Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.3-General-trinomial/126-1.2.3.2-c

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May 18, 2024 Compiled on May 18, 2024 at 9:20pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**294**]. This is test number [126].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (294)	0.00 (0)
Mathematica	99.32 (292)	0.68 (2)
Maple	61.90 (182)	38.10 (112)
Fricas	61.56 (181)	38.44 (113)
Reduce	51.36 (151)	48.64 (143)
Giac	45.92 (135)	54.08 (159)
Mupad	34.35 (101)	65.65 (193)
Maxima	28.23 (83)	71.77 (211)
Sympy	25.85 (76)	74.15 (218)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

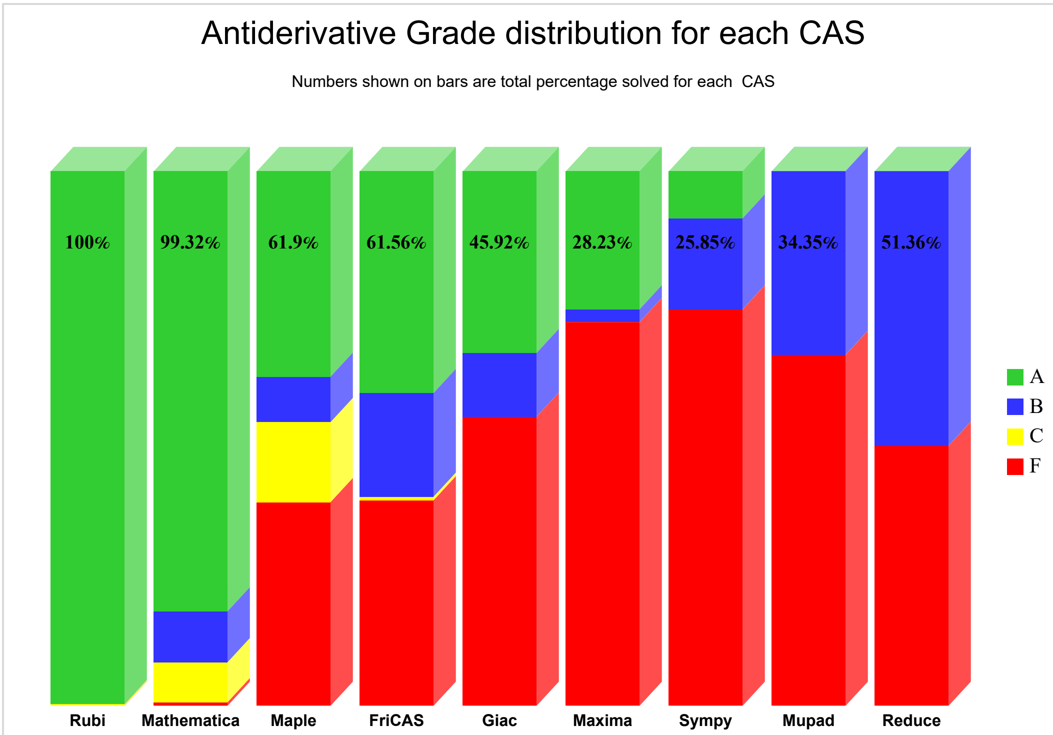
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

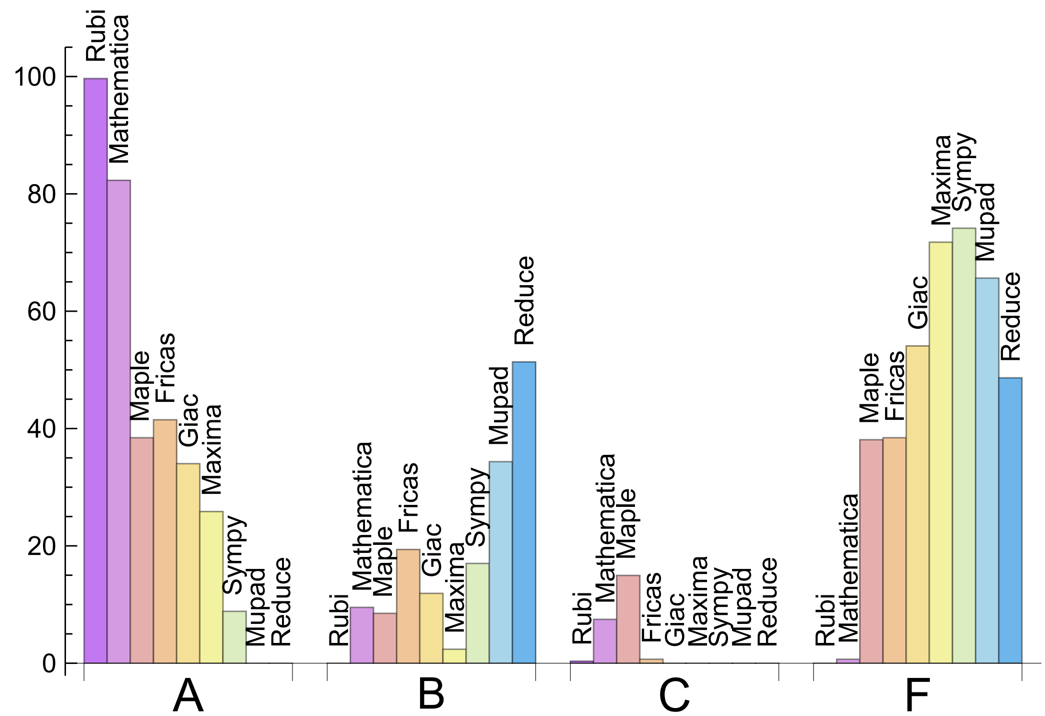
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.660	0.000	0.340	0.000
Mathematica	82.313	9.524	7.483	0.680
Fricas	41.497	19.388	0.680	38.435
Maple	38.435	8.503	14.966	38.095
Giac	34.014	11.905	0.000	54.082
Maxima	25.850	2.381	0.000	71.769
Sympy	8.844	17.007	0.000	74.150
Mupad	0.000	34.354	0.000	65.646
Reduce	0.000	51.361	0.000	48.639

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Maple	112	100.00	0.00	0.00
Fricas	113	51.33	20.35	28.32
Reduce	143	100.00	0.00	0.00
Giac	159	96.86	0.63	2.52
Mupad	193	0.00	100.00	0.00
Maxima	211	86.73	0.00	13.27
Sympy	218	63.76	33.94	2.29

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Giac	0.15
Fricas	0.18
Reduce	0.19
Maple	0.23
Rubi	0.37
Mathematica	0.67
Mupad	12.81
Sympy	14.88

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	81.69	0.93	52.00	0.78
Rubi	147.38	0.98	124.00	1.00
Mathematica	163.13	1.16	108.00	0.95
Maple	224.77	1.73	127.50	1.02
Reduce	246.26	1.91	92.00	0.98
Giac	562.78	3.18	120.00	1.04
Fricas	807.84	3.96	143.00	1.34
Sympy	1505.99	9.90	282.00	3.85
Mupad	2122.30	8.71	183.00	1.95

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

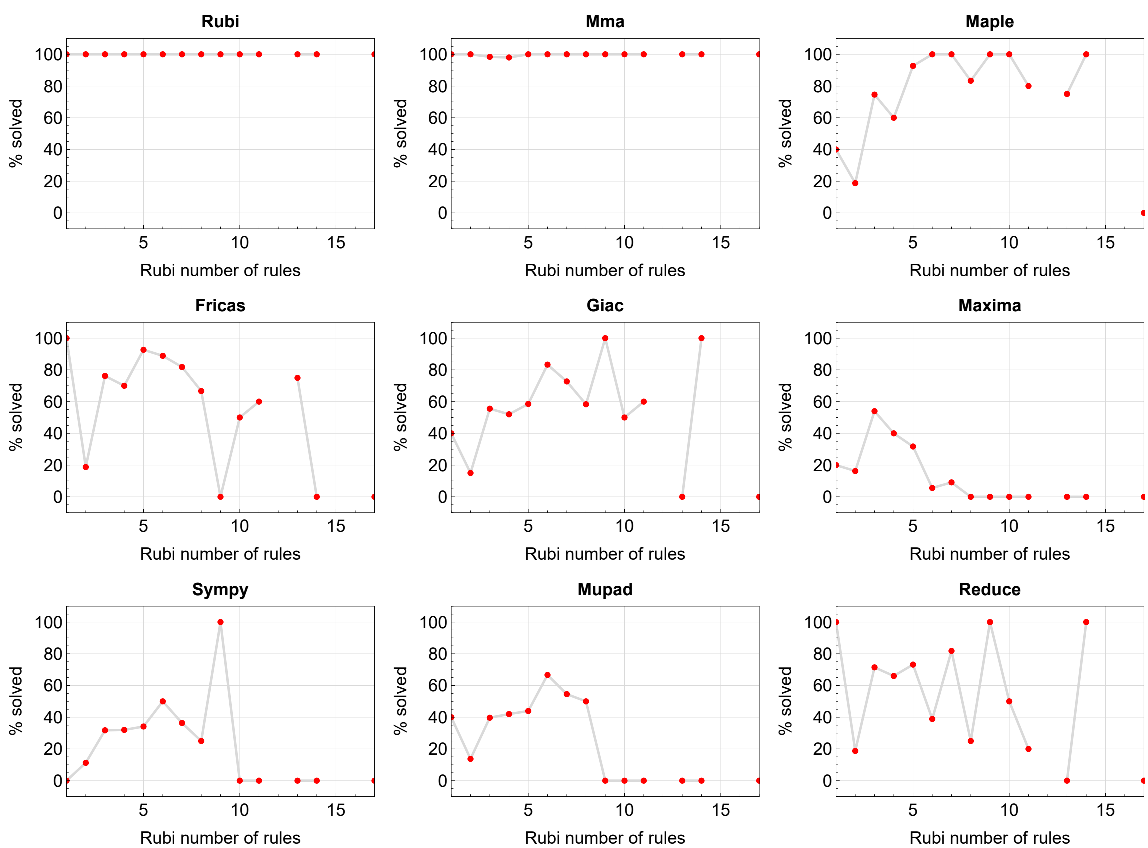


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

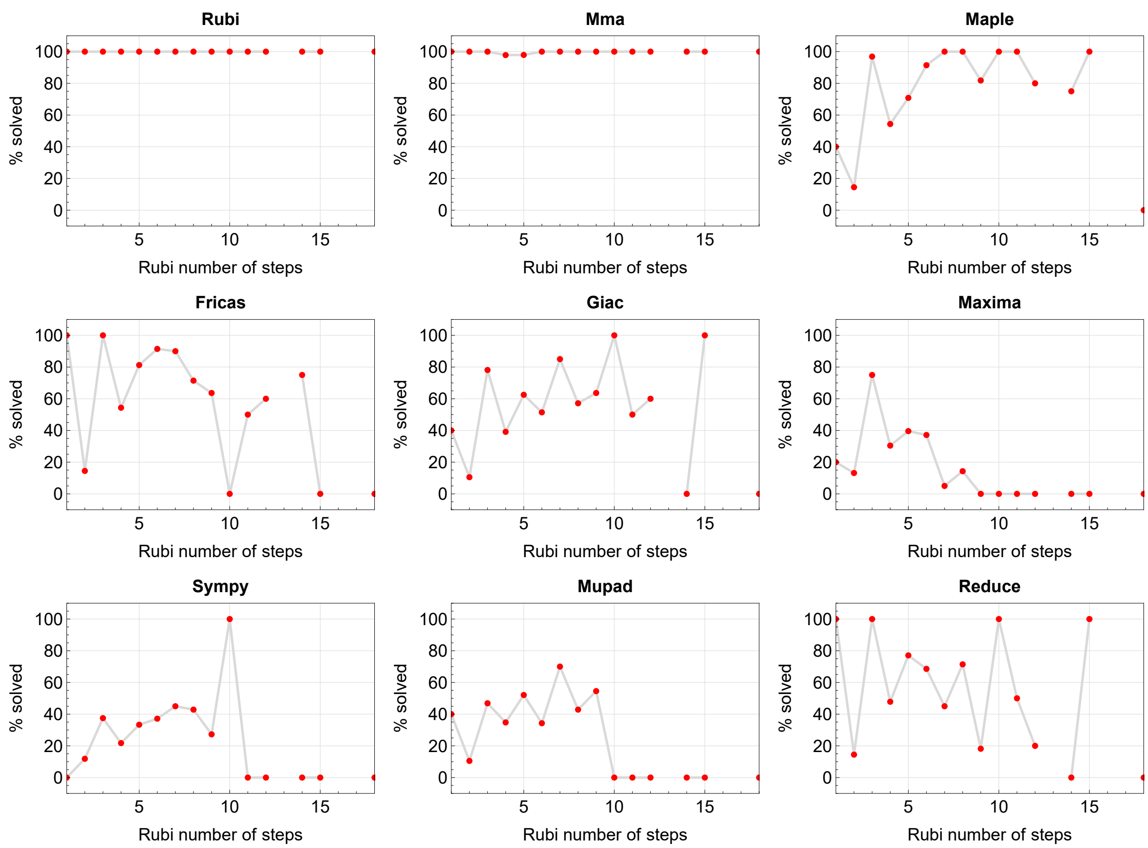


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

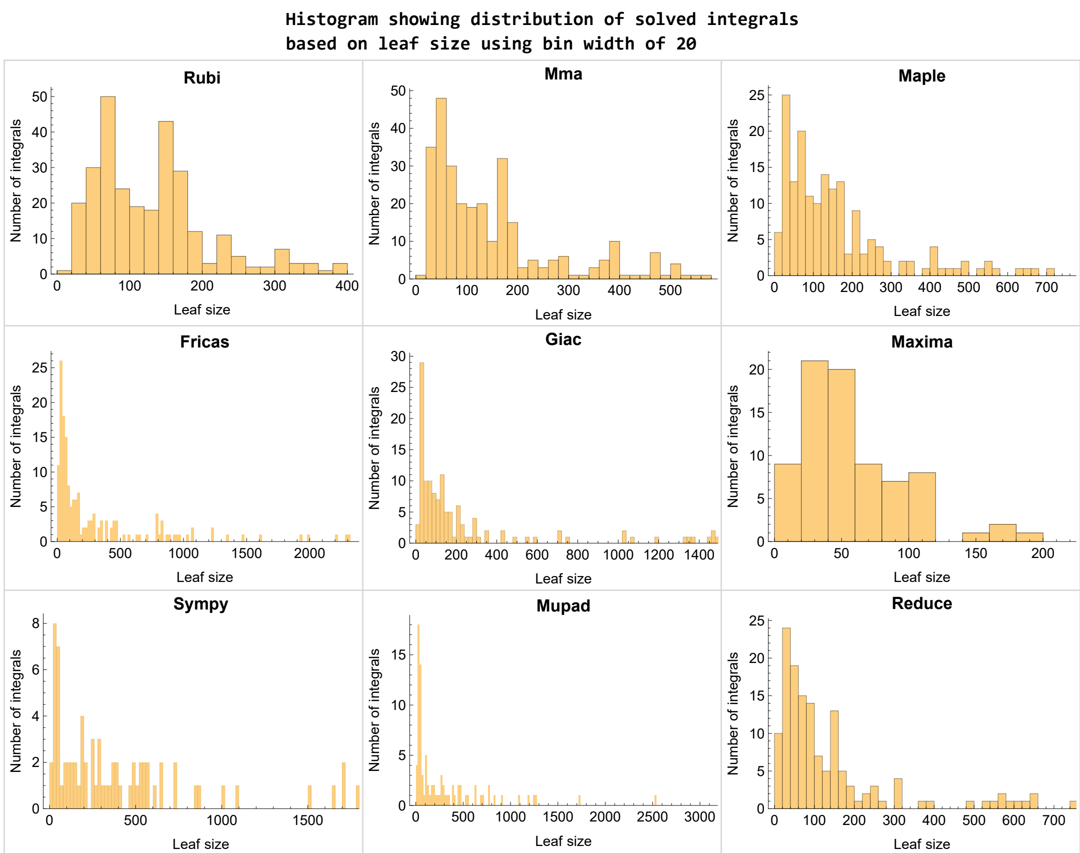


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

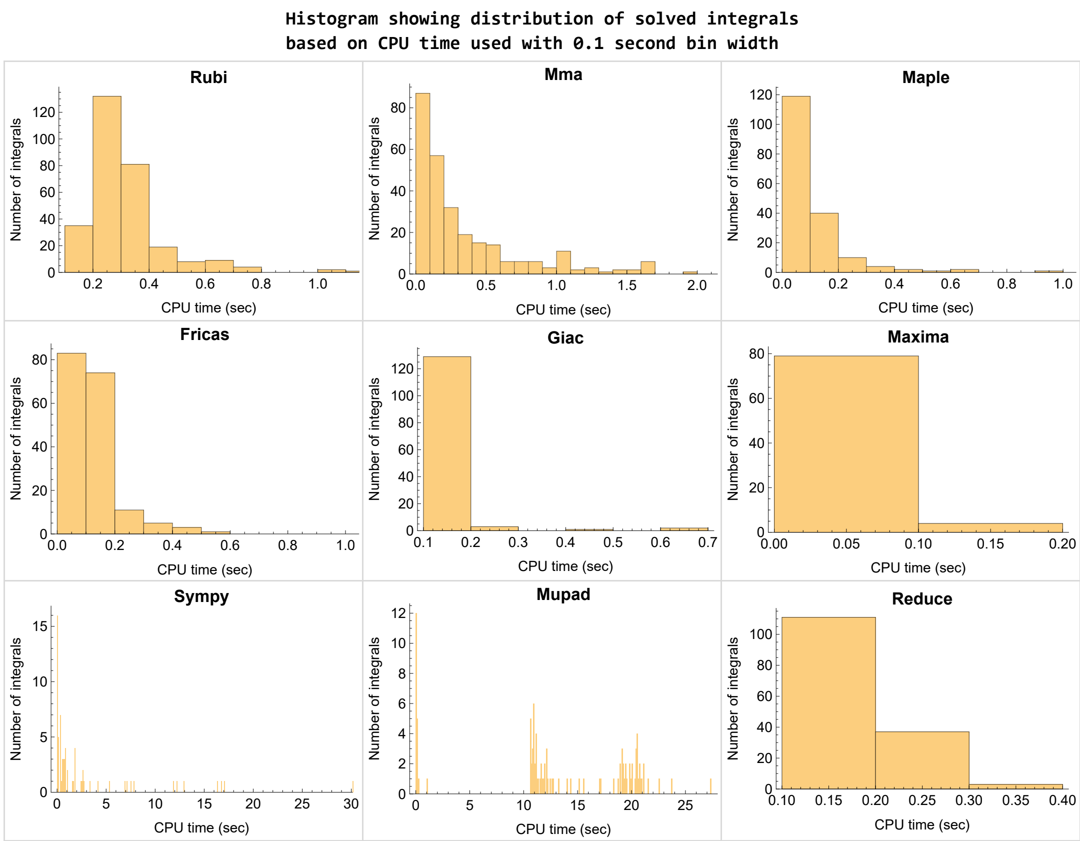


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

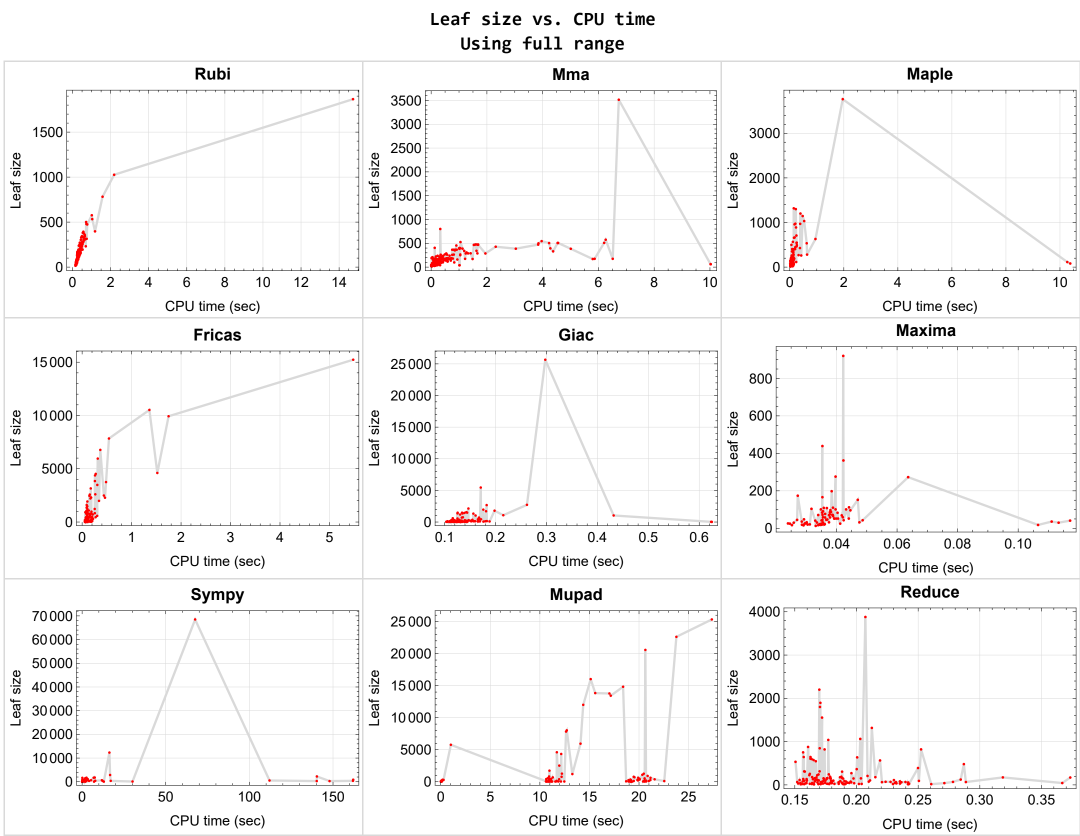


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {82, 83, 84, 98, 99, 100, 290, 291}

Mathematica {77, 93, 195, 196, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 243, 244, 245, 246, 247, 248}

Maple $\{1, 2, 3, 147, 148, 176, 177, 240, 241, 242, 292\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

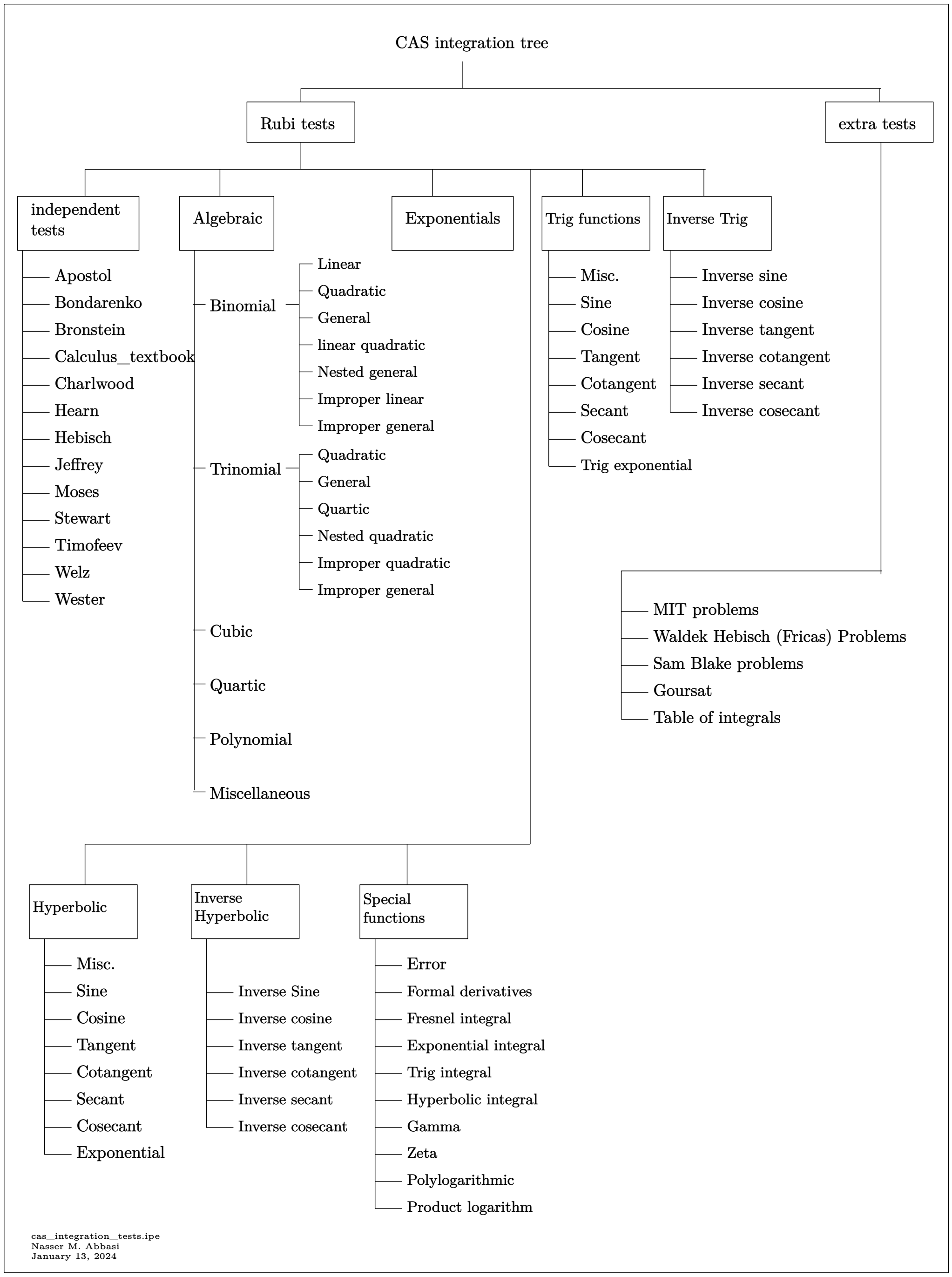
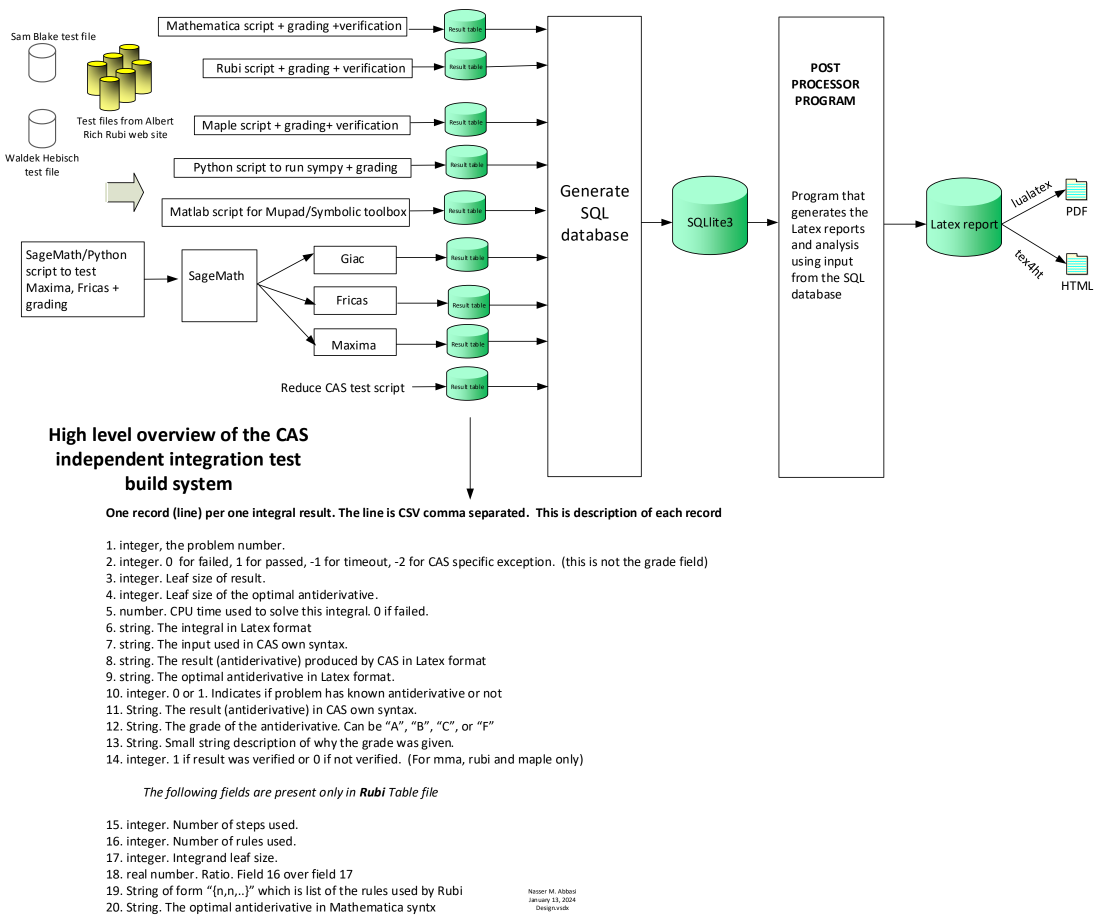


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

B grade { }

C grade { 107 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 3, 4, 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 216, 223, 226, 227, 228, 229, 230, 231, 232, 237, 240, 241, 242, 243, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292 }

B grade { 2, 193, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 224, 225, 233, 234, 235, 236, 238, 239, 244, 245, 246, 248, 264, 265, 266, 291 }

C grade { 6, 7, 8, 78, 79, 80, 81, 94, 95, 96, 97, 107, 115, 116, 117, 118, 119, 120, 175, 188, 189, 190 }

F normal fail { 293, 294 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 76, 108, 109, 110, 111, 112, 113, 114, 117, 118, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 144, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 199, 216, 223 }

B grade { 30, 31, 35, 56, 69, 151, 152, 158, 178, 179, 180, 181, 182, 183, 184, 194, 197, 198, 200, 201, 264, 265, 266, 290, 291 }

C grade { 1, 2, 3, 115, 116, 119, 120, 147, 148, 176, 177, 185, 186, 187, 188, 189, 190, 240, 241, 242, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292 }

F normal fail { 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 121, 122, 135, 136, 137, 139, 140, 141, 142, 143, 145, 146, 149, 150, 175, 191, 192, 193, 195, 196, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 293, 294 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 37, 38, 39, 40, 41, 42, 43, 44, 45, 49, 50, 51, 52, 56, 57, 58, 59, 64, 65, 66, 71, 72, 73, 86, 87, 88, 89, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 144, 147, 148, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 194, 197, 198, 199, 200, 201, 216, 223, 230, 268, 270, 271 }

B grade { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 70, 157, 181, 185, 186, 187, 188, 189, 190, 237, 240, 241, 242, 264, 265, 266, 267, 269, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292 }

C grade { 115, 120 }

F normal fail { 78, 79, 80, 81, 82, 83, 84, 90, 91, 92, 105, 106, 135, 136, 137, 139, 140, 141, 142, 143, 145, 146, 149, 150, 191, 192, 193, 195, 196, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 243, 244, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 293, 294 }

F(-1) timedout fail { 46, 47, 48, 53, 54, 55, 60, 61, 62, 63, 67, 68, 69, 74, 75, 76, 94, 95, 96, 97, 98, 99, 100 }

F(-2) exception fail { 77, 85, 93, 101, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 245, 246, 247, 248 }

Maxima

A grade { 1, 2, 4, 5, 6, 7, 37, 38, 39, 40, 41, 42, 43, 44, 45, 102, 103, 104, 108, 109, 110, 111, 112, 113, 114, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 144, 147, 148, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 197, 198, 199, 200, 201, 240, 241, 242, 292 }

B grade { 3, 157, 264, 265, 266, 290, 291 }

C grade { }

F normal fail { 8, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 105, 106, 107, 115, 116, 117, 118, 119, 120, 122, 135, 136, 137, 139, 140, 141, 142, 143, 145, 146, 149, 150, 175, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 293, 294 }

F(-1) timedout fail { }

F(-2) exception fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 89, 104, 111, 115, 116, 117, 123, 124, 125, 129, 130, 131, 148, 152, 153, 154, 155, 156, 157, 165, 166, 167, 170, 171, 172, 176, 181, 197, 198, 199, 268, 270, 290, 291 }

B grade { 86, 87, 88, 102, 103, 147, 151, 187, 240, 241, 242, 264, 265, 266, 267, 269, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292 }

C grade { }

F normal fail { 57, 58, 59, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 105, 106, 107, 108, 109, 110, 112, 113, 114, 118, 119, 120, 121, 122, 126, 127, 128, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 149, 150, 158, 159, 160, 161, 162, 163, 164, 168, 169, 173, 174, 175, 177, 178, 179, 180, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 194, 195, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 293, 294 }

F(-1) timedout fail { 94 }

F(-2) exception fail { 63, 76, 257, 272 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 56, 74, 75, 102, 103, 104, 107, 111, 151, 152, 153, 154, 155, 156, 157, 176, 181, 194, 197, 198, 199, 200, 201, 240, 241, 242, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292 }

C grade { }

F normal fail { }

F(-1) timedout fail { 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 105, 106, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 293, 294 }

F(-2) exception fail { }

Sympy

A grade { 4, 5, 6, 7, 8, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 60, 61, 62, 63, 108, 267, 269, 272, 274 }

B grade { 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 28, 29, 30, 31, 32, 33, 34, 109, 110, 111, 112, 197, 198, 199, 200, 201, 240, 241, 242, 264, 265, 266, 268, 270, 271, 273, 275, 276, 277, 278, 283, 284, 286, 290, 291, 292 }

C grade { }

F normal fail { 2, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 81, 85, 86, 87, 88, 89, 90, 91, 92, 97, 101, 102, 103, 104, 105, 106, 107, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 160, 161, 162, 166, 167, 168, 169, 172, 173, 175, 176, 185, 186, 187, 188, 189, 190, 191, 192, 193, 203, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 243, 245, 246, 247, 248, 254, 293, 294 }

F(-1) timedout fail { 1, 3, 17, 18, 25, 26, 27, 35, 36, 77, 78, 80, 82, 83, 93, 94, 95, 96, 98, 99, 114, 151, 156, 157, 158, 159, 163, 164, 165, 170, 171, 174, 177, 178, 179, 180, 181, 182, 183, 184, 194, 195, 196, 202, 204, 205, 207, 208, 209, 210, 211, 244, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 279, 280, 281, 282, 285, 287, 288, 289 }

F(-2) exception fail { 79, 84, 100, 113, 206 }

Reduce

A grade { }

B grade { 1, 2, 3, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 86, 87, 88, 89, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 144, 147, 148, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 194, 197, 198, 199, 200, 201, 240, 241, 242, 264, 265, 266, 290, 291, 292 }

C grade { }

F normal fail { 4, 5, 6, 7, 8, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 105, 106, 115, 116, 117, 118, 119, 120, 135, 136, 137, 139, 140, 141, 142, 143, 145, 146, 149, 150, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 293, 294 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	39	31	83	13	0	23	15	59
N.S.	1	1.06	0.58	0.46	1.24	0.19	0.00	0.34	0.22	0.88
time (sec)	N/A	0.206	1.019	0.126	0.040	0.062	0.000	0.131	0.155	20.549

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	45	454	26	96	11	0	28	11	109
N.S.	1	0.60	6.05	0.35	1.28	0.15	0.00	0.37	0.15	1.45
time (sec)	N/A	0.200	0.904	0.077	0.045	0.062	0.000	0.111	0.166	22.547

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	22	86	13	0	30	15	33
N.S.	1	1.00	0.95	0.56	2.21	0.33	0.00	0.77	0.38	0.85
time (sec)	N/A	0.190	1.014	0.083	0.038	0.066	0.000	0.127	0.155	20.573

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	39	37	31	30	30	37	30	30	32
N.S.	1	1.05	1.00	0.84	0.81	0.81	1.00	0.81	0.81	0.86
time (sec)	N/A	0.202	0.015	0.037	0.113	0.066	0.083	0.623	0.165	0.069

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	16	20
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.70	0.87
time (sec)	N/A	0.172	0.008	0.037	0.107	0.061	0.066	0.625	0.170	0.053

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	197	31	36	32	41	33	12	34
N.S.	1	1.10	5.05	0.79	0.92	0.82	1.05	0.85	0.31	0.87
time (sec)	N/A	0.215	0.046	0.054	0.111	0.065	0.090	0.131	0.159	20.411

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	50	208	38	41	49	48	45	14	41
N.S.	1	1.04	4.33	0.79	0.85	1.02	1.00	0.94	0.29	0.85
time (sec)	N/A	0.229	0.047	0.060	0.117	0.064	0.105	0.113	0.162	20.709

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	197	31	0	32	41	33	12	34
N.S.	1	1.10	5.05	0.79	0.00	0.82	1.05	0.85	0.31	0.87
time (sec)	N/A	0.228	0.023	0.046	0.000	0.063	0.090	0.103	0.164	0.033

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	140	164	0	466	605	145	312	183
N.S.	1	1.00	0.95	1.12	0.00	3.17	4.12	0.99	2.12	1.24
time (sec)	N/A	0.341	0.125	0.096	0.000	0.081	0.784	0.128	0.158	20.706

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	112	128	0	383	498	113	255	151
N.S.	1	1.00	0.95	1.08	0.00	3.25	4.22	0.96	2.16	1.28
time (sec)	N/A	0.301	0.081	0.079	0.000	0.077	0.646	0.130	0.162	0.123

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	84	98	0	297	381	86	180	112
N.S.	1	1.00	0.94	1.10	0.00	3.34	4.28	0.97	2.02	1.26
time (sec)	N/A	0.269	0.098	0.062	0.000	0.081	0.572	0.110	0.164	0.133

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	75	0	235	306	67	136	172
N.S.	1	1.00	1.04	1.07	0.00	3.36	4.37	0.96	1.94	2.46
time (sec)	N/A	0.239	0.063	0.040	0.000	0.076	0.352	0.127	0.157	20.557

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	185	216	55	84	112
N.S.	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	1.50	2.00
time (sec)	N/A	0.224	0.032	0.035	0.000	0.070	0.188	0.120	0.166	20.477

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	38	35	0	120	124	34	46	46
N.S.	1	1.06	1.12	1.03	0.00	3.53	3.65	1.00	1.35	1.35
time (sec)	N/A	0.186	0.008	0.028	0.000	0.066	0.143	0.116	0.159	0.050

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	63	61	61	0	211	564	62	96	213
N.S.	1	1.02	0.98	0.98	0.00	3.40	9.10	1.00	1.55	3.44
time (sec)	N/A	0.247	0.068	0.046	0.000	0.077	2.548	0.123	0.173	19.894

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	85	77	81	0	269	862	79	158	339
N.S.	1	1.05	0.95	1.00	0.00	3.32	10.64	0.98	1.95	4.19
time (sec)	N/A	0.296	0.077	0.056	0.000	0.095	162.070	0.116	0.176	20.062

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	113	102	128	0	358	0	105	225	447
N.S.	1	1.09	0.98	1.23	0.00	3.44	0.00	1.01	2.16	4.30
time (sec)	N/A	0.336	0.127	0.053	0.000	0.098	0.000	0.133	0.162	20.057

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	140	131	157	0	445	0	136	308	524
N.S.	1	1.02	0.96	1.15	0.00	3.25	0.00	0.99	2.25	3.82
time (sec)	N/A	0.390	0.097	0.079	0.000	0.142	0.000	0.129	0.158	20.564

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	186	163	238	0	1029	1012	188	846	382
N.S.	1	0.95	0.83	1.21	0.00	5.25	5.16	0.96	4.32	1.95
time (sec)	N/A	0.415	0.233	0.105	0.000	0.085	1.511	0.112	0.170	21.575

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	154	132	198	0	837	842	161	752	261
N.S.	1	1.03	0.88	1.32	0.00	5.58	5.61	1.07	5.01	1.74
time (sec)	N/A	0.360	0.178	0.089	0.000	0.082	1.077	0.111	0.157	21.146

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	125	109	169	0	635	729	125	547	279
N.S.	1	1.10	0.96	1.48	0.00	5.57	6.39	1.10	4.80	2.45
time (sec)	N/A	0.320	0.145	0.085	0.000	0.101	0.838	0.105	0.167	19.837

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	81	97	0	387	280	88	243	135
N.S.	1	1.06	1.21	1.45	0.00	5.78	4.18	1.31	3.63	2.01
time (sec)	N/A	0.225	0.088	0.050	0.000	0.073	0.355	0.114	0.178	0.108

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	338	253	76	223	110
N.S.	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	3.38	1.67
time (sec)	N/A	0.219	0.064	0.053	0.000	0.090	0.321	0.109	0.168	20.985

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	341	265	76	241	119
N.S.	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	3.65	1.80
time (sec)	N/A	0.215	0.073	0.046	0.000	0.107	0.335	0.115	0.169	19.469

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	139	107	177	0	781	0	126	644	620
N.S.	1	1.29	0.99	1.64	0.00	7.23	0.00	1.17	5.96	5.74
time (sec)	N/A	0.360	0.175	0.060	0.000	0.142	0.000	0.111	0.157	21.159

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	173	131	205	0	975	0	171	877	775
N.S.	1	1.17	0.89	1.39	0.00	6.59	0.00	1.16	5.93	5.24
time (sec)	N/A	0.412	0.251	0.061	0.000	0.189	0.000	0.112	0.161	19.463

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	213	175	255	0	1226	0	229	1039	914
N.S.	1	1.05	0.87	1.26	0.00	6.07	0.00	1.13	5.14	4.52
time (sec)	N/A	0.467	0.369	0.078	0.000	0.254	0.000	0.134	0.177	20.868

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	257	260	401	0	1926	1714	282	1896	705
N.S.	1	1.08	1.09	1.68	0.00	8.09	7.20	1.18	7.97	2.96
time (sec)	N/A	0.530	0.343	0.118	0.000	0.106	2.610	0.114	0.171	19.538

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	221	221	357	0	1603	1510	245	1555	620
N.S.	1	1.16	1.16	1.88	0.00	8.44	7.95	1.29	8.18	3.26
time (sec)	N/A	0.461	0.293	0.128	0.000	0.092	1.892	0.113	0.172	19.921

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	122	174	260	0	953	547	202	644	343
N.S.	1	1.14	1.63	2.43	0.00	8.91	5.11	1.89	6.02	3.21
time (sec)	N/A	0.287	0.167	0.072	0.000	0.077	0.864	0.122	0.163	19.135

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	118	126	223	0	872	513	163	604	271
N.S.	1	1.10	1.18	2.08	0.00	8.15	4.79	1.52	5.64	2.53
time (sec)	N/A	0.272	0.190	0.069	0.000	0.084	0.738	0.119	0.163	0.151

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	129	131	210	0	887	570	154	820	313
N.S.	1	1.12	1.14	1.83	0.00	7.71	4.96	1.34	7.13	2.72
time (sec)	N/A	0.292	0.136	0.075	0.000	0.104	0.858	0.113	0.174	0.193

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	115	102	118	0	788	481	135	575	253
N.S.	1	1.12	0.99	1.15	0.00	7.65	4.67	1.31	5.58	2.46
time (sec)	N/A	0.262	0.095	0.061	0.000	0.084	0.652	0.119	0.165	18.944

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	112	97	116	0	785	474	136	598	285
N.S.	1	1.09	0.94	1.13	0.00	7.62	4.60	1.32	5.81	2.77
time (sec)	N/A	0.256	0.097	0.072	0.000	0.086	0.706	0.116	0.164	19.175

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	236	178	352	0	1985	0	239	1801	1089
N.S.	1	1.28	0.96	1.90	0.00	10.73	0.00	1.29	9.74	5.89
time (sec)	N/A	0.498	0.343	0.082	0.000	0.341	0.000	0.116	0.170	20.333

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	280	221	404	0	2280	0	309	2201	1255
N.S.	1	1.17	0.92	1.69	0.00	9.54	0.00	1.29	9.21	5.25
time (sec)	N/A	0.565	0.399	0.089	0.000	0.462	0.000	0.119	0.170	20.479

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	40	27	30	30	34	32	30	26
N.S.	1	1.05	1.00	0.68	0.75	0.75	0.85	0.80	0.75	0.65
time (sec)	N/A	0.203	0.008	0.027	0.026	0.059	0.068	0.111	0.165	0.048

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	35	33	22	25	25	27	27	25	21
N.S.	1	1.06	1.00	0.67	0.76	0.76	0.82	0.82	0.76	0.64
time (sec)	N/A	0.203	0.006	0.029	0.025	0.064	0.074	0.122	0.162	0.037

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	20	20	20	22	20	16
N.S.	1	1.08	1.00	0.65	0.77	0.77	0.77	0.85	0.77	0.62
time (sec)	N/A	0.196	0.006	0.025	0.030	0.057	0.078	0.105	0.164	0.080

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	23	21	14	17	17	17	19	17	13
N.S.	1	1.10	1.00	0.67	0.81	0.81	0.81	0.90	0.81	0.62
time (sec)	N/A	0.189	0.004	0.023	0.034	0.059	0.051	0.123	0.169	0.071

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	25	21	14	17	17	15	19	17	8
N.S.	1	1.19	1.00	0.67	0.81	0.81	0.71	0.90	0.81	0.38
time (sec)	N/A	0.190	0.004	0.025	0.025	0.061	0.058	0.130	0.158	19.224

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	29	27	18	21	21	24	24	21	17
N.S.	1	1.07	1.00	0.67	0.78	0.78	0.89	0.89	0.78	0.63
time (sec)	N/A	0.196	0.007	0.034	0.031	0.070	0.081	0.109	0.156	19.104

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	34	27	26	30	31	29	30	22
N.S.	1	1.06	1.00	0.79	0.76	0.88	0.91	0.85	0.88	0.65
time (sec)	N/A	0.199	0.006	0.033	0.024	0.070	0.085	0.135	0.173	19.228

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	43	41	31	31	39	36	34	39	26
N.S.	1	1.05	1.00	0.76	0.76	0.95	0.88	0.83	0.95	0.63
time (sec)	N/A	0.202	0.006	0.034	0.034	0.073	0.094	0.112	0.159	0.047

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	50	48	36	36	44	41	39	44	32
N.S.	1	1.04	1.00	0.75	0.75	0.92	0.85	0.81	0.92	0.67
time (sec)	N/A	0.212	0.006	0.035	0.029	0.067	0.109	0.124	0.164	19.318

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	195	113	151	0	0	182	109	116	0
N.S.	1	1.12	0.65	0.87	0.00	0.00	1.05	0.63	0.67	0.00
time (sec)	N/A	0.364	0.325	0.095	0.000	0.000	0.550	0.152	0.167	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	125	89	99	0	0	153	81	80	0
N.S.	1	1.08	0.77	0.85	0.00	0.00	1.32	0.70	0.69	0.00
time (sec)	N/A	0.279	0.248	0.076	0.000	0.000	0.389	0.166	0.156	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	62	50	0	0	119	52	43	72
N.S.	1	1.00	1.11	0.89	0.00	0.00	2.12	0.93	0.77	1.29
time (sec)	N/A	0.210	0.177	0.069	0.000	0.000	0.365	0.141	0.155	18.703

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	0	19	0	25	27	0
N.S.	1	1.00	1.00	0.80	0.00	0.76	0.00	1.00	1.08	0.00
time (sec)	N/A	0.157	0.113	0.078	0.000	0.092	0.000	0.139	0.158	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	90	48	67	0	42	0	84	67	0
N.S.	1	1.07	0.57	0.80	0.00	0.50	0.00	1.00	0.80	0.00
time (sec)	N/A	0.230	0.133	0.083	0.000	0.100	0.000	0.123	0.152	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	160	72	119	0	64	0	146	105	0
N.S.	1	1.13	0.51	0.84	0.00	0.45	0.00	1.03	0.74	0.00
time (sec)	N/A	0.321	0.147	0.081	0.000	0.100	0.000	0.140	0.169	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	230	96	171	0	86	0	208	141	0
N.S.	1	1.15	0.48	0.86	0.00	0.43	0.00	1.04	0.70	0.00
time (sec)	N/A	0.433	0.160	0.087	0.000	0.102	0.000	0.125	0.173	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	231	137	227	0	0	0	148	189	0
N.S.	1	1.14	0.68	1.12	0.00	0.00	0.00	0.73	0.94	0.00
time (sec)	N/A	0.695	0.531	0.088	0.000	0.000	0.000	0.183	0.188	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	157	113	175	0	0	0	120	153	0
N.S.	1	1.09	0.78	1.22	0.00	0.00	0.00	0.83	1.06	0.00
time (sec)	N/A	0.428	0.399	0.080	0.000	0.000	0.000	0.180	0.176	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	87	84	124	0	0	0	92	113	0
N.S.	1	1.06	1.02	1.51	0.00	0.00	0.00	1.12	1.38	0.00
time (sec)	N/A	0.241	0.282	0.079	0.000	0.000	0.000	0.188	0.154	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	31	45	0	36	0	34	33	40
N.S.	1	1.00	1.24	1.80	0.00	1.44	0.00	1.36	1.32	1.60
time (sec)	N/A	0.158	0.158	0.076	0.000	0.098	0.000	0.143	0.153	18.953

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	85	55	46	0	63	0	0	76	0
N.S.	1	1.39	0.90	0.75	0.00	1.03	0.00	0.00	1.25	0.00
time (sec)	N/A	0.236	0.203	0.086	0.000	0.101	0.000	0.000	0.172	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	155	81	98	0	87	0	0	114	0
N.S.	1	1.30	0.68	0.82	0.00	0.73	0.00	0.00	0.96	0.00
time (sec)	N/A	0.326	0.212	0.091	0.000	0.104	0.000	0.000	0.160	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	225	105	150	0	109	0	0	152	0
N.S.	1	1.27	0.59	0.85	0.00	0.62	0.00	0.00	0.86	0.00
time (sec)	N/A	0.432	0.231	0.091	0.000	0.113	0.000	0.000	0.179	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	231	126	177	0	0	199	123	134	0
N.S.	1	1.13	0.62	0.87	0.00	0.00	0.98	0.60	0.66	0.00
time (sec)	N/A	0.395	0.298	0.084	0.000	0.000	1.801	0.158	0.160	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	161	102	125	0	0	170	95	98	0
N.S.	1	1.10	0.70	0.86	0.00	0.00	1.16	0.65	0.67	0.00
time (sec)	N/A	0.311	0.272	0.079	0.000	0.000	0.550	0.169	0.160	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	93	80	74	0	0	143	67	62	0
N.S.	1	1.07	0.92	0.85	0.00	0.00	1.64	0.77	0.71	0.00
time (sec)	N/A	0.230	0.251	0.076	0.000	0.000	0.353	0.141	0.171	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	40	32	0	0	99	0	27	0
N.S.	1	1.00	1.18	0.94	0.00	0.00	2.91	0.00	0.79	0.00
time (sec)	N/A	0.173	0.154	0.078	0.000	0.000	0.470	0.000	0.176	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	35	41	0	29	0	53	41	0
N.S.	1	1.00	0.65	0.76	0.00	0.54	0.00	0.98	0.76	0.00
time (sec)	N/A	0.184	0.158	0.080	0.000	0.100	0.000	0.134	0.171	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	124	59	93	0	50	0	115	81	0
N.S.	1	1.11	0.53	0.83	0.00	0.45	0.00	1.03	0.72	0.00
time (sec)	N/A	0.265	0.178	0.083	0.000	0.109	0.000	0.123	0.166	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	194	83	145	0	72	0	177	117	0
N.S.	1	1.14	0.49	0.85	0.00	0.42	0.00	1.04	0.69	0.00
time (sec)	N/A	0.368	0.190	0.086	0.000	0.113	0.000	0.143	0.168	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	194	124	201	0	0	0	134	171	0
N.S.	1	1.11	0.71	1.16	0.00	0.00	0.00	0.77	0.98	0.00
time (sec)	N/A	0.525	0.593	0.083	0.000	0.000	0.000	0.167	0.167	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	119	100	148	0	0	0	106	135	0
N.S.	1	1.03	0.87	1.29	0.00	0.00	0.00	0.92	1.17	0.00
time (sec)	N/A	0.311	0.421	0.082	0.000	0.000	0.000	0.165	0.176	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	62	72	101	0	0	0	71	89	0
N.S.	1	1.03	1.20	1.68	0.00	0.00	0.00	1.18	1.48	0.00
time (sec)	N/A	0.215	0.234	0.082	0.000	0.000	0.000	0.176	0.190	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	46	25	0	54	0	26	55	0
N.S.	1	1.00	1.53	0.83	0.00	1.80	0.00	0.87	1.83	0.00
time (sec)	N/A	0.175	0.186	0.086	0.000	0.122	0.000	0.131	0.180	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	119	70	72	0	79	0	0	94	0
N.S.	1	1.34	0.79	0.81	0.00	0.89	0.00	0.00	1.06	0.00
time (sec)	N/A	0.281	0.222	0.085	0.000	0.111	0.000	0.000	0.157	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	189	96	124	0	101	0	0	132	0
N.S.	1	1.29	0.65	0.84	0.00	0.69	0.00	0.00	0.90	0.00
time (sec)	N/A	0.374	0.243	0.089	0.000	0.123	0.000	0.000	0.165	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	259	120	176	0	123	0	0	170	0
N.S.	1	1.26	0.59	0.86	0.00	0.60	0.00	0.00	0.83	0.00
time (sec)	N/A	0.490	0.255	0.090	0.000	0.145	0.000	0.000	0.373	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	75	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.23	0.69
time (sec)	N/A	0.203	10.027	0.000	0.000	0.000	0.000	0.000	0.489	11.186

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	15	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.25	0.69
time (sec)	N/A	0.205	10.023	0.000	0.000	0.000	0.000	0.000	0.244	11.242

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	111	105	84	0	0	0	0	821	0
N.S.	1	1.05	0.99	0.79	0.00	0.00	0.00	0.00	7.75	0.00
time (sec)	N/A	0.293	0.248	0.017	0.000	0.000	0.000	0.000	0.252	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	155	183	0	0	0	0	0	0	0
N.S.	1	0.97	1.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.341	0.624	0.000	0.000	0.000	0.000	0.000	0.480	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	807	782	170	0	0	0	0	0	0	0
N.S.	1	0.97	0.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.576	5.796	0.000	0.000	0.000	0.000	0.000	0.330	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	471	479	170	0	0	0	0	0	0	0
N.S.	1	1.02	0.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.727	1.487	0.000	0.000	0.000	0.000	0.000	0.296	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	271	170	0	0	0	0	0	0	0
N.S.	1	1.06	0.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.381	0.611	0.000	0.000	0.000	0.000	0.000	0.294	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	164	165	0	0	0	0	0	361	0
N.S.	1	1.02	1.02	0.00	0.00	0.00	0.00	0.00	2.24	0.00
time (sec)	N/A	0.251	0.281	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	169	166	0	0	0	0	0	189	0
N.S.	1	1.04	1.02	0.00	0.00	0.00	0.00	0.00	1.17	0.00
time (sec)	N/A	0.315	0.452	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	186	173	0	0	0	0	0	516	0
N.S.	1	1.07	0.99	0.00	0.00	0.00	0.00	0.00	2.97	0.00
time (sec)	N/A	0.312	0.546	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	186	173	0	0	0	0	0	1214	0
N.S.	1	1.07	0.99	0.00	0.00	0.00	0.00	0.00	6.98	0.00
time (sec)	N/A	0.307	0.665	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	73	68	0	0	0	0	0	0	0
N.S.	1	0.95	0.88	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.243	0.110	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	353	303	184	0	0	474	0	1184	533	0
N.S.	1	0.86	0.52	0.00	0.00	1.34	0.00	3.35	1.51	0.00
time (sec)	N/A	0.435	0.300	0.000	0.000	0.114	0.000	0.130	0.151	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	263	235	140	0	0	295	0	702	313	0
N.S.	1	0.89	0.53	0.00	0.00	1.12	0.00	2.67	1.19	0.00
time (sec)	N/A	0.366	0.293	0.000	0.000	0.105	0.000	0.134	0.170	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	171	113	0	0	160	0	348	153	0
N.S.	1	0.99	0.65	0.00	0.00	0.92	0.00	2.01	0.88	0.00
time (sec)	N/A	0.306	0.147	0.000	0.000	0.094	0.000	0.129	0.166	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	83	54	0	0	65	0	119	58	0
N.S.	1	1.12	0.73	0.00	0.00	0.88	0.00	1.61	0.78	0.00
time (sec)	N/A	0.225	0.103	0.000	0.000	0.095	0.000	0.138	0.168	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	0	0	0	0	0	105	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	1.62	0.00
time (sec)	N/A	0.211	0.103	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	71	61	0	0	0	0	0	232	0
N.S.	1	1.04	0.90	0.00	0.00	0.00	0.00	0.00	3.41	0.00
time (sec)	N/A	0.214	0.112	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	71	61	0	0	0	0	0	575	0
N.S.	1	1.04	0.90	0.00	0.00	0.00	0.00	0.00	8.46	0.00
time (sec)	N/A	0.215	0.111	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	159	187	0	0	0	0	0	0	0
N.S.	1	0.98	1.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	0.729	0.000	0.000	0.000	0.000	0.000	1.007	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	2032	1866	174	0	0	0	0	0	20	0
N.S.	1	0.92	0.09	0.00	0.00	0.00	0.00	0.00	0.01	0.00
time (sec)	N/A	14.730	6.513	0.000	0.000	0.000	0.000	0.000	200.018	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1092	1026	174	0	0	0	0	0	0	0
N.S.	1	0.94	0.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.185	5.864	0.000	0.000	0.000	0.000	0.000	0.365	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	499	501	174	0	0	0	0	0	0	0
N.S.	1	1.00	0.35	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.714	1.218	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	169	0	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.344	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	178	175	0	0	0	0	0	94	0
N.S.	1	1.04	1.02	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.311	0.461	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	192	180	0	0	0	0	0	1166	0
N.S.	1	1.08	1.01	0.00	0.00	0.00	0.00	0.00	6.55	0.00
time (sec)	N/A	0.319	0.647	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	182	194	182	0	0	0	0	0	0	0
N.S.	1	1.07	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	0.733	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	77	68	0	0	0	0	0	0	0
N.S.	1	0.95	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	0.122	0.000	0.000	0.000	0.000	0.000	0.354	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	436	330	207	0	362	579	0	1564	636	777
N.S.	1	0.76	0.47	0.00	0.83	1.33	0.00	3.59	1.46	1.78
time (sec)	N/A	0.455	0.364	0.000	0.042	0.305	0.000	0.143	0.201	12.189

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	292	239	143	0	198	297	0	745	306	390
N.S.	1	0.82	0.49	0.00	0.68	1.02	0.00	2.55	1.05	1.34
time (sec)	N/A	0.353	0.288	0.000	0.038	0.221	0.000	0.136	0.188	11.942

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	138	83	0	77	110	0	229	98	138
N.S.	1	0.97	0.58	0.00	0.54	0.77	0.00	1.61	0.69	0.97
time (sec)	N/A	0.276	0.033	0.000	0.036	0.135	0.000	0.149	0.181	11.691

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	0	0	0	0	0	66	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.96	0.00
time (sec)	N/A	0.214	0.111	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	75	61	0	0	0	0	0	249	0
N.S.	1	1.04	0.85	0.00	0.00	0.00	0.00	0.00	3.46	0.00
time (sec)	N/A	0.215	0.123	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	A	F	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	162	101	0	0	82	0	0	87	69
N.S.	1	1.11	0.69	0.00	0.00	0.56	0.00	0.00	0.60	0.47
time (sec)	N/A	0.315	0.415	0.000	0.000	0.204	0.000	0.000	0.187	11.674

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	38	47	45	38	63	0	38	0
N.S.	1	0.89	0.83	1.02	0.98	0.83	1.37	0.00	0.83	0.00
time (sec)	N/A	0.196	0.058	0.127	0.027	0.154	12.964	0.000	0.180	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	26	27	31	32	24	48	0	24	0
N.S.	1	0.93	0.96	1.11	1.14	0.86	1.71	0.00	0.86	0.00
time (sec)	N/A	0.183	0.048	0.110	0.030	0.069	7.124	0.000	0.190	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	19	15	46	0	15	0
N.S.	1	1.00	1.00	1.20	1.27	1.00	3.07	0.00	1.00	0.00
time (sec)	N/A	0.155	0.026	0.105	0.029	0.093	1.852	0.000	0.190	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	26	25	26	27	22	54	25	22	20
N.S.	1	1.13	1.09	1.13	1.17	0.96	2.35	1.09	0.96	0.87
time (sec)	N/A	0.163	0.044	0.101	0.030	0.090	1.667	0.124	0.204	10.940

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	54	48	58	58	59	97	0	59	0
N.S.	1	0.95	0.84	1.02	1.02	1.04	1.70	0.00	1.04	0.00
time (sec)	N/A	0.207	0.107	0.105	0.036	0.071	30.131	0.000	0.196	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	70	63	75	71	72	0	0	72	0
N.S.	1	0.92	0.83	0.99	0.93	0.95	0.00	0.00	0.95	0.00
time (sec)	N/A	0.210	0.123	0.109	0.039	0.075	0.000	0.000	0.180	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	84	76	90	84	85	0	0	85	0
N.S.	1	0.90	0.82	0.97	0.90	0.91	0.00	0.00	0.91	0.00
time (sec)	N/A	0.224	0.147	0.113	0.035	0.081	0.000	0.000	0.180	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	259	34	54	0	270	0	203	25	0
N.S.	1	1.48	0.19	0.31	0.00	1.54	0.00	1.16	0.14	0.00
time (sec)	N/A	0.461	0.053	0.143	0.000	0.083	0.000	0.142	0.182	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	34	54	0	212	0	136	25	0
N.S.	1	1.00	0.21	0.34	0.00	1.32	0.00	0.85	0.16	0.00
time (sec)	N/A	0.339	0.050	0.137	0.000	0.086	0.000	0.123	0.193	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	48	32	79	0	143	0	38	25	0
N.S.	1	0.96	0.64	1.58	0.00	2.86	0.00	0.76	0.50	0.00
time (sec)	N/A	0.189	0.049	0.130	0.000	0.131	0.000	0.139	0.187	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	34	97	0	161	0	0	47	0
N.S.	1	1.03	0.50	1.43	0.00	2.37	0.00	0.00	0.69	0.00
time (sec)	N/A	0.216	0.052	0.136	0.000	0.180	0.000	0.000	0.188	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	178	34	73	0	171	0	0	47	0
N.S.	1	1.01	0.19	0.41	0.00	0.97	0.00	0.00	0.27	0.00
time (sec)	N/A	0.362	0.053	0.141	0.000	0.086	0.000	0.000	0.184	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	277	34	73	0	249	0	0	47	0
N.S.	1	1.45	0.18	0.38	0.00	1.30	0.00	0.00	0.25	0.00
time (sec)	N/A	0.492	0.054	0.149	0.000	0.086	0.000	0.000	0.173	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	38	0	43	59	0	0	41	0
N.S.	1	1.00	1.03	0.00	1.16	1.59	0.00	0.00	1.11	0.00
time (sec)	N/A	0.181	0.053	0.000	0.049	0.077	0.000	0.000	0.189	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	43	0	0	59	0	0	45	0
N.S.	1	1.00	1.13	0.00	0.00	1.55	0.00	0.00	1.18	0.00
time (sec)	N/A	0.183	0.116	0.000	0.000	0.080	0.000	0.000	0.180	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	62	46	61	25	28	0	53	24	0
N.S.	1	0.67	0.49	0.66	0.27	0.30	0.00	0.57	0.26	0.00
time (sec)	N/A	0.232	0.059	0.039	0.041	0.075	0.000	0.119	0.186	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	62	46	61	25	28	0	53	24	0
N.S.	1	0.67	0.49	0.66	0.27	0.30	0.00	0.57	0.26	0.00
time (sec)	N/A	0.222	0.048	0.023	0.035	0.073	0.000	0.134	0.177	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	57	39	56	19	20	0	25	17	0
N.S.	1	0.65	0.44	0.64	0.22	0.23	0.00	0.28	0.19	0.00
time (sec)	N/A	0.206	0.015	0.022	0.035	0.070	0.000	0.117	0.171	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	54	38	54	13	15	0	0	15	0
N.S.	1	0.64	0.45	0.64	0.15	0.18	0.00	0.00	0.18	0.00
time (sec)	N/A	0.221	0.031	0.025	0.033	0.078	0.000	0.000	0.190	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	63	42	61	22	23	0	0	20	0
N.S.	1	0.67	0.45	0.65	0.23	0.24	0.00	0.00	0.21	0.00
time (sec)	N/A	0.226	0.049	0.026	0.034	0.111	0.000	0.000	0.178	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	65	47	61	22	23	0	0	24	0
N.S.	1	0.68	0.49	0.64	0.23	0.24	0.00	0.00	0.25	0.00
time (sec)	N/A	0.230	0.048	0.026	0.036	0.115	0.000	0.000	0.182	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	111	123	146	108	144	0	292	151	0
N.S.	1	0.52	0.58	0.69	0.51	0.68	0.00	1.38	0.71	0.00
time (sec)	N/A	0.272	0.125	0.031	0.037	0.171	0.000	0.140	0.179	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	110	124	145	109	145	0	292	151	0
N.S.	1	0.52	0.59	0.69	0.52	0.69	0.00	1.38	0.72	0.00
time (sec)	N/A	0.267	0.102	0.026	0.036	0.082	0.000	0.147	0.176	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	105	122	138	101	130	0	263	146	0
N.S.	1	0.51	0.59	0.67	0.49	0.63	0.00	1.28	0.71	0.00
time (sec)	N/A	0.258	0.118	0.024	0.035	0.078	0.000	0.126	0.189	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	89	67	127	43	44	0	0	44	0
N.S.	1	0.45	0.34	0.65	0.22	0.22	0.00	0.00	0.22	0.00
time (sec)	N/A	0.246	0.067	0.027	0.042	0.082	0.000	0.000	0.194	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	111	124	147	101	131	0	0	148	0
N.S.	1	0.52	0.58	0.69	0.48	0.62	0.00	0.00	0.70	0.00
time (sec)	N/A	0.278	0.133	0.033	0.040	0.110	0.000	0.000	0.198	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	117	124	145	101	134	0	0	152	0
N.S.	1	0.54	0.57	0.67	0.46	0.61	0.00	0.00	0.70	0.00
time (sec)	N/A	0.278	0.134	0.030	0.043	0.119	0.000	0.000	0.173	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	71	53	0	0	0	0	0	15	0
N.S.	1	1.11	0.83	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.214	0.059	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	71	53	0	0	0	0	0	13	0
N.S.	1	1.11	0.83	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.205	0.039	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	62	44	0	0	0	0	0	11	0
N.S.	1	1.13	0.80	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.189	0.034	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	64	45	66	27	22	0	0	22	0
N.S.	1	0.75	0.53	0.78	0.32	0.26	0.00	0.00	0.26	0.00
time (sec)	N/A	0.202	0.062	0.027	0.030	0.075	0.000	0.000	0.191	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	72	51	0	0	0	0	0	18	0
N.S.	1	1.11	0.78	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.202	0.047	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	74	53	0	0	0	0	0	18	0
N.S.	1	1.10	0.79	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.203	0.047	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	73	55	0	0	0	0	0	41	0
N.S.	1	1.14	0.86	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.202	0.060	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	73	55	0	0	0	0	0	39	0
N.S.	1	1.14	0.86	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.200	0.038	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	64	46	0	0	0	0	0	37	0
N.S.	1	1.16	0.84	0.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.188	0.032	0.000	0.000	0.000	0.000	0.000	0.284	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	95	79	104	70	106	0	0	123	0
N.S.	1	0.60	0.50	0.65	0.44	0.67	0.00	0.00	0.77	0.00
time (sec)	N/A	0.257	0.124	0.031	0.038	0.077	0.000	0.000	0.284	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	74	53	0	0	0	0	0	50	0
N.S.	1	1.14	0.82	0.00	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.205	0.053	0.000	0.000	0.000	0.000	0.000	0.275	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	76	55	0	0	0	0	0	50	0
N.S.	1	1.13	0.82	0.00	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.206	0.051	0.000	0.000	0.000	0.000	0.000	0.287	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	247	137	90	499	276	390	0	2719	480	0
N.S.	1	0.55	0.36	2.02	1.12	1.58	0.00	11.01	1.94	0.00
time (sec)	N/A	0.304	0.167	0.060	0.040	0.133	0.000	0.262	0.287	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	111	77	55	99	47	57	0	173	41	0
N.S.	1	0.69	0.50	0.89	0.42	0.51	0.00	1.56	0.37	0.00
time (sec)	N/A	0.236	0.066	0.043	0.037	0.179	0.000	0.125	0.271	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	83	62	0	0	0	0	0	19	0
N.S.	1	1.09	0.82	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.221	0.057	0.000	0.000	0.000	0.000	0.000	0.276	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	85	61	0	0	0	0	0	45	0
N.S.	1	1.12	0.80	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.215	0.059	0.000	0.000	0.000	0.000	0.000	0.286	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	30	204	71	71	0	119	69	41
N.S.	1	1.00	0.70	4.74	1.65	1.65	0.00	2.77	1.60	0.95
time (sec)	N/A	0.190	0.055	0.059	0.034	0.110	0.000	0.140	0.278	11.061

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	30	132	45	45	0	77	43	41
N.S.	1	1.00	0.70	3.07	1.05	1.05	0.00	1.79	1.00	0.95
time (sec)	N/A	0.181	0.041	0.032	0.037	0.188	0.000	0.114	0.367	10.751

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	61	19	19	0	35	17	38
N.S.	1	1.00	0.73	1.49	0.46	0.46	0.00	0.85	0.41	0.93
time (sec)	N/A	0.182	0.026	0.028	0.034	0.073	0.000	0.129	0.261	10.626

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	35	38	19	15	0	26	15	31
N.S.	1	1.00	0.76	0.83	0.41	0.33	0.00	0.57	0.33	0.67
time (sec)	N/A	0.185	0.038	0.039	0.035	0.107	0.000	0.131	0.242	11.559

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	30	29	31	31	0	27	30	39
N.S.	1	1.00	0.70	0.67	0.72	0.72	0.00	0.63	0.70	0.91
time (sec)	N/A	0.184	0.038	0.033	0.036	0.104	0.000	0.132	0.235	11.868

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	30	29	59	59	0	27	56	39
N.S.	1	1.00	0.70	0.67	1.37	1.37	0.00	0.63	1.30	0.91
time (sec)	N/A	0.183	0.047	0.049	0.036	0.160	0.000	0.123	0.242	12.123

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	30	29	87	87	0	27	82	39
N.S.	1	1.00	0.70	0.67	2.02	2.02	0.00	0.63	1.91	0.91
time (sec)	N/A	0.181	0.048	0.048	0.038	0.069	0.000	0.125	0.235	12.467

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	96	208	74	74	0	0	72	0
N.S.	1	0.97	1.22	2.63	0.94	0.94	0.00	0.00	0.91	0.00
time (sec)	N/A	0.217	0.090	0.052	0.038	0.125	0.000	0.000	0.224	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	70	135	48	48	0	0	46	0
N.S.	1	0.97	0.89	1.71	0.61	0.61	0.00	0.00	0.58	0.00
time (sec)	N/A	0.210	0.066	0.032	0.029	0.190	0.000	0.000	0.231	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	44	64	22	22	0	0	20	0
N.S.	1	0.97	0.57	0.83	0.29	0.29	0.00	0.00	0.26	0.00
time (sec)	N/A	0.210	0.041	0.027	0.029	0.075	0.000	0.000	0.230	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	78	47	71	32	24	0	0	24	0
N.S.	1	0.98	0.59	0.89	0.40	0.30	0.00	0.00	0.30	0.00
time (sec)	N/A	0.217	0.069	0.037	0.038	0.114	0.000	0.000	0.243	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	75	40	37	41	41	0	0	35	0
N.S.	1	1.56	0.83	0.77	0.85	0.85	0.00	0.00	0.73	0.00
time (sec)	N/A	0.210	0.074	0.033	0.033	0.166	0.000	0.000	0.232	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	40	37	69	69	0	0	66	0
N.S.	1	0.97	0.51	0.47	0.87	0.87	0.00	0.00	0.84	0.00
time (sec)	N/A	0.211	0.084	0.040	0.042	0.085	0.000	0.000	0.289	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	40	37	97	97	0	0	92	0
N.S.	1	0.97	0.51	0.47	1.23	1.23	0.00	0.00	1.16	0.00
time (sec)	N/A	0.211	0.077	0.037	0.038	0.105	0.000	0.000	0.251	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	116	100	200	72	75	0	123	75	0
N.S.	1	0.38	0.33	0.66	0.24	0.25	0.00	0.41	0.25	0.00
time (sec)	N/A	0.273	0.102	0.055	0.039	0.094	0.000	0.156	0.210	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	88	73	128	45	48	0	80	48	0
N.S.	1	0.45	0.37	0.66	0.23	0.25	0.00	0.41	0.25	0.00
time (sec)	N/A	0.245	0.077	0.033	0.035	0.076	0.000	0.142	0.179	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	57	44	57	16	21	0	33	21	0
N.S.	1	0.70	0.54	0.70	0.20	0.26	0.00	0.40	0.26	0.00
time (sec)	N/A	0.215	0.047	0.033	0.029	0.103	0.000	0.134	0.202	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	76	58	101	42	37	0	0	38	0
N.S.	1	0.55	0.42	0.74	0.31	0.27	0.00	0.00	0.28	0.00
time (sec)	N/A	0.241	0.078	0.036	0.035	0.184	0.000	0.000	0.198	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	111	106	141	91	139	0	0	153	0
N.S.	1	0.45	0.43	0.58	0.37	0.57	0.00	0.00	0.63	0.00
time (sec)	N/A	0.279	0.168	0.041	0.036	0.092	0.000	0.000	0.179	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	118	100	200	72	77	0	125	77	0
N.S.	1	0.39	0.33	0.66	0.24	0.25	0.00	0.41	0.25	0.00
time (sec)	N/A	0.272	0.106	0.056	0.035	0.103	0.000	0.160	0.192	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	87	72	127	46	51	0	83	51	0
N.S.	1	0.45	0.37	0.65	0.24	0.26	0.00	0.43	0.26	0.00
time (sec)	N/A	0.240	0.077	0.033	0.037	0.182	0.000	0.145	0.188	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	64	20	20	0	37	22	0
N.S.	1	1.00	0.96	1.39	0.43	0.43	0.00	0.80	0.48	0.00
time (sec)	N/A	0.192	0.046	0.030	0.036	0.071	0.000	0.139	0.189	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	92	78	138	58	59	0	0	59	0
N.S.	1	0.48	0.41	0.73	0.31	0.31	0.00	0.00	0.31	0.00
time (sec)	N/A	0.247	0.112	0.044	0.038	0.117	0.000	0.000	0.185	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	132	126	181	110	160	0	0	176	0
N.S.	1	0.44	0.42	0.60	0.36	0.53	0.00	0.00	0.58	0.00
time (sec)	N/A	0.292	0.178	0.039	0.039	0.200	0.000	0.000	0.187	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	127	75	0	0	165	0	0	98	0
N.S.	1	0.98	0.58	0.00	0.00	1.28	0.00	0.00	0.76	0.00
time (sec)	N/A	0.283	0.148	0.000	0.000	0.078	0.000	0.000	0.179	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	32	81	32	41	0	64	43	43
N.S.	1	1.09	0.74	1.88	0.74	0.95	0.00	1.49	1.00	1.00
time (sec)	N/A	0.194	0.041	10.381	0.048	0.109	0.000	0.133	0.175	10.638

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	85	87	54	113	59	78	0	0	76	0
N.S.	1	1.02	0.64	1.33	0.69	0.92	0.00	0.00	0.89	0.00
time (sec)	N/A	0.231	0.088	10.271	0.042	0.173	0.000	0.000	0.195	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	103	97	973	0	353	0	0	207	0
N.S.	1	0.93	0.87	8.77	0.00	3.18	0.00	0.00	1.86	0.00
time (sec)	N/A	0.287	0.367	0.206	0.000	0.085	0.000	0.000	0.209	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	82	82	664	0	285	0	0	155	0
N.S.	1	0.94	0.94	7.63	0.00	3.28	0.00	0.00	1.78	0.00
time (sec)	N/A	0.258	0.266	0.178	0.000	0.122	0.000	0.000	0.188	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	66	66	402	0	231	0	0	97	0
N.S.	1	0.97	0.97	5.91	0.00	3.40	0.00	0.00	1.43	0.00
time (sec)	N/A	0.236	0.183	0.164	0.000	0.187	0.000	0.000	0.175	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	43	113	0	159	0	39	51	39
N.S.	1	1.00	1.10	2.90	0.00	4.08	0.00	1.00	1.31	1.00
time (sec)	N/A	0.187	0.103	0.134	0.000	0.106	0.000	0.114	0.193	11.198

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	99	90	658	0	333	0	0	189	0
N.S.	1	1.01	0.92	6.71	0.00	3.40	0.00	0.00	1.93	0.00
time (sec)	N/A	0.306	0.357	0.155	0.000	0.121	0.000	0.000	0.179	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	129	115	958	0	429	0	0	271	0
N.S.	1	1.02	0.91	7.60	0.00	3.40	0.00	0.00	2.15	0.00
time (sec)	N/A	0.358	0.590	0.176	0.000	0.200	0.000	0.000	0.196	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	158	147	1300	0	522	0	0	364	0
N.S.	1	0.96	0.90	7.93	0.00	3.18	0.00	0.00	2.22	0.00
time (sec)	N/A	0.412	0.489	0.223	0.000	0.145	0.000	0.000	0.200	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	329	340	280	0	3481	0	0	28	0
N.S.	1	0.93	0.96	0.79	0.00	9.86	0.00	0.00	0.08	0.00
time (sec)	N/A	0.560	1.186	0.640	0.000	0.307	0.000	0.000	0.182	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	533	526	260	0	2461	0	0	28	0
N.S.	1	0.87	0.86	0.43	0.00	4.03	0.00	0.00	0.05	0.00
time (sec)	N/A	1.035	1.052	0.424	0.000	0.147	0.000	0.000	0.207	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	167	145	114	0	801	0	1037	28	0
N.S.	1	0.99	0.86	0.67	0.00	4.74	0.00	6.14	0.17	0.00
time (sec)	N/A	0.284	0.462	0.238	0.000	0.092	0.000	0.432	0.182	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	206	127	268	0	1229	0	0	29	0
N.S.	1	1.00	0.62	1.31	0.00	6.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.402	0.351	0.326	0.000	0.105	0.000	0.000	0.195	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	699	576	127	534	0	3155	0	0	29	0
N.S.	1	0.82	0.18	0.76	0.00	4.51	0.00	0.00	0.04	0.00
time (sec)	N/A	1.012	0.269	0.626	0.000	0.175	0.000	0.000	0.187	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	372	127	630	0	4375	0	0	29	0
N.S.	1	0.90	0.31	1.52	0.00	10.57	0.00	0.00	0.07	0.00
time (sec)	N/A	0.603	0.302	0.950	0.000	0.261	0.000	0.000	0.189	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	149	265	0	0	0	0	0	22	0
N.S.	1	1.06	1.89	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.268	1.045	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	144	263	0	0	0	0	0	20	0
N.S.	1	1.06	1.93	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.249	0.812	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	133	261	0	0	0	0	0	18	0
N.S.	1	1.07	2.10	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.238	0.572	0.000	0.000	0.000	0.000	0.000	0.314	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	75	72	397	0	259	0	0	111	224
N.S.	1	1.01	0.97	5.36	0.00	3.50	0.00	0.00	1.50	3.03
time (sec)	N/A	0.260	0.257	0.119	0.000	0.084	0.000	0.000	0.230	10.927

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	151	240	0	0	0	0	0	28	0
N.S.	1	1.06	1.69	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.254	0.661	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	150	258	0	0	0	0	0	28	0
N.S.	1	1.07	1.84	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.262	0.758	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	62	41	248	52	76	284	50	63	50
N.S.	1	0.97	0.64	3.88	0.81	1.19	4.44	0.78	0.98	0.78
time (sec)	N/A	0.209	0.214	0.095	0.039	0.114	147.922	0.141	0.232	10.802

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	62	41	248	52	76	284	50	63	50
N.S.	1	0.97	0.64	3.88	0.81	1.19	4.44	0.78	0.98	0.78
time (sec)	N/A	0.202	0.001	0.072	0.039	0.188	140.262	0.145	0.221	0.001

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	60	55	53	52	67	377	49	61	52
N.S.	1	0.97	0.89	0.85	0.84	1.08	6.08	0.79	0.98	0.84
time (sec)	N/A	0.208	0.173	0.105	0.040	0.088	17.098	0.118	0.236	10.904

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	60	40	249	52	81	376	0	67	59
N.S.	1	0.97	0.65	4.02	0.84	1.31	6.06	0.00	1.08	0.95
time (sec)	N/A	0.209	0.321	0.089	0.044	0.088	12.215	0.000	0.240	10.911

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	62	43	249	52	87	384	0	70	65
N.S.	1	0.97	0.67	3.89	0.81	1.36	6.00	0.00	1.09	1.02
time (sec)	N/A	0.207	0.236	0.092	0.040	0.080	161.919	0.000	0.228	10.842

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	171	288	0	0	0	0	0	26	0
N.S.	1	1.06	1.78	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.282	1.659	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	171	288	0	0	0	0	0	24	0
N.S.	1	1.06	1.78	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.263	1.323	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	167	286	0	0	0	0	0	31	0
N.S.	1	1.06	1.81	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.258	1.240	0.000	0.000	0.000	0.000	0.000	0.355	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	167	287	0	0	0	0	0	34	0
N.S.	1	1.06	1.82	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.266	1.946	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	171	290	0	0	0	0	0	40	0
N.S.	1	1.06	1.79	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.267	1.417	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	330	313	507	0	0	0	0	0	59	0
N.S.	1	0.95	1.54	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.649	4.537	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	330	313	507	0	0	0	0	0	57	0
N.S.	1	0.95	1.54	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.634	4.554	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	302	505	0	0	0	0	0	70	0
N.S.	1	0.93	1.55	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.628	4.231	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	322	298	499	0	0	0	0	0	76	0
N.S.	1	0.93	1.55	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.604	3.849	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	328	306	472	0	0	0	0	0	88	0
N.S.	1	0.93	1.44	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.612	3.840	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	365	0	0	0	0	0	288	0
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	1.95	0.00
time (sec)	N/A	0.359	0.917	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	366	0	0	0	0	0	288	0
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	1.95	0.00
time (sec)	N/A	0.336	0.880	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	364	0	0	0	0	0	280	0
N.S.	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	1.89	0.00
time (sec)	N/A	0.296	0.825	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	351	0	0	0	0	0	274	0
N.S.	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	1.97	0.00
time (sec)	N/A	0.281	0.851	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	115	113	125	0	658	0	0	110	0
N.S.	1	0.97	0.95	1.05	0.00	5.53	0.00	0.00	0.92	0.00
time (sec)	N/A	0.301	0.416	0.049	0.000	0.162	0.000	0.000	0.217	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	365	0	0	0	0	0	352	0
N.S.	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	2.36	0.00
time (sec)	N/A	0.330	0.756	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	365	0	0	0	0	0	360	0
N.S.	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	2.38	0.00
time (sec)	N/A	0.328	0.812	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	469	0	0	0	0	0	0	0
N.S.	1	1.00	3.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	1.684	0.000	0.000	0.000	0.000	0.000	0.376	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	475	0	0	0	0	0	0	0
N.S.	1	1.00	3.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	1.682	0.000	0.000	0.000	0.000	0.000	0.349	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	471	0	0	0	0	0	0	0
N.S.	1	1.00	3.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.291	1.662	0.000	0.000	0.000	0.000	0.000	0.350	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	466	0	0	0	0	0	0	0
N.S.	1	1.00	3.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	1.531	0.000	0.000	0.000	0.000	0.000	0.347	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	177	159	209	0	827	0	0	209	0
N.S.	1	1.02	0.92	1.21	0.00	4.78	0.00	0.00	1.21	0.00
time (sec)	N/A	0.393	0.940	0.052	0.000	0.257	0.000	0.000	0.278	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	477	0	0	0	0	0	0	0
N.S.	1	1.00	3.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.344	1.608	0.000	0.000	0.000	0.000	0.000	0.347	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	471	0	0	0	0	0	0	0
N.S.	1	1.00	3.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.327	1.568	0.000	0.000	0.000	0.000	0.000	0.364	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	37	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.342	0.301	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	37	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.328	0.290	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	35	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.302	0.344	0.000	0.000	0.000	0.000	0.000	0.334	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	166	0	0	0	0	0	34	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.270	0.191	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	0	0	148	0	0	38	0
N.S.	1	1.00	0.98	0.00	0.00	3.15	0.00	0.00	0.81	0.00
time (sec)	N/A	0.192	0.205	0.000	0.000	0.199	0.000	0.000	0.218	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	173	0	0	0	0	0	44	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.329	0.293	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	175	0	0	0	0	0	44	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.333	0.312	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	70	0
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.343	1.040	0.000	0.000	0.000	0.000	0.000	0.261	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	70	0
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.333	1.036	0.000	0.000	0.000	0.000	0.000	0.288	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	68	0
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.304	1.011	0.000	0.000	0.000	0.000	0.000	0.296	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	384	0	0	0	0	0	67	0
N.S.	1	1.00	2.70	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.286	1.015	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	96	98	0	0	449	0	0	74	0
N.S.	1	0.98	1.00	0.00	0.00	4.58	0.00	0.00	0.76	0.00
time (sec)	N/A	0.252	0.829	0.000	0.000	0.273	0.000	0.000	0.199	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	395	0	0	0	0	0	86	0
N.S.	1	1.00	2.60	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.337	1.008	0.000	0.000	0.000	0.000	0.000	0.270	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	399	0	0	0	0	0	86	0
N.S.	1	1.00	2.59	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.342	1.011	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	200	182	137	3765	273	2303	68491	25656	3880	1734
N.S.	1	0.91	0.68	18.82	1.36	11.52	342.46	128.28	19.40	8.67
time (sec)	N/A	0.379	0.704	1.961	0.064	0.183	67.560	0.298	0.207	10.965

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	129	117	86	1032	152	706	12323	5454	1063	543
N.S.	1	0.91	0.67	8.00	1.18	5.47	95.53	42.28	8.24	4.21
time (sec)	N/A	0.274	0.239	0.533	0.047	0.214	16.311	0.171	0.203	10.789

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	58	41	172	65	142	1096	557	151	83
N.S.	1	0.91	0.64	2.69	1.02	2.22	17.12	8.70	2.36	1.30
time (sec)	N/A	0.206	0.117	0.141	0.039	0.076	3.366	0.125	0.204	10.618

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	307	0	0	0	0	0	26	0
N.S.	1	1.00	1.75	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.320	1.272	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	328	314	3515	0	0	0	0	0	59	0
N.S.	1	0.96	10.72	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.740	6.730	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	545	0	0	0	0	0	0	0
N.S.	1	1.00	3.39	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.345	3.963	0.000	0.000	0.000	0.000	0.000	0.491	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	388	0	0	0	0	0	543	0
N.S.	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	3.39	0.00
time (sec)	N/A	0.340	1.125	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	183	0	0	0	0	0	41	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.334	0.541	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	428	0	0	0	0	0	74	0
N.S.	1	1.00	2.63	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.344	2.316	0.000	0.000	0.000	0.000	0.000	0.827	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	181	0	0	0	0	0	533	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	3.37	0.00
time (sec)	N/A	0.324	0.693	0.000	0.000	0.000	0.000	0.000	0.274	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	173	0	0	0	0	0	314	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	2.15	0.00
time (sec)	N/A	0.309	0.520	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	173	0	0	0	0	0	314	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	2.15	0.00
time (sec)	N/A	0.307	0.450	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	173	0	0	0	0	0	306	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	2.10	0.00
time (sec)	N/A	0.286	0.437	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	164	0	0	0	0	0	283	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	2.07	0.00
time (sec)	N/A	0.291	0.425	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	166	0	0	0	0	0	116	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.325	0.568	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	171	0	0	0	0	0	376	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	2.56	0.00
time (sec)	N/A	0.313	0.454	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	173	0	0	0	0	0	385	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	2.58	0.00
time (sec)	N/A	0.309	0.489	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	179	0	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.328	0.777	0.000	0.000	0.000	0.000	0.000	0.723	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	186	0	0	0	0	0	630	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	3.94	0.00
time (sec)	N/A	0.331	0.591	0.000	0.000	0.000	0.000	0.000	0.337	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	183	0	0	0	0	0	597	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	3.83	0.00
time (sec)	N/A	0.328	0.556	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	164	0	0	0	0	0	283	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	2.07	0.00
time (sec)	N/A	0.273	0.301	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	186	0	0	0	0	0	992	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	5.70	0.00
time (sec)	N/A	0.334	0.578	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	186	0	0	0	0	0	1043	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	5.99	0.00
time (sec)	N/A	0.336	0.585	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	186	0	0	0	0	0	1043	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	5.99	0.00
time (sec)	N/A	0.338	0.592	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	44	154	179	166	166	240	204	178	164
N.S.	1	0.80	2.80	3.25	3.02	3.02	4.36	3.71	3.24	2.98
time (sec)	N/A	0.207	0.066	0.102	0.035	0.076	0.035	0.131	0.215	0.053

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	84	405	566	439	439	722	597	565	419
N.S.	1	0.81	3.89	5.44	4.22	4.22	6.94	5.74	5.43	4.03
time (sec)	N/A	0.286	0.128	0.115	0.035	0.167	0.069	0.116	0.219	10.641

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	124	801	1318	920	920	1654	1330	1317	825
N.S.	1	0.78	5.04	8.29	5.79	5.79	10.40	8.36	8.28	5.19
time (sec)	N/A	0.333	0.328	0.147	0.042	0.065	0.137	0.156	0.212	10.928

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	222	162	0	1346	219	1366	307	4605
N.S.	1	1.00	1.10	0.80	0.00	6.66	1.08	6.76	1.52	22.80
time (sec)	N/A	0.390	0.168	0.150	0.000	0.091	1.826	0.125	0.209	11.710

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	82	80	154	0	446	332	158	226	287
N.S.	1	0.94	0.92	1.77	0.00	5.13	3.82	1.82	2.60	3.30
time (sec)	N/A	0.272	0.057	0.102	0.000	0.114	1.022	0.132	0.228	0.281

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	173	178	143	0	799	124	1443	227	683
N.S.	1	1.02	1.05	0.84	0.00	4.70	0.73	8.49	1.34	4.02
time (sec)	N/A	0.292	0.111	0.083	0.000	0.124	0.809	0.139	0.200	11.302

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	130	0	274	189	61	145	477
N.S.	1	1.00	1.07	2.95	0.00	6.23	4.30	1.39	3.30	10.84
time (sec)	N/A	0.211	0.020	0.075	0.000	0.098	0.606	0.118	0.212	10.890

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	95	131	173	0	474	348	291	102	2520
N.S.	1	0.92	1.27	1.68	0.00	4.60	3.38	2.83	0.99	24.47
time (sec)	N/A	0.294	0.102	0.118	0.000	0.201	4.147	0.154	0.200	11.958

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	191	209	172	0	1477	258	0	138	4339
N.S.	1	0.94	1.02	0.84	0.00	7.24	1.26	0.00	0.68	21.27
time (sec)	N/A	0.339	0.410	0.125	0.000	0.105	2.478	0.000	0.204	12.152

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	121	157	217	0	828	532	356	174	5947
N.S.	1	0.91	1.18	1.63	0.00	6.23	4.00	2.68	1.31	44.71
time (sec)	N/A	0.345	0.166	0.140	0.000	0.185	112.038	0.155	0.218	14.097

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	226	238	192	0	2212	411	1353	210	5771
N.S.	1	0.96	1.01	0.81	0.00	9.37	1.74	5.73	0.89	24.45
time (sec)	N/A	0.436	0.231	0.165	0.000	0.168	7.843	0.135	0.213	1.020

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	258	266	327	0	2578	641	1474	0	8025
N.S.	1	0.92	0.95	1.17	0.00	9.24	2.30	5.28	0.00	28.76
time (sec)	N/A	0.392	0.490	0.131	0.000	0.155	5.346	0.134	0.306	12.713

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	98	103	280	0	1077	556	202	0	460
N.S.	1	0.95	1.00	2.72	0.00	10.46	5.40	1.96	0.00	4.47
time (sec)	N/A	0.264	0.139	0.140	0.000	0.135	2.701	0.153	0.505	11.139

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	244	250	323	0	2600	646	1482	0	7835
N.S.	1	0.93	0.95	1.23	0.00	9.89	2.46	5.63	0.00	29.79
time (sec)	N/A	0.354	1.044	0.123	0.000	0.266	11.832	0.144	0.273	12.641

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	96	99	272	0	1066	525	202	0	442
N.S.	1	0.98	1.01	2.78	0.00	10.88	5.36	2.06	0.00	4.51
time (sec)	N/A	0.252	0.154	0.136	0.000	0.107	2.605	0.122	0.247	11.006

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	187	238	403	0	2486	0	489	399	13434
N.S.	1	1.07	1.37	2.32	0.00	14.29	0.00	2.81	2.29	77.21
time (sec)	N/A	0.427	0.430	0.198	0.000	0.440	0.000	0.169	0.234	17.150

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	342	342	445	0	4520	0	1031	480	12008
N.S.	1	0.95	0.95	1.24	0.00	12.56	0.00	2.86	1.33	33.36
time (sec)	N/A	0.650	1.626	0.203	0.000	0.275	0.000	0.161	0.213	14.374

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	229	287	466	0	4604	0	705	561	14830
N.S.	1	1.00	1.26	2.04	0.00	20.19	0.00	3.09	2.46	65.04
time (sec)	N/A	0.483	0.542	0.253	0.000	1.520	0.000	0.168	0.222	18.393

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	397	387	493	0	5954	0	2137	642	13781
N.S.	1	0.94	0.91	1.17	0.00	14.08	0.00	5.05	1.52	32.58
time (sec)	N/A	1.181	3.037	0.263	0.000	0.316	0.000	0.147	0.395	17.013

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	343	331	708	0	6770	2237	1958	0	13840
N.S.	1	0.97	0.94	2.01	0.00	19.18	6.34	5.55	0.00	39.21
time (sec)	N/A	0.494	4.375	0.246	0.000	0.368	140.359	0.176	1.816	15.582

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	163	149	548	0	3843	1794	431	0	1267
N.S.	1	1.03	0.94	3.45	0.00	24.17	11.28	2.71	0.00	7.97
time (sec)	N/A	0.317	0.209	0.257	0.000	0.258	7.540	0.172	1.409	12.223

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	381	385	889	0	7838	0	2679	0	16025
N.S.	1	1.02	1.03	2.37	0.00	20.90	0.00	7.14	0.00	42.73
time (sec)	N/A	0.538	5.008	0.231	0.000	0.543	0.000	0.183	0.409	15.125

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	158	148	543	0	3748	1707	429	0	1199
N.S.	1	1.03	0.97	3.55	0.00	24.50	11.16	2.80	0.00	7.84
time (sec)	N/A	0.304	0.227	0.250	0.000	0.483	6.903	0.165	0.945	13.276

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	300	394	970	0	9926	0	1077	1021	22621
N.S.	1	1.11	1.46	3.59	0.00	36.76	0.00	3.99	3.78	83.78
time (sec)	N/A	0.575	4.276	0.385	0.000	1.748	0.000	0.215	0.232	23.770

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	474	575	1201	0	10518	0	1704	1167	20580
N.S.	1	0.95	1.15	2.41	0.00	21.08	0.00	3.41	2.34	41.24
time (sec)	N/A	0.772	6.261	0.395	0.000	1.360	0.000	0.181	0.231	20.641

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	356	509	1145	0	15231	0	1791	1313	25334
N.S.	1	1.04	1.48	3.34	0.00	44.41	0.00	5.22	3.83	73.86
time (sec)	N/A	0.645	6.205	0.477	0.000	5.480	0.000	0.198	0.243	27.344

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	104	104	104	107	28	103	29
N.S.	1	1.00	1.00	3.06	3.06	3.06	3.15	0.82	3.03	0.85
time (sec)	N/A	0.193	0.012	0.098	0.032	0.060	0.041	0.120	0.211	0.070

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	48	188	174	174	174	187	46	173	46
N.S.	1	0.86	3.36	3.11	3.11	3.11	3.34	0.82	3.09	0.82
time (sec)	N/A	0.229	0.016	0.123	0.027	0.057	0.065	0.127	0.319	10.668

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	93	79	60	427	112	242	2853	1475	390	188
N.S.	1	0.85	0.65	4.59	1.20	2.60	30.68	15.86	4.19	2.02
time (sec)	N/A	0.249	0.273	0.376	0.044	0.081	16.726	0.126	0.250	11.107

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	338	0	0	0	0	0	0	207	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.527	0.000	0.000	0.000	0.000	0.000	0.000	0.577	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	398	393	0	0	0	0	0	0	209	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.569	0.000	0.000	0.000	0.000	0.000	0.000	0.581	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [8] had the largest ratio of [.800000000000000044]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.06	26	0.154
2	A	4	4	0.60	26	0.154
3	A	5	4	1.00	26	0.154
4	A	6	5	1.05	14	0.357
5	A	4	3	1.00	14	0.214
6	A	8	7	1.10	14	0.500
7	A	6	5	1.04	14	0.357
8	A	9	8	1.10	10	0.800
9	A	3	3	1.00	18	0.167
10	A	3	3	1.00	18	0.167
11	A	3	3	1.00	16	0.188
12	A	3	3	1.00	14	0.214
13	A	6	5	1.00	18	0.278
14	A	4	3	1.06	18	0.167
15	A	8	7	1.02	18	0.389
16	A	5	5	1.05	18	0.278
17	A	5	5	1.09	18	0.278
18	A	5	5	1.02	18	0.278
19	A	4	4	0.95	16	0.250
20	A	5	5	1.03	14	0.357
21	A	4	4	1.10	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	5	4	1.06	18	0.222
23	A	5	4	1.00	18	0.222
24	A	5	4	1.00	18	0.222
25	A	5	5	1.29	18	0.278
26	A	5	5	1.17	18	0.278
27	A	5	5	1.05	18	0.278
28	A	7	7	1.08	14	0.500
29	A	6	6	1.16	18	0.333
30	A	6	5	1.14	18	0.278
31	A	6	5	1.10	18	0.278
32	A	7	6	1.12	18	0.333
33	A	6	5	1.12	18	0.278
34	A	6	5	1.09	18	0.278
35	A	7	7	1.28	18	0.389
36	A	7	7	1.17	18	0.389
37	A	3	3	1.05	18	0.167
38	A	3	3	1.06	16	0.188
39	A	3	3	1.08	14	0.214
40	A	3	3	1.10	18	0.167
41	A	4	3	1.19	18	0.167
42	A	3	3	1.07	18	0.167
43	A	3	3	1.06	18	0.167
44	A	3	3	1.05	18	0.167
45	A	3	3	1.04	18	0.167
46	A	9	8	1.12	19	0.421
47	A	7	6	1.08	17	0.353
48	A	5	4	1.00	15	0.267
49	A	1	1	1.00	19	0.053
50	A	3	3	1.07	19	0.158
51	A	5	5	1.13	19	0.263
52	A	7	7	1.15	19	0.368
53	A	15	14	1.14	19	0.737

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	11	10	1.09	19	0.526
55	A	7	6	1.06	17	0.353
56	A	1	1	1.00	15	0.067
57	A	3	3	1.39	19	0.158
58	A	5	5	1.30	19	0.263
59	A	7	7	1.27	19	0.368
60	A	10	9	1.13	21	0.429
61	A	8	7	1.10	21	0.333
62	A	6	5	1.07	21	0.238
63	A	4	3	1.00	21	0.143
64	A	2	2	1.00	21	0.095
65	A	4	4	1.11	21	0.190
66	A	6	6	1.14	21	0.286
67	A	12	11	1.11	21	0.524
68	A	8	7	1.03	21	0.333
69	A	5	4	1.03	21	0.190
70	A	3	2	1.00	21	0.095
71	A	4	4	1.34	21	0.190
72	A	6	6	1.29	21	0.286
73	A	8	8	1.26	21	0.381
74	A	5	4	1.00	19	0.211
75	A	5	4	1.00	19	0.211
76	A	9	8	1.05	20	0.400
77	A	5	4	0.97	20	0.200
78	A	12	11	0.97	18	0.611
79	A	9	8	1.02	18	0.444
80	A	6	5	1.06	16	0.312
81	A	4	3	1.02	14	0.214
82	A	4	3	1.04	18	0.167
83	A	4	3	1.07	18	0.167
84	A	4	3	1.07	18	0.167
85	A	5	4	0.95	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	5	4	0.86	24	0.167
87	A	5	4	0.89	24	0.167
88	A	5	4	0.99	22	0.182
89	A	5	4	1.12	20	0.200
90	A	4	3	1.00	24	0.125
91	A	4	3	1.04	24	0.125
92	A	4	3	1.04	24	0.125
93	A	5	4	0.98	24	0.167
94	A	18	17	0.92	22	0.773
95	A	14	13	0.94	22	0.591
96	A	9	8	1.00	20	0.400
97	A	6	5	1.00	18	0.278
98	A	4	3	1.04	22	0.136
99	A	4	3	1.08	22	0.136
100	A	4	3	1.07	22	0.136
101	A	5	4	0.95	30	0.133
102	A	5	4	0.76	28	0.143
103	A	5	4	0.82	26	0.154
104	A	5	4	0.97	24	0.167
105	A	4	3	1.00	28	0.107
106	A	4	3	1.04	28	0.107
107	C	1	1	1.11	77	0.013
108	A	5	4	0.89	23	0.174
109	A	5	4	0.93	23	0.174
110	A	2	2	1.00	23	0.087
111	A	6	5	1.13	21	0.238
112	A	5	4	0.95	23	0.174
113	A	5	4	0.92	23	0.174
114	A	5	4	0.90	23	0.174
115	A	12	11	1.48	25	0.440
116	A	12	11	1.00	25	0.440
117	A	6	5	0.96	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	7	6	1.03	25	0.240
119	A	14	13	1.01	25	0.520
120	A	14	13	1.45	25	0.520
121	A	1	1	1.00	26	0.038
122	A	1	1	1.00	28	0.036
123	A	3	3	0.67	28	0.107
124	A	3	3	0.67	26	0.115
125	A	2	2	0.65	24	0.083
126	A	3	3	0.64	28	0.107
127	A	3	3	0.67	28	0.107
128	A	3	3	0.68	28	0.107
129	A	3	3	0.52	28	0.107
130	A	3	3	0.52	26	0.115
131	A	3	3	0.51	24	0.125
132	A	6	5	0.45	28	0.179
133	A	3	3	0.52	28	0.107
134	A	3	3	0.54	28	0.107
135	A	2	2	1.11	28	0.071
136	A	2	2	1.11	26	0.077
137	A	2	2	1.13	24	0.083
138	A	7	6	0.75	28	0.214
139	A	2	2	1.11	28	0.071
140	A	2	2	1.10	28	0.071
141	A	2	2	1.14	28	0.071
142	A	2	2	1.14	26	0.077
143	A	2	2	1.16	24	0.083
144	A	6	5	0.60	28	0.179
145	A	2	2	1.14	28	0.071
146	A	2	2	1.13	28	0.071
147	A	3	3	0.55	30	0.100
148	A	3	3	0.69	30	0.100
149	A	2	2	1.09	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	2	2	1.12	30	0.067
151	A	4	3	1.00	30	0.100
152	A	4	3	1.00	30	0.100
153	A	4	3	1.00	30	0.100
154	A	4	3	1.00	30	0.100
155	A	3	2	1.00	30	0.067
156	A	3	2	1.00	30	0.067
157	A	3	2	1.00	30	0.067
158	A	5	4	0.97	32	0.125
159	A	5	4	0.97	32	0.125
160	A	5	4	0.97	32	0.125
161	A	5	4	0.98	32	0.125
162	A	4	3	1.56	32	0.094
163	A	4	3	0.97	32	0.094
164	A	4	3	0.97	32	0.094
165	A	6	5	0.38	32	0.156
166	A	6	5	0.45	32	0.156
167	A	3	3	0.70	32	0.094
168	A	6	5	0.55	32	0.156
169	A	6	5	0.45	32	0.156
170	A	6	5	0.39	32	0.156
171	A	6	5	0.45	32	0.156
172	A	5	4	1.00	32	0.125
173	A	6	5	0.48	32	0.156
174	A	6	5	0.44	32	0.156
175	A	3	3	0.98	35	0.086
176	A	4	3	1.09	28	0.107
177	A	5	4	1.02	30	0.133
178	A	4	3	0.93	24	0.125
179	A	4	3	0.94	24	0.125
180	A	6	5	0.97	24	0.208
181	A	4	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	6	5	1.01	24	0.208
183	A	6	5	1.02	24	0.208
184	A	6	5	0.96	24	0.208
185	A	6	5	0.93	26	0.192
186	A	12	11	0.87	26	0.423
187	A	4	3	0.99	26	0.115
188	A	6	5	1.00	26	0.192
189	A	14	13	0.82	26	0.500
190	A	8	7	0.90	26	0.269
191	A	2	2	1.06	20	0.100
192	A	2	2	1.06	18	0.111
193	A	2	2	1.07	16	0.125
194	A	8	7	1.01	20	0.350
195	A	2	2	1.06	20	0.100
196	A	2	2	1.07	20	0.100
197	A	2	2	0.97	22	0.091
198	A	2	2	0.97	22	0.091
199	A	2	2	0.97	22	0.091
200	A	2	2	0.97	22	0.091
201	A	2	2	0.97	22	0.091
202	A	2	2	1.06	24	0.083
203	A	2	2	1.06	24	0.083
204	A	2	2	1.06	24	0.083
205	A	2	2	1.06	24	0.083
206	A	2	2	1.06	24	0.083
207	A	4	4	0.95	24	0.167
208	A	4	4	0.95	24	0.167
209	A	4	4	0.93	24	0.167
210	A	4	4	0.93	24	0.167
211	A	4	4	0.93	24	0.167
212	A	2	2	1.00	22	0.091
213	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	2	2	1.00	20	0.100
215	A	2	2	1.00	18	0.111
216	A	9	8	0.97	22	0.364
217	A	2	2	1.00	22	0.091
218	A	2	2	1.00	22	0.091
219	A	2	2	1.00	22	0.091
220	A	2	2	1.00	22	0.091
221	A	2	2	1.00	20	0.100
222	A	2	2	1.00	18	0.111
223	A	11	10	1.02	22	0.455
224	A	2	2	1.00	22	0.091
225	A	2	2	1.00	22	0.091
226	A	2	2	1.00	22	0.091
227	A	2	2	1.00	22	0.091
228	A	2	2	1.00	20	0.100
229	A	2	2	1.00	18	0.111
230	A	4	3	1.00	22	0.136
231	A	2	2	1.00	22	0.091
232	A	2	2	1.00	22	0.091
233	A	2	2	1.00	22	0.091
234	A	2	2	1.00	22	0.091
235	A	2	2	1.00	20	0.100
236	A	2	2	1.00	18	0.111
237	A	6	5	0.98	22	0.227
238	A	2	2	1.00	22	0.091
239	A	2	2	1.00	22	0.091
240	A	2	2	0.91	22	0.091
241	A	2	2	0.91	22	0.091
242	A	2	2	0.91	20	0.100
243	A	2	2	1.00	22	0.091
244	A	4	4	0.96	22	0.182
245	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	2	2	1.00	24	0.083
247	A	2	2	1.00	24	0.083
248	A	2	2	1.00	24	0.083
249	A	2	2	1.00	22	0.091
250	A	2	2	1.00	20	0.100
251	A	2	2	1.00	20	0.100
252	A	2	2	1.00	18	0.111
253	A	2	2	1.00	16	0.125
254	A	4	3	1.00	20	0.150
255	A	2	2	1.00	20	0.100
256	A	2	2	1.00	20	0.100
257	A	2	2	1.00	22	0.091
258	A	2	2	1.00	24	0.083
259	A	2	2	1.00	22	0.091
260	A	2	2	1.00	16	0.125
261	A	2	2	1.00	24	0.083
262	A	2	2	1.00	24	0.083
263	A	2	2	1.00	24	0.083
264	A	4	3	0.80	31	0.097
265	A	5	4	0.81	33	0.121
266	A	5	4	0.78	33	0.121
267	A	5	4	1.00	33	0.121
268	A	7	6	0.94	33	0.182
269	A	4	3	1.02	33	0.091
270	A	5	4	1.00	31	0.129
271	A	9	8	0.92	33	0.242
272	A	6	5	0.94	33	0.152
273	A	7	6	0.91	33	0.182
274	A	7	6	0.96	33	0.182
275	A	5	4	0.92	33	0.121
276	A	6	5	0.95	33	0.152
277	A	5	4	0.93	33	0.121

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	6	5	0.98	31	0.161
279	A	7	6	1.07	33	0.182
280	A	7	6	0.95	33	0.182
281	A	7	6	1.00	33	0.182
282	A	9	8	0.94	33	0.242
283	A	7	6	0.97	33	0.182
284	A	7	6	1.03	33	0.182
285	A	7	6	1.02	33	0.182
286	A	7	6	1.03	31	0.194
287	A	9	8	1.11	33	0.242
288	A	9	8	0.95	33	0.242
289	A	9	8	1.04	33	0.242
290	A	4	3	1.00	24	0.125
291	A	5	4	0.86	26	0.154
292	A	4	3	0.85	32	0.094
293	A	5	4	0.99	26	0.154
294	A	4	3	0.99	28	0.107

CHAPTER 3

Listing of integrals

3.1 $\int x^9\sqrt{a^2+2abx^5+b^2x^{10}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int x^9\sqrt{a^2+2abx^5+b^2x^{10}}\,dx = -\frac{a(a+bx^5)\sqrt{a^2+2abx^5+b^2x^{10}}}{10b^2}+\frac{(a^2+2abx^5+b^2x^{10})^{3/2}}{15b^2}$$

output
```
-1/10*a*(b*x^5+a)*((b*x^5+a)^2)^(1/2)/b^2+1/15*(b^2*x^10+2*a*b*x^5+a^2)^(3
/2)/b^2
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int x^9\sqrt{a^2+2abx^5+b^2x^{10}}\,dx = \frac{x^{10}\sqrt{(a+bx^5)^2}(3a+2bx^5)}{30(a+bx^5)}$$

input
```
Integrate[x^9*Sqrt[a^2 + 2*a*b*x^5 + b^2*x^10],x]
```

output
```
(x^10*Sqrt[(a + b*x^5)^2]*(3*a + 2*b*x^5))/(30*(a + b*x^5))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \sqrt{a^2+2abx^5+b^2x^{10}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{5}\int x^5\sqrt{b^2x^{10}+2abx^5+a^2}dx^5$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{5}\left(\frac{\left(a^2+2abx^5+b^2x^{10}\right)^{3/2}}{3b^2}-\frac{a\int \sqrt{b^2x^{10}+2abx^5+a^2}dx^5}{b}\right)$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{5}\left(\frac{\left(a^2+2abx^5+b^2x^{10}\right)^{3/2}}{3b^2}-\frac{a\sqrt{a^2+2abx^5+b^2x^{10}}\int \left(b^2x^5+ab\right)dx^5}{b^2\left(a+bx^5\right)}\right)$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{5}\left(\frac{\left(a^2+2abx^5+b^2x^{10}\right)^{3/2}}{3b^2}-\frac{a\left(a+bx^5\right)\sqrt{a^2+2abx^5+b^2x^{10}}}{2b^2}\right)$$

input
```
Int[x^9*Sqrt[a^2 + 2*a*b*x^5 + b^2*x^10],x]
```

output
```
(-1/2*(a*(a + b*x^5)*Sqrt[a^2 + 2*a*b*x^5 + b^2*x^10])/b^2 + (a^2 + 2*a*b*
x^5 + b^2*x^10)^(3/2)/(3*b^2))/5
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{5}+a\right) \left(b\,x^{5}+a\right)^{2} \left(-2 b\,x^{5}+a\right)}{30 b^{2}}$	31
gosper	$\frac{x^{10} \left(2 b\,x^{5}+3 a\right) \sqrt{\left(b\,x^{5}+a\right)^{2}}}{30 b\,x^{5}+30 a}$	36
default	$\frac{x^{10} \left(2 b\,x^{5}+3 a\right) \sqrt{\left(b\,x^{5}+a\right)^{2}}}{30 b\,x^{5}+30 a}$	36
orering	$\frac{x^{10} \left(2 b\,x^{5}+3 a\right) \sqrt{\left(b\,x^{5}+a\right)^{2}}}{30 b\,x^{5}+30 a}$	36
risch	$\frac{\sqrt{\left(b\,x^{5}+a\right)^{2}}\, a\,x^{10}}{10 b\,x^{5}+10 a}+\frac{\sqrt{\left(b\,x^{5}+a\right)^{2}}\, b\,x^{15}}{15 b\,x^{5}+15 a}$	54

input
```
int(x^9*((b*x^5+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*csgn(b*x^5+a)*(b*x^5+a)^2*(-2*b*x^5+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.19

$$\int x^9\sqrt{a^2+2abx^5+b^2x^{10}}\,dx=\frac{1}{15}\,bx^{15}+\frac{1}{10}\,ax^{10}$$

input
```
integrate(x^9*((b*x^5+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*b*x^15 + 1/10*a*x^10
```

Sympy [F(-1)]

Timed out.

$$\int x^9\sqrt{a^2+2abx^5+b^2x^{10}}\,dx=\text{Timed out}$$

input
```
integrate(x**9*((b*x**5+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int x^9\sqrt{a^2+2abx^5+b^2x^{10}}\,dx = -\frac{\sqrt{b^2x^{10}+2\,abx^5+a^2}ax^5}{10\,b} - \frac{\sqrt{b^2x^{10}+2\,abx^5+a^2}a^2}{10\,b^2} + \frac{\left(b^2x^{10}+2\,abx^5+a^2\right)^{\frac{3}{2}}}{15\,b^2}$$

input
```
integrate(x^9*((b*x^5+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/10*sqrt(b^2*x^10 + 2*a*b*x^5 + a^2)*a*x^5/b - 1/10*sqrt(b^2*x^10 + 2*a*
b*x^5 + a^2)*a^2/b^2 + 1/15*(b^2*x^10 + 2*a*b*x^5 + a^2)^(3/2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.34

$$\int x^9\sqrt{a^2+2abx^5+b^2x^{10}}\,dx = \frac{1}{30}\left(2\,bx^{15}+3\,ax^{10}\right)\mathrm{sgn}\left(bx^5+a\right)$$

input
```
integrate(x^9*((b*x^5+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/30*(2*b*x^15 + 3*a*x^10)*sgn(b*x^5 + a)
```

Mupad [B] (verification not implemented)

Time = 20.55 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int x^9\sqrt{a^2+2abx^5+b^2x^{10}}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^5+b^2\,x^{10}}\,\left(8\,b^2\,\left(a^2+b^2\,x^{10}\right)-12\,a^2\,b^2+4\,a\,b^3\,x^5\right)}{120\,b^4}$$

input
```
int(x^9*((a + b*x^5)^2)^(1/2),x)
```

output
```
((a^2 + b^2*x^10 + 2*a*b*x^5)^(1/2)*(8*b^2*(a^2 + b^2*x^10) - 12*a^2*b^2 +
 4*a*b^3*x^5))/(120*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.22

$$\int x^9\sqrt{a^2+2abx^5+b^2x^{10}}\,dx=\frac{x^{10}(2b\,x^5+3a)}{30}$$

input
```
int(x^9*((b*x^5+a)^2)^(1/2),x)
```

output
```
(x**10*(3*a + 2*b*x**5))/30
```

3.2 $\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x}\,dx = \frac{bx^5\sqrt{a^2+2abx^5+b^2x^{10}}}{5\left(a+bx^5\right)} + \frac{a\sqrt{a^2+2abx^5+b^2x^{10}}\log(x)}{a+bx^5}$$

output
```
b*x^5*((b*x^5+a)^2)^(1/2)/(5*b*x^5+5*a)+a*((b*x^5+a)^2)^(1/2)*ln(x)/(b*x^5
+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 454 vs. 2(75) = 150.

