2/c)/(4*a*c-b^2)^(
1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.14

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx$$
$$=\left[\frac{2\left(b^2c-4\,ac^2\right)x^4-\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-\left(b^3-4\,abc\right)\log\left(cx\cdots\right.}{8\left(b^2c^2-4\,ac^3\right)}\right.$$

input
```
integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(2*(b^2*c - 4*a*c^2)*x^4 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 +
 b*x^4 + a)) - (b^3 - 4*a*b*c)*log(c*x^8 + b*x^4 + a))/(b^2*c^2 - 4*a*c^3)
, 1/8*(2*(b^2*c - 4*a*c^2)*x^4 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*
x^8 + b*x^4 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(73) = 146$.

Time = 2.72 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.90

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \left(-\frac{b}{8c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)\log\left(x^4+\frac{-ab-16ac^2\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)+4b^2c\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}}{8c^2\cdot(4\ldots}\right.}{2ac-b^2}\right. + \left(-\frac{b}{8c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)\log\left(x^4+\frac{-ab-16ac^2\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)+4b^2c\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}}{8c^2\cdot(4\ldots}\right.}{2ac-b^2}\right. + \frac{x^4}{4c}$$

input
```
integrate(x**11/(c*x**8+b*x**4+a),x)
```

output
```
(-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))
*log(x**4 + (-a*b - 16*a*c**2*(-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c -
b**2)/(8*c**2*(4*a*c - b**2))) + 4*b**2*c*(-b/(8*c**2) - sqrt(-4*a*c + b**
2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(8*c**2)
 + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))*log(x**4 +
(-a*b - 16*a*c**2*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**
2*(4*a*c - b**2))) + 4*b**2*c*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c -
b**2)/(8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**4/(4*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \frac{x^4}{4\,c} - \frac{b\log\left(cx^8+bx^4+a\right)}{8\,c^2} + \frac{(b^2-2\,ac)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/4*x^4/c - 1/8*b*log(c*x^8 + b*x^4 + a)/c^2 + 1/4*(b^2 - 2*a*c)*arctan((2
*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 22.08 (sec) , antiderivative size = 3916, normalized size of antiderivative = 48.35

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int(x^11/(a + b*x^4 + c*x^8),x)
```

output
```
x^4/(4*c) + (log(a + b*x^4 + c*x^8)*(4*b^3 - 16*a*b*c))/(2*(64*a*c^3 - 16*
b^2*c^2)) - (atan((8*c^4*x^4*(((a*c - b^2)*(((((2*a*c - b^2)*((((448*b^4*c
^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*
b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3
- 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))
/(8*c^2*(4*a*c - b^2)^(1/2)) + (4*b^3*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/
((4*a*c - b^2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^
2)^(1/2)) - ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((448*b^4*c^6 - 384
*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)
)*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b
*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*
a*c^3 - 16*b^2*c^2)) + ((2*a*c - b^2)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a
^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (
256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 1
6*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^3 - 16*b^2*c^2)) +
(((8*a^3*c^5 - 20*b^6*c^2 + 48*a*b^4*c^3 - 36*a^2*b^2*c^4)/c^4 - ((4*b^3 -
 16*a*b*c)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 1
6*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b
*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3
- 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (b^3*(4*b^...
```

Reduce [F]

$$\int \frac{x^{11}}{a+bx^4+cx^8}\,dx=\int \frac{x^{11}}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^11/(c*x^8+b*x^4+a),x)
```

output
```
int(x^11/(c*x^8+b*x^4+a),x)
```

3.46 $\int \frac{x^9}{a+bx^4+cx^8}\,dx$

Optimal result 359
Mathematica [A] (verified) 359
Rubi [A] (verified) 360
Maple [C] (verified) 362
Fricas [B] (verification not implemented) 362
Sympy [A] (verification not implemented) 363
Maxima [F] 364
Giac [B] (verification not implemented) 364
Mupad [B] (verification not implemented) 365
Reduce [F] 366

Optimal result

Integrand size = 18, antiderivative size = 192

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx=\frac{x^2}{2c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x^2/c-1/4*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x
^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2)-1/4*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b+
(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.09

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx=\frac{2\sqrt{c}x^2-\frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

input
```
Integrate[x^9/(a + b*x^4 + c*x^8),x]
```

output
```
(2*Sqrt[c]*x^2 - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*S
qrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1695, 1442, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{a+bx^4+cx^8} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^8}{cx^8+bx^4+a}dx^2$$

$$\Big\downarrow \text{1442}$$

$$\frac{1}{2}\left(\frac{x^2}{c}-\frac{\int \frac{bx^4+a}{cx^8+bx^4+a}dx^2}{c}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(\frac{x^2}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^2}{c}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{x^2}{c}-\frac{-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}\right)$$

input
```
Int[x^9/(a + b*x^4 + c*x^8),x]
```

output
```
(x^2/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/c)/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x^2}{2c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(16a^2c^3-8a\,b^2c^2+b^4c\right)\_Z^4+\left(12a^2b\,c^2-7a\,b^3c+b^5\right)\_Z^2+c^2a^3\right)}\_R\ln\left(\left(-a^2c^2+a\,b^2c\right)x^2+\left(-4ab\,c^2+b^3c\right)\_R^3+\right.}{4c}$
default	$\frac{x^2}{2c}-\frac{\left(-b\sqrt{-4ac+b^2}-2ac+b^2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-b\sqrt{-4ac+b^2}+2ac-b^2\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)}}\right.}{4c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$

input
```
int(x^9/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c+1/4/c*sum(_R*ln((-a^2*c^2+a*b^2*c)*x^2+(-4*a*b*c^2+b^3*c)*_R^3+(
2*a^2*c^2-4*a*b^2*c+b^4)*_R),_R=RootOf((16*a^2*c^3-8*a*b^2*c^2+b^4*c)*_Z^4
+(12*a^2*b*c^2-7*a*b^3*c+b^5)*_Z^2+c^2*a^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. 2(150) = 300.

Time = 0.09 (sec) , antiderivative size = 1071, normalized size of antiderivative = 5.58

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx=\text{Too large to display}$$

input
```
integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2
 - a^2*c)*x^2 + 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*
a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3
 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/
(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b
^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt
(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c +
 a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*
x^2 + 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c
 - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7
)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 -
4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a
^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^...
```

Sympy [A] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.70

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx$$

$$= \operatorname{RootSum}\left(t^4\cdot\left(4096a^2c^5-2048ab^2c^4+256b^4c^3\right)+t^2\cdot\left(192a^2bc^2-112ab^3c+16b^5\right)+a^3,\left(t\mapsto t\log\left(\right.\right.\right.$$

$$+\frac{x^2}{2c}$$

input
```
integrate(x**9/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**4*(4096*a**2*c**5 - 2048*a*b**2*c**4 + 256*b**4*c**3) + _t**2*
(192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + a**3, Lambda(_t, _t*log(x**2
+ (256*_t**3*a*b*c**4 - 64*_t**3*b**3*c**3 - 8*_t*a**2*c**2 + 16*_t*a*b**2
*c - 4*_t*b**4)/(a**2*c - a*b**2)))) + x**2/(2*c)
```

Maxima [F]

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = \int \frac{x^9}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*x^2/c - integrate((b*x^4 + a)*x/(c*x^8 + b*x^4 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2043 vs. $2(150) = 300$.

Time = 1.24 (sec) , antiderivative size = 2043, normalized size of antiderivative = 10.64

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/2*x^2/c + 1/8*(2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3 - (sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2
 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a
^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*x^4*abs(c) + (
2*b^4*c^3 - 8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*
c^3 - 2*(b^2 - 4*a*c)*b^2*c^3)*x^4 - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*a*b^4*c + 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 16*a...
```

Mupad [B] (verification not implemented)

Time = 21.88 (sec) , antiderivative size = 5659, normalized size of antiderivative = 29.47

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int(x^9/(a + b*x^4 + c*x^8),x)
```

output
```
atan(((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2
*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a
^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*
(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5
 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c
- b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2
*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/
2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(a^2*b^6 - 2
*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(
16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*(a*b^8 + 4*a^5*c^4
- 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^
3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*
c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(1
6*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1...
```

Reduce [F]

$$\int \frac{x^9}{a+bx^4+cx^8}\,dx=\int \frac{x^9}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^9/(c*x^8+b*x^4+a),x)
```

output
```
int(x^9/(c*x^8+b*x^4+a),x)
```

3.47 $\int \frac{x^7}{a+bx^4+cx^8}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

output
```
1/4*b*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)+1/8*ln(
c*x^8+b*x^4+a)/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx=\frac{-\frac{2b\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+bx^4+cx^8\right)}{8c}$$

input
```
Integrate[x^7/(a + b*x^4 + c*x^8),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[
a + b*x^4 + c*x^8])/(8*c)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1693, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{a+bx^4+cx^8}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^4}{cx^8+bx^4+a}dx^4$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4}\left(\frac{\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4}{2c}-\frac{b\int \frac{1}{cx^8+bx^4+a}dx^4}{2c}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(\frac{b\int \frac{1}{-x^8+b^2-4ac}d(2cx^4+b)}{c}+\frac{\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4}{2c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4}{2c}+\frac{b\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(\frac{b\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log(a+bx^4+cx^8)}{2c}\right)$$

input
```
Int[x^7/(a + b*x^4 + c*x^8),x]
```

output
```
((b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^4 + c*x^8]/(2*c))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
default	$\frac{\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{8c}-\frac{b\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{4c\sqrt{4ac-b^{2}}}$
risch	$\frac{\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}(4ac-b^{2})}\,b\right)x^{4}+2\sqrt{-b^{2}(4ac-b^{2})}\,a\right)a}{8ac-2b^{2}}-\frac{\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}(4ac-b^{2})}\,b\right)x^{4}+2\sqrt{-b^{2}(4ac-b^{2})}\,a\right)b^{2}}{8c(4ac-b^{2})}+$

input
```
int(x^7/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(c*x^8+b*x^4+a)/c-1/4*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*
c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.13

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)+(b^2-4\,ac)\log\left(cx^8+bx^4+a\right)}{8\,(b^2c-4\,ac^2)},\ \frac{2\,\sqrt{-b^2+4\,ac}b}{}\right.$$

input
```
integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*
x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) + (b^2 - 4*a*c)*log(c*x^8
 + b*x^4 + a))/(b^2*c - 4*a*c^2), 1/8*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c
*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^8 + b*
x^4 + a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54) = 108$.

Time = 1.58 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.54

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{8c\,(4ac-b^2)}+\frac{1}{8c}\right)\log\left(x^4+\frac{-16ac\left(-\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)+2a+4b^2\left(-\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{8c\,(4ac-b^2)}+\frac{1}{8c}\right)\log\left(x^4+\frac{-16ac\left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)+2a+4b^2\left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)}{b}\right)$$

input
```
integrate(x**7/(c*x**8+b*x**4+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*
c*(-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(
-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b) + (b*sqrt(-4*a*
c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*c*(b*sqrt(-4*a
*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(b*sqrt(-4*a*c +
 b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = -\frac{b\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}c} + \frac{\log\left(cx^8+bx^4+a\right)}{8\,c}$$

input
```
integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1
/8*log(c*x^8 + b*x^4 + a)/c
```

Mupad [B] (verification not implemented)

Time = 21.22 (sec) , antiderivative size = 2654, normalized size of antiderivative = 42.13

$$\int \frac{x^7}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int(x^7/(a + b*x^4 + c*x^8),x)
```

output
```
(log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - (b*a
tan((8*x^4*(((a*c - b^2)*(((((16*a*c - 4*b^2)*((b*(448*b^3*c^3 - (256*b^3*
c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) -
(32*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2)))
)/(2*(64*a*c^2 - 16*b^2*c)) - (b*(144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c
^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2
 - 16*b^2*c))))/(8*c*(4*a*c - b^2)^(1/2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2
- 16*b^2*c)) - (b*((b*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))/(6
4*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32*b^4*c^3*(16*a*c - 4*
b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/
2)) - (4*b^5*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))
/(8*c*(4*a*c - b^2)^(1/2)) + (b*(20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3
- (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/
(2*(64*a*c^2 - 16*b^2*c)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c))))/(
8*c*(4*a*c - b^2)^(1/2)) + (b^6*c*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*
c)*(4*a*c - b^2)^(3/2))))/(8*a^3*c^2) + ((b^3 - 3*a*b*c)*(b^7/(8*(4*a*c -
b^2)^2) + b^3 - ((20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c^4*
(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 -
16*b^2*c)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))*(16*a*c - 4*b^2))
/(2*(64*a*c^2 - 16*b^2*c)) + ((16*a*c - 4*b^2)*((b*((b*(448*b^3*c^3 - (...
```

Reduce [F]

$$\int \frac{x^7}{a + bx^4 + cx^8} \, dx = \int \frac{x^7}{c\,x^8 + b\,x^4 + a} dx$$

input
```
int(x^7/(c*x^8+b*x^4+a),x)
```

output
```
int(x^7/(c*x^8+b*x^4+a),x)
```

3.48 $\int \frac{x^5}{a+bx^4+cx^8}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 159

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

output
```
-1/4*(b-(-4*a*c+b^2)^(1/2))^(1/2)*arctan(2^(1/2)*c^(1/2)*x^2/(b-(-4*a*c+b^
2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/4*(b+(-4*a*c+b^2)^(1
/2))^(1/2)*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2
)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = \frac{\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

input
```
Integrate[x^5/(a + b*x^4 + c*x^8),x]
```

output
```
((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan
[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sq
rt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1695, 1450, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^4}{cx^8+bx^4+a}dx^2$$

$$\Big\downarrow \text{1450}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^2\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

input
```
Int[x^5/(a + b*x^4 + c*x^8),x]
```

output
```
(((1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((1 + b/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^2c^3-8a\,b^2c^2+b^4c\right)\_Z^4+\left(-4abc+b^3\right)\_Z^2+a\right)}\_R\ln\left(\left(\left(-4a\,c^2+b^2c\right)\_R^2+b\right)x^2+\left(4ab\,c^2-b^3c\right)\_R^3+\left(2ac-b^2\right.\right.\right.}{4}$
default	$2c\left(-\frac{\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)$

input
```
int(x^5/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(((-4*a*c^2+b^2*c)*_R^2+b)*x^2+(4*a*b*c^2-b^3*c)*_R^3+(2*a*c-
b^2)*_R),_R=RootOf((16*a^2*c^3-8*a*b^2*c^2+b^4*c)*_Z^4+(-4*a*b*c+b^3)*_Z^2
+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(119) = 238$.

Time = 0.08 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.57

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2+\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right) - \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right) - \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2+\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(x^2-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}\right)$$

input
```
integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
 - 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a
*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3))
 - 1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^
2*c - 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c -
4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^
3)) - 1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/
(b^2*c - 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c
 - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a
*c^3)) + 1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3
))/(b^2*c - 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^
2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 -
4*a*c^3))
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(4096a^2c^3-2048ab^2c^2+256b^4c\right)+t^2\left(-64abc+16b^3\right)+a,\left(t\mapsto t\log\left(512t^3ac^2-128t^3b\right.\right.\right.$$

input
```
integrate(x**5/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**4*(4096*a**2*c**3 - 2048*a*b**2*c**2 + 256*b**4*c) + _t**2*(-6
4*a*b*c + 16*b**3) + a, Lambda(_t, _t*log(512*_t**3*a*c**2 - 128*_t**3*b**
2*c - 4*_t*b + x**2)))
```

Maxima [F]

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = \int \frac{x^5}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^5/(c*x^8 + b*x^4 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. $2(119) = 238$.

Time = 1.26 (sec) , antiderivative size = 1034, normalized size of antiderivative = 6.50

$$\int \frac{x^5}{a + bx^4 + cx^8} \, dx = \text{Too large to display}$$

input
```
integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2
 - 4*a*c)*b*c^2)*x^4*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^2 - 4*a*c))/c
))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^
2 - 4*a^2*c^3)*abs(c)) + 1/8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 -
4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 +
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sq...
```

Mupad [B] (verification not implemented)

Time = 21.03 (sec) , antiderivative size = 1220, normalized size of antiderivative = 7.67

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int(x^5/(a + b*x^4 + c*x^8),x)
```

output
```
atan((x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + b^3*
x^2*1i - a*b*c*x^2*4i)/(8*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*
b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1
/2) + 128*b^5*c*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -
 b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 64*a^2*c
^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*
c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2) - 1024*a*b^3*c^2*(((b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^
4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 2048*a^2*b*c^3*(((b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512
*a^2*c^3 - 256*a*b^2*c^2))^(3/2) - 48*a*b^2*c*(((b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 25
6*a*b^2*c^2))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)*2i -
 atan((x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - b^3
*x^2*1i + a*b*c*x^2*4i)/(8*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^
(1/2) + 128*b^5*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 64*a^
2*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -...
```

Reduce [F]

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = \int \frac{x^5}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^5/(c*x^8+b*x^4+a),x)
```

output
```
int(x^5/(c*x^8+b*x^4+a),x)
```

3.49 $\int \frac{x^3}{a+bx^4+cx^8} \, dx$

Optimal result 383
Mathematica [A] (verified) 383
Rubi [A] (verified) 384
Maple [A] (verified) 385
Fricas [A] (verification not implemented) 385
Sympy [B] (verification not implemented) 386
Maxima [F(-2)] 386
Giac [A] (verification not implemented) 387
Mupad [B] (verification not implemented) 387
Reduce [F] 388

Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{x^3}{a+bx^4+cx^8} \, dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

output
```
-1/2*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{a+bx^4+cx^8} \, dx = \frac{\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}}$$

input
```
Integrate[x^3/(a + b*x^4 + c*x^8),x]
```

output
```
ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]]/(2*Sqrt[-b^2 + 4*a*c])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1690, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+bx^4+cx^8} \, dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{4}\int \frac{1}{cx^8+bx^4+a}dx^4$$

$$\Big\downarrow \text{1083}$$

$$-\frac{1}{2}\int \frac{1}{-x^8+b^2-4ac}d(2cx^4+b)$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

input
```
Int[x^3/(a + b*x^4 + c*x^8),x]
```

output
```
-1/2*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}}$	37
risch	$-\frac{\ln\left(\left(-b+\sqrt{-4ac+b^{2}}\right)x^{4}-2a\right)}{4\sqrt{-4ac+b^{2}}}+\frac{\ln\left(\left(b+\sqrt{-4ac+b^{2}}\right)x^{4}+2a\right)}{4\sqrt{-4ac+b^{2}}}$	70

input
```
int(x^3/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.39

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac-(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{4\,\sqrt{b^2-4\,ac}},\right.$$
$$\left.-\frac{\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^4+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{2\,(b^2-4\,ac)}\right]$$

input
```
integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c - (2*c*x^4 + b)*sqrt(b^2 - 4
*a*c))/(c*x^8 + b*x^4 + a))/sqrt(b^2 - 4*a*c), -1/2*sqrt(-b^2 + 4*a*c)*arc
tan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(36) = 72$.

Time = 0.46 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.45

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx = -\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4} + \frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4}$$

input
```
integrate(x**3/(c*x**8+b*x**4+a),x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x**4 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2
*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/4 + sqrt(-1/(4*a*c - b**2))*log(x**4
+ (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c)
)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx=\frac{\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}}$$

input
```
integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/2*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 18.96 (sec) , antiderivative size = 260, normalized size of antiderivative = 6.84

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx=$$

$$-\frac{\mathrm{atan}\left(\frac{{\left(4\,a\,c-b^2\right)}^2\,\left(\frac{\left(\frac{4\,a\,c^4}{4\,a\,c-b^2}-\frac{4\,a\,b^2\,c^4}{{\left(4\,a\,c-b^2\right)}^2}\right)\,\left(b^3-3\,a\,b\,c\right)}{8\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}}-x^4\,\left(\frac{\left(\frac{2\,c^4}{\sqrt{4\,a\,c-b^2}}-\frac{6\,b^2\,c^4}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(a\,c-b^2\right)}{8\,a^3\,c^2}-\frac{\left(b^3-3\,a\,b\,c\right)\,\left(\frac{6\,b\,c^4}{4\,a\,c-b^2}-\frac{2\,b^3\,c^4}{(4\,a\,c-b^2)\cdots}\right.}{8\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}}\cdots}{2\,c^4}\right)}{2\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^3/(a + b*x^4 + c*x^8),x)
```

output
```
-atan(((4*a*c - b^2)^2*((((4*a*c^4)/(4*a*c - b^2) - (4*a*b^2*c^4)/(4*a*c -
 b^2)^2)*(b^3 - 3*a*b*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)) - x^4*((((2*c^4)
/(4*a*c - b^2)^(1/2) - (6*b^2*c^4)/(4*a*c - b^2)^(3/2))*(a*c - b^2))/(8*a^
3*c^2) - ((b^3 - 3*a*b*c)*((6*b*c^4)/(4*a*c - b^2) - (2*b^3*c^4)/(4*a*c -
b^2)^2))/(8*a^3*c^2*(4*a*c - b^2)^(1/2))) + (b*c^2*(a*c - b^2))/(a^2*(4*a*
c - b^2)^(3/2))))/(2*c^4))/(2*(4*a*c - b^2)^(1/2))
```

Reduce [F]

$$\int \frac{x^3}{a+bx^4+cx^8} \, dx = \int \frac{x^3}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^3/(c*x^8+b*x^4+a),x)
```

output
```
int(x^3/(c*x^8+b*x^4+a),x)
```

3.50 $\int \frac{x}{a+bx^4+cx^8}\,dx$

Optimal result 389
Mathematica [A] (verified) 389
Rubi [A] (verified) 390
Maple [C] (verified) 391
Fricas [B] (verification not implemented) 392
Sympy [A] (verification not implemented) 393
Maxima [F] 394
Giac [B] (verification not implemented) 394
Mupad [B] (verification not implemented) 395
Reduce [F] 396

Optimal result

Integrand size = 16, antiderivative size = 154

$$\int \frac{x}{a+bx^4+cx^8}\,dx=\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*c^(1/2)*arctan(2^(1/2)*c^(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/
2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*c^(1/2)*arctan(2^(1
/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.86

$$\int \frac{x}{a+bx^4+cx^8}\,dx=\frac{\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}}$$

input
```
Integrate[x/(a + b*x^4 + c*x^8),x]
```

output
```
(Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/Sqrt[b
 - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[2]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1695, 1406, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{cx^8+bx^4+a}dx^2$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{2}\left(\frac{c\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^2}{\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

input
```
Int[x/(a + b*x^4 + c*x^8),x]
```

output
```
((Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcT
an[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.64

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16c^2a^3-8c\,a^2b^2+a\,b^4\right)\_Z^4+\left(-4abc+b^3\right)\_Z^2+c\right)}\_R\ln\left(\left(\left(4abc-b^3\right)\_R^2-c\right)x^2+\left(4a^2bc-b^3a\right)\_R^3-2ac\_R\right)\right)}{4}$
default	$2c\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)$

input
```
int(x/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(((4*a*b*c-b^3)*_R^2-c)*x^2+(4*a^2*b*c-a*b^3)*_R^3-2*a*c*_R),
_R=RootOf((16*a^3*c^2-8*a^2*b^2*c+a*b^4)*_Z^4+(-4*a*b*c+b^3)*_Z^2+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(119) = 238$.

Time = 0.09 (sec) , antiderivative size = 619, normalized size of antiderivative = 4.02

$$\int \frac{x}{a+bx^4+cx^8}\,dx = -\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2 + \frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2 - \frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) - \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2 + \frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) + \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(cx^2 - \frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)$$

input
```
integrate(x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^
2 - 4*a^2*c))*log(c*x^2 + 1/2*sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)
/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^
3*c))/(a*b^2 - 4*a^2*c))) + 1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqr
t(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 -
4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4
*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/4*sqrt(1/2)*sqrt(
-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*
x^2 + 1/2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^
3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2
*c))) + 1/4*sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)
)/(a*b^2 - 4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a
^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2
 - 4*a^3*c))/(a*b^2 - 4*a^2*c)))
```

Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.57

$$\int \frac{x}{a + bx^4 + cx^8}\, dx$$
$$= \text{RootSum}\left(t^4 \cdot \left(4096a^3c^2 - 2048a^2b^2c + 256ab^4\right) + t^2\left(-64abc + 16b^3\right) + c, \left(t \mapsto t\log\left(x^2 + \frac{256t^3a^2bc}{\vphantom{x}}\right.\right.\right.$$

input
```
integrate(x/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**4*(4096*a**3*c**2 - 2048*a**2*b**2*c + 256*a*b**4) + _t**2*(-6
4*a*b*c + 16*b**3) + c, Lambda(_t, _t*log(x**2 + (256*_t**3*a**2*b*c - 64*
_t**3*a*b**3 + 8*_t*a*c - 4*_t*b**2)/c)))
```

Maxima [F]

$$\int \frac{x}{a+bx^4+cx^8}\,dx = \int \frac{x}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x/(c*x^8 + b*x^4 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(119) = 238$.

Time = 1.28 (sec) , antiderivative size = 1028, normalized size of antiderivative = 6.68

$$\int \frac{x}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2
 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^2 - 4*a*c))/c))/(
(a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 -
4*a^2*c^3)*abs(c)) + 1/8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b...
```

Mupad [B] (verification not implemented)

Time = 19.89 (sec) , antiderivative size = 1105, normalized size of antiderivative = 7.18

$$\int \frac{x}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int(x/(a + b*x^4 + c*x^8),x)
```

output
```
atan((b^4*x^2*1i + b*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
(1/2)*1i + a^2*c^2*x^2*8i - a*b^2*c*x^2*6i)/(128*a^2*b^5*(-(b^3 + (b^6 - 6
4*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*
a^3*c^2 - 256*a^2*b^2*c))^(3/2) - 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 25
6*a^2*b^2*c))^(1/2) + 16*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2
*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*
c))^(1/2) - 1024*a^3*b^3*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2
) + 2048*a^4*b*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2)))*(-(b
^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32
*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2)*2i + atan((b^4*x^2*1i - b*x^2
*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x^2*8
i - a*b^2*c*x^2*6i)/(128*a^2*b^5*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))
^(3/2) - 64*a^3*c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) + 16*a
^2*b^2*c*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) - 1024*a^3*b^3...
```

Reduce [F]

$$\int \frac{x}{a+bx^4+cx^8}\,dx=\int \frac{x}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x/(c*x^8+b*x^4+a),x)
```

output
```
int(x/(c*x^8+b*x^4+a),x)
```

3.51 $\int \frac{1}{x(a+bx^4+cx^8)}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{1}{x(a+bx^4+cx^8)}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log(a+bx^4+cx^8)}{8a}$$

output
```
1/4*b*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)+ln(x)/a
-1/8*ln(c*x^8+b*x^4+a)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^4+cx^8)}\,dx=\frac{\log(x)}{a}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b+2c\#1^4}\&\right]}{4a}$$

input
```
Integrate[1/(x*(a + b*x^4 + c*x^8)),x]
```

output
```
Log[x]/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]*
#1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1693, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(a+b x^4+c x^8\right)} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4} \int \frac{1}{x^4 \left(c x^8+b x^4+a\right)}dx^4$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{4} \left(\frac{\int -\frac{c x^4+b}{c x^8+b x^4+a}dx^4}{a}+\frac{\log \left(x^4\right)}{a}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4} \left(\frac{\log \left(x^4\right)}{a}-\frac{\int \frac{c x^4+b}{c x^8+b x^4+a}dx^4}{a}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4} \left(\frac{\log \left(x^4\right)}{a}-\frac{\frac{1}{2} b \int \frac{1}{c x^8+b x^4+a}dx^4+\frac{1}{2} \int \frac{2 c x^4+b}{c x^8+b x^4+a}dx^4}{a}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4} \left(\frac{\log \left(x^4\right)}{a}-\frac{\frac{1}{2} \int \frac{2 c x^4+b}{c x^8+b x^4+a}dx^4-b \int \frac{1}{-x^8+b^2-4 a c}d\left(2 c x^4+b\right)}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)}{a}-\frac{\frac{1}{2}\int\frac{2cx^4+b}{cx^8+bx^4+a}dx^4-\frac{b\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)}{a}-\frac{\frac{1}{2}\log\left(a+bx^4+cx^8\right)-\frac{b\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

input
```
Int[1/(x*(a + b*x^4 + c*x^8)),x]
```

output
```
(Log[x^4]/a - (-((b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4
*a*c]) + Log[a + b*x^4 + c*x^8]/2)/a)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{\frac{\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4}+\frac{b\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}}}{2a}+\frac{\ln\left(x\right)}{a}$	66
risch	$\frac{\ln\left(x\right)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{2}c-b^{2}a\right)\_Z^{2}+\left(4ac-b^{2}\right)\_Z+c\right)}\_R\ln\left(\left(\left(18ac-5b^{2}\right)\_R+9c\right)x^{4}-\_Rab+4b\right)\right)}{4}$	77

input
```
int(1/x/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*(1/4*ln(c*x^8+b*x^4+a)+1/2*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(
4*a*c-b^2)^(1/2)))+ln(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.23

$$\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^8+bx^4+a\right)+8\left(b^2-4\,ac\right)\mathrm{lo}}{8\left(ab^2-4\,a^2c\right)}\right.$$

input
```
integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*
x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^2 - 4*a*c)*log(c*x^8
 + b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/8*(2*sqrt(-b^
2 + 4*a*c)*b*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^
2 - 4*a*c)*log(c*x^8 + b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2
*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.

Time = 9.57 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.67

$$\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx=\left(-\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)\log\left(x^4+\frac{-16a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)+4ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)-2ac+b^2}{bc}\right)$$
$$+\left(\frac{b\sqrt{-4ac+b^2}}{8a\left(4ac-b^2\right)}-\frac{1}{8a}\right)\log\left(x^4+\frac{-16a^2c\left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)+4ab^2\left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)-2ac+b^2}{bc}\right)$$
$$+\frac{\log\left(x\right)}{a}$$

input
```
integrate(1/x/(c*x**8+b*x**4+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-16*a*
*2*c*(-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2*(-
b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b*c
)) + (b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-1
6*a**2*c*(b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2
*(b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b
*c)) + log(x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx = -\frac{b\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}a} - \frac{\log\left(cx^8+bx^4+a\right)}{8\,a} + \frac{\log\left(x^4\right)}{4\,a}$$

input
```
integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1
/8*log(c*x^8 + b*x^4 + a)/a + 1/4*log(x^4)/a
```

Mupad [B] (verification not implemented)

Time = 19.91 (sec) , antiderivative size = 1690, normalized size of antiderivative = 24.49

$$\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(x*(a + b*x^4 + c*x^8)),x)
```

output
```
log(x)/a + (log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2
*c)) - (b*atan((4*(4*a*c - b^2)^2*(5*b^6 - 18*a^3*c^3 + 61*a^2*b^2*c^2 - 3
4*a*b^4*c)*((b^9*c^4)/(128*a^4*(4*a*c - b^2)^(5/2)) + (2*b^5*c^4*(16*a*c -
 4*b^2)^4)/((16*a*b^2 - 64*a^2*c)^4*(4*a*c - b^2)^(1/2)) - (b*(16*a*c - 4*
b^2)^3*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*
c)))/(16*a*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)^(1/2)) + (b^3*(16*a*c - 4
*b^2)*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c
)))/(256*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2)) - (3*b^7*c^4*(16*a
*c - 4*b^2)^2)/(4*a^2*(16*a*b^2 - 64*a^2*c)^2*(4*a*c - b^2)^(3/2))))/(b^4*
c^8*(81*a*c - 20*b^2)) + (128*a^5*x^4*(((5*b^5 + 23*a^2*b*c^2 - 24*a*b^3*c
)*(((576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(
16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2)^4)/(16*(16*a*b^2 - 64*a^2*c)^4) +
(b^4*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*
(16*a*b^2 - 64*a^2*c))))/(4096*a^4*(4*a*c - b^2)^2) + (b^2*(1280*b^5*c^4 -
 4608*a*b^3*c^5)*(16*a*c - 4*b^2)^3)/(128*a^2*(16*a*b^2 - 64*a^2*c)^3*(4*a
*c - b^2)) - (3*b^2*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*
c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2)^2)/(128*a^2*(16*a*
b^2 - 64*a^2*c)^2*(4*a*c - b^2)) - (b^4*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(1
6*a*c - 4*b^2))/(2048*a^4*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^2)))/(32*a^5
*c^4*(81*a*c - 20*b^2)) + ((5*b^6 - 18*a^3*c^3 + 61*a^2*b^2*c^2 - 34*a*...
```

Reduce [F]

$$\int \frac{1}{x\,(a+bx^4+cx^8)}\,dx = \int \frac{1}{x\,(c\,x^8+b\,x^4+a)}dx$$

input
```
int(1/x/(c*x^8+b*x^4+a),x)
```

output
```
int(1/x/(c*x^8+b*x^4+a),x)
```

3.52 $\int \frac{1}{x^3(a+bx^4+cx^8)}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 184

$$\int \frac{1}{x^3\left(a+bx^4+cx^8\right)}\,dx = -\frac{1}{2ax^2}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/2/a/x^2-1/4*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2
/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*
c^(1/2)*(1-b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2
)^(1/2))^(1/2))*2^(1/2)/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^3\left(a+bx^4+cx^8\right)}\,dx = -\frac{1}{2ax^2}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b\#1^2+2c\#1^6}\&\right]}{4a}$$

input
```
Integrate[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/2*1/(a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x
- #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1695, 1443, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b x^4+c x^8\right)} \, dx$$

$$\Big\downarrow \text{ 1695}$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(c x^8+b x^4+a\right)}dx^2$$

$$\Big\downarrow \text{ 1443}$$

$$\frac{1}{2} \left(\frac{\int -\frac{c x^4+b}{c x^8+b x^4+a}dx^2}{a}-\frac{1}{a x^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2} \left(-\frac{\int \frac{c x^4+b}{c x^8+b x^4+a}dx^2}{a}-\frac{1}{a x^2}\right)$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{1}{2} \left(-\frac{\frac{1}{2} c \left(\frac{b}{\sqrt{b^2-4 a c}}+1\right) \int \frac{1}{c x^4+\frac{1}{2} \left(b-\sqrt{b^2-4 a c}\right)}dx^2+\frac{1}{2} c \left(1-\frac{b}{\sqrt{b^2-4 a c}}\right) \int \frac{1}{c x^4+\frac{1}{2} \left(b+\sqrt{b^2-4 a c}\right)}dx^2}{a}-\frac{1}{a x^2}\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2}\left(-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{ax^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

output
```
(-(1/(a*x^2)) - ((Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

method	result
default	$-\frac{1}{2a\,x^2}-\frac{2c\left(-\frac{\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{a}$
risch	$-\frac{1}{2a\,x^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16c^2a^5-8a^4b^2c+a^3b^4\right)\_Z^4+\left(12a^2b\,c^2-7a\,b^3c+b^5\right)\_Z^2+c^3\right)}\_R\ln\left(\left(\left(-72c^2a^5+38a^4b^2c-5a^3b^4\right)\_R^4\right.\right.\right.}{4}$

input
```
int(1/x^3/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2-2/a*c*(-1/8*(b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))+1/8*(-b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(141) = 282$.

Time = 0.11 (sec) , antiderivative size = 1134, normalized size of antiderivative = 6.16

$$\int \frac{1}{x^3\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b
^2*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*
b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^
2*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2
- 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c +
8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b
^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b
^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b
*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7
*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5
 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*
b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3
*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^
4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a
^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 - 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*...
```

Sympy [A] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (a + bx^4 + cx^8)} dx$$
$$= \mathrm{RootSum}\left(t^4 \cdot (4096a^5c^2 - 2048a^4b^2c + 256a^3b^4) + t^2 \cdot (192a^2bc^2 - 112ab^3c + 16b^5) + c^3, \left(t \mapsto t \log\left(\right.\right.\right.$$
$$- \frac{1}{2ax^2}$$

input
```
integrate(1/x**3/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**4*(4096*a**5*c**2 - 2048*a**4*b**2*c + 256*a**3*b**4) + _t**2*
(192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + c**3, Lambda(_t, _t*log(x**2
+ (-512*_t**3*a**5*c**2 + 384*_t**3*a**4*b**2*c - 64*_t**3*a**3*b**4 - 20*
_t*a**2*b*c**2 + 20*_t*a*b**3*c - 4*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(
2*a*x**2)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^4 + cx^8)} dx = \int \frac{1}{(cx^8 + bx^4 + a)x^3} dx$$

input
```
integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((c*x^4 + b)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. 2(141) = 282.

Time = 1.25 (sec) , antiderivative size = 2055, normalized size of antiderivative = 11.17

$$\int \frac{1}{x^3 (a + bx^4 + cx^8)} dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/8*(2*a*b^4*c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 -
 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 2*b^4*c^2 + 16*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 16
*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 32*a^2*c^4
+ 2*(b^2 - 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*c^3)*x^4*abs(a) + (2*a*b^3*c
^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3
 - 2*(b^2 - 4*a*c)*a*b*c^3)*x^4 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - ...
```

Mupad [B] (verification not implemented)

Time = 20.04 (sec) , antiderivative size = 5451, normalized size of antiderivative = 29.62

$$\int \frac{1}{x^3 (a + bx^4 + cx^8)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^4 + c*x^8)),x)
```

output
```
- atan((((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^1
4*b^2*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5
 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3
*c^6) + x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^
2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
 a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^
(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) - ((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)
)/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)
)/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32
768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024*a...
```

Reduce [F]

$$\int \frac{1}{x^3 (a + bx^4 + cx^8)} \, dx = \int \frac{1}{x^3 (c\,x^8 + b\,x^4 + a)} dx$$

input
```
int(1/x^3/(c*x^8+b*x^4+a),x)
```

output
```
int(1/x^3/(c*x^8+b*x^4+a),x)
```

3.53 $\int \frac{1}{x^5(a+bx^4+cx^8)}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{1}{x^5(a+bx^4+cx^8)}\,dx = -\frac{1}{4ax^4} - \frac{(b^2-2ac)\,\mathrm{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^4+cx^8)}{8a^2}$$

output
```
-1/4/a/x^4-1/4*(-2*a*c+b^2)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a^2/(-
4*a*c+b^2)^(1/2)-b*ln(x)/a^2+1/8*b*ln(c*x^8+b*x^4+a)/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5(a+bx^4+cx^8)}\,dx = -\frac{1}{4ax^4} - \frac{b\log(x)}{a^2} + \frac{\mathrm{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^2\log(x-\#1)-ac\log(x-\#1)+bc\log(x-\#1)\#1^4}{b+2c\#1^4}\&\right]}{4a^2}$$

input
```
Integrate[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/4*1/(a*x^4) - (b*Log[x])/a^2 + RootSum[a + b*#1^4 + c*#1^8 & , (b^2*Log
[x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a
^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^4+cx^8)} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4} \int \frac{1}{x^8 (cx^8+bx^4+a)} dx^4$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{4} \left(\frac{\int -\frac{cx^4+b}{x^4 (cx^8+bx^4+a)} dx^4}{a} - \frac{1}{ax^4} \right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4} \left(-\frac{\int \frac{cx^4+b}{x^4 (cx^8+bx^4+a)} dx^4}{a} - \frac{1}{ax^4} \right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{4} \left(-\frac{\int \left(\frac{b}{ax^4} + \frac{-bcx^4-b^2+ac}{a(cx^8+bx^4+a)} \right) dx^4}{a} - \frac{1}{ax^4} \right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log(a+bx^4+cx^8)}{2a}+\frac{b\log(x^4)}{a}}{a}-\frac{1}{ax^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

output
```
(-(1/(a*x^4)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/
(a*Sqrt[b^2 - 4*a*c]) + (b*Log[x^4])/a - (b*Log[a + b*x^4 + c*x^8])/(2*a))
/a)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

method	result
default	$-\frac{-\frac{b\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4}+\frac{\left(ac-\frac{b^{2}}{2}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{2}}-\frac{1}{4a\,x^{4}}-\frac{b\ln(x)}{a^{2}}$
risch	$-\frac{1}{4a\,x^{4}}-\frac{b\ln(x)}{a^{2}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(-4abc+b^{3}\right)\_Z+c^{2}\right)}\_R\ln\left(\left(\left(18a^{3}c-5a^{2}b^{2}\right)\_R^{2}-8\_Rabc+4c^{2}\right)x^{4}-\right.\right.}{4}$

input
```
int(1/x^5/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2*(-1/4*b*ln(c*x^8+b*x^4+a)+(a*c-1/2*b^2)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^4+b)/(4*a*c-b^2)^(1/2)))-1/4/a/x^4-b*ln(x)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.29

$$\int \frac{1}{x^5\left(a+bx^4+cx^8\right)}\,dx$$

$$= \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x^4\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-(b^3-4\,abc)x^4\log\left(cx^8+bx^4+a\right)}{8\,(a^2b^2-4\,a^3c)x^4}\right.$$

$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x^4\arctan\left(-\frac{(2\,cx^4+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^3-4\,abc)x^4\log\left(cx^8+bx^4+a\right)+8\,(b^3-}{8\,(a^2b^2-4\,a^3c)x^4}$$

input
```
integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[-1/8*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^8 + 2*b*c*x^4 + b^
2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^3 -
 4*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*
b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4), -1/8*(2*(b^2 - 2*a*c)*sqrt(-b^2
+ 4*a*c)*x^4*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^
3 - 4*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2
*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a + bx^4 + cx^8)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**5/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 (a + bx^4 + cx^8)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^5 (a + bx^4 + cx^8)} \, dx = \frac{b \log (cx^8 + bx^4 + a)}{8 \, a^2} - \frac{b \log (x^4)}{4 \, a^2} + \frac{(b^2 - 2 \, ac) \arctan \left(\frac{2 \, cx^4 + b}{\sqrt{-b^2 + 4 \, ac}} \right)}{4 \sqrt{-b^2 + 4 \, ac} a^2} + \frac{bx^4 - a}{4 \, a^2 x^4}$$

input
```
integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/8*b*log(c*x^8 + b*x^4 + a)/a^2 - 1/4*b*log(x^4)/a^2 + 1/4*(b^2 - 2*a*c)*
arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/4*(b
*x^4 - a)/(a^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 20.55 (sec) , antiderivative size = 8817, normalized size of antiderivative = 99.07

$$\int \frac{1}{x^5 (a + bx^4 + cx^8)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^5*(a + b*x^4 + c*x^8)),x)
```

output
```
(atan((4*a^5*(4*a*c - b^2)^2*(5*b^7 - 23*a^3*b*c^3 + 66*a^2*b^3*c^2 - 35*a
*b^5*c)*(((4*b^3 - 16*a*b*c)*((((((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^
5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b
^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c -
 b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*
a^2*(4*a*c - b^2)^(1/2)) - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/
(a^3*(4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c
- b^2)^(1/2)) - (b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(4*a^5*(4*a*c
 - b^2)^(3/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - ((4
*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((256*a^4*b^5
*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c
 - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(
4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2
*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6
)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (12
8*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 1
6*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 1
6*a^2*b^2)) - (((16*a^3*b*c^7 - 96*a^2*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*
((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^
5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a...
```

Reduce [F]

$$\int \frac{1}{x^5 (a + bx^4 + cx^8)}\, dx = \int \frac{1}{x^5 (c\,x^8 + b\,x^4 + a)} dx$$

input
```
int(1/x^5/(c*x^8+b*x^4+a),x)
```

output
```
int(1/x^5/(c*x^8+b*x^4+a),x)
```

3.54 $\int \frac{x^{10}}{a+bx^4+cx^8}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 381

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \frac{x^3}{3c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
1/3*x^3/c-1/4*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x
/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(1/2))^(1
/4)-1/4*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+(
-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/
4*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*
c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/4*(b-
(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2
)^(1/2))^(1/4))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.18

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \frac{4x^3-3\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{12c}$$

input
```
Integrate[x^10/(a + b*x^4 + c*x^8),x]
```

output
```
(4*x^3 - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*
#1^4)/(b*#1 + 2*c*#1^5) & ])/(12*c)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1703, 27, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \; 1703$$

$$\frac{x^3}{3c}-\frac{\int \frac{3x^2\left(bx^4+a\right)}{cx^8+bx^4+a}dx}{3c}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3}{3c}-\frac{\int \frac{x^2\left(bx^4+a\right)}{cx^8+bx^4+a}dx}{c}$$

$$\Big\downarrow \; 1834$$

$$\frac{x^3}{3c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3}{3c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{c}$$

$$\Big\downarrow \; 827$$

$$\frac{x^3}{3c}-$$

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{2\sqrt{2}\sqrt{c}}-\cdots\right.}{c}$$

$$\Big\downarrow \; 218$$

$$\frac{x^3}{3c}-$$

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\cdots}{\sqrt[4]{\sqrt{b^2-4a\cdots}}}\right.}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-\cdots}}}\right.}{c}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{x^3}{3c} - \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}} - \frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) + \left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{}{\sqrt[4]{}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{}}\right.}{c}$$

input
```
Int[x^10/(a + b*x^4 + c*x^8),x]
```

output
```
x^3/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4
)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4
*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)
]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (b - (b^2 - 2*a*c)
/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(
1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4
)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt
[b^2 - 4*a*c])^(1/4))))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{x^3}{3c}-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(\_R^6b+\_R^2a\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4c}$	63
risch	$\frac{x^3}{3c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(-\_R^6b-\_R^2a\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4c}$	65

input
```
int(x^10/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/c-1/4/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_
Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7003 vs. $2(299) = 598$.

Time = 0.84 (sec) , antiderivative size = 7003, normalized size of antiderivative = 18.38

$$\int \frac{x^{10}}{a + bx^4 + cx^8}\, dx = \text{Too large to display}$$

input
```
integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{10}}{a + bx^4 + cx^8}\, dx = \text{Timed out}$$

input
```
integrate(x**10/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx=\int \frac{x^{10}}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
1/3*x^3/c - integrate((b*x^6 + a*x^2)/(c*x^8 + b*x^4 + a), x)/c
```

Giac [F]

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx=\int \frac{x^{10}}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^10/(c*x^8 + b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 21.36 (sec) , antiderivative size = 12709, normalized size of antiderivative = 33.36

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx=\text{Too large to display}$$

input
```
int(x^10/(a + b*x^4 + c*x^8),x)
```

output
```
atan((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^
3*c^5)/c^3 - (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 +
 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b
^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b
^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8
 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*
c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c -
b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(
-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^
4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/
4) + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 28
0*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*
c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10))
)^(1/4)*1i - (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192
*a^5*b^3*c^5)/c^3 + (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*
b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*
a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/...
```

Reduce [F]

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx=\int \frac{x^{10}}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^10/(c*x^8+b*x^4+a),x)
```

output
```
int(x^10/(c*x^8+b*x^4+a),x)
```

3.55 $\int \frac{x^8}{a+bx^4+cx^8}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 376

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \frac{x}{c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
x/c+1/4*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b-(
-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/
4*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c
+b^2)^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*(b+(
-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)
^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*(b-(-2*a*
c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2
))^(1/4))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.19

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx=\frac{x}{c}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

input
```
Integrate[x^8/(a + b*x^4 + c*x^8),x]
```

output
```
x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)
/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1703, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx$$
$$\Big\downarrow \; 1703$$

$$\frac{x}{c}-\frac{\int \frac{bx^4+a}{cx^8+bx^4+a}dx}{c}$$

$\big\downarrow$ 1752

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$\big\downarrow$ 756

$$\frac{x}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{\sqrt{b^2-4ac}-b}}\cdots\right.}{c}$$

$\big\downarrow$ 218

$$\frac{x}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}\cdots}}{\sqrt{\sqrt{b^2-4\cdots}}}\cdots\right.}{c}$$

$\big\downarrow$ 221

$$\frac{x}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\text{arcta}\cdots}{\cdots}\right.}{c}$$

input
```
Int[x^8/(a + b*x^4 + c*x^8),x]
```

output
```
x/c - (((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c]
)^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2
^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + ((b - (b^2 - 2*a*c)/S
qrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(
1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)
*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^
2 - 4*a*c])^(3/4))))/2)/c
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8} c+\_Z^{4} b+a\right)}\frac{\left(-\_R^{4} b-a\right)\ln\left(x-\_R\right)}{2\_R^{7} c+\_R^{3} b}}{4c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8} c+\_Z^{4} b+a\right)}\frac{\left(-\_R^{4} b-a\right)\ln\left(x-\_R\right)}{2\_R^{7} c+\_R^{3} b}}{4c}$	59

input
```
int(x^8/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4
*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4001 vs. $2(296) = 592$.

Time = 0.22 (sec) , antiderivative size = 4001, normalized size of antiderivative = 10.64

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x**8/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \int \frac{x^8}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
x/c - integrate((b*x^4 + a)/(c*x^8 + b*x^4 + a), x)/c
```

Giac [F]

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \int \frac{x^8}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^8/(c*x^8 + b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 21.49 (sec) , antiderivative size = 10382, normalized size of antiderivative = 27.61

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int(x^8/(a + b*x^4 + c*x^8),x)
```

output
```
atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x
*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1
20*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4
- 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*
c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4
*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2)
+ 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^
4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)
*1i - (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x
*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1
20*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4
- 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2...
```

Reduce [F]

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \int \frac{x^8}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^8/(c*x^8+b*x^4+a),x)
```

output
```
int(x^8/(c*x^8+b*x^4+a),x)
```

3.56 $\int \frac{x^6}{a+bx^4+cx^8}\,dx$

Optimal result 436
Mathematica [C] (verified) 437
Rubi [A] (verified) 437
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Reduce [F] 443

Optimal result

Integrand size = 18, antiderivative size = 325

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = -\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}} + \frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}} + \frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}} - \frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

output
```
-1/4*(-b-(-4*a*c+b^2)^(1/2))^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^
2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)+1/4*(-b+(-4*a*c+b^2)^(
1/2))^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4
)/c^(3/4)/(-4*a*c+b^2)^(1/2)+1/4*(-b-(-4*a*c+b^2)^(1/2))^(3/4)*arctanh(2^(
1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)
^(1/2)-1/4*(-b+(-4*a*c+b^2)^(1/2))^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4
*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.14

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)\#1^3}{b+2c\#1^4}\&\right]$$

input
```
Integrate[x^6/(a + b*x^4 + c*x^8),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1^3)/(b + 2*c*#1^4) & ]/4
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1710, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \; 1710$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+$$
$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \; 27$$

$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \; 827$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+$$
$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)$$

$$\Big\downarrow \; 218$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+$$
$$\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)$$

$$\Big\downarrow \; 221$$

$$\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)$$

input
```
Int[x^6/(a + b*x^4 + c*x^8),x]
```

output
```
(1 + b/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a
*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(
2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b
- Sqrt[b^2 - 4*a*c])^(1/4))) + (1 - b/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*
c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b
^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])
^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\_R^{6}\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	43
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\_R^{6}\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	43

input
```
int(x^6/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4433 vs. $2(245) = 490$.

Time = 0.13 (sec) , antiderivative size = 4433, normalized size of antiderivative = 13.64

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 48.43 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.71

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx$$
$$= \text{RootSum}\left(t^8\cdot\left(16777216a^4c^7-16777216a^3b^2c^6+6291456a^2b^4c^5-1048576ab^6c^4+65536b^8c^3\right)+t^4\left(-\right.\right.$$

input
```
integrate(x**6/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**8*(16777216*a**4*c**7 - 16777216*a**3*b**2*c**6 + 6291456*a**2
*b**4*c**5 - 1048576*a*b**6*c**4 + 65536*b**8*c**3) + _t**4*(-12288*a**3*b
*c**3 + 10240*a**2*b**3*c**2 - 2816*a*b**5*c + 256*b**7) + a**3, Lambda(_t
, _t*log(x + (2097152*_t**7*a**4*c**7 - 2621440*_t**7*a**3*b**2*c**6 + 117
9648*_t**7*a**2*b**4*c**5 - 229376*_t**7*a*b**6*c**4 + 16384*_t**7*b**8*c*
*3 - 1280*_t**3*a**3*b*c**3 + 1600*_t**3*a**2*b**3*c**2 - 576*_t**3*a*b**5
*c + 64*_t**3*b**7)/(a**3*c - a**2*b**2))))
```

Maxima [F]

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = \int \frac{x^6}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = \int \frac{x^6}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 21.31 (sec) , antiderivative size = 8033, normalized size of antiderivative = 24.72

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int(x^6/(a + b*x^4 + c*x^8),x)
```

output
```
atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c
^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^
3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5
 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256
*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^
5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3
*b^2*c^6)))^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c
^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256
*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3
/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-
(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*
a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 -
 16384*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a
*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a...
```

Reduce [F]

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx=\int \frac{x^6}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^6/(c*x^8+b*x^4+a),x)
```

output
```
int(x^6/(c*x^8+b*x^4+a),x)
```

3.57 $\int \frac{x^4}{a+bx^4+cx^8}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 325

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\,\mathrm{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\,\mathrm{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

output
```
1/4*(-b-(-4*a*c+b^2)^(1/2))^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2
)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)-1/4*(-b+(-4*a*c+b^2)^(1
/2))^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)
/c^(1/4)/(-4*a*c+b^2)^(1/2)+1/4*(-b-(-4*a*c+b^2)^(1/2))^(1/4)*arctanh(2^(1
/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^
(1/2)-1/4*(-b+(-4*a*c+b^2)^(1/2))^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*
a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.13

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)\#1}{b+2c\#1^4}\&\right]$$

input
```
Integrate[x^4/(a + b*x^4 + c*x^8),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1)/(b + 2*c*#1^4) & ]/4
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1710, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+$$
$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx$$

$$\Big\downarrow \text{ 756}$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+$$
$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{\sqrt{\sqrt{b^2-4ac}-b}}\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+$$
$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+$$
$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)$$

input
```
Int[x^4/(a + b*x^4 + c*x^8),x]
```

output
```
((1 + b/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 -
4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b
- Sqrt[b^2 - 4*a*c])^(3/4))))/2 + ((1 - b/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^
(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sq
rt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	43

input
```
int(x^4/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2141 vs. 2(245) = 490.

Time = 0.10 (sec) , antiderivative size = 2141, normalized size of antiderivative = 6.59

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)))*log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sq
rt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1/4*sqrt(sqrt(1/2)*
sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3
+ 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x
 - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64
*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1/4*sqrt(-sqrt(1/2)*sqrt(-(b + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x + (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*sqrt(-sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
4 - 64*a^3*c^5)) - 1/4*sqrt(-sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x - (b^4*c - 8*a*b^2*c^2 + 16*a^2...
```

Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.39

$$\int \frac{x^4}{a + bx^4 + cx^8}\,dx$$
$$= \text{RootSum}\left(t^8 \cdot \left(16777216a^4c^5 - 16777216a^3b^2c^4 + 6291456a^2b^4c^3 - 1048576ab^6c^2 + 65536b^8c\right) + t^4 \cdot (40$$

input
```
integrate(x**4/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**8*(16777216*a**4*c**5 - 16777216*a**3*b**2*c**4 + 6291456*a**2
*b**4*c**3 - 1048576*a*b**6*c**2 + 65536*b**8*c) + _t**4*(4096*a**2*b*c**2
 - 2048*a*b**3*c + 256*b**5) + a, Lambda(_t, _t*log(-32768*_t**5*a**2*c**3
 + 16384*_t**5*a*b**2*c**2 - 2048*_t**5*b**4*c - 4*_t*b + x)))
```

Maxima [F]

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \int \frac{x^4}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \int \frac{x^4}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 21.34 (sec) , antiderivative size = 8169, normalized size of antiderivative = 25.14

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int(x^4/(a + b*x^4 + c*x^8),x)
```

output
```
- atan((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(51
2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))
)^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(5
12*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)
))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a
^4*b^2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(
-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c +
 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) +
64*a^3*b*c^4 - 16*a^2*b^3*c^3) - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (
-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*
c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - ((-(b^
5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256
*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b
^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 25
6*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(2621
44*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) -
x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a
*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 -
 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 -
 16*a^2*b^3*c^3) + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b...
```

Reduce [F]

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = \int \frac{x^4}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^4/(c*x^8+b*x^4+a),x)
```

output
```
int(x^4/(c*x^8+b*x^4+a),x)
```

3.58 $\int \frac{x^2}{a+bx^4+cx^8}\,dx$

Optimal result 452
Mathematica [C] (verified) 453
Rubi [A] (verified) 453
Maple [C] (verified) 456
Fricas [B] (verification not implemented) 457
Sympy [A] (verification not implemented) 457
Maxima [F] 458
Giac [F] 458
Mupad [B] (verification not implemented) 458
Reduce [F] 459

Optimal result

Integrand size = 18, antiderivative size = 315

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = -\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
-1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/
4)/(-4*a*c+b^2)^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/2*c^(1/4)*arctan(2^(
1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)^(1/2)/(
-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/2*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4
*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)^(1/2)/(-b-(-4*a*c+b^2)^(1/2))
^(1/4)-1/2*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
)*2^(1/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.14

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx=\frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)}{b\#1+2c\#1^5}\&\right]$$

input
```
Integrate[x^2/(a + b*x^4 + c*x^8),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1 + 2*c*#1^5) & ]/4
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1711, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{c\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow 27$$

$$\frac{2c\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow 827$$

$$\frac{2c\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow 218$$

$$\frac{2c\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow 221$$

$$\frac{2c\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[x^2/(a + b*x^4 + c*x^8),x]
```

output
```
(-2*c*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/
4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])
^(1/4))))/Sqrt[b^2 - 4*a*c] + (2*c*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[
b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) -
ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(
3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	43

input
```
int(x^2/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3193 vs. $2(245) = 490$.

Time = 0.11 (sec) , antiderivative size = 3193, normalized size of antiderivative = 10.14

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx$$
$$= \text{RootSum}\left(t^8\cdot\left(16777216a^5c^4 - 16777216a^4b^2c^3 + 6291456a^3b^4c^2 - 1048576a^2b^6c + 65536ab^8\right) + t^4\cdot\left(4\right.\right.$$

input
```
integrate(x**2/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**8*(16777216*a**5*c**4 - 16777216*a**4*b**2*c**3 + 6291456*a**3
*b**4*c**2 - 1048576*a**2*b**6*c + 65536*a*b**8) + _t**4*(4096*a**2*b*c**2
 - 2048*a*b**3*c + 256*b**5) + c, Lambda(_t, _t*log(x + (1048576*_t**7*a**
4*b*c**3 - 786432*_t**7*a**3*b**3*c**2 + 196608*_t**7*a**2*b**5*c - 16384*
_t**7*a*b**7 - 512*_t**3*a**2*c**2 + 384*_t**3*a*b**2*c - 64*_t**3*b**4)/c
)))
```

Maxima [F]

$$\int \frac{x^2}{a + bx^4 + cx^8}\,dx = \int \frac{x^2}{cx^8 + bx^4 + a}\,dx$$

input
```
integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^2}{a + bx^4 + cx^8}\,dx = \int \frac{x^2}{cx^8 + bx^4 + a}\,dx$$

input
```
integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 19.98 (sec) , antiderivative size = 6067, normalized size of antiderivative = 19.26

$$\int \frac{x^2}{a + bx^4 + cx^8}\,dx = \text{Too large to display}$$

input
```
int(x^2/(a + b*x^4 + c*x^8),x)
```

output
```
2*atan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(51
2*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))
)^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c +
 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4
 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i - 4*a*
b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(51
2*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))
)^(1/4) - ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(
512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3
)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^
(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c
 + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c
^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i + 4*
a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(
512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3
)))^(1/4))/(((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)
/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c
^3)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5
)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*...
```

Reduce [F]

$$\int \frac{x^2}{a + bx^4 + cx^8} \, dx = \int \frac{x^2}{c\,x^8 + b\,x^4 + a} dx$$

input
```
int(x^2/(c*x^8+b*x^4+a),x)
```

output
```
int(x^2/(c*x^8+b*x^4+a),x)
```

3.59 $\int \frac{1}{a+bx^4+cx^8}\,dx$

Optimal result 460
Mathematica [C] (verified) 461
Rubi [A] (verified) 461
Maple [C] (verified) 463
Fricas [B] (verification not implemented) 464
Sympy [A] (verification not implemented) 464
Maxima [F] 465
Giac [F] 465
Mupad [B] (verification not implemented) 466
Reduce [F] 466

Optimal result

Integrand size = 14, antiderivative size = 315

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4
)/(-4*a*c+b^2)^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/2*c^(3/4)*arctan(2^(1
/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)^(1/2)/(-
b+(-4*a*c+b^2)^(1/2))^(3/4)+1/2*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*
a*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^
(3/4)-1/2*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))
*2^(3/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.14

$$\int \frac{1}{a+bx^4+cx^8}\,dx=\frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)}{b\#1^3+2c\#1^7}\&\right]$$

input
```
Integrate[(a + b*x^4 + c*x^8)^(-1),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1^3 + 2*c*#1^7) & ]/4
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1685, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^4+cx^8}dx$$

$$\Big\downarrow \text{1685}$$

$$\frac{c\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 756$$

$$\frac{c\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 218$$

$$\frac{c\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 221$$

$$\frac{c\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(a + b*x^4 + c*x^8)^(-1),x]
```

output
```
-((c*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4
)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4))))/Sqrt[b^2 - 4*a*c]) + (c*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b
^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - Ar
cTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)
*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	40

input
```
int(1/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3125 vs. $2(245) = 490$.

Time = 0.14 (sec) , antiderivative size = 3125, normalized size of antiderivative = 9.92

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.56

$$\int \frac{1}{a+bx^4+cx^8}\,dx$$
$$= \mathrm{RootSum}\Big(t^8 \cdot \left(16777216a^7c^4 - 16777216a^6b^2c^3 + 6291456a^5b^4c^2 - 1048576a^4b^6c + 65536a^3b^8\right) + t^4\big(-$$

input
```
integrate(1/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**8*(16777216*a**7*c**4 - 16777216*a**6*b**2*c**3 + 6291456*a**5
*b**4*c**2 - 1048576*a**4*b**6*c + 65536*a**3*b**8) + _t**4*(-12288*a**3*b
*c**3 + 10240*a**2*b**3*c**2 - 2816*a*b**5*c + 256*b**7) + c**3, Lambda(_t
, _t*log(x + (16384*_t**5*a**5*b*c**2 - 8192*_t**5*a**4*b**3*c + 1024*_t**
5*a**3*b**5 + 8*_t*a**2*c**2 - 16*_t*a*b**2*c + 4*_t*b**4)/(a*c**2 - b**2*
c))))
```

Maxima [F]

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \int \frac{1}{cx^8+bx^4+a}\,dx$$

input
```
integrate(1/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(1/(c*x^8 + b*x^4 + a), x)
```

Giac [F]

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \int \frac{1}{cx^8+bx^4+a}\,dx$$

input
```
integrate(1/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(1/(c*x^8 + b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 21.04 (sec) , antiderivative size = 10337, normalized size of antiderivative = 32.82

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*x^4 + c*x^8),x)
```

output
```
- atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3
*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 +
 ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 -
11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 1
6*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 -
262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c
^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*
(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*
c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 9
6*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 +
 b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
 - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*
c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4
*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^
5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 -
256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2...
```

Reduce [F]

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \int \frac{1}{c\,x^8+b\,x^4+a}dx$$

input
```
int(1/(c*x^8+b*x^4+a),x)
```

output
```
int(1/(c*x^8+b*x^4+a),x)
```

3.60 $\int \frac{1}{x^2(a+bx^4+cx^8)}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 363

$$\int \frac{1}{x^2(a+bx^4+cx^8)}\,dx = -\frac{1}{ax}-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
-1/a/x-1/4*c^(1/4)*(1-b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b-(
-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*c^(1
/4)*(1+b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/
2))^(1/4))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1/4)*(1-b/(-4*a*
c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(
1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1/4)*(1+b/(-4*a*c+b^2)^(1/2))*
arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-b+(-4
*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)} dx = -\frac{1}{ax} - \frac{\text{RootSum}\left[a + b\#1^4 + c\#1^8\&, \frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{4a}$$

input
```
Integrate[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

output
```
-(1/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1
]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1704, 25, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}dx$$

$$\Big\downarrow \; 1704$$

$$\frac{\int -\frac{x^2\left(cx^4+b\right)}{cx^8+bx^4+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{x^2\left(cx^4+b\right)}{cx^8+bx^4+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 1834$$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 27$$

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 827$$

$$-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}d\ldots}{2\sqrt{2}\sqrt{c}}\ldots\right.}{a}\ldots$$

$$\frac{1}{ax}$$

$$\Big\downarrow \; 218$$

$$-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\ldots}{\sqrt[4]{\sqrt{b^2\ldots}}}\right.}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b\ldots}}}\right.}{a}\ldots$$

$$\frac{1}{ax}$$

$$\Big\downarrow \; 221$$

$$-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\arctan\left(\cdots\right.}{2\,2^{3/4}c\cdots}\right.}{a}$$

$$\frac{1}{ax}$$

input
```
Int[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

output
```
-(1/(a*x)) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
- Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1
/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3
/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + c*(1 + b/Sqrt[b^2 - 4*a*c])
*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^
(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4
))))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.17

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8} c+\_Z^{4} b+a\right)}\frac{\left(\_R^{6} c+\_R^{2} b\right) \ln\left(x-\_R\right)}{2\_R^{7} c+\_R^{3} b}}{4a}-\frac{1}{ax}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(256a^{9}c^{4}-256b^{2}c^{3}a^{8}+96b^{4}c^{2}a^{7}-16b^{6}c\,a^{6}+a^{5}b^{8}\right)\_Z^{8}+\left(80a^{4}b\,c^{4}-120b^{3}a^{3}c^{3}+61a^{2}b^{5}c^{2}-13a\,b^{7}c+b^{9}\right)\_Z^{4}+c^{5}\right)}\right.}{}$

input
```
int(1/x^2/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^
4*b+a))-1/a/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5758 vs. 2(281) = 562.

Time = 0.37 (sec) , antiderivative size = 5758, normalized size of antiderivative = 15.86

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)} dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 121.14 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)} dx$$
$$= \text{RootSum}\left(t^8 \cdot \left(16777216a^9c^4 - 16777216a^8b^2c^3 + 6291456a^7b^4c^2 - 1048576a^6b^6c + 65536a^5b^8\right) + t^4 \cdot (\right.$$
$$- \frac{1}{ax}$$

input
```
integrate(1/x**2/(c*x**8+b*x**4+a),x)
```

output
```
RootSum(_t**8*(16777216*a**9*c**4 - 16777216*a**8*b**2*c**3 + 6291456*a**7
*b**4*c**2 - 1048576*a**6*b**6*c + 65536*a**5*b**8) + _t**4*(20480*a**4*b*
c**4 - 30720*a**3*b**3*c**3 + 15616*a**2*b**5*c**2 - 3328*a*b**7*c + 256*b
**9) + c**5, Lambda(_t, _t*log(x + (-2097152*_t**7*a**10*c**5 + 5767168*_t
**7*a**9*b**2*c**4 - 4587520*_t**7*a**8*b**4*c**3 + 1605632*_t**7*a**7*b**
6*c**2 - 262144*_t**7*a**6*b**8*c + 16384*_t**7*a**5*b**10 - 2304*_t**3*a*
*5*b*c**5 + 8256*_t**3*a**4*b**3*c**4 - 8832*_t**3*a**3*b**5*c**3 + 4032*_
t**3*a**2*b**7*c**2 - 832*_t**3*a*b**9*c + 64*_t**3*b**11)/(a**2*c**6 - 3*
a*b**2*c**5 + b**4*c**4))) - 1/(a*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)} \, dx = \int \frac{1}{(cx^8 + bx^4 + a)x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((c*x^6 + b*x^2)/(c*x^8 + b*x^4 + a), x)/a - 1/(a*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)} \, dx = \int \frac{1}{(cx^8 + bx^4 + a)x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^8 + b*x^4 + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 20.83 (sec) , antiderivative size = 10509, normalized size of antiderivative = 28.95

$$\int \frac{1}{x^2(a + bx^4 + cx^8)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^4 + c*x^8)),x)
```

output
```
2*atan((((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5
*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^
6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9
+ b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b
^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4
*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 1228
8*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^
8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i +
4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*
a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^
7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16
*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 + b^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 +
a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^
5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 25
6*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^
(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c
 - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(51...
```

Reduce [F]

$$\int \frac{1}{x^2(a + bx^4 + cx^8)}\,dx = \int \frac{1}{x^2(c\,x^8 + b\,x^4 + a)}dx$$

input
```
int(1/x^2/(c*x^8+b*x^4+a),x)
```

output
```
int(1/x^2/(c*x^8+b*x^4+a),x)
```

3.61 $\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 365

$$\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx = -\frac{1}{3ax^3} + \frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
-1/3/a/x^3+1/4*c^(3/4)*(1-b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(
-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*
c^(3/4)*(1+b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)
^(1/2))^(1/4))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^(3/4)*(1-b/(-
4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))
*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^(3/4)*(1+b/(-4*a*c+b^2)^(1/
2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-b
+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^4 (a+bx^4+cx^8)} dx = -\frac{1}{3ax^3} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4a}$$

input
```
Integrate[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/3*1/(a*x^3) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x
- #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*a)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1704, 27, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+bx^4+cx^8\right)}dx$$

$$\Big\downarrow \; 1704$$

$$\frac{\int -\frac{3\left(cx^4+b\right)}{cx^8+bx^4+a}dx}{3a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{cx^4+b}{cx^8+bx^4+a}dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \; 1752$$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \; 756$$

$$-\frac{\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\cdots}}\cdots}{\sqrt{\sqrt{b^2-4ac}\cdots}}\cdots\right)}{a}$$

$$\frac{1}{3ax^3}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-\cdots}}}\cdots}{\sqrt{\sqrt{\cdots}}}\cdots\right)}{a}$$

$$\frac{1}{3ax^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\Bigg(-}{a}}{\frac{1}{3ax^3}}$$

input

```
Int[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

output

```
-1/3*1/(a*x^3) - ((c*(1 - b/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*
x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c
])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(
2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + (c*(1 + b/Sqrt[b^2 -
 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^
(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*
x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c
])^(3/4))))/2)/a
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.17

method	result
default	$-\frac{1}{3 a \,x^{3}}+\frac{\sum_{\_R =\operatorname{RootOf}\left(\_Z^{8} c +\_Z^{4} b +a \right)}\frac{\left(-\_R^{4} c -b \right) \ln \left(x -\_R \right)}{2 \_R^{7} c +\_R^{3} b}}{4 a}$
risch	$-\frac{1}{3 a \,x^{3}}+\frac{\left(\sum_{\_R =\operatorname{RootOf}\left(\left(256 c^{4} a^{11}-256 a^{10} b^{2} c^{3}+96 a^{9} b^{4} c^{2}-16 a^{8} b^{6} c +a^{7} b^{8}\right) \_Z^{8}+\left(-112 b \,c^{5} a^{5}+280 b^{3} c^{4} a^{4}-231 b^{5} c^{3} a^{3}+86 b^{7} c^{2} a^{2}-15 b^{9}\right.\right.}\right.}{}$

input
```
int(1/x^4/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3+1/4/a*sum((-_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8
*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5030 vs. $2(281) = 562$.

Time = 0.46 (sec) , antiderivative size = 5030, normalized size of antiderivative = 13.78

$$\int \frac{1}{x^4 (a + bx^4 + cx^8)} dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^4 + cx^8)} dx = \text{Timed out}$$

input
```
integrate(1/x**4/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^4 (a + bx^4 + cx^8)} \, dx = \int \frac{1}{(cx^8 + bx^4 + a)x^4} \, dx$$

input
```
integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((c*x^4 + b)/(c*x^8 + b*x^4 + a), x)/a - 1/3/(a*x^3)
```

Giac [F]

$$\int \frac{1}{x^4 (a + bx^4 + cx^8)} \, dx = \int \frac{1}{(cx^8 + bx^4 + a)x^4} \, dx$$

input
```
integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^8 + b*x^4 + a)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 23.90 (sec) , antiderivative size = 16497, normalized size of antiderivative = 45.20

$$\int \frac{1}{x^4 (a + bx^4 + cx^8)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^4 + c*x^8)),x)
```

output
```
2*atan(-(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*
b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(
1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^
9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (
-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 2
31*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a
*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^
14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*
(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^
3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^
2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(
512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2
*c^3)))^(3/4)*1i - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i +
 x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) -
 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*
c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^...
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^4+cx^8)} \, dx = \int \frac{1}{x^4 (c\,x^8+b\,x^4+a)}dx$$

input
```
int(1/x^4/(c*x^8+b*x^4+a),x)
```

output
```
int(1/x^4/(c*x^8+b*x^4+a),x)
```

3.62 $\int \frac{1}{x^2(a+bx^4+cx^8)^2} dx$

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Optimal result

Integrand size = 18, antiderivative size = 553

$$\int \frac{1}{x^2 (a+bx^4+cx^8)^2} \, dx$$
$$= -\frac{5b^2-18ac}{4a^2(b^2-4ac)x} + \frac{b^2-2ac+bcx^4}{4a(b^2-4ac)x(a+bx^4+cx^8)}$$
$$+ \frac{\sqrt[4]{c}\left(5b^3-28abc-(5b^2-18ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\ 2^{3/4}a^2(b^2-4ac)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt[4]{c}\left(5b^3-28abc+(5b^2-18ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\ 2^{3/4}a^2(b^2-4ac)^{3/2}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt[4]{c}\left(5b^3-28abc-(5b^2-18ac)\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\ 2^{3/4}a^2(b^2-4ac)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\sqrt[4]{c}\left(5b^3-28abc+(5b^2-18ac)\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\ 2^{3/4}a^2(b^2-4ac)^{3/2}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output

```
-1/4*(-18*a*c+5*b^2)/a^2/(-4*a*c+b^2)/x+1/4*(b*c*x^4-2*a*c+b^2)/a/(-4*a*c+
b^2)/x/(c*x^8+b*x^4+a)+1/16*c^(1/4)*(5*b^3-28*a*b*c-(-18*a*c+5*b^2)*(-4*a*
c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1
/4)/a^2/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/16*c^(1/4)*(5*b
^3-28*a*b*c+(-18*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(
-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a^2/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b
^2)^(1/2))^(1/4)-1/16*c^(1/4)*(5*b^3-28*a*b*c-(-18*a*c+5*b^2)*(-4*a*c+b^2)
^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a
^2/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/16*c^(1/4)*(5*b^3-28
*a*b*c+(-18*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(
-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a^2/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^
(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^2 (a+bx^4+cx^8)^2} \, dx = -\frac{\frac{16}{x}+\frac{4x^3(b^3-3abc+b^2cx^4-2ac^2x^4)}{(b^2-4ac)(a+bx^4+cx^8)}+\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{5b^3\log(x-\#1)-23abc\log(x-\#1)+5b^2c\log(x-\#1)\#1^4-18ac}{b\#1+2c\#1^5}\right.}{b^2-4ac}}{16a^2}$$

input
```
Integrate[1/(x^2*(a + b*x^4 + c*x^8)^2),x]
```

output
```
-1/16*(16/x + (4*x^3*(b^3 - 3*a*b*c + b^2*c*x^4 - 2*a*c^2*x^4))/((b^2 - 4*
a*c)*(a + b*x^4 + c*x^8)) + RootSum[a + b*#1^4 + c*#1^8 & , (5*b^3*Log[x -
 #1] - 23*a*b*c*Log[x - #1] + 5*b^2*c*Log[x - #1]*#1^4 - 18*a*c^2*Log[x -
#1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(b^2 - 4*a*c))/a^2
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1702, 25, 1828, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^4+cx^8)^2} \, dx$$

$$\Big\downarrow \text{1702}$$

$$\frac{-2ac+b^2+bcx^4}{4ax(b^2-4ac)(a+bx^4+cx^8)}-\frac{\int -\frac{5bcx^4+5b^2-18ac}{x^2(cx^8+bx^4+a)}dx}{4a(b^2-4ac)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5bcx^4+5b^2-18ac}{x^2(cx^8+bx^4+a)}dx}{4a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^4}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\Big\downarrow$ 1828

$$\frac{-\frac{\int \frac{x^2\left(c\left(5b^2-18ac\right)x^4+b\left(5b^2-23ac\right)\right)}{cx^8+bx^4+a}dx}{a}-\frac{5b^2-18ac}{ax}}{4a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^4}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\Big\downarrow$ 1834

$$\frac{-\frac{\frac{1}{2}c\left(-\frac{28abc}{\sqrt{b^2-4ac}}+\frac{5b^3}{\sqrt{b^2-4ac}}-18ac+5b^2\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx-\frac{c\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{2\sqrt{b^2-4ac}}}{a}-\frac{5b^2-18ac}{ax}}{4a\left(b^2-4ac\right)}+$$
$$\frac{-2ac+b^2+bcx^4}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\Big\downarrow$ 27

$$\frac{-\frac{c\left(-\frac{28abc}{\sqrt{b^2-4ac}}+\frac{5b^3}{\sqrt{b^2-4ac}}-18ac+5b^2\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx-\frac{c\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}}{a}-\frac{5b^2-18ac}{ax}}{4a\left(b^2-4ac\right)}+$$
$$\frac{-2ac+b^2+bcx^4}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\Big\downarrow$ 827

$$\frac{-\frac{c\left(-\frac{28abc}{\sqrt{b^2-4ac}}+\frac{5b^3}{\sqrt{b^2-4ac}}-18ac+5b^2\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\sqrt{2}\sqrt{c}x^2}}dx}{2\sqrt{2}\sqrt{c}}\right)-\frac{c\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\left(\int \cdots\right.}{\cdots}}{a}}{4a\left(b^2-4ac\right)}$$
$$\frac{-2ac+b^2+bcx^4}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\Big\downarrow$ 218

$$-\frac{\frac{c\left(-\frac{28abc}{\sqrt{b^2-4ac}}+\frac{5b^3}{\sqrt{b^2-4ac}}-18ac+5b^2\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\int\frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)}{a}-\frac{c\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right.}{}}{4a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx^4}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\Big\downarrow$ 221

$$-\frac{\frac{c\left(-\frac{28abc}{\sqrt{b^2-4ac}}+\frac{5b^3}{\sqrt{b^2-4ac}}-18ac+5b^2\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a}-\frac{c\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-\right.}{}}{4a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx^4}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

input
```
Int[1/(x^2*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(b^2 - 2*a*c + b*c*x^4)/(4*a*(b^2 - 4*a*c)*x*(a + b*x^4 + c*x^8)) + (-((5*
b^2 - 18*a*c)/(a*x)) - (-((c*(5*b^3 - 28*a*b*c - (5*b^2 - 18*a*c)*Sqrt[b^2
 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*
2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)
*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*
a*c])^(1/4))))/Sqrt[b^2 - 4*a*c]) + c*(5*b^2 - 18*a*c + (5*b^3)/Sqrt[b^2 -
 4*a*c] - (28*a*b*c)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4
)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4
)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/a)/(4*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1702
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*
(p + 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && IGtQ[n, 0] && ILtQ[p, -1]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.31

method	result
default	$-\frac{\frac{\frac{c\left(2ac-b^{2}\right)x^{7}}{16ac-4b^{2}}+\frac{b\left(3ac-b^{2}\right)x^{3}}{16ac-4b^{2}}}{c\,x^{8}+b\,x^{4}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\left(c\left(18ac-5b^{2}\right)\_R^{6}+b\left(23ac-5b^{2}\right)\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{64ac-16b^{2}}}{a^{2}}-\frac{1}{a^{2}x}$
risch	$\frac{-\frac{c\left(18ac-5b^{2}\right)x^{8}}{4a^{2}\left(4ac-b^{2}\right)}-\frac{b\left(19ac-5b^{2}\right)x^{4}}{4\left(4ac-b^{2}\right)a^{2}}-\frac{1}{a}}{x\left(c\,x^{8}+b\,x^{4}+a\right)}+\frac{\Big(\sum_{\_R=\mathrm{RootOf}\left(\left(16777216a^{21}c^{12}-50331648a^{20}b^{2}c^{11}+69206016a^{19}b^{4}c^{10}-57671680a^{18}b^{6}c^{9}+32\right.\right.}}{}$

input
```
int(1/x^2/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*((1/4*c*(2*a*c-b^2)/(4*a*c-b^2)*x^7+1/4*b*(3*a*c-b^2)/(4*a*c-b^2)*x
^3)/(c*x^8+b*x^4+a)+1/16/(4*a*c-b^2)*sum((c*(18*a*c-5*b^2)*_R^6+b*(23*a*c-
5*b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a)))-1/a^2
/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15984 vs. $2(461) = 922$.