Time = 0.90 (sec) , antiderivative size = 454, normalized size of antiderivative = 6.05

$$\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x}\,dx$$
$$= \frac{-2a\sqrt{a^2}bx^5 - 2\sqrt{a^2}b^2x^{10} + 2abx^5\sqrt{\left(a+bx^5\right)^2} - 2a\left(a^2+abx^5-\sqrt{a^2}\sqrt{\left(a+bx^5\right)^2}\right)\operatorname{arctanh}\left(\frac{bx^5}{\sqrt{a^2}-\sqrt{(a\cdots}}\right.}{\cdots}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^5 + b^2*x^10]/x,x]
```

output
```
(-2*a*Sqrt[a^2]*b*x^5 - 2*Sqrt[a^2]*b^2*x^10 + 2*a*b*x^5*Sqrt[(a + b*x^5)^
2] - 2*a*(a^2 + a*b*x^5 - Sqrt[a^2]*Sqrt[(a + b*x^5)^2])*ArcTanh[(b*x^5)/(
Sqrt[a^2] - Sqrt[(a + b*x^5)^2])] - 2*((a^2)^(3/2) + a*Sqrt[a^2]*b*x^5 - a
^2*Sqrt[(a + b*x^5)^2])*Log[x^5] + (a^2)^(3/2)*Log[Sqrt[a^2] - b*x^5 - Sqr
t[(a + b*x^5)^2]] + a*Sqrt[a^2]*b*x^5*Log[Sqrt[a^2] - b*x^5 - Sqrt[(a + b*
x^5)^2]] - a^2*Sqrt[(a + b*x^5)^2]*Log[Sqrt[a^2] - b*x^5 - Sqrt[(a + b*x^5
)^2]] + (a^2)^(3/2)*Log[Sqrt[a^2] + b*x^5 - Sqrt[(a + b*x^5)^2]] + a*Sqrt[
a^2]*b*x^5*Log[Sqrt[a^2] + b*x^5 - Sqrt[(a + b*x^5)^2]] - a^2*Sqrt[(a + b*
x^5)^2]*Log[Sqrt[a^2] + b*x^5 - Sqrt[(a + b*x^5)^2]])/(10*(a^2 + a*b*x^5 -
 Sqrt[a^2]*Sqrt[(a + b*x^5)^2]))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^5+b^2x^{10}}\int \frac{b(bx^5+a)}{x}dx}{b(a+bx^5)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^5+b^2x^{10}}\int \frac{bx^5+a}{x}dx}{a+bx^5}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^5+b^2x^{10}}\int \left(bx^4+\frac{a}{x}\right)dx}{a+bx^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^5+b^2x^{10}}\left(a\log(x)+\frac{bx^5}{5}\right)}{a+bx^5}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^5 + b^2*x^10]/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^5 + b^2*x^10]*((b*x^5)/5 + a*Log[x]))/(a + b*x^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{5}+a\right)\left(b\,x^{5}+a+a\ln\left(b\,x^{5}\right)\right)}{5}$	26
default	$\frac{\sqrt{\left(b\,x^{5}+a\right)^{2}}\,\left(b\,x^{5}+5a\ln\left(x\right)\right)}{5b\,x^{5}+5a}$	34
risch	$\frac{b\,x^{5}\sqrt{\left(b\,x^{5}+a\right)^{2}}}{5b\,x^{5}+5a}+\frac{a\sqrt{\left(b\,x^{5}+a\right)^{2}}\,\ln\left(x\right)}{b\,x^{5}+a}$	52

input
```
int(((b*x^5+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/5*csgn(b*x^5+a)*(b*x^5+a+a*ln(b*x^5))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a^2 + 2abx^5 + b^2x^{10}}}{x} \, dx = \frac{1}{5} \, bx^5 + a \log(x)$$

input
```
integrate(((b*x^5+a)^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/5*b*x^5 + a*log(x)
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx^5 + b^2x^{10}}}{x} \, dx = \int \frac{\sqrt{(a + bx^5)^2}}{x} \, dx$$

input
```
integrate(((b*x**5+a)**2)**(1/2)/x,x)
```

output
```
Integral(sqrt((a + b*x**5)**2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x}\,dx = \frac{1}{5}\,(-1)^{2\,b^2x^5+2\,ab}\,a\log\left(2\,b^2x^5+2\,ab\right) - \frac{1}{5}\,(-1)^{2\,abx^5+2\,a^2}\,a\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^4|x|}\right) + \frac{1}{5}\,\sqrt{b^2x^{10}+2\,abx^5+a^2}$$

input
```
integrate(((b*x^5+a)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/5*(-1)^(2*b^2*x^5 + 2*a*b)*a*log(2*b^2*x^5 + 2*a*b) - 1/5*(-1)^(2*a*b*x^
5 + 2*a^2)*a*log(2*a*b*x/abs(x) + 2*a^2/(x^4*abs(x))) + 1/5*sqrt(b^2*x^10
+ 2*a*b*x^5 + a^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x}\,dx = \frac{1}{5}\,bx^5\mathrm{sgn}\left(bx^5+a\right) + a\log\left(|x|\right)\mathrm{sgn}\left(bx^5+a\right)$$

input
```
integrate(((b*x^5+a)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/5*b*x^5*sgn(b*x^5 + a) + a*log(abs(x))*sgn(b*x^5 + a)
```

Mupad [B] (verification not implemented)

Time = 22.55 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^5+b^2\,x^{10}}}{5} - \frac{\ln\left(a\,b+\frac{a^2}{x^5}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^5+b^2\,x^{10}}}{x^5}\right)\,\sqrt{a^2}}{5} + \frac{a\,b\,\ln\left(a\,b+\sqrt{\left(b\,x^5+a\right)^2}\,\sqrt{b^2}+b^2\,x^5\right)}{5\,\sqrt{b^2}}$$

input
```
int(((a + b*x^5)^2)^(1/2)/x,x)
```

output
```
(a^2 + b^2*x^10 + 2*a*b*x^5)^(1/2)/5 - (log(a*b + a^2/x^5 + ((a^2)^(1/2)*(
a^2 + b^2*x^10 + 2*a*b*x^5)^(1/2))/x^5)*(a^2)^(1/2))/5 + (a*b*log(a*b + ((
a + b*x^5)^2)^(1/2)*(b^2)^(1/2) + b^2*x^5))/(5*(b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x}\,dx = \log(x)\,a + \frac{b\,x^5}{5}$$

input
```
int(((b*x^5+a)^2)^(1/2)/x,x)
```

output
```
(5*log(x)*a + b*x**5)/5
```

3.3 $\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x^{11}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 39

$$\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x^{11}}\,dx=-\frac{(a+bx^5)\sqrt{a^2+2abx^5+b^2x^{10}}}{10ax^{10}}$$

output
```
-1/10*(b*x^5+a)*((b*x^5+a)^2)^(1/2)/a/x^10
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x^{11}}\,dx=-\frac{\sqrt{(a+bx^5)^2}(a+2bx^5)}{10x^{10}(a+bx^5)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^5 + b^2*x^10]/x^11,x]
```

output
```
-1/10*(Sqrt[(a + b*x^5)^2]*(a + 2*b*x^5))/(x^10*(a + b*x^5))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1693, 1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x^{11}}dx$$

$$\Big\downarrow \ 1693$$

$$\frac{1}{5}\int \frac{\sqrt{b^2x^{10}+2abx^5+a^2}}{x^{15}}dx^5$$

$$\Big\downarrow \ 1102$$

$$\frac{\sqrt{a^2+2abx^5+b^2x^{10}}\int \frac{b(bx^5+a)}{x^{15}}dx^5}{5b(a+bx^5)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^5+b^2x^{10}}\int \frac{bx^5+a}{x^{15}}dx^5}{5(a+bx^5)}$$

$$\Big\downarrow \ 48$$

$$-\frac{(a+bx^5)\sqrt{a^2+2abx^5+b^2x^{10}}}{10ax^{10}}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^5 + b^2*x^10]/x^11,x]
```

output
```
-1/10*((a + b*x^5)*Sqrt[a^2 + 2*a*b*x^5 + b^2*x^10])/(a*x^10)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{5}+a\right)\left(2 b\,x^{5}+a\right)}{10 x^{10}}$	22
gosper	$-\frac{\left(2 b\,x^{5}+a\right)\sqrt{\left(b\,x^{5}+a\right)^{2}}}{10 x^{10}\left(b\,x^{5}+a\right)}$	34
default	$-\frac{\left(2 b\,x^{5}+a\right)\sqrt{\left(b\,x^{5}+a\right)^{2}}}{10 x^{10}\left(b\,x^{5}+a\right)}$	34
orering	$-\frac{\left(2 b\,x^{5}+a\right)\sqrt{\left(b\,x^{5}+a\right)^{2}}}{10 x^{10}\left(b\,x^{5}+a\right)}$	34
risch	$\frac{\left(-\frac{b\,x^{5}}{5}-\frac{a}{10}\right)\sqrt{\left(b\,x^{5}+a\right)^{2}}}{x^{10}\left(b\,x^{5}+a\right)}$	35

input
```
int(((b*x^5+a)^2)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*csgn(b*x^5+a)*(2*b*x^5+a)/x^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x^{11}}\,dx = -\frac{2\,bx^5+a}{10\,x^{10}}$$

input
```
integrate(((b*x^5+a)^2)^(1/2)/x^11,x, algorithm="fricas")
```

output
```
-1/10*(2*b*x^5 + a)/x^10
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^5+b^2x^{10}}}{x^{11}}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**5+a)**2)**(1/2)/x**11,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(26) = 52$.

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{a^2 + 2abx^5 + b^2x^{10}}}{x^{11}}\,dx = \frac{\sqrt{b^2x^{10} + 2\,abx^5 + a^2}b^2}{10\,a^2} + \frac{\sqrt{b^2x^{10} + 2\,abx^5 + a^2}b}{10\,ax^5} - \frac{(b^2x^{10} + 2\,abx^5 + a^2)^{\frac{3}{2}}}{10\,a^2x^{10}}$$

input
```
integrate(((b*x^5+a)^2)^(1/2)/x^11,x, algorithm="maxima")
```

output
```
1/10*sqrt(b^2*x^10 + 2*a*b*x^5 + a^2)*b^2/a^2 + 1/10*sqrt(b^2*x^10 + 2*a*b
*x^5 + a^2)*b/(a*x^5) - 1/10*(b^2*x^10 + 2*a*b*x^5 + a^2)^(3/2)/(a^2*x^10)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a^2 + 2abx^5 + b^2x^{10}}}{x^{11}}\,dx = -\frac{2\,bx^5\mathrm{sgn}(bx^5 + a) + a\mathrm{sgn}(bx^5 + a)}{10\,x^{10}}$$

input
```
integrate(((b*x^5+a)^2)^(1/2)/x^11,x, algorithm="giac")
```

output
```
-1/10*(2*b*x^5*sgn(b*x^5 + a) + a*sgn(b*x^5 + a))/x^10
```

Mupad [B] (verification not implemented)

Time = 20.57 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a^2 + 2abx^5 + b^2x^{10}}}{x^{11}}\,dx = -\frac{(2\,b\,x^5 + a)\,\sqrt{(b\,x^5 + a)^2}}{10\,x^{10}\,(b\,x^5 + a)}$$

input
```
int(((a + b*x^5)^2)^(1/2)/x^11,x)
```

output
```
-((a + 2*b*x^5)*((a + b*x^5)^2)^(1/2))/(10*x^10*(a + b*x^5))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a^2 + 2abx^5 + b^2x^{10}}}{x^{11}} \, dx = \frac{-2b\,x^5 - a}{10x^{10}}$$

input
```
int(((b*x^5+a)^2)^(1/2)/x^11,x)
```

output
```
( - a - 2*b*x**5)/(10*x**10)
```

3.4 $\int \frac{x^9}{2+x^5+x^{10}} dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{x^9}{2+x^5+x^{10}} dx = -\frac{\arctan\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}} + \frac{1}{10}\log\left(2+x^5+x^{10}\right)$$

output
```
-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)+1/10*ln(x^10+x^5+2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{2+x^5+x^{10}} dx = -\frac{\arctan\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}} + \frac{1}{10}\log\left(2+x^5+x^{10}\right)$$

input
```
Integrate[x^9/(2 + x^5 + x^10),x]
```

output
```
-1/5*ArcTan[(1 + 2*x^5)/Sqrt[7]]/Sqrt[7] + Log[2 + x^5 + x^10]/10
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1693, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^{10}+x^5+2} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{5} \int \frac{x^5}{x^{10}+x^5+2} dx^5$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{5} \left(\frac{1}{2} \int \frac{2x^5+1}{x^{10}+x^5+2} dx^5 - \frac{1}{2} \int \frac{1}{x^{10}+x^5+2} dx^5\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{5} \left(\int \frac{1}{-x^{10}-7} d(2x^5+1) + \frac{1}{2} \int \frac{2x^5+1}{x^{10}+x^5+2} dx^5\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{5} \left(\frac{1}{2} \int \frac{2x^5+1}{x^{10}+x^5+2} dx^5 - \frac{\arctan\left(\frac{2x^5+1}{\sqrt{7}}\right)}{\sqrt{7}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{5} \left(\frac{1}{2} \log\left(x^{10}+x^5+2\right) - \frac{\arctan\left(\frac{2x^5+1}{\sqrt{7}}\right)}{\sqrt{7}}\right)$$

input
```
Int[x^9/(2 + x^5 + x^10),x]
```

output
```
(-(ArcTan[(1 + 2*x^5)/Sqrt[7]]/Sqrt[7]) + Log[2 + x^5 + x^10]/2)/5
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\arctan\left(\frac{\left(2x^{5}+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{35}+\frac{\ln\left(x^{10}+x^{5}+2\right)}{10}$	31
risch	$\frac{\ln\left(4x^{10}+4x^{5}+8\right)}{10}-\frac{\arctan\left(\frac{\left(2x^{5}+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	35

input
```
int(x^9/(x^10+x^5+2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)+1/10*ln(x^10+x^5+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = -\frac{1}{35}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^5+1)\right) + \frac{1}{10}\,\log\left(x^{10}+x^5+2\right)$$

input
```
integrate(x^9/(x^10+x^5+2),x, algorithm="fricas")
```

output
```
-1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = \frac{\log\left(x^{10}+x^5+2\right)}{10} - \frac{\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

input
```
integrate(x**9/(x**10+x**5+2),x)
```

output
```
log(x**10 + x**5 + 2)/10 - sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = -\frac{1}{35}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^5+1)\right)+\frac{1}{10}\,\log\left(x^{10}+x^5+2\right)$$

input
```
integrate(x^9/(x^10+x^5+2),x, algorithm="maxima")
```

output
```
-1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = -\frac{1}{35}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^5+1)\right)+\frac{1}{10}\,\log\left(x^{10}+x^5+2\right)$$

input
```
integrate(x^9/(x^10+x^5+2),x, algorithm="giac")
```

output
```
-1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = \frac{\ln\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{2\,\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

input
```
int(x^9/(x^5 + x^10 + 2),x)
```

output
```
log(x^5 + x^10 + 2)/10 - (7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35
```

Reduce [F]

$$\int \frac{x^9}{2+x^5+x^{10}}\,dx = -\frac{\left(\int \frac{x^4}{x^{10}+x^5+2}dx\right)}{2} + \frac{\log(x^{10}+x^5+2)}{10}$$

input
```
int(x^9/(x^10+x^5+2),x)
```

output
```
( - 5*int(x**4/(x**10 + x**5 + 2),x) + log(x**10 + x**5 + 2))/10
```

3.5 $\int \frac{x^4}{2+x^5+x^{10}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x^4}{2+x^5+x^{10}} \, dx = \frac{2 \arctan \left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}$$

output
```
2/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{2+x^5+x^{10}} \, dx = \frac{2 \arctan \left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}$$

input
```
Integrate[x^4/(2 + x^5 + x^10),x]
```

output
```
(2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1690, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^{10}+x^5+2}\,dx$$
$$\Big\downarrow \text{ 1690}$$
$$\frac{1}{5}\int \frac{1}{x^{10}+x^5+2}dx^5$$
$$\Big\downarrow \text{ 1083}$$
$$-\frac{2}{5}\int \frac{1}{-x^{10}-7}d\left(2x^5+1\right)$$
$$\Big\downarrow \text{ 217}$$
$$\frac{2\arctan\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

input
```
Int[x^4/(2 + x^5 + x^10),x]
```

output
```
(2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2 \arctan\left(\frac{\left(2x^5+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	19
risch	$\frac{2 \arctan\left(\frac{\left(2x^5+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	19

input
```
int(x^4/(x^10+x^5+2),x,method=_RETURNVERBOSE)
```

output
```
2/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2}{35}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x^5+1)\right)$$

input
```
integrate(x^4/(x^10+x^5+2),x, algorithm="fricas")
```

output
```
2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

input
```
integrate(x**4/(x**10+x**5+2),x)
```

output
```
2*sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x^5+1)\right)$$

input
```
integrate(x^4/(x^10+x^5+2),x, algorithm="maxima")
```

output
```
2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x^5+1)\right)$$

input
```
integrate(x^4/(x^10+x^5+2),x, algorithm="giac")
```

output
```
2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

input
```
int(x^4/(x^5 + x^10 + 2),x)
```

output
```
(2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35
```

Reduce [F]

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \int \frac{x^4}{x^{10}+x^5+2}dx$$

input
```
int(x^4/(x^10+x^5+2),x)
```

output
```
int(x**4/(x**10 + x**5 + 2),x)
```

3.6 $\int \frac{1}{x(1+x^5+x^{10})} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{1}{x(1+x^5+x^{10})} \, dx = -\frac{\arctan\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}} + \log(x) - \frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

output
```
-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/2)+ln(x)-1/10*ln(x^10+x^5+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.05

$$\int \frac{1}{x(1+x^5+x^{10})} \, dx = \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{5\sqrt{3}} + \log(x) - \frac{1}{10}\log\left(1+x+x^2\right) - \frac{1}{5}\text{RootSum}\Big[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7 +\#1^8\&, \frac{-\log(x-\#1)\#1+2\log(x-\#1)\#1^2-\log(x-\#1)\#1^3+3\log(x-\#1)\#1^4-\log(x-\#1)}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+8\#1^7}$$

input
```
Integrate[1/(x*(1 + x^5 + x^10)),x]
```

output
```
ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Roo
tSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*
Log[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^
5 - 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^
4 - 7*#1^6 + 8*#1^7) & ]/5
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1693, 1144, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(x^{10}+x^5+1\right)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{5}\int \frac{1}{x^5\left(x^{10}+x^5+1\right)}dx^5$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{5}\left(\int -\frac{x^5+1}{x^{10}+x^5+1}dx^5+\log\left(x^5\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\log\left(x^5\right)-\int \frac{x^5+1}{x^{10}+x^5+1}dx^5\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{5}\left(-\frac{1}{2}\int \frac{1}{x^{10}+x^5+1}dx^5-\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+1}dx^5+\log\left(x^5\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{5}\left(\int \frac{1}{-x^{10}-3}d\left(2x^5+1\right)-\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+1}dx^5+\log\left(x^5\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{5}\left(-\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+1}dx^5-\frac{\arctan\left(\frac{2x^5+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^5\right)\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{5}\left(-\frac{\arctan\left(\frac{2x^5+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^5\right)-\frac{1}{2}\log\left(x^{10}+x^5+1\right)\right)$$

input
```
Int[1/(x*(1 + x^5 + x^10)),x]
```

output
```
(-(ArcTan[(1 + 2*x^5)/Sqrt[3]]/Sqrt[3]) + Log[x^5] - Log[1 + x^5 + x^10]/2
)/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
risch	$\ln\left(x\right)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{5}+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{15}-\frac{\ln\left(x^{10}+x^{5}+1\right)}{10}$
default	$-\frac{\ln\left(x^{2}+x+1\right)}{10}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{15}-\frac{\left(\frac{i\sqrt{3}}{6}+\frac{1}{2}\right)\ln\left(2x^{4}+\left(-1+i\sqrt{3}\right)x^{3}+\left(-1-i\sqrt{3}\right)x^{2}+2x-1+i\sqrt{3}\right)}{5}-\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}\right)\ln($

input
```
int(1/x/(x^10+x^5+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/15*3^(1/2)*arctan(2/3*(x^5+1/2)*3^(1/2))-1/10*ln(x^10+x^5+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx = -\frac{1}{15}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^5+1)\right) - \frac{1}{10}\,\log\left(x^{10}+x^5+1\right)+\log\left(x\right)$$

input
```
integrate(1/x/(x^10+x^5+1),x, algorithm="fricas")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
 log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx = \log(x) - \frac{\log(x^{10}+x^5+1)}{10} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

input
```
integrate(1/x/(x**10+x**5+1),x)
```

output
```
log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3
)/3)/15
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx = -\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right) - \frac{1}{10}\log\left(x^{10}+x^5+1\right) + \frac{1}{5}\log\left(x^5\right)$$

input
```
integrate(1/x/(x^10+x^5+1),x, algorithm="maxima")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
 1/5*log(x^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx = -\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right) - \frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log\left(|x|\right)$$

input
```
integrate(1/x/(x^10+x^5+1),x, algorithm="giac")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
 log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 20.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx = \ln(x) - \frac{\ln(x^{10}+x^5+1)}{10} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

input
```
int(1/(x*(x^5 + x^10 + 1)),x)
```

output
```
log(x) - log(x^5 + x^10 + 1)/10 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5
)/3))/15
```

Reduce [F]

$$\int \frac{1}{x(1+x^5+x^{10})}\,dx = \int \frac{1}{x^{11}+x^6+x}dx$$

input
```
int(1/x/(x^10+x^5+1),x)
```

output
```
int(1/(x**11 + x**6 + x),x)
```

3.7 $\int \frac{1}{x^6(1+x^5+x^{10})}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{x^6(1+x^5+x^{10})}\,dx = -\frac{1}{5x^5} - \frac{\arctan\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}} - \log(x) + \frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

output
```
-1/5/x^5-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/2)-ln(x)+1/10*ln(x^10+x^5
+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 4.33

$$\int \frac{1}{x^6(1+x^5+x^{10})}\,dx = \frac{1}{30}\left(-\frac{6}{x^5} + 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 30\log(x) + 3\log\left(1+x+x^2\right) + 6\text{RootSum}\left[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7 + \#1^8\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1+\log(x-\#1)\#1^2-3\log(x-\#1)\#1^3+2\log(x-\#1)\#1^4}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+}\right.\right.$$

input
```
Integrate[1/(x^6*(1 + x^5 + x^10)),x]
```

output
```
(-6/x^5 + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 30*Log[x] + 3*Log[1 + x +
x^2] + 6*RootSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-Log[x - #
1] + Log[x - #1]*#1 + Log[x - #1]*#1^2 - 3*Log[x - #1]*#1^3 + 2*Log[x - #1
]*#1^4 + Log[x - #1]*#1^5 - 4*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 +
 3*#1^2 - 4*#1^3 + 5*#1^4 - 7*#1^6 + 8*#1^7) & ])/30
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(x^{10}+x^5+1\right)} \, dx$$

$$\Big\downarrow \ 1693$$

$$\frac{1}{5} \int \frac{1}{x^{10} \left(x^{10}+x^5+1\right)}dx^5$$

$$\Big\downarrow \ 1145$$

$$\frac{1}{5} \left(\int -\frac{x^5+1}{x^5 \left(x^{10}+x^5+1\right)}dx^5-\frac{1}{x^5}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{5} \left(-\int \frac{x^5+1}{x^5 \left(x^{10}+x^5+1\right)}dx^5-\frac{1}{x^5}\right)$$

$$\Big\downarrow \ 1200$$

$$\frac{1}{5} \left(-\int \left(\frac{1}{x^5}-\frac{x^5}{x^{10}+x^5+1}\right)dx^5-\frac{1}{x^5}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{5} \left(-\frac{\arctan\left(\frac{2 x^5+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{x^5}-\log \left(x^5\right)+\frac{1}{2} \log \left(x^{10}+x^5+1\right)\right)$$

input
```
Int[1/(x^6*(1 + x^5 + x^10)),x]
```

output
```
(-x^(-5) - ArcTan[(1 + 2*x^5)/Sqrt[3]]/Sqrt[3] - Log[x^5] + Log[1 + x^5 +
x^10]/2)/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{1}{5x^5}-\ln(x)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^5+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{15}+\frac{\ln(x^{10}+x^5+1)}{10}$
default	$\frac{\ln(x^2+x+1)}{10}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{15}+\frac{\left(\frac{i\sqrt{3}}{6}+\frac{1}{2}\right)\ln\left(2x^4+\left(-1-i\sqrt{3}\right)x^3+\left(-1+i\sqrt{3}\right)x^2+2x-1-i\sqrt{3}\right)}{5}+\frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}\right)\ln\left(2x\right.}{}$

input
```
int(1/x^6/(x^10+x^5+1),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^5-ln(x)-1/15*3^(1/2)*arctan(2/3*(x^5+1/2)*3^(1/2))+1/10*ln(x^10+x^5
+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^6(1+x^5+x^{10})}\,dx = -\frac{2\sqrt{3}x^5\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right)-3x^5\log\left(x^{10}+x^5+1\right)+30x^5\log(x)+6}{30x^5}$$

input
```
integrate(1/x^6/(x^10+x^5+1),x, algorithm="fricas")
```

output
```
-1/30*(2*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 3*x^5*log(x^10 + x^
5 + 1) + 30*x^5*log(x) + 6)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6\left(1+x^5+x^{10}\right)}\,dx=-\log\left(x\right)+\frac{\log\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}-\frac{1}{5x^5}$$

input
```
integrate(1/x**6/(x**10+x**5+1),x)
```

output
```
-log(x) + log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(
3)/3)/15 - 1/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^6\left(1+x^5+x^{10}\right)}\,dx=-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right)-\frac{1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{1}{5}\log\left(x^5\right)$$

input
```
integrate(1/x^6/(x^10+x^5+1),x, algorithm="maxima")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/5/x^5 + 1/10*log(x^10 +
x^5 + 1) - 1/5*log(x^5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^6\left(1+x^5+x^{10}\right)}\,dx=-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right)+\frac{x^5-1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\log\left(|x|\right)$$

input
```
integrate(1/x^6/(x^10+x^5+1),x, algorithm="giac")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) + 1/5*(x^5 - 1)/x^5 + 1/10*l
og(x^10 + x^5 + 1) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 20.71 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^6 (1+x^5+x^{10})} \, dx = \frac{\ln (x^{10}+x^5+1)}{10} - \ln (x) - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15} - \frac{1}{5\,x^5}$$

input
```
int(1/(x^6*(x^5 + x^10 + 1)),x)
```

output
```
log(x^5 + x^10 + 1)/10 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5
)/3))/15 - 1/(5*x^5)
```

Reduce [F]

$$\int \frac{1}{x^6 (1+x^5+x^{10})} \, dx = \int \frac{1}{x^{16}+x^{11}+x^6} dx$$

input
```
int(1/x^6/(x^10+x^5+1),x)
```

output
```
int(1/(x**16 + x**11 + x**6),x)
```

3.8 $\int \frac{1}{x+x^6+x^{11}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \frac{1}{x+x^6+x^{11}}\,dx = -\frac{\arctan\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

output
```
-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/2)+ln(x)-1/10*ln(x^10+x^5+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.05

$$\int \frac{1}{x+x^6+x^{11}}\,dx = \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x+x^2\right)$$
$$-\frac{1}{5}\text{RootSum}\Big[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7$$
$$+\#1^8\&,\frac{-\log(x-\#1)\#1+2\log(x-\#1)\#1^2-\log(x-\#1)\#1^3+3\log(x-\#1)\#1^4-\log(x-\#1)}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+8\#1^7}$$

input
```
Integrate[(x + x^6 + x^11)^(-1),x]
```

output
```
ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Roo
tSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*
Log[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^
5 - 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^
4 - 7*#1^6 + 8*#1^7) & ]/5
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {1949, 1693, 1144, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11}+x^6+x}\,dx$$

$$\Big\downarrow \text{1949}$$

$$\int \frac{1}{x\left(x^{10}+x^5+1\right)}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{5}\int \frac{1}{x^5\left(x^{10}+x^5+1\right)}dx^5$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{5}\left(\int -\frac{x^5+1}{x^{10}+x^5+1}dx^5+\log\left(x^5\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\log\left(x^5\right)-\int \frac{x^5+1}{x^{10}+x^5+1}dx^5\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{5}\left(-\frac{1}{2}\int \frac{1}{x^{10}+x^5+1}dx^5-\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+1}dx^5+\log\left(x^5\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{5}\left(\int \frac{1}{-x^{10}-3}d(2x^5+1)-\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+1}dx^5+\log\left(x^5\right)\right)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{5}\left(-\frac{1}{2}\int \frac{2x^5+1}{x^{10}+x^5+1}dx^5-\frac{\arctan\left(\frac{2x^5+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^5\right)\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{5}\left(-\frac{\arctan\left(\frac{2x^5+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^5\right)-\frac{1}{2}\log\left(x^{10}+x^5+1\right)\right)$$

input
```
Int[(x + x^6 + x^11)^(-1),x]
```

output
```
(-(ArcTan[(1 + 2*x^5)/Sqrt[3]]/Sqrt[3]) + Log[x^5] - Log[1 + x^5 + x^10]/2
)/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
risch	$\ln\left(x\right)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{5}+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{15}-\frac{\ln\left(x^{10}+x^{5}+1\right)}{10}$
default	$-\frac{\ln\left(x^{2}+x+1\right)}{10}+\frac{\arctan\left(\frac{\left(2x+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{15}-\frac{\left(\frac{i\sqrt{3}}{6}+\frac{1}{2}\right)\ln\left(2x^{4}+\left(-1+i\sqrt{3}\right)x^{3}+\left(-1-i\sqrt{3}\right)x^{2}+2x-1+i\sqrt{3}\right)}{5}-\frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}\right)\ln\left(\right.}{}$

input
```
int(1/(x^11+x^6+x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/15*3^(1/2)*arctan(2/3*(x^5+1/2)*3^(1/2))-1/10*ln(x^10+x^5+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x+x^6+x^{11}}\,dx = -\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right) - \frac{1}{10}\log\left(x^{10}+x^5+1\right) + \log(x)$$

input
```
integrate(1/(x^11+x^6+x),x, algorithm="fricas")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
 log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{x+x^6+x^{11}}\,dx = \log(x) - \frac{\log\left(x^{10}+x^5+1\right)}{10} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

input
```
integrate(1/(x**11+x**6+x),x)
```

output
```
log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3
)/3)/15
```

Maxima [F]

$$\int \frac{1}{x+x^6+x^{11}}\,dx = \int \frac{1}{x^{11}+x^6+x}\,dx$$

input
```
integrate(1/(x^11+x^6+x),x, algorithm="maxima")
```

output
```
1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/5*integrate((4*x^7 - 3*x^6
- x^5 + 3*x^4 - x^3 + 2*x^2 - x)/(x^8 - x^7 + x^5 - x^4 + x^3 - x + 1), x)
 - 1/10*log(x^2 + x + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{1}{x+x^6+x^{11}}\,dx = -\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right) - \frac{1}{10}\log\left(x^{10}+x^5+1\right) + \log\left(|x|\right)$$

input
```
integrate(1/(x^11+x^6+x),x, algorithm="giac")
```

output
```
-1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
 log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{x+x^6+x^{11}}\,dx = \ln(x) - \frac{\ln(x^{10}+x^5+1)}{10} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

input
```
int(1/(x + x^6 + x^11),x)
```

output
```
log(x) - log(x^5 + x^10 + 1)/10 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5
)/3))/15
```

Reduce [F]

$$\int \frac{1}{x+x^6+x^{11}}\,dx = \int \frac{1}{x^{11}+x^6+x}dx$$

input
```
int(1/(x^11+x^6+x),x)
```

output
```
int(1/(x**11 + x**6 + x),x)
```

3.9 $\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 147

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = -\frac{b(b^2-2ac)\,x}{c^4}+\frac{(b^2-ac)\,x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c} + \frac{b(b^4-5ab^2c+5a^2c^2)\,\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}} + \frac{(b^4-3ab^2c+a^2c^2)\log\left(a+bx+cx^2\right)}{2c^5}$$

output
```
-b*(-2*a*c+b^2)*x/c^4+1/2*(-a*c+b^2)*x^2/c^3-1/3*b*x^3/c^2+1/4*x^4/c+b*(5*
a^2*c^2-5*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b
^2)^(1/2)+1/2*(a^2*c^2-3*a*b^2*c+b^4)*ln(c*x^2+b*x+a)/c^5
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$

$$= \frac{cx(-12b^3+6b^2cx-4bc(-6a+cx^2)+3c^2x(-2a+cx^2)) - \frac{12b(b^4-5ab^2c+5a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + 6(b^4-3}{12c^5}$$

input
```
Integrate[x^3/(c + a/x^2 + b/x),x]
```

output
```
(c*x*(-12*b^3 + 6*b^2*c*x - 4*b*c*(-6*a + c*x^2) + 3*c^2*x*(-2*a + c*x^2))
 - (12*b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] + 6*(b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + x*(b + c*x
)])/(12*c^5)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1692, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\frac{a}{x^2}+\frac{b}{x}+c}\,dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x^5}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{1143}$$

$$\int \left(\frac{x\left(a^2c^2-3ab^2c+b^4\right)+ab\left(b^2-2ac\right)}{c^4\left(a+bx+cx^2\right)}-\frac{b\left(b^2-2ac\right)}{c^4}+\frac{x\left(b^2-ac\right)}{c^3}-\frac{bx^2}{c^2}+\frac{x^3}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(5a^2c^2 - 5ab^2c + b^4)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}} + \frac{(a^2c^2 - 3ab^2c + b^4)\log(a + bx + cx^2)}{2c^5} - \frac{bx(b^2 - 2ac)}{c^4} + \frac{x^2(b^2 - ac)}{2c^3} - \frac{bx^3}{3c^2} + \frac{x^4}{4c}$$

input
```
Int[x^3/(c + a/x^2 + b/x),x]
```

output
```
-((b*(b^2 - 2*a*c)*x)/c^4) + ((b^2 - a*c)*x^2)/(2*c^3) - (b*x^3)/(3*c^2) +
 x^4/(4*c) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) + ((b^4 - 3*a*b^2*c + a^2*c^2)*Log[a +
 b*x + c*x^2])/(2*c^5)
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.12

method	result
default	$\frac{\frac{1}{4}x^4c^3-\frac{1}{3}b\,x^3c^2-\frac{1}{2}a\,c^2x^2+\frac{1}{2}b^2c\,x^2+2abcx-b^3x}{c^4}+\frac{\frac{\left(a^2c^2-3a\,b^2c+b^4\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-2a^2bc+b^3a-\frac{\left(a^2c^2-3a\,b^2c+b^4\right)b}{2c}\right)\arctan}{\sqrt{4ac-b^2}}}{c^4}$
risch	Expression too large to display

input
```
int(x^3/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/4*x^4*c^3-1/3*b*x^3*c^2-1/2*a*c^2*x^2+1/2*b^2*c*x^2+2*a*b*c*x-b^3
*x)+1/c^4*(1/2*(a^2*c^2-3*a*b^2*c+b^4)/c*ln(c*x^2+b*x+a)+2*(-2*a^2*b*c+b^3
*a-1/2*(a^2*c^2-3*a*b^2*c+b^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.17

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$

$$= \left[\frac{3\,(b^2c^4-4\,ac^5)x^4-4\,(b^3c^3-4\,abc^4)x^3+6\,(b^4c^2-5\,ab^2c^3+4\,a^2c^4)x^2+6\,(b^5-5\,ab^3c+5\,a^2bc^2)\sqrt{b^2-}}{}\right.$$

input
```
integrate(x^3/(c+a/x^2+b/x),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3 + 6*(b^4*c^
2 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 6*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2
*c*x + b))/(c*x^2 + b*x + a)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x +
 6*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b
^2*c^5 - 4*a*c^6), 1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^
4)*x^3 + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 12*(b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b
^2 - 4*a*c)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(b^6 - 7*a*b^4
*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(144) = 288$.

Time = 0.78 (sec) , antiderivative size = 605, normalized size of antiderivative = 4.12

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$
$$= -\frac{bx^3}{3c^2} + x^2\left(-\frac{a}{2c^2}+\frac{b^2}{2c^3}\right) + x\left(\frac{2ab}{c^3}-\frac{b^3}{c^4}\right) + \left(-\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)} + \frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)\log\left(x + \frac{2a^3c^2-4a^2b^2c+ab^4-4ac^5\left(-\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)}+\frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)}{5a^2bc^2-5ab^3c+b^5}\right.$$
$$+ \left(\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)} + \frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)\log\left(x + \frac{2a^3c^2-4a^2b^2c+ab^4-4ac^5\left(\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)}+\frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)}{5a^2bc^2-5ab^3c+b^5}\right.$$
$$+ \frac{x^4}{4c}$$

input
```
integrate(x**3/(c+a/x**2+b/x),x)
```

output
```
-b*x**3/(3*c**2) + x**2*(-a/(2*c**2) + b**2/(2*c**3)) + x*(2*a*b/c**3 - b*
*3/c**4) + (-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c*
*5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5))*log(x + (2*
a**3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*a*c**5*(-b*sqrt(-4*a*c + b**2)*(5*a
**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b
**2*c + b**4)/(2*c**5)) + b**2*c**4*(-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 -
 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b*
*4)/(2*c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**5)) + (b*sqrt(-4*a*c + b**
2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2
- 3*a*b**2*c + b**4)/(2*c**5))*log(x + (2*a**3*c**2 - 4*a**2*b**2*c + a*b*
*4 - 4*a*c**5*(b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*
c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)) + b**2*c*
*4*(b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c
 - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)))/(5*a**2*b*c**2 - 5*
a*b**3*c + b**5)) + x**4/(4*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c+a/x^2+b/x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{3\,c^3x^4-4\,bc^2x^3+6\,b^2cx^2-6\,ac^2x^2-12\,b^3x+24\,abcx}{12\,c^4} + \frac{(b^4-3\,ab^2c+a^2c^2)\log\left(cx^2+bx+a\right)}{2\,c^5} - \frac{(b^5-5\,ab^3c+5\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^5}$$

input
```
integrate(x^3/(c+a/x^2+b/x),x, algorithm="giac")
```

output
```
1/12*(3*c^3*x^4 - 4*b*c^2*x^3 + 6*b^2*c*x^2 - 6*a*c^2*x^2 - 12*b^3*x + 24*
a*b*c*x)/c^4 + 1/2*(b^4 - 3*a*b^2*c + a^2*c^2)*log(c*x^2 + b*x + a)/c^5 -
(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sq
rt(-b^2 + 4*a*c)*c^5)
```

Mupad [B] (verification not implemented)

Time = 20.71 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.24

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = x\left(\frac{b\left(\frac{a}{c^2}-\frac{b^2}{c^3}\right)}{c}+\frac{a\,b}{c^3}\right)+\frac{x^4}{4\,c}-x^2\left(\frac{a}{2\,c^2}-\frac{b^2}{2\,c^3}\right) - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-4\,a^3\,c^3+13\,a^2\,b^2\,c^2-7\,a\,b^4\,c+b^6\right)}{2\,\left(4\,a\,c^6-b^2\,c^5\right)} - \frac{b\,x^3}{3\,c^2}-\frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(5\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{c^5\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^3/(c + a/x^2 + b/x),x)
```

output
```
x*((b*(a/c^2 - b^2/c^3))/c + (a*b)/c^3) + x^4/(4*c) - x^2*(a/(2*c^2) - b^2
/(2*c^3)) - (log(a + b*x + c*x^2)*(b^6 - 4*a^3*c^3 + 13*a^2*b^2*c^2 - 7*a*
b^4*c))/(2*(4*a*c^6 - b^2*c^5)) - (b*x^3)/(3*c^2) - (b*atan((b + 2*c*x)/(4
*a*c - b^2)^(1/2))*(b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c^5*(4*a*c - b^2)^(1/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.12

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{-60\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2b\,c^2+60\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^3c-12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b}{}$$

input
```
int(x^3/(c+a/x^2+b/x),x)
```

output
```
( - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2
 + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c - 1
2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**5 + 24*log(a
+ b*x + c*x**2)*a**3*c**3 - 78*log(a + b*x + c*x**2)*a**2*b**2*c**2 + 42*l
og(a + b*x + c*x**2)*a*b**4*c - 6*log(a + b*x + c*x**2)*b**6 + 96*a**2*b*c
**3*x - 24*a**2*c**4*x**2 - 72*a*b**3*c**2*x + 30*a*b**2*c**3*x**2 - 16*a*
b*c**4*x**3 + 12*a*c**5*x**4 + 12*b**5*c*x - 6*b**4*c**2*x**2 + 4*b**3*c**
3*x**3 - 3*b**2*c**4*x**4)/(12*c**5*(4*a*c - b**2))
```

3.10 $\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}} dx$

Optimal result

Integrand size = 18, antiderivative size = 118

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}} dx = \frac{(b^2-ac)\,x}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c} - \frac{(b^4-4ab^2c+2a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}} - \frac{b(b^2-2ac)\log(a+bx+cx^2)}{2c^4}$$

output
```
(-a*c+b^2)*x/c^3-1/2*b*x^2/c^2+1/3*x^3/c-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh
((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+b^2)^(1/2)-1/2*b*(-2*a*c+b^2)*l
n(c*x^2+b*x+a)/c^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}} dx = \frac{cx(6b^2-6ac-3bcx+2c^2x^2) + \frac{6(b^4-4ab^2c+2a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 3(b^3-2abc)\log(a+x(b+cx))}{6c^4}$$

input
```
Integrate[x^2/(c + a/x^2 + b/x),x]
```

output
```
(c*x*(6*b^2 - 6*a*c - 3*b*c*x + 2*c^2*x^2) + (6*(b^4 - 4*a*b^2*c + 2*a^2*c
^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 3*(b^3 -
2*a*b*c)*Log[a + x*(b + c*x)])/(6*c^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1692, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\frac{a}{x^2}+\frac{b}{x}+c}dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{x^4}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{ 1143}$$

$$\int \left(-\frac{bx\left(b^2-2ac\right)+a\left(b^2-ac\right)}{c^3\left(a+bx+cx^2\right)}+\frac{b^2-ac}{c^3}-\frac{bx}{c^2}+\frac{x^2}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2c^4}+\frac{x\left(b^2-ac\right)}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}$$

input
```
Int[x^2/(c + a/x^2 + b/x),x]
```

output
```
((b^2 - a*c)*x)/c^3 - (b*x^2)/(2*c^2) + x^3/(3*c) - ((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) -
 (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4)
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{-\frac{1}{3}c^2x^3+\frac{1}{2}bc\,x^2+xac-b^2x}{c^3}+\frac{\frac{\left(2abc-b^3\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(a^2c-b^2a-\frac{\left(2abc-b^3\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c^3}$	128
risch	Expression too large to display	1138

input
```
int(x^2/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/3*c^2*x^3+1/2*b*c*x^2+x*a*c-b^2*x)+1/c^3*(1/2*(2*a*b*c-b^3)/c*l
n(c*x^2+b*x+a)+2*(a^2*c-b^2*a-1/2*(2*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.25

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$
$$= \left[\frac{2\,(b^2c^3-4\,ac^4)x^3-3\,(b^3c^2-4\,abc^3)x^2+3\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-}}{cx^2+bx+a}\right.}{6\,(b^2c^4-4\,ac^5)}\right.$$

input
```
integrate(x^2/(c+a/x^2+b/x),x, algorithm="fricas")
```

output
```
[1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b*c^3)*x^2 + 3*(b^4 - 4
*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^4*c - 5*a*
b^2*c^2 + 4*a^2*c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x
 + a))/(b^2*c^4 - 4*a*c^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 -
4*a*b*c^3)*x^2 - 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*arctan
(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^4*c - 5*a*b^2*c^2 +
 4*a^2*c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a))/(b
^2*c^4 - 4*a*c^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(110) = 220$.

Time = 0.65 (sec) , antiderivative size = 498, normalized size of antiderivative = 4.22

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = -\frac{bx^2}{2c^2} + x\left(-\frac{a}{c^2}+\frac{b^2}{c^3}\right) + \left(\frac{b(2ac-b^2)}{2c^4} - \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)\log\left(x + \frac{-3a^2bc+ab^3+4ac^4\left(\frac{b(2ac-b^2)}{2c^4} - \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+}{2c^4\cdot(4ac-b^2)}\right.}{2a^2c^2-4ab^2}\right. + \left(\frac{b(2ac-b^2)}{2c^4} + \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)\log\left(x + \frac{-3a^2bc+ab^3+4ac^4\left(\frac{b(2ac-b^2)}{2c^4} + \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+}{2c^4\cdot(4ac-b^2)}\right.}{2a^2c^2-4ab^2}\right. + \frac{x^3}{3c}$$

input
```
integrate(x**2/(c+a/x**2+b/x),x)
```

output
```
-b*x**2/(2*c**2) + x*(-a/c**2 + b**2/c**3) + (b*(2*a*c - b**2)/(2*c**4) -
sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**
2)))*log(x + (-3*a**2*b*c + a*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) -
 sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b*
*2))) - b**2*c**3*(b*(2*a*c - b**2)/(2*c**4) - sqrt(-4*a*c + b**2)*(2*a**2
*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b*
*2*c + b**4)) + (b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c
**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)))*log(x + (-3*a**2*b*c + a
*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*
c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))) - b**2*c**3*(b*(2*a*c -
 b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2
*c**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c + b**4)) + x**3/(3*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c+a/x^2+b/x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{2\,c^2x^3-3\,bcx^2+6\,b^2x-6\,acx}{6\,c^3}-\frac{(b^3-2\,abc)\log\left(cx^2+bx+a\right)}{2\,c^4}+\frac{(b^4-4\,ab^2c+2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^4}$$

input
```
integrate(x^2/(c+a/x^2+b/x),x, algorithm="giac")
```

output
```
1/6*(2*c^2*x^3 - 3*b*c*x^2 + 6*b^2*x - 6*a*c*x)/c^3 - 1/2*(b^3 - 2*a*b*c)*
log(c*x^2 + b*x + a)/c^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b
)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{x^3}{3\,c}-x\left(\frac{a}{c^2}-\frac{b^2}{c^3}\right)-\frac{b\,x^2}{2\,c^2}+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(8\,a^2\,b\,c^2-6\,a\,b^3\,c+b^5\right)}{2\,\left(4\,a\,c^5-b^2\,c^4\right)} + \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c^4\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^2/(c + a/x^2 + b/x),x)
```

output
```
x^3/(3*c) - x*(a/c^2 - b^2/c^3) - (b*x^2)/(2*c^2) + (log(a + b*x + c*x^2)*
(b^5 + 8*a^2*b*c^2 - 6*a*b^3*c))/(2*(4*a*c^5 - b^2*c^4)) + (atan(b/(4*a*c
- b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))
/(c^4*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.16

$$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2c^2-24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2c+6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^4+}{}$$

input
```
int(x^2/(c+a/x^2+b/x),x)
```

output
```
(12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2 - 24
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c + 6*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4 + 24*log(a + b*x
+ c*x**2)*a**2*b*c**2 - 18*log(a + b*x + c*x**2)*a*b**3*c + 3*log(a + b*x
+ c*x**2)*b**5 - 24*a**2*c**3*x + 30*a*b**2*c**2*x - 12*a*b*c**3*x**2 + 8*
a*c**4*x**3 - 6*b**4*c*x + 3*b**3*c**2*x**2 - 2*b**2*c**3*x**3)/(6*c**4*(4
*a*c - b**2))
```

3.11 $\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}$$

output
```
-b*x/c^2+1/2*x^2/c+b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^
3/(-4*a*c+b^2)^(1/2)+1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/c^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{cx(-2b+cx)-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log(a+x(b+cx))}{2c^3}$$

input
```
Integrate[x/(c + a/x^2 + b/x),x]
```

output
```
(c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1692, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\frac{a}{x^2}+\frac{b}{x}+c}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x^3}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{1143}$$

$$\int \left(\frac{x\left(b^2-ac\right)+ab}{c^2\left(a+bx+cx^2\right)}-\frac{b}{c^2}+\frac{x}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(b^2-3ac\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

input
```
Int[x/(c + a/x^2 + b/x),x]
```

output
```
-((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*
c^3)
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

method	result
default	$-\frac{-\frac{1}{2}cx^2+bx}{c^2}+\frac{\frac{\left(-ac+b^2\right)\ln\left(cx^2+bx+a\right)}{2c}+\frac{2\left(ab-\frac{\left(-ac+b^2\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c^2}$
risch	$\frac{x^2}{2c}-\frac{bx}{c^2}-\frac{2\ln\left(12a^2bc^2-7ab^3c+b^5-2\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,cx-\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,b\right)a^2}{c(4ac-b^2)}+\frac{5\ln\left(12a^2bc^2-7ab^3\right.}{}$

input
```
int(x/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(-1/2*c*x^2+b*x)+1/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^2+b*x+a)+2*(a*b-1/2
*(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.34

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \left[\frac{(b^2c^2-4\,ac^3)x^2-(b^3-3\,abc)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^3c-4\,abc^2)x+}{2\,(b^2c^3-4\,ac^4)}\right.$$

input
```
integrate(x/(c+a/x^2+b/x),x, algorithm="fricas")
```

output
```
[1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c
^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b
*x + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x
^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3
- 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*
x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(83) = 166$.

Time = 0.57 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.28

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = -\frac{bx}{c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x+\frac{2a^2c-ab^2+4ac^3\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b}{2c^3}\right.}{3abc-b^3}\right.$$
$$+\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x+\frac{2a^2c-ab^2+4ac^3\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right)$$
$$+\frac{x^2}{2c}$$

input
```
integrate(x/(c+a/x**2+b/x),x)
```

output
```
-b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-
4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**
3)) - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b*
*2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*
(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2
*a**2*c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*
(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2
)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*
c - b**3)) + x**2/(2*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c+a/x^2+b/x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{cx^2-2\,bx}{2\,c^2}+\frac{(b^2-ac)\log\left(cx^2+bx+a\right)}{2\,c^3} - \frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate(x/(c+a/x^2+b/x),x, algorithm="giac")
```

output
```
1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3
- 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=\frac{x^2}{2\,c}-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(4\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{2\,\left(4\,a\,c^4-b^2\,c^3\right)}-\frac{b\,x}{c^2}+\frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(3\,a\,c-b^2\right)}{c^3\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x/(c + a/x^2 + b/x),x)
```

output
```
x^2/(2*c) - (log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c
^4 - b^2*c^3)) - (b*x)/c^2 + (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2))*(3*a
*c - b^2))/(c^3*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.02

$$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=\frac{6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abc-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^3-4\log(c\,x^2+bx+a)\,a^2c^2+5\log(c\,x^2+\cdots}{2c^3\left(4ac-b^2\right)}$$

input
```
int(x/(c+a/x^2+b/x),x)
```

output
```
(6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c - 2*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3 - 4*log(a + b*x +
c*x**2)*a**2*c**2 + 5*log(a + b*x + c*x**2)*a*b**2*c - log(a + b*x + c*x**
2)*b**4 - 8*a*b*c**2*x + 4*a*c**3*x**2 + 2*b**3*c*x - b**2*c**2*x**2)/(2*c
**3*(4*a*c - b**2))
```

3.12 $\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 70

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=\frac{x}{c}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}$$

output
```
x/c-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1
/2)-1/2*b*ln(c*x^2+b*x+a)/c^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx=\frac{x}{c}+\frac{(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}$$

input
```
Integrate[(c + a/x^2 + b/x)^(-1),x]
```

output
```
x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^
2 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1679, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{a}{x^2}+\frac{b}{x}+c}dx$$

$$\Big\downarrow \text{1679}$$

$$\int \frac{x^2}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{1143}$$

$$\int \left(\frac{1}{c}-\frac{a+bx}{c\left(a+bx+cx^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(b^2-2ac\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

input
```
Int[(c + a/x^2 + b/x)^(-1),x]
```

output
```
x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

method	result
default	$\frac{x}{c}+\frac{-\frac{b\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-a+\frac{b^{2}}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c}$
risch	$\frac{x}{c}-\frac{2\ln\left(-8a^{2}c^{2}+6a\,b^{2}c-b^{4}-2\sqrt{-(4ac-b^{2})(2ac-b^{2})^{2}}\,cx-\sqrt{-(4ac-b^{2})(2ac-b^{2})^{2}}\,b\right)ab}{c(4ac-b^{2})}+\frac{\ln\left(-8a^{2}c^{2}+6a\,b^{2}c-b^{4}-2\sqrt{-(4a\right.}{}$

input
```
int(1/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/c*(-1/2*b/c*ln(c*x^2+b*x+a)+2*(-a+1/2*b^2/c)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.36

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$$
$$=\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^2c-4\,ac^2)x+(b^3-4\,abc)\log\left(cx^2\right.}{2\,(b^2c^2-4\,ac^3)}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-2\,(b^2c-4\,ac^2)x+(b^3-4\,abc)\log\left(cx^2+bx+\right.}{2\,(b^2c^2-4\,ac^3)}$$

input
```
integrate(1/(c+a/x^2+b/x),x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*
a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c
^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2
*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(
b^2 - 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x +
a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(65) = 130$.

Time = 0.35 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.37

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-ab-4ac^2\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \frac{x}{c}$$

input
```
integrate(1/(c+a/x**2+b/x),x)
```

output
```
(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))
*log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2
)/(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*
a*c - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqr
t(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4
*a*c**2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c -
 b**2))) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**
2*(4*a*c - b**2))))/(2*a*c - b**2)) + x/c
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{x}{c} - \frac{b\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{(b^2-2\,ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(1/(c+a/x^2+b/x),x, algorithm="giac")
```

output
```
x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 20.56 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.46

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{x}{c}+\frac{b^3\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)}-\frac{2\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} +\frac{b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c-b^2}}-\frac{2\,a\,b\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^3-b^2\,c^2}$$

input
```
int(1/(c + a/x^2 + b/x),x)
```

output
```
x/c + (b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*atan(b/(4*
a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) +
 (b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c^2*(4*a
*c - b^2)^(1/2)) - (2*a*b*c*log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.94

$$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \frac{-4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ac+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2-4\log\left(c\,x^2+bx+a\right)abc+\log\left(c\,x^2+b\right.}{2c^2\left(4ac-b^2\right)}$$

input
```
int(1/(c+a/x^2+b/x),x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c + 2*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2 - 4*log(a + b*x +
 c*x**2)*a*b*c + log(a + b*x + c*x**2)*b**3 + 8*a*c**2*x - 2*b**2*c*x)/(2*
c**2*(4*a*c - b**2))
```

3.13 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 56

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx=\frac{b\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

output
```
b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)+1/2*ln(c*x^2+
b*x+a)/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx=\frac{-\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log(a+x(b+cx))}{2c}$$

input
```
Integrate[1/((c + a/x^2 + b/x)*x),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
+ x*(b + c*x)])/(2*c)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1692, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{x}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}-\frac{b\int \frac{1}{cx^2+bx+a}dx}{2c}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{c}+\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}+\frac{b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

input
```
Int[1/((c + a/x^2 + b/x)*x),x]
```

output
```
(b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
 b*x + c*x^2]/(2*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

method	result
default	$\frac{\ln\left(c\,x^{2}+b x+a\right)}{2c}-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{c\sqrt{4ac-b^{2}}}$
risch	$\frac{2\ln\left(-2\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,cx-4abc+b^{3}-\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,b\right)a}{4ac-b^{2}}-\frac{\ln\left(-2\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,cx-4abc+b^{3}-\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,b\right)b^{2}}{2c\left(4ac-b^{2}\right)}+\frac{\ln\left(-\right.}{}$

input
```
int(1/(c+a/x^2+b/x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.30

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx$$
$$= \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^2-4\,ac)\log\left(cx^2+bx+a\right)}{2\,(b^2c-4\,ac^2)}, \frac{2\,\sqrt{-b^2+4\,ac}b\,\mathrm{arc}}{}\right.$$

input
```
integrate(1/(c+a/x^2+b/x)/x,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x
 + a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 +
4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b
^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(49) = 98$.

Time = 0.19 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.86

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)$$

input
```
integrate(1/(c+a/x**2+b/x)/x,x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-
b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqr
t(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b*
*2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*
(4*a*c - b**2)) + 1/(2*c)))/b)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = -\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c} + \frac{\log\left(cx^2+bx+a\right)}{2\,c}$$

input
```
integrate(1/(c+a/x^2+b/x)/x,x, algorithm="giac")
```

output
```
-b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log
(c*x^2 + b*x + a)/c
```

Mupad [B] (verification not implemented)

Time = 20.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = \frac{2\,a\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^2-b^2\,c} - \frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} - \frac{b^2\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^2-b^2\,c\right)}$$

input
```
int(1/(x*(c + a/x^2 + b/x)),x)
```

output
```
(2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(
1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a
+ b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.50

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx = \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b+4\log(c\,x^2+bx+a)\,ac-\log(c\,x^2+bx+a)\,b^2}{2c\,(4ac-b^2)}$$

input
```
int(1/(c+a/x^2+b/x)/x,x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b + 4*log(a
+ b*x + c*x**2)*a*c - log(a + b*x + c*x**2)*b**2)/(2*c*(4*a*c - b**2))
```

3.14 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 34

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx=-\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
-2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx=\frac{2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

input
```
Integrate[1/((c + a/x^2 + b/x)*x^2),x]
```

output
```
(2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1690, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}dx$$

$$\Big\downarrow \text{1690}$$

$$-\int \frac{1}{\frac{a}{x^2}+c+\frac{b}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{1083}$$

$$2\int \frac{1}{b^2-4ac-\frac{1}{x^2}}d\left(\frac{2a}{x}+b\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[1/((c + a/x^2 + b/x)*x^2),x]
```

output
```
(2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	35
risch	$-\frac{\ln\left(2cx+\sqrt{-4ac+b^2}+b\right)}{\sqrt{-4ac+b^2}}+\frac{\ln\left(-2cx+\sqrt{-4ac+b^2}-b\right)}{\sqrt{-4ac+b^2}}$	61

input
```
int(1/(c+a/x^2+b/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.53

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}},\right.$$
$$\left.-\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

input
```
integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="fricas")
```

output
```
[log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(
c*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(34) = 68$.

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.65

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = -\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right) + \sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

input
```
integrate(1/(c+a/x**2+b/x)/x**2,x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sq
rt(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c
*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \frac{2\,\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="giac")
```

output
```
2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

input
```
int(1/(x^2*(c + a/x^2 + b/x)),x)
```

output
```
(2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = \frac{2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{4ac-b^2}$$

input
```
int(1/(c+a/x^2+b/x)/x^2,x)
```

output
```
(2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)))/(4*a*c - b**2)
```

3.15 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}$$

output
```
b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)+ln(x)/a-1/2*l
n(c*x^2+b*x+a)/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx=-\frac{\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2\log(x)+\log(a+x(b+cx))}{2a}$$

input
```
Integrate[1/((c + a/x^2 + b/x)*x^3),x]
```

output
```
-1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*
Log[x] + Log[a + x*(b + c*x)])/a
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1692, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}dx$$

$$\Big\downarrow \ 1692$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \ 1144$$

$$\frac{\int -\frac{b+cx}{cx^2+bx+a}dx}{a}+\frac{\log(x)}{a}$$

$$\Big\downarrow \ 25$$

$$\frac{\log(x)}{a}-\frac{\int \frac{b+cx}{cx^2+bx+a}dx}{a}$$

$$\Big\downarrow \ 1142$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}b\int \frac{1}{cx^2+bx+a}dx+\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx}{a}$$

$$\Big\downarrow \ 1083$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx-b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{a}$$

$$\Big\downarrow \ 219$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx-\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}$$

$$\Big\downarrow \ 1103$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\log\left(a+bx+cx^2\right)-\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}$$

input
```
Int[1/((c + a/x^2 + b/x)*x^3),x]
```

output
```
Log[x]/a - (-((b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
) + Log[a + b*x + c*x^2]/2)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
default	$\frac{-\frac{\ln\left(c\,x^2+bx+a\right)}{2}-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a}+\frac{\ln(x)}{a}$
risch	$-\frac{2\ln\left(\left(8a\,b^2c-2b^4+6\sqrt{-b^2(4ac-b^2)}\,ac-2\sqrt{-b^2(4ac-b^2)}\,b^2\right)x+12a^2bc-3b^3a-\sqrt{-b^2(4ac-b^2)}\,ab\right)c}{4ac-b^2}+\frac{\ln\left(\left(8a\,b^2c-2b^4+6\sqrt{-\right.\right.}{}$

input
```
int(1/(c+a/x^2+b/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/2*ln(c*x^2+b*x+a)-b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2)))+ln(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.40

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-(b^2-4\,ac)\log\left(cx^2+bx+a\right)+2\,(b^2-4\,ac)\log\left(\right.}{2\,(ab^2-4\,a^2c)}\right.$$

input
```
integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x
 + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*
c)*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)
*log(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(54) = 108$.

Time = 2.55 (sec) , antiderivative size = 564, normalized size of antiderivative = 9.10

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)} - \frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)}{9abc^2-}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)} - \frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-12a^3c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)+2a}{9abc^2-2b^3c}\right) + \frac{\log(x)}{a}$$

input
```
integrate(1/(c+a/x**2+b/x)/x**3,x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c
**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b
**2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3
*c**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**
4*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2
*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2
+ 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/
(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**
2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b*
*2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**
2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2
*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2
*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*
b*c**2 - 2*b**3*c)) + log(x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx = -\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a} - \frac{\log\left(cx^2+bx+a\right)}{2\,a} + \frac{\log\left(|x|\right)}{a}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="giac")
```

output
```
-b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log
(c*x^2 + b*x + a)/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 19.89 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.44

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx &= \frac{\ln(x)}{a} \\ &\quad - \ln\left(b\,c - \left(x\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\left(\frac{1}{2\,a}-\frac{b\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)\right. \\ &\quad \left. + 3\,c^2\,x\right)\left(\frac{1}{2\,a}-\frac{b\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right) \\ &\quad - \ln\left(\left(x\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\left(\frac{1}{2\,a}+\frac{b\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)\right. \\ &\quad \left. - b\,c - 3\,c^2\,x\right)\left(\frac{1}{2\,a}+\frac{b\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)\end{aligned}$$

input
```
int(1/(x^3*(c + a/x^2 + b/x)),x)
```

output
```
log(x)/a - log(b*c - (x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a) - (b*(b^2 -
4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) + 3*c^2*x)*(1/(2*a) - (b*(b^2 - 4*a*c
)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - log((x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(
2*a) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - b*c - 3*c^2*x)*(1/
(2*a) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$$
$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b-4\log(c\,x^2+bx+a)\,ac+\log(c\,x^2+bx+a)\,b^2+8\log(x)\,ac-2\log(x)\,b}{2a\left(4ac-b^2\right)}$$

input
```
int(1/(c+a/x^2+b/x)/x^3,x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b - 4*log(a
+ b*x + c*x**2)*a*c + log(a + b*x + c*x**2)*b**2 + 8*log(x)*a*c - 2*log(x)
*b**2)/(2*a*(4*a*c - b**2))
```

3.16 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = -\frac{1}{ax}-\frac{(b^2-2ac)\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2}+\frac{b\log(a+bx+cx^2)}{2a^2}$$

output
```
-1/a/x-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)
^(1/2)-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = \frac{-\frac{2a}{x}+\frac{2(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2b\log(x)+b\log(a+x(b+cx))}{2a^2}$$

input
```
Integrate[1/((c + a/x^2 + b/x)*x^4),x]
```

output
```
((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1692, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}dx$$

$$\Big\downarrow \; 1692$$

$$\int \frac{1}{x^2\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \; 1145$$

$$\frac{\int -\frac{b+cx}{x(cx^2+bx+a)}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{b+cx}{x(cx^2+bx+a)}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 1200$$

$$-\frac{\int \left(\frac{b}{ax}+\frac{-b^2-cxb+ac}{a(cx^2+bx+a)}\right)dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{(b^2-2ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log(a+bx+cx^2)}{2a}+\frac{b\log(x)}{a}}{a}-\frac{1}{ax}$$

input
```
Int[1/((c + a/x^2 + b/x)*x^4),x]
```

output
```
-(1/(a*x)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sq
rt[b^2 - 4*a*c]) + (b*Log[x])/a - (b*Log[a + b*x + c*x^2])/(2*a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\frac{b\ln\left(c\,x^{2}+bx+a\right)}{2}+\frac{2\left(-ac+\frac{b^{2}}{2}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a^{2}}-\frac{1}{ax}-\frac{b\ln\left(x\right)}{a^{2}}$	81
risch	Expression too large to display	1295

input
```
int(1/(c+a/x^2+b/x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2*b*ln(c*x^2+b*x+a)+2*(-a*c+1/2*b^2)/(4*a*c-b^2)^(1/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2)))-1/a/x-b*ln(x)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.32

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$$
$$= \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2\right.}{2\,(a^2b^2-4\,a^3c)x}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2+bx+\right.}{2\,(a^2b^2-4\,a^3c)x}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^
2*c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))
/((a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arcta
n(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^
3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^
2 - 4*a^3*c)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. $2(75) = 150$.

Time = 162.07 (sec) , antiderivative size = 862, normalized size of antiderivative = 10.64

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x**2+b/x)/x**4,x)
```

output
```
(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))*
log(x + (-28*a**6*b*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(2*a**2*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b/(2*a**2) - sqrt(-4*a*c + b
**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 4*a**5*c**3*(b/(2*a**2)
- sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 2*a**4*b**
5*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))
)**2 - 3*a**4*b**2*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
2*a**2*(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2
*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*a**2*b**3*c**2
- 14*a*b**5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3 - 12*a*b**4*c**2
+ 2*b**6*c)) + (b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4
*a*c - b**2)))*log(x + (-28*a**6*b*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b/(2*a**2) +
sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 4*a**5*c*
*3*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)
)) - 2*a**4*b**5*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*
(4*a*c - b**2)))**2 - 3*a**4*b**2*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(
2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*
a**2*b**3*c**2 - 14*a*b**5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = \frac{b\log\left(cx^2+bx+a\right)}{2\,a^2} - \frac{b\log\left(|x|\right)}{a^2} + \frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^2} - \frac{1}{ax}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="giac")
```

output
```
1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan(
(2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 20.06 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.19

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$$
$$= \frac{\ln\left(2\,a\,b^3+2\,b^4\,x-2\,a\,b^2\,\sqrt{b^2-4\,a\,c}+a^2\,c\,\sqrt{b^2-4\,a\,c}-2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\,c}{4\,a^3\,c-a^2\,b^2}$$
$$-\frac{1}{a\,x}$$
$$-\frac{\ln\left(2\,a\,b^3+2\,b^4\,x+2\,a\,b^2\,\sqrt{b^2-4\,a\,c}-a^2\,c\,\sqrt{b^2-4\,a\,c}+2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b}{4\,a^3\,c-a^2\,b^2}$$
$$-\frac{b\,\ln(x)}{a^2}$$

input
```
int(1/(x^4*(c + a/x^2 + b/x)),x)
```

output
```
(log(2*a*b^3 + 2*b^4*x - 2*a*b^2*(b^2 - 4*a*c)^(1/2) + a^2*c*(b^2 - 4*a*c)
^(1/2) - 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c
*x + 4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(a*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) - b
^3/2 + (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - 1/(a*x) - (log(
2*a*b^3 + 2*b^4*x + 2*a*b^2*(b^2 - 4*a*c)^(1/2) - a^2*c*(b^2 - 4*a*c)^(1/2
) + 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x -
4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(b^3/2 - a*(2*b*c + c*(b^2 - 4*a*c)^(1/2))
+ (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - (b*log(x))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.95

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$$
$$= \frac{-4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)acx+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2x+4\log(c\,x^2+bx+a)\,abcx-\log(c\,x^2}{2a^2x\,(4ac-b^2)}$$

input
```
int(1/(c+a/x^2+b/x)/x^4,x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*x + 2*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*x + 4*log(a + b
*x + c*x**2)*a*b*c*x - log(a + b*x + c*x**2)*b**3*x - 8*log(x)*a*b*c*x + 2
*log(x)*b**3*x - 8*a**2*c + 2*a*b**2)/(2*a**2*x*(4*a*c - b**2))
```

3.17 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 104

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2a^3}$$

output
```
-1/2/a/x^2+b/a^2/x+b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^
3/(-4*a*c+b^2)^(1/2)+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/a
^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = \frac{-\frac{a^2}{x^2}+\frac{2ab}{x}-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2(b^2-ac)\log(x)+(-b^2+ac)\log(a+x(b+cx))}{2a^3}$$

input
```
Integrate[1/((c + a/x^2 + b/x)*x^5),x]
```

output
```
(-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
+ 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a
+ x*(b + c*x)])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1692, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{1}{x^3\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \text{ 1145}$$

$$\frac{\int -\frac{b+cx}{x^2(cx^2+bx+a)}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{b+cx}{x^2(cx^2+bx+a)}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{ 1200}$$

$$-\frac{\int \left(\frac{b}{ax^2}+\frac{ac-b^2}{a^2x}+\frac{b(b^2-2ac)+c(b^2-ac)x}{a^2(cx^2+bx+a)}\right)dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{b(b^2-3ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^2}-\frac{\log(x)(b^2-ac)}{a^2}-\frac{b}{ax}}{a}-\frac{1}{2ax^2}$$

input
```
Int[1/((c + a/x^2 + b/x)*x^5),x]
```

output
```
-1/2*1/(a*x^2) - (-(b/(a*x)) - (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - a*c)*Log[x])/a^2 + ((b^2 -
 a*c)*Log[a + b*x + c*x^2])/(2*a^2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23

method	result
default	$\frac{\frac{\left(a\,c^2-b^2c\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(2abc-b^3-\frac{\left(a\,c^2-b^2c\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^3}-\frac{1}{2a\,x^2}+\frac{\left(-ac+b^2\right)\ln\left(x\right)}{a^3}+\frac{b}{a^2x}$
risch	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}-\frac{\ln\left(x\right)c}{a^2}+\frac{\ln\left(x\right)b^2}{a^3}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4c\,a^4-a^3b^2\right)\_Z^2+\left(-4a^2c^2+5a\,b^2c-b^4\right)\_Z+c^3\right)}\_R\ln\left(\left(\left(6a^5c-2a^4b\right.\right.\right.\right.$

input
```
int(1/(c+a/x^2+b/x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/2*(a*c^2-b^2*c)/c*ln(c*x^2+b*x+a)+2*(2*a*b*c-b^3-1/2*(a*c^2-b^2*c
)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))-1/2/a/x^2+(-
a*c+b^2)*ln(x)/a^3+b/a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.44

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$$
$$=\left[-\frac{(b^3-3\,abc)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+a^2b^2-4\,a^3c+(b^4-5\,ab^2c+4\,a^2c}{2\,(a^3b^2-4\,a^4c)x^2}\right.$$

input
```
integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="fricas")
```

output
```
[-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 -
4*a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4
- 5*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 -
 4*a^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt
(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a
*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*
c^2)*x^2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)/x**5,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = -\frac{(b^2-ac)\log\left(cx^2+bx+a\right)}{2\,a^3}+\frac{(b^2-ac)\log\left(|x|\right)}{a^3} - \frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="giac")
```

output
```
-1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 -
(b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*a^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 20.06 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.30

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$$

$$= \frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2+2\,a\,b^3\,\sqrt{b^2-4\,a\,c}+2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x-3\,a^2\,b\,c\,\sqrt{b^2-\right.}{}$$

$$-\frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2-2\,a\,b^3\,\sqrt{b^2-4\,a\,c}-2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x+3\,a^2\,b\,c\,\sqrt{b\right.}{}$$

$$-\frac{\frac{1}{2\,a}-\frac{b\,x}{a^2}}{x^2}-\frac{\ln\left(x\right)\,\left(a\,c-b^2\right)}{a^3}$$

input
```
int(1/(x^5*(c + a/x^2 + b/x)),x)
```

output
```
(log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^(1/2) + 2*b^4*x
*(b^2 - 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c
)^(1/2) + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*x*(b
^2 - 4*a*c)^(1/2))*(b^4/2 - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^(1/2))/2
) + (b^3*(b^2 - 4*a*c)^(1/2))/2 + 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (log(2
*a*b^4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^(1/2) - 2*b^4*x*(b^2
- 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^(1/2
) + 9*a^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c*x*(b^2 - 4
*a*c)^(1/2))*(a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^(1/2))/2) - b^4/2 + (b
^3*(b^2 - 4*a*c)^(1/2))/2 - 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (1/(2*a) - (
b*x)/a^2)/x^2 - (log(x)*(a*c - b^2))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.16

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$$
$$= \frac{6\sqrt{4ac-b^2}\, atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) abc\,x^2 - 2\sqrt{4ac-b^2}\, atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) b^3x^2 + 4\log(c\,x^2+bx+a)\,a^2c^2x^2 - 5\log}{}$$

input
```
int(1/(c+a/x^2+b/x)/x^5,x)
```

output
```
(6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*x**2 - 2*
sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*x**2 + 4*log(
a + b*x + c*x**2)*a**2*c**2*x**2 - 5*log(a + b*x + c*x**2)*a*b**2*c*x**2 +
 log(a + b*x + c*x**2)*b**4*x**2 - 8*log(x)*a**2*c**2*x**2 + 10*log(x)*a*b
**2*c*x**2 - 2*log(x)*b**4*x**2 - 4*a**3*c + a**2*b**2 + 8*a**2*b*c*x - 2*
a*b**3*x)/(2*a**3*x**2*(4*a*c - b**2))
```

3.18 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 137

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = & -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x} \\ & -\frac{(b^4-4ab^2c+2a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} \\ & -\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2a^4}\end{aligned}$$

output
```
-1/3/a/x^3+1/2*b/a^2/x^2-(-a*c+b^2)/a^3/x-(2*a^2*c^2-4*a*b^2*c+b^4)*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(1/2)-b*(-2*a*c+b^2)*ln(x
)/a^4+1/2*b*(-2*a*c+b^2)*ln(c*x^2+b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \frac{-\frac{2a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{6a(-b^2+ac)}{x}+\frac{6(b^4-4ab^2c+2a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-6(b^3-2abc)\log(x)+3(b^3-2abc)\log(a+}{6a^4}$$

input
```
Integrate[1/((c + a/x^2 + b/x)*x^6),x]
```

output
```
((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] -
 6*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*a^4
)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1692, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{x^4\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \text{1145}$$

$$\frac{\int -\frac{b+cx}{x^3(cx^2+bx+a)}dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b+cx}{x^3(cx^2+bx+a)}dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{ 1200}$$

$$-\frac{\int \left(\frac{b}{ax^3}+\frac{b^3-2abc}{a^3x}+\frac{-b^4+3acb^2-c(b^2-2ac)xb-a^2c^2}{a^3(cx^2+bx+a)}+\frac{ac-b^2}{a^2x^2}\right)dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^3}+\frac{b\log(x)(b^2-2ac)}{a^3}+\frac{b^2-ac}{a^2x}+\frac{(2a^2c^2-4ab^2c+b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-\frac{b}{2ax^2}}{a}-\frac{1}{3ax^3}$$

input
```
Int[1/((c + a/x^2 + b/x)*x^6),x]
```

output
```
-1/3*1/(a*x^3) - (-1/2*b/(a*x^2) + (b^2 - a*c)/(a^2*x) + ((b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*
c]) + (b*(b^2 - 2*a*c)*Log[x])/a^3 - (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2]
)/(2*a^3))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
default	$\frac{\frac{(-2ab\,c^2+b^3c)\ln(c\,x^2+bx+a)}{2c}+\frac{2\left(a^2c^2-3a\,b^2c+b^4-\frac{(-2ab\,c^2+b^3c)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^4}-\frac{1}{3a\,x^3}-\frac{-ac+b^2}{a^3x}+\frac{b(2ac-b^2)\ln(x)}{a^4}$
risch	Expression too large to display

input
```
int(1/(c+a/x^2+b/x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/2*(-2*a*b*c^2+b^3*c)/c*ln(c*x^2+b*x+a)+2*(a^2*c^2-3*a*b^2*c+b^4-1
/2*(-2*a*b*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2)))-1/3/a/x^3-(-a*c+b^2)/a^3/x+b*(2*a*c-b^2)/a^4*ln(x)+1/2*b/a^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 445, normalized size of antiderivative = 3.25

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$$
$$=\left[\frac{3\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{b^2-4\,ac}x^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,a^3b^2+8\,a^4c+3\,(b^5-6\right.$$
$$-\frac{6\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{-b^2+4\,ac}x^3\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,a^3b^2-8\,a^4c-3\,(b^5-6\,ab^3c+8}{6\,(}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="fricas")
```

output
```
[1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2
 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a
)) - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2
 + b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*
a^2*b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*
c)*x^3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arct
an(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c -
3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*
b^3*c + 8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2
- 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)/x**6,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \frac{(b^3-2\,abc)\log\left(cx^2+bx+a\right)}{2\,a^4} - \frac{(b^3-2\,abc)\log\left(|x|\right)}{a^4} + \frac{(b^4-4\,ab^2c+2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^4} + \frac{3\,a^2bx-2\,a^3-6\,(ab^2-a^2c)x^2}{6\,a^4x^3}$$

input
```
integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="giac")
```

output
```
1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))
/a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^
2)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 20.56 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.82

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx = \ln\Bigg(2\,a\,b^4\,\sqrt{b^2-4\,a\,c}-2\,b^6\,x-2\,a\,b^5+2\,b^5\,x\,\sqrt{b^2-4\,a\,c} + 11\,a^2\,b^3\,c-13\,a^3\,b\,c^2+2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c} - 17\,a^2\,b^2\,c^2\,x+12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c} - 8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\Bigg)\left(\frac{b^3}{2\,a^4} - \frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}+\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right) + \ln\Bigg(2\,a\,b^5+2\,b^6\,x+2\,a\,b^4\,\sqrt{b^2-4\,a\,c}+2\,b^5\,x\,\sqrt{b^2-4\,a\,c} - 11\,a^2\,b^3\,c+13\,a^3\,b\,c^2-2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c} + 17\,a^2\,b^2\,c^2\,x-12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c} - 8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\Bigg)\left(\frac{b^3}{2\,a^4} + \frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}-\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right) + \frac{\frac{x^2\,\left(a\,c-b^2\right)}{a^3}-\frac{1}{3\,a}+\frac{b\,x}{2\,a^2}}{x^3}+\frac{b\,\ln\left(x\right)\,\left(2\,a\,c-b^2\right)}{a^4}$$

input
```
int(1/(x^6*(c + a/x^2 + b/x)),x)
```

output
```
log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c
)^(1/2) + 11*a^2*b^3*c - 13*a^3*b*c^2 + 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c
)^(1/2) - 17*a^2*b^2*c^2*x + 12*a*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2
) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(
b^3/(2*a^4) - (b^2*(b^2 - 4*a*c)^(1/2))/(2*a^4) - (b*c)/a^3 + (a^2*c^2*(b^
2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(
b^2 - 4*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^3*c + 13*a^3*b
*c^2 - 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c^2*x - 12*a
*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/
2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/(2*a^4) + (b^2*(b^2 - 4*a*c)^
(1/2))/(2*a^4) - (b*c)/a^3 - (a^2*c^2*(b^2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*
b^2)) + ((x^2*(a*c - b^2))/a^3 - 1/(3*a) + (b*x)/(2*a^2))/x^3 + (b*log(x)*
(2*a*c - b^2))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.25

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$$
$$= \frac{12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2c^2x^3 - 24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2c\,x^3 + 6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{}$$

input
```
int(1/(c+a/x^2+b/x)/x^6,x)
```

output
```
(12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2*x**3
 - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*x**
3 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*x**3 -
24*log(a + b*x + c*x**2)*a**2*b*c**2*x**3 + 18*log(a + b*x + c*x**2)*a*b**
3*c*x**3 - 3*log(a + b*x + c*x**2)*b**5*x**3 + 48*log(x)*a**2*b*c**2*x**3
 - 36*log(x)*a*b**3*c*x**3 + 6*log(x)*b**5*x**3 - 8*a**4*c + 2*a**3*b**2 +
12*a**3*b*c*x + 24*a**3*c**2*x**2 - 3*a**2*b**3*x - 30*a**2*b**2*c*x**2 +
6*a*b**4*x**2)/(6*a**4*x**3*(4*a*c - b**2))
```

3.19 $\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 196

$$\begin{aligned}\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} dx =& -\frac{b(3b^2-11ac)x}{c^3(b^2-4ac)}+\frac{(3b^2-8ac)x^2}{2c^2(b^2-4ac)} \\ &-\frac{bx^3}{c(b^2-4ac)}+\frac{x^4(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} \\ &+\frac{b(3b^4-20ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4(b^2-4ac)^{3/2}} \\ &+\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2c^4}\end{aligned}$$

output
```
-b*(-11*a*c+3*b^2)*x/c^3/(-4*a*c+b^2)+1/2*(-8*a*c+3*b^2)*x^2/c^2/(-4*a*c+b
^2)-b*x^3/c/(-4*a*c+b^2)+x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(30*a^
2*c^2-20*a*b^2*c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+
b^2)^(3/2)+1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/c^4
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.83

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{-4bcx+c^2x^2+\frac{2(2a^3c^2+b^5x+ab^3(b-5cx)+a^2bc(-4b+5cx))}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(3b^4-20ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+(3b^2-2ac)\log}{2c^4}$$

input
```
Integrate[x/(c + a/x^2 + b/x)^2,x]
```

output
```
(-4*b*c*x + c^2*x^2 + (2*(2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*
(-4*b + 5*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^
2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(
3/2) + (3*b^2 - 2*a*c)*Log[a + x*(b + c*x)])/(2*c^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1692, 1164, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \ 1692$$

$$\int \frac{x^5}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \ 1164$$

$$\frac{x^4(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \frac{x^3(8a+3bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \textbf{1200}$$

$$\frac{x^4(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \left(\frac{3bx^2}{c}-\frac{(3b^2-8ac)x}{c^2}+\frac{b(3b^2-11ac)}{c^3}-\frac{ab(3b^2-11ac)+(b^2-4ac)(3b^2-2ac)x}{c^3(cx^2+bx+a)}\right)dx}{b^2-4ac}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{x^4(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{-\frac{b(30a^2c^2-20ab^2c+3b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{(b^2-4ac)(3b^2-2ac)\log(a+bx+cx^2)}{2c^4}+\frac{bx(3b^2-11ac)}{c^3}-\frac{x^2(3b^2-8ac)}{2c^2}+\frac{bx^3}{c}}{b^2-4ac}$$

input
```
Int[x/(c + a/x^2 + b/x)^2,x]
```

output
```
(x^4*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - ((b*(3*b^2 - 11*a*c)
*x)/c^3 - ((3*b^2 - 8*a*c)*x^2)/(2*c^2) + (b*x^3)/c - (b*(3*b^4 - 20*a*b^2
*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4
*a*c]) - ((b^2 - 4*a*c)*(3*b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4))/(b^
2 - 4*a*c)
```

Defintions of rubi rules used

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
 + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
 + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.21

method	result
default	$-\frac{-\frac{1}{2}c\,x^2+2bx}{c^3}+\frac{-\frac{b\left(5a^2c^2-5a\,b^2c+b^4\right)x}{c\left(4ac-b^2\right)}-\frac{a\left(2a^2c^2-4a\,b^2c+b^4\right)}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{\frac{\frac{\left(-8a^2c^2+14a\,b^2c-3b^4\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(11a^2bc-3b^3a-\frac{\left(-8a^2c^2 \cdots\right.}{\cdots}\right.}{\cdots}}{4ac-b^2}}{c^3}$
risch	Expression too large to display

input
```
int(x/(c+a/x^2+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/2*c*x^2+2*b*x)+1/c^3*((-b*(5*a^2*c^2-5*a*b^2*c+b^4)/c/(4*a*c-b^
2)*x-a/c*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2
)*(1/2*(-8*a^2*c^2+14*a*b^2*c-3*b^4)/c*ln(c*x^2+b*x+a)+2*(11*a^2*b*c-3*b^3
*a-1/2*(-8*a^2*c^2+14*a*b^2*c-3*b^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(188) = 376$.

Time = 0.08 (sec) , antiderivative size = 1029, normalized size of antiderivative = 5.25

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x/(c+a/x^2+b/x)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3 - 8*
a*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3
 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 - (3*a*b^5 -
 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^2
 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
 a)) + 2*(b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 -
 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64
*a^2*b^2*c^3 - 32*a^3*c^4)*x^2 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32
*a^3*b*c^3)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c
^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 1
6*a^2*b*c^6)*x), 1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3
 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 1
6*a^2*b*c^4)*x^3 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*
x^2 + 2*(3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 +
 30*a^2*b*c^3)*x^2 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(-b^2 +
4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^7 - 11
*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2*b^4*c + 64
*a^3*b^2*c^2 - 32*a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*
a^3*c^4)*x^2 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x)*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(180) = 360$.

Time = 1.51 (sec) , antiderivative size = 1012, normalized size of antiderivative = 5.16

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x/(c+a/x**2+b/x)**2,x)
```

output
```
-2*b*x/c**3 + (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3
*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) -
(2*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**2 - 17*a**2*b**2*c + 16*a**
2*c**5*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/
(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c
- 3*b**2)/(2*c**4)) + 3*a*b**4 - 8*a*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**3
)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b*
*2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + b**4*c**3*(-
b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(
64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)
/(2*c**4)))/(30*a**2*b*c**2 - 20*a*b**3*c + 3*b**5)) + (b*sqrt(-(4*a*c - b
**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))*log(x +
 (16*a**3*c**2 - 17*a**2*b**2*c + 16*a**2*c**5*(b*sqrt(-(4*a*c - b**2)**3)
*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + 3*a*b**4 - 8*
a*b**2*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b*
*4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*
a*c - 3*b**2)/(2*c**4)) + b**4*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c
**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 +...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.96

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = -\frac{(3\,b^5-20\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^4-4\,ac^5)\sqrt{-b^2+4\,ac}} + \frac{(3\,b^2-2\,ac)\log\left(cx^2+bx+a\right)}{2\,c^4} + \frac{c^2x^2-4\,bcx}{2\,c^4} + \frac{ab^4-4\,a^2b^2c+2\,a^3c^2+(b^5-5\,ab^3c+5\,a^2bc^2)x}{(cx^2+bx+a)(b^2-4\,ac)c^4}$$

input
```
integrate(x/(c+a/x^2+b/x)^2,x, algorithm="giac")
```

output
```
-(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*(3*b^2 - 2*a*c)*log(c*x^2
 + b*x + a)/c^4 + 1/2*(c^2*x^2 - 4*b*c*x)/c^4 + (a*b^4 - 4*a^2*b^2*c + 2*a
^3*c^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*
c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 21.57 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.95

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{x^2}{2\,c^2} - \frac{\frac{a\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}+\frac{b\,x\,\left(5\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}}{c^4\,x^2+b\,c^3\,x+a\,c^3} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(128\,a^4\,c^4-288\,a^3\,b^2\,c^3+168\,a^2\,b^4\,c^2-38\,a\,b^6\,c+3\,b^8\right)}{2\,\left(64\,a^3\,c^7-48\,a^2\,b^2\,c^6+12\,a\,b^4\,c^5-b^6\,c^4\right)} - \frac{2\,b\,x}{c^3} + \frac{b\,\mathrm{atan}\left(\frac{c^4\,\left(\frac{2\,b\,x\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b^4\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^3}-\frac{b\,\left(b^3\,c^3-4\,a\,b\,c^4\right)\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b^4\right)}{c^7\,{\left(4\,a\,c-b^2\right)}^4}\right)\,{\left(4\,a\,c-b^2\right)}^{5/2}}{30\,a^2\,b\,c^2-20\,a\,b^3\,c+3\,b^5}\right)\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b\right.}{c^4\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(x/(c + a/x^2 + b/x)^2,x)
```

output
```
x^2/(2*c^2) - ((a*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c*(4*a*c - b^2)) + (b*x*
(b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c*(4*a*c - b^2)))/(a*c^3 + c^4*x^2 + b*c^3
*x) - (log(a + b*x + c*x^2)*(3*b^8 + 128*a^4*c^4 + 168*a^2*b^4*c^2 - 288*a
^3*b^2*c^3 - 38*a*b^6*c))/(2*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2
*b^2*c^6)) - (2*b*x)/c^3 + (b*atan((c^4*((2*b*x*(3*b^4 + 30*a^2*c^2 - 20*a
*b^2*c))/(c^3*(4*a*c - b^2)^3) - (b*(b^3*c^3 - 4*a*b*c^4)*(3*b^4 + 30*a^2*
c^2 - 20*a*b^2*c))/(c^7*(4*a*c - b^2)^4))*(4*a*c - b^2)^(5/2))/(3*b^5 + 30
*a^2*b*c^2 - 20*a*b^3*c))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(c^4*(4*a*c -
 b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 846, normalized size of antiderivative = 4.32

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(x/(c+a/x^2+b/x)^2,x)
```

output
```
(60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2 -
40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c + 6
0*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c**2*x
 + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**3*
x**2 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**5 -
40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*c*x - 40
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c**2*x**2
+ 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**6*x + 6*sqr
t(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**5*c*x**2 - 32*log(
a + b*x + c*x**2)*a**4*c**3 + 64*log(a + b*x + c*x**2)*a**3*b**2*c**2 - 32
*log(a + b*x + c*x**2)*a**3*b*c**3*x - 32*log(a + b*x + c*x**2)*a**3*c**4*
x**2 - 26*log(a + b*x + c*x**2)*a**2*b**4*c + 64*log(a + b*x + c*x**2)*a**
2*b**3*c**2*x + 64*log(a + b*x + c*x**2)*a**2*b**2*c**3*x**2 + 3*log(a + b
*x + c*x**2)*a*b**6 - 26*log(a + b*x + c*x**2)*a*b**5*c*x - 26*log(a + b*x
 + c*x**2)*a*b**4*c**2*x**2 + 3*log(a + b*x + c*x**2)*b**7*x + 3*log(a + b
*x + c*x**2)*b**6*c*x**2 + 88*a**4*c**3 - 46*a**3*b**2*c**2 + 120*a**3*c**
4*x**2 + 6*a**2*b**4*c - 154*a**2*b**2*c**3*x**2 - 48*a**2*b*c**4*x**3 + 1
6*a**2*c**5*x**4 + 55*a*b**4*c**2*x**2 + 24*a*b**3*c**3*x**3 - 8*a*b**2*c*
*4*x**4 - 6*b**6*c*x**2 - 3*b**5*c**2*x**3 + b**4*c**3*x**4)/(2*c**4*(16*a
**3*c**2 - 8*a**2*b**2*c + 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**...
```

3.20 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{2(b^2-3ac)\,x}{c^2\,(b^2-4ac)}-\frac{bx^2}{c\,(b^2-4ac)}+\frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} -\frac{2(b^4-6ab^2c+6a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}}-\frac{b\log\left(a+bx+cx^2\right)}{c^3}$$

output
```
2*(-3*a*c+b^2)*x/c^2/(-4*a*c+b^2)-b*x^2/c/(-4*a*c+b^2)+x^3*(b*x+2*a)/(-4*a
*c+b^2)/(c*x^2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*
c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-b*ln(c*x^2+b*x+a)/c^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{cx+\frac{-b^4x-ab^2(b-4cx)+a^2c(3b-2cx)}{(b^2-4ac)(a+x(b+cx))}-\frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}-b\log(a+x(b+cx))}{c^3}$$

input
```
Integrate[(c + a/x^2 + b/x)^(-2),x]
```

output
```
(c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)
*(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1679, 1164, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \text{ 1679}$$

$$\int \frac{x^4}{\left(a+bx+cx^2\right)^2}dx$$

$$\Big\downarrow \text{ 1164}$$

$$\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{2x^2(3a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2\int \frac{x^2(3a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2\int \left(-\frac{b^2-3ac}{c^2}+\frac{bx}{c}+\frac{a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x}{c^2\left(cx^2+bx+a\right)}\right)dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-2\left(\frac{(6a^2c^2-6ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{b(b^2-4ac)\log(a+bx+cx^2)}{2c^3}-\frac{x(b^2-3ac)}{c^2}+\frac{bx^2}{2c}\right)}{b^2-4ac}$$

input

```
Int[(c + a/x^2 + b/x)^(-2),x]
```

output

```
(x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(-(((b^2 - 3*a*c)
*x)/c^2) + (b*x^2)/(2*c) + ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c
*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + (b*(b^2 - 4*a*c)*Log[a +
 b*x + c*x^2])/(2*c^3)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1679

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{x}{c^2}-\frac{\frac{-\frac{(2a^2c^2-4a\,b^2c+b^4)x}{c(4ac-b^2)}+\frac{ba(3ac-b^2)}{c(4ac-b^2)}}{c\,x^2+bx+a}+\frac{\frac{(4abc-b^3)\ln(c\,x^2+bx+a)}{c}+\frac{4\left(3a^2c-b^2a-\frac{(4abc-b^3)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{c^2}$	198
risch	Expression too large to display	1176

input
```
int(1/(c+a/x^2+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2-1/c^2*((-(2*a^2*c^2-4*a*b^2*c+b^4)/c/(4*a*c-b^2)*x+b*a/c*(3*a*c-b^2)
/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(4*a*b*c-b^3)/c*ln(c*x^2+b*
x+a)+2*(3*a^2*c-b^2*a-1/2*(4*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(146) = 292.

Time = 0.08 (sec) , antiderivative size = 837, normalized size of antiderivative = 5.58

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^2,x, algorithm="fricas")
```

output
```
[-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c +
 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*
a^2*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + s
qrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a
^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*
log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 -
 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)
, -(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*
c + 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c +
 6*a^2*b*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b
^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2
 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3
- 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 +
(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(141) = 282$.

Time = 1.08 (sec) , antiderivative size = 842, normalized size of antiderivative = 5.61

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x**2+b/x)**2,x)
```

output
```
(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**
3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a
**2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6
*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a
**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + s
qrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16
*a**2*c**4*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
 b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2
*a*b**3 + 8*a*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6))) - b**4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6
*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 +
x*(2*a**2*c**2 - 4*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*
a*c**5 - b**2*c**4) + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{2\,(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^3-4\,ac^4)\sqrt{-b^2+4\,ac}}+\frac{x}{c^2}-\frac{b\log\left(cx^2+bx+a\right)}{c^3}-\frac{\frac{(b^4-4\,ab^2c+2\,a^2c^2)x}{c}+\frac{ab^3-3\,a^2bc}{c}}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^2,x, algorithm="giac")
```

output
```
2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b
^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3
 - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b
*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 21.15 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.74

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{x}{c^2}+\frac{\frac{a\,(b^3-3\,a\,b\,c)}{c\,(4\,a\,c-b^2)}+\frac{x\,(2\,a^2\,c^2-4\,a\,b^2\,c+b^4)}{c\,(4\,a\,c-b^2)}}{c^3\,x^2+b\,c^2\,x+a\,c^2}+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-128\,a^3\,b\,c^3+96\,a^2\,b^3\,c^2-24\,a\,b^5\,c+2\,b^7\right)}{2\,\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5+12\,a\,b^4\,c^4-b^6\,c^3\right)}-\frac{2\,\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^2-4\,a\,b\,c^3}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(6\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(1/(c + a/x^2 + b/x)^2,x)
```

output
```
x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a
*b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c
*x^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^
6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*atan((2*c*x)/(4*a*c - b
^2)^(1/2) - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(b^4 + 6*a^2*
c^2 - 6*a*b^2*c))/(c^3*(4*a*c - b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 752, normalized size of antiderivative = 5.01

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \frac{-\log(c\,x^2+bx+a)\,a\,b^6-\log(c\,x^2+bx+a)\,b^7x+14a^3b^2c^2-24a^3c^4x^2-2a^2b^4c+2b^6c\,x^2+b^5c^2x^3-1}{}$$

input
```
int(1/(c+a/x^2+b/x)^2,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2
 + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c
- 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c**
2*x - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c*
*3*x**2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**5
 + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*c*x +
 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c**2*x*
*2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**6*x - 2*
sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**5*c*x**2 - 16*l
og(a + b*x + c*x**2)*a**3*b**2*c**2 + 8*log(a + b*x + c*x**2)*a**2*b**4*c
- 16*log(a + b*x + c*x**2)*a**2*b**3*c**2*x - 16*log(a + b*x + c*x**2)*a**
2*b**2*c**3*x**2 - log(a + b*x + c*x**2)*a*b**6 + 8*log(a + b*x + c*x**2)*
a*b**5*c*x + 8*log(a + b*x + c*x**2)*a*b**4*c**2*x**2 - log(a + b*x + c*x*
*2)*b**7*x - log(a + b*x + c*x**2)*b**6*c*x**2 - 24*a**4*c**3 + 14*a**3*b*
*2*c**2 - 24*a**3*c**4*x**2 - 2*a**2*b**4*c + 42*a**2*b**2*c**3*x**2 + 16*
a**2*b*c**4*x**3 - 17*a*b**4*c**2*x**2 - 8*a*b**3*c**3*x**3 + 2*b**6*c*x**
2 + b**5*c**2*x**3)/(b*c**3*(16*a**3*c**2 - 8*a**2*b**2*c + 16*a**2*b*c**2
*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x - 8*a*b**2*c**2*x**2 + b**5
*x + b**4*c*x**2))
```

3.21 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 114

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x} \, dx = -\frac{bx}{c(b^2-4ac)}+\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}}+\frac{\log(a+bx+cx^2)}{2c^2}$$

output
```
-b*x/c/(-4*a*c+b^2)+x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2
)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*ln(c*x^
2+b*x+a)/c^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x} \, dx = \frac{\frac{2(-2a^2c+b^3x+ab(b-3cx))}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+\log(a+x(b+cx))}{2c^2}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x),x]
```

output
```
((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))
) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a
*c)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1692, 1164, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \; 1692$$

$$\int \frac{x^3}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \; 1164$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \frac{x(4a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \; 1200$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \left(\frac{b}{c}-\frac{ab+(b^2-4ac)x}{c(cx^2+bx+a)}\right)dx}{b^2-4ac}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{-\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(b^2-4ac)\log(a+bx+cx^2)}{2c^2}+\frac{bx}{c}}{b^2-4ac}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x),x]
```

output
```
(x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - ((b*x)/c - (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) -
((b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*c^2))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.48

method	result
default	$\frac{\frac{b(3ac-b^2)x}{c^2(4ac-b^2)}+\frac{(2ac-b^2)a}{c^2(4ac-b^2)}}{cx^2+bx+a}+\frac{\frac{(4ac-b^2)\ln(cx^2+bx+a)}{2c}+\frac{2\left(-ab-\frac{(4ac-b^2)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c(4ac-b^2)}$
risch	$\frac{\frac{b(3ac-b^2)x}{c^2(4ac-b^2)}+\frac{(2ac-b^2)a}{c^2(4ac-b^2)}}{cx^2+bx+a}+\frac{8\ln\left(-24a^2bc^2+10ab^3c-b^5-2\sqrt{-b^2(4ac-b^2)(6ac-b^2)^2}\,cx-\sqrt{-b^2(4ac-b^2)(6ac-b^2)^2}\,b\right)a^2}{(4ac-b^2)^2}-$

input
```
int(1/(c+a/x^2+b/x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
(b/c^2*(3*a*c-b^2)/(4*a*c-b^2)*x+1/c^2*(2*a*c-b^2)*a/(4*a*c-b^2))/(c*x^2+b
*x+a)+1/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^2+b*x+a)+2*(-a*b-1/2*(4*a*
c-b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(108) = 216.

Time = 0.10 (sec) , antiderivative size = 635, normalized size of antiderivative = 5.57

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx$$
$$=\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+(ab^3-6\,a^2bc+(b^3c-6\,abc^2)x^2+(b^4-6\,ab^2c)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,b\right.}{2\,(ab^4c^2-8\,a^2b^2c}\right.$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^
4)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (
b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2
*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(
a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c
^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(104) = 208$.

Time = 0.84 (sec) , antiderivative size = 729, normalized size of antiderivative = 6.39

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\, dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x,x)
```

output
```
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-
b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*
c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4
*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*
c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**
2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*
a*b*c - b**3))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) +
x*(4*a*b*c**3 - b**3*c**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} + \frac{\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{ab^2-2\,a^2c+(b^3-3\,abc)x}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="giac")
```

output
```
-(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^
3)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c +
 (b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 19.84 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.45

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = \frac{\frac{a\,(2\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}+\frac{b\,x\,(3\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}{2\,\left(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2\right)} + \frac{b\,\mathrm{atan}\left(\frac{c^2\,{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(\frac{2\,b\,x\,\left(6\,a\,c-b^2\right)}{c\,{\left(4\,a\,c-b^2\right)}^3}+\frac{b^2\,\left(4\,a\,c^2-b^2\,c\right)\,\left(6\,a\,c-b^2\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^4}\right)}{b^3-6\,a\,b\,c}\right)\,\left(6\,a\,c-b^2\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(1/(x*(c + a/x^2 + b/x)^2),x)
```

output
```
((a*(2*a*c - b^2))/(c^2*(4*a*c - b^2)) + (b*x*(3*a*c - b^2))/(c^2*(4*a*c -
 b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c))/(2*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2
*b^2*c^4)) + (b*atan((c^2*(4*a*c - b^2)^(5/2)*((2*b*x*(6*a*c - b^2))/(c*(4
*a*c - b^2)^3) + (b^2*(4*a*c^2 - b^2*c)*(6*a*c - b^2))/(c^3*(4*a*c - b^2)^
4)))/(b^3 - 6*a*b*c))*(6*a*c - b^2))/(c^2*(4*a*c - b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 547, normalized size of antiderivative = 4.80

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = \frac{-12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2bc+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^3-12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2c}{}$$

input
```
int(1/(c+a/x^2+b/x)^2/x,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c +
2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3 - 12*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*x - 12*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**2*x**2 + 2*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*x + 2*sqrt(4*a*c -
b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c*x**2 + 16*log(a + b*x +
c*x**2)*a**3*c**2 - 8*log(a + b*x + c*x**2)*a**2*b**2*c + 16*log(a + b*x +
 c*x**2)*a**2*b*c**2*x + 16*log(a + b*x + c*x**2)*a**2*c**3*x**2 + log(a +
 b*x + c*x**2)*a*b**4 - 8*log(a + b*x + c*x**2)*a*b**3*c*x - 8*log(a + b*x
 + c*x**2)*a*b**2*c**2*x**2 + log(a + b*x + c*x**2)*b**5*x + log(a + b*x +
 c*x**2)*b**4*c*x**2 - 8*a**3*c**2 + 2*a**2*b**2*c - 24*a**2*c**3*x**2 + 1
4*a*b**2*c**2*x**2 - 2*b**4*c*x**2)/(2*c**2*(16*a**3*c**2 - 8*a**2*b**2*c
+ 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x - 8*a*b**2*
c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.22 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2} \, dx = \frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}+\frac{4a\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*a*arctanh((2*c*x+b)/(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2} \, dx = \frac{b^2x+a(b-2cx)}{c(-b^2+4ac)(a+x(b+cx))}+\frac{4a\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x^2),x]
```

output
```
(b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan
[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1690, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \ 1690$$

$$-\int \frac{1}{\left(\frac{a}{x^2}+c+\frac{b}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \ 1086$$

$$\frac{2a\int \frac{1}{\frac{a}{x^2}+c+\frac{b}{x}}d\frac{1}{x}}{b^2-4ac}+\frac{\frac{2a}{x}+b}{(b^2-4ac)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}$$

$$\Big\downarrow \ 1083$$

$$\frac{\frac{2a}{x}+b}{(b^2-4ac)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\int \frac{1}{b^2-4ac-\frac{1}{x^2}}d\left(\frac{2a}{x}+b\right)}{b^2-4ac}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{2a}{x}+b}{(b^2-4ac)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\text{arctanh}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x^2),x]
```

output
```
(b + (2*a)/x)/((b^2 - 4*a*c)*(c + a/x^2 + b/x)) - (4*a*ArcTanh[(b + (2*a)/
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.45

method	result
default	$\frac{-\frac{\left(2ac-b^2\right)x}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ac-b^2\right)x}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2a\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{2a\ln\left(\left(8a\,c^2-2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b\right.}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$

input
```
int(1/(c+a/x^2+b/x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/c*(2*a*c-b^2)/(4*a*c-b^2)*x+1/c*a*b/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*
a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(63) = 126$.

Time = 0.07 (sec) , antiderivative size = 387, normalized size of antiderivative = 5.78

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx$$

$$= \left[-\frac{ab^3 - 4a^2bc + 2(ac^2x^2 + abcx + a^2c)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right) + (b^4 - 6ab^2}{ab^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^2 + (b^5c - 8ab^3c^2 + 16a^2bc^3)x}\right.$$

$$-\frac{ab^3 - 4a^2bc - 4(ac^2x^2 + abcx + a^2c)\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right) + (b^4 - 6ab^2c + 8a^2c^2)}{ab^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^2 + (b^5c - 8ab^3c^2 + 16a^2bc^3)x}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="fricas")
```

output
```
[-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^
2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x +
a^2*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*
a*c)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3
*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 1
6*a^2*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(61) = 122$.

Time = 0.35 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.18

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx = -2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right) +2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right) + \frac{ab+x(-2ac+b^2)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x(4abc^2-b^3c)}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x**2,x)
```

output
```
-2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b*
*2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*
a*c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x +
 (32*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c
- b**2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + (a*b
 + x*(-2*a*c + b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2
) + x*(4*a*b*c**2 - b**3*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx=-\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{b^2x-2\,acx+ab}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="giac")
```

output
```
-4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a
*c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.01

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx=-\frac{\frac{x\,(2\,a\,c-b^2)}{c\,(4\,a\,c-b^2)}-\frac{a\,b}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a}-\frac{4\,a\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,a\,c\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,a}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

input
```
int(1/(x^2*(c + a/x^2 + b/x)^2),x)
```

output
```
- ((x*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (a*b)/(c*(4*a*c - b^2)))/(a + b*x
 + c*x^2) - (4*a*atan((((2*a*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*a*c
*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a)))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.63

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\, dx$$

$$= \frac{4\sqrt{4ac-b^2}\, atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) a^2b+4\sqrt{4ac-b^2}\, atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) a\, b^2x+4\sqrt{4ac-b^2}\, atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) abc\, x^2+}{b\,(16a^2c^3x^2-8a\, b^2c^2x^2+b^4c\, x^2+16a^2b\, c^2x-8a\, b^3cx+b^5x+16a^3c^2-8}$$

input
```
int(1/(c+a/x^2+b/x)^2/x^2,x)
```

output
```
(4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b + 4*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*x + 4*sqrt(4*a*
c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*x**2 + 8*a**3*c - 2*a
**2*b**2 + 8*a**2*c**2*x**2 - 6*a*b**2*c*x**2 + b**4*x**2)/(b*(16*a**3*c**
2 - 8*a**2*b**2*c + 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b*
*3*c*x - 8*a*b**2*c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.23 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3} \, dx = \frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)} - \frac{2b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)-2*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3} \, dx = \frac{2a+bx}{(b^2-4ac)(a+x(b+cx))} - \frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x^3),x]
```

output
```
(2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1692, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{x}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 1159}$$

$$\frac{b\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}+\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x^3),x]
```

output
```
(2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result	si
default	$\frac{-bx-2a}{(4ac-b^2)(c\,x^2+bx+a)}-\frac{2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	70
risch	$\frac{-\frac{bx}{4ac-b^2}-\frac{2a}{4ac-b^2}}{c\,x^2+bx+a}+\frac{b\ln\left((-8a\,c^2+2b^2c)x-(-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{b\ln\left((8a\,c^2-2b^2c)x-(-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}$	14

input
```
int(1/(c+a/x^2+b/x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(62) = 124$.

Time = 0.09 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.12

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx$$
$$= \left[\frac{2\,ab^2 - 8\,a^2c - (bcx^2 + b^2x + ab)\sqrt{b^2 - 4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right) + (b^3 - 4\,abc)x}{ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + (b^4c - 8\,ab^2c^2 + 16\,a^2c^3)x^2 + (b^5 - 8\,ab^3c + 16\,a^2bc^2)x}, \frac{2\,a}{}\right.$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="fricas")
```

output
```
[(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b
^2 - 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-
b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^
2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.

Time = 0.32 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.83

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx$$
$$= b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)$$
$$-b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)$$
$$+\frac{-2a-bx}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x(4abc-b^3)}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x**3,x)
```

output
```
b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**
2)**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*
c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) +
b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c
- a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx = \frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}+\frac{bx+2\,a}{(cx^2+bx+a)(b^2-4\,ac)}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="giac")
```

output
```
2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 20.98 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.67

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx$$
$$= -\frac{\frac{2\,a}{4\,a\,c-b^2}+\frac{b\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{2\,b\,\mathrm{atan}\left(\frac{\left(\frac{b^2}{\left(4\,a\,c-b^2\right)^{3/2}}+\frac{2\,b\,c\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\left(4\,a\,c-b^2\right)}{b}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

input
```
int(1/(x^3*(c + a/x^2 + b/x)^2),x)
```

output
```
- ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*ata
n(((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2)
)/b))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.38

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx$$
$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ab-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2x-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)bc\,x^2-4}{16a^2c^3x^2-8a\,b^2c^2x^2+b^4c\,x^2+16a^2b\,c^2x-8a\,b^3cx+b^5x+16a^3c^2-8a^2b^2c+}$$

input
```
int(1/(c+a/x^2+b/x)^2/x^3,x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b - 2*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*x - 2*sqrt(4*a*c
- b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*x**2 - 4*a**2*c + a*b**2
+ 4*a*c**2*x**2 - b**2*c*x**2)/(16*a**3*c**2 - 8*a**2*b**2*c + 16*a**2*b*c
**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x - 8*a*b**2*c**2*x**2 + b
**5*x + b**4*c*x**2)
```

3.24 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} dx = -\frac{b+2cx}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{4c\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
-(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} dx = -\frac{\frac{b+2cx}{a+x(b+cx)}+\frac{4c\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{b^2-4ac}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x^4),x]
```

output
```
-(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a
*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1692, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{1086}$$

$$-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x^4),x]
```

output
```
-((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result	si
default	$\frac{2cx+b}{\left(4ac-b^2\right)\left(c x^2+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}$	68
risch	$\frac{\frac{2cx}{4ac-b^2}+\frac{b}{4ac-b^2}}{c x^2+bx+a}+\frac{2c\ln\left(\left(-8a c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{2c\ln\left(\left(8a c^2-2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$	14

input
```
int(1/(c+a/x^2+b/x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(62) = 124$.

Time = 0.11 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.17

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx$$
$$= \left[-\frac{b^3-4\,abc+2\,(c^2x^2+bcx+ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x},\right.$$
$$\left.-\frac{b^3-4\,abc-4\,(c^2x^2+bcx+ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x}\right]$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="fricas")
```

output
```
[-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
+ a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b
^3 - 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-
b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 -
 8*a*b^3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(61) = 122$.

Time = 0.33 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.02

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx = -2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right) + 2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right) + \frac{b+2cx}{4a^2c-ab^2+x^2\cdot\left(4ac^2-b^2c\right)+x\left(4abc-b^3\right)}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x**4,x)
```

output
```
-2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*
a*c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x
+ (32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c
 - b**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b
 + 2*c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**
3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx = -\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,cx+b}{(cx^2+bx+a)(b^2-4\,ac)}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="giac")
```

output
```
-4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a
*c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 19.47 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.80

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx = \frac{\frac{b}{4\,a\,c-b^2}+\frac{2\,c\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a} - \frac{4\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,c^2\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\left(4\,a\,c-b^2\right)}{2\,c}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

input
```
int(1/(x^4*(c + a/x^2 + b/x)^2),x)
```

output
```
(b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan(((
(2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))
*(4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.65

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx$$
$$= \frac{4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abc+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2cx+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b\,c^2x^2-}{b\left(16a^2c^3x^2-8a\,b^2c^2x^2+b^4c\,x^2+16a^2b\,c^2x-8a\,b^3cx+b^5x+16a^3c^2-8a^2\right.}$$

input

```
int(1/(c+a/x^2+b/x)^2/x^4,x)
```

output

```
(4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c + 4*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*x + 4*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**2*x**2 - 8*a**2*c**2 +
6*a*b**2*c - 8*a*c**3*x**2 - b**4 + 2*b**2*c**2*x**2)/(b*(16*a**3*c**2 - 8
*a**2*b**2*c + 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*
x - 8*a*b**2*c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.25 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} dx$

Optimal result

Integrand size = 18, antiderivative size = 108

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} dx = \frac{b^2-2ac+bcx}{a(b^2-4ac)(a+bx+cx^2)} + \frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2-4ac)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log(a+bx+cx^2)}{2a^2}$$

output
```
(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)*arctanh((2*c
*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/2*ln(c*x^2+b*
x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5} dx = \frac{\frac{2a(b^2-2ac+bcx)}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + 2\log(x) - \log(a+x(b+cx))}{2a^2}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x^5),x]
```

output
```
((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*Log[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1692, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{x \left(a+bx+cx^2\right)^2}dx$$

$$\Big\downarrow \text{1165}$$

$$\frac{-2ac+b^2+bcx}{a \left(b^2-4ac\right) \left(a+bx+cx^2\right)}-\frac{\int -\frac{b^2+cxb-4ac}{x(cx^2+bx+a)}dx}{a \left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2+cxb-4ac}{x(cx^2+bx+a)}dx}{a \left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{a \left(b^2-4ac\right) \left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{\int \left(\frac{b^2-4ac}{ax}+\frac{-b(b^2-5ac)-c(b^2-4ac)x}{a(cx^2+bx+a)}\right)dx}{a \left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{a \left(b^2-4ac\right) \left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} - \frac{(b^2-4ac)\log(a+bx+cx^2)}{2a} + \frac{\log(x)(b^2-4ac)}{a}}{a\left(b^2-4ac\right)} + \frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x^5),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((b*(b^2 - 6*a
*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2
- 4*a*c)*Log[x])/a - ((b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*a))/(a*(b^2 -
 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.64

method	result
default	$-\frac{\frac{\frac{abcx}{4ac-b^2}-\frac{a(2ac-b^2)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{(4a\,c^2-b^2c)\ln(c\,x^2+bx+a)}{2c}+\frac{2\left(5abc-b^3-\frac{(4a\,c^2-b^2c)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^2}+\frac{\ln(x)}{a^2}$
risch	$\frac{-\frac{bcx}{a(4ac-b^2)}+\frac{2ac-b^2}{a(4ac-b^2)}}{c\,x^2+bx+a}+\frac{\ln(x)}{a^2}+\left(\sum_{\_R=\mathrm{RootOf}\left((64a^5c^3-48a^4b^2c^2+12a^3b^4c-a^2b^6)\_Z^2+(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right.}\right.$

input
```
int(1/(c+a/x^2+b/x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*((a*b*c/(4*a*c-b^2)*x-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4
*a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^2+b*x+a)+2*(5*a*b*c-b^3-1/2*(4*a*c
^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))+ln(
x)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(102) = 204$.

Time = 0.14 (sec) , antiderivative size = 781, normalized size of antiderivative = 7.23

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2x^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x
^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*
c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6
*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^
2 - 4*a*c)) + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*
c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2)*x)*log(c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*l
og(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 +
16*a^4*c^3)*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x**5,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{abcx+ab^2-2\,a^2c}{(cx^2+bx+a)(b^2-4\,ac)a^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="giac")
```

output
```
-(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*
c)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 +
(a*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

Mupad [B] (verification not implemented)

Time = 21.16 (sec) , antiderivative size = 620, normalized size of antiderivative = 5.74

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx = \frac{\ln(x)}{a^2} + \frac{\frac{2\,a\,c-b^2}{a\,(4\,a\,c-b^2)} - \frac{b\,c\,x}{a\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} + \frac{\ln\Big(2\,a\,b^6+2\,b^7\,x-96\,a^4\,c^3+2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3}-23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3}+84\,a^3\,b^2\,c^2}{} + \frac{\ln\Big(96\,a^4\,c^3-2\,b^7\,x-2\,a\,b^6+2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3}+23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3}-84\,a^3\,b^2\,c^2}{}$$

input
```
int(1/(x^5*(c + a/x^2 + b/x)^2),x)
```

output
```
log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))
/(a + b*x + c*x^2) + (log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*
c - b^2)^3)^(1/2) - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) + 84*a
^3*b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 24
*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*x - 12*a*b
^2*c*x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3
)^(1/2) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))
/(2*a^2*(4*a*c - b^2)^3) + (log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(
-(4*a*c - b^2)^3)^(1/2) + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2)
- 84*a^3*b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2
) + 24*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) + 120*a^3*b*c^3*x -
12*a*b^2*c*x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c -
b^2)^3)^(1/2) + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^(
1/2)))/(2*a^2*(4*a*c - b^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 644, normalized size of antiderivative = 5.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx$$

$$= \frac{-14a^2b^2c+8a^2c^3x^2-12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2cx-12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2bc+2\sqrt{4ac-}}{}$$

input
```
int(1/(c+a/x^2+b/x)^2/x^5,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c +
2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3 - 12*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*x - 12*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**2*x**2 + 2*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*x + 2*sqrt(4*a*c -
b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c*x**2 - 16*log(a + b*x +
c*x**2)*a**3*c**2 + 8*log(a + b*x + c*x**2)*a**2*b**2*c - 16*log(a + b*x +
 c*x**2)*a**2*b*c**2*x - 16*log(a + b*x + c*x**2)*a**2*c**3*x**2 - log(a +
 b*x + c*x**2)*a*b**4 + 8*log(a + b*x + c*x**2)*a*b**3*c*x + 8*log(a + b*x
 + c*x**2)*a*b**2*c**2*x**2 - log(a + b*x + c*x**2)*b**5*x - log(a + b*x +
 c*x**2)*b**4*c*x**2 + 32*log(x)*a**3*c**2 - 16*log(x)*a**2*b**2*c + 32*lo
g(x)*a**2*b*c**2*x + 32*log(x)*a**2*c**3*x**2 + 2*log(x)*a*b**4 - 16*log(x
)*a*b**3*c*x - 16*log(x)*a*b**2*c**2*x**2 + 2*log(x)*b**5*x + 2*log(x)*b**
4*c*x**2 + 24*a**3*c**2 - 14*a**2*b**2*c + 8*a**2*c**3*x**2 + 2*a*b**4 - 2
*a*b**2*c**2*x**2)/(2*a**2*(16*a**3*c**2 - 8*a**2*b**2*c + 16*a**2*b*c**2*
x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x - 8*a*b**2*c**2*x**2 + b**5*
x + b**4*c*x**2))
```

3.26 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 148

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6} \, dx = -\frac{2(b^2-3ac)}{a^2(b^2-4ac)\,x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)\,x\,(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx+cx^2)}{a^3}$$

output
```
(6*a*c-2*b^2)/a^2/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^2
+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/
a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+b*ln(c*x^2+b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6} \, dx = -\frac{\frac{a}{x}+\frac{a(b^3-3abc+b^2cx-2ac^2x)}{(b^2-4ac)(a+x(b+cx))}+\frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2b\log(x)-b\log(a+x(b+cx))}{a^3}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x^6),x]
```

output
```
-((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(
b + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^
3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1692, 1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2} \, dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{1}{x^2 \left(a+bx+cx^2\right)^2}dx$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{-2ac+b^2+bcx}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int -\frac{2\left(b^2+cxb-3ac\right)}{x^2\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow 27$$

$$\frac{2\int \frac{b^2+cxb-3ac}{x^2(cx^2+bx+a)}dx}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow 1200$$

$$\frac{2\int \left(\frac{b^2-3ac}{ax^2}+\frac{4abc-b^3}{a^2x}+\frac{b^4-5acb^2+c(b^2-4ac)xb+3a^2c^2}{a^2(cx^2+bx+a)}\right)dx}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow 2009$$

$$\frac{2\left(-\frac{(6a^2c^2-6ab^2c+b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b(b^2-4ac)\log(a+bx+cx^2)}{2a^2}-\frac{b\log(x)(b^2-4ac)}{a^2}-\frac{b^2-3ac}{ax}\right)}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x^6),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) + (2*(-((b^2 -
 3*a*c)/(a*x)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)*Log[x])/a^2 + (b*
(b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*a^2)))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\frac{\frac{ac\left(2ac-b^2\right)x}{4ac-b^2}+\frac{ab\left(3ac-b^2\right)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{\left(-4ab\,c^2+b^3c\right)\ln\left(c\,x^2+bx+a\right)}{c}+\frac{4\left(3a^2c^2-5a\,b^2c+b^4-\frac{\left(-4ab\,c^2+b^3c\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^3}-\frac{1}{a^2x}-$
risch	$\frac{-\frac{2c\left(3ac-b^2\right)x^2}{a^2\left(4ac-b^2\right)}-\frac{b\left(7ac-2b^2\right)x}{a^2\left(4ac-b^2\right)}-\frac{1}{a}}{x(c\,x^2+bx+a)}-\frac{2b\ln(x)}{a^3}+2\left(\sum_{\_R=\text{RootOf}\left(\left(64a^6c^3-48a^5b^2c^2+12a^4b^4c-a^3b^6\right)\_Z^2+\left(-64a^3b\,c^3+48a^2b\right.\right.}\right.$

input
```
int(1/(c+a/x^2+b/x)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x+a*b*(3*a*c-b^2)/(4*a*c-b^2))/(c*x^2
+b*x+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3*c)/c*ln(c*x^2+b*x+a)+2*(3*a^2*c
^2-5*a*b^2*c+b^4-1/2*(-4*a*b*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))))-1/a^2/x-2*b*ln(x)/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(144) = 288$.