Time = 23.94 (sec) , antiderivative size = 15984, normalized size of antiderivative = 28.90

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)^2} \, dx = \int \frac{1}{(cx^8 + bx^4 + a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*((5*b^2*c - 18*a*c^2)*x^8 + (5*b^3 - 19*a*b*c)*x^4 + 4*a*b^2 - 16*a^2
*c)/((a^2*b^2*c - 4*a^3*c^2)*x^9 + (a^2*b^3 - 4*a^3*b*c)*x^5 + (a^3*b^2 -
4*a^4*c)*x) + 1/4*integrate(-((5*b^2*c - 18*a*c^2)*x^6 + (5*b^3 - 23*a*b*c
)*x^2)/(c*x^8 + b*x^4 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)^2} \, dx = \int \frac{1}{(cx^8 + bx^4 + a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
integrate(1/((c*x^8 + b*x^4 + a)^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 27.30 (sec) , antiderivative size = 31085, normalized size of antiderivative = 56.21

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^4 + c*x^8)^2),x)
```

output
```
atan((((-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*
b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^
4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11
*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^1
0*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)
^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 18
1990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c -
 b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(131072*(a^9*b^
24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12
*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^1
2*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6
*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(x*(-(625*
b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 63847
5*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*
a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 1352482
5600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 1
2575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625
*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c
^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2
) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(131072*(a^9*b^24 + 167772...
```

Reduce [F]

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)^2} \, dx = \int \frac{1}{x^2 (c\,x^8 + b\,x^4 + a)^2} dx$$

input
```
int(1/x^2/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(1/x^2/(c*x^8+b*x^4+a)^2,x)
```

3.63 $\int \frac{x^{11}}{1+x^4+x^8}\,dx$

Optimal result 495
Mathematica [A] (verified) 495
Rubi [A] (verified) 496
Maple [A] (verified) 497
Fricas [A] (verification not implemented) 497
Sympy [A] (verification not implemented) 498
Maxima [A] (verification not implemented) 498
Giac [A] (verification not implemented) 498
Mupad [B] (verification not implemented) 499
Reduce [B] (verification not implemented) 499

Optimal result

Integrand size = 14, antiderivative size = 44

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{x^4}{4} - \frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1+x^4+x^8\right)$$

output
```
1/4*x^4-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)-1/8*ln(x^8+x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{x^4}{4} - \frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1+x^4+x^8\right)$$

input
```
Integrate[x^11/(1 + x^4 + x^8),x]
```

output
```
x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^8+x^4+1}\,dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{1}{4}\int \frac{x^8}{x^8+x^4+1}dx^4$$
$$\Big\downarrow \text{1143}$$
$$\frac{1}{4}\int \left(1-\frac{x^4+1}{x^8+x^4+1}\right)dx^4$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}\left(-\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{\sqrt{3}}+x^4-\frac{1}{2}\log\left(x^8+x^4+1\right)\right)$$

input
```
Int[x^11/(1 + x^4 + x^8),x]
```

output
```
(x^4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] - Log[1 + x^4 + x^8]/2)/4
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^4}{4}-\frac{\arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln\left(x^8+x^4+1\right)}{8}$	36
risch	$\frac{x^4}{4}-\frac{\arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln\left(4x^8+4x^4+4\right)}{8}$	40

input
```
int(x^11/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)-1/8*ln(x^8+x^4+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx=\frac{1}{4}\,x^4-\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right)-\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

input
```
integrate(x^11/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{x^4}{4} - \frac{\log\left(x^8+x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**11/(x**8+x**4+1),x)
```

output
```
x**4/4 - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/
3)/12
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4+1\right)\right) - \frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

input
```
integrate(x^11/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4+1\right)\right) - \frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

input
```
integrate(x^11/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
 + 1)
```

Mupad [B] (verification not implemented)

Time = 19.67 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{x^4}{4} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12} - \frac{\ln\left(x^8+x^4+1\right)}{8}$$

input
```
int(x^11/(x^4 + x^8 + 1),x)
```

output
```
x^4/4 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3))/12 - log(x^4 + x^8 +
 1)/8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.34

$$\int \frac{x^{11}}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\,atan\left(\sqrt{3}-2x\right)}{12} + \frac{\sqrt{3}\,atan\left(\sqrt{3}+2x\right)}{12} + \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{12} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{12} - \frac{\log(x^2-x+1)}{8} - \frac{\log(x^2+x+1)}{8} - \frac{\log\left(-\sqrt{3}\,x+x^2+1\right)}{8} - \frac{\log\left(\sqrt{3}\,x+x^2+1\right)}{8} + \frac{x^4}{4}$$

input
```
int(x^11/(x^8+x^4+1),x)
```

output
```
(2*sqrt(3)*atan(sqrt(3) - 2*x) + 2*sqrt(3)*atan(sqrt(3) + 2*x) + 2*sqrt(3)
*atan((2*x - 1)/sqrt(3)) - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) - 3*log(x**2
- x + 1) - 3*log(x**2 + x + 1) - 3*log( - sqrt(3)*x + x**2 + 1) - 3*log(sq
rt(3)*x + x**2 + 1) + 6*x**4)/24
```

3.64 $\int \frac{x^9}{1+x^4+x^8} \, dx$

Optimal result 500
Mathematica [C] (verified) 500
Rubi [A] (verified) 501
Maple [A] (verified) 503
Fricas [A] (verification not implemented) 503
Sympy [A] (verification not implemented) 503
Maxima [A] (verification not implemented) 504
Giac [A] (verification not implemented) 504
Mupad [B] (verification not implemented) 505
Reduce [B] (verification not implemented) 505

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \frac{x^9}{1+x^4+x^8} \, dx = \frac{x^2}{2} + \frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
1/2*x^2+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(2*x^2+1
)*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.81

$$\int \frac{x^9}{1+x^4+x^8} \, dx = \frac{x^2}{2} - \frac{\left(i+\sqrt{3}\right) \arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x^2\right)}{2\sqrt{6+6i\sqrt{3}}} - \frac{\left(-i+\sqrt{3}\right) \arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x^2\right)}{2\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[x^9/(1 + x^4 + x^8),x]
```

output
```
x^2/2 - ((I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 + (6*I)*S
qrt[3]]) - ((-I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 - (6*I
)*Sqrt[3]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1695, 1442, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^8+x^4+1}dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^8}{x^8+x^4+1}dx^2$$

$$\Big\downarrow \text{1442}$$

$$\frac{1}{2}\left(x^2-\int \frac{x^4+1}{x^8+x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx^2-\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx^2+x^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\int \frac{1}{-x^4-3}d(2x^2-1)+\int \frac{1}{-x^4-3}d(2x^2+1)+x^2\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}+x^2\right)$$

input
```
Int[x^9/(1 + x^4 + x^8),x]
```

output
```
(x^2 - ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x^2)/Sqrt[3]]/
Sqrt[3])/2
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x^2}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{6}-\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	43
risch	$\frac{x^2}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^2}{3}\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^6}{3}+\frac{2\sqrt{3}\,x^2}{3}\right)}{6}$	44

input
```
int(x^9/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/6*arctan(1/3*(2*x^2+1)
*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int \frac{x^9}{1+x^4+x^8}\,dx = \frac{1}{2}\,x^2-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x^2\right)-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(x^6+2\,x^2\right)\right)$$

input
```
integrate(x^9/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x^2) - 1/6*sqrt(3)*arctan(1/3*sqr
t(3)*(x^6 + 2*x^2))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{x^9}{1+x^4+x^8}\,dx = \frac{x^2}{2}+\frac{\sqrt{3}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^2}{3}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^6}{3}+\frac{2\sqrt{3}x^2}{3}\right)\right)}{12}$$

input
```
integrate(x**9/(x**8+x**4+1),x)
```

output
```
x**2/2 + sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt
(3)*x**2/3))/12
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{x^9}{1+x^4+x^8}\,dx = \frac{1}{2}\,x^2 - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right)$$

input
```
integrate(x^9/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{x^9}{1+x^4+x^8}\,dx = \frac{1}{2}\,x^2 - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right) - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right)$$

input
```
integrate(x^9/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{x^9}{1+x^4+x^8}\,dx = \frac{x^2}{2} - \frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^6}{3}+\frac{2\,\sqrt{3}\,x^2}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^2}{3}\right)\right)}{12}$$

input
```
int(x^9/(x^4 + x^8 + 1),x)
```

output
```
x^2/2 - (3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^
(1/2)*x^2)/3)))/12
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{x^9}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\,atan\left(\sqrt{3}-2x\right)}{6} + \frac{\sqrt{3}\,atan\left(\sqrt{3}+2x\right)}{6} - \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6} + \frac{x^2}{2}$$

input
```
int(x^9/(x^8+x^4+1),x)
```

output
```
(sqrt(3)*atan(sqrt(3) - 2*x) + sqrt(3)*atan(sqrt(3) + 2*x) - sqrt(3)*atan(
(2*x - 1)/sqrt(3)) + sqrt(3)*atan((2*x + 1)/sqrt(3)) + 3*x**2)/6
```

3.65 $\int \frac{x^7}{1+x^4+x^8} \, dx$

Optimal result 506
Mathematica [A] (verified) 506
Rubi [A] (verified) 507
Maple [A] (verified) 508
Fricas [A] (verification not implemented) 509
Sympy [A] (verification not implemented) 509
Maxima [A] (verification not implemented) 510
Giac [A] (verification not implemented) 510
Mupad [B] (verification not implemented) 510
Reduce [B] (verification not implemented) 511

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{x^7}{1+x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1+x^4+x^8\right)$$

output
```
-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)+1/8*ln(x^8+x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{1+x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1+x^4+x^8\right)$$

input
```
Integrate[x^7/(1 + x^4 + x^8),x]
```

output
```
-1/4*ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 + x^4 + x^8]/8
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1693, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^8+x^4+1}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^4}{x^8+x^4+1}dx^4$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{2x^4+1}{x^8+x^4+1}dx^4-\frac{1}{2}\int \frac{1}{x^8+x^4+1}dx^4\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(\int \frac{1}{-x^8-3}d(2x^4+1)+\frac{1}{2}\int \frac{2x^4+1}{x^8+x^4+1}dx^4\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{2x^4+1}{x^8+x^4+1}dx^4-\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(\frac{1}{2}\log\left(x^8+x^4+1\right)-\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

input
```
Int[x^7/(1 + x^4 + x^8),x]
```

output
```
(-(ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3]) + Log[1 + x^4 + x^8]/2)/4
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln(x^8+x^4+1)}{8}$	31
risch	$-\frac{\arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln(4x^8+4x^4+4)}{8}$	35

input
```
int(x^7/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)+1/8*ln(x^8+x^4+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{1+x^4+x^8}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right) + \frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

input
```
integrate(x^7/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{1+x^4+x^8}\,dx = \frac{\log\left(x^8+x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**7/(x**8+x**4+1),x)
```

output
```
log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/12
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{1+x^4+x^8}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right)+\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

input
```
integrate(x^7/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{1+x^4+x^8}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right)+\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

input
```
integrate(x^7/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{1+x^4+x^8}\,dx = \frac{\ln\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
int(x^7/(x^4 + x^8 + 1),x)
```

output
```
log(x^4 + x^8 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3))/12
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.65

$$\int \frac{x^7}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\,atan(\sqrt{3}-2x)}{12} + \frac{\sqrt{3}\,atan(\sqrt{3}+2x)}{12} + \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{12} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{12} + \frac{\log(x^2-x+1)}{8} + \frac{\log(x^2+x+1)}{8} + \frac{\log(-\sqrt{3}\,x+x^2+1)}{8} + \frac{\log(\sqrt{3}\,x+x^2+1)}{8}$$

input
```
int(x^7/(x^8+x^4+1),x)
```

output
```
(2*sqrt(3)*atan(sqrt(3) - 2*x) + 2*sqrt(3)*atan(sqrt(3) + 2*x) + 2*sqrt(3)
*atan((2*x - 1)/sqrt(3)) - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) + 3*log(x**2
- x + 1) + 3*log(x**2 + x + 1) + 3*log( - sqrt(3)*x + x**2 + 1) + 3*log(sq
rt(3)*x + x**2 + 1))/24
```

3.66 $\int \frac{x^5}{1+x^4+x^8} \, dx$

Optimal result 512
Mathematica [C] (verified) 512
Rubi [A] (verified) 513
Maple [A] (verified) 516
Fricas [A] (verification not implemented) 516
Sympy [A] (verification not implemented) 517
Maxima [A] (verification not implemented) 517
Giac [A] (verification not implemented) 518
Mupad [B] (verification not implemented) 518
Reduce [B] (verification not implemented) 519

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{x^5}{1+x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\operatorname{arctanh}\left(\frac{x^2}{1+x^4}\right)$$

output
```
-1/12*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(
1/2))*3^(1/2)-1/4*arctanh(x^2/(x^4+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.49

$$\int \frac{x^5}{1+x^4+x^8} \, dx = \frac{\sqrt{1-i\sqrt{3}}(-i+\sqrt{3}) \arctan\left(\frac{1}{2}(-i+\sqrt{3})\,x^2\right) + \sqrt{1+i\sqrt{3}}(i+\sqrt{3}) \arctan\left(\frac{1}{2}(i+\sqrt{3})\,x^2\right)}{4\sqrt{6}}$$

input
```
Integrate[x^5/(1 + x^4 + x^8),x]
```

output
```
(Sqrt[1 - I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2] + Sqrt[
1 + I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(4*Sqrt[6])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1695, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^8+x^4+1}dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^4}{x^8+x^4+1}dx^2$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{x^4+1}{x^8+x^4+1}dx^2-\frac{1}{2}\int \frac{1-x^4}{x^8+x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx^2+\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx^2\right)-\frac{1}{2}\int \frac{1-x^4}{x^8+x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\int \frac{1}{-x^4-3}d(2x^2-1)-\int \frac{1}{-x^4-3}d(2x^2+1)\right)-\frac{1}{2}\int \frac{1-x^4}{x^8+x^4+1}dx^2\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-x^4}{x^8+x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x^2}{x^4-x^2+1}dx^2+\frac{1}{2}\int -\frac{2x^2+1}{x^4+x^2+1}dx^2\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2-\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\log\left(x^4-x^2+1\right)-\frac{1}{2}\log\left(x^4+x^2+1\right)\right)\right)$$

input
```
Int[x^5/(1 + x^4 + x^8),x]
```

output
```
((ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[
3])/2 + (Log[1 - x^2 + x^4]/2 - Log[1 + x^2 + x^4]/2)/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\ln(x^4-x^2+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(x^4+x^2+1)}{8}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	62
risch	$\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln(4x^4+4x^2+4)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(4x^4-4x^2+4)}{8}$	68

input
```
int(x^5/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/8*ln(x^4+x^
2+1)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) - \frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x^5/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \frac{\log\left(x^4-x^2+1\right)}{8} - \frac{\log\left(x^4+x^2+1\right)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**5/(x**8+x**4+1),x)
```

output
```
log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x
**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) - \frac{1}{8}\log\left(x^4+x^2+1\right) + \frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x^5/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) - \frac{1}{8}\log\left(x^4+x^2+1\right) + \frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x^5/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 19.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{1+x^4+x^8}\,dx = \operatorname{atanh}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \operatorname{atanh}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

input
```
int(x^5/(x^4 + x^8 + 1),x)
```

output
```
atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3
^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\int \frac{x^5}{1+x^4+x^8}\,dx = -\frac{\sqrt{3}\,atan\left(\sqrt{3}-2x\right)}{12} - \frac{\sqrt{3}\,atan\left(\sqrt{3}+2x\right)}{12} + \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{12} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{12} - \frac{\log(x^2-x+1)}{8} - \frac{\log(x^2+x+1)}{8} + \frac{\log\left(-\sqrt{3}\,x+x^2+1\right)}{8} + \frac{\log\left(\sqrt{3}\,x+x^2+1\right)}{8}$$

input
```
int(x^5/(x^8+x^4+1),x)
```

output
```
( - 2*sqrt(3)*atan(sqrt(3) - 2*x) - 2*sqrt(3)*atan(sqrt(3) + 2*x) + 2*sqrt
(3)*atan((2*x - 1)/sqrt(3)) - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) - 3*log(x*
*2 - x + 1) - 3*log(x**2 + x + 1) + 3*log( - sqrt(3)*x + x**2 + 1) + 3*log
(sqrt(3)*x + x**2 + 1))/24
```

3.67 $\int \frac{x^3}{1+x^4+x^8} dx$

Optimal result 520
Mathematica [A] (verified) 520
Rubi [A] (verified) 521
Maple [A] (verified) 522
Fricas [A] (verification not implemented) 522
Sympy [A] (verification not implemented) 523
Maxima [A] (verification not implemented) 523
Giac [A] (verification not implemented) 523
Mupad [B] (verification not implemented) 524
Reduce [B] (verification not implemented) 524

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x^3}{1+x^4+x^8} \, dx = \frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
1/6*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+x^4+x^8} \, dx = \frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Integrate[x^3/(1 + x^4 + x^8),x]
```

output
```
ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1690, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8+x^4+1}dx$$
$$\Big\downarrow \text{ 1690}$$
$$\frac{1}{4}\int \frac{1}{x^8+x^4+1}dx^4$$
$$\Big\downarrow \text{ 1083}$$
$$-\frac{1}{2}\int \frac{1}{-x^8-3}d\left(2x^4+1\right)$$
$$\Big\downarrow \text{ 217}$$
$$\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[x^3/(1 + x^4 + x^8),x]
```

output
```
ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	19
risch	$\frac{\arctan\left(\frac{\left(2x^4+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	19

input
```
int(x^3/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right)$$

input
```
integrate(x^3/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**3/(x**8+x**4+1),x)
```

output
```
sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right)$$

input
```
integrate(x^3/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4+1)\right)$$

input
```
integrate(x^3/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\left(\frac{2\,x^4}{3}+\frac{1}{3}\right)\right)}{6}$$

input
```
int(x^3/(x^4 + x^8 + 1),x)
```

output
```
(3^(1/2)*atan(3^(1/2)*((2*x^4)/3 + 1/3)))/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\left(-atan\left(\sqrt{3}-2x\right)-atan\left(\sqrt{3}+2x\right)-atan\left(\frac{2x-1}{\sqrt{3}}\right)+atan\left(\frac{2x+1}{\sqrt{3}}\right)\right)}{6}$$

input
```
int(x^3/(x^8+x^4+1),x)
```

output
```
(sqrt(3)*( - atan(sqrt(3) - 2*x) - atan(sqrt(3) + 2*x) - atan((2*x - 1)/sq
rt(3)) + atan((2*x + 1)/sqrt(3))))/6
```

3.68 $\int \frac{x}{1+x^4+x^8} \, dx$

Optimal result 525
Mathematica [C] (verified) 525
Rubi [A] (verified) 526
Maple [A] (verified) 528
Fricas [A] (verification not implemented) 529
Sympy [A] (verification not implemented) 529
Maxima [A] (verification not implemented) 530
Giac [A] (verification not implemented) 530
Mupad [B] (verification not implemented) 531
Reduce [B] (verification not implemented) 531

Optimal result

Integrand size = 12, antiderivative size = 63

$$\int \frac{x}{1+x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\operatorname{arctanh}\left(\frac{x^2}{1+x^4}\right)$$

output
```
-1/12*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(
1/2))*3^(1/2)+1/4*arctanh(x^2/(x^4+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25

$$\int \frac{x}{1+x^4+x^8} \, dx = \frac{i\left(\sqrt{1-i\sqrt{3}}\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x^2\right) - \sqrt{1+i\sqrt{3}}\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x^2\right)\right)}{2\sqrt{6}}$$

input
```
Integrate[x/(1 + x^4 + x^8),x]
```

output
```
((I/2)*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x^2)/2] - Sqrt[1 + I*Sq
rt[3]]*ArcTan[((I + Sqrt[3])*x^2)/2]))/Sqrt[6]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {1695, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^8+x^4+1} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{x^8+x^4+1}dx^2$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx^2+\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx^2\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx^2-\frac{1}{2}\int -\frac{1-2x^2}{x^4-x^2+1}dx^2\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx^2+\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx^2+\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx^2+\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2-\int \frac{1}{-x^4-3}d(2x^2-1)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2-\int \frac{1}{-x^4-3}d(2x^2+1)\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2+\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^4-x^2+1\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^4+x^2+1\right)\right)\right)$$

input
```
Int[x/(1 + x^4 + x^8),x]
```

output
```
((ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3] - Log[1 - x^2 + x^4]/2)/2 + (ArcTan
[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3] + Log[1 + x^2 + x^4]/2)/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{\ln\left(x^4-x^2+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(x^4+x^2+1\right)}{8}+\frac{\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	62
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(4x^4-4x^2+4\right)}{8}+\frac{\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln\left(4x^4+4x^2+4\right)}{8}$	68

input
```
int(x/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))+1/8*ln(x^4+x
^2+1)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{x}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{x}{1+x^4+x^8}\,dx = -\frac{\log\left(x^4-x^2+1\right)}{8}+\frac{\log\left(x^4+x^2+1\right)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x/(x**8+x**4+1),x)
```

output
```
-log(x**4 - x**2 + 1)/8 + log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*
x**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{x}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{x}{1+x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 19.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{x}{1+x^4+x^8}\,dx = \operatorname{atan}\left(\frac{\sqrt{3}\,x^2}{2}-\frac{x^2\,1\mathrm{i}}{2}\right)\,\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right) + \operatorname{atan}\left(\frac{\sqrt{3}\,x^2}{2}+\frac{x^2\,1\mathrm{i}}{2}\right)\,\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int(x/(x^4 + x^8 + 1),x)
```

output
```
atan((3^(1/2)*x^2)/2 - (x^2*1i)/2)*(3^(1/2)/12 + 1i/4) + atan((3^(1/2)*x^2
)/2 + (x^2*1i)/2)*(3^(1/2)/12 - 1i/4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\int \frac{x}{1+x^4+x^8}\,dx = -\frac{\sqrt{3}\,atan\left(\sqrt{3}-2x\right)}{12} - \frac{\sqrt{3}\,atan\left(\sqrt{3}+2x\right)}{12} + \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{12} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{12} + \frac{\log(x^2-x+1)}{8} + \frac{\log(x^2+x+1)}{8} - \frac{\log\left(-\sqrt{3}\,x+x^2+1\right)}{8} - \frac{\log\left(\sqrt{3}\,x+x^2+1\right)}{8}$$

input
```
int(x/(x^8+x^4+1),x)
```

output
```
( - 2*sqrt(3)*atan(sqrt(3) - 2*x) - 2*sqrt(3)*atan(sqrt(3) + 2*x) + 2*sqrt
(3)*atan((2*x - 1)/sqrt(3)) - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) + 3*log(x*
*2 - x + 1) + 3*log(x**2 + x + 1) - 3*log( - sqrt(3)*x + x**2 + 1) - 3*log
(sqrt(3)*x + x**2 + 1))/24
```

3.69 $\int \frac{1}{x(1+x^4+x^8)} dx$

Optimal result 532
Mathematica [C] (verified) 532
Rubi [A] (verified) 533
Maple [A] (verified) 535
Fricas [A] (verification not implemented) 536
Sympy [A] (verification not implemented) 536
Maxima [A] (verification not implemented) 536
Giac [A] (verification not implemented) 537
Mupad [B] (verification not implemented) 537
Reduce [B] (verification not implemented) 538

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{1}{x(1+x^4+x^8)} dx = -\frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \log(x) - \frac{1}{8}\log\left(1+x^4+x^8\right)$$

output
```
-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)+ln(x)-1/8*ln(x^8+x^4+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.41

$$\int \frac{1}{x(1+x^4+x^8)} dx = \frac{1}{24}\left(2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + 24\log(x) - \sqrt{3}\left(i+\sqrt{3}\right)\log\left(i+\sqrt{3}-2ix^2\right) - \sqrt{3}\left(-i+\sqrt{3}\right)\log\left(-i+\sqrt{3}+2ix^2\right) - 3\log\left(1-x+x^2\right) - 3\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[1/(x*(1 + x^4 + x^8)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]
] + 24*Log[x] - Sqrt[3]*(I + Sqrt[3])*Log[I + Sqrt[3] - (2*I)*x^2] - Sqrt[
3]*(-I + Sqrt[3])*Log[-I + Sqrt[3] + (2*I)*x^2] - 3*Log[1 - x + x^2] - 3*L
og[1 + x + x^2])/24
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1693, 1144, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^8+x^4+1)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{1}{x^4(x^8+x^4+1)}dx^4$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{4}\left(\int -\frac{x^4+1}{x^8+x^4+1}dx^4+\log(x^4)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\log(x^4)-\int \frac{x^4+1}{x^8+x^4+1}dx^4\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{x^8+x^4+1}dx^4-\frac{1}{2}\int \frac{2x^4+1}{x^8+x^4+1}dx^4+\log(x^4)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(\int \frac{1}{-x^8-3}d(2x^4+1)-\frac{1}{2}\int \frac{2x^4+1}{x^8+x^4+1}dx^4+\log(x^4)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{2x^4+1}{x^8+x^4+1}dx^4-\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^4\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(-\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^4\right)-\frac{1}{2}\log\left(x^8+x^4+1\right)\right)$$

input
```
Int[1/(x*(1 + x^4 + x^8)),x]
```

output
```
(-(ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3]) + Log[x^4] - Log[1 + x^4 + x^8]/2)
/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
risch	$\ln\left(x\right)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{4}+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^{8}+x^{4}+1\right)}{8}$
default	$-\frac{\ln\left(x^{2}-x+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x-1\right)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^{2}+x+1\right)}{8}-\frac{\arctan\left(\frac{\left(2x+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln\left(x^{4}-x^{2}+1\right)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^{2}-}{3}\right.}{12}$ [cut off at page edge]

input
```
int(1/x/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/12*3^(1/2)*arctan(2/3*(x^4+1/2)*3^(1/2))-1/8*ln(x^8+x^4+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(1+x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) - \frac{1}{8}\log\left(x^8+x^4+1\right) + \log(x)$$

input
```
integrate(1/x/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + l
og(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(1+x^4+x^8)}\,dx = \log(x) - \frac{\log(x^8+x^4+1)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(1/x/(x**8+x**4+1),x)
```

output
```
log(x) - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/
3)/12
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(1+x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) - \frac{1}{8}\log\left(x^8+x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1
/4*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(1+x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) - \frac{1}{8}\log\left(x^8+x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1
/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 18.94 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(1+x^4+x^8)}\,dx = \ln(x) - \frac{\ln(x^8+x^4+1)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
int(1/(x*(x^4 + x^8 + 1)),x)
```

output
```
log(x) - log(x^4 + x^8 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/
3))/12
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.56

$$\int \frac{1}{x(1+x^4+x^8)}\,dx = \frac{\sqrt{3}\,atan(\sqrt{3}-2x)}{12} + \frac{\sqrt{3}\,atan(\sqrt{3}+2x)}{12} + \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{12} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{12} - \frac{\log(x^2-x+1)}{8} - \frac{\log(x^2+x+1)}{8} - \frac{\log(-\sqrt{3}\,x+x^2+1)}{8} - \frac{\log(\sqrt{3}\,x+x^2+1)}{8} + \log(x)$$

input
```
int(1/x/(x^8+x^4+1),x)
```

output
```
(2*sqrt(3)*atan(sqrt(3) - 2*x) + 2*sqrt(3)*atan(sqrt(3) + 2*x) + 2*sqrt(3)
*atan((2*x - 1)/sqrt(3)) - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) - 3*log(x**2
- x + 1) - 3*log(x**2 + x + 1) - 3*log( - sqrt(3)*x + x**2 + 1) - 3*log(sq
rt(3)*x + x**2 + 1) + 24*log(x))/24
```

3.70 $\int \frac{1}{x^3(1+x^4+x^8)} \, dx$

Optimal result 539
Mathematica [C] (verified) 539
Rubi [A] (verified) 540
Maple [A] (verified) 542
Fricas [A] (verification not implemented) 542
Sympy [A] (verification not implemented) 543
Maxima [A] (verification not implemented) 543
Giac [A] (verification not implemented) 543
Mupad [B] (verification not implemented) 544
Reduce [B] (verification not implemented) 544

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \frac{1}{x^3 (1 + x^4 + x^8)} \, dx = -\frac{1}{2x^2} + \frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
-1/2/x^2+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(2*x^2+
1)*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.85

$$\int \frac{1}{x^3 (1 + x^4 + x^8)} \, dx = \frac{1}{12}\left(-\frac{6}{x^2} - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - i\sqrt{3}\log\left(i+\sqrt{3}-2ix^2\right) + i\sqrt{3}\log\left(-i+\sqrt{3}+2ix^2\right)\right)$$

input
```
Integrate[1/(x^3*(1 + x^4 + x^8)),x]
```

output
```
(-6/x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x
)/Sqrt[3]] - I*Sqrt[3]*Log[I + Sqrt[3] - (2*I)*x^2] + I*Sqrt[3]*Log[-I + S
qrt[3] + (2*I)*x^2])/12
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1695, 1443, 25, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^8+x^4+1)} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2} \int \frac{1}{x^4 (x^8+x^4+1)} dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2} \left(\int -\frac{x^4+1}{x^8+x^4+1} dx^2 - \frac{1}{x^2} \right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(-\int \frac{x^4+1}{x^8+x^4+1} dx^2 - \frac{1}{x^2} \right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2} \left(-\frac{1}{2} \int \frac{1}{x^4-x^2+1} dx^2 - \frac{1}{2} \int \frac{1}{x^4+x^2+1} dx^2 - \frac{1}{x^2} \right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2} \left(\int \frac{1}{-x^4-3} d(2x^2-1) + \int \frac{1}{-x^4-3} d(2x^2+1) - \frac{1}{x^2} \right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2} \left(-\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{x^2} \right)$$

input
```
Int[1/(x^3*(1 + x^4 + x^8)),x]
```

output
```
(-x^(-2) - ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x^2)/Sqrt[
3]]/Sqrt[3])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{1}{2x^{2}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^{2}}{3}\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^{6}}{3}+\frac{2\sqrt{3}\,x^{2}}{3}\right)}{6}$	44
default	$-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{1}{2x^{2}}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{2}-1\right)\sqrt{3}}{3}\right)}{6}$	57

input
```
int(1/x^3/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-1/6*3^(1/2)*arctan(1/3*3^(1/2)*x^2)-1/6*3^(1/2)*arctan(1/3*3^(1/2
)*x^6+2/3*3^(1/2)*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3\left(1+x^4+x^8\right)}\,dx=-\frac{\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}x^2\right)+\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(x^6+2\,x^2)\right)+3}{6\,x^2}$$

input
```
integrate(1/x^3/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(3)*x^2*arctan(1/3*sqrt(3)*x^2) + sqrt(3)*x^2*arctan(1/3*sqrt(3)
*(x^6 + 2*x^2)) + 3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^3 (1 + x^4 + x^8)} \, dx = \frac{\sqrt{3} \left(-2 \operatorname{atan} \left(\frac{\sqrt{3} x^2}{3} \right) - 2 \operatorname{atan} \left(\frac{\sqrt{3} x^6}{3} + \frac{2\sqrt{3} x^2}{3} \right) \right)}{12} - \frac{1}{2x^2}$$

input
```
integrate(1/x**3/(x**8+x**4+1),x)
```

output
```
sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(3)*x**2/
3))/12 - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3 (1 + x^4 + x^8)} \, dx = -\frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2 x^2 + 1) \right) - \frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2 x^2 - 1) \right) - \frac{1}{2 x^2}$$

input
```
integrate(1/x^3/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/2/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3 (1 + x^4 + x^8)} \, dx = -\frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2 x^2 + 1) \right) - \frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2 x^2 - 1) \right) - \frac{1}{2 x^2}$$

input
```
integrate(1/x^3/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/2/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^3 (1+x^4+x^8)}\,dx = -\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^6}{3}+\frac{2\,\sqrt{3}\,x^2}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^2}{3}\right)\right)}{12}-\frac{1}{2\,x^2}$$

input
```
int(1/(x^3*(x^4 + x^8 + 1)),x)
```

output
```
- (3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^(1/2)*
x^2)/3)))/12 - 1/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^3 (1+x^4+x^8)}\,dx = \frac{\sqrt{3}\,atan\left(\sqrt{3}-2x\right)x^2+\sqrt{3}\,atan\left(\sqrt{3}+2x\right)x^2-\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^2+\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^2-3}{6x^2}$$

input
```
int(1/x^3/(x^8+x^4+1),x)
```

output
```
(sqrt(3)*atan(sqrt(3) - 2*x)*x**2 + sqrt(3)*atan(sqrt(3) + 2*x)*x**2 - sqr
t(3)*atan((2*x - 1)/sqrt(3))*x**2 + sqrt(3)*atan((2*x + 1)/sqrt(3))*x**2 -
 3)/(6*x**2)
```

3.71 $\int \frac{1}{x^5(1+x^4+x^8)}\,dx$

Optimal result 545
Mathematica [C] (verified) 545
Rubi [A] (verified) 546
Maple [A] (verified) 548
Fricas [A] (verification not implemented) 548
Sympy [A] (verification not implemented) 549
Maxima [A] (verification not implemented) 549
Giac [A] (verification not implemented) 549
Mupad [B] (verification not implemented) 550
Reduce [B] (verification not implemented) 550

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{x^5\left(1+x^4+x^8\right)}\,dx=-\frac{1}{4x^4}-\frac{\arctan\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\log(x)+\frac{1}{8}\log\left(1+x^4+x^8\right)$$

output
```
-1/4/x^4-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)-ln(x)+1/8*ln(x^8+x^4+1
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.83

$$\begin{aligned}\int \frac{1}{x^5\left(1+x^4+x^8\right)}\,dx&=\frac{1}{24}\left(-\frac{6}{x^4}+2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)\right.\\&\quad-24\log(x)+\sqrt{3}\left(-i+\sqrt{3}\right)\log\left(i+\sqrt{3}-2ix^2\right)\\&\quad+\sqrt{3}\left(i+\sqrt{3}\right)\log\left(-i+\sqrt{3}+2ix^2\right)+3\log\left(1-x+x^2\right)\\&\quad\left.+3\log\left(1+x+x^2\right)\right)\end{aligned}$$

input
```
Integrate[1/(x^5*(1 + x^4 + x^8)),x]
```

output
```
(-6/x^4 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x
)/Sqrt[3]] - 24*Log[x] + Sqrt[3]*(-I + Sqrt[3])*Log[I + Sqrt[3] - (2*I)*x^
2] + Sqrt[3]*(I + Sqrt[3])*Log[-I + Sqrt[3] + (2*I)*x^2] + 3*Log[1 - x + x
^2] + 3*Log[1 + x + x^2])/24
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^8+x^4+1)} \, dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{4} \int \frac{1}{x^8 (x^8+x^4+1)}dx^4$$

$$\Big\downarrow \text{ 1145}$$

$$\frac{1}{4} \left(\int -\frac{x^4+1}{x^4 (x^8+x^4+1)}dx^4-\frac{1}{x^4}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4} \left(-\int \frac{x^4+1}{x^4 (x^8+x^4+1)}dx^4-\frac{1}{x^4}\right)$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{4} \left(-\int \left(\frac{1}{x^4}-\frac{x^4}{x^8+x^4+1}\right)dx^4-\frac{1}{x^4}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4} \left(-\frac{\arctan\left(\frac{2x^4+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{x^4}-\log\left(x^4\right)+\frac{1}{2} \log\left(x^8+x^4+1\right)\right)$$

input
```
Int[1/(x^5*(1 + x^4 + x^8)),x]
```

output
```
(-x^(-4) - ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] - Log[x^4] + Log[1 + x^4 +
x^8]/2)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{1}{4x^4}-\ln(x)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^4+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^8+x^4+1)}{8}$
default	$\frac{\ln(x^2-x+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln(x^4-x^2+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{}}{3}\right.}{12}$

input
```
int(1/x^5/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4-ln(x)-1/12*3^(1/2)*arctan(2/3*(x^4+1/2)*3^(1/2))+1/8*ln(x^8+x^4+1
)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^5(1+x^4+x^8)}\,dx = -\frac{2\sqrt{3}x^4\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right)-3x^4\log(x^8+x^4+1)+24x^4\log(x)+6}{24x^4}$$

input
```
integrate(1/x^5/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 3*x^4*log(x^8 + x^4
 + 1) + 24*x^4*log(x) + 6)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\left(1+x^4+x^8\right)}\,dx = -\log\left(x\right)+\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}-\frac{1}{4x^4}$$

input
```
integrate(1/x**5/(x**8+x**4+1),x)
```

output
```
-log(x) + log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)
/3)/12 - 1/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5\left(1+x^4+x^8\right)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x^5/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/4/x^4 + 1/8*log(x^8 + x^
4 + 1) - 1/4*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^5\left(1+x^4+x^8\right)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)+\frac{x^4-1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x^5/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/4*(x^4 - 1)/x^4 + 1/8*lo
g(x^8 + x^4 + 1) - 1/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5(1+x^4+x^8)}\,dx = \frac{\ln(x^8+x^4+1)}{8} - \ln(x) - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12} - \frac{1}{4\,x^4}$$

input
```
int(1/(x^5*(x^4 + x^8 + 1)),x)
```

output
```
log(x^4 + x^8 + 1)/8 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/
3))/12 - 1/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.81

$$\int \frac{1}{x^5(1+x^4+x^8)}\,dx = \frac{2\sqrt{3}\,atan\left(\sqrt{3}-2x\right)x^4 + 2\sqrt{3}\,atan\left(\sqrt{3}+2x\right)x^4 + 2\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^4 - 2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^4 + 3\log(x}{}$$

input
```
int(1/x^5/(x^8+x^4+1),x)
```

output
```
(2*sqrt(3)*atan(sqrt(3) - 2*x)*x***4 + 2*sqrt(3)*atan(sqrt(3) + 2*x)*x***4 +
 2*sqrt(3)*atan((2*x - 1)/sqrt(3))*x***4 - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)
)*x***4 + 3*log(x***2 - x + 1)*x***4 + 3*log(x***2 + x + 1)*x***4 + 3*log( - sq
rt(3)*x + x***2 + 1)*x***4 + 3*log(sqrt(3)*x + x***2 + 1)*x***4 - 24*log(x)*x**
*4 - 6)/(24*x***4)
```

3.72 $\int \frac{1}{x^7(1+x^4+x^8)}\,dx$

Optimal result 551
Mathematica [A] (verified) 551
Rubi [A] (verified) 552
Maple [A] (verified) 555
Fricas [A] (verification not implemented) 556
Sympy [A] (verification not implemented) 556
Maxima [A] (verification not implemented) 557
Giac [A] (verification not implemented) 557
Mupad [B] (verification not implemented) 558
Reduce [B] (verification not implemented) 558

Optimal result

Integrand size = 14, antiderivative size = 77

$$\int \frac{1}{x^7(1+x^4+x^8)}\,dx = -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\operatorname{arctanh}\left(\frac{x^2}{1+x^4}\right)$$

output
```
-1/6/x^6+1/2/x^2-1/12*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)+1/12*arctan(1
/3*(2*x^2+1)*3^(1/2))*3^(1/2)-1/4*arctanh(x^2/(x^4+1))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^7(1+x^4+x^8)}\,dx = \frac{1}{24}\left(-\frac{4}{x^6}+\frac{12}{x^2}-2\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)-3\log\left(1-x+x^2\right)-3\log\left(1+x+x^2\right)+3\log\left(1-x^2+x^4\right)\right)$$

input
```
Integrate[1/(x^7*(1 + x^4 + x^8)),x]
```

output
```
(-4/x^6 + 12/x^2 - 2*Sqrt[3]*ArcTan[(1 - 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[
(1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 - 2*x^2)/Sqrt[3]] - 3*Log[1 - x +
 x^2] - 3*Log[1 + x + x^2] + 3*Log[1 - x^2 + x^4])/24
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {1695, 1443, 27, 1604, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (x^8+x^4+1)} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2} \int \frac{1}{x^8 (x^8+x^4+1)}dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2} \left(\frac{1}{3} \int -\frac{3(x^4+1)}{x^4 (x^8+x^4+1)}dx^2-\frac{1}{3x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\int \frac{x^4+1}{x^4 (x^8+x^4+1)}dx^2-\frac{1}{3x^6}\right)$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{2} \left(\int \frac{x^4}{x^8+x^4+1}dx^2-\frac{1}{3x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2} \left(-\frac{1}{2} \int \frac{1-x^4}{x^8+x^4+1}dx^2+\frac{1}{2} \int \frac{x^4+1}{x^8+x^4+1}dx^2-\frac{1}{3x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{x^4-x^2+1}dx^2+\frac{1}{2}\int\frac{1}{x^4+x^2+1}dx^2\right)-\frac{1}{2}\int\frac{1-x^4}{x^8+x^4+1}dx^2-\frac{1}{3x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\int\frac{1}{-x^4-3}d(2x^2-1)-\int\frac{1}{-x^4-3}d(2x^2+1)\right)-\frac{1}{2}\int\frac{1-x^4}{x^8+x^4+1}dx^2-\frac{1}{3x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-x^4}{x^8+x^4+1}dx^2+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{3x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int-\frac{1-2x^2}{x^4-x^2+1}dx^2+\frac{1}{2}\int-\frac{2x^2+1}{x^4+x^2+1}dx^2\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{3x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-2x^2}{x^4-x^2+1}dx^2-\frac{1}{2}\int\frac{2x^2+1}{x^4+x^2+1}dx^2\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{3x^6}+\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{3x^6}+\frac{1}{x^2}+\frac{1}{2}\left(\frac{1}{2}\log\left(x^4-x^2+1\right)-\frac{1}{2}\log\left(x^4+x^2+1\right)\right)\right)$$

input
```
Int[1/(x^7*(1 + x^4 + x^8)),x]
```

output
```
(-1/3*1/x^6 + x^(-2) + (ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 +
 2*x^2)/Sqrt[3]]/Sqrt[3])/2 + (Log[1 - x^2 + x^4]/2 - Log[1 + x^2 + x^4]/2
)/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\frac{x^4}{2}-\frac{1}{6}}{x^6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^2-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(x^4-x^2+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\left(x^2+\frac{1}{2}\right)}{3}\right)}{12}-\frac{\ln\left(x^4+x^2+1\right)}{8}$
default	$-\frac{\ln\left(x^2-x+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln\left(x^4-x^2+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-}{3}\right.}{12}$

input
```
int(1/x^7/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(1/2*x^4-1/6)/x^6+1/12*3^(1/2)*arctan(2/3*(x^2-1/2)*3^(1/2))+1/8*ln(x^4-x^
2+1)+1/12*3^(1/2)*arctan(2/3*3^(1/2)*(x^2+1/2))-1/8*ln(x^4+x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^7(1+x^4+x^8)}\,dx = \frac{2\sqrt{3}x^6\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)+2\sqrt{3}x^6\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)-3x^6\log\left(x^4+x^2+1\right)+3x^6\log\left(x^4\right.}{24x^6}$$

input
```
integrate(1/x^7/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/24*(2*sqrt(3)*x^6*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 2*sqrt(3)*x^6*arctan
(1/3*sqrt(3)*(2*x^2 - 1)) - 3*x^6*log(x^4 + x^2 + 1) + 3*x^6*log(x^4 - x^2
 + 1) + 12*x^4 - 4)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^7(1+x^4+x^8)}\,dx = \frac{\log(x^4-x^2+1)}{8}-\frac{\log(x^4+x^2+1)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}+\frac{3x^4-1}{6x^6}$$

input
```
integrate(1/x**7/(x**8+x**4+1),x)
```

output
```
log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x
**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12 + (3
*x**4 - 1)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^7(1+x^4+x^8)}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \frac{3x^4-1}{6x^6} - \frac{1}{8}\log\left(x^4+x^2+1\right) + \frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(1/x^7/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log
(x^4 - x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^7(1+x^4+x^8)}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \frac{3x^4-1}{6x^6} - \frac{1}{8}\log\left(x^4+x^2+1\right) + \frac{1}{8}\log\left(x^4-x^2+1\right)$$

input
```
integrate(1/x^7/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log
(x^4 - x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 19.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^7\,\left(1+x^4+x^8\right)}\,dx = \mathrm{atanh}\left(\frac{2\,x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \mathrm{atanh}\left(\frac{2\,x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{\frac{x^4}{2}-\frac{1}{6}}{x^6}$$

input
```
int(1/(x^7*(x^4 + x^8 + 1)),x)
```

output
```
atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3
^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) + (x^4/2 - 1/6)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.73

$$\int \frac{1}{x^7\,\left(1+x^4+x^8\right)}\,dx = \frac{-2\sqrt{3}\,atan\left(\sqrt{3}-2x\right)x^6-2\sqrt{3}\,atan\left(\sqrt{3}+2x\right)x^6+2\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^6-2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^6-3\log(}{24}$$

input
```
int(1/x^7/(x^8+x^4+1),x)
```

output
```
( - 2*sqrt(3)*atan(sqrt(3) - 2*x)*x**6 - 2*sqrt(3)*atan(sqrt(3) + 2*x)*x**
6 + 2*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**6 - 2*sqrt(3)*atan((2*x + 1)/sqrt
(3))*x**6 - 3*log(x**2 - x + 1)*x**6 - 3*log(x**2 + x + 1)*x**6 + 3*log( -
 sqrt(3)*x + x**2 + 1)*x**6 + 3*log(sqrt(3)*x + x**2 + 1)*x**6 + 12*x**4 -
 4)/(24*x**6)
```

3.73 $\int \frac{x^8}{1+x^4+x^8}\,dx$

Optimal result 559
Mathematica [C] (verified) 560
Rubi [A] (verified) 560
Maple [C] (verified) 563
Fricas [A] (verification not implemented) 564
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Maxima [F] 565
Giac [A] (verification not implemented) 566
Mupad [B] (verification not implemented) 566
Reduce [B] (verification not implemented) 567

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \frac{x^8}{1+x^4+x^8}\,dx = x+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} -\frac{1}{4}\arctan\left(\sqrt{3}+2x\right)-\frac{1}{4}\text{arctanh}\left(\frac{x}{1+x^2}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{4\sqrt{3}}$$

output
```
x+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/4*arctan(-3^(1/2)+2*x)-1/12*a
rctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*arctan(3^(1/2)+2*x)-1/4*arctanh(x/(
x^2+1))-1/12*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.26

$$\int \frac{x^8}{1+x^4+x^8}\,dx = -\frac{i\arctan\left(\frac{1}{2}(1-i\sqrt{3})\,x\right)}{\sqrt{-6+6i\sqrt{3}}}+\frac{i\arctan\left(\frac{1}{2}(1+i\sqrt{3})\,x\right)}{\sqrt{-6-6i\sqrt{3}}}+\frac{1}{24}\left(24x-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+3\log\left(1-x+x^2\right)-3\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[x^8/(1 + x^4 + x^8),x]
```

output
```
((-I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[-6 + (6*I)*Sqrt[3]] + (I*ArcTan[
((1 + I*Sqrt[3])*x)/2])/Sqrt[-6 - (6*I)*Sqrt[3]] + (24*x - 2*Sqrt[3]*ArcTa
n[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*Log[1 - x
+ x^2] - 3*Log[1 + x + x^2])/24
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.56, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1703, 1749, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^8+x^4+1}\,dx$$

$$\Big\downarrow \text{1703}$$

$$x-\int \frac{x^4+1}{x^8+x^4+1}dx$$

$$\Big\downarrow \text{1749}$$

$$-\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx+x$$

$\Big\downarrow$ 1407

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-x}{x^2-x+1}dx-\frac{1}{2}\int \frac{x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+x$$

$\Big\downarrow$ 1142

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+$$
$$\frac{1}{2}\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+x$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+$$
$$\frac{1}{2}\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+x$$

$\Big\downarrow$ 1083

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}\right.\right.$$
$$\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)}{2\sqrt{3}}\right)+$$
$$x$$

$\Big\downarrow$ 217

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+$$
$$\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx+\sqrt{3}\arctan\left(2x+\sqrt{3}\right)}{2\sqrt{3}}\right)+$$
$$x$$

$\Big\downarrow$ 1103

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2+x+1\right)\right)\right)+$$
$$\frac{1}{2}\left(-\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\sqrt{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)+$$
$$x$$

input
```
Int[x^8/(1 + x^4 + x^8),x]
```

output
```
x + ((-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2)/2 + (-(A
rcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x + x^2]/2)/2)/2 + (-1/2*(-(Sq
rt[3]*ArcTan[Sqrt[3] - 2*x]) - Log[1 - Sqrt[3]*x + x^2]/2)/Sqrt[3] - (Sqrt
[3]*ArcTan[Sqrt[3] + 2*x] + Log[1 + Sqrt[3]*x + x^2]/2)/(2*Sqrt[3]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

method	result
risch	$x - \frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} + \frac{\ln(4x^2-4x+4)}{8} + \frac{\left(\sum_{\_R=\mathrm{RootOf}(9\_Z^4+3\_Z^2+1)} \_R \ln\left(3\_R^3-\_R+x\right)\right)}{4} - \frac{\arctan\left(\frac{(2x\ldots}{}\right)}{12\ldots}$
default	$x + \frac{\ln(x^2-x+1)}{8} - \frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} - \frac{\ln(x^2+x+1)}{8} - \frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12} - \frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{24} - \frac{\arctan\left(\sqrt{\ldots}\right)}{4\ldots}$

input
```
int(x^8/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
x-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/8*ln(4*x^2-4*x+4)+1/4*sum(_R*
ln(3*_R^3-_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))-1/12*arctan(1/3*(2*x+1)*3^(1/2
))*3^(1/2)-1/8*ln(4*x^2+4*x+4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{x^8}{1+x^4+x^8}\,dx =& -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)\\&-\frac{1}{24}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)\\&+x-\frac{1}{4}\,\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{4}\,\arctan\left(-2\,x+\sqrt{3}\right)\\&-\frac{1}{8}\,\log\left(x^2+x+1\right)+\frac{1}{8}\,\log\left(x^2-x+1\right)\end{aligned}$$

input
```
integrate(x^8/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) + x - 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(-2*x + s
qrt(3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.75

$$\int \frac{x^8}{1+x^4+x^8}\,dx = x + \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) + \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}-9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right) + \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) + \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-9216t^5-8t+x\right)\right)\right)$$

input
```
integrate(x**8/(x**8+x**4+1),x)
```

output
```
x + (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/2
4)**5) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + s
qrt(3)*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 +
 sqrt(3)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3
)*I/24)**5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t,
_t*log(-9216*_t**5 - 8*_t + x)))
```

Maxima [F]

$$\int \frac{x^8}{1+x^4+x^8}\,dx = \int \frac{x^8}{x^8+x^4+1}\,dx$$

input
```
integrate(x^8/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + x - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x
 + 1) + 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.99

$$\int \frac{x^8}{1+x^4+x^8}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{24}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) + \frac{1}{24}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + x - \frac{1}{4}\,\arctan\left(2\,x+\sqrt{3}\right) - \frac{1}{4}\,\arctan\left(2\,x-\sqrt{3}\right) - \frac{1}{8}\,\log\left(x^2+x+1\right) + \frac{1}{8}\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^8/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) + x - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sq
rt(3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int \frac{x^8}{1+x^4+x^8}\,dx = x - \mathrm{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int(x^8/(x^4 + x^8 + 1),x)
```

output
```
x - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) - atan((2*x)/(3^(
1/2)*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(
1/2)/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90

$$\int \frac{x^8}{1+x^4+x^8}\,dx = \frac{atan(\sqrt{3}-2x)}{4} - \frac{atan(\sqrt{3}+2x)}{4} - \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{12} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{12} + \frac{\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)}{24} - \frac{\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)}{24} + \frac{\log(x^2-x+1)}{8} - \frac{\log(x^2+x+1)}{8} + x$$

input
```
int(x^8/(x^8+x^4+1),x)
```

output
```
(6*atan(sqrt(3) - 2*x) - 6*atan(sqrt(3) + 2*x) - 2*sqrt(3)*atan((2*x - 1)/
sqrt(3)) - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) + sqrt(3)*log( - sqrt(3)*x +
x**2 + 1) - sqrt(3)*log(sqrt(3)*x + x**2 + 1) + 3*log(x**2 - x + 1) - 3*lo
g(x**2 + x + 1) + 24*x)/24
```

3.74 $\int \frac{x^6}{1+x^4+x^8} \, dx$

Optimal result 568
Mathematica [A] (verified) 568
Rubi [A] (verified) 569
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Reduce [B] (verification not implemented) 574

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{x^6}{1+x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)-1/6*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{x^6}{1+x^4+x^8} \, dx = \frac{2\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 2\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + \log\left(-1+\sqrt{3}x-x^2\right) - \log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

input
```
Integrate[x^6/(1 + x^4 + x^8),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[-1 + Sqr
t[3]*x - x^2] - Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1708, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^8+x^4+1}dx$$

$$\Big\downarrow \text{1708}$$

$$\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[x^6/(1 + x^4 + x^8),x]
```

output
```
(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3])/2
 + (Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(2*Sqr
t[3]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1708
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Simp[1/(2*c*r)   Int[x^
(m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Simp[1/(2*c
*r)   Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]]
 /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2,
 0] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{12}+\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{12}$	67
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{3}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^{3}}{3}+\frac{2\sqrt{3}\,x}{3}\right)}{6}+\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{12}$	68

input
```
int(x^6/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*arctan(1/3*(2*x+1)*3^(1/2))*3^
(1/2)-1/12*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/12*3^(1/2)*ln(x^2-3^(1/2)*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x^3+2\,x)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{x^4+5\,x^2-2\,\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)$$

input
```
integrate(x^6/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^
2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}$$

input
```
integrate(x**6/(x**8+x**4+1),x)
```

output
```
sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12
+ sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)
/12
```

Maxima [F]

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \int \frac{x^6}{x^8+x^4+1}\,dx$$

input
```
integrate(x^6/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) + \frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)$$

input
```
integrate(x^6/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2
 - sqrt(3)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int \frac{x^6}{1+x^4+x^8}\,dx = -\frac{\sqrt{3}\left(\mathrm{atan}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3}-\frac{2}{3}\right)}\right) + \mathrm{atanh}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)\right)}{6}$$

input
```
int(x^6/(x^4 + x^8 + 1),x)
```

output
```
-(3^(1/2)*(atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))) + atanh((2*3^(1/2)*x)
/(3*((2*x^2)/3 + 2/3)))))/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{x^6}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\left(2atan\left(\frac{2x-1}{\sqrt{3}}\right)+2atan\left(\frac{2x+1}{\sqrt{3}}\right)+\log\left(-\sqrt{3}\,x+x^2+1\right)-\log\left(\sqrt{3}\,x+x^2+1\right)\right)}{12}$$

input
```
int(x^6/(x^8+x^4+1),x)
```

output
```
(sqrt(3)*(2*atan((2*x - 1)/sqrt(3)) + 2*atan((2*x + 1)/sqrt(3)) + log( - s
qrt(3)*x + x**2 + 1) - log(sqrt(3)*x + x**2 + 1)))/12
```

3.75 $\int \frac{x^4}{1+x^4+x^8}\,dx$

Optimal result 575
Mathematica [C] (verified) 576
Rubi [A] (verified) 576
Maple [C] (verified) 579
Fricas [A] (verification not implemented) 580
Sympy [C] (verification not implemented) 581
Maxima [F] 581
Giac [A] (verification not implemented) 582
Mupad [B] (verification not implemented) 582
Reduce [B] (verification not implemented) 583

Optimal result

Integrand size = 14, antiderivative size = 109

$$\int \frac{x^4}{1+x^4+x^8}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\arctan\left(\sqrt{3}-2x\right) - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\arctan\left(\sqrt{3}+2x\right) + \frac{1}{4}\operatorname{arctanh}\left(\frac{x}{1+x^2}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{4\sqrt{3}}$$

output
```
1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/4*arctan(-3^(1/2)+2*x)-1/12*arc
tan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*arctan(3^(1/2)+2*x)+1/4*arctanh(x/(x^
2+1))-1/12*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.24

$$\int \frac{x^4}{1+x^4+x^8}\,dx = \frac{1}{24}\left(-2i\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)\right.$$
$$+2i\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)$$
$$-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)$$
$$\left.-3\log\left(1-x+x^2\right)+3\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[x^4/(1 + x^4 + x^8),x]
```

output
```
((-2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] + (2*I)*Sqr
t[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*ArcTan[(-1
 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 - x + x^2
] + 3*Log[1 + x + x^2])/24
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.47, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1709, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^8+x^4+1}\,dx$$
$$\Big\downarrow \text{1709}$$
$$\frac{1}{2}\int \frac{x^2}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{x^2}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{x^2+1}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)+ \frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{3}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)+ \frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{-(2x-1)^2-3}d(2x-1)+\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)\right)+ \frac{1}{2}\left(\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)-\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)+ \frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int -\frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+ \frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+ \frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)\right)$$

input
```
Int[x^4/(1 + x^4 + x^8),x]
```

output
```
((-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3
])/2 + (-1/2*Log[1 - x + x^2] + Log[1 + x + x^2]/2)/2)/2 + ((-ArcTan[Sqrt[
3] - 2*x] + ArcTan[Sqrt[3] + 2*x])/2 + (Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3
]) - Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3]))/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(4x^2-4x+4)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln(4x^2+4x+4)}{8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}-\right.}{4}$
default	$-\frac{\ln(x^2-x+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\sqrt{3}\left(\frac{\ln\left(x^2+\sqrt{3}\,x+1\right)}{2}-\sqrt{3}\,\arctan\left(\sqrt{3}\right.\right.}{12}$

input
```
int(x^4/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/8*ln(4*x^2-4*x+4)-1/12*arctan(
1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/8*ln(4*x^2+4*x+4)+1/4*sum(_R*ln(6*_R^3+_R+x
),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{x^4}{1+x^4+x^8}\,dx =& -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)\\&-\frac{1}{24}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)\\&+\frac{1}{4}\,\arctan\left(2\,x+\sqrt{3}\right)-\frac{1}{4}\,\arctan\left(-2\,x+\sqrt{3}\right)\\&+\frac{1}{8}\,\log\left(x^2+x+1\right)-\frac{1}{8}\,\log\left(x^2-x+1\right)\end{aligned}$$

input
```
integrate(x^4/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(-2*x + sqrt(
3)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.81

$$\int \frac{x^4}{1+x^4+x^8}\,dx = \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right) + \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}-18432\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right) + \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-18432t^5-4t+x\right)\right)\right)$$

input
```
integrate(x**4/(x**8+x**4+1),x)
```

output
```
(1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24
)**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 -
 sqrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-
1/8 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8
+ sqrt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lam
bda(_t, _t*log(-18432*_t**5 - 4*_t + x)))
```

Maxima [F]

$$\int \frac{x^4}{1+x^4+x^8}\,dx = \int \frac{x^4}{x^8+x^4+1}\,dx$$

input
```
integrate(x^4/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x +
 1) - 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \frac{x^4}{1+x^4+x^8}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) + \frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \frac{1}{4}\arctan\left(2x+\sqrt{3}\right) + \frac{1}{4}\arctan\left(2x-\sqrt{3}\right) + \frac{1}{8}\log\left(x^2+x+1\right) - \frac{1}{8}\log\left(x^2-x+1\right)$$

input
```
integrate(x^4/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3
)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 18.92 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{1+x^4+x^8}\,dx = -\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int(x^4/(x^4 + x^8 + 1),x)
```

output
```
- atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) - atan((2*x)/(3^(1/
2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/
2)/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 + 1i/4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{1+x^4+x^8}\,dx = -\frac{atan(\sqrt{3}-2x)}{4} + \frac{atan(\sqrt{3}+2x)}{4} - \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{12} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{12} + \frac{\sqrt{3}\log(-\sqrt{3}\,x+x^2+1)}{24} - \frac{\sqrt{3}\log(\sqrt{3}\,x+x^2+1)}{24} - \frac{\log(x^2-x+1)}{8} + \frac{\log(x^2+x+1)}{8}$$

input
```
int(x^4/(x^8+x^4+1),x)
```

output
```
( - 6*atan(sqrt(3) - 2*x) + 6*atan(sqrt(3) + 2*x) - 2*sqrt(3)*atan((2*x -
1)/sqrt(3)) - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) + sqrt(3)*log( - sqrt(3)*x
 + x**2 + 1) - sqrt(3)*log(sqrt(3)*x + x**2 + 1) - 3*log(x**2 - x + 1) + 3
*log(x**2 + x + 1))/24
```

3.76 $\int \frac{x^2}{1+x^4+x^8}\,dx$

Optimal result 584
Mathematica [C] (verified) 585
Rubi [A] (verified) 585
Maple [C] (verified) 588
Fricas [A] (verification not implemented) 589
Sympy [C] (verification not implemented) 589
Maxima [F] 591
Giac [A] (verification not implemented) 591
Mupad [B] (verification not implemented) 592
Reduce [B] (verification not implemented) 592

Optimal result

Integrand size = 14, antiderivative size = 109

$$\int \frac{x^2}{1+x^4+x^8}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\arctan\left(\sqrt{3}-2x\right) - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\arctan\left(\sqrt{3}+2x\right) - \frac{1}{4}\operatorname{arctanh}\left(\frac{x}{1+x^2}\right) + \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{4\sqrt{3}}$$

output
```
1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/4*arctan(-3^(1/2)+2*x)-1/12*arc
tan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*arctan(3^(1/2)+2*x)-1/4*arctanh(x/(x^
2+1))+1/12*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.24

$$\int \frac{x^2}{1+x^4+x^8}\,dx = \frac{1}{48}\left(4i\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right) - 4i\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right) - 4\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 4\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + 6\log\left(1-x+x^2\right) - 6\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[x^2/(1 + x^4 + x^8),x]
```

output
```
((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (4*I)*Sqrt
[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 4*Sqrt[3]*ArcTan[(-1
+ 2*x)/Sqrt[3]] - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 6*Log[1 - x + x^2]
 - 6*Log[1 + x + x^2])/48
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.57, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1709, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^8+x^4+1}\,dx$$

$$\Big\downarrow \text{1709}$$

$$\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx - \frac{1}{2}\int \frac{1}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{ 1407}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-x}{x^2-x+1}dx-\frac{1}{2}\int \frac{x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)\right)+\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}d\right.\right.$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d(2x-\sqrt{3})}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d(2x+\sqrt{3})}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx+\sqrt{3}\arctan\left(2x+\sqrt{3}\right)}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2+x+1\right)\right)\right)+\frac{1}{2}\left(\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}+\frac{\sqrt{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)$$

input

```
Int[x^2/(1 + x^4 + x^8),x]
```

output

```
((-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2)/2 + (-(ArcTa
n[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x + x^2]/2)/2)/2 + ((-(Sqrt[3]*Arc
Tan[Sqrt[3] - 2*x]) - Log[1 - Sqrt[3]*x + x^2]/2)/(2*Sqrt[3]) + (Sqrt[3]*A
rcTan[Sqrt[3] + 2*x] + Log[1 + Sqrt[3]*x + x^2]/2)/(2*Sqrt[3]))/2
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln\left(4x^2+4x+4\right)}{8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-3\_R^3+\_R+x\right)\right)}{4}-\frac{\sqrt{3}\arctan\left(\frac{(2\right.}{12}$
default	$\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\sqrt{3}\ln\left(x^2+\sqrt{3}x+1\right)}{24}+\frac{\arctan\left(\sqrt{3}+2x\right.}{4}$

input
```
int(x^2/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/12*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/8*ln(4*x^2+4*x+4)+1/4*sum(_R*l
n(-3*_R^3+_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))-1/12*3^(1/2)*arctan(1/3*(2*x-1
)*3^(1/2))+1/8*ln(4*x^2-4*x+4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{1+x^4+x^8}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{24}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{24}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \frac{1}{4}\,\arctan\left(2\,x+\sqrt{3}\right) - \frac{1}{4}\,\arctan\left(-2\,x+\sqrt{3}\right) - \frac{1}{8}\,\log\left(x^2+x+1\right) + \frac{1}{8}\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^2/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(-2*x + sqrt(
3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.96

$$\int \frac{x^2}{1+x^4+x^8}\,dx = \left(-\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)\log\left(x+442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-192\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right) + \left(-\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)\log\left(x-192\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3+442368\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right) + \left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)\log\left(x+442368\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-192\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right) + \left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)\log\left(x-192\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3+442368\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right) + \mathrm{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(442368t^7-192t^3+x\right)\right)\right)$$

input
```
integrate(x**2/(x**8+x**4+1),x)
```

output
```
(-1/8 - sqrt(3)*I/24)*log(x + 442368*(-1/8 - sqrt(3)*I/24)**7 - 192*(-1/8
- sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 192*(-1/8 + sqrt(3)*I/
24)**3 + 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x + 4
42368*(1/8 - sqrt(3)*I/24)**7 - 192*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt
(3)*I/24)*log(x - 192*(1/8 + sqrt(3)*I/24)**3 + 442368*(1/8 + sqrt(3)*I/24
)**7) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(442368*_t**7
- 192*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^2}{1+x^4+x^8}\,dx = \int \frac{x^2}{x^8+x^4+1}\,dx$$

input
```
integrate(x^2/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x + 1
) + 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^2}{1+x^4+x^8}\,dx = &-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\\ &+\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)\\ &+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)\\ &-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)\end{aligned}$$

input
```
integrate(x^2/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3
)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{1+x^4+x^8}\,dx = \operatorname{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \operatorname{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right) - \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int(x^2/(x^4 + x^8 + 1),x)
```

output
```
atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) + atan((2*x)/(3^(1/2)
*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)
/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{1+x^4+x^8}\,dx = -\frac{atan\left(\sqrt{3}-2x\right)}{4}+\frac{atan\left(\sqrt{3}+2x\right)}{4}-\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{12} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{12}-\frac{\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)}{24} + \frac{\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)}{24}+\frac{\log(x^2-x+1)}{8}-\frac{\log(x^2+x+1)}{8}$$

input
```
int(x^2/(x^8+x^4+1),x)
```

output
```
( - 6*atan(sqrt(3) - 2*x) + 6*atan(sqrt(3) + 2*x) - 2*sqrt(3)*atan((2*x -
1)/sqrt(3)) - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) - sqrt(3)*log( - sqrt(3)*x
 + x**2 + 1) + sqrt(3)*log(sqrt(3)*x + x**2 + 1) + 3*log(x**2 - x + 1) - 3
*log(x**2 + x + 1))/24
```

3.77 $\int \frac{1}{1+x^4+x^8} \, dx$

Optimal result 594
Mathematica [A] (verified) 594
Rubi [A] (verified) 595
Maple [A] (verified) 597
Fricas [A] (verification not implemented) 598
Sympy [A] (verification not implemented) 598
Maxima [F] 599
Giac [A] (verification not implemented) 599
Mupad [B] (verification not implemented) 599
Reduce [B] (verification not implemented) 600

Optimal result

Integrand size = 10, antiderivative size = 67

$$\int \frac{1}{1+x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)+1/6*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{1}{1+x^4+x^8} \, dx = \frac{2\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-\log\left(-1+\sqrt{3}x-x^2\right)+\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

input
```
Integrate[(1 + x^4 + x^8)^(-1),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[-1 + Sqr
t[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {1684, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8+x^4+1}\,dx$$

$$\Big\downarrow \text{1684}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(-\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[(1 + x^4 + x^8)^(-1),x]
```

output
```
(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3])/2
 + (-1/2*Log[1 - Sqrt[3]*x + x^2]/Sqrt[3] + Log[1 + Sqrt[3]*x + x^2]/(2*Sq
rt[3]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1684
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x^(
n/2))/(q - r*x^(n/2) + x^n), x], x] + Simp[1/(2*c*q*r)   Int[(r + x^(n/2))/
(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{12}$	67
risch	$\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{3}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^{3}}{3}+\frac{2\sqrt{3}\,x}{3}\right)}{6}$	68

input
```
int(1/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*arctan(1/3*(2*x+1)*3^(1/2))*3^
(1/2)+1/12*3^(1/2)*ln(x^2+3^(1/2)*x+1)-1/12*3^(1/2)*ln(x^2-3^(1/2)*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{1}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x^3+2\,x)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) + \frac{1}{12}\sqrt{3}\log\left(\frac{x^4+5\,x^2+2\,\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)$$

input
```
integrate(1/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^
2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int \frac{1}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{12} - \frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}$$

input
```
integrate(1/(x**8+x**4+1),x)
```

output
```
sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12
- sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)
/12
```

Maxima [F]

$$\int \frac{1}{1+x^4+x^8}\,dx = \int \frac{1}{x^8+x^4+1}\,dx$$

input
```
integrate(1/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{1}{1+x^4+x^8}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)$$

input
```
integrate(1/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2
 - sqrt(3)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

$$\int \frac{1}{1+x^4+x^8}\,dx = -\frac{\sqrt{3}\left(\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}-\frac{2}{3}\right)}\right) - \mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)\right)}{6}$$

input
```
int(1/(x^4 + x^8 + 1),x)
```

output
```
-(3^(1/2)*(atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))) - atanh((2*3^(1/2)*x)
/(3*((2*x^2)/3 + 2/3)))))/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{1}{1+x^4+x^8}\,dx = \frac{\sqrt{3}\left(2atan\left(\frac{2x-1}{\sqrt{3}}\right)+2atan\left(\frac{2x+1}{\sqrt{3}}\right)-\log\left(-\sqrt{3}\,x+x^2+1\right)+\log\left(\sqrt{3}\,x+x^2+1\right)\right)}{12}$$

input
```
int(1/(x^8+x^4+1),x)
```

output
```
(sqrt(3)*(2*atan((2*x - 1)/sqrt(3)) + 2*atan((2*x + 1)/sqrt(3)) - log( - s
qrt(3)*x + x**2 + 1) + log(sqrt(3)*x + x**2 + 1)))/12
```

3.78 $\int \frac{1}{x^2(1+x^4+x^8)}\,dx$

Optimal result 601
Mathematica [C] (verified) 601
Rubi [A] (verified) 602
Maple [C] (verified) 606
Fricas [A] (verification not implemented) 606
Sympy [C] (verification not implemented) 607
Maxima [F] 608
Giac [A] (verification not implemented) 608
Mupad [B] (verification not implemented) 609
Reduce [B] (verification not implemented) 609

Optimal result

Integrand size = 14, antiderivative size = 114

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx = -\frac{1}{x}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\arctan\left(\sqrt{3}+2x\right)+\frac{1}{4}\text{arctanh}\left(\frac{x}{1+x^2}\right)+\frac{\text{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{4\sqrt{3}}$$

output
```
-1/x+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/4*arctan(-3^(1/2)+2*x)-1/1
2*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*arctan(3^(1/2)+2*x)+1/4*arctanh(
x/(x^2+1))+1/12*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx = \frac{1}{24}\left(-\frac{24}{x}+2i\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)\right.$$
$$-2i\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)$$
$$-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)$$
$$\left.-3\log\left(1-x+x^2\right)+3\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[1/(x^2*(1 + x^4 + x^8)),x]
```

output
```
(-24/x + (2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (2
*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*Arc
Tan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 -
x + x^2] + 3*Log[1 + x + x^2])/24
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.45, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {1704, 25, 1830, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(x^8+x^4+1)}dx$$

$$\Big\downarrow \text{ 1704}$$

$$\int -\frac{x^2(x^4+1)}{x^8+x^4+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{x^2(x^4+1)}{x^8+x^4+1}dx-\frac{1}{x}$$

$$\Big\downarrow \ 1830$$

$$-\frac{1}{2}\int \frac{x^2}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{x^2}{x^4+x^2+1}dx-\frac{1}{x}$$

$$\Big\downarrow \ 1447$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4-x^2+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx\right)-\frac{1}{x}$$

$$\Big\downarrow \ 1475$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{3}x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx\right)-\frac{1}{x}$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{-(2x-1)^2-3}d(2x-1)+\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)+\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)\right)\right)-\frac{1}{x}$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)-\frac{1}{x}$$

$$\Big\downarrow \ 1478$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int -\frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)\right)-\frac{1}{x}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx+\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)\right)-\frac{1}{x}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}\right)\right)-\frac{1}{x}$$

input
```
Int[1/(x^2*(1 + x^4 + x^8)),x]
```

output
```
-x^(-1) + ((-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[
3]]/Sqrt[3])/2 + (-1/2*Log[1 - x + x^2] + Log[1 + x + x^2]/2)/2)/2 + ((Arc
Tan[Sqrt[3] - 2*x] - ArcTan[Sqrt[3] + 2*x])/2 + (-1/2*Log[1 - Sqrt[3]*x +
x^2]/Sqrt[3] + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3]))/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1830
```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Simp[c/(2*q*r)   Int[(f*x)^m*(Simp[d*r - (c*d - e*q)*x^(n/2), x]
/(q - r*x^(n/2) + c*x^n)), x], x] + Simp[c/(2*q*r)   Int[(f*x)^m*(Simp[d*r
+ (c*d - e*q)*x^(n/2), x]/(q + r*x^(n/2) + c*x^n)), x], x]] /;  !LtQ[2*c*q
- b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a
*c, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-6\_R^3-\_R+x\right)\right)}{4}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(4x^2-4x+4)}{8}-\frac{\arctan$
default	$-\frac{\ln(x^2-x+1)}{8}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\sqrt{3}\left(\frac{\ln\left(x^2+\sqrt{3}\,x+1\right)}{2}-\sqrt{3}\arctan\left(\sqrt{3}\right.\right.}{12}$

input
```
int(1/x^2/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/4*sum(_R*ln(-6*_R^3-_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))-1/12*3^(1/2)*
arctan(1/3*(2*x-1)*3^(1/2))-1/8*ln(4*x^2-4*x+4)-1/12*arctan(1/3*(2*x+1)*3^
(1/2))*3^(1/2)+1/8*ln(4*x^2+4*x+4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\left(1+x^4+x^8\right)}\,dx =$$
$$-\frac{2\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+2\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\sqrt{3}x\log\left(x^2+\sqrt{3}x+1\right)+\sqrt{3}x\log\left(\right.}{}$$

input
```
integrate(1/x^2/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/24*(2*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*sqrt(3)*x*arctan(1/3*
sqrt(3)*(2*x - 1)) - sqrt(3)*x*log(x^2 + sqrt(3)*x + 1) + sqrt(3)*x*log(x^
2 - sqrt(3)*x + 1) + 6*x*arctan(2*x + sqrt(3)) - 6*x*arctan(-2*x + sqrt(3)
) - 3*x*log(x^2 + x + 1) + 3*x*log(x^2 - x + 1) + 24)/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.91

$$\begin{aligned}
&\int \frac{1}{x^2(1+x^4+x^8)}\,dx \\
&= \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-384\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right) \\
&\quad+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-384\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3-442368\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right) \\
&\quad+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-442368\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-384\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right) \\
&\quad+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-384\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3-442368\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right) \\
&\quad+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-442368t^7-384t^3+x\right)\right)\right)-\frac{1}{x}
\end{aligned}$$

input
```
integrate(1/x**2/(x**8+x**4+1),x)
```

output
```
(-1/8 - sqrt(3)*I/24)*log(x - 442368*(-1/8 - sqrt(3)*I/24)**7 - 384*(-1/8
- sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 384*(-1/8 + sqrt(3)*I/
24)**3 - 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x - 4
42368*(1/8 - sqrt(3)*I/24)**7 - 384*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt
(3)*I/24)*log(x - 384*(1/8 + sqrt(3)*I/24)**3 - 442368*(1/8 + sqrt(3)*I/24
)**7) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-442368*_t**7
 - 384*_t**3 + x))) - 1/x
```

Maxima [F]

$$\int \frac{1}{x^2 (1 + x^4 + x^8)}\, dx = \int \frac{1}{(x^8 + x^4 + 1)x^2}\, dx$$

input
```
integrate(1/x^2/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/x - 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2
 + x + 1) - 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{1}{x^2 (1 + x^4 + x^8)}\, dx = &-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\\&-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)\\&-\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{1}{x}-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)\\&-\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)\end{aligned}$$

input
```
integrate(1/x^2/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) - 1/x - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x -
sqrt(3)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx = \operatorname{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \operatorname{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right) - \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right) - \frac{1}{x}$$

input
```
int(1/(x^2*(x^4 + x^8 + 1)),x)
```

output
```
atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x)/(3^(1/2)
*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)
/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 + 1i/4) - 1/x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2(1+x^4+x^8)}\,dx = \frac{6atan(\sqrt{3}-2x)\,x-6atan(\sqrt{3}+2x)\,x-2\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x-2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x-\sqrt{3}\log(-\sqrt{3}\,x+}{24x}$$

input
```
int(1/x^2/(x^8+x^4+1),x)
```

output
```
(6*atan(sqrt(3) - 2*x)*x - 6*atan(sqrt(3) + 2*x)*x - 2*sqrt(3)*atan((2*x -
 1)/sqrt(3))*x - 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*x - sqrt(3)*log( - sqrt
(3)*x + x**2 + 1)*x + sqrt(3)*log(sqrt(3)*x + x**2 + 1)*x - 3*log(x**2 - x
 + 1)*x + 3*log(x**2 + x + 1)*x - 24)/(24*x)
```

3.79 $\int \frac{1}{x^4(1+x^4+x^8)}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 116

$$\int \frac{1}{x^4(1+x^4+x^8)}\,dx = -\frac{1}{3x^3}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} -\frac{1}{4}\arctan\left(\sqrt{3}+2x\right)-\frac{1}{4}\operatorname{arctanh}\left(\frac{x}{1+x^2}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{4\sqrt{3}}$$

output
```
-1/3/x^3+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/4*arctan(-3^(1/2)+2*x)
-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*arctan(3^(1/2)+2*x)-1/4*arct
anh(x/(x^2+1))-1/12*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^4(1+x^4+x^8)}\,dx = \frac{1}{24}\left(-\frac{8}{x^3} - \frac{4i\arctan\left(\frac{1}{2}(1-i\sqrt{3})\,x\right)}{\sqrt{\frac{1}{6}i\,(i+\sqrt{3})}} + \frac{4i\arctan\left(\frac{1}{2}(1+i\sqrt{3})\,x\right)}{\sqrt{-\frac{1}{6}i\,(-i+\sqrt{3})}} - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + 3\log\left(1-x+x^2\right) - 3\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[1/(x^4*(1 + x^4 + x^8)),x]
```

output
```
(-8/x^3 - ((4*I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[(I/6)*(I + Sqrt[3])]
+ ((4*I)*ArcTan[((1 + I*Sqrt[3])*x)/2])/Sqrt[(-1/6*I)*(-I + Sqrt[3])] - 2*
Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] +
 3*Log[1 - x + x^2] - 3*Log[1 + x + x^2])/24
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.53, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {1704, 27, 1749, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(x^8+x^4+1)}\,dx$$

$$\Big\downarrow \; 1704$$

$$\frac{1}{3}\int -\frac{3(x^4+1)}{x^8+x^4+1}dx - \frac{1}{3x^3}$$

$$\Big\downarrow \; 27$$

$$-\int \frac{x^4+1}{x^8+x^4+1}dx-\frac{1}{3x^3}$$

$\big\downarrow$ 1749

$$-\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx-\frac{1}{2}\int\frac{1}{x^4+x^2+1}dx-\frac{1}{3x^3}$$

$\big\downarrow$ 1407

$$\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-x}{x^2-x+1}dx-\frac{1}{2}\int\frac{x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\int\frac{\sqrt{3}-x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int\frac{x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)-\frac{1}{3x^3}$$

$\big\downarrow$ 1142

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int\frac{1}{x^2-x+1}dx\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{x^2+x+1}dx-\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx\right)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int-\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)-\frac{1}{3x^3}$$

$\big\downarrow$ 25

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{x^2+x+1}dx-\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx\right)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)-\frac{1}{3x^3}$$

$\big\downarrow$ 1083

$$\frac{1}{2}\left(\frac{1}{2}\left(\int\frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\int\frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx\right)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\int\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\int\frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d(2x-\sqrt{3})}{2\sqrt{3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\sqrt{3}\int\frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d(2x+\sqrt{3})}{2\sqrt{3}}\right)-\frac{1}{3x^3}$$

$\big\downarrow$ 217

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx+\sqrt{3}\arctan\left(2x+\sqrt{3}\right)}{2\sqrt{3}}\right)-\frac{1}{3x^3}$$

$\Big\downarrow$ 1103

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2+x+1\right)\right)\right)+\frac{1}{2}\left(-\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\sqrt{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)-\frac{1}{3x^3}$$

input
```
Int[1/(x^4*(1 + x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 + ((-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2)
/2 + (-(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x + x^2]/2)/2)/2 + (-
1/2*(-(Sqrt[3]*ArcTan[Sqrt[3] - 2*x]) - Log[1 - Sqrt[3]*x + x^2]/2)/Sqrt[3
] - (Sqrt[3]*ArcTan[Sqrt[3] + 2*x] + Log[1 + Sqrt[3]*x + x^2]/2)/(2*Sqrt[3
]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1749
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e
+ q*x^(n/2) + x^n, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x^(n/2) +
 x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c,
0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(3\_R^3-\_R+x\right)\right)}{4}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln\left(4x^2+4x+4\right)}{8}-\frac{\sqrt{3}\,\mathrm{arc}}{}$
default	$-\frac{1}{3x^3}+\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\sqrt{3}\ln\left(x^2+\sqrt{3}\,x+1\right)}{24}-\frac{\mathrm{arct}}{}$

input
```
int(1/x^4/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+1/4*sum(_R*ln(3*_R^3-_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))-1/12*arcta
n(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/8*ln(4*x^2+4*x+4)-1/12*3^(1/2)*arctan(1/3
*(2*x-1)*3^(1/2))+1/8*ln(4*x^2-4*x+4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^4\left(1+x^4+x^8\right)}\,dx =$$
$$-\frac{2\sqrt{3}x^3\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+2\sqrt{3}x^3\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\sqrt{3}x^3\log\left(x^2+\sqrt{3}x+1\right)-\sqrt{3}x^3}{}$$

input
```
integrate(1/x^4/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/24*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*sqrt(3)*x^3*arctan(
1/3*sqrt(3)*(2*x - 1)) + sqrt(3)*x^3*log(x^2 + sqrt(3)*x + 1) - sqrt(3)*x^
3*log(x^2 - sqrt(3)*x + 1) + 6*x^3*arctan(2*x + sqrt(3)) - 6*x^3*arctan(-2
*x + sqrt(3)) + 3*x^3*log(x^2 + x + 1) - 3*x^3*log(x^2 - x + 1) + 8)/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^4(1+x^4+x^8)}\,dx = \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) + \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}-9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right) + \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right) + \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-9216t^5-8t+x\right)\right)\right)-\frac{1}{3x^3}$$

input

```
integrate(1/x**4/(x**8+x**4+1),x)
```

output

```
(1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/24)**
5) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + sqrt(
3)*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 + sqr
t(3)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/
24)**5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*l
og(-9216*_t**5 - 8*_t + x))) - 1/(3*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (1 + x^4 + x^8)} \, dx = \int \frac{1}{(x^8 + x^4 + 1)x^4} \, dx$$

input
```
integrate(1/x^4/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/3/x^3 - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x
^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{1}{x^4 (1 + x^4 + x^8)} \, dx = & -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) \\ & - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) \\ & + \frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) - \frac{1}{3x^3} - \frac{1}{4}\arctan\left(2x+\sqrt{3}\right) \\ & - \frac{1}{4}\arctan\left(2x-\sqrt{3}\right) - \frac{1}{8}\log\left(x^2+x+1\right) + \frac{1}{8}\log\left(x^2-x+1\right)\end{aligned}$$

input
```
integrate(1/x^4/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) - 1/3/x^3 - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*
x - sqrt(3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4 (1 + x^4 + x^8)} \, dx = -\operatorname{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \operatorname{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right) - \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right) - \frac{1}{3\,x^3}$$

input
```
int(1/(x^4*(x^4 + x^8 + 1)),x)
```

output
```
- atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) - atan((2*x)/(3^(1/
2)*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/
2)/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4) - 1/(3*x
^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (1 + x^4 + x^8)} \, dx = \frac{6atan(\sqrt{3}-2x)\,x^3 - 6atan(\sqrt{3}+2x)\,x^3 - 2\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^3 - 2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^3 + \sqrt{3}\log(-\sqrt{3}\,x}{24x^3}$$

input
```
int(1/x^4/(x^8+x^4+1),x)
```

output
```
(6*atan(sqrt(3) - 2*x)*x**3 - 6*atan(sqrt(3) + 2*x)*x**3 - 2*sqrt(3)*atan(
(2*x - 1)/sqrt(3))*x**3 - 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*x**3 + sqrt(3)
*log( - sqrt(3)*x + x**2 + 1)*x**3 - sqrt(3)*log(sqrt(3)*x + x**2 + 1)*x**
3 + 3*log(x**2 - x + 1)*x**3 - 3*log(x**2 + x + 1)*x**3 - 8)/(24*x**3)
```

3.80 $\int \frac{1}{x^6(1+x^4+x^8)}\,dx$

Optimal result 620
Mathematica [A] (verified) 620
Rubi [A] (verified) 621
Maple [A] (verified) 624
Fricas [A] (verification not implemented) 625
Sympy [A] (verification not implemented) 625
Maxima [F] 626
Giac [A] (verification not implemented) 626
Mupad [B] (verification not implemented) 627
Reduce [B] (verification not implemented) 627

Optimal result

Integrand size = 14, antiderivative size = 77

$$\int \frac{1}{x^6(1+x^4+x^8)}\,dx = -\frac{1}{5x^5}+\frac{1}{x}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{2\sqrt{3}}$$

output
```
-1/5/x^5+1/x-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x
)*3^(1/2))*3^(1/2)-1/6*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^6(1+x^4+x^8)}\,dx = \frac{1}{60}\left(-\frac{12}{x^5}+\frac{60}{x}+10\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+10\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+5\sqrt{3}\log\left(-1+\sqrt{3}x-x^2\right)-5\sqrt{3}\log\left(1+\sqrt{3}x+x^2\right)\right)$$

input
```
Integrate[1/(x^6*(1 + x^4 + x^8)),x]
```

output
```
(-12/x^5 + 60/x + 10*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Sqrt[3]*ArcTa
n[(1 + 2*x)/Sqrt[3]] + 5*Sqrt[3]*Log[-1 + Sqrt[3]*x - x^2] - 5*Sqrt[3]*Log
[1 + Sqrt[3]*x + x^2])/60
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {1704, 27, 1828, 1708, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (x^8+x^4+1)} \, dx$$

$$\Big\downarrow \text{ 1704}$$

$$\frac{1}{5} \int -\frac{5 (x^4+1)}{x^2 (x^8+x^4+1)}dx-\frac{1}{5 x^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{x^4+1}{x^2 (x^8+x^4+1)}dx-\frac{1}{5 x^5}$$

$$\Big\downarrow \text{ 1828}$$

$$\int \frac{x^6}{x^8+x^4+1}dx-\frac{1}{5 x^5}+\frac{1}{x}$$

$$\Big\downarrow \text{ 1708}$$

$$-\frac{1}{2} \int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2} \int \frac{x^2+1}{x^4+x^2+1}dx-\frac{1}{5 x^5}+\frac{1}{x}$$

$$\Big\downarrow \text{ 1475}$$

$$\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{x^2-x+1}dx+\frac{1}{2} \int \frac{1}{x^2+x+1}dx\right)-\frac{1}{2} \int \frac{1-x^2}{x^4-x^2+1}dx-\frac{1}{5 x^5}+\frac{1}{x}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(-\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)-\frac{1}{5x^5}+\frac{1}{x}$$

$$\Big\downarrow \; 217$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{5x^5}+\frac{1}{x}$$

$$\Big\downarrow \; 1478$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{5x^5}+\frac{1}{x}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{5x^5}+\frac{1}{x}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{5x^5}+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)+\frac{1}{x}$$

input
```
Int[1/(x^6*(1 + x^4 + x^8)),x]
```

output
```
-1/5*1/x^5 + x^(-1) + (ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*
x)/Sqrt[3]]/Sqrt[3])/2 + (Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) - Log[1 + S
qrt[3]*x + x^2]/(2*Sqrt[3]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1708
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Simp[1/(2*c*r)   Int[x^
(m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Simp[1/(2*c
*r)   Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]
 /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2,
 0] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{12}+\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{1}{5x^5}+\frac{1}{x}$	75
risch	$\frac{x^4-\frac{1}{5}}{x^5}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{3}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\sqrt{3}\,x}{3}\right)}{6}+\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{12}$	77

input
```
int(1/x^6/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*arctan(1/3*(2*x+1)*3^(1/2))*3^
(1/2)-1/12*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/12*3^(1/2)*ln(x^2-3^(1/2)*x+1)-1/
5/x^5+1/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^6 (1+x^4+x^8)} \, dx = \frac{10 \sqrt{3} x^5 \arctan\left(\frac{1}{3} \sqrt{3} (x^3+2x)\right) + 10 \sqrt{3} x^5 \arctan\left(\frac{1}{3} \sqrt{3} x\right) + 5 \sqrt{3} x^5 \log\left(\frac{x^4+5x^2-2\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right) + 60 x^4}{60 x^5}$$

input
```
integrate(1/x^6/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
1/60*(10*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 10*sqrt(3)*x^5*arct
an(1/3*sqrt(3)*x) + 5*sqrt(3)*x^5*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) +
 1)/(x^4 - x^2 + 1)) + 60*x^4 - 12)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^6 (1+x^4+x^8)} \, dx = \frac{\sqrt{3} \cdot \left(2 \operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{2\sqrt{3}x}{3}\right)\right)}{12} + \frac{\sqrt{3} \log\left(x^2 - \sqrt{3}x + 1\right)}{12} - \frac{\sqrt{3} \log\left(x^2 + \sqrt{3}x + 1\right)}{12} + \frac{5x^4 - 1}{5x^5}$$

input
```
integrate(1/x**6/(x**8+x**4+1),x)
```

output
```
sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12
+ sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)
/12 + (5*x**4 - 1)/(5*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6\left(1+x^4+x^8\right)}\,dx = \int \frac{1}{(x^8+x^4+1)x^6}\,dx$$

input
```
integrate(1/x^6/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/5*(5*x^4 - 1)/x^5 + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1
), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\begin{aligned}\int \frac{1}{x^6\left(1+x^4+x^8\right)}\,dx &= \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\\ &\quad-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)\\ &\quad+\frac{5x^4-1}{5x^5}+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)\end{aligned}$$

input
```
integrate(1/x^6/(x^8+x^4+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^2
 - sqrt(3)*x + 1) + 1/5*(5*x^4 - 1)/x^5 + 1/4*arctan(2*x + sqrt(3)) + 1/4*
arctan(2*x - sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^6 \left(1+x^4+x^8\right)}\, dx = \frac{x^4-\frac{1}{5}}{x^5} - \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2\,x^2}{3}+\frac{2}{3}\right)}\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2\,x^2}{3}-\frac{2}{3}\right)}\right)}{6}$$

input
```
int(1/(x^6*(x^4 + x^8 + 1)),x)
```

output
```
(x^4 - 1/5)/x^5 - (3^(1/2)*atanh((2*3^(1/2)*x)/(3*((2*x^2)/3 + 2/3))))/6 -
 (3^(1/2)*atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))))/6
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^6 \left(1+x^4+x^8\right)}\, dx$$
$$= \frac{10\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^5 + 10\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^5 + 5\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)x^5 - 5\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)}{60x^5}$$

input
```
int(1/x^6/(x^8+x^4+1),x)
```

output
```
(10*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**5 + 10*sqrt(3)*atan((2*x + 1)/sqrt(
3))*x**5 + 5*sqrt(3)*log( - sqrt(3)*x + x**2 + 1)*x**5 - 5*sqrt(3)*log(sqr
t(3)*x + x**2 + 1)*x**5 + 60*x**4 - 12)/(60*x**5)
```

3.81 $\int \frac{1}{x^8(1+x^4+x^8)}\,dx$

Optimal result 628
Mathematica [C] (verified) 629
Rubi [A] (verified) 629
Maple [C] (verified) 634
Fricas [A] (verification not implemented) 634
Sympy [C] (verification not implemented) 635
Maxima [F] 636
Giac [A] (verification not implemented) 636
Mupad [B] (verification not implemented) 637
Reduce [B] (verification not implemented) 637

Optimal result

Integrand size = 14, antiderivative size = 123

$$\int \frac{1}{x^8(1+x^4+x^8)}\,dx = -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\arctan\left(\sqrt{3}-2x\right)-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\arctan\left(\sqrt{3}+2x\right)+\frac{1}{4}\operatorname{arctanh}\left(\frac{x}{1+x^2}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{4\sqrt{3}}$$

output
```
-1/7/x^7+1/3/x^3+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/4*arctan(-3^(1
/2)+2*x)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*arctan(3^(1/2)+2*x)+
1/4*arctanh(x/(x^2+1))-1/12*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^8\left(1+x^4+x^8\right)}\,dx = -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{2\sqrt{-6+6i\sqrt{3}}} + \frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{2\sqrt{-6-6i\sqrt{3}}} - \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)$$

input
```
Integrate[1/(x^8*(1 + x^4 + x^8)),x]
```

output
```
-1/7*1/x^7 + 1/(3*x^3) + ((I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2])/(2*
Sqrt[-6 + (6*I)*Sqrt[3]]) + ((-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])
/(2*Sqrt[-6 - (6*I)*Sqrt[3]]) - ArcTan[(-1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - A
rcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x + x^2]/8 + Log[1 + x + x^
2]/8
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.41, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {1704, 27, 1828, 27, 1709, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\left(x^8+x^4+1\right)}dx$$

$$\Big\downarrow \ 1704$$

$$\frac{1}{7}\int -\frac{7\left(x^4+1\right)}{x^4\left(x^8+x^4+1\right)}dx-\frac{1}{7x^7}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^4+1}{x^4 (x^8+x^4+1)}dx-\frac{1}{7x^7}$$

$$\Big\downarrow \text{1828}$$

$$\frac{1}{3}\int \frac{3x^4}{x^8+x^4+1}dx-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^4}{x^8+x^4+1}dx-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{1709}$$

$$\frac{1}{2}\int \frac{x^2}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{x^2}{x^4+x^2+1}dx-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{x^2+1}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)+ \frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx\right)-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{3}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)+ \frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)+\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx\right)-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{-(2x-1)^2-3}d(2x-1)+\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)\right)+ \frac{1}{2}\left(\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)-\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)- \frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx\right)+ \frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int -\frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)\right)-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)\right)-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)\right)-\frac{1}{7x^7}+\frac{1}{3x^3}$$

input
```
Int[1/(x^8*(1 + x^4 + x^8)),x]
```

output
```
-1/7*1/x^7 + 1/(3*x^3) + ((-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) - ArcTan[
(1 + 2*x)/Sqrt[3]]/Sqrt[3])/2 + (-1/2*Log[1 - x + x^2] + Log[1 + x + x^2]/
2)/2)/2 + ((-ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x])/2 + (Log[1 - S
qrt[3]*x + x^2]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3]))/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

method	result
risch	$\frac{\frac{x^4}{3}-\frac{1}{7}}{x^7}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(4x^2-4x+4\right)}{8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(6\_R^3+\_R+x\right)\right)}{4}-\frac{\sqrt{3}\ \mathrm{ar}}{}$
default	$-\frac{\ln\left(x^2-x+1\right)}{8}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\sqrt{3}\left(\frac{\ln\left(x^2+\sqrt{3}\,x+1\right)}{2}-\sqrt{3}\arctan\left(\sqrt{3}\right.\right.}{12}$

input
```
int(1/x^8/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(1/3*x^4-1/7)/x^7-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/8*ln(4*x^2-4*
x+4)+1/4*sum(_R*ln(6*_R^3+_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))-1/12*3^(1/2)*a
rctan(2/3*3^(1/2)*(x+1/2))+1/8*ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^8\left(1+x^4+x^8\right)}\,dx = -\frac{14\sqrt{3}x^7\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+14\sqrt{3}x^7\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+7\sqrt{3}x^7\log\left(x^2+\sqrt{3}x+1\right)-7}{}$$

input
```
integrate(1/x^8/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
-1/168*(14*sqrt(3)*x^7*arctan(1/3*sqrt(3)*(2*x + 1)) + 14*sqrt(3)*x^7*arct
an(1/3*sqrt(3)*(2*x - 1)) + 7*sqrt(3)*x^7*log(x^2 + sqrt(3)*x + 1) - 7*sqr
t(3)*x^7*log(x^2 - sqrt(3)*x + 1) - 42*x^7*arctan(2*x + sqrt(3)) + 42*x^7*
arctan(-2*x + sqrt(3)) - 21*x^7*log(x^2 + x + 1) + 21*x^7*log(x^2 - x + 1)
 - 56*x^4 + 24)/x^7
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^8(1+x^4+x^8)}\,dx = \left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right) + \left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right) + \left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}-18432\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right) + \operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-18432t^5-4t+x\right)\right)\right)+\frac{7x^4-3}{21x^7}$$

input
```
integrate(1/x**8/(x**8+x**4+1),x)
```

output
```
(1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24
)**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 -
 sqrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-
1/8 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8
+ sqrt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lam
bda(_t, _t*log(-18432*_t**5 - 4*_t + x))) + (7*x**4 - 3)/(21*x**7)
```

Maxima [F]

$$\int \frac{1}{x^8\left(1+x^4+x^8\right)}\,dx = \int \frac{1}{(x^8+x^4+1)x^8}\,dx$$

input
```
integrate(1/x^8/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) + 1/21*(7*x^4 - 3)/x^7 + 1/2*integrate(x^2/(x^4 - x^2 + 1),
 x) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{1}{x^8\left(1+x^4+x^8\right)}\,dx &= -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) \\ &\quad -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) \\ &\quad +\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{7x^4-3}{21x^7}+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right) \\ &\quad +\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)\end{aligned}$$

input
```
integrate(1/x^8/(x^8+x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(
x^2 - sqrt(3)*x + 1) + 1/21*(7*x^4 - 3)/x^7 + 1/4*arctan(2*x + sqrt(3)) +
1/4*arctan(2*x - sqrt(3)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 18.96 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^8\,(1+x^4+x^8)}\,dx = \frac{\frac{x^4}{3}-\frac{1}{7}}{x^7} - \operatorname{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right) - \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right) - \operatorname{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

input
```
int(1/(x^8*(x^4 + x^8 + 1)),x)
```

output
```
(x^4/3 - 1/7)/x^7 - atan((2*x)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) -
 atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/12 - 1i/4) - atan((x*2i)/(3^(1/2)*
1i + 1))*(3^(1/2)/12 + 1i/4) - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/
12 + 1/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^8\,(1+x^4+x^8)}\,dx = \frac{-42atan\left(\sqrt{3}-2x\right)x^7+42atan\left(\sqrt{3}+2x\right)x^7-14\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^7-14\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^7+7\sqrt{3}\log}{}$$

input
```
int(1/x^8/(x^8+x^4+1),x)
```

output
```
( - 42*atan(sqrt(3) - 2*x)*x**7 + 42*atan(sqrt(3) + 2*x)*x**7 - 14*sqrt(3)
*atan((2*x - 1)/sqrt(3))*x**7 - 14*sqrt(3)*atan((2*x + 1)/sqrt(3))*x**7 +
7*sqrt(3)*log( - sqrt(3)*x + x**2 + 1)*x**7 - 7*sqrt(3)*log(sqrt(3)*x + x*
*2 + 1)*x**7 - 21*log(x**2 - x + 1)*x**7 + 21*log(x**2 + x + 1)*x**7 + 56*
x**4 - 24)/(168*x**7)
```

3.82 $\int \frac{x^{11}}{1-x^4+x^8} \, dx$

Optimal result 639
Mathematica [A] (verified) 639
Rubi [A] (verified) 640
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Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{x^{11}}{1-x^4+x^8} \, dx = \frac{x^4}{4}+\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)$$

output
```
1/4*x^4+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)+1/8*ln(x^8-x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{1-x^4+x^8} \, dx = \frac{x^4}{4}-\frac{\arctan\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)$$

input
```
Integrate[x^11/(1 - x^4 + x^8),x]
```

output
```
x^4/4 - ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^8}{x^8-x^4+1}dx^4$$

$$\Big\downarrow \text{1143}$$

$$\frac{1}{4}\int \left(1-\frac{1-x^4}{x^8-x^4+1}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{\sqrt{3}}+x^4+\frac{1}{2}\log\left(x^8-x^4+1\right)\right)$$

input
```
Int[x^11/(1 - x^4 + x^8),x]
```

output
```
(x^4 + ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 - x^4 + x^8]/2)/4
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^4}{4}+\frac{\ln(x^8-x^4+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	38
risch	$\frac{x^4}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(4x^8-4x^4+4)}{8}$	40

input
```
int(x^11/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{1}{4}\,x^4-\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^11/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{x^4}{4} + \frac{\log\left(x^8-x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**11/(x**8-x**4+1),x)
```

output
```
x**4/4 + log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/
3)/12
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) + \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^11/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) + \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^11/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
 + 1)
```

Mupad [B] (verification not implemented)

Time = 19.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx=\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}+\frac{x^4}{4}$$

input
```
int(x^11/(x^8 - x^4 + 1),x)
```

output
```
log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12 +
x^4/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 333, normalized size of antiderivative = 7.24

$$\int \frac{x^{11}}{1-x^4+x^8}\,dx = \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} + \frac{\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} + \frac{\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{8} + \frac{\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{8} + \frac{x^4}{4}$$

input
```
int(x^11/(x^8-x^4+1),x)
```

output

```
(sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( - sq
rt(3) + 2))) + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(2) - 4*
x)/(2*sqrt( - sqrt(3) + 2))) + sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6)
+ sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + 3*sqrt( - sqrt(3) + 2)*sqrt(2
)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - sqrt( - sqrt(
3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) -
 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(
6) + sqrt(2))) - sqrt( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2)
 + 4*x)/(sqrt(6) + sqrt(2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt
( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) + 3*log( - sqrt( - sqrt(3) +
2)*x + x**2 + 1) + 3*log(sqrt( - sqrt(3) + 2)*x + x**2 + 1) + 3*log(( - sq
rt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) + 3*log((sqrt(6)*x + sqrt(2)*x + 2*x*
*2 + 2)/2) + 6*x**4)/24
```

3.83 $\int \frac{x^9}{1-x^4+x^8}\,dx$

Optimal result 646
Mathematica [A] (verified) 646
Rubi [A] (verified) 647
Maple [A] (verified) 648
Fricas [A] (verification not implemented) 649
Sympy [A] (verification not implemented) 649
Maxima [F] 650
Giac [B] (verification not implemented) 650
Mupad [B] (verification not implemented) 651
Reduce [B] (verification not implemented) 651

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{x^9}{1-x^4+x^8}\,dx=\frac{x^2}{2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x^2}{1+x^4}\right)}{2\sqrt{3}}$$

output
```
1/2*x^2-1/6*arctanh(3^(1/2)*x^2/(x^4+1))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{x^9}{1-x^4+x^8}\,dx=\frac{1}{12}\left(6x^2+\sqrt{3}\log\left(-1+\sqrt{3}x^2-x^4\right)-\sqrt{3}\log\left(1+\sqrt{3}x^2+x^4\right)\right)$$

input
```
Integrate[x^9/(1 - x^4 + x^8),x]
```

output
```
(6*x^2 + Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] - Sqrt[3]*Log[1 + Sqrt[3]*x^2
 + x^4])/12
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1695, 1442, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{ 1695}$$

$$\frac{1}{2}\int \frac{x^8}{x^8-x^4+1}dx^2$$

$$\Big\downarrow \text{ 1442}$$

$$\frac{1}{2}\left(x^2-\int \frac{1-x^4}{x^8-x^4+1}dx^2\right)$$

$$\Big\downarrow \text{ 1478}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+x^2\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}-\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+x^2\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(x^2+\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{2\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[x^9/(1 - x^4 + x^8),x]
```

output
```
(x^2 + Log[1 - Sqrt[3]*x^2 + x^4]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]
/(2*Sqrt[3]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{x^{2}}{2}-\frac{\sqrt{3}\, \ln\left(x^{4}+\sqrt{3}\, x^{2}+1\right)}{12}+\frac{\sqrt{3}\, \ln\left(x^{4}-\sqrt{3}\, x^{2}+1\right)}{12}$	44
risch	$\frac{x^{2}}{2}-\frac{\sqrt{3}\, \ln\left(x^{4}+\sqrt{3}\, x^{2}+1\right)}{12}+\frac{\sqrt{3}\, \ln\left(x^{4}-\sqrt{3}\, x^{2}+1\right)}{12}$	44

input
```
int(x^9/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-1/12*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)+1/12*3^(1/2)*ln(x^4-3^(1/2)*x^2
+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int \frac{x^9}{1-x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{12}\,\sqrt{3}\log\left(\frac{x^8+5\,x^4-2\,\sqrt{3}(x^6+x^2)+1}{x^8-x^4+1}\right)$$

input
```
integrate(x^9/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/2*x^2 + 1/12*sqrt(3)*log((x^8 + 5*x^4 - 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8
- x^4 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{x^9}{1-x^4+x^8}\,dx = \frac{x^2}{2} + \frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12} - \frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

input
```
integrate(x**9/(x**8-x**4+1),x)
```

output
```
x**2/2 + sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 - sqrt(3)*log(x**4 + sqrt
(3)*x**2 + 1)/12
```

Maxima [F]

$$\int \frac{x^9}{1-x^4+x^8}\,dx=\int \frac{x^9}{x^8-x^4+1}\,dx$$

input
```
integrate(x^9/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
1/2*x^2 + integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(26)=52$.

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.91

$$\begin{aligned}\int \frac{x^9}{1-x^4+x^8}\,dx &= \frac{1}{2}\,x^2+\frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2+\sqrt{3}\right)\\ &\quad+\frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2-\sqrt{3}\right)\\ &\quad+\frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4+\sqrt{3}x^2+1\right)\\ &\quad-\frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4-\sqrt{3}x^2+1\right)\end{aligned}$$

input
```
integrate(x^9/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/2*x^2 + 1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) + 1/4*(x^4 - 1)*arctan(2*x
^2 - sqrt(3)) + 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) -
1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{x^9}{1-x^4+x^8}\,dx = \frac{x^2}{2} - \frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{2\sqrt{3}\,x^2}{9\left(\frac{2x^4}{9}+\frac{2}{9}\right)}\right)}{6}$$

input
```
int(x^9/(x^8 - x^4 + 1),x)
```

output
```
x^2/2 - (3^(1/2)*atanh((2*3^(1/2)*x^2)/(9*((2*x^4)/9 + 2/9))))/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.50

$$\int \frac{x^9}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{12} - \frac{\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{12} + \frac{\sqrt{3}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{12} + \frac{\sqrt{3}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{12} + \frac{x^2}{2}$$

input
```
int(x^9/(x^8-x^4+1),x)
```

output
```
( - sqrt(3)*log( - sqrt( - sqrt(3) + 2)*x + x**2 + 1) - sqrt(3)*log(sqrt(
- sqrt(3) + 2)*x + x**2 + 1) + sqrt(3)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x
**2 + 2)/2) + sqrt(3)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2) + 6*x**2
)/12
```

3.84 $\int \frac{x^7}{1-x^4+x^8} \, dx$

Optimal result 652
Mathematica [A] (verified) 652
Rubi [A] (verified) 653
Maple [A] (verified) 655
Fricas [A] (verification not implemented) 655
Sympy [A] (verification not implemented) 655
Maxima [A] (verification not implemented) 656
Giac [A] (verification not implemented) 656
Mupad [B] (verification not implemented) 656
Reduce [B] (verification not implemented) 657

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{x^7}{1-x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1-x^4+x^8\right)$$

output
```
-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)+1/8*ln(x^8-x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{1-x^4+x^8} \, dx = \frac{\arctan\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8}\log\left(1-x^4+x^8\right)$$

input
```
Integrate[x^7/(1 - x^4 + x^8),x]
```

output
```
ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1693, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^4}{x^8-x^4+1}dx^4$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^8-x^4+1}dx^4+\frac{1}{2}\int -\frac{1-2x^4}{x^8-x^4+1}dx^4\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^8-x^4+1}dx^4-\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(-\int \frac{1}{-x^8-3}d\left(2x^4-1\right)-\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^8-x^4+1\right)\right)$$

input
```
Int[x^7/(1 - x^4 + x^8),x]
```

output
```
(ArcTan[(-1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 - x^4 + x^8]/2)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln(x^8-x^4+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	33
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(4x^8-4x^4+4)}{8}$	35

input
```
int(x^7/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1-x^4+x^8}\,dx=\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^7/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^7}{1-x^4+x^8}\,dx=\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**7/(x**8-x**4+1),x)
```

output
```
log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1-x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^7/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1-x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^7/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 18.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^7}{1-x^4+x^8}\,dx = \frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12}$$

input
```
int(x^7/(x^8 - x^4 + 1),x)
```

output
```
log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 328, normalized size of antiderivative = 8.41

$$\int \frac{x^7}{1-x^4+x^8}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} + \frac{\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} + \frac{\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{8} + \frac{\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{8}$$

input
```
int(x^7/(x^8-x^4+1),x)
```

output
```
( - sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( -
 sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(2) -
 4*x)/(2*sqrt( - sqrt(3) + 2))) - sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(
6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqr
t(2)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + sqrt( - sq
rt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))
) + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sq
rt(6) + sqrt(2))) + sqrt( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) +
 2) + 4*x)/(sqrt(6) + sqrt(2))) + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*s
qrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) + 3*log( - sqrt( - sqrt(3)
 + 2)*x + x**2 + 1) + 3*log(sqrt( - sqrt(3) + 2)*x + x**2 + 1) + 3*log(( -
 sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) + 3*log((sqrt(6)*x + sqrt(2)*x + 2
*x**2 + 2)/2))/24
```

3.85 $\int \frac{x^5}{1-x^4+x^8} dx$

Optimal result 659
Mathematica [C] (verified) 659
Rubi [A] (verified) 660
Maple [C] (verified) 662
Fricas [A] (verification not implemented) 663
Sympy [A] (verification not implemented) 663
Maxima [F] 664
Giac [A] (verification not implemented) 664
Mupad [B] (verification not implemented) 665
Reduce [B] (verification not implemented) 666

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{x^5}{1-x^4+x^8} dx = -\frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\arctan\left(\sqrt{3}+2x^2\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x^2}{1+x^4}\right)}{4\sqrt{3}}$$

output
```
1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(3^(1/2)+2*x^2)-1/12*arctanh(3^(1/2)*x
^2/(x^4+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.66

$$\int \frac{x^5}{1-x^4+x^8} dx = \frac{\sqrt{-1-i\sqrt{3}}\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right)+\sqrt{-1+i\sqrt{3}}\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)}{4\sqrt{6}}$$

input
```
Integrate[x^5/(1 - x^4 + x^8),x]
```

output
```
(Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x^2)/2] + Sqrt
[-1 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x^2)/2])/(4*Sqrt[6
])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1695, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^4}{x^8-x^4+1}dx^2$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{x^4+1}{x^8-x^4+1}dx^2-\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-\sqrt{3}x^2+1}dx^2+\frac{1}{2}\int \frac{1}{x^4+\sqrt{3}x^2+1}dx^2\right)-\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\int \frac{1}{-x^4-1}d\left(2x^2-\sqrt{3}\right)-\int \frac{1}{-x^4-1}d\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(2x^2+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x^2\right)\right)-\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x^2+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x^2\right)\right)\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}-\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x^2+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x^2\right)\right)\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(2x^2+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x^2\right)\right)+\frac{1}{2}\left(\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{2\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{2\sqrt{3}}\right)\right)$$

input
```
Int[x^5/(1 - x^4 + x^8),x]
```

output
```
((-ArcTan[Sqrt[3] - 2*x^2] + ArcTan[Sqrt[3] + 2*x^2])/2 + (Log[1 - Sqrt[3]
*x^2 + x^4]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]/(2*Sqrt[3]))/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^{4}+3\_Z^{2}+1\right)}\_R\ln\left(6\_R^{3}+x^{2}+\_R\right)\right)}{4}$	32
default	$-\frac{\sqrt{3}\left(\frac{\ln\left(x^{4}+\sqrt{3}\,x^{2}+1\right)}{2}-\sqrt{3}\,\arctan\left(2x^{2}+\sqrt{3}\right)\right)}{12}-\frac{\sqrt{3}\left(-\frac{\ln\left(x^{4}-\sqrt{3}\,x^{2}+1\right)}{2}-\sqrt{3}\,\arctan\left(2x^{2}-\sqrt{3}\right)\right)}{12}$	77

input
```
int(x^5/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(6*_R^3+x^2+_R),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{1-x^4+x^8}\,dx = -\frac{1}{24}\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right) + \frac{1}{4}\arctan\left(2\,x^2+\sqrt{3}\right)-\frac{1}{4}\arctan\left(-2\,x^2+\sqrt{3}\right)$$

input
```
integrate(x^5/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/24*sqrt(3)*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*sqrt(3)*log(x^4 - sqrt(3)*
x^2 + 1) + 1/4*arctan(2*x^2 + sqrt(3)) - 1/4*arctan(-2*x^2 + sqrt(3))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}-\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24} + \frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}+\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

input
```
integrate(x**5/(x**8-x**4+1),x)
```

output
```
sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 - sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

Maxima [F]

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \int \frac{x^5}{x^8-x^4+1}\,dx$$

input
```
integrate(x^5/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^5/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{x^5}{1-x^4+x^8}\,dx = \frac{1}{24}\sqrt{3}x^4\log\left(x^4+\sqrt{3}x^2+1\right) - \frac{1}{24}\sqrt{3}x^4\log\left(x^4-\sqrt{3}x^2+1\right) + \frac{1}{4}x^4\arctan\left(2x^2+\sqrt{3}\right) + \frac{1}{4}x^4\arctan\left(2x^2-\sqrt{3}\right)$$

input
```
integrate(x^5/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/24*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*x^4*log(x^4 - s
qrt(3)*x^2 + 1) + 1/4*x^4*arctan(2*x^2 + sqrt(3)) + 1/4*x^4*arctan(2*x^2 -
 sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 18.68 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{1-x^4+x^8}\,dx = -\mathrm{atan}\left(\frac{2\,x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{2\,x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

input

```
int(x^5/(x^8 - x^4 + 1),x)
```

output

```
- atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) - atan((2*x^2)/(3
^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 336, normalized size of antiderivative = 5.69

$$\int \frac{x^5}{1-x^4+x^8}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} - \frac{\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} + \frac{\sqrt{3}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24} + \frac{\sqrt{3}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24}$$

input
```
int(x^5/(x^8-x^4+1),x)
```

output
```
( - 3*sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt(
 - sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(2)
 - 4*x)/(2*sqrt( - sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(6)*atan((s
qrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)
*sqrt(2)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 3*sqrt
( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sq
rt(2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*
x)/(sqrt(6) + sqrt(2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - s
qrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*a
tan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) - sqrt(3)*log( - s
qrt( - sqrt(3) + 2)*x + x**2 + 1) - sqrt(3)*log(sqrt( - sqrt(3) + 2)*x + x
**2 + 1) + sqrt(3)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) + sqrt(3
)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2))/24
```

3.86 $\int \frac{x^3}{1-x^4+x^8} dx$

Optimal result 668
Mathematica [A] (verified) 668
Rubi [A] (verified) 669
Maple [A] (verified) 670
Fricas [A] (verification not implemented) 670
Sympy [A] (verification not implemented) 671
Maxima [A] (verification not implemented) 671
Giac [A] (verification not implemented) 671
Mupad [B] (verification not implemented) 672
Reduce [B] (verification not implemented) 672

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^3}{1-x^4+x^8} \, dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1-x^4+x^8} \, dx = \frac{\arctan\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Integrate[x^3/(1 - x^4 + x^8),x]
```

output
```
ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1690, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8-x^4+1} \, dx$$

$$\Big\downarrow \text{ 1690}$$

$$\frac{1}{4} \int \frac{1}{x^8-x^4+1}dx^4$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{1}{2} \int \frac{1}{-x^8-3}d\left(2x^4-1\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[x^3/(1 - x^4 + x^8),x]
```

output
```
ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{6}$	19
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{6}$	19

input
```
int(x^3/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)$$

input
```
integrate(x^3/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{1-x^4+x^8}\,dx=\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**3/(x**8-x**4+1),x)
```

output
```
sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1-x^4+x^8}\,dx=\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)$$

input
```
integrate(x^3/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1-x^4+x^8}\,dx=\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)$$

input
```
integrate(x^3/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\,x^4}{3}-\frac{1}{3}\right)\right)}{6}$$

input
```
int(x^3/(x^8 - x^4 + 1),x)
```

output
```
(3^(1/2)*atan(3^(1/2)*((2*x^4)/3 - 1/3)))/6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 202, normalized size of antiderivative = 8.78

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\left(-\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)-3atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)-\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)-3atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right.}{12}$$

input
```
int(x^3/(x^8-x^4+1),x)
```

output
```
(sqrt( - sqrt(3) + 2)*sqrt(2)*( - sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(
2*sqrt( - sqrt(3) + 2))) - 3*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( - sqr
t(3) + 2))) - sqrt(3)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) +
2))) - 3*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + sqrt(3
)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) + 3*atan((2*sqr
t( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) + sqrt(3)*atan((2*sqrt( - sq
rt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) + 3*atan((2*sqrt( - sqrt(3) + 2) +
4*x)/(sqrt(6) + sqrt(2)))))/12
```

3.87 $\int \frac{x}{1-x^4+x^8} \, dx$

Optimal result . . . 673
Mathematica [C] (verified) . . . 673
Rubi [A] (verified) . . . 674
Maple [C] (verified) . . . 676
Fricas [A] (verification not implemented) . . . 677
Sympy [A] (verification not implemented) . . . 677
Maxima [F] . . . 678
Giac [A] (verification not implemented) . . . 678
Mupad [B] (verification not implemented) . . . 678
Reduce [B] (verification not implemented) . . . 679

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \frac{x}{1-x^4+x^8} \, dx = -\frac{1}{4} \arctan\left(\sqrt{3}-2x^2\right) + \frac{1}{4} \arctan\left(\sqrt{3}+2x^2\right) + \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x^2}{1+x^4}\right)}{4\sqrt{3}}$$

output
```
1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(3^(1/2)+2*x^2)+1/12*arctanh(3^(1/2)*x
^2/(x^4+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

$$\int \frac{x}{1-x^4+x^8} \, dx = \frac{i\left(\sqrt{-1-i\sqrt{3}} \arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right) - \sqrt{-1+i\sqrt{3}} \arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)\right)}{2\sqrt{6}}$$

input
```
Integrate[x/(1 - x^4 + x^8),x]
```

output
```
((I/2)*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x^2)/2] - Sqrt[-1 + I
*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x^2)/2]))/Sqrt[6]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.71, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1695, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{x^8-x^4+1}dx^2$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\int \frac{x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4-\sqrt{3}x^2+1}dx^2-\frac{1}{2}\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4+\sqrt{3}x^2+1}dx^2+\frac{1}{2}\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4-\sqrt{3}x^2+1}dx^2+\frac{1}{2}\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4+\sqrt{3}x^2+1}dx^2+\frac{1}{2}\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2-\sqrt{3}\int \frac{1}{-x^4-1}d\left(2x^2-\sqrt{3}\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2-\sqrt{3}\int \frac{1}{-x^4-1}d\left(2x^2+\sqrt{3}\right)}{2\sqrt{3}}\right)$$

$$\Big\downarrow \, 217$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2-\sqrt{3}\arctan\left(\sqrt{3}-2x^2\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2+\sqrt{3}\arctan\left(2x^2+\sqrt{3}\right)}{2\sqrt{3}}\right)$$

$$\Big\downarrow \, 1103$$

$$\frac{1}{2}\left(\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x^2\right)-\frac{1}{2}\log\left(x^4-\sqrt{3}x^2+1\right)}{2\sqrt{3}}+\frac{\sqrt{3}\arctan\left(2x^2+\sqrt{3}\right)+\frac{1}{2}\log\left(x^4+\sqrt{3}x^2+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[x/(1 - x^4 + x^8),x]
```

output
```
((-(Sqrt[3]*ArcTan[Sqrt[3] - 2*x^2]) - Log[1 - Sqrt[3]*x^2 + x^4]/2)/(2*Sq
rt[3]) + (Sqrt[3]*ArcTan[Sqrt[3] + 2*x^2] + Log[1 + Sqrt[3]*x^2 + x^4]/2)/
(2*Sqrt[3]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^{4}+3\_Z^{2}+1\right)}\_R\ln\left(-3\_R^{3}+x^{2}+\_R\right)\right)}{4}$	32
default	$\frac{\sqrt{3}\,\ln\left(x^{4}+\sqrt{3}\,x^{2}+1\right)}{24}+\frac{\arctan\left(2x^{2}+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\,\ln\left(x^{4}-\sqrt{3}\,x^{2}+1\right)}{24}+\frac{\arctan\left(2x^{2}-\sqrt{3}\right)}{4}$	65

input
```
int(x/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(-3*_R^3+x^2+_R),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{x}{1-x^4+x^8}\,dx = \frac{1}{24}\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right) - \frac{1}{24}\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right) + \frac{1}{4}\arctan\left(2\,x^2+\sqrt{3}\right) - \frac{1}{4}\arctan\left(-2\,x^2+\sqrt{3}\right)$$

input
```
integrate(x/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/24*sqrt(3)*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*log(x^4 - sqrt(3)*x
^2 + 1) + 1/4*arctan(2*x^2 + sqrt(3)) - 1/4*arctan(-2*x^2 + sqrt(3))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int \frac{x}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24} + \frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24} + \frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4} + \frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

input
```
integrate(x/(x**8-x**4+1),x)
```

output
```
-sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
 + 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

Maxima [F]

$$\int \frac{x}{1-x^4+x^8}\,dx = \int \frac{x}{x^8-x^4+1}\,dx$$

input
```
integrate(x/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(x/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{x}{1-x^4+x^8}\,dx = \frac{1}{24}\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right) - \frac{1}{24}\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right) + \frac{1}{4}\arctan\left(2\,x^2+\sqrt{3}\right) + \frac{1}{4}\arctan\left(2\,x^2-\sqrt{3}\right)$$

input
```
integrate(x/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/24*sqrt(3)*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*log(x^4 - sqrt(3)*x
^2 + 1) + 1/4*arctan(2*x^2 + sqrt(3)) + 1/4*arctan(2*x^2 - sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 19.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{x}{1-x^4+x^8}\,dx = -\mathrm{atan}\left(-\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

input
```
int(x/(x^8 - x^4 + 1),x)
```

output
```
- atan((3^(1/2)*x^2*1i)/2 - x^2/2)*((3^(1/2)*1i)/12 + 1/4) - atan((3^(1/2)
*x^2*1i)/2 + x^2/2)*((3^(1/2)*1i)/12 - 1/4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 336, normalized size of antiderivative = 5.69

$$\int \frac{x}{1-x^4+x^8}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} + \frac{\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} - \frac{\sqrt{3}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24} - \frac{\sqrt{3}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24}$$

input
```
int(x/(x^8-x^4+1),x)
```

output
```
( - 3*sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt(
 - sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(2)
 - 4*x)/(2*sqrt( - sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(6)*atan((s
qrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)
*sqrt(2)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 3*sqrt
( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sq
rt(2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*
x)/(sqrt(6) + sqrt(2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - s
qrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*a
tan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) + sqrt(3)*log( - s
qrt( - sqrt(3) + 2)*x + x**2 + 1) + sqrt(3)*log(sqrt( - sqrt(3) + 2)*x + x
**2 + 1) - sqrt(3)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) - sqrt(3
)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2))/24
```

3.88 $\int \frac{1}{x(1-x^4+x^8)}\,dx$

Optimal result 681
Mathematica [C] (verified) 681
Rubi [A] (verified) 682
Maple [A] (verified) 684
Fricas [A] (verification not implemented) 684
Sympy [A] (verification not implemented) 685
Maxima [A] (verification not implemented) 685
Giac [A] (verification not implemented) 686
Mupad [B] (verification not implemented) 686
Reduce [B] (verification not implemented) 687

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x(1-x^4+x^8)}\,dx=-\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)$$

output
```
-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)+ln(x)-1/8*ln(x^8-x^4+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{1}{x(1-x^4+x^8)}\,dx=\log(x)-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-1+2\#1^4}\&\right]$$

input
```
Integrate[1/(x*(1 - x^4 + x^8)),x]
```

output
```
Log[x] - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-1
 + 2*#1^4) & ]/4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1693, 1144, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (x^8-x^4+1)} dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4} \int \frac{1}{x^4 (x^8-x^4+1)}dx^4$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{4} \left(\int \frac{1-x^4}{x^8-x^4+1}dx^4+\log \left(x^4\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4} \left(\frac{1}{2} \int \frac{1}{x^8-x^4+1}dx^4-\frac{1}{2} \int -\frac{1-2x^4}{x^8-x^4+1}dx^4+\log \left(x^4\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4} \left(\frac{1}{2} \int \frac{1}{x^8-x^4+1}dx^4+\frac{1}{2} \int \frac{1-2x^4}{x^8-x^4+1}dx^4+\log \left(x^4\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4} \left(-\int \frac{1}{-x^8-3}d(2x^4-1)+\frac{1}{2} \int \frac{1-2x^4}{x^8-x^4+1}dx^4+\log \left(x^4\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4} \left(\frac{1}{2} \int \frac{1-2x^4}{x^8-x^4+1}dx^4+\frac{\arctan \left(\frac{2x^4-1}{\sqrt{3}}\right)}{\sqrt{3}}+\log \left(x^4\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^4\right)-\frac{1}{2}\log\left(x^8-x^4+1\right)\right)$$

input
```
Int[1/(x*(1 - x^4 + x^8)),x]
```

output
```
(ArcTan[(-1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[x^4] - Log[1 - x^4 + x^8]/2)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln\left(x\right)+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{4}-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^{8}-x^{4}+1\right)}{8}$	33
default	$-\frac{\ln\left(x^{8}-x^{4}+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)}{12}+\ln\left(x\right)$	35

input
```
int(1/x/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/12*3^(1/2)*arctan(2/3*(x^4-1/2)*3^(1/2))-1/8*ln(x^8-x^4+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx=\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^{4}-1)\right)-\frac{1}{8}\,\log\left(x^{8}-x^{4}+1\right)+\log\left(x\right)$$

input
```
integrate(1/x/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + lo
g(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx = \log\left(x\right) - \frac{\log\left(x^8-x^4+1\right)}{8} + \frac{\sqrt{3}\,\text{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(1/x/(x**8-x**4+1),x)
```

output
```
log(x) - log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/
3)/12
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right) - \frac{1}{8}\log\left(x^8-x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) - \frac{1}{8}\,\log\left(x^8-x^4+1\right) + \frac{1}{4}\,\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx = \ln\left(x\right) - \frac{\ln\left(x^8-x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}$$

input
```
int(1/(x*(x^8 - x^4 + 1)),x)
```

output
```
log(x) - log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/
3))/12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 330, normalized size of antiderivative = 8.05

$$\int \frac{1}{x(1-x^4+x^8)}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} - \frac{\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} - \frac{\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{8} - \frac{\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{8} + \log(x)$$

input
```
int(1/x/(x^8-x^4+1),x)
```

output

```
( - sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( -
 sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(2) -
 4*x)/(2*sqrt( - sqrt(3) + 2))) - sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(
6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqr
t(2)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + sqrt( - sq
rt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))
) + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sq
rt(6) + sqrt(2))) + sqrt( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) +
 2) + 4*x)/(sqrt(6) + sqrt(2))) + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*s
qrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) - 3*log( - sqrt( - sqrt(3)
 + 2)*x + x**2 + 1) - 3*log(sqrt( - sqrt(3) + 2)*x + x**2 + 1) - 3*log(( -
 sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) - 3*log((sqrt(6)*x + sqrt(2)*x + 2
*x**2 + 2)/2) + 24*log(x))/24
```

3.89 $\int \frac{1}{x^3(1-x^4+x^8)}\,dx$

Optimal result 689
Mathematica [A] (verified) 689
Rubi [A] (verified) 690
Maple [A] (verified) 691
Fricas [A] (verification not implemented) 692
Sympy [A] (verification not implemented) 692
Maxima [F] 693
Giac [B] (verification not implemented) 693
Mupad [B] (verification not implemented) 694
Reduce [B] (verification not implemented) 694

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx=-\frac{1}{2x^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x^2}{1+x^4}\right)}{2\sqrt{3}}$$

output
```
-1/2/x^2+1/6*arctanh(3^(1/2)*x^2/(x^4+1))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx=\frac{1}{12}\left(-\frac{6}{x^2}-\sqrt{3}\log\left(-1+\sqrt{3}x^2-x^4\right)+\sqrt{3}\log\left(1+\sqrt{3}x^2+x^4\right)\right)$$

input
```
Integrate[1/(x^3*(1 - x^4 + x^8)),x]
```

output
```
(-6/x^2 - Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] + Sqrt[3]*Log[1 + Sqrt[3]*x^
2 + x^4])/12
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1695, 1443, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^8-x^4+1)} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2} \int \frac{1}{x^4 (x^8-x^4+1)}dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2} \left(\int \frac{1-x^4}{x^8-x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2} \left(-\frac{\int -\frac{\sqrt{3}-2 x^2}{x^4-\sqrt{3} x^2+1}dx^2}{2 \sqrt{3}}-\frac{\int -\frac{2 x^2+\sqrt{3}}{x^4+\sqrt{3} x^2+1}dx^2}{2 \sqrt{3}}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\frac{\int \frac{\sqrt{3}-2 x^2}{x^4-\sqrt{3} x^2+1}dx^2}{2 \sqrt{3}}+\frac{\int \frac{2 x^2+\sqrt{3}}{x^4+\sqrt{3} x^2+1}dx^2}{2 \sqrt{3}}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2} \left(-\frac{1}{x^2}-\frac{\log \left(x^4-\sqrt{3} x^2+1\right)}{2 \sqrt{3}}+\frac{\log \left(x^4+\sqrt{3} x^2+1\right)}{2 \sqrt{3}}\right)$$

input
```
Int[1/(x^3*(1 - x^4 + x^8)),x]
```

output
```
(-x^(-2) - Log[1 - Sqrt[3]*x^2 + x^4]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 +
x^4]/(2*Sqrt[3]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{1}{2x^{2}}+\frac{\sqrt{3}\,\ln\left(x^{4}+\sqrt{3}\,x^{2}+1\right)}{12}-\frac{\sqrt{3}\,\ln\left(x^{4}-\sqrt{3}\,x^{2}+1\right)}{12}$	44
risch	$-\frac{1}{2x^{2}}+\frac{\sqrt{3}\,\ln\left(x^{4}+\sqrt{3}\,x^{2}+1\right)}{12}-\frac{\sqrt{3}\,\ln\left(x^{4}-\sqrt{3}\,x^{2}+1\right)}{12}$	44

input
```
int(1/x^3/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+1/12*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)-1/12*3^(1/2)*ln(x^4-3^(1/2)*x^
2+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^3 (1-x^4+x^8)}\,dx = \frac{\sqrt{3}x^2 \log\left(\frac{x^8+5\,x^4+2\,\sqrt{3}\,(x^6+x^2)+1}{x^8-x^4+1}\right) - 6}{12\,x^2}$$

input
```
integrate(1/x^3/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/12*(sqrt(3)*x^2*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4
 + 1)) - 6)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^3 (1-x^4+x^8)}\,dx = -\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12} + \frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12} - \frac{1}{2x^2}$$

input
```
integrate(1/x**3/(x**8-x**4+1),x)
```

output
```
-sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2
 + 1)/12 - 1/(2*x**2)
```

Maxima [F]

$$\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx=\int \frac{1}{(x^8-x^4+1)x^3}\,dx$$

input
```
integrate(1/x^3/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/2/x^2 - integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(26) = 52$.

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.91

$$\begin{aligned}\int \frac{1}{x^3\left(1-x^4+x^8\right)}\,dx=&-\frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2+\sqrt{3}\right)\\&-\frac{1}{4}\left(x^4-1\right)\arctan\left(2\,x^2-\sqrt{3}\right)\\&-\frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4+\sqrt{3}x^2+1\right)\\&+\frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4-\sqrt{3}x^2+1\right)-\frac{1}{2\,x^2}\end{aligned}$$

input
```
integrate(1/x^3/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) - 1/4*(x^4 - 1)*arctan(2*x^2 - sqrt
(3)) - 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*(sqr
t(3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1) - 1/2/x^2
```

Mupad [B] (verification not implemented)

Time = 19.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 (1-x^4+x^8)} \, dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\,\sqrt{3}\,x^2}{9\,\left(\frac{2\,x^4}{9}+\frac{2}{9}\right)}\right)}{6} - \frac{1}{2\,x^2}$$

input
```
int(1/(x^3*(x^8 - x^4 + 1)),x)
```

output
```
(3^(1/2)*atanh((2*3^(1/2)*x^2)/(9*((2*x^4)/9 + 2/9))))/6 - 1/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.82

$$\int \frac{1}{x^3 (1-x^4+x^8)} \, dx$$
$$= \frac{\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)x^2+\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)x^2-\sqrt{3}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2}{12x^2}$$

input
```
int(1/x^3/(x^8-x^4+1),x)
```

output
```
(sqrt(3)*log( - sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x**2 + sqrt(3)*log(sqrt
( - sqrt(3) + 2)*x + x**2 + 1)*x**2 - sqrt(3)*log(( - sqrt(6)*x - sqrt(2)*
x + 2*x**2 + 2)/2)*x**2 - sqrt(3)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)
/2)*x**2 - 6)/(12*x**2)
```

3.90 $\int \frac{1}{x^5(1-x^4+x^8)}\,dx$

Optimal result 695
Mathematica [C] (verified) 695
Rubi [A] (verified) 696
Maple [A] (verified) 697
Fricas [A] (verification not implemented) 698
Sympy [A] (verification not implemented) 698
Maxima [A] (verification not implemented) 698
Giac [A] (verification not implemented) 699
Mupad [B] (verification not implemented) 699
Reduce [B] (verification not implemented) 700

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{x^5\left(1-x^4+x^8\right)}\,dx=-\frac{1}{4x^4}+\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)$$

output
```
-1/4/x^4+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)+ln(x)-1/8*ln(x^8-x^4+
1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^5\left(1-x^4+x^8\right)}\,dx=-\frac{1}{4x^4}+\log(x)-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^4}{-1+2\#1^4}\&\right]$$

input
```
Integrate[1/(x^5*(1 - x^4 + x^8)),x]
```

output
```
-1/4*1/x^4 + Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 +
 2*#1^4) & ]/4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1693, 1145, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^8-x^4+1)} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{1}{x^8 (x^8-x^4+1)}dx^4$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{4}\left(\int \frac{1-x^4}{x^4 (x^8-x^4+1)}dx^4-\frac{1}{x^4}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{4}\left(\int \left(\frac{1}{x^4}-\frac{x^4}{x^8-x^4+1}\right)dx^4-\frac{1}{x^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{x^4}+\log\left(x^4\right)-\frac{1}{2}\log\left(x^8-x^4+1\right)\right)$$

input
```
Int[1/(x^5*(1 - x^4 + x^8)),x]
```

output
```
(-x^(-4) + ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[x^4] - Log[1 - x^4 +
x^8]/2)/4
```

Defintions of rubi rules used

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{1}{4x^4}+\ln\left(x\right)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^4-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^8-x^4+1\right)}{8}$	38
default	$-\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12}-\frac{1}{4x^4}+\ln\left(x\right)$	40

input
```
int(1/x^5/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4+ln(x)-1/12*3^(1/2)*arctan(2/3*(x^4-1/2)*3^(1/2))-1/8*ln(x^8-x^4+1
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^5 (1-x^4+x^8)} \, dx = -\frac{2 \sqrt{3} x^4 \arctan \left(\frac{1}{3} \sqrt{3} (2 x^4-1)\right) + 3 x^4 \log \left(x^8-x^4+1\right) - 24 x^4 \log \left(x\right) + 6}{24 x^4}$$

input
```
integrate(1/x^5/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 3*x^4*log(x^8 - x^4
 + 1) - 24*x^4*log(x) + 6)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (1-x^4+x^8)} \, dx = \log \left(x\right) - \frac{\log \left(x^8-x^4+1\right)}{8} - \frac{\sqrt{3} \operatorname{atan} \left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12} - \frac{1}{4x^4}$$

input
```
integrate(1/x**5/(x**8-x**4+1),x)
```

output
```
log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/
3)/12 - 1/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^5 (1-x^4+x^8)} \, dx = -\frac{1}{12} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2 x^4-1)\right) - \frac{1}{4 x^4} - \frac{1}{8} \log \left(x^8-x^4+1\right) + \frac{1}{4} \log \left(x^4\right)$$

input
```
integrate(1/x^5/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4/x^4 - 1/8*log(x^8 - x^
4 + 1) + 1/4*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\left(1-x^4+x^8\right)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right)-\frac{x^4+1}{4x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x^5/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4*(x^4 + 1)/x^4 - 1/8*lo
g(x^8 - x^4 + 1) + 1/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5\left(1-x^4+x^8\right)}\,dx = \ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12}-\frac{1}{4x^4}$$

input
```
int(1/(x^5*(x^8 - x^4 + 1)),x)
```

output
```
log(x) - log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/
3))/12 - 1/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 375, normalized size of antiderivative = 7.81

$$\int \frac{1}{x^5 (1 - x^4 + x^8)} \, dx$$
$$= \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x^4 + 3\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x^4 + \sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{}}{2\sqrt{-}}\right.}{}$$

input
```
int(1/x^5/(x^8-x^4+1),x)
```

output
```
(sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( - sq
rt(3) + 2)))*x**4 + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(2)
 - 4*x)/(2*sqrt( - sqrt(3) + 2)))*x**4 + sqrt( - sqrt(3) + 2)*sqrt(6)*atan
((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*x**4 + 3*sqrt( - sqrt
(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*
x**4 - sqrt( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(s
qrt(6) + sqrt(2)))*x**4 - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt( - s
qrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2)))*x**4 - sqrt( - sqrt(3) + 2)*sqrt(6
)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2)))*x**4 - 3*sqrt(
- sqrt(3) + 2)*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt
(2)))*x**4 - 3*log( - sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x**4 - 3*log(sqrt
( - sqrt(3) + 2)*x + x**2 + 1)*x**4 - 3*log(( - sqrt(6)*x - sqrt(2)*x + 2*
x**2 + 2)/2)*x**4 - 3*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2)*x**4 + 2
4*log(x)*x**4 - 6)/(24*x**4)
```

3.91 $\int \frac{1}{x^7(1-x^4+x^8)}\,dx$

Optimal result 701
Mathematica [C] (verified) 701
Rubi [A] (verified) 702
Maple [C] (verified) 705
Fricas [A] (verification not implemented) 706
Sympy [A] (verification not implemented) 707
Maxima [F] 707
Giac [A] (verification not implemented) 707
Mupad [B] (verification not implemented) 708
Reduce [B] (verification not implemented) 708

Optimal result

Integrand size = 16, antiderivative size = 73

$$\int \frac{1}{x^7(1-x^4+x^8)}\,dx=-\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\arctan\left(\sqrt{3}+2x^2\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x^2}{1+x^4}\right)}{4\sqrt{3}}$$

output
```
-1/6/x^6-1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(3^(1/2)+2*x^2)+1/12*
arctanh(3^(1/2)*x^2/(x^4+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^7(1-x^4+x^8)}\,dx=-\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^4}\&\right]$$

input
```
Integrate[1/(x^7*(1 - x^4 + x^8)),x]
```

output
```
-1/6*1/x^6 - 1/(2*x^2) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-
1 + 2*#1^4) & ]/4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {1695, 1443, 27, 1604, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(x^8-x^4+1\right)} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2} \int \frac{1}{x^8 \left(x^8-x^4+1\right)}dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2} \left(\frac{1}{3} \int \frac{3 \left(1-x^4\right)}{x^4 \left(x^8-x^4+1\right)}dx^2-\frac{1}{3 x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\int \frac{1-x^4}{x^4 \left(x^8-x^4+1\right)}dx^2-\frac{1}{3 x^6}\right)$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{2} \left(-\int \frac{x^4}{x^8-x^4+1}dx^2-\frac{1}{3 x^6}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2} \left(\frac{1}{2} \int \frac{1-x^4}{x^8-x^4+1}dx^2-\frac{1}{2} \int \frac{x^4+1}{x^8-x^4+1}dx^2-\frac{1}{3 x^6}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2} \left(\frac{1}{2} \left(-\frac{1}{2} \int \frac{1}{x^4-\sqrt{3} x^2+1}dx^2-\frac{1}{2} \int \frac{1}{x^4+\sqrt{3} x^2+1}dx^2\right)+\frac{1}{2} \int \frac{1-x^4}{x^8-x^4+1}dx^2-\frac{1}{3 x^6}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{1}{-x^4-1}d\left(2x^2-\sqrt{3}\right)+\int \frac{1}{-x^4-1}d\left(2x^2+\sqrt{3}\right)\right)+\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2-\frac{1}{3x^6}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{3x^6}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \ 1478$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}-\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{3x^6}-\frac{1}{x^2}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{3x^6}-\frac{1}{x^2}\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{3x^6}-\frac{1}{x^2}+\frac{1}{2}\left(\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{2\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right.}{2\sqrt{3}}\right.\right.$$

input
```
Int[1/(x^7*(1 - x^4 + x^8)),x]
```

output
```
(-1/3*1/x^6 - x^(-2) + (ArcTan[Sqrt[3] - 2*x^2] - ArcTan[Sqrt[3] + 2*x^2])
/2 + (-1/2*Log[1 - Sqrt[3]*x^2 + x^4]/Sqrt[3] + Log[1 + Sqrt[3]*x^2 + x^4]
/(2*Sqrt[3]))/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{-\frac{x^4}{2}-\frac{1}{6}}{x^6}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-6\_R^3+x^2-\_R\right)\right)}{4}$	46
default	$\frac{\sqrt{3}\left(\frac{\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{2}-\sqrt{3}\,\arctan\left(2x^2+\sqrt{3}\right)\right)}{12}+\frac{\sqrt{3}\left(-\frac{\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{2}-\sqrt{3}\,\arctan\left(2x^2-\sqrt{3}\right)\right)}{12}-\frac{1}{6x^6}-\frac{1}{2x^2}$	87

input
```
int(1/x^7/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-1/2*x^4-1/6)/x^6+1/4*sum(_R*ln(-6*_R^3+x^2-_R),_R=RootOf(9*_Z^4+3*_Z^2+1
))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^7\left(1-x^4+x^8\right)}\,dx$$
$$= \frac{\sqrt{3}x^6\log\left(x^4+\sqrt{3}x^2+1\right)-\sqrt{3}x^6\log\left(x^4-\sqrt{3}x^2+1\right)-6\,x^6\arctan\left(2\,x^2+\sqrt{3}\right)+6\,x^6\arctan\left(-2\,x^2\right.}{24\,x^6}$$

input
```
integrate(1/x^7/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/24*(sqrt(3)*x^6*log(x^4 + sqrt(3)*x^2 + 1) - sqrt(3)*x^6*log(x^4 - sqrt(
3)*x^2 + 1) - 6*x^6*arctan(2*x^2 + sqrt(3)) + 6*x^6*arctan(-2*x^2 + sqrt(3
)) - 12*x^4 - 4)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^7 (1 - x^4 + x^8)} dx = -\frac{\sqrt{3}\log\left(x^4 - \sqrt{3}x^2 + 1\right)}{24} + \frac{\sqrt{3}\log\left(x^4 + \sqrt{3}x^2 + 1\right)}{24} - \frac{\operatorname{atan}\left(2x^2 - \sqrt{3}\right)}{4} - \frac{\operatorname{atan}\left(2x^2 + \sqrt{3}\right)}{4} + \frac{-3x^4 - 1}{6x^6}$$

input
```
integrate(1/x**7/(x**8-x**4+1),x)
```

output
```
-sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
 + 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 + (-3*x**4
- 1)/(6*x**6)
```

Maxima [F]

$$\int \frac{1}{x^7 (1 - x^4 + x^8)} dx = \int \frac{1}{(x^8 - x^4 + 1)x^7} dx$$

input
```
integrate(1/x^7/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/6*(3*x^4 + 1)/x^6 - integrate(x^5/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^7 (1 - x^4 + x^8)} dx = -\frac{1}{12}\sqrt{3}x^4 \log\left(x^4 + \sqrt{3}x^2 + 1\right) + \frac{1}{12}\sqrt{3}x^4 \log\left(x^4 - \sqrt{3}x^2 + 1\right) - \frac{3x^4 + 1}{6x^6}$$

input
```
integrate(1/x^7/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/12*sqrt(3)*x^4*log(x^4 -
sqrt(3)*x^2 + 1) - 1/6*(3*x^4 + 1)/x^6
```

Mupad [B] (verification not implemented)

Time = 19.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^7 (1-x^4+x^8)} \, dx = \operatorname{atan}\left(\frac{2\,x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \operatorname{atan}\left(\frac{2\,x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \frac{\frac{x^4}{2}+\frac{1}{6}}{x^6}$$

input
```
int(1/(x^7*(x^8 - x^4 + 1)),x)
```

output
```
atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x^2)/(3^(
1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - (x^4/2 + 1/6)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 381, normalized size of antiderivative = 5.22

$$\int \frac{1}{x^7 (1-x^4+x^8)} \, dx = \frac{3\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x^6 + 3\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x^6 + 3\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}}{2}\right.}{}$$

input
```
int(1/x^7/(x^8-x^4+1),x)
```

output
```
(3*sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( -
sqrt(3) + 2)))*x**6 + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(
2) - 4*x)/(2*sqrt( - sqrt(3) + 2)))*x**6 + 3*sqrt( - sqrt(3) + 2)*sqrt(6)*
atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*x**6 + 3*sqrt( -
sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2
)))*x**6 + 3*sqrt( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4
*x)/(sqrt(6) + sqrt(2)))*x**6 + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqr
t( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2)))*x**6 + 3*sqrt( - sqrt(3) + 2
)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2)))*x**6 +
3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6
) + sqrt(2)))*x**6 + sqrt(3)*log( - sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x**
6 + sqrt(3)*log(sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x**6 - sqrt(3)*log(( -
sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2)*x**6 - sqrt(3)*log((sqrt(6)*x + sqr
t(2)*x + 2*x**2 + 2)/2)*x**6 - 12*x**4 - 4)/(24*x**6)
```

3.92 $\int \frac{x^8}{1-x^4+x^8}\,dx$

Optimal result 710
Mathematica [C] (verified) 711
Rubi [A] (verified) 711
Maple [C] (verified) 715
Fricas [A] (verification not implemented) 715
Sympy [A] (verification not implemented) 716
Maxima [F] 716
Giac [A] (verification not implemented) 717
Mupad [B] (verification not implemented) 718
Reduce [B] (verification not implemented) 719

Optimal result

Integrand size = 16, antiderivative size = 286

$$\begin{aligned}\int \frac{x^8}{1-x^4+x^8}\,dx &= x+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\ &\quad-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\ &\quad-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)\\ &\quad+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)\\ &\quad+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\text{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right)\\ &\quad-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\text{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)\end{aligned}$$

output
```
x+1/4*(1/2*2^(1/2)+1/6*6^(1/2))*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1/2*
6^(1/2)+1/2*2^(1/2)))-1/4*(1/2*2^(1/2)-1/6*6^(1/2))*arctan((1/2*6^(1/2)+1/
2*2^(1/2)-2*x)/(1/2*6^(1/2)-1/2*2^(1/2)))-1/4*(1/2*2^(1/2)+1/6*6^(1/2))*ar
ctan((1/2*6^(1/2)-1/2*2^(1/2)+2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))+1/4*(1/2*2^(
1/2)-1/6*6^(1/2))*arctan((1/2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)-1/2*2^
(1/2)))+1/4*(1/2*2^(1/2)-1/6*6^(1/2))*arctanh((1/2*6^(1/2)-1/2*2^(1/2))*x/
(x^2+1))-1/4*(1/2*2^(1/2)+1/6*6^(1/2))*arctanh((1/2*6^(1/2)+1/2*2^(1/2))*x
/(x^2+1))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.21

$$\int \frac{x^8}{1-x^4+x^8}\,dx = x+\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[x^8/(1 - x^4 + x^8),x]
```

output
```
x + RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 +
 2*#1^7) & ]/4
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1703, 1751, 25, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^8-x^4+1}\,dx$$

$$\Big\downarrow \text{ 1703}$$

$$x-\int \frac{1-x^4}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{ 1751}$$

$$\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}+x$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}+x$$

$$\Big\downarrow \text{ 1483}$$

$$-\frac{\frac{\int \frac{\left(2-\sqrt{3}\right)x+\sqrt{3\left(2-\sqrt{3}\right)}}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(2-\sqrt{3}\right)x}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(2+\sqrt{3}\right)x+\sqrt{3\left(2+\sqrt{3}\right)}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}+x$$

$$\Big\downarrow \text{ 1142}$$

$$-\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\left(2-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\left(2+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}+x$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}+x$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{\cdots}{-\left(2x\cdots\right.}}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1\cdots}{-\left(2x+\sqrt{2+\cdots}\right.}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$x$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}+x$$

$$\Big\downarrow \; 1103$$

$$-\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}+x$$

input
```
Int[x^8/(1 - x^4 + x^8),x]
```

output
```
x - ((ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] + ((2 - Sqrt[3]
)*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]) + (ArcTan[(
Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] - ((2 - Sqrt[3])*Log[1 + Sqrt[
2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]))/(2*Sqrt[3]) - ((-ArcTan[(
-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((2 + Sqrt[3])*Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (-ArcTan[(Sqrt[2 + Sqrt
[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + ((2 + Sqrt[3])*Log[1 + Sqrt[2 + Sqrt[3]]*
x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1751
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x
^(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Simp[e/(2*c*q)   Int[(q +
2*x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGt
Q[n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.15

method	result	size
default	$x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44
risch	$x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44

input
```
int(x^8/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
x+1/4*sum((_R^4-1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.18

$$\int \frac{x^8}{1-x^4+x^8}\,dx=\text{Too large to display}$$

input
```
integrate(x^8/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/3)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/3)*(sqrt(-1/3
) - 1)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2)) + 2*x) - 1/4*sqrt(1/3)*sqrt(-sqrt
(3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3)*(sqrt(-1/3) - 1)*sqrt(-sqrt(3/2*s
qrt(-1/3) + 1/2)) + 2*x) - 1/4*sqrt(1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2)
)*log(3*sqrt(1/3)*(sqrt(-1/3) + 1)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2)) + 2*
x) + 1/4*sqrt(1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3)*(sq
rt(-1/3) + 1)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2)) + 2*x) + 1/4*sqrt(1/3)*(3
/2*sqrt(-1/3) + 1/2)^(1/4)*log(3*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4)*(s
qrt(-1/3) - 1) + 2*x) - 1/4*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*
sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4)*(sqrt(-1/3) - 1) + 2*x) - 1/4*sqrt(
1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1/4)*log(3*sqrt(1/3)*(sqrt(-1/3) + 1)*(-3/2*
sqrt(-1/3) + 1/2)^(1/4) + 2*x) + 1/4*sqrt(1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1/
4)*log(-3*sqrt(1/3)*(sqrt(-1/3) + 1)*(-3/2*sqrt(-1/3) + 1/2)^(1/4) + 2*x)
+ x
```

Sympy [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.09

$$\int \frac{x^8}{1-x^4+x^8}\,dx = x + \mathrm{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(9216t^5 - 8t + x\right)\right)\right)$$

input
```
integrate(x**8/(x**8-x**4+1),x)
```

output
```
x + RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 -
 8*_t + x)))
```

Maxima [F]

$$\int \frac{x^8}{1-x^4+x^8}\,dx = \int \frac{x^8}{x^8-x^4+1}\,dx$$

input
```
integrate(x^8/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
x + integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{1-x^4+x^8}\,dx = -\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) + x$$

input
```
integrate(x^8/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sq
rt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt
(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(
6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*
x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(s
qrt(6) - sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(
6) - sqrt(2)) + 1) + x
```

Mupad [B] (verification not implemented)

Time = 19.01 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.73

$$\int \frac{x^8}{1-x^4+x^8}\,dx = x+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}+\frac{\sqrt{3}\,x\,1{}\mathrm{i}}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1{}\mathrm{i}}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}-\frac{\sqrt{3}\,x}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{12} - \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12} - \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{12}$$

input
```
int(x^8/(x^8 - x^4 + 1),x)
```

output
```
x + (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8
i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (3^(1/2)*atan((x*1i)/(8 - 3^(1/2
)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4))/
12 - (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/
4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12
 - (2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1
/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.05

$$\int \frac{x^8}{1-x^4+x^8}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} + \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{24} - \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} + \frac{\sqrt{6}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{48} - \frac{\sqrt{6}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{48} + \frac{\sqrt{2}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{16} - \frac{\sqrt{2}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{16} + x$$

input
```
int(x^8/(x^8-x^4+1),x)
```

output

```
( - 4*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt(
 - sqrt(3) + 2))) + 4*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2)
 + 4*x)/(2*sqrt( - sqrt(3) + 2))) + 2*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2)
 - 4*x)/(sqrt(6) + sqrt(2))) + 6*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*
x)/(sqrt(6) + sqrt(2))) - 2*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(s
qrt(6) + sqrt(2))) - 6*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6
) + sqrt(2))) - 2*sqrt( - sqrt(3) + 2)*sqrt(3)*log( - sqrt( - sqrt(3) + 2)
*x + x**2 + 1) + 2*sqrt( - sqrt(3) + 2)*sqrt(3)*log(sqrt( - sqrt(3) + 2)*x
 + x**2 + 1) + sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) - sq
rt(6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2) + 3*sqrt(2)*log(( - sqrt
(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) - 3*sqrt(2)*log((sqrt(6)*x + sqrt(2)*x
+ 2*x**2 + 2)/2) + 48*x)/48
```

3.93 $\int \frac{x^6}{1-x^4+x^8} \, dx$

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Optimal result

Integrand size = 16, antiderivative size = 219

$$\int \frac{x^6}{1-x^4+x^8} \, dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right)}{2\sqrt{6}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)}{2\sqrt{6}}$$

output
```
-1/12*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1
/2)-1/12*arctan((1/2*6^(1/2)+1/2*2^(1/2)-2*x)/(1/2*6^(1/2)-1/2*2^(1/2)))*6
^(1/2)+1/12*arctan((1/2*6^(1/2)-1/2*2^(1/2)+2*x)/(1/2*6^(1/2)+1/2*2^(1/2))
)*6^(1/2)+1/12*arctan((1/2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)-1/2*2^(1/
2)))*6^(1/2)-1/12*arctanh((1/2*6^(1/2)-1/2*2^(1/2))*x/(x^2+1))*6^(1/2)-1/1
2*arctanh((1/2*6^(1/2)+1/2*2^(1/2))*x/(x^2+1))*6^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.19