Time = 0.19 (sec) , antiderivative size = 975, normalized size of antiderivative = 6.59

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6} \, dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="fricas")
```

output
```
[-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^
3*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6
*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c
^2 + 16*a^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 +
2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6
*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c
 + 6*a^3*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c
^2 + 16*a^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x**6,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \frac{2\,(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}} - \frac{2\,b^2cx^2-6\,ac^2x^2+2\,b^3x-7\,abcx+ab^2-4\,a^2c}{(a^2b^2-4\,a^3c)(cx^3+bx^2+ax)} + \frac{b\log\left(cx^2+bx+a\right)}{a^3} - \frac{2\,b\log\left(|x|\right)}{a^3}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="giac")
```

output
```
2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a
^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3
*x - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*
x)) + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

Mupad [B] (verification not implemented)

Time = 19.46 (sec) , antiderivative size = 775, normalized size of antiderivative = 5.24

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \ln\Bigg(2\,a\,b^7+2\,b^8\,x+2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-23\,a^2\,b^5\,c-108\,a^4\,b\,c^3+24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-9\,a^2\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+97\,a^2\,b^4\,c^2\,x-138\,a^3\,b^2\,c^3\,x-24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}\Bigg)\left(\frac{b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+6\,a^2\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-6\,a\,b^2\,c\,\sqrt{-(4\,a\,c-b^2)}}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}+\frac{b}{a^3}\right)-\frac{\frac{1}{a}-\frac{x\,\left(2\,b^3-7\,a\,b\,c\right)}{a^2\,\left(4\,a\,c-b^2\right)}+\frac{2\,c\,x^2\,\left(3\,a\,c-b^2\right)}{a^2\,\left(4\,a\,c-b^2\right)}}{c\,x^3+b\,x^2+a\,x}-\ln\Bigg(2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-2\,b^8\,x-2\,a\,b^7+23\,a^2\,b^5\,c+108\,a^4\,b\,c^3-24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-9\,a^2\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-97\,a^2\,b^4\,c^2\,x+138\,a^3\,b^2\,c^3\,x+24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}\Bigg)\left(\frac{b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+6\,a^2\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-6\,a\,b^2\,c\,\sqrt{-(4\,a\,c-b^2)}}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}-\frac{b}{a^3}\right)-\frac{2\,b\,\ln\left(x\right)}{a^3}$$

input
```
int(1/(x^6*(c + a/x^2 + b/x)^2),x)
```

output
```
log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 23*a^2*b^5*c -
108*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) + 87*a^3*b
^3*c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^
3)^(1/2) + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*
c*x*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((
b^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2) + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c))/(a^2*(4*a*c - b^2)) + (
2*c*x^2*(3*a*c - b^2))/(a^2*(4*a*c - b^2)))/(a*x + b*x^2 + c*x^3) - log(2*
a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*b^8*x - 2*a*b^7 + 23*a^2*b^5*c + 108*a^
4*b*c^3 - 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) - 87*a^3*b^3*c^2
 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^(1/
2) - 97*a^2*b^4*c^2*x + 138*a^3*b^2*c^3*x + 24*a*b^6*c*x - 12*a*b^3*c*x*(-
(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(-
(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2) - b/a^3) - (2*b*log(x))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 877, normalized size of antiderivative = 5.93

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \text{Too large to display}$$

input
```
int(1/(c+a/x^2+b/x)^2/x^6,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2
*x + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*
c*x - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2
*c**2*x**2 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*2*b*c**3*x**3 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a*b**5*x + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b
**4*c*x**2 + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
b**3*c**2*x**3 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*b**6*x**2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**
5*c*x**3 + 16*log(a + b*x + c*x**2)*a**3*b**2*c**2*x - 8*log(a + b*x + c*x
**2)*a**2*b**4*c*x + 16*log(a + b*x + c*x**2)*a**2*b**3*c**2*x**2 + 16*log
(a + b*x + c*x**2)*a**2*b**2*c**3*x**3 + log(a + b*x + c*x**2)*a*b**6*x -
8*log(a + b*x + c*x**2)*a*b**5*c*x**2 - 8*log(a + b*x + c*x**2)*a*b**4*c**
2*x**3 + log(a + b*x + c*x**2)*b**7*x**2 + log(a + b*x + c*x**2)*b**6*c*x*
*3 - 32*log(x)*a**3*b**2*c**2*x + 16*log(x)*a**2*b**4*c*x - 32*log(x)*a**2
*b**3*c**2*x**2 - 32*log(x)*a**2*b**2*c**3*x**3 - 2*log(x)*a*b**6*x + 16*l
og(x)*a*b**5*c*x**2 + 16*log(x)*a*b**4*c**2*x**3 - 2*log(x)*b**7*x**2 - 2*
log(x)*b**6*c*x**3 - 16*a**4*b*c**2 + 24*a**4*c**3*x + 8*a**3*b**3*c - 42*
a**3*b**2*c**2*x + 24*a**3*c**4*x**3 - a**2*b**5 + 17*a**2*b**4*c*x - 14*a
**2*b**2*c**3*x**3 - 2*a*b**6*x + 2*a*b**4*c**2*x**3)/(a**3*b*x*(16*a**...
```

3.27 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 202

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7} \, dx = & -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2}+\frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} \\ & +\frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} \\ & +\frac{b(3b^4-20ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}} \\ & +\frac{(3b^2-2ac)\log(x)}{a^4}-\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}\end{aligned}$$

output
```
-1/2*(-8*a*c+3*b^2)/a^2/(-4*a*c+b^2)/x^2+b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b^2
)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^2+b*x+a)+b*(30*a^2*c^2-20*a*
b^2*c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+
(-2*a*c+3*b^2)*ln(x)/a^4-1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx$$
$$= \frac{-\frac{a^2}{x^2}+\frac{4ab}{x}+\frac{2a(b^4-4ab^2c+2a^2c^2+b^3cx-3abc^2x)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(3b^4-20ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2(3b^2-2ac)\log(x)+(}{2a^4}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^2*x^7),x]
```

output
```
(-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*
a*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c +
 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
+ 2*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4
)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1692, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}dx$$
$$\Big\downarrow \text{1692}$$
$$\int \frac{1}{x^3\left(a+bx+cx^2\right)^2}dx$$
$$\Big\downarrow \text{1165}$$
$$\frac{-2ac+b^2+bcx}{ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int -\frac{3b^2+3cxb-8ac}{x^3(cx^2+bx+a)}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3b^2+3cxb-8ac}{x^3(cx^2+bx+a)}dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{\int \left(\frac{3b^2-8ac}{ax^3}+\frac{(b^2-4ac)(3b^2-2ac)}{a^3x}+\frac{-b(3b^4-17acb^2+19a^2c^2)-c(3b^4-14acb^2+8a^2c^2)x}{a^3(cx^2+bx+a)}+\frac{b(11ac-3b^2)}{a^2x^2}\right)dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(b^2-4ac)(3b^2-2ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-4ac)(3b^2-2ac)}{a^3}+\frac{b(3b^2-11ac)}{a^2x}+\frac{b(30a^2c^2-20ab^2c+3b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-3}{a(b^2-4ac)}\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[1/((c + a/x^2 + b/x)^2*x^7),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)) + (-1/2*(3*b
^2 - 8*a*c)/(a*x^2) + (b*(3*b^2 - 11*a*c))/(a^2*x) + (b*(3*b^4 - 20*a*b^2*
c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*
a*c]) + ((b^2 - 4*a*c)*(3*b^2 - 2*a*c)*Log[x])/a^3 - ((b^2 - 4*a*c)*(3*b^2
 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^3))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.26

method	result
default	$\frac{\frac{\frac{acb\left(3ac-b^2\right)x}{4ac-b^2}-\frac{a\left(2a^2c^2-4a\,b^2c+b^4\right)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{\left(8a^2c^3-14a\,b^2c^2+3b^4c\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(19a^2b\,c^2-17a\,b^3c+3b^5-\frac{\left(8a^2c^3-14a\,b^2c^2+3b^4c\right)b}{2c}\right)\text{ar}\cdots}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^4}$
risch	$\frac{\frac{bc\left(11ac-3b^2\right)x^3}{a^3\left(4ac-b^2\right)}-\frac{\left(8a^2c^2-25a\,b^2c+6b^4\right)x^2}{2a^3\left(4ac-b^2\right)}+\frac{3bx}{2a^2}-\frac{1}{2a}}{(c\,x^2+bx+a)x^2}-\frac{2\ln(x)c}{a^3}+\frac{3\ln(x)b^2}{a^4}+\left(\sum_{\_R=\text{RootOf}\left(\left(64a^7c^3-48a^6b^2c^2+12a^5b^4c-a\cdots\right.\right.}\right.$

input
```
int(1/(c+a/x^2+b/x)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*((a*c*b*(3*a*c-b^2)/(4*a*c-b^2)*x-a*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c
-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)/c
*ln(c*x^2+b*x+a)+2*(19*a^2*b*c^2-17*a*b^3*c+3*b^5-1/2*(8*a^2*c^3-14*a*b^2*
c^2+3*b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))-
1/2/a^2/x^2+(-2*a*c+3*b^2)*ln(x)/a^4+2/a^3*b/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(194) = 388$.

Time = 0.25 (sec) , antiderivative size = 1226, normalized size of antiderivative = 6.07

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="fricas")
```

output
```
[-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2
+ 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c
^3)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4
*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)
*(2*c*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2
)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7
 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4
*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c
- 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c +
64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*
c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x
^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c +
 16*a^7*c^2)*x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c
 - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*
b^2*c^2 - 32*a^4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4
 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 3
0*a^3*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b
)/(b^2 - 4*a*c)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c
- 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)**2/x**7,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.13

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = -\frac{(3\,b^5-20\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^4b^2-4\,a^5c)\sqrt{-b^2+4\,ac}} - \frac{(3\,b^2-2\,ac)\log\left(cx^2+bx+a\right)}{2\,a^4} + \frac{(3\,b^2-2\,ac)\log\left(|x|\right)}{a^4} - \frac{a^3b^2-4\,a^4c-2\,(3\,ab^3c-11\,a^2bc^2)x^3-(6\,ab^4-25\,a^2b^2c+8\,a^3c^2)x^2-3\,(a^2b^3-4\,a^3bc)x}{2\,(cx^2+bx+a)(b^2-4\,ac)a^4x^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="giac")
```

output
```
-(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2
 + b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c
 - 2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)
*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2
)
```

Mupad [B] (verification not implemented)

Time = 20.87 (sec) , antiderivative size = 914, normalized size of antiderivative = 4.52

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx$$
$$= \frac{\ln\left(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4-6\,a\,b^5\,\sqrt{-(4\,a\,c-b^2)^3}-73\,a^2\,b^6\,c-6\,b^6\,x\,\sqrt{-(4\,a\,c-b^2)^3}+307\,a^3\,b^4\,c^2\right.}{}$$
$$-\frac{\ln(x)\,(2\,a\,c-3\,b^2)}{a^4}-\frac{\frac{1}{2\,a}-\frac{3\,b\,x}{2\,a^2}+\frac{x^2\,(8\,a^2\,c^2-25\,a\,b^2\,c+6\,b^4)}{2\,a^3\,(4\,a\,c-b^2)}-\frac{b\,c\,x^3\,(11\,a\,c-3\,b^2)}{a^3\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^3+a\,x^2}$$
$$+\frac{\ln\left(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4+6\,a\,b^5\,\sqrt{-(4\,a\,c-b^2)^3}-73\,a^2\,b^6\,c+6\,b^6\,x\,\sqrt{-(4\,a\,c-b^2)^3}+307\,a^3\,b^4\right.}{}$$

input
```
int(1/(x^7*(c + a/x^2 + b/x)^2),x)
```

output
```
(log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^(1/2) -
73*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b^4*c^2 - 492*a^
4*b^2*c^3 + 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 27*a^3*b*c^2*(-(4*a*c
- b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x + 24*a^3*c^3*x*(-(
4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x + 40*a*b^4*c*x*(-(4
*a*c - b^2)^3)^(1/2) - 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(3*b^8 +
 128*a^4*c^4 - 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2 - 288*a^3*
b^2*c^3 - 38*a*b^6*c - 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b^3*c*
(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3) - (log(x)*(2*a*c - 3*b^
2))/a^4 - (1/(2*a) - (3*b*x)/(2*a^2) + (x^2*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*
c))/(2*a^3*(4*a*c - b^2)) - (b*c*x^3*(11*a*c - 3*b^2))/(a^3*(4*a*c - b^2))
)/(a*x^2 + b*x^3 + c*x^4) + (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 + 6*a*b^5
*(-(4*a*c - b^2)^3)^(1/2) - 73*a^2*b^6*c + 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2
) + 307*a^3*b^4*c^2 - 492*a^4*b^2*c^3 - 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1
/2) + 27*a^3*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*
b^3*c^3*x - 24*a^3*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4
*b*c^4*x - 40*a*b^4*c*x*(-(4*a*c - b^2)^3)^(1/2) + 69*a^2*b^2*c^2*x*(-(4*a
*c - b^2)^3)^(1/2))*(3*b^8 + 128*a^4*c^4 + 3*b^5*(-(4*a*c - b^2)^3)^(1/2)
+ 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3 - 38*a*b^6*c + 30*a^2*b*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 20*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1039, normalized size of antiderivative = 5.14

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7} \, dx = \text{Too large to display}$$

input
```
int(1/(c+a/x^2+b/x)^2/x^7,x)
```

output
```
(60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2*x*
*2 - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*
c*x**2 + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b
**2*c**2*x**3 + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a**2*b*c**3*x**4 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**
2))*a*b**5*x**2 - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2
))*a*b**4*c*x**3 - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**
2))*a*b**3*c**2*x**4 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*b**6*x**3 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2
))*b**5*c*x**4 + 32*log(a + b*x + c*x**2)*a**4*c**3*x**2 - 64*log(a + b*x
+ c*x**2)*a**3*b**2*c**2*x**2 + 32*log(a + b*x + c*x**2)*a**3*b*c**3*x**3
+ 32*log(a + b*x + c*x**2)*a**3*c**4*x**4 + 26*log(a + b*x + c*x**2)*a**2*
b**4*c*x**2 - 64*log(a + b*x + c*x**2)*a**2*b**3*c**2*x**3 - 64*log(a + b*
x + c*x**2)*a**2*b**2*c**3*x**4 - 3*log(a + b*x + c*x**2)*a*b**6*x**2 + 26
*log(a + b*x + c*x**2)*a*b**5*c*x**3 + 26*log(a + b*x + c*x**2)*a*b**4*c**
2*x**4 - 3*log(a + b*x + c*x**2)*b**7*x**3 - 3*log(a + b*x + c*x**2)*b**6*
c*x**4 - 64*log(x)*a**4*c**3*x**2 + 128*log(x)*a**3*b**2*c**2*x**2 - 64*lo
g(x)*a**3*b*c**3*x**3 - 64*log(x)*a**3*c**4*x**4 - 52*log(x)*a**2*b**4*c*x
**2 + 128*log(x)*a**2*b**3*c**2*x**3 + 128*log(x)*a**2*b**2*c**3*x**4 + 6*
log(x)*a*b**6*x**2 - 52*log(x)*a*b**5*c*x**3 - 52*log(x)*a*b**4*c**2*x*...
```

3.28 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 238

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3} dx = \frac{3(b^4-7ab^2c+10a^2c^2)\,x}{c^3\,(b^2-4ac)^2} - \frac{3b(b^2-6ac)\,x^2}{2c^2\,(b^2-4ac)^2} + \frac{x^5(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2} + \frac{x^3(a(b^2-10ac)+b(b^2-7ac)\,x)}{c\,(b^2-4ac)^2\,(a+bx+cx^2)} - \frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\,(b^2-4ac)^{5/2}} - \frac{3b\log\left(a+bx+cx^2\right)}{2c^4}$$

output
```
3*(10*a^2*c^2-7*a*b^2*c+b^4)*x/c^3/(-4*a*c+b^2)^2-3/2*b*(-6*a*c+b^2)*x^2/c
^2/(-4*a*c+b^2)^2+1/2*x^5*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+x^3*(a*(-
10*a*c+b^2)+b*(-7*a*c+b^2)*x)/c/(-4*a*c+b^2)^2/(c*x^2+b*x+a)-3*(-20*a^3*c^
3+30*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4
/(-4*a*c+b^2)^(5/2)-3/2*b*ln(c*x^2+b*x+a)/c^4
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \frac{2c^2x+\frac{b^7-14ab^5c+61a^2b^3c^2-78a^3bc^3-6b^6cx+48ab^4c^2x-102a^2b^2c^3x+36a^3c^4x}{(b^2-4ac)^2(a+x(b+cx))}+\frac{-b^6x+a^2b^2c(5b-9cx)-ab^4(b-6cx)+a^3c^2(-5b+2cx)}{(b^2-4ac)(a+x(b+cx))^2}}{2c^5}$$

input
```
Integrate[(c + a/x^2 + b/x)^(-3),x]
```

output
```
(2*c^2*x + (b^7 - 14*a*b^5*c + 61*a^2*b^3*c^2 - 78*a^3*b*c^3 - 6*b^6*c*x +
 48*a*b^4*c^2*x - 102*a^2*b^2*c^3*x + 36*a^3*c^4*x)/((b^2 - 4*a*c)^2*(a +
x*(b + c*x))) + (-(b^6*x) + a^2*b^2*c*(5*b - 9*c*x) - a*b^4*(b - 6*c*x) +
a^3*c^2*(-5*b + 2*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (6*c*(b^6 -
10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4
*a*c]])/(-b^2 + 4*a*c)^(5/2) - 3*b*c*Log[a + x*(b + c*x)])/(2*c^5)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1679, 1164, 27, 1233, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{ 1679}$$

$$\int \frac{x^6}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{ 1164}$$

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int \frac{2x^4(5a+bx)}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}$$

$\big\downarrow$ 27

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int \frac{x^4(5a+bx)}{(cx^2+bx+a)^2}dx}{b^2-4ac}$$

$\big\downarrow$ 1233

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{\int \frac{3x^2\left(a\left(b^2-10ac\right)+b\left(b^2-6ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^3(bx(b^2-7ac)+a(b^2-10ac))}{c(b^2-4ac)(a+bx+cx^2)}}{b^2-4ac}$$

$\big\downarrow$ 27

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{3\int \frac{x^2\left(a\left(b^2-10ac\right)+b\left(b^2-6ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^3(bx(b^2-7ac)+a(b^2-10ac))}{c(b^2-4ac)(a+bx+cx^2)}}{b^2-4ac}$$

$\big\downarrow$ 1200

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{3\int \left(-\frac{b^4-7acb^2+10a^2c^2}{c^2}+\frac{b\left(b^2-6ac\right)x}{c}+\frac{bx\left(b^2-4ac\right)^2+a\left(b^4-7acb^2+10a^2c^2\right)}{c^2\left(cx^2+bx+a\right)}\right)dx}{c(b^2-4ac)}-\frac{x^3(bx(b^2-7ac)+a(b^2-10ac))}{c(b^2-4ac)(a+bx+cx^2)}}{b^2-4ac}$$

$\big\downarrow$ 2009

$$\frac{x^5(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{3\left(-\frac{x\left(10a^2c^2-7ab^2c+b^4\right)}{c^2}+\frac{\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{b\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)}{2c^3}+\frac{bx^2\left(b^2-6ac\right)}{2c}\right)}{c(b^2-4ac)}-\frac{x^3(bx(b^2-7}{c(b^2-4a}}{b^2-4ac}$$

input
```
Int[(c + a/x^2 + b/x)^(-3),x]
```

output
```
(x^5*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (-((x^3*(a*(b^2
- 10*a*c) + b*(b^2 - 7*a*c)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2))) + (3*
(-(((b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/c^2) + (b*(b^2 - 6*a*c)*x^2)/(2*c) +
 ((b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqr
t[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + (b*(b^2 - 4*a*c)^2*Log[a + b*x
+ c*x^2])/(2*c^3)))/(c*(b^2 - 4*a*c)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.68

method	result
default	$\frac{x}{c^3} - \frac{\frac{-\frac{3\left(6a^3c^3-17a^2b^2c^2+8a\,b^4c-b^6\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{b\left(42a^3c^3+41a^2b^2c^2-34a\,b^4c+5b^6\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{a\left(14a^3c^3-71a^2b^2c^2+38a\,b^4c-5b^6\right)x}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{b\,a^2\left(58a^2c^2-3\cdots\right.}{2c\left(16a^2c^2-8\cdots\right.}}{\left(c\,x^2+bx+a\right)^2}}{c^3}$
risch	Expression too large to display

input
```
int(1/(c+a/x^2+b/x)^3,x,method=_RETURNVERBOSE)
```

output
```
x/c^3-1/c^3*((-3*(6*a^3*c^3-17*a^2*b^2*c^2+8*a*b^4*c-b^6)/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^3+1/2*b*(42*a^3*c^3+41*a^2*b^2*c^2-34*a*b^4*c+5*b^6)/(16*a^2*
c^2-8*a*b^2*c+b^4)/c*x^2-a/c*(14*a^3*c^3-71*a^2*b^2*c^2+38*a*b^4*c-5*b^6)/
(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/2*b*a^2/c*(58*a^2*c^2-36*a*b^2*c+5*b^4)/(16
*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2
*(16*a^2*b*c^2-8*a*b^3*c+b^5)/c*ln(c*x^2+b*x+a)+2*(10*c^2*a^3-7*c*a^2*b^2+
a*b^4-1/2*(16*a^2*b*c^2-8*a*b^3*c+b^5)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(228) = 456$.

Time = 0.11 (sec) , antiderivative size = 1926, normalized size of antiderivative = 8.09

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^3,x, algorithm="fricas")
```

output
```
[-1/2*(5*a^2*b^7 - 56*a^3*b^5*c + 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6
*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a
*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2
+ 123*a^2*b^4*c^3 - 254*a^3*b^2*c^4 + 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7
*c + 225*a^2*b^5*c^2 - 314*a^3*b^3*c^3 + 88*a^4*b*c^4)*x^2 + 3*(a^2*b^6 -
10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*
a^2*b^2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 -
 20*a^3*b*c^4)*x^3 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 -
40*a^4*c^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)
*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 -
4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a
^3*b^4*c^2 - 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c
+ 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4
 - 64*a^3*b*c^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b
^2*c^4)*x^3 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^
4*b*c^4)*x^2 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*
x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 -
64*a^5*c^7 + (b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 +
2*(b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4
- 10*a*b^6*c^5 + 24*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1714 vs. $2(236) = 472$.

Time = 2.61 (sec) , antiderivative size = 1714, normalized size of antiderivative = 7.20

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x**2+b/x)**3,x)
```

output
```
(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c
**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (
-66*a**3*b*c**2 - 64*a**3*c**6*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)
*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**
5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 2
0*a*b**8*c - b**10))) + 27*a**2*b**3*c + 48*a**2*b**2*c**5*(-3*b/(2*c**4)
- 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4
*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c
**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 3*a*b**5 - 12*a*b**4*c
**4*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b*
*2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**
4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + b**
6*c**3*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2
*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*
c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))))/(
60*a**3*c**3 - 90*a**2*b**2*c**2 + 30*a*b**4*c - 3*b**6)) + (-3*b/(2*c**4)
 + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**
4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*
c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66*a**3*b*...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx$$
$$=\frac{3\,(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4c^4-8\,ab^2c^5+16\,a^2c^6)\sqrt{-b^2+4\,ac}}+\frac{x}{c^3}-\frac{3\,b\log\left(cx^2+bx+a\right)}{2\,c^4}$$
$$-\frac{5\,a^2b^5-36\,a^3b^3c+58\,a^4bc^2+6\,(b^6c-8\,ab^4c^2+17\,a^2b^2c^3-6\,a^3c^4)x^3+(5\,b^7-34\,ab^5c+41\,a^2b^3c^2+}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2c^4}$$

input
```
integrate(1/(c+a/x^2+b/x)^3,x, algorithm="giac")
```

output
```
3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c))
+ x/c^3 - 3/2*b*log(c*x^2 + b*x + a)/c^4 - 1/2*(5*a^2*b^5 - 36*a^3*b^3*c +
 58*a^4*b*c^2 + 6*(b^6*c - 8*a*b^4*c^2 + 17*a^2*b^2*c^3 - 6*a^3*c^4)*x^3 +
 (5*b^7 - 34*a*b^5*c + 41*a^2*b^3*c^2 + 42*a^3*b*c^3)*x^2 + 2*(5*a*b^6 - 3
8*a^2*b^4*c + 71*a^3*b^2*c^2 - 14*a^4*c^3)*x)/((c*x^2 + b*x + a)^2*(b^2 -
4*a*c)^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 19.54 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx=\frac{x}{c^3}$$
$$-\frac{\frac{3\,x^3\,\left(-6\,a^3\,c^3+17\,a^2\,b^2\,c^2-8\,a\,b^4\,c+b^6\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{x^2\,\left(42\,a^3\,b\,c^3+41\,a^2\,b^3\,c^2-34\,a\,b^5\,c+5\,b^7\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(58\,a^2\,b\,c^2-36\,a\,b^3\,c+5\,b^5\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x\,\left(-14\,a^3\,c^3\right.}{c\,(16}}{a^2\,c^3+c^5\,x^4+x^2\,\left(b^2\,c^3+2\,a\,c^4\right)+2\,b\,c^4\,x^3+2\,a\,b\,c^3\,x}$$
$$+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-3072\,a^5\,b\,c^5+3840\,a^4\,b^3\,c^4-1920\,a^3\,b^5\,c^3+480\,a^2\,b^7\,c^2-60\,a\,b^9\,c+3\,b^{11}\right)}{2\,\left(1024\,a^5\,c^9-1280\,a^4\,b^2\,c^8+640\,a^3\,b^4\,c^7-160\,a^2\,b^6\,c^6+20\,a\,b^8\,c^5-b^{10}\,c^4\right)}$$
$$+\frac{3\,\mathrm{atan}\left(\frac{\left(\frac{3\,x\,\left(-20\,a^3\,c^3+30\,a^2\,b^2\,c^2-10\,a\,b^4\,c+b^6\right)}{c^3\,\left(4\,a\,c-b^2\right)^5}+\frac{3\,\left(16\,a^2\,b\,c^5-8\,a\,b^3\,c^4+b^5\,c^3\right)\,\left(-20\,a^3\,c^3+30\,a^2\,b^2\,c^2-10\,a\,b^4\,c+b^6\right)}{2\,c^7\,\left(4\,a\,c-b^2\right)^5\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\,\left(32\,a^2\,c^6\,\left(4\,a\,c-b^2\right)^{5/}\right.}{-60\,a^3\,c^3+90\,a^2\,b^2\,c^2-30\,a\,b^4\,c+3\,b^6}\right.}{c^4\,\left(4\,a\,c-b^2\right)^{5/2}}$$

input
```
int(1/(c + a/x^2 + b/x)^3,x)
```

output
```
x/c^3 - ((3*x^3*(b^6 - 6*a^3*c^3 + 17*a^2*b^2*c^2 - 8*a*b^4*c))/(b^4 + 16*
a^2*c^2 - 8*a*b^2*c) + (x^2*(5*b^7 + 42*a^3*b*c^3 + 41*a^2*b^3*c^2 - 34*a*
b^5*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(5*b^5 + 58*a^2*b*c^2
- 36*a*b^3*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(5*b^6 - 14*a^3
*c^3 + 71*a^2*b^2*c^2 - 38*a*b^4*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(
a^2*c^3 + c^5*x^4 + x^2*(2*a*c^4 + b^2*c^3) + 2*b*c^4*x^3 + 2*a*b*c^3*x) +
 (log(a + b*x + c*x^2)*(3*b^11 - 3072*a^5*b*c^5 + 480*a^2*b^7*c^2 - 1920*a
^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 60*a*b^9*c))/(2*(1024*a^5*c^9 - b^10*c^4 +
 20*a*b^8*c^5 - 160*a^2*b^6*c^6 + 640*a^3*b^4*c^7 - 1280*a^4*b^2*c^8)) + (
3*atan((((3*x*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^3*(4*a*
c - b^2)^5) + (3*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*(b^6 - 20*a^3*c^3
+ 30*a^2*b^2*c^2 - 10*a*b^4*c))/(2*c^7*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c)))*(32*a^2*c^6*(4*a*c - b^2)^(5/2) + 2*b^4*c^4*(4*a*c - b^2)^(5
/2) - 16*a*b^2*c^5*(4*a*c - b^2)^(5/2)))/(3*b^6 - 60*a^3*c^3 + 90*a^2*b^2*
c^2 - 30*a*b^4*c))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^4*
(4*a*c - b^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1896, normalized size of antiderivative = 7.97

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(c+a/x^2+b/x)^3,x)
```

output
```
( - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b*c**
3 + 180*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**3*
c**2 - 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*
*2*c**3*x - 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*4*b*c**4*x**2 - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a**3*b**5*c + 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a**3*b**4*c**2*x + 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*a**3*b**3*c**3*x**2 - 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(
4*a*c - b**2))*a**3*b**2*c**4*x**3 - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*
x)/sqrt(4*a*c - b**2))*a**3*b*c**5*x**4 + 6*sqrt(4*a*c - b**2)*atan((b + 2
*c*x)/sqrt(4*a*c - b**2))*a**2*b**7 - 120*sqrt(4*a*c - b**2)*atan((b + 2*c
*x)/sqrt(4*a*c - b**2))*a**2*b**6*c*x + 60*sqrt(4*a*c - b**2)*atan((b + 2*
c*x)/sqrt(4*a*c - b**2))*a**2*b**5*c**2*x**2 + 360*sqrt(4*a*c - b**2)*atan
((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**4*c**3*x**3 + 180*sqrt(4*a*c - b*
*2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c**4*x**4 + 12*sqrt(4*a
*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**8*x - 48*sqrt(4*a*c -
 b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**7*c*x**2 - 120*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**6*c**2*x**3 - 60*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**5*c**3*x**4 + 6*sqr
t(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**9*x**2 + 12*sqr...
```

3.29 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 190

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = -\frac{b(b^2-7ac)\,x}{c^2\left(b^2-4ac\right)^2}+\frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{x^2(a(b^2-16ac)+b(b^2-10ac)\,x)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{b(b^4-10ab^2c+30a^2c^2)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{5/2}} + \frac{\log\left(a+bx+cx^2\right)}{2c^3}$$

output
```
-b*(-7*a*c+b^2)*x/c^2/(-4*a*c+b^2)^2+1/2*x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2
+b*x+a)^2+1/2*x^2*(a*(-16*a*c+b^2)+b*(-10*a*c+b^2)*x)/c/(-4*a*c+b^2)^2/(c*
x^2+b*x+a)+b*(30*a^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/c^3/(-4*a*c+b^2)^(5/2)+1/2*ln(c*x^2+b*x+a)/c^3
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$$
$$=\frac{\frac{-b^6+11ab^4c-39a^2b^2c^2+32a^3c^3+4b^5cx-30ab^3c^2x+50a^2bc^3x}{(b^2-4ac)^2(a+x(b+cx))}+\frac{2a^3c^2+b^5x+ab^3(b-5cx)+a^2bc(-4b+5cx)}{(b^2-4ac)(a+x(b+cx))^2}-\frac{2bc(b^4-10ab^2c+30a^2c^2)\,\text{arct}}{(-b^2+4ac)^{5/}}}{2c^4}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x),x]
```

output
```
((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x - 30*a*b^3*c
^2*x + 50*a^2*b*c^3*x)/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (2*a^3*c^2 +
b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(-4*b + 5*c*x))/((b^2 - 4*a*c)*(a + x*
(b + c*x))^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + x*(b + c*x)])/(2*c^4
)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1692, 1164, 1233, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$
$$\Big\downarrow \text{ 1692}$$
$$\int \frac{x^5}{(a+bx+cx^2)^3}dx$$
$$\Big\downarrow \text{ 1164}$$

$$\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int \frac{x^3(8a+bx)}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \text{1233}$$

$$\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{\int \frac{2x\left(a\left(b^2-16ac\right)+b\left(b^2-7ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{c(b^2-4ac)(a+bx+cx^2)}}{2(b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{2\int \frac{x\left(a\left(b^2-16ac\right)+b\left(b^2-7ac\right)x\right)}{cx^2+bx+a}dx}{c(b^2-4ac)}-\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{c(b^2-4ac)(a+bx+cx^2)}}{2(b^2-4ac)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{2\int \left(-b\left(7a-\frac{b^2}{c}\right)-\frac{x\left(b^2-4ac\right)^2+ab\left(b^2-7ac\right)}{c\left(cx^2+bx+a\right)}\right)dx}{c(b^2-4ac)}-\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{c(b^2-4ac)(a+bx+cx^2)}}{2(b^2-4ac)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}-\frac{\frac{2\left(-\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)}{2c^2}-bx\left(7a-\frac{b^2}{c}\right)\right)}{c(b^2-4ac)}-\frac{x^2\left(bx\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{c(b^2-4ac)(a+bx+cx^2)}}{2(b^2-4ac)}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x),x]
```

output
```
(x^4*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (-((x^2*(a*(b^2
- 16*a*c) + b*(b^2 - 10*a*c)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2))) + (2
*(-(b*(7*a - b^2/c)*x) - (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - 4*a*c)^2*Log[a
 + b*x + c*x^2])/(2*c^2)))/(c*(b^2 - 4*a*c)))/(2*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.88

method	result
default	$\frac{\frac{b\left(25a^{2}c^{2}-15a\,b^{2}c+2b^{4}\right)x^{3}}{c^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(32a^{3}c^{3}+11a^{2}b^{2}c^{2}-19a\,b^{4}c+3b^{6}\right)x^{2}}{2c^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{ba\left(31a^{2}c^{2}-22a\,b^{2}c+3b^{4}\right)x}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{3}}+\frac{3a^{2}\left(8a^{2}c^{2}-7a\,b^{2}c+b^{4}\right)}{2c^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{2}+bx+a\right)^{2}}+\frac{\left(16a^{2}c^{2}-8a\right.}{}$
risch	Expression too large to display

input
```
int(1/(c+a/x^2+b/x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
(1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*
(32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^2+b*a*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+3
/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*
x+a)^2+1/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*a^2*c^2-8*a*b^2*c+b^4)/c*
ln(c*x^2+b*x+a)+2*(-7*a^2*b*c+b^3*a-1/2*(16*a^2*c^2-8*a*b^2*c+b^4)*b/c)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. 2(180) = 360.

Time = 0.09 (sec) , antiderivative size = 1603, normalized size of antiderivative = 8.44

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="fricas")
```

output
```
[1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c
 - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 31*a*b^6*
c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3
*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b
^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*
c^2 + 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqr
t(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*
(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*
c^2 - 124*a^4*b*c^3)*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5
*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c
^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^
6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*
b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 3
2*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*
b^3*c^5 - 64*a^4*b*c^6)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^
2 - 96*a^5*c^3 + 2*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^
4)*x^3 + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1510 vs. $2(180) = 360$.

Time = 1.89 (sec) , antiderivative size = 1510, normalized size of antiderivative = 7.95

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x,x)
```

output
```
(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(
1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*
c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(-b*sqrt
(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**
5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 2
0*a*b**8*c - b**10)) + 1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(-b*
sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024
*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10)) + 1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(-
b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(10
24*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) + 1/(2*c**3)) + a*b**4 + b**6*c**2*(-b*sqrt(-(4
*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*
b**8*c - b**10)) + 1/(2*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (
b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(10
24*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c
**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 2...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$$
$$= -\frac{(b^5-10\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4c^3-8\,ab^2c^4+16\,a^2c^5)\sqrt{-b^2+4\,ac}}+\frac{\log\left(cx^2+bx+a\right)}{2\,c^3}$$
$$+\frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\,(2\,b^5c-15\,ab^3c^2+25\,a^2bc^3)x^3+(3\,b^6-19\,ab^4c+11\,a^2b^2c^2+32\,a^3c^3)}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2c^3}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="giac")
```

output
```
-(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2
+ b*x + a)/c^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 2*(2*b^5*c -
 15*a*b^3*c^2 + 25*a^2*b*c^3)*x^3 + (3*b^6 - 19*a*b^4*c + 11*a^2*b^2*c^2 +
 32*a^3*c^3)*x^2 + 2*(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2)*x)/((c*x^2 +
b*x + a)^2*(b^2 - 4*a*c)^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 19.92 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.26

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$$
$$= \frac{\frac{3\,a^2\,(8\,a^2\,c^2-7\,a\,b^2\,c+b^4)}{2\,c^3\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^2\,(32\,a^3\,c^3+11\,a^2\,b^2\,c^2-19\,a\,b^4\,c+3\,b^6)}{2\,c^3\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{b\,x^3\,(25\,a^2\,c^2-15\,a\,b^2\,c+2\,b^4)}{c^2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{a\,b\,x\,(31\,a^2\,c^2-22\,a\,b^2\,c+3\,b^4)}{c^3\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}$$
$$-\frac{\ln\left(c\,x^2+b\,x+a\right)\,(-1024\,a^5\,c^5+1280\,a^4\,b^2\,c^4-640\,a^3\,b^4\,c^3+160\,a^2\,b^6\,c^2-20\,a\,b^8\,c+b^{10})}{2\,(1024\,a^5\,c^8-1280\,a^4\,b^2\,c^7+640\,a^3\,b^4\,c^6-160\,a^2\,b^6\,c^5+20\,a\,b^8\,c^4-b^{10}\,c^3)}$$
$$-\frac{b\,\mathrm{atan}\left(\frac{\left(\frac{b\,x\,(30\,a^2\,c^2-10\,a\,b^2\,c+b^4)}{c^2\,(4\,a\,c-b^2)^5}+\frac{b^2\,(16\,a^2\,c^4-8\,a\,b^2\,c^3+b^4\,c^2)\,(30\,a^2\,c^2-10\,a\,b^2\,c+b^4)}{2\,c^5\,(4\,a\,c-b^2)^5\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}\right)\left(32\,a^2\,c^5\,(4\,a\,c-b^2)^{5/2}+2\,b^4\,c^3\,(4\,a\,c-b^2)^{5/2}-\right.}{30\,a^2\,b\,c^2-10\,a\,b^3\,c+b^5}\right.}{c^3\,(4\,a\,c-b^2)^{5/2}}$$

input
```
int(1/(x*(c + a/x^2 + b/x)^3),x)
```

output
```
((3*a^2*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)) + (x^2*(3*b^6 + 32*a^3*c^3 + 11*a^2*b^2*c^2 - 19*a*b^4*c))/(2*c^3*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^3*(2*b^4 + 25*a^2*c^2 - 15*a*b^2*c))/(c
^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*b*x*(3*b^4 + 31*a^2*c^2 - 22*a*b^2
*c))/(c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*
x^4 + 2*a*b*x + 2*b*c*x^3) - (log(a + b*x + c*x^2)*(b^10 - 1024*a^5*c^5 +
160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 20*a*b^8*c))/(2*(10
24*a^5*c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 + 640*a^3*b^4*c^6 -
 1280*a^4*b^2*c^7)) - (b*atan((((b*x*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(c^2
*(4*a*c - b^2)^5) + (b^2*(16*a^2*c^4 + b^4*c^2 - 8*a*b^2*c^3)*(b^4 + 30*a^
2*c^2 - 10*a*b^2*c))/(2*c^5*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
))*(32*a^2*c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*
b^2*c^4*(4*a*c - b^2)^(5/2)))/(b^5 + 30*a^2*b*c^2 - 10*a*b^3*c))*(b^4 + 30
*a^2*c^2 - 10*a*b^2*c))/(c^3*(4*a*c - b^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1555, normalized size of antiderivative = 8.18

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\, dx = \text{Too large to display}$$

input
```
int(1/(c+a/x^2+b/x)^3/x,x)
```

output

```
( - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c**2
 + 20*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**3*c
- 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c*
*2*x - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*
c**3*x**2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2
*b**5 + 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*
*4*c*x - 20*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b
**3*c**2*x**2 - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a**2*b**2*c**3*x**3 - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a**2*b*c**4*x**4 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a
*c - b**2))*a*b**6*x + 16*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
 b**2))*a*b**5*c*x**2 + 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a*b**4*c**2*x**3 + 20*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a*b**3*c**3*x**4 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt
(4*a*c - b**2))*b**7*x**2 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a
*c - b**2))*b**6*c*x**3 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
 - b**2))*b**5*c**2*x**4 + 64*log(a + b*x + c*x**2)*a**5*c**3 - 48*log(a +
 b*x + c*x**2)*a**4*b**2*c**2 + 128*log(a + b*x + c*x**2)*a**4*b*c**3*x +
128*log(a + b*x + c*x**2)*a**4*c**4*x**2 + 12*log(a + b*x + c*x**2)*a**3*b
**4*c - 96*log(a + b*x + c*x**2)*a**3*b**3*c**2*x - 32*log(a + b*x + c*...
```

3.30 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2} \, dx = \frac{x^3(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2} - \frac{3ax(2a+bx)}{(b^2-4ac)^2(a+bx+cx^2)} - \frac{12a^2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output

```
1/2*x^3*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2-3*a*x*(b*x+2*a)/(-4*a*c+b^2
)^2/(c*x^2+b*x+a)-12*a^2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.63

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx=\frac{1}{2}\left(\frac{b^5-8ab^3c+22a^2bc^2-2b^4cx+16ab^2c^2x-20a^2c^3x}{c^3\left(b^2-4ac\right)^2\left(a+x(b+cx)\right)}+\frac{b^4x+ab^2(b-4cx)+a^2c(-3b+2cx)}{c^3\left(-b^2+4ac\right)\left(a+x(b+cx)\right)^2}+\frac{24a^2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}\right)$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^2),x]
```

output
```
((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x + 16*a*b^2*c^2*x - 20*a^2*c^3
*x)/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (b^4*x + a*b^2*(b - 4*c*x) +
 a^2*c*(-3*b + 2*c*x))/(c^3*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + (24*a^2*
ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1690, 1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{1690}$$

$$-\int \frac{1}{\left(\frac{a}{x^2}+c+\frac{b}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{1086}$$

$$\frac{3a\int \frac{1}{\left(\frac{a}{x^2}+c+\frac{b}{x}\right)^2}d\frac{1}{x}}{b^2-4ac}+\frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{3a\left(-\frac{2a\int \frac{1}{\frac{a}{x^2}+c+\frac{b}{x}}d\frac{1}{x}}{b^2-4ac}-\frac{\frac{2a}{x}+b}{\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}\right)}{b^2-4ac}+\frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3a\left(\frac{4a\int \frac{1}{b^2-4ac-\frac{1}{x^2}}d\left(\frac{2a}{x}+b\right)}{b^2-4ac}-\frac{\frac{2a}{x}+b}{\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}\right)}{b^2-4ac}+\frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3a\left(\frac{4a\text{arctanh}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{\frac{2a}{x}+b}{\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}\right)}{b^2-4ac}+\frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^2),x]
```

output
```
(b + (2*a)/x)/(2*(b^2 - 4*a*c)*(c + a/x^2 + b/x)^2) + (3*a*(-((b + (2*a)/x
)/((b^2 - 4*a*c)*(c + a/x^2 + b/x))) + (4*a*ArcTanh[(b + (2*a)/x)/Sqrt[b^2
 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(101) = 202$.

Time = 0.07 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.43

method	result
default	$\frac{-\frac{(10a^2c^2-8a\,b^2c+b^4)x^3}{c(16a^2c^2-8a\,b^2c+b^4)}+\frac{b(2a^2c^2+8a\,b^2c-b^4)x^2}{2c^2(16a^2c^2-8a\,b^2c+b^4)}-\frac{a(6a^2c^2-10a\,b^2c+b^4)x}{(16a^2c^2-8a\,b^2c+b^4)c^2}+\frac{b\,a^2(10ac-b^2)}{2c^2(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^2+bx+a)^2}+\frac{12a^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2-8a\,b^2c+b^4)\sqrt{4ac-\ldots}}$
risch	$\frac{-\frac{(10a^2c^2-8a\,b^2c+b^4)x^3}{c(16a^2c^2-8a\,b^2c+b^4)}+\frac{b(2a^2c^2+8a\,b^2c-b^4)x^2}{2c^2(16a^2c^2-8a\,b^2c+b^4)}-\frac{a(6a^2c^2-10a\,b^2c+b^4)x}{(16a^2c^2-8a\,b^2c+b^4)c^2}+\frac{b\,a^2(10ac-b^2)}{2c^2(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^2+bx+a)^2}-\frac{6a^2\ln\left(\left(32a^2c^3-16a\,b^2c^2+\ldots\right.\right.}{\ldots}$

input
```
int(1/(c+a/x^2+b/x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*b*(2*a
^2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-a*(6*a^2*c^2-10*a
*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x+1/2*b*a^2*(10*a*c-b^2)/c^2/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+12*a^2/(16*a^2*c^2-8*a*b^2*c+b^4
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(101) = 202$.

Time = 0.08 (sec) , antiderivative size = 953, normalized size of antiderivative = 8.91

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="fricas")
```

output
```
[-1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 4
2*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a
^3*b*c^3)*x^2 - 12*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x + a^4*c^
2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*
c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*
a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a
^2*b^2*c^6 - 64*a^3*c^7)*x^4 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5
- 64*a^3*b*c^6)*x^3 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^
2*c^5 - 128*a^4*c^6)*x^2 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4
- 64*a^4*b*c^5)*x), -1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c
 - 12*a*b^4*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 3
0*a^2*b^3*c^2 + 8*a^3*b*c^3)*x^2 + 24*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a
^3*b*c^2*x + a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*a
rctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^6 - 14*a^2*b
^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*
a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a
^3*c^7)*x^4 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x
^3 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c
^6)*x^2 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(100) = 200$.

Time = 0.86 (sec) , antiderivative size = 547, normalized size of antiderivative = 5.11

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx =$$

$$-6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}+}{12a^2c}\right.$$

$$+6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}-6}{12a^2c}\right.$$

$$+\frac{10a^3bc-a^2b^3+x^3(-20a^2c^3+16ab^2c^2-2b^4c)+x^2\cdot(2a^2bc^2+8ab^3c-}{32a^4c^4-16a^3b^2c^3+2a^2b^4c^2+x^4\cdot(32a^2c^6-16ab^2c^5+2b^4c^4)+x^3\cdot(64a^2bc^5-32ab^3c^4+4b^5c^3)+x}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x**2,x)
```

output
```
-6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**5*c**3*sqrt(-1/(4*a*c
 - b**2)**5) + 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 72*a**3*b**4
*c*sqrt(-1/(4*a*c - b**2)**5) + 6*a**2*b**6*sqrt(-1/(4*a*c - b**2)**5) + 6
*a**2*b)/(12*a**2*c)) + 6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**
5*c**3*sqrt(-1/(4*a*c - b**2)**5) - 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b*
*2)**5) + 72*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 6*a**2*b**6*sqrt(-1/
(4*a*c - b**2)**5) + 6*a**2*b)/(12*a**2*c)) + (10*a**3*b*c - a**2*b**3 + x
**3*(-20*a**2*c**3 + 16*a*b**2*c**2 - 2*b**4*c) + x**2*(2*a**2*b*c**2 + 8*
a*b**3*c - b**5) + x*(-12*a**3*c**2 + 20*a**2*b**2*c - 2*a*b**4))/(32*a**4
*c**4 - 16*a**3*b**2*c**3 + 2*a**2*b**4*c**2 + x**4*(32*a**2*c**6 - 16*a*b
**2*c**5 + 2*b**4*c**4) + x**3*(64*a**2*b*c**5 - 32*a*b**3*c**4 + 4*b**5*c
**3) + x**2*(64*a**3*c**5 - 12*a*b**4*c**3 + 2*b**6*c**2) + x*(64*a**3*b*c
**4 - 32*a**2*b**3*c**3 + 4*a*b**5*c**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = \frac{12\,a^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{2\,b^4cx^3-16\,ab^2c^2x^3+20\,a^2c^3x^3+b^5x^2-8\,ab^3cx^2-2\,a^2bc^2x^2+2\,ab^4x-20\,a^2b^2cx+12\,a^3c^2x+a^2b^3}{2\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)(cx^2+bx+a)^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="giac")
```

output
```
12*a^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^4*c*x^3 - 16*a*b^2*c^2*x^3 + 20*a^2*c^3
*x^3 + b^5*x^2 - 8*a*b^3*c*x^2 - 2*a^2*b*c^2*x^2 + 2*a*b^4*x - 20*a^2*b^2*
c*x + 12*a^3*c^2*x + a^2*b^3 - 10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*(c*x^2 + b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 19.13 (sec) , antiderivative size = 343, normalized size of antiderivative = 3.21

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = \frac{12\,a^2\,\mathrm{atan}\left(\frac{\left(\frac{6\,a^2\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{12\,a^2\,c\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,a^2}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}} - \frac{\frac{x^3\,\left(10\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(b^3-10\,a\,b\,c\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{x^2\,\left(2\,a^2\,b\,c^2+8\,a\,b^3\,c-b^5\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x\,\left(6\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}$$

input
```
int(1/(x^2*(c + a/x^2 + b/x)^3),x)
```

output
```
(12*a^2*atan((((6*a^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/
2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (12*a^2*c*x)/(4*a*c - b^2)^(5/2))*(b^
4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*a^2)))/(4*a*c - b^2)^(5/2) - ((x^3*(b^4 +
10*a^2*c^2 - 8*a*b^2*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(b^3 -
10*a*b*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^2*(2*a^2*b*c^2 - b^
5 + 8*a*b^3*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(b^4 + 6*a^2
*c^2 - 10*a*b^2*c))/(c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^
2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 644, normalized size of antiderivative = 6.02

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = \frac{24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^4bc+48\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^3b^2cx+48\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^3}{2bc\left(64a^3c^5x^4-48a^2b^2c^4x^4+12a\,b^4\right.}$$

input
```
int(1/(c+a/x^2+b/x)^3/x^2,x)
```

output
```
(24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c + 48*
sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c*x + 48
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2*x**2
+ 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c*x
**2 + 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2
*c**2*x**3 + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*2*b*c**3*x**4 + 40*a**5*c**2 - 2*a**4*b**2*c + 32*a**4*b*c**2*x + 80*a**4
*c**3*x**2 - 2*a**3*b**4 + 8*a**3*b**3*c*x - 36*a**3*b**2*c**2*x**2 + 40*a
**3*c**4*x**4 - 4*a**2*b**5*x + 12*a**2*b**4*c*x**2 - 42*a**2*b**2*c**3*x*
*4 - 2*a*b**6*x**2 + 12*a*b**4*c**2*x**4 - b**6*c*x**4)/(2*b*c*(64*a**5*c*
*3 - 48*a**4*b**2*c**2 + 128*a**4*b*c**3*x + 128*a**4*c**4*x**2 + 12*a**3*
b**4*c - 96*a**3*b**3*c**2*x - 32*a**3*b**2*c**3*x**2 + 128*a**3*b*c**4*x*
*3 + 64*a**3*c**5*x**4 - a**2*b**6 + 24*a**2*b**5*c*x - 24*a**2*b**4*c**2*
x**2 - 96*a**2*b**3*c**3*x**3 - 48*a**2*b**2*c**4*x**4 - 2*a*b**7*x + 10*a
*b**6*c*x**2 + 24*a*b**5*c**2*x**3 + 12*a*b**4*c**3*x**4 - b**8*x**2 - 2*b
**7*c*x**3 - b**6*c**2*x**4))
```

3.31 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} \, dx = -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{6ab\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

output
```
-1/2*x^3*(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+3/2*b*x*(b*x+2*a)/(-4*a*c+
b^2)^2/(c*x^2+b*x+a)+6*a*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} \, dx = -\frac{8a^3c+b^4x^2+abx(2b^2+bcx+6c^2x^2)+a^2(b^2+10bcx+16c^2x^2)}{2c\left(b^2-4ac\right)^2(a+x(b+cx))^2} - \frac{6ab\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^3),x]
```

output
```
-1/2*(8*a^3*c + b^4*x^2 + a*b*x*(2*b^2 + b*c*x + 6*c^2*x^2) + a^2*(b^2 + 1
0*b*c*x + 16*c^2*x^2))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) - (6*a*b*Ar
cTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1692, 1156, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x^3}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{1156}$$

$$\frac{3b\int \frac{x^2}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)} - \frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1153}$$

$$\frac{3b\left(\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2a\int\frac{1}{cx^2+bx+a}dx}{b^2-4ac}\right)}{2(b^2-4ac)}-\frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3b\left(\frac{4a\int\frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}-\frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3b\left(\frac{4a\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}-\frac{x^3(b+2cx)}{2(b^2-4ac)(a+bx+cx^2)^2}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^3),x]
```

output
```
-1/2*(x^3*(b + 2*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*((x*(2*a
 + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1156
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] + Simp[m*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c)))   Int[
(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(99) = 198$.

Time = 0.07 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.08

method	result
default	$\frac{-\frac{3abc\,x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{\left(16a^2c^2+a\,b^2c+b^4\right)x^2}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{\left(5ac+b^2\right)abx}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a^2\left(8ac+b^2\right)}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}-\frac{6ab\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}$
risch	$\frac{-\frac{3abc\,x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{\left(16a^2c^2+a\,b^2c+b^4\right)x^2}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{\left(5ac+b^2\right)abx}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a^2\left(8ac+b^2\right)}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}-\frac{3ba\ln\left(\left(32a^2c^3-16a\,b^2c^2+2b^4c\right)x\right.}{(-4a}$

input
```
int(1/(c+a/x^2+b/x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(16*a^2*c^2+a*b^2*c+b^4)/c/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^2-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b^4)*
x-1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2-6*a*b/
(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(99) = 198$.

Time = 0.08 (sec) , antiderivative size = 872, normalized size of antiderivative = 8.15

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\, dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="fricas")
```

output
```
[-1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^
3 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^2 - 6*(a*b*c^3*x^4 +
 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^2)*
sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*
c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*
x)/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 -
12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^
3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^
4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 +
48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x), -1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^
2 + 6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^3 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 -
64*a^3*c^3)*x^2 - 12*(a*b*c^3*x^4 + 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*
b*c + (a*b^3*c + 2*a^2*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x
)/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 1
2*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3
 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4
*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 4
8*a^3*b^3*c^3 - 64*a^4*b*c^4)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(102) = 204$.

Time = 0.74 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.79

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx$$
$$= 3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3}{6abc}\right.$$
$$-3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}}-}{6abc}\right.$$
$$+\frac{-8a^3c-a^2b^2-6abc^2x^3+x^2(-16a^2c^2-ab^2c-b^4)+x(}{32a^4c^3-16a^3b^2c^2+2a^2b^4c+x^4\cdot(32a^2c^5-16ab^2c^4+2b^4c^3)+x^3\cdot(64a^2bc^4-32ab^3c^3+4b^5c^2)+x^2}$$

input

```
integrate(1/(c+a/x**2+b/x)**3/x**3,x)
```

output

```
3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**4*b*c**3*sqrt(-1/(4*a*c
- b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**2*b**5
*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5) + 3*a*
b**2)/(6*a*b*c)) - 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**4*b*c*
*3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)*
*5) + 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*c
- b**2)**5) + 3*a*b**2)/(6*a*b*c)) + (-8*a**3*c - a**2*b**2 - 6*a*b*c**2*x
**3 + x**2*(-16*a**2*c**2 - a*b**2*c - b**4) + x*(-10*a**2*b*c - 2*a*b**3)
)/(32*a**4*c**3 - 16*a**3*b**2*c**2 + 2*a**2*b**4*c + x**4*(32*a**2*c**5 -
 16*a*b**2*c**4 + 2*b**4*c**3) + x**3*(64*a**2*b*c**4 - 32*a*b**3*c**3 + 4
*b**5*c**2) + x**2*(64*a**3*c**4 - 12*a*b**4*c**2 + 2*b**6*c) + x*(64*a**3
*b*c**3 - 32*a**2*b**3*c**2 + 4*a*b**5*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.52

$$\begin{aligned}
&\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx \\
&= -\frac{6\,ab\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} \\
&\quad -\frac{6\,abc^2x^3+b^4x^2+ab^2cx^2+16\,a^2c^2x^2+2\,ab^3x+10\,a^2bcx+a^2b^2+8\,a^3c}{2\,(b^4c-8\,ab^2c^2+16\,a^2c^3)(cx^2+bx+a)^2}
\end{aligned}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="giac")
```

output
```
-6*a*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*a*b*c^2*x^3 + b^4*x^2 + a*b^2*c*x^2 + 16*
a^2*c^2*x^2 + 2*a*b^3*x + 10*a^2*b*c*x + a^2*b^2 + 8*a^3*c)/((b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*(c*x^2 + b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.53

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} \, dx = -\frac{\frac{x^2 (16 a^2 c^2+a b^2 c+b^4)}{2 c (16 a^2 c^2-8 a b^2 c+b^4)}+\frac{a^2 (b^2+8 a c)}{2 c (16 a^2 c^2-8 a b^2 c+b^4)}+\frac{3 a b c x^3}{16 a^2 c^2-8 a b^2 c+b^4}+\frac{a b x (b^2+5 a c)}{c (16 a^2 c^2-8 a b^2 c+b^4)}}{x^2 (b^2+2 a c)+a^2+c^2 x^4+2 a b x+2 b c x^3} - \frac{6 a b \operatorname{atan}\left(\frac{\left(\frac{3 a b^2}{(4 a c-b^2)^{5/2}}+\frac{6 a b c x}{(4 a c-b^2)^{5/2}}\right) (16 a^2 c^2-8 a b^2 c+b^4)}{3 a b}\right)}{(4 a c-b^2)^{5/2}}$$

input
```
int(1/(x^3*(c + a/x^2 + b/x)^3),x)
```

output
```
- ((x^2*(b^4 + 16*a^2*c^2 + a*b^2*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
 + (a^2*(8*a*c + b^2))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*b*c*x^3
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (a*b*x*(5*a*c + b^2))/(c*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^
3) - (6*a*b*atan((((3*a*b^2)/(4*a*c - b^2)^(5/2) + (6*a*b*c*x)/(4*a*c - b^
2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(3*a*b)))/(4*a*c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 604, normalized size of antiderivative = 5.64

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} \, dx = \frac{-12\sqrt{4ac-b^2}\, atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) a^3bc-24\sqrt{4ac-b^2}\, atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right) a^2b^2cx-24\sqrt{4ac-b^2}\, atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{2c\left(64a^3c^5x^4-48a^2b^2c^4x^4+12a\,b^4c^3x^4-b^6c^2x^4+\right.}$$

input
```
int(1/(c+a/x^2+b/x)^3/x^3,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c -
24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c*x -
 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2*x*
*2 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c*x
**2 - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*
*2*x**3 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c
**3*x**4 - 20*a**4*c**2 + a**3*b**2*c - 16*a**3*b*c**2*x - 40*a**3*c**3*x*
*2 + a**2*b**4 - 4*a**2*b**3*c*x + 18*a**2*b**2*c**2*x**2 + 12*a**2*c**4*x
**4 + 2*a*b**5*x - 6*a*b**4*c*x**2 - 3*a*b**2*c**3*x**4 + b**6*x**2)/(2*c*
(64*a**5*c**3 - 48*a**4*b**2*c**2 + 128*a**4*b*c**3*x + 128*a**4*c**4*x**2
 + 12*a**3*b**4*c - 96*a**3*b**3*c**2*x - 32*a**3*b**2*c**3*x**2 + 128*a**
3*b*c**4*x**3 + 64*a**3*c**5*x**4 - a**2*b**6 + 24*a**2*b**5*c*x - 24*a**2
*b**4*c**2*x**2 - 96*a**2*b**3*c**3*x**3 - 48*a**2*b**2*c**4*x**4 - 2*a*b*
*7*x + 10*a*b**6*c*x**2 + 24*a*b**5*c**2*x**3 + 12*a*b**4*c**3*x**4 - b**8
*x**2 - 2*b**7*c*x**3 - b**6*c**2*x**4))
```

3.32 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 115

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4} \, dx = \frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{2(b^2+2ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

output
```
1/2*x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+(3*a*b+(2*a*c+b^2)*x)/(-4*a*c
+b^2)^2/(c*x^2+b*x+a)-2*(2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \frac{1}{2}\left(\frac{b^2x+a(b-2cx)}{c\,(-b^2+4ac)\,(a+x(b+cx))^2} + \frac{(b^2+2ac)\,(b+2cx)}{c\,(b^2-4ac)^2\,(a+x(b+cx))} + \frac{4(b^2+2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^4),x]
```

output
```
((b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + ((b^2 +
2*a*c)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (4*(b^2 + 2*a*
c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1692, 1164, 27, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x^2}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{1164}$$

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int \frac{2(a-bx)}{(cx^2+bx+a)^2}dx}{2\left(b^2-4ac\right)}$$

$\Big\downarrow$ 27

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int \frac{a-bx}{(cx^2+bx+a)^2}dx}{b^2-4ac}$$

$\Big\downarrow$ 1159

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{-\frac{\left(2ac+b^2\right)\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{b^2-4ac}$$

$\Big\downarrow$ 1083

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\frac{2\left(2ac+b^2\right)\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{b^2-4ac}$$

$\Big\downarrow$ 219

$$\frac{x(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\frac{2\left(2ac+b^2\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{x\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}}{b^2-4ac}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^4),x]
```

output
```
(x*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (-((3*a*b + (b^2 +
 2*a*c)*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (2*(b^2 + 2*a*c)*ArcTanh[(
b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.83

method	result
default	$\frac{\frac{c\left(2ac+b^2\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{3b\left(2ac+b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(2ac-5b^2\right)x}{16a^2c^2-8a\,b^2c+b^4}+\frac{3a^2b}{16a^2c^2-8a\,b^2c+b^4}}{\left(c\,x^2+bx+a\right)^2}+\frac{2(2ac+b^2)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2-8a\,b^2c+b^4)\sqrt{4ac-b^2}}$
risch	$\frac{\frac{c\left(2ac+b^2\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{3b\left(2ac+b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(2ac-5b^2\right)x}{16a^2c^2-8a\,b^2c+b^4}+\frac{3a^2b}{16a^2c^2-8a\,b^2c+b^4}}{\left(c\,x^2+bx+a\right)^2}-\frac{2\ln\Big((32a^2c^3-16a\,b^2c^2+2b^4c)x+(-4ac+b^2)}{(-4ac+b^2)^{\frac{5}{2}}}$

input
```
int(1/(c+a/x^2+b/x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
(c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(2*a*c+b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^2-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+3*a^2*b/
(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+2*(2*a*c+b^2)/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(109) = 218$.

Time $= 0.10$ (sec) , antiderivative size $= 887$, normalized size of antiderivative $= 7.71$

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\, dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="fricas")
```

output
```
[1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3
*(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^
2 + 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^
2 + 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x +
b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b
^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 1
0*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7
- 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), 1/2*(6*a^2*b^3 - 24*a^
3*b*c + 2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*(b^5 - 2*a*b^3*c - 8*a
^2*b*c^2)*x^2 - 4*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2 + 2*a^3*c + 2*(b^3*c
+ 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 + 2*(a*b^3 + 2*a^2*b*
c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*
x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(107) = 214$.

Time = 0.86 (sec) , antiderivative size = 570, normalized size of antiderivative = 4.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = -\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot (2ac$$
$$+b^2)\log\left(x+\frac{-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)-12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}}{4ac^2+2b^2c}\right.$$
$$+\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac$$
$$+b^2)\log\left(x+\frac{64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)-48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}}{4ac^2+2b^2c}\right.$$
$$+\frac{6a^2b+x^3\cdot(4ac^2+2b^2c)+x^2\cdot(6abc+3b^3)+x(-4a^2c}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(6}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x**4,x)
```

output
```
-sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (-64*a**3*c**3*sqrt(-1/
(4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**
2)**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b*
*2) + 2*a*b*c + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + b**3)/(4*
a*c**2 + 2*b**2*c)) + sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (6
4*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**2*
sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b
**2)**5)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c
 + b**2) + b**3)/(4*a*c**2 + 2*b**2*c)) + (6*a**2*b + x**3*(4*a*c**2 + 2*b
**2*c) + x**2*(6*a*b*c + 3*b**3) + x*(-4*a**2*c + 10*a*b**2))/(32*a**4*c**
2 - 16*a**3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2
*b**4*c**2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64
*a**3*c**3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c +
4*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.34

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \frac{2\,(b^2+2\,ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{2\,b^2cx^3+4\,ac^2x^3+3\,b^3x^2+6\,abcx^2+10\,ab^2x-4\,a^2cx+6\,a^2b}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="giac")
```

output
```
2*(b^2 + 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*b^2*c*x^3 + 4*a*c^2*x^3 + 3*b^3*
x^2 + 6*a*b*c*x^2 + 10*a*b^2*x - 4*a^2*c*x + 6*a^2*b)/((b^4 - 8*a*b^2*c +
16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.72

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \frac{\frac{3\,a^2\,b}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{a\,x\,\left(2\,a\,c-5\,b^2\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{3\,b\,x^2\,\left(b^2+2\,a\,c\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{c\,x^3\,\left(b^2+2\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} + \frac{2\,\mathrm{atan}\left(\frac{\left(\frac{\left(b^2+2\,a\,c\right)\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{2\,c\,x\,\left(b^2+2\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{b^2+2\,a\,c}\right)\,\left(b^2+2\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

input
```
int(1/(x^4*(c + a/x^2 + b/x)^3),x)
```

output
```
((3*a^2*b)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (a*x*(2*a*c - 5*b^2))/(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c) + (3*b*x^2*(2*a*c + b^2))/(2*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (c*x^3*(2*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*
a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (2*atan(((((2*a*c + b^
2)*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) + (2*c*x*(2*a*c + b^2))/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c))/(2*a*c + b^2))*(2*a*c + b^2))/(4*a*c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 820, normalized size of antiderivative = 7.13

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \frac{8\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^3bc+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2b^3+16\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2b^2cx}{}$$

input
```
int(1/(c+a/x^2+b/x)^3/x^4,x)
```

output
```
(8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c + 4*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3 + 16*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c*x + 16*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2*x**2 + 8*s
qrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*x + 16*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c*x**2 + 16*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c**2*x**3 + 8*s
qrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**3*x**4 + 4*s
qrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**5*x**2 + 8*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*c*x**3 + 4*sqrt(4*
a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c**2*x**4 - 8*a**4*c
**2 + 22*a**3*b**2*c - 32*a**3*b*c**2*x - 16*a**3*c**3*x**2 - 5*a**2*b**4
+ 40*a**2*b**3*c*x + 12*a**2*b**2*c**2*x**2 - 8*a**2*c**4*x**4 - 8*a*b**5*
x + 6*a*b**4*c*x**2 - 2*a*b**2*c**3*x**4 - 2*b**6*x**2 + b**4*c**2*x**4)/(
2*b*(64*a**5*c**3 - 48*a**4*b**2*c**2 + 128*a**4*b*c**3*x + 128*a**4*c**4*
x**2 + 12*a**3*b**4*c - 96*a**3*b**3*c**2*x - 32*a**3*b**2*c**3*x**2 + 128
*a**3*b*c**4*x**3 + 64*a**3*c**5*x**4 - a**2*b**6 + 24*a**2*b**5*c*x - 24*
a**2*b**4*c**2*x**2 - 96*a**2*b**3*c**3*x**3 - 48*a**2*b**2*c**4*x**4 - 2*
a*b**7*x + 10*a*b**6*c*x**2 + 24*a*b**5*c**2*x**3 + 12*a*b**4*c**3*x**4 -
b**8*x**2 - 2*b**7*c*x**3 - b**6*c**2*x**4))
```

3.33 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5} dx$

Optimal result

Integrand size = 18, antiderivative size = 103

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5} dx = \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{6bc\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

output
```
1/2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2-3/2*b*(2*c*x+b)/(-4*a*c+b^2)^2/
(c*x^2+b*x+a)+6*b*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/
2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5} dx = \frac{\frac{(b^2-4ac)(2a+bx)}{(a+x(b+cx))^2} - \frac{3b(b+2cx)}{a+x(b+cx)} - \frac{12bc\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2\left(b^2-4ac\right)^2}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^5),x]
```

output
```
(((b^2 - 4*a*c)*(2*a + b*x))/(a + x*(b + c*x))^2 - (3*b*(b + 2*c*x))/(a +
x*(b + c*x)) - (12*b*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
 4*a*c])/(2*(b^2 - 4*a*c)^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1692, 1159, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{3b\int \frac{1}{(cx^2+bx+a)^2}dx}{2(b^2-4ac)}+\frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{3b\left(-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}+\frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3b\left(\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}+\frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3b\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}+\frac{2a+bx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^5),x]
```

output
```
(2*a + b*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*(-((b + 2*c*x)/((
b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

method	result
default	$\frac{-bx-2a}{2(4ac-b^{2})(c\,x^{2}+bx+a)^{2}}-\frac{3b\left(\frac{2cx+b}{\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}\right)}{2(4ac-b^{2})}$
risch	$\frac{-\frac{3b\,c^{2}x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{9b^{2}c\,x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5ac+b^{2}\right)bx}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{a\left(8ac+b^{2}\right)}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{2}+bx+a\right)^{2}}-\frac{3cb\ln\left(\left(32a^{2}c^{3}-16a\,b^{2}c^{2}+2b^{4}c\right)x-(-4\right.}{(-4ac+b^{2}}$

input
```
int(1/(c+a/x^2+b/x)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)^2-3/2*b/(4*a*c-b^2)*((2*c*x+b)/(4
*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(95) = 190.

Time = 0.08 (sec) , antiderivative size = 788, normalized size of antiderivative = 7.65

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="fricas")
```

output
```
[-1/2*(a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*
(b^4*c - 4*a*b^2*c^2)*x^2 - 6*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a
^2*b*c + (b^3*c + 2*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c
*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(
b^5 + a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
- 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
+ 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 -
 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(a*b^4 + 4*a^2*b^2
*c - 32*a^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*(b^4*c - 4*a*b^2*c^2)*x^
2 - 12*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2*b*c + (b^3*c + 2*a*b
*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c +
48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*
x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x
^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(95) = 190$.

Time = 0.65 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.67

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx$$

$$= 3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+3}{6bc^2}\right.$$

$$-3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-3}{6bc^2}\right.$$

$$+\frac{-8a^2c-ab^2-9b^2cx^2-6bc^2x^3+x(-10abc-2b}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(6}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x**5,x)
```

output
```
3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**3*b*c**4*sqrt(-1/(4*a*c
- b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5*c*
*2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b*
*2*c)/(6*b*c**2)) - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**3*b*c
**4*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)
**5) + 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c
 - b**2)**5) + 3*b**2*c)/(6*b*c**2)) + (-8*a**2*c - a*b**2 - 9*b**2*c*x**2
 - 6*b*c**2*x**3 + x*(-10*a*b*c - 2*b**3))/(32*a**4*c**2 - 16*a**3*b**2*c
+ 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*
(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b
**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx = -\frac{6\,bc\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,bc^2x^3+9\,b^2cx^2+2\,b^3x+10\,abcx+ab^2+8\,a^2c}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="giac")
```

output
```
-6*b*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*b*c^2*x^3 + 9*b^2*c*x^2 + 2*b^3*x + 10*a*
b*c*x + a*b^2 + 8*a^2*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)
^2)
```

Mupad [B] (verification not implemented)

Time = 18.94 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx = -\frac{\frac{8\,c\,a^2+a\,b^2}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{9\,b^2\,c\,x^2}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,b\,c^2\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{b\,x\,(b^2+5\,a\,c)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} - \frac{6\,b\,c\,\mathrm{atan}\left(\frac{\left(\frac{3\,b^2\,c}{(4\,a\,c-b^2)^{5/2}}+\frac{6\,b\,c^2\,x}{(4\,a\,c-b^2)^{5/2}}\right)(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}{3\,b\,c}\right)}{(4\,a\,c-b^2)^{5/2}}$$

input
```
int(1/(x^5*(c + a/x^2 + b/x)^3),x)
```

output
```
- ((a*b^2 + 8*a^2*c)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^2*c*x^2)/(2
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b
^2*c) + (b*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c +
b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (6*b*c*atan((((3*b^2*c)/(4*a
*c - b^2)^(5/2) + (6*b*c^2*x)/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c))/(3*b*c)))/(4*a*c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 575, normalized size of antiderivative = 5.58

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx$$
$$= \frac{-12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2bc-24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2cx-24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)c}{128a^3c^5x^4-96a^2b^2c^4x^4+24a\,b^4c^3x^4-2b^6c^2x^4+256a^3b\,c^4x^3}$$

input
```
int(1/(c+a/x^2+b/x)^3/x^5,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c -
24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*x - 24
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**2*x**2 - 1
2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c*x**2 - 24
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c**2*x**3 -
12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**3*x**4 - 2
0*a**3*c**2 + a**2*b**2*c - 16*a**2*b*c**2*x + 24*a**2*c**3*x**2 + a*b**4
- 4*a*b**3*c*x - 30*a*b**2*c**2*x**2 + 12*a*c**4*x**4 + 2*b**5*x + 6*b**4*
c*x**2 - 3*b**2*c**3*x**4)/(2*(64*a**5*c**3 - 48*a**4*b**2*c**2 + 128*a**4
*b*c**3*x + 128*a**4*c**4*x**2 + 12*a**3*b**4*c - 96*a**3*b**3*c**2*x - 32
*a**3*b**2*c**3*x**2 + 128*a**3*b*c**4*x**3 + 64*a**3*c**5*x**4 - a**2*b**
6 + 24*a**2*b**5*c*x - 24*a**2*b**4*c**2*x**2 - 96*a**2*b**3*c**3*x**3 - 4
8*a**2*b**2*c**4*x**4 - 2*a*b**7*x + 10*a*b**6*c*x**2 + 24*a*b**5*c**2*x**
3 + 12*a*b**4*c**3*x**4 - b**8*x**2 - 2*b**7*c*x**3 - b**6*c**2*x**4))
```

3.34 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6} dx$

Optimal result

Integrand size = 18, antiderivative size = 103

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6} dx = \frac{-b-2cx}{2(b^2-4ac)(a+bx+cx^2)^2} + \frac{3c(b+2cx)}{(b^2-4ac)^2(a+bx+cx^2)} - \frac{12c^2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
1/2*(-2*c*x-b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+3*c*(2*c*x+b)/(-4*a*c+b^2)^2/(
c*x^2+b*x+a)-12*c^2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/
2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6} dx = \frac{-\frac{(b+2cx)(b^2-6bcx-2c(5a+3cx^2))}{(a+x(b+cx))^2} + \frac{24c^2 \arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2-4ac)^2}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^6),x]
```

output
```
(-(((b + 2*c*x)*(b^2 - 6*b*c*x - 2*c*(5*a + 3*c*x^2)))/(a + x*(b + c*x))^2
) + (24*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2
*(b^2 - 4*a*c)^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1692, 1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{1086}$$

$$-\frac{3c\int \frac{1}{(cx^2+bx+a)^2}dx}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1086}$$

$$-\frac{3c\left(-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{3c\left(\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^6),x]
```

output
```
-1/2*(b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*c*(-((b + 2*c*x)
/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

method	result
default	$\frac{2cx+b}{2(4ac-b^2)(c\,x^2+bx+a)^2}+\frac{3c\left(\frac{2cx+b}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}\right)}{4ac-b^2}$
risch	$\frac{\frac{6c^3x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{9b\,c^2x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2\left(5ac+b^2\right)cx}{16a^2c^2-8a\,b^2c+b^4}+\frac{b\left(10ac-b^2\right)}{32a^2c^2-16a\,b^2c+2b^4}}{(c\,x^2+bx+a)^2}-\frac{6c^2\ln\left((32a^2c^3-16a\,b^2c^2+2b^4c)x+(-4ac+b^2\right.}{(-4ac+b^2)^{\frac{5}{2}}}$

input
```
int(1/(c+a/x^2+b/x)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+3*c/(4*a*c-b^2)*((2*c*x+b)/(4*a*
c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(95) = 190$.

Time = 0.09 (sec) , antiderivative size = 785, normalized size of antiderivative = 7.62

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="fricas")
```

output
```
[-1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(
b^3*c^2 - 4*a*b*c^3)*x^2 - 12*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c
^2 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x +
 b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 4*(b^4*
c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 +
2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a
*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(b^5 - 14*a*b^3*c +
40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b^3*c^2 - 4*a*b*c^3)*x^2 +
 24*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^2 + 2*a*c^3)*x
^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) - 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3
*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 +
 (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 +
2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(95) = 190$.

Time = 0.71 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.60

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx =$$

$$-6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+}{12c^3}\right.$$

$$+6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-6}{12c^3}\right.$$

$$+\frac{10abc-b^3+18bc^2x^2+12c^3x^3+x(20ac^2+4b^2c}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(6}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x**6,x)
```

output
```
-6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**3*c**5*sqrt(-1/(4*a*c
- b**2)**5) + 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 72*a*b**4*c*
*3*sqrt(-1/(4*a*c - b**2)**5) + 6*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 6
*b*c**2)/(12*c**3)) + 6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**3*
c**5*sqrt(-1/(4*a*c - b**2)**5) - 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2
)**5) + 72*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 6*b**6*c**2*sqrt(-1/(4
*a*c - b**2)**5) + 6*b*c**2)/(12*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**
2 + 12*c**3*x**3 + x*(20*a*c**2 + 4*b**2*c))/(32*a**4*c**2 - 16*a**3*b**2*
c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**
3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a
*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\, dx = \frac{12\, c^2 \arctan\left(\frac{2\, cx+b}{\sqrt{-b^2+4\, ac}}\right)}{(b^4-8\, ab^2c+16\, a^2c^2)\sqrt{-b^2+4\, ac}} + \frac{12\, c^3x^3+18\, bc^2x^2+4\, b^2cx+20\, ac^2x-b^3+10\, abc}{2\,(b^4-8\, ab^2c+16\, a^2c^2)(cx^2+bx+a)^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="giac")
```

output
```
12*c^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*x^3 + 18*b*c^2*x^2 + 4*b^2*c*x + 20*
a*c^2*x - b^3 + 10*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a
)^2)
```

Mupad [B] (verification not implemented)

Time = 19.17 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.77

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx = \frac{\frac{6\,c^3\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{b^3-10\,a\,b\,c}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{9\,b\,c^2\,x^2}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{2\,c\,x\,(b^2+5\,a\,c)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} + \frac{12\,c^2\,\mathrm{atan}\left(\frac{\left(\frac{12\,c^3\,x}{(4\,a\,c-b^2)^{5/2}}+\frac{6\,c^2\,(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5)}{(4\,a\,c-b^2)^{5/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}\right)(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}{6\,c^2}\right)}{(4\,a\,c-b^2)^{5/2}}$$

input
```
int(1/(x^6*(c + a/x^2 + b/x)^3),x)
```

output
```
((6*c^3*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3 - 10*a*b*c)/(2*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*x^2)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (
2*c*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2) +
a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (12*c^2*atan((((12*c^3*x)/(4*a*c -
b^2)^(5/2) + (6*c^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2))
)/(4*a*c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 598, normalized size of antiderivative = 5.81

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx$$
$$= \frac{24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2b\,c^2+48\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2c^2x+48\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{2b\left(64a^3c^5x^4-48a^2b^2c^4x^4+12a\,b^4c^3x^4-b^6c^2x^4+12\right.}$$

input
```
int(1/(c+a/x^2+b/x)^3/x^6,x)
```

output
```
(24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2 +
48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c**2*x +
 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**3*x**2
+ 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c**2*x**
2 + 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c**3*x
**3 + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**4*x*
*4 - 24*a**3*c**3 + 46*a**2*b**2*c**2 + 32*a**2*b*c**3*x - 48*a**2*c**4*x*
*2 - 14*a*b**4*c + 8*a*b**3*c**2*x + 60*a*b**2*c**3*x**2 - 24*a*c**5*x**4
+ b**6 - 4*b**5*c*x - 12*b**4*c**2*x**2 + 6*b**2*c**4*x**4)/(2*b*(64*a**5*
c**3 - 48*a**4*b**2*c**2 + 128*a**4*b*c**3*x + 128*a**4*c**4*x**2 + 12*a**
3*b**4*c - 96*a**3*b**3*c**2*x - 32*a**3*b**2*c**3*x**2 + 128*a**3*b*c**4*
x**3 + 64*a**3*c**5*x**4 - a**2*b**6 + 24*a**2*b**5*c*x - 24*a**2*b**4*c**
2*x**2 - 96*a**2*b**3*c**3*x**3 - 48*a**2*b**2*c**4*x**4 - 2*a*b**7*x + 10
*a*b**6*c*x**2 + 24*a*b**5*c**2*x**3 + 12*a*b**4*c**3*x**4 - b**8*x**2 - 2
*b**7*c*x**3 - b**6*c**2*x**4))
```

3.35 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 185

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7} \, dx = \frac{b^2-2ac+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)x}{2a^2(b^2-4ac)^2(a+bx+cx^2)} + \frac{b(b^4-10ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{5/2}} + \frac{\log(x)}{a^3} - \frac{\log(a+bx+cx^2)}{2a^3}$$

output
```
1/2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+1/2*(2*b^4-15*a*b^2*c
+16*c^2*a^2+2*b*c*(-7*a*c+b^2)*x)/a^2/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+b*(30*a
^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b
^2)^(5/2)+ln(x)/a^3-1/2*ln(c*x^2+b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx$$
$$=\frac{\frac{a^2(b^2-2ac+bcx)}{(b^2-4ac)(a+x(b+cx))^2}+\frac{a(2b^4-15ab^2c+16a^2c^2+2b^3cx-14abc^2x)}{(b^2-4ac)^2(a+x(b+cx))}-\frac{2b(b^4-10ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}+2\log(x)-\log}{2a^3}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^7),x]
```

output
```
((a^2*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (a*(2*b
^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x - 14*a*b*c^2*x))/((b^2 - 4*a*c)^2
*(a + x*(b + c*x))) - (2*b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c
*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + 2*Log[x] - Log[a + x*(b +
c*x)])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1692, 1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{x(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{1165}$$

$$\frac{-2ac+b^2+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2}-\frac{\int -\frac{2(b^2-4ac)+3bcx}{x(cx^2+bx+a)^2}dx}{2a(b^2-4ac)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2(b^2-4ac)+3bcx}{x(cx^2+bx+a)^2}dx}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \; 1235$$

$$\frac{\frac{16a^2c^2+2bcx(b^2-7ac)-15ab^2c+2b^4}{a(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{2\left(\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)x\right)}{x\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\int \frac{\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)x}{x\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}+\frac{16a^2c^2+2bcx(b^2-7ac)-15ab^2c+2b^4}{a(b^2-4ac)(a+bx+cx^2)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \; 1200$$

$$\frac{\frac{2\int \left(\frac{\left(4ac-b^2\right)^2}{ax}+\frac{-cx\left(b^2-4ac\right)^2-b\left(b^4-9acb^2+23a^2c^2\right)}{a\left(cx^2+bx+a\right)}\right)dx}{a(b^2-4ac)}+\frac{16a^2c^2+2bcx(b^2-7ac)-15ab^2c+2b^4}{a(b^2-4ac)(a+bx+cx^2)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{2\left(\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)\left(b^2-4ac\right)^2}{a}\right)}{a(b^2-4ac)}+\frac{16a^2c^2+2bcx(b^2-7ac)-15ab^2c+2b^4}{a(b^2-4ac)(a+bx+cx^2)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{2a(b^2-4ac)(a+bx+cx^2)^2}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^7),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + ((2*b^4 -
15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x)/(a*(b^2 - 4*a*c)*(a + b*x
 + c*x^2)) + (2*((b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)^2*Log[x])/a - ((b
^2 - 4*a*c)^2*Log[a + b*x + c*x^2])/(2*a)))/(a*(b^2 - 4*a*c)))/(2*a*(b^2 -
 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(175) = 350$.

Time = 0.08 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.90

method	result
default	$-\frac{\frac{ab\,c^2\left(7ac-b^2\right)x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{ac\left(16a^2c^2-29a\,b^2c+4b^4\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{ab\left(a^2c^2+6a\,b^2c-b^4\right)x}{16a^2c^2-8a\,b^2c+b^4}-\frac{3a^2\left(8a^2c^2-7a\,b^2c+b^4\right)}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}+\frac{\left(16a^2c^3-8a\,b^2c^2+b^4c\right)\ln\left(c\,x^2+bx\right.}{2c}\ldots}{a^3}$
risch	$\frac{-\frac{b\,c^2\left(7ac-b^2\right)x^3}{a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{c\left(16a^2c^2-29a\,b^2c+4b^4\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)a^2}-\frac{b\left(a^2c^2+6a\,b^2c-b^4\right)x}{a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{12a^2c^2-\frac{21}{2}a\,b^2c+\frac{3}{2}b^4}{a\left(16a^2c^2-8a\,b^2c+b^4\right)}}{(c\,x^2+bx+a)^2}+\frac{\ln(x)}{a^3}+\Bigg(\sum_{\_R=\operatorname{RootOf}(\ldots}$

input
```
int(1/(c+a/x^2+b/x)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*((a*b*c^2*(7*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*a*c*(16*a^
2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+a*b*(a^2*c^2+6*a*b^
2*c-b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-3/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/
2*(16*a^2*c^3-8*a*b^2*c^2+b^4*c)/c*ln(c*x^2+b*x+a)+2*(23*a^2*b*c^2-9*a*b^3
*c+b^5-1/2*(16*a^2*c^3-8*a*b^2*c^2+b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))))+ln(x)/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(175) = 350$.

Time = 0.34 (sec) , antiderivative size = 1985, normalized size of antiderivative = 10.73

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="fricas")
```

output
```
[1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c
^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 13
2*a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 +
 (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 3
0*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2
 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c
^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b
*x + a)) + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b
^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*x)*log(c*x^2 + b*x + a) + 2*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5
)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^
8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a
*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(x))/(a^5*b^6 -
 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^
3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^4 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*
a^5*b^3*c^3 - 64*a^6*b*c^4)*x^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x**7,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx$$
$$= -\frac{(b^5-10\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^4-8\,a^4b^2c+16\,a^5c^2)\sqrt{-b^2+4\,ac}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a^3} + \frac{\log\left(|x|\right)}{a^3}$$
$$+ \frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\,(ab^3c^2-7\,a^2bc^3)x^3+(4\,ab^4c-29\,a^2b^2c^2+16\,a^3c^3)x^2+2\,(ab^5-6\,a^2b^3}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2a^3}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="giac")
```

output
```
-(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2
+ b*x + a)/a^3 + log(abs(x))/a^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*
c^2 + 2*(a*b^3*c^2 - 7*a^2*b*c^3)*x^3 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a
^3*c^3)*x^2 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2)*x)/((c*x^2 + b*x + a)^2*
(b^2 - 4*a*c)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 20.33 (sec) , antiderivative size = 1089, normalized size of antiderivative = 5.89

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7} \, dx = \text{Too large to display}$$

input
```
int(1/(x^7*(c + a/x^2 + b/x)^3),x)
```

output
```
log(x)/a^3 + ((3*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*a*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) + (x^2*(4*b^4*c + 16*a^2*c^3 - 29*a*b^2*c^2))/(2*a^2*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) - (b*x*(a^2*c^2 - b^4 + 6*a*b^2*c))/(a^2*(b^4 + 16*a
^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^3*(7*a*c - b^2))/(a^2*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (
log(1536*a^6*c^5 - 2*b^11*x - 2*a*b^10 + 2*a*b^5*(-(4*a*c - b^2)^5)^(1/2)
+ 39*a^2*b^8*c + 2*b^6*x*(-(4*a*c - b^2)^5)^(1/2) - 303*a^3*b^6*c^2 + 1160
*a^4*b^4*c^3 - 2160*a^5*b^2*c^4 - 17*a^2*b^3*c*(-(4*a*c - b^2)^5)^(1/2) +
39*a^3*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 321*a^2*b^7*c^2*x + 1286*a^3*b^5*c
^3*x - 2560*a^4*b^3*c^4*x - 48*a^3*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b
^9*c*x + 2016*a^5*b*c^5*x - 20*a*b^4*c*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2
*b^2*c^2*x*(-(4*a*c - b^2)^5)^(1/2))*(1024*a^5*c^5 - b^10 + b^5*(-(4*a*c -
 b^2)^5)^(1/2) - 160*a^2*b^6*c^2 + 640*a^3*b^4*c^3 - 1280*a^4*b^2*c^4 + 20
*a*b^8*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a*b^3*c*(-(4*a*c - b
^2)^5)^(1/2)))/(2*a^3*(4*a*c - b^2)^5) + (log(2*a*b^10 + 2*b^11*x - 1536*a
^6*c^5 + 2*a*b^5*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^8*c + 2*b^6*x*(-(4*a*
c - b^2)^5)^(1/2) + 303*a^3*b^6*c^2 - 1160*a^4*b^4*c^3 + 2160*a^5*b^2*c^4
- 17*a^2*b^3*c*(-(4*a*c - b^2)^5)^(1/2) + 39*a^3*b*c^2*(-(4*a*c - b^2)^5)^
(1/2) + 321*a^2*b^7*c^2*x - 1286*a^3*b^5*c^3*x + 2560*a^4*b^3*c^4*x - 48*a
^3*c^3*x*(-(4*a*c - b^2)^5)^(1/2) - 40*a*b^9*c*x - 2016*a^5*b*c^5*x - 2...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1801, normalized size of antiderivative = 9.74

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7} \, dx = \text{Too large to display}$$

input
```
int(1/(c+a/x^2+b/x)^3/x^7,x)
```

output
```
( - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c**2
 + 20*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**3*c
- 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c*
*2*x - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*
c**3*x**2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2
*b**5 + 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*
*4*c*x - 20*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b
**3*c**2*x**2 - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a**2*b**2*c**3*x**3 - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a**2*b*c**4*x**4 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a
*c - b**2))*a*b**6*x + 16*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
 b**2))*a*b**5*c*x**2 + 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a*b**4*c**2*x**3 + 20*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a*b**3*c**3*x**4 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt
(4*a*c - b**2))*b**7*x**2 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a
*c - b**2))*b**6*c*x**3 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
 - b**2))*b**5*c**2*x**4 - 64*log(a + b*x + c*x**2)*a**5*c**3 + 48*log(a +
 b*x + c*x**2)*a**4*b**2*c**2 - 128*log(a + b*x + c*x**2)*a**4*b*c**3*x -
128*log(a + b*x + c*x**2)*a**4*c**4*x**2 - 12*log(a + b*x + c*x**2)*a**3*b
**4*c + 96*log(a + b*x + c*x**2)*a**3*b**3*c**2*x + 32*log(a + b*x + c*...
```

3.36 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 239

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8} \, dx = & -\frac{3(b^2-5ac)(b^2-2ac)}{a^3(b^2-4ac)^2 x}+\frac{b^2-2ac+bcx}{2a(b^2-4ac)x(a+bx+cx^2)^2} \\ & +\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)x}{2a^2(b^2-4ac)^2 x(a+bx+cx^2)} \\ & -\frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{5/2}} \\ & -\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx+cx^2)}{2a^4}\end{aligned}$$

output
```
-3*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/x+1/2*(b*c*x-2*a*c+b^2)/a/
(-4*a*c+b^2)/x/(c*x^2+b*x+a)^2+1/2*(3*b^4-20*a*b^2*c+20*c^2*a^2+3*b*c*(-6*
a*c+b^2)*x)/a^2/(-4*a*c+b^2)^2/x/(c*x^2+b*x+a)-3*(-20*a^3*c^3+30*a^2*b^2*c
^2-10*a*b^4*c+b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^
(5/2)-3*b*ln(x)/a^4+3/2*b*ln(c*x^2+b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx$$
$$= \frac{-\frac{2a}{x}+\frac{a^2(b^3-3abc+b^2cx-2ac^2x)}{(-b^2+4ac)(a+x(b+cx))^2}-\frac{a(4b^5-29ab^3c+46a^2bc^2+4b^4cx-26ab^2c^2x+28a^2c^3x)}{(b^2-4ac)^2(a+x(b+cx))}+\frac{6(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\arctan\left(\cdots\right)}{(-b^2+4ac)^{5/2}}\cdots}{2a^4}$$

input
```
Integrate[1/((c + a/x^2 + b/x)^3*x^8),x]
```

output
```
((-2*a)/x + (a^2*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((-b^2 + 4*a*c)*(a
 + x*(b + c*x))^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x - 2
6*a*b^2*c^2*x + 28*a^2*c^3*x))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (6*(b
^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^
2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) - 6*b*Log[x] + 3*b*Log[a + x*(b + c*x)])
/(2*a^4)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1692, 1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^3}dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{1}{x^2\left(a+bx+cx^2\right)^3}dx$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int -\frac{3b^2+4cxb-10ac}{x^2\left(cx^2+bx+a\right)^2}dx}{2a\left(b^2-4ac\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{3b^2+4cxb-10ac}{x^2\left(cx^2+bx+a\right)^2}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 1235

$$\frac{\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{ax(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{6\left(\left(b^2-5ac\right)\left(b^2-2ac\right)+bc\left(b^2-6ac\right)x\right)}{x^2\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{6\int \frac{\left(b^2-5ac\right)\left(b^2-2ac\right)+bc\left(b^2-6ac\right)x}{x^2\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}+\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{ax(b^2-4ac)(a+bx+cx^2)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 1200

$$\frac{\frac{6\int \left(-\frac{b\left(4ac-b^2\right)^2}{a^2x}+\frac{b^6-9acb^4+23a^2c^2b^2+c\left(b^2-4ac\right)^2xb-10a^3c^3}{a^2\left(cx^2+bx+a\right)}+\frac{\left(b^2-5ac\right)\left(b^2-2ac\right)}{ax^2}\right)dx}{a(b^2-4ac)}+\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{ax(b^2-4ac)(a+bx+cx^2)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{20a^2c^2+3bcx\left(b^2-6ac\right)-20ab^2c+3b^4}{ax(b^2-4ac)(a+bx+cx^2)}+\frac{6\left(\frac{b\left(b^2-4ac\right)^2\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)\left(b^2-4ac\right)^2}{a^2}-\frac{\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}\right.}{a(b^2-4ac)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{2ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

input
```
Int[1/((c + a/x^2 + b/x)^3*x^8),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + ((3*b^4
- 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(a*(b^2 - 4*a*c)*x*(a +
 b*x + c*x^2)) + (6*(-(((b^2 - 5*a*c)*(b^2 - 2*a*c))/(a*x)) - ((b^6 - 10*a
*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)^2*Log[x])/a^2 + (b*(b^2 - 4
*a*c)^2*Log[a + b*x + c*x^2])/(2*a^2)))/(a*(b^2 - 4*a*c)))/(2*a*(b^2 - 4*a
*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.69

method	result
default	$-\frac{\frac{\frac{a c^2\left(14a^2c^2-13a b^2c+2b^4\right)x^3}{16a^2c^2-8a b^2c+b^4}+\frac{abc\left(74a^2c^2-55a b^2c+8b^4\right)x^2}{32a^2c^2-16a b^2c+2b^4}+\frac{a\left(18a^3c^3+7a^2b^2c^2-12a b^4c+2b^6\right)x}{16a^2c^2-8a b^2c+b^4}+\frac{a^2b\left(58a^2c^2-36a b^2c+5b^4\right)}{32a^2c^2-16a b^2c+2b^4}}{\left(c x^2+bx+a\right)^2}+}{a^4}$ 3(−16
risch	$\frac{-\frac{3c^2\left(10a^2c^2-7a b^2c+b^4\right)x^4}{a^3\left(16a^2c^2-8a b^2c+b^4\right)}-\frac{3bc\left(46a^2c^2-29a b^2c+4b^4\right)x^3}{2\left(16a^2c^2-8a b^2c+b^4\right)a^3}-\frac{\left(50a^3c^3+7a^2b^2c^2-18a b^4c+3b^6\right)x^2}{a^3\left(16a^2c^2-8a b^2c+b^4\right)}-\frac{b\left(122a^2c^2-68a b^2c+9b^4\right)x}{2a^2\left(16a^2c^2-8a b^2c+b^4\right)}-\frac{1}{a}}{x(c x^2+bx+a)^2}-$ 3b

input
```
int(1/(c+a/x^2+b/x)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/a^4*((a*c^2*(14*a^2*c^2-13*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
3+1/2*a*b*c*(74*a^2*c^2-55*a*b^2*c+8*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+a
*(18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+
1/2*a^2*b*(58*a^2*c^2-36*a*b^2*c+5*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2
+b*x+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(-16*a^2*b*c^3+8*a*b^3*c^2-b^5
*c)/c*ln(c*x^2+b*x+a)+2*(10*a^3*c^3-23*a^2*b^2*c^2+9*a*b^4*c-b^6-1/2*(-16*
a^2*b*c^3+8*a*b^3*c^2-b^5*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))))-1/a^3/x-3*b*ln(x)/a^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. $2(229) = 458$.

Time = 0.46 (sec) , antiderivative size = 2280, normalized size of antiderivative = 9.54

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="fricas")
```

output
```
[-1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*
c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a*b^7*c - 4
5*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8 - 30*a^2
*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 3*((b^6*c^2
- 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a*b^5*c^
2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2
 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3
*c^2 - 20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a
^5*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt
(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (9*a^2*b^7 - 104*a^3*b^5*c
 + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^
2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 -
 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3
 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*
b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)
*log(c*x^2 + b*x + a) + 6*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a
^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)
*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4
)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (
a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(x))/((a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx=\text{Timed out}$$

input
```
integrate(1/(c+a/x**2+b/x)**3/x**8,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \frac{3\,(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^4b^4-8\,a^5b^2c+16\,a^6c^2)\sqrt{-b^2+4\,ac}} + \frac{3\,b\log\left(cx^2+bx+a\right)}{2\,a^4} - \frac{3\,b\log\left(|x|\right)}{a^4} - \frac{2\,a^3b^4-16\,a^4b^2c+32\,a^5c^2+6\,(ab^4c^2-7\,a^2b^2c^3+10\,a^3c^4)x^4+3\,(4\,ab^5c-29\,a^2b^3c^2+46\,a^3bc^3)x^3+2}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2}$$

input
```
integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="giac")
```

output
```
3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c))
+ 3/2*b*log(c*x^2 + b*x + a)/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(2*a^3*b^4 -
16*a^4*b^2*c + 32*a^5*c^2 + 6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*x^4
 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*x^3 + 2*(3*a*b^6 - 18*a^2
*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122
*a^4*b*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*a^4*x)
```

Mupad [B] (verification not implemented)

Time = 20.48 (sec) , antiderivative size = 1255, normalized size of antiderivative = 5.25

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \text{Too large to display}$$

input
```
int(1/(x^8*(c + a/x^2 + b/x)^3),x)
```

output
```
- (1/a + (x^2*(3*b^6 + 50*a^3*c^3 + 7*a^2*b^2*c^2 - 18*a*b^4*c))/(a^3*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(9*b^5 + 122*a^2*b*c^2 - 68*a*b^3*c))/(2*
a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^3*(4*b^5*c - 29*a*b^3*c^2 + 46*
a^2*b*c^3))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^2*x^4*(b^4 + 10*
a^2*c^2 - 7*a*b^2*c))/(a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^3*(2*a*c +
b^2) + a^2*x + c^2*x^5 + 2*a*b*x^2 + 2*b*c*x^4) - (3*b*log(x))/a^4 - (3*lo
g(2*a*b^11 + 2*b^12*x + 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c -
1696*a^6*b*c^5 + 320*a^6*c^6*x + 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a^
3*b^7*c^2 - 1170*a^4*b^5*c^3 + 2240*a^5*b^3*c^4 - 10*a^4*c^3*(-(4*a*c - b^
2)^5)^(1/2) - 17*a^2*b^4*c*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^8*c^2*x -
1296*a^3*b^6*c^3*x + 2660*a^4*b^4*c^4*x - 2336*a^5*b^2*c^5*x - 40*a*b^10*c
*x + 39*a^3*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 20*a*b^5*c*x*(-(4*a*c - b^2
)^5)^(1/2) - 58*a^3*b*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2*b^3*c^2*x*(-
(4*a*c - b^2)^5)^(1/2))*(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 1024*a^5*b*
c^5 + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 20*a^3*c^3*(-
(4*a*c - b^2)^5)^(1/2) - 20*a*b^9*c + 30*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1
/2) - 10*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2)))/(2*a^4*(4*a*c - b^2)^5) - (3*l
og(2*a*b^11 + 2*b^12*x - 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c -
 1696*a^6*b*c^5 + 320*a^6*c^6*x - 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a
^3*b^7*c^2 - 1170*a^4*b^5*c^3 + 2240*a^5*b^3*c^4 + 10*a^4*c^3*(-(4*a*c ...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2201, normalized size of antiderivative = 9.21

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \text{Too large to display}$$

input
```
int(1/(c+a/x^2+b/x)^3/x^8,x)
```

output

```
( - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**5*b*c**
3*x + 180*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b**
3*c**2*x - 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**
4*b**2*c**3*x**2 - 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*a**4*b*c**4*x**3 - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a**3*b**5*c*x + 360*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*
c - b**2))*a**3*b**4*c**2*x**2 + 240*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/s
qrt(4*a*c - b**2))*a**3*b**3*c**3*x**3 - 240*sqrt(4*a*c - b**2)*atan((b +
2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c**4*x**4 - 120*sqrt(4*a*c - b**2)*at
an((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**5*x**5 + 6*sqrt(4*a*c - b**2)
*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**7*x - 120*sqrt(4*a*c - b**2)
*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**6*c*x**2 + 60*sqrt(4*a*c - b
**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**5*c**2*x**3 + 360*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**4*c**3*x**4 + 18
0*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c**4*x
**5 + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**8*x*
*2 - 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**7*c*x
**3 - 120*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**6*c
**2*x**4 - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*
*5*c**3*x**5 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)...
```

3.37 $\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 40

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx = \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{16}{567}\log(2+3x) + \frac{\log(1+5x)}{4375}$$

output
```
139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx = \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{16}{567}\log(2+3x) + \frac{\log(1+5x)}{4375}$$

input
```
Integrate[x^2/(15 + 2/x^2 + 13/x),x]
```

output
```
(139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x
]/4375
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\frac{2}{x^2}+\frac{13}{x}+15} \, dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{x^4}{15x^2+13x+2}dx$$

$$\Big\downarrow \text{ 1141}$$

$$15\int \left(\frac{x^2}{225}-\frac{13x}{3375}-\frac{16}{2835(3x+2)}+\frac{1}{13125(5x+1)}+\frac{139}{50625}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$15\left(\frac{x^3}{675}-\frac{13x^2}{6750}+\frac{139x}{50625}-\frac{16\log(3x+2)}{8505}+\frac{\log(5x+1)}{65625}\right)$$

input
```
Int[x^2/(15 + 2/x^2 + 13/x),x]
```

output
```
15*((139*x)/50625 - (13*x^2)/6750 + x^3/675 - (16*Log[2 + 3*x])/8505 + Log
[1 + 5*x]/65625)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}+\frac{\ln\left(x+\frac{1}{5}\right)}{4375}-\frac{16\ln\left(x+\frac{2}{3}\right)}{567}$	27
default	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(2+3x)}{567}+\frac{\ln(1+5x)}{4375}$	31
norman	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(2+3x)}{567}+\frac{\ln(1+5x)}{4375}$	31
risch	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(2+3x)}{567}+\frac{\ln(1+5x)}{4375}$	31

input
```
int(x^2/(15+2/x^2+13/x),x,method=_RETURNVERBOSE)
```

output
```
1/45*x^3-13/450*x^2+139/3375*x+1/4375*ln(x+1/5)-16/567*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^2/(15+2/x^2+13/x),x, algorithm="fricas")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x
+ 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x+\frac{1}{5}\right)}{4375} - \frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

input
```
integrate(x**2/(15+2/x**2+13/x),x)
```

output
```
x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/5
67
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log\left(5\,x+1\right) - \frac{16}{567}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^2/(15+2/x^2+13/x),x, algorithm="maxima")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x
+ 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log\left(|5\,x+1|\right) - \frac{16}{567}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(x^2/(15+2/x^2+13/x),x, algorithm="giac")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log
(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{139\,x}{3375}-\frac{16\,\ln\left(x+\frac{2}{3}\right)}{567}+\frac{\ln\left(x+\frac{1}{5}\right)}{4375}-\frac{13\,x^2}{450}+\frac{x^3}{45}$$

input
```
int(x^2/(13/x + 2/x^2 + 15),x)
```

output
```
(139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 +
x^3/45
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{\log(5x+1)}{4375}-\frac{16\log(3x+2)}{567}+\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}$$

input
```
int(x^2/(15+2/x^2+13/x),x)
```

output
```
(162*log(5*x + 1) - 20000*log(3*x + 2) + 15750*x**3 - 20475*x**2 + 29190*x
)/708750
```

3.38 $\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=-\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

output
```
-13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=-\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

input
```
Integrate[x/(15 + 2/x^2 + 13/x),x]
```

output
```
(-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\frac{2}{x^2}+\frac{13}{x}+15}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x^3}{15x^2+13x+2}dx$$

$$\Big\downarrow \text{1141}$$

$$15\int \left(\frac{x}{225}+\frac{8}{945(3x+2)}-\frac{1}{2625(5x+1)}-\frac{13}{3375}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(\frac{x^2}{450}-\frac{13x}{3375}+\frac{8\log(3x+2)}{2835}-\frac{\log(5x+1)}{13125}\right)$$

input
```
Int[x/(15 + 2/x^2 + 13/x),x]
```

output
```
15*((-13*x)/3375 + x^2/450 + (8*Log[2 + 3*x])/2835 - Log[1 + 5*x]/13125)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{x^2}{30}-\frac{13x}{225}-\frac{\ln\left(x+\frac{1}{5}\right)}{875}+\frac{8\ln\left(x+\frac{2}{3}\right)}{189}$	22
default	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(2+3x)}{189}-\frac{\ln(1+5x)}{875}$	26
norman	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(2+3x)}{189}-\frac{\ln(1+5x)}{875}$	26
risch	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(2+3x)}{189}-\frac{\ln(1+5x)}{875}$	26

input
```
int(x/(15+2/x^2+13/x),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^2-13/225*x-1/875*ln(x+1/5)+8/189*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

input
```
integrate(x/(15+2/x^2+13/x),x, algorithm="fricas")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{x^2}{30} - \frac{13x}{225} - \frac{\log\left(x+\frac{1}{5}\right)}{875} + \frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

input
```
integrate(x/(15+2/x**2+13/x),x)
```

output
```
x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(5\,x+1\right) + \frac{8}{189}\,\log\left(3\,x+2\right)$$

input
```
integrate(x/(15+2/x^2+13/x),x, algorithm="maxima")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(|5\,x+1|\right) + \frac{8}{189}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(x/(15+2/x^2+13/x),x, algorithm="giac")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{8\,\ln\left(x+\frac{2}{3}\right)}{189}-\frac{13\,x}{225}-\frac{\ln\left(x+\frac{1}{5}\right)}{875}+\frac{x^2}{30}$$

input
```
int(x/(13/x + 2/x^2 + 15),x)
```

output
```
(8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=-\frac{\log(5x+1)}{875}+\frac{8\log(3x+2)}{189}+\frac{x^2}{30}-\frac{13x}{225}$$

input
```
int(x/(15+2/x^2+13/x),x)
```

output
```
( - 54*log(5*x + 1) + 2000*log(3*x + 2) + 1575*x**2 - 2730*x)/47250
```

3.39 $\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}} dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}} dx = \frac{x}{15} - \frac{4}{63}\log(2+3x) + \frac{1}{175}\log(1+5x)$$

output
```
1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}} dx = \frac{x}{15} - \frac{4}{63}\log(2+3x) + \frac{1}{175}\log(1+5x)$$

input
```
Integrate[(15 + 2/x^2 + 13/x)^(-1),x]
```

output
```
x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1679, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{2}{x^2}+\frac{13}{x}+15} \, dx$$
$$\Big\downarrow \text{1679}$$
$$\int \frac{x^2}{15x^2+13x+2}dx$$
$$\Big\downarrow \text{1141}$$
$$15\int \left(\frac{1}{525(5x+1)}+\frac{1}{225}-\frac{4}{315(3x+2)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$15\left(\frac{x}{225}-\frac{4}{945}\log(3x+2)+\frac{\log(5x+1)}{2625}\right)$$

input
```
Int[(15 + 2/x^2 + 13/x)^(-1),x]
```

output
```
15*(x/225 - (4*Log[2 + 3*x])/945 + Log[1 + 5*x]/2625)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{x}{15}+\frac{\ln\left(x+\frac{1}{5}\right)}{175}-\frac{4\ln\left(x+\frac{2}{3}\right)}{63}$	17
default	$\frac{x}{15}-\frac{4\ln(2+3x)}{63}+\frac{\ln(1+5x)}{175}$	21
norman	$\frac{x}{15}-\frac{4\ln(2+3x)}{63}+\frac{\ln(1+5x)}{175}$	21
risch	$\frac{x}{15}-\frac{4\ln(2+3x)}{63}+\frac{\ln(1+5x)}{175}$	21

input
```
int(1/(15+2/x^2+13/x),x,method=_RETURNVERBOSE)
```

output
```
1/15*x+1/175*ln(x+1/5)-4/63*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(15+2/x^2+13/x),x, algorithm="fricas")
```

output
```
1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{x}{15}+\frac{\log\left(x+\frac{1}{5}\right)}{175}-\frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

input
```
integrate(1/(15+2/x**2+13/x),x)
```

output
```
x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(15+2/x^2+13/x),x, algorithm="maxima")
```

output
```
1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx = \frac{1}{15}\,x+\frac{1}{175}\,\log\left(|5\,x+1|\right)-\frac{4}{63}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(1/(15+2/x^2+13/x),x, algorithm="giac")
```

output
```
1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{x}{15}-\frac{4\,\ln\left(x+\frac{2}{3}\right)}{63}+\frac{\ln\left(x+\frac{1}{5}\right)}{175}$$

input
```
int(1/(13/x + 2/x^2 + 15),x)
```

output
```
x/15 - (4*log(x + 2/3))/63 + log(x + 1/5)/175
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx=\frac{\log(5x+1)}{175}-\frac{4\log(3x+2)}{63}+\frac{x}{15}$$

input
```
int(1/(15+2/x^2+13/x),x)
```

output
```
(9*log(5*x + 1) - 100*log(3*x + 2) + 105*x)/1575
```

3.40 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx=\frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

output
```
2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx=\frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

input
```
Integrate[1/((15 + 2/x^2 + 13/x)*x),x]
```

output
```
(2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{2}{x^2}+\frac{13}{x}+15\right)x}dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{x}{15x^2+13x+2}dx$$

$$\Big\downarrow \text{1141}$$

$$15\int \left(\frac{2}{105(3x+2)}-\frac{1}{105(5x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(\frac{2}{315}\log(3x+2)-\frac{1}{525}\log(5x+1)\right)$$

input
```
Int[1/((15 + 2/x^2 + 13/x)*x),x]
```

output
```
15*((2*Log[2 + 3*x])/315 - Log[1 + 5*x]/525)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{\ln\left(x+\frac{1}{5}\right)}{35}+\frac{2\ln\left(x+\frac{2}{3}\right)}{21}$	14
default	$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$	18
norman	$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$	18
risch	$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$	18

input
```
int(1/(15+2/x^2+13/x)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/35*ln(x+1/5)+2/21*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx=-\frac{1}{35}\log\left(5x+1\right)+\frac{2}{21}\log\left(3x+2\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x,x, algorithm="fricas")
```

output
```
-1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right) x}\, dx = -\frac{\log\left(x+\frac{1}{5}\right)}{35} + \frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

input
```
integrate(1/(15+2/x**2+13/x)/x,x)
```

output
```
-log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right) x}\, dx = -\frac{1}{35}\log\left(5x+1\right) + \frac{2}{21}\log\left(3x+2\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x,x, algorithm="maxima")
```

output
```
-1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right) x}\, dx = -\frac{1}{35}\log\left(|5x+1|\right) + \frac{2}{21}\log\left(|3x+2|\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x,x, algorithm="giac")
```

output
```
-1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx = \frac{2\ \ln\left(x+\frac{2}{3}\right)}{21}-\frac{\ln\left(x+\frac{1}{5}\right)}{35}$$

input
```
int(1/(x*(13/x + 2/x^2 + 15)),x)
```

output
```
(2*log(x + 2/3))/21 - log(x + 1/5)/35
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx = -\frac{\log(5x+1)}{35}+\frac{2\log(3x+2)}{21}$$

input
```
int(1/(15+2/x^2+13/x)/x,x)
```

output
```
( - 3*log(5*x + 1) + 10*log(3*x + 2))/105
```

3.41 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=-\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

output
```
-1/7*ln(2+3*x)+1/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=-\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

input
```
Integrate[1/((15 + 2/x^2 + 13/x)*x^2),x]
```

output
```
-1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1690, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{2}{x^2}+\frac{13}{x}+15\right)x^2}dx$$

$$\Big\downarrow \text{ 1690}$$

$$-\int \frac{1}{15+\frac{13}{x}+\frac{2}{x^2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 1081}$$

$$-2\int \left(\frac{1}{7\left(3+\frac{2}{x}\right)}-\frac{1}{14\left(5+\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$-2\left(\frac{1}{14}\log\left(\frac{2}{x}+3\right)-\frac{1}{14}\log\left(\frac{1}{x}+5\right)\right)$$

input
```
Int[1/((15 + 2/x^2 + 13/x)*x^2),x]
```

output
```
-2*(-1/14*Log[5 + x^(-1)] + Log[3 + 2/x]/14)
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
 + c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln\left(x+\frac{1}{5}\right)}{7}-\frac{\ln\left(x+\frac{2}{3}\right)}{7}$	14
default	$-\frac{\ln(2+3x)}{7}+\frac{\ln(1+5x)}{7}$	18
norman	$-\frac{\ln(2+3x)}{7}+\frac{\ln(1+5x)}{7}$	18
risch	$-\frac{\ln(2+3x)}{7}+\frac{\ln(1+5x)}{7}$	18

input
```
int(1/(15+2/x^2+13/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*ln(x+1/5)-1/7*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx=\frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="fricas")
```

output
```
1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx = \frac{\log\left(x+\frac{1}{5}\right)}{7} - \frac{\log\left(x+\frac{2}{3}\right)}{7}$$

input
```
integrate(1/(15+2/x**2+13/x)/x**2,x)
```

output
```
log(x + 1/5)/7 - log(x + 2/3)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx = \frac{1}{7}\log\left(5x+1\right) - \frac{1}{7}\log\left(3x+2\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="maxima")
```

output
```
1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx = \frac{1}{7}\log\left(\left|5x+1\right|\right) - \frac{1}{7}\log\left(\left|3x+2\right|\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="giac")
```

output
```
1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 19.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{30\,x}{7}+\frac{13}{7}\right)}{7}$$

input
```
int(1/(x^2*(13/x + 2/x^2 + 15)),x)
```

output
```
-(2*atanh((30*x)/7 + 13/7))/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx = \frac{\log(5x+1)}{7}-\frac{\log(3x+2)}{7}$$

input
```
int(1/(15+2/x^2+13/x)/x^2,x)
```

output
```
(log(5*x + 1) - log(3*x + 2))/7
```

3.42 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=\frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)$$

output
```
1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=\frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)$$

input
```
Integrate[1/((15 + 2/x^2 + 13/x)*x^3),x]
```

output
```
Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{2}{x^2}+\frac{13}{x}+15\right) x^3} \, dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{x \left(15 x^2+13 x+2\right)}dx$$

$$\Big\downarrow \text{1141}$$

$$15 \int \left(\frac{3}{70 (3 x+2)}-\frac{5}{21 (5 x+1)}+\frac{1}{30 x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15 \left(\frac{\log (x)}{30}+\frac{1}{70} \log (3 x+2)-\frac{1}{21} \log (5 x+1)\right)$$

input
```
Int[1/((15 + 2/x^2 + 13/x)*x^3),x]
```

output
```
15*(Log[x]/30 + Log[2 + 3*x]/70 - Log[1 + 5*x]/21)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln(x)}{2}-\frac{5\ln\left(x+\frac{1}{5}\right)}{7}+\frac{3\ln\left(x+\frac{2}{3}\right)}{14}$	18
default	$\frac{\ln(x)}{2}+\frac{3\ln(2+3x)}{14}-\frac{5\ln(1+5x)}{7}$	22
norman	$\frac{\ln(x)}{2}+\frac{3\ln(2+3x)}{14}-\frac{5\ln(1+5x)}{7}$	22
risch	$\frac{\ln(x)}{2}+\frac{3\ln(2+3x)}{14}-\frac{5\ln(1+5x)}{7}$	22

input
```
int(1/(15+2/x^2+13/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)-5/7*ln(x+1/5)+3/14*ln(x+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx = -\frac{5}{7}\log\left(5x+1\right)+\frac{3}{14}\log\left(3x+2\right)+\frac{1}{2}\log\left(x\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="fricas")
```

output
```
-5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx = \frac{\log{(x)}}{2} - \frac{5\log{\left(x+\frac{1}{5}\right)}}{7} + \frac{3\log{\left(x+\frac{2}{3}\right)}}{14}$$

input
```
integrate(1/(15+2/x**2+13/x)/x**3,x)
```

output
```
log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx = -\frac{5}{7}\log\left(5x+1\right) + \frac{3}{14}\log\left(3x+2\right) + \frac{1}{2}\log\left(x\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="maxima")
```

output
```
-5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx = -\frac{5}{7}\log\left(|5x+1|\right) + \frac{3}{14}\log\left(|3x+2|\right) + \frac{1}{2}\log\left(|x|\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="giac")
```

output
```
-5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 19.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=\frac{3\ \ln\left(x+\frac{2}{3}\right)}{14}-\frac{5\ \ln\left(x+\frac{1}{5}\right)}{7}+\frac{\ln\left(x\right)}{2}$$

input
```
int(1/(x^3*(13/x + 2/x^2 + 15)),x)
```

output
```
(3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx=-\frac{5\log(5x+1)}{7}+\frac{3\log(3x+2)}{14}+\frac{\log(x)}{2}$$

input
```
int(1/(15+2/x^2+13/x)/x^3,x)
```

output
```
( - 10*log(5*x + 1) + 3*log(3*x + 2) + 7*log(x))/14
```

3.43 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 34

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

output
```
-1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

input
```
Integrate[1/((15 + 2/x^2 + 13/x)*x^4),x]
```

output
```
-1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{2}{x^2}+\frac{13}{x}+15\right)x^4}dx$$

$$\Big\downarrow \text{ 1692}$$

$$\int \frac{1}{x^2\left(15x^2+13x+2\right)}dx$$

$$\Big\downarrow \text{ 1141}$$

$$15\int \left(-\frac{9}{140(3x+2)}+\frac{25}{21(5x+1)}-\frac{13}{60x}+\frac{1}{30x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$15\left(-\frac{1}{30x}-\frac{13\log(x)}{60}-\frac{3}{140}\log(3x+2)+\frac{5}{21}\log(5x+1)\right)$$

input
```
Int[1/((15 + 2/x^2 + 13/x)*x^4),x]
```

output
```
15*(-1/30*1/x - (13*Log[x])/60 - (3*Log[2 + 3*x])/140 + (5*Log[1 + 5*x])/2
1)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(2+3x)}{28}+\frac{25\ln(1+5x)}{7}$	27
norman	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(2+3x)}{28}+\frac{25\ln(1+5x)}{7}$	27
risch	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(2+3x)}{28}+\frac{25\ln(1+5x)}{7}$	27
parallelrisch	$-\frac{91\ln(x)x-100\ln\left(x+\frac{1}{5}\right)x+9\ln\left(x+\frac{2}{3}\right)x+14}{28x}$	27

input
```
int(1/(15+2/x^2+13/x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log\left(x\right)-14}{28\,x}$$

input
```
integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="fricas")
```

output
```
1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=-\frac{13\log{(x)}}{4}+\frac{25\log{\left(x+\frac{1}{5}\right)}}{7}-\frac{9\log{\left(x+\frac{2}{3}\right)}}{28}-\frac{1}{2x}$$

input
```
integrate(1/(15+2/x**2+13/x)/x**4,x)
```

output
```
-13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(5\,x+1\right)-\frac{9}{28}\,\log\left(3\,x+2\right)-\frac{13}{4}\,\log\left(x\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="maxima")
```

output
```
-1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx=-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(|5\,x+1|\right)-\frac{9}{28}\,\log\left(|3\,x+2|\right)-\frac{13}{4}\,\log\left(|x|\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="giac")
```

output
```
-1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x)
)
```

Mupad [B] (verification not implemented)

Time = 19.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx = \frac{25\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{9\,\ln\left(x+\frac{2}{3}\right)}{28}-\frac{13\,\ln\left(x\right)}{4}-\frac{1}{2\,x}$$

input
```
int(1/(x^4*(13/x + 2/x^2 + 15)),x)
```

output
```
(25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx = \frac{100\log(5x+1)\,x-9\log(3x+2)\,x-91\log(x)\,x-14}{28x}$$

input
```
int(1/(15+2/x^2+13/x)/x^4,x)
```

output
```
(100*log(5*x + 1)*x - 9*log(3*x + 2)*x - 91*log(x)*x - 14)/(28*x)
```

3.44 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx=-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

output
```
-1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx=-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

input
```
Integrate[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

output
```
-1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
 + 5*x])/7
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{2}{x^2}+\frac{13}{x}+15\right)x^5}dx$$

$$\Big\downarrow \ 1692$$

$$\int \frac{1}{x^3\left(15x^2+13x+2\right)}dx$$

$$\Big\downarrow \ 1141$$

$$15\int \left(\frac{27}{280(3x+2)}-\frac{125}{21(5x+1)}+\frac{139}{120x}-\frac{13}{60x^2}+\frac{1}{30x^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$15\left(-\frac{1}{60x^2}+\frac{13}{60x}+\frac{139\log(x)}{120}+\frac{9}{280}\log(3x+2)-\frac{25}{21}\log(5x+1)\right)$$

input
```
Int[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

output
```
15*(-1/60*1/x^2 + 13/(60*x) + (139*Log[x])/120 + (9*Log[2 + 3*x])/280 - (2
5*Log[1 + 5*x])/21)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\frac{13x}{4}-\frac{1}{4}}{x^{2}}+\frac{139\ln(x)}{8}+\frac{27\ln(2+3x)}{56}-\frac{125\ln(1+5x)}{7}$	31
default	$-\frac{1}{4x^{2}}+\frac{13}{4x}+\frac{139\ln(x)}{8}+\frac{27\ln(2+3x)}{56}-\frac{125\ln(1+5x)}{7}$	32
parallelrisch	$\frac{973\ln(x)x^{2}-1000\ln\left(x+\frac{1}{5}\right)x^{2}+27\ln\left(x+\frac{2}{3}\right)x^{2}-14+182x}{56x^{2}}$	36
norman	$\frac{-\frac{1}{4}x^{2}+\frac{13}{4}x^{3}}{x^{4}}+\frac{139\ln(x)}{8}+\frac{27\ln(2+3x)}{56}-\frac{125\ln(1+5x)}{7}$	37

input
```
int(1/(15+2/x^2+13/x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
(13/4*x-1/4)/x^2+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = -\frac{1000\,x^{2}\log\left(5\,x+1\right)-27\,x^{2}\log\left(3\,x+2\right)-973\,x^{2}\log\left(x\right)-182\,x+14}{56\,x^{2}}$$

input
```
integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="fricas")
```

output
```
-1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*
x + 14)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx=\frac{139\log\left(x\right)}{8}-\frac{125\log\left(x+\frac{1}{5}\right)}{7}+\frac{27\log\left(x+\frac{2}{3}\right)}{56}+\frac{13x-1}{4x^2}$$

input
```
integrate(1/(15+2/x**2+13/x)/x**5,x)
```

output
```
139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**
2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx=\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\,\log\left(5\,x+1\right)+\frac{27}{56}\,\log\left(3\,x+2\right)+\frac{139}{8}\,\log\left(x\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="maxima")
```

output
```
1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x
)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = \frac{13\,x-1}{4\,x^2} - \frac{125}{7}\,\log\left(|5\,x+1|\right) + \frac{27}{56}\,\log\left(|3\,x+2|\right) + \frac{139}{8}\,\log\left(|x|\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="giac")
```

output
```
1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 1
39/8*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = \frac{27\,\ln\left(x+\frac{2}{3}\right)}{56} - \frac{125\,\ln\left(x+\frac{1}{5}\right)}{7} + \frac{139\,\ln\left(x\right)}{8} + \frac{\frac{13\,x}{4}-\frac{1}{4}}{x^2}$$

input
```
int(1/(x^5*(13/x + 2/x^2 + 15)),x)
```

output
```
(27*log(x + 2/3))/56 - (125*log(x + 1/5))/7 + (139*log(x))/8 + ((13*x)/4 -
 1/4)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = \frac{-1000\log(5x+1)\,x^2+27\log(3x+2)\,x^2+973\log(x)\,x^2+182x-14}{56x^2}$$

input
```
int(1/(15+2/x^2+13/x)/x^5,x)
```

output
```
( - 1000*log(5*x + 1)*x**2 + 27*log(3*x + 2)*x**2 + 973*log(x)*x**2 + 182*
x - 14)/(56*x**2)
```

3.45 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx=-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

output
```
-1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx=-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

input
```
Integrate[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

output
```
-1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])
/112 + (625*Log[1 + 5*x])/7
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1692, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{2}{x^2}+\frac{13}{x}+15\right) x^6} \, dx$$

$$\Big\downarrow \text{1692}$$

$$\int \frac{1}{x^4 \left(15 x^2+13 x+2\right)}dx$$

$$\Big\downarrow \text{1141}$$

$$15 \int \left(-\frac{81}{560(3x+2)}+\frac{625}{21(5x+1)}-\frac{1417}{240x}+\frac{139}{120x^2}-\frac{13}{60x^3}+\frac{1}{30x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15 \left(-\frac{1}{90x^3}+\frac{13}{120x^2}-\frac{139}{120x}-\frac{1417\log(x)}{240}-\frac{27}{560}\log(3x+2)+\frac{125}{21}\log(5x+1)\right)$$

input
```
Int[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

output
```
15*(-1/90*1/x^3 + 13/(120*x^2) - 139/(120*x) - (1417*Log[x])/240 - (27*Log
[2 + 3*x])/560 + (125*Log[1 + 5*x])/21)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1692
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{-\frac{139}{8}x^2+\frac{13}{8}x-\frac{1}{6}}{x^3}-\frac{1417\ln(x)}{16}-\frac{81\ln(2+3x)}{112}+\frac{625\ln(1+5x)}{7}$	36
default	$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\ln(x)}{16}-\frac{81\ln(2+3x)}{112}+\frac{625\ln(1+5x)}{7}$	37
parallelrisch	$-\frac{29757\ln(x)x^3-30000\ln\left(x+\frac{1}{5}\right)x^3+243\ln\left(x+\frac{2}{3}\right)x^3+56+5838x^2-546x}{336x^3}$	41
norman	$\frac{-\frac{1}{6}x^2+\frac{13}{8}x^3-\frac{139}{8}x^4}{x^5}-\frac{1417\ln(x)}{16}-\frac{81\ln(2+3x)}{112}+\frac{625\ln(1+5x)}{7}$	42

input
```
int(1/(15+2/x^2+13/x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
(-139/8*x^2+13/8*x-1/6)/x^3-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = \frac{30000\,x^3\log\left(5\,x+1\right)-243\,x^3\log\left(3\,x+2\right)-29757\,x^3\log\left(x\right)-5838\,x^2+546\,x-56}{336\,x^3}$$

input
```
integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="fricas")
```

output
```
1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) -
5838*x^2 + 546*x - 56)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = -\frac{1417\log\left(x\right)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}+\frac{-417x^2+39x-4}{24x^3}$$

input
```
integrate(1/(15+2/x**2+13/x)/x**6,x)
```

output
```
-1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 + (-417*x**2 +
39*x - 4)/(24*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = -\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(5\,x+1\right) - \frac{81}{112}\,\log\left(3\,x+2\right)-\frac{1417}{16}\,\log\left(x\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="maxima")
```

output
```
-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2)
- 1417/16*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = -\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(|5\,x+1|\right) - \frac{81}{112}\,\log\left(|3\,x+2|\right)-\frac{1417}{16}\,\log\left(|x|\right)$$

input
```
integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="giac")
```

output
```
-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(
3*x + 2)) - 1417/16*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 19.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx=\frac{625\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{81\,\ln\left(x+\frac{2}{3}\right)}{112}-\frac{1417\,\ln\left(x\right)}{16}-\frac{\frac{139\,x^2}{8}-\frac{13\,x}{8}+\frac{1}{6}}{x^3}$$

input
```
int(1/(x^6*(13/x + 2/x^2 + 15)),x)
```

output
```
(625*log(x + 1/5))/7 - (81*log(x + 2/3))/112 - (1417*log(x))/16 - ((139*x^
2)/8 - (13*x)/8 + 1/6)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx = \frac{30000\log(5x+1)\,x^3-243\log(3x+2)\,x^3-29757\log(x)\,x^3-5838x^2+546x-56}{336x^3}$$

input
```
int(1/(15+2/x^2+13/x)/x^6,x)
```

output
```
(30000*log(5*x + 1)*x**3 - 243*log(3*x + 2)*x**3 - 29757*log(x)*x**3 - 583
8*x**2 + 546*x - 56)/(336*x**3)
```

3.46 $\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 174

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx=\frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5}-\frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4}+\frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3}-\frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2}+\frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a}-\frac{63b^5\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{64a^{11/2}}$$

output
```
63/64*b^4*(b*x^(1/2)+a*x)^(1/2)/a^5-21/32*b^3*x^(1/2)*(b*x^(1/2)+a*x)^(1/2
)/a^4+21/40*b^2*x*(b*x^(1/2)+a*x)^(1/2)/a^3-9/20*b*x^(3/2)*(b*x^(1/2)+a*x)
^(1/2)/a^2+2/5*x^2*(b*x^(1/2)+a*x)^(1/2)/a-63/64*b^5*arctanh(a^(1/2)*x^(1/
2)/(b*x^(1/2)+a*x)^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx=\frac{\sqrt{b\sqrt{x}+ax}\left(315b^4-210ab^3\sqrt{x}+168a^2b^2x-144a^3bx^{3/2}+128a^4x^2\right)}{320a^5}-\frac{63b^5\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{64a^{11/2}}$$

input
```
Integrate[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(315*b^4 - 210*a*b^3*Sqrt[x] + 168*a^2*b^2*x - 144*
a^3*b*x^(3/2) + 128*a^4*x^2))/(320*a^5) - (63*b^5*ArcTanh[(Sqrt[a]*Sqrt[b*
Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(64*a^(11/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1924, 1134, 1134, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x^{5/2}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\int \frac{x^2}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{10a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\int \frac{x^{3/2}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{8a}\right)}{10a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\int\frac{x}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{6a}\right)}{8a}\right)}{10a}\right)$$

$\downarrow$ 1134

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\int\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}\right)}{6a}\right)}{8a}\right)}{10a}\right)$$

$\downarrow$ 1160

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int\frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}\right)}{4a}\right)}{6a}\right)}{8a}\right)}{10a}\right)$$

$\downarrow$ 1091

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int\frac{1}{1-ax}\,d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}}{a}\right)}{4a}\right)}{6a}\right)}{8a}\right)}{10a}\right)$$

↓ 219

$$2\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}\right)}{4a}\right)}{6a}\right)}{8a}\right)}{10a}\right)$$

input
```
Int[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
2*((x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a) - (9*b*((x^(3/2)*Sqrt[b*Sqrt[x] + a*x
])/(4*a) - (7*b*((x*Sqrt[b*Sqrt[x] + a*x])/(3*a) - (5*b*((Sqrt[x]*Sqrt[b*S
qrt[x] + a*x])/(2*a) - (3*b*(Sqrt[b*Sqrt[x] + a*x]/a - (b*ArcTanh[(Sqrt[a]
*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(3/2)))/(4*a)))/(6*a)))/(8*a)))/(10*a)
)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{2x^2\sqrt{b\sqrt{x}+xa}}{5a}-\frac{9b\left(\frac{x^{\frac{3}{2}}\sqrt{b\sqrt{x}+xa}}{4a}-\frac{7b\left(\frac{x\sqrt{b\sqrt{x}+xa}}{3a}-\frac{5b\left(\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+xa}}{2a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+xa}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+\sqrt{x}\,a}{\sqrt{a}}+\sqrt{b\sqrt{x}+xa}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{6a}\right)}{8a}\right)}{5a}$
default	$-\frac{\sqrt{b\sqrt{x}+xa}\left(544\sqrt{x}\,a^{\frac{7}{2}}\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}b-256x\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}a^{\frac{9}{2}}+1300\sqrt{x}\,a^{\frac{5}{2}}\sqrt{b\sqrt{x}+xa}\,b^{3}-880a^{\frac{5}{2}}\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}b^{2}+64\cdots\right.}{640\cdots}$

input
```
int(x^2/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^2*(b*x^(1/2)+x*a)^(1/2)/a-9/5*b/a*(1/4*x^(3/2)*(b*x^(1/2)+x*a)^(1/2)
/a-7/8*b/a*(1/3*x*(b*x^(1/2)+x*a)^(1/2)/a-5/6*b/a*(1/2*x^(1/2)*(b*x^(1/2)+
x*a)^(1/2)/a-3/4*b/a*((b*x^(1/2)+x*a)^(1/2)/a-1/2*b/a^(3/2)*ln((1/2*b+x^(1
/2)*a)/a^(1/2)+(b*x^(1/2)+x*a)^(1/2))))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases}\sqrt{ax+b\sqrt{x}}\left(\frac{x^2}{5a}-\frac{9bx^{\frac{3}{2}}}{40a^2}+\frac{21b^2x}{80a^3}-\frac{21b^3\sqrt{x}}{64a^4}+\frac{63b^4}{128a^5}\right)-\frac{63b^5\left(\begin{cases}\frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } \frac{b^2}{a}\neq 0\\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherwise}\end{cases}\right)}{256a^5} \\ \frac{2\left(b\sqrt{x}\right)^{\frac{11}{2}}}{11b^6} \\ \tilde{\infty}x^3\end{cases}\right)$$

input
```
integrate(x**2/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise((sqrt(a*x + b*sqrt(x))*(x**2/(5*a) - 9*b*x**(3/2)/(40*a**2) +
21*b**2*x/(80*a**3) - 21*b**3*sqrt(x)/(64*a**4) + 63*b**4/(128*a**5)) - 63
*b**5*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(x) + b)/sq
rt(a), Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a))/sqrt(a*
(sqrt(x) + b/(2*a))**2), True))/(256*a**5), Ne(a, 0)), (2*(b*sqrt(x))**(11
/2)/(11*b**6), Ne(b, 0)), (zoo*x**3, True))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^2}{\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{320}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\sqrt{x}\left(\frac{8\sqrt{x}}{a}-\frac{9b}{a^2}\right)+\frac{21b^2}{a^3}\right)\sqrt{x}-\frac{105b^3}{a^4}\right)\sqrt{x}+\frac{315b^4}{a^5}\right) + \frac{63b^5\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{128a^{\frac{11}{2}}}$$

input
```
integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/320*sqrt(a*x + b*sqrt(x))*(2*(4*(2*sqrt(x)*(8*sqrt(x)/a - 9*b/a^2) + 21*
b^2/a^3)*sqrt(x) - 105*b^3/a^4)*sqrt(x) + 315*b^4/a^5) + 63/128*b^5*log(ab
s(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/a^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^2}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(x^2/(a*x + b*x^(1/2))^(1/2),x)
```

output
```
int(x^2/(a*x + b*x^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx$$
$$= \frac{-144x^{\frac{7}{4}}\sqrt{\sqrt{x}\,a+b}\,a^4b-210x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a^2b^3+128x^{\frac{9}{4}}\sqrt{\sqrt{x}\,a+b}\,a^5+168x^{\frac{5}{4}}\sqrt{\sqrt{x}\,a+b}\,a^3b^2+315x}{320a^6}$$

input
```
int(x^2/(b*x^(1/2)+a*x)^(1/2),x)
```

output
```
( - 144*x**(3/4)*sqrt(sqrt(x)*a + b)*a**4*b*x - 210*x**(3/4)*sqrt(sqrt(x)*
a + b)*a**2*b**3 + 128*x**(1/4)*sqrt(sqrt(x)*a + b)*a**5*x**2 + 168*x**(1/
4)*sqrt(sqrt(x)*a + b)*a**3*b**2*x + 315*x**(1/4)*sqrt(sqrt(x)*a + b)*a*b*
*4 - 315*sqrt(a)*log((sqrt(sqrt(x)*a + b) + x**(1/4)*sqrt(a))/sqrt(b))*b**
5)/(320*a**6)
```

3.47 $\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 116

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^{7/2}}$$

output
```
5/4*b^2*(b*x^(1/2)+a*x)^(1/2)/a^3-5/6*b*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^2+
2/3*x*(b*x^(1/2)+a*x)^(1/2)/a-5/4*b^3*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a
*x)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(15b^2-10ab\sqrt{x}+8a^2x\right)}{12a^3} - \frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{4a^{7/2}}$$

input
```
Integrate[x/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(15*b^2 - 10*a*b*Sqrt[x] + 8*a^2*x))/(12*a^3) - (5*
b^3*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(4*a^(7/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1924, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x^{3/2}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\int \frac{x}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{6a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}\right)}{6a}\right)$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}\right)}{4a}\right)}{6a}\right)$$

$$\Big\downarrow \text{1091}$$

$$2\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int\frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}}{a}\right)}{4a}\right)}{6a}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}\right)}{4a}\right)}{6a}\right)$$

input
```
Int[x/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
2*((x*Sqrt[b*Sqrt[x] + a*x])/(3*a) - (5*b*((Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])
/(2*a) - (3*b*(Sqrt[b*Sqrt[x] + a*x]/a - (b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt
[b*Sqrt[x] + a*x]])/a^(3/2)))/(4*a)))/(6*a))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{2x\sqrt{b\sqrt{x}+xa}}{3a}-\frac{5b\left(\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+xa}}{2a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+xa}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+\sqrt{x}\,a}{\sqrt{a}}+\sqrt{b\sqrt{x}+xa}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{3a}$
default	$\frac{\sqrt{b\sqrt{x}+xa}\left(16\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}a^{\frac{5}{2}}-36\sqrt{b\sqrt{x}+xa}\,\sqrt{x}\,a^{\frac{5}{2}}b-18\sqrt{b\sqrt{x}+xa}\,a^{\frac{3}{2}}b^{2}+48a^{\frac{3}{2}}\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}\,b^{2}-24a\ln\left(\frac{2\sqrt{x}\,a\right.}{}\right.}{24a^{\frac{9}{2}}\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}}$

input
```
int(x/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x*(b*x^(1/2)+x*a)^(1/2)/a-5/3*b/a*(1/2*x^(1/2)*(b*x^(1/2)+x*a)^(1/2)/a
-3/4*b/a*((b*x^(1/2)+x*a)^(1/2)/a-1/2*b/a^(3/2)*ln((1/2*b+x^(1/2)*a)/a^(1/
2)+(b*x^(1/2)+x*a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.32

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \sqrt{ax+b\sqrt{x}}\left(\frac{x}{3a}-\frac{5b\sqrt{x}}{12a^2}+\frac{5b^2}{8a^3}\right)-\frac{5b^3\left(\begin{cases}\frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } \frac{b^2}{a}\neq 0\\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherwise}\end{cases}\right)}{16a^3} & \text{for } a\neq 0\\ \frac{2\left(b\sqrt{x}\right)^{\frac{7}{2}}}{7b^4} & \text{for } b\neq 0\\ \tilde{\infty}x^2 & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise((sqrt(a*x + b*sqrt(x))*(x/(3*a) - 5*b*sqrt(x)/(12*a**2) + 5*b*
*2/(8*a**3)) - 5*b**3*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a
*sqrt(x) + b)/sqrt(a), Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) +
b/(2*a))/sqrt(a*(sqrt(x) + b/(2*a))**2), True))/(16*a**3), Ne(a, 0)), (2*(
b*sqrt(x))**(7/2)/(7*b**4), Ne(b, 0)), (zoo*x**2, True))
```

Maxima [F]

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx=\int \frac{x}{\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx=\frac{1}{12}\sqrt{ax+b\sqrt{x}}\left(2\sqrt{x}\left(\frac{4\sqrt{x}}{a}-\frac{5b}{a^2}\right)+\frac{15b^2}{a^3}\right)+\frac{5b^3\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{8a^{\frac{7}{2}}}$$

input
```
integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)*(4*sqrt(x)/a - 5*b/a^2) + 15*b^2/a^3
) + 5/8*b^3*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) +
b))/a^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(x/(a*x + b*x^(1/2))^(1/2),x)
```

output
```
int(x/(a*x + b*x^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{-10x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a^2b+8x^{\frac{5}{4}}\sqrt{\sqrt{x}\,a+b}\,a^3+15x^{\frac{1}{4}}\sqrt{\sqrt{x}\,a+b}\,a\,b^2-15\sqrt{a}\log\left(\frac{\sqrt{\sqrt{x}\,a+b}+x^{\frac{1}{4}}\sqrt{a}}{\sqrt{b}}\right)b^3}{12a^4}$$

input
```
int(x/(b*x^(1/2)+a*x)^(1/2),x)
```

output
```
( - 10*x**(3/4)*sqrt(sqrt(x)*a + b)*a**2*b + 8*x**(1/4)*sqrt(sqrt(x)*a + b
)*a**3*x + 15*x**(1/4)*sqrt(sqrt(x)*a + b)*a*b**2 - 15*sqrt(a)*log((sqrt(s
qrt(x)*a + b) + x**(1/4)*sqrt(a))/sqrt(b))*b**3)/(12*a**4)
```

3.48 $\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{2b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}$$

output
```
2*(b*x^(1/2)+a*x)^(1/2)/a-2*b*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2
))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{2b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{a^{3/2}}$$

input
```
Integrate[1/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
(2*Sqrt[b*Sqrt[x] + a*x])/a - (2*b*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])
/(b + a*Sqrt[x])])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1916, 1919, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1916}$$

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}b+ax}}dx}{2a}$$

$$\Big\downarrow \text{1919}$$

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a}$$

$$\Big\downarrow \text{1091}$$

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{2b\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}$$

input
```
Int[1/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
(2*Sqrt[b*Sqrt[x] + a*x])/a - (2*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x
] + a*x]])/a^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1916
```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Simp[a*((2*n - j - 2)/(b*(n -
2)))   Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
 LtQ[2*(n - 1), j, n]
```

rule 1919
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
 && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2\sqrt{b\sqrt{x}+xa}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+\sqrt{x}\,a}{\sqrt{a}}+\sqrt{b\sqrt{x}+xa}\right)}{a^{\frac{3}{2}}}$	50
default	$-\frac{\sqrt{b\sqrt{x}+xa}\left(b\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{\sqrt{x}\,(\sqrt{x}\,a+b)}\,\sqrt{a}+b}{2\sqrt{a}}\right)-2\sqrt{\sqrt{x}\,(\sqrt{x}\,a+b)}\,\sqrt{a}\right)}{\sqrt{\sqrt{x}\,(\sqrt{x}\,a+b)}\,a^{\frac{3}{2}}}$	83

input
```
int(1/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b*x^(1/2)+x*a)^(1/2)/a-b/a^(3/2)*ln((1/2*b+x^(1/2)*a)/a^(1/2)+(b*x^(1/2
)+x*a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.12

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} -\frac{b\left(\begin{cases} \frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } \frac{b^2}{a}\neq 0 \\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherwise} \end{cases}\right)}{2a}+\frac{\sqrt{ax+b\sqrt{x}}}{a} & \text{for } a\neq 0 \\ \frac{2\left(b\sqrt{x}\right)^{\frac{3}{2}}}{3b^2} & \text{for } b\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}\right)$$

input
```
integrate(1/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise((-b*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(
x) + b)/sqrt(a), Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a
))/sqrt(a*(sqrt(x) + b/(2*a))**2), True))/(2*a) + sqrt(a*x + b*sqrt(x))/a,
 Ne(a, 0)), (2*(b*sqrt(x))**(3/2)/(3*b**2), Ne(b, 0)), (zoo*x, True))
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{b\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{a^{\frac{3}{2}}} + \frac{2\sqrt{ax+b\sqrt{x}}}{a}$$

input
```
integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
b*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/a^(3/2
) + 2*sqrt(a*x + b*sqrt(x))/a
```

Mupad [B] (verification not implemented)

Time = 18.70 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\,x\left(\frac{3\sqrt{b}\,\sqrt{b+a\sqrt{x}}}{2\,a\sqrt{x}} + \frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,x^{1/4}\,1\mathrm{i}}{\sqrt{b}}\right)3\mathrm{i}}{2\,a^{3/2}\,x^{3/4}}\right)\sqrt{\frac{a\sqrt{x}}{b}+1}}{3\sqrt{a\,x+b\sqrt{x}}}$$

input
```
int(1/(a*x + b*x^(1/2))^(1/2),x)
```

output
```
(4*x*((3*b^(1/2)*(b + a*x^(1/2))^(1/2))/(2*a*x^(1/2)) + (b^(3/2)*asin((a^(
1/2)*x^(1/4)*1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/4)))*((a*x^(1/2))/b + 1)^(1/
2))/(3*(a*x + b*x^(1/2))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{2x^{\frac{1}{4}}\sqrt{\sqrt{x}\,a+b}\,a-2\sqrt{a}\log\left(\frac{\sqrt{\sqrt{x}\,a+b}+x^{\frac{1}{4}}\sqrt{a}}{\sqrt{b}}\right)b}{a^2}$$

input
```
int(1/(b*x^(1/2)+a*x)^(1/2),x)
```

output
```
(2*(x**(1/4)*sqrt(sqrt(x)*a + b)*a - sqrt(a)*log((sqrt(sqrt(x)*a + b) + x*
*(1/4)*sqrt(a))/sqrt(b))*b))/a**2
```

3.49 $\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

output
```
-4*(b*x^(1/2)+a*x)^(1/2)/b/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

input
```
Integrate[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax+b\sqrt{x}}}dx$$
$$\Big\downarrow \text{ 1920}$$
$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

input
```
Int[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+xa}}{b\sqrt{x}}$
default	$-\frac{\sqrt{b\sqrt{x}+xa}\left(4\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}\sqrt{a}-2\sqrt{b\sqrt{x}+xa}\,a^{\frac{3}{2}}x-2\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}\,a^{\frac{3}{2}}x-\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{b\sqrt{x}+xa}\,\sqrt{a}+b}{2\sqrt{a}}\right)abx+\ln(\right.}{\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}\,b^{2}x\sqrt{a}}$

input
```
int(1/x/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*(b*x^(1/2)+x*a)^(1/2)/b/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

input
```
integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-4*sqrt(a*x + b*sqrt(x))/(b*sqrt(x))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x}\,dx$$

input
```
integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4}{\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}}$$

input
```
integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x*(a*x + b*x^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = \frac{-4x^{\frac{1}{4}}\sqrt{\sqrt{x}\,a+b}-4\sqrt{x}\,\sqrt{a}}{\sqrt{x}\,b}$$

input
```
int(1/x/(b*x^(1/2)+a*x)^(1/2),x)
```

output
```
( - 4*(x**(1/4)*sqrt(sqrt(x)*a + b) + sqrt(x)*sqrt(a)))/(sqrt(x)*b)
```

3.50 $\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}+\frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x}-\frac{32a^2\sqrt{b\sqrt{x}+ax}}{15b^3\sqrt{x}}$$

output
```
-4/5*(b*x^(1/2)+a*x)^(1/2)/b/x^(3/2)+16/15*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x-3
2/15*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}\left(3b^2-4ab\sqrt{x}+8a^2x\right)}{15b^3x^{3/2}}$$

input
```
Integrate[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(3*b^2 - 4*a*b*Sqrt[x] + 8*a^2*x))/(15*b^3*x^(3/
2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{4a\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

input
```
Int[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x]
)/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])))/(5*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+xa}}{5b\,x^{\frac{3}{2}}}-\frac{8a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+xa}}{3b^{2}\sqrt{x}}\right)}{5b}$
default	$-\frac{\sqrt{b\sqrt{x}+xa}\left(60\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}x^{\frac{5}{2}}a^{\frac{5}{2}}-30\sqrt{b\sqrt{x}+xa}\,x^{\frac{7}{2}}a^{\frac{7}{2}}-15x^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{b\sqrt{x}+xa}\,\sqrt{a}+b}{2\sqrt{a}}\right)a^{3}b-30x^{\frac{7}{2}}a^{\frac{7}{2}}\sqrt{\sqrt{x}\,(}\right.}{15\sqrt{\sqrt{x}\,\left(\sqrt{x}\,a+b\right)}}$

input
```
int(1/x^2/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/5*(b*x^(1/2)+x*a)^(1/2)/b/x^(3/2)-8/5*a/b*(-2/3*(b*x^(1/2)+x*a)^(1/2)/b
/x+4/3*a*(b*x^(1/2)+x*a)^(1/2)/b^2/x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(4\,abx-(8\,a^2x+3\,b^2)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{15\,b^3x^2}$$

input
```
integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
4/15*(4*a*b*x - (8*a^2*x + 3*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^3*x^2)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^2\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x**2/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(20\,a\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+15\,\sqrt{a}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+3\,b^2\right)}{15\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5}$$

input
```
integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/15*(20*a*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 15*sqrt(a)*b*(sqr
t(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 3*b^2)/(sqrt(a)*sqrt(x) - sqrt(a*x
 + b*sqrt(x)))^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^2\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^2*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\frac{16x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,ab}{15}-\frac{32x^{\frac{5}{4}}\sqrt{\sqrt{x}\,a+b}\,a^2}{15}-\frac{4x^{\frac{1}{4}}\sqrt{\sqrt{x}\,a+b}\,b^2}{5}+\frac{32\sqrt{x}\,\sqrt{a}\,a^2x}{15}}{\sqrt{x}\,b^3x}$$

input

```
int(1/x^2/(b*x^(1/2)+a*x)^(1/2),x)
```

output

```
(4*(4*x**(3/4)*sqrt(sqrt(x)*a + b)*a*b - 8*x**(1/4)*sqrt(sqrt(x)*a + b)*a*
*2*x - 3*x**(1/4)*sqrt(sqrt(x)*a + b)*b**2 + 8*sqrt(x)*sqrt(a)*a**2*x))/(1
5*sqrt(x)*b**3*x)
```

3.51 $\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2}-\frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}}+\frac{256a^3\sqrt{b\sqrt{x}+ax}}{315b^4x}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{315b^5\sqrt{x}}$$

output
```
-4/9*(b*x^(1/2)+a*x)^(1/2)/b/x^(5/2)+32/63*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^2
-64/105*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(3/2)+256/315*a^3*(b*x^(1/2)+a*x)^
(1/2)/b^4/x-512/315*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(35b^4-40ab^3\sqrt{x}+48a^2b^2x-64a^3bx^{3/2}+128a^4x^2\right)}{315b^5x^{5/2}}$$

input
```
Integrate[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(35*b^4 - 40*a*b^3*Sqrt[x] + 48*a^2*b^2*x - 64*a
^3*b*x^(3/2) + 128*a^4*x^2))/(315*b^5*x^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{8a\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{8a\left(-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{8a\left(-\frac{6a\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

input

```
Int[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

output

```
(-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(5/2)) - (8*a*((-4*Sqrt[b*Sqrt[x] + a*x]
)/(7*b*x^2) - (6*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) - (4*a*((-4*S
qrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x]
)))/(5*b)))/(7*b)))/(9*b)
```

Defintions of rubi rules used

rule 1920

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+xa}}{9b\,x^{\frac{5}{2}}}-\frac{16a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{7b\,x^{2}}-\frac{6a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+xa}}{3b^{2}\sqrt{x}}\right)}{5b}\right)}{7b}\right)}{9b}$
default	$-\frac{\sqrt{b\sqrt{x}+xa}\left(1260(b\sqrt{x}+xa)^{\frac{3}{2}}x^{\frac{9}{2}}a^{\frac{9}{2}}-630\sqrt{b\sqrt{x}+xa}\,x^{\frac{11}{2}}a^{\frac{11}{2}}-315x^{\frac{11}{2}}\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{b\sqrt{x}+xa}\,\sqrt{a}+b}{2\sqrt{a}}\right)a^{5}b-630x^{\frac{11}{2}}a\right.}{}$

input
```
int(1/x^3/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/9*(b*x^(1/2)+x*a)^(1/2)/b/x^(5/2)-16/9*a/b*(-2/7*(b*x^(1/2)+x*a)^(1/2)/
b/x^2-6/7*a/b*(-2/5*(b*x^(1/2)+x*a)^(1/2)/b/x^(3/2)-4/5*a/b*(-2/3*(b*x^(1/
2)+x*a)^(1/2)/b/x+4/3*a*(b*x^(1/2)+x*a)^(1/2)/b^2/x^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(64\,a^3bx^2+40\,ab^3x-(128\,a^4x^2+48\,a^2b^2x+35\,b^4)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{315\,b^5x^3}$$

input
```
integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
4/315*(64*a^3*b*x^2 + 40*a*b^3*x - (128*a^4*x^2 + 48*a^2*b^2*x + 35*b^4)*s
qrt(x))*sqrt(a*x + b*sqrt(x))/(b^5*x^3)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^3\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x**3/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(1008\,a^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^4+1680\,a^{\frac{3}{2}}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3+1080\,ab^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+\right.\right.}{315\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^9}$$

input
```
integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/315*(1008*a^2*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 1680*a^(3/2)
*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 1080*a*b^2*(sqrt(a)*sqrt(
x) - sqrt(a*x + b*sqrt(x)))^2 + 315*sqrt(a)*b^3*(sqrt(a)*sqrt(x) - sqrt(a*
x + b*sqrt(x))) + 35*b^4)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^3\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\frac{256x^{\frac{7}{4}}\sqrt{\sqrt{x}\,a+b}\,a^3b}{315}+\frac{32x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a\,b^3}{63}-\frac{512x^{\frac{9}{4}}\sqrt{\sqrt{x}\,a+b}\,a^4}{315}-\frac{64x^{\frac{5}{4}}\sqrt{\sqrt{x}\,a+b}\,a^2b^2}{105}-\frac{4x^{\frac{1}{4}}\sqrt{\sqrt{x}\,a+b}\,b^4}{9}+\frac{512\sqrt{x}\,\sqrt{a}\,a^4x^2}{315}}{\sqrt{x}\,b^5x^2}$$

input
```
int(1/x^3/(b*x^(1/2)+a*x)^(1/2),x)
```

output
```
(4*(64*x**(3/4)*sqrt(sqrt(x)*a + b)*a**3*b*x + 40*x**(3/4)*sqrt(sqrt(x)*a
+ b)*a*b**3 - 128*x**(1/4)*sqrt(sqrt(x)*a + b)*a**4*x**2 - 48*x**(1/4)*sqr
t(sqrt(x)*a + b)*a**2*b**2*x - 35*x**(1/4)*sqrt(sqrt(x)*a + b)*b**4 + 128*
sqrt(x)*sqrt(a)*a**4*x**2))/(315*sqrt(x)*b**5*x**2)
```

3.52 $\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 200

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}}+\frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3}-\frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}}+\frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{1001b^5x^{3/2}}+\frac{2048a^5\sqrt{b\sqrt{x}+ax}}{3003b^6x}-\frac{4096a^6\sqrt{b\sqrt{x}+ax}}{3003b^7\sqrt{x}}$$

output

```
-4/13*(b*x^(1/2)+a*x)^(1/2)/b/x^(7/2)+48/143*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x
^3-160/429*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(5/2)+1280/3003*a^3*(b*x^(1/2)+
a*x)^(1/2)/b^4/x^2-512/1001*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(3/2)+2048/300
3*a^5*(b*x^(1/2)+a*x)^(1/2)/b^6/x-4096/3003*a^6*(b*x^(1/2)+a*x)^(1/2)/b^7/
x^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(231b^6-252ab^5\sqrt{x}+280a^2b^4x-320a^3b^3x^{3/2}+384a^4b^2x^2-512a^5bx^{5/2}+1024a^6x^3\right)}{3003b^7x^{7/2}}$$

input
```
Integrate[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(231*b^6 - 252*a*b^5*Sqrt[x] + 280*a^2*b^4*x - 3
20*a^3*b^3*x^(3/2) + 384*a^4*b^2*x^2 - 512*a^5*b*x^(5/2) + 1024*a^6*x^3))/
(3003*b^7*x^(7/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1922, 1922, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{12a\int \frac{1}{x^{7/2}\sqrt{\sqrt{x}b+ax}}dx}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{12a\left(-\frac{10a\int \frac{1}{x^3\sqrt{\sqrt{x}b+ax}}dx}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{12a\left(-\frac{10a\left(-\frac{8a\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

$\big\downarrow$ 1922

$$-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

$\big\downarrow$ 1922

$$-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

$\big\downarrow$ 1922

$$-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int\frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^{2}}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^{3}}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

$\downarrow$ 1920

$$-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

input
```
Int[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(13*b*x^(7/2)) - (12*a*((-4*Sqrt[b*Sqrt[x] + a*
x])/(11*b*x^3) - (10*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(5/2)) - (8*a*((
-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) - (6*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(5*
b*x^(3/2)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x
] + a*x])/(3*b^2*Sqrt[x])))/(5*b)))/(7*b)))/(9*b)))/(11*b)))/(13*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+xa}}{13b\,x^{\frac{7}{2}}}-\frac{24a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{11b\,x^{3}}-\frac{10a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{9b\,x^{\frac{5}{2}}}-\frac{8a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{7b\,x^{2}}-\frac{6a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{3bx}+\frac{4a\cdots}{\cdots}\right)}{5b}\right)}{7b}\right)}{9b}\right)}{11b}\right)}{13b}$
default	$-\frac{\sqrt{b\sqrt{x}+xa}\left(12012(b\sqrt{x}+xa)^{\frac{3}{2}}x^{\frac{13}{2}}a^{\frac{13}{2}}-6006\sqrt{b\sqrt{x}+xa}\,x^{\frac{15}{2}}a^{\frac{15}{2}}-3003x^{\frac{15}{2}}\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{b\sqrt{x}+xa}\,\sqrt{a}+b}{2\sqrt{a}}\right)a^{7}b-60\cdots\right)}{\cdots}$

input
```
int(1/x^4/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/13*(b*x^(1/2)+x*a)^(1/2)/b/x^(7/2)-24/13*a/b*(-2/11*(b*x^(1/2)+x*a)^(1/
2)/b/x^3-10/11*a/b*(-2/9*(b*x^(1/2)+x*a)^(1/2)/b/x^(5/2)-8/9*a/b*(-2/7*(b*
x^(1/2)+x*a)^(1/2)/b/x^2-6/7*a/b*(-2/5*(b*x^(1/2)+x*a)^(1/2)/b/x^(3/2)-4/5
*a/b*(-2/3*(b*x^(1/2)+x*a)^(1/2)/b/x+4/3*a*(b*x^(1/2)+x*a)^(1/2)/b^2/x^(1/
2))))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(512\,a^5bx^3+320\,a^3b^3x^2+252\,ab^5x-\left(1024\,a^6x^3+384\,a^4b^2x^2+280\,a^2b^4x+231\,b^6\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3003\,b^7x^4}$$

input
```
integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
4/3003*(512*a^5*b*x^3 + 320*a^3*b^3*x^2 + 252*a*b^5*x - (1024*a^6*x^3 + 38
4*a^4*b^2*x^2 + 280*a^2*b^4*x + 231*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b
^7*x^4)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^4\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x**4/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(27456\,a^3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^6+72072\,a^{\frac{5}{2}}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5+80080\,a^2b^2\left(\sqrt{a}\sqrt{x}-\sqrt{a\right.\right.}{}$$

input
```
integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/3003*(27456*a^3*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^6 + 72072*a^(5
/2)*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 80080*a^2*b^2*(sqrt(a)
*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 48048*a^(3/2)*b^3*(sqrt(a)*sqrt(x) -
 sqrt(a*x + b*sqrt(x)))^3 + 16380*a*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sq
rt(x)))^2 + 3003*sqrt(a)*b^5*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 2
31*b^6)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^13
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx=\int \frac{1}{x^4\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^4*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^4*(a*x + b*x^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\frac{2048x^{\frac{11}{4}}\sqrt{\sqrt{x}\,a+b}\,a^5b}{3003}+\frac{1280x^{\frac{7}{4}}\sqrt{\sqrt{x}\,a+b}\,a^3b^3}{3003}+\frac{48x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a\,b^5}{143}-\frac{4096x^{\frac{13}{4}}\sqrt{\sqrt{x}\,a+b}\,a^6}{3003}-\frac{512x^{\frac{9}{4}}\sqrt{\sqrt{x}\,a+b}\,a^4b^2}{1001}-\frac{160x^{\frac{5}{4}}\sqrt{\cdots}}{4\cdots}}{\sqrt{x}\,b^7x^3}$$

input
```
int(1/x^4/(b*x^(1/2)+a*x)^(1/2),x)
```

output
```
(4*(512*x**(3/4)*sqrt(sqrt(x)*a + b)*a**5*b*x**2 + 320*x**(3/4)*sqrt(sqrt(
x)*a + b)*a**3*b**3*x + 252*x**(3/4)*sqrt(sqrt(x)*a + b)*a*b**5 - 1024*x**
(1/4)*sqrt(sqrt(x)*a + b)*a**6*x**3 - 384*x**(1/4)*sqrt(sqrt(x)*a + b)*a**
4*b**2*x**2 - 280*x**(1/4)*sqrt(sqrt(x)*a + b)*a**2*b**4*x - 231*x**(1/4)*
sqrt(sqrt(x)*a + b)*b**6 + 1024*sqrt(x)*sqrt(a)*a**6*x**3))/(3003*sqrt(x)*
b**7*x**3)
```

3.53 $\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 202

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4b^5\sqrt{x}}{a^6\sqrt{b\sqrt{x}+ax}}+\frac{437b^4\sqrt{b\sqrt{x}+ax}}{64a^6}-\frac{103b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5}+\frac{71b^2x\sqrt{b\sqrt{x}+ax}}{40a^4}-\frac{19bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3}+\frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a^2}-\frac{693b^5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{64a^{13/2}}$$

output
```
4*b^5*x^(1/2)/a^6/(b*x^(1/2)+a*x)^(1/2)+437/64*b^4*(b*x^(1/2)+a*x)^(1/2)/a
^6-103/32*b^3*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^5+71/40*b^2*x*(b*x^(1/2)+a*x
)^(1/2)/a^4-19/20*b*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^3+2/5*x^2*(b*x^(1/2)+a
*x)^(1/2)/a^2-693/64*b^5*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^
(13/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(3465b^5+1155ab^4\sqrt{x}-462a^2b^3x+264a^3b^2x^{3/2}-176a^4bx^2+128a^5x\right.}{320a^6\left(b+a\sqrt{x}\right)}$$
$$-\frac{693b^5\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{64a^{13/2}}$$

input
```
Integrate[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(3465*b^5 + 1155*a*b^4*Sqrt[x] - 462*a^2*b^3*x + 26
4*a^3*b^2*x^(3/2) - 176*a^4*b*x^2 + 128*a^5*x^(5/2)))/(320*a^6*(b + a*Sqrt
[x])) - (693*b^5*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])
/(64*a^(13/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {1924, 1124, 25, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(ax+b\sqrt{x}\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 1924}$$
$$2\int \frac{x^{7/2}}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$
$$\Big\downarrow \text{ 1124}$$
$$2\left(\frac{\int -\frac{-x^{5/2}a^5+bx^2a^4-b^2x^{3/2}a^3+b^3xa^2-b^4\sqrt{x}a+b^5}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^6}+\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \ 25$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\int \frac{-x^{5/2}a^5+bx^2a^4-b^2x^{3/2}a^3+b^3xa^2-b^4\sqrt{x}a+b^5}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^6}\right)$$

$$\Big\downarrow \ 2192$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\int \frac{19bx^2a^5-10b^2x^{3/2}a^4+10b^3xa^3-10b^4\sqrt{x}a^2+10b^5a}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{5a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right)$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\int \frac{19bx^2a^5-10b^2x^{3/2}a^4+10b^3xa^3-10b^4\sqrt{x}a^2+10b^5a}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right)$$

$$\Big\downarrow \ 2192$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\int \frac{-213b^2x^{3/2}a^5+80b^3xa^4-80b^4\sqrt{x}a^3+80b^5a^2}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right)$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\int \frac{-213b^2x^{3/2}a^5+80b^3xa^4-80b^4\sqrt{x}a^3+80b^5a^2}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right)$$

$$\Big\downarrow \ 2192$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\frac{\int \frac{15\left(103b^3xa^5-32b^4\sqrt{x}a^4+32b^5a^3\right)}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{3a}-71a^4b^2x\sqrt{ax+b\sqrt{x}}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right)$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\frac{5\int \frac{103b^3xa^5-32b^4\sqrt{x}a^4+32b^5a^3}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}-71a^4b^2x\sqrt{ax+b\sqrt{x}}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+b\sqrt{x}}}{a^6}\right)$$

$$\Big\downarrow \ 2192$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\frac{5\left(\frac{\int \frac{a^4b^4(128b-437a\sqrt{x})}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}+\frac{103}{2}a^4b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}\right)}{2a}-71a^4b^2x\sqrt{ax+b\sqrt{x}}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+}}{a^6}\right.$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\frac{5\left(\frac{1}{4}a^3b^4\int \frac{128b-437a\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}+\frac{103}{2}a^4b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}\right)}{2a}-71a^4b^2x\sqrt{ax+b\sqrt{x}}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-\frac{1}{5}a^4x^2\sqrt{ax+b}}{a^6}\right.$$

$$\Big\downarrow \ 1160$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\frac{5\left(\frac{1}{4}a^3b^4\left(\frac{693}{2}b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}-437\sqrt{ax+b\sqrt{x}}\right)+\frac{103}{2}a^4b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}\right)}{2a}-71a^4b^2x\sqrt{ax+b\sqrt{x}}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-}{a^6}\right.$$

$$\Big\downarrow \ 1091$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\frac{5\left(\frac{1}{4}a^3b^4\left(693b\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}-437\sqrt{ax+b\sqrt{x}}\right)+\frac{103}{2}a^4b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}\right)}{2a}-71a^4b^2x\sqrt{ax+b\sqrt{x}}}{8a}+\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}}{10a}-}{a^6}\right.$$

$$\Big\downarrow \ 219$$

$$2\left(\frac{2b^5\sqrt{x}}{a^6\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{19}{4}a^4bx^{3/2}\sqrt{ax+b\sqrt{x}}+\frac{\frac{5\left(\frac{103}{2}a^4b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}+\frac{1}{4}a^3b^4\left(\frac{693b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}-437\sqrt{ax+b\sqrt{x}}\right)\right)}{2a}-71a^4b^2x\sqrt{ax+}}{8a}}{10a}}{a^6}\right.$$

input
```
Int[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
2*((2*b^5*Sqrt[x])/(a^6*Sqrt[b*Sqrt[x] + a*x]) - (-1/5*(a^4*x^2*Sqrt[b*Sqr
t[x] + a*x]) + ((19*a^4*b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/4 + (-71*a^4*b^2*
x*Sqrt[b*Sqrt[x] + a*x] + (5*((103*a^4*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/
2 + (a^3*b^4*(-437*Sqrt[b*Sqrt[x] + a*x] + (693*b*ArcTanh[(Sqrt[a]*Sqrt[x]
)/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]))/4))/(2*a))/(8*a))/(10*a))/a^6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{2x^{3}}{5a\sqrt{b\sqrt{x}+xa}}-\frac{11b\left(\frac{x^{\frac{5}{2}}}{4a\sqrt{b\sqrt{x}+xa}}-\frac{9b\left(\frac{x^{2}}{3a\sqrt{b\sqrt{x}+xa}}-\frac{7b\left(\frac{x^{\frac{3}{2}}}{2a\sqrt{b\sqrt{x}+xa}}-\frac{5b\left(\frac{x}{a\sqrt{b\sqrt{x}+xa}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+xa}}-\frac{b\left(-\frac{\cdots}{a\sqrt{b\cdots}}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{6a\cdots}\right.}{8a\cdots}\right.}{5a\cdots}$
default	$-\frac{\sqrt{b\sqrt{x}+xa}\left(352\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}x^{\frac{3}{2}}a^{\frac{11}{2}}b-256\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}x^{2}a^{\frac{13}{2}}-528\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}x\,a^{\frac{9}{2}}b^{2}+4060\sqrt{b\sqrt{x}+xa}\,x^{\frac{3}{2}}a^{\frac{9}{2}}b^{3}\cdots\right.}{\cdots}$

input
```
int(x^3/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^3/a/(b*x^(1/2)+x*a)^(1/2)-11/5*b/a*(1/4*x^(5/2)/a/(b*x^(1/2)+x*a)^(1
/2)-9/8*b/a*(1/3*x^2/a/(b*x^(1/2)+x*a)^(1/2)-7/6*b/a*(1/2*x^(3/2)/a/(b*x^(
1/2)+x*a)^(1/2)-5/4*b/a*(x/a/(b*x^(1/2)+x*a)^(1/2)-3/2*b/a*(-x^(1/2)/a/(b*
x^(1/2)+x*a)^(1/2)-1/2*b/a*(-1/a/(b*x^(1/2)+x*a)^(1/2)+1/b/a*(b+2*x^(1/2)*
a)/(b*x^(1/2)+x*a)^(1/2))+1/a^(3/2)*ln((1/2*b+x^(1/2)*a)/a^(1/2)+(b*x^(1/2
)+x*a)^(1/2))))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^3}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(x**3/(a*x + b*sqrt(x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(a*x + b*sqrt(x))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{1}{320}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\sqrt{x}\left(\frac{8\sqrt{x}}{a^2}-\frac{19\,b}{a^3}\right)+\frac{71\,b^2}{a^4}\right)\sqrt{x}-\frac{515\,b^3}{a^5}\right)\sqrt{x}+\frac{2185}{a^6}\right.$$
$$+\frac{693\,b^5\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{128\,a^{\frac{13}{2}}}$$
$$+\frac{4\,b^6}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{13}{2}}}$$

input
```
integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/320*sqrt(a*x + b*sqrt(x))*(2*(4*(2*sqrt(x)*(8*sqrt(x)/a^2 - 19*b/a^3) +
71*b^2/a^4)*sqrt(x) - 515*b^3/a^5)*sqrt(x) + 2185*b^4/a^6) + 693/128*b^5*l
og(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(13/2)
 + 4*b^6/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*a^(13/2)
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(x^3/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
int(x^3/(a*x + b*x^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{512x^{\frac{11}{4}}\sqrt{\sqrt{x}\,a+b}\,a^6+1056x^{\frac{7}{4}}\sqrt{\sqrt{x}\,a+b}\,a^4b^2+4620x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a^2b^4-704x^{\frac{9}{4}}}{}$$

input
```
int(x^3/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
(512*x**(3/4)*sqrt(sqrt(x)*a + b)*a**6*x**2 + 1056*x**(3/4)*sqrt(sqrt(x)*a
 + b)*a**4*b**2*x + 4620*x**(3/4)*sqrt(sqrt(x)*a + b)*a**2*b**4 - 704*x**(
1/4)*sqrt(sqrt(x)*a + b)*a**5*b*x**2 - 1848*x**(1/4)*sqrt(sqrt(x)*a + b)*a
**3*b**3*x + 13860*x**(1/4)*sqrt(sqrt(x)*a + b)*a*b**5 - 13860*sqrt(x)*sqr
t(a)*log((sqrt(sqrt(x)*a + b) + x**(1/4)*sqrt(a))/sqrt(b))*a*b**5 + 8085*s
qrt(x)*sqrt(a)*a*b**5 - 13860*sqrt(a)*log((sqrt(sqrt(x)*a + b) + x**(1/4)*
sqrt(a))/sqrt(b))*b**6 + 8085*sqrt(a)*b**6)/(1280*a**7*(sqrt(x)*a + b))
```

3.54 $\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 144

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4b^3\sqrt{x}}{a^4\sqrt{b\sqrt{x}+ax}}+\frac{19b^2\sqrt{b\sqrt{x}+ax}}{4a^4} -\frac{11b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3}+\frac{2x\sqrt{b\sqrt{x}+ax}}{3a^2}-\frac{35b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^{9/2}}$$

output
```
4*b^3*x^(1/2)/a^4/(b*x^(1/2)+a*x)^(1/2)+19/4*b^2*(b*x^(1/2)+a*x)^(1/2)/a^4
-11/6*b*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^3+2/3*x*(b*x^(1/2)+a*x)^(1/2)/a^2-
35/4*b^3*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(105b^3+35ab^2\sqrt{x}-14a^2bx+8a^3x^{3/2}\right)}{12a^4\left(b+a\sqrt{x}\right)} -\frac{35b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{4a^{9/2}}$$

input
```
Integrate[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(105*b^3 + 35*a*b^2*Sqrt[x] - 14*a^2*b*x + 8*a^3*x^
(3/2)))/(12*a^4*(b + a*Sqrt[x])) - (35*b^3*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x]
 + a*x])/(b + a*Sqrt[x])])/(4*a^(9/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1924, 1124, 25, 2192, 27, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x^{5/2}}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1124}$$

$$2\left(\frac{\int -\frac{-x^{3/2}a^3+bxa^2-b^2\sqrt{x}a+b^3}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^4}+\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\int \frac{-x^{3/2}a^3+bxa^2-b^2\sqrt{x}a+b^3}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^4}\right)$$

$$\Big\downarrow \text{2192}$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\int \frac{11bxa^3-6b^2\sqrt{x}a^2+6b^3a}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{3a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\int \frac{11bxa^3-6b^2\sqrt{x}a^2+6b^3a}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{6a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

$$\Big\downarrow \; 2192$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{\int \frac{3a^2b^2(8b-19a\sqrt{x})}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}+\frac{11}{2}a^2b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{3}{4}ab^2\int \frac{8b-19a\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}+\frac{11}{2}a^2b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

$$\Big\downarrow \; 1160$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{3}{4}ab^2\left(\frac{35}{2}b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}-19\sqrt{ax+b\sqrt{x}}\right)+\frac{11}{2}a^2b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

$$\Big\downarrow \; 1091$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{3}{4}ab^2\left(35b\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}-19\sqrt{ax+b\sqrt{x}}\right)+\frac{11}{2}a^2b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

$$\Big\downarrow \; 219$$

$$2\left(\frac{2b^3\sqrt{x}}{a^4\sqrt{ax+b\sqrt{x}}}-\frac{\frac{\frac{11}{2}a^2b\sqrt{x}\sqrt{ax+b\sqrt{x}}+\frac{3}{4}ab^2\left(\frac{35b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}-19\sqrt{ax+b\sqrt{x}}\right)}{6a}-\frac{1}{3}a^2x\sqrt{ax+b\sqrt{x}}}{a^4}\right)$$

input
```
Int[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
2*((2*b^3*Sqrt[x])/(a^4*Sqrt[b*Sqrt[x] + a*x]) - (-1/3*(a^2*x*Sqrt[b*Sqrt[
x] + a*x]) + ((11*a^2*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/2 + (3*a*b^2*(-19*S
qrt[b*Sqrt[x] + a*x] + (35*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*
x]])/Sqrt[a]))/4)/(6*a))/a^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{2x^{2}}{3a\sqrt{b\sqrt{x}+xa}}-\frac{7b\left(\frac{x^{\frac{3}{2}}}{2a\sqrt{b\sqrt{x}+xa}}-\frac{5b\left(\frac{x}{a\sqrt{b\sqrt{x}+xa}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+xa}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+xa}}+\frac{b+2\sqrt{x}\,a}{ba\sqrt{b\sqrt{x}+xa}}\right)}{2a}+\frac{\ln\left(\frac{\frac{b}{2}+\sqrt{x}\,a}{\sqrt{a}}+\cdots\right)}{a^{\frac{3}{2}}\cdots}\right)}{2a}\right)}{4a}\right)}{3a}$
default	$\frac{\sqrt{b\sqrt{x}+xa}\left(16x\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}a^{\frac{9}{2}}-60\sqrt{b\sqrt{x}+xa}\,x^{\frac{3}{2}}a^{\frac{9}{2}}b+32\sqrt{x}\,a^{\frac{7}{2}}\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}b-150\sqrt{b\sqrt{x}+xa}\,x\,a^{\frac{7}{2}}b^{2}+240x\,a^{\frac{7}{2}}\sqrt{\cdots}\right.}{\cdots}$

input
```
int(x^2/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^2/a/(b*x^(1/2)+x*a)^(1/2)-7/3*b/a*(1/2*x^(3/2)/a/(b*x^(1/2)+x*a)^(1/
2)-5/4*b/a*(x/a/(b*x^(1/2)+x*a)^(1/2)-3/2*b/a*(-x^(1/2)/a/(b*x^(1/2)+x*a)^
(1/2)-1/2*b/a*(-1/a/(b*x^(1/2)+x*a)^(1/2)+1/b/a*(b+2*x^(1/2)*a)/(b*x^(1/2)
+x*a)^(1/2))+1/a^(3/2)*ln((1/2*b+x^(1/2)*a)/a^(1/2)+(b*x^(1/2)+x*a)^(1/2))
)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(x**2/(a*x + b*sqrt(x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(a*x + b*sqrt(x))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{1}{12}\sqrt{ax+b\sqrt{x}}\left(2\sqrt{x}\left(\frac{4\sqrt{x}}{a^2}-\frac{11\,b}{a^3}\right)+\frac{57\,b^2}{a^4}\right) + \frac{35\,b^3\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{8\,a^{\frac{9}{2}}} + \frac{4\,b^4}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{9}{2}}}$$

input
```
integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/12*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)*(4*sqrt(x)/a^2 - 11*b/a^3) + 57*b^2/
a^4) + 35/8*b^3*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)
)) - b))/a^(9/2) + 4*b^4/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)
)) + b)*a^(9/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(x^2/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
int(x^2/(a*x + b*x^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{64x^{\frac{7}{4}}\sqrt{\sqrt{x}\,a+b}\,a^4+280x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a^2b^2-112x^{\frac{5}{4}}\sqrt{\sqrt{x}\,a+b}\,a^3b+840x^{\frac{1}{4}}\sqrt{\sqrt{x}}}{}$$

input
```
int(x^2/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
(64*x**(3/4)*sqrt(sqrt(x)*a + b)*a**4*x + 280*x**(3/4)*sqrt(sqrt(x)*a + b)
*a**2*b**2 - 112*x**(1/4)*sqrt(sqrt(x)*a + b)*a**3*b*x + 840*x**(1/4)*sqrt
(sqrt(x)*a + b)*a*b**3 - 840*sqrt(x)*sqrt(a)*log((sqrt(sqrt(x)*a + b) + x*
*(1/4)*sqrt(a))/sqrt(b))*a*b**3 + 525*sqrt(x)*sqrt(a)*a*b**3 - 840*sqrt(a)
*log((sqrt(sqrt(x)*a + b) + x**(1/4)*sqrt(a))/sqrt(b))*b**4 + 525*sqrt(a)*
b**4)/(96*a**5*(sqrt(x)*a + b))
```

3.55 $\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 82

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}} dx = \frac{4b\sqrt{x}}{a^2\sqrt{b\sqrt{x}+ax}} + \frac{2\sqrt{b\sqrt{x}+ax}}{a^2} - \frac{6b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{5/2}}$$

output
```
4*b*x^(1/2)/a^2/(b*x^(1/2)+a*x)^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)/a^2-6*b*arct
anh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}} dx = \frac{2\left(3b+a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{a^2\left(b+a\sqrt{x}\right)} - \frac{6b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{a^{5/2}}$$

input
```
Integrate[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
(2*(3*b + a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(a^2*(b + a*Sqrt[x])) - (6*b*A
rcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1924, 1124, 25, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(ax+b\sqrt{x}\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1924$$

$$2\int \frac{x^{3/2}}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \; 1124$$

$$2\left(\frac{\int -\frac{b-a\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^2}+\frac{2b\sqrt{x}}{a^2\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \; 25$$

$$2\left(\frac{2b\sqrt{x}}{a^2\sqrt{ax+b\sqrt{x}}}-\frac{\int \frac{b-a\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^2}\right)$$

$$\Big\downarrow \; 1160$$

$$2\left(\frac{2b\sqrt{x}}{a^2\sqrt{ax+b\sqrt{x}}}-\frac{\frac{3}{2}b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}-\sqrt{ax+b\sqrt{x}}}{a^2}\right)$$

$$\Big\downarrow \; 1091$$

$$2\left(\frac{2b\sqrt{x}}{a^2\sqrt{ax+b\sqrt{x}}}-\frac{3b\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}-\sqrt{ax+b\sqrt{x}}}{a^2}\right)$$

$$\Big\downarrow \; 219$$

$$2\left(\frac{2b\sqrt{x}}{a^2\sqrt{ax+b\sqrt{x}}}-\frac{\frac{3b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}-\sqrt{ax+b\sqrt{x}}}{a^2}\right)$$

input
```
Int[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
2*((2*b*Sqrt[x])/(a^2*Sqrt[b*Sqrt[x] + a*x]) - (-Sqrt[b*Sqrt[x] + a*x] + (
3*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a])/a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{2x}{a\sqrt{b\sqrt{x}+xa}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+xa}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+xa}}+\frac{b+2\sqrt{x}\,a}{ba\sqrt{b\sqrt{x}+xa}}\right)}{2a}+\frac{\ln\left(\frac{\frac{b}{2}+\sqrt{x}\,a}{\sqrt{a}}+\sqrt{b\sqrt{x}+xa}\right)}{a^{\frac{3}{2}}}\right)}{a}$
default	$\frac{\sqrt{b\sqrt{x}+xa}\left(6x\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}\,a^{\frac{5}{2}}-3x\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}\sqrt{a}+b}{2\sqrt{a}}\right)a^{2}b+12\sqrt{x}\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}\,a^{\frac{3}{2}}b-6\sqrt{x}\,\ln\left(\cdots\right.}{a^{\frac{5}{2}}\sqrt{\sqrt{x}\left(\sqrt{\cdots\right.}}}$

input
```
int(x/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/a/(b*x^(1/2)+x*a)^(1/2)-3*b/a*(-x^(1/2)/a/(b*x^(1/2)+x*a)^(1/2)-1/2*b/
a*(-1/a/(b*x^(1/2)+x*a)^(1/2)+1/b/a*(b+2*x^(1/2)*a)/(b*x^(1/2)+x*a)^(1/2))
+1/a^(3/2)*ln((1/2*b+x^(1/2)*a)/a^(1/2)+(b*x^(1/2)+x*a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(x/(a*x + b*sqrt(x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(a*x + b*sqrt(x))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{3\,b\log\left(\left|-2\,\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{a^{\frac{5}{2}}}+\frac{2\,\sqrt{ax+b\sqrt{x}}}{a^{2}}+\frac{4\,b^{2}}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{5}{2}}}$$

input
```
integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
3*b*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(
5/2) + 2*sqrt(a*x + b*sqrt(x))/a^2 + 4*b^2/((sqrt(a)*(sqrt(a)*sqrt(x) - sq
rt(a*x + b*sqrt(x))) + b)*a^(5/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(x/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
int(x/(a*x + b*x^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.38

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a^2+12x^{\frac{1}{4}}\sqrt{\sqrt{x}\,a+b}\,ab-12\sqrt{x}\,\sqrt{a}\log\left(\frac{\sqrt{\sqrt{x}\,a+b}+x^{\frac{1}{4}}\sqrt{a}}{\sqrt{b}}\right)ab+9\sqrt{x}\cdots}{2a^3\left(\sqrt{x}\,a+b\right)}$$

input
```
int(x/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
(4*x**(3/4)*sqrt(sqrt(x)*a + b)*a**2 + 12*x**(1/4)*sqrt(sqrt(x)*a + b)*a*b
 - 12*sqrt(x)*sqrt(a)*log((sqrt(sqrt(x)*a + b) + x**(1/4)*sqrt(a))/sqrt(b)
)*a*b + 9*sqrt(x)*sqrt(a)*a*b - 12*sqrt(a)*log((sqrt(sqrt(x)*a + b) + x**(
1/4)*sqrt(a))/sqrt(b))*b**2 + 9*sqrt(a)*b**2)/(2*a**3*(sqrt(x)*a + b))
```

3.56 $\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\sqrt{x}}{b\sqrt{b\sqrt{x}+ax}}$$

output
```
4*x^(1/2)/b/(b*x^(1/2)+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\sqrt{b\sqrt{x}+ax}}{b\left(b+a\sqrt{x}\right)}$$

input
```
Integrate[(b*Sqrt[x] + a*x)^(-3/2),x]
```

output
```
(4*Sqrt[b*Sqrt[x] + a*x])/(b*(b + a*Sqrt[x]))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1906$$

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[(b*Sqrt[x] + a*x)^(-3/2),x]
```

output
```
(4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(19) = 38$.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

method	result
derivativedivides	$-\frac{2}{a\sqrt{b\sqrt{x}+xa}}+\frac{2b+4\sqrt{x}\,a}{ba\sqrt{b\sqrt{x}+xa}}$
default	$\frac{\sqrt{b\sqrt{x}+xa}\left(2\sqrt{b\sqrt{x}+xa}\,x\,a^{\frac{5}{2}}+x\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{b\sqrt{x}+xa}\,\sqrt{a}+b}{2\sqrt{a}}\right)a^{2}b+2x\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}\,a^{\frac{5}{2}}-x\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{\sqrt{x}\left(\sqrt{x}\,a-\right.}}{2\sqrt{a}}\right.}{}$

input
```
int(1/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a/(b*x^(1/2)+x*a)^(1/2)+2/b/a*(b+2*x^(1/2)*a)/(b*x^(1/2)+x*a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\sqrt{ax+b\sqrt{x}}\left(a\sqrt{x}-b\right)}{a^2bx-b^3}$$

input
```
integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
4*sqrt(a*x + b*sqrt(x))*(a*sqrt(x) - b)/(a^2*b*x - b^3)
```

Sympy [F]

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral((a*x + b*sqrt(x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + b*sqrt(x))^(-3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)\sqrt{a}}$$

input
```
integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
4/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*sqrt(a))
```

Mupad [B] (verification not implemented)

Time = 18.95 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\,x\,\left(\frac{b}{a\,\sqrt{x}}+1\right)}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}\,\left(\sqrt{\frac{b}{a\,\sqrt{x}}+1}+1\right)}$$

input
```
int(1/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
-(4*x*(b/(a*x^(1/2)) + 1))/((a*x + b*x^(1/2))^(3/2)*((b/(a*x^(1/2)) + 1)^(
1/2) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4\sqrt{a}\,\sqrt{\sqrt{x}\,a+b}+4x^{\frac{1}{4}}a}{\sqrt{\sqrt{x}\,a+b}\,ab}$$

input
```
int(1/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
(4*(sqrt(a)*sqrt(sqrt(x)*a + b) + x**(1/4)*a))/(sqrt(sqrt(x)*a + b)*a*b)
```

3.57 $\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{16a\left(b+2a\sqrt{x}\right)}{3b^3\sqrt{b\sqrt{x}+ax}}-\frac{4}{3b\sqrt{x}\sqrt{b\sqrt{x}+ax}}$$

output
```
16/3*a*(b+2*a*x^(1/2))/b^3/(b*x^(1/2)+a*x)^(1/2)-4/3/b/x^(1/2)/(b*x^(1/2)+
a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}\left(b^2-4ab\sqrt{x}-8a^2x\right)}{3b^3\left(b+a\sqrt{x}\right)x}$$

input
```
Integrate[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(b^2 - 4*a*b*Sqrt[x] - 8*a^2*x))/(3*b^3*(b + a*S
qrt[x])*x)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{4\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{b}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{4\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{b}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{b}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
4/(b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]) + (4*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*
x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])))/b
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{16a\left(b+2\sqrt{x}\,a\right)}{3b^{3}\sqrt{b\sqrt{x}+xa}}-\frac{4}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+xa}}$
default	$\frac{\sqrt{b\sqrt{x}+xa}\,\left(24\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}x^{\frac{5}{2}}a^{\frac{7}{2}}-6\sqrt{b\sqrt{x}+xa}\,x^{\frac{7}{2}}a^{\frac{9}{2}}+3x^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{\sqrt{x}\,\left(\sqrt{x}\,a+b\right)}\,\sqrt{a}+b}{2\sqrt{a}}\right)a^{4}b-6x^{\frac{7}{2}}a^{\frac{9}{2}}\sqrt{\sqrt{x}\,(\sqrt{}}$

input
```
int(1/x/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
16/3*a*(b+2*x^(1/2)*a)/b^3/(b*x^(1/2)+x*a)^(1/2)-4/3/b/x^(1/2)/(b*x^(1/2)+
x*a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\left(4\,a^2bx-b^3-\left(8\,a^3x-5\,ab^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3\left(a^2b^3x^2-b^5x\right)}$$

input
```
integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-4/3*(4*a^2*b*x - b^3 - (8*a^3*x - 5*a*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))
/(a^2*b^3*x^2 - b^5*x)
```

Sympy [F]

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(x*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x*(a*x + b*x^(1/2))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{\frac{16\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,ab}{3}+\frac{32\sqrt{\sqrt{x}\,a+b}\,a^2x}{3}-\frac{4\sqrt{\sqrt{x}\,a+b}\,b^2}{3}-\frac{32x^{\frac{3}{4}}\sqrt{a}\,ab}{3}-\frac{32x^{\frac{5}{4}}\sqrt{a}\,a^2}{3}}{x^{\frac{1}{4}}b^3\left(\sqrt{x}\,b+ax\right)}$$

input
```
int(1/x/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
(4*(4*sqrt(x)*sqrt(sqrt(x)*a + b)*a*b + 8*sqrt(sqrt(x)*a + b)*a**2*x - sqr
t(sqrt(x)*a + b)*b**2 - 8*x**(3/4)*sqrt(a)*a*b - 8*x**(1/4)*sqrt(a)*a**2*x
))/(3*x**(1/4)*b**3*(sqrt(x)*b + a*x))
```

3.58 $\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 119

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{256a^3\left(b+2a\sqrt{x}\right)}{35b^5\sqrt{b\sqrt{x}+ax}}-\frac{4}{7bx^{3/2}\sqrt{b\sqrt{x}+ax}}+\frac{32a}{35b^2x\sqrt{b\sqrt{x}+ax}}-\frac{64a^2}{35b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}$$

output
```
256/35*a^3*(b+2*a*x^(1/2))/b^5/(b*x^(1/2)+a*x)^(1/2)-4/7/b/x^(3/2)/(b*x^(1
/2)+a*x)^(1/2)+32/35*a/b^2/x/(b*x^(1/2)+a*x)^(1/2)-64/35*a^2/b^3/x^(1/2)/(
b*x^(1/2)+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}\left(5b^4-8ab^3\sqrt{x}+16a^2b^2x-64a^3bx^{3/2}-128a^4x^2\right)}{35b^5\left(b+a\sqrt{x}\right)x^2}$$

input
```
Integrate[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(5*b^4 - 8*a*b^3*Sqrt[x] + 16*a^2*b^2*x - 64*a^3
*b*x^(3/2) - 128*a^4*x^2))/(35*b^5*(b + a*Sqrt[x])*x^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1921, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(ax+b\sqrt{x}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{8\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{b}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{8\left(-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{b}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{8\left(-\frac{6a\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{b}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{8\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int\frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{b}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{8\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{b}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
4/(b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x]) + (8*((-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*
x^2) - (6*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) - (4*a*((-4*Sqrt[b*S
qrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])))/(5*
b))/(7*b)))/b
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

method	result
derivativedivides	$-\frac{4}{7b\,x^{\frac{3}{2}}\sqrt{b\sqrt{x}+xa}}-\frac{16a\left(-\frac{2}{5bx\sqrt{b\sqrt{x}+xa}}-\frac{6a\left(-\frac{2}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+xa}}+\frac{8a\left(b+2\sqrt{x}\,a\right)}{3b^{3}\sqrt{b\sqrt{x}+xa}}\right)}{5b}\right)}{7b}$
default	$\frac{\sqrt{b\sqrt{x}+xa}\left(560\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}x^{\frac{9}{2}}a^{\frac{11}{2}}-210\sqrt{b\sqrt{x}+xa}\,x^{\frac{11}{2}}a^{\frac{13}{2}}+105x^{\frac{11}{2}}\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{\sqrt{x}\,\left(\sqrt{x}\,a+b\right)}\,\sqrt{a}+b}{2\sqrt{a}}\right)a^{6}b-210x^{\frac{11}{2}}\right.}{}$

input
```
int(1/x^2/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/7/b/x^(3/2)/(b*x^(1/2)+x*a)^(1/2)-16/7*a/b*(-2/5/b/x/(b*x^(1/2)+x*a)^(1
/2)-6/5*a/b*(-2/3/b/x^(1/2)/(b*x^(1/2)+x*a)^(1/2)+8/3*a*(b+2*x^(1/2)*a)/b^
3/(b*x^(1/2)+x*a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\left(64\,a^4bx^2-24\,a^2b^3x-5\,b^5-\left(128\,a^5x^2-80\,a^3b^2x-13\,ab^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\left(a^2b^5x^3-b^7x^2\right)}$$

input
```
integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-4/35*(64*a^4*b*x^2 - 24*a^2*b^3*x - 5*b^5 - (128*a^5*x^2 - 80*a^3*b^2*x -
 13*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^5*x^3 - b^7*x^2)
```

Sympy [F]

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**2*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

input
```
int(1/(x^2*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x^2*(a*x + b*x^(1/2))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2 \left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{\frac{256\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a^3bx}{35}+\frac{32\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a\,b^3}{35}+\frac{512\sqrt{\sqrt{x}\,a+b}\,a^4x^2}{35}-\frac{64\sqrt{\sqrt{x}\,a+b}\,a^2b^2x}{35}-\frac{4\sqrt{\sqrt{x}\,a+}}{7}}{x^{\frac{5}{4}}b^5\left(\sqrt{x}\,b+ax\right)}$$

input
```
int(1/x^2/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
(4*(64*sqrt(x)*sqrt(sqrt(x)*a + b)*a**3*b*x + 8*sqrt(x)*sqrt(sqrt(x)*a + b
)*a*b**3 + 128*sqrt(sqrt(x)*a + b)*a**4*x**2 - 16*sqrt(sqrt(x)*a + b)*a**2
*b**2*x - 5*sqrt(sqrt(x)*a + b)*b**4 - 128*x**(3/4)*sqrt(a)*a**3*b*x - 128
*x**(1/4)*sqrt(a)*a**4*x**2))/(35*x**(1/4)*b**5*x*(sqrt(x)*b + a*x))
```

3.59 $\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 177

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{2048a^5\left(b+2a\sqrt{x}\right)}{231b^7\sqrt{b\sqrt{x}+ax}} - \frac{4}{11bx^{5/2}\sqrt{b\sqrt{x}+ax}} + \frac{16a}{33b^2x^2\sqrt{b\sqrt{x}+ax}} - \frac{160a^2}{231b^3x^{3/2}\sqrt{b\sqrt{x}+ax}} + \frac{256a^3}{231b^4x\sqrt{b\sqrt{x}+ax}} - \frac{512a^4}{231b^5\sqrt{x}\sqrt{b\sqrt{x}+ax}}$$

output
```
2048/231*a^5*(b+2*a*x^(1/2))/b^7/(b*x^(1/2)+a*x)^(1/2)-4/11/b/x^(5/2)/(b*x
^(1/2)+a*x)^(1/2)+16/33*a/b^2/x^2/(b*x^(1/2)+a*x)^(1/2)-160/231*a^2/b^3/x^
(3/2)/(b*x^(1/2)+a*x)^(1/2)+256/231*a^3/b^4/x/(b*x^(1/2)+a*x)^(1/2)-512/23
1*a^4/b^5/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(21b^6-28ab^5\sqrt{x}+40a^2b^4x-64a^3b^3x^{3/2}+128a^4b^2x^2-512a^5bx^{5/2}-1024a^6x^3\right)}{231b^7\left(b+a\sqrt{x}\right)x^3}$$

input
```
Integrate[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(21*b^6 - 28*a*b^5*Sqrt[x] + 40*a^2*b^4*x - 64*a
^3*b^3*x^(3/2) + 128*a^4*b^2*x^2 - 512*a^5*b*x^(5/2) - 1024*a^6*x^3))/(231
*b^7*(b + a*Sqrt[x])*x^3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1921, 1922, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{12\int \frac{1}{x^{7/2}\sqrt{\sqrt{x}b+ax}}dx}{b}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{12\left(-\frac{10a\int \frac{1}{x^3\sqrt{\sqrt{x}b+ax}}dx}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{b}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{12\left(-\frac{10a\left(-\frac{8a\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{b}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \; 1922$$

$$\frac{12\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\int \frac{1}{x^{2}\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^{2}}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{b}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \; 1922$$

$$\frac{12\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^{2}}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{b}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \; 1922$$

$$\frac{12\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int\frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{b}+$$

$$\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \; 1920$$

$$\frac{12\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{b}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
4/(b*x^(5/2)*Sqrt[b*Sqrt[x] + a*x]) + (12*((-4*Sqrt[b*Sqrt[x] + a*x])/(11*
b*x^3) - (10*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(5/2)) - (8*a*((-4*Sqrt[
b*Sqrt[x] + a*x])/(7*b*x^2) - (6*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2
)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x]
)/(3*b^2*Sqrt[x])))/(5*b)))/(7*b)))/(9*b)))/(11*b)))/b
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

method	result
derivativedivides	$-\frac{4}{11 b\, x^{\frac{5}{2}} \sqrt{b\sqrt{x}+xa}}-\frac{24 a \left(-\frac{2}{9 b\, x^{2} \sqrt{b\sqrt{x}+xa}}-\frac{10 a \left(-\frac{2}{7 b\, x^{\frac{3}{2}} \sqrt{b\sqrt{x}+xa}}-\frac{8 a \left(-\frac{2}{5 b x \sqrt{b\sqrt{x}+xa}}-\frac{6 a \left(-\frac{2}{3 b \sqrt{x}\, \sqrt{b\sqrt{x}+xa}}+\frac{8}{3\cdots}\right.}{5 b}\right.}{7 b}\right.}{9 b}\right.}{11 b}$
default	$\frac{\sqrt{b\sqrt{x}+xa}\left(2048 \left(b\sqrt{x}+xa\right)^{\frac{3}{2}} x^{\frac{11}{2}} a^{\frac{11}{2}} b^{2}+8716 \left(b\sqrt{x}+xa\right)^{\frac{3}{2}} x^{6} a^{\frac{13}{2}} b-4620 \sqrt{b\sqrt{x}+xa}\, x^{7} a^{\frac{15}{2}} b-4620 x^{7} a^{\frac{15}{2}} \sqrt{\sqrt{x}\,(\sqrt{x}\cdots\right.}{\cdots}$

input
```
int(1/x^3/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/11/b/x^(5/2)/(b*x^(1/2)+x*a)^(1/2)-24/11*a/b*(-2/9/b/x^2/(b*x^(1/2)+x*a
)^(1/2)-10/9*a/b*(-2/7/b/x^(3/2)/(b*x^(1/2)+x*a)^(1/2)-8/7*a/b*(-2/5/b/x/(
b*x^(1/2)+x*a)^(1/2)-6/5*a/b*(-2/3/b/x^(1/2)/(b*x^(1/2)+x*a)^(1/2)+8/3*a*(
b+2*x^(1/2)*a)/b^3/(b*x^(1/2)+x*a)^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx =$$
$$-\frac{4\left(512\,a^6bx^3-192\,a^4b^3x^2-68\,a^2b^5x-21\,b^7-\left(1024\,a^7x^3-640\,a^5b^2x^2-104\,a^3b^4x-49\,ab^6\right)\sqrt{x}\right)\sqrt{ax+}}{231\left(a^2b^7x^4-b^9x^3\right)}$$

input
```
integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-4/231*(512*a^6*b*x^3 - 192*a^4*b^3*x^2 - 68*a^2*b^5*x - 21*b^7 - (1024*a^
7*x^3 - 640*a^5*b^2*x^2 - 104*a^3*b^4*x - 49*a*b^6)*sqrt(x))*sqrt(a*x + b*
sqrt(x))/(a^2*b^7*x^4 - b^9*x^3)
```

Sympy [F]

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**3*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^3\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x^3*(a*x + b*x^(1/2))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3 \left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{\frac{2048\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a^5 b\,x^2}{231} + \frac{256\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a^3 b^3 x}{231} + \frac{16\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a\,b^5}{33} + \frac{4096\sqrt{\sqrt{x}\,a+b}\,a^6 x^3}{231} - 5}{x^{\frac{9}{4}} b^7 \left(\sqrt{x}\,b\right.}$$

input
```
int(1/x^3/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
(4*(512*sqrt(x)*sqrt(sqrt(x)*a + b)*a**5*b*x**2 + 64*sqrt(x)*sqrt(sqrt(x)*
a + b)*a**3*b**3*x + 28*sqrt(x)*sqrt(sqrt(x)*a + b)*a*b**5 + 1024*sqrt(sqr
t(x)*a + b)*a**6*x**3 - 128*sqrt(sqrt(x)*a + b)*a**4*b**2*x**2 - 40*sqrt(s
qrt(x)*a + b)*a**2*b**4*x - 21*sqrt(sqrt(x)*a + b)*b**6 - 1024*x**(3/4)*sq
rt(a)*a**5*b*x**2 - 1024*x**(1/4)*sqrt(a)*a**6*x**3))/(231*x**(1/4)*b**7*x
**2*(sqrt(x)*b + a*x))
```

3.60 $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 204

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} -\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}-\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} +\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}+\frac{231b^6\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{256a^{13/2}}$$

output
```
-231/256*b^5*(b*x^(1/2)+a*x)^(1/2)/a^6+77/128*b^4*x^(1/2)*(b*x^(1/2)+a*x)^
(1/2)/a^5-77/160*b^3*x*(b*x^(1/2)+a*x)^(1/2)/a^4+33/80*b^2*x^(3/2)*(b*x^(1
/2)+a*x)^(1/2)/a^3-11/30*b*x^2*(b*x^(1/2)+a*x)^(1/2)/a^2+1/3*x^(5/2)*(b*x^
(1/2)+a*x)^(1/2)/a+231/256*b^6*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/
2))/a^(13/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.62

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(-3465b^5+2310ab^4\sqrt{x}-1848a^2b^3x+1584a^3b^2x^{3/2}-1408a^4bx^2+128\right.}{3840a^6} + \frac{231b^6\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{256a^{13/2}}$$

input
```
Integrate[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(-3465*b^5 + 2310*a*b^4*Sqrt[x] - 1848*a^2*b^3*x +
1584*a^3*b^2*x^(3/2) - 1408*a^4*b*x^2 + 1280*a^5*x^(5/2)))/(3840*a^6) + (2
31*b^6*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(256*a^(1
3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1924, 1134, 1134, 1134, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

$$\Big\downarrow \; 1924$$

$$2\int \frac{x^3}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}$$

$$\Big\downarrow \; 1134$$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\int \frac{x^{5/2}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{12a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\int\frac{x^2}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{10a}\right)}{12a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\int\frac{x^{3/2}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{8a}\right)}{10a}\right)}{12a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\int\frac{x}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{6a}\right)}{8a}\right)}{10a}\right)}{12a}\right)$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\left(\frac{x^{2}\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\int\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}\right)}{6a}\right)}{8a}\right)}{10a}\right)}{12a}\right)$$

↓ 1160

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\left(\frac{x^{2}\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int\frac{1}{\sqrt{\sqrt{x}b+ax}}\,d\sqrt{x}}{2a}\right.}{4a}\right.}{6a}\right.}{8a}\right.}{10a}\right.}{12a}\right.$$

↓ 1091

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a}-\frac{11b\left(\frac{x^{2}\sqrt{ax+b\sqrt{x}}}{5a}-\frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int\frac{1}{1-ax}\,d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+a\cdots}}\cdots}{a}\cdots\right.}{4a}\cdots\right.}{6a}\cdots\right.}{8a}\cdots\right.}{10a}\cdots\right.}{12a}\cdots\right.$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{6a} - \frac{11b\left(\frac{x^2\sqrt{ax+b\sqrt{x}}}{5a} - \frac{9b\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a} - \frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a} - \frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a} - \frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}\cdots}{\sqrt{ax+\cdots}}\right)}{a^{3/2}}\right)}{4a}\right)}{6a}\right)}{8a}\right)}{10a}\right)}{12a}\right)$$

input
```
Int[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
2*((x^(5/2)*Sqrt[b*Sqrt[x] + a*x])/(6*a) - (11*b*((x^2*Sqrt[b*Sqrt[x] + a*
x])/(5*a) - (9*b*((x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(4*a) - (7*b*((x*Sqrt[b*
Sqrt[x] + a*x])/(3*a) - (5*b*((Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(2*a) - (3*b
*(Sqrt[b*Sqrt[x] + a*x]/a - (b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] +
a*x]])/a^(3/2)))/(4*a)))/(6*a)))/(8*a)))/(10*a)))/(12*a))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}\sqrt{b\sqrt{x}+xa}}{3a}-\frac{11b\left(\frac{x^{2}\sqrt{b\sqrt{x}+xa}}{5a}-\frac{9b\left(\frac{x^{\frac{3}{2}}\sqrt{b\sqrt{x}+xa}}{4a}-\frac{7b\left(\frac{x\sqrt{b\sqrt{x}+xa}}{3a}-\frac{5b\left(\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+xa}}{2a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+xa}}{a}-\frac{b\ln\left(\frac{b}{2}+\right.}{\sqrt{}}\right.}{4a}\right.}{6a}\right.}{8a}\right.}{10a}\right.}{6a}$
default	$\frac{\sqrt{b\sqrt{x}+xa}\,\left(2560x^{\frac{3}{2}}\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}a^{\frac{11}{2}}+8544\sqrt{x}\,\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}a^{\frac{7}{2}}b^{2}-5376\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}a^{\frac{9}{2}}bx+16860\sqrt{x}\,\sqrt{b\sqrt{x}+xa}\,a^{\frac{5}{2}}\right.}{}$

input
```
int(x^(5/2)/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(5/2)*(b*x^(1/2)+x*a)^(1/2)/a-11/6*b/a*(1/5*x^2*(b*x^(1/2)+x*a)^(1/2
)/a-9/10*b/a*(1/4*x^(3/2)*(b*x^(1/2)+x*a)^(1/2)/a-7/8*b/a*(1/3*x*(b*x^(1/2
)+x*a)^(1/2)/a-5/6*b/a*(1/2*x^(1/2)*(b*x^(1/2)+x*a)^(1/2)/a-3/4*b/a*((b*x^
(1/2)+x*a)^(1/2)/a-1/2*b/a^(3/2)*ln((1/2*b+x^(1/2)*a)/a^(1/2)+(b*x^(1/2)+x
*a)^(1/2))))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.98

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \sqrt{ax+b\sqrt{x}}\left(\frac{x^{\frac{5}{2}}}{6a} - \frac{11bx^2}{60a^2} + \frac{33b^2x^{\frac{3}{2}}}{160a^3} - \frac{77b^3x}{320a^4} + \frac{77b^4\sqrt{x}}{256a^5} - \frac{231b^5}{512a^6}\right) + \frac{231b^6\left(\begin{cases} \frac{\log\left(2\sqrt{a}\sqrt{ax \dots}\right.}{\dots} \\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log \dots}{\sqrt{a\left(\sqrt{x} \dots\right.}} \end{cases}\right.}{\dots} \\ \frac{2\left(b\sqrt{x}\right)^{\frac{13}{2}}}{13b^7} \\ \tilde{\infty}x^{\frac{7}{2}} \end{cases}\right.$$

input
```
integrate(x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise((sqrt(a*x + b*sqrt(x))*(x**(5/2)/(6*a) - 11*b*x**2/(60*a**2) +
 33*b**2*x**(3/2)/(160*a**3) - 77*b**3*x/(320*a**4) + 77*b**4*sqrt(x)/(256
*a**5) - 231*b**5/(512*a**6)) + 231*b**6*Piecewise((log(2*sqrt(a)*sqrt(a*x
 + b*sqrt(x)) + 2*a*sqrt(x) + b)/sqrt(a), Ne(b**2/a, 0)), ((sqrt(x) + b/(2
*a))*log(sqrt(x) + b/(2*a))/sqrt(a*(sqrt(x) + b/(2*a))**2), True))/(1024*a
**6), Ne(a, 0)), (2*(b*sqrt(x))**(13/2)/(13*b**7), Ne(b, 0)), (zoo*x**(7/2
), True))
```

Maxima [F]

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.60

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{3840}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\left(8\sqrt{x}\left(\frac{10\sqrt{x}}{a}-\frac{11\,b}{a^2}\right)+\frac{99\,b^2}{a^3}\right)\sqrt{x}-\frac{231\,b^3}{a^4}\right)\sqrt{x}+11\cdots\right.\right.$$
$$-\frac{231\,b^6\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{512\,a^{\frac{13}{2}}}$$

input
```
integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/3840*sqrt(a*x + b*sqrt(x))*(2*(4*(2*(8*sqrt(x)*(10*sqrt(x)/a - 11*b/a^2)
 + 99*b^2/a^3)*sqrt(x) - 231*b^3/a^4)*sqrt(x) + 1155*b^4/a^5)*sqrt(x) - 34
65*b^5/a^6) - 231/512*b^6*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x +
b*sqrt(x))) + b))/a^(13/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^{5/2}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(x^(5/2)/(a*x + b*x^(1/2))^(1/2),x)
```

output
```
int(x^(5/2)/(a*x + b*x^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.66

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1280x^{\frac{11}{4}}\sqrt{\sqrt{x}\,a+b}\,a^6+1584x^{\frac{7}{4}}\sqrt{\sqrt{x}\,a+b}\,a^4b^2+2310x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a^2b^4-1408x^{\frac{9}{4}}}{}$$

input
```
int(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

output
```
(1280*x**(3/4)*sqrt(sqrt(x)*a + b)*a**6*x**2 + 1584*x**(3/4)*sqrt(sqrt(x)*
a + b)*a**4*b**2*x + 2310*x**(3/4)*sqrt(sqrt(x)*a + b)*a**2*b**4 - 1408*x*
*(1/4)*sqrt(sqrt(x)*a + b)*a**5*b*x**2 - 1848*x**(1/4)*sqrt(sqrt(x)*a + b)
*a**3*b**3*x - 3465*x**(1/4)*sqrt(sqrt(x)*a + b)*a*b**5 + 3465*sqrt(a)*log
((sqrt(sqrt(x)*a + b) + x**(1/4)*sqrt(a))/sqrt(b))*b**6)/(3840*a**7)
```

3.61 $\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} -\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2}+\frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a}+\frac{35b^4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{32a^{9/2}}$$

output
```
-35/32*b^3*(b*x^(1/2)+a*x)^(1/2)/a^4+35/48*b^2*x^(1/2)*(b*x^(1/2)+a*x)^(1/
2)/a^3-7/12*b*x*(b*x^(1/2)+a*x)^(1/2)/a^2+1/2*x^(3/2)*(b*x^(1/2)+a*x)^(1/2
)/a+35/32*b^4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(-105b^3+70ab^2\sqrt{x}-56a^2bx+48a^3x^{3/2}\right)}{96a^4} + \frac{35b^4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{32a^{9/2}}$$

input
```
Integrate[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(-105*b^3 + 70*a*b^2*Sqrt[x] - 56*a^2*b*x + 48*a^3*
x^(3/2)))/(96*a^4) + (35*b^4*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b +
a*Sqrt[x])])/(32*a^(9/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1924, 1134, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{ 1924}$$

$$2\int \frac{x^2}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1134}$$

$$2\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\int \frac{x^{3/2}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{8a}\right)$$

$$\Big\downarrow \text{ 1134}$$

$$2\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\int \frac{x}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{6a}\right)}{8a}\right)$$

$$\Big\downarrow \text{ 1134}$$

$$2\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\int\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}\right)}{6a}\right)}{8a}\right)$$

$\big\downarrow$ 1160

$$2\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int\frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}\right)}{4a}\right)}{6a}\right)}{8a}\right)$$

$\big\downarrow$ 1091

$$2\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int\frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}}{a}\right)}{4a}\right)}{6a}\right)}{8a}\right)$$

$\downarrow$ 219

$$2\left(\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{4a}-\frac{7b\left(\frac{x\sqrt{ax+b\sqrt{x}}}{3a}-\frac{5b\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}\right)}{4a}\right)}{6a}\right)}{8a}\right)$$

input
```
Int[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
2*((x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(4*a) - (7*b*((x*Sqrt[b*Sqrt[x] + a*x])
/(3*a) - (5*b*((Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(2*a) - (3*b*(Sqrt[b*Sqrt[x
] + a*x]/a - (b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(3/2))
)/(4*a)))/(6*a)))/(8*a))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}\sqrt{b\sqrt{x}+xa}}{2a}-\frac{7b\left(\frac{x\sqrt{b\sqrt{x}+xa}}{3a}-\frac{5b\left(\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+xa}}{2a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+xa}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+\sqrt{x}\,a}{\sqrt{a}}+\sqrt{b\sqrt{x}+xa}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{6a}\right)}{4a}$
default	$\frac{\sqrt{b\sqrt{x}+xa}\left(96\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}\sqrt{x}\,a^{\frac{7}{2}}+348\sqrt{x}\,\sqrt{b\sqrt{x}+xa}\,a^{\frac{5}{2}}b^{2}-208\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}a^{\frac{5}{2}}b+174\sqrt{b\sqrt{x}+xa}\,a^{\frac{3}{2}}b^{3}-384a^{\frac{3}{2}}\sqrt{\cdots}\right.}{192\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}\,a^{\cdots}}$

input
```
int(x^(3/2)/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(3/2)*(b*x^(1/2)+x*a)^(1/2)/a-7/4*b/a*(1/3*x*(b*x^(1/2)+x*a)^(1/2)/a
-5/6*b/a*(1/2*x^(1/2)*(b*x^(1/2)+x*a)^(1/2)/a-3/4*b/a*((b*x^(1/2)+x*a)^(1/
2)/a-1/2*b/a^(3/2)*ln((1/2*b+x^(1/2)*a)/a^(1/2)+(b*x^(1/2)+x*a)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx=\text{Timed out}$$

input
```
integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\begin{cases} \sqrt{ax+b\sqrt{x}}\left(\frac{x^{\frac{3}{2}}}{4a} - \frac{7bx}{24a^2} + \frac{35b^2\sqrt{x}}{96a^3} - \frac{35b^3}{64a^4}\right) + \frac{35b^4\left(\begin{cases} \frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for} \\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{oth} \end{cases}\right)}{128a^4} \\ \frac{2\left(b\sqrt{x}\right)^{\frac{9}{2}}}{9b^5} \\ \tilde{\infty}x^{\frac{5}{2}} \end{cases}$$

input
```
integrate(x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise((sqrt(a*x + b*sqrt(x))*(x**(3/2)/(4*a) - 7*b*x/(24*a**2) + 35*
b**2*sqrt(x)/(96*a**3) - 35*b**3/(64*a**4)) + 35*b**4*Piecewise((log(2*sqr
t(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(x) + b)/sqrt(a), Ne(b**2/a, 0)), ((s
qrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a))/sqrt(a*(sqrt(x) + b/(2*a))**2), T
rue))/(128*a**4), Ne(a, 0)), (2*(b*sqrt(x))**(9/2)/(9*b**5), Ne(b, 0)), (z
oo*x**(5/2), True))
```

Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{96}\sqrt{ax+b\sqrt{x}}\left(2\left(4\sqrt{x}\left(\frac{6\sqrt{x}}{a}-\frac{7\,b}{a^2}\right)+\frac{35\,b^2}{a^3}\right)\sqrt{x}-\frac{105\,b^3}{a^4}\right) - \frac{35\,b^4\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{64\,a^{\frac{9}{2}}}$$

input
```
integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/96*sqrt(a*x + b*sqrt(x))*(2*(4*sqrt(x)*(6*sqrt(x)/a - 7*b/a^2) + 35*b^2/
a^3)*sqrt(x) - 105*b^3/a^4) - 35/64*b^4*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x)
 - sqrt(a*x + b*sqrt(x))) + b))/a^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^{3/2}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(x^(3/2)/(a*x + b*x^(1/2))^(1/2),x)
```

output
```
int(x^(3/2)/(a*x + b*x^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.67

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{48x^{\frac{7}{4}}\sqrt{\sqrt{x}\,a+b}\,a^4+70x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a^2b^2-56x^{\frac{5}{4}}\sqrt{\sqrt{x}\,a+b}\,a^3b-105x^{\frac{1}{4}}\sqrt{\sqrt{x}\,a+}}{96a^5}$$

input
```
int(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

output
```
(48*x**(3/4)*sqrt(sqrt(x)*a + b)*a**4*x + 70*x**(3/4)*sqrt(sqrt(x)*a + b)*
a**2*b**2 - 56*x**(1/4)*sqrt(sqrt(x)*a + b)*a**3*b*x - 105*x**(1/4)*sqrt(s
qrt(x)*a + b)*a*b**3 + 105*sqrt(a)*log((sqrt(sqrt(x)*a + b) + x**(1/4)*sqr
t(a))/sqrt(b))*b**4)/(96*a**5)
```

3.62 $\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}} dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}} dx = -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{3b^2 \operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{5/2}}$$

output
```
-3/2*b*(b*x^(1/2)+a*x)^(1/2)/a^2+x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a+3/2*b^2*a
rctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}} dx = \frac{\left(-3b+2a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{3b^2 \operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{2a^{5/2}}$$

input
```
Integrate[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
((-3*b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (3*b^2*ArcTanh[(Sqr
t[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(2*a^(5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1924, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}\right)$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}\right)}{4a}\right)$$

$$\Big\downarrow \text{1091}$$

$$2\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}}{a}\right)}{4a}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-\frac{3b\left(\frac{\sqrt{ax+b\sqrt{x}}}{a}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}\right)}{4a}\right)$$

input
```
Int[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

output
```
2*((Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(2*a) - (3*b*(Sqrt[b*Sqrt[x] + a*x]/a -
 (b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(3/2)))/(4*a))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+xa}}{a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+xa}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+\sqrt{x}\,a}{\sqrt{a}}+\sqrt{b\sqrt{x}+xa}\right)}{2a^{\frac{3}{2}}}\right)}{2a}$
default	$\frac{\sqrt{b\sqrt{x}+xa}\left(4\sqrt{x}\,\sqrt{b\sqrt{x}+xa}\,a^{\frac{5}{2}}+2\sqrt{b\sqrt{x}+xa}\,a^{\frac{3}{2}}b-8\sqrt{\sqrt{x}\,(\sqrt{x}\,a+b)}\,a^{\frac{3}{2}}b+4a\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{\sqrt{x}\,(\sqrt{x}\,a+b)}\,\sqrt{a}+b}{2\sqrt{a}}\right)b^{2}-}{4\sqrt{\sqrt{x}\,(\sqrt{x}\,a+b)}\,a^{\frac{7}{2}}}$

input
```
int(x^(1/2)/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)*(b*x^(1/2)+x*a)^(1/2)/a-3/2*b/a*((b*x^(1/2)+x*a)^(1/2)/a-1/2*b/a^(
3/2)*ln((1/2*b+x^(1/2)*a)/a^(1/2)+(b*x^(1/2)+x*a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx=\text{Timed out}$$

input
```
integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \left(\frac{\sqrt{x}}{2a}-\frac{3b}{4a^2}\right)\sqrt{ax+b\sqrt{x}}+\frac{3b^2\left(\begin{cases}\frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } \frac{b^2}{a}\neq 0\\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherwise}\end{cases}\right)}{8a^2} & \text{for } a\neq 0\\ \frac{2\left(b\sqrt{x}\right)^{\frac{5}{2}}}{5b^3} & \text{for } b\neq 0\\ \tilde{\infty}x^{\frac{3}{2}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise(((sqrt(x)/(2*a) - 3*b/(4*a**2))*sqrt(a*x + b*sqrt(x)) + 3*b**2
*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(x) + b)/sqrt(a)
, Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a))/sqrt(a*(sqrt
(x) + b/(2*a))**2), True))/(8*a**2), Ne(a, 0)), (2*(b*sqrt(x))**(5/2)/(5*b
**3), Ne(b, 0)), (zoo*x**(3/2), True))
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{2}\sqrt{ax+b\sqrt{x}}\left(\frac{2\sqrt{x}}{a}-\frac{3\,b}{a^2}\right) - \frac{3\,b^2\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{4\,a^{\frac{5}{2}}}$$

input
```
integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)/a - 3*b/a^2) - 3/4*b^2*log(abs(2*sqrt
(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{\sqrt{x}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(x^(1/2)/(a*x + b*x^(1/2))^(1/2),x)
```

output
```
int(x^(1/2)/(a*x + b*x^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{2x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a^2-3x^{\frac{1}{4}}\sqrt{\sqrt{x}\,a+b}\,ab+3\sqrt{a}\log\left(\frac{\sqrt{\sqrt{x}\,a+b}+x^{\frac{1}{4}}\sqrt{a}}{\sqrt{b}}\right)b^2}{2a^3}$$

input
```
int(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

output
```
(2*x**(3/4)*sqrt(sqrt(x)*a + b)*a**2 - 3*x**(1/4)*sqrt(sqrt(x)*a + b)*a*b
+ 3*sqrt(a)*log((sqrt(sqrt(x)*a + b) + x**(1/4)*sqrt(a))/sqrt(b))*b**2)/(2
*a**3)
```

3.63 $\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx=\frac{4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{\sqrt{a}}$$

output
```
4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx=\frac{4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(4*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1919, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1919}$$

$$2\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}$$

$$\Big\downarrow \text{1091}$$

$$4\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}$$

$$\Big\downarrow \text{219}$$

$$\frac{4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1919
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
 && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2\ln\left(\frac{\frac{b}{2}+\sqrt{x}\,a}{\sqrt{a}}+\sqrt{b\sqrt{x}+xa}\right)}{\sqrt{a}}$
default	$\frac{\sqrt{b\sqrt{x}+xa}\left(2\sqrt{b\sqrt{x}+xa}\,\sqrt{a}+b\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{b\sqrt{x}+xa}\,\sqrt{a}+b}{2\sqrt{a}}\right)-2\sqrt{\sqrt{x}\,(\sqrt{x}\,a+b)}\,\sqrt{a}+b\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{\sqrt{x}\,(\sqrt{x}\,a+b)}\,\sqrt{a}+}{2\sqrt{a}}\right.\right.}{\sqrt{\sqrt{x}\,(\sqrt{x}\,a+b)}\,b\sqrt{a}}$

input
```
int(1/x^(1/2)/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*ln((1/2*b+x^(1/2)*a)/a^(1/2)+(b*x^(1/2)+x*a)^(1/2))/a^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.91

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } a\neq 0\wedge \frac{b^2}{a}\neq 0 \\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{for } a\neq 0 \\ \frac{2\sqrt{b\sqrt{x}}}{b} & \text{for } b\neq 0 \\ \tilde{\infty}\sqrt{x} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
2*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(x) + b)/sqrt(a
), Ne(a, 0) & Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a))/
sqrt(a*(sqrt(x) + b/(2*a))**2), Ne(a, 0)), (2*sqrt(b*sqrt(x))/b, Ne(b, 0))
, (zoo*sqrt(x), True))
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*sqrt(x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^(1/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\sqrt{a}\log\left(\frac{\sqrt{\sqrt{x}\,a+b}+x^{\frac{1}{4}}\sqrt{a}}{\sqrt{b}}\right)}{a}$$

input
```
int(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

output
```
(4*sqrt(a)*log((sqrt(sqrt(x)*a + b) + x**(1/4)*sqrt(a))/sqrt(b)))/a
```

3.64 $\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{8a\sqrt{b\sqrt{x}+ax}}{3b^2\sqrt{x}}$$

output
```
-4/3*(b*x^(1/2)+a*x)^(1/2)/b/x+8/3*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\left(b-2a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{3b^2x}$$

input
```
Integrate[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*(b - 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

$$\Big\downarrow \text{1920}$$

$$\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

input
```
Int[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sq
rt[x])
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+xa}}{3bx}+\frac{8a\sqrt{b\sqrt{x}+xa}}{3b^{2}\sqrt{x}}$
default	$-\frac{\sqrt{b\sqrt{x}+xa}\left(6x^{\frac{5}{2}}\sqrt{b\sqrt{x}+xa}\,a^{\frac{5}{2}}+6x^{\frac{5}{2}}a^{\frac{5}{2}}\sqrt{\sqrt{x}\,(\sqrt{x}\,a+b)}+3x^{\frac{5}{2}}\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{b\sqrt{x}+xa}\,\sqrt{a}+b}{2\sqrt{a}}\right)a^{2}b-3x^{\frac{5}{2}}\ln\left(\frac{2\sqrt{x}\,a+2}{}\right.\right.}{3\sqrt{\sqrt{x}\,(\sqrt{x}\,a+b)}\,b^{3}x^{\frac{5}{2}}\sqrt{a}}$

input
```
int(1/x^(3/2)/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(b*x^(1/2)+x*a)^(1/2)/b/x+8/3*a*(b*x^(1/2)+x*a)^(1/2)/b^2/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx=\frac{4\sqrt{ax+b\sqrt{x}}\,(2\,a\sqrt{x}-b)}{3\,b^{2}x}$$

input
```
integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
4/3*sqrt(a*x + b*sqrt(x))*(2*a*sqrt(x) - b)/(b^2*x)
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**(3/2)*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(3\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)}{3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3}$$

input
```
integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/3*(3*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)/(sqrt(a)*sqr
t(x) - sqrt(a*x + b*sqrt(x)))^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{3/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^(3/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^(3/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\frac{8x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a}{3}-\frac{4x^{\frac{1}{4}}\sqrt{\sqrt{x}\,a+b}\,b}{3}-\frac{8\sqrt{a}\,ax}{3}}{b^2x}$$

input
```
int(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

output
```
(4*(2*x**(3/4)*sqrt(sqrt(x)*a + b)*a - x**(1/4)*sqrt(sqrt(x)*a + b)*b - 2*
sqrt(a)*a*x))/(3*b**2*x)
```

3.65 $\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2}+\frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} -\frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x}+\frac{64a^3\sqrt{b\sqrt{x}+ax}}{35b^4\sqrt{x}}$$

output
```
-4/7*(b*x^(1/2)+a*x)^(1/2)/b/x^2+24/35*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(3/2)
-32/35*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x+64/35*a^3*(b*x^(1/2)+a*x)^(1/2)/b^4
/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(5b^3-6ab^2\sqrt{x}+8a^2bx-16a^3x^{3/2}\right)}{35b^4x^2}$$

input
```
Integrate[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(5*b^3 - 6*a*b^2*Sqrt[x] + 8*a^2*b*x - 16*a^3*x^
(3/2)))/(35*b^4*x^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{6a\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{6a\left(-\frac{4a\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

$$\Big\downarrow \text{ 1920}$$

$$-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

input
```
Int[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) - (6*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(5
*b*x^(3/2)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[
x] + a*x])/(3*b^2*Sqrt[x])))/(5*b)))/(7*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+xa}}{7b\,x^{2}}-\frac{12a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+xa}}{3b^{2}\sqrt{x}}\right)}{5b}\right)}{7b}$
default	$-\frac{\sqrt{b\sqrt{x}+xa}\left(70x^{\frac{9}{2}}\sqrt{b\sqrt{x}+xa}\,a^{\frac{9}{2}}+70x^{\frac{9}{2}}a^{\frac{9}{2}}\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}-140x^{\frac{7}{2}}\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}a^{\frac{7}{2}}+35x^{\frac{9}{2}}\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{b\sqrt{x}+xa}\,\sqrt{}}{2\sqrt{a}}\right.\right.}{35\sqrt{\sqrt{}}}$

input
```
int(1/x^(5/2)/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/7*(b*x^(1/2)+x*a)^(1/2)/b/x^2-12/7*a/b*(-2/5*(b*x^(1/2)+x*a)^(1/2)/b/x^
(3/2)-4/5*a/b*(-2/3*(b*x^(1/2)+x*a)^(1/2)/b/x+4/3*a*(b*x^(1/2)+x*a)^(1/2)/
b^2/x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\left(8\,a^2bx+5\,b^3-2\left(8\,a^3x+3\,ab^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\,b^4x^2}$$

input
```
integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-4/35*(8*a^2*b*x + 5*b^3 - 2*(8*a^3*x + 3*a*b^2)*sqrt(x))*sqrt(a*x + b*sqr
t(x))/(b^4*x^2)
```

Sympy [F]

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**(5/2)*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(70\,a^{\frac{3}{2}}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3+84\,ab\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+35\,\sqrt{a}b^2\left(\cdots\right.\right.}{35\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^7}$$

input
```
integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/35*(70*a^(3/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 84*a*b*(sqr
t(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 35*sqrt(a)*b^2*(sqrt(a)*sqrt(x)
- sqrt(a*x + b*sqrt(x))) + 5*b^3)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))
)^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{5/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^(5/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^(5/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\frac{64x^{\frac{7}{4}}\sqrt{\sqrt{x}\,a+b}\,a^3}{35}+\frac{24x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a\,b^2}{35}-\frac{32x^{\frac{5}{4}}\sqrt{\sqrt{x}\,a+b}\,a^2b}{35}-\frac{4x^{\frac{1}{4}}\sqrt{\sqrt{x}\,a+b}\,b^3}{7}-\frac{64\sqrt{a}\,a^3x^2}{35}}{b^4x^2}$$

input
```
int(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

output
```
(4*(16*x**(3/4)*sqrt(sqrt(x)*a + b)*a**3*x + 6*x**(3/4)*sqrt(sqrt(x)*a + b
)*a*b**2 - 8*x**(1/4)*sqrt(sqrt(x)*a + b)*a**2*b*x - 5*x**(1/4)*sqrt(sqrt(
x)*a + b)*b**3 - 16*sqrt(a)*a**3*x**2))/(35*b**4*x**2)
```

3.66 $\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3}+\frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}}-\frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2}+\frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{693b^5x}+\frac{1024a^5\sqrt{b\sqrt{x}+ax}}{693b^6\sqrt{x}}$$

output
```
-4/11*(b*x^(1/2)+a*x)^(1/2)/b/x^3+40/99*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(5/2
)-320/693*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^2+128/231*a^3*(b*x^(1/2)+a*x)^(1
/2)/b^4/x^(3/2)-512/693*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x+1024/693*a^5*(b*x^
(1/2)+a*x)^(1/2)/b^6/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(63b^5-70ab^4\sqrt{x}+80a^2b^3x-96a^3b^2x^{3/2}+128a^4bx^2-256a^5x^{5/2}\right)}{693b^6x^3}$$

input
```
Integrate[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(63*b^5 - 70*a*b^4*Sqrt[x] + 80*a^2*b^3*x - 96*a
^3*b^2*x^(3/2) + 128*a^4*b*x^2 - 256*a^5*x^(5/2)))/(693*b^6*x^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1922, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt{ax+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{1922}$$

$$-\frac{10a\int \frac{1}{x^3\sqrt{\sqrt{x}b+ax}}dx}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{10a\left(-\frac{8a\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{10a\left(-\frac{8a\left(-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

$$\Big\downarrow \ 1922$$

$$-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

$$\Big\downarrow \ 1920$$

$$-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

input
```
Int[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(11*b*x^3) - (10*a*((-4*Sqrt[b*Sqrt[x] + a*x])/
(9*b*x^(5/2)) - (8*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) - (6*a*((-4*Sqr
t[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*
x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])))/(5*b)))/(7*b)))/(9*b)))
/(11*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+xa}}{11b\,x^{3}}-\frac{20a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{9b\,x^{\frac{5}{2}}}-\frac{8a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{7b\,x^{2}}-\frac{6a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+xa}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+xa}}{3b^{2}\sqrt{x}}\right)}{5b}\right)}{7b}\right)}{9b}\right)}{11b}$
default	$-\frac{\sqrt{b\sqrt{x}+xa}\left(1386x^{\frac{13}{2}}\sqrt{b\sqrt{x}+xa}\,a^{\frac{13}{2}}+1386x^{\frac{13}{2}}a^{\frac{13}{2}}\sqrt{\sqrt{x}\,(\sqrt{x}\,a+b)}-2772x^{\frac{11}{2}}(b\sqrt{x}+xa)^{\frac{3}{2}}a^{\frac{11}{2}}+693x^{\frac{13}{2}}\ln\left(\frac{2\sqrt{x}\cdots}{}\right.\right.}{}$

input
```
int(1/x^(7/2)/(b*x^(1/2)+x*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/11*(b*x^(1/2)+x*a)^(1/2)/b/x^3-20/11*a/b*(-2/9*(b*x^(1/2)+x*a)^(1/2)/b/
x^(5/2)-8/9*a/b*(-2/7*(b*x^(1/2)+x*a)^(1/2)/b/x^2-6/7*a/b*(-2/5*(b*x^(1/2)
+x*a)^(1/2)/b/x^(3/2)-4/5*a/b*(-2/3*(b*x^(1/2)+x*a)^(1/2)/b/x+4/3*a*(b*x^(
1/2)+x*a)^(1/2)/b^2/x^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\left(128\,a^{4}bx^{2}+80\,a^{2}b^{3}x+63\,b^{5}-2\left(128\,a^{5}x^{2}+48\,a^{3}b^{2}x+35\,ab^{4}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{693\,b^{6}x^{3}}$$

input
```
integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-4/693*(128*a^4*b*x^2 + 80*a^2*b^3*x + 63*b^5 - 2*(128*a^5*x^2 + 48*a^3*b^
2*x + 35*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^6*x^3)
```

Sympy [F]

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{\frac{7}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

input
```
integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

output
```
Integral(1/(x**(7/2)*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x + b*sqrt(x))*x^(7/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(3696\,a^{\frac{5}{2}}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{5}+7920\,a^{2}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{4}+693\right.}{}$$

input
```
integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

output
```
4/693*(3696*a^(5/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 7920*a^2
*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 6930*a^(3/2)*b^2*(sqrt(a)
*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 3080*a*b^3*(sqrt(a)*sqrt(x) - sqrt(a
*x + b*sqrt(x)))^2 + 693*sqrt(a)*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(
x))) + 63*b^5)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^11
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{7/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(x^(7/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

output
```
int(1/(x^(7/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\frac{1024x^{\frac{11}{4}}\sqrt{\sqrt{x}\,a+b}\,a^5}{693}+\frac{128x^{\frac{7}{4}}\sqrt{\sqrt{x}\,a+b}\,a^3b^2}{231}+\frac{40x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a\,b^4}{99}-\frac{512x^{\frac{9}{4}}\sqrt{\sqrt{x}\,a+b}\,a^4b}{693}-\frac{320x^{\frac{5}{4}}\sqrt{}}{}}{b^6x^3}$$

input
```
int(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

output
```
(4*(256*x**(3/4)*sqrt(sqrt(x)*a + b)*a**5*x**2 + 96*x**(3/4)*sqrt(sqrt(x)*
a + b)*a**3*b**2*x + 70*x**(3/4)*sqrt(sqrt(x)*a + b)*a*b**4 - 128*x**(1/4)
*sqrt(sqrt(x)*a + b)*a**4*b*x**2 - 80*x**(1/4)*sqrt(sqrt(x)*a + b)*a**2*b*
*3*x - 63*x**(1/4)*sqrt(sqrt(x)*a + b)*b**5 - 256*sqrt(a)*a**5*x**3))/(693
*b**6*x**3)
```

3.67 $\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4b^4\sqrt{x}}{a^5\sqrt{b\sqrt{x}+ax}}-\frac{187b^3\sqrt{b\sqrt{x}+ax}}{32a^5}+\frac{41b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4}-\frac{5bx\sqrt{b\sqrt{x}+ax}}{4a^3}+\frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2}+\frac{315b^4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{32a^{11/2}}$$

output
```
-4*b^4*x^(1/2)/a^5/(b*x^(1/2)+a*x)^(1/2)-187/32*b^3*(b*x^(1/2)+a*x)^(1/2)/
a^5+41/16*b^2*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^4-5/4*b*x*(b*x^(1/2)+a*x)^(1
/2)/a^3+1/2*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^2+315/32*b^4*arctanh(a^(1/2)*x
^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.71