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^3}{-1+2\#1^4}\&\right]$$

input
```
Integrate[x^6/(1 - x^4 + x^8),x]
```

output
```
RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/4
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1708, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{ 1708}$$

$$\frac{\int \frac{\sqrt{3}x^2+1}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int \frac{1-\sqrt{3}x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{\frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\left(1-\sqrt{3}\right)x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(1+\sqrt{3}\right)x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{-\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-\cdots}\cdots}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}\cdots$$

$$\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-\cdots}\cdots}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

input
```
Int[x^6/(1 - x^4 + x^8),x]
```

output
```
((Sqrt[2/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]
]] - ((1 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqr
t[3]]) + (Sqrt[2/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 +
Sqrt[3]]] + ((1 - Sqrt[3])*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[
2 - Sqrt[3]]))/(2*Sqrt[3]) - ((-(Sqrt[2/(2 - Sqrt[3])]*ArcTan[(-Sqrt[2 + S
qrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]) - ((1 + Sqrt[3])*Log[1 - Sqrt[2 + Sqrt[
3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (-(Sqrt[2/(2 - Sqrt[3])]*ArcTan[(
Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]) + ((1 + Sqrt[3])*Log[1 + Sqrt
[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1708
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Simp[1/(2*c*r)   Int[x^
(m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Simp[1/(2*c
*r)   Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]]
 /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2,
 0] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	32
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	32

input
```
int(x^6/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.59

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \frac{1}{4}\sqrt{\frac{2}{3}}\arctan\left(4\,x^2+3\,\sqrt{\frac{2}{3}}(x^3+2\,x)+3\right) + \frac{1}{4}\sqrt{\frac{2}{3}}\arctan\left(-4\,x^2+3\,\sqrt{\frac{2}{3}}(x^3+2\,x)-3\right) + \frac{1}{4}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{2}{3}}x+\frac{1}{3}\right) + \frac{1}{4}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{2}{3}}x-\frac{1}{3}\right) - \frac{1}{8}\sqrt{\frac{2}{3}}\log\left(x^4+3\,x^2+3\,\sqrt{\frac{2}{3}}(x^3+x)+1\right) + \frac{1}{8}\sqrt{\frac{2}{3}}\log\left(x^4+3\,x^2-3\,\sqrt{\frac{2}{3}}(x^3+x)+1\right)$$

input
```
integrate(x^6/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(2/3)*arctan(4*x^2 + 3*sqrt(2/3)*(x^3 + 2*x) + 3) + 1/4*sqrt(2/3)*
arctan(-4*x^2 + 3*sqrt(2/3)*(x^3 + 2*x) - 3) + 1/4*sqrt(2/3)*arctan(sqrt(2
/3)*x + 1/3) + 1/4*sqrt(2/3)*arctan(sqrt(2/3)*x - 1/3) - 1/8*sqrt(2/3)*log
(x^4 + 3*x^2 + 3*sqrt(2/3)*(x^3 + x) + 1) + 1/8*sqrt(2/3)*log(x^4 + 3*x^2
- 3*sqrt(2/3)*(x^3 + x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.75

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} + \frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} + \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} - \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

input
```
integrate(x**6/(x**8-x**4+1),x)
```

output
```
sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3
+ 4*x**2 + 2*sqrt(6)*x + 3))/24 + sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2
 - sqrt(6)*x + 1)/24 - sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*
x + 1)/24
```

Maxima [F]

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \int \frac{x^6}{x^8-x^4+1}\,dx$$

input
```
integrate(x^6/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^6/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{x^6}{1-x^4+x^8}\,dx &= \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &-\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &+\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &-\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\ &+\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

input
```
integrate(x^6/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(
6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*ar
ctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*sqrt(6)*log(x^2
 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6)
+ sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) +
1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.24

$$\int \frac{x^6}{1-x^4+x^8}\,dx = \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right) + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)$$

input
```
int(x^6/(x^8 - x^4 + 1),x)
```

output
```
- 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12)
 - 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.44

$$\int \frac{x^6}{1-x^4+x^8}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} - \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} + \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} - \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{12} + \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{12} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} + \frac{\sqrt{6}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24} - \frac{\sqrt{6}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24}$$

input
```
int(x^6/(x^8-x^4+1),x)
```

output
```
( - 2*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt(
 - sqrt(3) + 2))) - 6*sqrt( - sqrt(3) + 2)*atan((sqrt(6) + sqrt(2) - 4*x)/
(2*sqrt( - sqrt(3) + 2))) + 2*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) +
 sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + 6*sqrt( - sqrt(3) + 2)*atan((s
qrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 2*sqrt(6)*atan((2*sqrt
( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) + 2*sqrt(6)*atan((2*sqrt( - s
qrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) + sqrt( - sqrt(3) + 2)*sqrt(3)*log
( - sqrt( - sqrt(3) + 2)*x + x**2 + 1) - sqrt( - sqrt(3) + 2)*sqrt(3)*log(
sqrt( - sqrt(3) + 2)*x + x**2 + 1) + 3*sqrt( - sqrt(3) + 2)*log( - sqrt( -
 sqrt(3) + 2)*x + x**2 + 1) - 3*sqrt( - sqrt(3) + 2)*log(sqrt( - sqrt(3) +
 2)*x + x**2 + 1) + sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2)
 - sqrt(6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2))/24
```

3.94 $\int \frac{x^4}{1-x^4+x^8} dx$

Optimal result . . . 731
Mathematica [C] (verified) . . . 732
Rubi [A] (verified) . . . 732
Maple [C] (verified) . . . 736
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Optimal result

Integrand size = 16, antiderivative size = 273

$$\int \frac{x^4}{1-x^4+x^8} dx = \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

output
```
1/4*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2
^(1/2)+1/2*6^(1/2))-1/4*arctan((1/2*6^(1/2)+1/2*2^(1/2)-2*x)/(1/2*6^(1/2)-
1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/4*arctan((1/2*6^(1/2)-1/2*2^(1/2
)+2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((1/
2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(
1/2))+1/4*arctanh((1/2*6^(1/2)-1/2*2^(1/2))*x/(x^2+1))/(3/2*2^(1/2)-1/2*6^
(1/2))-1/4*arctanh((1/2*6^(1/2)+1/2*2^(1/2))*x/(x^2+1))/(3/2*2^(1/2)+1/2*6
^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.14

$$\int \frac{x^4}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1}{-1+2\#1^4}\&\right]$$

input
```
Integrate[x^4/(1 - x^4 + x^8),x]
```

output
```
RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) & ]/4
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.33, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1709, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{ 1709}$$

$$\frac{\int \frac{x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int \frac{x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 1447}$$

$$\frac{\frac{1}{2}\int \frac{x^2+1}{x^4-\sqrt{3}x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\int \frac{x^2+1}{x^4+\sqrt{3}x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 1475}$$

$$\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$\downarrow$ 1083

$$\frac{\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)-\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x+\sqrt{2+\sqrt{3}}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\frac{\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)-\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x+\sqrt{2-\sqrt{3}}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$\downarrow$ 217

$$\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$\downarrow$ 1478

$$\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int -\frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int -\frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}-\frac{\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}-\frac{\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}$$

input
```
Int[x^4/(1 - x^4 + x^8),x]
```

output
```
-1/2*((ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[2 + Sqrt[
3]] + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[2 + Sqrt[3]
])/2 + (Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2 - Sqrt[3]]) - Log[1 +
 Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2 - Sqrt[3]]))/2)/Sqrt[3] + ((ArcTan[(
-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2 - Sqrt[3]] + ArcTan[(S
qrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2 - Sqrt[3]])/2 + (Log[1 -
 Sqrt[2 + Sqrt[3]]*x + x^2]/(2*Sqrt[2 + Sqrt[3]]) - Log[1 + Sqrt[2 + Sqrt[
3]]*x + x^2]/(2*Sqrt[2 + Sqrt[3]]))/2)/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	40

input
```
int(x^4/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^4/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.12

$$\int \frac{x^4}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{4}\sqrt{\frac{1}{3}}\sqrt{-\sqrt{\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}\log\left(3\sqrt{\frac{1}{3}}\sqrt{-\frac{1}{3}}\sqrt{-\sqrt{\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}+x\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{3}}\sqrt{-\sqrt{\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}\log\left(-3\sqrt{\frac{1}{3}}\sqrt{-\frac{1}{3}}\sqrt{-\sqrt{\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}+x\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{3}}\sqrt{-\sqrt{-\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}\log\left(3\sqrt{\frac{1}{3}}\sqrt{-\frac{1}{3}}\sqrt{-\sqrt{-\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}+x\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{3}}\sqrt{-\sqrt{-\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}\log\left(-3\sqrt{\frac{1}{3}}\sqrt{-\frac{1}{3}}\sqrt{-\sqrt{-\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}+x\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{3}}\left(\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}\right)^{\frac{1}{4}}\log\left(3\sqrt{\frac{1}{3}}\sqrt{-\frac{1}{3}}\left(\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}\right)^{\frac{1}{4}}+x\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{3}}\left(\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}\right)^{\frac{1}{4}}\log\left(-3\sqrt{\frac{1}{3}}\sqrt{-\frac{1}{3}}\left(\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}\right)^{\frac{1}{4}}+x\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{3}}\left(-\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}\right)^{\frac{1}{4}}\log\left(3\sqrt{\frac{1}{3}}\sqrt{-\frac{1}{3}}\left(-\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}\right)^{\frac{1}{4}}+x\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{3}}\left(-\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}\right)^{\frac{1}{4}}\log\left(-3\sqrt{\frac{1}{3}}\sqrt{-\frac{1}{3}}\left(-\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}\right)^{\frac{1}{4}}+x\right)$$

input
```
integrate(x^4/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/4*sqrt(1/3)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/3)*sqrt(-1/3
)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2)) + x) + 1/4*sqrt(1/3)*sqrt(-sqrt(3/2*sq
rt(-1/3) + 1/2))*log(-3*sqrt(1/3)*sqrt(-1/3)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1
/2)) + x) + 1/4*sqrt(1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/
3)*sqrt(-1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2)) + x) - 1/4*sqrt(1/3)*sqrt
(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3)*sqrt(-1/3)*sqrt(-sqrt(-3/2
*sqrt(-1/3) + 1/2)) + x) - 1/4*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4)*log(
3*sqrt(1/3)*sqrt(-1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4) + x) + 1/4*sqrt(1/3)*(
3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*sqrt(-1/3)*(3/2*sqrt(-1/3) +
1/2)^(1/4) + x) + 1/4*sqrt(1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1/4)*log(3*sqrt(1
/3)*sqrt(-1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1/4) + x) - 1/4*sqrt(1/3)*(-3/2*sq
rt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*sqrt(-1/3)*(-3/2*sqrt(-1/3) + 1/2)^
(1/4) + x)
```

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.09

$$\int \frac{x^4}{1-x^4+x^8}\,dx$$
$$= \text{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-18432t^5 + 4t + x\right)\right)\right)$$

input
```
integrate(x**4/(x**8-x**4+1),x)
```

output
```
RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4
*_t + x)))
```

Maxima [F]

$$\int \frac{x^4}{1-x^4+x^8}\,dx = \int \frac{x^4}{x^8-x^4+1}\,dx$$

input
```
integrate(x^4/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^4/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{1-x^4+x^8}\,dx = \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

input

```
integrate(x^4/(x^8-x^4+1),x, algorithm="giac")
```

output

```
1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(
2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6
) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sq
rt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6
) - sqrt(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 18.67 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.74

$$\int \frac{x^4}{1 - x^4 + x^8} \, dx = \text{Too large to display}$$

input
```
int(x^4/(x^8 - x^4 + 1),x)
```

output
```
(2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(
1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) -
(2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1
)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i
+ 1)^(1/4)*1i)/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/
2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) - (3^(1/2)*x*
(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3
^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan((x*(8 - 3
^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)
*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3
^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4
)*1i)/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*
((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1
/2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(
1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(
3^(1/2)*1i + 1)^(1/4))/12
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.49

$$\int \frac{x^4}{1-x^4+x^8}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{6} - \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{6} + \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} - \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{24} - \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{12} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{12} - \frac{\sqrt{-\sqrt{3}+2}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} - \frac{\sqrt{6}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{48} + \frac{\sqrt{6}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{48} + \frac{\sqrt{2}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{16} - \frac{\sqrt{2}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{16}$$

input
```
int(x^4/(x^8-x^4+1),x)
```

output
```
( - 8*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt(
 - sqrt(3) + 2))) - 12*sqrt( - sqrt(3) + 2)*atan((sqrt(6) + sqrt(2) - 4*x)
/(2*sqrt( - sqrt(3) + 2))) + 8*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6)
+ sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + 12*sqrt( - sqrt(3) + 2)*atan(
(sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 2*sqrt(6)*atan((2*sq
rt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) + 6*sqrt(2)*atan((2*sqrt( -
 sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) + 2*sqrt(6)*atan((2*sqrt( - sqrt
(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) - 6*sqrt(2)*atan((2*sqrt( - sqrt(3) +
 2) + 4*x)/(sqrt(6) + sqrt(2))) - 4*sqrt( - sqrt(3) + 2)*sqrt(3)*log( - sq
rt( - sqrt(3) + 2)*x + x**2 + 1) + 4*sqrt( - sqrt(3) + 2)*sqrt(3)*log(sqrt
( - sqrt(3) + 2)*x + x**2 + 1) - 6*sqrt( - sqrt(3) + 2)*log( - sqrt( - sqr
t(3) + 2)*x + x**2 + 1) + 6*sqrt( - sqrt(3) + 2)*log(sqrt( - sqrt(3) + 2)*
x + x**2 + 1) - sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) + s
qrt(6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2) + 3*sqrt(2)*log(( - sqr
t(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) - 3*sqrt(2)*log((sqrt(6)*x + sqrt(2)*x
 + 2*x**2 + 2)/2))/48
```

3.95 $\int \frac{x^2}{1-x^4+x^8}\,dx$

Optimal result 743
Mathematica [C] (verified) 744
Rubi [A] (verified) 744
Maple [C] (verified) 747
Fricas [A] (verification not implemented) 748
Sympy [A] (verification not implemented) 748
Maxima [F] 749
Giac [A] (verification not implemented) 749
Mupad [B] (verification not implemented) 750
Reduce [B] (verification not implemented) 751

Optimal result

Integrand size = 16, antiderivative size = 273

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

output
```
1/4*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2
^(1/2)+1/2*6^(1/2))-1/4*arctan((1/2*6^(1/2)+1/2*2^(1/2)-2*x)/(1/2*6^(1/2)-
1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/4*arctan((1/2*6^(1/2)-1/2*2^(1/2
)+2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((1/
2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(
1/2))-1/4*arctanh((1/2*6^(1/2)-1/2*2^(1/2))*x/(x^2+1))/(3/2*2^(1/2)-1/2*6^
(1/2))+1/4*arctanh((1/2*6^(1/2)+1/2*2^(1/2))*x/(x^2+1))/(3/2*2^(1/2)+1/2*6
^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1+2\#1^5}\&\right]$$

input
```
Integrate[x^2/(1 - x^4 + x^8),x]
```

output
```
RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1 + 2*#1^5) & ]/4
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.49, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1709, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{1709}$$

$$\frac{\int \frac{1}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int \frac{1}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{1407}$$

$$\frac{\frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d(2x+}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d(2x+}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{\frac{\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

input
```
Int[x^2/(1 - x^4 + x^8),x]
```

output
```
-1/2*((Sqrt[(2 - Sqrt[3])/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)
/Sqrt[2 + Sqrt[3]]] - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/2)/(2*Sqrt[2 - Sq
rt[3]]) + (Sqrt[(2 - Sqrt[3])/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]] + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/2)/(2*Sqrt[2 -
 Sqrt[3]]))/Sqrt[3] + ((Sqrt[(2 + Sqrt[3])/(2 - Sqrt[3])]*ArcTan[(-Sqrt[2
+ Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/
2)/(2*Sqrt[2 + Sqrt[3]]) + (Sqrt[(2 + Sqrt[3])/(2 - Sqrt[3])]*ArcTan[(Sqrt
[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^
2]/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	40

input
```
int(x^2/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^2/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/3)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/3)*sqrt(3/2*s
qrt(-1/3) + 1/2)*(sqrt(-1/3) + 1)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2)) + 2*x)
 - 1/4*sqrt(1/3)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3)*sqrt(3
/2*sqrt(-1/3) + 1/2)*(sqrt(-1/3) + 1)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2)) +
2*x) - 1/4*sqrt(1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/3)*(s
qrt(-1/3) - 1)*sqrt(-3/2*sqrt(-1/3) + 1/2)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/
2)) + 2*x) + 1/4*sqrt(1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(
1/3)*(sqrt(-1/3) - 1)*sqrt(-3/2*sqrt(-1/3) + 1/2)*sqrt(-sqrt(-3/2*sqrt(-1/
3) + 1/2)) + 2*x) - 1/4*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4)*log(3*sqrt(
1/3)*(3/2*sqrt(-1/3) + 1/2)^(3/4)*(sqrt(-1/3) + 1) + 2*x) + 1/4*sqrt(1/3)*
(3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(3/4)
*(sqrt(-1/3) + 1) + 2*x) + 1/4*sqrt(1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1/4)*log
(3*sqrt(1/3)*(sqrt(-1/3) - 1)*(-3/2*sqrt(-1/3) + 1/2)^(3/4) + 2*x) - 1/4*s
qrt(1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*(sqrt(-1/3) - 1)*(
-3/2*sqrt(-1/3) + 1/2)^(3/4) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-442368t^7-192t^3+x\right)\right)\right)$$

input
```
integrate(x**2/(x**8-x**4+1),x)
```

output
```
RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 -
192*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \int \frac{x^2}{x^8-x^4+1}\,dx$$

input
```
integrate(x^2/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^2/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{x^2}{1-x^4+x^8}\,dx =& \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\ &+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

input
```
integrate(x^2/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(
2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6
) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x
*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sq
rt(6) - sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6
) - sqrt(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 19.10 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}+\frac{\sqrt{3}\,x\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{\sqrt{3}\,x\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} - \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{2^{3/4}\,\sqrt{3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}+\frac{2^{3/4}\,\sqrt{3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(x^2/(x^8 - x^4 + 1),x)
```

output
```
(3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)) - (3^(1/2
)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1/4))/
12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)) + (3^(1
/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1
/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*(
3^(1/2)*1i - 1)) - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/
2)*1i - 1)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4
)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) + (2^(3/4)*3^(1/2)*x*(
3^(1/2)*1i + 1)^(1/4))/(2*(3^(1/2)*1i - 1)))*(3^(1/2)*1i + 1)^(1/4))/12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.49

$$\int \frac{x^2}{1-x^4+x^8}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{6} - \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{6} + \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} - \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{24} - \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{12} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{12} + \frac{\sqrt{-\sqrt{3}+2}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} + \frac{\sqrt{6}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{48} - \frac{\sqrt{6}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{48} - \frac{\sqrt{2}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{16} + \frac{\sqrt{2}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{16}$$

input
```
int(x^2/(x^8-x^4+1),x)
```

output
```
( - 8*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt(
 - sqrt(3) + 2))) - 12*sqrt( - sqrt(3) + 2)*atan((sqrt(6) + sqrt(2) - 4*x)
/(2*sqrt( - sqrt(3) + 2))) + 8*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6)
+ sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + 12*sqrt( - sqrt(3) + 2)*atan(
(sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 2*sqrt(6)*atan((2*sq
rt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) + 6*sqrt(2)*atan((2*sqrt( -
 sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) + 2*sqrt(6)*atan((2*sqrt( - sqrt
(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) - 6*sqrt(2)*atan((2*sqrt( - sqrt(3) +
 2) + 4*x)/(sqrt(6) + sqrt(2))) + 4*sqrt( - sqrt(3) + 2)*sqrt(3)*log( - sq
rt( - sqrt(3) + 2)*x + x**2 + 1) - 4*sqrt( - sqrt(3) + 2)*sqrt(3)*log(sqrt
( - sqrt(3) + 2)*x + x**2 + 1) + 6*sqrt( - sqrt(3) + 2)*log( - sqrt( - sqr
t(3) + 2)*x + x**2 + 1) - 6*sqrt( - sqrt(3) + 2)*log(sqrt( - sqrt(3) + 2)*
x + x**2 + 1) + sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) - s
qrt(6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2) - 3*sqrt(2)*log(( - sqr
t(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) + 3*sqrt(2)*log((sqrt(6)*x + sqrt(2)*x
 + 2*x**2 + 2)/2))/48
```

3.96 $\int \frac{1}{1-x^4+x^8}\,dx$

Optimal result . . . 753
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Rubi [A] (verified) . . . 754
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Optimal result

Integrand size = 12, antiderivative size = 219

$$\int \frac{1}{1-x^4+x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right)}{2\sqrt{6}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)}{2\sqrt{6}}$$

output
```
-1/12*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1
/2)-1/12*arctan((1/2*6^(1/2)+1/2*2^(1/2)-2*x)/(1/2*6^(1/2)-1/2*2^(1/2)))*6
^(1/2)+1/12*arctan((1/2*6^(1/2)-1/2*2^(1/2)+2*x)/(1/2*6^(1/2)+1/2*2^(1/2))
)*6^(1/2)+1/12*arctan((1/2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)-1/2*2^(1/
2)))*6^(1/2)+1/12*arctanh((1/2*6^(1/2)-1/2*2^(1/2))*x/(x^2+1))*6^(1/2)+1/1
2*arctanh((1/2*6^(1/2)+1/2*2^(1/2))*x/(x^2+1))*6^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.19

$$\int \frac{1}{1-x^4+x^8}\,dx=\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 - x^4 + x^8)^(-1),x]
```

output
```
RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {1684, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{ 1684}$$

$$\frac{\int \frac{\sqrt{3}-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{\frac{\int \frac{\left(1-\sqrt{3}\right)x+\sqrt{3\left(2-\sqrt{3}\right)}}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(1-\sqrt{3}\right)x}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(1+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(1+\sqrt{3}\right)x+\sqrt{3\left(2+\sqrt{3}\right)}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{\frac{-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\cdots}}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$
$$\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-\cdots}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

input
```
Int[(1 - x^4 + x^8)^(-1),x]
```

output
```
((Sqrt[2/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]
]] + ((1 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqr
t[3]]) + (Sqrt[2/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 +
Sqrt[3]]] - ((1 - Sqrt[3])*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[
2 - Sqrt[3]]))/(2*Sqrt[3]) + ((Sqrt[2/(2 - Sqrt[3])]*ArcTan[(-Sqrt[2 + Sqr
t[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((1 + Sqrt[3])*Log[1 - Sqrt[2 + Sqrt[3]]
*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (Sqrt[2/(2 - Sqrt[3])]*ArcTan[(Sqrt[
2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + ((1 + Sqrt[3])*Log[1 + Sqrt[2 + S
qrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1684
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x^(
n/2))/(q - r*x^(n/2) + x^n), x], x] + Simp[1/(2*c*q*r)   Int[(r + x^(n/2))/
(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	30
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	30

input
```
int(1/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.59

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{1}{4}\sqrt{\frac{2}{3}}\arctan\left(4\,x^2+3\,\sqrt{\frac{2}{3}}\left(x^3+2\,x\right)+3\right) + \frac{1}{4}\sqrt{\frac{2}{3}}\arctan\left(-4\,x^2+3\,\sqrt{\frac{2}{3}}\left(x^3+2\,x\right)-3\right) + \frac{1}{4}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{2}{3}}x+\frac{1}{3}\right) + \frac{1}{4}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{2}{3}}x-\frac{1}{3}\right) + \frac{1}{8}\sqrt{\frac{2}{3}}\log\left(x^4+3\,x^2+3\,\sqrt{\frac{2}{3}}\left(x^3+x\right)+1\right) - \frac{1}{8}\sqrt{\frac{2}{3}}\log\left(x^4+3\,x^2-3\,\sqrt{\frac{2}{3}}\left(x^3+x\right)+1\right)$$

input
```
integrate(1/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(2/3)*arctan(4*x^2 + 3*sqrt(2/3)*(x^3 + 2*x) + 3) + 1/4*sqrt(2/3)*
arctan(-4*x^2 + 3*sqrt(2/3)*(x^3 + 2*x) - 3) + 1/4*sqrt(2/3)*arctan(sqrt(2
/3)*x + 1/3) + 1/4*sqrt(2/3)*arctan(sqrt(2/3)*x - 1/3) + 1/8*sqrt(2/3)*log
(x^4 + 3*x^2 + 3*sqrt(2/3)*(x^3 + x) + 1) - 1/8*sqrt(2/3)*log(x^4 + 3*x^2
- 3*sqrt(2/3)*(x^3 + x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.75

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} + \frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} - \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} + \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

input
```
integrate(1/(x**8-x**4+1),x)
```

output
```
sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3
+ 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2
 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*
x + 1)/24
```

Maxima [F]

$$\int \frac{1}{1-x^4+x^8}\,dx = \int \frac{1}{x^8-x^4+1}\,dx$$

input
```
integrate(1/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94

$$
\begin{aligned}
\int \frac{1}{1-x^4+x^8}\,dx = {} & \frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\
& + \frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\
& + \frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\
& + \frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\
& + \frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\
& - \frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\
& + \frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right) \\
& - \frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right)
\end{aligned}
$$

input
```
integrate(1/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(
6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*ar
ctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2
 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6)
+ sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) -
1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.24

$$\int \frac{1}{1-x^4+x^8} \, dx = \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right) + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

input

```
int(1/(x^8 - x^4 + 1),x)
```

output

```
- 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/12)
 - 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/12
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.44

$$\int \frac{1}{1-x^4+x^8} \, dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} - \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} + \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} - \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{12} + \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{12} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} - \frac{\sqrt{6}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24} + \frac{\sqrt{6}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24}$$

input
```
int(1/(x^8-x^4+1),x)
```

output
```
( - 2*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt(
 - sqrt(3) + 2))) - 6*sqrt( - sqrt(3) + 2)*atan((sqrt(6) + sqrt(2) - 4*x)/
(2*sqrt( - sqrt(3) + 2))) + 2*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) +
 sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + 6*sqrt( - sqrt(3) + 2)*atan((s
qrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 2*sqrt(6)*atan((2*sqrt
( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) + 2*sqrt(6)*atan((2*sqrt( - s
qrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) - sqrt( - sqrt(3) + 2)*sqrt(3)*log
( - sqrt( - sqrt(3) + 2)*x + x**2 + 1) + sqrt( - sqrt(3) + 2)*sqrt(3)*log(
sqrt( - sqrt(3) + 2)*x + x**2 + 1) - 3*sqrt( - sqrt(3) + 2)*log( - sqrt( -
 sqrt(3) + 2)*x + x**2 + 1) + 3*sqrt( - sqrt(3) + 2)*log(sqrt( - sqrt(3) +
 2)*x + x**2 + 1) - sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2)
 + sqrt(6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2))/24
```

3.97 $\int \frac{1}{x^2(1-x^4+x^8)}\,dx$

Optimal result 763
Mathematica [C] (verified) 764
Rubi [A] (verified) 764
Maple [C] (verified) 769
Fricas [A] (verification not implemented) 769
Sympy [A] (verification not implemented) 770
Maxima [F] 771
Giac [A] (verification not implemented) 771
Mupad [B] (verification not implemented) 772
Reduce [B] (verification not implemented) 773

Optimal result

Integrand size = 16, antiderivative size = 290

$$
\begin{aligned}
\int \frac{1}{x^2(1-x^4+x^8)}\,dx = & -\frac{1}{x}+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) \\
& -\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) \\
& -\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right) \\
& +\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right) \\
& -\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right) \\
& +\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)
\end{aligned}
$$

output
```
-1/x+1/4*(1/2*2^(1/2)+1/6*6^(1/2))*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1
/2*6^(1/2)+1/2*2^(1/2)))-1/4*(1/2*2^(1/2)-1/6*6^(1/2))*arctan((1/2*6^(1/2)
+1/2*2^(1/2)-2*x)/(1/2*6^(1/2)-1/2*2^(1/2)))-1/4*(1/2*2^(1/2)+1/6*6^(1/2))
*arctan((1/2*6^(1/2)-1/2*2^(1/2)+2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))+1/4*(1/2*
2^(1/2)-1/6*6^(1/2))*arctan((1/2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)-1/2
*2^(1/2)))-1/4*(1/2*2^(1/2)-1/6*6^(1/2))*arctanh((1/2*6^(1/2)-1/2*2^(1/2))
*x/(x^2+1))+1/4*(1/2*2^(1/2)+1/6*6^(1/2))*arctanh((1/2*6^(1/2)+1/2*2^(1/2)
)*x/(x^2+1))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^2 (1 - x^4 + x^8)} dx = -\frac{1}{x} - \frac{1}{4}\text{RootSum}\left[1 - \#1^4 + \#1^8\&, \frac{-\log(x - \#1) + \log(x - \#1)\#1^4}{-\#1 + 2\#1^5}\&\right]$$

input
```
Integrate[1/(x^2*(1 - x^4 + x^8)),x]
```

output
```
-x^(-1) - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-
#1 + 2*#1^5) & ]/4
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.26, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1704, 1830, 1602, 25, 1483, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^8 - x^4 + 1)} dx$$

$$\Big\downarrow \text{1704}$$

$$\int \frac{x^2 \left(1-x^4\right)}{x^8-x^4+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{1830}$$

$$\frac{\int \frac{x^2 \left(\sqrt{3}-2 x^2\right)}{x^4-\sqrt{3} x^2+1}dx}{2 \sqrt{3}}+\frac{\int \frac{x^2 \left(2 x^2+\sqrt{3}\right)}{x^4+\sqrt{3} x^2+1}dx}{2 \sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{1602}$$

$$\frac{-\int -\frac{2-\sqrt{3} x^2}{x^4-\sqrt{3} x^2+1}dx-2 x}{2 \sqrt{3}}+\frac{2 x-\int \frac{\sqrt{3} x^2+2}{x^4+\sqrt{3} x^2+1}dx}{2 \sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2-\sqrt{3} x^2}{x^4-\sqrt{3} x^2+1}dx-2 x}{2 \sqrt{3}}+\frac{2 x-\int \frac{\sqrt{3} x^2+2}{x^4+\sqrt{3} x^2+1}dx}{2 \sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{1483}$$

$$\frac{-\frac{\int \frac{2 \sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right) x}{x^2-\sqrt{2-\sqrt{3}} x+1}dx}{2 \sqrt{2-\sqrt{3}}}-\frac{\int \frac{\sqrt{2-\sqrt{3}} \left(\sqrt{2-\sqrt{3}} x+2\right)}{x^2+\sqrt{2-\sqrt{3}} x+1}dx}{2 \sqrt{2-\sqrt{3}}}+2 x}{2 \sqrt{3}}+\frac{\frac{\int \frac{2 \sqrt{2+\sqrt{3}}-\left(2+\sqrt{3}\right) x}{x^2-\sqrt{2+\sqrt{3}} x+1}dx}{2 \sqrt{2+\sqrt{3}}}+\frac{\int \frac{\sqrt{2+\sqrt{3}} \left(\sqrt{2+\sqrt{3}} x+2\right)}{x^2+\sqrt{2+\sqrt{3}} x+1}dx}{2 \sqrt{2+\sqrt{3}}}-2 x}{2 \sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{2 \sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right) x}{x^2-\sqrt{2-\sqrt{3}} x+1}dx}{2 \sqrt{2-\sqrt{3}}}-\frac{1}{2} \int \frac{\sqrt{2-\sqrt{3}} x+2}{x^2+\sqrt{2-\sqrt{3}} x+1}dx+2 x}{2 \sqrt{3}}+\frac{\frac{\int \frac{2 \sqrt{2+\sqrt{3}}-\left(2+\sqrt{3}\right) x}{x^2-\sqrt{2+\sqrt{3}} x+1}dx}{2 \sqrt{2+\sqrt{3}}}+\frac{1}{2} \int \frac{\sqrt{2+\sqrt{3}} x+2}{x^2+\sqrt{2+\sqrt{3}} x+1}dx-2 x}{2 \sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{1142}$$

$$\frac{-\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{2x+\cdots}{x^2+\sqrt{\cdots}}\cdots\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\left(2+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\cdots}}{x^2+\sqrt{2+\sqrt{\cdots}}}\cdots\right.}{2\sqrt{3}}$$

$$\frac{1}{x}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{2x+\sqrt{\cdots}}{x^2+\sqrt{2-\cdots}}\cdots\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\sqrt{\cdots}}}{x^2+\sqrt{2+\sqrt{3}}x\cdots}\cdots\right.}{2\sqrt{3}}$$

$$\frac{1}{x}$$

$$\Big\downarrow \; 1083$$

$$\frac{-\frac{\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(\left(2+\sqrt{3}\right)\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x\cdots\right.\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\left(2-\sqrt{\cdots}\right.\right.}{2\sqrt{3}}$$

$$\frac{1}{x}$$

$$\Big\downarrow \; 217$$

$$\frac{-\frac{\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2-\cdots}}{\sqrt{2+\sqrt{3}}}\right.\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)\cdots\right.}{2\sqrt{3}}$$

$$\frac{1}{x}$$

$\Big\downarrow$ 1103

$$\frac{-\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\sqrt{2+\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\sqrt{2-\sqrt{3}}\log\left(x^2+\sqrt{2}\right.\right.}{2\sqrt{3}}$$

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\sqrt{2+\sqrt{3}}\log\left(x^2+\sqrt{2+}\right.\right.}{2\sqrt{3}}$$

$$\frac{1}{x}$$

input
```
Int[1/(x^2*(1 - x^4 + x^8)),x]
```

output
```
-x^(-1) + (2*x - (ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] - (
(2 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]])
 + (-(Sqrt[2 + Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]
]) - (Sqrt[2 - Sqrt[3]]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/2)/(2*Sqrt[
3]) + (-2*x + (ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((2
+ Sqrt[3])*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) +
(Sqrt[2 - Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + (
Sqrt[2 + Sqrt[3]]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/2)/2)/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1830
```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Simp[c/(2*q*r)   Int[(f*x)^m*(Simp[d*r - (c*d - e*q)*x^(n/2), x]
/(q - r*x^(n/2) + c*x^n)), x], x] + Simp[c/(2*q*r)   Int[(f*x)^m*(Simp[d*r
+ (c*d - e*q)*x^(n/2), x]/(q + r*x^(n/2) + c*x^n)), x], x]] /;  !LtQ[2*c*q
- b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a
*c, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.14

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(81\_Z^{8}-9\_Z^{4}+1\right)}\_R\ln\left(-27\_R^{7}+6\_R^{3}+x\right)\right)}{4}$	40
default	$-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{6}-\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}-\frac{1}{x}$	52

input
```
int(1/x^2/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/4*sum(_R*ln(-27*_R^7+6*_R^3+x),_R=RootOf(81*_Z^8-9*_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^2\left(1-x^4+x^8\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/4*(sqrt(1/3)*x*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/3)*sqrt(3/
2*sqrt(-1/3) + 1/2)*(sqrt(-1/3) - 1)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2)) + 2
*x) - sqrt(1/3)*x*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3)*sqrt(
3/2*sqrt(-1/3) + 1/2)*(sqrt(-1/3) - 1)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2)) +
 2*x) - sqrt(1/3)*x*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/3)*(sq
rt(-1/3) + 1)*sqrt(-3/2*sqrt(-1/3) + 1/2)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2
)) + 2*x) + sqrt(1/3)*x*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3
)*(sqrt(-1/3) + 1)*sqrt(-3/2*sqrt(-1/3) + 1/2)*sqrt(-sqrt(-3/2*sqrt(-1/3)
+ 1/2)) + 2*x) - sqrt(1/3)*x*(3/2*sqrt(-1/3) + 1/2)^(1/4)*log(3*sqrt(1/3)*
(3/2*sqrt(-1/3) + 1/2)^(3/4)*(sqrt(-1/3) - 1) + 2*x) + sqrt(1/3)*x*(3/2*sq
rt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(3/4)*(sqrt(
-1/3) - 1) + 2*x) + sqrt(1/3)*x*(-3/2*sqrt(-1/3) + 1/2)^(1/4)*log(3*sqrt(1
/3)*(sqrt(-1/3) + 1)*(-3/2*sqrt(-1/3) + 1/2)^(3/4) + 2*x) - sqrt(1/3)*x*(-
3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*(sqrt(-1/3) + 1)*(-3/2*sqrt(-
1/3) + 1/2)^(3/4) + 2*x) - 4)/x
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^2(1-x^4+x^8)}\,dx$$
$$= \text{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-442368t^7+384t^3+x\right)\right)\right)-\frac{1}{x}$$

input
```
integrate(1/x**2/(x**8-x**4+1),x)
```

output
```
RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 +
384*_t**3 + x))) - 1/x
```

Maxima [F]

$$\int \frac{1}{x^2(1-x^4+x^8)}\,dx = \int \frac{1}{(x^8-x^4+1)x^2}\,dx$$

input
```
integrate(1/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/x - integrate((x^6 - x^2)/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{1}{x^2(1-x^4+x^8)}\,dx = & -\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\ & -\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\ & -\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\ & -\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\ & +\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\ & -\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\ & +\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) \\ & -\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{x}\end{aligned}$$

input
```
integrate(1/x^2/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sq
rt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt
(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(
6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*
x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(s
qrt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(
6) - sqrt(2)) + 1) - 1/x
```

Mupad [B] (verification not implemented)

Time = 19.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2(1-x^4+x^8)}\,dx = -\frac{1}{x} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} + \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}-\frac{2^{3/4}\sqrt{3}\,x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{12} - \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}+\frac{2^{3/4}\sqrt{3}\,x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(1/(x^2*(x^8 - x^4 + 1)),x)
```

output
```
(3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)) + (3^(1/2)*x
*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4)*1
i)/12 - 1/x - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i -
 1)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/
2)*8i)^(1/4))/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x)/(2*(3^(1/2)*1i + 1)^(
3/4)) - (2^(3/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1
)^(1/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*1i)/(2*(3^(1/2)*1i + 1)^
(3/4)) + (2^(3/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^
(1/4))/12
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(1-x^4+x^8)}\,dx = \frac{-4\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x+4\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)x+2\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)\cdots}{\cdots}$$

input
```
int(1/x^2/(x^8-x^4+1),x)
```

output
```
( - 4*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt(
 - sqrt(3) + 2)))*x + 4*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(
2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*x + 2*sqrt(6)*atan((2*sqrt( - sqrt(3)
+ 2) - 4*x)/(sqrt(6) + sqrt(2)))*x + 6*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2
) - 4*x)/(sqrt(6) + sqrt(2)))*x - 2*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) +
 4*x)/(sqrt(6) + sqrt(2)))*x - 6*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) + 4*
x)/(sqrt(6) + sqrt(2)))*x + 2*sqrt( - sqrt(3) + 2)*sqrt(3)*log( - sqrt( -
sqrt(3) + 2)*x + x**2 + 1)*x - 2*sqrt( - sqrt(3) + 2)*sqrt(3)*log(sqrt( -
sqrt(3) + 2)*x + x**2 + 1)*x - sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x
**2 + 2)/2)*x + sqrt(6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2)*x - 3*
sqrt(2)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2)*x + 3*sqrt(2)*log((
sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2)*x - 48)/(48*x)
```

3.98 $\int \frac{1}{x^4(1-x^4+x^8)}\,dx$

Optimal result 774
Mathematica [C] (verified) 775
Rubi [A] (verified) 775
Maple [C] (verified) 779
Fricas [A] (verification not implemented) 780
Sympy [A] (verification not implemented) 780
Maxima [F] 781
Giac [A] (verification not implemented) 781
Mupad [B] (verification not implemented) 782
Reduce [B] (verification not implemented) 783

Optimal result

Integrand size = 16, antiderivative size = 292

$$\begin{aligned}\int \frac{1}{x^4(1-x^4+x^8)}\,dx &= -\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\
&\quad+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\
&\quad+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)\\
&\quad-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)\\
&\quad-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\text{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right)\\
&\quad+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\text{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)\end{aligned}$$

output
```
-1/3/x^3-1/4*(1/2*2^(1/2)+1/6*6^(1/2))*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x
)/(1/2*6^(1/2)+1/2*2^(1/2)))+1/4*(1/2*2^(1/2)-1/6*6^(1/2))*arctan((1/2*6^(
1/2)+1/2*2^(1/2)-2*x)/(1/2*6^(1/2)-1/2*2^(1/2)))+1/4*(1/2*2^(1/2)+1/6*6^(1
/2))*arctan((1/2*6^(1/2)-1/2*2^(1/2)+2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))-1/4*(
1/2*2^(1/2)-1/6*6^(1/2))*arctan((1/2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)
-1/2*2^(1/2)))-1/4*(1/2*2^(1/2)-1/6*6^(1/2))*arctanh((1/2*6^(1/2)-1/2*2^(1
/2))*x/(x^2+1))+1/4*(1/2*2^(1/2)+1/6*6^(1/2))*arctanh((1/2*6^(1/2)+1/2*2^(
1/2))*x/(x^2+1))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^4 (1-x^4+x^8)} dx = -\frac{1}{3x^3} - \frac{1}{4}\text{RootSum}\left[1-\#1^4 + \#1^8\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[1/(x^4*(1 - x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)
/(-#1^3 + 2*#1^7) & ]/4
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1704, 27, 1751, 25, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^8-x^4+1)} dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{3}\int \frac{3\left(1-x^4\right)}{x^8-x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1-x^4}{x^8-x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1751}$$

$$-\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1483}$$

$$\frac{\frac{\int \frac{\left(2-\sqrt{3}\right)x+\sqrt{3\left(2-\sqrt{3}\right)}}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(2-\sqrt{3}\right)x}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(2+\sqrt{3}\right)x+\sqrt{3\left(2+\sqrt{3}\right)}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\left(2-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\left(2+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{3x^3}$$

$\downarrow$ 1083

$$\frac{\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{}{-\left(2x+\sqrt{\cdots}\right.}}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x+\sqrt{2+\cdots}\right.}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{1}{3x^3}$$

$\downarrow$ 217

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{3x^3}$$

$\downarrow$ 1103

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{3x^3}$$

input

```
Int[1/(x^4*(1 - x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 + ((ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] + ((2
- Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]) +
(ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] - ((2 - Sqrt[3])*Log[
1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]))/(2*Sqrt[3]) + ((
-ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((2 + Sqrt[3])*Log
[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (-ArcTan[(Sqrt
[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + ((2 + Sqrt[3])*Log[1 + Sqrt[2 +
Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1751
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x
^(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Simp[e/(2*c*q)   Int[(q +
2*x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGt
Q[n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.13

method	result	size
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(-9\_R^5+2\_R+x\right)\right)}{4}$	38
default	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	50

input
```
int(1/x^4/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+1/4*sum(_R*ln(-9*_R^5+2*_R+x),_R=RootOf(81*_Z^8-9*_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^4 (1-x^4+x^8)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/12*(3*sqrt(1/3)*x^3*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/3)*(
sqrt(-1/3) - 1)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2)) + 2*x) - 3*sqrt(1/3)*x^3
*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3)*(sqrt(-1/3) - 1)*sqrt(
-sqrt(3/2*sqrt(-1/3) + 1/2)) + 2*x) - 3*sqrt(1/3)*x^3*sqrt(-sqrt(-3/2*sqrt
(-1/3) + 1/2))*log(3*sqrt(1/3)*(sqrt(-1/3) + 1)*sqrt(-sqrt(-3/2*sqrt(-1/3)
 + 1/2)) + 2*x) + 3*sqrt(1/3)*x^3*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(-
3*sqrt(1/3)*(sqrt(-1/3) + 1)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2)) + 2*x) + 3
*sqrt(1/3)*x^3*(3/2*sqrt(-1/3) + 1/2)^(1/4)*log(3*sqrt(1/3)*(3/2*sqrt(-1/3
) + 1/2)^(1/4)*(sqrt(-1/3) - 1) + 2*x) - 3*sqrt(1/3)*x^3*(3/2*sqrt(-1/3) +
 1/2)^(1/4)*log(-3*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4)*(sqrt(-1/3) - 1)
 + 2*x) - 3*sqrt(1/3)*x^3*(-3/2*sqrt(-1/3) + 1/2)^(1/4)*log(3*sqrt(1/3)*(s
qrt(-1/3) + 1)*(-3/2*sqrt(-1/3) + 1/2)^(1/4) + 2*x) + 3*sqrt(1/3)*x^3*(-3/
2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*(sqrt(-1/3) + 1)*(-3/2*sqrt(-1/
3) + 1/2)^(1/4) + 2*x) + 4)/x^3
```

Sympy [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^4 (1-x^4+x^8)} \, dx = \operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(-9216t^5 + 8t + x\right)\right)\right) - \frac{1}{3x^3}$$

input
```
integrate(1/x**4/(x**8-x**4+1),x)
```

output
```
RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-9216*_t**5 + 8*
_t + x))) - 1/(3*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (1 - x^4 + x^8)} \, dx = \int \frac{1}{(x^8 - x^4 + 1)x^4} \, dx$$

input
```
integrate(1/x^4/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/3/x^3 - integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{1}{x^4 (1 - x^4 + x^8)} \, dx = & \frac{1}{24} \left(\sqrt{6} + 3\sqrt{2}\right) \arctan\left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) \\ & + \frac{1}{24} \left(\sqrt{6} + 3\sqrt{2}\right) \arctan\left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) \\ & + \frac{1}{24} \left(\sqrt{6} - 3\sqrt{2}\right) \arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) \\ & + \frac{1}{24} \left(\sqrt{6} - 3\sqrt{2}\right) \arctan\left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) \\ & + \frac{1}{48} \left(\sqrt{6} + 3\sqrt{2}\right) \log\left(x^2 + \frac{1}{2}x\left(\sqrt{6} + \sqrt{2}\right) + 1\right) \\ & - \frac{1}{48} \left(\sqrt{6} + 3\sqrt{2}\right) \log\left(x^2 - \frac{1}{2}x\left(\sqrt{6} + \sqrt{2}\right) + 1\right) \\ & + \frac{1}{48} \left(\sqrt{6} - 3\sqrt{2}\right) \log\left(x^2 + \frac{1}{2}x\left(\sqrt{6} - \sqrt{2}\right) + 1\right) \\ & - \frac{1}{48} \left(\sqrt{6} - 3\sqrt{2}\right) \log\left(x^2 - \frac{1}{2}x\left(\sqrt{6} - \sqrt{2}\right) + 1\right) - \frac{1}{3x^3}\end{aligned}$$

input
```
integrate(1/x^4/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(
2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6
) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sq
rt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6
) - sqrt(2)) + 1) - 1/3/x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^4 (1-x^4+x^8)} \, dx = -\frac{1}{3\,x^3} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}+\frac{\sqrt{3}\,x\,1{}\mathrm{i}}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1{}\mathrm{i}}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}-\frac{\sqrt{3}\,x}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{12}$$

input
```
int(1/(x^4*(x^8 - x^4 + 1)),x)
```

output
```
(2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^
(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3
^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/
4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (3^(1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^
(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4))/12 - 1
/(3*x^3) + (2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4))
 + (2^(1/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))
/12
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^4 (1 - x^4 + x^8)}\, dx$$
$$= \frac{4\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x^3 - 4\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)x^3 - 2\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{}$$

input
```
int(1/x^4/(x^8-x^4+1),x)
```

output
```
(4*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( -
sqrt(3) + 2)))*x**3 - 4*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(
2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*x**3 - 2*sqrt(6)*atan((2*sqrt( - sqrt(
3) + 2) - 4*x)/(sqrt(6) + sqrt(2)))*x**3 - 6*sqrt(2)*atan((2*sqrt( - sqrt(
3) + 2) - 4*x)/(sqrt(6) + sqrt(2)))*x**3 + 2*sqrt(6)*atan((2*sqrt( - sqrt(
3) + 2) + 4*x)/(sqrt(6) + sqrt(2)))*x**3 + 6*sqrt(2)*atan((2*sqrt( - sqrt(
3) + 2) + 4*x)/(sqrt(6) + sqrt(2)))*x**3 + 2*sqrt( - sqrt(3) + 2)*sqrt(3)*
log( - sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x**3 - 2*sqrt( - sqrt(3) + 2)*sq
rt(3)*log(sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x**3 - sqrt(6)*log(( - sqrt(6
)*x - sqrt(2)*x + 2*x**2 + 2)/2)*x**3 + sqrt(6)*log((sqrt(6)*x + sqrt(2)*x
 + 2*x**2 + 2)/2)*x**3 - 3*sqrt(2)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2
+ 2)/2)*x**3 + 3*sqrt(2)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2)*x**3
- 16)/(48*x**3)
```

3.99 $\int \frac{1}{x^6(1-x^4+x^8)}\,dx$

Optimal result 784
Mathematica [C] (verified) 785
Rubi [A] (verified) 785
Maple [C] (verified) 789
Fricas [A] (verification not implemented) 790
Sympy [A] (verification not implemented) 790
Maxima [F] 791
Giac [A] (verification not implemented) 792
Mupad [B] (verification not implemented) 793
Reduce [B] (verification not implemented) 793

Optimal result

Integrand size = 16, antiderivative size = 231

$$\int \frac{1}{x^6\left(1-x^4+x^8\right)}\,dx = -\frac{1}{5x^5}-\frac{1}{x}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} +\frac{\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right)}{2\sqrt{6}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)}{2\sqrt{6}}$$

output
```
-1/5/x^5-1/x+1/12*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1/2*6^(1/2)+1/2*2^
(1/2)))*6^(1/2)+1/12*arctan((1/2*6^(1/2)+1/2*2^(1/2)-2*x)/(1/2*6^(1/2)-1/2
*2^(1/2)))*6^(1/2)-1/12*arctan((1/2*6^(1/2)-1/2*2^(1/2)+2*x)/(1/2*6^(1/2)+
1/2*2^(1/2)))*6^(1/2)-1/12*arctan((1/2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/
2)-1/2*2^(1/2)))*6^(1/2)+1/12*arctanh((1/2*6^(1/2)-1/2*2^(1/2))*x/(x^2+1))
*6^(1/2)+1/12*arctanh((1/2*6^(1/2)+1/2*2^(1/2))*x/(x^2+1))*6^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^6 (1 - x^4 + x^8)} \, dx = -\frac{1}{5x^5} - \frac{1}{x} - \frac{1}{4}\text{RootSum}\left[1 - \#1^4 + \#1^8\&, \frac{\log(x - \#1)\#1^3}{-1 + 2\#1^4}\&\right]$$

input
```
Integrate[1/(x^6*(1 - x^4 + x^8)),x]
```

output
```
-1/5*1/x^5 - x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 +
 2*#1^4) & ]/4
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1704, 27, 1828, 1708, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (x^8 - x^4 + 1)} \, dx$$

$$\Big\downarrow \text{ 1704}$$

$$\frac{1}{5} \int \frac{5(1 - x^4)}{x^2 (x^8 - x^4 + 1)} dx - \frac{1}{5x^5}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1 - x^4}{x^2 (x^8 - x^4 + 1)} dx - \frac{1}{5x^5}$$

$$\Big\downarrow \text{ 1828}$$

$$-\int \frac{x^6}{x^8 - x^4 + 1} dx - \frac{1}{5x^5} - \frac{1}{x}$$

$$\Big\downarrow \text{ 1708}$$

$$\frac{\int \frac{1-\sqrt{3}x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int \frac{\sqrt{3}x^2+1}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{x}$$

$\downarrow$ 1483

$$-\frac{\frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\left(1-\sqrt{3}\right)x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(1+\sqrt{3}\right)x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{x}$$

$\downarrow$ 1142

$$-\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{x}$$

$\downarrow$ 25

$$-\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{x}$$

$\downarrow$ 1083

$$-\frac{\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}}}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{1}{5x^5}-\frac{1}{x}$$

$\downarrow$ 217

$$-\frac{\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int\frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int\frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{x}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{x}$$

input
```
Int[1/(x^6*(1 - x^4 + x^8)),x]
```

output
```
-1/5*1/x^5 - x^(-1) - ((Sqrt[2/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] +
 2*x)/Sqrt[2 + Sqrt[3]]] - ((1 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^
2])/2)/(2*Sqrt[2 - Sqrt[3]]) + (Sqrt[2/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqr
t[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] + ((1 - Sqrt[3])*Log[1 + Sqrt[2 - Sqrt[3]]
*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]))/(2*Sqrt[3]) + ((-(Sqrt[2/(2 - Sqrt[3]
)]*ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]) - ((1 + Sqrt[3])*
Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (-(Sqrt[2/(
2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]) + ((1 +
 Sqrt[3])*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2
*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1708
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Simp[1/(2*c*r)   Int[x^
(m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Simp[1/(2*c
*r)   Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]
 /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2,
 0] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.19

method	result	size
default	$-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^{4}+1\right)}\_R\ln\left(9x\_R^{3}-3\_R^{2}+x^{2}\right)\right)}{4}-\frac{1}{5x^{5}}-\frac{1}{x}$	43
risch	$\frac{-x^{4}-\frac{1}{5}}{x^{5}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^{4}+1\right)}\_R\ln\left(-9x\_R^{3}-3\_R^{2}+x^{2}\right)\right)}{4}$	44

input
```
int(1/x^6/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))-1/5/x^5-1/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^6\left(1-x^4+x^8\right)}\,dx = -\frac{10\sqrt{\frac{2}{3}}x^5\arctan\left(4\,x^2+3\sqrt{\frac{2}{3}}(x^3+2\,x)+3\right)+10\sqrt{\frac{2}{3}}x^5\arctan\left(-4\,x^2+3\sqrt{\frac{2}{3}}(x^3+2\,x)-3\right)+10}{}$$

input
```
integrate(1/x^6/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/40*(10*sqrt(2/3)*x^5*arctan(4*x^2 + 3*sqrt(2/3)*(x^3 + 2*x) + 3) + 10*s
qrt(2/3)*x^5*arctan(-4*x^2 + 3*sqrt(2/3)*(x^3 + 2*x) - 3) + 10*sqrt(2/3)*x
^5*arctan(sqrt(2/3)*x + 1/3) + 10*sqrt(2/3)*x^5*arctan(sqrt(2/3)*x - 1/3)
- 5*sqrt(2/3)*x^5*log(x^4 + 3*x^2 + 3*sqrt(2/3)*(x^3 + x) + 1) + 5*sqrt(2/
3)*x^5*log(x^4 + 3*x^2 - 3*sqrt(2/3)*(x^3 + x) + 1) + 40*x^4 + 8)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \frac{1}{x^6\left(1-x^4+x^8\right)}\,dx &= \frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}\\ &\quad+\frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24}\\ &\quad-\frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24}\\ &\quad+\frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}+\frac{-5x^4-1}{5x^5}\end{aligned}$$

input
```
integrate(1/x**6/(x**8-x**4+1),x)
```

output
```
sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqr
t(6)*x - 3))/24 + sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*x**
3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x*
*2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6
)*x + 1)/24 + (-5*x**4 - 1)/(5*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (1-x^4+x^8)} \, dx = \int \frac{1}{(x^8-x^4+1)x^6} \, dx$$

input
```
integrate(1/x^6/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/5*(5*x^4 + 1)/x^5 - integrate(x^6/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.94

$$
\begin{aligned}
\int \frac{1}{x^6 (1 - x^4 + x^8)} \, dx = & -\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) \\
& - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) \\
& - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) \\
& - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) \\
& + \frac{1}{24}\sqrt{6}\log\left(x^2 + \frac{1}{2}x\left(\sqrt{6} + \sqrt{2}\right) + 1\right) \\
& - \frac{1}{24}\sqrt{6}\log\left(x^2 - \frac{1}{2}x\left(\sqrt{6} + \sqrt{2}\right) + 1\right) \\
& + \frac{1}{24}\sqrt{6}\log\left(x^2 + \frac{1}{2}x\left(\sqrt{6} - \sqrt{2}\right) + 1\right) \\
& - \frac{1}{24}\sqrt{6}\log\left(x^2 - \frac{1}{2}x\left(\sqrt{6} - \sqrt{2}\right) + 1\right) - \frac{5x^4 + 1}{5x^5}
\end{aligned}
$$

input

```
integrate(1/x^6/(x^8-x^4+1),x, algorithm="giac")
```

output

```
-1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12
*sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt
(6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*a
rctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^
2 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6)
 + sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) -
 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/5*(5*x^4 + 1)/x
^5
```

Mupad [B] (verification not implemented)

Time = 19.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^6 (1-x^4+x^8)} \, dx = -\frac{x^4+\frac{1}{5}}{x^5} + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right) + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

input
```
int(1/(x^6*(x^8 - x^4 + 1)),x)
```

output
```
6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) +
 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12)
- (x^4 + 1/5)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.57

$$\int \frac{1}{x^6 (1-x^4+x^8)} \, dx = \frac{10\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x^5 + 30\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x^5 - 10\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}}{2\sqrt{}}\right.}{}$$

input
```
int(1/x^6/(x^8-x^4+1),x)
```

output
```
(10*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( -
 sqrt(3) + 2)))*x**5 + 30*sqrt( - sqrt(3) + 2)*atan((sqrt(6) + sqrt(2) - 4
*x)/(2*sqrt( - sqrt(3) + 2)))*x**5 - 10*sqrt( - sqrt(3) + 2)*sqrt(3)*atan(
(sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*x**5 - 30*sqrt( - sqrt
(3) + 2)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*x**5 + 1
0*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2)))*x**5 -
10*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2)))*x**5 -
 5*sqrt( - sqrt(3) + 2)*sqrt(3)*log( - sqrt( - sqrt(3) + 2)*x + x**2 + 1)*
x**5 + 5*sqrt( - sqrt(3) + 2)*sqrt(3)*log(sqrt( - sqrt(3) + 2)*x + x**2 +
1)*x**5 - 15*sqrt( - sqrt(3) + 2)*log( - sqrt( - sqrt(3) + 2)*x + x**2 + 1
)*x**5 + 15*sqrt( - sqrt(3) + 2)*log(sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x*
*5 - 5*sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2)*x**5 + 5*sqr
t(6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2)*x**5 - 120*x**4 - 24)/(12
0*x**5)
```

3.100 $\int \frac{1}{x^8(1-x^4+x^8)}\,dx$

Optimal result 795
Mathematica [C] (verified) 796
Rubi [A] (verified) 796
Maple [C] (verified) 801
Fricas [A] (verification not implemented) 801
Sympy [A] (verification not implemented) 802
Maxima [F] 802
Giac [A] (verification not implemented) 803
Mupad [B] (verification not implemented) 804
Reduce [B] (verification not implemented) 804

Optimal result

Integrand size = 16, antiderivative size = 287

$$\int \frac{1}{x^8(1-x^4+x^8)}\,dx = -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\text{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\text{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

output

```
-1/7/x^7-1/3/x^3-1/4*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1/2*6^(1/2)+1/2
*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((1/2*6^(1/2)+1/2*2^(1/2)-2
*x)/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))+1/4*arctan((1/2*6
^(1/2)-1/2*2^(1/2)+2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2
))-1/4*arctan((1/2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/
2*2^(1/2)-1/2*6^(1/2))-1/4*arctanh((1/2*6^(1/2)-1/2*2^(1/2))*x/(x^2+1))/(3
/2*2^(1/2)-1/2*6^(1/2))+1/4*arctanh((1/2*6^(1/2)+1/2*2^(1/2))*x/(x^2+1))/(
3/2*2^(1/2)+1/2*6^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^8 (1-x^4+x^8)} \, dx = -\frac{1}{7x^7} - \frac{1}{3x^3} - \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)\#1}{-1+2\#1^4}\&\right]$$

input
```
Integrate[1/(x^8*(1 - x^4 + x^8)),x]
```

output
```
-1/7*1/x^7 - 1/(3*x^3) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1
+ 2*#1^4) & ]/4
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.31, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {1704, 27, 1828, 27, 1709, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (x^8-x^4+1)} \, dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{7}\int \frac{7(1-x^4)}{x^4 (x^8-x^4+1)}dx - \frac{1}{7x^7}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1-x^4}{x^4 (x^8-x^4+1)}dx - \frac{1}{7x^7}$$

$$\Big\downarrow \text{1828}$$

$$-\frac{1}{3}\int \frac{3x^4}{x^8-x^4+1}dx - \frac{1}{7x^7} - \frac{1}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^4}{x^8-x^4+1}dx-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1709}$$

$$-\frac{\int \frac{x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1447}$$

$$-\frac{\frac{1}{2}\int \frac{x^2+1}{x^4-\sqrt{3}x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{x^2+1}{x^4+\sqrt{3}x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1475}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+$$
$$\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)-\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x+\sqrt{2-\sqrt{3}}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}}{2\sqrt{3}}$$
$$\frac{\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)-\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x+\sqrt{2+\sqrt{3}}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}}{2\sqrt{3}}$$
$$\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+$$
$$\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1478}$$

$$\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int -\frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int -\frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}-\frac{\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}-\frac{\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{1}{7x^7}-\frac{1}{3x^3}$$

input
```
Int[1/(x^8*(1 - x^4 + x^8)),x]
```

output
```
-1/7*1/x^7 - 1/(3*x^3) + ((ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt
[3]]]/Sqrt[2 + Sqrt[3]] + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3
]]]/Sqrt[2 + Sqrt[3]])/2 + (Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2 -
 Sqrt[3]]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2 - Sqrt[3]]))/2)/
(2*Sqrt[3]) - ((ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[
2 - Sqrt[3]] + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2
- Sqrt[3]])/2 + (Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(2*Sqrt[2 + Sqrt[3]])
- Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(2*Sqrt[2 + Sqrt[3]]))/2)/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.15

method	result	size
risch	$\frac{-\frac{x^4}{3}-\frac{1}{7}}{x^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(18\_R^5-\_R+x\right)\right)}{4}$	44
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}-\frac{1}{7x^7}-\frac{1}{3x^3}$	51

input
```
int(1/x^8/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*x^4-1/7)/x^7+1/4*sum(_R*ln(18*_R^5-_R+x),_R=RootOf(81*_Z^8-9*_Z^4+1)
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^8\left(1-x^4+x^8\right)}\,dx$$
$$= \frac{21\sqrt{\frac{1}{3}}x^7\sqrt{-\sqrt{\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}\log\left(3\sqrt{\frac{1}{3}}\sqrt{-\frac{1}{3}}\sqrt{-\sqrt{\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}+x\right)-21\sqrt{\frac{1}{3}}x^7\sqrt{-\sqrt{\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}\log\left(\right.}{}$$

input
```
integrate(1/x^8/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/84*(21*sqrt(1/3)*x^7*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/3)*s
qrt(-1/3)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2)) + x) - 21*sqrt(1/3)*x^7*sqrt(-
sqrt(3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3)*sqrt(-1/3)*sqrt(-sqrt(3/2*sqr
t(-1/3) + 1/2)) + x) - 21*sqrt(1/3)*x^7*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))
*log(3*sqrt(1/3)*sqrt(-1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2)) + x) + 21*s
qrt(1/3)*x^7*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3)*sqrt(-1/3
)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2)) + x) + 21*sqrt(1/3)*x^7*(3/2*sqrt(-1/
3) + 1/2)^(1/4)*log(3*sqrt(1/3)*sqrt(-1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4) +
x) - 21*sqrt(1/3)*x^7*(3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*sqrt(-
1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4) + x) - 21*sqrt(1/3)*x^7*(-3/2*sqrt(-1/3)
 + 1/2)^(1/4)*log(3*sqrt(1/3)*sqrt(-1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1/4) + x
) + 21*sqrt(1/3)*x^7*(-3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*sqrt(-
1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1/4) + x) - 28*x^4 - 12)/x^7
```

Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

$$\int \frac{1}{x^8(1-x^4+x^8)}\,dx$$
$$= \operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(18432t^5-4t+x\right)\right)\right)+\frac{-7x^4-3}{21x^7}$$

input
```
integrate(1/x**8/(x**8-x**4+1),x)
```

output
```
RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(18432*_t**5 - 4*
_t + x))) + (-7*x**4 - 3)/(21*x**7)
```

Maxima [F]

$$\int \frac{1}{x^8(1-x^4+x^8)}\,dx = \int \frac{1}{(x^8-x^4+1)x^8}\,dx$$

input
```
integrate(1/x^8/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/21*(7*x^4 + 3)/x^7 - integrate(x^4/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^8(1-x^4+x^8)}\,dx = -\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{7x^4+3}{21x^7}$$

input
```
integrate(1/x^8/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sq
rt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt
(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(
6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*
x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(s
qrt(6) - sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(
6) - sqrt(2)) + 1) - 1/21*(7*x^4 + 3)/x^7
```

Mupad [B] (verification not implemented)

Time = 19.07 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^8 (1 - x^4 + x^8)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^8*(x^8 - x^4 + 1)),x)
```

output
```
(3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/
2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*
1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4))
)*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (x^4/3 + 1/7)/x^7 + (3^(1/2)*atan((x*(8
- 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3
^(1/2)*8i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8
- 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(
1/4))/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2
^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)
*1i)/2)) - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(
1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(
3^(1/2)*1i + 1)^(1/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1
i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)
*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4
))/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i +
 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)^(1/4))/12
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.62

$$\int \frac{1}{x^8 (1 - x^4 + x^8)} \, dx = \frac{56\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x^7 + 84\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x^7 - 56\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}}{2\sqrt{}}\right.}{}$$

input
```
int(1/x^8/(x^8-x^4+1),x)
```

output
```
(56*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( -
 sqrt(3) + 2)))*x**7 + 84*sqrt( - sqrt(3) + 2)*atan((sqrt(6) + sqrt(2) - 4
*x)/(2*sqrt( - sqrt(3) + 2)))*x**7 - 56*sqrt( - sqrt(3) + 2)*sqrt(3)*atan(
(sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*x**7 - 84*sqrt( - sqrt
(3) + 2)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*x**7 + 1
4*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2)))*x**7 -
42*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2)))*x**7 -
 14*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2)))*x**7
+ 42*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2)))*x**7
 + 28*sqrt( - sqrt(3) + 2)*sqrt(3)*log( - sqrt( - sqrt(3) + 2)*x + x**2 +
1)*x**7 - 28*sqrt( - sqrt(3) + 2)*sqrt(3)*log(sqrt( - sqrt(3) + 2)*x + x**
2 + 1)*x**7 + 42*sqrt( - sqrt(3) + 2)*log( - sqrt( - sqrt(3) + 2)*x + x**2
 + 1)*x**7 - 42*sqrt( - sqrt(3) + 2)*log(sqrt( - sqrt(3) + 2)*x + x**2 + 1
)*x**7 + 7*sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2)*x**7 - 7
*sqrt(6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2)*x**7 - 21*sqrt(2)*log
(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2)*x**7 + 21*sqrt(2)*log((sqrt(6)
*x + sqrt(2)*x + 2*x**2 + 2)/2)*x**7 - 112*x**4 - 48)/(336*x**7)
```

3.101 $\int \frac{x^{11}}{1+3x^4+x^8}\,dx$

Optimal result 806
Mathematica [A] (verified) 806
Rubi [A] (verified) 807
Maple [A] (verified) 808
Fricas [A] (verification not implemented) 809
Sympy [A] (verification not implemented) 809
Maxima [A] (verification not implemented) 810
Giac [A] (verification not implemented) 810
Mupad [B] (verification not implemented) 810
Reduce [F] 811

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx=\frac{x^4}{4}-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

output
```
1/4*x^4-1/40*(15-7*5^(1/2))*ln(3-5^(1/2)+2*x^4)-1/40*(15+7*5^(1/2))*ln(3+5
^(1/2)+2*x^4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx=\frac{1}{40}\left(10x^4+\left(-15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right)-\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)\right)$$

input
```
Integrate[x^11/(1 + 3*x^4 + x^8),x]
```

output
```
(10*x^4 + (-15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] - (15 + 7*Sqrt[5])*L
og[3 + Sqrt[5] + 2*x^4])/40
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^8+3x^4+1} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^8}{x^8+3x^4+1}dx^4$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{4}\int \left(-\frac{15+7\sqrt{5}}{5\left(2x^4+\sqrt{5}+3\right)}+1-\frac{15-7\sqrt{5}}{5\left(2x^4-\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(x^4-\frac{1}{10}\left(15-7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{10}\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)\right)$$

input
```
Int[x^11/(1 + 3*x^4 + x^8),x]
```

output
```
(x^4 - ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/10 - ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] + 2*x^4])/10)/4
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{x^4}{4}-\frac{3\ln\left(x^8+3x^4+1\right)}{8}-\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)}{20}$	38
risch	$\frac{x^4}{4}-\frac{3\ln\left(3-\sqrt{5}+2x^4\right)}{8}+\frac{7\ln\left(3-\sqrt{5}+2x^4\right)\sqrt{5}}{40}-\frac{3\ln\left(3+\sqrt{5}+2x^4\right)}{8}-\frac{7\ln\left(3+\sqrt{5}+2x^4\right)\sqrt{5}}{40}$	69

input
```
int(x^11/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4-3/8*ln(x^8+3*x^4+1)-7/20*5^(1/2)*arctanh(1/5*(2*x^4+3)*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^8+6\,x^4-\sqrt{5}(2\,x^4+3)+7}{x^8+3\,x^4+1}\right) - \frac{3}{8}\,\log\left(x^8+3\,x^4+1\right)$$

input
```
integrate(x^11/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8
+ 3*x^4 + 1)) - 3/8*log(x^8 + 3*x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{x^4}{4} + \left(-\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right) + \left(-\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

input
```
integrate(x**11/(x**8+3*x**4+1),x)
```

output
```
x**4/4 + (-3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (-7*sqrt(5)/4
0 - 3/8)*log(x**4 + sqrt(5)/2 + 3/2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^4 - \sqrt{5} + 3}{2\,x^4 + \sqrt{5} + 3}\right) - \frac{3}{8}\,\log\left(x^8 + 3\,x^4 + 1\right)$$

input
```
integrate(x^11/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) -
3/8*log(x^8 + 3*x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^4 - \sqrt{5} + 3}{2\,x^4 + \sqrt{5} + 3}\right) - \frac{3}{8}\,\log\left(x^8 + 3\,x^4 + 1\right)$$

input
```
integrate(x^11/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) -
3/8*log(x^8 + 3*x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 18.67 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{7\,\sqrt{5}\,\ln\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{40} - \frac{3\,\ln\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{8} - \frac{3\,\ln\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{8} - \frac{7\,\sqrt{5}\,\ln\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{40} + \frac{x^4}{4}$$

input
```
int(x^11/(3*x^4 + x^8 + 1),x)
```

output
```
(7*5^(1/2)*log(x^4 - 5^(1/2)/2 + 3/2))/40 - (3*log(5^(1/2)/2 + x^4 + 3/2))
/8 - (3*log(x^4 - 5^(1/2)/2 + 3/2))/8 - (7*5^(1/2)*log(5^(1/2)/2 + x^4 + 3
/2))/40 + x^4/4
```

Reduce [F]

$$\int \frac{x^{11}}{1+3x^4+x^8}\,dx = \frac{7\left(\int \frac{x^3}{x^8+3x^4+1}dx\right)}{2} - \frac{3\log(x^8+3x^4+1)}{8} + \frac{x^4}{4}$$

input
```
int(x^11/(x^8+3*x^4+1),x)
```

output
```
(28*int(x**3/(x**8 + 3*x**4 + 1),x) - 3*log(x**8 + 3*x**4 + 1) + 2*x**4)/8
```

3.102 $\int \frac{x^9}{1+3x^4+x^8} \, dx$

Optimal result . . . 812
Mathematica [A] (verified) . . . 812
Rubi [A] (verified) . . . 813
Maple [C] (verified) . . . 815
Fricas [A] (verification not implemented) . . . 815
Sympy [A] (verification not implemented) . . . 816
Maxima [F] . . . 816
Giac [A] (verification not implemented) . . . 816
Mupad [B] (verification not implemented) . . . 817
Reduce [F] . . . 817

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{x^9}{1+3x^4+x^8} \, dx = \frac{x^2}{2} - \frac{\arctan\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x^2\right)}{2\sqrt{5\left(9-4\sqrt{5}\right)}} + \frac{\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{2\sqrt{5\left(9+4\sqrt{5}\right)}}$$

output
```
1/2*x^2-1/2*arctan((1/2*5^(1/2)-1/2)*x^2)/(5-2*5^(1/2))+1/2*arctan((1/2+1/
2*5^(1/2))*x^2)/(5+2*5^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{x^9}{1+3x^4+x^8} \, dx = \frac{1}{40}\left(20x^2 - \sqrt{6-2\sqrt{5}}\left(15+7\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \sqrt{2\left(3+\sqrt{5}\right)}\left(-15+7\sqrt{5}\right)\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)\right)$$

input
```
Integrate[x^9/(1 + 3*x^4 + x^8),x]
```

output
```
(20*x^2 - Sqrt[6 - 2*Sqrt[5]]*(15 + 7*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])
]*x^2] + Sqrt[2*(3 + Sqrt[5])]*(-15 + 7*Sqrt[5])*ArcTan[Sqrt[(3 + Sqrt[5])
/2]*x^2])/40
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1695, 1442, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^8}{x^8+3x^4+1}dx^2$$

$$\Big\downarrow \text{1442}$$

$$\frac{1}{2}\left(x^2-\int \frac{3x^4+1}{x^8+3x^4+1}dx^2\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx^2-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx^2+x^2\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(-\frac{\left(15+7\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}-\frac{1}{10}\left(15-7\sqrt{5}\right)\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)+x^2\right)$$

input
```
Int[x^9/(1 + 3*x^4 + x^8),x]
```

output
```
(x^2 - ((15 + 7*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/(5*Sqrt[2*(3 +
 Sqrt[5])]) - ((15 - 7*Sqrt[5])*Sqrt[(3 + Sqrt[5])/2]*ArcTan[Sqrt[(3 + Sqr
t[5])/2]*x^2])/10)/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

method	result	size
risch	$\frac{x^2}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+90\_Z^2+1\right)}\_R\ln\left(15\_R^3+8x^2+47\_R\right)\right)}{4}$	42
default	$\frac{x^2}{2}-\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5\left(-2+2\sqrt{5}\right)}-\frac{\sqrt{5}\left(7+3\sqrt{5}\right)\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5\left(2+2\sqrt{5}\right)}$	79

input
```
int(x^9/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/4*sum(_R*ln(15*_R^3+8*x^2+47*_R),_R=RootOf(25*_Z^4+90*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \frac{1}{2}\,x^2-\frac{1}{10}\left(2\sqrt{5}-5\right)\arctan\left(\frac{1}{2}\sqrt{5}x^2+\frac{1}{2}\,x^2\right) - \frac{1}{10}\left(2\sqrt{5}+5\right)\arctan\left(\frac{1}{2}\sqrt{5}x^2-\frac{1}{2}\,x^2\right)$$

input
```
integrate(x^9/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/10*(2*sqrt(5) - 5)*arctan(1/2*sqrt(5)*x^2 + 1/2*x^2) - 1/10*(2
*sqrt(5) + 5)*arctan(1/2*sqrt(5)*x^2 - 1/2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \frac{x^2}{2} + 2\cdot\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right) - 2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

input
```
integrate(x**9/(x**8+3*x**4+1),x)
```

output
```
x**2/2 + 2*(1/4 - sqrt(5)/10)*atan(2*x**2/(-1 + sqrt(5))) - 2*(sqrt(5)/10
+ 1/4)*atan(2*x**2/(1 + sqrt(5)))
```

Maxima [F]

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \int \frac{x^9}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^9/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - integrate((3*x^4 + 1)*x/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = \frac{1}{2}\,x^2 - \frac{1}{20}\left(3\,x^4\left(\sqrt{5}-5\right)+\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right) - \frac{1}{20}\left(3\,x^4\left(\sqrt{5}+5\right)+\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

input
```
integrate(x^9/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/20*(3*x^4*(sqrt(5) - 5) + sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) +
 1)) - 1/20*(3*x^4*(sqrt(5) + 5) + sqrt(5) + 5)*arctan(2*x^2/(sqrt(5) - 1)
)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.48

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = 2\,\mathrm{atanh}\left(\frac{1280\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}-192}+\frac{768\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}-192}\right)\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}} - 2\,\mathrm{atanh}\left(\frac{1280\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}+192} - \frac{768\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\,\sqrt{5}+192}\right)\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}+\frac{x^2}{2}$$

input
```
int(x^9/(3*x^4 + x^8 + 1),x)
```

output
```
2*atanh((1280*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192) + (768*5^(
1/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192))*(5^(1/2)/20 - 9/80
)^(1/2) - 2*atanh((1280*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192
) - (768*5^(1/2)*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192))*(- 5
^(1/2)/20 - 9/80)^(1/2) + x^2/2
```

Reduce [F]

$$\int \frac{x^9}{1+3x^4+x^8}\,dx = -3\left(\int \frac{x^5}{x^8+3x^4+1}dx\right)-\left(\int \frac{x}{x^8+3x^4+1}dx\right)+\frac{x^2}{2}$$

input
```
int(x^9/(x^8+3*x^4+1),x)
```

output
```
( - 6*int(x**5/(x**8 + 3*x**4 + 1),x) - 2*int(x/(x**8 + 3*x**4 + 1),x) + x
**2)/2
```

3.103 $\int \frac{x^7}{1+3x^4+x^8} \, dx$

Optimal result 819
Mathematica [A] (verified) 819
Rubi [A] (verified) 820
Maple [A] (verified) 821
Fricas [A] (verification not implemented) 821
Sympy [A] (verification not implemented) 822
Maxima [A] (verification not implemented) 822
Giac [A] (verification not implemented) 823
Mupad [B] (verification not implemented) 823
Reduce [F] 824

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{x^7}{1+3x^4+x^8} \, dx = \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

output
```
1/40*(5-3*5^(1/2))*ln(3-5^(1/2)+2*x^4)+1/40*(5+3*5^(1/2))*ln(3+5^(1/2)+2*x
^4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{1+3x^4+x^8} \, dx = \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right) + \frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

input
```
Integrate[x^7/(1 + 3*x^4 + x^8),x]
```

output
```
((5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 +
Sqrt[5] + 2*x^4])/40
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^8+3x^4+1} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^4}{x^8+3x^4+1}dx^4$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{4}\int \left(\frac{5+3\sqrt{5}}{5\left(2x^4+\sqrt{5}+3\right)}+\frac{5-3\sqrt{5}}{5\left(2x^4-\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{10}\left(5-3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)\right)$$

input
```
Int[x^7/(1 + 3*x^4 + x^8),x]
```

output
```
(((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/10 + ((5 + 3*Sqrt[5])*Log[3 +
Sqrt[5] + 2*x^4])/10)/4
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\ln(x^8+3x^4+1)}{8}+\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)}{20}$	33
risch	$\frac{\ln\left(3+\sqrt{5}+2x^4\right)}{8}+\frac{3\ln\left(3+\sqrt{5}+2x^4\right)\sqrt{5}}{40}+\frac{\ln\left(3-\sqrt{5}+2x^4\right)}{8}-\frac{3\ln\left(3-\sqrt{5}+2x^4\right)\sqrt{5}}{40}$	64

input
```
int(x^7/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(x^8+3*x^4+1)+3/20*5^(1/2)*arctanh(1/5*(2*x^4+3)*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{x^7}{1+3x^4+x^8}\,dx=\frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^8+6\,x^4+\sqrt{5}(2\,x^4+3)+7}{x^8+3\,x^4+1}\right)+\frac{1}{8}\log\left(x^8+3\,x^4+1\right)$$

input
```
integrate(x^7/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 +
1)) + 1/8*log(x^8 + 3*x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = \left(\frac{1}{8}-\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right) + \left(\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

input
```
integrate(x**7/(x**8+3*x**4+1),x)
```

output
```
(1/8 - 3*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (1/8 + 3*sqrt(5)/40)*lo
g(x**4 + sqrt(5)/2 + 3/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = -\frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)+\frac{1}{8}\log\left(x^8+3\,x^4+1\right)$$

input
```
integrate(x^7/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
-3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x
^8 + 3*x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = -\frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)+\frac{1}{8}\log\left(x^8+3\,x^4+1\right)$$

input
```
integrate(x^7/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
-3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x
^8 + 3*x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = \frac{\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}+\frac{\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8} - \frac{3\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}+\frac{3\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}$$

input
```
int(x^7/(3*x^4 + x^8 + 1),x)
```

output
```
log(x^4 - 5^(1/2)/2 + 3/2)/8 + log(5^(1/2)/2 + x^4 + 3/2)/8 - (3*5^(1/2)*l
og(x^4 - 5^(1/2)/2 + 3/2))/40 + (3*5^(1/2)*log(5^(1/2)/2 + x^4 + 3/2))/40
```

Reduce [F]

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = -\frac{3\left(\int \frac{x^3}{x^8+3x^4+1}dx\right)}{2} + \frac{\log(x^8+3x^4+1)}{8}$$

input
```
int(x^7/(x^8+3*x^4+1),x)
```

output
```
( - 12*int(x**3/(x**8 + 3*x**4 + 1),x) + log(x**8 + 3*x**4 + 1))/8
```

3.104 $\int \frac{x^5}{1+3x^4+x^8}\,dx$

Optimal result 825
Mathematica [A] (verified) 825
Rubi [A] (verified) 826
Maple [C] (verified) 827
Fricas [A] (verification not implemented) 828
Sympy [A] (verification not implemented) 828
Maxima [F] 828
Giac [A] (verification not implemented) 829
Mupad [B] (verification not implemented) 829
Reduce [F] 830

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x^2\right) - \frac{\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

output
```
1/2*(1/2+1/10*5^(1/2))*arctan((1/2*5^(1/2)-1/2)*x^2)-arctan((1/2+1/2*5^(1/
2))*x^2)/(5+5^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \frac{2\sqrt{5}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\left(5-3\sqrt{5}\right)\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{10\sqrt{6-2\sqrt{5}}}$$

input
```
Integrate[x^5/(1 + 3*x^4 + x^8),x]
```

output
```
(2*Sqrt[5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2] + (5 - 3*Sqrt[5])*ArcTan[Sqrt
[(3 + Sqrt[5])/2]*x^2])/(10*Sqrt[6 - 2*Sqrt[5]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1695, 1450, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^8+3x^4+1}\,dx$$
$$\Big\downarrow \text{ 1695}$$
$$\frac{1}{2}\int \frac{x^4}{x^8+3x^4+1}dx^2$$
$$\Big\downarrow \text{ 1450}$$
$$\frac{1}{2}\left(\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx^2+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx^2\right)$$
$$\Big\downarrow \text{ 216}$$
$$\frac{1}{2}\left(\frac{\left(5+3\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}+\frac{1}{10}\left(5-3\sqrt{5}\right)\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)\right)$$

input
```
Int[x^5/(1 + 3*x^4 + x^8),x]
```

output
```
(((5 + 3*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/(5*Sqrt[2*(3 + Sqrt[5
])]) + ((5 - 3*Sqrt[5])*Sqrt[(3 + Sqrt[5])/2]*ArcTan[Sqrt[(3 + Sqrt[5])/2]
*x^2])/10)/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^{4}+15\_Z^{2}+1\right)}\_R\ln\left(-10\_R^{3}+x^{2}-3\_R\right)\right)}{4}$	34
default	$\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\arctan\left(\frac{4x^{2}}{-2+2\sqrt{5}}\right)}{-10+10\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^{2}}{2+2\sqrt{5}}\right)}{10+10\sqrt{5}}$	70

input
```
int(x^5/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(-10*_R^3+x^2-3*_R),_R=RootOf(25*_Z^4+15*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \frac{1}{20}\left(\sqrt{5}-5\right)\arctan\left(\frac{1}{2}\sqrt{5}x^2+\frac{1}{2}x^2\right) + \frac{1}{20}\left(\sqrt{5}+5\right)\arctan\left(\frac{1}{2}\sqrt{5}x^2-\frac{1}{2}x^2\right)$$

input
```
integrate(x^5/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/20*(sqrt(5) - 5)*arctan(1/2*sqrt(5)*x^2 + 1/2*x^2) + 1/20*(sqrt(5) + 5)*
arctan(1/2*sqrt(5)*x^2 - 1/2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = -2\cdot\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

input
```
integrate(x**5/(x**8+3*x**4+1),x)
```

output
```
-2*(1/8 - sqrt(5)/40)*atan(2*x**2/(-1 + sqrt(5))) + 2*(sqrt(5)/40 + 1/8)*a
tan(2*x**2/(1 + sqrt(5)))
```