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{\sqrt{b\sqrt{x}+ax}\left(-315b^4-105ab^3\sqrt{x}+42a^2b^2x-24a^3bx^{3/2}+16a^4x^2\right)}{32a^5\left(b+a\sqrt{x}\right)}+\frac{315b^4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{32a^{11/2}}$$

input
```
Integrate[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(-315*b^4 - 105*a*b^3*Sqrt[x] + 42*a^2*b^2*x - 24*a
^3*b*x^(3/2) + 16*a^4*x^2))/(32*a^5*(b + a*Sqrt[x])) + (315*b^4*ArcTanh[(S
qrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(32*a^(11/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {1924, 1124, 2192, 27, 2192, 27, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x^3}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1124}$$

$$2\left(\frac{\int \frac{x^2a^4-bx^{3/2}a^3+b^2xa^2-b^3\sqrt{x}a+b^4}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{2192}$$

$$2\left(\frac{\frac{\int \frac{-15bx^{3/2}a^4+8b^2xa^3-8b^3\sqrt{x}a^2+8b^4a}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{4a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\frac{\int \frac{-15bx^{3/2}a^4+8b^2xa^3-8b^3\sqrt{x}a^2+8b^4a}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \; 2192$$

$$2\left(\frac{\frac{\frac{\int \frac{3\left(41b^2xa^4-16b^3\sqrt{x}a^3+16b^4a^2\right)}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{3a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{\frac{\int \frac{41b^2xa^4-16b^3\sqrt{x}a^3+16b^4a^2}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \; 2192$$

$$2\left(\frac{\frac{\frac{\frac{\int \frac{a^3b^3\left(64b-187a\sqrt{x}\right)}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}+\frac{41}{2}a^3b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{\frac{\frac{1}{4}a^2b^3\int \frac{64b-187a\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}+\frac{41}{2}a^3b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \; 1160$$

$$2\left(\frac{\frac{\frac{\frac{1}{4}a^2b^3\left(\frac{315}{2}b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}-187\sqrt{ax+b\sqrt{x}}\right)+\frac{41}{2}a^3b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \; 1091$$

$$2\left(\frac{\frac{\frac{\frac{1}{4}a^2b^3\left(315b\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}-187\sqrt{ax+b\sqrt{x}}\right)+\frac{41}{2}a^3b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{2a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}+\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b\sqrt{x}}}\right)$$

$\downarrow$ 219

$$2\left(\frac{\frac{1}{4}a^3x^{3/2}\sqrt{ax+b\sqrt{x}}+\frac{\frac{\frac{41}{2}a^3b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}+\frac{1}{4}a^2b^3\left(\frac{315b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}-187\sqrt{ax+b\sqrt{x}}\right)}{2a}-5a^3bx\sqrt{ax+b\sqrt{x}}}{8a}}{a^5}-\frac{2b^4\sqrt{x}}{a^5\sqrt{ax+b}}\right.$$

input
```
Int[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
2*((-2*b^4*Sqrt[x])/(a^5*Sqrt[b*Sqrt[x] + a*x]) + ((a^3*x^(3/2)*Sqrt[b*Sqr
t[x] + a*x])/4 + (-5*a^3*b*x*Sqrt[b*Sqrt[x] + a*x] + ((41*a^3*b^2*Sqrt[x]*
Sqrt[b*Sqrt[x] + a*x])/2 + (a^2*b^3*(-187*Sqrt[b*Sqrt[x] + a*x] + (315*b*A
rcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]))/4)/(2*a))/(8*a)
)/a^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}}{2a\sqrt{b\sqrt{x}+xa}}-\frac{9b\left(\frac{x^{2}}{3a\sqrt{b\sqrt{x}+xa}}-\frac{7b\left(\frac{x^{\frac{3}{2}}}{2a\sqrt{b\sqrt{x}+xa}}-\frac{5b\left(\frac{x}{a\sqrt{b\sqrt{x}+xa}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+xa}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+xa}}+\frac{b+2\sqrt{x}\,a}{ba\sqrt{b\sqrt{x}+x}}\right.}{2a}\right.}{2a}\right.}{4a}\right.}{6a}\right.}{4a}$
default	$\frac{\sqrt{b\sqrt{x}+xa}\left(32x^{\frac{3}{2}}\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}a^{\frac{11}{2}}+276x^{\frac{3}{2}}\sqrt{b\sqrt{x}+xa}\,a^{\frac{9}{2}}b^{2}-48\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}a^{\frac{9}{2}}bx+690x\sqrt{b\sqrt{x}+xa}\,a^{\frac{7}{2}}b^{3}-768x\,a^{\frac{7}{2}}\right.}{}$

input

```
int(x^(5/2)/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/2*x^(5/2)/a/(b*x^(1/2)+x*a)^(1/2)-9/4*b/a*(1/3*x^2/a/(b*x^(1/2)+x*a)^(1/
2)-7/6*b/a*(1/2*x^(3/2)/a/(b*x^(1/2)+x*a)^(1/2)-5/4*b/a*(x/a/(b*x^(1/2)+x*
a)^(1/2)-3/2*b/a*(-x^(1/2)/a/(b*x^(1/2)+x*a)^(1/2)-1/2*b/a*(-1/a/(b*x^(1/2
)+x*a)^(1/2)+1/b/a*(b+2*x^(1/2)*a)/(b*x^(1/2)+x*a)^(1/2))+1/a^(3/2)*ln((1/
2*b+x^(1/2)*a)/a^(1/2)+(b*x^(1/2)+x*a)^(1/2))))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(x**(5/2)/(a*x + b*sqrt(x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/(a*x + b*sqrt(x))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.77

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{1}{32}\sqrt{ax+b\sqrt{x}}\left(2\left(4\sqrt{x}\left(\frac{2\sqrt{x}}{a^2}-\frac{5\,b}{a^3}\right)+\frac{41\,b^2}{a^4}\right)\sqrt{x}-\frac{187\,b^3}{a^5}\right) - \frac{315\,b^4\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{64\,a^{\frac{11}{2}}} - \frac{4\,b^5}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{11}{2}}}$$

input
```
integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/32*sqrt(a*x + b*sqrt(x))*(2*(4*sqrt(x)*(2*sqrt(x)/a^2 - 5*b/a^3) + 41*b^
2/a^4)*sqrt(x) - 187*b^3/a^5) - 315/64*b^4*log(abs(-2*sqrt(a)*(sqrt(a)*sqr
t(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(11/2) - 4*b^5/((sqrt(a)*(sqrt(a)*sq
rt(x) - sqrt(a*x + b*sqrt(x))) + b)*a^(11/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^{5/2}}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(x^(5/2)/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
int(x^(5/2)/(a*x + b*x^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{-24x^{\frac{7}{4}}\sqrt{\sqrt{x}\,a+b}\,a^4b-105x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a^2b^3+16x^{\frac{9}{4}}\sqrt{\sqrt{x}\,a+b}\,a^5+42x^{\frac{5}{4}}\sqrt{\sqrt{x}}}{}$$

input
```
int(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
( - 24*x**(3/4)*sqrt(sqrt(x)*a + b)*a**4*b*x - 105*x**(3/4)*sqrt(sqrt(x)*a
 + b)*a**2*b**3 + 16*x**(1/4)*sqrt(sqrt(x)*a + b)*a**5*x**2 + 42*x**(1/4)*
sqrt(sqrt(x)*a + b)*a**3*b**2*x - 315*x**(1/4)*sqrt(sqrt(x)*a + b)*a*b**4
+ 315*sqrt(x)*sqrt(a)*log((sqrt(sqrt(x)*a + b) + x**(1/4)*sqrt(a))/sqrt(b)
)*a*b**4 - 189*sqrt(x)*sqrt(a)*a*b**4 + 315*sqrt(a)*log((sqrt(sqrt(x)*a +
b) + x**(1/4)*sqrt(a))/sqrt(b))*b**5 - 189*sqrt(a)*b**5)/(32*a**6*(sqrt(x)
*a + b))
```

3.68 $\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4b^2\sqrt{x}}{a^3\sqrt{b\sqrt{x}+ax}} - \frac{7b\sqrt{b\sqrt{x}+ax}}{2a^3} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2} + \frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{7/2}}$$

output
```
-4*b^2*x^(1/2)/a^3/(b*x^(1/2)+a*x)^(1/2)-7/2*b*(b*x^(1/2)+a*x)^(1/2)/a^3+x
^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^2+15/2*b^2*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/
2)+a*x)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(-15b^2-5ab\sqrt{x}+2a^2x\right)}{2a^3\left(b+a\sqrt{x}\right)} + \frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{2a^{7/2}}$$

input
```
Integrate[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
(Sqrt[b*Sqrt[x] + a*x]*(-15*b^2 - 5*a*b*Sqrt[x] + 2*a^2*x))/(2*a^3*(b + a*
Sqrt[x])) + (15*b^2*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x]
)])/(2*a^(7/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1924, 1124, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x^2}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1124}$$

$$2\left(\frac{\int \frac{xa^2-b\sqrt{x}a+b^2}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a^3}-\frac{2b^2\sqrt{x}}{a^3\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{2192}$$

$$2\left(\frac{\frac{\int \frac{ab\left(4b-7a\sqrt{x}\right)}{2\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{2a}+\frac{1}{2}a\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^3}-\frac{2b^2\sqrt{x}}{a^3\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\frac{1}{4}b\int \frac{4b-7a\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}+\frac{1}{2}a\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^3}-\frac{2b^2\sqrt{x}}{a^3\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{\frac{1}{4}b\left(\frac{15}{2}b\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}-7\sqrt{ax+b\sqrt{x}}\right)+\frac{1}{2}a\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^3}-\frac{2b^2\sqrt{x}}{a^3\sqrt{ax+b\sqrt{x}}}\right)$$

$\big\downarrow$ 1091

$$2\left(\frac{\frac{1}{4}b\left(15b\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}-7\sqrt{ax+b\sqrt{x}}\right)+\frac{1}{2}a\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^3}-\frac{2b^2\sqrt{x}}{a^3\sqrt{ax+b\sqrt{x}}}\right)$$

$\big\downarrow$ 219

$$2\left(\frac{\frac{1}{4}b\left(\frac{15b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}-7\sqrt{ax+b\sqrt{x}}\right)+\frac{1}{2}a\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^3}-\frac{2b^2\sqrt{x}}{a^3\sqrt{ax+b\sqrt{x}}}\right)$$

input
```
Int[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
2*((-2*b^2*Sqrt[x])/(a^3*Sqrt[b*Sqrt[x] + a*x]) + ((a*Sqrt[x]*Sqrt[b*Sqrt[
x] + a*x])/2 + (b*(-7*Sqrt[b*Sqrt[x] + a*x] + (15*b*ArcTanh[(Sqrt[a]*Sqrt[
x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]))/4)/a^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{a\sqrt{b\sqrt{x}+xa}}-\frac{5b\left(\frac{x}{a\sqrt{b\sqrt{x}+xa}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+xa}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+xa}}+\frac{b+2\sqrt{x}\,a}{ba\sqrt{b\sqrt{x}+xa}}\right)}{2a}+\frac{\ln\left(\frac{\frac{b}{2}+\sqrt{x}\,a}{\sqrt{a}}+\sqrt{b\sqrt{x}+xa}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)}{2a}$
default	$\frac{\sqrt{b\sqrt{x}+xa}\left(4x^{\frac{3}{2}}\sqrt{b\sqrt{x}+xa}\,a^{\frac{9}{2}}+10x\sqrt{b\sqrt{x}+xa}\,a^{\frac{7}{2}}b-32x\,a^{\frac{7}{2}}\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}\,b+16x\,a^{3}\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}\,\sqrt{a}}{2\sqrt{a}}\right.\right.}{}$

input
```
int(x^(3/2)/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^(3/2)/a/(b*x^(1/2)+x*a)^(1/2)-5/2*b/a*(x/a/(b*x^(1/2)+x*a)^(1/2)-3/2*b/a
*(-x^(1/2)/a/(b*x^(1/2)+x*a)^(1/2)-1/2*b/a*(-1/a/(b*x^(1/2)+x*a)^(1/2)+1/b
/a*(b+2*x^(1/2)*a)/(b*x^(1/2)+x*a)^(1/2))+1/a^(3/2)*ln((1/2*b+x^(1/2)*a)/a
^(1/2)+(b*x^(1/2)+x*a)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(x**(3/2)/(a*x + b*sqrt(x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/(a*x + b*sqrt(x))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{1}{2}\sqrt{ax+b\sqrt{x}}\left(\frac{2\sqrt{x}}{a^2}-\frac{7b}{a^3}\right)-\frac{15\,b^2\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{4\,a^{\frac{7}{2}}}-\frac{4\,b^3}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{7}{2}}}$$

input
```
integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/2*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)/a^2 - 7*b/a^3) - 15/4*b^2*log(abs(-2*
sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(7/2) - 4*b^3/((
sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*a^(7/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^{3/2}}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(x^(3/2)/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
int(x^(3/2)/(a*x + b*x^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{-5x^{\frac{3}{4}}\sqrt{\sqrt{x}\,a+b}\,a^2b+2x^{\frac{5}{4}}\sqrt{\sqrt{x}\,a+b}\,a^3-15x^{\frac{1}{4}}\sqrt{\sqrt{x}\,a+b}\,a\,b^2+15\sqrt{x}\,\sqrt{a}\log\Big(}{2a^4\left(\sqrt{x}\,a\right.}$$

input
```
int(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
( - 5*x**(3/4)*sqrt(sqrt(x)*a + b)*a**2*b + 2*x**(1/4)*sqrt(sqrt(x)*a + b)
*a**3*x - 15*x**(1/4)*sqrt(sqrt(x)*a + b)*a*b**2 + 15*sqrt(x)*sqrt(a)*log(
(sqrt(sqrt(x)*a + b) + x**(1/4)*sqrt(a))/sqrt(b))*a*b**2 - 10*sqrt(x)*sqrt
(a)*a*b**2 + 15*sqrt(a)*log((sqrt(sqrt(x)*a + b) + x**(1/4)*sqrt(a))/sqrt(
b))*b**3 - 10*sqrt(a)*b**3)/(2*a**4*(sqrt(x)*a + b))
```

3.69 $\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}+\frac{4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}$$

output
```
-4*x^(1/2)/a/(b*x^(1/2)+a*x)^(1/2)+4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*
x)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}}{a\left(b+a\sqrt{x}\right)}+\frac{4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{a^{3/2}}$$

input
```
Integrate[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x])/(a*(b + a*Sqrt[x])) + (4*ArcTanh[(Sqrt[a]*Sqrt[
b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1924, 1124, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1924}$$

$$2\int \frac{x}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1124}$$

$$2\left(\frac{\int \frac{1}{\sqrt{\sqrt{x}b+ax}}d\sqrt{x}}{a}-\frac{2\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{1091}$$

$$2\left(\frac{2\int \frac{1}{1-ax}d\frac{\sqrt{x}}{\sqrt{\sqrt{x}b+ax}}}{a}-\frac{2\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}-\frac{2\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}\right)$$

input
```
Int[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

output
```
2*((-2*Sqrt[x])/(a*Sqrt[b*Sqrt[x] + a*x]) + (2*ArcTanh[(Sqrt[a]*Sqrt[x])/S
qrt[b*Sqrt[x] + a*x]])/a^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

method	result
derivativedivides	$-\frac{2\sqrt{x}}{a\sqrt{b\sqrt{x}+xa}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+xa}}+\frac{b+2\sqrt{x}\,a}{ba\sqrt{b\sqrt{x}+xa}}\right)}{a}+\frac{2\ln\left(\frac{\frac{b}{2}+\sqrt{x}\,a}{\sqrt{a}}+\sqrt{b\sqrt{x}+xa}\right)}{a^{\frac{3}{2}}}$
default	$-\frac{2\sqrt{b\sqrt{x}+xa}\left(2x\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}\,a^{\frac{5}{2}}-x\ln\left(\frac{2\sqrt{x}\,a+2\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}\,\sqrt{a}+b}{2\sqrt{a}}\right)a^{2}b+4\sqrt{x}\,\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}\,a^{\frac{3}{2}}b-2\sqrt{x}\,\ln\left(\right.}{a^{\frac{3}{2}}\sqrt{\sqrt{x}\left(\sqrt{\right.}}}$

input
```
int(x^(1/2)/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x^(1/2)/a/(b*x^(1/2)+x*a)^(1/2)-b/a*(-1/a/(b*x^(1/2)+x*a)^(1/2)+1/b/a*(
b+2*x^(1/2)*a)/(b*x^(1/2)+x*a)^(1/2))+2/a^(3/2)*ln((1/2*b+x^(1/2)*a)/a^(1/
2)+(b*x^(1/2)+x*a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(sqrt(x)/(a*x + b*sqrt(x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/(a*x + b*sqrt(x))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{2\,\log\left(\left|2\,\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{a^{\frac{3}{2}}} - \frac{4\,b}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{3}{2}}}$$

input
```
integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-2*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/a^(3/
2) - 4*b/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*a^(3/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{\sqrt{x}}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

input
```
int(x^(1/2)/(a*x + b*x^(1/2))^(3/2),x)
```

output
```
int(x^(1/2)/(a*x + b*x^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{-4x^{\frac{1}{4}}\sqrt{\sqrt{x}\,a+b}\,a+4\sqrt{x}\,\sqrt{a}\log\left(\frac{\sqrt{\sqrt{x}\,a+b}+x^{\frac{1}{4}}\sqrt{a}}{\sqrt{b}}\right)a-4\sqrt{x}\,\sqrt{a}\,a+4\sqrt{a}\log\left(\frac{\sqrt{\sqrt{x}}{}\right.}{a^{2}\left(\sqrt{x}\,a+b\right)}$$

input
```
int(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
(4*( - x**(1/4)*sqrt(sqrt(x)*a + b)*a + sqrt(x)*sqrt(a)*log((sqrt(sqrt(x)*
a + b) + x**(1/4)*sqrt(a))/sqrt(b))*a - sqrt(x)*sqrt(a)*a + sqrt(a)*log((s
qrt(sqrt(x)*a + b) + x**(1/4)*sqrt(a))/sqrt(b))*b - sqrt(a)*b))/(a**2*(sqr
t(x)*a + b))
```

3.70 $\int \frac{1}{\sqrt{x}(b\sqrt{x}+ax)^{3/2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}} \, dx = -\frac{4\left(b+2a\sqrt{x}\right)}{b^2\sqrt{b\sqrt{x}+ax}}$$

output
```
(-4*b-8*a*x^(1/2))/b^2/(b*x^(1/2)+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}} \, dx = -\frac{4\left(b+2a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{b^2\left(b+a\sqrt{x}\right)\sqrt{x}}$$

input
```
Integrate[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*(b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(b^2*(b + a*Sqrt[x])*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1919, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(ax+b\sqrt{x}\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 1919}$$
$$2\int \frac{1}{\left(\sqrt{x}b+ax\right)^{3/2}}d\sqrt{x}$$
$$\Big\downarrow \text{ 1088}$$
$$-\frac{4\left(2a\sqrt{x}+b\right)}{b^2\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*(b + 2*a*Sqrt[x]))/(b^2*Sqrt[b*Sqrt[x] + a*x])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1919
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
 && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{4(b+2\sqrt{x}\,a)}{b^{2}\sqrt{b\sqrt{x}+xa}}$	25
default	$-\frac{4\sqrt{b\sqrt{x}+xa}\,\left(x\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}a^{2}+2\sqrt{x}\,\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}ab-\left(\sqrt{x}\,\left(\sqrt{x}\,a+b\right)\right)^{\frac{3}{2}}a^{2}x+\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}b^{2}\right)}{\sqrt{\sqrt{x}\,\left(\sqrt{x}\,a+b\right)}\,b^{3}x\left(\sqrt{x}\,a+b\right)^{2}}$	111

input
```
int(1/x^(1/2)/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4*(b+2*x^(1/2)*a)/b^2/(b*x^(1/2)+x*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\left(abx-\left(2\,a^{2}x-b^{2}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{a^{2}b^{2}x^{2}-b^{4}x}$$

input
```
integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
4*(a*b*x - (2*a^2*x - b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^2*x^2 - b
^4*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\sqrt{x}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(sqrt(x)*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\left(\frac{2a\sqrt{x}}{b^2}+\frac{1}{b}\right)}{\sqrt{ax+b\sqrt{x}}}$$

input
```
integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
-4*(2*a*sqrt(x)/b^2 + 1/b)/sqrt(a*x + b*sqrt(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\sqrt{x}\,\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^(1/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x^(1/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.83

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{-8\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a-4\sqrt{\sqrt{x}\,a+b}\,b-8x^{\frac{3}{4}}\sqrt{a}\,a-8x^{\frac{1}{4}}\sqrt{a}\,b}{x^{\frac{1}{4}}b^{2}\left(\sqrt{x}\,a+b\right)}$$

input
```
int(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
(4*( - 2*sqrt(x)*sqrt(sqrt(x)*a + b)*a - sqrt(sqrt(x)*a + b)*b - 2*x**(3/4
)*sqrt(a)*a - 2*x**(1/4)*sqrt(a)*b))/(x**(1/4)*b**2*(sqrt(x)*a + b))
```

3.71 $\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{32a^2\left(b+2a\sqrt{x}\right)}{5b^4\sqrt{b\sqrt{x}+ax}}-\frac{4}{5bx\sqrt{b\sqrt{x}+ax}}+\frac{8a}{5b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}$$

output
```
-32/5*a^2*(b+2*a*x^(1/2))/b^4/(b*x^(1/2)+a*x)^(1/2)-4/5/b/x/(b*x^(1/2)+a*x
)^(1/2)+8/5*a/b^2/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(b^3-2ab^2\sqrt{x}+8a^2bx+16a^3x^{3/2}\right)}{5b^4\left(b+a\sqrt{x}\right)x^{3/2}}$$

input
```
Integrate[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(b^3 - 2*a*b^2*Sqrt[x] + 8*a^2*b*x + 16*a^3*x^(3
/2)))/(5*b^4*(b + a*Sqrt[x])*x^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.34, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1921, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(ax+b\sqrt{x}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{6\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{b}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{6\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{b}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{6\left(-\frac{4a\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{b}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{6\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{b}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
4/(b*x*Sqrt[b*Sqrt[x] + a*x]) + (6*((-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2
)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x]
)/(3*b^2*Sqrt[x])))/(5*b)))/b
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

method	result
derivativedivides	$-\frac{4}{5bx\sqrt{b\sqrt{x}+xa}}-\frac{12a\left(-\frac{2}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+xa}}+\frac{8a\left(b+2\sqrt{x}\,a\right)}{3b^{3}\sqrt{b\sqrt{x}+xa}}\right)}{5b}$
default	$\frac{2\sqrt{b\sqrt{x}+xa}\,\left(10x^{\frac{9}{2}}\sqrt{b\sqrt{x}+xa}\,a^{\frac{11}{2}}+10x^{\frac{9}{2}}a^{\frac{11}{2}}\sqrt{\sqrt{x}\,\left(\sqrt{x}\,a+b\right)}-30x^{\frac{7}{2}}\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}a^{\frac{9}{2}}+10x^{\frac{7}{2}}a^{\frac{9}{2}}\left(\sqrt{x}\,\left(\sqrt{x}\,a+b\right)\right)^{\frac{3}{2}}+1\right.}{}$

input
```
int(1/x^(3/2)/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/5/b/x/(b*x^(1/2)+x*a)^(1/2)-12/5*a/b*(-2/3/b/x^(1/2)/(b*x^(1/2)+x*a)^(1
/2)+8/3*a*(b+2*x^(1/2)*a)/b^3/(b*x^(1/2)+x*a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4\left(8\,a^3bx^2 - 3\,ab^3x - (16\,a^4x^2 - 10\,a^2b^2x - b^4)\sqrt{x}\right)\sqrt{ax + b\sqrt{x}}}{5\left(a^2b^4x^3 - b^6x^2\right)}$$

input
```
integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
4/5*(8*a^3*b*x^2 - 3*a*b^3*x - (16*a^4*x^2 - 10*a^2*b^2*x - b^4)*sqrt(x))*
sqrt(a*x + b*sqrt(x))/(a^2*b^4*x^3 - b^6*x^2)
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**(3/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^{3/2}\,\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^(3/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x^(3/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{-\frac{64\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a^{3}x}{5}+\frac{8\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a\,b^{2}}{5}-\frac{32\sqrt{\sqrt{x}\,a+b}\,a^{2}bx}{5}-\frac{4\sqrt{\sqrt{x}\,a+b}\,b^{3}}{5}+\frac{64x^{\frac{7}{4}}\sqrt{a}\,a^{3}}{5}+}{x^{\frac{5}{4}}b^{4}\left(\sqrt{x}\,a+b\right)}$$

input
```
int(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
(4*( - 16*sqrt(x)*sqrt(sqrt(x)*a + b)*a**3*x + 2*sqrt(x)*sqrt(sqrt(x)*a +
b)*a*b**2 - 8*sqrt(sqrt(x)*a + b)*a**2*b*x - sqrt(sqrt(x)*a + b)*b**3 + 16
*x**(3/4)*sqrt(a)*a**3*x + 16*x**(1/4)*sqrt(a)*a**2*b*x))/(5*x**(1/4)*b**4
*x*(sqrt(x)*a + b))
```

3.72 $\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{512a^4\left(b+2a\sqrt{x}\right)}{63b^6\sqrt{b\sqrt{x}+ax}}-\frac{4}{9bx^2\sqrt{b\sqrt{x}+ax}}+\frac{40a}{63b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}-\frac{64a^2}{63b^3x\sqrt{b\sqrt{x}+ax}}+\frac{128a^3}{63b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}$$

output
```
-512/63*a^4*(b+2*a*x^(1/2))/b^6/(b*x^(1/2)+a*x)^(1/2)-4/9/b/x^2/(b*x^(1/2)
+a*x)^(1/2)+40/63*a/b^2/x^(3/2)/(b*x^(1/2)+a*x)^(1/2)-64/63*a^2/b^3/x/(b*x
^(1/2)+a*x)^(1/2)+128/63*a^3/b^4/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}\left(7b^5-10ab^4\sqrt{x}+16a^2b^3x-32a^3b^2x^{3/2}+128a^4bx^2+256a^5x^{5/2}\right)}{63b^6\left(b+a\sqrt{x}\right)x^{5/2}}$$

input
```
Integrate[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(7*b^5 - 10*a*b^4*Sqrt[x] + 16*a^2*b^3*x - 32*a^
3*b^2*x^(3/2) + 128*a^4*b*x^2 + 256*a^5*x^(5/2)))/(63*b^6*(b + a*Sqrt[x])*
x^(5/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1921, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{10\int \frac{1}{x^3\sqrt{\sqrt{x}b+ax}}dx}{b}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(-\frac{8a\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{b}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(-\frac{8a\left(-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{b}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{10\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\int \frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{b}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

$\downarrow$ 1922

$$\frac{10\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int \frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{b}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

$\downarrow$ 1920

$$\frac{10\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{b}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

input
```
Int[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
4/(b*x^2*Sqrt[b*Sqrt[x] + a*x]) + (10*((-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(
5/2)) - (8*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) - (6*a*((-4*Sqrt[b*Sqrt
[x] + a*x])/(5*b*x^(3/2)) - (4*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*
a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])))/(5*b)))/(7*b)))/(9*b)))/b
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{4}{9 b\, x^{2} \sqrt{b\sqrt{x}+xa}}-\frac{20 a \left(-\frac{2}{7 b\, x^{\frac{3}{2}} \sqrt{b\sqrt{x}+xa}}-\frac{8 a \left(-\frac{2}{5 b x \sqrt{b\sqrt{x}+xa}}-\frac{6 a \left(-\frac{2}{3 b \sqrt{x}\,\sqrt{b\sqrt{x}+xa}}+\frac{8 a (b+2\sqrt{x}\,a)}{3 b^{3}\sqrt{b\sqrt{x}+xa}}\right)}{5 b}\right)}{7 b}\right)}{9 b}$
default	$\frac{4\sqrt{b\sqrt{x}+xa}\left(126 x^{\frac{13}{2}}\sqrt{b\sqrt{x}+xa}\,a^{\frac{15}{2}}+126 x^{\frac{13}{2}} a^{\frac{15}{2}}\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}-315 x^{\frac{11}{2}}\left(b\sqrt{x}+xa\right)^{\frac{3}{2}} a^{\frac{13}{2}}+63 x^{\frac{11}{2}} a^{\frac{13}{2}}\left(\sqrt{x}\left(\sqrt{x}\,a \cdots\right.\right.\right.}{\cdots}$

input
```
int(1/x^(5/2)/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/9/b/x^2/(b*x^(1/2)+x*a)^(1/2)-20/9*a/b*(-2/7/b/x^(3/2)/(b*x^(1/2)+x*a)^
(1/2)-8/7*a/b*(-2/5/b/x/(b*x^(1/2)+x*a)^(1/2)-6/5*a/b*(-2/3/b/x^(1/2)/(b*x
^(1/2)+x*a)^(1/2)+8/3*a*(b+2*x^(1/2)*a)/b^3/(b*x^(1/2)+x*a)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\left(128\,a^5bx^3-48\,a^3b^3x^2-17\,ab^5x-\left(256\,a^6x^3-160\,a^4b^2x^2-26\,a^2b^4x-7\,b^6 \cdots\right.\right.}{63\left(a^2b^6x^4-b^8x^3\right)}$$

input
```
integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
4/63*(128*a^5*b*x^3 - 48*a^3*b^3*x^2 - 17*a*b^5*x - (256*a^6*x^3 - 160*a^4
*b^2*x^2 - 26*a^2*b^4*x - 7*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^6*x
^4 - b^8*x^3)
```

Sympy [F]

$$\int \frac{1}{x^{5/2} \left(b\sqrt{x} + ax\right)^{3/2}}\, dx = \int \frac{1}{x^{\frac{5}{2}} \left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**(5/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^{5/2} \left(b\sqrt{x} + ax\right)^{3/2}}\, dx = \int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}} x^{\frac{5}{2}}}\, dx$$

input
```
integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{5/2} \left(b\sqrt{x} + ax\right)^{3/2}}\, dx = \int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}} x^{\frac{5}{2}}}\, dx$$

input
```
integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2} \left(b\sqrt{x} + ax\right)^{3/2}}\, dx = \int \frac{1}{x^{5/2} \left(a\, x + b\, \sqrt{x}\right)^{3/2}}\, dx$$

input
```
int(1/(x^(5/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x^(5/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^{5/2} \left(b\sqrt{x} + ax\right)^{3/2}}\, dx = \frac{-\frac{1024\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a^5x^2}{63} + \frac{128\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a^3b^2x}{63} + \frac{40\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a\,b^4}{63} - \frac{512\sqrt{\sqrt{x}\,a+b}\,a^4b\,x^2}{63} - }{x^{\frac{9}{4}} b^6 \left(\sqrt{x}\,a + b\right)}$$

input
```
int(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
(4*( - 256*sqrt(x)*sqrt(sqrt(x)*a + b)*a**5*x**2 + 32*sqrt(x)*sqrt(sqrt(x)
*a + b)*a**3*b**2*x + 10*sqrt(x)*sqrt(sqrt(x)*a + b)*a*b**4 - 128*sqrt(sqr
t(x)*a + b)*a**4*b*x**2 - 16*sqrt(sqrt(x)*a + b)*a**2*b**3*x - 7*sqrt(sqrt
(x)*a + b)*b**5 + 256*x**(3/4)*sqrt(a)*a**5*x**2 + 256*x**(1/4)*sqrt(a)*a*
*4*b*x**2))/(63*x**(1/4)*b**6*x**2*(sqrt(x)*a + b))
```

3.73 $\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4096a^6\left(b+2a\sqrt{x}\right)}{429b^8\sqrt{b\sqrt{x}+ax}} - \frac{4}{13bx^3\sqrt{b\sqrt{x}+ax}} + \frac{56a}{143b^2x^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{224a^2}{429b^3x^2\sqrt{b\sqrt{x}+ax}} + \frac{320a^3}{429b^4x^{3/2}\sqrt{b\sqrt{x}+ax}} - \frac{512a^4}{429b^5x\sqrt{b\sqrt{x}+ax}} + \frac{1024a^5}{429b^6\sqrt{x}\sqrt{b\sqrt{x}+ax}}$$

output

```
-4096/429*a^6*(b+2*a*x^(1/2))/b^8/(b*x^(1/2)+a*x)^(1/2)-4/13/b/x^3/(b*x^(1
/2)+a*x)^(1/2)+56/143*a/b^2/x^(5/2)/(b*x^(1/2)+a*x)^(1/2)-224/429*a^2/b^3/
x^2/(b*x^(1/2)+a*x)^(1/2)+320/429*a^3/b^4/x^(3/2)/(b*x^(1/2)+a*x)^(1/2)-51
2/429*a^4/b^5/x/(b*x^(1/2)+a*x)^(1/2)+1024/429*a^5/b^6/x^(1/2)/(b*x^(1/2)+
a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^{7/2} \left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(33b^7-42ab^6\sqrt{x}+56a^2b^5x-80a^3b^4x^{3/2}+128a^4b^3x^2-256a^5b^2x^{5/2}+1024a^6bx^3+2048a\right.}{429b^8\left(b+a\sqrt{x}\right)x^{7/2}}$$

input
```
Integrate[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
(-4*Sqrt[b*Sqrt[x] + a*x]*(33*b^7 - 42*a*b^6*Sqrt[x] + 56*a^2*b^5*x - 80*a
^3*b^4*x^(3/2) + 128*a^4*b^3*x^2 - 256*a^5*b^2*x^(5/2) + 1024*a^6*b*x^3 +
2048*a^7*x^(7/2)))/(429*b^8*(b + a*Sqrt[x])*x^(7/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1921, 1922, 1922, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\left(ax+b\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1921}$$

$$\frac{14\int \frac{1}{x^4\sqrt{\sqrt{x}b+ax}}dx}{b}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{14\left(-\frac{12a\int \frac{1}{x^{7/2}\sqrt{\sqrt{x}b+ax}}dx}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}\right)}{b}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{14\left(-\frac{12a\left(-\frac{10a\int \frac{1}{x^3\sqrt{\sqrt{x}b+ax}}dx}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}\right)}{b}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{14\left(-\frac{12a\left(-\frac{10a\left(-\frac{8a\int \frac{1}{x^{5/2}\sqrt{\sqrt{x}b+ax}}dx}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}\right)}{b}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{14\left(-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\int \frac{1}{x^2\sqrt{\sqrt{x}b+ax}}dx}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}\right)}{b}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{14\left(-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\int\frac{1}{x^{3/2}\sqrt{\sqrt{x}b+ax}}dx}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}\right)}{b}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

$\downarrow$ 1922

$$14\left(-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(-\frac{2a\int\frac{1}{x\sqrt{\sqrt{x}b+ax}}dx}{3b}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+}}{13bx^7}\right.$$

$$\Big\downarrow \text{1920}$$

$$\frac{14\left(-\frac{12a\left(-\frac{10a\left(-\frac{8a\left(-\frac{6a\left(-\frac{4a\left(\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}\right)}{5b}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}\right)}{7b}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}\right)}{9b}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}\right)}{11b}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}\right)}{13b}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}\right)}{b}-\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

input

```
Int[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

output
```
4/(b*x^3*Sqrt[b*Sqrt[x] + a*x]) + (14*((-4*Sqrt[b*Sqrt[x] + a*x])/(13*b*x^
(7/2)) - (12*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(11*b*x^3) - (10*a*((-4*Sqrt[b*
Sqrt[x] + a*x])/(9*b*x^(5/2)) - (8*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2)
 - (6*a*((-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) - (4*a*((-4*Sqrt[b*Sqrt[
x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqrt[x])))/(5*b)))
/(7*b)))/(9*b)))/(11*b)))/(13*b)))/b
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.86

method	result
derivativedivides	$-\frac{4}{13b\,x^{3}\sqrt{b\sqrt{x}+xa}}-\frac{28a\left(-\frac{2}{11b\,x^{\frac{5}{2}}\sqrt{b\sqrt{x}+xa}}-\frac{12a\left(-\frac{2}{9b\,x^{2}\sqrt{b\sqrt{x}+xa}}-\frac{10a\left(-\frac{2}{7b\,x^{\frac{3}{2}}\sqrt{b\sqrt{x}+xa}}-\frac{8a\left(-\frac{2}{5bx\sqrt{b\sqrt{x}+xa}}-\right.}{}\right)}{9b}\right)}{11b}\right)}{13b}$
default	$\frac{2\sqrt{b\sqrt{x}+xa}\left(2574x^{\frac{17}{2}}\sqrt{b\sqrt{x}+xa}\,a^{\frac{19}{2}}+2574x^{\frac{17}{2}}a^{\frac{19}{2}}\sqrt{\sqrt{x}\left(\sqrt{x}\,a+b\right)}-6006x^{\frac{15}{2}}\left(b\sqrt{x}+xa\right)^{\frac{3}{2}}a^{\frac{17}{2}}+858x^{\frac{15}{2}}a^{\frac{17}{2}}\left(\sqrt{x}\right.\right.}{}$

input
```
int(1/x^(7/2)/(b*x^(1/2)+x*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/13/b/x^3/(b*x^(1/2)+x*a)^(1/2)-28/13*a/b*(-2/11/b/x^(5/2)/(b*x^(1/2)+x*
a)^(1/2)-12/11*a/b*(-2/9/b/x^2/(b*x^(1/2)+x*a)^(1/2)-10/9*a/b*(-2/7/b/x^(3
/2)/(b*x^(1/2)+x*a)^(1/2)-8/7*a/b*(-2/5/b/x/(b*x^(1/2)+x*a)^(1/2)-6/5*a/b*
(-2/3/b/x^(1/2)/(b*x^(1/2)+x*a)^(1/2)+8/3*a*(b+2*x^(1/2)*a)/b^3/(b*x^(1/2)
+x*a)^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\left(1024\,a^7bx^4-384\,a^5b^3x^3-136\,a^3b^5x^2-75\,ab^7x-(2048\,a^8x^4-1280\,a^6b^2x^3\right.}{429\left(a^2b^8x^5-b^{10}x^4\right)}$$

input
```
integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

output
```
4/429*(1024*a^7*b*x^4 - 384*a^5*b^3*x^3 - 136*a^3*b^5*x^2 - 75*a*b^7*x - (
2048*a^8*x^4 - 1280*a^6*b^2*x^3 - 208*a^4*b^4*x^2 - 98*a^2*b^6*x - 33*b^8)
*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^8*x^5 - b^10*x^4)
```

Sympy [F]

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^{\frac{7}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

output
```
Integral(1/(x**(7/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^{7/2}\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^(7/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

output
```
int(1/(x^(7/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{-\frac{8192\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a^{7}x^{3}}{429}+\frac{1024\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a^{5}b^{2}x^{2}}{429}+\frac{320\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a^{3}b^{4}x}{429}+\frac{56\sqrt{x}\,\sqrt{\sqrt{x}\,a+b}\,a}{143}}{}$$

input
```
int(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

output
```
(4*( - 2048*sqrt(x)*sqrt(sqrt(x)*a + b)*a**7*x**3 + 256*sqrt(x)*sqrt(sqrt(
x)*a + b)*a**5*b**2*x**2 + 80*sqrt(x)*sqrt(sqrt(x)*a + b)*a**3*b**4*x + 42
*sqrt(x)*sqrt(sqrt(x)*a + b)*a*b**6 - 1024*sqrt(sqrt(x)*a + b)*a**6*b*x**3
 - 128*sqrt(sqrt(x)*a + b)*a**4*b**3*x**2 - 56*sqrt(sqrt(x)*a + b)*a**2*b*
*5*x - 33*sqrt(sqrt(x)*a + b)*b**7 + 2048*x**(3/4)*sqrt(a)*a**7*x**3 + 204
8*x**(1/4)*sqrt(a)*a**6*b*x**3))/(429*x**(1/4)*b**8*x**3*(sqrt(x)*a + b))
```

3.74 $\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\frac{9\sqrt[3]{1+\frac{b\sqrt[3]{x}}{a}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{11}{3},-\frac{b\sqrt[3]{x}}{a}\right)}{8\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

output
```
9/8*(1+b*x^(1/3)/a)^(1/3)*x*hypergeom([1/3, 8/3],[11/3],-b*x^(1/3)/a)/(a*x
^(1/3)+b*x^(2/3))^(1/3)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\frac{9\sqrt[3]{1+\frac{b\sqrt[3]{x}}{a}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{11}{3},-\frac{b\sqrt[3]{x}}{a}\right)}{8\sqrt[3]{\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}}$$

input
```
Integrate[(a*x^(1/3) + b*x^(2/3))^(-1/3),x]
```

output
```
(9*(1 + (b*x^(1/3))/a)^(1/3)*x*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^(1
/3))/a)])/(8*((a + b*x^(1/3))*x^(1/3))^(1/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1917, 864, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{\sqrt[9]{x}\sqrt[3]{a+b\sqrt[3]{x}}\int \frac{1}{\sqrt[3]{a+b\sqrt[3]{x}}\sqrt[9]{x}}dx}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$\Big\downarrow \text{864}$$

$$\frac{3\sqrt[9]{x}\sqrt[3]{a+b\sqrt[3]{x}}\int \frac{x^{5/9}}{\sqrt[3]{a+b\sqrt[3]{x}}}d\sqrt[3]{x}}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$\Big\downarrow \text{76}$$

$$\frac{3\sqrt[9]{x}\sqrt[3]{\frac{b\sqrt[3]{x}}{a}+1}\int \frac{x^{5/9}}{\sqrt[3]{\frac{\sqrt[3]{x}b}{a}+1}}d\sqrt[3]{x}}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$\Big\downarrow \text{74}$$

$$\frac{9x\sqrt[3]{\frac{b\sqrt[3]{x}}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{11}{3},-\frac{b\sqrt[3]{x}}{a}\right)}{8\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

input
```
Int[(a*x^(1/3) + b*x^(2/3))^(-1/3),x]
```

output
```
(9*(1 + (b*x^(1/3))/a)^(1/3)*x*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^(1
/3))/a)])/(8*(a*x^(1/3) + b*x^(2/3))^(1/3))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{1}{\left(a\,x^{\frac{1}{3}}+b\,x^{\frac{2}{3}}\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x)
```

output
```
int(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{\frac{2}{3}}}}\,dx$$

input
```
integrate(1/(a*x**(1/3)+b*x**(2/3))**(1/3),x)
```

output
```
Integral((a*x**(1/3) + b*x**(2/3))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^(2/3) + a*x^(1/3))^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^(2/3) + a*x^(1/3))^(-1/3), x)
```

Mupad [B] (verification not implemented)

Time = 11.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\frac{9\,x\,\left(\frac{b\,x^{1/3}}{a}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{8}{3};\ \frac{11}{3};\ -\frac{b\,x^{1/3}}{a}\right)}{8\,\left(a\,x^{1/3}+b\,x^{2/3}\right)^{1/3}}$$

input
```
int(1/(a*x^(1/3) + b*x^(2/3))^(1/3),x)
```

output
```
(9*x*((b*x^(1/3))/a + 1)^(1/3)*hypergeom([1/3, 8/3], 11/3, -(b*x^(1/3))/a)
)/(8*(a*x^(1/3) + b*x^(2/3))^(1/3))
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx = \left(\int \frac{x^{\frac{8}{9}}}{x^{\frac{2}{3}}\left(x^{\frac{1}{3}}b+a\right)^{\frac{1}{3}}a+\left(x^{\frac{1}{3}}b+a\right)^{\frac{1}{3}}bx}dx\right)b + \left(\int \frac{x^{\frac{5}{9}}}{x^{\frac{2}{3}}\left(x^{\frac{1}{3}}b+a\right)^{\frac{1}{3}}a+\left(x^{\frac{1}{3}}b+a\right)^{\frac{1}{3}}bx}dx\right)a$$

input
```
int(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x)
```

output
```
int(x**(8/9)/(x**(2/3)*(x**(1/3)*b + a)**(1/3)*a + (x**(1/3)*b + a)**(1/3)
*b*x),x)*b + int(x**(5/9)/(x**(2/3)*(x**(1/3)*b + a)**(1/3)*a + (x**(1/3)*
b + a)**(1/3)*b*x),x)*a
```

3.75 $\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\frac{9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2/3}x\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},-\frac{b\sqrt[3]{x}}{a}\right)}{7\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

output
```
9/7*(1+b*x^(1/3)/a)^(2/3)*x*hypergeom([2/3, 7/3],[10/3],-b*x^(1/3)/a)/(a*x
^(1/3)+b*x^(2/3))^(2/3)
```

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\frac{9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2/3}x\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},-\frac{b\sqrt[3]{x}}{a}\right)}{7\left(\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}\right)^{2/3}}$$

input
```
Integrate[(a*x^(1/3) + b*x^(2/3))^(-2/3),x]
```

output
```
(9*(1 + (b*x^(1/3))/a)^(2/3)*x*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^(1
/3))/a)])/(7*((a + b*x^(1/3))*x^(1/3))^(2/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1917, 864, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{x^{2/9}\left(a+b\sqrt[3]{x}\right)^{2/3}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}}dx}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$\Big\downarrow \text{864}$$

$$\frac{3x^{2/9}\left(a+b\sqrt[3]{x}\right)^{2/3}\int \frac{x^{4/9}}{\left(a+b\sqrt[3]{x}\right)^{2/3}}d\sqrt[3]{x}}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$\Big\downarrow \text{76}$$

$$\frac{3x^{2/9}\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2/3}\int \frac{x^{4/9}}{\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2/3}}d\sqrt[3]{x}}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$\Big\downarrow \text{74}$$

$$\frac{9x\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},-\frac{b\sqrt[3]{x}}{a}\right)}{7\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

input
```
Int[(a*x^(1/3) + b*x^(2/3))^(-2/3),x]
```

output
```
(9*(1 + (b*x^(1/3))/a)^(2/3)*x*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^(1
/3))/a)])/(7*(a*x^(1/3) + b*x^(2/3))^(2/3))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{1}{\left(a\,x^{\frac{1}{3}}+b\,x^{\frac{2}{3}}\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x)
```

output
```
int(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\int \frac{1}{\left(a\sqrt[3]{x}+bx^{\frac{2}{3}}\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a*x**(1/3)+b*x**(2/3))**(2/3),x)
```

output
```
Integral((a*x**(1/3) + b*x**(2/3))**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^(2/3) + a*x^(1/3))^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^(2/3) + a*x^(1/3))^(-2/3), x)
```

Mupad [B] (verification not implemented)

Time = 11.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\frac{9\,x\,\left(\frac{b\,x^{1/3}}{a}+1\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{2}{3},\frac{7}{3};\ \frac{10}{3};\ -\frac{b\,x^{1/3}}{a}\right)}{7\,\left(a\,x^{1/3}+b\,x^{2/3}\right)^{2/3}}$$

input
```
int(1/(a*x^(1/3) + b*x^(2/3))^(2/3),x)
```

output
```
(9*x*((b*x^(1/3))/a + 1)^(2/3)*hypergeom([2/3, 7/3], 10/3, -(b*x^(1/3))/a)
)/(7*(a*x^(1/3) + b*x^(2/3))^(2/3))
```

Reduce [F]

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\int \frac{1}{x^{\frac{2}{9}}\left(x^{\frac{1}{3}}b+a\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x)
```

output
```
int(1/(x**(2/9)*(x**(1/3)*b + a)**(2/3)),x)
```

3.76 $\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 106

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = 2\sqrt{a+b\sqrt{x}+cx}-2\sqrt{a}\text{arctanh}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)+\frac{b\text{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

output
```
2*(a+b*x^(1/2)+c*x)^(1/2)-2*a^(1/2)*arctanh(1/2*(2*a+b*x^(1/2))/a^(1/2)/(a
+b*x^(1/2)+c*x)^(1/2))+b*arctanh(1/2*(b+2*c*x^(1/2))/c^(1/2)/(a+b*x^(1/2)+
c*x)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = 2\sqrt{a+b\sqrt{x}+cx}+4\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{x}-\sqrt{a+b\sqrt{x}+cx}}{\sqrt{a}}\right)-\frac{b\log\left(b+2c\sqrt{x}-2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}\right)}{\sqrt{c}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

output
```
2*Sqrt[a + b*Sqrt[x] + c*x] + 4*Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[x] - Sqrt[a
+ b*Sqrt[x] + c*x])/Sqrt[a]] - (b*Log[b + 2*c*Sqrt[x] - 2*Sqrt[c]*Sqrt[a +
 b*Sqrt[x] + c*x]])/Sqrt[c]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1693, 1162, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}dx$$

$$\Big\downarrow \text{1693}$$

$$2\int \frac{\sqrt{a+cx+b\sqrt{x}}}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{1162}$$

$$2\left(\sqrt{a+b\sqrt{x}+cx}-\frac{1}{2}\int -\frac{2a+b\sqrt{x}}{\sqrt{x}\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{1}{2}\int \frac{2a+b\sqrt{x}}{\sqrt{x}\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}+\sqrt{a+b\sqrt{x}+cx}\right)$$

$$\Big\downarrow \text{1269}$$

$$2\left(\frac{1}{2}\left(b\int \frac{1}{\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}+2a\int \frac{1}{\sqrt{x}\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}\right)+\sqrt{a+b\sqrt{x}+cx}\right)$$

$$\Big\downarrow \text{1092}$$

$$2\left(\frac{1}{2}\left(2b\int \frac{1}{4c-x}d\frac{b+2c\sqrt{x}}{\sqrt{a+cx+b\sqrt{x}}}+2a\int \frac{1}{\sqrt{x}\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}\right)+\sqrt{a+b\sqrt{x}+cx}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{1}{2}\left(2a\int \frac{1}{\sqrt{x}\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}+\frac{b\text{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}\right)+\sqrt{a+b\sqrt{x}+cx}\right)$$

$$\Big\downarrow \text{1154}$$

$$2\left(\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}-4a\int \frac{1}{4a-x}d\frac{2a+b\sqrt{x}}{\sqrt{a+cx+b\sqrt{x}}}\right)+\sqrt{a+b\sqrt{x}+cx}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}-2\sqrt{a}\text{arctanh}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)\right)+\sqrt{a+b\sqrt{x}+cx}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

output
```
2*(Sqrt[a + b*Sqrt[x] + c*x] + (-2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])/(2*Sq
rt[a]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c
]*Sqrt[a + b*Sqrt[x] + c*x])])/Sqrt[c])/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$2\sqrt{a+b\sqrt{x}+cx}+\frac{b\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+b\sqrt{x}+cx}\right)}{\sqrt{c}}-2\sqrt{a}\,\ln\left(\frac{2a+b\sqrt{x}+2\sqrt{a}\,\sqrt{a+b\sqrt{x}+cx}}{\sqrt{x}}\right)$	84
default	$2\sqrt{a+b\sqrt{x}+cx}+\frac{b\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+b\sqrt{x}+cx}\right)}{\sqrt{c}}-2\sqrt{a}\,\ln\left(\frac{2a+b\sqrt{x}+2\sqrt{a}\,\sqrt{a+b\sqrt{x}+cx}}{\sqrt{x}}\right)$	84

input
```
int((a+b*x^(1/2)+c*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(a+b*x^(1/2)+c*x)^(1/2)+b*ln((1/2*b+c*x^(1/2))/c^(1/2)+(a+b*x^(1/2)+c*x)
^(1/2))/c^(1/2)-2*a^(1/2)*ln((2*a+b*x^(1/2)+2*a^(1/2)*(a+b*x^(1/2)+c*x)^(1
/2))/x^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x^(1/2)+c*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$$

input
```
integrate((a+b*x**(1/2)+c*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*sqrt(x) + c*x)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \int \frac{\sqrt{cx+b\sqrt{x}+a}}{x}\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x + b*sqrt(x) + a)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^(1/2)+c*x)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \int \frac{\sqrt{a+c\,x+b\,\sqrt{x}}}{x}\,dx$$

input
```
int((a + c*x + b*x^(1/2))^(1/2)/x,x)
```

output
```
int((a + c*x + b*x^(1/2))^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 821, normalized size of antiderivative = 7.75

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = \text{Too large to display}$$

input
```
int((a+b*x^(1/2)+c*x)^(1/2)/x,x)
```

output
```
( - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqr
t(sqrt(x)*b + a + c*x) + 2*sqrt(x)*c + b)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c
 - b**2))*b*c - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2
*sqrt(c)*sqrt(sqrt(x)*b + a + c*x) + 2*sqrt(x)*c + b)/sqrt(4*sqrt(c)*sqrt(
a)*b - 4*a*c - b**2))*a*c + 8*sqrt(sqrt(x)*b + a + c*x)*a*c**2 - 2*sqrt(sq
rt(x)*b + a + c*x)*b**2*c - sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**
2)*log(2*sqrt(c)*sqrt(sqrt(x)*b + a + c*x) - sqrt(4*sqrt(c)*sqrt(a)*b + 4*
a*c + b**2) + 2*sqrt(x)*c + b)*b*c + sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*
a*c + b**2)*log(2*sqrt(c)*sqrt(sqrt(x)*b + a + c*x) + sqrt(4*sqrt(c)*sqrt(
a)*b + 4*a*c + b**2) + 2*sqrt(x)*c + b)*b*c + 2*sqrt(c)*sqrt(4*sqrt(c)*sqr
t(a)*b + 4*a*c + b**2)*log(2*sqrt(c)*sqrt(sqrt(x)*b + a + c*x) - sqrt(4*sq
rt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(x)*c + b)*a*c - 2*sqrt(c)*sqrt(4*
sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(2*sqrt(c)*sqrt(sqrt(x)*b + a + c*x)
+ sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(x)*c + b)*a*c - 4*sqrt
(a)*log(8*sqrt(x)*sqrt(c)*sqrt(sqrt(x)*b + a + c*x)*c + 4*sqrt(c)*sqrt(sqr
t(x)*b + a + c*x)*b + 4*sqrt(c)*sqrt(a)*b + 8*sqrt(x)*b*c + 8*c**2*x)*a*c*
*2 + sqrt(a)*log(8*sqrt(x)*sqrt(c)*sqrt(sqrt(x)*b + a + c*x)*c + 4*sqrt(c)
*sqrt(sqrt(x)*b + a + c*x)*b + 4*sqrt(c)*sqrt(a)*b + 8*sqrt(x)*b*c + 8*c**
2*x)*b**2*c + 4*sqrt(a)*log(2*sqrt(c)*sqrt(sqrt(x)*b + a + c*x) - sqrt(4*s
qrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(x)*c + b)*a*c**2 - sqrt(a)*lo...
```

3.77 $\int (dx)^m \left(a+b\sqrt{x}+cx\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 159

$$\int (dx)^m \left(a+b\sqrt{x}+cx\right)^p \, dx = \frac{\left(1+\frac{2c\sqrt{x}}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2c\sqrt{x}}{b+\sqrt{b^2-4ac}}\right)^{-p}(dx)^{1+m}\left(a+b\sqrt{x}+cx\right)^p \operatorname{AppellF1}\left(2(1+m),-p,-p,3+2m,-\right.}{d(1+m)}$$

output
```
(d*x)^(1+m)*(a+b*x^(1/2)+c*x)^p*AppellF1(2+2*m,-p,-p,3+2*m,-2*c*x^(1/2)/(b
-(-4*a*c+b^2)^(1/2)),-2*c*x^(1/2)/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*
x^(1/2)/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^(1/2)/(b+(-4*a*c+b^2)^(1/2)))
^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.15

$$\int (dx)^m \left(a+b\sqrt{x}+cx\right)^p \, dx = \frac{\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt{x}}{b+\sqrt{b^2-4ac}}\right)^{-p} x(dx)^m\left(a+b\sqrt{x}+cx\right)^p \operatorname{AppellF1}\left(2(1+m),-p,-p,3+2m,\right.}{1+m}$$

input
```
Integrate[(d*x)^m*(a + b*Sqrt[x] + c*x)^p,x]
```

output
```
(x*(d*x)^m*(a + b*Sqrt[x] + c*x)^p*AppellF1[2*(1 + m), -p, -p, 3 + 2*m, (-
2*c*Sqrt[x])/(b + Sqrt[b^2 - 4*a*c]), (2*c*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c
])])/((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*Sqrt[x])/(b - Sqrt[b^2 - 4*a*c
]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*Sqrt[x])/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1716, 1715, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\sqrt{x}+cx\right)^p dx$$

$$\Big\downarrow \text{ 1716}$$

$$x^{-m}(dx)^m \int x^m \left(a+cx+b\sqrt{x}\right)^p dx$$

$$\Big\downarrow \text{ 1715}$$

$$2x^{-m}(dx)^m \int x^{\frac{1}{2}(2m+1)} \left(a+cx+b\sqrt{x}\right)^p d\sqrt{x}$$

$$\Big\downarrow \text{ 1179}$$

$$2x^{-m}(dx)^m \left(\frac{2c\sqrt{x}}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2c\sqrt{x}}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+b\sqrt{x}+cx\right)^p \int \left(\frac{2\sqrt{x}c}{b-\sqrt{b^2-4ac}}+1\right)^p \left(\frac{}{b+}\right.$$

$$\Big\downarrow \text{ 150}$$

$$\frac{x(dx)^m \left(\frac{2c\sqrt{x}}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2c\sqrt{x}}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+b\sqrt{x}+cx\right)^p \text{AppellF1}\left(2(m+1),-p,-p,2m+3,-\frac{2c\sqrt{x}}{b-\sqrt{b^2-4}}\right.}{m+1}$$

input
```
Int[(d*x)^m*(a + b*Sqrt[x] + c*x)^p,x]
```

output
```
(x*(d*x)^m*(a + b*Sqrt[x] + c*x)^p*AppellF1[2*(1 + m), -p, -p, 3 + 2*m, (-
2*c*Sqrt[x])/(b - Sqrt[b^2 - 4*a*c]), (-2*c*Sqrt[x])/(b + Sqrt[b^2 - 4*a*c
])])/((1 + m)*(1 + (2*c*Sqrt[x])/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*Sqrt
[x])/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1715
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*
x^(k*n) + c*x^(2*k*n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, m, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && FractionQ[n]
```

rule 1716
```
Int[((d_)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Simp[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[n2, 2
*n] && NeQ[b^2 - 4*a*c, 0] && FractionQ[n]
```

Maple [F]

$$\int (dx)^m \left(a+b\sqrt{x}+cx\right)^p\,dx$$

input
```
int((d*x)^m*(a+b*x^(1/2)+c*x)^p,x)
```

output
```
int((d*x)^m*(a+b*x^(1/2)+c*x)^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m \left(a+b\sqrt{x}+cx\right)^p\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b*x^(1/2)+c*x)^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\sqrt{x}+cx\right)^p\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*x**(1/2)+c*x)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (dx)^m \left(a+b\sqrt{x}+cx\right)^p\, dx = \int \left(cx+b\sqrt{x}+a\right)^p (dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*x^(1/2)+c*x)^p,x, algorithm="maxima")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a+b\sqrt{x}+cx\right)^p\, dx = \int \left(cx+b\sqrt{x}+a\right)^p (dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*x^(1/2)+c*x)^p,x, algorithm="giac")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\sqrt{x}+cx\right)^p\, dx = \int (d\,x)^m \left(a+c\,x+b\,\sqrt{x}\right)^p dx$$

input
```
int((d*x)^m*(a + c*x + b*x^(1/2))^p,x)
```

output
```
int((d*x)^m*(a + c*x + b*x^(1/2))^p, x)
```

Reduce [F]

$$\int (dx)^m \left(a+b\sqrt{x}+cx\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x)^m*(a+b*x^(1/2)+c*x)^p,x)
```

output
```
(d**m*(4*x**((2*m + 1)/2)*(sqrt(x)*b + a + c*x)**p*b*m*p + 2*x**((2*m + 1)
/2)*(sqrt(x)*b + a + c*x)**p*b*p**2 - 4*x**m*(sqrt(x)*b + a + c*x)**p*a*m*
p - 2*x**m*(sqrt(x)*b + a + c*x)**p*a*p + 8*x**m*(sqrt(x)*b + a + c*x)**p*
c*m**2*x + 12*x**m*(sqrt(x)*b + a + c*x)**p*c*m*p*x + 4*x**m*(sqrt(x)*b +
a + c*x)**p*c*m*x + 4*x**m*(sqrt(x)*b + a + c*x)**p*c*p**2*x + 2*x**m*(sqr
t(x)*b + a + c*x)**p*c*p*x - 16*int((x**((2*m + 1)/2)*(sqrt(x)*b + a + c*x
)**p)/(4*a**2*m**3*x + 10*a**2*m**2*p*x + 6*a**2*m**2*x + 8*a**2*m*p**2*x
+ 9*a**2*m*p*x + 2*a**2*m*x + 2*a**2*p**3*x + 3*a**2*p**2*x + a**2*p*x + 8
*a*c*m**3*x**2 + 20*a*c*m**2*p*x**2 + 12*a*c*m**2*x**2 + 16*a*c*m*p**2*x**
2 + 18*a*c*m*p*x**2 + 4*a*c*m*x**2 + 4*a*c*p**3*x**2 + 6*a*c*p**2*x**2 + 2
*a*c*p*x**2 - 4*b**2*m**3*x**2 - 10*b**2*m**2*p*x**2 - 6*b**2*m**2*x**2 -
8*b**2*m*p**2*x**2 - 9*b**2*m*p*x**2 - 2*b**2*m*x**2 - 2*b**2*p**3*x**2 -
3*b**2*p**2*x**2 - b**2*p*x**2 + 4*c**2*m**3*x**3 + 10*c**2*m**2*p*x**3 +
6*c**2*m**2*x**3 + 8*c**2*m*p**2*x**3 + 9*c**2*m*p*x**3 + 2*c**2*m*x**3 +
2*c**2*p**3*x**3 + 3*c**2*p**2*x**3 + c**2*p*x**3),x)*a**2*b*m**5*p - 40*i
nt((x**((2*m + 1)/2)*(sqrt(x)*b + a + c*x)**p)/(4*a**2*m**3*x + 10*a**2*m*
*2*p*x + 6*a**2*m**2*x + 8*a**2*m*p**2*x + 9*a**2*m*p*x + 2*a**2*m*x + 2*a
**2*p**3*x + 3*a**2*p**2*x + a**2*p*x + 8*a*c*m**3*x**2 + 20*a*c*m**2*p*x*
*2 + 12*a*c*m**2*x**2 + 16*a*c*m*p**2*x**2 + 18*a*c*m*p*x**2 + 4*a*c*m*x**
2 + 4*a*c*p**3*x**2 + 6*a*c*p**2*x**2 + 2*a*c*p*x**2 - 4*b**2*m**3*x**2...
```

3.78 $\int x^3\left(a+b\sqrt{x}+cx\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 807

$$\int x^3\left(a+b\sqrt{x}+cx\right)^p\,dx =$$

$$-\frac{(48a^3c^3(105+142p+60p^2+8p^3)-12a^2b^2c^2(2520+3634p+1831p^2+386p^3+29p^4)+10ab^4c(2520\cdots}{}$$

$$+\frac{(24a^2c^2(35+24p+4p^2)-6ab^2c(420+304p+69p^2+5p^3)+b^4(840+638p+179p^2+22p^3+p^4))\,x\cdots}{4c^5(2+p)(3+p)(4+p)(5+2p)(7+2p)}$$

$$+\frac{b(6+p)\left(2ac(35+11p)-b^2(35+12p+p^2)\right)x^{3/2}\left(a+b\sqrt{x}+cx\right)^{1+p}}{2c^4(3+p)(4+p)(5+2p)(7+2p)}$$

$$-\frac{\left(6ac(7+2p)-b^2(42+13p+p^2)\right)x^2\left(a+b\sqrt{x}+cx\right)^{1+p}}{2c^3(3+p)(4+p)(7+2p)}$$

$$-\frac{b(7+p)x^{5/2}\left(a+b\sqrt{x}+cx\right)^{1+p}}{c^2(4+p)(7+2p)}+\frac{x^3\left(a+b\sqrt{x}+cx\right)^{1+p}}{c(4+p)}$$

$$-\frac{2^{-2+p}b(840a^3c^3-420a^2b^2c^2(5+p)+42ab^4c(30+11p+p^2)-b^6(210+107p+18p^2+p^3))\left(-\frac{b-\sqrt{b^2-\cdots}}{\sqrt{b\cdots}}\right.\cdots}{c^7\sqrt{b^2-4ac}(1+p)(3+2p)\cdots}$$

output
```
-1/8*(48*a^3*c^3*(8*p^3+60*p^2+142*p+105)-12*a^2*b^2*c^2*(29*p^4+386*p^3+1
831*p^2+3634*p+2520)+10*a*b^4*c*(4*p^5+79*p^4+602*p^3+2201*p^2+3834*p+2520
)-b^6*(p^6+27*p^5+295*p^4+1665*p^3+5104*p^2+8028*p+5040)+2*b*c*(p+1)*(5+p)
*(12*a^2*c^2*(19*p^2+100*p+126)-12*a*b^2*c*(3*p^3+37*p^2+142*p+168)+b^4*(p
^4+20*p^3+145*p^2+450*p+504))*x^(1/2))*(a+b*x^(1/2)+c*x)^(p+1)/c^7/(p+1)/(
2+p)/(3+p)/(4+p)/(3+2*p)/(5+2*p)/(7+2*p)+1/4*(24*a^2*c^2*(4*p^2+24*p+35)-6
*a*b^2*c*(5*p^3+69*p^2+304*p+420)+b^4*(p^4+22*p^3+179*p^2+638*p+840))*x*(a
+b*x^(1/2)+c*x)^(p+1)/c^5/(2+p)/(3+p)/(4+p)/(5+2*p)/(7+2*p)+1/2*b*(6+p)*(2
*a*c*(35+11*p)-b^2*(p^2+12*p+35))*x^(3/2)*(a+b*x^(1/2)+c*x)^(p+1)/c^4/(3+p
)/(4+p)/(5+2*p)/(7+2*p)-1/2*(6*a*c*(7+2*p)-b^2*(p^2+13*p+42))*x^2*(a+b*x^(
1/2)+c*x)^(p+1)/c^3/(3+p)/(4+p)/(7+2*p)-b*(7+p)*x^(5/2)*(a+b*x^(1/2)+c*x)^
(p+1)/c^2/(4+p)/(7+2*p)+x^3*(a+b*x^(1/2)+c*x)^(p+1)/c/(4+p)-2^(-2+p)*b*(84
0*a^3*c^3-420*a^2*b^2*c^2*(5+p)+42*a*b^4*c*(p^2+11*p+30)-b^6*(p^3+18*p^2+1
07*p+210))*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x^(1/2))/(-4*a*c+b^2)^(1/2))^(-1-p)
*(a+b*x^(1/2)+c*x)^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+(-4*a*c+b^2)^(1/
2)+2*c*x^(1/2))/(-4*a*c+b^2)^(1/2))/c^7/(-4*a*c+b^2)^(1/2)/(p+1)/(3+2*p)/(
5+2*p)/(7+2*p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 5.80 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.21

$$\int x^3\left(a+b\sqrt{x}+cx\right)^p\,dx = \frac{1}{4}\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt{x}}{b+\sqrt{b^2-4ac}}\right)^{-p}x^4\left(a+b\sqrt{x}+cx\right)^p\operatorname{AppellF1}\left(8,-p,-p,9,-\frac{2c\sqrt{x}}{b+\sqrt{b^2-4ac}},\frac{2c\sqrt{x}}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^3*(a + b*Sqrt[x] + c*x)^p,x]
```

output
```
(x^4*(a + b*Sqrt[x] + c*x)^p*AppellF1[8, -p, -p, 9, (-2*c*Sqrt[x])/(b + Sq
rt[b^2 - 4*a*c]), (2*c*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])])/(4*((b - Sqrt[b
^2 - 4*a*c] + 2*c*Sqrt[x])/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a
*c] + 2*c*Sqrt[x])/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 782, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {1693, 1166, 25, 1236, 25, 1236, 1236, 25, 1236, 1225, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sqrt{x}+cx\right)^p\,dx$$

$$\Big\downarrow \text{1693}$$

$$2\int x^{7/2}\left(a+cx+b\sqrt{x}\right)^p d\sqrt{x}$$

$$\Big\downarrow \text{1166}$$

$$2\left(\frac{\int -\left(\left(6a+b(p+7)\sqrt{x}\right)x^{5/2}\left(a+cx+b\sqrt{x}\right)^p\right)d\sqrt{x}}{2c(p+4)}+\frac{x^3\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+4)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{x^3\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+4)}-\frac{\int \left(6a+b(p+7)\sqrt{x}\right)x^{5/2}\left(a+cx+b\sqrt{x}\right)^p d\sqrt{x}}{2c(p+4)}\right)$$

$$\Big\downarrow \text{1236}$$

$$2\left(\frac{x^3\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+4)}-\frac{\frac{\int -\left(\left(5ab(p+7)-\left(6ac(2p+7)-b^2\left(p^2+13p+42\right)\right)\sqrt{x}\right)x^2\left(a+cx+b\sqrt{x}\right)^p\right)d\sqrt{x}}{c(2p+7)}+\frac{b(p+7)x^{5/2}\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+7)}}{2c(p+4)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{x^3\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+4)}-\frac{\frac{b(p+7)x^{5/2}\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+7)}-\frac{\int \left(5ab(p+7)-\left(6ac(2p+7)-b^2\left(p^2+13p+42\right)\right)\sqrt{x}\right)x^2\left(a+cx+b\sqrt{x}\right)^p d\sqrt{x}}{c(2p+7)}}{2c(p+4)}\right)$$

$$\Big\downarrow \text{1236}$$

$$2\left(\frac{x^3\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+4)}-\frac{\frac{b(p+7)x^{5/2}\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+7)}-\frac{\frac{\int\left(b(p+6)\sqrt{x}\left(2ac(11p+35)-b^2\left(p^2+12p+35\right)\right)+4a\left(6ac(2p+7)-b^2\left(p^2+13p+42\right)\right)\cdots}{2c(p+3)}}{c(2p+\cdots}}{2c(p+4)}\right.$$

$\downarrow$ 1236

$$2\left(\frac{x^3\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+4)}-\frac{\frac{b(p+7)x^{5/2}\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+7)}-\frac{\frac{\int-\left(\left(3ab(p+6)\left(2ac(11p+35)-b^2\left(p^2+12p+35\right)\right)-\left(\left(p^4+22p^3+179p^2+638p+840\cdots\right.\right.\right.\right.}{c(\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 25

$$2\left(\frac{x^3\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+4)}-\frac{\frac{b(p+7)x^{5/2}\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+7)}-\frac{\frac{b(p+6)x^{3/2}\left(2ac(11p+35)-b^2\left(p^2+12p+35\right)\right)\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+5)}-\frac{\int\left(3ab(p+6)\left(2ac\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 1236

$$2\left(\frac{x^3\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+4)}-\frac{\frac{b(p+7)x^{5/2}\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+7)}-\frac{\frac{b(p+6)x^{3/2}\left(2ac(11p+35)-b^2\left(p^2+12p+35\right)\right)\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+5)}-\frac{\int\left(b(p+5)\sqrt{x}\left(\left(p\cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 1225

$$2\left(\frac{x^3\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+4)}-\frac{\frac{b(p+7)x^{5/2}\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+7)}-\frac{\frac{b(p+6)x^{3/2}\left(2ac(11p+35)-b^2\left(p^2+12p+35\right)\right)\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+5)}-\frac{\frac{\left(48a^3c^3\left(8p^3+6\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 1096

$$2\left(\frac{x^3\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+4)}-\frac{\frac{b(p+7)x^{5/2}\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+7)}-\frac{\frac{b(p+6)x^{3/2}\left(2ac(11p+35)-b^2\left(p^2+12p+35\right)\right)\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+5)}-\frac{b2^p\left(p^3+9p^2+2\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

input
```
Int[x^3*(a + b*Sqrt[x] + c*x)^p,x]
```

output
```
2*((x^3*(a + b*Sqrt[x] + c*x)^(1 + p))/(2*c*(4 + p)) - ((b*(7 + p)*x^(5/2)
*(a + b*Sqrt[x] + c*x)^(1 + p))/(c*(7 + 2*p)) - (-1/2*((6*a*c*(7 + 2*p) -
b^2*(42 + 13*p + p^2))*x^2*(a + b*Sqrt[x] + c*x)^(1 + p))/(c*(3 + p)) + ((
b*(6 + p)*(2*a*c*(35 + 11*p) - b^2*(35 + 12*p + p^2))*x^(3/2)*(a + b*Sqrt[
x] + c*x)^(1 + p))/(c*(5 + 2*p)) - (-1/2*((24*a^2*c^2*(35 + 24*p + 4*p^2)
- 6*a*b^2*c*(420 + 304*p + 69*p^2 + 5*p^3) + b^4*(840 + 638*p + 179*p^2 +
22*p^3 + p^4))*x*(a + b*Sqrt[x] + c*x)^(1 + p))/(c*(2 + p)) + (((48*a^3*c^
3*(105 + 142*p + 60*p^2 + 8*p^3) - 12*a^2*b^2*c^2*(2520 + 3634*p + 1831*p^
2 + 386*p^3 + 29*p^4) + 10*a*b^4*c*(2520 + 3834*p + 2201*p^2 + 602*p^3 + 7
9*p^4 + 4*p^5) - b^6*(5040 + 8028*p + 5104*p^2 + 1665*p^3 + 295*p^4 + 27*p
^5 + p^6) + 2*b*c*(1 + p)*(5 + p)*(12*a^2*c^2*(126 + 100*p + 19*p^2) - 12*
a*b^2*c*(168 + 142*p + 37*p^2 + 3*p^3) + b^4*(504 + 450*p + 145*p^2 + 20*p
^3 + p^4))*Sqrt[x])*(a + b*Sqrt[x] + c*x)^(1 + p))/(2*c^2*(1 + p)*(3 + 2*p
)) + (2^p*b*(24 + 26*p + 9*p^2 + p^3)*(840*a^3*c^3 - 420*a^2*b^2*c^2*(5 +
p) + 42*a*b^4*c*(30 + 11*p + p^2) - b^6*(210 + 107*p + 18*p^2 + p^3))*(-((
b - Sqrt[b^2 - 4*a*c] + 2*c*Sqrt[x])/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*S
qrt[x] + c*x)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 -
4*a*c] + 2*c*Sqrt[x])/(2*Sqrt[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 +
p)*(3 + 2*p)))/(2*c*(2 + p)))/(c*(5 + 2*p)))/(2*c*(3 + p)))/(c*(7 + 2*p)))
/(2*c*(4 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^3 \left(a + b\sqrt{x} + cx\right)^p \, dx$$

input
```
int(x^3*(a+b*x^(1/2)+c*x)^p,x)
```

output
```
int(x^3*(a+b*x^(1/2)+c*x)^p,x)
```

Fricas [F]

$$\int x^3 \left(a + b\sqrt{x} + cx\right)^p \, dx = \int \left(cx + b\sqrt{x} + a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(a+b*x^(1/2)+c*x)^p,x, algorithm="fricas")
```

output
```
integral((c*x + b*sqrt(x) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b\sqrt{x} + cx\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*x**(1/2)+c*x)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3 \left(a+b\sqrt{x}+cx\right)^p\,dx=\int \left(cx+b\sqrt{x}+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b*x^(1/2)+c*x)^p,x, algorithm="maxima")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p*x^3, x)
```

Giac [F]

$$\int x^3 \left(a+b\sqrt{x}+cx\right)^p\,dx=\int \left(cx+b\sqrt{x}+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b*x^(1/2)+c*x)^p,x, algorithm="giac")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\sqrt{x}+cx\right)^p\,dx=\int x^3\,\left(a+c\,x+b\,\sqrt{x}\right)^p\,dx$$

input
```
int(x^3*(a + c*x + b*x^(1/2))^p,x)
```

output
```
int(x^3*(a + c*x + b*x^(1/2))^p, x)
```

Reduce [F]

$$\int x^3\left(a+b\sqrt{x}+cx\right)^p\,dx=\text{too large to display}$$

input
```
int(x^3*(a+b*x^(1/2)+c*x)^p,x)
```

output
```
( - 1824*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a**3*b*c**3*p**5 - 21312*sqrt(x)
*(sqrt(x)*b + a + c*x)**p*a**3*b*c**3*p**4 - 84576*sqrt(x)*(sqrt(x)*b + a
+ c*x)**p*a**3*b*c**3*p**3 - 134208*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a**3*
b*c**3*p**2 - 70560*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a**3*b*c**3*p + 288*s
qrt(x)*(sqrt(x)*b + a + c*x)**p*a**2*b**3*c**2*p**6 + 5976*sqrt(x)*(sqrt(x
)*b + a + c*x)**p*a**2*b**3*c**2*p**5 + 45648*sqrt(x)*(sqrt(x)*b + a + c*x
)**p*a**2*b**3*c**2*p**4 + 159624*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a**2*b*
*3*c**2*p**3 + 252144*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a**2*b**3*c**2*p**2
 + 141120*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a**2*b**3*c**2*p + 704*sqrt(x)*
(sqrt(x)*b + a + c*x)**p*a**2*b*c**4*p**6*x + 9696*sqrt(x)*(sqrt(x)*b + a
+ c*x)**p*a**2*b*c**4*p**5*x + 44288*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a**2
*b*c**4*p**4*x + 84384*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a**2*b*c**4*p**3*x
 + 65888*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a**2*b*c**4*p**2*x + 16800*sqrt(
x)*(sqrt(x)*b + a + c*x)**p*a**2*b*c**4*p*x - 8*sqrt(x)*(sqrt(x)*b + a + c
*x)**p*a*b**5*c*p**7 - 288*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a*b**5*c*p**6
- 3740*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a*b**5*c*p**5 - 23400*sqrt(x)*(sqr
t(x)*b + a + c*x)**p*a*b**5*c*p**4 - 75452*sqrt(x)*(sqrt(x)*b + a + c*x)**
p*a*b**5*c*p**3 - 118872*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a*b**5*c*p**2 -
70560*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a*b**5*c*p - 32*sqrt(x)*(sqrt(x)*b
+ a + c*x)**p*a*b**3*c**3*p**7*x - 928*sqrt(x)*(sqrt(x)*b + a + c*x)**p...
```

3.79 $\int x^2 \left(a+b\sqrt{x}+cx\right)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 471

$$\int x^2 \left(a+b\sqrt{x}+cx\right)^p \, dx =$$
$$-\frac{\left(b^2(2+p)(4+p)\left(2ac(15+7p)-b^2(15+8p+p^2)\right)-2ac(3+2p)\left(4ac(5+2p)-b^2(20+9p+p^2)\right)\cdots\right.}{4c^5(1+p)(2+p)(3+p)(3+2p\cdots}$$
$$-\frac{\left(4ac(5+2p)-b^2(20+9p+p^2)\right)x\left(a+b\sqrt{x}+cx\right)^{1+p}}{2c^3(2+p)(3+p)(5+2p)}$$
$$-\frac{b(5+p)x^{3/2}\left(a+b\sqrt{x}+cx\right)^{1+p}}{c^2(3+p)(5+2p)}+\frac{x^2\left(a+b\sqrt{x}+cx\right)^{1+p}}{c(3+p)}$$
$$+\frac{2^{-1+p}b\left(60a^2c^2-20ab^2c(4+p)+b^4(20+9p+p^2)\right)\left(-\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+b\sqrt{x}+cx\right)^{1+p}\text{Hyper}\cdots}{c^5\sqrt{b^2-4ac}(1+p)(3+2p)(5+2p)}$$

output
```
-1/4*(b^2*(2+p)*(4+p)*(2*a*c*(15+7*p)-b^2*(p^2+8*p+15))-2*a*c*(3+2*p)*(4*a
*c*(5+2*p)-b^2*(p^2+9*p+20))-2*b*c*(p+1)*(4+p)*(2*a*c*(15+7*p)-b^2*(p^2+8*
p+15))*x^(1/2))*(a+b*x^(1/2)+c*x)^(p+1)/c^5/(p+1)/(2+p)/(3+p)/(3+2*p)/(5+2
*p)-1/2*(4*a*c*(5+2*p)-b^2*(p^2+9*p+20))*x*(a+b*x^(1/2)+c*x)^(p+1)/c^3/(2+
p)/(3+p)/(5+2*p)-b*(5+p)*x^(3/2)*(a+b*x^(1/2)+c*x)^(p+1)/c^2/(3+p)/(5+2*p)
+x^2*(a+b*x^(1/2)+c*x)^(p+1)/c/(3+p)+2^(-1+p)*b*(60*c^2*a^2-20*a*b^2*c*(4+
p)+b^4*(p^2+9*p+20))*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x^(1/2))/(-4*a*c+b^2)^(1/
2))^(-1-p)*(a+b*x^(1/2)+c*x)^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+(-4*a*
c+b^2)^(1/2)+2*c*x^(1/2))/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)/(p+1)
/(3+2*p)/(5+2*p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.49 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.36

$$\int x^2\left(a+b\sqrt{x}+cx\right)^p\,dx = \frac{1}{3}\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt{x}}{b+\sqrt{b^2-4ac}}\right)^{-p}x^3\left(a+b\sqrt{x}+cx\right)^p\operatorname{AppellF1}\left(6,-p,-p,7,-\frac{2c\sqrt{x}}{b+\sqrt{b^2-4ac}},\frac{2c\sqrt{x}}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^2*(a + b*Sqrt[x] + c*x)^p,x]
```

output
```
(x^3*(a + b*Sqrt[x] + c*x)^p*AppellF1[6, -p, -p, 7, (-2*c*Sqrt[x])/(b + Sq
rt[b^2 - 4*a*c]), (2*c*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])])/(3*((b - Sqrt[b
^2 - 4*a*c] + 2*c*Sqrt[x])/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a
*c] + 2*c*Sqrt[x])/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1693, 1166, 25, 1236, 25, 1236, 1225, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt{x}+cx\right)^p\,dx$$

$$\Big\downarrow \text{1693}$$

$$2\int x^{5/2}\left(a+cx+b\sqrt{x}\right)^p\,d\sqrt{x}$$

$$\Big\downarrow \text{1166}$$

$$2\left(\frac{\int -\left(\left(4a+b(p+5)\sqrt{x}\right)x^{3/2}\left(a+cx+b\sqrt{x}\right)^p\right)d\sqrt{x}}{2c(p+3)}+\frac{x^2\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+3)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{x^2\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+3)}-\frac{\int \left(4a+b(p+5)\sqrt{x}\right)x^{3/2}\left(a+cx+b\sqrt{x}\right)^p d\sqrt{x}}{2c(p+3)}\right)$$

$$\Big\downarrow \text{1236}$$

$$2\left(\frac{x^2\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+3)}-\frac{\frac{\int -\left(\left(3ab(p+5)-\left(4ac(2p+5)-b^2\left(p^2+9p+20\right)\right)\sqrt{x}\right)x\left(a+cx+b\sqrt{x}\right)^p\right)d\sqrt{x}}{c(2p+5)}+\frac{b(p+5)x^{3/2}\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+5)}}{2c(p+3)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{x^2\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+3)}-\frac{\frac{b(p+5)x^{3/2}\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+5)}-\frac{\int \left(3ab(p+5)-\left(4ac(2p+5)-b^2\left(p^2+9p+20\right)\right)\sqrt{x}\right)x\left(a+cx+b\sqrt{x}\right)^p d\sqrt{x}}{c(2p+5)}}{2c(p+3)}\right)$$

$$\Big\downarrow \text{1236}$$

$$2\left(\frac{x^2\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+3)}-\frac{\frac{b(p+5)x^{3/2}\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+5)}-\frac{\frac{\int\left(b(p+4)\sqrt{x}\left(2ac(7p+15)-b^2\left(p^2+8p+15\right)\right)+2a\left(4ac(2p+5)-b^2\left(p^2+9p+20\right)\right)\right)\sqrt{x}\cdots}{2c(p+2)}\cdots}{c(2p+5)}\cdots}{2c(p+3)}\cdots$$

$\big\downarrow$ 1225

$$2\left(\frac{x^2\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+3)}-\frac{\frac{b(p+5)x^{3/2}\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+5)}-\frac{-\frac{b\left(p^2+5p+6\right)\left(60a^2c^2-20ab^2c(p+4)+b^4\left(p^2+9p+20\right)\right)\int\left(a+cx+b\sqrt{x}\right)^pd\sqrt{x}}{2c^2(2p+3)}-\left(-\cdots\right.}{\cdots}}{\cdots}\cdots$$

$\big\downarrow$ 1096

$$2\left(\frac{x^2\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+3)}-\frac{\frac{b(p+5)x^{3/2}\left(a+b\sqrt{x}+cx\right)^{p+1}}{c(2p+5)}-\frac{\frac{b2^p\left(p^2+5p+6\right)\left(60a^2c^2-20ab^2c(p+4)+b^4\left(p^2+9p+20\right)\right)\left(-\frac{-\sqrt{b^2-4ac}+b+2c\sqrt{x}}{\sqrt{b^2-4ac}}\cdots\right.}{c^2(p+1)(2p+\cdots}\cdots}{\cdots}}{\cdots}\cdots$$

input
```
Int[x^2*(a + b*Sqrt[x] + c*x)^p,x]
```

output
```
2*((x^2*(a + b*Sqrt[x] + c*x)^(1 + p))/(2*c*(3 + p)) - ((b*(5 + p)*x^(3/2)
*(a + b*Sqrt[x] + c*x)^(1 + p))/(c*(5 + 2*p)) - (-1/2*((4*a*c*(5 + 2*p) -
b^2*(20 + 9*p + p^2))*x*(a + b*Sqrt[x] + c*x)^(1 + p))/(c*(2 + p)) + (-1/2
*((b^2*(2 + p)*(4 + p)*(2*a*c*(15 + 7*p) - b^2*(15 + 8*p + p^2)) - 2*a*c*(
3 + 2*p)*(4*a*c*(5 + 2*p) - b^2*(20 + 9*p + p^2)) - 2*b*c*(1 + p)*(4 + p)*
(2*a*c*(15 + 7*p) - b^2*(15 + 8*p + p^2))*Sqrt[x])*(a + b*Sqrt[x] + c*x)^(
1 + p))/(c^2*(1 + p)*(3 + 2*p)) + (2^p*b*(6 + 5*p + p^2)*(60*a^2*c^2 - 20*
a*b^2*c*(4 + p) + b^4*(20 + 9*p + p^2))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*Sq
rt[x])/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*Sqrt[x] + c*x)^(1 + p)*Hypergeo
metric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*Sqrt[x])/(2*Sqrt[
b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p)))/(2*c*(2 + p)))/
(c*(5 + 2*p)))/(2*c*(3 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^2 (a+b\sqrt{x}+cx)^p \, dx$$

input
```
int(x^2*(a+b*x^(1/2)+c*x)^p,x)
```

output
```
int(x^2*(a+b*x^(1/2)+c*x)^p,x)
```

Fricas [F]

$$\int x^2 \left(a+b\sqrt{x}+cx\right)^p \, dx = \int \left(cx+b\sqrt{x}+a\right)^p x^2 \, dx$$

input
```
integrate(x^2*(a+b*x^(1/2)+c*x)^p,x, algorithm="fricas")
```

output
```
integral((c*x + b*sqrt(x) + a)^p*x^2, x)
```

Sympy [F(-2)]

Exception generated.

$$\int x^2 \left(a+b\sqrt{x}+cx\right)^p \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**2*(a+b*x**(1/2)+c*x)**p,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int x^2\left(a+b\sqrt{x}+cx\right)^p\,dx=\int\left(cx+b\sqrt{x}+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*x^(1/2)+c*x)^p,x, algorithm="maxima")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p*x^2, x)
```

Giac [F]

$$\int x^2\left(a+b\sqrt{x}+cx\right)^p\,dx=\int\left(cx+b\sqrt{x}+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*x^(1/2)+c*x)^p,x, algorithm="giac")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\sqrt{x}+cx\right)^p\,dx=\int x^2\,\left(a+c\,x+b\,\sqrt{x}\right)^p\,dx$$

input
```
int(x^2*(a + c*x + b*x^(1/2))^p,x)
```

output
```
int(x^2*(a + c*x + b*x^(1/2))^p, x)
```

Reduce [F]

$$\int x^2 \left(a+b\sqrt{x}+cx\right)^p \, dx = \text{too large to display}$$

input
```
int(x^2*(a+b*x^(1/2)+c*x)^p,x)
```

output
```
(112*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a**2*b*c**2*p**4 + 864*sqrt(x)*(sqrt
(x)*b + a + c*x)**p*a**2*b*c**2*p**3 + 1904*sqrt(x)*(sqrt(x)*b + a + c*x)*
*p*a**2*b*c**2*p**2 + 1200*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a**2*b*c**2*p
- 8*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a*b**3*c*p**5 - 140*sqrt(x)*(sqrt(x)*
b + a + c*x)**p*a*b**3*c*p**4 - 784*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a*b**
3*c*p**3 - 1708*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a*b**3*c*p**2 - 1200*sqrt
(x)*(sqrt(x)*b + a + c*x)**p*a*b**3*c*p - 32*sqrt(x)*(sqrt(x)*b + a + c*x)
**p*a*b*c**3*p**5*x - 336*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a*b*c**3*p**4*x
 - 928*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a*b*c**3*p**3*x - 864*sqrt(x)*(sqr
t(x)*b + a + c*x)**p*a*b*c**3*p**2*x - 240*sqrt(x)*(sqrt(x)*b + a + c*x)**
p*a*b*c**3*p*x + 2*sqrt(x)*(sqrt(x)*b + a + c*x)**p*b**5*p**5 + 28*sqrt(x)
*(sqrt(x)*b + a + c*x)**p*b**5*p**4 + 142*sqrt(x)*(sqrt(x)*b + a + c*x)**p
*b**5*p**3 + 308*sqrt(x)*(sqrt(x)*b + a + c*x)**p*b**5*p**2 + 240*sqrt(x)*
(sqrt(x)*b + a + c*x)**p*b**5*p + 8*sqrt(x)*(sqrt(x)*b + a + c*x)**p*b**3*
c**2*p**5*x + 84*sqrt(x)*(sqrt(x)*b + a + c*x)**p*b**3*c**2*p**4*x + 272*s
qrt(x)*(sqrt(x)*b + a + c*x)**p*b**3*c**2*p**3*x + 276*sqrt(x)*(sqrt(x)*b
+ a + c*x)**p*b**3*c**2*p**2*x + 80*sqrt(x)*(sqrt(x)*b + a + c*x)**p*b**3*
c**2*p*x + 32*sqrt(x)*(sqrt(x)*b + a + c*x)**p*b*c**4*p**5*x**2 + 160*sqrt
(x)*(sqrt(x)*b + a + c*x)**p*b*c**4*p**4*x**2 + 280*sqrt(x)*(sqrt(x)*b + a
 + c*x)**p*b*c**4*p**3*x**2 + 200*sqrt(x)*(sqrt(x)*b + a + c*x)**p*b*c*...
```

3.80 $\int x\left(a+b\sqrt{x}+cx\right)^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 256

$$\int x\left(a+b\sqrt{x}+cx\right)^p\,dx = \frac{\left(b^2(2+p)(3+p)-2ac(3+2p)-2bc(1+p)(3+p)\sqrt{x}\right)\left(a+b\sqrt{x}+cx\right)^{1+p}}{2c^3(1+p)(2+p)(3+2p)} + \frac{x\left(a+b\sqrt{x}+cx\right)^{1+p}}{c(2+p)} - \frac{2^p b(6ac-b^2(3+p))\left(-\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+b\sqrt{x}+cx\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2+\right.}{c^3\sqrt{b^2-4ac}(1+p)(3+2p)}$$

output
```
1/2*(b^2*(2+p)*(3+p)-2*a*c*(3+2*p)-2*b*c*(p+1)*(3+p)*x^(1/2))*(a+b*x^(1/2)
+c*x)^(p+1)/c^3/(p+1)/(2+p)/(3+2*p)+x*(a+b*x^(1/2)+c*x)^(p+1)/c/(2+p)-2^p*
b*(6*a*c-b^2*(3+p))*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x^(1/2))/(-4*a*c+b^2)^(1/2
))^(-1-p)*(a+b*x^(1/2)+c*x)^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+(-4*a*c
+b^2)^(1/2)+2*c*x^(1/2))/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)/(p+1)/
(3+2*p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.61 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.66

$$\int x\left(a+b\sqrt{x}+cx\right)^p\,dx = \frac{1}{2}\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt{x}}{b+\sqrt{b^2-4ac}}\right)^{-p}x^2\left(a+b\sqrt{x}+cx\right)^p\operatorname{AppellF1}\left(4,-p,-p,5,-\frac{2c\sqrt{x}}{b+\sqrt{b^2-4ac}},\frac{2c\sqrt{x}}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x*(a + b*Sqrt[x] + c*x)^p,x]
```

output
```
(x^2*(a + b*Sqrt[x] + c*x)^p*AppellF1[4, -p, -p, 5, (-2*c*Sqrt[x])/(b + Sq
rt[b^2 - 4*a*c]), (2*c*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])])/(2*((b - Sqrt[b
^2 - 4*a*c] + 2*c*Sqrt[x])/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a
*c] + 2*c*Sqrt[x])/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1693, 1166, 25, 1225, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt{x}+cx\right)^p\,dx$$

$$\Big\downarrow \text{1693}$$

$$2\int x^{3/2}\left(a+cx+b\sqrt{x}\right)^p\,d\sqrt{x}$$

$$\Big\downarrow \text{1166}$$

$$2\left(\frac{\int -\left(\left(2a+b(p+3)\sqrt{x}\right)\sqrt{x}\left(a+cx+b\sqrt{x}\right)^p\right)d\sqrt{x}}{2c(p+2)}+\frac{x\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+2)}\right)$$

$$\Big\downarrow \textbf{25}$$

$$2\left(\frac{x\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+2)}-\frac{\int \left(2a+b(p+3)\sqrt{x}\right)\sqrt{x}\left(a+cx+b\sqrt{x}\right)^p d\sqrt{x}}{2c(p+2)}\right)$$

$$\Big\downarrow \textbf{1225}$$

$$2\left(\frac{x\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+2)}-\frac{-\frac{b(p+2)\left(6ac-b^2(p+3)\right)\int \left(a+cx+b\sqrt{x}\right)^p d\sqrt{x}}{2c^2(2p+3)}-\frac{\left(-2ac(2p+3)+b^2(p+2)(p+3)-2bc(p+1)(p+3)\sqrt{x}\right)\left(a+b\sqrt{\right.}}{2c^2(p+1)(2p+3)}}{2c(p+2)}\right.$$

$$\Big\downarrow \textbf{1096}$$

$$2\left(\frac{x\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+2)}-\frac{\frac{b2^p(p+2)\left(6ac-b^2(p+3)\right)\left(-\frac{-\sqrt{b^2-4ac}+b+2c\sqrt{x}}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+b\sqrt{x}+cx\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\right.}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}}}{2c(p+2)}\right.$$

input
```
Int[x*(a + b*Sqrt[x] + c*x)^p,x]
```

output
```
2*((x*(a + b*Sqrt[x] + c*x)^(1 + p))/(2*c*(2 + p)) - (-1/2*((b^2*(2 + p)*(
3 + p) - 2*a*c*(3 + 2*p) - 2*b*c*(1 + p)*(3 + p)*Sqrt[x])*(a + b*Sqrt[x] +
 c*x)^(1 + p))/(c^2*(1 + p)*(3 + 2*p)) + (2^p*b*(2 + p)*(6*a*c - b^2*(3 +
p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*Sqrt[x])/Sqrt[b^2 - 4*a*c]))^(-1 - p)*
(a + b*Sqrt[x] + c*x)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqr
t[b^2 - 4*a*c] + 2*c*Sqrt[x])/(2*Sqrt[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*
c]*(1 + p)*(3 + 2*p)))/(2*c*(2 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x\left(a+b\sqrt{x}+cx\right)^p dx$$

input
```
int(x*(a+b*x^(1/2)+c*x)^p,x)
```

output
```
int(x*(a+b*x^(1/2)+c*x)^p,x)
```

Fricas [F]

$$\int x\left(a+b\sqrt{x}+cx\right)^p\,dx=\int\left(cx+b\sqrt{x}+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*x^(1/2)+c*x)^p,x, algorithm="fricas")
```

output
```
integral((c*x + b*sqrt(x) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\sqrt{x}+cx\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*x**(1/2)+c*x)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+b\sqrt{x}+cx\right)^p\,dx=\int \left(cx+b\sqrt{x}+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*x^(1/2)+c*x)^p,x, algorithm="maxima")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+b\sqrt{x}+cx\right)^p\,dx=\int \left(cx+b\sqrt{x}+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*x^(1/2)+c*x)^p,x, algorithm="giac")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\sqrt{x}+cx\right)^p\,dx=\int x\,\left(a+c\,x+b\,\sqrt{x}\right)^p\,dx$$

input
```
int(x*(a + c*x + b*x^(1/2))^p,x)
```

output
```
int(x*(a + c*x + b*x^(1/2))^p, x)
```

Reduce [F]

$$\int x\left(a+b\sqrt{x}+cx\right)^p\,dx=\text{too large to display}$$

input
```
int(x*(a+b*x^(1/2)+c*x)^p,x)
```

output
```
( - 8*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a*b*c*p**3 - 40*sqrt(x)*(sqrt(x)*b
+ a + c*x)**p*a*b*c*p**2 - 36*sqrt(x)*(sqrt(x)*b + a + c*x)**p*a*b*c*p + 2
*sqrt(x)*(sqrt(x)*b + a + c*x)**p*b**3*p**3 + 10*sqrt(x)*(sqrt(x)*b + a +
c*x)**p*b**3*p**2 + 12*sqrt(x)*(sqrt(x)*b + a + c*x)**p*b**3*p + 8*sqrt(x)
*(sqrt(x)*b + a + c*x)**p*b*c**2*p**3*x + 12*sqrt(x)*(sqrt(x)*b + a + c*x)
**p*b*c**2*p**2*x + 4*sqrt(x)*(sqrt(x)*b + a + c*x)**p*b*c**2*p*x + 8*(sqr
t(x)*b + a + c*x)**p*a**2*c*p**2 + 40*(sqrt(x)*b + a + c*x)**p*a**2*c*p +
36*(sqrt(x)*b + a + c*x)**p*a**2*c - 2*(sqrt(x)*b + a + c*x)**p*a*b**2*p**
2 - 10*(sqrt(x)*b + a + c*x)**p*a*b**2*p - 12*(sqrt(x)*b + a + c*x)**p*a*b
**2 + 16*(sqrt(x)*b + a + c*x)**p*a*c**2*p**3*x + 32*(sqrt(x)*b + a + c*x)
**p*a*c**2*p**2*x + 12*(sqrt(x)*b + a + c*x)**p*a*c**2*p*x - 4*(sqrt(x)*b
+ a + c*x)**p*b**2*c*p**3*x - 14*(sqrt(x)*b + a + c*x)**p*b**2*c*p**2*x -
6*(sqrt(x)*b + a + c*x)**p*b**2*c*p*x + 16*(sqrt(x)*b + a + c*x)**p*c**3*p
**3*x**2 + 48*(sqrt(x)*b + a + c*x)**p*c**3*p**2*x**2 + 44*(sqrt(x)*b + a
+ c*x)**p*c**3*p*x**2 + 12*(sqrt(x)*b + a + c*x)**p*c**3*x**2 - 96*int((sq
rt(x)*b + a + c*x)**p/(4*sqrt(x)*b*p**2 + 8*sqrt(x)*b*p + 3*sqrt(x)*b + 4*
a*p**2 + 8*a*p + 3*a + 4*c*p**2*x + 8*c*p*x + 3*c*x),x)*a**2*c**2*p**5 - 4
80*int((sqrt(x)*b + a + c*x)**p/(4*sqrt(x)*b*p**2 + 8*sqrt(x)*b*p + 3*sqrt
(x)*b + 4*a*p**2 + 8*a*p + 3*a + 4*c*p**2*x + 8*c*p*x + 3*c*x),x)*a**2*c**
2*p**4 - 840*int((sqrt(x)*b + a + c*x)**p/(4*sqrt(x)*b*p**2 + 8*sqrt(x)...
```

3.81 $\int \left(a+b\sqrt{x}+cx\right)^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 161

$$\int \left(a+b\sqrt{x}+cx\right)^p \, dx = \frac{\left(a+b\sqrt{x}+cx\right)^{1+p}}{c(1+p)} + \frac{2^{1+p}b\left(-\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+b\sqrt{x}+cx\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-4ac}+2c\sqrt{}}{2\sqrt{b^2-4ac}}\right.}{c\sqrt{b^2-4ac}(1+p)}$$

output
```
(a+b*x^(1/2)+c*x)^(p+1)/c/(p+1)+2^(p+1)*b*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x^(1
/2))/(-4*a*c+b^2)^(1/2))^(-1-p)*(a+b*x^(1/2)+c*x)^(p+1)*hypergeom([-p, p+1
],[2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x^(1/2))/(-4*a*c+b^2)^(1/2))/c/(-4*a
*c+b^2)^(1/2)/(p+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\int \left(a+b\sqrt{x}+cx\right)^p\,dx = \left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt{x}}{b+\sqrt{b^2-4ac}}\right)^{-p} x\left(a+b\sqrt{x}+cx\right)^p \text{AppellF1}\left(2,-p,-p,3,-\frac{2c\sqrt{x}}{b+\sqrt{b^2-4ac}},\frac{2c\sqrt{x}}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^p,x]
```

output
```
(x*(a + b*Sqrt[x] + c*x)^p*AppellF1[2, -p, -p, 3, (-2*c*Sqrt[x])/(b + Sqrt
[b^2 - 4*a*c]), (2*c*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 -
 4*a*c] + 2*c*Sqrt[x])/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c]
+ 2*c*Sqrt[x])/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1680, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}+cx\right)^p\,dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \sqrt{x}\left(a+cx+b\sqrt{x}\right)^p d\sqrt{x}$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{\left(a+b\sqrt{x}+cx\right)^{p+1}}{2c(p+1)}-\frac{b\int \left(a+cx+b\sqrt{x}\right)^p d\sqrt{x}}{2c}\right)$$

$$\Big\downarrow \text{1096}$$

$$2\left(\frac{b2^p\left(a+b\sqrt{x}+cx\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2c\sqrt{x}}{\sqrt{b^2-4ac}}\right)^{-p-1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+\sqrt{b^2-4ac}+2c\sqrt{x}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}+\right.$$

input
```
Int[(a + b*Sqrt[x] + c*x)^p,x]
```

output
```
2*((a + b*Sqrt[x] + c*x)^(1 + p)/(2*c*(1 + p)) + (2^p*b*(-((b - Sqrt[b^2 -
 4*a*c] + 2*c*Sqrt[x])/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*Sqrt[x] + c*x)^
(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*S
qrt[x])/(2*Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p)))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [F]

$$\int \left(a+b\sqrt{x}+cx\right)^p\,dx$$

input
```
int((a+b*x^(1/2)+c*x)^p,x)
```

output
```
int((a+b*x^(1/2)+c*x)^p,x)
```

Fricas [F]

$$\int \left(a+b\sqrt{x}+cx\right)^p\,dx=\int \left(cx+b\sqrt{x}+a\right)^p\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^p,x, algorithm="fricas")
```

output
```
integral((c*x + b*sqrt(x) + a)^p, x)
```

Sympy [F]

$$\int \left(a+b\sqrt{x}+cx\right)^p\,dx=\int \left(a+b\sqrt{x}+cx\right)^p\,dx$$

input
```
integrate((a+b*x**(1/2)+c*x)**p,x)
```

output
```
Integral((a + b*sqrt(x) + c*x)**p, x)
```

Maxima [F]

$$\int \left(a+b\sqrt{x}+cx\right)^p\,dx=\int \left(cx+b\sqrt{x}+a\right)^p\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^p,x, algorithm="maxima")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p, x)
```

Giac [F]

$$\int \left(a+b\sqrt{x}+cx\right)^p\,dx=\int \left(cx+b\sqrt{x}+a\right)^p\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^p,x, algorithm="giac")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt{x}+cx\right)^p\,dx=\int \left(a+c\,x+b\,\sqrt{x}\right)^p\,dx$$

input
```
int((a + c*x + b*x^(1/2))^p,x)
```

output
```
int((a + c*x + b*x^(1/2))^p, x)
```

Reduce [F]

$$\int \left(a+b\sqrt{x}+cx\right)^p\,dx$$
$$=\frac{2\sqrt{x}\left(\sqrt{x}\,b+a+cx\right)^p bp-2\left(\sqrt{x}\,b+a+cx\right)^p a+4\left(\sqrt{x}\,b+a+cx\right)^p cpx+2\left(\sqrt{x}\,b+a+cx\right)^p cx+8\Big(}{}$$

input
```
int((a+b*x^(1/2)+c*x)^p,x)
```

output
```
(2*sqrt(x)*(sqrt(x)*b + a + c*x)**p*b*p - 2*(sqrt(x)*b + a + c*x)**p*a + 4
*(sqrt(x)*b + a + c*x)**p*c*p*x + 2*(sqrt(x)*b + a + c*x)**p*c*x + 8*int((
sqrt(x)*b + a + c*x)**p/(2*sqrt(x)*b*p + sqrt(x)*b + 2*a*p + a + 2*c*p*x +
 c*x),x)*a*c*p**3 + 12*int((sqrt(x)*b + a + c*x)**p/(2*sqrt(x)*b*p + sqrt(
x)*b + 2*a*p + a + 2*c*p*x + c*x),x)*a*c*p**2 + 4*int((sqrt(x)*b + a + c*x
)**p/(2*sqrt(x)*b*p + sqrt(x)*b + 2*a*p + a + 2*c*p*x + c*x),x)*a*c*p - 2*
int((sqrt(x)*b + a + c*x)**p/(2*sqrt(x)*b*p + sqrt(x)*b + 2*a*p + a + 2*c*
p*x + c*x),x)*b**2*p**3 - 3*int((sqrt(x)*b + a + c*x)**p/(2*sqrt(x)*b*p +
sqrt(x)*b + 2*a*p + a + 2*c*p*x + c*x),x)*b**2*p**2 - int((sqrt(x)*b + a +
 c*x)**p/(2*sqrt(x)*b*p + sqrt(x)*b + 2*a*p + a + 2*c*p*x + c*x),x)*b**2*p
)/(2*c*(2*p**2 + 3*p + 1))
```

3.82 $\int \frac{(a+b\sqrt{x}+cx)^p}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 162

$$\int \frac{(a+b\sqrt{x}+cx)^p}{x}\,dx = \frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}(a+b\sqrt{x}+cx)^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2c\sqrt{x}}\right.}{p}$$

output
```
4^p*(a+b*x^(1/2)+c*x)^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*a*c+b^2)^(1/
2))/c/x^(1/2),-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^(1/2))/p/(((b-(-4*a*c+b^2)^(
1/2)+2*c*x^(1/2))/c/x^(1/2))^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^(1/2))/c/x^(
1/2))^p)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\sqrt{x}+cx)^p}{x}\,dx = \frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}(a+b\sqrt{x}+cx)^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{-b-\sqrt{b^2-4ac}}{2c\sqrt{x}},\right.}{p}$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^p/x,x]
```

output
```
(4^p*(a + b*Sqrt[x] + c*x)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (-b - Sqrt[b^
2 - 4*a*c])/(2*c*Sqrt[x]), (-b + Sqrt[b^2 - 4*a*c])/(2*c*Sqrt[x])]])/(p*((b
 - Sqrt[b^2 - 4*a*c] + 2*c*Sqrt[x])/(c*Sqrt[x]))^p*((b + Sqrt[b^2 - 4*a*c]
 + 2*c*Sqrt[x])/(c*Sqrt[x]))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sqrt{x}+cx)^p}{x} \, dx$$

$$\Big\downarrow \text{1693}$$

$$2\int \frac{(a+cx+b\sqrt{x})^p}{\sqrt{x}} d\sqrt{x}$$

$$\Big\downarrow \text{1178}$$

$$-2^{2p+1}x^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}(a+b\sqrt{x}+cx)^p\int \left(\frac{b-\sqrt{b^2-4ac}}{2c\sqrt{x}}+\right.$$

$$\Big\downarrow \text{150}$$

$$\frac{2^{2p}x^{-2p}\left(\frac{-\sqrt{b^2-4ac}+b+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}(a+b\sqrt{x}+cx)^p\operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4a}}{2c\sqrt{x}}\right.}{p}$$

input
```
Int[(a + b*Sqrt[x] + c*x)^p/x,x]
```

output
```
(2^(2*p)*(a + b*Sqrt[x] + c*x)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b -
 Sqrt[b^2 - 4*a*c])/(c*Sqrt[x]), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*Sqrt[x])]
)/(p*((b - Sqrt[b^2 - 4*a*c] + 2*c*Sqrt[x])/(c*Sqrt[x]))^p*((b + Sqrt[b^2
- 4*a*c] + 2*c*Sqrt[x])/(c*Sqrt[x]))^p*x^(2*p))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(a + b\sqrt{x} + cx)^p}{x} dx$$

input
```
int((a+b*x^(1/2)+c*x)^p/x,x)
```

output
```
int((a+b*x^(1/2)+c*x)^p/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x}\,dx = \int \frac{\left(cx+b\sqrt{x}+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^p/x,x, algorithm="fricas")
```

output
```
integral((c*x + b*sqrt(x) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**(1/2)+c*x)**p/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x}\,dx = \int \frac{\left(cx+b\sqrt{x}+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^p/x,x, algorithm="maxima")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p/x, x)
```

Giac [F]

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x}\,dx=\int \frac{\left(cx+b\sqrt{x}+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^p/x,x, algorithm="giac")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x}\,dx=\int \frac{\left(a+c\,x+b\,\sqrt{x}\right)^p}{x}\,dx$$

input
```
int((a + c*x + b*x^(1/2))^p/x,x)
```

output
```
int((a + c*x + b*x^(1/2))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x}\,dx$$
$$=\frac{2\left(\sqrt{x}\,b+a+cx\right)^p+\left(\int \frac{\left(\sqrt{x}\,b+a+cx\right)^p}{c^2x^2+2acx-b^2x+a^2}dx\right)acp+\left(\int \frac{\left(\sqrt{x}\,b+a+cx\right)^p}{c^2x^3+2ac\,x^2-b^2x^2+a^2x}dx\right)a^2p-\left(\int \frac{\left(\sqrt{x}\,b+a+cx\right)^p}{\sqrt{x}\,b+a+cx}dx\right)}{p}$$

input
```
int((a+b*x^(1/2)+c*x)^p/x,x)
```

output
```
(2*(sqrt(x)*b + a + c*x)**p + int((sqrt(x)*b + a + c*x)**p/(a**2 + 2*a*c*x
 - b**2*x + c**2*x**2),x)*a*c*p + int((sqrt(x)*b + a + c*x)**p/(a**2*x + 2
*a*c*x**2 - b**2*x**2 + c**2*x**3),x)*a**2*p - int((sqrt(x)*b + a + c*x)**
p/(sqrt(x)*b + a + c*x),x)*c*p - int((sqrt(x)*(sqrt(x)*b + a + c*x)**p)/(a
**2*x + 2*a*c*x**2 - b**2*x**2 + c**2*x**3),x)*a*b*p)/p
```

3.83 $\int \frac{(a+b\sqrt{x}+cx)^p}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 174

$$\int \frac{(a+b\sqrt{x}+cx)^p}{x^2}\,dx = -\frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}(a+b\sqrt{x}+cx)^p\,\text{AppellF1}\left(2(1-p),-p,-p,3-2p,-\frac{b-}{}\right.}{(1-p)x}$$

output
```
-4^p*(a+b*x^(1/2)+c*x)^p*AppellF1(2-2*p,-p,-p,3-2*p,-1/2*(b-(-4*a*c+b^2)^(
1/2))/c/x^(1/2),-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^(1/2))/(1-p)/(((b-(-4*a*c+
b^2)^(1/2)+2*c*x^(1/2))/c/x^(1/2))^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^(1/2))
/c/x^(1/2))^p)/x
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\sqrt{x}+cx)^p}{x^2}\,dx = \frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}(a+b\sqrt{x}+cx)^p\,\text{AppellF1}\left(2-2p,-p,-p,3-2p,\frac{-b-\sqrt{b^2-4}}{2c\sqrt{x}}\right.}{(-1+p)x}$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^p/x^2,x]
```

output
```
(4^p*(a + b*Sqrt[x] + c*x)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (-b - Sqrt
[b^2 - 4*a*c])/(2*c*Sqrt[x]), (-b + Sqrt[b^2 - 4*a*c])/(2*c*Sqrt[x])]])/((-
1 + p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*Sqrt[x])/(c*Sqrt[x]))^p*((b + Sqrt[b^
2 - 4*a*c] + 2*c*Sqrt[x])/(c*Sqrt[x]))^p*x)
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x^2}dx$$

$$\Big\downarrow \text{ 1693}$$

$$2\int \frac{\left(a+cx+b\sqrt{x}\right)^p}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1178}$$

$$-2^{2p+1}x^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(a+b\sqrt{x}+cx\right)^p\int \left(\frac{b-\sqrt{b^2-4ac}}{2c\sqrt{x}}+$$

$$\Big\downarrow \text{ 150}$$

$$-\frac{2^{2p}x^{\frac{1}{2}(2-2p)-p}\left(\frac{-\sqrt{b^2-4ac}+b+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(a+b\sqrt{x}+cx\right)^p\text{AppellF1}\left(2-2p,-p,-p,3-2p,\right.}{1-p}$$

input
```
Int[(a + b*Sqrt[x] + c*x)^p/x^2,x]
```

output
```
-((2^(2*p)*x^((2 - 2*p)/2 - p)*(a + b*Sqrt[x] + c*x)^p*AppellF1[2 - 2*p, -
p, -p, 3 - 2*p, -1/2*(b - Sqrt[b^2 - 4*a*c])/(c*Sqrt[x]), -1/2*(b + Sqrt[b
^2 - 4*a*c])/(c*Sqrt[x])]])/((1 - p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*Sqrt[x])
/(c*Sqrt[x]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*Sqrt[x])/(c*Sqrt[x]))^p))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^{p}}{x^{2}}dx$$

input
```
int((a+b*x^(1/2)+c*x)^p/x^2,x)
```

output
```
int((a+b*x^(1/2)+c*x)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b\sqrt{x}+cx)^p}{x^2}\,dx = \int \frac{(cx+b\sqrt{x}+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^p/x^2,x, algorithm="fricas")
```

output
```
integral((c*x + b*sqrt(x) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sqrt{x}+cx)^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**(1/2)+c*x)**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sqrt{x}+cx)^p}{x^2}\,dx = \int \frac{(cx+b\sqrt{x}+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x^2}\,dx=\int \frac{\left(cx+b\sqrt{x}+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^p/x^2,x, algorithm="giac")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x^2}\,dx=\int \frac{\left(a+c\,x+b\,\sqrt{x}\right)^p}{x^2}\,dx$$

input
```
int((a + c*x + b*x^(1/2))^p/x^2,x)
```

output
```
int((a + c*x + b*x^(1/2))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x^2}\,dx$$
$$=\frac{-2\left(\sqrt{x}\,b+a+cx\right)^p-2\left(\int \frac{\left(\sqrt{x}\,b+a+cx\right)^p}{c^2p\,x^3+2acp\,x^2-b^2p\,x^2-2c^2x^3+a^2px-4ac\,x^2+2b^2x^2-2a^2x}dx\right)acpx-2\left(\int \frac{(}{c^2p\,x^2+2acpx-b^2p}\right.}{}$$

input
```
int((a+b*x^(1/2)+c*x)^p/x^2,x)
```

output
```
( - 2*(sqrt(x)*b + a + c*x)**p - 2*int((sqrt(x)*b + a + c*x)**p/(a**2*p*x
- 2*a**2*x + 2*a*c*p*x**2 - 4*a*c*x**2 - b**2*p*x**2 + 2*b**2*x**2 + c**2*
p*x**3 - 2*c**2*x**3),x)*a*c*p*x - 2*int((sqrt(x)*b + a + c*x)**p/(a**2*p
- 2*a**2 + 2*a*c*p*x - 4*a*c*x - b**2*p*x + 2*b**2*x + c**2*p*x**2 - 2*c**
2*x**2),x)*c**2*p*x + int((sqrt(x)*b + a + c*x)**p/(sqrt(x)*a*p*x - 2*sqrt
(x)*a*x + sqrt(x)*c*p*x**2 - 2*sqrt(x)*c*x**2 + b*p*x**2 - 2*b*x**2),x)*b*
p**2*x - 2*int((sqrt(x)*b + a + c*x)**p/(sqrt(x)*a*p*x - 2*sqrt(x)*a*x + s
qrt(x)*c*p*x**2 - 2*sqrt(x)*c*x**2 + b*p*x**2 - 2*b*x**2),x)*b*p*x + 2*int
((sqrt(x)*(sqrt(x)*b + a + c*x)**p)/(a**2*p*x - 2*a**2*x + 2*a*c*p*x**2 -
4*a*c*x**2 - b**2*p*x**2 + 2*b**2*x**2 + c**2*p*x**3 - 2*c**2*x**3),x)*b*c
*p*x + 2*int((sqrt(x)*(sqrt(x)*b + a + c*x)**p)/(sqrt(x)*a*p*x - 2*sqrt(x)
*a*x + sqrt(x)*c*p*x**2 - 2*sqrt(x)*c*x**2 + b*p*x**2 - 2*b*x**2),x)*c*p**
2*x - 2*int((sqrt(x)*(sqrt(x)*b + a + c*x)**p)/(sqrt(x)*a*p*x - 2*sqrt(x)*
a*x + sqrt(x)*c*p*x**2 - 2*sqrt(x)*c*x**2 + b*p*x**2 - 2*b*x**2),x)*c*p*x)
/(2*x)
```

3.84 $\int \frac{(a+b\sqrt{x}+cx)^p}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 174

$$\int \frac{(a+b\sqrt{x}+cx)^p}{x^3} dx = -\frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}(a+b\sqrt{x}+cx)^p \operatorname{AppellF1}\left(2(2-p),-p,-p,5-2p,-\frac{b-}{}\right.}{(2-p)x^2}$$

output
```
-4^p*(a+b*x^(1/2)+c*x)^p*AppellF1(4-2*p,-p,-p,5-2*p,-1/2*(b-(-4*a*c+b^2)^(
1/2))/c/x^(1/2),-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^(1/2))/(2-p)/(((b-(-4*a*c+
b^2)^(1/2)+2*c*x^(1/2))/c/x^(1/2))^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^(1/2))
/c/x^(1/2))^p)/x^2
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\sqrt{x}+cx)^p}{x^3} dx = \frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}(a+b\sqrt{x}+cx)^p \operatorname{AppellF1}\left(4-2p,-p,-p,5-2p,\frac{-b-\sqrt{b^2-4}}{2c\sqrt{x}}\right.}{(-2+p)x^2}$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^p/x^3,x]
```

output
```
(4^p*(a + b*Sqrt[x] + c*x)^p*AppellF1[4 - 2*p, -p, -p, 5 - 2*p, (-b - Sqrt
[b^2 - 4*a*c])/(2*c*Sqrt[x]), (-b + Sqrt[b^2 - 4*a*c])/(2*c*Sqrt[x])])/((-
2 + p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*Sqrt[x])/(c*Sqrt[x]))^p*((b + Sqrt[b^
2 - 4*a*c] + 2*c*Sqrt[x])/(c*Sqrt[x]))^p*x^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x^3}dx$$

$$\Big\downarrow \text{1693}$$

$$2\int \frac{\left(a+cx+b\sqrt{x}\right)^p}{x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1178}$$

$$-2^{2p+1}x^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(a+b\sqrt{x}+cx\right)^p\int \left(\frac{b-\sqrt{b^2-4ac}}{2c\sqrt{x}}+1\right.$$

$$\Big\downarrow \text{150}$$

$$-\frac{2^{2p}x^{\frac{1}{2}(4-2p)-p}\left(\frac{-\sqrt{b^2-4ac}+b+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2c\sqrt{x}}{c\sqrt{x}}\right)^{-p}\left(a+b\sqrt{x}+cx\right)^p\text{AppellF1}\left(4-2p,-p,-p,5-2p,-\right.}{2-p}$$

input
```
Int[(a + b*Sqrt[x] + c*x)^p/x^3,x]
```

output
```
-((2^(2*p)*x^((4 - 2*p)/2 - p)*(a + b*Sqrt[x] + c*x)^p*AppellF1[4 - 2*p, -
p, -p, 5 - 2*p, -1/2*(b - Sqrt[b^2 - 4*a*c])/(c*Sqrt[x]), -1/2*(b + Sqrt[b
^2 - 4*a*c])/(c*Sqrt[x])])/((2 - p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*Sqrt[x])
/(c*Sqrt[x]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*Sqrt[x])/(c*Sqrt[x]))^p))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(a + b\sqrt{x} + cx)^p}{x^3} dx$$

input
```
int((a+b*x^(1/2)+c*x)^p/x^3,x)
```

output
```
int((a+b*x^(1/2)+c*x)^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x^3}\,dx = \int \frac{\left(cx+b\sqrt{x}+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^p/x^3,x, algorithm="fricas")
```

output
```
integral((c*x + b*sqrt(x) + a)^p/x^3, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**(1/2)+c*x)**p/x**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x^3}\,dx = \int \frac{\left(cx+b\sqrt{x}+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x^3}\,dx=\int \frac{\left(cx+b\sqrt{x}+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^p/x^3,x, algorithm="giac")
```

output
```
integrate((c*x + b*sqrt(x) + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x^3}\,dx=\int \frac{\left(a+c\,x+b\,\sqrt{x}\right)^p}{x^3}\,dx$$

input
```
int((a + c*x + b*x^(1/2))^p/x^3,x)
```

output
```
int((a + c*x + b*x^(1/2))^p/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sqrt{x}+cx\right)^p}{x^3}\,dx=\text{Too large to display}$$

input
```
int((a+b*x^(1/2)+c*x)^p/x^3,x)
```

output

```
( - 2*(sqrt(x)*b + a + c*x)**p*a*p**2 + 14*(sqrt(x)*b + a + c*x)**p*a*p -
24*(sqrt(x)*b + a + c*x)**p*a + 4*(sqrt(x)*b + a + c*x)**p*c*p**2*x - 12*(
sqrt(x)*b + a + c*x)**p*c*p*x + int((sqrt(x)*b + a + c*x)**p/(sqrt(x)*a*p*
x**2 - 4*sqrt(x)*a*x**2 + sqrt(x)*c*p*x**3 - 4*sqrt(x)*c*x**3 + b*p*x**3 -
 4*b*x**3),x)*a*b*p**4*x**2 - 11*int((sqrt(x)*b + a + c*x)**p/(sqrt(x)*a*p
*x**2 - 4*sqrt(x)*a*x**2 + sqrt(x)*c*p*x**3 - 4*sqrt(x)*c*x**3 + b*p*x**3
- 4*b*x**3),x)*a*b*p**3*x**2 + 40*int((sqrt(x)*b + a + c*x)**p/(sqrt(x)*a*
p*x**2 - 4*sqrt(x)*a*x**2 + sqrt(x)*c*p*x**3 - 4*sqrt(x)*c*x**3 + b*p*x**3
 - 4*b*x**3),x)*a*b*p**2*x**2 - 48*int((sqrt(x)*b + a + c*x)**p/(sqrt(x)*a
*p*x**2 - 4*sqrt(x)*a*x**2 + sqrt(x)*c*p*x**3 - 4*sqrt(x)*c*x**3 + b*p*x**
3 - 4*b*x**3),x)*a*b*p*x**2 - 2*int((sqrt(x)*b + a + c*x)**p/(sqrt(x)*a*p*
x - 4*sqrt(x)*a*x + sqrt(x)*c*p*x**2 - 4*sqrt(x)*c*x**2 + b*p*x**2 - 4*b*x
**2),x)*b*c*p**4*x**2 + 18*int((sqrt(x)*b + a + c*x)**p/(sqrt(x)*a*p*x - 4
*sqrt(x)*a*x + sqrt(x)*c*p*x**2 - 4*sqrt(x)*c*x**2 + b*p*x**2 - 4*b*x**2),
x)*b*c*p**3*x**2 - 52*int((sqrt(x)*b + a + c*x)**p/(sqrt(x)*a*p*x - 4*sqrt
(x)*a*x + sqrt(x)*c*p*x**2 - 4*sqrt(x)*c*x**2 + b*p*x**2 - 4*b*x**2),x)*b*
c*p**2*x**2 + 48*int((sqrt(x)*b + a + c*x)**p/(sqrt(x)*a*p*x - 4*sqrt(x)*a
*x + sqrt(x)*c*p*x**2 - 4*sqrt(x)*c*x**2 + b*p*x**2 - 4*b*x**2),x)*b*c*p*x
**2 + 2*int((sqrt(x)*(sqrt(x)*b + a + c*x)**p)/(sqrt(x)*a*p*x**2 - 4*sqrt(
x)*a*x**2 + sqrt(x)*c*p*x**3 - 4*sqrt(x)*c*x**3 + b*p*x**3 - 4*b*x**3),...
```

3.85 $\int (dx)^m \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 77

$$\int (dx)^m \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx = \frac{\left(1+\frac{c\sqrt{x}}{a}\right)^{-2p} (dx)^{1+m} \left(a^2+2ac\sqrt{x}+c^2x\right)^p \text{Hypergeometric2F1}\left(2(1+m),-2p,3+2m,-\frac{c\sqrt{x}}{a}\right)}{d(1+m)}$$

output
```
(d*x)^(1+m)*(a^2+2*a*c*x^(1/2)+c^2*x)^p*hypergeom([-2*p, 2+2*m],[3+2*m],-c
*x^(1/2)/a)/d/(1+m)/((1+c*x^(1/2)/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int (dx)^m \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx = \frac{\left(\left(a+c\sqrt{x}\right)^2\right)^p \left(1+\frac{c\sqrt{x}}{a}\right)^{-2p} x(dx)^m \text{Hypergeometric2F1}\left(2(1+m),-2p,1+2(1+m),-\frac{c\sqrt{x}}{a}\right)}{1+m}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p,x]
```

output
```
(((a + c*Sqrt[x])^2)^p*x*(d*x)^m*Hypergeometric2F1[2*(1 + m), -2*p, 1 + 2*
(1 + m), -((c*Sqrt[x])/a)])/((1 + m)*(1 + (c*Sqrt[x])/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1385, 866, 864, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a^2+2ac\sqrt{x}+c^2x\right)^p dx$$

$$\Big\downarrow \ 1385$$

$$\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p} \left(a^2+2ac\sqrt{x}+c^2x\right)^p \int \left(\frac{\sqrt{x}c}{a}+1\right)^{2p} (dx)^m dx$$

$$\Big\downarrow \ 866$$

$$x^{-m}(dx)^m \left(\frac{c\sqrt{x}}{a}+1\right)^{-2p} \left(a^2+2ac\sqrt{x}+c^2x\right)^p \int \left(\frac{\sqrt{x}c}{a}+1\right)^{2p} x^m dx$$

$$\Big\downarrow \ 864$$

$$2x^{-m}(dx)^m \left(\frac{c\sqrt{x}}{a}+1\right)^{-2p} \left(a^2+2ac\sqrt{x}+c^2x\right)^p \int \left(\frac{\sqrt{x}c}{a}+1\right)^{2p} x^{\frac{1}{2}(2m+1)} d\sqrt{x}$$

$$\Big\downarrow \ 74$$

$$\frac{x(dx)^m \left(\frac{c\sqrt{x}}{a}+1\right)^{-2p} \left(a^2+2ac\sqrt{x}+c^2x\right)^p \text{Hypergeometric2F1}\left(2(m+1),-2p,2m+3,-\frac{c\sqrt{x}}{a}\right)}{m+1}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p,x]
```

output
```
(x*(d*x)^m*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p*Hypergeometric2F1[2*(1 + m), -2
*p, 3 + 2*m, -((c*Sqrt[x])/a)])/((1 + m)*(1 + (c*Sqrt[x])/a)^(2*p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int (dx)^m \left(a^2+2ac\sqrt{x}+c^2x\right)^p dx$$

input
```
int((d*x)^m*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

output
```
int((d*x)^m*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int (dx)^m \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx = \int (dx)^m \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx$$

input
```
integrate((d*x)**m*(a**2+2*a*c*x**(1/2)+c**2*x)**p,x)
```

output
```
Integral((d*x)**m*(a**2 + 2*a*c*sqrt(x) + c**2*x)**p, x)
```

Maxima [F]

$$\int (dx)^m \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx = \int \left(c^2x+2\,ac\sqrt{x}+a^2\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="maxima")
```

output
```
integrate((c^2*x + 2*a*c*sqrt(x) + a^2)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx = \int \left(c^2x+2\,ac\sqrt{x}+a^2\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="giac")
```

output
```
integrate((c^2*x + 2*a*c*sqrt(x) + a^2)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx = \int (d\,x)^m \left(c^2\,x+a^2+2\,a\,c\,\sqrt{x}\right)^p dx$$

input
```
int((d*x)^m*(c^2*x + a^2 + 2*a*c*x^(1/2))^p,x)
```

output
```
int((d*x)^m*(c^2*x + a^2 + 2*a*c*x^(1/2))^p, x)
```

Reduce [F]

$$\int (dx)^m \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x)^m*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

output
```
(d**m*(2*x**((2*m + 1)/2)*(2*sqrt(x)*a*c + a**2 + c**2*x)**p*a*c*m*p + 2*x
**((2*m + 1)/2)*(2*sqrt(x)*a*c + a**2 + c**2*x)**p*a*c*p**2 - 2*x**m*(2*sq
rt(x)*a*c + a**2 + c**2*x)**p*a**2*m*p - x**m*(2*sqrt(x)*a*c + a**2 + c**2
*x)**p*a**2*p + 2*x**m*(2*sqrt(x)*a*c + a**2 + c**2*x)**p*c**2*m**2*x + 4*
x**m*(2*sqrt(x)*a*c + a**2 + c**2*x)**p*c**2*m*p*x + x**m*(2*sqrt(x)*a*c +
 a**2 + c**2*x)**p*c**2*m*x + 2*x**m*(2*sqrt(x)*a*c + a**2 + c**2*x)**p*c*
*2*p**2*x + x**m*(2*sqrt(x)*a*c + a**2 + c**2*x)**p*c**2*p*x + 4*int((x**m
*(2*sqrt(x)*a*c + a**2 + c**2*x)**p)/(2*a**2*m**3*x + 6*a**2*m**2*p*x + 3*
a**2*m**2*x + 6*a**2*m*p**2*x + 6*a**2*m*p*x + a**2*m*x + 2*a**2*p**3*x +
3*a**2*p**2*x + a**2*p*x - 2*c**2*m**3*x**2 - 6*c**2*m**2*p*x**2 - 3*c**2*
m**2*x**2 - 6*c**2*m*p**2*x**2 - 6*c**2*m*p*x**2 - c**2*m*x**2 - 2*c**2*p*
*3*x**2 - 3*c**2*p**2*x**2 - c**2*p*x**2),x)*a**4*m**5*p + 12*int((x**m*(2
*sqrt(x)*a*c + a**2 + c**2*x)**p)/(2*a**2*m**3*x + 6*a**2*m**2*p*x + 3*a**
2*m**2*x + 6*a**2*m*p**2*x + 6*a**2*m*p*x + a**2*m*x + 2*a**2*p**3*x + 3*a
**2*p**2*x + a**2*p*x - 2*c**2*m**3*x**2 - 6*c**2*m**2*p*x**2 - 3*c**2*m**
2*x**2 - 6*c**2*m*p**2*x**2 - 6*c**2*m*p*x**2 - c**2*m*x**2 - 2*c**2*p**3*
x**2 - 3*c**2*p**2*x**2 - c**2*p*x**2),x)*a**4*m**4*p**2 + 8*int((x**m*(2*
sqrt(x)*a*c + a**2 + c**2*x)**p)/(2*a**2*m**3*x + 6*a**2*m**2*p*x + 3*a**2
*m**2*x + 6*a**2*m*p**2*x + 6*a**2*m*p*x + a**2*m*x + 2*a**2*p**3*x + 3*a*
*2*p**2*x + a**2*p*x - 2*c**2*m**3*x**2 - 6*c**2*m**2*p*x**2 - 3*c**2*m...
```

3.86 $\int x^3 \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 353

$$\begin{aligned}\int x^3 \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx = & -\frac{2a^7\left(a+c\sqrt{x}\right)\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^8(1+2p)} \\ & +\frac{7a^6\left(a+c\sqrt{x}\right)^2\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^8(1+p)} \\ & -\frac{42a^5\left(a+c\sqrt{x}\right)^3\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^8(3+2p)} \\ & +\frac{35a^4\left(a+c\sqrt{x}\right)^4\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^8(2+p)} \\ & -\frac{70a^3\left(a+c\sqrt{x}\right)^5\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^8(5+2p)} \\ & +\frac{21a^2\left(a+c\sqrt{x}\right)^6\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^8(3+p)} \\ & -\frac{14a\left(a+c\sqrt{x}\right)^7\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^8(7+2p)} \\ & +\frac{\left(a+c\sqrt{x}\right)^8\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^8(4+p)}\end{aligned}$$

output
```
-2*a^7*(a+c*x^(1/2))*(a^2+2*a*c*x^(1/2)+c^2*x)^p/c^8/(1+2*p)+7*a^6*(a+c*x^
(1/2))^2*(a^2+2*a*c*x^(1/2)+c^2*x)^p/c^8/(p+1)-42*a^5*(a+c*x^(1/2))^3*(a^2
+2*a*c*x^(1/2)+c^2*x)^p/c^8/(3+2*p)+35*a^4*(a+c*x^(1/2))^4*(a^2+2*a*c*x^(1
/2)+c^2*x)^p/c^8/(2+p)-70*a^3*(a+c*x^(1/2))^5*(a^2+2*a*c*x^(1/2)+c^2*x)^p/
c^8/(5+2*p)+21*a^2*(a+c*x^(1/2))^6*(a^2+2*a*c*x^(1/2)+c^2*x)^p/c^8/(3+p)-1
4*a*(a+c*x^(1/2))^7*(a^2+2*a*c*x^(1/2)+c^2*x)^p/c^8/(7+2*p)+(a+c*x^(1/2))^
8*(a^2+2*a*c*x^(1/2)+c^2*x)^p/c^8/(4+p)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.52

$$\int x^3\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$
$$=\frac{\left(-\frac{2a^7}{1+2p}+\frac{7a^6(a+c\sqrt{x})}{1+p}-\frac{42a^5(a+c\sqrt{x})^2}{3+2p}+\frac{35a^4(a+c\sqrt{x})^3}{2+p}-\frac{70a^3(a+c\sqrt{x})^4}{5+2p}+\frac{21a^2(a+c\sqrt{x})^5}{3+p}-\frac{14a(a+c\sqrt{x})^6}{7+2p}+\frac{(a+c\sqrt{x})^7}{4+p}\right.}{c^8}$$

input
```
Integrate[x^3*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p,x]
```

output
```
(((-2*a^7)/(1 + 2*p) + (7*a^6*(a + c*Sqrt[x]))/(1 + p) - (42*a^5*(a + c*Sq
rt[x])^2)/(3 + 2*p) + (35*a^4*(a + c*Sqrt[x])^3)/(2 + p) - (70*a^3*(a + c*
Sqrt[x])^4)/(5 + 2*p) + (21*a^2*(a + c*Sqrt[x])^5)/(3 + p) - (14*a*(a + c*
Sqrt[x])^6)/(7 + 2*p) + (a + c*Sqrt[x])^7/(4 + p))*(a + c*Sqrt[x])*((a + c
*Sqrt[x])^2)^p)/c^8
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1385, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}x^3dx$$

$$\Big\downarrow \text{798}$$

$$2\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}x^{7/2}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int\left(-\frac{a^7\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}}{c^7}+\frac{7a^7\left(\frac{\sqrt{x}c}{a}+1\right)^{2p+1}}{c^7}-\frac{21a^7\left(\frac{\sqrt{x}c}{a}+1\right)^{2p+2}}{c^7}+\frac{35a}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{7a^8\left(\frac{c\sqrt{x}}{a}+1\right)^{2(p+1)}}{2c^8(p+1)}+\frac{35a^8\left(\frac{c\sqrt{x}}{a}+1\right)^{2(p+2)}}{2c^8(p+2)}+\frac{21a^8\left(\frac{c\sqrt{x}}{a}+1\right)^{2(p+3)}}{2c^8(p+3)}+\frac{a^8\left(\frac{c\sqrt{x}}{a}+1\right)^{2(p+4)}}{2c^8(p+4)}-\frac{a^8\left(\frac{c\sqrt{x}}{a}+1\right)^{2p}}{c^8(2p+1)}\right.$$

input
```
Int[x^3*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p,x]
```

output
```
(2*((7*a^8*(1 + (c*Sqrt[x])/a)^(2*(1 + p)))/(2*c^8*(1 + p)) + (35*a^8*(1 +
 (c*Sqrt[x])/a)^(2*(2 + p)))/(2*c^8*(2 + p)) + (21*a^8*(1 + (c*Sqrt[x])/a)
^(2*(3 + p)))/(2*c^8*(3 + p)) + (a^8*(1 + (c*Sqrt[x])/a)^(2*(4 + p)))/(2*c
^8*(4 + p)) - (a^8*(1 + (c*Sqrt[x])/a)^(1 + 2*p))/(c^8*(1 + 2*p)) - (21*a^
8*(1 + (c*Sqrt[x])/a)^(3 + 2*p))/(c^8*(3 + 2*p)) - (35*a^8*(1 + (c*Sqrt[x]
)/a)^(5 + 2*p))/(c^8*(5 + 2*p)) - (7*a^8*(1 + (c*Sqrt[x])/a)^(7 + 2*p))/(c
^8*(7 + 2*p)))*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p)/(1 + (c*Sqrt[x])/a)^(2*p)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^3 \left(a^2 + 2ac\sqrt{x} + c^2 x\right)^p dx$$

input
```
int(x^3*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

output
```
int(x^3*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.34

$$\int x^3\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx = -\frac{\left(630\,a^8-\left(16\,c^8p^7+224\,c^8p^6+1288\,c^8p^5+3920\,c^8p^4+6769\,c^8p^3+6566\,c^8p^2+3267\,c^8p+630\,c^8\right)x^4\right.}{}$$

input
```
integrate(x^3*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="fricas")
```

output
```
-(630*a^8 - (16*c^8*p^7 + 224*c^8*p^6 + 1288*c^8*p^5 + 3920*c^8*p^4 + 6769
*c^8*p^3 + 6566*c^8*p^2 + 3267*c^8*p + 630*c^8)*x^4 + 7*(8*a^2*c^6*p^6 + 6
0*a^2*c^6*p^5 + 170*a^2*c^6*p^4 + 225*a^2*c^6*p^3 + 137*a^2*c^6*p^2 + 30*a
^2*c^6*p)*x^3 + 105*(4*a^4*c^4*p^4 + 12*a^4*c^4*p^3 + 11*a^4*c^4*p^2 + 3*a
^4*c^4*p)*x^2 + 630*(2*a^6*c^2*p^2 + a^6*c^2*p)*x - 2*(630*a^7*c*p + (8*a*
c^7*p^7 + 84*a*c^7*p^6 + 350*a*c^7*p^5 + 735*a*c^7*p^4 + 812*a*c^7*p^3 + 4
41*a*c^7*p^2 + 90*a*c^7*p)*x^3 + 21*(4*a^3*c^5*p^5 + 20*a^3*c^5*p^4 + 35*a
^3*c^5*p^3 + 25*a^3*c^5*p^2 + 6*a^3*c^5*p)*x^2 + 210*(2*a^5*c^3*p^3 + 3*a^
5*c^3*p^2 + a^5*c^3*p)*x)*sqrt(x))*(c^2*x + 2*a*c*sqrt(x) + a^2)^p/(16*c^8
*p^8 + 288*c^8*p^7 + 2184*c^8*p^6 + 9072*c^8*p^5 + 22449*c^8*p^4 + 33642*c
^8*p^3 + 29531*c^8*p^2 + 13698*c^8*p + 2520*c^8)
```

Sympy [F]

$$\int x^3\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\int x^3\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$

input
```
integrate(x**3*(a**2+2*a*c*x**(1/2)+c**2*x)**p,x)
```

output
```
Integral(x**3*(a**2 + 2*a*c*sqrt(x) + c**2*x)**p, x)
```

Maxima [F]

$$\int x^3\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\int\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p x^3\,dx$$

input
```
integrate(x^3*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="maxima")
```

output
```
integrate((c^2*x + 2*a*c*sqrt(x) + a^2)^p*x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1184 vs. 2(321) = 642.

Time = 0.13 (sec) , antiderivative size = 1184, normalized size of antiderivative = 3.35

$$\int x^3\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="giac")
```

output
```
(16*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*c^8*p^7*x^4 + 16*(c^2*x + 2*a*c*sqrt(x
) + a^2)^p*a*c^7*p^7*x^(7/2) + 224*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*c^8*p^6
*x^4 + 168*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a*c^7*p^6*x^(7/2) - 56*(c^2*x +
 2*a*c*sqrt(x) + a^2)^p*a^2*c^6*p^6*x^3 + 1288*(c^2*x + 2*a*c*sqrt(x) + a^
2)^p*c^8*p^5*x^4 + 700*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a*c^7*p^5*x^(7/2) -
 420*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^2*c^6*p^5*x^3 + 3920*(c^2*x + 2*a*c
*sqrt(x) + a^2)^p*c^8*p^4*x^4 + 168*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^3*c^
5*p^5*x^(5/2) + 1470*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a*c^7*p^4*x^(7/2) - 1
190*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^2*c^6*p^4*x^3 + 6769*(c^2*x + 2*a*c*
sqrt(x) + a^2)^p*c^8*p^3*x^4 + 840*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^3*c^5
*p^4*x^(5/2) + 1624*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a*c^7*p^3*x^(7/2) - 42
0*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^4*c^4*p^4*x^2 - 1575*(c^2*x + 2*a*c*sq
rt(x) + a^2)^p*a^2*c^6*p^3*x^3 + 6566*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*c^8*
p^2*x^4 + 1470*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^3*c^5*p^3*x^(5/2) + 882*(
c^2*x + 2*a*c*sqrt(x) + a^2)^p*a*c^7*p^2*x^(7/2) - 1260*(c^2*x + 2*a*c*sqr
t(x) + a^2)^p*a^4*c^4*p^3*x^2 - 959*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^2*c^
6*p^2*x^3 + 3267*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*c^8*p*x^4 + 840*(c^2*x +
2*a*c*sqrt(x) + a^2)^p*a^5*c^3*p^3*x^(3/2) + 1050*(c^2*x + 2*a*c*sqrt(x) +
 a^2)^p*a^3*c^5*p^2*x^(5/2) + 180*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a*c^7*p*
x^(7/2) - 1155*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^4*c^4*p^2*x^2 - 210*(c...
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx=\int x^3\,{\left(c^2\,x+a^2+2\,a\,c\,\sqrt{x}\right)}^p \,d x$$

input
```
int(x^3*(c^2*x + a^2 + 2*a*c*x^(1/2))^p,x)
```

output
```
int(x^3*(c^2*x + a^2 + 2*a*c*x^(1/2))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.51

$$\int x^3\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$
$$=\frac{\left(2\sqrt{x}\,ac+a^2+c^2x\right)^p\left(630c^8x^4-630a^6c^2px-420a^4c^4p^4x^2-1260a^4c^4p^3x^2-1155a^4c^4p^2x^2-315a^4c^4p\right.}{}$$

input
```
int(x^3*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

output
```
((2*sqrt(x)*a*c + a**2 + c**2*x)**p*(1260*sqrt(x)*a**7*c*p + 840*sqrt(x)*a
**5*c**3*p**3*x + 1260*sqrt(x)*a**5*c**3*p**2*x + 420*sqrt(x)*a**5*c**3*p*
x + 168*sqrt(x)*a**3*c**5*p**5*x**2 + 840*sqrt(x)*a**3*c**5*p**4*x**2 + 14
70*sqrt(x)*a**3*c**5*p**3*x**2 + 1050*sqrt(x)*a**3*c**5*p**2*x**2 + 252*sq
rt(x)*a**3*c**5*p*x**2 + 16*sqrt(x)*a*c**7*p**7*x**3 + 168*sqrt(x)*a*c**7*
p**6*x**3 + 700*sqrt(x)*a*c**7*p**5*x**3 + 1470*sqrt(x)*a*c**7*p**4*x**3 +
 1624*sqrt(x)*a*c**7*p**3*x**3 + 882*sqrt(x)*a*c**7*p**2*x**3 + 180*sqrt(x
)*a*c**7*p*x**3 - 630*a**8 - 1260*a**6*c**2*p**2*x - 630*a**6*c**2*p*x - 4
20*a**4*c**4*p**4*x**2 - 1260*a**4*c**4*p**3*x**2 - 1155*a**4*c**4*p**2*x*
*2 - 315*a**4*c**4*p*x**2 - 56*a**2*c**6*p**6*x**3 - 420*a**2*c**6*p**5*x*
*3 - 1190*a**2*c**6*p**4*x**3 - 1575*a**2*c**6*p**3*x**3 - 959*a**2*c**6*p
**2*x**3 - 210*a**2*c**6*p*x**3 + 16*c**8*p**7*x**4 + 224*c**8*p**6*x**4 +
 1288*c**8*p**5*x**4 + 3920*c**8*p**4*x**4 + 6769*c**8*p**3*x**4 + 6566*c*
*8*p**2*x**4 + 3267*c**8*p*x**4 + 630*c**8*x**4))/(c**8*(16*p**8 + 288*p**
7 + 2184*p**6 + 9072*p**5 + 22449*p**4 + 33642*p**3 + 29531*p**2 + 13698*p
 + 2520))
```

3.87 $\int x^2 \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 263

$$\int x^2 \left(a^2+2ac\sqrt{x}+c^2x\right)^p \, dx = -\frac{2a^5\left(a+c\sqrt{x}\right)\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^6(1+2p)} + \frac{5a^4\left(a+c\sqrt{x}\right)^2\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^6(1+p)} - \frac{20a^3\left(a+c\sqrt{x}\right)^3\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^6(3+2p)} + \frac{10a^2\left(a+c\sqrt{x}\right)^4\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^6(2+p)} - \frac{10a\left(a+c\sqrt{x}\right)^5\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^6(5+2p)} + \frac{\left(a+c\sqrt{x}\right)^6\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^6(3+p)}$$

output
```
-2*a^5*(a+c*x^(1/2))*(a^2+2*a*c*x^(1/2)+c^2*x)^p/c^6/(1+2*p)+5*a^4*(a+c*x^
(1/2))^2*(a^2+2*a*c*x^(1/2)+c^2*x)^p/c^6/(p+1)-20*a^3*(a+c*x^(1/2))^3*(a^2
+2*a*c*x^(1/2)+c^2*x)^p/c^6/(3+2*p)+10*a^2*(a+c*x^(1/2))^4*(a^2+2*a*c*x^(1
/2)+c^2*x)^p/c^6/(2+p)-10*a*(a+c*x^(1/2))^5*(a^2+2*a*c*x^(1/2)+c^2*x)^p/c^
6/(5+2*p)+(a+c*x^(1/2))^6*(a^2+2*a*c*x^(1/2)+c^2*x)^p/c^6/(3+p)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.53

$$\int x^2\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx = \frac{\left(-\frac{2a^5}{1+2p}+\frac{5a^4(a+c\sqrt{x})}{1+p}-\frac{20a^3(a+c\sqrt{x})^2}{3+2p}+\frac{10a^2(a+c\sqrt{x})^3}{2+p}-\frac{10a(a+c\sqrt{x})^4}{5+2p}+\frac{(a+c\sqrt{x})^5}{3+p}\right)\left(a+c\sqrt{x}\right)\left(\left(a+c\sqrt{x}\right)^2\right)^p}{c^6}$$

input
```
Integrate[x^2*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p,x]
```

output
```
(((-2*a^5)/(1 + 2*p) + (5*a^4*(a + c*Sqrt[x]))/(1 + p) - (20*a^3*(a + c*Sq
rt[x])^2)/(3 + 2*p) + (10*a^2*(a + c*Sqrt[x])^3)/(2 + p) - (10*a*(a + c*Sq
rt[x])^4)/(5 + 2*p) + (a + c*Sqrt[x])^5/(3 + p))*(a + c*Sqrt[x])*((a + c*S
qrt[x])^2)^p)/c^6
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1385, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}x^2dx$$

$$\Big\downarrow \text{798}$$

$$2\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}x^{5/2}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int\left(-\frac{a^5\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}}{c^5}+\frac{5a^5\left(\frac{\sqrt{x}c}{a}+1\right)^{2p+1}}{c^5}-\frac{10a^5\left(\frac{\sqrt{x}c}{a}+1\right)^{2p+2}}{c^5}+\frac{10a\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{5a^6\left(\frac{c\sqrt{x}}{a}+1\right)^{2(p+1)}}{2c^6(p+1)}+\frac{5a^6\left(\frac{c\sqrt{x}}{a}+1\right)^{2(p+2)}}{c^6(p+2)}+\frac{a^6\left(\frac{c\sqrt{x}}{a}+1\right)^{2(p+3)}}{2c^6(p+3)}-\frac{a^6\left(\frac{c\sqrt{x}}{a}+1\right)^{2p+1}}{c^6(2p+1)}-\frac{10a^6\left(\frac{c\sqrt{x}}{a}+1\right)^{2p+3}}{c^6(2p+3)}\right.$$

input
```
Int[x^2*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p,x]
```

output
```
(2*((5*a^6*(1 + (c*Sqrt[x])/a)^(2*(1 + p)))/(2*c^6*(1 + p)) + (5*a^6*(1 +
(c*Sqrt[x])/a)^(2*(2 + p)))/(c^6*(2 + p)) + (a^6*(1 + (c*Sqrt[x])/a)^(2*(3
 + p)))/(2*c^6*(3 + p)) - (a^6*(1 + (c*Sqrt[x])/a)^(1 + 2*p))/(c^6*(1 + 2*
p)) - (10*a^6*(1 + (c*Sqrt[x])/a)^(3 + 2*p))/(c^6*(3 + 2*p)) - (5*a^6*(1 +
 (c*Sqrt[x])/a)^(5 + 2*p))/(c^6*(5 + 2*p)))*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^
p)/(1 + (c*Sqrt[x])/a)^(2*p)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^2\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$

input
```
int(x^2*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

output
```
int(x^2*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.12

$$\int x^2\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx = -\frac{(30\,a^6-(8\,c^6p^5+60\,c^6p^4+170\,c^6p^3+225\,c^6p^2+137\,c^6p+30\,c^6)x^3+5\,(4\,a^2c^4p^4+12\,a^2c^4p^3+11\,a^2$$

input
```
integrate(x^2*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="fricas")
```

output
```
-(30*a^6 - (8*c^6*p^5 + 60*c^6*p^4 + 170*c^6*p^3 + 225*c^6*p^2 + 137*c^6*p
 + 30*c^6)*x^3 + 5*(4*a^2*c^4*p^4 + 12*a^2*c^4*p^3 + 11*a^2*c^4*p^2 + 3*a^
2*c^4*p)*x^2 + 30*(2*a^4*c^2*p^2 + a^4*c^2*p)*x - 2*(30*a^5*c*p + (4*a*c^5
*p^5 + 20*a*c^5*p^4 + 35*a*c^5*p^3 + 25*a*c^5*p^2 + 6*a*c^5*p)*x^2 + 10*(2
*a^3*c^3*p^3 + 3*a^3*c^3*p^2 + a^3*c^3*p)*x)*sqrt(x))*(c^2*x + 2*a*c*sqrt(
x) + a^2)^p/(8*c^6*p^6 + 84*c^6*p^5 + 350*c^6*p^4 + 735*c^6*p^3 + 812*c^6*
p^2 + 441*c^6*p + 90*c^6)
```

Sympy [F]

$$\int x^2\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\int x^2\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$

input
```
integrate(x**2*(a**2+2*a*c*x**(1/2)+c**2*x)**p,x)
```

output
```
Integral(x**2*(a**2 + 2*a*c*sqrt(x) + c**2*x)**p, x)
```

Maxima [F]

$$\int x^2\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\int\left(c^2x+2\,ac\sqrt{x}+a^2\right)^px^2\,dx$$

input
```
integrate(x^2*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="maxima")
```

output
```
integrate((c^2*x + 2*a*c*sqrt(x) + a^2)^p*x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. 2(239) = 478.

Time = 0.13 (sec) , antiderivative size = 702, normalized size of antiderivative = 2.67

$$\int x^2\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\text{Too large to display}$$

input
```
integrate(x^2*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="giac")
```

output
```
(8*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*c^6*p^5*x^3 + 8*(c^2*x + 2*a*c*sqrt(x)
+ a^2)^p*a*c^5*p^5*x^(5/2) + 60*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*c^6*p^4*x^
3 + 40*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a*c^5*p^4*x^(5/2) - 20*(c^2*x + 2*a
*c*sqrt(x) + a^2)^p*a^2*c^4*p^4*x^2 + 170*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*
c^6*p^3*x^3 + 70*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a*c^5*p^3*x^(5/2) - 60*(c
^2*x + 2*a*c*sqrt(x) + a^2)^p*a^2*c^4*p^3*x^2 + 225*(c^2*x + 2*a*c*sqrt(x)
 + a^2)^p*c^6*p^2*x^3 + 40*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^3*c^3*p^3*x^(
3/2) + 50*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a*c^5*p^2*x^(5/2) - 55*(c^2*x +
2*a*c*sqrt(x) + a^2)^p*a^2*c^4*p^2*x^2 + 137*(c^2*x + 2*a*c*sqrt(x) + a^2)
^p*c^6*p*x^3 + 60*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^3*c^3*p^2*x^(3/2) + 12
*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a*c^5*p*x^(5/2) - 60*(c^2*x + 2*a*c*sqrt(
x) + a^2)^p*a^4*c^2*p^2*x - 15*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^2*c^4*p*x
^2 + 30*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*c^6*x^3 + 20*(c^2*x + 2*a*c*sqrt(x
) + a^2)^p*a^3*c^3*p*x^(3/2) - 30*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^4*c^2*
p*x + 60*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^5*c*p*sqrt(x) - 30*(c^2*x + 2*a
*c*sqrt(x) + a^2)^p*a^6)/(8*c^6*p^6 + 84*c^6*p^5 + 350*c^6*p^4 + 735*c^6*p
^3 + 812*c^6*p^2 + 441*c^6*p + 90*c^6)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\int x^2\,\left(c^2\,x+a^2+2\,a\,c\,\sqrt{x}\right)^p\,dx$$

input
```
int(x^2*(c^2*x + a^2 + 2*a*c*x^(1/2))^p,x)
```

output
```
int(x^2*(c^2*x + a^2 + 2*a*c*x^(1/2))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.19

$$\int x^2\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$
$$=\frac{\left(2\sqrt{x}\,ac+a^2+c^2x\right)^p\left(60\sqrt{x}\,a^5cp+40\sqrt{x}\,a^3c^3p^3x+60\sqrt{x}\,a^3c^3p^2x+20\sqrt{x}\,a^3c^3px+8\sqrt{x}\,a\,c^5p^5x^2+40\right.}{}$$

input
```
int(x^2*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

output
```
((2*sqrt(x)*a*c + a**2 + c**2*x)**p*(60*sqrt(x)*a**5*c*p + 40*sqrt(x)*a**3
*c**3*p**3*x + 60*sqrt(x)*a**3*c**3*p**2*x + 20*sqrt(x)*a**3*c**3*p*x + 8*
sqrt(x)*a*c**5*p**5*x**2 + 40*sqrt(x)*a*c**5*p**4*x**2 + 70*sqrt(x)*a*c**5
*p**3*x**2 + 50*sqrt(x)*a*c**5*p**2*x**2 + 12*sqrt(x)*a*c**5*p*x**2 - 30*a
**6 - 60*a**4*c**2*p**2*x - 30*a**4*c**2*p*x - 20*a**2*c**4*p**4*x**2 - 60
*a**2*c**4*p**3*x**2 - 55*a**2*c**4*p**2*x**2 - 15*a**2*c**4*p*x**2 + 8*c*
*6*p**5*x**3 + 60*c**6*p**4*x**3 + 170*c**6*p**3*x**3 + 225*c**6*p**2*x**3
 + 137*c**6*p*x**3 + 30*c**6*x**3))/(c**6*(8*p**6 + 84*p**5 + 350*p**4 + 7
35*p**3 + 812*p**2 + 441*p + 90))
```

3.88 $\int x\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 173

$$\int x\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=-\frac{2a^3\left(a+c\sqrt{x}\right)\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^4(1+2p)}+\frac{3a^2\left(a+c\sqrt{x}\right)^2\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^4(1+p)}-\frac{6a\left(a+c\sqrt{x}\right)^3\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^4(3+2p)}+\frac{\left(a+c\sqrt{x}\right)^4\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^4(2+p)}$$

output
```
-2*a^3*(a+c*x^(1/2))*(a^2+2*a*c*x^(1/2)+c^2*x)^p/c^4/(1+2*p)+3*a^2*(a+c*x^
(1/2))^2*(a^2+2*a*c*x^(1/2)+c^2*x)^p/c^4/(p+1)-6*a*(a+c*x^(1/2))^3*(a^2+2*
a*c*x^(1/2)+c^2*x)^p/c^4/(3+2*p)+(a+c*x^(1/2))^4*(a^2+2*a*c*x^(1/2)+c^2*x)
^p/c^4/(2+p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.65

$$\int x\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$
$$=\frac{\left(a+c\sqrt{x}\right)\left(\left(a+c\sqrt{x}\right)^2\right)^p\left(-3a^3+3a^2c(1+2p)\sqrt{x}-3ac^2(1+3p+2p^2)\,x+c^3(3+11p+12p^2+4p^3)\right.}{c^4(1+p)(2+p)(1+2p)(3+2p)}$$

input
```
Integrate[x*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p,x]
```

output
```
((a + c*Sqrt[x])*((a + c*Sqrt[x])^2)^p*(-3*a^3 + 3*a^2*c*(1 + 2*p)*Sqrt[x]
 - 3*a*c^2*(1 + 3*p + 2*p^2)*x + c^3*(3 + 11*p + 12*p^2 + 4*p^3)*x^(3/2)))
/(c^4*(1 + p)*(2 + p)*(1 + 2*p)*(3 + 2*p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1385, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$

$$\Big\downarrow \text{ 1385}$$

$$\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}xdx$$

$$\Big\downarrow \text{ 798}$$

$$2\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}x^{3/2}d\sqrt{x}$$

$$\Big\downarrow \text{ 53}$$

$$2\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int\left(-\frac{a^3\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}}{c^3}+\frac{3a^3\left(\frac{\sqrt{x}c}{a}+1\right)^{2p+1}}{c^3}-\frac{3a^3\left(\frac{\sqrt{x}c}{a}+1\right)^{2p+2}}{c^3}+\frac{a^3\left(\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{3a^4\left(\frac{c\sqrt{x}}{a}+1\right)^{2(p+1)}}{2c^4(p+1)}+\frac{a^4\left(\frac{c\sqrt{x}}{a}+1\right)^{2(p+2)}}{2c^4(p+2)}-\frac{a^4\left(\frac{c\sqrt{x}}{a}+1\right)^{2p+1}}{c^4(2p+1)}-\frac{3a^4\left(\frac{c\sqrt{x}}{a}+1\right)^{2p+3}}{c^4(2p+3)}\right)\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+\right.$$

input
```
Int[x*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p,x]
```

output
```
(2*((3*a^4*(1 + (c*Sqrt[x])/a)^(2*(1 + p)))/(2*c^4*(1 + p)) + (a^4*(1 + (c
*Sqrt[x])/a)^(2*(2 + p)))/(2*c^4*(2 + p)) - (a^4*(1 + (c*Sqrt[x])/a)^(1 +
2*p))/(c^4*(1 + 2*p)) - (3*a^4*(1 + (c*Sqrt[x])/a)^(3 + 2*p))/(c^4*(3 + 2*
p)))*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p)/(1 + (c*Sqrt[x])/a)^(2*p)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x\left(a^2+2ac\sqrt{x}+c^2x\right)^p dx$$

input
```
int(x*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

output
```
int(x*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.92

$$\int x\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx = -\frac{(3\,a^4-(4\,c^4p^3+12\,c^4p^2+11\,c^4p+3\,c^4)x^2+3\,(2\,a^2c^2p^2+a^2c^2p)x-2\,(3\,a^3cp+(2\,ac^3p^3+3\,ac^3p^2+}{4\,c^4p^4+20\,c^4p^3+35\,c^4p^2+25\,c^4p+6\,c^4}$$

input
```
integrate(x*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="fricas")
```

output
```
-(3*a^4 - (4*c^4*p^3 + 12*c^4*p^2 + 11*c^4*p + 3*c^4)*x^2 + 3*(2*a^2*c^2*p
^2 + a^2*c^2*p)*x - 2*(3*a^3*c*p + (2*a*c^3*p^3 + 3*a*c^3*p^2 + a*c^3*p)*x
)*sqrt(x))*(c^2*x + 2*a*c*sqrt(x) + a^2)^p/(4*c^4*p^4 + 20*c^4*p^3 + 35*c^
4*p^2 + 25*c^4*p + 6*c^4)
```

Sympy [F]

$$\int x\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\int x\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$

input
```
integrate(x*(a**2+2*a*c*x**(1/2)+c**2*x)**p,x)
```

output
```
Integral(x*(a**2 + 2*a*c*sqrt(x) + c**2*x)**p, x)
```

Maxima [F]

$$\int x\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\int\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p x\,dx$$

input
```
integrate(x*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="maxima")
```

output
```
integrate((c^2*x + 2*a*c*sqrt(x) + a^2)^p*x, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(157) = 314.

Time = 0.13 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.01

$$\int x\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$
$$=\frac{4\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p c^4p^3x^2+4\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p ac^3p^3x^{\frac{3}{2}}+12\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p c^4p^2x^2+6\left(c\right.}{}$$

input
```
integrate(x*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="giac")
```

output
```
(4*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*c^4*p^3*x^2 + 4*(c^2*x + 2*a*c*sqrt(x)
+ a^2)^p*a*c^3*p^3*x^(3/2) + 12*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*c^4*p^2*x^
2 + 6*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a*c^3*p^2*x^(3/2) - 6*(c^2*x + 2*a*c
*sqrt(x) + a^2)^p*a^2*c^2*p^2*x + 11*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*c^4*p
*x^2 + 2*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a*c^3*p*x^(3/2) - 3*(c^2*x + 2*a*
c*sqrt(x) + a^2)^p*a^2*c^2*p*x + 3*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*c^4*x^2
 + 6*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*a^3*c*p*sqrt(x) - 3*(c^2*x + 2*a*c*sq
rt(x) + a^2)^p*a^4)/(4*c^4*p^4 + 20*c^4*p^3 + 35*c^4*p^2 + 25*c^4*p + 6*c^
4)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\int x\left(c^2\,x+a^2+2\,a\,c\,\sqrt{x}\right)^p\,dx$$

input
```
int(x*(c^2*x + a^2 + 2*a*c*x^(1/2))^p,x)
```

output
```
int(x*(c^2*x + a^2 + 2*a*c*x^(1/2))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88

$$\int x\left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$
$$=\frac{\left(2\sqrt{x}\,ac+a^2+c^2x\right)^p\left(6\sqrt{x}\,a^3cp+4\sqrt{x}\,a\,c^3p^3x+6\sqrt{x}\,a\,c^3p^2x+2\sqrt{x}\,a\,c^3px-3a^4-6a^2c^2p^2x-3a^2c^2p\right.}{c^4\left(4p^4+20p^3+35p^2+25p+6\right)}$$

input
```
int(x*(a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

output
```
((2*sqrt(x)*a*c + a**2 + c**2*x)**p*(6*sqrt(x)*a**3*c*p + 4*sqrt(x)*a*c**3
*p**3*x + 6*sqrt(x)*a*c**3*p**2*x + 2*sqrt(x)*a*c**3*p*x - 3*a**4 - 6*a**2
*c**2*p**2*x - 3*a**2*c**2*p*x + 4*c**4*p**3*x**2 + 12*c**4*p**2*x**2 + 11
*c**4*p*x**2 + 3*c**4*x**2))/(c**4*(4*p**4 + 20*p**3 + 35*p**2 + 25*p + 6)
)
```

3.89 $\int \left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 74

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=-\frac{2a\left(a+c\sqrt{x}\right)\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^2(1+2p)}+\frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^{1+p}}{c^2(1+p)}$$

output
```
-2*a*(a+c*x^(1/2))*(a^2+2*a*c*x^(1/2)+c^2*x)^p/c^2/(1+2*p)+(a^2+2*a*c*x^(1
/2)+c^2*x)^(p+1)/c^2/(p+1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\frac{\left(a+c\sqrt{x}\right)\left(\left(a+c\sqrt{x}\right)^2\right)^p\left(-a+c(1+2p)\sqrt{x}\right)}{c^2(1+p)(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p,x]
```

output
```
((a + c*Sqrt[x])*((a + c*Sqrt[x])^2)^p*(-a + c*(1 + 2*p)*Sqrt[x]))/(c^2*(1
 + p)*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1680, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$

$$\Big\downarrow \; 1680$$

$$2\int \sqrt{x}\left(a^2+2c\sqrt{x}a+c^2x\right)^p\,d\sqrt{x}$$

$$\Big\downarrow \; 1100$$

$$2\left(\frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^{p+1}}{2c^2(p+1)}-\frac{a\int \left(a^2+2c\sqrt{x}a+c^2x\right)^p\,d\sqrt{x}}{c}\right)$$

$$\Big\downarrow \; 1079$$

$$2\left(\frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^{p+1}}{2c^2(p+1)}-\frac{a\left(ac+c^2\sqrt{x}\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int \left(\sqrt{x}c^2+ac\right)^{2p}d\sqrt{x}}{c}\right)$$

$$\Big\downarrow \; 17$$

$$2\left(\frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^{p+1}}{2c^2(p+1)}-\frac{a\left(ac+c^2\sqrt{x}\right)\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{c^3(2p+1)}\right)$$

input
```
Int[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p,x]
```

output
```
2*(-((a*(a*c + c^2*Sqrt[x])*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p)/(c^3*(1 + 2*p
))) + (a^2 + 2*a*c*Sqrt[x] + c^2*x)^(1 + p)/(2*c^2*(1 + p)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [F]

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^p dx$$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

output
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\frac{\left(2\,acp\sqrt{x}-a^2+(2\,c^2p+c^2)x\right)\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p}{2\,c^2p^2+3\,c^2p+c^2}$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="fricas")
```

output
```
(2*a*c*p*sqrt(x) - a^2 + (2*c^2*p + c^2)*x)*(c^2*x + 2*a*c*sqrt(x) + a^2)^
p/(2*c^2*p^2 + 3*c^2*p + c^2)
```

Sympy [F]

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\int \left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$

input
```
integrate((a**2+2*a*c*x**(1/2)+c**2*x)**p,x)
```

output
```
Integral((a**2 + 2*a*c*sqrt(x) + c**2*x)**p, x)
```

Maxima [F]

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\int \left(c^2x+2\,ac\sqrt{x}+a^2\right)^p\,dx$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="maxima")
```

output
```
integrate((c^2*x + 2*a*c*sqrt(x) + a^2)^p, x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx$$
$$=\frac{2\left(c^2x+2\,ac\sqrt{x}+a^2\right)^pc^2px+2\left(c^2x+2\,ac\sqrt{x}+a^2\right)^pacp\sqrt{x}+\left(c^2x+2\,ac\sqrt{x}+a^2\right)^pc^2x-\left(c^2x+2\,ac\right.}{2\,c^2p^2+3\,c^2p+c^2}$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^p,x, algorithm="giac")
```

output
```
(2*(c^2*x + 2*a*c*sqrt(x) + a^2)^p*c^2*p*x + 2*(c^2*x + 2*a*c*sqrt(x) + a^
2)^p*a*c*p*sqrt(x) + (c^2*x + 2*a*c*sqrt(x) + a^2)^p*c^2*x - (c^2*x + 2*a*
c*sqrt(x) + a^2)^p*a^2)/(2*c^2*p^2 + 3*c^2*p + c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\int {\left(c^2\,x+a^2+2\,a\,c\,\sqrt{x}\right)}^p\,dx$$

input
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^p,x)
```

output
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^p\,dx=\frac{\left(2\sqrt{x}\,ac+a^2+c^2x\right)^p\left(2\sqrt{x}\,acp-a^2+2c^2px+c^2x\right)}{c^2\left(2p^2+3p+1\right)}$$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^p,x)
```

output
```
((2*sqrt(x)*a*c + a**2 + c**2*x)**p*(2*sqrt(x)*a*c*p - a**2 + 2*c**2*p*x +
 c**2*x))/(c**2*(2*p**2 + 3*p + 1))
```

3.90 $\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x}\,dx = -\frac{2\left(a+c\sqrt{x}\right)\left(a^2+2ac\sqrt{x}+c^2x\right)^p \text{Hypergeometric2F1}\left(1,1+2p,2(1+p),1+\frac{c\sqrt{x}}{a}\right)}{a(1+2p)}$$

output
```
-2*(a+c*x^(1/2))*(a^2+2*a*c*x^(1/2)+c^2*x)^p*hypergeom([1, 1+2*p],[2*p+2],
1+c*x^(1/2)/a)/a/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x}\,dx = -\frac{2\left(a+c\sqrt{x}\right)\left(\left(a+c\sqrt{x}\right)^2\right)^p \text{Hypergeometric2F1}\left(1,1+2p,2+2p,1+\frac{c\sqrt{x}}{a}\right)}{a(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p/x,x]
```

output
```
(-2*(a + c*Sqrt[x])*((a + c*Sqrt[x])^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2
+ 2*p, 1 + (c*Sqrt[x])/a])/(a*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{1385, 798, 75\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x}\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int \frac{\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$2\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int \frac{\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{75}$$

$$-\frac{2\left(\frac{c\sqrt{x}}{a}+1\right)\left(a^2+2ac\sqrt{x}+c^2x\right)^p \text{Hypergeometric2F1}\left(1,2p+1,2(p+1),\frac{\sqrt{x}c}{a}+1\right)}{2p+1}$$

input
```
Int[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p/x,x]
```

output
```
(-2*(1 + (c*Sqrt[x])/a)*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p*Hypergeometric2F1[
1, 1 + 2*p, 2*(1 + p), 1 + (c*Sqrt[x])/a])/(1 + 2*p)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x}dx$$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^p/x,x)
```

output
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^p/x,x)
```

Fricas [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x}\,dx=\int \frac{\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p}{x}\,dx$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^p/x,x, algorithm="fricas")
```

output
```
integral((c^2*x + 2*a*c*sqrt(x) + a^2)^p/x, x)
```

Sympy [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x}\,dx=\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x}\,dx$$

input
```
integrate((a**2+2*a*c*x**(1/2)+c**2*x)**p/x,x)
```

output
```
Integral((a**2 + 2*a*c*sqrt(x) + c**2*x)**p/x, x)
```

Maxima [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x}\,dx=\int \frac{\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p}{x}\,dx$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^p/x,x, algorithm="maxima")
```

output
```
integrate((c^2*x + 2*a*c*sqrt(x) + a^2)^p/x, x)
```

Giac [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x}\,dx=\int \frac{\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p}{x}\,dx$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^p/x,x, algorithm="giac")
```

output
```
integrate((c^2*x + 2*a*c*sqrt(x) + a^2)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x}\,dx=\int \frac{{\left(c^2\,x+a^2+2\,a\,c\,\sqrt{x}\right)}^p}{x} \,d x$$

input
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^p/x,x)
```

output
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x}\,dx = \frac{\left(2\sqrt{x}\,ac+a^2+c^2x\right)^p+\left(\int \frac{\left(2\sqrt{x}\,ac+a^2+c^2x\right)^p}{-c^2x^2+a^2x}dx\right)a^2p-\left(\int \frac{\left(2\sqrt{x}\,ac+a^2+c^2x\right)^p}{\sqrt{x}\,a^2-\sqrt{x}\,c^2x}dx\right)acp}{p}$$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^p/x,x)
```

output
```
((2*sqrt(x)*a*c + a**2 + c**2*x)**p + int((2*sqrt(x)*a*c + a**2 + c**2*x)*
*p/(a**2*x - c**2*x**2),x)*a**2*p - int((2*sqrt(x)*a*c + a**2 + c**2*x)**p
/(sqrt(x)*a**2 - sqrt(x)*c**2*x),x)*a*c*p)/p
```

3.91 $\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^2}\,dx = -\frac{2c^2\left(a+c\sqrt{x}\right)\left(a^2+2ac\sqrt{x}+c^2x\right)^p \text{Hypergeometric2F1}\left(3,1+2p,2(1+p),1+\frac{c\sqrt{x}}{a}\right)}{a^3(1+2p)}$$

output
```
-2*c^2*(a+c*x^(1/2))*(a^2+2*a*c*x^(1/2)+c^2*x)^p*hypergeom([3, 1+2*p],[2*p
+2],1+c*x^(1/2)/a)/a^3/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^2}\,dx = -\frac{2c^2\left(a+c\sqrt{x}\right)\left(\left(a+c\sqrt{x}\right)^2\right)^p \text{Hypergeometric2F1}\left(3,1+2p,2+2p,1+\frac{c\sqrt{x}}{a}\right)}{a^3(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p/x^2,x]
```

output
```
(-2*c^2*(a + c*Sqrt[x])*((a + c*Sqrt[x])^2)^p*Hypergeometric2F1[3, 1 + 2*p
, 2 + 2*p, 1 + (c*Sqrt[x])/a])/(a^3*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1385, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^2}\,dx$$

$$\Big\downarrow \text{ 1385}$$

$$\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int \frac{\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}}{x^2}dx$$

$$\Big\downarrow \text{ 798}$$

$$2\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int \frac{\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{2c^2\left(\frac{c\sqrt{x}}{a}+1\right)\left(a^2+2ac\sqrt{x}+c^2x\right)^p\text{Hypergeometric2F1}\left(3,2p+1,2(p+1),\frac{\sqrt{x}c}{a}+1\right)}{a^2(2p+1)}$$

input
```
Int[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p/x^2,x]
```

output
```
(-2*c^2*(1 + (c*Sqrt[x])/a)*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p*Hypergeometric
2F1[3, 1 + 2*p, 2*(1 + p), 1 + (c*Sqrt[x])/a])/(a^2*(1 + 2*p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{\left(a^2 + 2ac\sqrt{x} + c^2 x\right)^p}{x^2} dx$$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^p/x^2,x)
```

output
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a^2 + 2ac\sqrt{x} + c^2 x\right)^p}{x^2} dx = \int \frac{\left(c^2 x + 2\,ac\sqrt{x} + a^2\right)^p}{x^2} dx$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^p/x^2,x, algorithm="fricas")
```

output
```
integral((c^2*x + 2*a*c*sqrt(x) + a^2)^p/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^2}\,dx=\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^2}\,dx$$

input
```
integrate((a**2+2*a*c*x**(1/2)+c**2*x)**p/x**2,x)
```

output
```
Integral((a**2 + 2*a*c*sqrt(x) + c**2*x)**p/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^2}\,dx=\int \frac{\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p}{x^2}\,dx$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c^2*x + 2*a*c*sqrt(x) + a^2)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^2}\,dx=\int \frac{\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p}{x^2}\,dx$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^p/x^2,x, algorithm="giac")
```

output
```
integrate((c^2*x + 2*a*c*sqrt(x) + a^2)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^2}\,dx=\int \frac{\left(c^2\,x+a^2+2\,a\,c\,\sqrt{x}\right)^p}{x^2}\,dx$$

input
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^p/x^2,x)
```

output
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^2}\,dx$$
$$=\frac{-2\sqrt{x}\left(2\sqrt{x}\,ac+a^2+c^2x\right)^p cp-\left(2\sqrt{x}\,ac+a^2+c^2x\right)^p a+2\left(\int \frac{(2\sqrt{x}\,ac+a^2+c^2x)^p}{-c^2x^2+a^2x}dx\right) a\,c^2p^2x-\left(\int \frac{(2\sqrt{x}\,ac}{-c^2}\right.}{ax}$$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^p/x^2,x)
```

output
```
( - 2*sqrt(x)*(2*sqrt(x)*a*c + a**2 + c**2*x)**p*c*p - (2*sqrt(x)*a*c + a*
*2 + c**2*x)**p*a + 2*int((2*sqrt(x)*a*c + a**2 + c**2*x)**p/(a**2*x - c**
2*x**2),x)*a*c**2*p**2*x - int((2*sqrt(x)*a*c + a**2 + c**2*x)**p/(a**2*x
- c**2*x**2),x)*a*c**2*p*x - 2*int((sqrt(x)*(2*sqrt(x)*a*c + a**2 + c**2*x
)**p)/(a**2*x - c**2*x**2),x)*c**3*p**2*x + int((sqrt(x)*(2*sqrt(x)*a*c +
a**2 + c**2*x)**p)/(a**2*x - c**2*x**2),x)*c**3*p*x)/(a*x)
```

3.92 $\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^3}\,dx = -\frac{2c^4\left(a+c\sqrt{x}\right)\left(a^2+2ac\sqrt{x}+c^2x\right)^p \text{Hypergeometric2F1}\left(5,1+2p,2(1+p),1+\frac{c\sqrt{x}}{a}\right)}{a^5(1+2p)}$$

output
```
-2*c^4*(a+c*x^(1/2))*(a^2+2*a*c*x^(1/2)+c^2*x)^p*hypergeom([5, 1+2*p],[2*p
+2],1+c*x^(1/2)/a)/a^5/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^3}\,dx = -\frac{2c^4\left(a+c\sqrt{x}\right)\left(\left(a+c\sqrt{x}\right)^2\right)^p \text{Hypergeometric2F1}\left(5,1+2p,2+2p,1+\frac{c\sqrt{x}}{a}\right)}{a^5(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p/x^3,x]
```

output
```
(-2*c^4*(a + c*Sqrt[x])*((a + c*Sqrt[x])^2)^p*Hypergeometric2F1[5, 1 + 2*p
, 2 + 2*p, 1 + (c*Sqrt[x])/a])/(a^5*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1385, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^3}\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int \frac{\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}}{x^3}dx$$

$$\Big\downarrow \text{798}$$

$$2\left(\frac{c\sqrt{x}}{a}+1\right)^{-2p}\left(a^2+2ac\sqrt{x}+c^2x\right)^p\int \frac{\left(\frac{\sqrt{x}c}{a}+1\right)^{2p}}{x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{75}$$

$$-\frac{2c^4\left(\frac{c\sqrt{x}}{a}+1\right)\left(a^2+2ac\sqrt{x}+c^2x\right)^p \text{Hypergeometric2F1}\left(5,2p+1,2(p+1),\frac{\sqrt{x}c}{a}+1\right)}{a^4(2p+1)}$$

input
```
Int[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p/x^3,x]
```

output
```
(-2*c^4*(1 + (c*Sqrt[x])/a)*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^p*Hypergeometric
2F1[5, 1 + 2*p, 2*(1 + p), 1 + (c*Sqrt[x])/a])/(a^4*(1 + 2*p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{\left(a^{2}+2ac\sqrt{x}+c^{2}x\right)^{p}}{x^{3}}dx$$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^p/x^3,x)
```

output
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^3}\,dx=\int \frac{\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p}{x^3}\,dx$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^p/x^3,x, algorithm="fricas")
```

output
```
integral((c^2*x + 2*a*c*sqrt(x) + a^2)^p/x^3, x)
```

Sympy [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^3}\,dx=\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^3}\,dx$$

input
```
integrate((a**2+2*a*c*x**(1/2)+c**2*x)**p/x**3,x)
```

output
```
Integral((a**2 + 2*a*c*sqrt(x) + c**2*x)**p/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^3}\,dx=\int \frac{\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p}{x^3}\,dx$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((c^2*x + 2*a*c*sqrt(x) + a^2)^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^p}{x^3}\,dx=\int \frac{\left(c^2x+2\,ac\sqrt{x}+a^2\right)^p}{x^3}\,dx$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^p/x^3,x, algorithm="giac")
```

output
```
integrate((c^2*x + 2*a*c*sqrt(x) + a^2)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2ac\sqrt{x} + c^2x)^p}{x^3}\,dx = \int \frac{(c^2\,x + a^2 + 2\,a\,c\,\sqrt{x})^p}{x^3}\,dx$$

input
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^p/x^3,x)
```

output
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^p/x^3, x)
```

Reduce [F]

$$\int \frac{(a^2 + 2ac\sqrt{x} + c^2x)^p}{x^3}\,dx$$
$$= \frac{-2\sqrt{x}\,(2\sqrt{x}\,ac + a^2 + c^2x)^p\,a^2cp - 4\sqrt{x}\,(2\sqrt{x}\,ac + a^2 + c^2x)^p\,c^3p^3x + 10\sqrt{x}\,(2\sqrt{x}\,ac + a^2 + c^2x)^p\,c^3p^2}{}$$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^p/x^3,x)
```

output
```
( - 2*sqrt(x)*(2*sqrt(x)*a*c + a**2 + c**2*x)**p*a**2*c*p - 4*sqrt(x)*(2*s
qrt(x)*a*c + a**2 + c**2*x)**p*c**3*p**3*x + 10*sqrt(x)*(2*sqrt(x)*a*c + a
**2 + c**2*x)**p*c**3*p**2*x - 6*sqrt(x)*(2*sqrt(x)*a*c + a**2 + c**2*x)**
p*c**3*p*x - 3*(2*sqrt(x)*a*c + a**2 + c**2*x)**p*a**3 - 2*(2*sqrt(x)*a*c
+ a**2 + c**2*x)**p*a*c**2*p**2*x + 3*(2*sqrt(x)*a*c + a**2 + c**2*x)**p*a
*c**2*p*x + 4*int((2*sqrt(x)*a*c + a**2 + c**2*x)**p/(a**2*x - c**2*x**2),
x)*a*c**4*p**4*x**2 - 12*int((2*sqrt(x)*a*c + a**2 + c**2*x)**p/(a**2*x -
c**2*x**2),x)*a*c**4*p**3*x**2 + 11*int((2*sqrt(x)*a*c + a**2 + c**2*x)**p
/(a**2*x - c**2*x**2),x)*a*c**4*p**2*x**2 - 3*int((2*sqrt(x)*a*c + a**2 +
c**2*x)**p/(a**2*x - c**2*x**2),x)*a*c**4*p*x**2 - 4*int((sqrt(x)*(2*sqrt(
x)*a*c + a**2 + c**2*x)**p)/(a**2*x - c**2*x**2),x)*c**5*p**4*x**2 + 12*in
t((sqrt(x)*(2*sqrt(x)*a*c + a**2 + c**2*x)**p)/(a**2*x - c**2*x**2),x)*c**
5*p**3*x**2 - 11*int((sqrt(x)*(2*sqrt(x)*a*c + a**2 + c**2*x)**p)/(a**2*x
- c**2*x**2),x)*c**5*p**2*x**2 + 3*int((sqrt(x)*(2*sqrt(x)*a*c + a**2 + c*
*2*x)**p)/(a**2*x - c**2*x**2),x)*c**5*p*x**2)/(6*a**3*x**2)
```

3.93 $\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p (dx)^m\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p (dx)^m\,dx=\frac{\left(1+\frac{2c\sqrt[3]{x}}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2c\sqrt[3]{x}}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p (dx)^{1+m}\operatorname{AppellF1}\left(3(1+\right.}{d(1+m)}$$

output
```
(a+b*x^(1/3)+c*x^(2/3))^p*(d*x)^(1+m)*AppellF1(3+3*m,-p,-p,4+3*m,-2*c*x^(1
/3)/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^(1/3)/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((
1+2*c*x^(1/3)/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^(1/3)/(b+(-4*a*c+b^2)^(
1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.15

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p (dx)^m\,dx=\frac{\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x(dx)^m\operatorname{AppellF1}\left(3\right.}{1+m}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^p*(d*x)^m,x]
```

output
```
((a + b*x^(1/3) + c*x^(2/3))^p*x*(d*x)^m*AppellF1[3*(1 + m), -p, -p, 4 + 3
*m, (-2*c*x^(1/3))/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^(1/3))/(-b + Sqrt[b^2 -
 4*a*c])])/((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(b - Sqrt[b^2 -
 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(b + Sqrt[b^2 - 4*a*c])
)^p)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1716, 1715, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p dx$$

$$\Big\downarrow \text{1716}$$

$$x^{-m}(dx)^m \int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x^m dx$$

$$\Big\downarrow \text{1715}$$

$$3x^{-m}(dx)^m \int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x^{\frac{1}{3}(3m+2)}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1179}$$

$$3x^{-m}(dx)^m \left(\frac{2c\sqrt[3]{x}}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2c\sqrt[3]{x}}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p \int \left(\frac{2\sqrt[3]{x}c}{b-\sqrt{b^2-4ac}}+1\right)^p \Bigg(\frac{}{b}$$

$$\Big\downarrow \text{150}$$

$$\frac{x(dx)^m \left(\frac{2c\sqrt[3]{x}}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2c\sqrt[3]{x}}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p \text{AppellF1}\left(3(m+1),-p,-p,3m+4,-\frac{2}{b-}\right.}{m+1}$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^p*(d*x)^m,x]
```

output
```
((a + b*x^(1/3) + c*x^(2/3))^p*x*(d*x)^m*AppellF1[3*(1 + m), -p, -p, 4 + 3
*m, (-2*c*x^(1/3))/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^(1/3))/(b + Sqrt[b^2 -
 4*a*c])])/((1 + m)*(1 + (2*c*x^(1/3))/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*
c*x^(1/3))/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1715
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*
x^(k*n) + c*x^(2*k*n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, m, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && FractionQ[n]
```

rule 1716
```
Int[((d_)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Simp[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[n2, 2
*n] && NeQ[b^2 - 4*a*c, 0] && FractionQ[n]
```

Maple [F]

$$\int \left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{p} (dx)^{m}\,dx$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p*(d*x)^m,x)
```

output
```
int((a+b*x^(1/3)+c*x^(2/3))^p*(d*x)^m,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p (dx)^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p*(d*x)^m,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p (dx)^m\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**p*(d*x)**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p (dx)^m \, dx = \int (dx)^m \left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p*(d*x)^m,x, algorithm="maxima")
```

output
```
integrate((d*x)^m*(c*x^(2/3) + b*x^(1/3) + a)^p, x)
```

Giac [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p (dx)^m \, dx = \int (dx)^m \left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p*(d*x)^m,x, algorithm="giac")
```

output
```
integrate((d*x)^m*(c*x^(2/3) + b*x^(1/3) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p (dx)^m \, dx = \int (d\,x)^m \left(a+b\,x^{1/3}+c\,x^{2/3}\right)^p dx$$

input
```
int((d*x)^m*(a + b*x^(1/3) + c*x^(2/3))^p,x)
```

output
```
int((d*x)^m*(a + b*x^(1/3) + c*x^(2/3))^p, x)
```

Reduce [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p (dx)^m\,dx = \text{too large to display}$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p*(d*x)^m,x)
```

output
```
(d**m*(27*x**((3*m + 2)/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b**2*c*m**2*p
+ 27*x**((3*m + 2)/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b**2*c*m*p**2 + 9*x
**((3*m + 2)/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b**2*c*m*p + 6*x**((3*m +
 2)/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b**2*c*p**3 + 3*x**((3*m + 2)/3)*(
x**(2/3)*c + x**(1/3)*b + a)**p*b**2*c*p**2 + 54*x**((3*m + 1)/3)*(x**(2/3
)*c + x**(1/3)*b + a)**p*a*b*c*m**2*p + 54*x**((3*m + 1)/3)*(x**(2/3)*c +
x**(1/3)*b + a)**p*a*b*c*m*p**2 + 36*x**((3*m + 1)/3)*(x**(2/3)*c + x**(1/
3)*b + a)**p*a*b*c*m*p + 12*x**((3*m + 1)/3)*(x**(2/3)*c + x**(1/3)*b + a)
**p*a*b*c*p**3 + 12*x**((3*m + 1)/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a*b*
c*p**2 - 27*x**((3*m + 1)/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b**3*m**2*p
- 18*x**((3*m + 1)/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b**3*m*p**2 - 18*x*
*((3*m + 1)/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b**3*m*p - 3*x**((3*m + 1)
/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b**3*p**3 - 6*x**((3*m + 1)/3)*(x**(2
/3)*c + x**(1/3)*b + a)**p*b**3*p**2 - 54*x**m*(x**(2/3)*c + x**(1/3)*b +
a)**p*a**2*c*m**2*p - 36*x**m*(x**(2/3)*c + x**(1/3)*b + a)**p*a**2*c*m*p*
*2 - 54*x**m*(x**(2/3)*c + x**(1/3)*b + a)**p*a**2*c*m*p - 12*x**m*(x**(2/
3)*c + x**(1/3)*b + a)**p*a**2*c*p**2 - 12*x**m*(x**(2/3)*c + x**(1/3)*b +
 a)**p*a**2*c*p + 27*x**m*(x**(2/3)*c + x**(1/3)*b + a)**p*a*b**2*m**2*p +
 9*x**m*(x**(2/3)*c + x**(1/3)*b + a)**p*a*b**2*m*p**2 + 27*x**m*(x**(2/3)
*c + x**(1/3)*b + a)**p*a*b**2*m*p + 3*x**m*(x**(2/3)*c + x**(1/3)*b + ...
```

3.94 $\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 2032

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^3\,dx=\text{Too large to display}$$

output
```
-3/64*(3840*a^5*c^5*(32*p^5+560*p^4+3760*p^3+12040*p^2+18258*p+10395)-720*
a^4*b^2*c^4*(281*p^6+7181*p^5+73495*p^4+383815*p^3+1072824*p^2+1511604*p+8
31600)+840*a^3*b^4*c^3*(58*p^7+1983*p^6+28135*p^5+213945*p^4+937507*p^3+23
54472*p^2+3116700*p+1663200)-56*a^2*b^6*c^2*(67*p^8+2920*p^7+54178*p^6+557
500*p^5+3469063*p^4+13316140*p^3+30647892*p^2+38438640*p+19958400)+18*a*b^
8*c*(6*p^9+323*p^8+7544*p^7+100142*p^6+830774*p^5+4455227*p^4+15396956*p^3
+32941908*p^2+39406320*p+19958400)-b^10*(p^10+65*p^9+1860*p^8+30810*p^7+32
6613*p^6+2310945*p^5+11028590*p^4+34967140*p^3+70290936*p^2+80627040*p+399
16800)+2*b*c*(p+1)*(7+p)*(240*a^4*c^4*(437*p^4+6321*p^3+33088*p^2+74004*p+
59400)-720*a^3*b^2*c^3*(52*p^5+1243*p^4+11339*p^3+49232*p^2+101484*p+79200
)+420*a^2*b^4*c^2*(8*p^6+271*p^5+3694*p^4+25847*p^3+97548*p^2+187452*p+142
560)-8*a*b^6*c*(13*p^7+579*p^6+10759*p^5+107877*p^4+628768*p^3+2124444*p^2
+3839760*p+2851200)+b^8*(p^8+56*p^7+1342*p^6+17948*p^5+146293*p^4+743036*p
^3+2293164*p^2+3926160*p+2851200))*x^(1/3))*(a+b*x^(1/3)+c*x^(2/3))^(p+1)/
c^11/(p+1)/(2+p)/(3+p)/(4+p)/(5+p)/(6+p)/(3+2*p)/(5+2*p)/(7+2*p)/(9+2*p)/(
11+2*p)+3/32*(1920*a^4*c^4*(16*p^4+256*p^3+1496*p^2+3776*p+3465)-120*a^3*b
^2*c^3*(203*p^5+5340*p^4+54185*p^3+264300*p^2+618452*p+554400)+60*a^2*b^4*
c^2*(47*p^6+1737*p^5+26105*p^4+203595*p^3+865928*p^2+1896348*p+1663200)-14
*a*b^6*c*(7*p^7+337*p^6+6835*p^5+75595*p^4+491518*p^3+1874548*p^2+3871320*
p+3326400)+b^8*(p^8+60*p^7+1554*p^6+22680*p^5+203889*p^4+1155420*p^3+40...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 6.51 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.09

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^3\,dx=\frac{1}{4}\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^4\,\text{AppellF1}\left(12,-p,-p,13,-\frac{2c\sqrt[3]{x}}{b+\sqrt{b^2-4ac}},\frac{2c\sqrt[3]{x}}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^p*x^3,x]
```

output
```
((a + b*x^(1/3) + c*x^(2/3))^p*x^4*AppellF1[12, -p, -p, 13, (-2*c*x^(1/3))
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^(1/3))/(-b + Sqrt[b^2 - 4*a*c])])/(4*((b
- Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b
^2 - 4*a*c] + 2*c*x^(1/3))/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 14.73 (sec) , antiderivative size = 1866, normalized size of antiderivative = 0.92, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {1693, 1166, 25, 1236, 25, 1236, 1236, 25, 1236, 1236, 25, 1236, 1236, 25, 1236, 1225, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p dx$$

$$\Big\downarrow \text{1693}$$

$$3\int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x^{11/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1166}$$

$$3\left(\frac{\int -\left(\left(10a+b(p+11)\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x^3\right)d\sqrt[3]{x}}{2c(p+6)}+\frac{x^{10/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+6)}\right)$$

$$\Big\downarrow \text{25}$$

$$3\left(\frac{x^{10/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+6)}-\frac{\int \left(10a+b(p+11)\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x^3d\sqrt[3]{x}}{2c(p+6)}\right)$$

$$\Big\downarrow \text{1236}$$

$$3\left(\frac{x^{10/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+6)}-\frac{\frac{\int -\left(\left(9ab(p+11)-\left(10ac(2p+11)-b^2\left(p^2+21p+110\right)\right)\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x^{8/3}\right)d\sqrt[3]{x}}{c(2p+11)}+\frac{b(p+}{}}{2c(p+6)}\right.$$

$$\Big\downarrow \text{25}$$

$$3\left(\frac{x^{10/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+6)}-\frac{\frac{b(p+11)x^3\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+11)}-\frac{\int\left(9ab(p+11)-\left(10ac(2p+11)-b^2\left(p^2+21p+110\right)\right)\sqrt[3]{x}\right)\left(a+cx\right.}{c(2p+11)}}{2c(p+6)}\right.$$

$\downarrow$ 1236

$$3\left(\frac{x^{10/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+6)}-\frac{\frac{b(p+11)x^3\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+11)}-\frac{\int\left(b(p+10)\sqrt[3]{x}\left(2ac(19p+99)-b^2\left(p^2+20p+99\right)\right)+8a\left(10ac(2p+11)-\right.\right.}{2c(p+5)}}{2c}\right.$$

$\downarrow$ 1236

$$3\left(\frac{x^{10/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+6)}-\frac{\frac{b(p+11)x^3\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+11)}-\frac{\int-\left(\left(7ab(p+10)\left(2ac(19p+99)-b^2\left(p^2+20p+99\right)\right)-\left(\left(p^4+38p^3+539\right.\right.\right.\right.}{}}{}\right.$$

$\downarrow$ 25

$$3\left(\frac{x^{10/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+6)}-\frac{\frac{b(p+11)x^3\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+11)}-\frac{\frac{b(p+10)x^{7/3}\left(2ac(19p+99)-b^2\left(p^2+20p+99\right)\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+}}{c(2p+9)}}{}}{}\right.$$

$\downarrow$ 1236

$$3\left(\frac{x^{10/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+6)}-\frac{\frac{b(p+11)x^3\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+11)}-\frac{\frac{b(p+10)x^{7/3}\left(2ac(19p+99)-b^2\left(p^2+20p+99\right)\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+}}{c(2p+9)}}{}}{}\right.$$

$\downarrow$ 1236

$$3\left(\frac{x^{10/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+6)}-\frac{\frac{b(p+11)x^3\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+11)}-\frac{\frac{b(p+10)x^{7/3}\left(2ac(19p+99)-b^2\left(p^2+20p+99\right)\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+}}{c(2p+9)}}{}}{}\right.$$

$\downarrow$ 25

$$3\left(\frac{x^{10/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+6)}-\frac{\frac{b(p+11)x^3\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+11)}-\frac{\frac{b(p+10)x^{7/3}\left(2ac(19p+99)-b^2\left(p^2+20p+99\right)\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+}}{c(2p+9)}}{}}{}\right.$$

$\downarrow$ 1236

$$3\left(\frac{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^{10/3}}{2c(p+6)}-\frac{\frac{b(p+11)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^3}{c(2p+11)}-\frac{\frac{b(p+10)\left(2ac(19p+99)-b^2\left(p^2+20p+99\right)\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^7}{c(2p+9)}}{}}{}\right.$$

$\downarrow$ 1236

$$3\left(\frac{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^{10/3}}{2c(p+6)}-\frac{\frac{b(p+11)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^3}{c(2p+11)}-\frac{\frac{b(p+10)\left(2ac(19p+99)-b^2\left(p^2+20p+99\right)\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^7}{c(2p+9)}}{}}{}\right.$$

$\downarrow$ 25

$$3\left(\frac{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^{10/3}}{2c(p+6)}-\frac{\frac{b(p+11)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^3}{c(2p+11)}-\frac{\frac{b(p+10)\left(2ac(19p+99)-b^2\left(p^2+20p+99\right)\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^7}{c(2p+9)}}{}}{}\right.$$

$\downarrow$ 1236

$$3\left(\frac{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^{10/3}}{2c(p+6)}-\frac{\frac{b(p+11)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^3}{c(2p+11)}-\frac{\frac{b(p+10)\left(2ac(19p+99)-b^2\left(p^2+20p+99\right)\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^7}{c(2p+9)}}{}}{}\right.$$

$\downarrow$ 1225

$$3\left(\frac{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^{10/3}}{2c(p+6)}-\frac{\frac{b(p+11)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^3}{c(2p+11)}-\frac{\frac{b(p+10)\left(2ac(19p+99)-b^2\left(p^2+20p+99\right)\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^7}{c(2p+9)}}{}}{}\right.$$

$$\Big\downarrow \text{1096}$$

$$3\left(\frac{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^{10/3}}{2c(p+6)}-\frac{\frac{b(p+11)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^3}{c(2p+11)}-\frac{\frac{b(p+10)\left(2ac(19p+99)-b^2\left(p^2+20p+99\right)\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^7}{c(2p+9)}}{}}{}\right.$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^p*x^3,x]
```

output
```
3*(((a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x^(10/3))/(2*c*(6 + p)) - ((b*(11
+ p)*(a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x^3)/(c*(11 + 2*p)) - (-1/2*((10*
a*c*(11 + 2*p) - b^2*(110 + 21*p + p^2))*(a + b*x^(1/3) + c*x^(2/3))^(1 +
p)*x^(8/3))/(c*(5 + p)) + ((b*(10 + p)*(2*a*c*(99 + 19*p) - b^2*(99 + 20*p
 + p^2))*(a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x^(7/3))/(c*(9 + 2*p)) - (-1/
2*((80*a^2*c^2*(99 + 40*p + 4*p^2) - 6*a*b^2*c*(3960 + 1646*p + 215*p^2 +
9*p^3) + b^4*(7920 + 3382*p + 539*p^2 + 38*p^3 + p^4))*(a + b*x^(1/3) + c*
x^(2/3))^(1 + p)*x^2)/(c*(4 + p)) + ((b*(9 + p)*(12*a^2*c^2*(1540 + 666*p
+ 71*p^2) - 4*a*b^2*c*(6160 + 2746*p + 383*p^2 + 17*p^3) + b^4*(6160 + 282
6*p + 481*p^2 + 36*p^3 + p^4))*(a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x^(5/3)
)/(c*(7 + 2*p)) - (-1/2*((480*a^3*c^3*(693 + 478*p + 108*p^2 + 8*p^3) - 60
*a^2*b^2*c^2*(33264 + 23478*p + 5951*p^2 + 642*p^3 + 25*p^4) + 10*a*b^4*c*
(166320 + 119982*p + 33535*p^2 + 4570*p^3 + 305*p^4 + 8*p^5) - b^6*(332640
 + 245004*p + 74524*p^2 + 11985*p^3 + 1075*p^4 + 51*p^5 + p^6))*(a + b*x^(
1/3) + c*x^(2/3))^(1 + p)*x^(4/3))/(c*(3 + p)) + ((b*(8 + p)*(120*a^3*c^3*
(6930 + 5233*p + 1286*p^2 + 103*p^3) - 20*a^2*b^2*c^2*(103950 + 80517*p +
22261*p^2 + 2603*p^3 + 109*p^4) + 30*a*b^4*c*(41580 + 32988*p + 10073*p^2
+ 1489*p^3 + 107*p^4 + 3*p^5) - b^6*(207900 + 168720*p + 56113*p^2 + 9792*
p^3 + 946*p^4 + 48*p^5 + p^6))*(a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x)/(c*(
5 + 2*p)) - (-1/2*((1920*a^4*c^4*(3465 + 3776*p + 1496*p^2 + 256*p^3 + ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{p} x^{3}dx$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p*x^3,x)
```

output
```
int((a+b*x^(1/3)+c*x^(2/3))^p*x^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^3\,dx=\text{Timed out}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p*x^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^3\,dx=\text{Timed out}$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**p*x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^3\,dx=\int \left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p x^3\,dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p*x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^(2/3) + b*x^(1/3) + a)^p*x^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^3\,dx=\text{Timed out}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p*x^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^3\,dx=\int x^3\,\left(a+b\,x^{1/3}+c\,x^{2/3}\right)^p\,dx$$

input
```
int(x^3*(a + b*x^(1/3) + c*x^(2/3))^p,x)
```

output
```
int(x^3*(a + b*x^(1/3) + c*x^(2/3))^p, x)
```

Reduce [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^3\,dx=\int \left(x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a\right)^p x^3dx$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p*x^3,x)
```

output
```
int((a+b*x^(1/3)+c*x^(2/3))^p*x^3,x)
```

3.95 $\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1092

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^2\,dx=\text{Too large to display}$$

output
```
3/16*(b*(24*a^3*c^3*(93*p^4+1186*p^3+5367*p^2+10154*p+6720)-20*a^2*b^2*c^2
*(37*p^5+718*p^4+5351*p^3+19022*p^2+31992*p+20160)+6*a*b^4*c*(9*p^6+241*p^
5+2605*p^4+14495*p^3+43586*p^2+66744*p+40320)-b^6*(p^7+35*p^6+511*p^5+4025
*p^4+18424*p^3+48860*p^2+69264*p+40320))-2*c*(p+1)*(840*a^3*c^3*(p^3+9*p^2
+26*p+24)-12*a^2*b^2*c^2*(47*p^4+774*p^3+4573*p^2+11406*p+10080)+10*a*b^4*
c*(5*p^5+122*p^4+1159*p^3+5338*p^2+11856*p+10080)-b^6*(p^6+33*p^5+445*p^4+
3135*p^3+12154*p^2+24552*p+20160))*x^(1/3))*(a+b*x^(1/3)+c*x^(2/3))^(p+1)/
c^8/(p+1)/(2+p)/(3+p)/(4+p)/(3+2*p)/(5+2*p)/(7+2*p)/(9+2*p)-3/8*b*(6+p)*(1
2*a^2*c^2*(29*p^2+183*p+280)-4*a*b^2*c*(11*p^3+164*p^2+769*p+1120)+b^4*(p^
4+24*p^3+211*p^2+804*p+1120))*(a+b*x^(1/3)+c*x^(2/3))^(p+1)*x^(2/3)/c^6/(2
+p)/(3+p)/(4+p)/(5+2*p)/(7+2*p)/(9+2*p)+3/4*(140*a^2*c^2*(p^2+7*p+12)-12*a
*b^2*c*(3*p^3+49*p^2+256*p+420)+b^4*(p^4+26*p^3+251*p^2+1066*p+1680))*(a+b
*x^(1/3)+c*x^(2/3))^(p+1)*x/c^5/(3+p)/(4+p)/(5+2*p)/(7+2*p)/(9+2*p)+3/4*b*
(7+p)*(2*a*c*(48+13*p)-b^2*(p^2+14*p+48))*(a+b*x^(1/3)+c*x^(2/3))^(p+1)*x^
(4/3)/c^4/(3+p)/(4+p)/(7+2*p)/(9+2*p)-3/2*(14*a*c*(4+p)-b^2*(p^2+15*p+56))
*(a+b*x^(1/3)+c*x^(2/3))^(p+1)*x^(5/3)/c^3/(4+p)/(7+2*p)/(9+2*p)-3/2*b*(8+
p)*(a+b*x^(1/3)+c*x^(2/3))^(p+1)*x^2/c^2/(4+p)/(9+2*p)+3*(a+b*x^(1/3)+c*x^
(2/3))^(p+1)*x^(7/3)/c/(9+2*p)-3*2^(-3+p)*(1680*a^4*c^4-3360*a^3*b^2*c^3*(
5+p)+840*a^2*b^4*c^2*(p^2+11*p+30)-56*a*b^6*c*(p^3+18*p^2+107*p+210)+b^8*(
p^4+26*p^3+251*p^2+1066*p+1680))*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x^(1/3))/(...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 5.86 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.16

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^2\,dx=\frac{1}{3}\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^3\,\text{AppellF1}\left(9,-p,-p,10,-\frac{2c\sqrt[3]{x}}{b+\sqrt{b^2-4ac}},\frac{2c\sqrt[3]{x}}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^p*x^2,x]
```

output
```
((a + b*x^(1/3) + c*x^(2/3))^p*x^3*AppellF1[9, -p, -p, 10, (-2*c*x^(1/3))/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^(1/3))/(-b + Sqrt[b^2 - 4*a*c])])/(3*((b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^
2 - 4*a*c] + 2*c*x^(1/3))/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 1026, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {1693, 1166, 25, 1236, 25, 1236, 1236, 25, 1236, 1236, 25, 1225, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p \, dx$$

$$\Big\downarrow \text{1693}$$

$$3\int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x^{8/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1166}$$

$$3\left(\frac{\int -\left(\left(7a+b(p+8)\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x^2\right)d\sqrt[3]{x}}{c(2p+9)}+\frac{x^{7/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+9)}\right)$$

$$\Big\downarrow \text{25}$$

$$3\left(\frac{x^{7/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+9)}-\frac{\int \left(7a+b(p+8)\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x^2d\sqrt[3]{x}}{c(2p+9)}\right)$$

$$\Big\downarrow \text{1236}$$

$$3\left(\frac{x^{7/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+9)}-\frac{\frac{\int -\left(\left(6ab(p+8)-\left(14ac(p+4)-b^2\left(p^2+15p+56\right)\right)\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x^{5/3}\right)d\sqrt[3]{x}}{2c(p+4)}+\frac{b(p+8)x^2\left(\right.}{}}{c(2p+9)}\right.$$

$$\Big\downarrow \text{25}$$

$$3\left(\frac{x^{7/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+9)}-\frac{\frac{b(p+8)x^2\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+4)}-\frac{\int\left(6ab(p+8)-\left(14ac(p+4)-b^2\left(p^2+15p+56\right)\right)\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\right.}{2c(p+4)}}{c(2p+9)}\right.$$

$\downarrow$ 1236

$$3\left(\frac{x^{7/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+9)}-\frac{\frac{b(p+8)x^2\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+4)}-\frac{\int\left(b(p+7)\sqrt[3]{x}\left(2ac(13p+48)-b^2\left(p^2+14p+48\right)\right)+5a\left(14ac(p+4)-b^2\left(p^2\right.\right.\right.}{c(2p+7)}}{c(2p+}\right.$$

$\downarrow$ 1236

$$3\left(\frac{x^{7/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+9)}-\frac{\frac{b(p+8)x^2\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+4)}-\frac{\int-\left(\left(4ab(p+7)\left(2ac(13p+48)-b^2\left(p^2+14p+48\right)\right)-\left(\left(p^4+26p^3+251p^2+\right.\right.\right.\right.}{}}{}\right.$$

$\downarrow$ 25

$$3\left(\frac{x^{7/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+9)}-\frac{\frac{b(p+8)x^2\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+4)}-\frac{\frac{b(p+7)x^{4/3}\left(2ac(13p+48)-b^2\left(p^2+14p+48\right)\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+3)}-}{}}{}\right.$$

$\downarrow$ 1236

$$3\left(\frac{x^{7/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+9)}-\frac{\frac{b(p+8)x^2\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+4)}-\frac{\frac{b(p+7)x^{4/3}\left(2ac(13p+48)-b^2\left(p^2+14p+48\right)\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+3)}-}{}}{}\right.$$

$\downarrow$ 1236

$$3\left(\frac{x^{7/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+9)}-\frac{\frac{b(p+8)x^2\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+4)}-\frac{\frac{b(p+7)x^{4/3}\left(2ac(13p+48)-b^2\left(p^2+14p+48\right)\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+3)}-}{}}{}\right.$$

$\downarrow$ 25

$$3\left(\frac{x^{7/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+9)}-\frac{\frac{b(p+8)x^2\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+4)}-\frac{\frac{b(p+7)x^{4/3}\left(2ac(13p+48)-b^2\left(p^2+14p+48\right)\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+3)}-}{}}{}\right.$$

$\downarrow$ 1225

$$3\left(\frac{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^{7/3}}{c(2p+9)}-\frac{\frac{b(p+8)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^2}{2c(p+4)}-\frac{\frac{b(p+7)\left(2ac(13p+48)-b^2\left(p^2+14p+48\right)\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^{4/3}}{2c(p+3)}-}{}}{}\right.$$

$\downarrow$ 1096

$$3\left(\frac{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^{7/3}}{c(2p+9)}-\frac{\frac{b(p+8)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^2}{2c(p+4)}-\frac{\frac{b(p+7)\left(2ac(13p+48)-b^2\left(p^2+14p+48\right)\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{p+1}x^{4/3}}{2c(p+3)}-\cdots}{\cdots}}{\cdots}\right.$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^p*x^2,x]
```

output
```
3*(((a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x^(7/3))/(c*(9 + 2*p)) - ((b*(8 +
p)*(a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x^2)/(2*c*(4 + p)) - (-(((14*a*c*(4
 + p) - b^2*(56 + 15*p + p^2))*(a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x^(5/3)
)/(c*(7 + 2*p))) + ((b*(7 + p)*(2*a*c*(48 + 13*p) - b^2*(48 + 14*p + p^2))
*(a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x^(4/3))/(2*c*(3 + p)) - (-(((140*a^2
*c^2*(12 + 7*p + p^2) - 12*a*b^2*c*(420 + 256*p + 49*p^2 + 3*p^3) + b^4*(1
680 + 1066*p + 251*p^2 + 26*p^3 + p^4))*(a + b*x^(1/3) + c*x^(2/3))^(1 + p
)*x)/(c*(5 + 2*p))) + ((b*(6 + p)*(12*a^2*c^2*(280 + 183*p + 29*p^2) - 4*a
*b^2*c*(1120 + 769*p + 164*p^2 + 11*p^3) + b^4*(1120 + 804*p + 211*p^2 + 2
4*p^3 + p^4))*(a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x^(2/3))/(2*c*(2 + p)) -
 (((b*(24*a^3*c^3*(6720 + 10154*p + 5367*p^2 + 1186*p^3 + 93*p^4) - 20*a^2
*b^2*c^2*(20160 + 31992*p + 19022*p^2 + 5351*p^3 + 718*p^4 + 37*p^5) + 6*a
*b^4*c*(40320 + 66744*p + 43586*p^2 + 14495*p^3 + 2605*p^4 + 241*p^5 + 9*p
^6) - b^6*(40320 + 69264*p + 48860*p^2 + 18424*p^3 + 4025*p^4 + 511*p^5 +
35*p^6 + p^7)) - 2*c*(1 + p)*(840*a^3*c^3*(24 + 26*p + 9*p^2 + p^3) - 12*a
^2*b^2*c^2*(10080 + 11406*p + 4573*p^2 + 774*p^3 + 47*p^4) + 10*a*b^4*c*(1
0080 + 11856*p + 5338*p^2 + 1159*p^3 + 122*p^4 + 5*p^5) - b^6*(20160 + 245
52*p + 12154*p^2 + 3135*p^3 + 445*p^4 + 33*p^5 + p^6))*x^(1/3))*(a + b*x^(
1/3) + c*x^(2/3))^(1 + p))/(2*c^2*(1 + p)*(3 + 2*p)) - (2^p*(24 + 26*p + 9
*p^2 + p^3)*(1680*a^4*c^4 - 3360*a^3*b^2*c^3*(5 + p) + 840*a^2*b^4*c^2*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{p} x^{2}dx$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p*x^2,x)
```

output
```
int((a+b*x^(1/3)+c*x^(2/3))^p*x^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^2\,dx=\text{Timed out}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p*x^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^2\,dx=\text{Timed out}$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**p*x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^2\,dx=\int \left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p x^2\,dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p*x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^(2/3) + b*x^(1/3) + a)^p*x^2, x)
```

Giac [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^2\,dx=\int \left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p x^2\,dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p*x^2,x, algorithm="giac")
```

output
```
integrate((c*x^(2/3) + b*x^(1/3) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^2\,dx=\int x^2\,\left(a+b\,x^{1/3}+c\,x^{2/3}\right)^p\,dx$$

input
```
int(x^2*(a + b*x^(1/3) + c*x^(2/3))^p,x)
```

output
```
int(x^2*(a + b*x^(1/3) + c*x^(2/3))^p, x)
```

Reduce [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^2\,dx = \text{too large to display}$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p*x^2,x)
```

output
```
( - 8352*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**3*b**2*c**4*p**6 - 1
24560*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**3*b**2*c**4*p**5 - 6566
40*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**3*b**2*c**4*p**4 - 1508400
*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**3*b**2*c**4*p**3 - 1451808*x
**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**3*b**2*c**4*p**2 - 423360*x**(
2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**3*b**2*c**4*p + 1056*x**(2/3)*(x*
*(2/3)*c + x**(1/3)*b + a)**p*a**2*b**4*c**3*p**7 + 27576*x**(2/3)*(x**(2/
3)*c + x**(1/3)*b + a)**p*a**2*b**4*c**3*p**6 + 270660*x**(2/3)*(x**(2/3)*
c + x**(1/3)*b + a)**p*a**2*b**4*c**3*p**5 + 1260720*x**(2/3)*(x**(2/3)*c
+ x**(1/3)*b + a)**p*a**2*b**4*c**3*p**4 + 2858124*x**(2/3)*(x**(2/3)*c +
x**(1/3)*b + a)**p*a**2*b**4*c**3*p**3 + 2839464*x**(2/3)*(x**(2/3)*c + x*
*(1/3)*b + a)**p*a**2*b**4*c**3*p**2 + 846720*x**(2/3)*(x**(2/3)*c + x**(1
/3)*b + a)**p*a**2*b**4*c**3*p - 2688*x**(2/3)*(x**(2/3)*c + x**(1/3)*b +
a)**p*a**2*b*c**6*p**7*x - 32256*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p
*a**2*b*c**6*p**6*x - 149856*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**
2*b*c**6*p**5*x - 342720*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**2*b*
c**6*p**4*x - 403872*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**2*b*c**6
*p**3*x - 229824*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**2*b*c**6*p**
2*x - 48384*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**2*b*c**6*p*x - 24
*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a*b**6*c**2*p**8 - 1068*x**(...
```

3.96 $\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x\,dx$

Optimal result

Integrand size = 20, antiderivative size = 499

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x\,dx = -\frac{3\left(b^2(2+p)(4+p)\left(2ac(15+7p)-b^2(15+8p+p^2)\right)-2ac(3+2p)\left(4ac(5+2p)-b^2(20+9p+p^2)\right)-\right.}{8c^5(1+p)(2+p)(3+p)(3+2p)}$$

$$-\frac{3(4ac(5+2p)-b^2(20+9p+p^2))\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{1+p}x^{2/3}}{4c^3(2+p)(3+p)(5+2p)}$$

$$-\frac{3b(5+p)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{1+p}x}{2c^2(3+p)(5+2p)}+\frac{3\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{1+p}x^{4/3}}{2c(3+p)}$$

$$+\frac{3\,2^{-2+p}b(60a^2c^2-20ab^2c(4+p)+b^4(20+9p+p^2))\left(-\frac{b-\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{1+p}\text{Hy}}{c^5\sqrt{b^2-4ac}(1+p)(3+2p)(5+2p)}$$

output
```
-3/8*(b^2*(2+p)*(4+p)*(2*a*c*(15+7*p)-b^2*(p^2+8*p+15))-2*a*c*(3+2*p)*(4*a
*c*(5+2*p)-b^2*(p^2+9*p+20))-2*b*c*(p+1)*(4+p)*(2*a*c*(15+7*p)-b^2*(p^2+8*
p+15))*x^(1/3))*(a+b*x^(1/3)+c*x^(2/3))^(p+1)/c^5/(p+1)/(2+p)/(3+p)/(3+2*p
)/(5+2*p)-3/4*(4*a*c*(5+2*p)-b^2*(p^2+9*p+20))*(a+b*x^(1/3)+c*x^(2/3))^(p+
1)*x^(2/3)/c^3/(2+p)/(3+p)/(5+2*p)-3/2*b*(5+p)*(a+b*x^(1/3)+c*x^(2/3))^(p+
1)*x/c^2/(3+p)/(5+2*p)+3/2*(a+b*x^(1/3)+c*x^(2/3))^(p+1)*x^(4/3)/c/(3+p)+3
*2^(-2+p)*b*(60*c^2*a^2-20*a*b^2*c*(4+p)+b^4*(p^2+9*p+20))*(-(b-(-4*a*c+b^
2)^(1/2)+2*c*x^(1/3))/(-4*a*c+b^2)^(1/2))^(-1-p)*(a+b*x^(1/3)+c*x^(2/3))^(
p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x^(1/3))/(-4*
a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)/(p+1)/(3+2*p)/(5+2*p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.22 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.35

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x\,dx=\frac{1}{2}\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x^2\,\text{AppellF1}\left(6,-p,-p,7,-\frac{2c\sqrt[3]{x}}{b+\sqrt{b^2-4ac}},\frac{2c\sqrt[3]{x}}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^p*x,x]
```

output
```
((a + b*x^(1/3) + c*x^(2/3))^p*x^2*AppellF1[6, -p, -p, 7, (-2*c*x^(1/3))/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^(1/3))/(-b + Sqrt[b^2 - 4*a*c])])/(2*((b -
Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2
 - 4*a*c] + 2*c*x^(1/3))/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1693, 1166, 25, 1236, 25, 1236, 1225, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p dx$$

$$\Big\downarrow \text{ 1693}$$

$$3\int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x^{5/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 1166}$$

$$3\left(\frac{\int -\left(\left(4a+b(p+5)\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x\right)d\sqrt[3]{x}}{2c(p+3)}+\frac{x^{4/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+3)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$3\left(\frac{x^{4/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+3)}-\frac{\int \left(4a+b(p+5)\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p xd\sqrt[3]{x}}{2c(p+3)}\right)$$

$$\Big\downarrow \text{ 1236}$$

$$3\left(\frac{x^{4/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+3)}-\frac{\frac{\int -\left(\left(3ab(p+5)-\left(4ac(2p+5)-b^2\left(p^2+9p+20\right)\right)\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x^{2/3}\right)d\sqrt[3]{x}}{c(2p+5)}+\frac{b(p+5)x\left(a\right.}{}}{2c(p+3)}\right.$$

$$\Big\downarrow \text{ 25}$$

$$3\left(\frac{x^{4/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+3)}-\frac{\frac{b(p+5)x\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+5)}-\frac{\int \left(3ab(p+5)-\left(4ac(2p+5)-b^2\left(p^2+9p+20\right)\right)\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right.}{c(2p+5)}}{2c(p+3)}\right.$$

$$\Big\downarrow \text{ 1236}$$

$$3\left(\frac{x^{4/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+3)}-\frac{\frac{b(p+5)x\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+5)}-\frac{\frac{\int\left(b(p+4)\sqrt[3]{x}\left(2ac(7p+15)-b^2\left(p^2+8p+15\right)\right)+2a\left(4ac(2p+5)-b^2\left(p^2+9\right.\right.\right.}{2c(p+2)}}{}}{2c(p+3}\right.$$

$\downarrow$ 1225

$$3\left(\frac{x^{4/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+3)}-\frac{\frac{b(p+5)x\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+5)}-\frac{-\frac{b\left(p^2+5p+6\right)\left(60a^2c^2-20ab^2c(p+4)+b^4\left(p^2+9p+20\right)\right)\int\left(a+cx^{2/3}+b\sqrt[3]{}\right.}{2c^2(2p+3)}}{}}{}\right.$$

$\downarrow$ 1096

$$3\left(\frac{x^{4/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+3)}-\frac{\frac{b(p+5)x\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+5)}-\frac{\frac{b2^p\left(p^2+5p+6\right)\left(60a^2c^2-20ab^2c(p+4)+b^4\left(p^2+9p+20\right)\right)\left(-\frac{-\sqrt{b^2-4ac}-}{\sqrt{b^2-}}\right.}{}}{}}{}\right.$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^p*x,x]
```

output
```
3*(((a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x^(4/3))/(2*c*(3 + p)) - ((b*(5 +
p)*(a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x)/(c*(5 + 2*p)) - (-1/2*((4*a*c*(5
 + 2*p) - b^2*(20 + 9*p + p^2))*(a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x^(2/3
))/(c*(2 + p)) + (-1/2*((b^2*(2 + p)*(4 + p)*(2*a*c*(15 + 7*p) - b^2*(15 +
 8*p + p^2)) - 2*a*c*(3 + 2*p)*(4*a*c*(5 + 2*p) - b^2*(20 + 9*p + p^2)) -
2*b*c*(1 + p)*(4 + p)*(2*a*c*(15 + 7*p) - b^2*(15 + 8*p + p^2))*x^(1/3))*(
a + b*x^(1/3) + c*x^(2/3))^(1 + p))/(c^2*(1 + p)*(3 + 2*p)) + (2^p*b*(6 +
5*p + p^2)*(60*a^2*c^2 - 20*a*b^2*c*(4 + p) + b^4*(20 + 9*p + p^2))*(-((b
- Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^(
1/3) + c*x^(2/3))^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^
2 - 4*a*c] + 2*c*x^(1/3))/(2*Sqrt[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(
1 + p)*(3 + 2*p)))/(2*c*(2 + p)))/(c*(5 + 2*p)))/(2*c*(3 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{p} x dx$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p*x,x)
```

output
```
int((a+b*x^(1/3)+c*x^(2/3))^p*x,x)
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x\,dx = \text{Timed out}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p*x,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**p*x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x\,dx = \int \left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p x\,dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p*x,x, algorithm="maxima")
```

output
```
integrate((c*x^(2/3) + b*x^(1/3) + a)^p*x, x)
```

Giac [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x\,dx = \int \left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p x\,dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p*x,x, algorithm="giac")
```

output
```
integrate((c*x^(2/3) + b*x^(1/3) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x\,dx = \int x\left(a+b\,x^{1/3}+c\,x^{2/3}\right)^p\,dx$$

input
```
int(x*(a + b*x^(1/3) + c*x^(2/3))^p,x)
```

output
```
int(x*(a + b*x^(1/3) + c*x^(2/3))^p, x)
```

Reduce [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x\,dx = \text{too large to display}$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p*x,x)
```

output
```
( - 192*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**2*c**3*p**4 - 864*x**
(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**2*c**3*p**3 - 1104*x**(2/3)*(x**
(2/3)*c + x**(1/3)*b + a)**p*a**2*c**3*p**2 - 360*x**(2/3)*(x**(2/3)*c + x
**(1/3)*b + a)**p*a**2*c**3*p + 24*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)*
*p*a*b**2*c**2*p**5 + 348*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a*b**2
*c**2*p**4 + 1488*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a*b**2*c**2*p*
*3 + 2100*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a*b**2*c**2*p**2 + 720
*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a*b**2*c**2*p - 6*x**(2/3)*(x**
(2/3)*c + x**(1/3)*b + a)**p*b**4*c*p**5 - 75*x**(2/3)*(x**(2/3)*c + x**(1
/3)*b + a)**p*b**4*c*p**4 - 318*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*
b**4*c*p**3 - 501*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b**4*c*p**2 -
180*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b**4*c*p + 48*x**(2/3)*(x**(
2/3)*c + x**(1/3)*b + a)**p*b*c**4*p**5*x + 240*x**(2/3)*(x**(2/3)*c + x**
(1/3)*b + a)**p*b*c**4*p**4*x + 420*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)
**p*b*c**4*p**3*x + 300*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b*c**4*p
**2*x + 72*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b*c**4*p*x + 168*x**(
1/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a**2*b*c**2*p**4 + 1296*x**(1/3)*(x*
*(2/3)*c + x**(1/3)*b + a)**p*a**2*b*c**2*p**3 + 2856*x**(1/3)*(x**(2/3)*c
 + x**(1/3)*b + a)**p*a**2*b*c**2*p**2 + 1800*x**(1/3)*(x**(2/3)*c + x**(1
/3)*b + a)**p*a**2*b*c**2*p - 12*x**(1/3)*(x**(2/3)*c + x**(1/3)*b + a)...
```

3.97 $\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 238

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p \, dx = -\frac{3b(2+p)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{1+p}}{2c^2(1+p)(3+2p)}+\frac{3\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{1+p}\sqrt[3]{x}}{c(3+2p)} + \frac{3\, 2^p(2ac-b^2(2+p))\left(-\frac{b-\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2\right.}{c^2\sqrt{b^2-4ac}(1+p)(3+2p)}$$

output
```
-3/2*b*(2+p)*(a+b*x^(1/3)+c*x^(2/3))^(p+1)/c^2/(p+1)/(3+2*p)+3*(a+b*x^(1/3
)+c*x^(2/3))^(p+1)*x^(1/3)/c/(3+2*p)+3*2^p*(2*a*c-b^2*(2+p))*(-(b-(-4*a*c+
b^2)^(1/2)+2*c*x^(1/3))/(-4*a*c+b^2)^(1/2))^(-1-p)*(a+b*x^(1/3)+c*x^(2/3))
^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x^(1/3))/(-
4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)/(p+1)/(3+2*p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.34 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\,dx=\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p x\,\text{AppellF1}\left(3,-p,-p,4,-\frac{2c\sqrt[3]{x}}{b+\sqrt{b^2-4ac}},\frac{2c\sqrt[3]{x}}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^p,x]
```

output
```
((a + b*x^(1/3) + c*x^(2/3))^p*x*AppellF1[3, -p, -p, 4, (-2*c*x^(1/3))/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^(1/3))/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt
[b^2 - 4*a*c] + 2*c*x^(1/3))/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4
*a*c] + 2*c*x^(1/3))/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1680, 1166, 25, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\,dx$$

$$\Big\downarrow \text{1680}$$

$$3\int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1166}$$

$$3\left(\frac{\int -\left(\left(a+b(p+2)\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p\right)d\sqrt[3]{x}}{c(2p+3)}+\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+3)}\right)$$

$\Big\downarrow$ 25

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+3)}-\frac{\int \left(a+b(p+2)\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p d\sqrt[3]{x}}{c(2p+3)}\right)$$

$\Big\downarrow$ 1160

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+3)}-\frac{\frac{\left(2ac-b^2(p+2)\right)\int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p d\sqrt[3]{x}}{2c}+\frac{b(p+2)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+1)}}{c(2p+3)}\right)$$

$\Big\downarrow$ 1096

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{c(2p+3)}-\frac{\frac{b(p+2)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{p+1}}{2c(p+1)}-\frac{2^p\left(2ac-b^2(p+2)\right)\left(-\frac{-\sqrt{b^2-4ac}+b+2c\sqrt[3]{x}}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)}{c(p+1)\sqrt{b^2}}}{c(2p+3)}\right.$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^p,x]
```

output
```
3*(((a + b*x^(1/3) + c*x^(2/3))^(1 + p)*x^(1/3))/(c*(3 + 2*p)) - ((b*(2 +
p)*(a + b*x^(1/3) + c*x^(2/3))^(1 + p))/(2*c*(1 + p)) - (2^p*(2*a*c - b^2*
(2 + p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/Sqrt[b^2 - 4*a*c]))^(-1
- p)*(a + b*x^(1/3) + c*x^(2/3))^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 +
p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(2*Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b
^2 - 4*a*c]*(1 + p)))/(c*(3 + 2*p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [F]

$$\int \left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{p}dx$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p,x)
```

output
```
int((a+b*x^(1/3)+c*x^(2/3))^p,x)
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\,dx=\int \left(a+b\sqrt[3]{x}+cx^{\frac{2}{3}}\right)^p\,dx$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**p,x)
```

output
```
Integral((a + b*x**(1/3) + c*x**(2/3))**p, x)
```

Maxima [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\,dx=\int \left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p\,dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2/3) + b*x^(1/3) + a)^p, x)
```

Giac [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\,dx=\int \left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p\,dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2/3) + b*x^(1/3) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\,dx=\int \left(a+b\,x^{1/3}+c\,x^{2/3}\right)^p\,dx$$

input
```
int((a + b*x^(1/3) + c*x^(2/3))^p,x)
```

output
```
int((a + b*x^(1/3) + c*x^(2/3))^p, x)
```

Reduce [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\,dx=\text{too large to display}$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p,x)
```

output
```
(6*x**(2/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b**2*c*p**2 + 3*x**(2/3)*(x**
(2/3)*c + x**(1/3)*b + a)**p*b**2*c*p + 12*x**(1/3)*(x**(2/3)*c + x**(1/3)
*b + a)**p*a*b*c*p**2 + 12*x**(1/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a*b*c
*p - 3*x**(1/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*b**3*p**2 - 6*x**(1/3)*(x
**(2/3)*c + x**(1/3)*b + a)**p*b**3*p - 12*(x**(2/3)*c + x**(1/3)*b + a)**
p*a**2*c*p - 12*(x**(2/3)*c + x**(1/3)*b + a)**p*a**2*c + 3*(x**(2/3)*c +
x**(1/3)*b + a)**p*a*b**2*p + 6*(x**(2/3)*c + x**(1/3)*b + a)**p*a*b**2 +
12*(x**(2/3)*c + x**(1/3)*b + a)**p*b*c**2*p**2*x + 18*(x**(2/3)*c + x**(1
/3)*b + a)**p*b*c**2*p*x + 6*(x**(2/3)*c + x**(1/3)*b + a)**p*b*c**2*x + 3
2*int((x**(2/3)*c + x**(1/3)*b + a)**p/(4*x**(2/3)*b*p**2 + 8*x**(2/3)*b*p
 + 3*x**(2/3)*b + 4*x**(1/3)*a*p**2 + 8*x**(1/3)*a*p + 3*x**(1/3)*a + 4*c*
p**2*x + 8*c*p*x + 3*c*x),x)*a**2*c**2*p**4 + 96*int((x**(2/3)*c + x**(1/3
)*b + a)**p/(4*x**(2/3)*b*p**2 + 8*x**(2/3)*b*p + 3*x**(2/3)*b + 4*x**(1/3
)*a*p**2 + 8*x**(1/3)*a*p + 3*x**(1/3)*a + 4*c*p**2*x + 8*c*p*x + 3*c*x),x
)*a**2*c**2*p**3 + 88*int((x**(2/3)*c + x**(1/3)*b + a)**p/(4*x**(2/3)*b*p
**2 + 8*x**(2/3)*b*p + 3*x**(2/3)*b + 4*x**(1/3)*a*p**2 + 8*x**(1/3)*a*p +
 3*x**(1/3)*a + 4*c*p**2*x + 8*c*p*x + 3*c*x),x)*a**2*c**2*p**2 + 24*int((
x**(2/3)*c + x**(1/3)*b + a)**p/(4*x**(2/3)*b*p**2 + 8*x**(2/3)*b*p + 3*x*
*(2/3)*b + 4*x**(1/3)*a*p**2 + 8*x**(1/3)*a*p + 3*x**(1/3)*a + 4*c*p**2*x
+ 8*c*p*x + 3*c*x),x)*a**2*c**2*p - 16*int((x**(2/3)*c + x**(1/3)*b + a...
```

3.98 $\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 171

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x}\,dx=\frac{3\ 2^{-1+2p}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\text{App}}{p}$$

output
```
3*2^(-1+2*p)*(a+b*x^(1/3)+c*x^(2/3))^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(
-4*a*c+b^2)^(1/2))/c/x^(1/3),-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^(1/3))/p/(((b
-(-4*a*c+b^2)^(1/2)+2*c*x^(1/3))/c/x^(1/3))^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c
*x^(1/3))/c/x^(1/3))^p)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x}\,dx=\frac{3\ 2^{-1+2p}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\text{App}}{p}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^p/x,x]
```

output
```
(3*2^(-1 + 2*p)*(a + b*x^(1/3) + c*x^(2/3))^p*AppellF1[-2*p, -p, -p, 1 - 2
*p, (-b - Sqrt[b^2 - 4*a*c])/(2*c*x^(1/3)), (-b + Sqrt[b^2 - 4*a*c])/(2*c*
x^(1/3))])/(p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(c*x^(1/3)))^p*((b +
Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(c*x^(1/3)))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x}\,dx$$

$$\Big\downarrow \; 1693$$

$$3\int \frac{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \; 1178$$

$$-34^p x^{-2p/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\left(\frac{-\sqrt{b^2-4ac}+b+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\int \left(\frac{b-\sqrt{b^2-4a}}{2c\sqrt[3]{x}}\right.$$

$$\Big\downarrow \; 150$$

$$\frac{3\ 2^{2p-1}x^{-4p/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\left(\frac{-\sqrt{b^2-4ac}+b+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\operatorname{AppellF1}\left(-2p,-p,-p,1-2\right.}{p}$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^p/x,x]
```

output
```
(3*2^(-1 + 2*p)*(a + b*x^(1/3) + c*x^(2/3))^p*AppellF1[-2*p, -p, -p, 1 - 2
*p, -1/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^(1/3)), -1/2*(b + Sqrt[b^2 - 4*a*c])
/(c*x^(1/3))])/(p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(c*x^(1/3)))^p*((
b + Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(c*x^(1/3)))^p*x^((4*p)/3))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a + b\, x^{\frac{1}{3}} + c\, x^{\frac{2}{3}}\right)^{p}}{x}dx$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p/x,x)
```

output
```
int((a+b*x^(1/3)+c*x^(2/3))^p/x,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p/x,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**p/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x}\,dx = \int \frac{\left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p/x,x, algorithm="maxima")
```

output
```
integrate((c*x^(2/3) + b*x^(1/3) + a)^p/x, x)
```

Giac [F]

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x}\,dx=\int \frac{\left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p/x,x, algorithm="giac")
```

output
```
integrate((c*x^(2/3) + b*x^(1/3) + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x}\,dx=\int \frac{\left(a+b\,x^{1/3}+c\,x^{2/3}\right)^p}{x}\,dx$$

input
```
int((a + b*x^(1/3) + c*x^(2/3))^p/x,x)
```

output
```
int((a + b*x^(1/3) + c*x^(2/3))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x}\,dx=\frac{3\left(x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a\right)^p-\left(\int \frac{\left(x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a\right)^p}{x^{\frac{2}{3}}b+x^{\frac{1}{3}}a+cx}dx\right)cp+\left(\int \frac{\left(x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a\right)^p}{x^{\frac{5}{3}}c+x^{\frac{4}{3}}b+ax}dx\right)ap}{p}$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p/x,x)
```

output
```
(3*(x**(2/3)*c + x**(1/3)*b + a)**p - int((x**(2/3)*c + x**(1/3)*b + a)**p
/(x**(2/3)*b + x**(1/3)*a + c*x),x)*c*p + int((x**(2/3)*c + x**(1/3)*b + a
)**p/(x**(2/3)*c*x + x**(1/3)*b*x + a*x),x)*a*p)/p
```

3.99 $\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 178

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^2}\,dx =$$
$$-\frac{3\,4^p\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\text{AppellF1}\left(3-2p,-p,-p,2(2-p)\right.}{(3-2p)x}$$

output
```
-3*4^p*(a+b*x^(1/3)+c*x^(2/3))^p*AppellF1(3-2*p,-p,-p,4-2*p,-1/2*(b-(-4*a*
c+b^2)^(1/2))/c/x^(1/3),-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^(1/3))/(3-2*p)/(((
b-(-4*a*c+b^2)^(1/2)+2*c*x^(1/3))/c/x^(1/3))^p)/(((b+(-4*a*c+b^2)^(1/2)+2*
c*x^(1/3))/c/x^(1/3))^p)/x
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^2}\,dx = \frac{3\,4^p\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\text{AppellF1}}{(-3+2p)x}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^p/x^2,x]
```

output
```
(3*4^p*(a + b*x^(1/3) + c*x^(2/3))^p*AppellF1[3 - 2*p, -p, -p, 4 - 2*p, (-
b - Sqrt[b^2 - 4*a*c])/(2*c*x^(1/3)), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^(1/3
))])/((-3 + 2*p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(c*x^(1/3)))^p*((b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(c*x^(1/3)))^p*x)
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^2}\,dx$$

$$\Big\downarrow \text{ 1693}$$

$$3\int \frac{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 1178}$$

$$-34^px^{-2p/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\left(\frac{-\sqrt{b^2-4ac}+b+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\int \left(\frac{b-\sqrt{b^2-4a}}{2c\sqrt[3]{x}}\right.$$

$$\Big\downarrow \text{ 150}$$

$$-\frac{3\ 4^px^{\frac{1}{3}(3-2p)-\frac{2p}{3}}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\left(\frac{-\sqrt{b^2-4ac}+b+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\text{AppellF1}\left(3-2p,-p,-p\right.}{3-2p}$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^p/x^2,x]
```

output
```
(-3*4^p*(a + b*x^(1/3) + c*x^(2/3))^p*x^((3 - 2*p)/3 - (2*p)/3)*AppellF1[3
 - 2*p, -p, -p, 2*(2 - p), -1/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^(1/3)), -1/2*
(b + Sqrt[b^2 - 4*a*c])/(c*x^(1/3))])/((3 - 2*p)*((b - Sqrt[b^2 - 4*a*c] +
 2*c*x^(1/3))/(c*x^(1/3)))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(c*x^(
1/3)))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{p}}{x^{2}}dx$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p/x^2,x)
```

output
```
int((a+b*x^(1/3)+c*x^(2/3))^p/x^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p/x^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^2}\,dx = \int \frac{\left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^(2/3) + b*x^(1/3) + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^2}\,dx=\int \frac{\left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^(2/3) + b*x^(1/3) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,x^{1/3}+c\,x^{2/3}\right)^p}{x^2}\,dx$$

input
```
int((a + b*x^(1/3) + c*x^(2/3))^p/x^2,x)
```

output
```
int((a + b*x^(1/3) + c*x^(2/3))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^2}\,dx=\text{Too large to display}$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p/x^2,x)
```

output