Maxima [F]

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \int \frac{x^5}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^5/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^5/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \frac{1}{20}\,x^4\left(\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right) + \frac{1}{20}\,x^4\left(\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

input
```
integrate(x^5/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/20*x^4*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*x^4*(sqrt(5) + 5
)*arctan(2*x^2/(sqrt(5) - 1))
```

Mupad [B] (verification not implemented)

Time = 18.90 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.52

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = 2\,\mathrm{atanh}\left(\frac{60\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3}+\frac{28\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3}\right)\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}} - 2\,\mathrm{atanh}\left(\frac{60\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}-3} - \frac{28\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}-3}\right)\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}$$

input
```
int(x^5/(3*x^4 + x^8 + 1),x)
```

output
```
2*atanh((60*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) + 3) + (28*5^(1/2)*x
^2*(5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) + 3))*(5^(1/2)/160 - 3/160)^(1/2)
 - 2*atanh((60*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) - 3) - (28*5^(1
/2)*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) - 3))*(- 5^(1/2)/160 - 3/1
60)^(1/2)
```

Reduce [F]

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \int \frac{x^5}{x^8+3x^4+1}dx$$

input
```
int(x^5/(x^8+3*x^4+1),x)
```

output
```
int(x**5/(x**8 + 3*x**4 + 1),x)
```

3.105 $\int \frac{x^3}{1+3x^4+x^8}\,dx$

Optimal result 831
Mathematica [A] (verified) 831
Rubi [A] (verified) 832
Maple [A] (verified) 833
Fricas [A] (verification not implemented) 833
Sympy [A] (verification not implemented) 834
Maxima [A] (verification not implemented) 834
Giac [A] (verification not implemented) 834
Mupad [B] (verification not implemented) 835
Reduce [F] 835

Optimal result

Integrand size = 16, antiderivative size = 47

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{\log\left(3-\sqrt{5}+2x^4\right)}{4\sqrt{5}} - \frac{\log\left(3+\sqrt{5}+2x^4\right)}{4\sqrt{5}}$$

output
```
1/20*ln(3-5^(1/2)+2*x^4)*5^(1/2)-1/20*ln(3+5^(1/2)+2*x^4)*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{\log\left(-3+\sqrt{5}-2x^4\right)-\log\left(3+\sqrt{5}+2x^4\right)}{4\sqrt{5}}$$

input
```
Integrate[x^3/(1 + 3*x^4 + x^8),x]
```

output
```
(Log[-3 + Sqrt[5] - 2*x^4] - Log[3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1690, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{4}\int \frac{1}{x^8+3x^4+1}dx^4$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{4}\int \left(\frac{2}{\sqrt{5}\left(2x^4-\sqrt{5}+3\right)}-\frac{2}{\sqrt{5}\left(2x^4+\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{\log\left(2x^4-\sqrt{5}+3\right)}{\sqrt{5}}-\frac{\log\left(2x^4+\sqrt{5}+3\right)}{\sqrt{5}}\right)$$

input
```
Int[x^3/(1 + 3*x^4 + x^8),x]
```

output
```
(Log[3 - Sqrt[5] + 2*x^4]/Sqrt[5] - Log[3 + Sqrt[5] + 2*x^4]/Sqrt[5])/4
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.40

method	result	size
default	$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^{4}+3\right)\sqrt{5}}{5}\right)}{10}$	19
risch	$\frac{\ln\left(3-\sqrt{5}+2x^{4}\right)\sqrt{5}}{20}-\frac{\ln\left(3+\sqrt{5}+2x^{4}\right)\sqrt{5}}{20}$	36

input
```
int(x^3/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/10*5^(1/2)*arctanh(1/5*(2*x^4+3)*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{1}{20}\,\sqrt{5}\log\left(\frac{2\,x^8+6\,x^4-\sqrt{5}(2\,x^4+3)+7}{x^8+3\,x^4+1}\right)$$

input
```
integrate(x^3/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 +
1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{\sqrt{5}\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20} - \frac{\sqrt{5}\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20}$$

input
```
integrate(x**3/(x**8+3*x**4+1),x)
```

output
```
sqrt(5)*log(x**4 - sqrt(5)/2 + 3/2)/20 - sqrt(5)*log(x**4 + sqrt(5)/2 + 3/
2)/20
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)$$

input
```
integrate(x^3/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)$$

input
```
integrate(x^3/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

Mupad [B] (verification not implemented)

Time = 18.88 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{1+3x^4+x^8} \, dx = \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{8\,\sqrt{5}\,x^4+3\,\sqrt{5}}{18\,x^4+7}\right)}{10}$$

input
```
int(x^3/(3*x^4 + x^8 + 1),x)
```

output
```
(5^(1/2)*atanh((3*5^(1/2) + 8*5^(1/2)*x^4)/(18*x^4 + 7)))/10
```

Reduce [F]

$$\int \frac{x^3}{1+3x^4+x^8} \, dx = \int \frac{x^3}{x^8+3x^4+1}dx$$

input
```
int(x^3/(x^8+3*x^4+1),x)
```

output
```
int(x**3/(x**8 + 3*x**4 + 1),x)
```

3.106 $\int \frac{x}{1+3x^4+x^8}\,dx$

Optimal result . . . 836
Mathematica [A] (verified) . . . 836
Rubi [A] (verified) . . . 837
Maple [C] (verified) . . . 838
Fricas [A] (verification not implemented) . . . 839
Sympy [A] (verification not implemented) . . . 839
Maxima [F] . . . 839
Giac [A] (verification not implemented) . . . 840
Mupad [B] (verification not implemented) . . . 840
Reduce [F] . . . 841

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{x}{1+3x^4+x^8}\,dx = -\frac{\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}+\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

output
```
-arctan(2^(1/2)/(3+5^(1/2))^(1/2)*x^2)/(5+5^(1/2))+1/2*(1/2+1/10*5^(1/2))*
arctan((1/2+1/2*5^(1/2))*x^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{x}{1+3x^4+x^8}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{3-\sqrt{5}}}x^2\right)}{\sqrt{10\left(3-\sqrt{5}\right)}}-\frac{\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

input
```
Integrate[x/(1 + 3*x^4 + x^8),x]
```

output
```
ArcTan[Sqrt[2/(3 - Sqrt[5])]*x^2]/Sqrt[10*(3 - Sqrt[5])] - ArcTan[Sqrt[2/(
3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1695, 1406, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^8+3x^4+1}\,dx$$
$$\Big\downarrow \ 1695$$
$$\frac{1}{2}\int \frac{1}{x^8+3x^4+1}dx^2$$
$$\Big\downarrow \ 1406$$
$$\frac{1}{2}\left(\frac{\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx^2}{\sqrt{5}}-\frac{\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx^2}{\sqrt{5}}\right)$$
$$\Big\downarrow \ 216$$
$$\frac{1}{2}\left(\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)\right)$$

input
```
Int[x/(1 + 3*x^4 + x^8),x]
```

output
```
(-(Sqrt[2/(5*(3 + Sqrt[5]))]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2]) + Sqrt[(3
+ Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.45

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^{4}+15\_Z^{2}+1\right)}\_R\ln\left(-15\_R^{3}+x^{2}-7\_R\right)\right)}{4}$	34
default	$\frac{2\sqrt{5}\,\arctan\left(\frac{4x^{2}}{-2+2\sqrt{5}}\right)}{5\left(-2+2\sqrt{5}\right)}-\frac{2\sqrt{5}\,\arctan\left(\frac{4x^{2}}{2+2\sqrt{5}}\right)}{5\left(2+2\sqrt{5}\right)}$	60

input
```
int(x/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(-15*_R^3+x^2-7*_R),_R=RootOf(25*_Z^4+15*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60

$$\int \frac{x}{1+3x^4+x^8}\,dx = \frac{1}{20}\left(\sqrt{5}+5\right)\arctan\left(\frac{1}{2}\sqrt{5}x^2+\frac{1}{2}x^2\right) + \frac{1}{20}\left(\sqrt{5}-5\right)\arctan\left(\frac{1}{2}\sqrt{5}x^2-\frac{1}{2}x^2\right)$$

input
```
integrate(x/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/20*(sqrt(5) + 5)*arctan(1/2*sqrt(5)*x^2 + 1/2*x^2) + 1/20*(sqrt(5) - 5)*
arctan(1/2*sqrt(5)*x^2 - 1/2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.65

$$\int \frac{x}{1+3x^4+x^8}\,dx = 2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right) - 2\cdot\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

input
```
integrate(x/(x**8+3*x**4+1),x)
```

output
```
2*(sqrt(5)/40 + 1/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(1/8 - sqrt(5)/40)*at
an(2*x**2/(1 + sqrt(5)))
```

Maxima [F]

$$\int \frac{x}{1+3x^4+x^8}\,dx = \int \frac{x}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

$$\int \frac{x}{1+3x^4+x^8}\,dx = \frac{1}{20}\left(\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\left(\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

input
```
integrate(x/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/20*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*(sqrt(5) + 5)*arctan
(2*x^2/(sqrt(5) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.67

$$\int \frac{x}{1+3x^4+x^8}\,dx = 2\,\mathrm{atanh}\left(\frac{160\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}-18} - \frac{72\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}-18}\right)\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}} - 2\,\mathrm{atanh}\left(\frac{160\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}+18} + \frac{72\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}+18}\right)\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}$$

input
```
int(x/(3*x^4 + x^8 + 1),x)
```

output
```
2*atanh((160*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) - 18) - (72*5^(1/
2)*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) - 18))*(5^(1/2)/160 - 3/160
)^(1/2) - 2*atanh((160*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) + 18)
 + (72*5^(1/2)*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) + 18))*(- 5^(
1/2)/160 - 3/160)^(1/2)
```

Reduce [F]

$$\int \frac{x}{1+3x^4+x^8}\,dx = \int \frac{x}{x^8+3x^4+1}dx$$

input
```
int(x/(x^8+3*x^4+1),x)
```

output
```
int(x/(x**8 + 3*x**4 + 1),x)
```

3.107 $\int \frac{1}{x(1+3x^4+x^8)} \, dx$

Optimal result . . . 842
Mathematica [A] (verified) . . . 842
Rubi [A] (verified) . . . 843
Maple [A] (verified) . . . 844
Fricas [A] (verification not implemented) . . . 845
Sympy [A] (verification not implemented) . . . 845
Maxima [A] (verification not implemented) . . . 846
Giac [A] (verification not implemented) . . . 846
Mupad [B] (verification not implemented) . . . 847
Reduce [F] . . . 847

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{x(1+3x^4+x^8)} \, dx = \log(x) - \frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right) - \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

output
```
ln(x)-1/40*(5+3*5^(1/2))*ln(3-5^(1/2)+2*x^4)-1/40*(5-3*5^(1/2))*ln(3+5^(1/
2)+2*x^4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(1+3x^4+x^8)} \, dx = \log(x) + \frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right) + \frac{1}{40}\left(-5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

input
```
Integrate[1/(x*(1 + 3*x^4 + x^8)),x]
```

output
```
Log[x] + ((-5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((-5 + 3*Sqrt[5
])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^8+3x^4+1)}dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{1}{4}\int \frac{1}{x^4(x^8+3x^4+1)}dx^4$$
$$\Big\downarrow \text{1141}$$
$$\frac{1}{4}\int \left(\frac{2}{(5-3\sqrt{5})x^4-7\sqrt{5}+15}+\frac{1}{x^4}+\frac{4}{\sqrt{5}(3+\sqrt{5})(2x^4+\sqrt{5}+3)}\right)dx^4$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}\left(\log(x^4)+\frac{2\log(2x^4+\sqrt{5}+3)}{\sqrt{5}(3+\sqrt{5})}+\frac{2\log((5-3\sqrt{5})x^4-7\sqrt{5}+15)}{5-3\sqrt{5}}\right)$$

input
```
Int[1/(x*(1 + 3*x^4 + x^8)),x]
```

output
```
(Log[x^4] + (2*Log[3 + Sqrt[5] + 2*x^4])/(Sqrt[5]*(3 + Sqrt[5])) + (2*Log[
15 - 7*Sqrt[5] + (5 - 3*Sqrt[5])*x^4])/(5 - 3*Sqrt[5]))/4
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

method	result	size
default	$\ln(x)-\frac{\ln(x^8+3x^4+1)}{8}+\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)}{20}$	35
risch	$\ln(x)+\frac{3\ln\left(3x^4+\frac{3\sqrt{5}}{2}+\frac{9}{2}\right)\sqrt{5}}{40}-\frac{\ln\left(3x^4+\frac{3\sqrt{5}}{2}+\frac{9}{2}\right)}{8}-\frac{\ln\left(3x^4+\frac{9}{2}-\frac{3\sqrt{5}}{2}\right)}{8}-\frac{3\ln\left(3x^4+\frac{9}{2}-\frac{3\sqrt{5}}{2}\right)\sqrt{5}}{40}$	70

input
```
int(1/x/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/8*ln(x^8+3*x^4+1)+3/20*5^(1/2)*arctanh(1/5*(2*x^4+3)*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^8+6\,x^4+\sqrt{5}(2\,x^4+3)+7}{x^8+3\,x^4+1}\right) - \frac{1}{8}\log\left(x^8+3\,x^4+1\right)+\log\left(x\right)$$

input
```
integrate(1/x/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 +
1)) - 1/8*log(x^8 + 3*x^4 + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = \log\left(x\right)+\left(-\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right) + \left(-\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

input
```
integrate(1/x/(x**8+3*x**4+1),x)
```

output
```
log(x) + (-3*sqrt(5)/40 - 1/8)*log(x**4 - sqrt(5)/2 + 3/2) + (-1/8 + 3*sqr
t(5)/40)*log(x**4 + sqrt(5)/2 + 3/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = -\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right) - \frac{1}{8}\log\left(x^8+3x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
-3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x
^8 + 3*x^4 + 1) + 1/4*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = -\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right) - \frac{1}{8}\log\left(x^8+3x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
-3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x
^8 + 3*x^4 + 1) + 1/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = \ln(x) - \ln\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40} + \frac{1}{8}\right) + \ln\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40} - \frac{1}{8}\right)$$

input
```
int(1/(x*(3*x^4 + x^8 + 1)),x)
```

output
```
log(x) - log(x^4 - 5^(1/2)/2 + 3/2)*((3*5^(1/2))/40 + 1/8) + log(5^(1/2)/2
 + x^4 + 3/2)*((3*5^(1/2))/40 - 1/8)
```

Reduce [F]

$$\int \frac{1}{x(1+3x^4+x^8)}\,dx = \int \frac{1}{x^9+3x^5+x}dx$$

input
```
int(1/x/(x^8+3*x^4+1),x)
```

output
```
int(1/(x**9 + 3*x**5 + x),x)
```

3.108 $\int \frac{1}{x^3(1+3x^4+x^8)}\,dx$

Optimal result 848
Mathematica [C] (verified) 848
Rubi [A] (verified) 849
Maple [C] (verified) 851
Fricas [A] (verification not implemented) 851
Sympy [A] (verification not implemented) 852
Maxima [F] 852
Giac [A] (verification not implemented) 852
Mupad [B] (verification not implemented) 853
Reduce [F] 854

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{1}{x^3(1+3x^4+x^8)}\,dx = -\frac{1}{2x^2}+\frac{\arctan\left(\sqrt{\frac{1}{2}(3-\sqrt{5})}x^2\right)}{2\sqrt{5(9+4\sqrt{5})}}-\frac{\arctan\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x^2\right)}{2\sqrt{5(9-4\sqrt{5})}}$$

output
```
-1/2/x^2+1/2*arctan((1/2*5^(1/2)-1/2)*x^2)/(5+2*5^(1/2))-1/2*arctan((1/2+1
/2*5^(1/2))*x^2)/(5-2*5^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3(1+3x^4+x^8)}\,dx = -\frac{1}{2x^2}-\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{3\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^2+2\#1^6}\&\right]$$

input
```
Integrate[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

output
```
-1/2*1/x^2 - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1
^4)/(3*#1^2 + 2*#1^6) & ]/4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1695, 1443, 25, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(x^8+3 x^4+1\right)} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(x^8+3 x^4+1\right)}dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2} \left(\int -\frac{x^4+3}{x^8+3 x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(-\int \frac{x^4+3}{x^8+3 x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2} \left(-\frac{1}{10} \left(5+3 \sqrt{5}\right) \int \frac{1}{x^4+\frac{1}{2} \left(3-\sqrt{5}\right)}dx^2-\frac{1}{10} \left(5-3 \sqrt{5}\right) \int \frac{1}{x^4+\frac{1}{2} \left(3+\sqrt{5}\right)}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2} \left(-\frac{\left(5-3 \sqrt{5}\right) \arctan \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2\right)}{5 \sqrt{2 \left(3+\sqrt{5}\right)}}-\frac{1}{10} \sqrt{\frac{1}{2} \left(3+\sqrt{5}\right)} \left(5+3 \sqrt{5}\right) \arctan \left(\sqrt{\frac{1}{2} \left(3+\sqrt{5}\right)} x^2\right)-\frac{1}{x^2}\right)$$

input
```
Int[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

output
```
(-x^(-2) - ((5 - 3*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/(5*Sqrt[2*(
3 + Sqrt[5])]) - (Sqrt[(3 + Sqrt[5])/2]*(5 + 3*Sqrt[5])*ArcTan[Sqrt[(3 + S
qrt[5])/2]*x^2])/10)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+90\_Z^2+1\right)}\_R\ln\left(35\_R^3+8x^2+123\_R\right)\right)}{4}$	42
default	$-\frac{1}{2x^2}-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5\left(-2+2\sqrt{5}\right)}-\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5\left(2+2\sqrt{5}\right)}$	75

input
```
int(1/x^3/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+1/4*sum(_R*ln(35*_R^3+8*x^2+123*_R),_R=RootOf(25*_Z^4+90*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3\left(1+3x^4+x^8\right)}\,dx = -\frac{\left(2\sqrt{5}x^2+5\,x^2\right)\arctan\left(\frac{1}{2}\sqrt{5}x^2+\frac{1}{2}\,x^2\right)+\left(2\sqrt{5}x^2-5\,x^2\right)\arctan\left(\frac{1}{2}\sqrt{5}x^2-\frac{1}{2}\,x^2\right)+5}{10\,x^2}$$

input
```
integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
-1/10*((2*sqrt(5)*x^2 + 5*x^2)*arctan(1/2*sqrt(5)*x^2 + 1/2*x^2) + (2*sqrt
(5)*x^2 - 5*x^2)*arctan(1/2*sqrt(5)*x^2 - 1/2*x^2) + 5)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^3 (1 + 3x^4 + x^8)} dx = -2\left(\frac{\sqrt{5}}{10} + \frac{1}{4}\right) \operatorname{atan}\left(\frac{2x^2}{-1 + \sqrt{5}}\right) + 2 \cdot \left(\frac{1}{4} - \frac{\sqrt{5}}{10}\right) \operatorname{atan}\left(\frac{2x^2}{1 + \sqrt{5}}\right) - \frac{1}{2x^2}$$

input
```
integrate(1/x**3/(x**8+3*x**4+1),x)
```

output
```
-2*(sqrt(5)/10 + 1/4)*atan(2*x**2/(-1 + sqrt(5))) + 2*(1/4 - sqrt(5)/10)*a
tan(2*x**2/(1 + sqrt(5))) - 1/(2*x**2)
```

Maxima [F]

$$\int \frac{1}{x^3 (1 + 3x^4 + x^8)} dx = \int \frac{1}{(x^8 + 3x^4 + 1)x^3} dx$$

input
```
integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
-1/2/x^2 - integrate((x^4 + 3)*x/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3 (1 + 3x^4 + x^8)} dx = -\frac{1}{20} \left(x^4 \left(\sqrt{5} - 5\right) + 3\sqrt{5} - 15\right) \arctan\left(\frac{2x^2}{\sqrt{5} + 1}\right) - \frac{1}{20} \left(x^4 \left(\sqrt{5} + 5\right) + 3\sqrt{5} + 15\right) \arctan\left(\frac{2x^2}{\sqrt{5} - 1}\right) - \frac{1}{2x^2}$$

input
```
integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
-1/20*(x^4*(sqrt(5) - 5) + 3*sqrt(5) - 15)*arctan(2*x^2/(sqrt(5) + 1)) - 1
/20*(x^4*(sqrt(5) + 5) + 3*sqrt(5) + 15)*arctan(2*x^2/(sqrt(5) - 1)) - 1/2
/x^2
```

Mupad [B] (verification not implemented)

Time = 18.86 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^3\left(1+3x^4+x^8\right)}\,dx = 2\,\mathrm{atanh}\left(\frac{26880\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}+7872} + \frac{12032\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}+7872}\right)\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}} - 2\,\mathrm{atanh}\left(\frac{26880\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}-7872} - \frac{12032\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\,\sqrt{5}-7872}\right)\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}} - \frac{1}{2\,x^2}$$

input
```
int(1/(x^3*(3*x^4 + x^8 + 1)),x)
```

output
```
2*atanh((26880*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872) + (1
2032*5^(1/2)*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872))*(- 5^
(1/2)/20 - 9/80)^(1/2) - 2*atanh((26880*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(35
20*5^(1/2) - 7872) - (12032*5^(1/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(3520*5
^(1/2) - 7872))*(5^(1/2)/20 - 9/80)^(1/2) - 1/(2*x^2)
```

Reduce [F]

$$\int \frac{1}{x^3 (1 + 3x^4 + x^8)} \, dx = \frac{-2\left(\int \frac{x^5}{x^8+3x^4+1}dx\right) x^2 - 6\left(\int \frac{x}{x^8+3x^4+1}dx\right) x^2 - 1}{2x^2}$$

input
```
int(1/x^3/(x^8+3*x^4+1),x)
```

output
```
( - 2*int(x**5/(x**8 + 3*x**4 + 1),x)*x**2 - 6*int(x/(x**8 + 3*x**4 + 1),x
)*x**2 - 1)/(2*x**2)
```

3.109 $\int \frac{1}{x^5(1+3x^4+x^8)}\,dx$

Optimal result . . . 855
Mathematica [A] (verified) . . . 855
Rubi [A] (verified) . . . 856
Maple [A] (verified) . . . 857
Fricas [A] (verification not implemented) . . . 858
Sympy [A] (verification not implemented) . . . 858
Maxima [A] (verification not implemented) . . . 859
Giac [A] (verification not implemented) . . . 859
Mupad [B] (verification not implemented) . . . 860
Reduce [F] . . . 860

Optimal result

Integrand size = 16, antiderivative size = 83

$$\int \frac{1}{x^5(1+3x^4+x^8)}\,dx = -\frac{1}{4x^4} - 3\log(x) - \frac{\log\left(3+\sqrt{5}+2x^4\right)}{\sqrt{5}\left(3+\sqrt{5}\right)^2} - \frac{\log\left(-2\left(20-9\sqrt{5}\right)-\left(15-7\sqrt{5}\right)x^4\right)}{2\left(15-7\sqrt{5}\right)}$$

output
```
-1/4/x^4-3*ln(x)-1/5*ln(3+5^(1/2)+2*x^4)*5^(1/2)/(3+5^(1/2))^2-ln(-40+18*5
^(1/2)-(15-7*5^(1/2))*x^4)/(30-14*5^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^5(1+3x^4+x^8)}\,dx = \frac{1}{40}\left(-\frac{10}{x^4} - 120\log(x) + \left(15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right) + \left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)\right)$$

input
```
Integrate[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

output
```
(-10/x^4 - 120*Log[x] + (15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] + (15 -
 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^8+3x^4+1)} dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4} \int \frac{1}{x^8 (x^8+3x^4+1)} dx^4$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{4} \int \left(-\frac{2}{(15-7\sqrt{5})\, x^4+2\,(20-9\sqrt{5})} - \frac{3}{x^4} + \frac{1}{x^8} - \frac{8}{\sqrt{5}\,(3+\sqrt{5})^2\,(2x^4+\sqrt{5}+3)}\right) dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{1}{x^4} - 3\log(x^4) - \frac{4\log(2x^4+\sqrt{5}+3)}{\sqrt{5}\,(3+\sqrt{5})^2} - \frac{2\log\left(-\left((15-7\sqrt{5})\, x^4\right) - 2(20-9\sqrt{5})\right)}{15-7\sqrt{5}}\right)$$

input
```
Int[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

output
```
(-x^(-4) - 3*Log[x^4] - (4*Log[3 + Sqrt[5] + 2*x^4])/(Sqrt[5]*(3 + Sqrt[5]
)^2) - (2*Log[-2*(20 - 9*Sqrt[5]) - (15 - 7*Sqrt[5])*x^4])/(15 - 7*Sqrt[5]
))/4
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.51

method	result
default	$-\frac{1}{4x^4}-3\ln\left(x\right)+\frac{3\ln\left(x^8+3x^4+1\right)}{8}-\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)}{20}$
risch	$-\frac{1}{4x^4}-3\ln\left(x\right)+\frac{3\ln\left(7x^4+\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)}{8}+\frac{7\ln\left(7x^4+\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40}+\frac{3\ln\left(7x^4+\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)}{8}-\frac{7\ln\left(7x^4+\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40}$

input
```
int(1/x^5/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4-3*ln(x)+3/8*ln(x^8+3*x^4+1)-7/20*5^(1/2)*arctanh(1/5*(2*x^4+3)*5^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^5(1+3x^4+x^8)}\,dx = \frac{7\sqrt{5}x^4\log\left(\frac{2x^8+6x^4-\sqrt{5}(2x^4+3)+7}{x^8+3x^4+1}\right)+15x^4\log(x^8+3x^4+1)-120x^4\log(x)-10}{40x^4}$$

input
```
integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/40*(7*sqrt(5)*x^4*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3
*x^4 + 1)) + 15*x^4*log(x^8 + 3*x^4 + 1) - 120*x^4*log(x) - 10)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^5(1+3x^4+x^8)}\,dx = -3\log(x)+\left(\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(\frac{3}{8}-\frac{7\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)-\frac{1}{4x^4}$$

input
```
integrate(1/x**5/(x**8+3*x**4+1),x)
```

output
```
-3*log(x) + (3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (3/8 - 7*sq
rt(5)/40)*log(x**4 + sqrt(5)/2 + 3/2) - 1/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^5 (1 + 3x^4 + x^8)} \, dx = \frac{7}{40} \sqrt{5} \log \left(\frac{2 x^4 - \sqrt{5} + 3}{2 x^4 + \sqrt{5} + 3} \right) - \frac{1}{4 x^4} + \frac{3}{8} \log \left(x^8 + 3 x^4 + 1 \right) - \frac{3}{4} \log \left(x^4 \right)$$

input
```
integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/4/x^4 +
3/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^5 (1 + 3x^4 + x^8)} \, dx = \frac{7}{40} \sqrt{5} \log \left(\frac{2 x^4 - \sqrt{5} + 3}{2 x^4 + \sqrt{5} + 3} \right) + \frac{3 x^4 - 1}{4 x^4} + \frac{3}{8} \log \left(x^8 + 3 x^4 + 1 \right) - \frac{3}{4} \log \left(x^4 \right)$$

input
```
integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/4*(3*x^4
 - 1)/x^4 + 3/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^5(1+3x^4+x^8)}\,dx = \ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}+\frac{3}{8}\right)-\frac{1}{4\,x^4} -3\,\ln(x)-\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)$$

input
```
int(1/(x^5*(3*x^4 + x^8 + 1)),x)
```

output
```
log(x^4 - 5^(1/2)/2 + 3/2)*((7*5^(1/2))/40 + 3/8) - 1/(4*x^4) - 3*log(x) -
 log(5^(1/2)/2 + x^4 + 3/2)*((7*5^(1/2))/40 - 3/8)
```

Reduce [F]

$$\int \frac{1}{x^5(1+3x^4+x^8)}\,dx = \int \frac{1}{x^{13}+3x^9+x^5}dx$$

input
```
int(1/x^5/(x^8+3*x^4+1),x)
```

output
```
int(1/(x**13 + 3*x**9 + x**5),x)
```

3.110 $\int \frac{1}{x^7(1+3x^4+x^8)}\,dx$

Optimal result 861
Mathematica [C] (verified) 861
Rubi [A] (verified) 862
Maple [C] (verified) 864
Fricas [A] (verification not implemented) 865
Sympy [A] (verification not implemented) 865
Maxima [F] 866
Giac [A] (verification not implemented) 866
Mupad [B] (verification not implemented) 867
Reduce [F] 867

Optimal result

Integrand size = 16, antiderivative size = 99

$$\int \frac{1}{x^7(1+3x^4+x^8)}\,dx = -\frac{1}{6x^6}+\frac{3}{2x^2} - \frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x^2\right) + \frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

output
```
-1/6/x^6+3/2/x^2-1/2*(5/2-11/10*5^(1/2))*arctan((1/2*5^(1/2)-1/2)*x^2)+1/2
*(5/2+11/10*5^(1/2))*arctan((1/2+1/2*5^(1/2))*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^7(1+3x^4+x^8)}\,dx = -\frac{1}{6x^6}+\frac{3}{2x^2}+\frac{1}{4}\text{RootSum}\left[1+3\#1^4 + \#1^8\&,\frac{8\log(x-\#1)+3\log(x-\#1)\#1^4}{3\#1^2+2\#1^6}\&\right]$$

input
```
Integrate[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

output
```
-1/6*1/x^6 + 3/(2*x^2) + RootSum[1 + 3*#1^4 + #1^8 & , (8*Log[x - #1] + 3*
Log[x - #1]*#1^4)/(3*#1^2 + 2*#1^6) & ]/4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1695, 1443, 27, 1604, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (x^8+3x^4+1)} \, dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2} \int \frac{1}{x^8 (x^8+3x^4+1)} dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2} \left(\frac{1}{3} \int -\frac{3(x^4+3)}{x^4 (x^8+3x^4+1)} dx^2 - \frac{1}{3x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\int \frac{x^4+3}{x^4 (x^8+3x^4+1)} dx^2 - \frac{1}{3x^6}\right)$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{2} \left(\int \frac{3x^4+8}{x^8+3x^4+1} dx^2 - \frac{1}{3x^6} + \frac{3}{x^2}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2} \left(\frac{1}{10}\left(15+7\sqrt{5}\right) \int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)} dx^2 + \frac{1}{10}\left(15-7\sqrt{5}\right) \int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)} dx^2 - \frac{1}{3x^6} + \frac{3}{x^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{(15-7\sqrt{5})\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}+\frac{1}{10}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\left(15+7\sqrt{5}\right)\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{3x^6}+\frac{3}{x^2}\right)$$

input
```
Int[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

output
```
(-1/3*1/x^6 + 3/x^2 + ((15 - 7*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])
/(5*Sqrt[2*(3 + Sqrt[5])]) + (Sqrt[(3 + Sqrt[5])/2]*(15 + 7*Sqrt[5])*ArcTa
n[Sqrt[(3 + Sqrt[5])/2]*x^2])/10)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

method	result	size
risch	$\frac{\frac{3x^4}{2}-\frac{1}{6}}{x^6}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+615\_Z^2+1\right)}\_R\ln\left(-90\_R^3+55x^2-2207\_R\right)\right)}{4}$	48
default	$-\frac{1}{6x^6}+\frac{3}{2x^2}+\frac{\sqrt{5}\left(7+3\sqrt{5}\right)\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{-10+10\sqrt{5}}+\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{10+10\sqrt{5}}$	84

input
```
int(1/x^7/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(3/2*x^4-1/6)/x^6+1/4*sum(_R*ln(-90*_R^3+55*x^2-2207*_R),_R=RootOf(25*_Z^4
+615*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^7 (1+3x^4+x^8)} dx = \frac{90 x^4 + 3 \left(11 \sqrt{5}x^6 + 25 x^6\right) \arctan\left(\frac{1}{2} \sqrt{5}x^2 + \frac{1}{2} x^2\right) + 3 \left(11 \sqrt{5}x^6 - 25 x^6\right) \arctan\left(\frac{1}{2} \sqrt{5}x^2 - \frac{1}{2} x^2\right) - 10}{60 x^6}$$

input
```
integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/60*(90*x^4 + 3*(11*sqrt(5)*x^6 + 25*x^6)*arctan(1/2*sqrt(5)*x^2 + 1/2*x^
2) + 3*(11*sqrt(5)*x^6 - 25*x^6)*arctan(1/2*sqrt(5)*x^2 - 1/2*x^2) - 10)/x
^6
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^7 (1+3x^4+x^8)} dx = 2 \cdot \left(\frac{11\sqrt{5}}{40} + \frac{5}{8}\right) \operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right) - 2 \cdot \left(\frac{5}{8} - \frac{11\sqrt{5}}{40}\right) \operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right) + \frac{9x^4-1}{6x^6}$$

input
```
integrate(1/x**7/(x**8+3*x**4+1),x)
```

output
```
2*(11*sqrt(5)/40 + 5/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(5/8 - 11*sqrt(5)/
40)*atan(2*x**2/(1 + sqrt(5))) + (9*x**4 - 1)/(6*x**6)
```

Maxima [F]

$$\int \frac{1}{x^7 (1+3x^4+x^8)} \, dx = \int \frac{1}{(x^8+3\,x^4+1)x^7} \, dx$$

input
```
integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
1/6*(9*x^4 - 1)/x^6 + integrate((3*x^4 + 8)*x/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7 (1+3x^4+x^8)} \, dx = \frac{1}{20} \left(3\,x^4\left(\sqrt{5}-5\right)+8\,\sqrt{5}-40\right) \arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right) + \frac{1}{20} \left(3\,x^4\left(\sqrt{5}+5\right)+8\,\sqrt{5}+40\right) \arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right) + \frac{9\,x^4-1}{6\,x^6}$$

input
```
integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/20*(3*x^4*(sqrt(5) - 5) + 8*sqrt(5) - 40)*arctan(2*x^2/(sqrt(5) + 1)) +
1/20*(3*x^4*(sqrt(5) + 5) + 8*sqrt(5) + 40)*arctan(2*x^2/(sqrt(5) - 1)) +
1/6*(9*x^4 - 1)/x^6
```

Mupad [B] (verification not implemented)

Time = 18.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^7(1+3x^4+x^8)}\,dx = 2\,\mathrm{atanh}\left(\frac{3327500\,x^2\sqrt{\frac{11\sqrt{5}}{32}-\frac{123}{160}}}{1140425\sqrt{5}-2550075} - \frac{1488300\sqrt{5}\,x^2\sqrt{\frac{11\sqrt{5}}{32}-\frac{123}{160}}}{1140425\sqrt{5}-2550075}\right)\sqrt{\frac{11\sqrt{5}}{32}-\frac{123}{160}} - 2\,\mathrm{atanh}\left(\frac{3327500\,x^2\sqrt{-\frac{11\sqrt{5}}{32}-\frac{123}{160}}}{1140425\sqrt{5}+2550075} + \frac{1488300\sqrt{5}\,x^2\sqrt{-\frac{11\sqrt{5}}{32}-\frac{123}{160}}}{1140425\sqrt{5}+2550075}\right)\sqrt{-\frac{11\sqrt{5}}{32}-\frac{123}{160}} + \frac{\frac{3x^4}{2}-\frac{1}{6}}{x^6}$$

input
```
int(1/(x^7*(3*x^4 + x^8 + 1)),x)
```

output
```
2*atanh((3327500*x^2*((11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) -
 2550075) - (1488300*5^(1/2)*x^2*((11*5^(1/2))/32 - 123/160)^(1/2))/(11404
25*5^(1/2) - 2550075))*((11*5^(1/2))/32 - 123/160)^(1/2) - 2*atanh((332750
0*x^2*(- (11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) + 2550075) + (
1488300*5^(1/2)*x^2*(- (11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2)
+ 2550075))*(- (11*5^(1/2))/32 - 123/160)^(1/2) + ((3*x^4)/2 - 1/6)/x^6
```

Reduce [F]

$$\int \frac{1}{x^7(1+3x^4+x^8)}\,dx = \frac{18\left(\int \frac{x^5}{x^8+3x^4+1}dx\right)x^6 + 48\left(\int \frac{x}{x^8+3x^4+1}dx\right)x^6 + 9x^4 - 1}{6x^6}$$

input
```
int(1/x^7/(x^8+3*x^4+1),x)
```

output

```
(18*int(x**5/(x**8 + 3*x**4 + 1),x)*x**6 + 48*int(x/(x**8 + 3*x**4 + 1),x)
*x**6 + 9*x**4 - 1)/(6*x**6)
```

3.111 $\int \frac{x^8}{1+3x^4+x^8} \, dx$

Optimal result 870
Mathematica [C] (verified) 871
Rubi [A] (verified) 871
Maple [C] (verified) 878
Fricas [A] (verification not implemented) 879
Sympy [A] (verification not implemented) 880
Maxima [F] 880
Giac [A] (verification not implemented) 881
Mupad [B] (verification not implemented) 882
Reduce [F] 883

Optimal result

Integrand size = 16, antiderivative size = 342

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = x-\frac{\sqrt[4]{123-55\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123+55\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123+55\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\operatorname{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}x}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123+55\sqrt{5}}\operatorname{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}x}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\right)}{2\,2^{3/4}\sqrt{5}}$$

output

```
x+1/20*(123-55*5^(1/2))^(1/4)*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(1/
4)*5^(1/2)+1/20*(123-55*5^(1/2))^(1/4)*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4
))*2^(1/4)*5^(1/2)-1/20*(123+55*5^(1/2))^(1/4)*arctan(-1+2^(3/4)*x/(3+5^(1
/2))^(1/4))*2^(1/4)*5^(1/2)-1/20*(123+55*5^(1/2))^(1/4)*arctan(1+2^(3/4)*x
/(3+5^(1/2))^(1/4))*2^(1/4)*5^(1/2)+1/20*(123-55*5^(1/2))^(1/4)*arctanh(2^
(3/4)*(3-5^(1/2))^(1/4)*x/(1/2*10^(1/2)-1/2*2^(1/2)+x^2*2^(1/2)))*2^(1/4)*
5^(1/2)-1/20*(123+55*5^(1/2))^(1/4)*arctanh(2^(3/4)*(3+5^(1/2))^(1/4)*x/(1
/2*10^(1/2)+1/2*2^(1/2)+x^2*2^(1/2)))*2^(1/4)*5^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.17

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = x-\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)+3\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[x^8/(1 + 3*x^4 + x^8),x]
```

output
```
x - RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1] + 3*Log[x - #1]*#1^4)/(3*#1
^3 + 2*#1^7) & ]/4
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.40, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1703, 1752, 755, 27, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1703}$$

$$x-\int \frac{3x^4+1}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1752}$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx+x$$

$$\Big\downarrow \text{755}$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\int\frac{2\left(\sqrt{3-\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}+\frac{\int\frac{2\left(\sqrt{2}x^2+\sqrt{3-\sqrt{5}}\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\int\frac{2\left(\sqrt{3+\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}+\frac{\int\frac{2\left(\sqrt{2}x^2+\sqrt{3+\sqrt{5}}\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}\right)+x$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\int\frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\int\frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)+x$$

$$\Big\downarrow \text{1476}$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\frac{\int\frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int\frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3-\sqrt{5}}}+\frac{\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\frac{\int\frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int\frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3+\sqrt{5}}}+\frac{\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)+x$$

$$\Big\downarrow \text{1082}$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\cdots}}\right.}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\cdots}}\right.}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}\right.$$

$$x$$

$$\downarrow 217$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}\right)-$$

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}\right)+x$$

$\downarrow$ 1479

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{2x+\sqrt[4]{2}\cdots}{x^2+\sqrt[4]{2\left(3-\sqrt{\cdots}\right.}\cdots}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{-\frac{\int -\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int -\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+\cdots\right.}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}\cdots}{\cdots}\right.$$

x

$\downarrow$ 25

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx+\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{2x+\sqrt[4]{2\left(3-\right.}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\frac{\int\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-}{\sqrt{3-}}\right.$$

$$x$$

$\Big\downarrow$ 1103

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}+\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\right.}{}\right.$$

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(\right.}{}}{\sqrt{3+\sqrt{5}}}\right.$$

$$x$$

input
```
Int[x^8/(1 + 3*x^4 + x^8),x]
```

output
```
x - ((15 - 7*Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(
3/4)*(3 - Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(
2^(3/4)*(3 - Sqrt[5])^(1/4)))/Sqrt[3 - Sqrt[5]] + (-1/4*(((3 + Sqrt[5])/2)
^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) +
 (((3 + Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^
(1/4)*x + 2*x^2])/4)/Sqrt[3 - Sqrt[5]]))/10 - ((15 + 7*Sqrt[5])*((-(ArcTan
[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4))) + Arc
Tan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sq
rt[3 + Sqrt[5]] + (-1/2*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1
/4)*x + 2*x^2]/(2^(3/4)*(3 + Sqrt[5])^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] +
 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqr
t[3 + Sqrt[5]]))/10
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1703

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.13

method	result	size
default	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(-3\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	46
risch	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(-3\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	46

input
```
int(x^8/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
x+1/4*sum((-3*_R^4-1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{1+3x^4+x^8}\,dx$$
$$= \frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{\frac{55}{2}\sqrt{5}-\frac{123}{2}}}\log\left(\sqrt{\frac{1}{5}}\left(3\sqrt{5}+5\right)\sqrt{-\sqrt{\frac{55}{2}\sqrt{5}-\frac{123}{2}}}+2x\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{\frac{55}{2}\sqrt{5}-\frac{123}{2}}}\log\left(-\sqrt{\frac{1}{5}}\left(3\sqrt{5}+5\right)\sqrt{-\sqrt{\frac{55}{2}\sqrt{5}-\frac{123}{2}}}+2x\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{-\frac{55}{2}\sqrt{5}-\frac{123}{2}}}\log\left(\sqrt{\frac{1}{5}}\left(3\sqrt{5}-5\right)\sqrt{-\sqrt{-\frac{55}{2}\sqrt{5}-\frac{123}{2}}}+2x\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{-\frac{55}{2}\sqrt{5}-\frac{123}{2}}}\log\left(-\sqrt{\frac{1}{5}}\left(3\sqrt{5}-5\right)\sqrt{-\sqrt{-\frac{55}{2}\sqrt{5}-\frac{123}{2}}}+2x\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{5}}\left(\frac{55}{2}\sqrt{5}-\frac{123}{2}\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{1}{5}}\left(\frac{55}{2}\sqrt{5}-\frac{123}{2}\right)^{\frac{1}{4}}\left(3\sqrt{5}+5\right)+2x\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{5}}\left(\frac{55}{2}\sqrt{5}-\frac{123}{2}\right)^{\frac{1}{4}}\log\left(-\sqrt{\frac{1}{5}}\left(\frac{55}{2}\sqrt{5}-\frac{123}{2}\right)^{\frac{1}{4}}\left(3\sqrt{5}+5\right)+2x\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{5}}\left(-\frac{55}{2}\sqrt{5}-\frac{123}{2}\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{1}{5}}\left(3\sqrt{5}-5\right)\left(-\frac{55}{2}\sqrt{5}-\frac{123}{2}\right)^{\frac{1}{4}}+2x\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{5}}\left(-\frac{55}{2}\sqrt{5}-\frac{123}{2}\right)^{\frac{1}{4}}\log\left(-\sqrt{\frac{1}{5}}\left(3\sqrt{5}-5\right)\left(-\frac{55}{2}\sqrt{5}-\frac{123}{2}\right)^{\frac{1}{4}}+2x\right)+x$$

input
```
integrate(x^8/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/5)*sqrt(-sqrt(55/2*sqrt(5) - 123/2))*log(sqrt(1/5)*(3*sqrt(5) +
 5)*sqrt(-sqrt(55/2*sqrt(5) - 123/2)) + 2*x) - 1/4*sqrt(1/5)*sqrt(-sqrt(55
/2*sqrt(5) - 123/2))*log(-sqrt(1/5)*(3*sqrt(5) + 5)*sqrt(-sqrt(55/2*sqrt(5
) - 123/2)) + 2*x) - 1/4*sqrt(1/5)*sqrt(-sqrt(-55/2*sqrt(5) - 123/2))*log(
sqrt(1/5)*(3*sqrt(5) - 5)*sqrt(-sqrt(-55/2*sqrt(5) - 123/2)) + 2*x) + 1/4*
sqrt(1/5)*sqrt(-sqrt(-55/2*sqrt(5) - 123/2))*log(-sqrt(1/5)*(3*sqrt(5) - 5
)*sqrt(-sqrt(-55/2*sqrt(5) - 123/2)) + 2*x) + 1/4*sqrt(1/5)*(55/2*sqrt(5)
- 123/2)^(1/4)*log(sqrt(1/5)*(55/2*sqrt(5) - 123/2)^(1/4)*(3*sqrt(5) + 5)
+ 2*x) - 1/4*sqrt(1/5)*(55/2*sqrt(5) - 123/2)^(1/4)*log(-sqrt(1/5)*(55/2*s
qrt(5) - 123/2)^(1/4)*(3*sqrt(5) + 5) + 2*x) - 1/4*sqrt(1/5)*(-55/2*sqrt(5
) - 123/2)^(1/4)*log(sqrt(1/5)*(3*sqrt(5) - 5)*(-55/2*sqrt(5) - 123/2)^(1/
4) + 2*x) + 1/4*sqrt(1/5)*(-55/2*sqrt(5) - 123/2)^(1/4)*log(-sqrt(1/5)*(3*
sqrt(5) - 5)*(-55/2*sqrt(5) - 123/2)^(1/4) + 2*x) + x
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.08

$$\int \frac{x^8}{1+3x^4+x^8}\,dx$$
$$= x + \operatorname{RootSum}\left(40960000t^8 + 787200t^4 + 1, \left(t \mapsto t\log\left(\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right)$$

input
```
integrate(x**8/(x**8+3*x**4+1),x)
```

output
```
x + RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(15360*_t*
*5/11 + 1288*_t/55 + x)))
```

Maxima [F]

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = \int \frac{x^8}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^8/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
x - integrate((3*x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.70

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = -\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}+55} + \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}+55} + \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}-55} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}-55} - \frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(722500\left(x+\sqrt{\sqrt{5}+1}\right)^2+722500\,x^2\right) + \frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(722500\left(x-\sqrt{\sqrt{5}+1}\right)^2+722500\,x^2\right) + \frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(2992900\left(x+\sqrt{\sqrt{5}-1}\right)^2+2992900\,x^2\right) - \frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(2992900\left(x-\sqrt{\sqrt{5}-1}\right)^2+2992900\,x^2\right) + x$$

input
```
integrate(x^8/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
-1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/8
0*(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/80*(
pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/80*(pi +
 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/40*sqrt(25*
sqrt(5) + 55)*log(722500*(x + sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1/40*sq
rt(25*sqrt(5) + 55)*log(722500*(x - sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1
/40*sqrt(25*sqrt(5) - 55)*log(2992900*(x + sqrt(sqrt(5) - 1))^2 + 2992900*
x^2) - 1/40*sqrt(25*sqrt(5) - 55)*log(2992900*(x - sqrt(sqrt(5) - 1))^2 +
2992900*x^2) + x
```

Mupad [B] (verification not implemented)

Time = 18.97 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.63

$$\int \frac{x^8}{1+3x^4+x^8}\,dx$$
$$= x - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{1/4}\,x}{2\left(-55\sqrt{5}-123\right)^{1/4}}+\frac{2^{1/4}\sqrt{5}\,x}{2\left(-55\sqrt{5}-123\right)^{1/4}}\right)\left(-55\sqrt{5}-123\right)^{1/4}}{20}$$
$$+ \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{1/4}\,x}{2\left(55\sqrt{5}-123\right)^{1/4}}-\frac{2^{1/4}\sqrt{5}\,x}{2\left(55\sqrt{5}-123\right)^{1/4}}\right)\left(55\sqrt{5}-123\right)^{1/4}}{20}$$
$$+ \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,3\mathrm{i}}{2\left(-55\sqrt{5}-123\right)^{1/4}}+\frac{2^{1/4}\sqrt{5}\,x\,1\mathrm{i}}{2\left(-55\sqrt{5}-123\right)^{1/4}}\right)\left(-55\sqrt{5}-123\right)^{1/4}\,1\mathrm{i}}{20}$$
$$- \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,3\mathrm{i}}{2\left(55\sqrt{5}-123\right)^{1/4}}-\frac{2^{1/4}\sqrt{5}\,x\,1\mathrm{i}}{2\left(55\sqrt{5}-123\right)^{1/4}}\right)\left(55\sqrt{5}-123\right)^{1/4}\,1\mathrm{i}}{20}$$

input
```
int(x^8/(3*x^4 + x^8 + 1),x)
```

output
```
x - (2^(3/4)*5^(1/2)*atan((3*2^(1/4)*x)/(2*(- 55*5^(1/2) - 123)^(1/4)) + (
2^(1/4)*5^(1/2)*x)/(2*(- 55*5^(1/2) - 123)^(1/4)))*(- 55*5^(1/2) - 123)^(1
/4))/20 + (2^(3/4)*5^(1/2)*atan((3*2^(1/4)*x)/(2*(55*5^(1/2) - 123)^(1/4))
 - (2^(1/4)*5^(1/2)*x)/(2*(55*5^(1/2) - 123)^(1/4)))*(55*5^(1/2) - 123)^(1
/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(1/4)*x*3i)/(2*(- 55*5^(1/2) - 123)^(1/
4)) + (2^(1/4)*5^(1/2)*x*1i)/(2*(- 55*5^(1/2) - 123)^(1/4)))*(- 55*5^(1/2)
 - 123)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(1/4)*x*3i)/(2*(55*5^(1/2)
 - 123)^(1/4)) - (2^(1/4)*5^(1/2)*x*1i)/(2*(55*5^(1/2) - 123)^(1/4)))*(55*
5^(1/2) - 123)^(1/4)*1i)/20
```

Reduce [F]

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = -3\left(\int \frac{x^4}{x^8+3x^4+1}dx\right)-\left(\int \frac{1}{x^8+3x^4+1}dx\right)+x$$

input
```
int(x^8/(x^8+3*x^4+1),x)
```

output
```
 - 3*int(x**4/(x**8 + 3*x**4 + 1),x) - int(1/(x**8 + 3*x**4 + 1),x) + x
```

3.112 $\int \frac{x^6}{1+3x^4+x^8}\,dx$

Optimal result 885
Mathematica [C] (verified) 886
Rubi [A] (verified) 886
Maple [C] (verified) 893
Fricas [A] (verification not implemented) 894
Sympy [A] (verification not implemented) 895
Maxima [F] 895
Giac [A] (verification not implemented) 896
Mupad [B] (verification not implemented) 897
Reduce [F] 897

Optimal result

Integrand size = 16, antiderivative size = 320

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \frac{\sqrt[4]{9-4\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} - \frac{\sqrt[4]{9-4\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}} + \frac{\sqrt[4]{9-4\sqrt{5}}\,\text{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}x}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\right)}{2\sqrt{10}} - \frac{\left(3+\sqrt{5}\right)^{3/4}\text{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}x}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\right)}{4\sqrt[4]{2}\sqrt{5}}$$

output

```
-1/20*(9-4*5^(1/2))^(1/4)*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*10^(1/2)-
1/20*(9-4*5^(1/2))^(1/4)*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*10^(1/2)+1/
40*(3+5^(1/2))^(3/4)*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*2^(3/4)*5^(1/2
)+1/40*(3+5^(1/2))^(3/4)*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*2^(3/4)*5^(
1/2)+1/20*(9-4*5^(1/2))^(1/4)*arctanh(2^(3/4)*(3-5^(1/2))^(1/4)*x/(1/2*10^
(1/2)-1/2*2^(1/2)+x^2*2^(1/2)))*10^(1/2)-1/40*(3+5^(1/2))^(3/4)*arctanh(2^
(3/4)*(3+5^(1/2))^(1/4)*x/(1/2*10^(1/2)+1/2*2^(1/2)+x^2*2^(1/2)))*2^(3/4)*
5^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.13

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^3}{3+2\#1^4}\&\right]$$

input
```
Integrate[x^6/(1 + 3*x^4 + x^8),x]
```

output
```
RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1^3)/(3 + 2*#1^4) & ]/4
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.45, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1710, 27, 826, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{2x^2}{2x^4-\sqrt{5}+3}dx+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{2x^2}{2x^4+\sqrt{5}+3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\int \frac{x^2}{2x^4-\sqrt{5}+3}dx+\frac{1}{5}\left(5+3\sqrt{5}\right)\int \frac{x^2}{2x^4+\sqrt{5}+3}dx$$

$$\Big\downarrow \text{826}$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)+\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)+\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x}{2x^4-\sqrt{5}+3}}{2\sqrt{2}}\right.$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x}{2x^4+\sqrt{5}+3}}{2\sqrt{2}}\right.$$

$$\Big\downarrow \ 217$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)+$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)$$

$\downarrow$ 1479

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int-\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3\right.}}\cdots}{\cdots}\right.$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{-\frac{\int-\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int-\frac{\cdots}{x^2+\sqrt[4]{\cdots}}}{\cdots}}{2\sqrt{2}}\right.$$

$\downarrow$ 25

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}\cdots}{\cdots}\right.$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{\int\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int\frac{2x+\sqrt[4]{\cdots}}{x^2+\sqrt[4]{2\left(3\cdots\right.}}\cdots}{2\,2^{3\cdots}}}{2\sqrt{2}}\right.$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\cdots\right.}{\cdots}\right.$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x^2\cdots\right.}{\cdots}}{2\sqrt{2}}\right.$$

input
```
Int[x^6/(1 + 3*x^4 + x^8),x]
```

output
```
((5 - 3*Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)*
(3 - Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/
4)*(3 - Sqrt[5])^(1/4)))/(2*Sqrt[2]) - (-1/4*(((3 + Sqrt[5])/2)^(1/4)*Log[
Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) + (((3 + Sqr
t[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2
*x^2])/4)/(2*Sqrt[2])))/5 + ((5 + 3*Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3
 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)
/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4)))/(2*Sqrt[2]) - (-1/2*L
og[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2^(3/4)*(
3 + Sqrt[5])^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4
)*x + 2*x^2]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4)))/(2*Sqrt[2])))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\_R^{6}\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\_R^{6}\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	40

input
```
int(x^6/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^6/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.20

$$
\begin{aligned}
&\int \frac{x^6}{1+3x^4+x^8}\,dx \\
&= \frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{4\sqrt{5}-9}}\log\left(\sqrt{\frac{1}{5}}\left(7\sqrt{5}+15\right)\sqrt{4\sqrt{5}-9}\sqrt{-\sqrt{4\sqrt{5}-9}}+2x\right) \\
&\quad - \frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{4\sqrt{5}-9}}\log\left(-\sqrt{\frac{1}{5}}\left(7\sqrt{5}+15\right)\sqrt{4\sqrt{5}-9}\sqrt{-\sqrt{4\sqrt{5}-9}}+2x\right) \\
&\quad - \frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{-4\sqrt{5}-9}}\log\left(\sqrt{\frac{1}{5}}\left(7\sqrt{5}-15\right)\sqrt{-4\sqrt{5}-9}\sqrt{-\sqrt{-4\sqrt{5}-9}}\right. \\
&\qquad \left. +2x\right) \\
&\quad + \frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{-4\sqrt{5}-9}}\log\left(-\sqrt{\frac{1}{5}}\left(7\sqrt{5}-15\right)\sqrt{-4\sqrt{5}-9}\sqrt{-\sqrt{-4\sqrt{5}-9}}\right. \\
&\qquad \left. +2x\right) - \frac{1}{4}\sqrt{\frac{1}{5}}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{1}{5}}\left(7\sqrt{5}+15\right)\left(4\sqrt{5}-9\right)^{\frac{3}{4}}+2x\right) \\
&\quad + \frac{1}{4}\sqrt{\frac{1}{5}}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(-\sqrt{\frac{1}{5}}\left(7\sqrt{5}+15\right)\left(4\sqrt{5}-9\right)^{\frac{3}{4}}+2x\right) \\
&\quad + \frac{1}{4}\sqrt{\frac{1}{5}}\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{1}{5}}\left(7\sqrt{5}-15\right)\left(-4\sqrt{5}-9\right)^{\frac{3}{4}}+2x\right) \\
&\quad - \frac{1}{4}\sqrt{\frac{1}{5}}\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(-\sqrt{\frac{1}{5}}\left(7\sqrt{5}-15\right)\left(-4\sqrt{5}-9\right)^{\frac{3}{4}}+2x\right)
\end{aligned}
$$

input
```
integrate(x^6/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/5)*sqrt(-sqrt(4*sqrt(5) - 9))*log(sqrt(1/5)*(7*sqrt(5) + 15)*sq
rt(4*sqrt(5) - 9)*sqrt(-sqrt(4*sqrt(5) - 9)) + 2*x) - 1/4*sqrt(1/5)*sqrt(-
sqrt(4*sqrt(5) - 9))*log(-sqrt(1/5)*(7*sqrt(5) + 15)*sqrt(4*sqrt(5) - 9)*s
qrt(-sqrt(4*sqrt(5) - 9)) + 2*x) - 1/4*sqrt(1/5)*sqrt(-sqrt(-4*sqrt(5) - 9
))*log(sqrt(1/5)*(7*sqrt(5) - 15)*sqrt(-4*sqrt(5) - 9)*sqrt(-sqrt(-4*sqrt(
5) - 9)) + 2*x) + 1/4*sqrt(1/5)*sqrt(-sqrt(-4*sqrt(5) - 9))*log(-sqrt(1/5)
*(7*sqrt(5) - 15)*sqrt(-4*sqrt(5) - 9)*sqrt(-sqrt(-4*sqrt(5) - 9)) + 2*x)
- 1/4*sqrt(1/5)*(4*sqrt(5) - 9)^(1/4)*log(sqrt(1/5)*(7*sqrt(5) + 15)*(4*sq
rt(5) - 9)^(3/4) + 2*x) + 1/4*sqrt(1/5)*(4*sqrt(5) - 9)^(1/4)*log(-sqrt(1/
5)*(7*sqrt(5) + 15)*(4*sqrt(5) - 9)^(3/4) + 2*x) + 1/4*sqrt(1/5)*(-4*sqrt(
5) - 9)^(1/4)*log(sqrt(1/5)*(7*sqrt(5) - 15)*(-4*sqrt(5) - 9)^(3/4) + 2*x)
 - 1/4*sqrt(1/5)*(-4*sqrt(5) - 9)^(1/4)*log(-sqrt(1/5)*(7*sqrt(5) - 15)*(-
4*sqrt(5) - 9)^(3/4) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.08

$$\int \frac{x^6}{1+3x^4+x^8}\,dx$$
$$= \mathrm{RootSum}\left(40960000t^8+115200t^4+1,\left(t\mapsto t\log\left(-1792000t^7-4920t^3+x\right)\right)\right)$$

input
```
integrate(x**6/(x**8+3*x**4+1),x)
```

output
```
RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-1792000*_t**
7 - 4920*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \int \frac{x^6}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^6/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^6/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.75

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}+20} - \frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}-20} + \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}-20} - \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x+\sqrt{\sqrt{5}+1}\right)^2+400x^2\right) + \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x-\sqrt{\sqrt{5}+1}\right)^2+400x^2\right) + \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x+\sqrt{\sqrt{5}-1}\right)^2+10000x^2\right) - \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x-\sqrt{\sqrt{5}-1}\right)^2+10000x^2\right)$$

input
```
integrate(x^6/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) + 20) - 1/80
*(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) + 20) - 1/80*(p
i + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) - 20) + 1/80*(pi +
4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) - 20) - 1/40*sqrt(10*s
qrt(5) + 20)*log(400*(x + sqrt(sqrt(5) + 1))^2 + 400*x^2) + 1/40*sqrt(10*s
qrt(5) + 20)*log(400*(x - sqrt(sqrt(5) + 1))^2 + 400*x^2) + 1/40*sqrt(10*s
qrt(5) - 20)*log(10000*(x + sqrt(sqrt(5) - 1))^2 + 10000*x^2) - 1/40*sqrt(
10*sqrt(5) - 20)*log(10000*(x - sqrt(sqrt(5) - 1))^2 + 10000*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.47

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{8\,\sqrt{5}+24}\right)\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\left(4\,\sqrt{5}-9\right)^{1/4}}{8\,\sqrt{5}-24}\right)\,\left(4\,\sqrt{5}-9\right)^{1/4}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,16\mathrm{i}}{8\,\sqrt{5}+24}\right)\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\left(4\,\sqrt{5}-9\right)^{1/4}\,16\mathrm{i}}{8\,\sqrt{5}-24}\right)\,\left(4\,\sqrt{5}-9\right)^{1/4}\,1\mathrm{i}}{10}$$

input
```
int(x^6/(3*x^4 + x^8 + 1),x)
```

output
```
(5^(1/2)*atan((16*x*(- 4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) + 24))*(- 4*5^(1/2
) - 9)^(1/4))/10 + (5^(1/2)*atan((16*x*(4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) -
 24))*(4*5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((x*(- 4*5^(1/2) - 9)^(1/4)
*16i)/(8*5^(1/2) + 24))*(- 4*5^(1/2) - 9)^(1/4)*1i)/10 + (5^(1/2)*atan((x*
(4*5^(1/2) - 9)^(1/4)*16i)/(8*5^(1/2) - 24))*(4*5^(1/2) - 9)^(1/4)*1i)/10
```

Reduce [F]

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = \int \frac{x^6}{x^8+3x^4+1}dx$$

input
```
int(x^6/(x^8+3*x^4+1),x)
```

output
```
int(x**6/(x**8 + 3*x**4 + 1),x)
```

3.113 $\int \frac{x^4}{1+3x^4+x^8} \, dx$

Optimal result . . . 899
Mathematica [C] (verified) . . . 900
Rubi [A] (verified) . . . 900
Maple [C] (verified) . . . 907
Fricas [A] (verification not implemented) . . . 908
Sympy [A] (verification not implemented) . . . 909
Maxima [F] . . . 909
Giac [A] (verification not implemented) . . . 910
Mupad [B] (verification not implemented) . . . 911
Reduce [F] . . . 912

Optimal result

Integrand size = 16, antiderivative size = 335

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}}\operatorname{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}x}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\operatorname{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}x}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\right)}{2\,2^{3/4}\sqrt{5}}$$

output

```
-1/20*(3-5^(1/2))^(1/4)*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(1/4)*5^(
1/2)-1/20*(3-5^(1/2))^(1/4)*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(1/4)*
5^(1/2)+1/20*(3+5^(1/2))^(1/4)*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*2^(1
/4)*5^(1/2)+1/20*(3+5^(1/2))^(1/4)*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*2
^(1/4)*5^(1/2)-1/20*(3-5^(1/2))^(1/4)*arctanh(2^(3/4)*(3-5^(1/2))^(1/4)*x/
(1/2*10^(1/2)-1/2*2^(1/2)+x^2*2^(1/2)))*2^(1/4)*5^(1/2)+1/20*(3+5^(1/2))^(
1/4)*arctanh(2^(3/4)*(3+5^(1/2))^(1/4)*x/(1/2*10^(1/2)+1/2*2^(1/2)+x^2*2^(
1/2)))*2^(1/4)*5^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.12

$$\int \frac{x^4}{1+3x^4+x^8}\,dx=\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1}{3+2\#1^4}\&\right]$$

input
```
Integrate[x^4/(1 + 3*x^4 + x^8),x]
```

output
```
RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1)/(3 + 2*#1^4) & ]/4
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.43, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1710, 755, 27, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{2\left(\sqrt{3-\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3-\sqrt{5}}\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{2\left(\sqrt{3+\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3+\sqrt{5}}\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\cdots\right.}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}\cdots}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\cdots\right.}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}\cdots}{\sqrt{3+\sqrt{5}}}\right.$$

$\downarrow$ 217

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}\right)+$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}\right)$$

$\downarrow$ 1479

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{2x+\sqrt[4]{2\left(3\right.}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right.}}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{-\frac{\int -\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int -\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right.}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{\ }}\right.$$

$\downarrow$ 25

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx+\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{2x+\sqrt[4]{2\left(3-\sqrt{5}\right.}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\ldots}}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\ldots}{\sqrt{3+\ldots}}\right.$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}+\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x\ldots\right.}{\ldots}\right.$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x\ldots\right.}{\ldots}}{\sqrt{3+\sqrt{5}}}\right.$$

input
```
Int[x^4/(1 + 3*x^4 + x^8),x]
```

output
```
((5 - 3*Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)*
(3 - Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/
4)*(3 - Sqrt[5])^(1/4)))/Sqrt[3 - Sqrt[5]] + (-1/4*(((3 + Sqrt[5])/2)^(1/4
)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) + (((3
 + Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)
*x + 2*x^2])/4)/Sqrt[3 - Sqrt[5]]))/10 + ((5 + 3*Sqrt[5])*((-(ArcTan[1 - (
2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4))) + ArcTan[1
+ (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqrt[3 +
 Sqrt[5]] + (-1/2*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x
+ 2*x^2]/(2^(3/4)*(3 + Sqrt[5])^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*
(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqrt[3 +
Sqrt[5]]))/10
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^8+3\_Z^4+1)}\frac{\_R^4\ln(x-\_R)}{2\_R^7+3\_R^3}}{4}$	40
risch	$\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^8+3\_Z^4+1)}\frac{\_R^4\ln(x-\_R)}{2\_R^7+3\_R^3}}{4}$	40

input
```
int(x^4/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^4/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.90

$$
\begin{aligned}
\int \frac{x^4}{1+3x^4+x^8}\,dx = & -\frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{\frac{1}{2}\sqrt{5}-\frac{3}{2}}}\log\left(\sqrt{5}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{\frac{1}{2}\sqrt{5}-\frac{3}{2}}}+x\right) \\
& +\frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{\frac{1}{2}\sqrt{5}-\frac{3}{2}}}\log\left(-\sqrt{5}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{\frac{1}{2}\sqrt{5}-\frac{3}{2}}}+x\right) \\
& +\frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{-\frac{1}{2}\sqrt{5}-\frac{3}{2}}}\log\left(\sqrt{5}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{-\frac{1}{2}\sqrt{5}-\frac{3}{2}}}+x\right) \\
& -\frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{-\frac{1}{2}\sqrt{5}-\frac{3}{2}}}\log\left(-\sqrt{5}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{-\frac{1}{2}\sqrt{5}-\frac{3}{2}}}+x\right) \\
& -\frac{1}{4}\sqrt{\frac{1}{5}}\left(\frac{1}{2}\sqrt{5}-\frac{3}{2}\right)^{\frac{1}{4}}\log\left(\sqrt{5}\sqrt{\frac{1}{5}}\left(\frac{1}{2}\sqrt{5}-\frac{3}{2}\right)^{\frac{1}{4}}+x\right) \\
& +\frac{1}{4}\sqrt{\frac{1}{5}}\left(\frac{1}{2}\sqrt{5}-\frac{3}{2}\right)^{\frac{1}{4}}\log\left(-\sqrt{5}\sqrt{\frac{1}{5}}\left(\frac{1}{2}\sqrt{5}-\frac{3}{2}\right)^{\frac{1}{4}}+x\right) \\
& +\frac{1}{4}\sqrt{\frac{1}{5}}\left(-\frac{1}{2}\sqrt{5}-\frac{3}{2}\right)^{\frac{1}{4}}\log\left(\sqrt{5}\sqrt{\frac{1}{5}}\left(-\frac{1}{2}\sqrt{5}-\frac{3}{2}\right)^{\frac{1}{4}}+x\right) \\
& -\frac{1}{4}\sqrt{\frac{1}{5}}\left(-\frac{1}{2}\sqrt{5}-\frac{3}{2}\right)^{\frac{1}{4}}\log\left(-\sqrt{5}\sqrt{\frac{1}{5}}\left(-\frac{1}{2}\sqrt{5}-\frac{3}{2}\right)^{\frac{1}{4}}+x\right)
\end{aligned}
$$

input
```
integrate(x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*sqrt(1/5)*sqrt(-sqrt(1/2*sqrt(5) - 3/2))*log(sqrt(5)*sqrt(1/5)*sqrt(-
sqrt(1/2*sqrt(5) - 3/2)) + x) + 1/4*sqrt(1/5)*sqrt(-sqrt(1/2*sqrt(5) - 3/2
))*log(-sqrt(5)*sqrt(1/5)*sqrt(-sqrt(1/2*sqrt(5) - 3/2)) + x) + 1/4*sqrt(1
/5)*sqrt(-sqrt(-1/2*sqrt(5) - 3/2))*log(sqrt(5)*sqrt(1/5)*sqrt(-sqrt(-1/2*
sqrt(5) - 3/2)) + x) - 1/4*sqrt(1/5)*sqrt(-sqrt(-1/2*sqrt(5) - 3/2))*log(-
sqrt(5)*sqrt(1/5)*sqrt(-sqrt(-1/2*sqrt(5) - 3/2)) + x) - 1/4*sqrt(1/5)*(1/
2*sqrt(5) - 3/2)^(1/4)*log(sqrt(5)*sqrt(1/5)*(1/2*sqrt(5) - 3/2)^(1/4) + x
) + 1/4*sqrt(1/5)*(1/2*sqrt(5) - 3/2)^(1/4)*log(-sqrt(5)*sqrt(1/5)*(1/2*sq
rt(5) - 3/2)^(1/4) + x) + 1/4*sqrt(1/5)*(-1/2*sqrt(5) - 3/2)^(1/4)*log(sqr
t(5)*sqrt(1/5)*(-1/2*sqrt(5) - 3/2)^(1/4) + x) - 1/4*sqrt(1/5)*(-1/2*sqrt(
5) - 3/2)^(1/4)*log(-sqrt(5)*sqrt(1/5)*(-1/2*sqrt(5) - 3/2)^(1/4) + x)
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.07

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = \operatorname{RootSum}\left(40960000t^8+19200t^4+1,\left(t\mapsto t\log\left(-51200t^5-12t+x\right)\right)\right)$$

input
```
integrate(x**4/(x**8+3*x**4+1),x)
```

output
```
RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-51200*_t**5 -
 12*_t + x)))
```

Maxima [F]

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = \int \frac{x^4}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^4/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^4/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.71

$$
\begin{aligned}
\int \frac{x^4}{1+3x^4+x^8}\,dx = {} & \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}+5} \\
& -\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}+5} \\
& -\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}-5} \\
& +\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}-5} \\
& +\frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(625\left(x+\sqrt{\sqrt{5}+1}\right)^2+625\,x^2\right) \\
& -\frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(625\left(x-\sqrt{\sqrt{5}+1}\right)^2+625\,x^2\right) \\
& -\frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(4225\left(x+\sqrt{\sqrt{5}-1}\right)^2+4225\,x^2\right) \\
& +\frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(4225\left(x-\sqrt{\sqrt{5}-1}\right)^2+4225\,x^2\right)
\end{aligned}
$$

input
```
integrate(x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(
pi + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi +
4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arct
an(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) +
5)*log(625*(x + sqrt(sqrt(5) + 1))^2 + 625*x^2) - 1/40*sqrt(5*sqrt(5) + 5)
*log(625*(x - sqrt(sqrt(5) + 1))^2 + 625*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*l
og(4225*(x + sqrt(sqrt(5) - 1))^2 + 4225*x^2) + 1/40*sqrt(5*sqrt(5) - 5)*l
og(4225*(x - sqrt(sqrt(5) - 1))^2 + 4225*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.36

$$\int \frac{x^4}{1+3x^4+x^8}\,dx$$
$$= \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{3/4}\,x\left(-\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}-\frac{2^{3/4}\sqrt{5}\,x\left(-\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}}{20}$$
$$-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(-\sqrt{5}-3\right)^{1/4}3\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}-\frac{2^{3/4}\sqrt{5}\,x\left(-\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\sqrt{5}\sqrt{-\sqrt{5}-3}}{2}\right)}\right)\left(-\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{20}$$
$$-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{3/4}\,x\left(\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}+\frac{2^{3/4}\sqrt{5}\,x\left(\sqrt{5}-3\right)^{1/4}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}}{20}$$
$$+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(\sqrt{5}-3\right)^{1/4}3\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}+\frac{2^{3/4}\sqrt{5}\,x\left(\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{2\left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}-3}}{2}\right)}\right)\left(\sqrt{5}-3\right)^{1/4}1\mathrm{i}}{20}$$

input
```
int(x^4/(3*x^4 + x^8 + 1),x)
```

output
```
(2^(3/4)*5^(1/2)*atan((3*2^(3/4)*x*(- 5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(
- 5^(1/2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)) - (2
^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(- 5^(1/2) - 3)^(1/
2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)))*(- 5^(1/2) - 3)^(1/4)
)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*3i)/(2*((3*2
^(1/2)*(- 5^(1/2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/
2)) - (2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*1i)/(2*((3*2^(1/2)*(- 5^(1/
2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)))*(- 5^(1/2)
 - 3)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((3*2^(3/4)*x*(5^(1/2) - 3)^(1/4
))/(2*((3*2^(1/2)*(5^(1/2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^
(1/2))/2)) + (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(5^(1/
2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)))*(5^(1/2) - 3
)^(1/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*
((3*2^(1/2)*(5^(1/2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))
/2)) + (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*1i)/(2*((3*2^(1/2)*(5^(1/2)
- 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)))*(5^(1/2) - 3)^(
1/4)*1i)/20
```

Reduce [F]

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = \int \frac{x^4}{x^8+3x^4+1}dx$$

input
```
int(x^4/(x^8+3*x^4+1),x)
```

output
```
int(x**4/(x**8 + 3*x**4 + 1),x)
```

3.114 $\int \frac{x^2}{1+3x^4+x^8} \, dx$

Optimal result 914
Mathematica [C] (verified) 915
Rubi [A] (verified) 915
Maple [C] (verified) 922
Fricas [A] (verification not implemented) 923
Sympy [A] (verification not implemented) 923
Maxima [F] 924
Giac [A] (verification not implemented) 924
Mupad [B] (verification not implemented) 925
Reduce [F] 926

Optimal result

Integrand size = 16, antiderivative size = 320

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = -\frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} + \frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}} - \frac{\sqrt[4]{3+\sqrt{5}}\operatorname{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}x}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\right)}{2\ 2^{3/4}\sqrt{5}} + \frac{\operatorname{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}x}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

output

```
1/20*(3+5^(1/2))^(1/4)*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(1/4)*5^(1
/2)+1/20*(3+5^(1/2))^(1/4)*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(1/4)*5
^(1/2)-1/10*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*5^(1/2)/(6+2*5^(1/2))^(
1/4)-1/10*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*5^(1/2)/(6+2*5^(1/2))^(1/4
)-1/20*(3+5^(1/2))^(1/4)*arctanh(2^(3/4)*(3-5^(1/2))^(1/4)*x/(1/2*10^(1/2)
-1/2*2^(1/2)+x^2*2^(1/2)))*2^(1/4)*5^(1/2)+1/10*arctanh(2^(3/4)*(3+5^(1/2)
)^(1/4)*x/(1/2*10^(1/2)+1/2*2^(1/2)+x^2*2^(1/2)))*5^(1/2)/(6+2*5^(1/2))^(1
/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.12

$$\int \frac{x^2}{1+3x^4+x^8}\,dx=\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)}{3\#1+2\#1^5}\&\right]$$

input
```
Integrate[x^2/(1 + 3*x^4 + x^8),x]
```

output
```
RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1 + 2*#1^5) & ]/4
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.41, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1711, 27, 826, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^8+3x^4+1}dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{\int \frac{2x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{5}}-\frac{\int \frac{2x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{5}}-\frac{2\int \frac{x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{826}$$

$$\frac{2\left(\frac{\int \frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}-\frac{2\left(\frac{\int \frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}-\frac{2\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}-\frac{2\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}$$

$$\Big\downarrow \ 217$$

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}-$$

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)}{\sqrt{5}}$$

$$\Big\downarrow \ 1479$$

$$2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int-\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\right.}}{2\sqrt{2}}\right.$$
$$\overline{\sqrt{5}}$$

$$2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{-\frac{\int-\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int-\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\right.}}}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}\right.$$
$$\overline{\sqrt{5}}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx+\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right.}}{2\sqrt{2}}\right.}{\sqrt{5}}$$

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{\int\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}\right.}{\sqrt{5}}$$

↓ 1103

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3\right.} \cdots}{2\sqrt{2}} \cdots\right)}{\sqrt{5}}$$

$$\frac{2\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3\right.} \cdots}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}} \cdots\right)}{\sqrt{5}}$$

input
```
Int[x^2/(1 + 3*x^4 + x^8),x]
```

output
```
(2*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)*(3 - Sqrt[5])^
(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)*(3 - Sqrt[5
])^(1/4)))/(2*Sqrt[2]) - (-1/4*(((3 + Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sq
rt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) + (((3 + Sqrt[5])/2)^(1/4)
*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/4)/(2*S
qrt[2])))/Sqrt[5] - (2*((-(ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^
(3/4)*(3 + Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/
(2^(3/4)*(3 + Sqrt[5])^(1/4)))/(2*Sqrt[2]) - (-1/2*Log[Sqrt[2*(3 + Sqrt[5]
)] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2^(3/4)*(3 + Sqrt[5])^(1/4)) +
Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2*2^(3/4
)*(3 + Sqrt[5])^(1/4)))/(2*Sqrt[2])))/Sqrt[5]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	40

input
```
int(x^2/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^2/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/5)*sqrt(-sqrt(1/2*sqrt(5) - 3/2))*log(sqrt(1/5)*(3*sqrt(5) + 5)
*sqrt(1/2*sqrt(5) - 3/2)*sqrt(-sqrt(1/2*sqrt(5) - 3/2)) + 2*x) - 1/4*sqrt(
1/5)*sqrt(-sqrt(1/2*sqrt(5) - 3/2))*log(-sqrt(1/5)*(3*sqrt(5) + 5)*sqrt(1/
2*sqrt(5) - 3/2)*sqrt(-sqrt(1/2*sqrt(5) - 3/2)) + 2*x) - 1/4*sqrt(1/5)*sqr
t(-sqrt(-1/2*sqrt(5) - 3/2))*log(sqrt(1/5)*(3*sqrt(5) - 5)*sqrt(-1/2*sqrt(
5) - 3/2)*sqrt(-sqrt(-1/2*sqrt(5) - 3/2)) + 2*x) + 1/4*sqrt(1/5)*sqrt(-sqr
t(-1/2*sqrt(5) - 3/2))*log(-sqrt(1/5)*(3*sqrt(5) - 5)*sqrt(-1/2*sqrt(5) -
3/2)*sqrt(-sqrt(-1/2*sqrt(5) - 3/2)) + 2*x) - 1/4*sqrt(1/5)*(1/2*sqrt(5) -
 3/2)^(1/4)*log(sqrt(1/5)*(3*sqrt(5) + 5)*(1/2*sqrt(5) - 3/2)^(3/4) + 2*x)
 + 1/4*sqrt(1/5)*(1/2*sqrt(5) - 3/2)^(1/4)*log(-sqrt(1/5)*(3*sqrt(5) + 5)*
(1/2*sqrt(5) - 3/2)^(3/4) + 2*x) + 1/4*sqrt(1/5)*(-1/2*sqrt(5) - 3/2)^(1/4
)*log(sqrt(1/5)*(3*sqrt(5) - 5)*(-1/2*sqrt(5) - 3/2)^(3/4) + 2*x) - 1/4*sq
rt(1/5)*(-1/2*sqrt(5) - 3/2)^(1/4)*log(-sqrt(1/5)*(3*sqrt(5) - 5)*(-1/2*sq
rt(5) - 3/2)^(3/4) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.08

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \operatorname{RootSum}\left(40960000t^8+19200t^4+1,\left(t\mapsto t\log\left(-6144000t^7-2240t^3+x\right)\right)\right)$$

input
```
integrate(x**2/(x**8+3*x**4+1),x)
```

output
```
RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-6144000*_t**7
 - 2240*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \int \frac{x^2}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^2/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^2/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{x^2}{1+3x^4+x^8}\,dx &= \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}+5} \\ &\quad -\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}+5} \\ &\quad -\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}-5} \\ &\quad +\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}-5} \\ &\quad +\frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x+\sqrt{\sqrt{5}+1}\right)^2+16900\,x^2\right) \\ &\quad -\frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x-\sqrt{\sqrt{5}+1}\right)^2+16900\,x^2\right) \\ &\quad -\frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x+\sqrt{\sqrt{5}-1}\right)^2+2500\,x^2\right) \\ &\quad +\frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x-\sqrt{\sqrt{5}-1}\right)^2+2500\,x^2\right)\end{aligned}$$

input
```
integrate(x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(
pi + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi +
4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arct
an(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) -
5)*log(16900*(x + sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5)
- 5)*log(16900*(x - sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5
) + 5)*log(2500*(x + sqrt(sqrt(5) - 1))^2 + 2500*x^2) + 1/40*sqrt(5*sqrt(5
) + 5)*log(2500*(x - sqrt(sqrt(5) - 1))^2 + 2500*x^2)
```

Mupad [B] (verification not implemented)

Time = 18.83 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{3\,2^{3/4}\,\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}-7\right)}\right)\,{\left(\sqrt{5}-3\right)}^{1/4}}{20} + \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(\sqrt{5}-3\right)^{1/4}\,7\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{2^{3/4}\,\sqrt{5}\,x\,\left(\sqrt{5}-3\right)^{1/4}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}\right)\,{\left(\sqrt{5}-3\right)}^{1/4}\,1\mathrm{i}}{20} + \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{3\,2^{3/4}\,\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}}{2\,\left(3\,\sqrt{5}+7\right)}\right)\,{\left(-\sqrt{5}-3\right)}^{1/4}}{20} + \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,\left(-\sqrt{5}-3\right)^{1/4}\,7\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{2^{3/4}\,\sqrt{5}\,x\,\left(-\sqrt{5}-3\right)^{1/4}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}\right)\,{\left(-\sqrt{5}-3\right)}^{1/4}\,1\mathrm{i}}{20}$$

input
```
int(x^2/(3*x^4 + x^8 + 1),x)
```

output
```
(2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7)
) - (3*2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7)))*(5^(1/2
) - 3)^(1/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2) - 3)^(1/4)*7i
)/(2*(3*5^(1/2) - 7)) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*(3*5
^(1/2) - 7)))*(5^(1/2) - 3)^(1/4)*1i)/20 + (2^(3/4)*5^(1/2)*atan((7*2^(3/4
)*x*(- 5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) + 7)) + (3*2^(3/4)*5^(1/2)*x*(- 5
^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) + 7)))*(- 5^(1/2) - 3)^(1/4))/20 + (2^(3/
4)*5^(1/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*7i)/(2*(3*5^(1/2) + 7)) +
 (2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*3i)/(2*(3*5^(1/2) + 7)))*(- 5^(1
/2) - 3)^(1/4)*1i)/20
```

Reduce [F]

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \int \frac{x^2}{x^8+3x^4+1}dx$$

input
```
int(x^2/(x^8+3*x^4+1),x)
```

output
```
int(x**2/(x**8 + 3*x**4 + 1),x)
```

3.115 $\int \frac{1}{1+3x^4+x^8}\,dx$

Optimal result 927
Mathematica [C] (verified) 928
Rubi [A] (verified) 928
Maple [C] (verified) 933
Fricas [A] (verification not implemented) 935
Sympy [A] (verification not implemented) 936
Maxima [F] 937
Giac [A] (verification not implemented) 937
Mupad [B] (verification not implemented) 938
Reduce [F] 939

Optimal result

Integrand size = 12, antiderivative size = 304

$$
\begin{aligned}
\int \frac{1}{1+3x^4+x^8}\,dx = & -\frac{\sqrt[4]{9+4\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} \\
& +\frac{\sqrt[4]{9+4\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}} \\
& +\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}-\frac{\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}} \\
& +\frac{\sqrt[4]{9+4\sqrt{5}}\operatorname{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}x}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\right)}{2\sqrt{10}} \\
& -\frac{\operatorname{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}x}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}
\end{aligned}
$$

output
```
1/20*(9+4*5^(1/2))^(1/4)*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*10^(1/2)+1
/20*(9+4*5^(1/2))^(1/4)*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*10^(1/2)-1/5
*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*5^(1/2)/(6+2*5^(1/2))^(3/4)-1/5*ar
ctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*5^(1/2)/(6+2*5^(1/2))^(3/4)+1/20*(9+4*
5^(1/2))^(1/4)*arctanh(2^(3/4)*(3-5^(1/2))^(1/4)*x/(1/2*10^(1/2)-1/2*2^(1/
2)+x^2*2^(1/2)))*10^(1/2)-1/5*arctanh(2^(3/4)*(3+5^(1/2))^(1/4)*x/(1/2*10^
(1/2)+1/2*2^(1/2)+x^2*2^(1/2)))*5^(1/2)/(6+2*5^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.14

$$\int \frac{1}{1+3x^4+x^8}\,dx=\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)}{3\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 + 3*x^4 + x^8)^(-1),x]
```

output
```
RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1^3 + 2*#1^7) & ]/4
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.53, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {1685, 755, 27, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8+3x^4+1}\,dx$$

$$\Big\downarrow \ 1685$$

$$\frac{\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx}{\sqrt{5}}-\frac{\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{\int \frac{2\left(\sqrt{3-\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3-\sqrt{5}}\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}}{\sqrt{5}}-\frac{\frac{\int \frac{2\left(\sqrt{3+\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3+\sqrt{5}}\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}}{\sqrt{5}}-\frac{\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}}{\sqrt{5}}-\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}}{\sqrt{5}}-\frac{\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}}{\sqrt{5}}-\frac{\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int -\frac{2x+\sqrt[4]{2\left(3-\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{\sqrt{3-\sqrt{5}}}+\frac{\arctan\left(\cdots\right)}{2^{3/\cdots}}\cdots}{\sqrt{5}}$$

$$\frac{\frac{-\frac{\int -\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int -\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\cdots\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}\cdots}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx+\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int \frac{2x+\sqrt[4]{2\left(3-\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}\cdots}{\sqrt[4]{3\cdots}}\right.}{2^{3/4}\sqrt[4]{3\cdots}}\cdots}{\cdots}}{\sqrt{5}}$$

$$\frac{\frac{\frac{\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int \frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

$\downarrow$ 1103

$$\frac{\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}+\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\cdots\right.}\cdots}{\sqrt{3-\sqrt{5}}}}{\sqrt{5}}$$

$$\frac{\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\cdots\right.}\right.}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\cdots}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}$$

input
```
Int[(1 + 3*x^4 + x^8)^(-1),x]
```

output
```
((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)*(3 - Sqrt[5])^(1/
4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2^(3/4)*(3 - Sqrt[5])^
(1/4)))/Sqrt[3 - Sqrt[5]] + (-1/4*(((3 + Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 -
 Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) + (((3 + Sqrt[5])/2)^(1
/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/4)/S
qrt[3 - Sqrt[5]])/Sqrt[5] - ((-(ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)
]/(2^(3/4)*(3 + Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1
/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqrt[3 + Sqrt[5]] + (-1/2*Log[Sqrt[2*(
3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2^(3/4)*(3 + Sqrt[5]
)^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2
]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4)))/Sqrt[3 + Sqrt[5]])/Sqrt[5]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	37
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	37

input
```
int(1/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.15

$$
\begin{aligned}
\int \frac{1}{1+3x^4+x^8}\,dx = &-\frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{4\sqrt{5}-9}}\log\left(\sqrt{\frac{1}{5}}\left(3\sqrt{5}+5\right)\sqrt{-\sqrt{4\sqrt{5}-9}}+2x\right) \\
&+\frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{4\sqrt{5}-9}}\log\left(-\sqrt{\frac{1}{5}}\left(3\sqrt{5}+5\right)\sqrt{-\sqrt{4\sqrt{5}-9}}+2x\right) \\
&+\frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{-4\sqrt{5}-9}}\log\left(\sqrt{\frac{1}{5}}\left(3\sqrt{5}-5\right)\sqrt{-\sqrt{-4\sqrt{5}-9}}+2x\right) \\
&-\frac{1}{4}\sqrt{\frac{1}{5}}\sqrt{-\sqrt{-4\sqrt{5}-9}}\log\left(-\sqrt{\frac{1}{5}}\left(3\sqrt{5}-5\right)\sqrt{-\sqrt{-4\sqrt{5}-9}}+2x\right) \\
&-\frac{1}{4}\sqrt{\frac{1}{5}}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{1}{5}}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\left(3\sqrt{5}+5\right)+2x\right) \\
&+\frac{1}{4}\sqrt{\frac{1}{5}}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(-\sqrt{\frac{1}{5}}\left(4\sqrt{5}-9\right)^{\frac{1}{4}}\left(3\sqrt{5}+5\right)+2x\right) \\
&+\frac{1}{4}\sqrt{\frac{1}{5}}\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{1}{5}}\left(3\sqrt{5}-5\right)\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}+2x\right) \\
&-\frac{1}{4}\sqrt{\frac{1}{5}}\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}\log\left(-\sqrt{\frac{1}{5}}\left(3\sqrt{5}-5\right)\left(-4\sqrt{5}-9\right)^{\frac{1}{4}}+2x\right)
\end{aligned}
$$

input
```
integrate(1/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*sqrt(1/5)*sqrt(-sqrt(4*sqrt(5) - 9))*log(sqrt(1/5)*(3*sqrt(5) + 5)*sq
rt(-sqrt(4*sqrt(5) - 9)) + 2*x) + 1/4*sqrt(1/5)*sqrt(-sqrt(4*sqrt(5) - 9))
*log(-sqrt(1/5)*(3*sqrt(5) + 5)*sqrt(-sqrt(4*sqrt(5) - 9)) + 2*x) + 1/4*sq
rt(1/5)*sqrt(-sqrt(-4*sqrt(5) - 9))*log(sqrt(1/5)*(3*sqrt(5) - 5)*sqrt(-sq
rt(-4*sqrt(5) - 9)) + 2*x) - 1/4*sqrt(1/5)*sqrt(-sqrt(-4*sqrt(5) - 9))*log
(-sqrt(1/5)*(3*sqrt(5) - 5)*sqrt(-sqrt(-4*sqrt(5) - 9)) + 2*x) - 1/4*sqrt(
1/5)*(4*sqrt(5) - 9)^(1/4)*log(sqrt(1/5)*(4*sqrt(5) - 9)^(1/4)*(3*sqrt(5)
+ 5) + 2*x) + 1/4*sqrt(1/5)*(4*sqrt(5) - 9)^(1/4)*log(-sqrt(1/5)*(4*sqrt(5
) - 9)^(1/4)*(3*sqrt(5) + 5) + 2*x) + 1/4*sqrt(1/5)*(-4*sqrt(5) - 9)^(1/4)
*log(sqrt(1/5)*(3*sqrt(5) - 5)*(-4*sqrt(5) - 9)^(1/4) + 2*x) - 1/4*sqrt(1/
5)*(-4*sqrt(5) - 9)^(1/4)*log(-sqrt(1/5)*(3*sqrt(5) - 5)*(-4*sqrt(5) - 9)^
(1/4) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.09

$$\int \frac{1}{1+3x^4+x^8}\,dx = \operatorname{RootSum}\left(40960000t^8+115200t^4+1,\left(t\mapsto t\log\left(-9600t^5-\frac{47t}{2}+x\right)\right)\right)$$

input
```
integrate(1/(x**8+3*x**4+1),x)
```

output
```
RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-9600*_t**5 -
 47*_t/2 + x)))
```

Maxima [F]

$$\int \frac{1}{1+3x^4+x^8}\,dx = \int \frac{1}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(1/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \frac{1}{1+3x^4+x^8}\,dx &= \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}+20} \\ &- \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}+20} \\ &- \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}-20} \\ &+ \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}-20} \\ &- \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x+\sqrt{\sqrt{5}+1}\right)^2+10000\,x^2\right) \\ &+ \frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x-\sqrt{\sqrt{5}+1}\right)^2+10000\,x^2\right) \\ &+ \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x+\sqrt{\sqrt{5}-1}\right)^2+400\,x^2\right) \\ &- \frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x-\sqrt{\sqrt{5}-1}\right)^2+400\,x^2\right)\end{aligned}$$

input
```
integrate(1/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) + 20) - 1/80
*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) + 20) - 1/80*(p
i + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) - 20) + 1/80*(pi +
4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) - 20) - 1/40*sqrt(10*s
qrt(5) - 20)*log(10000*(x + sqrt(sqrt(5) + 1))^2 + 10000*x^2) + 1/40*sqrt(
10*sqrt(5) - 20)*log(10000*(x - sqrt(sqrt(5) + 1))^2 + 10000*x^2) + 1/40*s
qrt(10*sqrt(5) + 20)*log(400*(x + sqrt(sqrt(5) - 1))^2 + 400*x^2) - 1/40*s
qrt(10*sqrt(5) + 20)*log(400*(x - sqrt(sqrt(5) - 1))^2 + 400*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.33

$$\int \frac{1}{1+3x^4+x^8}\,dx$$
$$= \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{144\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}+\frac{64\,\sqrt{5}\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}\right)\,\left(-4\,\sqrt{5}-9\right)^{1/4}}{10}$$
$$+ \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{144\,x\,\left(4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}-\frac{64\,\sqrt{5}\,x\,\left(4\,\sqrt{5}-9\right)^{1/4}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}\right)\,\left(4\,\sqrt{5}-9\right)^{1/4}}{10}$$
$$- \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,144\mathrm{i}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}+\frac{\sqrt{5}\,x\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,64\mathrm{i}}{24\,\sqrt{5}\,\sqrt{-4\,\sqrt{5}-9}+56\,\sqrt{-4\,\sqrt{5}-9}}\right)\,\left(-4\,\sqrt{5}-9\right)^{1/4}\,1\mathrm{i}}{10}$$
$$- \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\left(4\,\sqrt{5}-9\right)^{1/4}\,144\mathrm{i}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}-\frac{\sqrt{5}\,x\,\left(4\,\sqrt{5}-9\right)^{1/4}\,64\mathrm{i}}{24\,\sqrt{5}\,\sqrt{4\,\sqrt{5}-9}-56\,\sqrt{4\,\sqrt{5}-9}}\right)\,\left(4\,\sqrt{5}-9\right)^{1/4}\,1\mathrm{i}}{10}$$

input
```
int(1/(3*x^4 + x^8 + 1),x)
```

output
```
(5^(1/2)*atan((144*x*(- 4*5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(- 4*5^(1/2) - 9
)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)) + (64*5^(1/2)*x*(- 4*5^(1/2) - 9)^(1
/4))/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)))*(-
 4*5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((144*x*(4*5^(1/2) - 9)^(1/4))/(2
4*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)) - (64*5^(1/2)*
x*(4*5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2)
 - 9)^(1/2)))*(4*5^(1/2) - 9)^(1/4))/10 - (5^(1/2)*atan((x*(- 4*5^(1/2) -
9)^(1/4)*144i)/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^
(1/2)) + (5^(1/2)*x*(- 4*5^(1/2) - 9)^(1/4)*64i)/(24*5^(1/2)*(- 4*5^(1/2)
- 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)))*(- 4*5^(1/2) - 9)^(1/4)*1i)/10 -
 (5^(1/2)*atan((x*(4*5^(1/2) - 9)^(1/4)*144i)/(24*5^(1/2)*(4*5^(1/2) - 9)^
(1/2) - 56*(4*5^(1/2) - 9)^(1/2)) - (5^(1/2)*x*(4*5^(1/2) - 9)^(1/4)*64i)/
(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)))*(4*5^(1/2)
- 9)^(1/4)*1i)/10
```

Reduce [F]

$$\int \frac{1}{1+3x^4+x^8}\,dx = \int \frac{1}{x^8+3x^4+1}dx$$

input
```
int(1/(x^8+3*x^4+1),x)
```

output
```
int(1/(x**8 + 3*x**4 + 1),x)
```

3.116 $\int \frac{1}{x^2(1+3x^4+x^8)}\,dx$

Optimal result 941
Mathematica [C] (verified) 942
Rubi [A] (verified) 942
Maple [C] (verified) 949
Fricas [A] (verification not implemented) 950
Sympy [A] (verification not implemented) 950
Maxima [F] 951
Giac [A] (verification not implemented) 952
Mupad [B] (verification not implemented) 953
Reduce [F] 954

Optimal result

Integrand size = 16, antiderivative size = 340

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx = -\frac{1}{x}+\frac{\left(3+\sqrt{5}\right)^{5/4}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{5/4}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{123-55\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{123-55\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{5/4}\operatorname{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}x}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{123-55\sqrt{5}}\operatorname{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}x}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\right)}{2\,2^{3/4}\sqrt{5}}$$

output
```
-1/x-1/40*(3+5^(1/2))^(5/4)*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(1/4)
*5^(1/2)-1/40*(3+5^(1/2))^(5/4)*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(1
/4)*5^(1/2)+1/20*(123-55*5^(1/2))^(1/4)*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1
/4))*2^(1/4)*5^(1/2)+1/20*(123-55*5^(1/2))^(1/4)*arctan(1+2^(3/4)*x/(3+5^(
1/2))^(1/4))*2^(1/4)*5^(1/2)+1/40*(3+5^(1/2))^(5/4)*arctanh(2^(3/4)*(3-5^(
1/2))^(1/4)*x/(1/2*10^(1/2)-1/2*2^(1/2)+x^2*2^(1/2)))*2^(1/4)*5^(1/2)-1/20
*(123-55*5^(1/2))^(1/4)*arctanh(2^(3/4)*(3+5^(1/2))^(1/4)*x/(1/2*10^(1/2)+
1/2*2^(1/2)+x^2*2^(1/2)))*2^(1/4)*5^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx = -\frac{1}{x} - \frac{1}{4}\text{RootSum}\left[1+3\#1^4 + \#1^8\&, \frac{3\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1+2\#1^5}\&\right]$$

input
```
Integrate[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

output
```
-x^(-1) - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^4)
/(3*#1 + 2*#1^5) & ]/4
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.38, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1704, 25, 1834, 27, 826, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(x^8+3x^4+1)}\,dx$$

$$\Big\downarrow \text{1704}$$

$$\int -\frac{x^2(x^4+3)}{x^8+3x^4+1}dx - \frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{x^2(x^4+3)}{x^8+3x^4+1}dx - \frac{1}{x}$$

$$\Big\downarrow \text{1834}$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{2x^2}{2x^4-\sqrt{5}+3}dx-\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{2x^2}{2x^4+\sqrt{5}+3}dx-\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\int \frac{x^2}{2x^4-\sqrt{5}+3}dx-\frac{1}{5}\left(5-3\sqrt{5}\right)\int \frac{x^2}{2x^4+\sqrt{5}+3}dx-\frac{1}{x}$$

$$\Big\downarrow \text{826}$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)-\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)-\frac{1}{x}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)-\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)-\frac{1}{x}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3-\sqrt{5}}-\cdots}{2x^4-\sqrt{5}+\cdots}}{2\sqrt{2}}\right.$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x\cdots}{2x^4+\sqrt{5}+3\cdots}}{2\sqrt{2}}\right.$$

$$\frac{1}{x}$$

$$\downarrow 217$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{2\sqrt{2}}\right)-$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{2\sqrt{2}}\right)-\frac{1}{x}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int-\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x\ldots}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\ldots}}\ldots}{\ldots}\right.$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{-\frac{\int-\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\;2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int-\ldots}{x^2+\sqrt[4]{\ldots}}\ldots}{2\sqrt{2}}\right.$$

$$\frac{1}{x}$$

$$\downarrow 25$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\ldots\right.}}\ldots}{\ldots}\right.$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{\int\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int\frac{2x+\sqrt[4]{\ldots}}{x^2+\sqrt[4]{2\left(3\ldots\right.}\ldots}\ldots}{2\,2^{3\ldots}}}{2\sqrt{2}}\right.$$

$$\frac{1}{x}$$

$$\downarrow 1103$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x\right.}{}\right.$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{2\sqrt{2}}-\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\,2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x^2\right.}{}}{2\sqrt{2}}\right.$$

$$\frac{1}{x}$$

input
```
Int[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

output
```
-x^(-1) - ((5 + 3*Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
/(2^(3/4)*(3 - Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/
4)]/(2^(3/4)*(3 - Sqrt[5])^(1/4)))/(2*Sqrt[2]) - (-1/4*(((3 + Sqrt[5])/2)^
(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2]) +
(((3 + Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(
1/4)*x + 2*x^2])/4)/(2*Sqrt[2])))/5 - ((5 - 3*Sqrt[5])*((-(ArcTan[1 - (2^(
3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4))) + ArcTan[1 + (
2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4)))/(2*Sqrt[2])
 - (-1/2*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/
(2^(3/4)*(3 + Sqrt[5])^(1/4)) + Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt
[5]))^(1/4)*x + 2*x^2]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4)))/(2*Sqrt[2])))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.12

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(625\_Z^{8}+3075\_Z^{4}+1\right)}\_R\ln\left(1175\_R^{7}+5778\_R^{3}+11x\right)\right)}{4}$	42
default	$-\frac{1}{x}-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(\_R^{6}+3\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	52

input
```
int(1/x^2/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/4*sum(_R*ln(1175*_R^7+5778*_R^3+11*x),_R=RootOf(625*_Z^8+3075*_Z^4+
1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^2\left(1+3x^4+x^8\right)}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(1/5)*x*sqrt(-sqrt(55/2*sqrt(5) - 123/2))*log(sqrt(1/5)*(47*sqrt
(5) + 105)*sqrt(55/2*sqrt(5) - 123/2)*sqrt(-sqrt(55/2*sqrt(5) - 123/2)) +
2*x) - sqrt(1/5)*x*sqrt(-sqrt(55/2*sqrt(5) - 123/2))*log(-sqrt(1/5)*(47*sq
rt(5) + 105)*sqrt(55/2*sqrt(5) - 123/2)*sqrt(-sqrt(55/2*sqrt(5) - 123/2))
+ 2*x) - sqrt(1/5)*x*sqrt(-sqrt(-55/2*sqrt(5) - 123/2))*log(sqrt(1/5)*(47*
sqrt(5) - 105)*sqrt(-55/2*sqrt(5) - 123/2)*sqrt(-sqrt(-55/2*sqrt(5) - 123/
2)) + 2*x) + sqrt(1/5)*x*sqrt(-sqrt(-55/2*sqrt(5) - 123/2))*log(-sqrt(1/5)
*(47*sqrt(5) - 105)*sqrt(-55/2*sqrt(5) - 123/2)*sqrt(-sqrt(-55/2*sqrt(5) -
 123/2)) + 2*x) - sqrt(1/5)*x*(55/2*sqrt(5) - 123/2)^(1/4)*log(sqrt(1/5)*(
47*sqrt(5) + 105)*(55/2*sqrt(5) - 123/2)^(3/4) + 2*x) + sqrt(1/5)*x*(55/2*
sqrt(5) - 123/2)^(1/4)*log(-sqrt(1/5)*(47*sqrt(5) + 105)*(55/2*sqrt(5) - 1
23/2)^(3/4) + 2*x) + sqrt(1/5)*x*(-55/2*sqrt(5) - 123/2)^(1/4)*log(sqrt(1/
5)*(47*sqrt(5) - 105)*(-55/2*sqrt(5) - 123/2)^(3/4) + 2*x) - sqrt(1/5)*x*(
-55/2*sqrt(5) - 123/2)^(1/4)*log(-sqrt(1/5)*(47*sqrt(5) - 105)*(-55/2*sqrt
(5) - 123/2)^(3/4) + 2*x) + 4)/x
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^2\left(1+3x^4+x^8\right)}\,dx = \operatorname{RootSum}\left(40960000t^8+787200t^4+1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}+\frac{369792t^3}{11}+x\right)\right)\right) - \frac{1}{x}$$

input
```
integrate(1/x**2/(x**8+3*x**4+1),x)
```

output
```
RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(19251200*_t**
7/11 + 369792*_t**3/11 + x))) - 1/x
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+3x^4+x^8\right)}\,dx=\int \frac{1}{(x^8+3\,x^4+1)x^2}\,dx$$

input
```
integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
-1/x - integrate((x^6 + 3*x^2)/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx = -\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}+55} + \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}+55} + \frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}-55} - \frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}-55} - \frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(748225\left(x+\sqrt{\sqrt{5}+1}\right)^2 + 748225\,x^2\right) + \frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(748225\left(x-\sqrt{\sqrt{5}+1}\right)^2 + 748225\,x^2\right) + \frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(180625\left(x+\sqrt{\sqrt{5}-1}\right)^2 + 180625\,x^2\right) - \frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(180625\left(x-\sqrt{\sqrt{5}-1}\right)^2 + 180625\,x^2\right) - \frac{1}{x}$$

input
```
integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
-1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) + 55) + 1/8
0*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) + 55) + 1/80*(
pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) - 55) - 1/80*(pi +
 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) - 55) - 1/40*sqrt(25*
sqrt(5) - 55)*log(748225*(x + sqrt(sqrt(5) + 1))^2 + 748225*x^2) + 1/40*sq
rt(25*sqrt(5) - 55)*log(748225*(x - sqrt(sqrt(5) + 1))^2 + 748225*x^2) + 1
/40*sqrt(25*sqrt(5) + 55)*log(180625*(x + sqrt(sqrt(5) - 1))^2 + 180625*x^
2) - 1/40*sqrt(25*sqrt(5) + 55)*log(180625*(x - sqrt(sqrt(5) - 1))^2 + 180
625*x^2) - 1/x
```

Mupad [B] (verification not implemented)

Time = 20.14 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx = -\frac{1}{x} - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2585\,2^{3/4}\,x\left(-55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}+6765\right)} + \frac{1155\,2^{3/4}\sqrt{5}\,x\left(-55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}+6765\right)}\right)\left(-55\sqrt{5}-123\right)^{1/4}}{20} - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2585\,2^{3/4}\,x\left(55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}-6765\right)} - \frac{1155\,2^{3/4}\sqrt{5}\,x\left(55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}-6765\right)}\right)\left(55\sqrt{5}-123\right)^{1/4}}{20} - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(-55\sqrt{5}-123\right)^{1/4}\,2585\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)} + \frac{2^{3/4}\sqrt{5}\,x\left(-55\sqrt{5}-123\right)^{1/4}\,1155\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}\right)\left(-55\sqrt{5}-123\right)^{1/4}\,1\mathrm{i}}{20} - \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(55\sqrt{5}-123\right)^{1/4}\,2585\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)} - \frac{2^{3/4}\sqrt{5}\,x\left(55\sqrt{5}-123\right)^{1/4}\,1155\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}\right)\left(55\sqrt{5}-123\right)^{1/4}\,1\mathrm{i}}{20}$$

input
```
int(1/(x^2*(3*x^4 + x^8 + 1)),x)
```

output
```
- 1/x - (2^(3/4)*5^(1/2)*atan((2585*2^(3/4)*x*(- 55*5^(1/2) - 123)^(1/4))/
(2*(3025*5^(1/2) + 6765)) + (1155*2^(3/4)*5^(1/2)*x*(- 55*5^(1/2) - 123)^(
1/4))/(2*(3025*5^(1/2) + 6765)))*(- 55*5^(1/2) - 123)^(1/4))/20 - (2^(3/4)
*5^(1/2)*atan((2585*2^(3/4)*x*(55*5^(1/2) - 123)^(1/4))/(2*(3025*5^(1/2) -
 6765)) - (1155*2^(3/4)*5^(1/2)*x*(55*5^(1/2) - 123)^(1/4))/(2*(3025*5^(1/
2) - 6765)))*(55*5^(1/2) - 123)^(1/4))/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)
*x*(- 55*5^(1/2) - 123)^(1/4)*2585i)/(2*(3025*5^(1/2) + 6765)) + (2^(3/4)*
5^(1/2)*x*(- 55*5^(1/2) - 123)^(1/4)*1155i)/(2*(3025*5^(1/2) + 6765)))*(-
55*5^(1/2) - 123)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(55*5^(1
/2) - 123)^(1/4)*2585i)/(2*(3025*5^(1/2) - 6765)) - (2^(3/4)*5^(1/2)*x*(55
*5^(1/2) - 123)^(1/4)*1155i)/(2*(3025*5^(1/2) - 6765)))*(55*5^(1/2) - 123)
^(1/4)*1i)/20
```

Reduce [F]

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx = \frac{-\left(\int \frac{x^6}{x^8+3x^4+1}dx\right)x-3\left(\int \frac{x^2}{x^8+3x^4+1}dx\right)x-1}{x}$$

input
```
int(1/x^2/(x^8+3*x^4+1),x)
```

output
```
( - int(x**6/(x**8 + 3*x**4 + 1),x)*x - 3*int(x**2/(x**8 + 3*x**4 + 1),x)*
x - 1)/x
```

3.117 $\int \frac{1}{x^4(1+3x^4+x^8)}\,dx$

Optimal result 956
Mathematica [C] (verified) 957
Rubi [A] (verified) 957
Maple [C] (verified) 964
Fricas [A] (verification not implemented) 965
Sympy [A] (verification not implemented) 966
Maxima [F] 966
Giac [A] (verification not implemented) 967
Mupad [B] (verification not implemented) 968
Reduce [F] 969

Optimal result

Integrand size = 16, antiderivative size = 348

$$\int \frac{1}{x^4(1+3x^4+x^8)}\,dx = -\frac{1}{3x^3}+\frac{\sqrt[4]{843+377\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{843+377\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{843-377\sqrt{5}}\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{843-377\sqrt{5}}\arctan\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\ 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{843+377\sqrt{5}}\,\text{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}x}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\right)}{2\ 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{843-377\sqrt{5}}\,\text{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}x}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}\right)}{2\ 2^{3/4}\sqrt{5}}$$

output
```
-1/3/x^3-1/20*(843+377*5^(1/2))^(1/4)*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4
))*2^(1/4)*5^(1/2)-1/20*(843+377*5^(1/2))^(1/4)*arctan(1+2^(3/4)*x/(3-5^(1
/2))^(1/4))*2^(1/4)*5^(1/2)+1/20*(843-377*5^(1/2))^(1/4)*arctan(-1+2^(3/4)
*x/(3+5^(1/2))^(1/4))*2^(1/4)*5^(1/2)+1/20*(843-377*5^(1/2))^(1/4)*arctan(
1+2^(3/4)*x/(3+5^(1/2))^(1/4))*2^(1/4)*5^(1/2)-1/20*(843+377*5^(1/2))^(1/4
)*arctanh(2^(3/4)*(3-5^(1/2))^(1/4)*x/(1/2*10^(1/2)-1/2*2^(1/2)+x^2*2^(1/2
)))*2^(1/4)*5^(1/2)+1/20*(843-377*5^(1/2))^(1/4)*arctanh(2^(3/4)*(3+5^(1/2
))^(1/4)*x/(1/2*10^(1/2)+1/2*2^(1/2)+x^2*2^(1/2)))*2^(1/4)*5^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^4 (1+3x^4+x^8)} dx = -\frac{1}{3x^3} - \frac{1}{4}\text{RootSum}\Big[1+3\#1^4 + \#1^8\&, \frac{3\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\Big]$$

input
```
Integrate[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1
^4)/(3*#1^3 + 2*#1^7) & ]/4
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.40, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1704, 27, 1752, 755, 27, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^8+3x^4+1)} dx$$

$$\Big\downarrow \text{ 1704}$$

$$\frac{1}{3}\int -\frac{3(x^4+3)}{x^8+3x^4+1}dx - \frac{1}{3x^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{x^4+3}{x^8+3x^4+1}dx - \frac{1}{3x^3}$$

$$\Big\downarrow \text{ 1752}$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3-\sqrt{5}\right)}dx-\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(3+\sqrt{5}\right)}dx-\frac{1}{3x^3}$$

$\downarrow$ 755

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{2\left(\sqrt{3-\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3-\sqrt{5}}\right)}{2x^4-\sqrt{5}+3}dx}{2\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{2\left(\sqrt{3+\sqrt{5}}-\sqrt{2}x^2\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}+\frac{\int \frac{2\left(\sqrt{2}x^2+\sqrt{3+\sqrt{5}}\right)}{2x^4+\sqrt{5}+3}dx}{2\sqrt{3+\sqrt{5}}}\right)-\frac{1}{3x^3}$$

$\downarrow$ 27

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)-\frac{1}{3x^3}$$

$\downarrow$ 1476

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3-\sqrt{5}}}+\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int \frac{1}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\sqrt{2}}}{\sqrt{3+\sqrt{5}}}+\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}\right)-\frac{1}{3x^3}$$

$\downarrow$ 1082

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right.}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)^2-1}d\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int \frac{1}{-\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)^2-1}d\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right.}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}\right.$$

$$\frac{1}{3x^3}$$

$$\Big\downarrow 217$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\int\frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{2x^4-\sqrt{5}+3}dx}{\sqrt{3-\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}\right)-$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\int\frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{2x^4+\sqrt{5}+3}dx}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}\right)-\frac{1}{3x^3}$$

$$\downarrow 1479$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int-\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx-\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int-\frac{2x+\sqrt[4]{2\left(\right.}}{x^2+\sqrt[4]{2\left(3-\right.}}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{-\frac{\int-\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\int-\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right.}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{}}\right.$$

$$\frac{1}{3x^3}$$

$\downarrow$ 25

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}}dx+\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\int\frac{2x+\sqrt[4]{2\left(3-\sqrt{\right.}}}{x^2+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+}}{\sqrt{3-\sqrt{5}}}\right.$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\int\frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}-2x}{x^2-\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}+\frac{\int\frac{2x+\sqrt[4]{2\left(3+\sqrt{5}\right)}}{x^2+\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}}dx}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-}{\sqrt{3+}}\right.$$

$$\frac{1}{3x^3}$$

$\downarrow$ 1103

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3-\sqrt{5}}}}{\sqrt{3-\sqrt{5}}}+\frac{\frac{1}{4}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x\right.}{}\right.$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(\frac{\frac{\arctan\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}}}{\sqrt{3+\sqrt{5}}}+\frac{\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\log\left(2x\right.}{}}{\sqrt{3+\sqrt{5}}}\right.$$

$$\frac{1}{3x^3}$$

input
```
Int[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 - ((5 + 3*Sqrt[5])*((-(ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/
4)]/(2^(3/4)*(3 - Sqrt[5])^(1/4))) + ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^
(1/4)]/(2^(3/4)*(3 - Sqrt[5])^(1/4)))/Sqrt[3 - Sqrt[5]] + (-1/4*(((3 + Sqr
t[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2
*x^2]) + (((3 + Sqrt[5])/2)^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sq
rt[5]))^(1/4)*x + 2*x^2])/4)/Sqrt[3 - Sqrt[5]]))/10 - ((5 - 3*Sqrt[5])*((-
(ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/4))
) + ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2^(3/4)*(3 + Sqrt[5])^(1/
4)))/Sqrt[3 + Sqrt[5]] + (-1/2*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[
5]))^(1/4)*x + 2*x^2]/(2^(3/4)*(3 + Sqrt[5])^(1/4)) + Log[Sqrt[2*(3 + Sqrt
[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4
)))/Sqrt[3 + Sqrt[5]]))/10
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.11

method	result	size
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(625\_Z^8+21075\_Z^4+1\right)}\_R\ln\left(175\_R^5+5778\_R+377x\right)\right)}{4}$	40
default	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\left(-\_R^4-3\right)\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	50

input
```
int(1/x^4/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+1/4*sum(_R*ln(175*_R^5+5778*_R+377*x),_R=RootOf(625*_Z^8+21075*_Z
^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (1+3x^4+x^8)}\,dx = \frac{3\sqrt{\frac{1}{5}}x^3\sqrt{-\sqrt{\frac{377}{2}\sqrt{5}-\frac{843}{2}}}\log\left(\sqrt{\frac{1}{5}}(7\sqrt{5}+15)\sqrt{-\sqrt{\frac{377}{2}\sqrt{5}-\frac{843}{2}}}+2x\right)-3\sqrt{\frac{1}{5}}x^3\sqrt{-\sqrt{\frac{377}{2}\sqrt{5}-\frac{8}{}}}}{}$$

input
```
integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
1/12*(3*sqrt(1/5)*x^3*sqrt(-sqrt(377/2*sqrt(5) - 843/2))*log(sqrt(1/5)*(7*
sqrt(5) + 15)*sqrt(-sqrt(377/2*sqrt(5) - 843/2)) + 2*x) - 3*sqrt(1/5)*x^3*
sqrt(-sqrt(377/2*sqrt(5) - 843/2))*log(-sqrt(1/5)*(7*sqrt(5) + 15)*sqrt(-s
qrt(377/2*sqrt(5) - 843/2)) + 2*x) - 3*sqrt(1/5)*x^3*sqrt(-sqrt(-377/2*sqr
t(5) - 843/2))*log(sqrt(1/5)*(7*sqrt(5) - 15)*sqrt(-sqrt(-377/2*sqrt(5) -
843/2)) + 2*x) + 3*sqrt(1/5)*x^3*sqrt(-sqrt(-377/2*sqrt(5) - 843/2))*log(-
sqrt(1/5)*(7*sqrt(5) - 15)*sqrt(-sqrt(-377/2*sqrt(5) - 843/2)) + 2*x) + 3*
sqrt(1/5)*x^3*(377/2*sqrt(5) - 843/2)^(1/4)*log(sqrt(1/5)*(377/2*sqrt(5) -
 843/2)^(1/4)*(7*sqrt(5) + 15) + 2*x) - 3*sqrt(1/5)*x^3*(377/2*sqrt(5) - 8
43/2)^(1/4)*log(-sqrt(1/5)*(377/2*sqrt(5) - 843/2)^(1/4)*(7*sqrt(5) + 15)
+ 2*x) - 3*sqrt(1/5)*x^3*(-377/2*sqrt(5) - 843/2)^(1/4)*log(sqrt(1/5)*(7*s
qrt(5) - 15)*(-377/2*sqrt(5) - 843/2)^(1/4) + 2*x) + 3*sqrt(1/5)*x^3*(-377
/2*sqrt(5) - 843/2)^(1/4)*log(-sqrt(1/5)*(7*sqrt(5) - 15)*(-377/2*sqrt(5)
- 843/2)^(1/4) + 2*x) - 4)/x^3
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^4 (1+3x^4+x^8)} \, dx = \text{RootSum}\left(40960000t^8+5395200t^4+1, \left(t \mapsto t\log\left(\frac{179200t^5}{377}+\frac{23112t}{377}+x\right)\right)\right) - \frac{1}{3x^3}$$

input
```
integrate(1/x**4/(x**8+3*x**4+1),x)
```

output
```
RootSum(40960000*_t**8 + 5395200*_t**4 + 1, Lambda(_t, _t*log(179200*_t**5
/377 + 23112*_t/377 + x))) - 1/(3*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (1+3x^4+x^8)} \, dx = \int \frac{1}{(x^8+3\,x^4+1)x^4} \, dx$$

input
```
integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
-1/3/x^3 - integrate((x^4 + 3)/(x^8 + 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^4 (1+3x^4+x^8)}\,dx = -\frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{65\sqrt{5}+145} + \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{65\sqrt{5}+145} + \frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{65\sqrt{5}-145} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{65\sqrt{5}-145} + \frac{1}{40}\sqrt{65\sqrt{5}-145}\log\left(93122500\left(x+\sqrt{\sqrt{5}+1}\right)^2 + 93122500\,x^2\right) - \frac{1}{40}\sqrt{65\sqrt{5}-145}\log\left(93122500\left(x-\sqrt{\sqrt{5}+1}\right)^2 + 93122500\,x^2\right) - \frac{1}{40}\sqrt{65\sqrt{5}+145}\log\left(53728900\left(x+\sqrt{\sqrt{5}-1}\right)^2 + 53728900\,x^2\right) + \frac{1}{40}\sqrt{65\sqrt{5}+145}\log\left(53728900\left(x-\sqrt{\sqrt{5}-1}\right)^2 + 53728900\,x^2\right) - \frac{1}{3\,x^3}$$

input
```
integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
-1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/
80*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/80
*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) + 1/40*sqrt
(65*sqrt(5) - 145)*log(93122500*(x + sqrt(sqrt(5) + 1))^2 + 93122500*x^2)
- 1/40*sqrt(65*sqrt(5) - 145)*log(93122500*(x - sqrt(sqrt(5) + 1))^2 + 931
22500*x^2) - 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x + sqrt(sqrt(5) -
1))^2 + 53728900*x^2) + 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x - sqrt
(sqrt(5) - 1))^2 + 53728900*x^2) - 1/3/x^3
```

Mupad [B] (verification not implemented)

Time = 20.26 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^4 (1+3x^4+x^8)} dx = \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{46371\,2^{3/4}\,x\left(377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)} - \frac{20735\,2^{3/4}\sqrt{5}\,x\left(377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}\right)\left(}{20}$$
$$- \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{46371\,2^{3/4}\,x\left(-377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-843}\right)} + \frac{20735\,2^{3/4}\sqrt{5}\,x\left(-377\sqrt{5}-843\right)^{1/4}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-}\right.}}{20}$$
$$- \frac{1}{3\,x^3}$$
$$+ \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(-377\sqrt{5}-843\right)^{1/4}\,46371\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-843}\right)} + \frac{2^{3/4}\sqrt{5}\,x\left(-377\sqrt{5}-843\right)^{1/4}\,20735\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{-377\sqrt{5}-843}+1508\sqrt{2}\sqrt{5}\sqrt{-377\sqrt{5}-}\right.}}{20}$$
$$- \frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(377\sqrt{5}-843\right)^{1/4}\,46371\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)} - \frac{2^{3/4}\sqrt{5}\,x\left(377\sqrt{5}-843\right)^{1/4}\,20735\mathrm{i}}{2\left(3393\sqrt{2}\sqrt{377\sqrt{5}-843}-1508\sqrt{2}\sqrt{5}\sqrt{377\sqrt{5}-843}\right)}\right)}{20}$$

input
```
int(1/(x^4*(3*x^4 + x^8 + 1)),x)
```

output
```
(2^(3/4)*5^(1/2)*atan((46371*2^(3/4)*x*(377*5^(1/2) - 843)^(1/4))/(2*(3393
*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*(377*5^(1/2) - 8
43)^(1/2))) - (20735*2^(3/4)*5^(1/2)*x*(377*5^(1/2) - 843)^(1/4))/(2*(3393
*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*(377*5^(1/2) - 8
43)^(1/2))))*(377*5^(1/2) - 843)^(1/4))/20 - (2^(3/4)*5^(1/2)*atan((46371*
2^(3/4)*x*(- 377*5^(1/2) - 843)^(1/4))/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 8
43)^(1/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))) + (20735*2^
(3/4)*5^(1/2)*x*(- 377*5^(1/2) - 843)^(1/4))/(2*(3393*2^(1/2)*(- 377*5^(1/
2) - 843)^(1/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))))*(- 3
77*5^(1/2) - 843)^(1/4))/20 - 1/(3*x^3) + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x
*(- 377*5^(1/2) - 843)^(1/4)*46371i)/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843
)^(1/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))) + (2^(3/4)*5^
(1/2)*x*(- 377*5^(1/2) - 843)^(1/4)*20735i)/(2*(3393*2^(1/2)*(- 377*5^(1/2
) - 843)^(1/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))))*(- 37
7*5^(1/2) - 843)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(377*5^(1
/2) - 843)^(1/4)*46371i)/(2*(3393*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508
*2^(1/2)*5^(1/2)*(377*5^(1/2) - 843)^(1/2))) - (2^(3/4)*5^(1/2)*x*(377*5^(
1/2) - 843)^(1/4)*20735i)/(2*(3393*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 150
8*2^(1/2)*5^(1/2)*(377*5^(1/2) - 843)^(1/2))))*(377*5^(1/2) - 843)^(1/4)*1
i)/20
```

Reduce [F]

$$\int \frac{1}{x^4(1+3x^4+x^8)}\,dx = \frac{-3\left(\int \frac{x^4}{x^8+3x^4+1}dx\right)x^3 - 9\left(\int \frac{1}{x^8+3x^4+1}dx\right)x^3 - 1}{3x^3}$$

input
```
int(1/x^4/(x^8+3*x^4+1),x)
```

output
```
( - 3*int(x**4/(x**8 + 3*x**4 + 1),x)*x**3 - 9*int(1/(x**8 + 3*x**4 + 1),x
)*x**3 - 1)/(3*x**3)
```

3.118 $\int \frac{x^{11}}{1-3x^4+x^8}\,dx$

Optimal result . . . 970
Mathematica [A] (verified) . . . 970
Rubi [A] (verified) . . . 971
Maple [A] (verified) . . . 972
Fricas [A] (verification not implemented) . . . 973
Sympy [A] (verification not implemented) . . . 973
Maxima [A] (verification not implemented) . . . 974
Giac [A] (verification not implemented) . . . 974
Mupad [B] (verification not implemented) . . . 974
Reduce [F] . . . 975

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

output
```
1/4*x^4+1/40*(15-7*5^(1/2))*ln(3-5^(1/2)-2*x^4)+1/40*(15+7*5^(1/2))*ln(3+5
^(1/2)-2*x^4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{40}\left(10x^4+\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)+\left(15-7\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)\right)$$

input
```
Integrate[x^11/(1 - 3*x^4 + x^8),x]
```

output
```
(10*x^4 + (15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] + (15 - 7*Sqrt[5])*Log
[-3 + Sqrt[5] + 2*x^4])/40
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^8-3x^4+1}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^8}{x^8-3x^4+1}dx^4$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{4}\int \left(-\frac{15+7\sqrt{5}}{5\left(-2x^4+\sqrt{5}+3\right)}+1-\frac{15-7\sqrt{5}}{5\left(-2x^4-\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(x^4+\frac{1}{10}\left(15-7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{10}\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)\right)$$

input
```
Int[x^11/(1 - 3*x^4 + x^8),x]
```

output
```
(x^4 + ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/10 + ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] - 2*x^4])/10)/4
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{x^4}{4}+\frac{3\ln\left(x^8-3x^4+1\right)}{8}-\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^4-3\right)\sqrt{5}}{5}\right)}{20}$	38
risch	$\frac{x^4}{4}+\frac{3\ln\left(2x^4-\sqrt{5}-3\right)}{8}+\frac{7\ln\left(2x^4-\sqrt{5}-3\right)\sqrt{5}}{40}+\frac{3\ln\left(2x^4+\sqrt{5}-3\right)}{8}-\frac{7\ln\left(2x^4+\sqrt{5}-3\right)\sqrt{5}}{40}$	69

input
```
int(x^11/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+3/8*ln(x^8-3*x^4+1)-7/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}(2\,x^4-3)+7}{x^8-3\,x^4+1}\right) + \frac{3}{8}\,\log\left(x^8-3\,x^4+1\right)$$

input
```
integrate(x^11/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8
- 3*x^4 + 1)) + 3/8*log(x^8 - 3*x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{x^4}{4} + \left(\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right) + \left(\frac{3}{8}-\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

input
```
integrate(x**11/(x**8-3*x**4+1),x)
```

output
```
x**4/4 + (3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (3/8 - 7*sqrt(
5)/40)*log(x**4 - 3/2 + sqrt(5)/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}-3}{2\,x^4+\sqrt{5}-3}\right) + \frac{3}{8}\,\log\left(x^8-3\,x^4+1\right)$$

input
```
integrate(x^11/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) +
3/8*log(x^8 - 3*x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{1}{4}\,x^4 + \frac{7}{40}\,\sqrt{5}\log\left(\frac{\left|2\,x^4-\sqrt{5}-3\right|}{\left|2\,x^4+\sqrt{5}-3\right|}\right) + \frac{3}{8}\,\log\left(\left|x^8-3\,x^4+1\right|\right)$$

input
```
integrate(x^11/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/4*x^4 + 7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) -
3)) + 3/8*log(abs(x^8 - 3*x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = \frac{3\,\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8} + \frac{3\,\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8} + \frac{7\,\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40} - \frac{7\,\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40} + \frac{x^4}{4}$$

input
```
int(x^11/(x^8 - 3*x^4 + 1),x)
```

output
```
(3*log(x^4 - 5^(1/2)/2 - 3/2))/8 + (3*log(5^(1/2)/2 + x^4 - 3/2))/8 + (7*5
^(1/2)*log(x^4 - 5^(1/2)/2 - 3/2))/40 - (7*5^(1/2)*log(5^(1/2)/2 + x^4 - 3
/2))/40 + x^4/4
```

Reduce [F]

$$\int \frac{x^{11}}{1-3x^4+x^8}\,dx = -\frac{7\sqrt{5}\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{20} - \frac{7\sqrt{5}\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{20} + \frac{7\sqrt{5}\log\left(\sqrt{5}+2x^2+1\right)}{20} - 7\left(\int \frac{x}{x^8-3x^4+1}dx\right) + \frac{5\log(x^4-x^2-1)}{4} - \frac{\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{2} - \frac{\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{2} - \frac{\log\left(\sqrt{5}+2x^2+1\right)}{2} + \frac{x^4}{4}$$

input
```
int(x^11/(x^8-3*x^4+1),x)
```

output
```
( - 7*sqrt(5)*log( - sqrt(sqrt(5) - 1) + sqrt(2)*x) - 7*sqrt(5)*log(sqrt(s
qrt(5) - 1) + sqrt(2)*x) + 7*sqrt(5)*log(sqrt(5) + 2*x**2 + 1) - 140*int(x
/(x**8 - 3*x**4 + 1),x) + 25*log(x**4 - x**2 - 1) - 10*log( - sqrt(sqrt(5)
 - 1) + sqrt(2)*x) - 10*log(sqrt(sqrt(5) - 1) + sqrt(2)*x) - 10*log(sqrt(5
) + 2*x**2 + 1) + 5*x**4)/20
```

3.119 $\int \frac{x^9}{1-3x^4+x^8} \, dx$

Optimal result . . . 976
Mathematica [A] (verified) . . . 976
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Reduce [F] . . . 982

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{x^9}{1-3x^4+x^8} \, dx = \frac{x^2}{2} - \frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x^2\right)}{2\sqrt{5\left(9-4\sqrt{5}\right)}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{2\sqrt{5\left(9+4\sqrt{5}\right)}}$$

output
```
1/2*x^2-1/2*arctanh((1/2*5^(1/2)-1/2)*x^2)/(5-2*5^(1/2))+1/2*arctanh((1/2+
1/2*5^(1/2))*x^2)/(5+2*5^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17

$$\int \frac{x^9}{1-3x^4+x^8} \, dx = \frac{1}{20}\left(10x^2 + \left(-5+2\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right) + \left(5+2\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right) + \left(5-2\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right) - \left(5+2\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

input
```
Integrate[x^9/(1 - 3*x^4 + x^8),x]
```

output
```
(10*x^2 + (-5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log
[1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + 2
*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1695, 1442, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^8-3x^4+1}dx$$
$$\Big\downarrow \text{1695}$$
$$\frac{1}{2}\int \frac{x^8}{x^8-3x^4+1}dx^2$$
$$\Big\downarrow \text{1442}$$
$$\frac{1}{2}\left(x^2-\int \frac{1-3x^4}{x^8-3x^4+1}dx^2\right)$$
$$\Big\downarrow \text{1480}$$
$$\frac{1}{2}\left(\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx^2+\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx^2+x^2\right)$$
$$\Big\downarrow \text{220}$$
$$\frac{1}{2}\left(-\frac{\left(15+7\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}-\frac{1}{10}\left(15-7\sqrt{5}\right)\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)+x^2\right)$$

input
```
Int[x^9/(1 - 3*x^4 + x^8),x]
```

output
```
(x^2 - ((15 + 7*Sqrt[5])*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/(5*Sqrt[2*(3
+ Sqrt[5])]) - ((15 - 7*Sqrt[5])*Sqrt[(3 + Sqrt[5])/2]*ArcTanh[Sqrt[(3 + S
qrt[5])/2]*x^2])/10)/2
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

method	result
default	$\frac{x^2}{2}+\frac{\ln(x^4-x^2-1)}{4}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{5}-\frac{\ln(x^4+x^2-1)}{4}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{5}$
risch	$\frac{x^2}{2}-\frac{\ln\left(2x^2-\sqrt{5}+1\right)}{4}+\frac{\ln\left(2x^2-\sqrt{5}+1\right)\sqrt{5}}{10}-\frac{\ln\left(2x^2+\sqrt{5}+1\right)}{4}-\frac{\ln\left(2x^2+\sqrt{5}+1\right)\sqrt{5}}{10}+\frac{\ln\left(2x^2-\sqrt{5}-1\right)}{4}+\frac{\ln\left(2x^2-\sqrt{}\right.}{10}$

input
```
int(x^9/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/4*ln(x^4-x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/4*l
n(x^4+x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(50) = 100.

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{1}{2}\,x^2+\frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^4+2\,x^2-\sqrt{5}(2\,x^2+1)+3}{x^4+x^2-1}\right) + \frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^4-2\,x^2-\sqrt{5}(2\,x^2-1)+3}{x^4-x^2-1}\right) - \frac{1}{4}\,\log\left(x^4+x^2-1\right)+\frac{1}{4}\,\log\left(x^4-x^2-1\right)$$

input
```
integrate(x^9/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/2*x^2 + 1/10*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4
+ x^2 - 1)) + 1/10*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(
x^4 - x^2 - 1)) - 1/4*log(x^4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(51) = 102$.

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.93

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{x^2}{2} + \left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47}{8}-\frac{47\sqrt{5}}{20}-120\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3\right) + \left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47}{8}-120\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)^3+\frac{47\sqrt{5}}{20}\right) + \left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47\sqrt{5}}{20}-120\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{47}{8}\right) + \left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-120\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3+\frac{47\sqrt{5}}{20}+\frac{47}{8}\right)$$

input
```
integrate(x**9/(x**8-3*x**4+1),x)
```

output
```
x**2/2 + (-1/4 - sqrt(5)/10)*log(x**2 - 47/8 - 47*sqrt(5)/20 - 120*(-1/4 -
 sqrt(5)/10)**3) + (-1/4 + sqrt(5)/10)*log(x**2 - 47/8 - 120*(-1/4 + sqrt(
5)/10)**3 + 47*sqrt(5)/20) + (1/4 - sqrt(5)/10)*log(x**2 - 47*sqrt(5)/20 -
 120*(1/4 - sqrt(5)/10)**3 + 47/8) + (sqrt(5)/10 + 1/4)*log(x**2 - 120*(sq
rt(5)/10 + 1/4)**3 + 47*sqrt(5)/20 + 47/8)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right) + \frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right) - \frac{1}{4}\,\log\left(x^4+x^2-1\right) + \frac{1}{4}\,\log\left(x^4-x^2-1\right)$$

input
```
integrate(x^9/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
1/2*x^2 + 1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) +
1/10*sqrt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/4*log(x^
4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{10}\,\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}+1\right|}{2\,x^2+\sqrt{5}+1}\right) + \frac{1}{10}\,\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}-1\right|}{\left|2\,x^2+\sqrt{5}-1\right|}\right) - \frac{1}{4}\,\log\left(\left|x^4+x^2-1\right|\right) + \frac{1}{4}\,\log\left(\left|x^4-x^2-1\right|\right)$$

input
```
integrate(x^9/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/2*x^2 + 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1))
 + 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1
/4*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = \frac{x^2}{2} - \operatorname{atanh}\left(\frac{64\,x^2}{64\,\sqrt{5}+192} + \frac{64\,\sqrt{5}\,x^2}{64\,\sqrt{5}+192}\right)\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right) - \operatorname{atanh}\left(\frac{64\,x^2}{64\,\sqrt{5}-192} - \frac{64\,\sqrt{5}\,x^2}{64\,\sqrt{5}-192}\right)\left(\frac{\sqrt{5}}{5}-\frac{1}{2}\right)$$

input
```
int(x^9/(x^8 - 3*x^4 + 1),x)
```

output
```
x^2/2 - atanh((64*x^2)/(64*5^(1/2) + 192) + (64*5^(1/2)*x^2)/(64*5^(1/2) +
 192))*(5^(1/2)/5 + 1/2) - atanh((64*x^2)/(64*5^(1/2) - 192) - (64*5^(1/2)
*x^2)/(64*5^(1/2) - 192))*(5^(1/2)/5 - 1/2)
```

Reduce [F]

$$\int \frac{x^9}{1-3x^4+x^8}\,dx = -4\left(\int \frac{x}{x^8-3x^4+1}dx\right) + \frac{3\log(x^4-x^2-1)}{4} - \frac{3\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{4} - \frac{3\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{4} - \frac{3\log\left(\sqrt{5}+2x^2+1\right)}{4} + \frac{x^2}{2}$$

input
```
int(x^9/(x^8-3*x^4+1),x)
```

output
```
( - 16*int(x/(x**8 - 3*x**4 + 1),x) + 3*log(x**4 - x**2 - 1) - 3*log( - sq
rt(sqrt(5) - 1) + sqrt(2)*x) - 3*log(sqrt(sqrt(5) - 1) + sqrt(2)*x) - 3*lo
g(sqrt(5) + 2*x**2 + 1) + 2*x**2)/4
```

3.120 $\int \frac{x^7}{1-3x^4+x^8}\,dx$

Optimal result 983
Mathematica [A] (verified) 983
Rubi [A] (verified) 984
Maple [A] (verified) 985
Fricas [A] (verification not implemented) 985
Sympy [A] (verification not implemented) 986
Maxima [A] (verification not implemented) 986
Giac [A] (verification not implemented) 987
Mupad [B] (verification not implemented) 987
Reduce [F] 988

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{x^7}{1-3x^4+x^8}\,dx=\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

output
```
1/40*(5-3*5^(1/2))*ln(3-5^(1/2)-2*x^4)+1/40*(5+3*5^(1/2))*ln(3+5^(1/2)-2*x
^4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{1-3x^4+x^8}\,dx=\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)+\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)$$

input
```
Integrate[x^7/(1 - 3*x^4 + x^8),x]
```

output
```
((5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((5 - 3*Sqrt[5])*Log[-3 +
Sqrt[5] + 2*x^4])/40
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^8-3x^4+1} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{x^4}{x^8-3x^4+1}dx^4$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{4}\int \left(-\frac{5+3\sqrt{5}}{5\left(-2x^4+\sqrt{5}+3\right)}-\frac{5-3\sqrt{5}}{5\left(-2x^4-\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{10}\left(5-3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)\right)$$

input
```
Int[x^7/(1 - 3*x^4 + x^8),x]
```

output
```
(((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/10 + ((5 + 3*Sqrt[5])*Log[3 +
Sqrt[5] - 2*x^4])/10)/4
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\ln(x^8-3x^4+1)}{8}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^4-3)\sqrt{5}}{5}\right)}{20}$	33
risch	$\frac{\ln\left(2x^4-\sqrt{5}-3\right)}{8}+\frac{3\ln\left(2x^4-\sqrt{5}-3\right)\sqrt{5}}{40}+\frac{\ln\left(2x^4+\sqrt{5}-3\right)}{8}-\frac{3\ln\left(2x^4+\sqrt{5}-3\right)\sqrt{5}}{40}$	64

input
```
int(x^7/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(x^8-3*x^4+1)-3/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}(2\,x^4-3)+7}{x^8-3\,x^4+1}\right)+\frac{1}{8}\log\left(x^8-3\,x^4+1\right)$$

input
```
integrate(x^7/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 +
1)) + 1/8*log(x^8 - 3*x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \left(\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right) + \left(\frac{1}{8}-\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

input
```
integrate(x**7/(x**8-3*x**4+1),x)
```

output
```
(1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (1/8 - 3*sqrt(5)/40)*lo
g(x**4 - 3/2 + sqrt(5)/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}-3}{2\,x^4+\sqrt{5}-3}\right) + \frac{1}{8}\log\left(x^8-3\,x^4+1\right)$$

input
```
integrate(x^7/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 1/8*log(x^
8 - 3*x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{|2x^4-\sqrt{5}-3|}{|2x^4+\sqrt{5}-3|}\right)+\frac{1}{8}\log\left(|x^8-3x^4+1|\right)$$

input
```
integrate(x^7/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/8*
log(abs(x^8 - 3*x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8} + \frac{3\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}-\frac{3\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}$$

input
```
int(x^7/(x^8 - 3*x^4 + 1),x)
```

output
```
log(x^4 - 5^(1/2)/2 - 3/2)/8 + log(5^(1/2)/2 + x^4 - 3/2)/8 + (3*5^(1/2)*l
og(x^4 - 5^(1/2)/2 - 3/2))/40 - (3*5^(1/2)*log(5^(1/2)/2 + x^4 - 3/2))/40
```

Reduce [F]

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = -\frac{3\sqrt{5}\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{20} - \frac{3\sqrt{5}\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{20} + \frac{3\sqrt{5}\log\left(\sqrt{5}+2x^2+1\right)}{20} - 3\left(\int \frac{x}{x^8-3x^4+1}dx\right) + \frac{\log(x^4-x^2-1)}{2} - \frac{\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{4} - \frac{\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{4} - \frac{\log\left(\sqrt{5}+2x^2+1\right)}{4}$$

input
```
int(x^7/(x^8-3*x^4+1),x)
```

output
```
( - 3*sqrt(5)*log( - sqrt(sqrt(5) - 1) + sqrt(2)*x) - 3*sqrt(5)*log(sqrt(s
qrt(5) - 1) + sqrt(2)*x) + 3*sqrt(5)*log(sqrt(5) + 2*x**2 + 1) - 60*int(x/
(x**8 - 3*x**4 + 1),x) + 10*log(x**4 - x**2 - 1) - 5*log( - sqrt(sqrt(5) -
 1) + sqrt(2)*x) - 5*log(sqrt(sqrt(5) - 1) + sqrt(2)*x) - 5*log(sqrt(5) +
2*x**2 + 1))/20
```

3.121 $\int \frac{x^5}{1-3x^4+x^8} \, dx$

Optimal result 989
Mathematica [A] (verified) 989
Rubi [A] (verified) 990
Maple [A] (verified) 991
Fricas [B] (verification not implemented) 992
Sympy [B] (verification not implemented) 992
Maxima [B] (verification not implemented) 993
Giac [B] (verification not implemented) 993
Mupad [B] (verification not implemented) 994
Reduce [F] 994

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{x^5}{1-3x^4+x^8} \, dx = -\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x^2\right) + \frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

output
```
-1/2*(1/2+1/10*5^(1/2))*arctanh((1/2*5^(1/2)-1/2)*x^2)+arctanh((1/2+1/2*5^
(1/2))*x^2)/(5+5^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \frac{x^5}{1-3x^4+x^8} \, dx = \frac{1}{40}\left(\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)+\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right) - \left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)-\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

input
```
Integrate[x^5/(1 - 3*x^4 + x^8),x]
```

output
```
((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + Sqrt[5])*Log[1 + Sqrt[5]
- 2*x^2] - (-5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + Sqrt[5])*Log[1
+ Sqrt[5] + 2*x^2])/40
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1695, 1450, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^8-3x^4+1}\,dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{x^4}{x^8-3x^4+1}dx^2$$

$$\Big\downarrow \text{1450}$$

$$\frac{1}{2}\left(\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx^2+\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx^2\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(-\frac{\left(5+3\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}-\frac{1}{10}\left(5-3\sqrt{5}\right)\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)\right)$$

input
```
Int[x^5/(1 - 3*x^4 + x^8),x]
```

output
```
(-1/5*((5 + 3*Sqrt[5])*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/Sqrt[2*(3 + Sqr
t[5])] - ((5 - 3*Sqrt[5])*Sqrt[(3 + Sqrt[5])/2]*ArcTanh[Sqrt[(3 + Sqrt[5])
/2]*x^2])/10)/2
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

method	result
default	$\frac{\ln\left(x^4-x^2-1\right)}{8}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2-1\right)\sqrt{5}}{5}\right)}{20}-\frac{\ln\left(x^4+x^2-1\right)}{8}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^2+1\right)\sqrt{5}}{5}\right)}{20}$
risch	$-\frac{\ln\left(2x^2-\sqrt{5}+1\right)}{8}+\frac{\ln\left(2x^2-\sqrt{5}+1\right)\sqrt{5}}{40}-\frac{\ln\left(2x^2+\sqrt{5}+1\right)}{8}-\frac{\ln\left(2x^2+\sqrt{5}+1\right)\sqrt{5}}{40}+\frac{\ln\left(2x^2-\sqrt{5}-1\right)}{8}+\frac{\ln\left(2x^2-\sqrt{5}-1\right)}{40}$

input
```
int(x^5/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(x^4-x^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/8*ln(x^4+x
^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(38) = 76$.

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.43

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^4+2\,x^2-\sqrt{5}(2\,x^2+1)+3}{x^4+x^2-1}\right) + \frac{1}{40}\sqrt{5}\log\left(\frac{2\,x^4-2\,x^2-\sqrt{5}(2\,x^2-1)+3}{x^4-x^2-1}\right) - \frac{1}{8}\log\left(x^4+x^2-1\right) + \frac{1}{8}\log\left(x^4-x^2-1\right)$$

input
```
integrate(x^5/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/40*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1)
) + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2
- 1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(48) = 96$.

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.17

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-\frac{3\sqrt{5}}{10}-640\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3\right) + \left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-640\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{3\sqrt{5}}{10}\right) + \left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3\sqrt{5}}{10}-640\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{3}{2}\right) + \left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-640\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3+\frac{3\sqrt{5}}{10}+\frac{3}{2}\right)$$

input
```
integrate(x**5/(x**8-3*x**4+1),x)
```

output
```
(-1/8 - sqrt(5)/40)*log(x**2 - 3/2 - 3*sqrt(5)/10 - 640*(-1/8 - sqrt(5)/40
)**3) + (-1/8 + sqrt(5)/40)*log(x**2 - 3/2 - 640*(-1/8 + sqrt(5)/40)**3 +
3*sqrt(5)/10) + (1/8 - sqrt(5)/40)*log(x**2 - 3*sqrt(5)/10 - 640*(1/8 - sq
rt(5)/40)**3 + 3/2) + (sqrt(5)/40 + 1/8)*log(x**2 - 640*(sqrt(5)/40 + 1/8)
**3 + 3*sqrt(5)/10 + 3/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(38) = 76$.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.14

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right) - \frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

input
```
integrate(x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqrt(
5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 -
1) + 1/8*log(x^4 - x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(38) = 76$.

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{40}\sqrt{5}\log\left(\frac{|2x^2-\sqrt{5}+1|}{2x^2+\sqrt{5}+1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{|2x^2-\sqrt{5}-1|}{|2x^2+\sqrt{5}-1|}\right) - \frac{1}{8}\log\left(|x^4+x^2-1|\right)+\frac{1}{8}\log\left(|x^4-x^2-1|\right)$$

input
```
integrate(x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sq
rt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(abs
(x^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = -\operatorname{atanh}\left(\frac{4\,x^2}{\sqrt{5}-3}-\frac{2\,\sqrt{5}\,x^2}{\sqrt{5}-3}\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right) - \operatorname{atanh}\left(\frac{4\,x^2}{\sqrt{5}+3}+\frac{2\,\sqrt{5}\,x^2}{\sqrt{5}+3}\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{4}\right)$$

input
```
int(x^5/(x^8 - 3*x^4 + 1),x)
```

output
```
- atanh((4*x^2)/(5^(1/2) - 3) - (2*5^(1/2)*x^2)/(5^(1/2) - 3))*(5^(1/2)/20
 + 1/4) - atanh((4*x^2)/(5^(1/2) + 3) + (2*5^(1/2)*x^2)/(5^(1/2) + 3))*(5^
(1/2)/20 - 1/4)
```

Reduce [F]

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = -\left(\int \frac{x}{x^8-3x^4+1}dx\right)+\frac{\log(x^4-x^2-1)}{4} - \frac{\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{4} - \frac{\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{4} - \frac{\log\left(\sqrt{5}+2x^2+1\right)}{4}$$

input
```
int(x^5/(x^8-3*x^4+1),x)
```

output
```
( - 4*int(x/(x**8 - 3*x**4 + 1),x) + log(x**4 - x**2 - 1) - log( - sqrt(sq
rt(5) - 1) + sqrt(2)*x) - log(sqrt(sqrt(5) - 1) + sqrt(2)*x) - log(sqrt(5)
 + 2*x**2 + 1))/4
```

3.122 $\int \frac{x^3}{1-3x^4+x^8}\,dx$

Optimal result 996
Mathematica [A] (verified) 996
Rubi [A] (verified) 997
Maple [A] (verified) 998
Fricas [A] (verification not implemented) 998
Sympy [A] (verification not implemented) 999
Maxima [A] (verification not implemented) 999
Giac [A] (verification not implemented) 999
Mupad [B] (verification not implemented) 1000
Reduce [F] 1000

Optimal result

Integrand size = 16, antiderivative size = 47

$$\int \frac{x^3}{1-3x^4+x^8}\,dx=-\frac{\log\left(3-\sqrt{5}-2x^4\right)}{4\sqrt{5}}+\frac{\log\left(3+\sqrt{5}-2x^4\right)}{4\sqrt{5}}$$

output
```
-1/20*ln(3-5^(1/2)-2*x^4)*5^(1/2)+1/20*ln(3+5^(1/2)-2*x^4)*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{1-3x^4+x^8}\,dx=\frac{\log\left(3+\sqrt{5}-2x^4\right)-\log\left(-3+\sqrt{5}+2x^4\right)}{4\sqrt{5}}$$

input
```
Integrate[x^3/(1 - 3*x^4 + x^8),x]
```

output
```
(Log[3 + Sqrt[5] - 2*x^4] - Log[-3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1690, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8-3x^4+1}dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{4}\int \frac{1}{x^8-3x^4+1}dx^4$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{4}\int \left(\frac{2}{\sqrt{5}\left(-2x^4-\sqrt{5}+3\right)}-\frac{2}{\sqrt{5}\left(-2x^4+\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{\log\left(-2x^4+\sqrt{5}+3\right)}{\sqrt{5}}-\frac{\log\left(-2x^4-\sqrt{5}+3\right)}{\sqrt{5}}\right)$$

input
```
Int[x^3/(1 - 3*x^4 + x^8),x]
```

output
```
(-(Log[3 - Sqrt[5] - 2*x^4]/Sqrt[5]) + Log[3 + Sqrt[5] - 2*x^4]/Sqrt[5])/4
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.40

method	result	size
default	$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^{4}-3\right)\sqrt{5}}{5}\right)}{10}$	19
risch	$\frac{\ln\left(2x^{4}-\sqrt{5}-3\right)\sqrt{5}}{20}-\frac{\ln\left(2x^{4}+\sqrt{5}-3\right)\sqrt{5}}{20}$	36

input
```
int(x^3/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/10*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{1-3x^4+x^8}\,dx=\frac{1}{20}\,\sqrt{5}\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}(2\,x^4-3)+7}{x^8-3\,x^4+1}\right)$$

input
```
integrate(x^3/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 +
1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{1-3x^4+x^8}\,dx = \frac{\sqrt{5}\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}{20} - \frac{\sqrt{5}\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)}{20}$$

input
```
integrate(x**3/(x**8-3*x**4+1),x)
```

output
```
sqrt(5)*log(x**4 - 3/2 - sqrt(5)/2)/20 - sqrt(5)*log(x**4 - 3/2 + sqrt(5)/
2)/20
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{1-3x^4+x^8}\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)$$

input
```
integrate(x^3/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
1/20*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{x^3}{1-3x^4+x^8}\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{|2x^4-\sqrt{5}-3|}{|2x^4+\sqrt{5}-3|}\right)$$

input
```
integrate(x^3/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/20*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3))
```

Mupad [B] (verification not implemented)

Time = 19.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{1-3x^4+x^8}\,dx=\frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{3\,\sqrt{5}-8\,\sqrt{5}\,x^4}{18\,x^4-7}\right)}{10}$$

input
```
int(x^3/(x^8 - 3*x^4 + 1),x)
```

output
```
(5^(1/2)*atanh((3*5^(1/2) - 8*5^(1/2)*x^4)/(18*x^4 - 7)))/10
```

Reduce [F]

$$\begin{aligned}\int \frac{x^3}{1-3x^4+x^8}\,dx=&-\frac{\sqrt{5}\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{10}-\frac{\sqrt{5}\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{10}\\&+\frac{\sqrt{5}\log(\sqrt{5}+2x^2+1)}{10}-2\left(\int\frac{x}{x^8-3x^4+1}dx\right)\\&+\frac{\log(x^4-x^2-1)}{4}-\frac{\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{4}\\&-\frac{\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{4}-\frac{\log(\sqrt{5}+2x^2+1)}{4}\end{aligned}$$

input
```
int(x^3/(x^8-3*x^4+1),x)
```

output
```
( - 2*sqrt(5)*log( - sqrt(sqrt(5) - 1) + sqrt(2)*x) - 2*sqrt(5)*log(sqrt(s
qrt(5) - 1) + sqrt(2)*x) + 2*sqrt(5)*log(sqrt(5) + 2*x**2 + 1) - 40*int(x/
(x**8 - 3*x**4 + 1),x) + 5*log(x**4 - x**2 - 1) - 5*log( - sqrt(sqrt(5) -
1) + sqrt(2)*x) - 5*log(sqrt(sqrt(5) - 1) + sqrt(2)*x) - 5*log(sqrt(5) + 2
*x**2 + 1))/20
```

3.123 $\int \frac{x}{1-3x^4+x^8}\,dx$

Optimal result . . . 1001
Mathematica [A] (verified) . . . 1001
Rubi [A] (verified) . . . 1002
Maple [A] (verified) . . . 1003
Fricas [B] (verification not implemented) . . . 1004
Sympy [B] (verification not implemented) . . . 1004
Maxima [B] (verification not implemented) . . . 1005
Giac [B] (verification not implemented) . . . 1005
Mupad [B] (verification not implemented) . . . 1006
Reduce [F] . . . 1006

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{x}{1-3x^4+x^8}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}} + \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

output
```
-arctanh(2^(1/2)/(3+5^(1/2))^(1/2)*x^2)/(5+5^(1/2))+1/2*(1/2+1/10*5^(1/2))
*arctanh((1/2+1/2*5^(1/2))*x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.21

$$\int \frac{x}{1-3x^4+x^8}\,dx = \frac{1}{40}\left(-\left(\left(5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)\right) - \left(-5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right) + \left(5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right) + \left(-5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

input
```
Integrate[x/(1 - 3*x^4 + x^8),x]
```

output
```
(-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2]) - (-5 + Sqrt[5])*Log[1 + Sqrt[
5] - 2*x^2] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + Sqrt[5])*Log
[1 + Sqrt[5] + 2*x^2])/40
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1695, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^8-3x^4+1}dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{x^8-3x^4+1}dx^2$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx^2}{\sqrt{5}}-\frac{\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx^2}{\sqrt{5}}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)\right)$$

input
```
Int[x/(1 - 3*x^4 + x^8),x]
```

output
```
(-(Sqrt[2/(5*(3 + Sqrt[5]))]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]) + Sqrt[(3
 + Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

method	result
default	$\frac{\ln\left(x^{4}-x^{2}-1\right)}{8}+\frac{\sqrt{5}\, \operatorname{arctanh}\left(\frac{\left(2 x^{2}-1\right) \sqrt{5}}{5}\right)}{20}-\frac{\ln\left(x^{4}+x^{2}-1\right)}{8}+\frac{\sqrt{5}\, \operatorname{arctanh}\left(\frac{\left(2 x^{2}+1\right) \sqrt{5}}{5}\right)}{20}$
risch	$-\frac{\ln\left(2 x^{2}+\sqrt{5}+1\right)}{8}+\frac{\ln\left(2 x^{2}+\sqrt{5}+1\right) \sqrt{5}}{40}-\frac{\ln\left(2 x^{2}-\sqrt{5}+1\right)}{8}-\frac{\ln\left(2 x^{2}-\sqrt{5}+1\right) \sqrt{5}}{40}+\frac{\ln\left(2 x^{2}+\sqrt{5}-1\right)}{8}+\frac{\ln\left(2 x^{2}+\sqrt{5}-1\right.}{40}$

input
```
int(x/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(x^4-x^2-1)+1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/8*ln(x^4+x
^2-1)+1/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(43) = 86$.

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int \frac{x}{1-3x^4+x^8}\,dx = \frac{1}{40}\,\sqrt{5}\log\left(\frac{2\,x^4+2\,x^2+\sqrt{5}(2\,x^2+1)+3}{x^4+x^2-1}\right) + \frac{1}{40}\,\sqrt{5}\log\left(\frac{2\,x^4-2\,x^2+\sqrt{5}(2\,x^2-1)+3}{x^4-x^2-1}\right) - \frac{1}{8}\,\log\left(x^4+x^2-1\right)+\frac{1}{8}\,\log\left(x^4-x^2-1\right)$$

input
```
integrate(x/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/40*sqrt(5)*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1)
) + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2
- 1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(51) = 102$.