```
( - 3*x**(1/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a*p**2 + 15*x**(1/3)*(x**(
2/3)*c + x**(1/3)*b + a)**p*a*p - 18*x**(1/3)*(x**(2/3)*c + x**(1/3)*b + a
)**p*a + 9*(x**(2/3)*c + x**(1/3)*b + a)**p*c*p**2*x - 18*(x**(2/3)*c + x*
*(1/3)*b + a)**p*c*p*x + x**(1/3)*int((x**(2/3)*c + x**(1/3)*b + a)**p/(x*
*(2/3)*a*p*x - 3*x**(2/3)*a*x + x**(1/3)*c*p*x**2 - 3*x**(1/3)*c*x**2 + b*
p*x**2 - 3*b*x**2),x)*a*b*p**4*x - 8*x**(1/3)*int((x**(2/3)*c + x**(1/3)*b
 + a)**p/(x**(2/3)*a*p*x - 3*x**(2/3)*a*x + x**(1/3)*c*p*x**2 - 3*x**(1/3)
*c*x**2 + b*p*x**2 - 3*b*x**2),x)*a*b*p**3*x + 21*x**(1/3)*int((x**(2/3)*c
 + x**(1/3)*b + a)**p/(x**(2/3)*a*p*x - 3*x**(2/3)*a*x + x**(1/3)*c*p*x**2
 - 3*x**(1/3)*c*x**2 + b*p*x**2 - 3*b*x**2),x)*a*b*p**2*x - 18*x**(1/3)*in
t((x**(2/3)*c + x**(1/3)*b + a)**p/(x**(2/3)*a*p*x - 3*x**(2/3)*a*x + x**(
1/3)*c*p*x**2 - 3*x**(1/3)*c*x**2 + b*p*x**2 - 3*b*x**2),x)*a*b*p*x - 3*x*
*(1/3)*int((x**(2/3)*c + x**(1/3)*b + a)**p/(x**(2/3)*c*p*x - 3*x**(2/3)*c
*x + x**(1/3)*b*p*x - 3*x**(1/3)*b*x + a*p*x - 3*a*x),x)*b*c*p**4*x + 18*x
**(1/3)*int((x**(2/3)*c + x**(1/3)*b + a)**p/(x**(2/3)*c*p*x - 3*x**(2/3)*
c*x + x**(1/3)*b*p*x - 3*x**(1/3)*b*x + a*p*x - 3*a*x),x)*b*c*p**3*x - 33*
x**(1/3)*int((x**(2/3)*c + x**(1/3)*b + a)**p/(x**(2/3)*c*p*x - 3*x**(2/3)
*c*x + x**(1/3)*b*p*x - 3*x**(1/3)*b*x + a*p*x - 3*a*x),x)*b*c*p**2*x + 18
*x**(1/3)*int((x**(2/3)*c + x**(1/3)*b + a)**p/(x**(2/3)*c*p*x - 3*x**(2/3
)*c*x + x**(1/3)*b*p*x - 3*x**(1/3)*b*x + a*p*x - 3*a*x),x)*b*c*p*x + 2...
```

3.100 $\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 182

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^3}\,dx = -\frac{3\,2^{-1+2p}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\operatorname{AppellF1}\left(2(3-p),-p,-p,7\right.}{(3-p)x^2}$$

output

```
-3*2^(-1+2*p)*(a+b*x^(1/3)+c*x^(2/3))^p*AppellF1(6-2*p,-p,-p,7-2*p,-1/2*(b
-(-4*a*c+b^2)^(1/2))/c/x^(1/3),-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^(1/3))/(3-p
)/(((b-(-4*a*c+b^2)^(1/2)+2*c*x^(1/3))/c/x^(1/3))^p)/(((b+(-4*a*c+b^2)^(1/
2)+2*c*x^(1/3))/c/x^(1/3))^p)/x^2
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^3}\,dx = \frac{3\,2^{-1+2p}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\left(\frac{b-\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\text{App}}{(-3+p)x^2}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^p/x^3,x]
```

output
```
(3*2^(-1 + 2*p)*(a + b*x^(1/3) + c*x^(2/3))^p*AppellF1[6 - 2*p, -p, -p, 7
- 2*p, (-b - Sqrt[b^2 - 4*a*c])/(2*c*x^(1/3)), (-b + Sqrt[b^2 - 4*a*c])/(2
*c*x^(1/3))])/((-3 + p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(c*x^(1/3))
)^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(c*x^(1/3)))^p*x^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^3}dx$$

$$\Big\downarrow \ 1693$$

$$3\int \frac{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^p}{x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \ 1178$$

$$-34^px^{-2p/3}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\left(\frac{-\sqrt{b^2-4ac}+b+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\int \left(\frac{b-\sqrt{b^2-4a}}{2c\sqrt[3]{x}}\right.$$

$$\Big\downarrow \ 150$$

$$-\frac{3\ 2^{2p-1}x^{\frac{1}{3}(6-2p)-\frac{2p}{3}}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p\left(\frac{-\sqrt{b^2-4ac}+b+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2c\sqrt[3]{x}}{c\sqrt[3]{x}}\right)^{-p}\text{AppellF1}\left(6-2p,-p,\right.}{3-p}$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^p/x^3,x]
```

output
```
(-3*2^(-1 + 2*p)*(a + b*x^(1/3) + c*x^(2/3))^p*x^((6 - 2*p)/3 - (2*p)/3)*A
ppellF1[6 - 2*p, -p, -p, 7 - 2*p, -1/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^(1/3))
, -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^(1/3))])/((3 - p)*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^(1/3))/(c*x^(1/3)))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^(1/3))/(
c*x^(1/3)))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{p}}{x^{3}}dx$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p/x^3,x)
```

output
```
int((a+b*x^(1/3)+c*x^(2/3))^p/x^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p/x^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^3}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**p/x**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^3}\,dx=\int \frac{\left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^(2/3) + b*x^(1/3) + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^3}\,dx=\int \frac{\left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^p/x^3,x, algorithm="giac")
```

output
```
integrate((c*x^(2/3) + b*x^(1/3) + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^3}\,dx=\int \frac{\left(a+b\,x^{1/3}+c\,x^{2/3}\right)^p}{x^3}\,dx$$

input
```
int((a + b*x^(1/3) + c*x^(2/3))^p/x^3,x)
```

output
```
int((a + b*x^(1/3) + c*x^(2/3))^p/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^p}{x^3}\,dx=\text{too large to display}$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^p/x^3,x)
```

output
```
( - 6*x**(1/3)*(x**(2/3)*c + x**(1/3)*b + a)**p*a*p**2 + 66*x**(1/3)*(x**(
2/3)*c + x**(1/3)*b + a)**p*a*p - 180*x**(1/3)*(x**(2/3)*c + x**(1/3)*b +
a)**p*a + 9*(x**(2/3)*c + x**(1/3)*b + a)**p*c*p**2*x - 45*(x**(2/3)*c + x
**(1/3)*b + a)**p*c*p*x + 2*x**(1/3)*int((x**(2/3)*c + x**(1/3)*b + a)**p/
(x**(2/3)*a*p*x**2 - 6*x**(2/3)*a*x**2 + x**(1/3)*c*p*x**3 - 6*x**(1/3)*c*
x**3 + b*p*x**3 - 6*b*x**3),x)*a*b*p**4*x**2 - 34*x**(1/3)*int((x**(2/3)*c
 + x**(1/3)*b + a)**p/(x**(2/3)*a*p*x**2 - 6*x**(2/3)*a*x**2 + x**(1/3)*c*
p*x**3 - 6*x**(1/3)*c*x**3 + b*p*x**3 - 6*b*x**3),x)*a*b*p**3*x**2 + 192*x
**(1/3)*int((x**(2/3)*c + x**(1/3)*b + a)**p/(x**(2/3)*a*p*x**2 - 6*x**(2/
3)*a*x**2 + x**(1/3)*c*p*x**3 - 6*x**(1/3)*c*x**3 + b*p*x**3 - 6*b*x**3),x
)*a*b*p**2*x**2 - 360*x**(1/3)*int((x**(2/3)*c + x**(1/3)*b + a)**p/(x**(2
/3)*a*p*x**2 - 6*x**(2/3)*a*x**2 + x**(1/3)*c*p*x**3 - 6*x**(1/3)*c*x**3 +
 b*p*x**3 - 6*b*x**3),x)*a*b*p*x**2 - 3*x**(1/3)*int((x**(2/3)*c + x**(1/3
)*b + a)**p/(x**(2/3)*c*p*x**2 - 6*x**(2/3)*c*x**2 + x**(1/3)*b*p*x**2 - 6
*x**(1/3)*b*x**2 + a*p*x**2 - 6*a*x**2),x)*b*c*p**4*x**2 + 45*x**(1/3)*int
((x**(2/3)*c + x**(1/3)*b + a)**p/(x**(2/3)*c*p*x**2 - 6*x**(2/3)*c*x**2 +
 x**(1/3)*b*p*x**2 - 6*x**(1/3)*b*x**2 + a*p*x**2 - 6*a*x**2),x)*b*c*p**3*
x**2 - 222*x**(1/3)*int((x**(2/3)*c + x**(1/3)*b + a)**p/(x**(2/3)*c*p*x**
2 - 6*x**(2/3)*c*x**2 + x**(1/3)*b*p*x**2 - 6*x**(1/3)*b*x**2 + a*p*x**2 -
 6*a*x**2),x)*b*c*p**2*x**2 + 360*x**(1/3)*int((x**(2/3)*c + x**(1/3)*b...
```

3.101 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^{1+m}\,\text{Hypergeometric2F1}\left(3(1+m),-2\right.}{d(1+m)}$$

output
```
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^(1+m)*hypergeom([-2*p, 3+3*m],[4+3
*m],-b*x^(1/3)/a)/d/(1+m)/((1+b*x^(1/3)/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.84

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p} x(dx)^m\,\text{Hypergeometric2F1}\left(3(1+m),-2p,1+3(\right.}{1+m}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
```

output
```
(((a + b*x^(1/3))^2)^p*x*(d*x)^m*Hypergeometric2F1[3*(1 + m), -2*p, 1 + 3*
(1 + m), -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1385, 866, 864, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}(dx)^mdx$$

$$\Big\downarrow \text{866}$$

$$x^{-m}(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}x^mdx$$

$$\Big\downarrow \text{864}$$

$$3x^{-m}(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}x^{\frac{1}{3}(3m+2)}d\sqrt[3]{x}$$

$$\Big\downarrow \text{74}$$

$$\frac{x(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\text{Hypergeometric2F1}\left(3(m+1),-2p,3m+4,-\frac{b\sqrt[3]{x}}{a}\right)}{m+1}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
```

output
```
((a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x*(d*x)^m*Hypergeometric2F1[3*(1 +
m), -2*p, 4 + 3*m, -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)^(2*p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \left(a^2 + 2ab\, x^{\frac{1}{3}} + b^2 x^{\frac{2}{3}}\right)^p (dx)^m \, dx$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx = \int (dx)^m \left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\,dx$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*(d*x)**m,x)
```

output
```
Integral((d*x)**m*((a + b*x**(1/3))**2)**p, x)
```

Maxima [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx = \int \left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p (dx)^m\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="maxima")
```

output
```
integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)
```

Giac [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\int \left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p (dx)^m\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\int (d\,x)^m\,\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p, x)
```

Reduce [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx=\text{too large to display}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x)
```

output
```
(3*d**m*(18*x**((3*m + 2)/3)*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*a*
b**2*m**2*p + 24*x**((3*m + 2)/3)*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)*
*p*a*b**2*m*p**2 + 6*x**((3*m + 2)/3)*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a*
*2)**p*a*b**2*m*p + 8*x**((3*m + 2)/3)*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a
**2)**p*a*b**2*p**3 + 4*x**((3*m + 2)/3)*(x**(2/3)*b**2 + 2*x**(1/3)*a*b +
 a**2)**p*a*b**2*p**2 - 18*x**((3*m + 1)/3)*(x**(2/3)*b**2 + 2*x**(1/3)*a*
b + a**2)**p*a**2*b*m**2*p - 12*x**((3*m + 1)/3)*(x**(2/3)*b**2 + 2*x**(1/
3)*a*b + a**2)**p*a**2*b*m*p**2 - 12*x**((3*m + 1)/3)*(x**(2/3)*b**2 + 2*x
**(1/3)*a*b + a**2)**p*a**2*b*m*p - 8*x**((3*m + 1)/3)*(x**(2/3)*b**2 + 2*
x**(1/3)*a*b + a**2)**p*a**2*b*p**2 + 18*x**m*(x**(2/3)*b**2 + 2*x**(1/3)*
a*b + a**2)**p*a**3*m**2*p + 18*x**m*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**
2)**p*a**3*m*p + 4*x**m*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*a**3*p
+ 27*x**m*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*b**3*m**3*x + 54*x**m
*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*b**3*m**2*p*x + 27*x**m*(x**(2
/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*b**3*m**2*x + 36*x**m*(x**(2/3)*b**2
+ 2*x**(1/3)*a*b + a**2)**p*b**3*m*p**2*x + 36*x**m*(x**(2/3)*b**2 + 2*x**
(1/3)*a*b + a**2)**p*b**3*m*p*x + 6*x**m*(x**(2/3)*b**2 + 2*x**(1/3)*a*b +
 a**2)**p*b**3*m*x + 8*x**m*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*b**
3*p**3*x + 12*x**m*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*b**3*p**2*x
+ 4*x**m*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*b**3*p*x - 1458*int...
```

3.102 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 436

$$\begin{aligned}\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx &= \frac{3a^8\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+2p)} \\ &- \frac{12a^7\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+p)} \\ &+ \frac{84a^6\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(3+2p)} \\ &- \frac{84a^5\left(a+b\sqrt[3]{x}\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2+p)} \\ &+ \frac{210a^4\left(a+b\sqrt[3]{x}\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(5+2p)} \\ &- \frac{84a^3\left(a+b\sqrt[3]{x}\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(3+p)} \\ &+ \frac{84a^2\left(a+b\sqrt[3]{x}\right)^7\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(7+2p)} \\ &- \frac{12a\left(a+b\sqrt[3]{x}\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(4+p)} \\ &+ \frac{3\left(a+b\sqrt[3]{x}\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(9+2p)}\end{aligned}$$

output
```
3*a^8*(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(1+2*p)-12*a^7*(
a+b*x^(1/3))^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(p+1)+84*a^6*(a+b*x^(
1/3))^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(3+2*p)-84*a^5*(a+b*x^(1/3))
^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(2+p)+210*a^4*(a+b*x^(1/3))^5*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(5+2*p)-84*a^3*(a+b*x^(1/3))^6*(a^2+2*a
*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(3+p)+84*a^2*(a+b*x^(1/3))^7*(a^2+2*a*b*x^(1
/3)+b^2*x^(2/3))^p/b^9/(7+2*p)-12*a*(a+b*x^(1/3))^8*(a^2+2*a*b*x^(1/3)+b^2
*x^(2/3))^p/b^9/(4+p)+3*(a+b*x^(1/3))^9*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/
b^9/(9+2*p)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.47

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx = \frac{3\left(\frac{a^8}{1+2p}-\frac{4a^7\left(a+b\sqrt[3]{x}\right)}{1+p}+\frac{28a^6\left(a+b\sqrt[3]{x}\right)^2}{3+2p}-\frac{28a^5\left(a+b\sqrt[3]{x}\right)^3}{2+p}+\frac{70a^4\left(a+b\sqrt[3]{x}\right)^4}{5+2p}-\frac{28a^3\left(a+b\sqrt[3]{x}\right.}{3+p}\right.}{b^9}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

output
```
(3*(a^8/(1 + 2*p) - (4*a^7*(a + b*x^(1/3)))/(1 + p) + (28*a^6*(a + b*x^(1/
3))^2)/(3 + 2*p) - (28*a^5*(a + b*x^(1/3))^3)/(2 + p) + (70*a^4*(a + b*x^(
1/3))^4)/(5 + 2*p) - (28*a^3*(a + b*x^(1/3))^5)/(3 + p) + (28*a^2*(a + b*x
^(1/3))^6)/(7 + 2*p) - (4*a*(a + b*x^(1/3))^7)/(4 + p) + (a + b*x^(1/3))^8
/(9 + 2*p))*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p)/b^9
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1385, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}x^2dx$$

$$\Big\downarrow \text{798}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}x^{8/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(\frac{a^8\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{b^8}-\frac{8a^8\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p+1}}{b^8}+\frac{28a^8\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p+2}}{b^8}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{4a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+1)}}{b^9(p+1)}-\frac{28a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+2)}}{b^9(p+2)}-\frac{28a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+3)}}{b^9(p+3)}-\frac{4a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+4)}}{b^9(p+4)}+\frac{a^9\left(\right.}{}\right.$$

input

```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

output
```
(3*((-4*a^9*(1 + (b*x^(1/3))/a)^(2*(1 + p)))/(b^9*(1 + p)) - (28*a^9*(1 +
(b*x^(1/3))/a)^(2*(2 + p)))/(b^9*(2 + p)) - (28*a^9*(1 + (b*x^(1/3))/a)^(2
*(3 + p)))/(b^9*(3 + p)) - (4*a^9*(1 + (b*x^(1/3))/a)^(2*(4 + p)))/(b^9*(4
 + p)) + (a^9*(1 + (b*x^(1/3))/a)^(1 + 2*p))/(b^9*(1 + 2*p)) + (28*a^9*(1
+ (b*x^(1/3))/a)^(3 + 2*p))/(b^9*(3 + 2*p)) + (70*a^9*(1 + (b*x^(1/3))/a)^
(5 + 2*p))/(b^9*(5 + 2*p)) + (28*a^9*(1 + (b*x^(1/3))/a)^(7 + 2*p))/(b^9*(
7 + 2*p)) + (a^9*(1 + (b*x^(1/3))/a)^(9 + 2*p))/(b^9*(9 + 2*p)))*(a^2 + 2*
a*b*x^(1/3) + b^2*x^(2/3))^p)/(1 + (b*x^(1/3))/a)^(2*p)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(a^2 + 2ab\,x^{\frac{1}{3}} + b^2 x^{\frac{2}{3}}\right)^p x^2 dx$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.33

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2 x^{2/3}\right)^p x^2\,dx = \frac{3\left(2520\,a^9 + (16\,b^9p^8 + 288\,b^9p^7 + 2184\,b^9p^6 + 9072\,b^9p^5 + 22449\,b^9p^4 + 33642\,b^9p^3 + 29\right.}{}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="fricas")
```

output
```
3*(2520*a^9 + (16*b^9*p^8 + 288*b^9*p^7 + 2184*b^9*p^6 + 9072*b^9*p^5 + 22
449*b^9*p^4 + 33642*b^9*p^3 + 29531*b^9*p^2 + 13698*b^9*p + 2520*b^9)*x^3
+ 28*(8*a^3*b^6*p^6 + 60*a^3*b^6*p^5 + 170*a^3*b^6*p^4 + 225*a^3*b^6*p^3 +
 137*a^3*b^6*p^2 + 30*a^3*b^6*p)*x^2 - 1680*(2*a^6*b^3*p^3 + 3*a^6*b^3*p^2
 + a^6*b^3*p)*x + (5040*a^7*b^2*p^2 + 2520*a^7*b^2*p + (16*a*b^8*p^8 + 224
*a*b^8*p^7 + 1288*a*b^8*p^6 + 3920*a*b^8*p^5 + 6769*a*b^8*p^4 + 6566*a*b^8
*p^3 + 3267*a*b^8*p^2 + 630*a*b^8*p)*x^2 - 168*(4*a^4*b^5*p^5 + 20*a^4*b^5
*p^4 + 35*a^4*b^5*p^3 + 25*a^4*b^5*p^2 + 6*a^4*b^5*p)*x)*x^(2/3) - 4*(1260
*a^8*b*p + 2*(8*a^2*b^7*p^7 + 84*a^2*b^7*p^6 + 350*a^2*b^7*p^5 + 735*a^2*b
^7*p^4 + 812*a^2*b^7*p^3 + 441*a^2*b^7*p^2 + 90*a^2*b^7*p)*x^2 - 105*(4*a^
5*b^4*p^4 + 12*a^5*b^4*p^3 + 11*a^5*b^4*p^2 + 3*a^5*b^4*p)*x)*x^(1/3))*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p^
7 + 37800*b^9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293
175*b^9*p^2 + 128322*b^9*p + 22680*b^9)
```

Sympy [F]

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p x^2\,dx = \int x^2\left(\left(a + b\sqrt[3]{x}\right)^2\right)^p\,dx$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x**2,x)
```

output
```
Integral(x**2*((a + b*x**(1/3))**2)**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.83

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p x^2\,dx = \frac{3\left((16\,p^8 + 288\,p^7 + 2184\,p^6 + 9072\,p^5 + 22449\,p^4 + 33642\,p^3 + 29531\,p^2 + 13698\,p + 25\right.}{}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="maxima")
```

output
```
3*((16*p^8 + 288*p^7 + 2184*p^6 + 9072*p^5 + 22449*p^4 + 33642*p^3 + 29531
*p^2 + 13698*p + 2520)*b^9*x^3 + (16*p^8 + 224*p^7 + 1288*p^6 + 3920*p^5 +
 6769*p^4 + 6566*p^3 + 3267*p^2 + 630*p)*a*b^8*x^(8/3) - 8*(8*p^7 + 84*p^6
 + 350*p^5 + 735*p^4 + 812*p^3 + 441*p^2 + 90*p)*a^2*b^7*x^(7/3) + 28*(8*p
^6 + 60*p^5 + 170*p^4 + 225*p^3 + 137*p^2 + 30*p)*a^3*b^6*x^2 - 168*(4*p^5
 + 20*p^4 + 35*p^3 + 25*p^2 + 6*p)*a^4*b^5*x^(5/3) + 420*(4*p^4 + 12*p^3 +
 11*p^2 + 3*p)*a^5*b^4*x^(4/3) - 1680*(2*p^3 + 3*p^2 + p)*a^6*b^3*x + 2520
*(2*p^2 + p)*a^7*b^2*x^(2/3) - 5040*a^8*b*p*x^(1/3) + 2520*a^9)*(b*x^(1/3)
 + a)^(2*p)/((32*p^9 + 720*p^8 + 6960*p^7 + 37800*p^6 + 126546*p^5 + 26932
5*p^4 + 361840*p^3 + 293175*p^2 + 128322*p + 22680)*b^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. $2(382) = 764$.

Time = 0.14 (sec) , antiderivative size = 1564, normalized size of antiderivative = 3.59

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p x^2\,dx = \text{Too large to display}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="giac")
```

output
```
3*(16*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^8*x^3 + 16*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^8*x^(8/3) + 288*(b^2*x^(2/3) + 2*a*b*x^(1
/3) + a^2)^p*b^9*p^7*x^3 + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8
*p^7*x^(8/3) - 64*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^7*x^(7/3
) + 2184*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^6*x^3 + 1288*(b^2*x^(
2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^6*x^(8/3) - 672*(b^2*x^(2/3) + 2*a*b
*x^(1/3) + a^2)^p*a^2*b^7*p^6*x^(7/3) + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
 a^2)^p*a^3*b^6*p^6*x^2 + 9072*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p
^5*x^3 + 3920*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^5*x^(8/3) - 28
00*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^5*x^(7/3) + 1680*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^5*x^2 + 22449*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*b^9*p^4*x^3 - 672*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^
p*a^4*b^5*p^5*x^(5/3) + 6769*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p
^4*x^(8/3) - 5880*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^4*x^(7/3
) + 4760*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^4*x^2 + 33642*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^3*x^3 - 3360*(b^2*x^(2/3) + 2*a*b
*x^(1/3) + a^2)^p*a^4*b^5*p^4*x^(5/3) + 6566*(b^2*x^(2/3) + 2*a*b*x^(1/3)
+ a^2)^p*a*b^8*p^3*x^(8/3) + 1680*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^
5*b^4*p^4*x^(4/3) - 6496*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^3
*x^(7/3) + 6300*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^3*x^2 +...
```

Mupad [B] (verification not implemented)

Time = 12.19 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.78

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx = \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,\left(\frac{3\,x^3\,\left(16\,p^8+288\,p^7+2184\,p^6+9072\,p^5+22449\,p^4+\right.}{32\,p^9+720\,p^8+6960\,p^7+37800\,p^6+126546\,p^5+269325\,p}\right.$$

input
```
int(x^2*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

output
```
(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^3*(13698*p + 29531*p^2 + 33642
*p^3 + 22449*p^4 + 9072*p^5 + 2184*p^6 + 288*p^7 + 16*p^8 + 2520))/(128322
*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*
p^7 + 720*p^8 + 32*p^9 + 22680) + (7560*a^9)/(b^9*(128322*p + 293175*p^2 +
 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 3
2*p^9 + 22680)) - (15120*a^8*p*x^(1/3))/(b^8*(128322*p + 293175*p^2 + 3618
40*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9
 + 22680)) + (3*a*p*x^(8/3)*(3267*p + 6566*p^2 + 6769*p^3 + 3920*p^4 + 128
8*p^5 + 224*p^6 + 16*p^7 + 630))/(b*(128322*p + 293175*p^2 + 361840*p^3 +
269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)
) + (84*a^3*p*x^2*(137*p + 225*p^2 + 170*p^3 + 60*p^4 + 8*p^5 + 30))/(b^3*
(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6
+ 6960*p^7 + 720*p^8 + 32*p^9 + 22680)) - (5040*a^6*p*x*(3*p + 2*p^2 + 1))
/(b^6*(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 3780
0*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)) - (24*a^2*p*x^(7/3)*(441*p +
 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90))/(b^2*(128322*p + 2931
75*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720
*p^8 + 32*p^9 + 22680)) + (7560*a^7*p*x^(2/3)*(2*p + 1))/(b^7*(128322*p +
293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 +
 720*p^8 + 32*p^9 + 22680)) + (1260*a^5*p*x^(4/3)*(11*p + 12*p^2 + 4*p^...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.46

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx = \frac{3\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)^p \left(16b^9p^8x^3+288b^9p^7x^3+2184b^9p^6x^3+9072b^9p^5x^3+22449b^9p^4\right.}{}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x)
```

output
```
(3*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*(5040*x**(2/3)*a**7*b**2*p**
2 + 2520*x**(2/3)*a**7*b**2*p - 672*x**(2/3)*a**4*b**5*p**5*x - 3360*x**(2
/3)*a**4*b**5*p**4*x - 5880*x**(2/3)*a**4*b**5*p**3*x - 4200*x**(2/3)*a**4
*b**5*p**2*x - 1008*x**(2/3)*a**4*b**5*p*x + 16*x**(2/3)*a*b**8*p**8*x**2
+ 224*x**(2/3)*a*b**8*p**7*x**2 + 1288*x**(2/3)*a*b**8*p**6*x**2 + 3920*x*
*(2/3)*a*b**8*p**5*x**2 + 6769*x**(2/3)*a*b**8*p**4*x**2 + 6566*x**(2/3)*a
*b**8*p**3*x**2 + 3267*x**(2/3)*a*b**8*p**2*x**2 + 630*x**(2/3)*a*b**8*p*x
**2 - 5040*x**(1/3)*a**8*b*p + 1680*x**(1/3)*a**5*b**4*p**4*x + 5040*x**(1
/3)*a**5*b**4*p**3*x + 4620*x**(1/3)*a**5*b**4*p**2*x + 1260*x**(1/3)*a**5
*b**4*p*x - 64*x**(1/3)*a**2*b**7*p**7*x**2 - 672*x**(1/3)*a**2*b**7*p**6*
x**2 - 2800*x**(1/3)*a**2*b**7*p**5*x**2 - 5880*x**(1/3)*a**2*b**7*p**4*x*
*2 - 6496*x**(1/3)*a**2*b**7*p**3*x**2 - 3528*x**(1/3)*a**2*b**7*p**2*x**2
 - 720*x**(1/3)*a**2*b**7*p*x**2 + 2520*a**9 - 3360*a**6*b**3*p**3*x - 504
0*a**6*b**3*p**2*x - 1680*a**6*b**3*p*x + 224*a**3*b**6*p**6*x**2 + 1680*a
**3*b**6*p**5*x**2 + 4760*a**3*b**6*p**4*x**2 + 6300*a**3*b**6*p**3*x**2 +
 3836*a**3*b**6*p**2*x**2 + 840*a**3*b**6*p*x**2 + 16*b**9*p**8*x**3 + 288
*b**9*p**7*x**3 + 2184*b**9*p**6*x**3 + 9072*b**9*p**5*x**3 + 22449*b**9*p
**4*x**3 + 33642*b**9*p**3*x**3 + 29531*b**9*p**2*x**3 + 13698*b**9*p*x**3
 + 2520*b**9*x**3))/(b**9*(32*p**9 + 720*p**8 + 6960*p**7 + 37800*p**6 + 1
26546*p**5 + 269325*p**4 + 361840*p**3 + 293175*p**2 + 128322*p + 22680...
```

3.103 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx$

Optimal result

Integrand size = 26, antiderivative size = 292

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx = -\frac{3a^5\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(1+2p)} + \frac{15a^4\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(1+p)} - \frac{30a^3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(3+2p)} + \frac{15a^2\left(a+b\sqrt[3]{x}\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2+p)} - \frac{15a\left(a+b\sqrt[3]{x}\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(5+2p)} + \frac{3\left(a+b\sqrt[3]{x}\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(3+p)}$$

output
```
-3*a^5*(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(1+2*p)+15/2*a^
4*(a+b*x^(1/3))^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(p+1)-30*a^3*(a+b*
x^(1/3))^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(3+2*p)+15*a^2*(a+b*x^(1/
3))^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(2+p)-15*a*(a+b*x^(1/3))^5*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(5+2*p)+3/2*(a+b*x^(1/3))^6*(a^2+2*a*b*
x^(1/3)+b^2*x^(2/3))^p/b^6/(3+p)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.49

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx=\frac{3\left(-\frac{2a^5}{1+2p}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)}{1+p}-\frac{20a^3\left(a+b\sqrt[3]{x}\right)^2}{3+2p}+\frac{10a^2\left(a+b\sqrt[3]{x}\right)^3}{2+p}-\frac{10a\left(a+b\sqrt[3]{x}\right)^4}{5+2p}+\frac{\left(a+b\sqrt[3]{x}\right)^5}{3+p}\right)}{2b^6}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

output
```
(3*((-2*a^5)/(1 + 2*p) + (5*a^4*(a + b*x^(1/3)))/(1 + p) - (20*a^3*(a + b*
x^(1/3))^2)/(3 + 2*p) + (10*a^2*(a + b*x^(1/3))^3)/(2 + p) - (10*a*(a + b*
x^(1/3))^4)/(5 + 2*p) + (a + b*x^(1/3))^5/(3 + p))*(a + b*x^(1/3))*((a + b
*x^(1/3))^2)^p)/(2*b^6)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1385, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}xdx$$

$$\Big\downarrow \text{798}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}x^{5/3}d\sqrt[3]{x}$$

$$\Big\downarrow \; 53$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int\left(-\frac{a^5\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{b^5}+\frac{5a^5\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p+1}}{b^5}-\frac{10a^5\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{b^5}\right.$$

$$\Big\downarrow \; 2009$$

$$3\left(\frac{5a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+1)}}{2b^6(p+1)}+\frac{5a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+2)}}{b^6(p+2)}+\frac{a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+3)}}{2b^6(p+3)}-\frac{a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2p+1}}{b^6(2p+1)}-\frac{10a^6\left(\frac{b\sqrt[3]{x}}{a}+\right.}{b^6(2p+}\right.$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

output
```
(3*((5*a^6*(1 + (b*x^(1/3))/a)^(2*(1 + p)))/(2*b^6*(1 + p)) + (5*a^6*(1 +
(b*x^(1/3))/a)^(2*(2 + p)))/(b^6*(2 + p)) + (a^6*(1 + (b*x^(1/3))/a)^(2*(3
 + p)))/(2*b^6*(3 + p)) - (a^6*(1 + (b*x^(1/3))/a)^(1 + 2*p))/(b^6*(1 + 2*
p)) - (10*a^6*(1 + (b*x^(1/3))/a)^(3 + 2*p))/(b^6*(3 + 2*p)) - (5*a^6*(1 +
 (b*x^(1/3))/a)^(5 + 2*p))/(b^6*(5 + 2*p)))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(
2/3))^p)/(1 + (b*x^(1/3))/a)^(2*p)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p x dx$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.02

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx =$$

$$-\frac{3\left(30\,a^6-(8\,b^6p^5+60\,b^6p^4+170\,b^6p^3+225\,b^6p^2+137\,b^6p+30\,b^6)x^2-20\,(2\,a^3b^3p^3+3\,a^3b^3p^2+a^3b^3p)\right.}{}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="fricas")
```

output
```
-3/2*(30*a^6 - (8*b^6*p^5 + 60*b^6*p^4 + 170*b^6*p^3 + 225*b^6*p^2 + 137*b
^6*p + 30*b^6)*x^2 - 20*(2*a^3*b^3*p^3 + 3*a^3*b^3*p^2 + a^3*b^3*p)*x + 2*
(30*a^4*b^2*p^2 + 15*a^4*b^2*p - (4*a*b^5*p^5 + 20*a*b^5*p^4 + 35*a*b^5*p^
3 + 25*a*b^5*p^2 + 6*a*b^5*p)*x)*x^(2/3) - 5*(12*a^5*b*p - (4*a^2*b^4*p^4
+ 12*a^2*b^4*p^3 + 11*a^2*b^4*p^2 + 3*a^2*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p
^3 + 812*b^6*p^2 + 441*b^6*p + 90*b^6)
```

Sympy [F]

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p x\,dx = \int x\left(\left(a + b\sqrt[3]{x}\right)^2\right)^p\,dx$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x,x)
```

output
```
Integral(x*((a + b*x**(1/3))**2)**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.68

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^p x\,dx = \frac{3\left((8\,p^5 + 60\,p^4 + 170\,p^3 + 225\,p^2 + 137\,p + 30)b^6x^2 + 2\,(4\,p^5 + 20\,p^4 + 35\,p^3 + 25\,p^2 + 6}{2\,(8\,p^6 +}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="maxima")
```

output
```
3/2*((8*p^5 + 60*p^4 + 170*p^3 + 225*p^2 + 137*p + 30)*b^6*x^2 + 2*(4*p^5
+ 20*p^4 + 35*p^3 + 25*p^2 + 6*p)*a*b^5*x^(5/3) - 5*(4*p^4 + 12*p^3 + 11*p
^2 + 3*p)*a^2*b^4*x^(4/3) + 20*(2*p^3 + 3*p^2 + p)*a^3*b^3*x - 30*(2*p^2 +
 p)*a^4*b^2*x^(2/3) + 60*a^5*b*p*x^(1/3) - 30*a^6)*(b*x^(1/3) + a)^(2*p)/(
(8*p^6 + 84*p^5 + 350*p^4 + 735*p^3 + 812*p^2 + 441*p + 90)*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(252) = 504$.

Time = 0.14 (sec) , antiderivative size = 745, normalized size of antiderivative = 2.55

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2 x^{2/3}\right)^p x\,dx = \text{Too large to display}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="giac")
```

output
```
3/2*(8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^5*x^2 + 8*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^5*x^(5/3) + 60*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^p*b^6*p^4*x^2 + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p
^4*x^(5/3) - 20*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^4*p^4*x^(4/3)
+ 170*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^3*x^2 + 70*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^3*x^(5/3) - 60*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^p*a^2*b^4*p^3*x^(4/3) + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^3*b^3*p^3*x + 225*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^2*x^2 + 5
0*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^2*x^(5/3) - 55*(b^2*x^(2/3
) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^4*p^2*x^(4/3) + 60*(b^2*x^(2/3) + 2*a*b*x
^(1/3) + a^2)^p*a^3*b^3*p^2*x + 137*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*
b^6*p*x^2 - 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p^2*x^(2/3) +
 12*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p*x^(5/3) - 15*(b^2*x^(2/3
) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^4*p*x^(4/3) + 20*(b^2*x^(2/3) + 2*a*b*x^(
1/3) + a^2)^p*a^3*b^3*p*x + 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*x
^2 - 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p*x^(2/3) + 60*(b^2*
x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b*p*x^(1/3) - 30*(b^2*x^(2/3) + 2*a*b
*x^(1/3) + a^2)^p*a^6)/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3
 + 812*b^6*p^2 + 441*b^6*p + 90*b^6)
```

Mupad [B] (verification not implemented)

Time = 11.94 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.34

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2 x^{2/3}\right)^p x\,dx = \left(a^2 + b^2\,x^{2/3} + 2\,a\,b\,x^{1/3}\right)^p \left(\frac{3\,x^2\,\left(8\,p^5 + 60\,p^4 + 170\,p^3 + 225\,p^2 + 137\,p + 30\right)}{2\,\left(8\,p^6 + 84\,p^5 + 350\,p^4 + 735\,p^3 + 812\,p^2 + 441\,p + 90\right)} - \frac{}{b^6}\right.$$

input
```
int(x*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

output
```
(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^2*(137*p + 225*p^2 + 170*p^3 +
 60*p^4 + 8*p^5 + 30))/(2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 +
8*p^6 + 90)) - (45*a^6)/(b^6*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5
 + 8*p^6 + 90)) + (90*a^5*p*x^(1/3))/(b^5*(441*p + 812*p^2 + 735*p^3 + 350
*p^4 + 84*p^5 + 8*p^6 + 90)) - (15*a^2*p*x^(4/3)*(11*p + 12*p^2 + 4*p^3 +
3))/(2*b^2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) +
(30*a^3*p*x*(3*p + 2*p^2 + 1))/(b^3*(441*p + 812*p^2 + 735*p^3 + 350*p^4 +
 84*p^5 + 8*p^6 + 90)) - (45*a^4*p*x^(2/3)*(2*p + 1))/(b^4*(441*p + 812*p^
2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) + (3*a*p*x^(5/3)*(25*p + 35*
p^2 + 20*p^3 + 4*p^4 + 6))/(b*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^
5 + 8*p^6 + 90)))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.05

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx = \frac{3\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)^p\left(-60x^{\frac{2}{3}}a^4b^2p^2-30x^{\frac{2}{3}}a^4b^2p+8x^{\frac{5}{3}}ab^5p^5+40x^{\frac{5}{3}}ab^5p^4+70x^{\frac{5}{3}}ab\right.}{}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x)
```

output
```
(3*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*( - 60*x**(2/3)*a**4*b**2*p*
*2 - 30*x**(2/3)*a**4*b**2*p + 8*x**(2/3)*a*b**5*p**5*x + 40*x**(2/3)*a*b*
*5*p**4*x + 70*x**(2/3)*a*b**5*p**3*x + 50*x**(2/3)*a*b**5*p**2*x + 12*x**
(2/3)*a*b**5*p*x + 60*x**(1/3)*a**5*b*p - 20*x**(1/3)*a**2*b**4*p**4*x - 6
0*x**(1/3)*a**2*b**4*p**3*x - 55*x**(1/3)*a**2*b**4*p**2*x - 15*x**(1/3)*a
**2*b**4*p*x - 30*a**6 + 40*a**3*b**3*p**3*x + 60*a**3*b**3*p**2*x + 20*a*
*3*b**3*p*x + 8*b**6*p**5*x**2 + 60*b**6*p**4*x**2 + 170*b**6*p**3*x**2 +
225*b**6*p**2*x**2 + 137*b**6*p*x**2 + 30*b**6*x**2))/(2*b**6*(8*p**6 + 84
*p**5 + 350*p**4 + 735*p**3 + 812*p**2 + 441*p + 90))
```

3.104 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+2p)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+p)}+\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(3+2p)}$$

output
```
3*a^2*(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(1+2*p)-3*a*(a+b
*x^(1/3))^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(p+1)+3*(a+b*x^(1/3))^3*
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(a^2-ab(1+2p)\sqrt[3]{x}+b^2\left(1+3p+2p^2\right)x^{2/3}\right)}{b^3(1+p)(1+2p)(3+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

output
```
(3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(a^2 - a*b*(1 + 2*p)*x^(1/3) + b^
2*(1 + 3*p + 2*p^2)*x^(2/3)))/(b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1385, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{774}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \left(\frac{a^2\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{b^2}-\frac{2a^2\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p+1}}{b^2}+\frac{a^2\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p+2}}{b^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+1)}}{b^3(p+1)}+\frac{a^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2p+1}}{b^3(2p+1)}+\frac{a^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2p+3}}{b^3(2p+3)}\right)\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right.$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

output
```
(3*(-((a^3*(1 + (b*x^(1/3))/a)^(2*(1 + p)))/(b^3*(1 + p))) + (a^3*(1 + (b*
x^(1/3))/a)^(1 + 2*p))/(b^3*(1 + 2*p)) + (a^3*(1 + (b*x^(1/3))/a)^(3 + 2*p
))/(b^3*(3 + 2*p)))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(1 + (b*x^(1/3)
)/a)^(2*p)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right)^{p}d x$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx = -\frac{3\left(2\,a^2bpx^{\frac{1}{3}}-a^3-(2\,b^3p^2+3\,b^3p+b^3)x-(2\,ab^2p^2+ab^2p)x^{\frac{2}{3}}\right)\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="fricas")
```

output
```
-3*(2*a^2*b*p*x^(1/3) - a^3 - (2*b^3*p^2 + 3*b^3*p + b^3)*x - (2*a*b^2*p^2
 + a*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(4*b^3*p^3 + 12
*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Sympy [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx = \int \left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p,x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.54

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx = \frac{3\left((2\,p^2+3\,p+1)b^3x+(2\,p^2+p)ab^2x^{\frac{2}{3}}-2\,a^2bpx^{\frac{1}{3}}+a^3\right)\left(bx^{\frac{1}{3}}+a\right)^{2\,p}}{(4\,p^3+12\,p^2+11\,p+3)b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="maxima")
```

output
```
3*((2*p^2 + 3*p + 1)*b^3*x + (2*p^2 + p)*a*b^2*x^(2/3) - 2*a^2*b*p*x^(1/3)
 + a^3)*(b*x^(1/3) + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.61

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3\left(2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pb^3p^2x+2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pab^2p^2x^{\frac{2}{3}}+3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+\right.\right.}{}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="giac")
```

output
```
3*(2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*p^2*x + 2*(b^2*x^(2/3) + 2*
a*b*x^(1/3) + a^2)^p*a*b^2*p^2*x^(2/3) + 3*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
a^2)^p*b^3*p*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^2*p*x^(2/3) - 2
*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b*p*x^(1/3) + (b^2*x^(2/3) + 2*
a*b*x^(1/3) + a^2)^p*b^3*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3)/(4
*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Mupad [B] (verification not implemented)

Time = 11.69 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\left(\frac{3\,x\,\left(2\,p^2+3\,p+1\right)}{4\,p^3+12\,p^2+11\,p+3}+\frac{3\,a^3}{b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\frac{}{b^2\,\left(4\,p^3\right.}\right.$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

output
```
(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x*(3*p + 2*p^2 + 1))/(11*p + 12*
p^2 + 4*p^3 + 3) + (3*a^3)/(b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (6*a^2*p*x^
(1/3))/(b^2*(11*p + 12*p^2 + 4*p^3 + 3)) + (3*a*p*x^(2/3)*(2*p + 1))/(b*(1
1*p + 12*p^2 + 4*p^3 + 3)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.69

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)^p\left(2x^{\frac{2}{3}}a\,b^2p^2+x^{\frac{2}{3}}a\,b^2p-2x^{\frac{1}{3}}a^2bp+a^3+2b^3p^2x+3b^3px+b^3x\right)}{b^3\left(4p^3+12p^2+11p+3\right)}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

output
```
(3*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*(2*x**(2/3)*a*b**2*p**2 + x*
*(2/3)*a*b**2*p - 2*x**(1/3)*a**2*b*p + a**3 + 2*b**3*p**2*x + 3*b**3*p*x
+ b**3*x))/(b**3*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.105 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx = -\frac{3\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \text{Hypergeometric2F1}\left(1,1+2p,2(1+p),1+\frac{b\sqrt[3]{x}}{a}\right)}{a(1+2p)}$$

output
```
-3*(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*hypergeom([1, 1+2*p],[2
*p+2],1+b*x^(1/3)/a)/a/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx = -\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p \text{Hypergeometric2F1}\left(1,1+2p,2+2p,1+\frac{b\sqrt[3]{x}}{a}\right)}{a(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
```

output
```
(-3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2
+ 2*p, 1 + (b*x^(1/3))/a])/(a*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1385, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}dx$$

$$\Big\downarrow \text{ 1385}$$

$$\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \frac{\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{x}dx$$

$$\Big\downarrow \text{ 798}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \frac{\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\text{Hypergeometric2F1}\left(1,2p+1,2(p+1),\frac{\sqrt[3]{x}b}{a}+1\right)}{2p+1}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
```

output
```
(-3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeometr
ic2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(1 + 2*p)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{\left(a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right)^{p}}{x}d x$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x)
```

Fricas [F]

$$\int \frac{(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{x}\,dx=\int \frac{\left(b^{2} x^{\frac{2}{3}}+2 \, a b x^{\frac{1}{3}}+a^{2}\right)^{p}}{x}\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="fricas")
```

output
```
integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

Sympy [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p}{x}\,dx$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x,x)
```

output
```
Integral(((a + b*x**(1/3))**2)**p/x, x)
```

Maxima [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x}\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="maxima")
```

output
```
integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

Giac [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x}\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\int \frac{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p}{x}\,dx$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x,x)
```

output
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx=\frac{3\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)^p+2\left(\int \frac{\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)^p}{x^{\frac{4}{3}}b+ax}dx\right)ap}{2p}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x)
```

output
```
(3*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p + 2*int((x**(2/3)*b**2 + 2*x
**(1/3)*a*b + a**2)**p/(x**(1/3)*b*x + a*x),x)*a*p)/(2*p)
```

3.106 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 72

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\frac{3b^3\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\text{Hypergeometric2F1}\left(4,1+2p,2(\right.}{a^4(1+2p)}$$

output
```
3*b^3*(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*hypergeom([4, 1+2*p]
,[2*p+2],1+b*x^(1/3)/a)/a^4/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\frac{3b^3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\text{Hypergeometric2F1}\left(4,1+2p,2+2p,1+\right.}{a^4(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
```

output
```
(3*b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[4, 1 + 2*p,
 2 + 2*p, 1 + (b*x^(1/3))/a])/(a^4*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1385, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \frac{\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{x^2}dx$$

$$\Big\downarrow \text{798}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \frac{\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{75}$$

$$\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \text{Hypergeometric2F1}\left(4,2p+1,2(p+1),\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
```

output
```
(3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeom
etric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 + 2*p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{\left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p}{x^2}dx$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{x^2}\,dx=\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\int \frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p}{x^2}\,dx$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2,x)
```

output
```
Integral(((a + b*x**(1/3))**2)**p/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\int \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{x^2}\,dx$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\int \frac{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p}{x^2}\,dx$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2,x)
```

output
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx=\frac{-6x^{\frac{2}{3}}\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)^p b^2p^2+6x^{\frac{2}{3}}\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)^p b^2p-3x^{\frac{1}{3}}\left(\right.}{}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x)
```

output
```
( - 6*x**(2/3)*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*b**2*p**2 + 6*x*
*(2/3)*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*b**2*p - 3*x**(1/3)*(x**
(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*a*b*p - 3*(x**(2/3)*b**2 + 2*x**(1/
3)*a*b + a**2)**p*a**2 + 4*int((x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p/
(x**(1/3)*b*x + a*x),x)*b**3*p**3*x - 6*int((x**(2/3)*b**2 + 2*x**(1/3)*a*
b + a**2)**p/(x**(1/3)*b*x + a*x),x)*b**3*p**2*x + 2*int((x**(2/3)*b**2 +
2*x**(1/3)*a*b + a**2)**p/(x**(1/3)*b*x + a*x),x)*b**3*p*x)/(3*a**2*x)
```

3.107 $\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)dx$

Optimal result

Integrand size = 77, antiderivative size = 146

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right) dx = -\frac{\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{ax} + \frac{b(1-p)\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{a^2x^{2/3}} - \frac{b^2(1-2p)(1-p)\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{a^3\sqrt[3]{x}}$$

output
```
-(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a/x+b*(1-p)*(a+b*x^(1/3))
*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^2/x^(2/3)-b^2*(1-2*p)*(1-p)*(a+b*x^(1
/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right) dx = \frac{b^3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(2p(1-3p+2p^2)\,\mathrm{Hy}\right.}{}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1
- p)*p*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

output
```
(b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(2*p*(1 - 3*p + 2*p^2)*Hypergeo
metric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a] + 3*Hypergeometric2F1[
4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a]))/(a^3*(a + 2*a*p))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.013, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2b^3(1-2p)(1-p)p\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \text{Hypergeometric2F1}\left(1,2p+1,2(p+1),\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}+\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \text{Hypergeometric2F1}\left(4,2p+1,2(p+1),\frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 - p)*p
*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

output
```
(2*b^3*(1 - 2*p)*(1 - p)*p*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*
x^(2/3))^p*Hypergeometric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a
^3*(1 + 2*p)) + (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2
/3))^p*Hypergeometric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(
1 + 2*p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{\left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p}{x^2}-\frac{2b^3(1-2p)\,(1-p)\,p\left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p}{3a^3x}\right)dx$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a
*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a
*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)dx = -\frac{\left(a^2bpx^{\frac{1}{3}}+a^3+(2\,b^3p^2-3\,b^3p+b^3)x+2\,(ab^2p^2-ab^2p)x^{\frac{2}{3}}\right)\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{a^3x}$$

input

```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a
^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="fricas")
```

output

```
-(a^2*b*p*x^(1/3) + a^3 + (2*b^3*p^2 - 3*b^3*p + b^3)*x + 2*(a*b^2*p^2 - a
*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(a^3*x)
```

Sympy [F]

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)dx = -\frac{\int\left(-\frac{3a^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x^2}\right)dx+\int\frac{2b^3p\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x}\,dx+\int\left(-\frac{6b^3p^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x}\right)dx+\int}{3a^3}$$

input

```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2-2/3*b**3*(1-2*p)*(1-
p)*p*(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/a**3/x,x)
```

output

```
-(Integral(-3*a**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x**2, x) + I
ntegral(2*b**3*p*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integr
al(-6*b**3*p**2*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integra
l(4*b**3*p**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x))/(3*a**3)
```

Maxima [F]

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right) dx = \int -\frac{2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p b^3(2\,p-1)(p-1)p}{3\,a^3x} +$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a
^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="maxima")
```

output
```
integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)
*p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Giac [F]

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right) dx = \int -\frac{2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p b^3(2\,p-1)(p-1)p}{3\,a^3x} +$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a
^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="giac")
```

output
```
integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)
*p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 11.67 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)\,dx = -\frac{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,\left(\frac{b^3\,x\,\left(2\,p^2-3\,p+1\right)}{a^3}+\frac{b\,p\,x^{1/3}}{a}+\frac{2\,b^2\,p\,x^{2/3}\,\left(p-1\right)}{a^2}+1\right)}{x}$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2 - (2*b^3*p*(2*p - 1)*(p - 1)
*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p)/(3*a^3*x),x)
```

output
```
-((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((b^3*x*(2*p^2 - 3*p + 1))/a^3 + (
b*p*x^(1/3))/a + (2*b^2*p*x^(2/3)*(p - 1))/a^2 + 1))/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.60

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)\,dx = \frac{\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)^p\left(-2x^{\frac{2}{3}}a\,b^2p^2+2x^{\frac{2}{3}}a\,b^2p-x\right.}{a^3x}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a
*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x)
```

output
```
((x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*( - 2*x**(2/3)*a*b**2*p**2 + 2
*x**(2/3)*a*b**2*p - x**(1/3)*a**2*b*p - a**3 - 2*b**3*p**2*x + 3*b**3*p*x
 - b**3*x))/(a**3*x)
```

3.108 $\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx=-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b^2\log(b+cx^n)}{c^3n}$$

output
```
-b*x^n/c^2/n+1/2*x^(2*n)/c/n+b^2*ln(b+c*x^n)/c^3/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx=\frac{cx^n(-2b+cx^n)+2b^2\log(b+cx^n)}{2c^3n}$$

input
```
Integrate[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(c*x^n*(-2*b + c*x^n) + 2*b^2*Log[b + c*x^n])/(2*c^3*n)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {10, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4n-1}}{bx^n + cx^{2n}} \, dx$$

$$\Big\downarrow \text{ 10}$$

$$\int \frac{x^{3n-1}}{b + cx^n} dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{x^{2n}}{cx^n+b} dx^n}{n}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(\frac{x^n}{c} + \frac{b^2}{c^2(cx^n+b)} - \frac{b}{c^2}\right) dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{b^2 \log(b+cx^n)}{c^3} - \frac{bx^n}{c^2} + \frac{x^{2n}}{2c}}{n}$$

input
```
Int[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-((b*x^n)/c^2) + x^(2*n)/(2*c) + (b^2*Log[b + c*x^n])/c^3)/n
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{x^{2n}}{2cn}-\frac{b\,x^{n}}{c^{2}n}+\frac{b^{2}\ln\left(x^{n}+\frac{b}{c}\right)}{c^{3}n}$	47
norman	$\left(\frac{\mathrm{e}^{3n\ln(x)}}{2cn}-\frac{b\,\mathrm{e}^{2n\ln(x)}}{c^{2}n}\right)\mathrm{e}^{-n\ln(x)}+\frac{b^{2}\ln(\mathrm{e}^{n\ln(x)}c+b)}{c^{3}n}$	62

input
```
int(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/2/c/n*(x^n)^2-b*x^n/c^2/n+b^2/c^3/n*ln(x^n+b/c)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\, dx = \frac{c^2x^{2n} - 2\,bcx^n + 2\,b^2\log\left(cx^n + b\right)}{2\,c^3n}$$

input
```
integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/2*(c^2*x^(2*n) - 2*b*c*x^n + 2*b^2*log(c*x^n + b))/(c^3*n)
```

Sympy [A] (verification not implemented)

Time = 12.96 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\, dx = \begin{cases} \frac{\log(x)}{b} & \text{for } c = 0 \wedge n = 0 \\ \frac{xx^{-n}x^{4n-1}}{3bn} & \text{for } c = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ \frac{b^2\log\left(\frac{b}{c}+x^n\right)}{c^3n} - \frac{bx^n}{c^2n} + \frac{x^{2n}}{2cn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+4*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (x*x**(4*n - 1)/(3*b*n*x**n), E
q(c, 0)), (log(x)/(b + c), Eq(n, 0)), (b**2*log(b/c + x**n)/(c**3*n) - b*x
**n/(c**2*n) + x**(2*n)/(2*c*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\,dx = \frac{b^2\log\left(\frac{cx^n+b}{c}\right)}{c^3n} + \frac{cx^{2n} - 2\,bx^n}{2\,c^2n}$$

input
```
integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
b^2*log((c*x^n + b)/c)/(c^3*n) + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n)
```

Giac [F]

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{4n-1}}{cx^{2n} + bx^n}\,dx$$

input
```
integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{4n-1}}{b\,x^n + c\,x^{2n}}\,dx$$

input
```
int(x^(4*n - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(4*n - 1)/(b*x^n + c*x^(2*n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}}\,dx = \frac{x^{2n}c^2 - 2x^n bc + 2\log(x^n c + b)\,b^2}{2c^3n}$$

input
```
int(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x)
```

output
```
(x**(2*n)*c**2 - 2*x**n*b*c + 2*log(x**n*c + b)*b**2)/(2*c**3*n)
```

3.109 $\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 28

$$\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx = \frac{x^n}{cn}-\frac{b\log(b+cx^n)}{c^2n}$$

output
```
x^n/c/n-b*ln(b+c*x^n)/c^2/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx = \frac{cx^n-b\log(cn(b+cx^n))}{c^2n}$$

input
```
Integrate[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(c*x^n - b*Log[c*n*(b + c*x^n)])/(c^2*n)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {10, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{ 10}$$

$$\int \frac{x^{2n-1}}{b+cx^n}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{x^n}{cx^n+b}dx^n}{n}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(\frac{1}{c}-\frac{b}{c(cx^n+b)}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{x^n}{c}-\frac{b\log(b+cx^n)}{c^2}}{n}$$

input
```
Int[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(x^n/c - (b*Log[b + c*x^n])/c^2)/n
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{x^{n}}{c n}-\frac{b \ln\left(x^{n}+\frac{b}{c}\right)}{c^{2} n}$	31
norman	$\frac{{\mathrm e}^{n \ln\left(x \right)}}{c n}-\frac{b \ln\left({\mathrm e}^{n \ln\left(x \right)} c +b \right)}{c^{2} n}$	33

input
```
int(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
x^n/c/n-b/c^2/n*ln(x^n+b/c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}}\,dx = \frac{cx^n - b\log\left(cx^n + b\right)}{c^2n}$$

input
```
integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
(c*x^n - b*log(c*x^n + b))/(c^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.

Time = 7.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}}\,dx = \begin{cases} \frac{\log(x)}{b} & \text{for}\ c = 0 \wedge n = 0 \\ \frac{xx^{-n}x^{3n-1}}{2bn} & \text{for}\ c = 0 \\ \frac{\log(x)}{b+c} & \text{for}\ n = 0 \\ -\frac{b\log\left(\frac{b}{c}+x^n\right)}{c^2n} + \frac{x^n}{cn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+3*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (x*x**(3*n - 1)/(2*b*n*x**n), E
q(c, 0)), (log(x)/(b + c), Eq(n, 0)), (-b*log(b/c + x**n)/(c**2*n) + x**n/
(c*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx=\frac{x^n}{cn}-\frac{b\log\left(\frac{cx^n+b}{c}\right)}{c^2n}$$

input
```
integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
x^n/(c*n) - b*log((c*x^n + b)/c)/(c^2*n)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{3\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

input
```
integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{3\,n-1}}{b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^(3*n - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(3*n - 1)/(b*x^n + c*x^(2*n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}} \, dx = \frac{x^n c - \log(x^n c + b) \, b}{c^2 n}$$

input
```
int(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x)
```

output
```
(x**n*c - log(x**n*c + b)*b)/(c**2*n)
```

3.110 $\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 15

$$\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx=\frac{\log(b+cx^n)}{cn}$$

output
```
ln(b+c*x^n)/c/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx=\frac{\log(b+cx^n)}{cn}$$

input
```
Integrate[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
Log[b + c*x^n]/(c*n)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {10, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{bx^n + cx^{2n}} \, dx$$
$$\Big\downarrow \; 10$$
$$\int \frac{x^{n-1}}{b + cx^n} dx$$
$$\Big\downarrow \; 792$$
$$\frac{\log(b + cx^n)}{cn}$$

input
```
Int[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
Log[b + c*x^n]/(c*n)
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
norman	$\frac{\ln(e^{n\ln(x)}c+b)}{cn}$	18
risch	$\frac{\ln\left(x^n+\frac{b}{c}\right)}{cn}$	18

input
```
int(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/c/n*ln(exp(n*ln(x))*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx=\frac{\log(cx^n+b)}{cn}$$

input
```
integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
log(c*x^n + b)/(c*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(10) = 20.

Time = 1.85 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.07

$$\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx=\begin{cases}\frac{\log(x)}{b} & \text{for } c=0\wedge n=0\\ \frac{xx^{-n}x^{2n-1}}{bn} & \text{for } c=0\\ \frac{\log(x)}{b+c} & \text{for } n=0\\ -\frac{\log(x)}{c}+\frac{\log\left(\frac{bx^n}{c}+x^{2n}\right)}{cn} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+2*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (x*x**(2*n - 1)/(b*n*x**n), Eq(
c, 0)), (log(x)/(b + c), Eq(n, 0)), (-log(x)/c + log(b*x**n/c + x**(2*n))/
(c*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{x^{-1+2n}}{bx^n + cx^{2n}}\,dx = \frac{\log\left(\frac{cx^n+b}{c}\right)}{cn}$$

input
```
integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
log((c*x^n + b)/c)/(c*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{2\,n-1}}{cx^{2\,n} + bx^n}\,dx$$

input
```
integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{2\,n-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

input
```
int(x^(2*n - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(2*n - 1)/(b*x^n + c*x^(2*n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+2n}}{bx^n + cx^{2n}}\,dx = \frac{\log(x^n c + b)}{cn}$$

input
```
int(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x)
```

output
```
log(x**n*c + b)/(c*n)
```

3.111 $\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx=\frac{\log(x)}{b}-\frac{\log(b+cx^n)}{bn}$$

output
```
ln(x)/b-ln(b+c*x^n)/b/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx=\frac{\log(x^n)-\log(bn(b+cx^n))}{bn}$$

input
```
Integrate[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(Log[x^n] - Log[b*n*(b + c*x^n)])/(b*n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {10, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{1}{x(b+cx^n)}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-n}}{cx^n+b}dx^n}{n}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{\int x^{-n}dx^n}{b}-\frac{c\int \frac{1}{cx^n+b}dx^n}{b}}{n}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{\log(x^n)}{b}-\frac{c\int \frac{1}{cx^n+b}dx^n}{b}}{n}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\log(x^n)}{b}-\frac{\log(b+cx^n)}{b}}{n}$$

input
```
Int[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(Log[x^n]/b - Log[b + c*x^n]/b)/n
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

method	result	size
norman	$\frac{\ln(x)}{b}-\frac{\ln(e^{n\ln(x)}c+b)}{bn}$	26
risch	$\frac{\ln(x)}{b}-\frac{\ln\left(x^{n}+\frac{b}{c}\right)}{bn}$	26

input
```
int(x^(-1+n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/b-1/b/n*ln(exp(n*ln(x))*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}} \, dx = \frac{n \log(x) - \log(cx^n + b)}{bn}$$

input
```
integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
(n*log(x) - log(c*x^n + b))/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(15) = 30$.

Time = 1.67 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}} \, dx = \begin{cases} \tilde{\infty} \log(x) & \text{for } b = 0 \wedge c = 0 \wedge n = 0 \\ -\frac{x x^{-2n} x^{n-1}}{cn} & \text{for } b = 0 \\ \frac{\log(x)}{b} & \text{for } c = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ \frac{2\log(x)}{b} - \frac{\log\left(\frac{bx^n}{c} + x^{2n}\right)}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((zoo*log(x), Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (-x*x**(n - 1)/(c*
n*x**(2*n)), Eq(b, 0)), (log(x)/b, Eq(c, 0)), (log(x)/(b + c), Eq(n, 0)),
(2*log(x)/b - log(b*x**n/c + x**(2*n))/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\,dx = \frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}$$

input
```
integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
log(x)/b - log((c*x^n + b)/c)/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\,dx = \frac{\log(|x|)}{b} - \frac{\log(|cx^n + b|)}{bn}$$

input
```
integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
log(abs(x))/b - log(abs(c*x^n + b))/(b*n)
```

Mupad [B] (verification not implemented)

Time = 10.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x^n}{b} + 1\right)}{b\,n}$$

input
```
int(x^(n - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
-(2*atanh((2*c*x^n)/b + 1))/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\, dx = \frac{-\log(x^n c + b) + \log(x)\, n}{bn}$$

input
```
int(x^(-1+n)/(b*x^n+c*x^(2*n)),x)
```

output
```
( - log(x**n*c + b) + log(x)*n)/(b*n)
```

3.112 $\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx=-\frac{x^{-2n}}{2bn}+\frac{cx^{-n}}{b^2n}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log\left(b+cx^n\right)}{b^3n}$$

output
```
-1/2/b/n/(x^(2*n))+c/b^2/n/(x^n)+c^2*ln(x)/b^3-c^2*ln(b+c*x^n)/b^3/n
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx=-\frac{bx^{-2n}(b-2cx^n)-2c^2\log\left(x^n\right)+2c^2\log\left(b+cx^n\right)}{2b^3n}$$

input
```
Integrate[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

output
```
-1/2*((b*(b - 2*c*x^n))/x^(2*n) - 2*c^2*Log[x^n] + 2*c^2*Log[b + c*x^n]])/(
b^3*n)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {10, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{x^{-2n-1}}{b+cx^n}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-3n}}{cx^n+b}dx^n}{n}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{x^{-3n}}{b}-\frac{cx^{-2n}}{b^2}+\frac{c^2x^{-n}}{b^3}-\frac{c^3}{b^3(cx^n+b)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c^2\log(x^n)}{b^3}-\frac{c^2\log(b+cx^n)}{b^3}+\frac{cx^{-n}}{b^2}-\frac{x^{-2n}}{2b}}{n}$$

input
```
Int[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-1/2*1/(b*x^(2*n)) + c/(b^2*x^n) + (c^2*Log[x^n])/b^3 - (c^2*Log[b + c*x^
n])/b^3)/n
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{c\,x^{-n}}{b^{2} n}-\frac{x^{-2 n}}{2 b n}+\frac{c^{2} \ln \left(x \right)}{b^{3}}-\frac{c^{2} \ln \left(x^{n}+\frac{b}{c}\right)}{b^{3} n}$	58
norman	$\left(\frac{c\,{\mathrm e}^{n \ln \left(x \right)}}{b^{2} n}-\frac{1}{2 b n}+\frac{c^{2} \ln \left(x \right) {\mathrm e}^{2 n \ln \left(x \right)}}{b^{3}}\right) {\mathrm e}^{-2 n \ln \left(x \right)}-\frac{c^{2} \ln \left({\mathrm e}^{n \ln \left(x \right)} c +b \right)}{b^{3} n}$	69

input
```
int(x^(-1-n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
c/b^2/n/(x^n)-1/2/b/n/(x^n)^2+c^2*ln(x)/b^3-c^2/b^3/n*ln(x^n+b/c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{x^{-1-n}}{bx^n + cx^{2n}}\,dx = \frac{2\,c^2nx^{2\,n}\log\left(x\right) - 2\,c^2x^{2\,n}\log\left(cx^n + b\right) + 2\,bcx^n - b^2}{2\,b^3nx^{2\,n}}$$

input
```
integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/2*(2*c^2*n*x^(2*n)*log(x) - 2*c^2*x^(2*n)*log(c*x^n + b) + 2*b*c*x^n - b
^2)/(b^3*n*x^(2*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(48) = 96.

Time = 30.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.70

$$\int \frac{x^{-1-n}}{bx^n + cx^{2n}}\,dx = \begin{cases} \tilde{\infty}\log\left(x\right) & \text{for}\: b = 0 \wedge c = 0 \wedge n = 0 \\ -\frac{xx^{-2n}x^{-n-1}}{3cn} & \text{for}\: b = 0 \\ -\frac{xx^{-n}x^{-n-1}}{2bn} & \text{for}\: c = 0 \\ \frac{\log\left(x\right)}{b+c} & \text{for}\: n = 0 \\ -\frac{x^{-2n}}{2bn} + \frac{cx^{-n}}{b^2n} + \frac{c^2\log\left(x\right)}{b^3} - \frac{c^2\log\left(\frac{b}{c}+x^n\right)}{b^3n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((zoo*log(x), Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (-x*x**(-n - 1)/(3
*c*n*x**(2*n)), Eq(b, 0)), (-x*x**(-n - 1)/(2*b*n*x**n), Eq(c, 0)), (log(x
)/(b + c), Eq(n, 0)), (-1/(2*b*n*x**(2*n)) + c/(b**2*n*x**n) + c**2*log(x)
/b**3 - c**2*log(b/c + x**n)/(b**3*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{x^{-1-n}}{bx^n + cx^{2n}}\,dx = \frac{c^2\log(x)}{b^3} - \frac{c^2\log\left(\frac{cx^n+b}{c}\right)}{b^3n} + \frac{2\,cx^n - b}{2\,b^2nx^{2\,n}}$$

input
```
integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
c^2*log(x)/b^3 - c^2*log((c*x^n + b)/c)/(b^3*n) + 1/2*(2*c*x^n - b)/(b^2*n
*x^(2*n))
```

Giac [F]

$$\int \frac{x^{-1-n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-n-1}}{cx^{2\,n} + bx^n}\,dx$$

input
```
integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{n+1}\,\left(b\,x^n + c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{x^{-1-n}}{bx^n + cx^{2n}}\,dx = \frac{-2x^{2n}\log(x^n c + b)\,c^2 + 2x^{2n}\log(x)\,c^2 n + 2x^n bc - b^2}{2x^{2n} b^3 n}$$

input
```
int(x^(-1-n)/(b*x^n+c*x^(2*n)),x)
```

output
```
( - 2*x**(2*n)*log(x**n*c + b)*c**2 + 2*x**(2*n)*log(x)*c**2*n + 2*x**n*b*
c - b**2)/(2*x**(2*n)*b**3*n)
```

3.113 $\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx=-\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log(b+cx^n)}{b^4n}$$

output
```
-1/3/b/n/(x^(3*n))+1/2*c/b^2/n/(x^(2*n))-c^2/b^3/n/(x^n)-c^3*ln(x)/b^4+c^3
*ln(b+c*x^n)/b^4/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx=-\frac{bx^{-3n}(2b^2-3bcx^n+6c^2x^{2n})+6c^3\log(x^n)-6c^3\log(b+cx^n)}{6b^4n}$$

input
```
Integrate[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
-1/6*((b*(2*b^2 - 3*b*c*x^n + 6*c^2*x^(2*n)))/x^(3*n) + 6*c^3*Log[x^n] - 6
*c^3*Log[b + c*x^n])/(b^4*n)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {10, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2n-1}}{bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{x^{-3n-1}}{b+cx^n}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-4n}}{cx^n+b}dx^n}{n}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{x^{-4n}}{b}-\frac{cx^{-3n}}{b^2}+\frac{c^2x^{-2n}}{b^3}-\frac{c^3x^{-n}}{b^4}+\frac{c^4}{b^4(cx^n+b)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{c^3\log(x^n)}{b^4}+\frac{c^3\log(b+cx^n)}{b^4}-\frac{c^2x^{-n}}{b^3}+\frac{cx^{-2n}}{2b^2}-\frac{x^{-3n}}{3b}}{n}$$

input

```
Int[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

output

```
(-1/3*1/(b*x^(3*n)) + c/(2*b^2*x^(2*n)) - c^2/(b^3*x^n) - (c^3*Log[x^n])/b
^4 + (c^3*Log[b + c*x^n])/b^4)/n
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

method	result	size
risch	$-\frac{c^{2} x^{-n}}{b^{3} n}+\frac{c \,x^{-2 n}}{2 b^{2} n}-\frac{x^{-3 n}}{3 b n}-\frac{c^{3} \ln \left(x \right)}{b^{4}}+\frac{c^{3} \ln \left(x^{n}+\frac{b}{c}\right)}{b^{4} n}$	75
norman	$\left(-\frac{1}{3 b n}+\frac{c \,{\mathrm e}^{n \ln \left(x \right)}}{2 b^{2} n}-\frac{c^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{b^{3} n}-\frac{c^{3} \ln \left(x \right) {\mathrm e}^{3 n \ln \left(x \right)}}{b^{4}}\right) {\mathrm e}^{-3 n \ln \left(x \right)}+\frac{c^{3} \ln \left({\mathrm e}^{n \ln \left(x \right)} c +b \right)}{b^{4} n}$	88

input
```
int(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-c^2/b^3/n/(x^n)+1/2*c/b^2/n/(x^n)^2-1/3/b/n/(x^n)^3-c^3*ln(x)/b^4+c^3/b^4
/n*ln(x^n+b/c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx = -\frac{6\,c^3nx^{3\,n}\log\left(x\right)-6\,c^3x^{3\,n}\log\left(cx^n+b\right)+6\,bc^2x^{2\,n}-3\,b^2cx^n+2\,b^3}{6\,b^4nx^{3\,n}}$$

input
```
integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
-1/6*(6*c^3*n*x^(3*n)*log(x) - 6*c^3*x^(3*n)*log(c*x^n + b) + 6*b*c^2*x^(2
*n) - 3*b^2*c*x^n + 2*b^3)/(b^4*n*x^(3*n))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-2*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx = -\frac{c^3\log\left(x\right)}{b^4}+\frac{c^3\log\left(\frac{cx^n+b}{c}\right)}{b^4n}-\frac{6\,c^2x^{2\,n}-3\,bcx^n+2\,b^2}{6\,b^3nx^{3\,n}}$$

input
```
integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-c^3*log(x)/b^4 + c^3*log((c*x^n + b)/c)/(b^4*n) - 1/6*(6*c^2*x^(2*n) - 3*
b*c*x^n + 2*b^2)/(b^3*n*x^(3*n))
```

Giac [F]

$$\int \frac{x^{-1-2n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-2\,n-1}}{cx^{2\,n} + bx^n}\,dx$$

input
```
integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{2\,n+1}\,\left(b\,x^n + c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1-2n}}{bx^n + cx^{2n}}\,dx = \frac{6x^{3n}\log(x^n c + b)\,c^3 - 6x^{3n}\log(x)\,c^3 n - 6x^{2n} b\,c^2 + 3x^n b^2 c - 2b^3}{6x^{3n} b^4 n}$$

input
```
int(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x)
```

output
```
(6*x**(3*n)*log(x**n*c + b)*c**3 - 6*x**(3*n)*log(x)*c**3*n - 6*x**(2*n)*b
*c**2 + 3*x**n*b**2*c - 2*b**3)/(6*x**(3*n)*b**4*n)
```

3.114 $\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx=-\frac{x^{-4n}}{4bn}+\frac{cx^{-3n}}{3b^2n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}+\frac{c^4\log(x)}{b^5}-\frac{c^4\log\left(b+cx^n\right)}{b^5n}$$

output
```
-1/4/b/n/(x^(4*n))+1/3*c/b^2/n/(x^(3*n))-1/2*c^2/b^3/n/(x^(2*n))+c^3/b^4/n
/(x^n)+c^4*ln(x)/b^5-c^4*ln(b+c*x^n)/b^5/n
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82

$$\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx = -\frac{bx^{-4n}\left(3b^3-4b^2cx^n+6bc^2x^{2n}-12c^3x^{3n}\right)-12c^4\log\left(x^n\right)+12c^4\log\left(b+cx^n\right)}{12b^5n}$$

input
```
Integrate[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
-1/12*((b*(3*b^3 - 4*b^2*c*x^n + 6*b*c^2*x^(2*n) - 12*c^3*x^(3*n)))/x^(4*n
) - 12*c^4*Log[x^n] + 12*c^4*Log[b + c*x^n])/(b^5*n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {10, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-3n-1}}{bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{ 10}$$

$$\int \frac{x^{-4n-1}}{b+cx^n}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{x^{-5n}}{cx^n+b}dx^n}{n}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\int \left(\frac{x^{-5n}}{b}-\frac{cx^{-4n}}{b^2}+\frac{c^2x^{-3n}}{b^3}-\frac{c^3x^{-2n}}{b^4}+\frac{c^4x^{-n}}{b^5}-\frac{c^5}{b^5(cx^n+b)}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{c^4\log(x^n)}{b^5}-\frac{c^4\log(b+cx^n)}{b^5}+\frac{c^3x^{-n}}{b^4}-\frac{c^2x^{-2n}}{2b^3}+\frac{cx^{-3n}}{3b^2}-\frac{x^{-4n}}{4b}}{n}$$

input
```
Int[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-1/4*1/(b*x^(4*n)) + c/(3*b^2*x^(3*n)) - c^2/(2*b^3*x^(2*n)) + c^3/(b^4*x
^n) + (c^4*Log[x^n])/b^5 - (c^4*Log[b + c*x^n])/b^5)/n
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{c^3 x^{-n}}{b^4 n}-\frac{c^2 x^{-2n}}{2b^3 n}+\frac{c\,x^{-3n}}{3b^2 n}-\frac{x^{-4n}}{4bn}+\frac{c^4 \ln(x)}{b^5}-\frac{c^4 \ln\left(x^n+\frac{b}{c}\right)}{b^5 n}$	90
norman	$\left(\frac{c^3 \mathrm{e}^{3n\ln(x)}}{b^4 n}-\frac{1}{4bn}+\frac{c\,\mathrm{e}^{n\ln(x)}}{3b^2 n}-\frac{c^2 \mathrm{e}^{2n\ln(x)}}{2b^3 n}+\frac{c^4 \ln(x)\mathrm{e}^{4n\ln(x)}}{b^5}\right)\mathrm{e}^{-4n\ln(x)}-\frac{c^4 \ln\left(\mathrm{e}^{n\ln(x)}c+b\right)}{b^5 n}$	105

input
```
int(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
c^3/b^4/n/(x^n)-1/2*c^2/b^3/n/(x^n)^2+1/3*c/b^2/n/(x^n)^3-1/4/b/n/(x^n)^4+
c^4*ln(x)/b^5-c^4/b^5/n*ln(x^n+b/c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \frac{12\,c^4nx^{4\,n}\log\left(x\right) - 12\,c^4x^{4\,n}\log\left(cx^n + b\right) + 12\,bc^3x^{3\,n} - 6\,b^2c^2x^{2\,n} + 4\,b^3cx^n - 3\,b^4}{12\,b^5nx^{4\,n}}$$

input
```
integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/12*(12*c^4*n*x^(4*n)*log(x) - 12*c^4*x^(4*n)*log(c*x^n + b) + 12*b*c^3*x
^(3*n) - 6*b^2*c^2*x^(2*n) + 4*b^3*c*x^n - 3*b^4)/(b^5*n*x^(4*n))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-3*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \frac{c^4\log\left(x\right)}{b^5} - \frac{c^4\log\left(\frac{cx^n+b}{c}\right)}{b^5n} + \frac{12\,c^3x^{3\,n} - 6\,bc^2x^{2\,n} + 4\,b^2cx^n - 3\,b^3}{12\,b^4nx^{4\,n}}$$

input
```
integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
c^4*log(x)/b^5 - c^4*log((c*x^n + b)/c)/(b^5*n) + 1/12*(12*c^3*x^(3*n) - 6
*b*c^2*x^(2*n) + 4*b^2*c*x^n - 3*b^3)/(b^4*n*x^(4*n))
```

Giac [F]

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-3\,n-1}}{cx^{2\,n} + bx^n}\,dx$$

input
```
integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{3\,n+1}\,\left(b\,x^n + c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}}\,dx = \frac{-12x^{4n}\log(x^n c + b)\,c^4 + 12x^{4n}\log(x)\,c^4 n + 12x^{3n} b\,c^3 - 6x^{2n} b^2 c^2 + 4x^n b^3 c - 3b^4}{12x^{4n} b^5 n}$$

input
```
int(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x)
```

output
```
( - 12*x**(4*n)*log(x**n*c + b)*c**4 + 12*x**(4*n)*log(x)*c**4*n + 12*x**(
3*n)*b*c**3 - 6*x**(2*n)*b**2*c**2 + 4*x**n*b**3*c - 3*b**4)/(12*x**(4*n)*
b**5*n)
```

3.115 $\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = -\frac{4x^{-3n/4}}{3bn}+\frac{\sqrt{2}c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} - \frac{\sqrt{2}c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} - \frac{\sqrt{2}c^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}}{\sqrt{b}+\sqrt{c}x^{n/2}}\right)}{b^{7/4}n}$$

output
```
-4/3/b/n/(x^(3/4*n))-2^(1/2)*c^(3/4)*arctan(-1+2^(1/2)*c^(1/4)*x^(1/4*n)/b
^(1/4))/b^(7/4)/n-2^(1/2)*c^(3/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/4*n)/b^(1/
4))/b^(7/4)/n-2^(1/2)*c^(3/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/4*n)/(b
^(1/2)+c^(1/2)*x^(1/2*n)))/b^(7/4)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.19

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx=-\frac{4x^{-3n/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\frac{cx^n}{b}\right)}{3bn}$$

input
```
Integrate[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-4*Hypergeometric2F1[-3/4, 1, 1/4, -((c*x^n)/b)])/(3*b*n*x^((3*n)/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.48, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {10, 886, 868, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{4}-1}}{bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{x^{-\frac{3n}{4}-1}}{b+cx^n}dx$$

$$\Big\downarrow \text{886}$$

$$-\frac{c\int \frac{x^{\frac{n-4}{4}}}{cx^n+b}dx}{b}-\frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \text{868}$$

$$-\frac{4c\int \frac{1}{cx^n+b}dx^{n/4}}{bn}-\frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \text{755}$$

$$-\frac{4c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x^{n/2}}{cx^n+b}dx^{n/4}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x^{n/2}+\sqrt{b}}{cx^n+b}dx^{n/4}}{2\sqrt{b}}\right)}{bn}-\frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \ 1476$$

$$-\frac{4c\left(\frac{\frac{\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}}dx^{n/4}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}}dx^{n/4}}{2\sqrt{c}}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{b}-\sqrt{c}x^{n/2}}{cx^n+b}dx^{n/4}}{2\sqrt{b}}\right)}{bn}-\frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \ 1082$$

$$-\frac{4c\left(\frac{\frac{\int \frac{1}{-x^{n/2}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x^{n/2}-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{b}-\sqrt{c}x^{n/2}}{cx^n+b}dx^{n/4}}{2\sqrt{b}}\right)}{bn}-\frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \ 217$$

$$-\frac{4c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x^{n/2}}{cx^n+b}dx^{n/4}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{bn}-\frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \ 1479$$

$$-\frac{4c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}x^{n/4}}{\sqrt[4]{c}\left(-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x^{n/4}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{bn}$$

$$\frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \; 25$$

$$-\frac{4c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}x^{n/4}}{\sqrt[4]{c}\left(-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x^{n/4}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{bn}-\frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \; 27$$

$$-\frac{4c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}x^{n/4}}{-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}}dx^{n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x^{n/4}+\sqrt[4]{b}}{\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{c}}+x^{n/2}+\frac{\sqrt{b}}{\sqrt{c}}}dx^{n/4}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{bn}-\frac{4x^{-3n/4}}{3bn}$$

$$\Big\downarrow \; 1103$$

$$-\frac{4c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{n/4}+\sqrt{b}+\sqrt{c}x^{n/2}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{bn}-\frac{4x^{-3n/4}}{3bn}$$

input
```
Int[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

output
```
-4/(3*b*n*x^((3*n)/4)) - (4*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x^(n/4))/b^(
1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x^(n/4))/b^
(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2
]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)]/(Sqrt[2]*b^(1/4)*c^(1/4)) + L
og[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)]/(2*Sqrt[2]
*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/(b*n)
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.31

method	result	size
risch	$-\frac{4x^{-\frac{3n}{4}}}{3bn}+\left(\sum_{\_R=\operatorname{RootOf}\left(b^{7}n^{4}\_Z^{4}+c^{3}\right)}\_R\ln\left(x^{\frac{n}{4}}-\frac{b^{2}n\_R}{c}\right)\right)$	54

input
```
int(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-4/3/b/n/(x^(1/4*n))^3+sum(_R*ln(x^(1/4*n)-b^2*n/c*_R),_R=RootOf(_Z^4*b^7*
n^4+c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.54

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx =$$

$$-\frac{3\,bnx^3x^{\frac{3}{4}n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2n\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}+cxx^{\frac{1}{4}n-1}}{x}\right)-3\,bnx^3x^{\frac{3}{4}n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\log\left(-\frac{b^2n\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}-cxx^{\frac{1}{4}n-}}{x}\right.}{}$$

input
```
integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
-1/3*(3*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*log((b^2*n*(-c^3/(b^7
*n^4))^(1/4) + c*x*x^(1/4*n - 1))/x) - 3*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*
n^4))^(1/4)*log(-(b^2*n*(-c^3/(b^7*n^4))^(1/4) - c*x*x^(1/4*n - 1))/x) + 3
*I*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*log((I*b^2*n*(-c^3/(b^7*n^
4))^(1/4) + c*x*x^(1/4*n - 1))/x) - 3*I*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n
^4))^(1/4)*log((-I*b^2*n*(-c^3/(b^7*n^4))^(1/4) + c*x*x^(1/4*n - 1))/x) +
4)/(b*n*x^3*x^(3/4*n - 3))
```

Sympy [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-n}x^{\frac{n}{4}-1}}{b+cx^n}\,dx$$

input
```
integrate(x**(-1+1/4*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(n/4 - 1)/(x**n*(b + c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^n}\,dx$$

input
```
integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-c*integrate(x^(1/4*n)/(b*c*x*x^n + b^2*x), x) - 4/3/(b*n*x^(3/4*n))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.16

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = -\frac{\frac{6\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,(x^n)^{\frac{1}{4}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{6\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,(x^n)^{\frac{1}{4}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{3\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(x^{\frac{1}{2}\,n}+\sqrt{2}(x^n)^{\frac{1}{4}}\left(\right.\right.}{b^2}}{6\,n}$$

input
```
integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
-1/6*(6*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*
(x^n)^(1/4))/(b/c)^(1/4))/b^2 + 6*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) - 2*(x^n)^(1/4))/(b/c)^(1/4))/b^2 + 3*sqrt(2)*(b*c^
3)^(1/4)*log(x^(1/2*n) + sqrt(2)*(x^n)^(1/4)*(b/c)^(1/4) + sqrt(b/c))/b^2
- 3*sqrt(2)*(b*c^3)^(1/4)*log(x^(1/2*n) - sqrt(2)*(x^n)^(1/4)*(b/c)^(1/4)
+ sqrt(b/c))/b^2 + 8/(b*x^(3/4*n)))/n
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{\frac{n}{4}-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

input
```
int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)), x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{\frac{n}{4}}}{x^{2n}cx + x^n bx}dx$$

input
```
int(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x)
```

output
```
int(x**(n/4)/(x**(2*n)*c*x + x**n*b*x),x)
```

3.116 $\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = -\frac{3x^{-2n/3}}{2bn}+\frac{\sqrt{3}c^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n} - \frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}$$

output
```
-3/2/b/n/(x^(2/3*n))+3^(1/2)*c^(2/3)*arctan(1/3*(b^(1/3)-2*c^(1/3)*x^(1/3*
n))*3^(1/2)/b^(1/3))/b^(5/3)/n-c^(2/3)*ln(b^(1/3)+c^(1/3)*x^(1/3*n))/b^(5/
3)/n+1/2*c^(2/3)*ln(b^(2/3)-b^(1/3)*c^(1/3)*x^(1/3*n)+c^(2/3)*x^(2/3*n))/b
^(5/3)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.21

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = -\frac{3x^{-2n/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},1,\frac{1}{3},-\frac{cx^n}{b}\right)}{2bn}$$

input
```
Integrate[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-3*Hypergeometric2F1[-2/3, 1, 1/3, -((c*x^n)/b)])/(2*b*n*x^((2*n)/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {10, 886, 868, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{3}-1}}{bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{x^{-\frac{2n}{3}-1}}{b+cx^n}dx$$

$$\Big\downarrow \text{886}$$

$$-\frac{c\int \frac{x^{\frac{n-3}{3}}}{cx^n+b}dx}{b}-\frac{3x^{-2n/3}}{2bn}$$

$$\Big\downarrow \text{868}$$

$$-\frac{3c\int \frac{1}{cx^n+b}dx^{n/3}}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$$\Big\downarrow \text{750}$$

$$-\frac{3c\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{c}x^{n/3}}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}}{3b^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{c}x^{n/3}+\sqrt[3]{b}}dx^{n/3}}{3b^{2/3}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$$\Big\downarrow \text{16}$$

$$-\frac{3c\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{c}x^{n/3}}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$\Big\downarrow$ 1142

$$-\frac{3c\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}-\frac{\int -\frac{\sqrt[3]{c}\left(\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}\right)}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}}{2\sqrt[3]{c}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$\Big\downarrow$ 25

$$-\frac{3c\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}+\frac{\int \frac{\sqrt[3]{c}\left(\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}\right)}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}}{2\sqrt[3]{c}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$\Big\downarrow$ 27

$$-\frac{3c\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}+\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$\Big\downarrow$ 1082

$$-\frac{3c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}+\frac{3\int \frac{1}{-x^{2n/3}-3}d\left(1-\frac{2\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b}}\right)}{\sqrt[3]{c}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$$\Big\downarrow \; 217$$

$$-\frac{3c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}+b^{2/3}}dx^{n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

$$\Big\downarrow \; 1103$$

$$-\frac{3c\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}x^{n/3}+c^{2/3}x^{2n/3}\right)}{2\sqrt[3]{c}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{c}x^{n/3}\right)}{3b^{2/3}\sqrt[3]{c}}\right)}{bn}-\frac{3x^{-2n/3}}{2bn}$$

input
```
Int[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

output
```
-3/(2*b*n*x^((2*n)/3)) - (3*c*(Log[b^(1/3) + c^(1/3)*x^(n/3)]/(3*b^(2/3)*c
^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*c^(1/3)*x^(n/3))/b^(1/3))/Sqrt[3]])/c
^(1/3)) - Log[b^(2/3) - b^(1/3)*c^(1/3)*x^(n/3) + c^(2/3)*x^((2*n)/3)]/(2*
c^(1/3)))/(3*b^(2/3))))/(b*n)
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

method	result	size
risch	$-\frac{3x^{-\frac{2n}{3}}}{2bn}+\left(\sum_{\_R=\operatorname{RootOf}\left(b^{5}n^{3}\_Z^{3}+c^{2}\right)}\_R\ln\left(x^{\frac{n}{3}}-\frac{b^{2}n\_R}{c}\right)\right)$	54

input
```
int(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-3/2/b/n/(x^(1/3*n))^2+sum(_R*ln(x^(1/3*n)-b^2*n/c*_R),_R=RootOf(_Z^3*b^5*
n^3+c^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.32

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$$
$$= \frac{2\sqrt{3}x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bxx^{\frac{1}{3}n-1}\left(-\frac{c^2}{b^2}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)+2x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{cxx^{\frac{1}{3}n-1}-b\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}}{x}\right)-x}{2bnx^2x^{\frac{2}{3}n-2}}$$

input
```
integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*x^2*x^(2/3*n - 2)*(-c^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*
x*x^(1/3*n - 1)*(-c^2/b^2)^(2/3) - sqrt(3)*c)/c) + 2*x^2*x^(2/3*n - 2)*(-c
^2/b^2)^(1/3)*log((c*x*x^(1/3*n - 1) - b*(-c^2/b^2)^(1/3))/x) - x^2*x^(2/3
*n - 2)*(-c^2/b^2)^(1/3)*log((c^2*x^2*x^(2/3*n - 2) + b*c*x*x^(1/3*n - 1)*
(-c^2/b^2)^(1/3) + b^2*(-c^2/b^2)^(2/3))/x^2) - 3)/(b*n*x^2*x^(2/3*n - 2))
```

Sympy [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-n}x^{\frac{n}{3}-1}}{b+cx^n}\,dx$$

input
```
integrate(x**(-1+1/3*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(n/3 - 1)/(x**n*(b + c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}} \, dx = \int \frac{x^{\frac{1}{3}\,n-1}}{cx^{2\,n} + bx^n} \, dx$$

input
```
integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-c*integrate(x^(1/3*n)/(b*c*x*x^n + b^2*x), x) - 3/2/(b*n*x^(2/3*n))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}} \, dx$$

$$= \frac{\frac{2\,c\left(-\frac{b}{c}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}\,n}-\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right|\right)}{b^2} - \frac{2\,\sqrt{3}\left(-bc^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}\,n}+\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{c}\right)^{\frac{1}{3}}}\right)}{b^2} - \frac{\left(-bc^2\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}\,n}\left(-\frac{b}{c}\right)^{\frac{1}{3}}+\left(x^n\right)^{\frac{2}{3}}+\left(-\frac{b}{c}\right)^{\frac{2}{3}}\right)}{b^2} - \frac{\ }{b}}{2\,n}$$

input
```
integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
1/2*(2*c*(-b/c)^(1/3)*log(abs(x^(1/3*n) - (-b/c)^(1/3)))/b^2 - 2*sqrt(3)*(
-b*c^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3*n) + (-b/c)^(1/3))/(-b/c)^(1/3)
)/b^2 - (-b*c^2)^(1/3)*log(x^(1/3*n)*(-b/c)^(1/3) + (x^n)^(2/3) + (-b/c)^(
2/3))/b^2 - 3/(b*(x^n)^(2/3)))/n
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{\frac{n}{3}-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

input
```
int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)), x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{\frac{n}{3}}}{x^{2n}cx + x^n bx}dx$$

input
```
int(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x)
```

output
```
int(x**(n/3)/(x**(2*n)*c*x + x**n*b*x),x)
```

3.117 $\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=-\frac{2x^{-n/2}}{bn}+\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}$$

output
```
-2/b/n/(x^(1/2*n))+2*c^(1/2)*arctan(b^(1/2)/c^(1/2)/(x^(1/2*n)))/b^(3/2)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx=-\frac{2x^{-n/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{cx^n}{b}\right)}{bn}$$

input
```
Integrate[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-2*Hypergeometric2F1[-1/2, 1, 1/2, -((c*x^n)/b)])/(b*n*x^(n/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {10, 868, 772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{2}-1}}{bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{x^{-\frac{n}{2}-1}}{b+cx^n}dx$$

$$\Big\downarrow \text{868}$$

$$-\frac{2\int \frac{1}{cx^n+b}dx^{-n/2}}{n}$$

$$\Big\downarrow \text{772}$$

$$-\frac{2\int \frac{x^{-n}}{bx^{-n}+c}dx^{-n/2}}{n}$$

$$\Big\downarrow \text{262}$$

$$-\frac{2\left(\frac{x^{-n/2}}{b}-\frac{c\int \frac{1}{bx^{-n}+c}dx^{-n/2}}{b}\right)}{n}$$

$$\Big\downarrow \text{218}$$

$$-\frac{2\left(\frac{x^{-n/2}}{b}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}}\right)}{n}$$

input
```
Int[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-2*(1/(b*x^(n/2)) - (Sqrt[c]*ArcTan[Sqrt[b]/(Sqrt[c]*x^(n/2))])/b^(3/2)))
/n
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

method	result	size
risch	$-\frac{2x^{-\frac{n}{2}}}{bn}+\frac{\sqrt{-bc}\,\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-bc}}{c}\right)}{b^{2}n}-\frac{\sqrt{-bc}\,\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-bc}}{c}\right)}{b^{2}n}$	79

input
```
int(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-2/b/n/(x^(1/2*n))+1/b^2*(-b*c)^(1/2)/n*ln(x^(1/2*n)-(-b*c)^(1/2)/c)-1/b^2
*(-b*c)^(1/2)/n*ln(x^(1/2*n)+(-b*c)^(1/2)/c)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.86

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}}\, dx = \left[\frac{xx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2x^{n-2} - 2\,bxx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}} - b}{cx^2x^{n-2}+b}\right) - 2}{bnxx^{\frac{1}{2}n-1}}, -\frac{2\left(xx^{\frac{1}{2}n-1}\sqrt{\frac{c}{b}}\arctan\left(xx^{\frac{1}{2}n-1}\sqrt{\frac{c}{b}}\right) + 1\right)}{bnxx^{\frac{1}{2}n-1}} \right]$$

input
```
integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[(x*x^(1/2*n - 1)*sqrt(-c/b)*log((c*x^2*x^(n - 2) - 2*b*x*x^(1/2*n - 1)*sq
rt(-c/b) - b)/(c*x^2*x^(n - 2) + b)) - 2)/(b*n*x*x^(1/2*n - 1)), -2*(x*x^(
1/2*n - 1)*sqrt(c/b)*arctan(x*x^(1/2*n - 1)*sqrt(c/b)) + 1)/(b*n*x*x^(1/2*
n - 1))]
```

Sympy [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}}\, dx = \int \frac{x^{-n}x^{\frac{n}{2}-1}}{b + cx^n}\, dx$$

input
```
integrate(x**(-1+1/2*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(n/2 - 1)/(x**n*(b + c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}}\, dx = \int \frac{x^{\frac{1}{2}\,n-1}}{cx^{2\,n} + bx^n}\, dx$$

input
```
integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-c*integrate(x^(1/2*n)/(b*c*x*x^n + b^2*x), x) - 2/(b*n*x^(1/2*n))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}}\, dx = -\frac{2\left(\frac{c\arctan\left(\frac{c\sqrt{x^n}}{\sqrt{bc}}\right)}{\sqrt{bc}b} + \frac{1}{b\sqrt{x^n}}\right)}{n}$$

input
```
integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
-2*(c*arctan(c*sqrt(x^n)/sqrt(b*c))/(sqrt(b*c)*b) + 1/(b*sqrt(x^n)))/n
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}}\, dx = \int \frac{x^{\frac{n}{2}-1}}{b\,x^n + c\,x^{2\,n}}\, dx$$

input
```
int(x^(n/2 - 1)/(b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(n/2 - 1)/(b*x^n + c*x^(2*n)), x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{\frac{n}{2}}}{x^{2n}cx + x^n bx}dx$$

input
```
int(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x)
```

output
```
int(x**(n/2)/(x**(2*n)*c*x + x**n*b*x),x)
```

3.118 $\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}} dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}} dx = -\frac{2x^{-3n/2}}{3bn} + \frac{2cx^{-n/2}}{b^2n} - \frac{2c^{3/2}\arctan\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}$$

output
```
-2/3/b/n/(x^(3/2*n))+2*c/b^2/n/(x^(1/2*n))-2*c^(3/2)*arctan(b^(1/2)/c^(1/2
)/(x^(1/2*n)))/b^(5/2)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}} dx = -\frac{2x^{-3n/2}\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{cx^n}{b}\right)}{3bn}$$

input
```
Integrate[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-2*Hypergeometric2F1[-3/2, 1, -1/2, -((c*x^n)/b)])/(3*b*n*x^((3*n)/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {10, 886, 868, 772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{2}-1}}{bx^n+cx^{2n}}dx$$
$$\Big\downarrow \text{10}$$
$$\int \frac{x^{-\frac{3n}{2}-1}}{b+cx^n}dx$$
$$\Big\downarrow \text{886}$$
$$-\frac{c\int \frac{x^{-\frac{n}{2}-1}}{cx^n+b}dx}{b}-\frac{2x^{-3n/2}}{3bn}$$
$$\Big\downarrow \text{868}$$
$$\frac{2c\int \frac{1}{cx^n+b}dx^{-n/2}}{bn}-\frac{2x^{-3n/2}}{3bn}$$
$$\Big\downarrow \text{772}$$
$$\frac{2c\int \frac{x^{-n}}{bx^{-n}+c}dx^{-n/2}}{bn}-\frac{2x^{-3n/2}}{3bn}$$
$$\Big\downarrow \text{262}$$
$$\frac{2c\left(\frac{x^{-n/2}}{b}-\frac{c\int \frac{1}{bx^{-n}+c}dx^{-n/2}}{b}\right)}{bn}-\frac{2x^{-3n/2}}{3bn}$$
$$\Big\downarrow \text{218}$$
$$\frac{2c\left(\frac{x^{-n/2}}{b}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}}\right)}{bn}-\frac{2x^{-3n/2}}{3bn}$$

input
```
Int[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
```

output
```
-2/(3*b*n*x^((3*n)/2)) + (2*c*(1/(b*x^(n/2)) - (Sqrt[c]*ArcTan[Sqrt[b]/(Sq
rt[c]*x^(n/2))])/b^(3/2)))/(b*n)
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.43

method	result	size
risch	$\frac{2c\,x^{-\frac{n}{2}}}{b^2n}-\frac{2x^{-\frac{3n}{2}}}{3bn}+\frac{\sqrt{-bc}\,c\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-bc}}{c}\right)}{b^3n}-\frac{\sqrt{-bc}\,c\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-bc}}{c}\right)}{b^3n}$	97

input
```
int(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
2*c/b^2/n/(x^(1/2*n))-2/3/b/n/(x^(1/2*n))^3+1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1
/2*n)+(-b*c)^(1/2)/c)-1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1/2*n)-(-b*c)^(1/2)/c)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.37

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx = \left[-\frac{2\,bx^3x^{-\frac{3}{2}\,n-3}-6\,cxx^{-\frac{1}{2}\,n-1}-3\,c\sqrt{-\frac{c}{b}}\log\left(\frac{bx^2x^{-n-2}-2\,bxx^{-\frac{1}{2}\,n-1}\sqrt{-\frac{c}{b}}-c}{bx^2x^{-n-2}+c}\right)}{3\,b^2n}, -\frac{2\left(bx^3x^{-\frac{3}{2}\,n-3}-3\,cxx^{-\frac{1}{2}\,n-1}+3\,c\sqrt{\frac{c}{b}}\arctan\left(\frac{bxx^{-\frac{1}{2}\,n-1}\sqrt{\frac{c}{b}}}{c}\right)\right)}{3\,b^2n}\right]$$

input
```
integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[-1/3*(2*b*x^3*x^(-3/2*n - 3) - 6*c*x*x^(-1/2*n - 1) - 3*c*sqrt(-c/b)*log(
(b*x^2*x^(-n - 2) - 2*b*x*x^(-1/2*n - 1)*sqrt(-c/b) - c)/(b*x^2*x^(-n - 2)
 + c)))/(b^2*n), -2/3*(b*x^3*x^(-3/2*n - 3) - 3*c*x*x^(-1/2*n - 1) + 3*c*s
qrt(c/b)*arctan(b*x*x^(-1/2*n - 1)*sqrt(c/b)/c))/(b^2*n)]
```

Sympy [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-n}x^{-\frac{n}{2}-1}}{b + cx^n}\,dx$$

input
```
integrate(x**(-1-1/2*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(-n/2 - 1)/(x**n*(b + c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n} + bx^n}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
c^2*integrate(x^(1/2*n)/(b^2*c*x*x^n + b^3*x), x) + 2/3*(3*c*x^n - b)/(b^2
*n*x^(3/2*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n} + bx^n}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{2}+1}\,(b\,x^n + c\,x^{2\,n})}\,dx$$

input
```
int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}}\,dx = \frac{-3x^{\frac{3n}{2}}\left(\int \frac{1}{x^{\frac{3n}{2}}cx+x^{\frac{n}{2}}bx}dx\right)cn - 2}{3x^{\frac{3n}{2}}bn}$$

input
```
int(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x)
```

output
```
( - 3*x**((3*n)/2)*int(1/(x**((3*n)/2)*c*x + x**(n/2)*b*x),x)*c*n - 2)/(3*
x**((3*n)/2)*b*n)
```

3.119 $\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = -\frac{3x^{-4n/3}}{4bn}+\frac{3cx^{-n/3}}{b^2n}+\frac{\sqrt{3}c^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{\sqrt{3}\sqrt[3]{c}}\right)}{b^{7/3}n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\log\left(c^{2/3}+b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}\right)}{2b^{7/3}n}$$

output
```
-3/4/b/n/(x^(4/3*n))+3*c/b^2/n/(x^(1/3*n))+3^(1/2)*c^(4/3)*arctan(1/3*(c^(
1/3)-2*b^(1/3)/(x^(1/3*n)))*3^(1/2)/c^(1/3))/b^(7/3)/n-c^(4/3)*ln(c^(1/3)+
b^(1/3)/(x^(1/3*n)))/b^(7/3)/n+1/2*c^(4/3)*ln(c^(2/3)+b^(2/3)/(x^(2/3*n))-
b^(1/3)*c^(1/3)/(x^(1/3*n)))/b^(7/3)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.19

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx=-\frac{3x^{-4n/3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},1,-\frac{1}{3},-\frac{cx^n}{b}\right)}{4bn}$$

input
```
Integrate[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-3*Hypergeometric2F1[-4/3, 1, -1/3, -((c*x^n)/b)])/(4*b*n*x^((4*n)/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {10, 886, 868, 772, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{3}-1}}{bx^n+cx^{2n}}dx$$

$$\Big\downarrow \; 10$$

$$\int \frac{x^{-\frac{4n}{3}-1}}{b+cx^n}dx$$

$$\Big\downarrow \; 886$$

$$-\frac{c\int \frac{x^{-\frac{n}{3}-1}}{cx^n+b}dx}{b}-\frac{3x^{-4n/3}}{4bn}$$

$$\Big\downarrow \; 868$$

$$\frac{3c\int \frac{1}{cx^n+b}dx^{-n/3}}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$$\Big\downarrow \; 772$$

$$\frac{3c\int \frac{x^{-n}}{bx^{-n}+c}dx^{-n/3}}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$\downarrow$ 843

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\int \frac{1}{bx^{-n}+c}dx^{-n/3}}{b}\right)}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$\downarrow$ 750

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{b}x^{-n/3}}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}}dx^{-n/3}}{3c^{2/3}}\right)}{b}\right)}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$\downarrow$ 16

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{b}x^{-n/3}}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right)}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$\downarrow$ 1142

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}\right)}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}}{2\sqrt[3]{b}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right.}{bn}$$

$$\frac{3x^{-4n/3}}{4bn}$$

$\downarrow$ 25

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}\right)}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}}{2\sqrt[3]{b}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right)}{bn}$$

$$\frac{3x^{-4n/3}}{4bn}$$

$\downarrow$ 27

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}+\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right)}{bn}$$

$$\frac{3x^{-4n/3}}{4bn}$$

$\downarrow$ 1082

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}+\frac{3\int \frac{1}{-x^{-2n/3}-3}d\left(1-\frac{2\sqrt[3]{b}x^{-n/3}}{\sqrt[3]{c}}\right)}{\sqrt[3]{b}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right)}{bn}-$$

$$\frac{3x^{-4n/3}}{4bn}$$

$\downarrow$ 217

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{b}x^{-n/3}}{b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}}dx^{-n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{-n/3}}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right)}{bn}-\frac{3x^{-4n/3}}{4bn}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3c\left(\frac{x^{-n/3}}{b}-\frac{c\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{-n/3}}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{\log\left(b^{2/3}x^{-2n/3}-\sqrt[3]{b}\sqrt[3]{c}x^{-n/3}+c^{2/3}\right)}{2\sqrt[3]{b}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{b}x^{-n/3}+\sqrt[3]{c}\right)}{3\sqrt[3]{b}c^{2/3}}\right)}{b}\right)}{bn}-\frac{3x^{-4n/3}}{4bn}$$

input
```
Int[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

output
```
-3/(4*b*n*x^((4*n)/3)) + (3*c*(1/(b*x^(n/3)) - (c*(Log[c^(1/3) + b^(1/3)/x
^(n/3)]/(3*b^(1/3)*c^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3))/(c^(1/3)
*x^(n/3)))/Sqrt[3]])/b^(1/3)) - Log[c^(2/3) + b^(2/3)/x^((2*n)/3) - (b^(1/
3)*c^(1/3))/x^(n/3)]/(2*b^(1/3)))/(3*c^(2/3))))/b))/(b*n)
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.41

method	result	size
risch	$\frac{3c\,x^{-\frac{n}{3}}}{b^2 n} - \frac{3x^{-\frac{4n}{3}}}{4bn} + \left(\sum_{\_R=\mathrm{RootOf}\left(b^7 n^3 \_Z^3+c^4\right)} \_R \ln\left(x^{\frac{n}{3}} + \frac{b^5 n^2 \_R^2}{c^3}\right)\right)$	73

input
```
int(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
3*c/b^2/n/(x^(1/3*n))-3/4/b/n/(x^(1/3*n))^4+sum(_R*ln(x^(1/3*n)+b^5*n^2/c^
3*_R^2),_R=RootOf(_Z^3*b^7*n^3+c^4))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = -\frac{3\,bx^4x^{-\frac{4}{3}\,n-4} - 12\,cxx^{-\frac{1}{3}\,n-1} - 4\,\sqrt{3}c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bxx^{-\frac{1}{3}\,n-1}\left(-\frac{c}{b}\right)^{\frac{2}{3}}-\sqrt{3}c}{3\,c}\right) - 4\,c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\log\left(\frac{xx^{-\frac{1}{3}\,n-1}}{\cdots}\right.}{4\,b^2n}$$

input
```
integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
-1/4*(3*b*x^4*x^(-4/3*n - 4) - 12*c*x*x^(-1/3*n - 1) - 4*sqrt(3)*c*(-c/b)^
(1/3)*arctan(1/3*(2*sqrt(3)*b*x*x^(-1/3*n - 1)*(-c/b)^(2/3) - sqrt(3)*c)/c
) - 4*c*(-c/b)^(1/3)*log((x*x^(-1/3*n - 1) - (-c/b)^(1/3))/x) + 2*c*(-c/b)
^(1/3)*log((x^2*x^(-2/3*n - 2) + x*x^(-1/3*n - 1)*(-c/b)^(1/3) + (-c/b)^(2
/3))/x^2))/(b^2*n)
```

Sympy [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-n}x^{-\frac{n}{3}-1}}{b + cx^n}\,dx$$

input
```
integrate(x**(-1-1/3*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(-n/3 - 1)/(x**n*(b + c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n} + bx^n}\,dx$$

input
```
integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
c^2*integrate(x^(2/3*n)/(b^2*c*x*x^n + b^3*x), x) + 3/4*(4*c*x^n - b)/(b^2
*n*x^(4/3*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n} + bx^n}\,dx$$

input
```
integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{3}+1}\,(b\,x^n + c\,x^{2\,n})}\,dx$$

input
```
int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}}\,dx = \frac{-4x^{\frac{4n}{3}}\left(\int \frac{1}{x^{\frac{4n}{3}}cx + x^{\frac{n}{3}}bx}dx\right)cn - 3}{4x^{\frac{4n}{3}}bn}$$

input
```
int(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x)
```

output
```
( - 4*x**((4*n)/3)*int(1/(x**((4*n)/3)*c*x + x**(n/3)*b*x),x)*c*n - 3)/(4*
x**((4*n)/3)*b*n)
```

3.120 $\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 191

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = -\frac{4x^{-5n/4}}{5bn}+\frac{4cx^{-n/4}}{b^2n}+\frac{\sqrt{2}c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n} - \frac{\sqrt{2}c^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n} - \frac{\sqrt{2}c^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}}{\sqrt{c}+\sqrt{b}x^{-n/2}}\right)}{b^{9/4}n}$$

output
```
-4/5/b/n/(x^(5/4*n))+4*c/b^2/n/(x^(1/4*n))+2^(1/2)*c^(5/4)*arctan(1-2^(1/2
)*b^(1/4)/c^(1/4)/(x^(1/4*n)))/b^(9/4)/n-2^(1/2)*c^(5/4)*arctan(1+2^(1/2)*
b^(1/4)/c^(1/4)/(x^(1/4*n)))/b^(9/4)/n-2^(1/2)*c^(5/4)*arctanh(2^(1/2)*b^(
1/4)*c^(1/4)/(x^(1/4*n))/(c^(1/2)+b^(1/2)/(x^(1/2*n))))/b^(9/4)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.18

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx=-\frac{4x^{-5n/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\frac{cx^n}{b}\right)}{5bn}$$

input
```
Integrate[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

output
```
(-4*Hypergeometric2F1[-5/4, 1, -1/4, -((c*x^n)/b)])/(5*b*n*x^((5*n)/4))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.45, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {10, 886, 868, 772, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{4}-1}}{bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{x^{-\frac{5n}{4}-1}}{b+cx^n}dx$$

$$\Big\downarrow \text{886}$$

$$-\frac{c\int \frac{x^{-\frac{n}{4}-1}}{cx^n+b}dx}{b}-\frac{4x^{-5n/4}}{5bn}$$

$$\Big\downarrow \text{868}$$

$$\frac{4c\int \frac{1}{cx^n+b}dx^{-n/4}}{bn}-\frac{4x^{-5n/4}}{5bn}$$

$$\Big\downarrow \text{772}$$

$$\frac{4c\int \frac{x^{-n}}{bx^{-n}+c}dx^{-n/4}}{bn}-\frac{4x^{-5n/4}}{5bn}$$

$$\Big\downarrow \text{843}$$

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\int \frac{1}{bx^{-n}+c}dx^{-n/4}}{b}\right)}{bn}-\frac{4x^{-5n/4}}{5bn}$$

$$\Big\downarrow \text{755}$$

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{b}x^{-n/2}}{bx^{-n}+c}dx^{-n/4}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{b}x^{-n/2}+\sqrt{c}}{bx^{-n}+c}dx^{-n/4}}{2\sqrt{c}}\right)}{b}\right)}{bn}-\frac{4x^{-5n/4}}{5bn}$$

$$\Big\downarrow \text{1476}$$

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{b}x^{-n/2}}{bx^{-n}+c}dx^{-n/4}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}}dx^{-n/4}}{2\sqrt{b}}+\frac{\int \frac{1}{x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}}dx^{-n/4}}{2\sqrt{b}}}{2\sqrt{c}}\right)}{b}\right)}{bn}-\frac{4x^{-5n/4}}{5bn}$$

$$\Big\downarrow \text{1082}$$

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\frac{\int \frac{1}{-x^{-n/2}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x^{-n/2}-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{b}x^{-n/2}}{bx^{-n}+c}dx^{-n/4}}{2\sqrt{c}}\right)}{b}\right)}{bn}-\frac{4x^{-5n/4}}{5bn}$$

$\Big\downarrow$ 217

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{b}x^{-n/2}}{bx^{-n}+c}dx^{-n/4}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}\right)}{bn}-\frac{4x^{-5n/4}}{5bn}$$

$\Big\downarrow$ 1479

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{b}\left(x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x^{-n/4}+\sqrt[4]{c}\right)}{\sqrt[4]{b}\left(x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\cdots}{2\cdots}\right)}{b}\cdots\right)}{bn}$$

$$\frac{4x^{-5n/4}}{5bn}$$

$$\Big\downarrow \; 25$$

$$\frac{4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{b}\left(x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x^{-n/4}+\sqrt[4]{c}\right)}{\sqrt[4]{b}\left(x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\text{arct}\cdots}{\cdots}}{2\sqrt{c}}\cdots\right)}{b}\cdots\right)}{bn}$$

$$\frac{4x^{-5n/4}}{5bn}$$

$$\Big\downarrow \; 27$$

$$4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{b}x^{-n/4}}{x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}}dx^{-n/4}}{2\sqrt{2}\sqrt{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}+\sqrt[4]{c}}{x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{c}x^{-n/4}}{\sqrt[4]{b}}+\frac{\sqrt{c}}{\sqrt{b}}}dx^{-n/4}}{2\sqrt{b}\sqrt[4]{c}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x\cdots}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\cdots}{2\sqrt{c}}\cdots\right)}{b}\cdots\right)$$
$$\overline{\qquad bn \qquad}$$

$$\frac{4x^{-5n/4}}{5bn}$$

$$\Big\downarrow \; 1103$$

$$4c\left(\frac{x^{-n/4}}{b}-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x^{-n/2}+\sqrt{c}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}+\sqrt{b}x\cdots\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\cdots}{2\sqrt{c}}\cdots\right)}{b}\cdots\right)$$
$$\overline{\qquad bn \qquad}$$

$$\frac{4x^{-5n/4}}{5bn}$$

input
```
Int[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

output
```
-4/(5*b*n*x^((5*n)/4)) + (4*c*(1/(b*x^(n/4)) - (c*((-(ArcTan[1 - (Sqrt[2]*
b^(1/4))/(c^(1/4)*x^(n/4))]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[
2]*b^(1/4))/(c^(1/4)*x^(n/4))]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) + (-
1/2*Log[Sqrt[c] + Sqrt[b]/x^(n/2) - (Sqrt[2]*b^(1/4)*c^(1/4))/x^(n/4)]/(Sq
rt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[c] + Sqrt[b]/x^(n/2) + (Sqrt[2]*b^(1/4)*
c^(1/4))/x^(n/4)]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/b))/(b*n)
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.38

method	result	size
risch	$\frac{4c\,x^{-\frac{n}{4}}}{b^{2}n}-\frac{4x^{-\frac{5n}{4}}}{5bn}+\left(\sum_{\_R=\operatorname{RootOf}\left(b^{9}n^{4}\_Z^{4}+c^{5}\right)}\_R\ln\left(x^{\frac{n}{4}}+\frac{b^{7}n^{3}\_R^{3}}{c^{4}}\right)\right)$	73

input
```
int(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
4*c/b^2/n/(x^(1/4*n))-4/5/b/n/(x^(1/4*n))^5+sum(_R*ln(x^(1/4*n)+b^7*n^3/c^
4*_R^3),_R=RootOf(_Z^4*b^9*n^4+c^5))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.30

$$\int\frac{x^{-1-\frac{n}{4}}}{bx^{n}+cx^{2n}}\,dx=$$

$$-\frac{4\,bx^{5}x^{-\frac{5}{4}n-5}+5\,b^{2}n\left(-\frac{c^{5}}{b^{9}n^{4}}\right)^{\frac{1}{4}}\log\left(\frac{b^{2}n\left(-\frac{c^{5}}{b^{9}n^{4}}\right)^{\frac{1}{4}}+cxx^{-\frac{1}{4}n-1}}{x}\right)-5\,b^{2}n\left(-\frac{c^{5}}{b^{9}n^{4}}\right)^{\frac{1}{4}}\log\left(-\frac{b^{2}n\left(-\frac{c^{5}}{b^{9}n^{4}}\right)^{\frac{1}{4}}-cxx^{-}}{x}\right.}{}$$

input
```
integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
-1/5*(4*b*x^5*x^(-5/4*n - 5) + 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log((b^2*n*(
-c^5/(b^9*n^4))^(1/4) + c*x*x^(-1/4*n - 1))/x) - 5*b^2*n*(-c^5/(b^9*n^4))^
(1/4)*log(-(b^2*n*(-c^5/(b^9*n^4))^(1/4) - c*x*x^(-1/4*n - 1))/x) + 5*I*b^
2*n*(-c^5/(b^9*n^4))^(1/4)*log((I*b^2*n*(-c^5/(b^9*n^4))^(1/4) + c*x*x^(-1
/4*n - 1))/x) - 5*I*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log((-I*b^2*n*(-c^5/(b^9*
n^4))^(1/4) + c*x*x^(-1/4*n - 1))/x) - 20*c*x*x^(-1/4*n - 1))/(b^2*n)
```

Sympy [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-n}x^{-\frac{n}{4}-1}}{b + cx^n}\,dx$$

input
```
integrate(x**(-1-1/4*n)/(b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(-n/4 - 1)/(x**n*(b + c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n} + bx^n}\,dx$$

input
```
integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
c^2*integrate(x^(3/4*n)/(b^2*c*x*x^n + b^3*x), x) + 4/5*(5*c*x^n - b)/(b^2
*n*x^(5/4*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n} + bx^n}\,dx$$

input
```
integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}} \, dx = \int \frac{1}{x^{\frac{n}{4}+1} \left(b\, x^n + c\, x^{2\,n}\right)} \, dx$$

input
```
int(1/(x^(n/4 + 1)*(b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n/4 + 1)*(b*x^n + c*x^(2*n))), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}} \, dx = \frac{-5x^{\frac{5n}{4}} \left(\int \frac{1}{x^{\frac{5n}{4}} cx + x^{\frac{n}{4}} bx} dx\right) cn - 4}{5x^{\frac{5n}{4}} bn}$$

input
```
int(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x)
```

output
```
( - 5*x**((5*n)/4)*int(1/(x**((5*n)/4)*c*x + x**(n/4)*b*x),x)*c*n - 4)/(5*
x**((5*n)/4)*b*n)
```

3.121 $\int x^{-1-n(-1+p)}(bx^n + cx^{2n})^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 37

$$\int x^{-1-n(-1+p)}\left(bx^n + cx^{2n}\right)^p \, dx = \frac{x^{-n(1+p)}\left(bx^n + cx^{2n}\right)^{1+p}}{cn(1+p)}$$

output
```
(b*x^n+c*x^(2*n))^(p+1)/c/n/(p+1)/(x^(n*(p+1)))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^{-1-n(-1+p)}\left(bx^n + cx^{2n}\right)^p \, dx = \frac{x^{-np}(b + cx^n)\left(x^n(b + cx^n)\right)^p}{cn(1+p)}$$

input
```
Integrate[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
```

output
```
((b + c*x^n)*(x^n*(b + c*x^n))^p)/(c*n*(1 + p)*x^(n*p))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p-1)-1}\left(bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{x^{-n(p+1)}\left(bx^n+cx^{2n}\right)^{p+1}}{cn(p+1)}$$

input
```
Int[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
```

output
```
(b*x^n + c*x^(2*n))^(1 + p)/(c*n*(1 + p)*x^(n*(1 + p)))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [F]

$$\int x^{-1-n(p-1)}\left(b\,x^n+c\,x^{2n}\right)^p dx$$

input
```
int(x^(-1-n*(p-1))*(b*x^n+c*x^(2*n))^p,x)
```

output
```
int(x^(-1-n*(p-1))*(b*x^n+c*x^(2*n))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\frac{\left(cxx^{-np+n-1}x^n+bxx^{-np+n-1}\right)\left(cx^{2n}+bx^n\right)^p}{(cnp+cn)x^n}$$

input
```
integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
(c*x*x^(-n*p + n - 1)*x^n + b*x*x^(-n*p + n - 1))*(c*x^(2*n) + b*x^n)^p/((
c*n*p + c*n)*x^n)
```