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.20

$$\int \frac{x}{1-3x^4+x^8}\,dx = \left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-\frac{7}{2}-\frac{7\sqrt{5}}{10}+960\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3\right) + \left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{7}{2}+960\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{7\sqrt{5}}{10}\right) + \left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{7\sqrt{5}}{10}+960\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{7}{2}\right) + \left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2+960\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{7\sqrt{5}}{10}+\frac{7}{2}\right)$$

input
```
integrate(x/(x**8-3*x**4+1),x)
```

output
```
(sqrt(5)/40 + 1/8)*log(x**2 - 7/2 - 7*sqrt(5)/10 + 960*(sqrt(5)/40 + 1/8)*
*3) + (1/8 - sqrt(5)/40)*log(x**2 - 7/2 + 960*(1/8 - sqrt(5)/40)**3 + 7*sq
rt(5)/10) + (-1/8 + sqrt(5)/40)*log(x**2 - 7*sqrt(5)/10 + 960*(-1/8 + sqrt
(5)/40)**3 + 7/2) + (-1/8 - sqrt(5)/40)*log(x**2 + 960*(-1/8 - sqrt(5)/40)
**3 + 7*sqrt(5)/10 + 7/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(43) = 86$.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int \frac{x}{1-3x^4+x^8}\,dx = -\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right) - \frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right) - \frac{1}{8}\log\left(x^4+x^2-1\right) + \frac{1}{8}\log\left(x^4-x^2-1\right)$$

input
```
integrate(x/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*sqrt
(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 -
 1) + 1/8*log(x^4 - x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(43) = 86$.

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{x}{1-3x^4+x^8}\,dx = -\frac{1}{40}\sqrt{5}\log\left(\frac{|2x^2-\sqrt{5}+1|}{2x^2+\sqrt{5}+1}\right) - \frac{1}{40}\sqrt{5}\log\left(\frac{|2x^2-\sqrt{5}-1|}{|2x^2+\sqrt{5}-1|}\right) - \frac{1}{8}\log\left(|x^4+x^2-1|\right) + \frac{1}{8}\log\left(|x^4-x^2-1|\right)$$

input
```
integrate(x/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*s
qrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(ab
s(x^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 20.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \frac{x}{1-3x^4+x^8}\,dx = \operatorname{atanh}\left(\frac{29\,x^2}{8\,\sqrt{5}-18}-\frac{13\,\sqrt{5}\,x^2}{8\,\sqrt{5}-18}\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{4}\right) + \operatorname{atanh}\left(\frac{29\,x^2}{8\,\sqrt{5}+18}+\frac{13\,\sqrt{5}\,x^2}{8\,\sqrt{5}+18}\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)$$

input
```
int(x/(x^8 - 3*x^4 + 1),x)
```

output
```
atanh((29*x^2)/(8*5^(1/2) - 18) - (13*5^(1/2)*x^2)/(8*5^(1/2) - 18))*(5^(1
/2)/20 - 1/4) + atanh((29*x^2)/(8*5^(1/2) + 18) + (13*5^(1/2)*x^2)/(8*5^(1
/2) + 18))*(5^(1/2)/20 + 1/4)
```

Reduce [F]

$$\int \frac{x}{1-3x^4+x^8}\,dx = \int \frac{x}{x^8-3x^4+1}dx$$

input
```
int(x/(x^8-3*x^4+1),x)
```

output
```
int(x/(x**8 - 3*x**4 + 1),x)
```

3.124 $\int \frac{1}{x(1-3x^4+x^8)}\,dx$

Optimal result . . . 1007
Mathematica [A] (verified) . . . 1007
Rubi [A] (verified) . . . 1008
Maple [A] (verified) . . . 1009
Fricas [A] (verification not implemented) . . . 1010
Sympy [A] (verification not implemented) . . . 1010
Maxima [A] (verification not implemented) . . . 1011
Giac [A] (verification not implemented) . . . 1011
Mupad [B] (verification not implemented) . . . 1011
Reduce [F] . . . 1012

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx = \log(x) - \frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right) - \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

output
```
ln(x)-1/40*(5+3*5^(1/2))*ln(3-5^(1/2)-2*x^4)-1/40*(5-3*5^(1/2))*ln(3+5^(1/
2)-2*x^4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx = \log(x) + \frac{1}{40}\left(-5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right) + \frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)$$

input
```
Integrate[1/(x*(1 - 3*x^4 + x^8)),x]
```

output
```
Log[x] + ((-5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((-5 - 3*Sqrt[5]
)*Log[-3 + Sqrt[5] + 2*x^4])/40
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^8-3x^4+1)}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{1}{4}\int \frac{1}{x^4(x^8-3x^4+1)}dx^4$$

$$\Big\downarrow \text{ 1141}$$

$$\frac{1}{4}\int \left(-\frac{4}{\sqrt{5}\left(3+\sqrt{5}\right)\left(-2x^4+\sqrt{5}+3\right)}+\frac{1}{x^4}+\frac{4}{\sqrt{5}\left(3-\sqrt{5}\right)\left(-2x^4-\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\log\left(x^4\right)-\frac{2\log\left(-2x^4-\sqrt{5}+3\right)}{\sqrt{5}\left(3-\sqrt{5}\right)}+\frac{2\log\left(-2x^4+\sqrt{5}+3\right)}{\sqrt{5}\left(3+\sqrt{5}\right)}\right)$$

input
```
Int[1/(x*(1 - 3*x^4 + x^8)),x]
```

output
```
(Log[x^4] - (2*Log[3 - Sqrt[5] - 2*x^4])/(Sqrt[5]*(3 - Sqrt[5])) + (2*Log[
3 + Sqrt[5] - 2*x^4])/(Sqrt[5]*(3 + Sqrt[5])))/4
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

method	result	size
default	$\ln(x)-\frac{\ln(x^4-x^2-1)}{8}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}-\frac{\ln(x^4+x^2-1)}{8}+\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{20}$	64
risch	$\ln(x)+\frac{3\ln\left(3x^4-\frac{3\sqrt{5}}{2}-\frac{9}{2}\right)\sqrt{5}}{40}-\frac{\ln\left(3x^4-\frac{3\sqrt{5}}{2}-\frac{9}{2}\right)}{8}-\frac{\ln\left(3x^4-\frac{9}{2}+\frac{3\sqrt{5}}{2}\right)}{8}-\frac{3\ln\left(3x^4-\frac{9}{2}+\frac{3\sqrt{5}}{2}\right)\sqrt{5}}{40}$	70

input
```
int(1/x/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/8*ln(x^4-x^2-1)-3/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/8*ln
(x^4+x^2-1)+3/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx = \frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}(2\,x^4-3)+7}{x^8-3\,x^4+1}\right) - \frac{1}{8}\log\left(x^8-3\,x^4+1\right)+\log\left(x\right)$$

input
```
integrate(1/x/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 +
1)) - 1/8*log(x^8 - 3*x^4 + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx = \log\left(x\right)+\left(-\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right) + \left(-\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

input
```
integrate(1/x/(x**8-3*x**4+1),x)
```

output
```
log(x) + (-1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-3*sqrt(5)/4
0 - 1/8)*log(x**4 - 3/2 + sqrt(5)/2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx=\frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}-3}{2\,x^4+\sqrt{5}-3}\right)-\frac{1}{8}\log\left(x^8-3\,x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

input
```
integrate(1/x/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/8*log(x^
8 - 3*x^4 + 1) + 1/4*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx=\frac{3}{40}\sqrt{5}\log\left(\frac{|2\,x^4-\sqrt{5}-3|}{|2\,x^4+\sqrt{5}-3|}\right)+\frac{1}{4}\log\left(x^4\right)-\frac{1}{8}\log\left(|x^8-3\,x^4+1|\right)$$

input
```
integrate(1/x/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/4*
log(x^4) - 1/8*log(abs(x^8 - 3*x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 20.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{x(1-3x^4+x^8)}\,dx=\ln(x)+\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)-\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}+\frac{1}{8}\right)$$

input
```
int(1/(x*(x^8 - 3*x^4 + 1)),x)
```

output
```
log(x) + log(x^4 - 5^(1/2)/2 - 3/2)*((3*5^(1/2))/40 - 1/8) - log(5^(1/2)/2
 + x^4 - 3/2)*((3*5^(1/2))/40 + 1/8)
```

Reduce [F]

$$\int \frac{1}{x\left(1-3x^4+x^8\right)}\,dx=\int \frac{1}{x^9-3x^5+x}dx$$

input
```
int(1/x/(x^8-3*x^4+1),x)
```

output
```
int(1/(x**9 - 3*x**5 + x),x)
```

3.125 $\int \frac{1}{x^3(1-3x^4+x^8)}\,dx$

Optimal result 1013
Mathematica [A] (verified) 1014
Rubi [A] (verified) 1014
Maple [A] (verified) 1016
Fricas [B] (verification not implemented) 1016
Sympy [B] (verification not implemented) 1017
Maxima [A] (verification not implemented) 1018
Giac [A] (verification not implemented) 1018
Mupad [B] (verification not implemented) 1019
Reduce [F] 1019

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx = -\frac{1}{2x^2} - \frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{2}(3-\sqrt{5})}x^2\right)}{2\sqrt{5(9+4\sqrt{5})}} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x^2\right)}{2\sqrt{5(9-4\sqrt{5})}}$$

output

```
-1/2/x^2-1/2*arctanh((1/2*5^(1/2)-1/2)*x^2)/(5+2*5^(1/2))+1/2*arctanh((1/2
+1/2*5^(1/2))*x^2)/(5-2*5^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^3 (1-3x^4+x^8)}\,dx = \frac{1}{20}\left(-\frac{10}{x^2} - \left(5+2\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right) + \left(5-2\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right) + \left(5+2\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right) + \left(-5+2\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

input
```
Integrate[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

output
```
(-10/x^2 - (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log
[1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 +
2*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1695, 1443, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^8-3x^4+1)}\,dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{x^4 (x^8-3x^4+1)}dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2}\left(\int \frac{3-x^4}{x^8-3x^4+1}dx^2 - \frac{1}{x^2}\right)$$

$$\Big\downarrow \; 1480$$

$$\frac{1}{2}\left(-\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx^2-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \; 220$$

$$\frac{1}{2}\left(\frac{\left(5-3\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}+\frac{1}{10}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\left(5+3\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{x^2}\right)$$

input
```
Int[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

output
```
(-x^(-2) + ((5 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/(5*Sqrt[2*
(3 + Sqrt[5])]) + (Sqrt[(3 + Sqrt[5])/2]*(5 + 3*Sqrt[5])*ArcTanh[Sqrt[(3 +
 Sqrt[5])/2]*x^2])/10)/2
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

method	result
default	$-\frac{1}{2x^2}+\frac{\ln(x^4-x^2-1)}{4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{5}-\frac{\ln(x^4+x^2-1)}{4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{5}$
risch	$-\frac{1}{2x^2}-\frac{\ln\left(4x^2+2+2\sqrt{5}\right)}{4}+\frac{\ln\left(4x^2+2+2\sqrt{5}\right)\sqrt{5}}{10}-\frac{\ln\left(4x^2+2-2\sqrt{5}\right)}{4}-\frac{\ln\left(4x^2+2-2\sqrt{5}\right)\sqrt{5}}{10}+\frac{\ln\left(4x^2-2+2\sqrt{5}\right)}{4}+$

input
```
int(1/x^3/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+1/4*ln(x^4-x^2-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/4*
ln(x^4+x^2-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(50) = 100$.

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx = \frac{2\sqrt{5}x^2\log\left(\frac{2x^4+2x^2+\sqrt{5}(2x^2+1)+3}{x^4+x^2-1}\right)+2\sqrt{5}x^2\log\left(\frac{2x^4-2x^2+\sqrt{5}(2x^2-1)+3}{x^4-x^2-1}\right)-5x^2\log\left(x^4+x^2-1\right)+5x^2\log}{20x^2}$$

input
```
integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/20*(2*sqrt(5)*x^2*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x
^2 - 1)) + 2*sqrt(5)*x^2*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^
4 - x^2 - 1)) - 5*x^2*log(x^4 + x^2 - 1) + 5*x^2*log(x^4 - x^2 - 1) - 10)/
x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(53) = 106$.

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.95

$$\int \frac{1}{x^3(1-3x^4+x^8)}\,dx = \left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-\frac{123}{8}-\frac{123\sqrt{5}}{20}+280\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3\right) + \left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{123}{8}+280\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{123\sqrt{5}}{20}\right) + \left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{123\sqrt{5}}{20}+280\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)^3+\frac{123}{8}\right) + \left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2+280\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{123\sqrt{5}}{20}+\frac{123}{8}\right) - \frac{1}{2x^2}$$

input
```
integrate(1/x**3/(x**8-3*x**4+1),x)
```

output
```
(sqrt(5)/10 + 1/4)*log(x**2 - 123/8 - 123*sqrt(5)/20 + 280*(sqrt(5)/10 + 1
/4)**3) + (1/4 - sqrt(5)/10)*log(x**2 - 123/8 + 280*(1/4 - sqrt(5)/10)**3
+ 123*sqrt(5)/20) + (-1/4 + sqrt(5)/10)*log(x**2 - 123*sqrt(5)/20 + 280*(-
1/4 + sqrt(5)/10)**3 + 123/8) + (-1/4 - sqrt(5)/10)*log(x**2 + 280*(-1/4 -
 sqrt(5)/10)**3 + 123*sqrt(5)/20 + 123/8) - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^3\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{10}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right) - \frac{1}{2\,x^2}-\frac{1}{4}\log\left(x^4+x^2-1\right)+\frac{1}{4}\log\left(x^4-x^2-1\right)$$

input
```
integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*sqrt
(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 - 1/4*log(x
^4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^3\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}+1\right|}{2\,x^2+\sqrt{5}+1}\right) - \frac{1}{10}\sqrt{5}\log\left(\frac{\left|2\,x^2-\sqrt{5}-1\right|}{\left|2\,x^2+\sqrt{5}-1\right|}\right)-\frac{1}{2\,x^2} - \frac{1}{4}\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{4}\log\left(\left|x^4-x^2-1\right|\right)$$

input
```
integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*s
qrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 -
1/4*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 19.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(1-3x^4+x^8\right)}\,dx = \operatorname{atanh}\left(\frac{12736\,x^2}{3520\,\sqrt{5}-7872}-\frac{5696\,\sqrt{5}\,x^2}{3520\,\sqrt{5}-7872}\right)\left(\frac{\sqrt{5}}{5}-\frac{1}{2}\right) + \operatorname{atanh}\left(\frac{12736\,x^2}{3520\,\sqrt{5}+7872}+\frac{5696\,\sqrt{5}\,x^2}{3520\,\sqrt{5}+7872}\right)\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right) - \frac{1}{2\,x^2}$$

input
```
int(1/(x^3*(x^8 - 3*x^4 + 1)),x)
```

output
```
atanh((12736*x^2)/(3520*5^(1/2) - 7872) - (5696*5^(1/2)*x^2)/(3520*5^(1/2)
 - 7872))*(5^(1/2)/5 - 1/2) + atanh((12736*x^2)/(3520*5^(1/2) + 7872) + (5
696*5^(1/2)*x^2)/(3520*5^(1/2) + 7872))*(5^(1/2)/5 + 1/2) - 1/(2*x^2)
```

Reduce [F]

$$\int \frac{1}{x^3\left(1-3x^4+x^8\right)}\,dx = \int \frac{1}{x^{11}-3x^7+x^3}dx$$

input
```
int(1/x^3/(x^8-3*x^4+1),x)
```

output
```
int(1/(x**11 - 3*x**7 + x**3),x)
```

3.126 $\int \frac{1}{x^5(1-3x^4+x^8)}\,dx$

Optimal result . . . 1020
Mathematica [A] (verified) . . . 1020
Rubi [A] (verified) . . . 1021
Maple [A] (verified) . . . 1022
Fricas [A] (verification not implemented) . . . 1022
Sympy [A] (verification not implemented) . . . 1023
Maxima [A] (verification not implemented) . . . 1023
Giac [A] (verification not implemented) . . . 1024
Mupad [B] (verification not implemented) . . . 1024
Reduce [F] . . . 1025

Optimal result

Integrand size = 16, antiderivative size = 73

$$\int \frac{1}{x^5(1-3x^4+x^8)}\,dx=-\frac{1}{4x^4}+3\log(x)-\frac{\log\left(3-\sqrt{5}-2x^4\right)}{\sqrt{5}\left(3-\sqrt{5}\right)^2}+\frac{\log\left(3+\sqrt{5}-2x^4\right)}{\sqrt{5}\left(3+\sqrt{5}\right)^2}$$

output
```
-1/4/x^4+3*ln(x)-1/5*ln(3-5^(1/2)-2*x^4)*5^(1/2)/(3-5^(1/2))^2+1/5*ln(3+5^
(1/2)-2*x^4)*5^(1/2)/(3+5^(1/2))^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^5(1-3x^4+x^8)}\,dx=\frac{1}{40}\left(-\frac{10}{x^4}+120\log(x)+\left(-15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)-\left(15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)\right)$$

input
```
Integrate[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

output
```
(-10/x^4 + 120*Log[x] + (-15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] - (15 +
 7*Sqrt[5])*Log[-3 + Sqrt[5] + 2*x^4])/40
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^8-3x^4+1)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{4}\int \frac{1}{x^8 (x^8-3x^4+1)}dx^4$$

$$\Big\downarrow \text{1141}$$

$$\frac{1}{4}\int \left(-\frac{8}{\sqrt{5}\left(3+\sqrt{5}\right)^2\left(-2x^4+\sqrt{5}+3\right)}+\frac{3}{x^4}+\frac{1}{x^8}+\frac{8}{\sqrt{5}\left(3-\sqrt{5}\right)^2\left(-2x^4-\sqrt{5}+3\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{1}{x^4}+3\log\left(x^4\right)-\frac{4\log\left(-2x^4-\sqrt{5}+3\right)}{\sqrt{5}\left(3-\sqrt{5}\right)^2}+\frac{4\log\left(-2x^4+\sqrt{5}+3\right)}{\sqrt{5}\left(3+\sqrt{5}\right)^2}\right)$$

input
```
Int[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

output
```
(-x^(-4) + 3*Log[x^4] - (4*Log[3 - Sqrt[5] - 2*x^4])/(Sqrt[5]*(3 - Sqrt[5]
)^2) + (4*Log[3 + Sqrt[5] - 2*x^4])/(Sqrt[5]*(3 + Sqrt[5])^2))/4
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

method	result
default	$-\frac{1}{4x^4}+3\ln(x)-\frac{3\ln(x^4-x^2-1)}{8}-\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}-\frac{3\ln(x^4+x^2-1)}{8}+\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{20}$
risch	$-\frac{1}{4x^4}+3\ln(x)-\frac{3\ln\left(7x^4-\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)}{8}+\frac{7\ln\left(7x^4-\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40}-\frac{3\ln\left(7x^4-\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)}{8}-\frac{7\ln\left(7x^4-\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40}$

input
```
int(1/x^5/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4+3*ln(x)-3/8*ln(x^4-x^2-1)-7/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1
/2))-3/8*ln(x^4+x^2-1)+7/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^5(1-3x^4+x^8)}\,dx = \frac{7\sqrt{5}x^4\log\left(\frac{2x^8-6x^4-\sqrt{5}(2x^4-3)+7}{x^8-3x^4+1}\right)-15x^4\log(x^8-3x^4+1)+120x^4\log(x)-10}{40x^4}$$

input
```
integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/40*(7*sqrt(5)*x^4*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3
*x^4 + 1)) - 15*x^4*log(x^8 - 3*x^4 + 1) + 120*x^4*log(x) - 10)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^5 (1-3x^4+x^8)} \, dx = 3\log(x) + \left(-\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right) + \left(-\frac{7\sqrt{5}}{40} - \frac{3}{8}\right)\log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right) - \frac{1}{4x^4}$$

input
```
integrate(1/x**5/(x**8-3*x**4+1),x)
```

output
```
3*log(x) + (-3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-7*sqrt(5)
/40 - 3/8)*log(x**4 - 3/2 + sqrt(5)/2) - 1/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^5 (1-3x^4+x^8)} \, dx = \frac{7}{40}\sqrt{5}\log\left(\frac{2x^4 - \sqrt{5} - 3}{2x^4 + \sqrt{5} - 3}\right) - \frac{1}{4x^4} - \frac{3}{8}\log\left(x^8 - 3x^4 + 1\right) + \frac{3}{4}\log\left(x^4\right)$$

input
```
integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/4/x^4 -
3/8*log(x^8 - 3*x^4 + 1) + 3/4*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^5 (1 - 3x^4 + x^8)} \, dx = \frac{7}{40}\sqrt{5}\log\left(\frac{|2x^4 - \sqrt{5} - 3|}{|2x^4 + \sqrt{5} - 3|}\right) - \frac{3x^4 + 1}{4x^4} + \frac{3}{4}\log(x^4) - \frac{3}{8}\log(|x^8 - 3x^4 + 1|)$$

input
```
integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) - 1/4*
(3*x^4 + 1)/x^4 + 3/4*log(x^4) - 3/8*log(abs(x^8 - 3*x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 19.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^5 (1 - 3x^4 + x^8)} \, dx = 3 \ln(x) - \frac{1}{4x^4} + \ln\left(x^4 - \frac{\sqrt{5}}{2} - \frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40} - \frac{3}{8}\right) - \ln\left(x^4 + \frac{\sqrt{5}}{2} - \frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40} + \frac{3}{8}\right)$$

input
```
int(1/(x^5*(x^8 - 3*x^4 + 1)),x)
```

output
```
3*log(x) - 1/(4*x^4) + log(x^4 - 5^(1/2)/2 - 3/2)*((7*5^(1/2))/40 - 3/8) -
 log(5^(1/2)/2 + x^4 - 3/2)*((7*5^(1/2))/40 + 3/8)
```

Reduce [F]

$$\int \frac{1}{x^5 (1-3x^4+x^8)} \, dx = \int \frac{1}{x^{13}-3x^9+x^5} dx$$

input
```
int(1/x^5/(x^8-3*x^4+1),x)
```

output
```
int(1/(x**13 - 3*x**9 + x**5),x)
```

3.127 $\int \frac{1}{x^7(1-3x^4+x^8)}\,dx$

Optimal result 1026
Mathematica [A] (verified) 1027
Rubi [A] (verified) 1027
Maple [A] (verified) 1030
Fricas [B] (verification not implemented) 1030
Sympy [B] (verification not implemented) 1031
Maxima [A] (verification not implemented) 1032
Giac [B] (verification not implemented) 1032
Mupad [B] (verification not implemented) 1033
Reduce [F] 1033

Optimal result

Integrand size = 16, antiderivative size = 99

$$\int \frac{1}{x^7\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{6x^6}-\frac{3}{2x^2} -\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x^2\right) +\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

output

```
-1/6/x^6-3/2/x^2-1/2*(5/2-11/10*5^(1/2))*arctanh((1/2*5^(1/2)-1/2)*x^2)+1/
2*(5/2+11/10*5^(1/2))*arctanh((1/2+1/2*5^(1/2))*x^2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^7(1-3x^4+x^8)}\,dx = \frac{1}{120}\left(-\frac{20}{x^6}-\frac{180}{x^2}-3\left(25+11\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)+3\left(25-11\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)+3\left(25+11\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)+3\left(-25+11\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

input
```
Integrate[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

output
```
(-20/x^6 - 180/x^2 - 3*(25 + 11*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + 3*(25
 - 11*Sqrt[5])*Log[1 + Sqrt[5] - 2*x^2] + 3*(25 + 11*Sqrt[5])*Log[-1 + Sqr
t[5] + 2*x^2] + 3*(-25 + 11*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/120
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1695, 1443, 27, 1604, 25, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7(x^8-3x^4+1)}\,dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{2}\int \frac{1}{x^8(x^8-3x^4+1)}dx^2$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{3(3-x^4)}{x^4(x^8-3x^4+1)}dx^2-\frac{1}{3x^6}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\left(\int \frac{3-x^4}{x^4\left(x^8-3x^4+1\right)}dx^2-\frac{1}{3x^6}\right)$$

$$\Big\downarrow 1604$$

$$\frac{1}{2}\left(-\int -\frac{8-3x^4}{x^8-3x^4+1}dx^2-\frac{1}{3x^6}-\frac{3}{x^2}\right)$$

$$\Big\downarrow 25$$

$$\frac{1}{2}\left(\int \frac{8-3x^4}{x^8-3x^4+1}dx^2-\frac{1}{3x^6}-\frac{3}{x^2}\right)$$

$$\Big\downarrow 1480$$

$$\frac{1}{2}\left(-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx^2-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx^2-\frac{1}{3x^6}-\frac{3}{x^2}\right)$$

$$\Big\downarrow 220$$

$$\frac{1}{2}\left(\frac{\left(15-7\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}+\frac{1}{10}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\left(15+7\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{3x^6}-\frac{3}{x^2}\right)$$

input
```
Int[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

output
```
(-1/3*1/x^6 - 3/x^2 + ((15 - 7*Sqrt[5])*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]
)/(5*Sqrt[2*(3 + Sqrt[5])]) + (Sqrt[(3 + Sqrt[5])/2]*(15 + 7*Sqrt[5])*ArcT
anh[Sqrt[(3 + Sqrt[5])/2]*x^2])/10)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

method	result
default	$-\frac{1}{6x^{6}}-\frac{3}{2x^{2}}+\frac{5\ln\left(x^{4}-x^{2}-1\right)}{8}+\frac{11\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^{2}-1\right)\sqrt{5}}{5}\right)}{20}-\frac{5\ln\left(x^{4}+x^{2}-1\right)}{8}+\frac{11\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2x^{2}+1\right)\sqrt{5}}{5}\right)}{20}$
risch	$\frac{-\frac{3x^{4}}{2}-\frac{1}{6}}{x^{6}}-\frac{5\ln\left(11x^{2}+\frac{11}{2}+\frac{11\sqrt{5}}{2}\right)}{8}+\frac{11\ln\left(11x^{2}+\frac{11}{2}+\frac{11\sqrt{5}}{2}\right)\sqrt{5}}{40}-\frac{5\ln\left(11x^{2}+\frac{11}{2}-\frac{11\sqrt{5}}{2}\right)}{8}-\frac{11\ln\left(11x^{2}+\frac{11}{2}-\frac{11\sqrt{5}}{2}\right)\sqrt{5}}{40}$

input
```
int(1/x^7/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^6-3/2/x^2+5/8*ln(x^4-x^2-1)+11/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(
1/2))-5/8*ln(x^4+x^2-1)+11/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(51) = 102.

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^7\left(1-3x^4+x^8\right)}\,dx$$
$$=\frac{33\sqrt{5}x^{6}\log\left(\frac{2x^{4}+2x^{2}+\sqrt{5}(2x^{2}+1)+3}{x^{4}+x^{2}-1}\right)+33\sqrt{5}x^{6}\log\left(\frac{2x^{4}-2x^{2}+\sqrt{5}(2x^{2}-1)+3}{x^{4}-x^{2}-1}\right)-75x^{6}\log\left(x^{4}+x^{2}-1\right)+75}{120x^{6}}$$

input
```
integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/120*(33*sqrt(5)*x^6*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 +
 x^2 - 1)) + 33*sqrt(5)*x^6*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/
(x^4 - x^2 - 1)) - 75*x^6*log(x^4 + x^2 - 1) + 75*x^6*log(x^4 - x^2 - 1) -
 180*x^4 - 20)/x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(66) = 132$.

Time = 0.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.01

$$\int \frac{1}{x^7 (1 - 3x^4 + x^8)} \, dx = \left(\frac{11\sqrt{5}}{40} + \frac{5}{8}\right) \log\left(x^2 - \frac{2207}{22} - \frac{2207\sqrt{5}}{50} + \frac{1152\left(\frac{11\sqrt{5}}{40} + \frac{5}{8}\right)^3}{11}\right) + \left(\frac{5}{8} - \frac{11\sqrt{5}}{40}\right) \log\left(x^2 - \frac{2207}{22} + \frac{1152\left(\frac{5}{8} - \frac{11\sqrt{5}}{40}\right)^3}{11} + \frac{2207\sqrt{5}}{50}\right) + \left(-\frac{5}{8} + \frac{11\sqrt{5}}{40}\right) \log\left(x^2 - \frac{2207\sqrt{5}}{50} + \frac{1152\left(-\frac{5}{8} + \frac{11\sqrt{5}}{40}\right)^3}{11} + \frac{2207}{22}\right) + \left(-\frac{5}{8} - \frac{11\sqrt{5}}{40}\right) \log\left(x^2 + \frac{1152\left(-\frac{5}{8} - \frac{11\sqrt{5}}{40}\right)^3}{11} + \frac{2207\sqrt{5}}{50} + \frac{2207}{22}\right) + \frac{-9x^4 - 1}{6x^6}$$

input
```
integrate(1/x**7/(x**8-3*x**4+1),x)
```

output
```
(11*sqrt(5)/40 + 5/8)*log(x**2 - 2207/22 - 2207*sqrt(5)/50 + 1152*(11*sqrt
(5)/40 + 5/8)**3/11) + (5/8 - 11*sqrt(5)/40)*log(x**2 - 2207/22 + 1152*(5/
8 - 11*sqrt(5)/40)**3/11 + 2207*sqrt(5)/50) + (-5/8 + 11*sqrt(5)/40)*log(x
**2 - 2207*sqrt(5)/50 + 1152*(-5/8 + 11*sqrt(5)/40)**3/11 + 2207/22) + (-5
/8 - 11*sqrt(5)/40)*log(x**2 + 1152*(-5/8 - 11*sqrt(5)/40)**3/11 + 2207*sq
rt(5)/50 + 2207/22) + (-9*x**4 - 1)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^7(1-3x^4+x^8)}\,dx = -\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{9x^4+1}{6x^6}-\frac{5}{8}\log\left(x^4+x^2-1\right)+\frac{5}{8}\log\left(x^4-x^2-1\right)$$

input
```
integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-11/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40*sq
rt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x^4 + 1)/x
^6 - 5/8*log(x^4 + x^2 - 1) + 5/8*log(x^4 - x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(51) = 102$.

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^7(1-3x^4+x^8)}\,dx = -\frac{11}{40}\sqrt{5}\log\left(\frac{|2x^2-\sqrt{5}+1|}{2x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{|2x^2-\sqrt{5}-1|}{|2x^2+\sqrt{5}-1|}\right)-\frac{9x^4+1}{6x^6}-\frac{5}{8}\log\left(|x^4+x^2-1|\right)+\frac{5}{8}\log\left(|x^4-x^2-1|\right)$$

input
```
integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-11/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40
*sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x
^4 + 1)/x^6 - 5/8*log(abs(x^4 + x^2 - 1)) + 5/8*log(abs(x^4 - x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^7(1-3x^4+x^8)}\,dx = \operatorname{atanh}\left(\frac{4126100\,x^2}{1140425\,\sqrt{5}-2550075} - \frac{1845250\,\sqrt{5}\,x^2}{1140425\,\sqrt{5}-2550075}\right)\left(\frac{11\,\sqrt{5}}{20}-\frac{5}{4}\right) + \operatorname{atanh}\left(\frac{4126100\,x^2}{1140425\,\sqrt{5}+2550075} + \frac{1845250\,\sqrt{5}\,x^2}{1140425\,\sqrt{5}+2550075}\right)\left(\frac{11\,\sqrt{5}}{20}+\frac{5}{4}\right) - \frac{\frac{3x^4}{2}+\frac{1}{6}}{x^6}$$

input
```
int(1/(x^7*(x^8 - 3*x^4 + 1)),x)
```

output
```
atanh((4126100*x^2)/(1140425*5^(1/2) - 2550075) - (1845250*5^(1/2)*x^2)/(1
140425*5^(1/2) - 2550075))*((11*5^(1/2))/20 - 5/4) + atanh((4126100*x^2)/(
1140425*5^(1/2) + 2550075) + (1845250*5^(1/2)*x^2)/(1140425*5^(1/2) + 2550
075))*((11*5^(1/2))/20 + 5/4) - ((3*x^4)/2 + 1/6)/x^6
```

Reduce [F]

$$\int \frac{1}{x^7(1-3x^4+x^8)}\,dx = \int \frac{1}{x^{15}-3x^{11}+x^7}dx$$

input
```
int(1/x^7/(x^8-3*x^4+1),x)
```

output
```
int(1/(x**15 - 3*x**11 + x**7),x)
```

3.128 $\int \frac{x^8}{1-3x^4+x^8}\,dx$

Optimal result 1034
Mathematica [A] (verified) 1035
Rubi [A] (verified) 1035
Maple [C] (verified) 1038
Fricas [A] (verification not implemented) 1039
Sympy [A] (verification not implemented) 1040
Maxima [F] 1040
Giac [A] (verification not implemented) 1041
Mupad [B] (verification not implemented) 1042
Reduce [F] 1043

Optimal result

Integrand size = 16, antiderivative size = 178

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

output
```
x-1/10*(123/2+55/2*5^(1/2))^(1/4)*arctan(2^(1/4)*(1/(3+5^(1/2)))^(1/4)*x)*
5^(1/2)+1/10*(123/2-55/2*5^(1/2))^(1/4)*arctan((3/2+1/2*5^(1/2))^(1/4)*x)*
5^(1/2)-1/10*(123/2+55/2*5^(1/2))^(1/4)*arctanh(2^(1/4)*(1/(3+5^(1/2)))^(1
/4)*x)*5^(1/2)+1/10*(123/2-55/2*5^(1/2))^(1/4)*arctanh((3/2+1/2*5^(1/2))^(
1/4)*x)*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.90

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = x+\frac{\left(-2+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\left(2+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(-2+\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\left(2+\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

input
```
Integrate[x^8/(1 - 3*x^4 + x^8),x]
```

output
```
x + ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5
])] - ((2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5]
)] + ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt
[5])] - ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt
[5])]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1703, 1752, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^8-3x^4+1}\,dx$$

$$\Big\downarrow \text{1703}$$

$$x-\int \frac{1-3x^4}{x^8-3x^4+1}dx$$

$$\Big\downarrow \text{1752}$$

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx+\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx+x$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{10}\left(15-7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{\sqrt{3-\sqrt{5}}}\right)+\frac{1}{10}\left(15+7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{\sqrt{3+\sqrt{5}}}\right)+x$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{10}\left(15-7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)+\frac{1}{10}\left(15+7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)+x$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{10}\left(15+7\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)+\frac{1}{10}\left(15-7\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)+x$$

input

```
Int[x^8/(1 - 3*x^4 + x^8),x]
```

output
```
x + ((15 + 7*Sqrt[5])*(-(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 + S
qrt[5])^(3/4))) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 + Sqrt[5]
)^(3/4))))/10 + ((15 - 7*Sqrt[5])*(-(ArcTan[((3 + Sqrt[5])/2)^(1/4)*x]/(2^
(1/4)*(3 - Sqrt[5])^(3/4))) - ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]/(2^(1/4)*
(3 - Sqrt[5])^(3/4))))/10
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.39

method	result
risch	$x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^{4}+55\_Z^{2}-1\right)}\_R\ln\left(15\_R^{3}+29\_R+5x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^{4}-55\_Z^{2}-1\right)}\_R\ln\left(-15\_R^{3}\right.\right.}{4}$
default	$x-\frac{\left(2+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}+\frac{\left(-2+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}+\frac{\left(-2+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}-\frac{\left(2+\sqrt{\right.}}{}$

input
```
int(x^8/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
x+1/4*sum(_R*ln(15*_R^3+29*_R+5*x),_R=RootOf(25*_Z^4+55*_Z^2-1))+1/4*sum(_
R*ln(-15*_R^3+29*_R+5*x),_R=RootOf(25*_Z^4-55*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.12

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}\sqrt{5}+\frac{11}{10}}\arctan\left(\left(2\sqrt{5}x-5x\right)\sqrt{\frac{1}{2}\sqrt{5}+\frac{11}{10}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\sqrt{5}-\frac{11}{10}}\arctan\left(\left(2\sqrt{5}x+5x\right)\sqrt{\frac{1}{2}\sqrt{5}-\frac{11}{10}}\right) - \frac{1}{4}\sqrt{\frac{1}{2}\sqrt{5}+\frac{11}{10}}\log\left(\left(3\sqrt{5}-5\right)\sqrt{\frac{1}{2}\sqrt{5}+\frac{11}{10}}+2x\right) + \frac{1}{4}\sqrt{\frac{1}{2}\sqrt{5}+\frac{11}{10}}\log\left(-\left(3\sqrt{5}-5\right)\sqrt{\frac{1}{2}\sqrt{5}+\frac{11}{10}}+2x\right) + \frac{1}{4}\sqrt{\frac{1}{2}\sqrt{5}-\frac{11}{10}}\log\left(\left(3\sqrt{5}+5\right)\sqrt{\frac{1}{2}\sqrt{5}-\frac{11}{10}}+2x\right) - \frac{1}{4}\sqrt{\frac{1}{2}\sqrt{5}-\frac{11}{10}}\log\left(-\left(3\sqrt{5}+5\right)\sqrt{\frac{1}{2}\sqrt{5}-\frac{11}{10}}+2x\right) + x$$

input
```
integrate(x^8/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2*sqrt(5) + 11/10)*arctan((2*sqrt(5)*x - 5*x)*sqrt(1/2*sqrt(5)
+ 11/10)) + 1/2*sqrt(1/2*sqrt(5) - 11/10)*arctan((2*sqrt(5)*x + 5*x)*sqrt(
1/2*sqrt(5) - 11/10)) - 1/4*sqrt(1/2*sqrt(5) + 11/10)*log((3*sqrt(5) - 5)*
sqrt(1/2*sqrt(5) + 11/10) + 2*x) + 1/4*sqrt(1/2*sqrt(5) + 11/10)*log(-(3*s
qrt(5) - 5)*sqrt(1/2*sqrt(5) + 11/10) + 2*x) + 1/4*sqrt(1/2*sqrt(5) - 11/1
0)*log((3*sqrt(5) + 5)*sqrt(1/2*sqrt(5) - 11/10) + 2*x) - 1/4*sqrt(1/2*sqr
t(5) - 11/10)*log(-(3*sqrt(5) + 5)*sqrt(1/2*sqrt(5) - 11/10) + 2*x) + x
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.33

$$\int \frac{x^8}{1-3x^4+x^8}\,dx$$
$$= x + \text{RootSum}\left(6400t^4 - 880t^2 - 1, \left(t \mapsto t\log\left(-\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right)$$
$$+ \text{RootSum}\left(6400t^4 + 880t^2 - 1, \left(t \mapsto t\log\left(-\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right)$$

input
```
integrate(x**8/(x**8-3*x**4+1),x)
```

output
```
x + RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(-15360*_t**5/11
+ 1288*_t/55 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*lo
g(-15360*_t**5/11 + 1288*_t/55 + x)))
```

Maxima [F]

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = \int \frac{x^8}{x^8 - 3\,x^4 + 1}\,dx$$

input
```
integrate(x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
x + 1/2*integrate((2*x^2 + 1)/(x^4 - x^2 - 1), x) - 1/2*integrate((2*x^2 -
 1)/(x^4 + x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = -\frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) + x$$

input
```
integrate(x^8/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/20*sqrt(50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt
(50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5
) + 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 11
0)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110)*log
(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) - 110)*log(abs(x
 - sqrt(1/2*sqrt(5) - 1/2))) + x
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.38

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = x - \frac{\operatorname{atan}\left(\frac{x\sqrt{-50\sqrt{5}-110}\,55\mathrm{i}}{2\left(275\sqrt{5}+605\right)}+\frac{\sqrt{5}\,x\sqrt{-50\sqrt{5}-110}\,33\mathrm{i}}{2\left(275\sqrt{5}+605\right)}\right)\sqrt{-50\sqrt{5}-110}\,1\mathrm{i}}{20} - \frac{\operatorname{atan}\left(\frac{x\sqrt{110-50\sqrt{5}}\,55\mathrm{i}}{2\left(275\sqrt{5}-605\right)}-\frac{\sqrt{5}\,x\sqrt{110-50\sqrt{5}}\,33\mathrm{i}}{2\left(275\sqrt{5}-605\right)}\right)\sqrt{110-50\sqrt{5}}\,1\mathrm{i}}{20} + \frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}-110}\,55\mathrm{i}}{2\left(275\sqrt{5}-605\right)}-\frac{\sqrt{5}\,x\sqrt{50\sqrt{5}-110}\,33\mathrm{i}}{2\left(275\sqrt{5}-605\right)}\right)\sqrt{50\sqrt{5}-110}\,1\mathrm{i}}{20} + \frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}+110}\,55\mathrm{i}}{2\left(275\sqrt{5}+605\right)}+\frac{\sqrt{5}\,x\sqrt{50\sqrt{5}+110}\,33\mathrm{i}}{2\left(275\sqrt{5}+605\right)}\right)\sqrt{50\sqrt{5}+110}\,1\mathrm{i}}{20}$$

input

```
int(x^8/(x^8 - 3*x^4 + 1),x)
```

output

```
x - (atan((x*(- 50*5^(1/2) - 110)^(1/2)*55i)/(2*(275*5^(1/2) + 605)) + (5^
(1/2)*x*(- 50*5^(1/2) - 110)^(1/2)*33i)/(2*(275*5^(1/2) + 605)))*(- 50*5^(
1/2) - 110)^(1/2)*1i)/20 - (atan((x*(110 - 50*5^(1/2))^(1/2)*55i)/(2*(275*
5^(1/2) - 605)) - (5^(1/2)*x*(110 - 50*5^(1/2))^(1/2)*33i)/(2*(275*5^(1/2)
 - 605)))*(110 - 50*5^(1/2))^(1/2)*1i)/20 + (atan((x*(50*5^(1/2) - 110)^(1
/2)*55i)/(2*(275*5^(1/2) - 605)) - (5^(1/2)*x*(50*5^(1/2) - 110)^(1/2)*33i
)/(2*(275*5^(1/2) - 605)))*(50*5^(1/2) - 110)^(1/2)*1i)/20 + (atan((x*(50*
5^(1/2) + 110)^(1/2)*55i)/(2*(275*5^(1/2) + 605)) + (5^(1/2)*x*(50*5^(1/2)
 + 110)^(1/2)*33i)/(2*(275*5^(1/2) + 605)))*(50*5^(1/2) + 110)^(1/2)*1i)/2
0
```

Reduce [F]

$$\int \frac{x^8}{1-3x^4+x^8}\,dx = \frac{3\sqrt{\sqrt{5}+1}\,\sqrt{10}\,atan\left(\frac{2x}{\sqrt{\sqrt{5}+1}\,\sqrt{2}}\right)}{20} - \frac{3\sqrt{\sqrt{5}+1}\,\sqrt{2}\,atan\left(\frac{2x}{\sqrt{\sqrt{5}+1}\,\sqrt{2}}\right)}{4} + \frac{3\sqrt{\sqrt{5}-1}\,\sqrt{10}\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{40} - \frac{3\sqrt{\sqrt{5}-1}\,\sqrt{10}\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{40} + \frac{3\sqrt{\sqrt{5}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{8} - \frac{3\sqrt{\sqrt{5}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{8} + 3\left(\int \frac{x^2}{x^8-3x^4+1}dx\right) + 2\left(\int \frac{1}{x^8-3x^4+1}dx\right) + x$$

input
```
int(x^8/(x^8-3*x^4+1),x)
```

output
```
(6*sqrt(sqrt(5) + 1)*sqrt(10)*atan((2*x)/(sqrt(sqrt(5) + 1)*sqrt(2))) - 30
*sqrt(sqrt(5) + 1)*sqrt(2)*atan((2*x)/(sqrt(sqrt(5) + 1)*sqrt(2))) + 3*sqr
t(sqrt(5) - 1)*sqrt(10)*log( - sqrt(sqrt(5) - 1) + sqrt(2)*x) - 3*sqrt(sqr
t(5) - 1)*sqrt(10)*log(sqrt(sqrt(5) - 1) + sqrt(2)*x) + 15*sqrt(sqrt(5) -
1)*sqrt(2)*log( - sqrt(sqrt(5) - 1) + sqrt(2)*x) - 15*sqrt(sqrt(5) - 1)*sq
rt(2)*log(sqrt(sqrt(5) - 1) + sqrt(2)*x) + 120*int(x**2/(x**8 - 3*x**4 + 1
),x) + 80*int(1/(x**8 - 3*x**4 + 1),x) + 40*x)/40
```

3.129 $\int \frac{x^6}{1-3x^4+x^8} \, dx$

Optimal result . . . 1044
Mathematica [A] (verified) . . . 1045
Rubi [A] (verified) . . . 1045
Maple [C] (verified) . . . 1048
Fricas [A] (verification not implemented) . . . 1048
Sympy [A] (verification not implemented) . . . 1049
Maxima [F] . . . 1049
Giac [A] (verification not implemented) . . . 1050
Mupad [B] (verification not implemented) . . . 1051
Reduce [F] . . . 1052

Optimal result

Integrand size = 16, antiderivative size = 167

$$\int \frac{x^6}{1-3x^4+x^8} \, dx = \frac{\left(3+\sqrt{5}\right)^{3/4} \arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{144-64\sqrt{5}} \arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{3/4} \operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{144-64\sqrt{5}} \operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt{5}}$$

output
```
1/20*(3+5^(1/2))^(3/4)*arctan(2^(1/4)*(1/(3+5^(1/2)))^(1/4)*x)*2^(1/4)*5^(
1/2)-1/20*(144-64*5^(1/2))^(1/4)*arctan((3/2+1/2*5^(1/2))^(1/4)*x)*5^(1/2)
-1/20*(3+5^(1/2))^(3/4)*arctanh(2^(1/4)*(1/(3+5^(1/2)))^(1/4)*x)*2^(1/4)*5
^(1/2)+1/20*(144-64*5^(1/2))^(1/4)*arctanh((3/2+1/2*5^(1/2))^(1/4)*x)*5^(1
/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{\frac{\left(-3+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{-1+\sqrt{5}}}+\frac{\left(3+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}-\frac{\left(-3+\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{-1+\sqrt{5}}}-\frac{\left(3+\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

input
```
Integrate[x^6/(1 - 3*x^4 + x^8),x]
```

output
```
(((-3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] + ((
3 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] - ((-3 + S
qrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((3 + Sqrt
[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1710, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^8-3x^4+1}\,dx$$

$$\Big\downarrow \text{ 1710}$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\int -\frac{2x^2}{-2x^4-\sqrt{5}+3}dx+\frac{1}{10}\left(5+3\sqrt{5}\right)\int -\frac{2x^2}{-2x^4+\sqrt{5}+3}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{5}\left(5-3\sqrt{5}\right)\int \frac{x^2}{-2x^4-\sqrt{5}+3}dx-\frac{1}{5}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-2x^4+\sqrt{5}+3}dx$$

$$\Big\downarrow \text{ 827}$$

$$-\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{2\sqrt{2}}\right)-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)-\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)$$

input
```
Int[x^6/(1 - 3*x^4 + x^8),x]
```

output
```
-1/5*((5 + 3*Sqrt[5])*(-1/2*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*(3
+ Sqrt[5])^(1/4)) + ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2*2^(3/4)*(3 + Sqr
t[5])^(1/4)))) - ((5 - 3*Sqrt[5])*(-1/2*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x]/
(2^(3/4)*(3 - Sqrt[5])^(1/4)) + ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]/(2*2^(3
/4)*(3 - Sqrt[5])^(1/4))))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.41

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^{4}-20\_Z^{2}-1\right)}\_R\ln\left(5\_R^{3}-7\_R+2x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^{4}+20\_Z^{2}-1\right)}\_R\ln\left(5\_R^{3}+7\_R+2x\right)\right)}{4}$
default	$-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}+\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\left(3+\sqrt{5}\right)\sqrt{5}}{10}$

input
```
int(x^6/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(5*_R^3-7*_R+2*x),_R=RootOf(25*_Z^4-20*_Z^2-1))+1/4*sum(_R*ln
(5*_R^3+7*_R+2*x),_R=RootOf(25*_Z^4+20*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

$$\begin{aligned}\int \frac{x^6}{1-3x^4+x^8}\,dx &= \frac{1}{2}\sqrt{\frac{1}{5}\sqrt{5}+\frac{2}{5}}\arctan\left(\frac{1}{2}\left(3\sqrt{5}x-5x\right)\sqrt{\frac{1}{5}\sqrt{5}+\frac{2}{5}}\right)\\ &\quad-\frac{1}{2}\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}\arctan\left(\frac{1}{2}\left(3\sqrt{5}x+5x\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}\right)\\ &\quad+\frac{1}{4}\sqrt{\frac{1}{5}\sqrt{5}+\frac{2}{5}}\log\left(\left(\sqrt{5}-5\right)\sqrt{\frac{1}{5}\sqrt{5}+\frac{2}{5}}+2x\right)\\ &\quad-\frac{1}{4}\sqrt{\frac{1}{5}\sqrt{5}+\frac{2}{5}}\log\left(-\left(\sqrt{5}-5\right)\sqrt{\frac{1}{5}\sqrt{5}+\frac{2}{5}}+2x\right)\\ &\quad+\frac{1}{4}\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}\log\left(\left(\sqrt{5}+5\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}+2x\right)\\ &\quad-\frac{1}{4}\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}\log\left(-\left(\sqrt{5}+5\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}+2x\right)\end{aligned}$$

input
```
integrate(x^6/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/5*sqrt(5) + 2/5)*arctan(1/2*(3*sqrt(5)*x - 5*x)*sqrt(1/5*sqrt(5
) + 2/5)) - 1/2*sqrt(1/5*sqrt(5) - 2/5)*arctan(1/2*(3*sqrt(5)*x + 5*x)*sqr
t(1/5*sqrt(5) - 2/5)) + 1/4*sqrt(1/5*sqrt(5) + 2/5)*log((sqrt(5) - 5)*sqrt
(1/5*sqrt(5) + 2/5) + 2*x) - 1/4*sqrt(1/5*sqrt(5) + 2/5)*log(-(sqrt(5) - 5
)*sqrt(1/5*sqrt(5) + 2/5) + 2*x) + 1/4*sqrt(1/5*sqrt(5) - 2/5)*log((sqrt(5
) + 5)*sqrt(1/5*sqrt(5) - 2/5) + 2*x) - 1/4*sqrt(1/5*sqrt(5) - 2/5)*log(-(
sqrt(5) + 5)*sqrt(1/5*sqrt(5) - 2/5) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.32

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \operatorname{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(-1792000t^7+4920t^3+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\left(-1792000t^7+4920t^3+x\right)\right)\right)$$

input
```
integrate(x**6/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(-1792000*_t**7 + 492
0*_t**3 + x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(-17
92000*_t**7 + 4920*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \int \frac{x^6}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^6/(x^8 - 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

input
```
integrate(x^6/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/10*sqrt(5*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(5*
sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) + 10
)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(ab
s(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x + sq
rt(1/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x - sqrt(1/2*s
qrt(5) - 1/2)))
```

Mupad [B] (verification not implemented)

Time = 19.53 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\sqrt{2-\sqrt{5}}}{8\,\sqrt{5}-24}\right)\,\sqrt{\sqrt{5}-2}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,\sqrt{-\sqrt{5}-2}}{8\,\sqrt{5}+24}\right)\,\sqrt{\sqrt{5}+2}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{2-\sqrt{5}}\,16\mathrm{i}}{8\,\sqrt{5}-24}\right)\,\sqrt{2-\sqrt{5}}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{-\sqrt{5}-2}\,16\mathrm{i}}{8\,\sqrt{5}+24}\right)\,\sqrt{-\sqrt{5}-2}\,1\mathrm{i}}{10}$$

input
```
int(x^6/(x^8 - 3*x^4 + 1),x)
```

output
```
(5^(1/2)*atan((16*x*(2 - 5^(1/2))^(1/2))/(8*5^(1/2) - 24))*(5^(1/2) - 2)^(
1/2)*1i)/10 + (5^(1/2)*atan((16*x*(- 5^(1/2) - 2)^(1/2))/(8*5^(1/2) + 24))
*(5^(1/2) + 2)^(1/2)*1i)/10 + (5^(1/2)*atan((x*(2 - 5^(1/2))^(1/2)*16i)/(8
*5^(1/2) - 24))*(2 - 5^(1/2))^(1/2)*1i)/10 + (5^(1/2)*atan((x*(- 5^(1/2) -
 2)^(1/2)*16i)/(8*5^(1/2) + 24))*(- 5^(1/2) - 2)^(1/2)*1i)/10
```

Reduce [F]

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{3\sqrt{\sqrt{5}+1}\,\sqrt{10}\,atan\left(\frac{2x}{\sqrt{\sqrt{5}+1}\,\sqrt{2}}\right)}{20} - \frac{\sqrt{\sqrt{5}+1}\,\sqrt{2}\,atan\left(\frac{2x}{\sqrt{\sqrt{5}+1}\,\sqrt{2}}\right)}{4} + \frac{3\sqrt{\sqrt{5}-1}\,\sqrt{10}\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{40} - \frac{3\sqrt{\sqrt{5}-1}\,\sqrt{10}\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{40} + \frac{\sqrt{\sqrt{5}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{8} - \frac{\sqrt{\sqrt{5}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{8} + 2\left(\int \frac{x^2}{x^8-3x^4+1}dx\right) + \int \frac{1}{x^8-3x^4+1}dx$$

input
```
int(x^6/(x^8-3*x^4+1),x)
```

output
```
(6*sqrt(sqrt(5) + 1)*sqrt(10)*atan((2*x)/(sqrt(sqrt(5) + 1)*sqrt(2))) - 10
*sqrt(sqrt(5) + 1)*sqrt(2)*atan((2*x)/(sqrt(sqrt(5) + 1)*sqrt(2))) + 3*sqr
t(sqrt(5) - 1)*sqrt(10)*log( - sqrt(sqrt(5) - 1) + sqrt(2)*x) - 3*sqrt(sqr
t(5) - 1)*sqrt(10)*log(sqrt(sqrt(5) - 1) + sqrt(2)*x) + 5*sqrt(sqrt(5) - 1
)*sqrt(2)*log( - sqrt(sqrt(5) - 1) + sqrt(2)*x) - 5*sqrt(sqrt(5) - 1)*sqrt
(2)*log(sqrt(sqrt(5) - 1) + sqrt(2)*x) + 80*int(x**2/(x**8 - 3*x**4 + 1),x
) + 40*int(1/(x**8 - 3*x**4 + 1),x))/40
```

3.130 $\int \frac{x^4}{1-3x^4+x^8} \, dx$

Optimal result 1053
Mathematica [A] (verified) 1054
Rubi [A] (verified) 1054
Maple [C] (verified) 1056
Fricas [A] (verification not implemented) 1057
Sympy [A] (verification not implemented) 1058
Maxima [F] 1058
Giac [A] (verification not implemented) 1059
Mupad [B] (verification not implemented) 1060
Reduce [F] 1061

Optimal result

Integrand size = 16, antiderivative size = 173

$$\int \frac{x^4}{1-3x^4+x^8} \, dx = -\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)} \arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)} \arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)} \operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)} \operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

output

```
-1/10*(3/2+1/2*5^(1/2))^(1/4)*arctan(2^(1/4)*(1/(3+5^(1/2)))^(1/4)*x)*5^(1
/2)+1/10*(3/2-1/2*5^(1/2))^(1/4)*arctan((3/2+1/2*5^(1/2))^(1/4)*x)*5^(1/2)
-1/10*(3/2+1/2*5^(1/2))^(1/4)*arctanh(2^(1/4)*(1/(3+5^(1/2)))^(1/4)*x)*5^(
1/2)+1/10*(3/2-1/2*5^(1/2))^(1/4)*arctanh((3/2+1/2*5^(1/2))^(1/4)*x)*5^(1/
2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \frac{\sqrt{-1+\sqrt{5}}\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)-\sqrt{1+\sqrt{5}}\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)+\sqrt{-1+\sqrt{5}}\text{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)-\sqrt{}}{2\sqrt{10}}$$

input
```
Integrate[x^4/(1 - 3*x^4 + x^8),x]
```

output
```
(Sqrt[-1 + Sqrt[5]]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x] - Sqrt[1 + Sqrt[5]]*A
rcTan[Sqrt[2/(1 + Sqrt[5])]*x] + Sqrt[-1 + Sqrt[5]]*ArcTanh[Sqrt[2/(-1 + S
qrt[5])]*x] - Sqrt[1 + Sqrt[5]]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[
10])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1710, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^8-3x^4+1}\,dx$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx+\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{\sqrt{3-\sqrt{5}}}\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{\sqrt{3+\sqrt{5}}}\right)$$

$\Big\downarrow$ 216

$$\frac{1}{10}\left(5-3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)+\frac{1}{10}\left(5-3\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)$$

input
```
Int[x^4/(1 - 3*x^4 + x^8),x]
```

output
```
((5 + 3*Sqrt[5])*(-(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 + Sqrt[5
])^(3/4))) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 + Sqrt[5])^(3/
4))))/10 + ((5 - 3*Sqrt[5])*(-(ArcTan[((3 + Sqrt[5])/2)^(1/4)*x]/(2^(1/4)*
(3 - Sqrt[5])^(3/4))) - ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]/(2^(1/4)*(3 - S
qrt[5])^(3/4))))/10
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+5\_Z^2-1\right)}\_R\ln\left(10\_R^3+\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-5\_Z^2-1\right)}\_R\ln\left(-10\_R^3+\_R+x\right)\right.}{4}$
default	$-\frac{\left(\sqrt{5}+1\right)\sqrt{5}\ \mathrm{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}+\frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\sqrt{5}\left(\sqrt{5}-1\right)\mathrm{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{\left(\sqrt{5}+1\right)\sqrt{5}}{10}$

input
```
int(x^4/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(10*_R^3+_R+x),_R=RootOf(25*_Z^4+5*_Z^2-1))+1/4*sum(_R*ln(-10
*_R^3+_R+x),_R=RootOf(25*_Z^4-5*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{x^4}{1-3x^4+x^8}\,dx = {} & \frac{1}{2}\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\arctan\left(\frac{1}{2}\left(\sqrt{5}x-5\,x\right)\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\right)\\
&+\frac{1}{2}\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\arctan\left(\frac{1}{2}\left(\sqrt{5}x+5\,x\right)\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\log\left(x+\sqrt{5}\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\log\left(x-\sqrt{5}\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\log\left(x+\sqrt{5}\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\log\left(x-\sqrt{5}\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\right)\end{aligned}$$

input
```
integrate(x^4/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/10*sqrt(5) + 1/10)*arctan(1/2*(sqrt(5)*x - 5*x)*sqrt(1/10*sqrt(
5) + 1/10)) + 1/2*sqrt(1/10*sqrt(5) - 1/10)*arctan(1/2*(sqrt(5)*x + 5*x)*s
qrt(1/10*sqrt(5) - 1/10)) - 1/4*sqrt(1/10*sqrt(5) + 1/10)*log(x + sqrt(5)*
sqrt(1/10*sqrt(5) + 1/10)) + 1/4*sqrt(1/10*sqrt(5) + 1/10)*log(x - sqrt(5)
*sqrt(1/10*sqrt(5) + 1/10)) + 1/4*sqrt(1/10*sqrt(5) - 1/10)*log(x + sqrt(5
)*sqrt(1/10*sqrt(5) - 1/10)) - 1/4*sqrt(1/10*sqrt(5) - 1/10)*log(x - sqrt(
5)*sqrt(1/10*sqrt(5) - 1/10))
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.28

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \text{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(-51200t^5+12t+x\right)\right)\right) +\text{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(-51200t^5+12t+x\right)\right)\right)$$

input
```
integrate(x**4/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 + 12*_t
+ x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5
 + 12*_t + x)))
```

Maxima [F]

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \int \frac{x^4}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^4/(x^8 - 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = -\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

input
```
integrate(x^4/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/20*sqrt(10*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(
10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5)
+ 10)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*l
og(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*log(abs(
x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) - 10)*log(abs(x - sqr
t(1/2*sqrt(5) - 1/2)))
```

Mupad [B] (verification not implemented)

Time = 19.51 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.55

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{2\left(\sqrt{5}-1\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3\mathrm{i}}{10\left(\sqrt{5}-1\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{20} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{2\left(\sqrt{5}+1\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3\mathrm{i}}{10\left(\sqrt{5}+1\right)}\right)\,\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{2\left(\sqrt{5}-1\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3\mathrm{i}}{10\left(\sqrt{5}-1\right)}\right)\,\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{2\left(\sqrt{5}+1\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3\mathrm{i}}{10\left(\sqrt{5}+1\right)}\right)\,\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{20}$$

input
```
int(x^4/(x^8 - 3*x^4 + 1),x)
```

output
```
(10^(1/2)*atan((10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*1i)/(2*(5^(1/2) - 1)) - (
5^(1/2)*10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(10*(5^(1/2) - 1)))*(- 5^(1/2
) - 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2))^(1/2)*1i)/(
2*(5^(1/2) + 1)) + (5^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*3i)/(10*(5^(1/2
) + 1)))*(1 - 5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2)
+ 1)^(1/2)*1i)/(2*(5^(1/2) - 1)) - (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)
*3i)/(10*(5^(1/2) - 1)))*(5^(1/2) + 1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(
1/2)*x*(5^(1/2) - 1)^(1/2)*1i)/(2*(5^(1/2) + 1)) + (5^(1/2)*10^(1/2)*x*(5^
(1/2) - 1)^(1/2)*3i)/(10*(5^(1/2) + 1)))*(5^(1/2) - 1)^(1/2)*1i)/20
```

Reduce [F]

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \frac{\sqrt{\sqrt{5}+1}\,\sqrt{10}\,atan\left(\frac{2x}{\sqrt{\sqrt{5}+1}\,\sqrt{2}}\right)}{20} - \frac{\sqrt{\sqrt{5}+1}\,\sqrt{2}\,atan\left(\frac{2x}{\sqrt{\sqrt{5}+1}\,\sqrt{2}}\right)}{4} + \frac{\sqrt{\sqrt{5}-1}\,\sqrt{10}\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{40} - \frac{\sqrt{\sqrt{5}-1}\,\sqrt{10}\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{40} + \frac{\sqrt{\sqrt{5}-1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{8} - \frac{\sqrt{\sqrt{5}-1}\,\sqrt{2}\log\left(\sqrt{\sqrt{5}-1}+\sqrt{2}\,x\right)}{8} + \int \frac{x^2}{x^8-3x^4+1}dx + \int \frac{1}{x^8-3x^4+1}dx$$

input
```
int(x^4/(x^8-3*x^4+1),x)
```

output
```
(2*sqrt(sqrt(5) + 1)*sqrt(10)*atan((2*x)/(sqrt(sqrt(5) + 1)*sqrt(2))) - 10
*sqrt(sqrt(5) + 1)*sqrt(2)*atan((2*x)/(sqrt(sqrt(5) + 1)*sqrt(2))) + sqrt(
sqrt(5) - 1)*sqrt(10)*log( - sqrt(sqrt(5) - 1) + sqrt(2)*x) - sqrt(sqrt(5)
 - 1)*sqrt(10)*log(sqrt(sqrt(5) - 1) + sqrt(2)*x) + 5*sqrt(sqrt(5) - 1)*sq
rt(2)*log( - sqrt(sqrt(5) - 1) + sqrt(2)*x) - 5*sqrt(sqrt(5) - 1)*sqrt(2)*
log(sqrt(sqrt(5) - 1) + sqrt(2)*x) + 40*int(x**2/(x**8 - 3*x**4 + 1),x) +
40*int(1/(x**8 - 3*x**4 + 1),x))/40
```

3.131 $\int \frac{x^2}{1-3x^4+x^8}\,dx$

Optimal result . . . 1062
Mathematica [A] (verified) . . . 1063
Rubi [A] (verified) . . . 1063
Maple [C] (verified) . . . 1065
Fricas [A] (verification not implemented) . . . 1066
Sympy [A] (verification not implemented) . . . 1067
Maxima [F] . . . 1067
Giac [A] (verification not implemented) . . . 1068
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Reduce [F] . . . 1069

Optimal result

Integrand size = 16, antiderivative size = 145

$$\begin{aligned}\int \frac{x^2}{1-3x^4+x^8}\,dx &= \frac{1}{20}\sqrt{-10+10\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{-2+2\sqrt{5}}x\right)\\ &\quad-\frac{1}{20}\sqrt{10+10\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}x\right)\\ &\quad-\frac{1}{20}\sqrt{-10+10\sqrt{5}}\,\text{arctanh}\left(\frac{1}{2}\sqrt{-2+2\sqrt{5}}x\right)\\ &\quad+\frac{1}{20}\sqrt{10+10\sqrt{5}}\,\text{arctanh}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}x\right)\end{aligned}$$

output
```
1/20*(-10+10*5^(1/2))^(1/2)*arctan(1/2*(-2+2*5^(1/2))^(1/2)*x)-1/20*(10+10
*5^(1/2))^(1/2)*arctan(1/2*(2+2*5^(1/2))^(1/2)*x)-1/20*(-10+10*5^(1/2))^(1
/2)*arctanh(1/2*(-2+2*5^(1/2))^(1/2)*x)+1/20*(10+10*5^(1/2))^(1/2)*arctanh
(1/2*(2+2*5^(1/2))^(1/2)*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = -\frac{\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

input
```
Integrate[x^2/(1 - 3*x^4 + x^8),x]
```

output
```
-(ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])]) + ArcTan[Sqrt[
2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])
]*x]/Sqrt[10*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1
 + Sqrt[5])]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1711, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^8-3x^4+1}dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{\int -\frac{2x^2}{-2x^4+\sqrt{5}+3}dx}{\sqrt{5}}-\frac{\int -\frac{2x^2}{-2x^4-\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{x^2}{-2x^4-\sqrt{5}+3}dx}{\sqrt{5}}-\frac{2\int \frac{x^2}{-2x^4+\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \ 827$$

$$\frac{2\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{2\sqrt{2}}\right)}{\sqrt{5}}-\frac{2\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{2\sqrt{2}}\right)}{\sqrt{5}}$$

$$\Big\downarrow \ 216$$

$$\frac{2\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}}-\frac{2\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}}$$

$$\Big\downarrow \ 219$$

$$\frac{2\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}}-\frac{2\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}}$$

input
```
Int[x^2/(1 - 3*x^4 + x^8),x]
```

output
```
(-2*(-1/2*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*(3 + Sqrt[5])^(1/4))
+ ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2*2^(3/4)*(3 + Sqrt[5])^(1/4))))/Sqr
t[5] + (2*(-1/2*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x]/(2^(3/4)*(3 - Sqrt[5])^(
1/4)) + ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]/(2*2^(3/4)*(3 - Sqrt[5])^(1/4))
))/Sqrt[5]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(25\_Z^4+5\_Z^2-1\right)} \_R\ln\left(-5\_R^3-3\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\text{RootOf}\left(25\_Z^4-5\_Z^2-1\right)} \_R\ln\left(-5\_R^3+3\_R+x\right.\right.}{4}$
default	$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}-\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}+\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}$

input
```
int(x^2/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(-5*_R^3-3*_R+x),_R=RootOf(25*_Z^4+5*_Z^2-1))+1/4*sum(_R*ln(-
5*_R^3+3*_R+x),_R=RootOf(25*_Z^4-5*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.23

$$\begin{aligned}\int \frac{x^2}{1-3x^4+x^8}\,dx = &-\frac{1}{2}\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\arctan\left(\sqrt{5}x\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\right)\\ &+\frac{1}{2}\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\arctan\left(\sqrt{5}x\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\right)\\ &-\frac{1}{4}\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\log\left(\left(\sqrt{5}-5\right)\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}+2\,x\right)\\ &+\frac{1}{4}\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\log\left(-\left(\sqrt{5}-5\right)\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}+2\,x\right)\\ &-\frac{1}{4}\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\log\left(\left(\sqrt{5}+5\right)\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}+2\,x\right)\\ &+\frac{1}{4}\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\log\left(-\left(\sqrt{5}+5\right)\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}+2\,x\right)\end{aligned}$$

input
```
integrate(x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/10*sqrt(5) + 1/10)*arctan(sqrt(5)*x*sqrt(1/10*sqrt(5) + 1/10))
 + 1/2*sqrt(1/10*sqrt(5) - 1/10)*arctan(sqrt(5)*x*sqrt(1/10*sqrt(5) - 1/10
)) - 1/4*sqrt(1/10*sqrt(5) + 1/10)*log((sqrt(5) - 5)*sqrt(1/10*sqrt(5) + 1
/10) + 2*x) + 1/4*sqrt(1/10*sqrt(5) + 1/10)*log(-(sqrt(5) - 5)*sqrt(1/10*s
qrt(5) + 1/10) + 2*x) - 1/4*sqrt(1/10*sqrt(5) - 1/10)*log((sqrt(5) + 5)*sq
rt(1/10*sqrt(5) - 1/10) + 2*x) + 1/4*sqrt(1/10*sqrt(5) - 1/10)*log(-(sqrt(
5) + 5)*sqrt(1/10*sqrt(5) - 1/10) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.37

$$\int \frac{x^2}{1-3x^4+x^8}\,dx$$
$$= \operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(6144000t^7-2240t^3+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(6144000t^7-2240t^3+x\right)\right)\right)$$

input
```
integrate(x**2/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(6144000*_t**7 - 2240*
_t**3 + x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(614400
0*_t**7 - 2240*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \int \frac{x^2}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(x^2/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^2/(x^8 - 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

input
```
integrate(x^2/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(1
0*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5) -
 10)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*lo
g(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x
 + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x - sqrt
(1/2*sqrt(5) - 1/2)))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.86

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{\sqrt{5}-1}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{\sqrt{5}+1}\,1\mathrm{i}}{20} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{1-\sqrt{5}}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,7\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1\mathrm{i}}{20}$$

input
```
int(x^2/(x^8 - 3*x^4 + 1),x)
```

output
```
(10^(1/2)*atan((10^(1/2)*x*(5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (
5^(1/2)*10^(1/2)*x*(5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) - 7)))*(5^(1/2)
- 1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(2*
(3*5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*7i)/(10*(3*5^(1
/2) + 7)))*(5^(1/2) + 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1/2)*x*(1 - 5^
(1/2))^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(
1/2)*7i)/(10*(3*5^(1/2) - 7)))*(1 - 5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan
((10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) + 7)) + (5^(1/2)*10^(
1/2)*x*(- 5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) + 7)))*(- 5^(1/2) - 1)^(1/
2)*1i)/20
```

Reduce [F]

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \int \frac{x^2}{x^8-3x^4+1}dx$$

input
```
int(x^2/(x^8-3*x^4+1),x)
```

output
```
int(x**2/(x**8 - 3*x**4 + 1),x)
```

3.132 $\int \frac{1}{1-3x^4+x^8}\,dx$

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Mupad [B] (verification not implemented) 1078
Reduce [F] 1078

Optimal result

Integrand size = 12, antiderivative size = 169

$$\int \frac{1}{1-3x^4+x^8}\,dx = -\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\,2^{3/4}\sqrt{5}}$$

output
```
-1/10*arctan(2^(1/4)*(1/(3+5^(1/2)))^(1/4)*x)*2^(3/4)*5^(1/2)/(3+5^(1/2))^
(3/4)+1/20*(3+5^(1/2))^(3/4)*arctan((3/2+1/2*5^(1/2))^(1/4)*x)*2^(1/4)*5^(
1/2)-1/10*arctanh(2^(1/4)*(1/(3+5^(1/2)))^(1/4)*x)*2^(3/4)*5^(1/2)/(3+5^(1
/2))^(3/4)+1/20*(3+5^(1/2))^(3/4)*arctanh((3/2+1/2*5^(1/2))^(1/4)*x)*2^(1/
4)*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{1}{1-3x^4+x^8}\,dx = \frac{\frac{\left(1+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{-1+\sqrt{5}}}-\frac{\left(-1+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}+\frac{\left(1+\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{\sqrt{-1+\sqrt{5}}}-\frac{\left(-1+\sqrt{5}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

input
```
Integrate[(1 - 3*x^4 + x^8)^(-1),x]
```

output
```
(((1 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-
1 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] + ((1 + Sq
rt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-1 + Sqrt
[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1685, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8-3x^4+1}\,dx$$

$$\Big\downarrow \; 1685$$

$$\frac{\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx}{\sqrt{5}}-\frac{\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx}{\sqrt{5}}$$

$$\Big\downarrow \; 756$$

$$\frac{-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{\sqrt{3+\sqrt{5}}}}{\sqrt{5}}-\frac{-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{\sqrt{3-\sqrt{5}}}}{\sqrt{5}}$$

$$\Big\downarrow \text{216}$$

$$\frac{-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}}{\sqrt{5}}-\frac{-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}}{\sqrt{5}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}}{\sqrt{5}}-\frac{-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}}{\sqrt{5}}$$

input
```
Int[(1 - 3*x^4 + x^8)^(-1),x]
```

output
```
(-(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 + Sqrt[5])^(3/4))) - ArcT
anh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 + Sqrt[5])^(3/4)))/Sqrt[5] - (-
(ArcTan[((3 + Sqrt[5])/2)^(1/4)*x]/(2^(1/4)*(3 - Sqrt[5])^(3/4))) - ArcTan
h[((3 + Sqrt[5])/2)^(1/4)*x]/(2^(1/4)*(3 - Sqrt[5])^(3/4)))/Sqrt[5]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^{4}-20\_Z^{2}-1\right)}\_R\ln\left(15\_R^{3}-11\_R+2x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^{4}+20\_Z^{2}-1\right)}\_R\ln\left(-15\_R^{3}-11\_\right.\right.}{4}$
default	$-\frac{\sqrt{5}\left(\sqrt{5}-1\right)\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}+\frac{\left(\sqrt{5}+1\right)\sqrt{5}\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\left(\sqrt{5}+1\right)\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{5}\left(\sqrt{5}-1\right)}{10}$

input
```
int(1/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(15*_R^3-11*_R+2*x),_R=RootOf(25*_Z^4-20*_Z^2-1))+1/4*sum(_R*
ln(-15*_R^3-11*_R+2*x),_R=RootOf(25*_Z^4+20*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.18

$$\int \frac{1}{1-3x^4+x^8}\,dx = -\frac{1}{2}\sqrt{\frac{1}{5}\sqrt{5}+\frac{2}{5}}\arctan\left(\frac{1}{2}\left(\sqrt{5}x-5\,x\right)\sqrt{\frac{1}{5}\sqrt{5}+\frac{2}{5}}\right) - \frac{1}{2}\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}\arctan\left(\frac{1}{2}\left(\sqrt{5}x+5\,x\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}\right) + \frac{1}{4}\sqrt{\frac{1}{5}\sqrt{5}+\frac{2}{5}}\log\left(\left(3\sqrt{5}-5\right)\sqrt{\frac{1}{5}\sqrt{5}+\frac{2}{5}}+2\,x\right) - \frac{1}{4}\sqrt{\frac{1}{5}\sqrt{5}+\frac{2}{5}}\log\left(-\left(3\sqrt{5}-5\right)\sqrt{\frac{1}{5}\sqrt{5}+\frac{2}{5}}+2\,x\right) - \frac{1}{4}\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}\log\left(\left(3\sqrt{5}+5\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}+2\,x\right) + \frac{1}{4}\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}\log\left(-\left(3\sqrt{5}+5\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}+2\,x\right)$$

input
```
integrate(1/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/5*sqrt(5) + 2/5)*arctan(1/2*(sqrt(5)*x - 5*x)*sqrt(1/5*sqrt(5)
 + 2/5)) - 1/2*sqrt(1/5*sqrt(5) - 2/5)*arctan(1/2*(sqrt(5)*x + 5*x)*sqrt(1
/5*sqrt(5) - 2/5)) + 1/4*sqrt(1/5*sqrt(5) + 2/5)*log((3*sqrt(5) - 5)*sqrt(
1/5*sqrt(5) + 2/5) + 2*x) - 1/4*sqrt(1/5*sqrt(5) + 2/5)*log(-(3*sqrt(5) -
5)*sqrt(1/5*sqrt(5) + 2/5) + 2*x) - 1/4*sqrt(1/5*sqrt(5) - 2/5)*log((3*sqr
t(5) + 5)*sqrt(1/5*sqrt(5) - 2/5) + 2*x) + 1/4*sqrt(1/5*sqrt(5) - 2/5)*log
(-(3*sqrt(5) + 5)*sqrt(1/5*sqrt(5) - 2/5) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.31

$$\int \frac{1}{1-3x^4+x^8}\,dx = \operatorname{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(9600t^5-\frac{47t}{2}+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\left(9600t^5-\frac{47t}{2}+x\right)\right)\right)$$

input
```
integrate(1/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5 - 47*_t/2
 + x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5
 - 47*_t/2 + x)))
```

Maxima [F]

$$\int \frac{1}{1-3x^4+x^8}\,dx = \int \frac{1}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(1/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^8 - 3*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.87

$$\int \frac{1}{1-3x^4+x^8}\,dx = -\frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

input
```
integrate(1/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/10*sqrt(5*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/10*sqrt(5
*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) - 1
0)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(a
bs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x + s
qrt(1/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x - sqrt(1/2*
sqrt(5) - 1/2)))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.45

$$\int \frac{1}{1-3x^4+x^8}\,dx = -\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{2-\sqrt{5}}\,144\mathrm{i}}{104\,\sqrt{5}-232}-\frac{\sqrt{5}\,x\,\sqrt{2-\sqrt{5}}\,64\mathrm{i}}{104\,\sqrt{5}-232}\right)\,\sqrt{2-\sqrt{5}}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{-\sqrt{5}-2}\,144\mathrm{i}}{104\,\sqrt{5}+232}+\frac{\sqrt{5}\,x\,\sqrt{-\sqrt{5}-2}\,64\mathrm{i}}{104\,\sqrt{5}+232}\right)\,\sqrt{-\sqrt{5}-2}\,1\mathrm{i}}{10} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{\sqrt{5}-2}\,144\mathrm{i}}{104\,\sqrt{5}-232}-\frac{\sqrt{5}\,x\,\sqrt{\sqrt{5}-2}\,64\mathrm{i}}{104\,\sqrt{5}-232}\right)\,\sqrt{\sqrt{5}-2}\,1\mathrm{i}}{10} - \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,\sqrt{\sqrt{5}+2}\,144\mathrm{i}}{104\,\sqrt{5}+232}+\frac{\sqrt{5}\,x\,\sqrt{\sqrt{5}+2}\,64\mathrm{i}}{104\,\sqrt{5}+232}\right)\,\sqrt{\sqrt{5}+2}\,1\mathrm{i}}{10}$$

input
```
int(1/(x^8 - 3*x^4 + 1),x)
```

output
```
(5^(1/2)*atan((x*(- 5^(1/2) - 2)^(1/2)*144i)/(104*5^(1/2) + 232) + (5^(1/2
)*x*(- 5^(1/2) - 2)^(1/2)*64i)/(104*5^(1/2) + 232))*(- 5^(1/2) - 2)^(1/2)*
1i)/10 - (5^(1/2)*atan((x*(2 - 5^(1/2))^(1/2)*144i)/(104*5^(1/2) - 232) -
(5^(1/2)*x*(2 - 5^(1/2))^(1/2)*64i)/(104*5^(1/2) - 232))*(2 - 5^(1/2))^(1/
2)*1i)/10 + (5^(1/2)*atan((x*(5^(1/2) - 2)^(1/2)*144i)/(104*5^(1/2) - 232)
 - (5^(1/2)*x*(5^(1/2) - 2)^(1/2)*64i)/(104*5^(1/2) - 232))*(5^(1/2) - 2)^
(1/2)*1i)/10 - (5^(1/2)*atan((x*(5^(1/2) + 2)^(1/2)*144i)/(104*5^(1/2) + 2
32) + (5^(1/2)*x*(5^(1/2) + 2)^(1/2)*64i)/(104*5^(1/2) + 232))*(5^(1/2) +
2)^(1/2)*1i)/10
```

Reduce [F]

$$\int \frac{1}{1-3x^4+x^8}\,dx = \int \frac{1}{x^8-3x^4+1}dx$$

input
```
int(1/(x^8-3*x^4+1),x)
```

output
```
int(1/(x**8 - 3*x**4 + 1),x)
```

3.133 $\int \frac{1}{x^2(1-3x^4+x^8)}\,dx$

Optimal result 1079
Mathematica [A] (verified) 1080
Rubi [A] (verified) 1080
Maple [C] (verified) 1083
Fricas [A] (verification not implemented) 1083
Sympy [A] (verification not implemented) 1084
Maxima [F] 1084
Giac [A] (verification not implemented) 1085
Mupad [B] (verification not implemented) 1086
Reduce [F] 1087

Optimal result

Integrand size = 16, antiderivative size = 180

$$\int \frac{1}{x^2(1-3x^4+x^8)}\,dx = -\frac{1}{x}+\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} - \frac{\left(3+\sqrt{5}\right)^{5/4}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\left(3+\sqrt{5}\right)^{5/4}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\sqrt[4]{2}\sqrt{5}}$$

output
```
-1/x+1/10*(123/2-55/2*5^(1/2))^(1/4)*arctan(2^(1/4)*(1/(3+5^(1/2)))^(1/4)*
x)*5^(1/2)-1/40*(3+5^(1/2))^(5/4)*arctan((3/2+1/2*5^(1/2))^(1/4)*x)*2^(3/4
)*5^(1/2)-1/10*(123/2-55/2*5^(1/2))^(1/4)*arctanh(2^(1/4)*(1/(3+5^(1/2)))^
(1/4)*x)*5^(1/2)+1/40*(3+5^(1/2))^(5/4)*arctanh((3/2+1/2*5^(1/2))^(1/4)*x)
*2^(3/4)*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2(1-3x^4+x^8)}\,dx = -\frac{1}{x} - \frac{(3+\sqrt{5})\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10(-1+\sqrt{5})}} - \frac{(-3+\sqrt{5})\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10(1+\sqrt{5})}} + \frac{(3+\sqrt{5})\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10(-1+\sqrt{5})}} + \frac{(-3+\sqrt{5})\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10(1+\sqrt{5})}}$$

input
```
Integrate[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

output
```
-x^(-1) - ((3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[10*(-1
+ Sqrt[5])]) - ((-3 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10
*(1 + Sqrt[5])]) + ((3 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sq
rt[10*(-1 + Sqrt[5])]) + ((-3 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])
/(2*Sqrt[10*(1 + Sqrt[5])])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1704, 1834, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(x^8-3x^4+1)}\,dx$$

$$\Big\downarrow \text{1704}$$

$$\int \frac{x^2\left(3-x^4\right)}{x^8-3x^4+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{1834}$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\int -\frac{2x^2}{-2x^4-\sqrt{5}+3}dx-\frac{1}{10}\left(5-3\sqrt{5}\right)\int -\frac{2x^2}{-2x^4+\sqrt{5}+3}dx-\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-2x^4-\sqrt{5}+3}dx+\frac{1}{5}\left(5-3\sqrt{5}\right)\int \frac{x^2}{-2x^4+\sqrt{5}+3}dx-\frac{1}{x}$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{2\sqrt{2}}\right)+\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{2\sqrt{2}}\right)-\frac{1}{x}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)+\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)-\frac{1}{x}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{5}\left(5-3\sqrt{5}\right)\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)+\frac{1}{5}\left(5+3\sqrt{5}\right)\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)-\frac{1}{x}$$

input
```
Int[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

output
```
-x^(-1) + ((5 - 3*Sqrt[5])*(-1/2*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4
)*(3 + Sqrt[5])^(1/4)) + ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2*2^(3/4)*(3
+ Sqrt[5])^(1/4))))/5 + ((5 + 3*Sqrt[5])*(-1/2*ArcTan[((3 + Sqrt[5])/2)^(1
/4)*x]/(2^(3/4)*(3 - Sqrt[5])^(1/4)) + ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]/
(2*2^(3/4)*(3 - Sqrt[5])^(1/4))))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.41

method	result
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+55\_Z^2-1\right)}\_R\ln\left(-20\_R^3-47\_R+5x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4-55\_Z^2-1\right)}\_R\ln\left(-20\_\right.\right.}{4}$
default	$-\frac{1}{x}+\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{\left(\sqrt{5}-\right.}{}$

input
```
int(1/x^2/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/4*sum(_R*ln(-20*_R^3-47*_R+5*x),_R=RootOf(25*_Z^4+55*_Z^2-1))+1/4*s
um(_R*ln(-20*_R^3+47*_R+5*x),_R=RootOf(25*_Z^4-55*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^2\left(1-3x^4+x^8\right)}\,dx =$$

$$-\frac{2\,x\sqrt{\frac{1}{2}\,\sqrt{5}+\frac{11}{10}}\arctan\left(\frac{1}{2}\left(3\,\sqrt{5}x-5\,x\right)\sqrt{\frac{1}{2}\,\sqrt{5}+\frac{11}{10}}\right)-2\,x\sqrt{\frac{1}{2}\,\sqrt{5}-\frac{11}{10}}\arctan\left(\frac{1}{2}\left(3\,\sqrt{5}x+5\,x\right)\sqrt{\frac{1}{2}\right.\right.}{}$$

input
```
integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/4*(2*x*sqrt(1/2*sqrt(5) + 11/10)*arctan(1/2*(3*sqrt(5)*x - 5*x)*sqrt(1/
2*sqrt(5) + 11/10)) - 2*x*sqrt(1/2*sqrt(5) - 11/10)*arctan(1/2*(3*sqrt(5)*
x + 5*x)*sqrt(1/2*sqrt(5) - 11/10)) + x*sqrt(1/2*sqrt(5) + 11/10)*log((2*s
qrt(5) - 5)*sqrt(1/2*sqrt(5) + 11/10) + x) - x*sqrt(1/2*sqrt(5) + 11/10)*l
og(-(2*sqrt(5) - 5)*sqrt(1/2*sqrt(5) + 11/10) + x) + x*sqrt(1/2*sqrt(5) -
11/10)*log((2*sqrt(5) + 5)*sqrt(1/2*sqrt(5) - 11/10) + x) - x*sqrt(1/2*sqr
t(5) - 11/10)*log(-(2*sqrt(5) + 5)*sqrt(1/2*sqrt(5) - 11/10) + x) + 4)/x
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^2(1-3x^4+x^8)}\,dx = \text{RootSum}\left(6400t^4-880t^2-1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right) + \text{RootSum}\left(6400t^4+880t^2-1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right)-\frac{1}{x}$$

input
```
integrate(1/x**2/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(19251200*_t**7/11 -
369792*_t**3/11 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t
*log(19251200*_t**7/11 - 369792*_t**3/11 + x))) - 1/x
```

Maxima [F]

$$\int \frac{1}{x^2(1-3x^4+x^8)}\,dx = \int \frac{1}{(x^8-3\,x^4+1)x^2}\,dx$$

input
```
integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-1/x - 1/2*integrate((x^2 + 2)/(x^4 + x^2 - 1), x) - 1/2*integrate((x^2 -
2)/(x^4 - x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2(1-3x^4+x^8)}\,dx = \frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{x}$$

input
```
integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/20*sqrt(50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(
50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5)
 - 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110
)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 110)*log(
abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) + 110)*log(abs(x
- sqrt(1/2*sqrt(5) - 1/2))) - 1/x
```

Mupad [B] (verification not implemented)

Time = 19.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^2 (1-3x^4+x^8)} \, dx$$
$$= -\frac{1}{x} - \frac{\operatorname{atan}\left(\frac{x\sqrt{-50\sqrt{5}-110}\,1155\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)} + \frac{\sqrt{5}\,x\sqrt{-50\sqrt{5}-110}\,517\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}\right)\sqrt{-50\sqrt{5}-110}\,1\mathrm{i}}{20}$$
$$+ \frac{\operatorname{atan}\left(\frac{x\sqrt{110-50\sqrt{5}}\,1155\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)} - \frac{\sqrt{5}\,x\sqrt{110-50\sqrt{5}}\,517\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}\right)\sqrt{110-50\sqrt{5}}\,1\mathrm{i}}{20}$$
$$+ \frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}-110}\,1155\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)} - \frac{\sqrt{5}\,x\sqrt{50\sqrt{5}-110}\,517\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}\right)\sqrt{50\sqrt{5}-110}\,1\mathrm{i}}{20}$$
$$- \frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}+110}\,1155\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)} + \frac{\sqrt{5}\,x\sqrt{50\sqrt{5}+110}\,517\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}\right)\sqrt{50\sqrt{5}+110}\,1\mathrm{i}}{20}$$

input
```
int(1/(x^2*(x^8 - 3*x^4 + 1)),x)
```

output
```
(atan((x*(110 - 50*5^(1/2))^(1/2)*1155i)/(2*(3025*5^(1/2) - 6765)) - (5^(1
/2)*x*(110 - 50*5^(1/2))^(1/2)*517i)/(2*(3025*5^(1/2) - 6765)))*(110 - 50*
5^(1/2))^(1/2)*1i)/20 - (atan((x*(- 50*5^(1/2) - 110)^(1/2)*1155i)/(2*(302
5*5^(1/2) + 6765)) + (5^(1/2)*x*(- 50*5^(1/2) - 110)^(1/2)*517i)/(2*(3025*
5^(1/2) + 6765)))*(- 50*5^(1/2) - 110)^(1/2)*1i)/20 + (atan((x*(50*5^(1/2)
 - 110)^(1/2)*1155i)/(2*(3025*5^(1/2) - 6765)) - (5^(1/2)*x*(50*5^(1/2) -
110)^(1/2)*517i)/(2*(3025*5^(1/2) - 6765)))*(50*5^(1/2) - 110)^(1/2)*1i)/2
0 - (atan((x*(50*5^(1/2) + 110)^(1/2)*1155i)/(2*(3025*5^(1/2) + 6765)) + (
5^(1/2)*x*(50*5^(1/2) + 110)^(1/2)*517i)/(2*(3025*5^(1/2) + 6765)))*(50*5^
(1/2) + 110)^(1/2)*1i)/20 - 1/x
```

Reduce [F]

$$\int \frac{1}{x^2 (1 - 3x^4 + x^8)} \, dx = \int \frac{1}{x^{10} - 3x^6 + x^2} dx$$

input
```
int(1/x^2/(x^8-3*x^4+1),x)
```

output
```
int(1/(x**10 - 3*x**6 + x**2),x)
```

3.134 $\int \frac{1}{x^4(1-3x^4+x^8)}\,dx$

Optimal result 1088
Mathematica [A] (verified) 1089
Rubi [A] (verified) 1089
Maple [C] (verified) 1092
Fricas [A] (verification not implemented) 1092
Sympy [A] (verification not implemented) 1093
Maxima [F] 1093
Giac [A] (verification not implemented) 1094
Mupad [B] (verification not implemented) 1095
Reduce [F] 1096

Optimal result

Integrand size = 16, antiderivative size = 182

$$\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{4\,2^{3/4}\sqrt{5}}$$

output
```
-1/3/x^3-1/10*(843/2-377/2*5^(1/2))^(1/4)*arctan(2^(1/4)*(1/(3+5^(1/2)))^(
1/4)*x)*5^(1/2)+1/40*(3+5^(1/2))^(7/4)*arctan((3/2+1/2*5^(1/2))^(1/4)*x)*2
^(1/4)*5^(1/2)-1/10*(843/2-377/2*5^(1/2))^(1/4)*arctanh(2^(1/4)*(1/(3+5^(1
/2)))^(1/4)*x)*5^(1/2)+1/40*(3+5^(1/2))^(7/4)*arctanh((3/2+1/2*5^(1/2))^(1
/4)*x)*2^(1/4)*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^4 (1 - 3x^4 + x^8)} \, dx = -\frac{1}{3x^3} + \frac{(2+\sqrt{5}) \arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}} x\right)}{\sqrt{10(-1+\sqrt{5})}} - \frac{(-2+\sqrt{5}) \arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}} x\right)}{\sqrt{10(1+\sqrt{5})}} + \frac{(2+\sqrt{5}) \operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}} x\right)}{\sqrt{10(-1+\sqrt{5})}} - \frac{(-2+\sqrt{5}) \operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}} x\right)}{\sqrt{10(1+\sqrt{5})}}$$

input
```
Integrate[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 + ((2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1
+ Sqrt[5])] - ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1
+ Sqrt[5])] + ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-
1 + Sqrt[5])] - ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*
(1 + Sqrt[5])]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1704, 27, 1752, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^8 - 3x^4 + 1)} \, dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{3}\int \frac{3(3-x^4)}{x^8-3x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{3-x^4}{x^8-3x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1752}$$

$$-\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{756}$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(5-3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{\sqrt{3+\sqrt{5}}}\right)-\frac{1}{3x^3}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)-\frac{1}{10}\left(5-3\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)-\frac{1}{3x^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{10}\left(5-3\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)-\frac{1}{10}\left(5+3\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)-\frac{1}{3x^3}$$

input
```
Int[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 - ((5 - 3*Sqrt[5])*(-(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4
)*(3 + Sqrt[5])^(3/4))) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*(3 +
 Sqrt[5])^(3/4))))/10 - ((5 + 3*Sqrt[5])*(-(ArcTan[((3 + Sqrt[5])/2)^(1/4)
*x]/(2^(1/4)*(3 - Sqrt[5])^(3/4))) - ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]/(2
^(1/4)*(3 - Sqrt[5])^(3/4))))/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.40

method	result
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+145\_Z^2-1\right)}\_R\ln\left(-35\_R^3-199\_R+13x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4-145\_Z^2-1\right)}\_R\ln\right.}{4}$
default	$-\frac{1}{3x^3}-\frac{\left(-2+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}+\frac{\left(2+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}+\frac{\left(2+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}-$

input
```
int(1/x^4/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+1/4*sum(_R*ln(-35*_R^3-199*_R+13*x),_R=RootOf(25*_Z^4+145*_Z^2-1)
)+1/4*sum(_R*ln(35*_R^3-199*_R+13*x),_R=RootOf(25*_Z^4-145*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx =$$
$$-\frac{6\,x^3\sqrt{\frac{13}{10}\sqrt{5}+\frac{29}{10}}\arctan\left(\left(2\sqrt{5}x-5\,x\right)\sqrt{\frac{13}{10}\sqrt{5}+\frac{29}{10}}\right)+6\,x^3\sqrt{\frac{13}{10}\sqrt{5}-\frac{29}{10}}\arctan\left(\left(2\sqrt{5}x+5\,x\right)\sqrt{\right.}}{}$$

input
```
integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/12*(6*x^3*sqrt(13/10*sqrt(5) + 29/10)*arctan((2*sqrt(5)*x - 5*x)*sqrt(1
3/10*sqrt(5) + 29/10)) + 6*x^3*sqrt(13/10*sqrt(5) - 29/10)*arctan((2*sqrt(
5)*x + 5*x)*sqrt(13/10*sqrt(5) - 29/10)) - 3*x^3*sqrt(13/10*sqrt(5) + 29/1
0)*log((7*sqrt(5) - 15)*sqrt(13/10*sqrt(5) + 29/10) + 2*x) + 3*x^3*sqrt(13
/10*sqrt(5) + 29/10)*log(-(7*sqrt(5) - 15)*sqrt(13/10*sqrt(5) + 29/10) + 2
*x) + 3*x^3*sqrt(13/10*sqrt(5) - 29/10)*log((7*sqrt(5) + 15)*sqrt(13/10*sq
rt(5) - 29/10) + 2*x) - 3*x^3*sqrt(13/10*sqrt(5) - 29/10)*log(-(7*sqrt(5)
+ 15)*sqrt(13/10*sqrt(5) - 29/10) + 2*x) + 4)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^4(1-3x^4+x^8)}\,dx = \operatorname{RootSum}\left(6400t^4-2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right) - \frac{1}{3x^3}$$

input
```
integrate(1/x**4/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 2320*_t**2 - 1, Lambda(_t, _t*log(179200*_t**5/377 -
23112*_t/377 + x))) + RootSum(6400*_t**4 + 2320*_t**2 - 1, Lambda(_t, _t*l
og(179200*_t**5/377 - 23112*_t/377 + x))) - 1/(3*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4(1-3x^4+x^8)}\,dx = \int \frac{1}{(x^8-3\,x^4+1)x^4}\,dx$$

input
```
integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-1/3/x^3 - 1/2*integrate((2*x^2 + 3)/(x^4 + x^2 - 1), x) + 1/2*integrate((
2*x^2 - 3)/(x^4 - x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4(1-3x^4+x^8)}\,dx = -\frac{1}{20}\sqrt{130\sqrt{5}-290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{20}\sqrt{130\sqrt{5}+290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{130\sqrt{5}-290}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{130\sqrt{5}-290}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{130\sqrt{5}+290}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{130\sqrt{5}+290}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{3x^3}$$

input
```
integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/20*sqrt(130*sqrt(5) - 290)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqr
t(130*sqrt(5) + 290)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(130*sqr
t(5) - 290)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5)
- 290)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5) + 290
)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(130*sqrt(5) + 290)*log
(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/3/x^3
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^4 (1-3x^4+x^8)}\,dx = \frac{\operatorname{atan}\left(\frac{x\,\sqrt{-130\,\sqrt{5}-290}\,20735\mathrm{i}}{2\,\left(87841\,\sqrt{5}+196417\right)}+\frac{\sqrt{5}\,x\,\sqrt{-130\,\sqrt{5}-290}\,46371\mathrm{i}}{10\,\left(87841\,\sqrt{5}+196417\right)}\right)\,\sqrt{-130\,\sqrt{5}-290}\,1\mathrm{i}}{20} + \frac{\operatorname{atan}\left(\frac{x\,\sqrt{290-130\,\sqrt{5}}\,20735\mathrm{i}}{2\,\left(87841\,\sqrt{5}-196417\right)}-\frac{\sqrt{5}\,x\,\sqrt{290-130\,\sqrt{5}}\,46371\mathrm{i}}{10\,\left(87841\,\sqrt{5}-196417\right)}\right)\,\sqrt{290-130\,\sqrt{5}}\,1\mathrm{i}}{20} - \frac{1}{3\,x^3} - \frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{13\,\sqrt{5}-29}\,20735\mathrm{i}}{2\,\left(87841\,\sqrt{5}-196417\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{13\,\sqrt{5}-29}\,46371\mathrm{i}}{10\,\left(87841\,\sqrt{5}-196417\right)}\right)\,\sqrt{13\,\sqrt{5}-29}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\operatorname{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{13\,\sqrt{5}+29}\,20735\mathrm{i}}{2\,\left(87841\,\sqrt{5}+196417\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{13\,\sqrt{5}+29}\,46371\mathrm{i}}{10\,\left(87841\,\sqrt{5}+196417\right)}\right)\,\sqrt{13\,\sqrt{5}+29}\,1\mathrm{i}}{20}$$

input
```
int(1/(x^4*(x^8 - 3*x^4 + 1)),x)
```

output
```
(atan((x*(- 130*5^(1/2) - 290)^(1/2)*20735i)/(2*(87841*5^(1/2) + 196417))
+ (5^(1/2)*x*(- 130*5^(1/2) - 290)^(1/2)*46371i)/(10*(87841*5^(1/2) + 1964
17)))*(- 130*5^(1/2) - 290)^(1/2)*1i)/20 + (atan((x*(290 - 130*5^(1/2))^(1
/2)*20735i)/(2*(87841*5^(1/2) - 196417)) - (5^(1/2)*x*(290 - 130*5^(1/2))^
(1/2)*46371i)/(10*(87841*5^(1/2) - 196417)))*(290 - 130*5^(1/2))^(1/2)*1i)
/20 - 1/(3*x^3) - (10^(1/2)*atan((10^(1/2)*x*(13*5^(1/2) - 29)^(1/2)*20735
i)/(2*(87841*5^(1/2) - 196417)) - (5^(1/2)*10^(1/2)*x*(13*5^(1/2) - 29)^(1
/2)*46371i)/(10*(87841*5^(1/2) - 196417)))*(13*5^(1/2) - 29)^(1/2)*1i)/20
- (10^(1/2)*atan((10^(1/2)*x*(13*5^(1/2) + 29)^(1/2)*20735i)/(2*(87841*5^(
1/2) + 196417)) + (5^(1/2)*10^(1/2)*x*(13*5^(1/2) + 29)^(1/2)*46371i)/(10*
(87841*5^(1/2) + 196417)))*(13*5^(1/2) + 29)^(1/2)*1i)/20
```

Reduce [F]

$$\int \frac{1}{x^4(1-3x^4+x^8)}\,dx = \int \frac{1}{x^{12}-3x^8+x^4}dx$$

input
```
int(1/x^4/(x^8-3*x^4+1),x)
```

output
```
int(1/(x**12 - 3*x**8 + x**4),x)
```

3.135 $\int \frac{1}{x^6(1-3x^4+x^8)}\,dx$

Optimal result . . . 1097
Mathematica [A] (verified) . . . 1098
Rubi [A] (verified) . . . 1098
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Maxima [F] . . . 1104
Giac [A] (verification not implemented) . . . 1104
Mupad [B] (verification not implemented) . . . 1105
Reduce [F] . . . 1106

Optimal result

Integrand size = 16, antiderivative size = 173

$$\int \frac{1}{x^6(1-3x^4+x^8)}\,dx = -\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{2889-1292\sqrt{5}}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{2889+1292\sqrt{5}}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{2889-1292\sqrt{5}}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{2889+1292\sqrt{5}}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

output
```
-1/5/x^5-3/x+1/10*(2889-1292*5^(1/2))^(1/4)*arctan(2^(1/4)*(1/(3+5^(1/2)))
^(1/4)*x)*5^(1/2)-1/10*(2889+1292*5^(1/2))^(1/4)*arctan((3/2+1/2*5^(1/2))^
(1/4)*x)*5^(1/2)-1/10*(2889-1292*5^(1/2))^(1/4)*arctanh(2^(1/4)*(1/(3+5^(1
/2)))^(1/4)*x)*5^(1/2)+1/10*(2889+1292*5^(1/2))^(1/4)*arctanh((3/2+1/2*5^(
1/2))^(1/4)*x)*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^6(1-3x^4+x^8)}\,dx = -\frac{1}{5x^5}-\frac{3}{x}+\frac{\left(-7-3\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}} + \frac{\left(7-3\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}} - \frac{\left(-7-3\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}} - \frac{\left(7-3\sqrt{5}\right)\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

input
```
Integrate[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

output
```
-1/5*1/x^5 - 3/x + ((-7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*
Sqrt[10*(-1 + Sqrt[5])]) + ((7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x
])/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqr
t[5])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(
1 + Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1704, 27, 1828, 25, 1834, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6(x^8-3x^4+1)}\,dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{5}\int \frac{5(3-x^4)}{x^2\left(x^8-3x^4+1\right)}dx-\frac{1}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{3-x^4}{x^2\left(x^8-3x^4+1\right)}dx-\frac{1}{5x^5}$$

$$\Big\downarrow \text{1828}$$

$$-\int -\frac{x^2(8-3x^4)}{x^8-3x^4+1}dx-\frac{1}{5x^5}-\frac{3}{x}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{x^2(8-3x^4)}{x^8-3x^4+1}dx-\frac{1}{5x^5}-\frac{3}{x}$$

$$\Big\downarrow \text{1834}$$

$$-\frac{1}{10}\left(15+7\sqrt{5}\right)\int -\frac{2x^2}{-2x^4-\sqrt{5}+3}dx-\frac{1}{10}\left(15-7\sqrt{5}\right)\int -\frac{2x^2}{-2x^4+\sqrt{5}+3}dx-\frac{1}{5x^5}-\frac{3}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(15+7\sqrt{5}\right)\int \frac{x^2}{-2x^4-\sqrt{5}+3}dx+\frac{1}{5}\left(15-7\sqrt{5}\right)\int \frac{x^2}{-2x^4+\sqrt{5}+3}dx-\frac{1}{5x^5}-\frac{3}{x}$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{5}\left(15+7\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{2\sqrt{2}}\right)+\frac{1}{5}\left(15-7\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{2\sqrt{2}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{2\sqrt{2}}\right)-\frac{1}{5x^5}-\frac{3}{x}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{5}\left(15+7\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3-\sqrt{5}-\sqrt{2}x^2}}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)+\frac{1}{5}\left(15-7\sqrt{5}\right)\left(\frac{\int \frac{1}{\sqrt{3+\sqrt{5}-\sqrt{2}x^2}}dx}{2\sqrt{2}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)-\frac{1}{5x^5}-\frac{3}{x}$$

$\Big\downarrow$ 219

$$\frac{1}{5}\left(15-7\sqrt{5}\right)\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\ 2^{3/4}\sqrt[4]{3+\sqrt{5}}}\right)+\frac{1}{5}\left(15+7\sqrt{5}\right)\left(\frac{\text{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\ 2^{3/4}\sqrt[4]{3-\sqrt{5}}}\right)-\frac{1}{5x^5}-\frac{3}{x}$$

input
```
Int[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

output
```
-1/5*1/x^5 - 3/x + ((15 - 7*Sqrt[5])*(-1/2*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*
x]/(2^(3/4)*(3 + Sqrt[5])^(1/4)) + ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2*2
^(3/4)*(3 + Sqrt[5])^(1/4))))/5 + ((15 + 7*Sqrt[5])*(-1/2*ArcTan[((3 + Sqr
t[5])/2)^(1/4)*x]/(2^(3/4)*(3 - Sqrt[5])^(1/4)) + ArcTanh[((3 + Sqrt[5])/2
)^(1/4)*x]/(2*2^(3/4)*(3 - Sqrt[5])^(1/4))))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.46

method	result
risch	$\frac{-3x^4-\frac{1}{5}}{x^5}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4-380\_Z^2-1\right)}\_R\ln\left(-55\_R^3+843\_R+34x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+380\_Z^2-1\right)}\_R\right.}{4}$
default	$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\ \operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{\sqrt{5}\left(7+3\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\sqrt{5}\left(7+3\sqrt{5}\right)\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right.}{10\sqrt{-2+2\sqrt{5}}}$

input
```
int(1/x^6/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-3*x^4-1/5)/x^5+1/4*sum(_R*ln(-55*_R^3+843*_R+34*x),_R=RootOf(25*_Z^4-380
*_Z^2-1))+1/4*sum(_R*ln(-55*_R^3-843*_R+34*x),_R=RootOf(25*_Z^4+380*_Z^2-1
))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^6\left(1-3x^4+x^8\right)}\,dx =$$
$$-\frac{10\,x^5\sqrt{\frac{17}{5}\sqrt{5}+\frac{38}{5}}\arctan\left(\frac{1}{2}\left(7\sqrt{5}x-15\,x\right)\sqrt{\frac{17}{5}\sqrt{5}+\frac{38}{5}}\right)-10\,x^5\sqrt{\frac{17}{5}\sqrt{5}-\frac{38}{5}}\arctan\left(\frac{1}{2}\left(7\sqrt{5}x+\right.\right.}{}$$

input
```
integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/20*(10*x^5*sqrt(17/5*sqrt(5) + 38/5)*arctan(1/2*(7*sqrt(5)*x - 15*x)*sq
rt(17/5*sqrt(5) + 38/5)) - 10*x^5*sqrt(17/5*sqrt(5) - 38/5)*arctan(1/2*(7*
sqrt(5)*x + 15*x)*sqrt(17/5*sqrt(5) - 38/5)) + 5*x^5*sqrt(17/5*sqrt(5) + 3
8/5)*log((11*sqrt(5) - 25)*sqrt(17/5*sqrt(5) + 38/5) + 2*x) - 5*x^5*sqrt(1
7/5*sqrt(5) + 38/5)*log(-(11*sqrt(5) - 25)*sqrt(17/5*sqrt(5) + 38/5) + 2*x
) + 5*x^5*sqrt(17/5*sqrt(5) - 38/5)*log((11*sqrt(5) + 25)*sqrt(17/5*sqrt(5
) - 38/5) + 2*x) - 5*x^5*sqrt(17/5*sqrt(5) - 38/5)*log(-(11*sqrt(5) + 25)*
sqrt(17/5*sqrt(5) - 38/5) + 2*x) + 60*x^4 + 4)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^6 (1 - 3x^4 + x^8)} dx = \operatorname{RootSum}\left(6400t^4 - 6080t^2 - 1, \left(t \mapsto t \log\left(\frac{215808000t^7}{323} - \frac{194833880t^3}{323} + x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4 + 6080t^2 - 1, \left(t \mapsto t \log\left(\frac{215808000t^7}{323} - \frac{194833880t^3}{323} + x\right)\right)\right) + \frac{-15x^4 - 1}{5x^5}$$

input
```
integrate(1/x**6/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 6080*_t**2 - 1, Lambda(_t, _t*log(215808000*_t**7/323
 - 194833880*_t**3/323 + x))) + RootSum(6400*_t**4 + 6080*_t**2 - 1, Lambd
a(_t, _t*log(215808000*_t**7/323 - 194833880*_t**3/323 + x))) + (-15*x**4
- 1)/(5*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (1-3x^4+x^8)} \, dx = \int \frac{1}{(x^8-3\,x^4+1)x^6} \, dx$$

input
```
integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-1/5*(15*x^4 + 1)/x^5 - 1/2*integrate((3*x^2 + 5)/(x^4 + x^2 - 1), x) - 1/
2*integrate((3*x^2 - 5)/(x^4 - x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{1}{x^6 (1-3x^4+x^8)} \, dx = {} & \frac{1}{10}\sqrt{85\sqrt{5}-190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) \\ & - \frac{1}{10}\sqrt{85\sqrt{5}+190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) \\ & - \frac{1}{20}\sqrt{85\sqrt{5}-190}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) \\ & + \frac{1}{20}\sqrt{85\sqrt{5}-190}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) \\ & + \frac{1}{20}\sqrt{85\sqrt{5}+190}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) \\ & - \frac{1}{20}\sqrt{85\sqrt{5}+190}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{15\,x^4+1}{5\,x^5}\end{aligned}$$

input
```
integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
1/10*sqrt(85*sqrt(5) - 190)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(
85*sqrt(5) + 190)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(85*sqrt(5)
 - 190)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) - 190
)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) + 190)*log(
abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/20*sqrt(85*sqrt(5) + 190)*log(abs(x
- sqrt(1/2*sqrt(5) - 1/2))) - 1/5*(15*x^4 + 1)/x^5
```

Mupad [B] (verification not implemented)

Time = 19.36 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^6(1-3x^4+x^8)}\,dx = -\frac{3\,x^4+\frac{1}{5}}{x^5} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-85\,\sqrt{5}-190}\,372096\mathrm{i}}{2550408\,\sqrt{5}+5702888} + \frac{\sqrt{5}\,x\,\sqrt{-85\,\sqrt{5}-190}\,832048\mathrm{i}}{5\,\left(2550408\,\sqrt{5}+5702888\right)}\right)\,\sqrt{-85\,\sqrt{5}-190}\,1\mathrm{i}}{10} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{190-85\,\sqrt{5}}\,372096\mathrm{i}}{2550408\,\sqrt{5}-5702888} - \frac{\sqrt{5}\,x\,\sqrt{190-85\,\sqrt{5}}\,832048\mathrm{i}}{5\,\left(2550408\,\sqrt{5}-5702888\right)}\right)\,\sqrt{190-85\,\sqrt{5}}\,1\mathrm{i}}{10} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{85\,\sqrt{5}-190}\,372096\mathrm{i}}{2550408\,\sqrt{5}-5702888} - \frac{\sqrt{5}\,x\,\sqrt{85\,\sqrt{5}-190}\,832048\mathrm{i}}{5\,\left(2550408\,\sqrt{5}-5702888\right)}\right)\,\sqrt{85\,\sqrt{5}-190}\,1\mathrm{i}}{10} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{85\,\sqrt{5}+190}\,372096\mathrm{i}}{2550408\,\sqrt{5}+5702888} + \frac{\sqrt{5}\,x\,\sqrt{85\,\sqrt{5}+190}\,832048\mathrm{i}}{5\,\left(2550408\,\sqrt{5}+5702888\right)}\right)\,\sqrt{85\,\sqrt{5}+190}\,1\mathrm{i}}{10}$$

input
```
int(1/(x^6*(x^8 - 3*x^4 + 1)),x)
```

output
```
(atan((x*(190 - 85*5^(1/2))^(1/2)*372096i)/(2550408*5^(1/2) - 5702888) - (
5^(1/2)*x*(190 - 85*5^(1/2))^(1/2)*832048i)/(5*(2550408*5^(1/2) - 5702888)
))*(190 - 85*5^(1/2))^(1/2)*1i)/10 - (atan((x*(- 85*5^(1/2) - 190)^(1/2)*3
72096i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(- 85*5^(1/2) - 190)^(1/2
)*832048i)/(5*(2550408*5^(1/2) + 5702888)))*(- 85*5^(1/2) - 190)^(1/2)*1i)
/10 + (atan((x*(85*5^(1/2) - 190)^(1/2)*372096i)/(2550408*5^(1/2) - 570288
8) - (5^(1/2)*x*(85*5^(1/2) - 190)^(1/2)*832048i)/(5*(2550408*5^(1/2) - 57
02888)))*(85*5^(1/2) - 190)^(1/2)*1i)/10 - (atan((x*(85*5^(1/2) + 190)^(1/
2)*372096i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(85*5^(1/2) + 190)^(1
/2)*832048i)/(5*(2550408*5^(1/2) + 5702888)))*(85*5^(1/2) + 190)^(1/2)*1i)
/10 - (3*x^4 + 1/5)/x^5
```

Reduce [F]

$$\int \frac{1}{x^6 (1 - 3x^4 + x^8)} \, dx = \int \frac{1}{x^{14} - 3x^{10} + x^6} dx$$

input
```
int(1/x^6/(x^8-3*x^4+1),x)
```

output
```
int(1/(x**14 - 3*x**10 + x**6),x)
```

3.136 $\int \frac{1}{x^8(1-3x^4+x^8)}\,dx$

Optimal result 1107
Mathematica [A] (verified) 1108
Rubi [A] (verified) 1108
Maple [C] (verified) 1112
Fricas [A] (verification not implemented) 1112
Sympy [A] (verification not implemented) 1113
Maxima [F] 1114
Giac [A] (verification not implemented) 1114
Mupad [B] (verification not implemented) 1115
Reduce [F] 1116

Optimal result

Integrand size = 16, antiderivative size = 189

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx = -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{5}}$$

output
```
-1/7/x^7-1/x^3-1/10*(39603/2-17711/2*5^(1/2))^(1/4)*arctan(2^(1/4)*(1/(3+5
^(1/2)))^(1/4)*x)*5^(1/2)+1/10*(39603/2+17711/2*5^(1/2))^(1/4)*arctan((3/2
+1/2*5^(1/2))^(1/4)*x)*5^(1/2)-1/10*(39603/2-17711/2*5^(1/2))^(1/4)*arctan
h(2^(1/4)*(1/(3+5^(1/2)))^(1/4)*x)*5^(1/2)+1/10*(39603/2+17711/2*5^(1/2))^
(1/4)*arctanh((3/2+1/2*5^(1/2))^(1/4)*x)*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx = -\frac{1}{7x^7}-\frac{1}{x^3}+\frac{(11+5\sqrt{5})\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10(-1+\sqrt{5})}}+\frac{(11-5\sqrt{5})\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10(1+\sqrt{5})}}-\frac{(-11-5\sqrt{5})\operatorname{arctanh}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)}{2\sqrt{10(-1+\sqrt{5})}}-\frac{(-11+5\sqrt{5})\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10(1+\sqrt{5})}}$$

input
```
Integrate[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

output
```
-1/7*1/x^7 - x^(-3) + ((11 + 5*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/
(2*Sqrt[10*(-1 + Sqrt[5])]) + ((11 - 5*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5]
)]*x])/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-11 - 5*Sqrt[5])*ArcTanh[Sqrt[2/(-1
+ Sqrt[5])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((-11 + 5*Sqrt[5])*ArcTanh[S
qrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1704, 27, 1828, 27, 1752, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8(x^8-3x^4+1)}\,dx$$

$$\Big\downarrow \text{1704}$$

$$\frac{1}{7}\int \frac{7(3-x^4)}{x^4\left(x^8-3x^4+1\right)}dx-\frac{1}{7x^7}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{3-x^4}{x^4\left(x^8-3x^4+1\right)}dx-\frac{1}{7x^7}$$

$$\Big\downarrow \text{1828}$$

$$-\frac{1}{3}\int -\frac{3(8-3x^4)}{x^8-3x^4+1}dx-\frac{1}{7x^7}-\frac{1}{x^3}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{8-3x^4}{x^8-3x^4+1}dx-\frac{1}{7x^7}-\frac{1}{x^3}$$

$$\Big\downarrow \text{1752}$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3-\sqrt{5}\right)}dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{x^4+\frac{1}{2}\left(-3+\sqrt{5}\right)}dx-\frac{1}{7x^7}-\frac{1}{x^3}$$

$$\Big\downarrow \text{756}$$

$$-\frac{1}{10}\left(15+7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3-\sqrt{5}}}dx}{\sqrt{3-\sqrt{5}}}\right)-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\int \frac{1}{\sqrt{2}x^2+\sqrt{3+\sqrt{5}}}dx}{\sqrt{3+\sqrt{5}}}\right)-\frac{1}{7x^7}-\frac{1}{x^3}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{10}\left(15+7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3-\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(-\frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx}{\sqrt{3+\sqrt{5}}}-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)-\frac{1}{7x^7}-\frac{1}{x^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{10}\left(15-7\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\left(3+\sqrt{5}\right)^{3/4}}\right)-\frac{1}{10}\left(15+7\sqrt{5}\right)\left(-\frac{\arctan\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt[4]{2}\left(3-\sqrt{5}\right)^{3/4}}\right)-\frac{1}{7x^7}-\frac{1}{x^3}$$

input
```
Int[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

output
```
-1/7*1/x^7 - x^(-3) - ((15 - 7*Sqrt[5])*(-(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*
x]/(2^(1/4)*(3 + Sqrt[5])^(3/4))) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^
(1/4)*(3 + Sqrt[5])^(3/4))))/10 - ((15 + 7*Sqrt[5])*(-(ArcTan[((3 + Sqrt[5
])/2)^(1/4)*x]/(2^(1/4)*(3 - Sqrt[5])^(3/4))) - ArcTanh[((3 + Sqrt[5])/2)^
(1/4)*x]/(2^(1/4)*(3 - Sqrt[5])^(3/4))))/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.42

method	result
risch	$\frac{-x^4-\frac{1}{7}}{x^7}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4-995\_Z^2-1\right)}\_R\ln\left(90\_R^3-3571\_R+89x\right)\right)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^4+995\_Z^2-1\right)}\_R\ln\right.}{4}$
default	$-\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\left(-11+5\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}+\frac{\sqrt{5}\left(11+5\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\sqrt{5}\left(11+5\sqrt{5}\right)\operatorname{arctanh}\left(\frac{2}{\sqrt{-2}}\right.}{10\sqrt{-2+2\sqrt{5}}}$

input
```
int(1/x^8/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(-x^4-1/7)/x^7+1/4*sum(_R*ln(90*_R^3-3571*_R+89*x),_R=RootOf(25*_Z^4-995*_
Z^2-1))+1/4*sum(_R*ln(-90*_R^3-3571*_R+89*x),_R=RootOf(25*_Z^4+995*_Z^2-1)
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^8\left(1-3x^4+x^8\right)}\,dx =$$

$$-\frac{14\,x^7\sqrt{\frac{89}{10}\,\sqrt{5}+\frac{199}{10}}\arctan\left(\frac{1}{2}\left(11\,\sqrt{5}x-25\,x\right)\sqrt{\frac{89}{10}\,\sqrt{5}+\frac{199}{10}}\right)+14\,x^7\sqrt{\frac{89}{10}\,\sqrt{5}-\frac{199}{10}}\arctan\left(\frac{1}{2}\left(11\,\sqrt{\right.\right.}{}$$

input
```
integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
-1/28*(14*x^7*sqrt(89/10*sqrt(5) + 199/10)*arctan(1/2*(11*sqrt(5)*x - 25*x
)*sqrt(89/10*sqrt(5) + 199/10)) + 14*x^7*sqrt(89/10*sqrt(5) - 199/10)*arct
an(1/2*(11*sqrt(5)*x + 25*x)*sqrt(89/10*sqrt(5) - 199/10)) - 7*x^7*sqrt(89
/10*sqrt(5) + 199/10)*log((9*sqrt(5) - 20)*sqrt(89/10*sqrt(5) + 199/10) +
x) + 7*x^7*sqrt(89/10*sqrt(5) + 199/10)*log(-(9*sqrt(5) - 20)*sqrt(89/10*s
qrt(5) + 199/10) + x) + 7*x^7*sqrt(89/10*sqrt(5) - 199/10)*log((9*sqrt(5)
+ 20)*sqrt(89/10*sqrt(5) - 199/10) + x) - 7*x^7*sqrt(89/10*sqrt(5) - 199/1
0)*log(-(9*sqrt(5) + 20)*sqrt(89/10*sqrt(5) - 199/10) + x) + 28*x^4 + 4)/x
^7
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^8 (1-3x^4+x^8)} \, dx = \operatorname{RootSum}\left(6400t^4-15920t^2-1, \left(t\mapsto t\log\left(\frac{460800t^5}{17711}-\frac{2842588t}{17711}+x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4+15920t^2-1, \left(t\mapsto t\log\left(\frac{460800t^5}{17711}-\frac{2842588t}{17711}+x\right)\right)\right) + \frac{-7x^4-1}{7x^7}$$

input
```
integrate(1/x**8/(x**8-3*x**4+1),x)
```

output
```
RootSum(6400*_t**4 - 15920*_t**2 - 1, Lambda(_t, _t*log(460800*_t**5/17711
 - 2842588*_t/17711 + x))) + RootSum(6400*_t**4 + 15920*_t**2 - 1, Lambda(
_t, _t*log(460800*_t**5/17711 - 2842588*_t/17711 + x))) + (-7*x**4 - 1)/(7
*x**7)
```

Maxima [F]

$$\int \frac{1}{x^8 (1-3x^4+x^8)} \, dx = \int \frac{1}{(x^8-3\,x^4+1)x^8} \, dx$$

input
```
integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
-1/7*(7*x^4 + 1)/x^7 - 1/2*integrate((5*x^2 + 8)/(x^4 + x^2 - 1), x) + 1/2
*integrate((5*x^2 - 8)/(x^4 - x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \frac{1}{x^8 (1-3x^4+x^8)} \, dx = & -\frac{1}{20}\,\sqrt{890\,\sqrt{5}-1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\,\sqrt{5}+\frac{1}{2}}}\right) \\ & +\frac{1}{20}\,\sqrt{890\,\sqrt{5}+1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\,\sqrt{5}-\frac{1}{2}}}\right) \\ & -\frac{1}{40}\,\sqrt{890\,\sqrt{5}-1990}\log\left(\left|x+\sqrt{\frac{1}{2}\,\sqrt{5}+\frac{1}{2}}\right|\right) \\ & +\frac{1}{40}\,\sqrt{890\,\sqrt{5}-1990}\log\left(\left|x-\sqrt{\frac{1}{2}\,\sqrt{5}+\frac{1}{2}}\right|\right) \\ & +\frac{1}{40}\,\sqrt{890\,\sqrt{5}+1990}\log\left(\left|x+\sqrt{\frac{1}{2}\,\sqrt{5}-\frac{1}{2}}\right|\right) \\ & -\frac{1}{40}\,\sqrt{890\,\sqrt{5}+1990}\log\left(\left|x-\sqrt{\frac{1}{2}\,\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{7\,x^4+1}{7\,x^7}\end{aligned}$$

input
```
integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
-1/20*sqrt(890*sqrt(5) - 1990)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sq
rt(890*sqrt(5) + 1990)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(890*s
qrt(5) - 1990)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(
5) - 1990)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(5) +
 1990)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(890*sqrt(5) + 199
0)*log(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/7*(7*x^4 + 1)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^8 (1 - 3x^4 + x^8)} dx = -\frac{x^4 + \frac{1}{7}}{x^7} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-89\sqrt{5}-199}\,6677047\mathrm{i}}{2\left(74049691\sqrt{5}+165580139\right)} + \frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-89\sqrt{5}-199}\,14930373\mathrm{i}}{10\left(74049691\sqrt{5}+165580139\right)}\right)\sqrt{-89\sqrt{5}-199}\,1\mathrm{i}}{20} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{199-89\sqrt{5}}\,6677047\mathrm{i}}{2\left(74049691\sqrt{5}-165580139\right)} - \frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{199-89\sqrt{5}}\,14930373\mathrm{i}}{10\left(74049691\sqrt{5}-165580139\right)}\right)\sqrt{199-89\sqrt{5}}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{89\sqrt{5}-199}\,6677047\mathrm{i}}{2\left(74049691\sqrt{5}-165580139\right)} - \frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{89\sqrt{5}-199}\,14930373\mathrm{i}}{10\left(74049691\sqrt{5}-165580139\right)}\right)\sqrt{89\sqrt{5}-199}\,1\mathrm{i}}{20} - \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{89\sqrt{5}+199}\,6677047\mathrm{i}}{2\left(74049691\sqrt{5}+165580139\right)} + \frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{89\sqrt{5}+199}\,14930373\mathrm{i}}{10\left(74049691\sqrt{5}+165580139\right)}\right)\sqrt{89\sqrt{5}+199}\,1\mathrm{i}}{20}$$

input
```
int(1/(x^8*(x^8 - 3*x^4 + 1)),x)
```

output
```
(10^(1/2)*atan((10^(1/2)*x*(- 89*5^(1/2) - 199)^(1/2)*6677047i)/(2*(740496
91*5^(1/2) + 165580139)) + (5^(1/2)*10^(1/2)*x*(- 89*5^(1/2) - 199)^(1/2)*
14930373i)/(10*(74049691*5^(1/2) + 165580139)))*(- 89*5^(1/2) - 199)^(1/2)
*1i)/20 - (x^4 + 1/7)/x^7 + (10^(1/2)*atan((10^(1/2)*x*(199 - 89*5^(1/2))^
(1/2)*6677047i)/(2*(74049691*5^(1/2) - 165580139)) - (5^(1/2)*10^(1/2)*x*(
199 - 89*5^(1/2))^(1/2)*14930373i)/(10*(74049691*5^(1/2) - 165580139)))*(1
99 - 89*5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(89*5^(1/2) - 1
99)^(1/2)*6677047i)/(2*(74049691*5^(1/2) - 165580139)) - (5^(1/2)*10^(1/2)
*x*(89*5^(1/2) - 199)^(1/2)*14930373i)/(10*(74049691*5^(1/2) - 165580139))
)*(89*5^(1/2) - 199)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(89*5^(1/2)
 + 199)^(1/2)*6677047i)/(2*(74049691*5^(1/2) + 165580139)) + (5^(1/2)*10^(
1/2)*x*(89*5^(1/2) + 199)^(1/2)*14930373i)/(10*(74049691*5^(1/2) + 1655801
39)))*(89*5^(1/2) + 199)^(1/2)*1i)/20
```

Reduce [F]

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx=\int \frac{1}{x^{16}-3x^{12}+x^8}dx$$

input
```
int(1/x^8/(x^8-3*x^4+1),x)
```

output
```
int(1/(x**16 - 3*x**12 + x**8),x)
```

3.137 $\int \frac{x^3}{2+3x^4+x^8} \, dx$

Optimal result 1117
Mathematica [A] (verified) 1117
Rubi [A] (verified) 1118
Maple [A] (verified) 1119
Fricas [A] (verification not implemented) 1119
Sympy [A] (verification not implemented) 1120
Maxima [A] (verification not implemented) 1120
Giac [A] (verification not implemented) 1120
Mupad [B] (verification not implemented) 1121
Reduce [B] (verification not implemented) 1121

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{x^3}{2+3x^4+x^8} \, dx = \frac{1}{4}\log\left(1+x^4\right)-\frac{1}{4}\log\left(2+x^4\right)$$

output
```
1/4*ln(x^4+1)-1/4*ln(x^4+2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{2+3x^4+x^8} \, dx = \frac{1}{4}\log\left(1+x^4\right)-\frac{1}{4}\log\left(2+x^4\right)$$

input
```
Integrate[x^3/(2 + 3*x^4 + x^8),x]
```

output
```
Log[1 + x^4]/4 - Log[2 + x^4]/4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1690, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^8+3x^4+2}dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{4}\int \frac{1}{x^8+3x^4+2}dx^4$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{4}\int \left(\frac{1}{x^4+1}+\frac{1}{-x^4-2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\log \left(x^4+1\right)-\log \left(x^4+2\right)\right)$$

input
```
Int[x^3/(2 + 3*x^4 + x^8),x]
```

output
```
(Log[1 + x^4] - Log[2 + x^4])/4
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\ln(x^4+1)}{4}-\frac{\ln(x^4+2)}{4}$	18
norman	$\frac{\ln(x^4+1)}{4}-\frac{\ln(x^4+2)}{4}$	18
risch	$\frac{\ln(x^4+1)}{4}-\frac{\ln(x^4+2)}{4}$	18
parallelrisch	$\frac{\ln(x^4+1)}{4}-\frac{\ln(x^4+2)}{4}$	18

input
```
int(x^3/(x^8+3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^4+1)-1/4*ln(x^4+2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{2+3x^4+x^8}\,dx=-\frac{1}{4}\log\left(x^4+2\right)+\frac{1}{4}\log\left(x^4+1\right)$$

input
```
integrate(x^3/(x^8+3*x^4+2),x, algorithm="fricas")
```

output
```
-1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = \frac{\log(x^4+1)}{4} - \frac{\log(x^4+2)}{4}$$

input
```
integrate(x**3/(x**8+3*x**4+2),x)
```

output
```
log(x**4 + 1)/4 - log(x**4 + 2)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = -\frac{1}{4}\log\left(x^4+2\right) + \frac{1}{4}\log\left(x^4+1\right)$$

input
```
integrate(x^3/(x^8+3*x^4+2),x, algorithm="maxima")
```

output
```
-1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = -\frac{1}{4}\log\left(x^4+2\right) + \frac{1}{4}\log\left(x^4+1\right)$$

input
```
integrate(x^3/(x^8+3*x^4+2),x, algorithm="giac")
```

output
```
-1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 19.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = -\frac{\operatorname{atanh}\left(\frac{256}{9\,(144\,x^4+160)}-\frac{7}{9}\right)}{2}$$

input
```
int(x^3/(3*x^4 + x^8 + 2),x)
```

output
```
-atanh(256/(9*(144*x^4 + 160)) - 7/9)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.81

$$\int \frac{x^3}{2+3x^4+x^8}\,dx = -\frac{\log\left(-\sqrt{2}\,2^{\frac{1}{4}}x+\sqrt{2}+x^2\right)}{4}+\frac{\log\left(-\sqrt{2}\,x+x^2+1\right)}{4} - \frac{\log\left(\sqrt{2}\,2^{\frac{1}{4}}x+\sqrt{2}+x^2\right)}{4}+\frac{\log\left(\sqrt{2}\,x+x^2+1\right)}{4}$$

input
```
int(x^3/(x^8+3*x^4+2),x)
```

output
```
( - log( - sqrt(2)*2**(1/4)*x + sqrt(2) + x**2) + log( - sqrt(2)*x + x**2
+ 1) - log(sqrt(2)*2**(1/4)*x + sqrt(2) + x**2) + log(sqrt(2)*x + x**2 + 1
))/4
```

3.138 $\int \frac{x^{11}}{2+3x^4+x^8} dx$

Optimal result 1122
Mathematica [A] (verified) 1122
Rubi [A] (verified) 1123
Maple [A] (verified) 1124
Fricas [A] (verification not implemented) 1124
Sympy [A] (verification not implemented) 1125
Maxima [A] (verification not implemented) 1125
Giac [A] (verification not implemented) 1125
Mupad [B] (verification not implemented) 1126
Reduce [B] (verification not implemented) 1126

Optimal result

Integrand size = 16, antiderivative size = 26

$$\int \frac{x^{11}}{2+3x^4+x^8} \, dx = \frac{x^4}{4} + \frac{1}{4}\log \left(1+x^4\right) - \log \left(2+x^4\right)$$

output
```
1/4*x^4+1/4*ln(x^4+1)-ln(x^4+2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{2+3x^4+x^8} \, dx = \frac{x^4}{4} + \frac{1}{4}\log \left(1+x^4\right) - \log \left(2+x^4\right)$$

input
```
Integrate[x^11/(2 + 3*x^4 + x^8),x]
```

output
```
x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1693, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{x^8+3x^4+2}dx$$
$$\Big\downarrow \text{ 1693}$$
$$\frac{1}{4}\int \frac{x^8}{x^8+3x^4+2}dx^4$$
$$\Big\downarrow \text{ 1141}$$
$$\frac{1}{4}\int \left(-\frac{4}{x^4+2}+1+\frac{1}{x^4+1}\right)dx^4$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{4}\left(x^4+\log\left(x^4+1\right)-4\log\left(x^4+2\right)\right)$$

input
```
Int[x^11/(2 + 3*x^4 + x^8),x]
```

output
```
(x^4 + Log[1 + x^4] - 4*Log[2 + x^4])/4
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23
norman	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23
risch	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23
parallelrisch	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23

input
```
int(x^11/(x^8+3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+1/4*ln(x^4+1)-ln(x^4+2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx=\frac{1}{4}\,x^4-\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

input
```
integrate(x^11/(x^8+3*x^4+2),x, algorithm="fricas")
```

output
```
1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = \frac{x^4}{4} + \frac{\log\left(x^4+1\right)}{4} - \log\left(x^4+2\right)$$

input
```
integrate(x**11/(x**8+3*x**4+2),x)
```

output
```
x**4/4 + log(x**4 + 1)/4 - log(x**4 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \log\left(x^4+2\right) + \frac{1}{4}\,\log\left(x^4+1\right)$$

input
```
integrate(x^11/(x^8+3*x^4+2),x, algorithm="maxima")
```

output
```
1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = \frac{1}{4}\,x^4 - \log\left(x^4+2\right) + \frac{1}{4}\,\log\left(x^4+1\right)$$

input
```
integrate(x^11/(x^8+3*x^4+2),x, algorithm="giac")
```

output
```
1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = \frac{\ln(x^4+1)}{4} - \ln(x^4+2) + \frac{x^4}{4}$$

input
```
int(x^11/(3*x^4 + x^8 + 2),x)
```

output
```
log(x^4 + 1)/4 - log(x^4 + 2) + x^4/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \frac{x^{11}}{2+3x^4+x^8}\,dx = -\log\left(-\sqrt{2}\,2^{\frac{1}{4}}x+\sqrt{2}+x^2\right)+\frac{\log\left(-\sqrt{2}\,x+x^2+1\right)}{4} - \log\left(\sqrt{2}\,2^{\frac{1}{4}}x+\sqrt{2}+x^2\right)+\frac{\log\left(\sqrt{2}\,x+x^2+1\right)}{4}+\frac{x^4}{4}$$

input
```
int(x^11/(x^8+3*x^4+2),x)
```

output
```
( - 4*log( - sqrt(2)*2**(1/4)*x + sqrt(2) + x**2) + log( - sqrt(2)*x + x**
2 + 1) - 4*log(sqrt(2)*2**(1/4)*x + sqrt(2) + x**2) + log(sqrt(2)*x + x**2
 + 1) + x**4)/4
```

3.139 $\int \frac{x^m}{a+bx^4+cx^8}\,dx$

Optimal result 1127
Mathematica [C] (warning: unable to verify) 1127
Rubi [A] (verified) 1128
Maple [F] 1130
Fricas [F] 1130
Sympy [F(-1)] 1130
Maxima [F] 1131
Giac [F] 1131
Mupad [F(-1)] 1131
Reduce [F] 1132

Optimal result

Integrand size = 18, antiderivative size = 163

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \frac{2cx^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)} - \frac{2cx^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)(1+m)}$$

output
```
2*c*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*c*x^4/(b-(-4*a*c+b^2)^
(1/2)))/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))/(1+m)-2*c*x^(1+m)*hyperg
eom([1, 1/4+1/4*m],[5/4+1/4*m],-2*c*x^4/(b+(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^
2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/(1+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.50

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \frac{x^m \text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1^3+2c\#1^7}\&\right]}{4m}$$

input
```
Integrate[x^m/(a + b*x^4 + c*x^8),x]
```

output
```
(x^m*RootSum[a + b*#1^4 + c*#1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1
/(x - #1))]/((x/(x - #1))^m*(b*#1^3 + 2*c*#1^7)) & ])/(4*m)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1711, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{c\int \frac{2x^m}{2cx^4+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{2x^m}{2cx^4+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2c\int \frac{x^m}{2cx^4+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x^m}{2cx^4+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2cx^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[x^m/(a + b*x^4 + c*x^8),x]
```

output
```
(2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b -
Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(1 + m)) -
 (2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b +
 Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{x^m}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^m/(c*x^8+b*x^4+a),x)
```

output
```
int(x^m/(c*x^8+b*x^4+a),x)
```

Fricas [F]

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx=\int \frac{x^m}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
integral(x^m/(c*x^8 + b*x^4 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx=\text{Timed out}$$

input
```
integrate(x**m/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \int \frac{x^m}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \int \frac{x^m}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \int \frac{x^m}{c\,x^8+b\,x^4+a}\,dx$$

input
```
int(x^m/(a + b*x^4 + c*x^8),x)
```

output
```
int(x^m/(a + b*x^4 + c*x^8), x)
```

Reduce [F]

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \int \frac{x^m}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^m/(c*x^8+b*x^4+a),x)
```

output
```
int(x**m/(a + b*x**4 + c*x**8),x)
```

3.140 $\int \frac{x^m}{1+x^4+x^8} \, dx$

Optimal result 1133
Mathematica [C] (warning: unable to verify) 1133
Rubi [A] (verified) 1134
Maple [F] 1136
Fricas [F] 1136
Sympy [F] 1136
Maxima [F] 1137
Giac [F] 1137
Mupad [F(-1)] 1137
Reduce [F] 1138

Optimal result

Integrand size = 14, antiderivative size = 127

$$\int \frac{x^m}{1+x^4+x^8} \, dx = \frac{2x^{1+m} \text{Hypergeometric2F1}\left(1, \frac{1+m}{4}, \frac{5+m}{4}, -\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)} - \frac{2x^{1+m} \text{Hypergeometric2F1}\left(1, \frac{1+m}{4}, \frac{5+m}{4}, -\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}$$

output

```
2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x^4/(1-I*3^(1/2)))*3^(
1/2)/(3^(1/2)+I)/(1+m)-2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2
*x^4/(1+I*3^(1/2)))*3^(1/2)/(I-3^(1/2))/(1+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.09 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.84

$$\int \frac{x^m}{1+x^4+x^8}\,dx$$

$$= \frac{x^m\left(-\frac{i\left(\left(\frac{x}{-\sqrt[3]{-1}+x}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,1-m,\frac{\sqrt[3]{-1}}{\sqrt[3]{-1}-x}\right)+\left(\frac{x}{-(-1)^{2/3}+x}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,1-m,\frac{\ldots}{(\ldots}\right.\right.}{\ldots}\right.}{\ldots}$$

input
```
Integrate[x^m/(1 + x^4 + x^8),x]
```

output
```
(x^m*(((-I)*(Hypergeometric2F1[-m, -m, 1 - m, (-1)^(1/3)/((-1)^(1/3) - x)]
/(x/(-(-1)^(1/3) + x))^m + Hypergeometric2F1[-m, -m, 1 - m, (-1)^(2/3)/((-
1)^(2/3) - x)]/(x/(-(-1)^(2/3) + x))^m - Hypergeometric2F1[-m, -m, 1 - m,
(-1)^(1/3)/((-1)^(1/3) + x)]/(x/((-1)^(1/3) + x))^m - Hypergeometric2F1[-m
, -m, 1 - m, (-1)^(2/3)/((-1)^(2/3) + x)]/(x/((-1)^(2/3) + x))^m))/Sqrt[3]
 + RootSum[1 - #1^2 + #1^4 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x -
#1))]/((x/(x - #1))^m*(-#1 + 2*#1^3)) & ] - RootSum[1 - #1^2 + #1^4 & , (m
*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeometric2F1[-m, -m, 1 - m,
 -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hypergeometric2F1[-m, -m, 1 -
 m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hypergeometric2F1[-m, -m,
1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2)/(x/#1)^m)/(-#1 + 2*
#1^3) & ]/(2 + 3*m + m^2)))/(4*m)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1711, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{x^8+x^4+1}\,dx$$

$$\Big\downarrow \; 1711$$

$$\frac{i\int \frac{2x^m}{2x^4+i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{i\int \frac{2x^m}{2x^4-i\sqrt{3}+1}dx}{\sqrt{3}}$$

$$\Big\downarrow \; 27$$

$$\frac{2i\int \frac{x^m}{2x^4+i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{2i\int \frac{x^m}{2x^4-i\sqrt{3}+1}dx}{\sqrt{3}}$$

$$\Big\downarrow \; 888$$

$$\frac{2ix^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(1+i\sqrt{3}\right)(m+1)}-\frac{2ix^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(1-i\sqrt{3}\right)(m+1)}$$

input
```
Int[x^m/(1 + x^4 + x^8),x]
```

output
```
((-2*I)*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 -
 I*Sqrt[3])])/(Sqrt[3]*(1 - I*Sqrt[3])*(1 + m)) + ((2*I)*x^(1 + m)*Hyperge
ometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(1
 + I*Sqrt[3])*(1 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{x^m}{x^8 + x^4 + 1} dx$$

input
```
int(x^m/(x^8+x^4+1),x)
```

output
```
int(x^m/(x^8+x^4+1),x)
```

Fricas [F]

$$\int \frac{x^m}{1 + x^4 + x^8} \, dx = \int \frac{x^m}{x^8 + x^4 + 1} \, dx$$

input
```
integrate(x^m/(x^8+x^4+1),x, algorithm="fricas")
```

output
```
integral(x^m/(x^8 + x^4 + 1), x)
```

Sympy [F]

$$\int \frac{x^m}{1 + x^4 + x^8} \, dx = \int \frac{x^m}{(x^2 - x + 1)(x^2 + x + 1)(x^4 - x^2 + 1)} \, dx$$

input
```
integrate(x**m/(x**8+x**4+1),x)
```

output
```
Integral(x**m/((x**2 - x + 1)*(x**2 + x + 1)*(x**4 - x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{x^m}{1+x^4+x^8}\,dx = \int \frac{x^m}{x^8+x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^m/(x^8 + x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^m}{1+x^4+x^8}\,dx = \int \frac{x^m}{x^8+x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+x^4+1),x, algorithm="giac")
```

output
```
integrate(x^m/(x^8 + x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1+x^4+x^8}\,dx = \int \frac{x^m}{x^8+x^4+1}\,dx$$

input
```
int(x^m/(x^4 + x^8 + 1),x)
```

output
```
int(x^m/(x^4 + x^8 + 1), x)
```

Reduce [F]

$$\int \frac{x^m}{1+x^4+x^8}\,dx = \int \frac{x^m}{x^8+x^4+1}dx$$

input
```
int(x^m/(x^8+x^4+1),x)
```

output
```
int(x**m/(x**8 + x**4 + 1),x)
```

3.141 $\int \frac{x^m}{1-x^4+x^8} dx$

Optimal result . . . 1139
Mathematica [C] (warning: unable to verify) . . . 1139
Rubi [A] (verified) . . . 1140
Maple [F] . . . 1142
Fricas [F] . . . 1142
Sympy [F] . . . 1142
Maxima [F] . . . 1143
Giac [F] . . . 1143
Mupad [F(-1)] . . . 1143
Reduce [F] . . . 1144

Optimal result

Integrand size = 16, antiderivative size = 127

$$\int \frac{x^m}{1-x^4+x^8} dx = \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)} - \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}$$

output
```
2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4/(1-I*3^(1/2)))*3^(1
/2)/(3^(1/2)+I)/(1+m)-2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x
^4/(1+I*3^(1/2)))*3^(1/2)/(I-3^(1/2))/(1+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.62

$$\int \frac{x^m}{1-x^4+x^8}\,dx$$
$$=\frac{x^m\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{-\#1^3+2\#1^7}\&\right]}{4m}$$

input
```
Integrate[x^m/(1 - x^4 + x^8),x]
```

output
```
(x^m*RootSum[1 - #1^4 + #1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x
- #1))]/((x/(x - #1))^m*(-#1^3 + 2*#1^7)) & ])/(4*m)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1711, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{x^8-x^4+1}\,dx$$
$$\Big\downarrow \ 1711$$
$$\frac{i\int -\frac{2x^m}{-2x^4-i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{i\int -\frac{2x^m}{-2x^4+i\sqrt{3}+1}dx}{\sqrt{3}}$$
$$\Big\downarrow \ 27$$
$$\frac{2i\int \frac{x^m}{-2x^4+i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{2i\int \frac{x^m}{-2x^4-i\sqrt{3}+1}dx}{\sqrt{3}}$$
$$\Big\downarrow \ 888$$

$$\frac{2ix^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(1+i\sqrt{3}\right)(m+1)}-\frac{2ix^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(1-i\sqrt{3}\right)(m+1)}$$

input
```
Int[x^m/(1 - x^4 + x^8),x]
```

output
```
((-2*I)*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(1 -
I*Sqrt[3])])/(Sqrt[3]*(1 - I*Sqrt[3])*(1 + m)) + ((2*I)*x^(1 + m)*Hypergeo
metric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(1 +
 I*Sqrt[3])*(1 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{x^m}{x^8 - x^4 + 1} dx$$

input
```
int(x^m/(x^8-x^4+1),x)
```

output
```
int(x^m/(x^8-x^4+1),x)
```

Fricas [F]

$$\int \frac{x^m}{1 - x^4 + x^8} dx = \int \frac{x^m}{x^8 - x^4 + 1} dx$$

input
```
integrate(x^m/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
integral(x^m/(x^8 - x^4 + 1), x)
```

Sympy [F]

$$\int \frac{x^m}{1 - x^4 + x^8} dx = \int \frac{x^m}{x^8 - x^4 + 1} dx$$

input
```
integrate(x**m/(x**8-x**4+1),x)
```

output
```
Integral(x**m/(x**8 - x**4 + 1), x)
```

Maxima [F]

$$\int \frac{x^m}{1-x^4+x^8}\,dx = \int \frac{x^m}{x^8-x^4+1}\,dx$$

input
```
integrate(x^m/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^m/(x^8 - x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^m}{1-x^4+x^8}\,dx = \int \frac{x^m}{x^8-x^4+1}\,dx$$

input
```
integrate(x^m/(x^8-x^4+1),x, algorithm="giac")
```

output
```
integrate(x^m/(x^8 - x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1-x^4+x^8}\,dx = \int \frac{x^m}{x^8-x^4+1}\,dx$$

input
```
int(x^m/(x^8 - x^4 + 1),x)
```

output
```
int(x^m/(x^8 - x^4 + 1), x)
```

Reduce [F]

$$\int \frac{x^m}{1-x^4+x^8}\,dx = \int \frac{x^m}{x^8-x^4+1}dx$$

input
```
int(x^m/(x^8-x^4+1),x)
```

output
```
int(x**m/(x**8 - x**4 + 1),x)
```

3.142 $\int \frac{x^m}{1+3x^4+x^8}\,dx$

Optimal result . . . 1145
Mathematica [C] (warning: unable to verify) . . . 1145
Rubi [A] (verified) . . . 1146
Maple [F] . . . 1148
Fricas [F] . . . 1148
Sympy [F] . . . 1148
Maxima [F] . . . 1149
Giac [F] . . . 1149
Mupad [F(-1)] . . . 1149
Reduce [F] . . . 1150

Optimal result

Integrand size = 16, antiderivative size = 117

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)} - \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}$$

output
```
2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x^4/(3-5^(1/2)))*5^(1/
2)/(3-5^(1/2))/(1+m)-2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x
^4/(3+5^(1/2)))*5^(1/2)/(3+5^(1/2))/(1+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \frac{x^m}{1+3x^4+x^8}\,dx$$
$$= \frac{x^m\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{3\#1^3+2\#1^7}\&\right]}{4m}$$

input
```
Integrate[x^m/(1 + 3*x^4 + x^8),x]
```

output
```
(x^m*RootSum[1 + 3*#1^4 + #1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(
x - #1))]/((x/(x - #1))^m*(3*#1^3 + 2*#1^7)) & ])/(4*m)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1711, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{x^8+3x^4+1}\,dx$$
$$\Big\downarrow \text{1711}$$
$$\frac{\int \frac{2x^m}{2x^4-\sqrt{5}+3}dx}{\sqrt{5}}-\frac{\int \frac{2x^m}{2x^4+\sqrt{5}+3}dx}{\sqrt{5}}$$
$$\Big\downarrow \text{27}$$
$$\frac{2\int \frac{x^m}{2x^4-\sqrt{5}+3}dx}{\sqrt{5}}-\frac{2\int \frac{x^m}{2x^4+\sqrt{5}+3}dx}{\sqrt{5}}$$
$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

input
```
Int[x^m/(1 + 3*x^4 + x^8),x]
```

output
```
(2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(3 - Sqrt
[5])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
 (1 + m)/4, (5 + m)/4, (-2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1
+ m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{x^m}{x^8+3x^4+1}dx$$

input
```
int(x^m/(x^8+3*x^4+1),x)
```

output
```
int(x^m/(x^8+3*x^4+1),x)
```

Fricas [F]

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+3*x^4+1),x, algorithm="fricas")
```

output
```
integral(x^m/(x^8 + 3*x^4 + 1), x)
```

Sympy [F]

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3x^4+1}\,dx$$

input
```
integrate(x**m/(x**8+3*x**4+1),x)
```

output
```
Integral(x**m/(x**8 + 3*x**4 + 1), x)
```

Maxima [F]

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^m/(x^8 + 3*x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8+3*x^4+1),x, algorithm="giac")
```

output
```
integrate(x^m/(x^8 + 3*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3\,x^4+1}\,dx$$

input
```
int(x^m/(3*x^4 + x^8 + 1),x)
```

output
```
int(x^m/(3*x^4 + x^8 + 1), x)
```

Reduce [F]

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \int \frac{x^m}{x^8+3x^4+1}dx$$

input
```
int(x^m/(x^8+3*x^4+1),x)
```

output
```
int(x**m/(x**8 + 3*x**4 + 1),x)
```

3.143 $\int \frac{x^m}{1-3x^4+x^8} dx$

Optimal result 1151
Mathematica [C] (warning: unable to verify) 1151
Rubi [A] (verified) 1152
Maple [F] 1154
Fricas [F] 1154
Sympy [F] 1154
Maxima [F] 1155
Giac [F] 1155
Mupad [F(-1)] 1155
Reduce [F] 1156

Optimal result

Integrand size = 16, antiderivative size = 117

$$\int \frac{x^m}{1-3x^4+x^8} dx = \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\,(3-\sqrt{5})\,(1+m)} - \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\,(3+\sqrt{5})\,(1+m)}$$

output
```
2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4/(3-5^(1/2)))*5^(1/2
)/(3-5^(1/2))/(1+m)-2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4
/(3+5^(1/2)))*5^(1/2)/(3+5^(1/2))/(1+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.97 (sec) , antiderivative size = 575, normalized size of antiderivative = 4.91

$$\int \frac{x^m}{1-3x^4+x^8}\,dx$$

$$= \frac{x^m\left(-\text{RootSum}\left[-1-\#1^2+\#1^4\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{-\#1+2\#1^3}\&\right]+\frac{\text{RootSum}\left[-\right.}{}\right.}{}$$

input
```
Integrate[x^m/(1 - 3*x^4 + x^8),x]
```

output
```
(x^m*(-RootSum[-1 - #1^2 + #1^4 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/
(x - #1))]/((x/(x - #1))^m*(-#1 + 2*#1^3)) & ] + (RootSum[-1 - #1^2 + #1^4
 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeometric2F1[-m, -m,
 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hypergeometric2F1[-m,
-m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hypergeometric2F1[-
m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2)/(x/#1)^m)/(-
#1 + 2*#1^3) & ] - (2 + 3*m + m^2)*RootSum[-1 + #1^2 + #1^4 & , Hypergeome
tric2F1[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(#1 + 2*#1^3)) & ]
- RootSum[-1 + #1^2 + #1^4 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2
*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (
3*m*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m
+ (m^2*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))
^m + (m*#1^2)/(x/#1)^m)/(#1 + 2*#1^3) & ])/(2 + 3*m + m^2)))/(4*m)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1711, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{x^8-3x^4+1} \, dx$$

$$\Big\downarrow \text{1711}$$

$$\frac{\int -\frac{2x^m}{-2x^4+\sqrt{5}+3}dx}{\sqrt{5}}-\frac{\int -\frac{2x^m}{-2x^4-\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{x^m}{-2x^4-\sqrt{5}+3}dx}{\sqrt{5}}-\frac{2\int \frac{x^m}{-2x^4+\sqrt{5}+3}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

input
```
Int[x^m/(1 - 3*x^4 + x^8),x]
```

output
```
(2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(3 - Sqrt[
5])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
(1 + m)/4, (5 + m)/4, (2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1 +
m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{x^m}{x^8 - 3x^4 + 1} dx$$

input
```
int(x^m/(x^8-3*x^4+1),x)
```

output
```
int(x^m/(x^8-3*x^4+1),x)
```

Fricas [F]

$$\int \frac{x^m}{1 - 3x^4 + x^8} dx = \int \frac{x^m}{x^8 - 3x^4 + 1} dx$$

input
```
integrate(x^m/(x^8-3*x^4+1),x, algorithm="fricas")
```

output
```
integral(x^m/(x^8 - 3*x^4 + 1), x)
```

Sympy [F]

$$\int \frac{x^m}{1 - 3x^4 + x^8} dx = \int \frac{x^m}{(x^4 - x^2 - 1)(x^4 + x^2 - 1)} dx$$

input
```
integrate(x**m/(x**8-3*x**4+1),x)
```

output
```
Integral(x**m/((x**4 - x**2 - 1)*(x**4 + x**2 - 1)), x)
```

Maxima [F]

$$\int \frac{x^m}{1-3x^4+x^8}\,dx=\int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8-3*x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^m}{1-3x^4+x^8}\,dx=\int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

input
```
integrate(x^m/(x^8-3*x^4+1),x, algorithm="giac")
```

output
```
integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{1-3x^4+x^8}\,dx=\int \frac{x^m}{x^8-3\,x^4+1}\,dx$$

input
```
int(x^m/(x^8 - 3*x^4 + 1),x)
```

output
```
int(x^m/(x^8 - 3*x^4 + 1), x)
```

Reduce [F]

$$\int \frac{x^m}{1-3x^4+x^8}\,dx = \int \frac{x^m}{x^8-3x^4+1}dx$$

input
```
int(x^m/(x^8-3*x^4+1),x)
```

output
```
int(x**m/(x**8 - 3*x**4 + 1),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 1157
4.2 Links to plain text integration problems used in this report for each CAS . 1175

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file