Sympy [F]

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int x^{-n(p-1)-1}\left(x^n(b+cx^n)\right)^p\,dx$$

input
```
integrate(x**(-1-n*(-1+p))*(b*x**n+c*x**(2*n))**p,x)
```

output
```
Integral(x**(-n*(p - 1) - 1)*(x**n*(b + c*x**n))**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^n+b)e^{\left(-np\log(x)+p\log(cx^n+b)+p\log(x^n)\right)}}{cn(p+1)}$$

input
```
integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
(c*x^n + b)*e^(-n*p*log(x) + p*log(c*x^n + b) + p*log(x^n))/(c*n*(p + 1))
```

Giac [F]

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2\,n}+bx^n\right)^p x^{-n(p-1)-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(p - 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int \frac{\left(b\,x^n+c\,x^{2\,n}\right)^p}{x^{n\,(p-1)+1}}\,dx$$

input
```
int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1),x)
```

output
```
int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\frac{\left(x^{2n}c+x^n b\right)^p\left(x^n c+b\right)}{x^{np}cn\left(p+1\right)}$$

input
```
int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
```

output
```
((x**(2*n)*c + x**n*b)**p*(x**n*c + b))/(x**(n*p)*c*n*(p + 1))
```

3.122 $\int x^{-1-n(1+2p)}(bx^n + cx^{2n})^p \, dx$

Optimal result

Integrand size = 28, antiderivative size = 38

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p \, dx = -\frac{x^{-2n(1+p)}\left(bx^n+cx^{2n}\right)^{1+p}}{bn(1+p)}$$

output
```
-(b*x^n+c*x^(2*n))^(p+1)/b/n/(p+1)/(x^(2*n*(p+1)))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p \, dx = -\frac{x^{-n(1+2p)}(b+cx^n)\left(x^n(b+cx^n)\right)^p}{bn(1+p)}$$

input
```
Integrate[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
```

output
```
-(((b + c*x^n)*(x^n*(b + c*x^n))^p)/(b*n*(1 + p)*x^(n*(1 + 2*p))))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(2p+1)-1}\left(bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \ 1920$$

$$-\frac{x^{-2n(p+1)}\left(bx^n+cx^{2n}\right)^{p+1}}{bn(p+1)}$$

input
```
Int[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
```

output
```
-((b*x^n + c*x^(2*n))^(1 + p)/(b*n*(1 + p)*x^(2*n*(1 + p))))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [F]

$$\int x^{-1-n(1+2p)}\left(b\,x^n+c\,x^{2n}\right)^p dx$$

input
```
int(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x)
```

output
```
int(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=-\frac{\left(cxx^{-2\,np-n-1}x^n+bxx^{-2\,np-n-1}\right)\left(cx^{2\,n}+bx^n\right)^p}{bnp+bn}$$

input
```
integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
-(c*x*x^(-2*n*p - n - 1)*x^n + b*x*x^(-2*n*p - n - 1))*(c*x^(2*n) + b*x^n)
^p/(b*n*p + b*n)
```

Sympy [F]

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int x^{-n(2p+1)-1}(x^n(b+cx^n))^p\,dx$$

input
```
integrate(x**(-1-n*(1+2*p))*(b*x**n+c*x**(2*n))**p,x)
```

output
```
Integral(x**(-n*(2*p + 1) - 1)*(x**n*(b + c*x**n))**p, x)
```

Maxima [F]

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2\,n}+bx^n\right)^p x^{-n(2\,p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

Giac [F]

$$\int x^{-1-n(1+2p)} \left(bx^n + cx^{2n}\right)^p \, dx = \int \left(cx^{2n} + bx^n\right)^p x^{-n(2p+1)-1} \, dx$$

input
```
integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(1+2p)} \left(bx^n + cx^{2n}\right)^p \, dx = \int \frac{\left(b\,x^n + c\,x^{2\,n}\right)^p}{x^{n\,(2\,p+1)+1}} \, dx$$

input
```
int((b*x^n + c*x^(2*n))^p/x^(n*(2*p + 1) + 1),x)
```

output
```
int((b*x^n + c*x^(2*n))^p/x^(n*(2*p + 1) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int x^{-1-n(1+2p)} \left(bx^n + cx^{2n}\right)^p \, dx = -\frac{\left(x^{2n}c + x^n b\right)^p \left(x^n c + b\right)}{x^{2np+n} bn \left(p+1\right)}$$

input
```
int(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x)
```

output
```
( - (x**(2*n)*c + x**n*b)**p*(x**n*c + b))/(x**(2*n*p + n)*b*n*(p + 1))
```

3.123 $\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 93

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}+\frac{b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)\left(ab+b^2x^n\right)}$$

output
```
a*x^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(3*a+3*b*x^n)+b^2*x^(3+n)*(a^2+2*a
*b*x^n+b^2*x^(2*n))^(1/2)/(3+n)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^3\sqrt{(a+bx^n)^2}(a(3+n)+3bx^n)}{3(3+n)\left(a+bx^n\right)}$$

input
```
Integrate[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^3*Sqrt[(a + b*x^n)^2]*(a*(3 + n) + 3*b*x^n))/(3*(3 + n)*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^2\left(b^2x^n+ab\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \ 802$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^{n+2}+abx^2\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(\frac{1}{3}abx^3+\frac{b^2x^{n+3}}{n+3}\right)}{ab+b^2x^n}$$

input
```
Int[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((a*b*x^3)/3 + (b^2*x^(3 + n))/(3 + n
)))/(a*b + b^2*x^n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,a\,x^3}{3a+3b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\,b\,x^3x^n}{(a+b\,x^n)(3+n)}$	61

input
```
int(x^2*(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x^3+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(3
+n)*x^3*x^n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.30

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{3\,bx^3x^n+(an+3\,a)x^3}{3\,(n+3)}$$

input
```
integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(3*b*x^3*x^n + (a*n + 3*a)*x^3)/(n + 3)
```

Sympy [F]

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^2\sqrt{(a+bx^n)^2}\,dx$$

input
```
integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**2*sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{3\,bx^3x^n+a(n+3)x^3}{3\,(n+3)}$$

input
```
integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
1/3*(3*b*x^3*x^n + a*(n + 3)*x^3)/(n + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$
$$=\frac{3\,bx^3x^n\mathrm{sgn}(bx^n+a)+anx^3\mathrm{sgn}(bx^n+a)+3\,ax^3\mathrm{sgn}(bx^n+a)}{3\,(n+3)}$$

input
```
integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
1/3*(3*b*x^3*x^n*sgn(b*x^n + a) + a*n*x^3*sgn(b*x^n + a) + 3*a*x^3*sgn(b*x
^n + a))/(n + 3)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^2\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

input
```
int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.26

$$\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^3(3x^nb+an+3a)}{3n+9}$$

input
```
int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
(x**3*(3*x**n*b + a*n + 3*a))/(3*(n + 3))
```

3.124 $\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 93

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2\left(a+bx^n\right)}+\frac{b^2x^{2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+n)\left(ab+b^2x^n\right)}$$

output
```
a*x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2*a+2*b*x^n)+b^2*x^(2+n)*(a^2+2*a
*b*x^n+b^2*x^(2*n))^(1/2)/(2+n)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^2\sqrt{\left(a+bx^n\right)^2}(a(2+n)+2bx^n)}{2(2+n)\left(a+bx^n\right)}$$

input
```
Integrate[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^2*Sqrt[(a + b*x^n)^2]*(a*(2 + n) + 2*b*x^n))/(2*(2 + n)*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x\left(b^2x^n+ab\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^{n+1}+abx\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(\frac{1}{2}abx^2+\frac{b^2x^{n+2}}{n+2}\right)}{ab+b^2x^n}$$

input
```
Int[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((a*b*x^2)/2 + (b^2*x^(2 + n))/(2 + n
)))/(a*b + b^2*x^n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,a\,x^{2}}{2a+2b\,x^{n}}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b\,x^{2}x^{n}}{(a+b\,x^{n})(2+n)}$	61

input
```
int(x*(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(2
+n)*x^2*x^n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.30

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{2\,bx^2x^n+(an+2\,a)x^2}{2\,(n+2)}$$

input
```
integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*b*x^2*x^n + (a*n + 2*a)*x^2)/(n + 2)
```

Sympy [F]

$$\int x\sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx = \int x\sqrt{(a + bx^n)^2}\,dx$$

input
```
integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x*sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

$$\int x\sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx = \frac{2\,bx^2x^n + a(n+2)x^2}{2\,(n+2)}$$

input
```
integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*b*x^2*x^n + a*(n + 2)*x^2)/(n + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int x\sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx = \frac{2\,bx^2x^n\mathrm{sgn}(bx^n + a) + anx^2\mathrm{sgn}(bx^n + a) + 2\,ax^2\mathrm{sgn}(bx^n + a)}{2\,(n+2)}$$

input
```
integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
1/2*(2*b*x^2*x^n*sgn(b*x^n + a) + a*n*x^2*sgn(b*x^n + a) + 2*a*x^2*sgn(b*x
^n + a))/(n + 2)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

input
```
int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.26

$$\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^2(2x^nb+an+2a)}{2n+4}$$

input
```
int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
(x**2*(2*x**n*b + a*n + 2*a))/(2*(n + 2))
```

3.125 $\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}+\frac{b^2x^{1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+n)\,(ab+b^2x^n)}$$

output
```
a*x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+b^2*x^(1+n)*(a^2+2*a*b*x^n
+b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x\sqrt{(a+bx^n)^2}(a+an+bx^n)}{(1+n)\,(a+bx^n)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x*Sqrt[(a + b*x^n)^2]*(a + a*n + b*x^n))/((1 + n)*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$
$$\Big\downarrow \text{1384}$$
$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^n+ab\right)dx}{ab+b^2x^n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(abx+\frac{b^2x^{n+1}}{n+1}\right)}{ab+b^2x^n}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(a*b*x + (b^2*x^(1 + n))/(1 + n)))/(a
*b + b^2*x^n)
```

Defintions of rubi rules used

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,a x}{a+b\,x^{n}}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b x\,x^{n}}{(a+b\,x^{n})(1+n)}$	56

input
```
int((a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(1+n)*x*
x^n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.23

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{bxx^n+(an+a)x}{n+1}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
(b*x*x^n + (a*n + a)*x)/(n + 1)
```

Sympy [F]

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.22

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{a(n+1)x+bxx^n}{n+1}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
(a*(n + 1)*x + b*x*x^n)/(n + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.28

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\left(ax+\frac{bx^{n+1}}{n+1}\right)\operatorname{sgn}(bx^n+a)$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
(a*x + b*x^(n + 1)/(n + 1))*sgn(b*x^n + a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int \sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.19

$$\int \sqrt{a^2 + 2abx^n + b^2x^{2n}}\,dx = \frac{x(x^n b + an + a)}{n + 1}$$

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
(x*(x**n*b + a*n + a))/(n + 1)
```

3.126 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx=\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{a+bx^n}$$

output
```
b^2*x^n*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+a*(a^2+2*a*b*x^n
+b^2*x^(2*n))^(1/2)*ln(x)/(a+b*x^n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx=\frac{\sqrt{(a+bx^n)^2}(bx^n+a\log(x^n))}{n(a+bx^n)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(b*x^n + a*Log[x^n]))/(n*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{b^2x^n+ab}{x}dx}{ab+b^2x^n}$$

$$\Big\downarrow \ 802$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^{n-1}+\frac{ab}{x}\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(ab\log(x)+\frac{b^2x^n}{n}\right)}{ab+b^2x^n}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((b^2*x^n)/n + a*b*Log[x]))/(a*b + b^
2*x^n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,a\ln(x)}{a+b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\,b\,x^n}{(a+b\,x^n)n}$	54

input
```
int((a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*ln(x)+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/n*x^
n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx=\frac{an\log(x)+bx^n}{n}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="fricas")
```

output
```
(a*n*log(x) + b*x^n)/n
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x}\,dx = \int \frac{\sqrt{(a + bx^n)^2}}{x}\,dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x,x)
```

output
```
Integral(sqrt((a + b*x**n)**2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x}\,dx = a\log(x) + \frac{bx^n}{n}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="maxima")
```

output
```
a*log(x) + b*x^n/n
```

Giac [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x}\,dx = \int \frac{\sqrt{b^2x^{2n} + 2abx^n + a^2}}{x}\,dx$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x}\, dx = \int \frac{\sqrt{a^2 + b^2\, x^{2\,n} + 2\,a\,b\,x^n}}{x}\, dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x,x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x}\, dx = \frac{x^n b + \log(x)\, an}{n}$$

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x)
```

output
```
(x**n*b + log(x)*a*n)/n
```

3.127 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx=-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)}-\frac{b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\,(ab+b^2x^n)}$$

output
```
-a*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x/(a+b*x^n)-b^2*x^(-1+n)*(a^2+2*a*b*x
^n+b^2*x^(2*n))^(1/2)/(1-n)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx=\frac{\sqrt{(a+bx^n)^2}(a-an+bx^n)}{(-1+n)x\,(a+bx^n)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(a - a*n + b*x^n))/((-1 + n)*x*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{b^2x^n+ab}{x^2}dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^{n-2}+\frac{ab}{x^2}\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{ab}{x}-\frac{b^2x^{n-1}}{1-n}\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}{ab+b^2x^n}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

output
```
((-((a*b)/x) - (b^2*x^(-1 + n))/(1 - n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n
)])/(a*b + b^2*x^n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^{n})^{2}}\,a}{(a+b\,x^{n})x}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b\,x^{n}}{(a+b\,x^{n})(-1+n)x}$	61

input
```
int((a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*b/
x*x^n
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx=-\frac{an-bx^n-a}{(n-1)x}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-(a*n - b*x^n - a)/((n - 1)*x)
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2}\,dx = \int \frac{\sqrt{(a + bx^n)^2}}{x^2}\,dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a + b*x**n)**2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2}\,dx = -\frac{a(n-1) - bx^n}{(n-1)x}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-(a*(n - 1) - b*x^n)/((n - 1)*x)
```

Giac [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2}\,dx = \int \frac{\sqrt{b^2x^{2n} + 2\,abx^n + a^2}}{x^2}\,dx$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2}\,dx = \int \frac{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}{x^2}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2,x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2}\,dx = \frac{x^n b - an + a}{x\,(n-1)}$$

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x)
```

output
```
(x**n*b - a*n + a)/(x*(n - 1))
```

3.128 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 96

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx = -\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\,(a+bx^n)} - \frac{b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\,(ab+b^2x^n)}$$

output
```
-1/2*a*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2/(a+b*x^n)-b^2*x^(-2+n)*(a^2+2
*a*b*x^n+b^2*x^(2*n))^(1/2)/(2-n)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx = \frac{\sqrt{(a+bx^n)^2}(-a(-2+n)+2bx^n)}{2(-2+n)x^2\,(a+bx^n)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(-(a*(-2 + n)) + 2*b*x^n))/(2*(-2 + n)*x^2*(a + b*x^n
))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{b^2x^n+ab}{x^3}dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^{n-3}+\frac{ab}{x^3}\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{ab}{2x^2}-\frac{b^2x^{n-2}}{2-n}\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}{ab+b^2x^n}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

output
```
((-1/2*(a*b)/x^2 - (b^2*x^(-2 + n))/(2 - n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^
(2*n)])/(a*b + b^2*x^n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a}{2\left(a+b\,x^{n}\right)x^{2}}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b\,x^{n}}{\left(a+b\,x^{n}\right)\left(-2+n\right)x^{2}}$	61

input
```
int((a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-2
+n)*b/x^2*x^n
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx=-\frac{an-2\,bx^n-2\,a}{2\,(n-2)x^2}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a*n - 2*b*x^n - 2*a)/((n - 2)*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^3}\,dx = \int \frac{\sqrt{(a + bx^n)^2}}{x^3}\,dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**3,x)
```

output
```
Integral(sqrt((a + b*x**n)**2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^3}\,dx = -\frac{a(n-2) - 2\,bx^n}{2\,(n-2)x^2}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(a*(n - 2) - 2*b*x^n)/((n - 2)*x^2)
```

Giac [F]

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^3}\,dx = \int \frac{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}{x^3}\,dx$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^3}\,dx = \int \frac{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}{x^3}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3,x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^3}\,dx = \frac{2x^n b - an + 2a}{2x^2\,(n-2)}$$

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x)
```

output
```
(2*x**n*b - a*n + 2*a)/(2*x**2*(n - 2))
```

3.129 $\int x^2(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 212

$$\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\,(a+bx^n)}+\frac{b^4x^{3(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(1+n)\,(ab+b^2x^n)}+\frac{3a^2b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)\,(ab+b^2x^n)}+\frac{3ab^3x^{3+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+2n)\,(ab+b^2x^n)}$$

output
```
a^3*x^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(3*a+3*b*x^n)+1/3*b^4*x^(3+3*n)*
(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)+3*a^2*b^2*x^(3+n)*(a
^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(3+n)/(a*b+b^2*x^n)+3*a*b^3*x^(3+2*n)*(a^2
+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(3+2*n)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.58

$$\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{x^3\sqrt{(a+bx^n)^2}\left(a^3(9+18n+11n^2+2n^3)+9a^2b(3+5n+2n^2)\,x^n+9ab^2(3+4n+n^2)\,x\right.}{3(1+n)(3+n)(3+2n)\,(a+bx^n)}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^3*Sqrt[(a + b*x^n)^2]*(a^3*(9 + 18*n + 11*n^2 + 2*n^3) + 9*a^2*b*(3 + 5
*n + 2*n^2)*x^n + 9*a*b^2*(3 + 4*n + n^2)*x^(2*n) + b^3*(9 + 9*n + 2*n^2)*
x^(3*n)))/(3*(1 + n)*(3 + n)*(3 + 2*n)*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.107$, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^2\left(b^2x^n+ab\right)^3dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(3ab^5x^{2(n+1)}+3a^2b^4x^{n+2}+b^6x^{3n+2}+a^3b^3x^2\right)dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(\frac{1}{3}a^3b^3x^3+\frac{3a^2b^4x^{n+3}}{n+3}+\frac{3ab^5x^{2n+3}}{2n+3}+\frac{b^6x^{3(n+1)}}{3(n+1)}\right)}{ab^3+b^4x^n}$$

input
```
Int[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((a^3*b^3*x^3)/3 + (b^6*x^(3*(1 + n))
)/(3*(1 + n)) + (3*a^2*b^4*x^(3 + n))/(3 + n) + (3*a*b^5*x^(3 + 2*n))/(3 +
 2*n)))/(a*b^3 + b^4*x^n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,a^3x^3}{3a+3b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\,b^3x^3x^{3n}}{3(a+b\,x^n)(1+n)}+\frac{3\sqrt{(a+b\,x^n)^2}\,b^2a\,x^3x^{2n}}{(a+b\,x^n)(3+2n)}+\frac{3\sqrt{(a+b\,x^n)^2}\,b\,a^2x^3x^n}{(a+b\,x^n)(3+n)}$	146

input
```
int(x^2*(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x^3+1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n
)*b^3*x^3/(1+n)*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2*a/(3+2*n)*x^3*
(x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b*a^2/(3+n)*x^3*x^n
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.68

$$\int x^2(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx=\frac{(2\,b^3n^2+9\,b^3n+9\,b^3)x^3x^{3\,n}+9\,(ab^2n^2+4\,ab^2n+3\,ab^2)x^3x^{2\,n}+9\,(2\,a^2bn^2+5\,a^2bn+3\,a}{3\,(2\,n^3+11\,n^2+18\,n+9)}$$

input
```
integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
1/3*((2*b^3*n^2 + 9*b^3*n + 9*b^3)*x^3*x^(3*n) + 9*(a*b^2*n^2 + 4*a*b^2*n
+ 3*a*b^2)*x^3*x^(2*n) + 9*(2*a^2*b*n^2 + 5*a^2*b*n + 3*a^2*b)*x^3*x^n + (
2*a^3*n^3 + 11*a^3*n^2 + 18*a^3*n + 9*a^3)*x^3)/(2*n^3 + 11*n^2 + 18*n + 9
)
```

Sympy [F]

$$\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int x^2\left((a+bx^n)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x**2*((a + b*x**n)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.51

$$\int x^2 \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx = \frac{(2\,n^2+9\,n+9)b^3x^3x^{3\,n}+9\,(n^2+4\,n+3)ab^2x^3x^{2\,n}+9\,(2\,n^2+5\,n+3)a^2bx^3x^n+(2\,n^3+ \ldots}{3\,(2\,n^3+11\,n^2+18\,n+9)}$$

input
```
integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
1/3*((2*n^2 + 9*n + 9)*b^3*x^3*x^(3*n) + 9*(n^2 + 4*n + 3)*a*b^2*x^3*x^(2*
n) + 9*(2*n^2 + 5*n + 3)*a^2*b*x^3*x^n + (2*n^3 + 11*n^2 + 18*n + 9)*a^3*x
^3)/(2*n^3 + 11*n^2 + 18*n + 9)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.38

$$\int x^2 \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx = \frac{2\,b^3n^2x^3x^{3\,n}\mathrm{sgn}(bx^n+a)+9\,ab^2n^2x^3x^{2\,n}\mathrm{sgn}(bx^n+a)+18\,a^2bn^2x^3x^n\mathrm{sgn}(bx^n+a)+2\,a^3 \ldots}{\ldots}$$

input
```
integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
1/3*(2*b^3*n^2*x^3*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^3*x^(2*n)*sgn(b*
x^n + a) + 18*a^2*b*n^2*x^3*x^n*sgn(b*x^n + a) + 2*a^3*n^3*x^3*sgn(b*x^n +
 a) + 9*b^3*n*x^3*x^(3*n)*sgn(b*x^n + a) + 36*a*b^2*n*x^3*x^(2*n)*sgn(b*x^
n + a) + 45*a^2*b*n*x^3*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^3*sgn(b*x^n + a)
 + 9*b^3*x^3*x^(3*n)*sgn(b*x^n + a) + 27*a*b^2*x^3*x^(2*n)*sgn(b*x^n + a)
+ 27*a^2*b*x^3*x^n*sgn(b*x^n + a) + 18*a^3*n*x^3*sgn(b*x^n + a) + 9*a^3*x^
3*sgn(b*x^n + a))/(2*n^3 + 11*n^2 + 18*n + 9)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} \, dx = \int x^2\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2} \,dx$$

input
```
int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.71

$$\int x^2 \left(a^2+2abx^n +b^2x^{2n}\right)^{3/2} \, dx = \frac{x^3(2x^{3n}b^3n^2+9x^{3n}b^3n+9x^{3n}b^3+9x^{2n}a\,b^2n^2+36x^{2n}a\,b^2n+27x^{2n}a\,b^2+18x^na^2b\,n^2+4}{6n^3+33n^2+54n+27}$$

input
```
int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
(x**3*(2*x**(3*n)*b**3*n**2 + 9*x**(3*n)*b**3*n + 9*x**(3*n)*b**3 + 9*x**(
2*n)*a*b**2*n**2 + 36*x**(2*n)*a*b**2*n + 27*x**(2*n)*a*b**2 + 18*x**n*a**
2*b*n**2 + 45*x**n*a**2*b*n + 27*x**n*a**2*b + 2*a**3*n**3 + 11*a**3*n**2
+ 18*a**3*n + 9*a**3))/(3*(2*n**3 + 11*n**2 + 18*n + 9))
```

3.130 $\int x(a^2 + 2abx^n + b^2x^{2n})^{3/2}\, dx$

Optimal result

Integrand size = 26, antiderivative size = 211

$$\int x\left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2}\, dx = \frac{a^3x^2\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2\left(a + bx^n\right)} + \frac{3ab^3x^{2(1+n)}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2(1+n)\left(ab + b^2x^n\right)} + \frac{3a^2b^2x^{2+n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(2+n)\left(ab + b^2x^n\right)} + \frac{b^4x^{2+3n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(2+3n)\left(ab + b^2x^n\right)}$$

output
```
a^3*x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2*a+2*b*x^n)+3/2*a*b^3*x^(2+2*n
)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)+3*a^2*b^2*x^(2+n)*
(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2+n)/(a*b+b^2*x^n)+b^4*x^(2+3*n)*(a^2+2
*a*b*x^n+b^2*x^(2*n))^(1/2)/(2+3*n)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.59

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}dx=\frac{x^2\sqrt{(a+bx^n)^2}\left(a^3(4+12n+11n^2+3n^3)+6a^2b(2+5n+3n^2)x^n+3ab^2(4+8n+3n^2)\right.}{2(1+n)(2+n)(2+3n)(a+bx^n)}$$

input
```
Integrate[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^2*Sqrt[(a + b*x^n)^2]*(a^3*(4 + 12*n + 11*n^2 + 3*n^3) + 6*a^2*b*(2 + 5
*n + 3*n^2)*x^n + 3*a*b^2*(4 + 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 + 3*n + n^2
)*x^(3*n)))/(2*(1 + n)*(2 + n)*(2 + 3*n)*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x\left(b^2x^n+ab\right)^3dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(3a^2b^4x^{n+1}+3ab^5x^{2n+1}+b^6x^{3n+1}+a^3b^3x\right)dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(\frac{1}{2}a^3b^3x^2+\frac{3a^2b^4x^{n+2}}{n+2}+\frac{3ab^5x^{2(n+1)}}{2(n+1)}+\frac{b^6x^{3n+2}}{3n+2}\right)}{ab^3+b^4x^n}$$

input
```
Int[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((a^3*b^3*x^2)/2 + (3*a*b^5*x^(2*(1 +
 n)))/(2*(1 + n)) + (3*a^2*b^4*x^(2 + n))/(2 + n) + (b^6*x^(2 + 3*n))/(2 +
 3*n)))/(a*b^3 + b^4*x^n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,a^3x^2}{2a+2b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\,b^3x^2x^{3n}}{(a+b\,x^n)(2+3n)}+\frac{3\sqrt{(a+b\,x^n)^2}\,b^2a\,x^2x^{2n}}{2(a+b\,x^n)(1+n)}+\frac{3\sqrt{(a+b\,x^n)^2}\,b\,a^2x^2x^n}{(a+b\,x^n)(2+n)}$	145

input
```
int(x*(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^
3/(2+3*n)*x^2*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2*a*x^2/(1+n)*(x
^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b*a^2/(2+n)*x^2*x^n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.69

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{2\,(b^3n^2+3\,b^3n+2\,b^3)x^2x^{3\,n}+3\,(3\,ab^2n^2+8\,ab^2n+4\,ab^2)x^2x^{2\,n}+6\,(3\,a^2bn^2+5\,a^2bn+2}{2\,(3\,n^3+11\,n^2+12\,n+4)}$$

input
```
integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^2*x^(3*n) + 3*(3*a*b^2*n^2 + 8*a*b^2*
n + 4*a*b^2)*x^2*x^(2*n) + 6*(3*a^2*b*n^2 + 5*a^2*b*n + 2*a^2*b)*x^2*x^n +
 (3*a^3*n^3 + 11*a^3*n^2 + 12*a^3*n + 4*a^3)*x^2)/(3*n^3 + 11*n^2 + 12*n +
 4)
```

Sympy [F]

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int x\left((a+bx^n)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x*((a + b*x**n)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.52

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{2\,(n^2+3\,n+2)b^3x^2x^{3\,n}+3\,(3\,n^2+8\,n+4)ab^2x^2x^{2\,n}+6\,(3\,n^2+5\,n+2)a^2bx^2x^n+(3\,n^3}{2\,(3\,n^3+11\,n^2+12\,n+4)}$$

input
```
integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*(n^2 + 3*n + 2)*b^3*x^2*x^(3*n) + 3*(3*n^2 + 8*n + 4)*a*b^2*x^2*x^(
2*n) + 6*(3*n^2 + 5*n + 2)*a^2*b*x^2*x^n + (3*n^3 + 11*n^2 + 12*n + 4)*a^3
*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.38

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{2\,b^3n^2x^2x^{3\,n}\mathrm{sgn}(bx^n+a)+9\,ab^2n^2x^2x^{2\,n}\mathrm{sgn}(bx^n+a)+18\,a^2bn^2x^2x^n\mathrm{sgn}(bx^n+a)+3\,a^3}{}$$

input
```
integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
1/2*(2*b^3*n^2*x^2*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^2*x^(2*n)*sgn(b*
x^n + a) + 18*a^2*b*n^2*x^2*x^n*sgn(b*x^n + a) + 3*a^3*n^3*x^2*sgn(b*x^n +
 a) + 6*b^3*n*x^2*x^(3*n)*sgn(b*x^n + a) + 24*a*b^2*n*x^2*x^(2*n)*sgn(b*x^
n + a) + 30*a^2*b*n*x^2*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^2*sgn(b*x^n + a)
 + 4*b^3*x^2*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*x^2*x^(2*n)*sgn(b*x^n + a)
+ 12*a^2*b*x^2*x^n*sgn(b*x^n + a) + 12*a^3*n*x^2*sgn(b*x^n + a) + 4*a^3*x^
2*sgn(b*x^n + a))/(3*n^3 + 11*n^2 + 12*n + 4)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int x\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}\,dx$$

input
```
int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.72

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{x^2(2x^{3n}b^3n^2+6x^{3n}b^3n+4x^{3n}b^3+9x^{2n}a\,b^2n^2+24x^{2n}a\,b^2n+12x^{2n}a\,b^2+18x^na^2b\,n^2+3}{6n^3+22n^2+24n+8}$$

input
```
int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
(x**2*(2*x**(3*n)*b**3*n**2 + 6*x**(3*n)*b**3*n + 4*x**(3*n)*b**3 + 9*x**(
2*n)*a*b**2*n**2 + 24*x**(2*n)*a*b**2*n + 12*x**(2*n)*a*b**2 + 18*x**n*a**
2*b*n**2 + 30*x**n*a**2*b*n + 12*x**n*a**2*b + 3*a**3*n**3 + 11*a**3*n**2
+ 12*a**3*n + 4*a**3))/(2*(3*n**3 + 11*n**2 + 12*n + 4))
```

3.131 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n} + \frac{3a^2b^2x^{1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+n)\left(ab+b^2x^n\right)} + \frac{3ab^3x^{1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+2n)\left(ab+b^2x^n\right)} + \frac{b^4x^{1+3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+3n)\left(ab+b^2x^n\right)}$$

output
```
a^3*x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+3*a^2*b^2*x^(1+n)*(a^2+2
*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)+3*a*b^3*x^(1+2*n)*(a^2+2*a
*b*x^n+b^2*x^(2*n))^(1/2)/(1+2*n)/(a*b+b^2*x^n)+b^4*x^(1+3*n)*(a^2+2*a*b*x
^n+b^2*x^(2*n))^(1/2)/(1+3*n)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.59

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{x\sqrt{(a+bx^n)^2}\left(a^3(1+6n+11n^2+6n^3)+3a^2b(1+5n+6n^2)\,x^n+3ab^2(1+4n+3n^2)\,x^{2n}\right.}{(1+n)(1+2n)(1+3n)\,(a+bx^n)}$$

input
```
Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x*Sqrt[(a + b*x^n)^2]*(a^3*(1 + 6*n + 11*n^2 + 6*n^3) + 3*a^2*b*(1 + 5*n
+ 6*n^2)*x^n + 3*a*b^2*(1 + 4*n + 3*n^2)*x^(2*n) + b^3*(1 + 3*n + 2*n^2)*x
^(3*n)))/((1 + n)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.51, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1384, 775, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^n+ab\right)^3dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{775}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(3a^2b^4x^n+3ab^5x^{2n}+b^6x^{3n}+a^3b^3\right)dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(a^3b^3x+\frac{3a^2b^4x^{n+1}}{n+1}+\frac{3ab^5x^{2n+1}}{2n+1}+\frac{b^6x^{3n+1}}{3n+1}\right)}{ab^3+b^4x^n}$$

input
```
Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(a^3*b^3*x + (3*a^2*b^4*x^(1 + n))/(1
 + n) + (3*a*b^5*x^(1 + 2*n))/(1 + 2*n) + (b^6*x^(1 + 3*n))/(1 + 3*n)))/(a
*b^3 + b^4*x^n)
```

Defintions of rubi rules used

rule 775
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{3}x}{a+b\,x^{n}}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{3}x\,x^{3n}}{\left(a+b\,x^{n}\right)\left(1+3n\right)}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{2}ax\,x^{2n}}{\left(a+b\,x^{n}\right)\left(1+2n\right)}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b\,a^{2}x\,x^{n}}{\left(a+b\,x^{n}\right)\left(1+n\right)}$	138

input
```
int((a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3/(1+3
*n)*x*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2*a/(1+2*n)*x*(x^n)^2+3*((
a+b*x^n)^2)^(1/2)/(a+b*x^n)*b*a^2/(1+n)*x*x^n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.63

$$\int \left(a^2+2abx^n +b^2x^{2n}\right)^{3/2}\,dx=\frac{(2\,b^3n^2+3\,b^3n+b^3)xx^{3\,n}+3\,(3\,ab^2n^2+4\,ab^2n+ab^2)xx^{2\,n}+3\,(6\,a^2bn^2+5\,a^2bn+a^2b)x}{6\,n^3+11\,n^2+6\,n+1}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
((2*b^3*n^2 + 3*b^3*n + b^3)*x*x^(3*n) + 3*(3*a*b^2*n^2 + 4*a*b^2*n + a*b^
2)*x*x^(2*n) + 3*(6*a^2*b*n^2 + 5*a^2*b*n + a^2*b)*x*x^n + (6*a^3*n^3 + 11
*a^3*n^2 + 6*a^3*n + a^3)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [F]

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.49

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{(2n^2+3n+1)b^3xx^{3n}+3(3n^2+4n+1)ab^2xx^{2n}+3(6n^2+5n+1)a^2bxx^n+(6n^3+}{6n^3+11n^2+6n+1}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
((2*n^2 + 3*n + 1)*b^3*x*x^(3*n) + 3*(3*n^2 + 4*n + 1)*a*b^2*x*x^(2*n) + 3
*(6*n^2 + 5*n + 1)*a^2*b*x*x^n + (6*n^3 + 11*n^2 + 6*n + 1)*a^3*x)/(6*n^3
+ 11*n^2 + 6*n + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.28

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{6a^3n^3x\operatorname{sgn}(bx^n+a)+2b^3n^2xx^{3n}\operatorname{sgn}(bx^n+a)+9ab^2n^2xx^{2n}\operatorname{sgn}(bx^n+a)+18a^2bn^2xx^{n}}{}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
(6*a^3*n^3*x*sgn(b*x^n + a) + 2*b^3*n^2*x*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2
*n^2*x*x^(2*n)*sgn(b*x^n + a) + 18*a^2*b*n^2*x*x^n*sgn(b*x^n + a) + 11*a^3
*n^2*x*sgn(b*x^n + a) + 3*b^3*n*x*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*n*x*x^
(2*n)*sgn(b*x^n + a) + 15*a^2*b*n*x*x^n*sgn(b*x^n + a) + 6*a^3*n*x*sgn(b*x
^n + a) + b^3*x*x^(3*n)*sgn(b*x^n + a) + 3*a*b^2*x*x^(2*n)*sgn(b*x^n + a)
+ 3*a^2*b*x*x^n*sgn(b*x^n + a) + a^3*x*sgn(b*x^n + a))/(6*n^3 + 11*n^2 + 6
*n + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \int \left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2} dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.71

$$\int \left(a^2+2abx^n +b^2x^{2n}\right)^{3/2} dx = \frac{x(2x^{3n}b^3n^2+3x^{3n}b^3n+x^{3n}b^3+9x^{2n}a\,b^2n^2+12x^{2n}a\,b^2n+3x^{2n}a\,b^2+18x^na^2b\,n^2+15x^n}{6n^3+11n^2+6n+1}$$

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
(x*(2*x**(3*n)*b**3*n**2 + 3*x**(3*n)*b**3*n + x**(3*n)*b**3 + 9*x**(2*n)*
a*b**2*n**2 + 12*x**(2*n)*a*b**2*n + 3*x**(2*n)*a*b**2 + 18*x**n*a**2*b*n*
*2 + 15*x**n*a**2*b*n + 3*x**n*a**2*b + 6*a**3*n**3 + 11*a**3*n**2 + 6*a**
3*n + a**3))/(6*n**3 + 11*n**2 + 6*n + 1)
```

3.132 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 196

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x}\,dx=\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\left(ab+b^2x^n\right)}+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\left(ab+b^2x^n\right)}+\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{a+bx^n}$$

output

```
3*a^2*b^2*x^n*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+3/2*a*b^3*
x^(2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+1/3*b^4*x^(3*n)*
(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+a^3*(a^2+2*a*b*x^n+b^2*x
^(2*n))^(1/2)*ln(x)/(a+b*x^n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.34

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x} dx = \frac{\sqrt{(a + bx^n)^2}(bx^n(18a^2 + 9abx^n + 2b^2x^{2n}) + 6a^3 \log (x^n))}{6n\,(a + bx^n)}$$

input
```
Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(b*x^n*(18*a^2 + 9*a*b*x^n + 2*b^2*x^(2*n)) + 6*a^3*L
og[x^n]))/(6*n*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1384, 798, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x} dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int \frac{(b^2x^n+ab)^3}{x}dx}{ab^3 + b^4x^n}$$

$$\Big\downarrow \ 798$$

$$\frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int b^3x^{-n}(bx^n + a)^3\,dx^n}{n\,(ab^3 + b^4x^n)}$$

$$\Big\downarrow \ 27$$

$$\frac{b^3\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int x^{-n}(bx^n + a)^3\,dx^n}{n\,(ab^3 + b^4x^n)}$$

$$\Big\downarrow \ 49$$

$$\frac{b^3\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(a^3x^{-n}+3ab^2x^n+b^3x^{2n}+3a^2b\right)dx^n}{n\left(ab^3+b^4x^n\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^3\sqrt{a^2+2abx^n+b^2x^{2n}}\left(a^3\log\left(x^n\right)+3a^2bx^n+\frac{3}{2}ab^2x^{2n}+\frac{1}{3}b^3x^{3n}\right)}{n\left(ab^3+b^4x^n\right)}$$

input
```
Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

output
```
(b^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(3*a^2*b*x^n + (3*a*b^2*x^(2*n))/
2 + (b^3*x^(3*n))/3 + a^3*Log[x^n]))/(n*(a*b^3 + b^4*x^n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{\sqrt{(a+b x^n)^2}\, a^3 \ln(x)}{a+b x^n}+\frac{\sqrt{(a+b x^n)^2}\, b^3 x^{3n}}{3(a+b x^n)n}+\frac{3\sqrt{(a+b x^n)^2}\, b^2 a\, x^{2n}}{2(a+b x^n)n}+\frac{3\sqrt{(a+b x^n)^2}\, b\, a^2 x^n}{(a+b x^n)n}$	127

input
```
int((a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*ln(x)+1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*
b^3/n*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2*a/n*(x^n)^2+3*((a+b*x^
n)^2)^(1/2)/(a+b*x^n)*b*a^2/n*x^n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x}\,dx=\frac{6\,a^3n\log(x)+2\,b^3x^{3\,n}+9\,ab^2x^{2\,n}+18\,a^2bx^n}{6\,n}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*n*log(x) + 2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

Sympy [F]

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x}\,dx=\int \frac{\left((a+bx^n)^2\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x,x)
```

output
```
Integral(((a + b*x**n)**2)**(3/2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x} dx = a^3 \log(x) + \frac{2 b^3 x^{3n} + 9 ab^2 x^{2n} + 18 a^2 b x^n}{6 n}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + 1/6*(2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

Giac [F]

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x} dx = \int \frac{(b^2 x^{2n} + 2 abx^n + a^2)^{\frac{3}{2}}}{x} dx$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x} dx = \int \frac{(a^2 + b^2 x^{2n} + 2 a b x^n)^{3/2}}{x} dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x,x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x}\,dx = \frac{2x^{3n}b^3 + 9x^{2n}a\,b^2 + 18x^na^2b + 6\log(x)\,a^3n}{6n}$$

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x)
```

output
```
(2*x**(3*n)*b**3 + 9*x**(2*n)*a*b**2 + 18*x**n*a**2*b + 6*log(x)*a**3*n)/(
6*n)
```

3.133 $\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 212

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^2} dx = -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x(a+bx^n)} - \frac{3a^2b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)(ab+b^2x^n)} - \frac{3ab^3x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)(ab+b^2x^n)} - \frac{b^4x^{-1+3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)(ab+b^2x^n)}$$

output
```
-a^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x/(a+b*x^n)-3*a^2*b^2*x^(-1+n)*(a^2
+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1-n)/(a*b+b^2*x^n)-3*a*b^3*x^(-1+2*n)*(a^2+
2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1-2*n)/(a*b+b^2*x^n)-b^4*x^(-1+3*n)*(a^2+2*a
*b*x^n+b^2*x^(2*n))^(1/2)/(1-3*n)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.58

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{(a+bx^n)^2}\left(a^3(1-6n+11n^2-6n^3)+3a^2b(1-5n+6n^2)\,x^n+3ab^2(1\right.}{(-1+n)(-1+2n)(-1+3n)x\,(a+}$$

input
```
Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(a^3*(1 - 6*n + 11*n^2 - 6*n^3) + 3*a^2*b*(1 - 5*n +
6*n^2)*x^n + 3*a*b^2*(1 - 4*n + 3*n^2)*x^(2*n) + b^3*(1 - 3*n + 2*n^2)*x^(
3*n)))/((-1 + n)*(-1 + 2*n)*(-1 + 3*n)*x*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{\left(b^2x^n+ab\right)^3}{x^2}dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \ 802$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(3a^2b^4x^{n-2}+3ab^5x^{2(n-1)}+b^6x^{3n-2}+\frac{a^3b^3}{x^2}\right)dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(-\frac{a^3b^3}{x}-\frac{3a^2b^4x^{n-1}}{1-n}-\frac{3ab^5x^{2n-1}}{1-2n}-\frac{b^6x^{3n-1}}{1-3n}\right)}{ab^3+b^4x^n}$$

input
```
Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(-((a^3*b^3)/x) - (3*a^2*b^4*x^(-1 +
n))/(1 - n) - (3*a*b^5*x^(-1 + 2*n))/(1 - 2*n) - (b^6*x^(-1 + 3*n))/(1 - 3
*n)))/(a*b^3 + b^4*x^n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{3}}{\left(a+b\,x^{n}\right)x}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{3}x^{3n}}{\left(a+b\,x^{n}\right)\left(-1+3n\right)x}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{2}a\,x^{2n}}{\left(a+b\,x^{n}\right)\left(-1+2n\right)x}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b\,a^{2}x^{n}}{\left(a+b\,x^{n}\right)\left(-1+n\right)x}$	147

input
```
int((a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+3*n
)*b^3/x*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+2*n)*b^2*a/x*(x^n)^2+3
*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*b*a^2/x*x^n
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.62

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2} dx = -\frac{6 a^3n^3 - 11 a^3n^2 + 6 a^3n - a^3 - (2 b^3n^2 - 3 b^3n + b^3)x^{3 n} - 3 (3 ab^2n^2 - 4 ab^2n + ab^2)x^{2 n} - 3 (6 a^2bn^2 -}{(6 n^3 - 11 n^2 + 6 n - 1)x}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-(6*a^3*n^3 - 11*a^3*n^2 + 6*a^3*n - a^3 - (2*b^3*n^2 - 3*b^3*n + b^3)*x^(
3*n) - 3*(3*a*b^2*n^2 - 4*a*b^2*n + a*b^2)*x^(2*n) - 3*(6*a^2*b*n^2 - 5*a^
2*b*n + a^2*b)*x^n)/((6*n^3 - 11*n^2 + 6*n - 1)*x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2} dx = \int \frac{\left((a + bx^n)^2\right)^{\frac{3}{2}}}{x^2} dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**2,x)
```

output
```
Integral(((a + b*x**n)**2)**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.48

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2} dx = \frac{(2 n^2 - 3 n + 1)b^3x^{3 n} + 3 (3 n^2 - 4 n + 1)ab^2x^{2 n} + 3 (6 n^2 - 5 n + 1)a^2bx}{(6 n^3 - 11 n^2 + 6 n - 1)x}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
((2*n^2 - 3*n + 1)*b^3*x^(3*n) + 3*(3*n^2 - 4*n + 1)*a*b^2*x^(2*n) + 3*(6*
n^2 - 5*n + 1)*a^2*b*x^n - (6*n^3 - 11*n^2 + 6*n - 1)*a^3)/((6*n^3 - 11*n^
2 + 6*n - 1)*x)
```

Giac [F]

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2} \, dx = \int \frac{(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{3}{2}}}{x^2} \, dx$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2} \, dx = \int \frac{(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n)^{3/2}}{x^2} \, dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2,x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.70

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2} \, dx = \frac{2x^{3n}b^3n^2 - 3x^{3n}b^3n + x^{3n}b^3 + 9x^{2n}a\,b^2n^2 - 12x^{2n}a\,b^2n + 3x^{2n}a\,b^2 + 18x^{n}}{x\,(6n^3 - 11n^2 + 6n}$$

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x)
```

output
```
(2*x**(3*n)*b**3*n**2 - 3*x**(3*n)*b**3*n + x**(3*n)*b**3 + 9*x**(2*n)*a*b
**2*n**2 - 12*x**(2*n)*a*b**2*n + 3*x**(2*n)*a*b**2 + 18*x**n*a**2*b*n**2
- 15*x**n*a**2*b*n + 3*x**n*a**2*b - 6*a**3*n**3 + 11*a**3*n**2 - 6*a**3*n
 + a**3)/(x*(6*n**3 - 11*n**2 + 6*n - 1))
```

3.134 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 218

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3}\,dx = -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)} - \frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\left(ab+b^2x^n\right)} - \frac{3a^2b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)} - \frac{b^4x^{-2+3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)\left(ab+b^2x^n\right)}$$

output
```
-1/2*a^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2/(a+b*x^n)-3/2*a*b^3*(a^2+2*
a*b*x^n+b^2*x^(2*n))^(1/2)/(1-n)/(x^(2-2*n))/(a*b+b^2*x^n)-3*a^2*b^2*x^(-2
+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2-n)/(a*b+b^2*x^n)-b^4*x^(-2+3*n)*(
a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2-3*n)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3}\,dx=\frac{\sqrt{(a+bx^n)^2}\left(a^3(4-12n+11n^2-3n^3)+6a^2b(2-5n+3n^2)\,x^n+3ab^2(\right.}{2(-2+n)(-1+n)(-2+3n)x^2\,(a+}$$

input
```
Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(a^3*(4 - 12*n + 11*n^2 - 3*n^3) + 6*a^2*b*(2 - 5*n +
 3*n^2)*x^n + 3*a*b^2*(4 - 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 - 3*n + n^2)*x^
(3*n)))/(2*(-2 + n)*(-1 + n)*(-2 + 3*n)*x^2*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.54, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{\left(b^2x^n+ab\right)^3}{x^3}dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(3a^2b^4x^{n-3}+b^6x^{3(n-1)}+3ab^5x^{2n-3}+\frac{a^3b^3}{x^3}\right)dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(-\frac{a^3b^3}{2x^2}-\frac{3a^2b^4x^{n-2}}{2-n}-\frac{3ab^5x^{-2(1-n)}}{2(1-n)}-\frac{b^6x^{3n-2}}{2-3n}\right)}{ab^3+b^4x^n}$$

input
```
Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(-1/2*(a^3*b^3)/x^2 - (3*a*b^5)/(2*(1
 - n)*x^(2*(1 - n))) - (3*a^2*b^4*x^(-2 + n))/(2 - n) - (b^6*x^(-2 + 3*n))
/(2 - 3*n)))/(a*b^3 + b^4*x^n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{3}}{2\left(a+b\,x^{n}\right)x^{2}}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{3}x^{3n}}{\left(a+b\,x^{n}\right)\left(-2+3n\right)x^{2}}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{2}a\,x^{2n}}{2\left(a+b\,x^{n}\right)\left(-1+n\right)x^{2}}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b\,a^{2}x^{n}}{\left(a+b\,x^{n}\right)\left(-2+n\right)x^{2}}$	145

input
```
int((a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(
-2+3*n)*b^3/x^2*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*b^2*a/x^2
*(x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-2+n)*b*a^2/x^2*x^n
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.61

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3}\,dx = -\frac{3\,a^3n^3 - 11\,a^3n^2 + 12\,a^3n - 4\,a^3 - 2\,(b^3n^2 - 3\,b^3n + 2\,b^3)x^{3\,n} - 3\,(3\,ab^2n^2 - 8\,ab^2n + 4\,ab^2)x^{2\,n} - 6\,(3\,a^2}{2\,(3\,n^3 - 11\,n^2 + 12\,n - 4)x^2}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(3*a^3*n^3 - 11*a^3*n^2 + 12*a^3*n - 4*a^3 - 2*(b^3*n^2 - 3*b^3*n + 2
*b^3)*x^(3*n) - 3*(3*a*b^2*n^2 - 8*a*b^2*n + 4*a*b^2)*x^(2*n) - 6*(3*a^2*b
*n^2 - 5*a^2*b*n + 2*a^2*b)*x^n)/((3*n^3 - 11*n^2 + 12*n - 4)*x^2)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3}\,dx = \int \frac{\left((a + bx^n)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**3,x)
```

output
```
Integral(((a + b*x**n)**2)**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.46

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3}\,dx = \frac{2\,(n^2 - 3\,n + 2)b^3x^{3\,n} + 3\,(3\,n^2 - 8\,n + 4)ab^2x^{2\,n} + 6\,(3\,n^2 - 5\,n + 2)a^2bx}{2\,(3\,n^3 - 11\,n^2 + 12\,n - 4)x^2}$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
1/2*(2*(n^2 - 3*n + 2)*b^3*x^(3*n) + 3*(3*n^2 - 8*n + 4)*a*b^2*x^(2*n) + 6
*(3*n^2 - 5*n + 2)*a^2*b*x^n - (3*n^3 - 11*n^2 + 12*n - 4)*a^3)/((3*n^3 -
11*n^2 + 12*n - 4)*x^2)
```

Giac [F]

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3}\,dx = \int \frac{(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3}\,dx = \int \frac{(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n)^{3/2}}{x^3}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3,x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.70

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3}\,dx = \frac{2x^{3n}b^3n^2 - 6x^{3n}b^3n + 4x^{3n}b^3 + 9x^{2n}a\,b^2n^2 - 24x^{2n}a\,b^2n + 12x^{2n}a\,b^2 + 18}{2x^2\,(3n^3 - 11n^2 +}$$

input
```
int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x)
```

output
```
(2*x**(3*n)*b**3*n**2 - 6*x**(3*n)*b**3*n + 4*x**(3*n)*b**3 + 9*x**(2*n)*a
*b**2*n**2 - 24*x**(2*n)*a*b**2*n + 12*x**(2*n)*a*b**2 + 18*x**n*a**2*b*n*
*2 - 30*x**n*a**2*b*n + 12*x**n*a**2*b - 3*a**3*n**3 + 11*a**3*n**2 - 12*a
**3*n + 4*a**3)/(2*x**2*(3*n**3 - 11*n**2 + 12*n - 4))
```

3.135 $\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 64

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^3(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/3*x^3*(a+b*x^n)*hypergeom([1, 3/n],[(3+n)/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+
b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^3(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},1+\frac{3}{n},-\frac{bx^n}{a}\right)}{3a\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, 1 + 3/n, -((b*x^n)/a)])/(3*a*Sq
rt[(a + b*x^n)^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$
$$\Big\downarrow \text{1384}$$
$$\frac{(ab+b^2x^n)\int \frac{x^2}{b^2x^n+ab}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$\Big\downarrow \text{888}$$
$$\frac{x^3(ab+b^2x^n)\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3ab\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^3*(a*b + b^2*x^n)*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(
3*a*b*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{x^2}{\sqrt{a^2 + 2x^n ab + b^2 x^{2n}}} dx$$

input
```
int(x^2/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x)
```

output
```
int(x^2/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a^2 + 2abx^n + b^2 x^{2n}}} dx = \int \frac{x^2}{\sqrt{b^2 x^{2n} + 2\,abx^n + a^2}} dx$$

input
```
integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a^2 + 2abx^n + b^2 x^{2n}}} dx = \int \frac{x^2}{\sqrt{(a + bx^n)^2}} dx$$

input
```
integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**2/sqrt((a + b*x**n)**2), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{x^2}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

input
```
integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{x^2}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

input
```
integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{x^2}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

input
```
int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{x^2}{x^nb+a}dx$$

input
```
int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
int(x**2/(x**n*b + a),x)
```

3.136 $\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 64

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^2(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/2*x^2*(a+b*x^n)*hypergeom([1, 2/n],[(2+n)/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+
b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^2(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},1+\frac{2}{n},-\frac{bx^n}{a}\right)}{2a\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a*Sq
rt[(a + b*x^n)^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{(ab+b^2x^n)\int \frac{x}{b^2x^n+ab}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^2(ab+b^2x^n)\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2ab\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^2*(a*b + b^2*x^n)*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(
2*a*b*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{x}{\sqrt{a^2 + 2x^n ab + b^2 x^{2n}}} dx$$

input
```
int(x/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x)
```

output
```
int(x/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} dx = \int \frac{x}{\sqrt{b^2x^{2n} + 2abx^n + a^2}} dx$$

input
```
integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{x}{\sqrt{(a + bx^n)^2}}\,dx$$

input
```
integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x/sqrt((a + b*x**n)**2), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{x}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

input
```
integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{x}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

input
```
integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\, dx = \int \frac{x}{\sqrt{a^2 + b^2\, x^{2\,n} + 2\,a\,b\,x^n}}\, dx$$

input
```
int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\, dx = \int \frac{x}{x^n b + a} dx$$

input
```
int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
int(x/(x**n*b + a),x)
```

3.137 $\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
x*(a+b*x^n)*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+b^2*x^(2
*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*
Sqrt[(a + b*x^n)^2])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1384, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$$
$$\Big\downarrow \text{1384}$$
$$\frac{(ab+b^2x^n)\int \frac{1}{b^2x^n+ab}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$\Big\downarrow \text{778}$$
$$\frac{x(ab+b^2x^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x*(a*b + b^2*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])
/(a*b*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{1}{\sqrt{a^2 + 2x^n ab + b^2 x^{2n}}} dx$$

input
```
int(1/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x)
```

output
```
int(1/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} dx = \int \frac{1}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}} dx$$

input
```
integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} dx = \int \frac{1}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} dx$$

input
```
integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(1/sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

input
```
integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\,dx$$

input
```
integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{1}{x^n b + a}dx$$

input
```
int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
int(1/(x**n*b + a),x)
```

3.138 $\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{(a+bx^n)\log(x)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log(a+bx^n)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
(a+b*x^n)*ln(x)/a/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-(a+b*x^n)*ln(a+b*x^n)/
a/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.53

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{(a+bx^n)\left(\log(x^n)-\log(an(a+bx^n))\right)}{an\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

output
```
((a + b*x^n)*(Log[x^n] - Log[a*n*(a + b*x^n)]))/(a*n*Sqrt[(a + b*x^n)^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1384, 798, 27, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{(ab+b^2x^n)\int \frac{1}{x(b^2x^n+ab)}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{(ab+b^2x^n)\int \frac{x^{-n}}{b(bx^n+a)}dx^n}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(ab+b^2x^n)\int \frac{x^{-n}}{bx^n+a}dx^n}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{(ab+b^2x^n)\left(\frac{\int x^{-n}dx^n}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}\right)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{(ab+b^2x^n)\left(\frac{\log(x^n)}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}\right)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{(ab+b^2x^n)\left(\frac{\log(x^n)}{a}-\frac{\log(a+bx^n)}{a}\right)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

output
```
((a*b + b^2*x^n)*(Log[x^n]/a - Log[a + b*x^n]/a))/(b*n*Sqrt[a^2 + 2*a*b*x^
n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,\ln(x)}{(a+b\,x^n)a} - \frac{\sqrt{(a+b\,x^n)^2}\,\ln\left(x^n+\frac{a}{b}\right)}{(a+b\,x^n)an}$	66

input
```
int(1/x/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*ln(x)/a-((a+b*x^n)^2)^(1/2)/(a+b*x^n)/a/n*ln
(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{n\log(x)-\log(bx^n+a)}{an}$$

input
```
integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
(n*log(x) - log(b*x^n + a))/(a*n)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{x\sqrt{(a+bx^n)^2}}\,dx$$

input
```
integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a + b*x**n)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.32

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{\log(x)}{a}-\frac{\log\left(\frac{bx^n+a}{b}\right)}{an}$$

input
```
integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
log(x)/a - log((b*x^n + a)/b)/(a*n)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}x}\,dx$$

input
```
integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{x\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

input
```
int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
```

output
```
int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

$$\int \frac{1}{x\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \frac{-\log(x^n b + a) + \log(x)\,n}{an}$$

input
```
int(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
( - log(x**n*b + a) + log(x)*n)/(a*n)
```

3.139 $\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-(a+b*x^n)*hypergeom([1, -1/n],[-(1-n)/n],-b*x^n/a)/a/x/(a^2+2*a*b*x^n+b^2
*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{ax\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

output
```
-(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), 1 - n^(-1), -((b*x^n)/a)])/(a
*x*Sqrt[(a + b*x^n)^2]))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{(ab+b^2x^n)\int \frac{1}{x^2(b^2x^n+ab)}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{(ab+b^2x^n)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{abx\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

output
```
-(((a*b + b^2*x^n)*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a
)])/(a*b*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{1}{x^2\sqrt{a^2 + 2x^n ab + b^2x^{2n}}}dx$$

input
```
int(1/x^2/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x)
```

output
```
int(1/x^2/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}x^2}\,dx$$

input
```
integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^2*x^2*x^(2*n) + 2*a*b*x^2*
x^n + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{1}{x^2\sqrt{(a + bx^n)^2}}\,dx$$

input
```
integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((a + b*x**n)**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^2}\,dx$$

input
```
integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^2}\,dx$$

input
```
integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{x^2\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{x^n b\,x^2+a\,x^2}dx$$

input
```
int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
int(1/(x**n*b*x**2 + a*x**2),x)
```

3.140 $\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 67

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-1/2*(a+b*x^n)*hypergeom([1, -2/n],[-(2-n)/n],-b*x^n/a)/a/x^2/(a^2+2*a*b*x
^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},1-\frac{2}{n},-\frac{bx^n}{a}\right)}{2ax^2\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

output
```
-1/2*((a + b*x^n)*Hypergeometric2F1[1, -2/n, 1 - 2/n, -((b*x^n)/a)])/(a*x^
2*Sqrt[(a + b*x^n)^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{(ab+b^2x^n)\int \frac{1}{x^3(b^2x^n+ab)}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{(ab+b^2x^n)\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2abx^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

output
```
-1/2*((a*b + b^2*x^n)*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a
)])/(a*b*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{1}{x^3\sqrt{a^2+2x^nab+b^2x^{2n}}}dx$$

input
```
int(1/x^3/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x)
```

output
```
int(1/x^3/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^3}\,dx$$

input
```
integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^2*x^3*x^(2*n) + 2*a*b*x^3*
x^n + a^2*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{x^3\sqrt{(a+bx^n)^2}}\,dx$$

input
```
integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt((a + b*x**n)**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^3}\,dx$$

input
```
integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^3}\,dx$$

input
```
integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{x^3\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

input
```
int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
```

output
```
int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\,dx = \int \frac{1}{x^n b\,x^3 + a\,x^3}dx$$

input
```
int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
int(1/(x**n*b*x**3 + a*x**3),x)
```

3.141 $\int \frac{x^2}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 64

$$\int \frac{x^2}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x^3(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/3*x^3*(a+b*x^n)*hypergeom([3, 3/n],[(3+n)/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^
n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x^3(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,\frac{3}{n},1+\frac{3}{n},-\frac{bx^n}{a}\right)}{3a^3\left((a+bx^n)^2\right)^{3/2}}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^3*(a + b*x^n)^3*Hypergeometric2F1[3, 3/n, 1 + 3/n, -((b*x^n)/a)])/(3*a^
3*((a + b*x^n)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{(ab^3+b^4x^n)\int \frac{x^2}{(b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^3(ab^3+b^4x^n)\,\text{Hypergeometric2F1}\left(3,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3a^3b^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^3*(a*b^3 + b^4*x^n)*Hypergeometric2F1[3, 3/n, (3 + n)/n, -((b*x^n)/a)])
/(3*a^3*b^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{x^2}{(a^2 + 2x^n ab + b^2 x^{2n})^{\frac{3}{2}}} dx$$

input
```
int(x^2/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x)
```

output
```
int(x^2/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{x^2}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2/(b^4*x^(4*n) + 4*a^2*b^2*
x^(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{x^2}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{x^2}{\left((a + bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x**2/((a + b*x**n)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{x^2}{(b^2x^{2n} + 2abx^n + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
(2*n^2 - 9*n + 9)*integrate(1/2*x^2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b
*(2*n - 3)*x^3*x^n + 3*a*(n - 1)*x^3)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x
^n + a^4*n^2)
```

Giac [F]

$$\int \frac{x^2}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{x^2}{(b^2x^{2n} + 2abx^n + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x^2}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}}\,dx$$

input
```
int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x^2}{x^{3n}b^3+3x^{2n}a\,b^2+3x^n a^2b+a^3}dx$$

input
```
int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
int(x**2/(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b + a**3),x)
```

3.142 $\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 64

$$\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x^2(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/2*x^2*(a+b*x^n)*hypergeom([3, 2/n],[(2+n)/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^
n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x^2(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,\frac{2}{n},1+\frac{2}{n},-\frac{bx^n}{a}\right)}{2a^3\left((a+bx^n)^2\right)^{3/2}}$$

input
```
Integrate[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^2*(a + b*x^n)^3*Hypergeometric2F1[3, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a^
3*((a + b*x^n)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{(ab^3+b^4x^n)\int \frac{x}{(b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2(ab^3+b^4x^n)\,\text{Hypergeometric2F1}\left(3,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a^3b^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^2*(a*b^3 + b^4*x^n)*Hypergeometric2F1[3, 2/n, (2 + n)/n, -((b*x^n)/a)])
/(2*a^3*b^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] && !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{x}{(a^2 + 2x^n ab + b^2 x^{2n})^{\frac{3}{2}}} dx$$

input
```
int(x/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x)
```

output
```
int(x/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x)
```

Fricas [F]

$$\int \frac{x}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}} dx = \int \frac{x}{(b^2x^{2n} + 2abx^n + a^2)^{\frac{3}{2}}} dx$$

input
```
integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x/(b^4*x^(4*n) + 4*a^2*b^2*x^
(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{x}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{x}{\left((a + bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x/((a + b*x**n)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{x}{(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
(n^2 - 3*n + 2)*integrate(x/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(2*b*(n -
1)*x^2*x^n + a*(3*n - 2)*x^2)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4
*n^2)
```

Giac [F]

$$\int \frac{x}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{x}{(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

input
```
int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x}{x^{3n}b^3+3x^{2n}a\,b^2+3x^na^2b+a^3}dx$$

input
```
int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
int(x/(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b + a**3),x)
```

3.143 $\int \frac{1}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{1}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x(a+bx^n)\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
x*(a+b*x^n)*hypergeom([3, 1/n],[1+1/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^n+b^2*x^
(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3\left((a+bx^n)^2\right)^{3/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
```

output
```
(x*(a + b*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(
a^3*((a + b*x^n)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1384, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^n+b^2x^{2n})^{3/2}} \, dx$$

$$\Big\downarrow \ 1384$$

$$\frac{(ab^3+b^4x^n)\int \frac{1}{(b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \ 778$$

$$\frac{x(ab^3+b^4x^n)\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3b^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
```

output
```
(x*(a*b^3 + b^4*x^n)*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)
])/(a^3*b^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{1}{(a^2 + 2x^n ab + b^2 x^{2n})^{\frac{3}{2}}} dx$$

input
```
int(1/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x)
```

output
```
int(1/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}} dx = \int \frac{1}{(b^2x^{2n} + 2abx^n + a^2)^{\frac{3}{2}}} dx$$

input
```
integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^(4*n) + 4*a^2*b^2*x^(2
*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
(2*n^2 - 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b*(2*
n - 1)*x*x^n + a*(3*n - 1)*x)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4
*n^2)
```

Giac [F]

$$\int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n)^{3/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{x^{3n}b^3 + 3x^{2n}a\,b^2 + 3x^n a^2 b + a^3}dx$$

input
```
int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
int(1/(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b + a**3),x)
```

3.144 $\int \frac{1}{x(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 159

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{(a+bx^n)\log(x)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log\left(a+bx^n\right)}{a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/a^2/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)+1/2/a/n/(a+b*x^n)/(a^2+2*a*b*x^n
+b^2*x^(2*n))^(1/2)+(a+b*x^n)*ln(x)/a^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-
(a+b*x^n)*ln(a+b*x^n)/a^3/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.50

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{a(3a+2bx^n)+2(a+bx^n)^2\log\left(x^n\right)-2(a+bx^n)^2\log\left(a+bx^n\right)}{2a^3n\left(a+bx^n\right)\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

output
```
(a*(3*a + 2*b*x^n) + 2*(a + b*x^n)^2*Log[x^n] - 2*(a + b*x^n)^2*Log[a + b*
x^n])/(2*a^3*n*(a + b*x^n)*Sqrt[(a + b*x^n)^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.60, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1384, 798, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\left(ab^3+b^4x^n\right)\int \frac{1}{x(b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\left(ab^3+b^4x^n\right)\int \frac{x^{-n}}{b^3(bx^n+a)^3}dx^n}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(ab^3+b^4x^n\right)\int \frac{x^{-n}}{(bx^n+a)^3}dx^n}{b^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\left(ab^3+b^4x^n\right)\int \left(\frac{x^{-n}}{a^3}-\frac{b}{a^3(bx^n+a)}-\frac{b}{a^2(bx^n+a)^2}-\frac{b}{a(bx^n+a)^3}\right)dx^n}{b^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(ab^3+b^4x^n\right)\left(-\frac{\log(a+bx^n)}{a^3}+\frac{\log(x^n)}{a^3}+\frac{1}{a^2(a+bx^n)}+\frac{1}{2a(a+bx^n)^2}\right)}{b^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

output
```
((a*b^3 + b^4*x^n)*(1/(2*a*(a + b*x^n)^2) + 1/(a^2*(a + b*x^n)) + Log[x^n]
/a^3 - Log[a + b*x^n]/a^3))/(b^3*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,\ln(x)}{(a+b\,x^{n})a^{3}}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,(2b\,x^{n}+3a)}{2(a+b\,x^{n})^{3}a^{2}n}-\frac{\sqrt{(a+b\,x^{n})^{2}}\,\ln\left(x^{n}+\frac{a}{b}\right)}{(a+b\,x^{n})a^{3}n}$	104

input
```
int(1/x/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*ln(x)/a^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^
3*(2*b*x^n+3*a)/a^2/n-((a+b*x^n)^2)^(1/2)/(a+b*x^n)/a^3/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.67

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \frac{2\,b^2nx^{2\,n}\log\left(x\right)+2\,a^2n\log\left(x\right)+3\,a^2+2\left(2\,abn\log\left(x\right)+ab\right)x^n-2\,(b^2}{2\left(a^3b^2nx^{2\,n}+2\,a^4bnx^n+a^5n\right)}$$

input
```
integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*n*x^(2*n)*log(x) + 2*a^2*n*log(x) + 3*a^2 + 2*(2*a*b*n*log(x) +
 a*b)*x^n - 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))/(a^3*b^2*n*x
^(2*n) + 2*a^4*b*n*x^n + a^5*n)
```

Sympy [F]

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{x\left(\left(a+bx^n\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(1/(x*((a + b*x**n)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.44

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \frac{2\,bx^n+3\,a}{2\,(a^2b^2nx^{2\,n}+2\,a^3bnx^n+a^4n)}+\frac{\log(x)}{a^3}-\frac{\log\left(\frac{bx^n+a}{b}\right)}{a^3n}$$

input
```
integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*b*x^n + 3*a)/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a^4*n) + log(x)/a
^3 - log((b*x^n + a)/b)/(a^3*n)
```

Giac [F]

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{x\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
```

output
```
int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \frac{-2x^{2n}\log(x^nb+a)\,b^2+2x^{2n}\log(x)\,b^2n-x^{2n}b^2-4x^n\log(x^nb+a)\,ab+}{2a^3n\left(x^{2n}b^2+2x^nab\right.}$$

input
```
int(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
( - 2*x**(2*n)*log(x**n*b + a)*b**2 + 2*x**(2*n)*log(x)*b**2*n - x**(2*n)*
b**2 - 4*x**n*log(x**n*b + a)*a*b + 4*x**n*log(x)*a*b*n - 2*log(x**n*b + a
)*a**2 + 2*log(x)*a**2*n + 2*a**2)/(2*a**3*n*(x**(2*n)*b**2 + 2*x**n*a*b +
 a**2))
```

3.145 $\int \frac{1}{x^2(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{1}{x^2(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(3,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-(a+b*x^n)*hypergeom([3, -1/n],[-(1-n)/n],-b*x^n/a)/a^3/x/(a^2+2*a*b*x^n+b
^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3x\left((a+bx^n)^2\right)^{3/2}}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

output
```
-(((a + b*x^n)^3*Hypergeometric2F1[3, -n^(-1), 1 - n^(-1), -((b*x^n)/a)])/
(a^3*x*((a + b*x^n)^2)^(3/2)))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^2+2abx^n+b^2x^{2n})^{3/2}} \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{(ab^3+b^4x^n) \int \frac{1}{x^2 (b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{(ab^3+b^4x^n) \text{Hypergeometric2F1}\left(3,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^3b^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

output
```
-(((a*b^3 + b^4*x^n)*Hypergeometric2F1[3, -n^(-1), -((1 - n)/n), -((b*x^n)
/a)])/(a^3*b^3*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{1}{x^2\left(a^2+2x^nab+b^2x^{2n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x)
```

output
```
int(1/x^2/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\int \frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^2*x^(4*n) + 4*a^2*b^2*
x^2*x^(2*n) + 4*a^3*b*x^2*x^n + a^4*x^2 + 2*(2*a*b^3*x^2*x^n + a^2*b^2*x^2
)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{x^2 \left((a + bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(1/(x**2*((a + b*x**n)**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}} x^2}\,dx$$

input
```
integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
(2*n^2 + 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^2*x^n + a^3*n^2*x^2), x) + 1/
2*(b*(2*n + 1)*x^n + a*(3*n + 1))/(a^2*b^2*n^2*x*x^(2*n) + 2*a^3*b*n^2*x*x
^n + a^4*n^2*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}} x^2}\,dx$$

input
```
integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2}} \, dx = \int \frac{1}{x^2 \left(a^2 + b^2 \, x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2}} \, dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2}} \, dx = \int \frac{1}{x^{3n}b^3x^2 + 3x^{2n}a\,b^2x^2 + 3x^n a^2b\,x^2 + a^3x^2} dx$$

input
```
int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
int(1/(x**(3*n)*b**3*x**2 + 3*x**(2*n)*a*b**2*x**2 + 3*x**n*a**2*b*x**2 +
a**3*x**2),x)
```

3.146 $\int \frac{1}{x^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 67

$$\int \frac{1}{x^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=-\frac{(a+bx^n)\,\text{Hypergeometric2F1}\left(3,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output

```
-1/2*(a+b*x^n)*hypergeom([3, -2/n],[-(2-n)/n],-b*x^n/a)/a^3/x^2/(a^2+2*a*b
*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=-\frac{(a+bx^n)^3\,\text{Hypergeometric2F1}\left(3,-\frac{2}{n},1-\frac{2}{n},-\frac{bx^n}{a}\right)}{2a^3x^2\left((a+bx^n)^2\right)^{3/2}}$$

input

```
Integrate[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

output

```
-1/2*((a + b*x^n)^3*Hypergeometric2F1[3, -2/n, 1 - 2/n, -((b*x^n)/a)])/(a^
3*x^2*((a + b*x^n)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^2+2abx^n+b^2x^{2n})^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{(ab^3+b^4x^n) \int \frac{1}{x^3 (b^2x^n+ab)^3} dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{(ab^3+b^4x^n) \text{Hypergeometric2F1}\left(3,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^3b^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

output
```
-1/2*((a*b^3 + b^4*x^n)*Hypergeometric2F1[3, -2/n, -((2 - n)/n), -((b*x^n)
/a)])/(a^3*b^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{1}{x^3\left(a^2+2x^n ab+b^2x^{2n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^3/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x)
```

output
```
int(1/x^3/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{x^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\int \frac{1}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^3*x^(4*n) + 4*a^2*b^2*
x^3*x^(2*n) + 4*a^3*b*x^3*x^n + a^4*x^3 + 2*(2*a*b^3*x^3*x^n + a^2*b^2*x^3
)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{1}{x^3 \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{x^3 \left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(1/(x**3*((a + b*x**n)**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{\left(b^2x^{2n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
(n^2 + 3*n + 2)*integrate(1/(a^2*b*n^2*x^3*x^n + a^3*n^2*x^3), x) + 1/2*(2
*b*(n + 1)*x^n + a*(3*n + 2))/(a^2*b^2*n^2*x^2*x^(2*n) + 2*a^3*b*n^2*x^2*x
^n + a^4*n^2*x^2)
```

Giac [F]

$$\int \frac{1}{x^3 \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \int \frac{1}{\left(b^2x^{2n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\int \frac{1}{x^3\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
```

output
```
int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\int \frac{1}{x^{3n}b^3x^3+3x^{2n}a\,b^2x^3+3x^na^2b\,x^3+a^3x^3}dx$$

input
```
int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
int(1/(x**(3*n)*b**3*x**3 + 3*x**(2*n)*a*b**2*x**3 + 3*x**n*a**2*b*x**3 +
a**3*x**3),x)
```

3.147 $\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$

Optimal result

Integrand size = 30, antiderivative size = 247

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{3a^2b^2x^n(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m+n)\left(ab+b^2x^n\right)} + \frac{3ab^3x^{2n}(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m+2n)\left(ab+b^2x^n\right)} + \frac{b^4x^{3n}(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m+3n)\left(ab+b^2x^n\right)}$$

output
```
a^3*(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/d/(1+m)/(a+b*x^n)+3*a^2*
b^2*x^n*(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/d/(1+m+n)/(a*b+b^2*x
^n)+3*a*b^3*x^(2*n)*(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/d/(1+m+2
*n)/(a*b+b^2*x^n)+b^4*x^(3*n)*(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2
)/d/(1+m+3*n)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.36

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{x(dx)^m \left((a+bx^n)^2\right)^{3/2} \left(\frac{a^3}{1+m}+\frac{3a^2bx^n}{1+m+n}+\frac{3ab^2x^{2n}}{1+m+2n}+\frac{b^3x^{3n}}{1+m+3n}\right)}{(a+bx^n)^3}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x*(d*x)^m*((a + b*x^n)^2)^(3/2)*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n)
+ (3*a*b^2*x^(2*n))/(1 + m + 2*n) + (b^3*x^(3*n))/(1 + m + 3*n)))/(a + b*x
^n)^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.55, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int (dx)^m \left(b^2x^n+ab\right)^3 dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{802}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int \left(3a^2b^4x^n(dx)^m+3ab^5x^{2n}(dx)^m+b^6x^{3n}(dx)^m+a^3b^3(dx)^m\right) dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(\frac{a^3b^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2b^4x^{n+1}(dx)^m}{m+n+1}+\frac{3ab^5x^{2n+1}(dx)^m}{m+2n+1}+\frac{b^6x^{3n+1}(dx)^m}{m+3n+1}\right)}{ab^3+b^4x^n}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((3*a^2*b^4*x^(1 + n)*(d*x)^m)/(1 + m
 + n) + (3*a*b^5*x^(1 + 2*n)*(d*x)^m)/(1 + m + 2*n) + (b^6*x^(1 + 3*n)*(d*
x)^m)/(1 + m + 3*n) + (a^3*b^3*(d*x)^(1 + m))/(d*(1 + m))))/(a*b^3 + b^4*x
^n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.02

method	result
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,x\,(6a^3n+3m\,a^3+12b^2a\,x^{2n}n+3b^3m^2n\,x^{3n}+2b^3m\,n^2x^{3n}+3a\,b^2m^3x^{2n}+6b^3mn\,x^{3n}+9a\,b^2m^2x^{2n}+9a\,b^2n^2x^{2n}+9m\,b}{}$

input
```
int((d*x)^m*(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x*(6*a^3*n+3*m*a^3+a^3+9*a*b^2*m^2*(x^n)^2+9
*a*b^2*n^2*(x^n)^2+9*a^2*b*m^2*x^n+18*a^2*b*n^2*x^n+9*m*b^2*a*(x^n)^2+12*b
^2*a*(x^n)^2*n+9*m*b*a^2*x^n+15*b*a^2*n*x^n+3*b^3*m^2*n*(x^n)^3+2*b^3*m*n^
2*(x^n)^3+3*a*b^2*m^3*(x^n)^2+6*b^3*m*n*(x^n)^3+3*a^2*b*m^3*x^n+6*a^3*n^3+
24*a*b^2*m*n*(x^n)^2+30*a^2*b*m*n*x^n+11*a^3*n^2+12*a*b^2*m^2*n*(x^n)^2+9*
a*b^2*m*n^2*(x^n)^2+15*a^2*b*m^2*n*x^n+18*a^2*b*m*n^2*x^n+3*b^2*a*(x^n)^2+
3*m*b^3*(x^n)^3+3*a^3*m^2+b^3*(x^n)^3+a^3*m^3+2*b^3*n^2*(x^n)^3+3*b^3*m^2*
(x^n)^3+3*b*a^2*x^n+b^3*m^3*(x^n)^3+3*b^3*(x^n)^3*n+6*a^3*m^2*n+11*a^3*m*n
^2+12*a^3*m*n)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)*x^m*d^m*exp(1/2*I*csgn(I*
d*x)*Pi*m*(csgn(I*d*x)-csgn(I*x))*(-csgn(I*d*x)+csgn(I*d)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.58

$$\int (dx)^m \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2} dx = \frac{(b^3m^3 + 3\,b^3m^2 + 3\,b^3m + b^3 + 2\,(b^3m + b^3)n^2 + 3\,(b^3m^2 + 2\,b^3m + b^3)n)xx^{3\,n}e^{(m\log(d)+m}}{}$$

input
```
integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3 + 2*(b^3*m + b^3)*n^2 + 3*(b^3*m^2 +
 2*b^3*m + b^3)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*(a*b^2*m^3 + 3*a*
b^2*m^2 + 3*a*b^2*m + a*b^2 + 3*(a*b^2*m + a*b^2)*n^2 + 4*(a*b^2*m^2 + 2*a
*b^2*m + a*b^2)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^3 + 3*a^
2*b*m^2 + 3*a^2*b*m + a^2*b + 6*(a^2*b*m + a^2*b)*n^2 + 5*(a^2*b*m^2 + 2*a
^2*b*m + a^2*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^3*m^3 + 6*a^3*n^3 +
3*a^3*m^2 + 3*a^3*m + a^3 + 11*(a^3*m + a^3)*n^2 + 6*(a^3*m^2 + 2*a^3*m +
a^3)*n)*x*e^(m*log(d) + m*log(x)))/(m^4 + 6*(m + 1)*n^3 + 4*m^3 + 11*(m^2
+ 2*m + 1)*n^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m + 1)
```

Sympy [F]

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \int (dx)^m \left((a+bx^n)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral((d*x)**m*((a + b*x**n)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a^2+2abx^n +b^2x^{2n}\right)^{3/2} dx = \frac{(m^3+3\,m^2(2\,n+1)+6\,n^3+(11\,n^2+12\,n+3)m+11\,n^2+6\,n+1)a^3d^mxx^m+(m^3+3}{}$$

input
```
integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
((m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1
)*a^3*d^m*x*x^m + (m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n
 + 1)*b^3*d^m*x*e^(m*log(x) + 3*n*log(x)) + 3*(m^3 + m^2*(4*n + 3) + (3*n^
2 + 8*n + 3)*m + 3*n^2 + 4*n + 1)*a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x)) +
3*(m^3 + m^2*(5*n + 3) + (6*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*a^2*b*d^m
*x*e^(m*log(x) + n*log(x)))/(m^4 + 2*m^3*(3*n + 2) + (11*n^2 + 18*n + 6)*m
^2 + 6*n^3 + 2*(3*n^3 + 11*n^2 + 9*n + 2)*m + 11*n^2 + 6*n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2719 vs. $2(239) = 478$.

Time = 0.26 (sec) , antiderivative size = 2719, normalized size of antiderivative = 11.01

$$\int (dx)^m \left(a^2 + 2abx^n + b^2x^{2n}\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
(b^3*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*
x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(3*n)*e^(
m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^(2*n)*e^(m*log(d) +
m*log(x))*sgn(b*x^n + a) + b^3*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b
*x^n + a) + 12*a*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a
) + 3*b^3*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2
*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^
(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*x^n*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^n*e^(m*log(d) + m*log(x))*
sgn(b*x^n + a) + b^3*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 15
*a^2*b*m^2*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*m^2*n
*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x^n*e^(m*log
(d) + m*log(x))*sgn(b*x^n + a) + 18*a^2*b*m*n^2*x*x^n*e^(m*log(d) + m*log(
x))*sgn(b*x^n + a) + 9*a*b^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + 2*b^3*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a^3*m^3
*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*e^(m*log(d) + m*
log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n +
 a) + b^3*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*m^2*n*x*e^(
m*log(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*m^2*n*x*e^(m*log(d) + m*log
(x))*sgn(b*x^n + a) + 12*a*b^2*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^...
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \int (d\,x)^m \left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2} dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.94

$$\int (dx)^m \left(a^2+2abx^n +b^2x^{2n}\right)^{3/2} dx = \frac{x^m d^m x(9x^n a^2bm + a^3m^3 + 3a^3m^2 + 3a^3m + 9x^n a^2b\,m^2 + 3x^{3n}b^3m^2n + x^{3n}b^3m^3 + 2x^{3n}b}{}$$

input
```
int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
(x**m*d**m*x*(x**(3*n)*b**3*m**3 + 3*x**(3*n)*b**3*m**2*n + 3*x**(3*n)*b**
3*m**2 + 2*x**(3*n)*b**3*m*n**2 + 6*x**(3*n)*b**3*m*n + 3*x**(3*n)*b**3*m
+ 2*x**(3*n)*b**3*n**2 + 3*x**(3*n)*b**3*n + x**(3*n)*b**3 + 3*x**(2*n)*a*
b**2*m**3 + 12*x**(2*n)*a*b**2*m**2*n + 9*x**(2*n)*a*b**2*m**2 + 9*x**(2*n
)*a*b**2*m*n**2 + 24*x**(2*n)*a*b**2*m*n + 9*x**(2*n)*a*b**2*m + 9*x**(2*n
)*a*b**2*n**2 + 12*x**(2*n)*a*b**2*n + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b*m
**3 + 15*x**n*a**2*b*m**2*n + 9*x**n*a**2*b*m**2 + 18*x**n*a**2*b*m*n**2 +
 30*x**n*a**2*b*m*n + 9*x**n*a**2*b*m + 18*x**n*a**2*b*n**2 + 15*x**n*a**2
*b*n + 3*x**n*a**2*b + a**3*m**3 + 6*a**3*m**2*n + 3*a**3*m**2 + 11*a**3*m
*n**2 + 12*a**3*m*n + 3*a**3*m + 6*a**3*n**3 + 11*a**3*n**2 + 6*a**3*n + a
**3))/(m**4 + 6*m**3*n + 4*m**3 + 11*m**2*n**2 + 18*m**2*n + 6*m**2 + 6*m*
n**3 + 22*m*n**2 + 18*m*n + 4*m + 6*n**3 + 11*n**2 + 6*n + 1)
```

3.148 $\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 111

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)}+\frac{b^2x^n(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m+n)\left(ab+b^2x^n\right)}$$

output
```
a*(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/d/(1+m)/(a+b*x^n)+b^2*x^n*
(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/d/(1+m+n)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.50

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x(dx)^m\sqrt{\left(a+bx^n\right)^2}(a(1+m+n)+b(1+m)x^n)}{(1+m)(1+m+n)\left(a+bx^n\right)}$$

input
```
Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x*(d*x)^m*Sqrt[(a + b*x^n)^2]*(a*(1 + m + n) + b*(1 + m)*x^n))/((1 + m)*(
1 + m + n)*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int (dx)^m \left(b^2x^n+ab\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \; 802$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^n(dx)^m+ab(dx)^m\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(\frac{ab(dx)^{m+1}}{d(m+1)}+\frac{b^2x^{n+1}(dx)^m}{m+n+1}\right)}{ab+b^2x^n}$$

input
```
Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*((b^2*x^(1 + n)*(d*x)^m)/(1 + m + n)
+ (a*b*(d*x)^(1 + m))/(d*(1 + m))))/(a*b + b^2*x^n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,x(mb\,x^n+am+an+b\,x^n+a)x^m d^m \mathrm{e}^{\frac{i\,\mathrm{csgn}(idx)\pi m(\mathrm{csgn}(idx)-\mathrm{csgn}(ix))(-\,\mathrm{csgn}(idx)+\mathrm{csgn}(id))}{2}}}{(a+b\,x^n)(1+m)(1+m+n)}$	99

input
```
int((d*x)^m*(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x*(m*b*x^n+a*m+a*n+b*x^n+a)/(1+m)/(1+m+n)*x^
m*d^m*exp(1/2*I*csgn(I*d*x)*Pi*m*(csgn(I*d*x)-csgn(I*x))*(-csgn(I*d*x)+csg
n(I*d)))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.51

$$\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx = \frac{(bm+b)xx^n e^{(m\log(d)+m\log(x))}+(am+an+a)xe^{(m\log(d)+m\log(x))}}{m^2+(m+1)n+2\,m+1}$$

input
```
integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
((b*m + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m + a*n + a)*x*e^(m*log(d) +
 m*log(x)))/(m^2 + (m + 1)*n + 2*m + 1)
```

Sympy [F]

$$\int (dx)^m \sqrt{a^2 + 2abx^n + b^2x^{2n}}\, dx = \int (dx)^m \sqrt{(a + bx^n)^2}\, dx$$

input
```
integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.42

$$\int (dx)^m \sqrt{a^2 + 2abx^n + b^2x^{2n}}\, dx = \frac{ad^m(m+n+1)xx^m + bd^m(m+1)xe^{(m\log(x)+n\log(x))}}{m^2 + m(n+2) + n + 1}$$

input
```
integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
(a*d^m*(m + n + 1)*x*x^m + b*d^m*(m + 1)*x*e^(m*log(x) + n*log(x)))/(m^2 +
 m*(n + 2) + n + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.56

$$\int (dx)^m \sqrt{a^2 + 2abx^n + b^2x^{2n}}\, dx$$
$$= \frac{bmxx^ne^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a) + amxe^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a) + bmxe^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx}{}$$

input
```
integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
(b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*m*x*e^(m*log(d) + m*
log(x))*sgn(b*x^n + a) + b*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*
n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*x^n*e^(m*log(d) + m*log(x
))*sgn(b*x^n + a) + a*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*e^(m*
log(d) + m*log(x))*sgn(b*x^n + a))/(m^2 + m*n + 2*m + n + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int (d\,x)^m\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.37

$$\int (dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^m d^m x(x^n bm+x^n b+am+an+a)}{m^2+mn+2m+n+1}$$

input
```
int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
(x**m*d**m*x*(x**n*b*m + x**n*b + a*m + a*n + a))/(m**2 + m*n + 2*m + n +
1)
```

3.149 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{(dx)^{1+m}(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
(d*x)^(1+m)*(a+b*x^n)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/d/(1+
m)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{x(dx)^m(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},1+\frac{1+m}{n},-\frac{bx^n}{a}\right)}{a(1+m)\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x*(d*x)^m*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, 1 + (1 + m)/n, -((b
*x^n)/a)])/(a*(1 + m)*Sqrt[(a + b*x^n)^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{(ab+b^2x^n)\int \frac{(dx)^m}{b^2x^n+ab}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(dx)^{m+1}(ab+b^2x^n)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{abd(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
((d*x)^(1 + m)*(a*b + b^2*x^n)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, -((b*x^n)/a)])/(a*b*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2x^n ab + b^2 x^{2n}}} dx$$

input
```
int((d*x)^m/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x)
```

output
```
int((d*x)^m/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} dx = \int \frac{(dx)^m}{\sqrt{b^2x^{2n} + 2abx^n + a^2}} dx$$

input
```
integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\, dx = \int \frac{(dx)^m}{\sqrt{(a + bx^n)^2}}\, dx$$

input
```
integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt((a + b*x**n)**2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\, dx = \int \frac{(dx)^m}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\, dx$$

input
```
integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}\, dx = \int \frac{(dx)^m}{\sqrt{b^2x^{2\,n} + 2\,abx^n + a^2}}\, dx$$

input
```
integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{(d\,x)^m}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=d^m\left(\int \frac{x^m}{x^nb+a}dx\right)$$

input
```
int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
d**m*int(x**m/(x**n*b + a),x)
```

3.150 $\int \frac{(dx)^m}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \frac{(dx)^m}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{(dx)^{1+m}\left(a+bx^n\right)\text{Hypergeometric2F1}\left(3,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
(d*x)^(1+m)*(a+b*x^n)*hypergeom([3, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^3/d/(
1+m)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \frac{(dx)^m}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x(dx)^m\left(a+bx^n\right)\text{Hypergeometric2F1}\left(3,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^3(1+m)\sqrt{\left(a+bx^n\right)^2}}$$

input
```
Integrate[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x*(d*x)^m*(a + b*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((b
*x^n)/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^n)^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1384, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{(ab^3+b^4x^n)\int \frac{(dx)^m}{(b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{888}$$

$$\frac{(dx)^{m+1}(ab^3+b^4x^n)\,\text{Hypergeometric2F1}\left(3,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{a^3b^3d(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
((d*x)^(1 + m)*(a*b^3 + b^4*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m +
n)/n, -((b*x^n)/a)])/(a^3*b^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)
])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{(dx)^m}{(a^2+2x^n ab+b^2x^{2n})^{\frac{3}{2}}}dx$$

input
```
int((d*x)^m/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x)
```

output
```
int((d*x)^m/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x)^m/(b^4*x^(4*n) + 4*a^2*
b^2*x^(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{\left((a + bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral((d*x)**m/((a + b*x**n)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
(m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*d^m*integrate(1/2*x^m/(a^2*b*n^2*x^n
 + a^3*n^2), x) - 1/2*(a*d^m*(m - 3*n + 1)*x*x^m + b*d^m*(m - 2*n + 1)*x*e
^(m*log(x) + n*log(x)))/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n^2)
```

Giac [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^{2\,n} + 2\,abx^n + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{(d\,x)^m}{\left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2}}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = d^m \left(\int \frac{x^m}{x^{3n}b^3 + 3x^{2n}a\,b^2 + 3x^n a^2 b + a^3} dx \right)$$

input
```
int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
d**m*int(x**m/(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b + a**3),x
)
```

3.151 $\int x^{-1+n}(a^2+2abx^n+b^2x^{2n})^{5/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 43

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{6bn}$$

output
```
1/6*(a+b*x^n)^5*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{(a+bx^n)\left((a+bx^n)^2\right)^{5/2}}{6bn}$$

input
```
Integrate[x^(-1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
((a + b*x^n)*((a + b*x^n)^2)^(5/2))/(6*b*n)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1690, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{\int \left(2abx^n+b^2x^{2n}+a^2\right)^{5/2}dx^n}{n}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^n+ab\right)^5dx^n}{b^5n\left(a+bx^n\right)}$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(a+bx^n\right)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{6bn}$$

input
```
Int[x^(-1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
((a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(6*b*n)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(39) = 78$.

Time = 0.06 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.74

method	result
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,b^5x^{6n}}{6(a+b\,x^n)n}+\frac{\sqrt{(a+b\,x^n)^2}\,a\,b^4x^{5n}}{(a+b\,x^n)n}+\frac{5\sqrt{(a+b\,x^n)^2}\,a^2b^3x^{4n}}{2(a+b\,x^n)n}+\frac{10\sqrt{(a+b\,x^n)^2}\,a^3b^2x^{3n}}{3(a+b\,x^n)n}+\frac{5\sqrt{(a+b\,x^n)^2}\,a^4b\,x^{2n}}{2(a+b\,x^n)n}+$

input
```
int(x^(-1+n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^5/n*(x^n)^6+((a+b*x^n)^2)^(1/2)/(a+b*x
^n)*a*b^4/n*(x^n)^5+5/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b^3/n*(x^n)^4+10
/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*b^2/n*(x^n)^3+5/2*((a+b*x^n)^2)^(1/2)
/(a+b*x^n)*a^4*b/n*(x^n)^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^5/n*x^n
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{b^5x^{6\,n}+6\,ab^4x^{5\,n}+15\,a^2b^3x^{4\,n}+20\,a^3b^2x^{3\,n}+15\,a^4bx^{2\,n}+6\,a^5x^n}{6\,n}$$

input
```
integrate(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="fricas"
)
```

output
```
1/6*(b^5*x^(6*n) + 6*a*b^4*x^(5*n) + 15*a^2*b^3*x^(4*n) + 20*a^3*b^2*x^(3*
n) + 15*a^4*b*x^(2*n) + 6*a^5*x^n)/n
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{b^5x^{6\,n}+6\,ab^4x^{5\,n}+15\,a^2b^3x^{4\,n}+20\,a^3b^2x^{3\,n}+15\,a^4bx^{2\,n}+6\,a^5x^n}{6\,n}$$

input
```
integrate(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxima"
)
```

output
```
1/6*(b^5*x^(6*n) + 6*a*b^4*x^(5*n) + 15*a^2*b^3*x^(4*n) + 20*a^3*b^2*x^(3*
n) + 15*a^4*b*x^(2*n) + 6*a^5*x^n)/n
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(39) = 78.

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.77

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{b^5x^{6\,n}\text{sgn}(bx^n+a)+6\,ab^4x^{5\,n}\text{sgn}(bx^n+a)+15\,a^2b^3x^{4\,n}\text{sgn}(bx^n+a)+20\,a^3b^2x^{3\,n}\text{sgn}(bx}{6\,n}$$

input
```
integrate(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac")
```

output
```
1/6*(b^5*x^(6*n)*sgn(b*x^n + a) + 6*a*b^4*x^(5*n)*sgn(b*x^n + a) + 15*a^2*
b^3*x^(4*n)*sgn(b*x^n + a) + 20*a^3*b^2*x^(3*n)*sgn(b*x^n + a) + 15*a^4*b*
x^(2*n)*sgn(b*x^n + a) + 6*a^5*x^n*sgn(b*x^n + a))/n
```

Mupad [B] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{\left(a\,b+b^2\,x^n\right)\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{5/2}}{6\,b^2\,n}$$

input
```
int(x^(n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
```

output
```
((a*b + b^2*x^n)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2))/(6*b^2*n)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{x^n\left(x^{5n}b^5+6x^{4n}a\,b^4+15x^{3n}a^2b^3+20x^{2n}a^3b^2+15x^na^4b+6a^5\right)}{6n}$$

input
```
int(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

output
```
(x**n*(x**(5*n)*b**5 + 6*x**(4*n)*a*b**4 + 15*x**(3*n)*a**2*b**3 + 20*x**(
2*n)*a**3*b**2 + 15*x**n*a**4*b + 6*a**5))/(6*n)
```

3.152 $\int x^{-1+n}(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 43

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{4bn}$$

output
```
1/4*(a+b*x^n)^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{(a+bx^n)\left((a+bx^n)^2\right)^{3/2}}{4bn}$$

input
```
Integrate[x^(-1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
((a + b*x^n)*((a + b*x^n)^2)^(3/2))/(4*b*n)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1690, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{\int \left(2abx^n+b^2x^{2n}+a^2\right)^{3/2}dx^n}{n}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^n+ab\right)^3dx^n}{b^3n\left(a+bx^n\right)}$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(a+bx^n\right)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{4bn}$$

input
```
Int[x^(-1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
((a + b*x^n)^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(4*b*n)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(39) = 78$.

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.07

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{3}x^{4n}}{4(a+b\,x^{n})n}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{2}a\,x^{3n}}{(a+b\,x^{n})n}+\frac{3\sqrt{(a+b\,x^{n})^{2}}\,b\,a^{2}x^{2n}}{2(a+b\,x^{n})n}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,a^{3}x^{n}}{(a+b\,x^{n})n}$	132

input
```
int(x^(-1+n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3/n*(x^n)^4+((a+b*x^n)^2)^(1/2)/(a+b*x
^n)*b^2*a/n*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b*a^2/n*(x^n)^2+((a+
b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/n*x^n
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{b^3x^{4\,n}+4\,ab^2x^{3\,n}+6\,a^2bx^{2\,n}+4\,a^3x^n}{4\,n}$$

input
```
integrate(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas"
)
```

output
```
1/4*(b^3*x^(4*n) + 4*a*b^2*x^(3*n) + 6*a^2*b*x^(2*n) + 4*a^3*x^n)/n
```

Sympy [F]

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int x^{n-1}\left((a+bx^n)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**(-1+n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x**(n - 1)*((a + b*x**n)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{b^3x^{4\,n}+4\,ab^2x^{3\,n}+6\,a^2bx^{2\,n}+4\,a^3x^n}{4\,n}$$

input
```
integrate(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima"
)
```

output
```
1/4*(b^3*x^(4*n) + 4*a*b^2*x^(3*n) + 6*a^2*b*x^(2*n) + 4*a^3*x^n)/n
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.79

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{b^3x^{4\,n}\mathrm{sgn}(bx^n+a)+4\,ab^2x^{3\,n}\mathrm{sgn}(bx^n+a)+6\,a^2bx^{2\,n}\mathrm{sgn}(bx^n+a)+4\,a^3x^n\mathrm{sgn}(bx^n+a)}{4\,n}$$

input
```
integrate(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
1/4*(b^3*x^(4*n)*sgn(b*x^n + a) + 4*a*b^2*x^(3*n)*sgn(b*x^n + a) + 6*a^2*b
*x^(2*n)*sgn(b*x^n + a) + 4*a^3*x^n*sgn(b*x^n + a))/n
```

Mupad [B] (verification not implemented)

Time = 10.75 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{\left(a\,b+b^2\,x^n\right)\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}{4\,b^2\,n}$$

input
```
int(x^(n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
((a*b + b^2*x^n)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2))/(4*b^2*n)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{x^n\left(x^{3n}b^3+4x^{2n}a\,b^2+6x^n a^2b+4a^3\right)}{4n}$$

input
```
int(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
(x**n*(x**(3*n)*b**3 + 4*x**(2*n)*a*b**2 + 6*x**n*a**2*b + 4*a**3))/(4*n)
```

3.153 $\int x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 41

$$\int x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}{2bn}$$

output
```
1/2*(a+b*x^n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{(a+bx^n)\sqrt{(a+bx^n)^2}}{2bn}$$

input
```
Integrate[x^(-1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
((a + b*x^n)*Sqrt[(a + b*x^n)^2])/(2*b*n)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1690, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{\int \sqrt{2abx^n+b^2x^{2n}+a^2}dx^n}{n}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(b^2x^n+ab\right)dx^n}{bn\left(a+bx^n\right)}$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}{2bn}$$

input
```
Int[x^(-1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
((a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*b*n)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

method	result	size
risch	$\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b\,x^{2 n}}{2\left(a+b\,x^{n}\right) n}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a\,x^{n}}{\left(a+b\,x^{n}\right) n}$	61

input
```
int(x^(-1+n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/n*(x^n)^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n
)*a/n*x^n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{bx^{2\,n}+2\,ax^n}{2\,n}$$

input
```
integrate(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas"
)
```

output
```
1/2*(b*x^(2*n) + 2*a*x^n)/n
```

Sympy [F]

$$\int x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^{n-1}\sqrt{(a+bx^n)^2}\,dx$$

input
```
integrate(x**(-1+n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**(n - 1)*sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{bx^{2n}+2\,ax^n}{2\,n}$$

input
```
integrate(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima"
)
```

output
```
1/2*(b*x^(2*n) + 2*a*x^n)/n
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{bx^{2n}\mathrm{sgn}(bx^n+a)+2\,ax^n\mathrm{sgn}(bx^n+a)}{2\,n}$$

input
```
integrate(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
1/2*(b*x^(2*n)*sgn(b*x^n + a) + 2*a*x^n*sgn(b*x^n + a))/n
```

Mupad [B] (verification not implemented)

Time = 10.63 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{\left(\frac{x^n}{2}+\frac{a}{2\,b}\right)\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}{n}$$

input
```
int(x^(n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
((x^n/2 + a/(2*b))*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2))/n
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.41

$$\int x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^n(x^nb+2a)}{2n}$$

input
```
int(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
(x**n*(x**n*b + 2*a))/(2*n)
```

3.154 $\int \frac{x^{-1+n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 46

$$\int \frac{x^{-1+n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{(a+bx^n)\log(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
(a+b*x^n)*ln(a+b*x^n)/b/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{x^{-1+n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{(a+bx^n)\log(a+bx^n)}{bn\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[x^(-1 + n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
((a + b*x^n)*Log[a + b*x^n])/(b*n*Sqrt[(a + b*x^n)^2])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1690, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$
$$\Big\downarrow \text{1690}$$
$$\frac{\int \frac{1}{\sqrt{2abx^n+b^2x^{2n}+a^2}}dx^n}{n}$$
$$\Big\downarrow \text{1079}$$
$$\frac{b(a+bx^n)\int \frac{1}{b^2x^n+ab}dx^n}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$\Big\downarrow \text{16}$$
$$\frac{(a+bx^n)\log(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x^(-1 + n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
((a + b*x^n)*Log[a + b*x^n])/(b*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,\ln\left(x^n+\frac{a}{b}\right)}{(a+b\,x^n)bn}$	38

input
```
int(x^(-1+n)/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)/b/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.33

$$\int \frac{x^{-1+n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{\log\left(bx^n+a\right)}{bn}$$

input
```
integrate(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas"
)
```

output
```
log(b*x^n + a)/(b*n)
```

Sympy [F]

$$\int \frac{x^{-1+n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^{n-1}}{\sqrt{(a+bx^n)^2}}\,dx$$

input
```
integrate(x**(-1+n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**(n - 1)/sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.41

$$\int \frac{x^{-1+n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{\log\left(\frac{bx^n+a}{b}\right)}{bn}$$

input
```
integrate(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima"
)
```

output
```
log((b*x^n + a)/b)/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \frac{x^{-1+n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{\log(|bx^n+a|)}{bn\mathrm{sgn}(bx^n+a)}$$

input
```
integrate(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
log(abs(b*x^n + a))/(b*n*sgn(b*x^n + a))
```

Mupad [B] (verification not implemented)

Time = 11.56 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \frac{x^{-1+n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{\mathrm{sign}(a+b\,x^n)\,\ln\left(a\,b+b^2\,x^n\right)\,\mathrm{sign}(b)}{n\,\sqrt{b^2}}$$

input
```
int(x^(n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
(sign(a + b*x^n)*log(a*b + b^2*x^n)*sign(b))/(n*(b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.33

$$\int \frac{x^{-1+n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{\log(x^n b+a)}{bn}$$

input
```
int(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
log(x**n*b + a)/(b*n)
```

3.155 $\int \frac{x^{-1+n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 43

$$\int \frac{x^{-1+n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=-\frac{1}{2bn\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-1/2/b/n/(a+b*x^n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{x^{-1+n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=-\frac{a+bx^n}{2bn\left(\left(a+bx^n\right)^2\right)^{3/2}}$$

input
```
Integrate[x^(-1 + n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
-1/2*(a + b*x^n)/(b*n*((a + b*x^n)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1690, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 1690}$$
$$\frac{\int \frac{1}{\left(2abx^n+b^2x^{2n}+a^2\right)^{3/2}}dx^n}{n}$$
$$\Big\downarrow \text{ 1078}$$
$$-\frac{1}{2bn\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x^(-1 + n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
-1/2*1/(b*n*(a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^{n})^{2}}}{2(a+b\,x^{n})^{3} b n}$	29

input
```
int(x^(-1+n)/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^3/b/n
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=-\frac{1}{2\,(b^3nx^{2\,n}+2\,ab^2nx^n+a^2bn)}$$

input
```
integrate(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas"
)
```

output
```
-1/2/(b^3*n*x^(2*n) + 2*a*b^2*n*x^n + a^2*b*n)
```

Sympy [F]

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x^{n-1}}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(-1+n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x**(n - 1)/((a + b*x**n)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{1}{2\,(b^3nx^{2\,n}+2\,ab^2nx^n+a^2bn)}$$

input
```
integrate(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima"
)
```

output
```
-1/2/(b^3*n*x^(2*n) + 2*a*b^2*n*x^n + a^2*b*n)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{1}{2\,(bx^n+a)^2bn\mathrm{sgn}\,(bx^n+a)}$$

input
```
integrate(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
-1/2/((b*x^n + a)^2*b*n*sgn(b*x^n + a))
```

Mupad [B] (verification not implemented)

Time = 11.87 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}{2\,b\,n\,(a+b\,x^n)^3}$$

input
```
int(x^(n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
-(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/(2*b*n*(a + b*x^n)^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{1}{2bn\,(x^{2n}b^2+2x^nab+a^2)}$$

input
```
int(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
( - 1)/(2*b*n*(x**(2*n)*b**2 + 2*x**n*a*b + a**2))
```

3.156 $\int \frac{x^{-1+n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 43

$$\int \frac{x^{-1+n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}\,dx = -\frac{1}{4bn\left(a+bx^n\right)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-1/4/b/n/(a+b*x^n)^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{x^{-1+n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}\,dx = -\frac{a+bx^n}{4bn\left((a+bx^n)^2\right)^{5/2}}$$

input
```
Integrate[x^(-1 + n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
-1/4*(a + b*x^n)/(b*n*((a + b*x^n)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1690, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{\int \frac{1}{(2abx^n+b^2x^{2n}+a^2)^{5/2}}dx^n}{n}$$

$$\Big\downarrow \text{1078}$$

$$-\frac{1}{4bn(a+bx^n)(a^2+2abx^n+b^2x^{2n})^{3/2}}$$

input
```
Int[x^(-1 + n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
-1/4*1/(b*n*(a + b*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^{n})^{2}}}{4(a+b\,x^{n})^{5} b n}$	29

input
```
int(x^(-1+n)/(a^2+2*x^n*a*b+b^2*x^(2*n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^5/b/n
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = -\frac{1}{4\,(b^5nx^{4\,n}+4\,ab^4nx^{3\,n}+6\,a^2b^3nx^{2\,n}+4\,a^3b^2nx^n+a^4bn)}$$

input
```
integrate(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="fricas"
)
```

output
```
-1/4/(b^5*n*x^(4*n) + 4*a*b^4*n*x^(3*n) + 6*a^2*b^3*n*x^(2*n) + 4*a^3*b^2*
n*x^n + a^4*b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = -\frac{1}{4\,(b^5nx^{4\,n}+4\,ab^4nx^{3\,n}+6\,a^2b^3nx^{2\,n}+4\,a^3b^2nx^n+a^4bn)}$$

input
```
integrate(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxima"
)
```

output
```
-1/4/(b^5*n*x^(4*n) + 4*a*b^4*n*x^(3*n) + 6*a^2*b^3*n*x^(2*n) + 4*a^3*b^2*
n*x^n + a^4*b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = -\frac{1}{4\,(bx^n+a)^4bn\mathrm{sgn}\,(bx^n+a)}$$

input
```
integrate(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac")
```

output
```
-1/4/((b*x^n + a)^4*b*n*sgn(b*x^n + a))
```

Mupad [B] (verification not implemented)

Time = 12.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = -\frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}{4\,b\,n\,(a+b\,x^n)^5}$$

input
```
int(x^(n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
```

output
```
-(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/(4*b*n*(a + b*x^n)^5)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = -\frac{1}{4bn\,(x^{4n}b^4+4x^{3n}a\,b^3+6x^{2n}a^2b^2+4x^na^3b+a^4)}$$

input
```
int(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

output
```
( - 1)/(4*b*n*(x**(4*n)*b**4 + 4*x**(3*n)*a*b**3 + 6*x**(2*n)*a**2*b**2 +
4*x**n*a**3*b + a**4))
```

3.157 $\int \frac{x^{-1+n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 43

$$\int \frac{x^{-1+n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx=-\frac{1}{6bn\,(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-1/6/b/n/(a+b*x^n)^5/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{x^{-1+n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx=-\frac{a+bx^n}{6bn\left((a+bx^n)^2\right)^{7/2}}$$

input
```
Integrate[x^(-1 + n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

output
```
-1/6*(a + b*x^n)/(b*n*((a + b*x^n)^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1690, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{\int \frac{1}{(2abx^n+b^2x^{2n}+a^2)^{7/2}}dx^n}{n}$$

$$\Big\downarrow \text{1078}$$

$$-\frac{1}{6bn\,(a+bx^n)\,(a^2+2abx^n+b^2x^{2n})^{5/2}}$$

input
```
Int[x^(-1 + n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

output
```
-1/6*1/(b*n*(a + b*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2))
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^{n})^{2}}}{6(a+b\,x^{n})^{7} b n}$	29

input
```
int(x^(-1+n)/(a^2+2*x^n*a*b+b^2*x^(2*n))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^7/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx =$$
$$-\frac{1}{6\,(b^7nx^{6\,n}+6\,ab^6nx^{5\,n}+15\,a^2b^5nx^{4\,n}+20\,a^3b^4nx^{3\,n}+15\,a^4b^3nx^{2\,n}+6\,a^5b^2nx^n+a^6bn)}$$

input
```
integrate(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="fricas"
)
```

output
```
-1/6/(b^7*n*x^(6*n) + 6*a*b^6*n*x^(5*n) + 15*a^2*b^5*n*x^(4*n) + 20*a^3*b^
4*n*x^(3*n) + 15*a^4*b^3*n*x^(2*n) + 6*a^5*b^2*n*x^n + a^6*b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(39) = 78$.

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = -\frac{1}{6\,(b^7nx^{6\,n}+6\,ab^6nx^{5\,n}+15\,a^2b^5nx^{4\,n}+20\,a^3b^4nx^{3\,n}+15\,a^4b^3nx^{2\,n}+6\,a^5b^2nx^n+a^6bn)}$$

input
```
integrate(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="maxima"
)
```

output
```
-1/6/(b^7*n*x^(6*n) + 6*a*b^6*n*x^(5*n) + 15*a^2*b^5*n*x^(4*n) + 20*a^3*b^
4*n*x^(3*n) + 15*a^4*b^3*n*x^(2*n) + 6*a^5*b^2*n*x^n + a^6*b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = -\frac{1}{6\,(bx^n+a)^6bn\mathrm{sgn}\,(bx^n+a)}$$

input
```
integrate(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="giac")
```

output
```
-1/6/((b*x^n + a)^6*b*n*sgn(b*x^n + a))
```

Mupad [B] (verification not implemented)

Time = 12.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx=-\frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}{6\,b\,n\,{\left(a+b\,x^n\right)}^7}$$

input
```
int(x^(n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2),x)
```

output
```
-(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/(6*b*n*(a + b*x^n)^7)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.91

$$\int \frac{x^{-1+n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = -\frac{1}{6bn\left(x^{6n}b^6+6x^{5n}a\,b^5+15x^{4n}a^2b^4+20x^{3n}a^3b^3+15x^{2n}a^4b^2+6x^n a^5b+a^6\right)}$$

input
```
int(x^(-1+n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x)
```

output
```
( - 1)/(6*b*n*(x**(6*n)*b**6 + 6*x**(5*n)*a*b**5 + 15*x**(4*n)*a**2*b**4 +
 20*x**(3*n)*a**3*b**3 + 15*x**(2*n)*a**4*b**2 + 6*x**n*a**5*b + a**6))
```

3.158 $\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^{5/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 79

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx = -\frac{a(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{6b^2n}+\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}{7b^2n}$$

output
```
-1/6*a*(a+b*x^n)^5*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/b^2/n+1/7*(a^2+2*a*b*
x^n+b^2*x^(2*n))^(7/2)/b^2/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.22

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx = \frac{x^{2n}\left((a+bx^n)^2\right)^{5/2}\left(21a^5+70a^4bx^n+105a^3b^2x^{2n}+84a^2b^3x^{3n}+35ab^4x^{4n}+6b^5x^{5n}\right)}{42n\,(a+bx^n)^5}$$

input
```
Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
(x^(2*n)*((a + b*x^n)^2)^(5/2)*(21*a^5 + 70*a^4*b*x^n + 105*a^3*b^2*x^(2*n
) + 84*a^2*b^3*x^(3*n) + 35*a*b^4*x^(4*n) + 6*b^5*x^(5*n)))/(42*n*(a + b*x
^n)^5)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int x^n\left(2abx^n+b^2x^{2n}+a^2\right)^{5/2}dx^n}{n}$$

$$\Big\downarrow \text{1100}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}{7b^2}-\frac{a\int\left(2abx^n+b^2x^{2n}+a^2\right)^{5/2}dx^n}{b}}{n}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}{7b^2}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(b^2x^n+ab\right)^5dx^n}{b^6(a+bx^n)}}{n}$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}{7b^2}-\frac{a(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{6b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
(-1/6*(a*(a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/b^2 + (a^2 + 2
*a*b*x^n + b^2*x^(2*n))^(7/2)/(7*b^2))/n
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(71) = 142$.

Time = 0.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.63

method	result
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,b^5x^{7n}}{7(a+b\,x^n)n}+\frac{5\sqrt{(a+b\,x^n)^2}\,a\,b^4x^{6n}}{6(a+b\,x^n)n}+\frac{2\sqrt{(a+b\,x^n)^2}\,a^2b^3x^{5n}}{(a+b\,x^n)n}+\frac{5\sqrt{(a+b\,x^n)^2}\,a^3b^2x^{4n}}{2(a+b\,x^n)n}+\frac{5\sqrt{(a+b\,x^n)^2}\,a^4b\,x^{3n}}{3(a+b\,x^n)n}+$

input
```
int(x^(-1+2*n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^5/n*(x^n)^7+5/6*((a+b*x^n)^2)^(1/2)/(a
+b*x^n)*a*b^4/n*(x^n)^6+2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b^3/n*(x^n)^5+
5/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*b^2/n*(x^n)^4+5/3*((a+b*x^n)^2)^(1/2
)/(a+b*x^n)*a^4*b/n*(x^n)^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^5/n*(x^n)^
2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{6\,b^5x^{7\,n}+35\,ab^4x^{6\,n}+84\,a^2b^3x^{5\,n}+105\,a^3b^2x^{4\,n}+70\,a^4bx^{3\,n}+21\,a^5x^{2\,n}}{42\,n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="frica
s")
```

output
```
1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*
x^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{6\,b^5x^{7\,n}+35\,ab^4x^{6\,n}+84\,a^2b^3x^{5\,n}+105\,a^3b^2x^{4\,n}+70\,a^4bx^{3\,n}+21\,a^5x^{2\,n}}{42\,n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxim
a")
```

output
```
1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*
x^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

Giac [F]

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\int\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{5}{2}}x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2)*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\int x^{2\,n-1}\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{5/2}\,dx$$

input
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
```

output
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{x^{2n}\left(6x^{5n}b^5+35x^{4n}a\,b^4+84x^{3n}a^2b^3+105x^{2n}a^3b^2+70x^na^4b+21a^5\right)}{42n}$$

input
```
int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

output
```
(x**(2*n)*(6*x**(5*n)*b**5 + 35*x**(4*n)*a*b**4 + 84*x**(3*n)*a**2*b**3 +
105*x**(2*n)*a**3*b**2 + 70*x**n*a**4*b + 21*a**5))/(42*n)
```

3.159 $\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 79

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx = -\frac{a(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{4b^2n}+\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}{5b^2n}$$

output
```
-1/4*a*(a+b*x^n)^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/b^2/n+1/5*(a^2+2*a*b*
x^n+b^2*x^(2*n))^(5/2)/b^2/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx = \frac{x^{2n}\left((a+bx^n)^2\right)^{3/2}\left(10a^3+20a^2bx^n+15ab^2x^{2n}+4b^3x^{3n}\right)}{20n\,(a+bx^n)^3}$$

input
```
Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(x^(2*n)*((a + b*x^n)^2)^(3/2)*(10*a^3 + 20*a^2*b*x^n + 15*a*b^2*x^(2*n) +
 4*b^3*x^(3*n)))/(20*n*(a + b*x^n)^3)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int x^n\left(2abx^n+b^2x^{2n}+a^2\right)^{3/2}dx^n}{n}$$

$$\Big\downarrow \text{1100}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}{5b^2}-\frac{a\int\left(2abx^n+b^2x^{2n}+a^2\right)^{3/2}dx^n}{b}}{n}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}{5b^2}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(b^2x^n+ab\right)^3dx^n}{b^4(a+bx^n)}}{n}$$

$$\Big\downarrow \text{17}$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}{5b^2}-\frac{a(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{4b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(-1/4*(a*(a + b*x^n)^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/b^2 + (a^2 + 2
*a*b*x^n + b^2*x^(2*n))^(5/2)/(5*b^2))/n
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.71

method	result	size
risch	$\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{3}x^{5n}}{5\left(a+b\,x^{n}\right)n}+\frac{3\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b^{2}a\,x^{4n}}{4\left(a+b\,x^{n}\right)n}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b\,a^{2}x^{3n}}{\left(a+b\,x^{n}\right)n}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a^{3}x^{2n}}{2\left(a+b\,x^{n}\right)n}$	135

input
```
int(x^(-1+2*n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3/n*(x^n)^5+3/4*((a+b*x^n)^2)^(1/2)/(a
+b*x^n)*b^2*a/n*(x^n)^4+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b*a^2/n*(x^n)^3+1/2*
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/n*(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.61

$$\int x^{-1+2n} \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} \, dx = \frac{4\, b^3 x^{5\, n} + 15\, a b^2 x^{4\, n} + 20\, a^2 b x^{3\, n} + 10\, a^3 x^{2\, n}}{20\, n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="frica
s")
```

output
```
1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n)
)/n
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+2n} \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.61

$$\int x^{-1+2n} \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} \, dx = \frac{4\, b^3 x^{5\, n} + 15\, a b^2 x^{4\, n} + 20\, a^2 b x^{3\, n} + 10\, a^3 x^{2\, n}}{20\, n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxim
a")
```

output
```
1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n)
)/n
```

Giac [F]

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int \left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{3}{2}}x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int x^{2\,n-1}\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}\,dx$$

input
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{x^{2n}(4x^{3n}b^3+15x^{2n}a\,b^2+20x^na^2b+10a^3)}{20n}$$

input
```
int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
(x**(2*n)*(4*x**(3*n)*b**3 + 15*x**(2*n)*a*b**2 + 20*x**n*a**2*b + 10*a**3
))/(20*n)
```

3.160 $\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 77

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=-\frac{a(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}{2b^2n}+\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{3b^2n}$$

output
```
-1/2*a*(a+b*x^n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/b^2/n+1/3*(a^2+2*a*b*x^
n+b^2*x^(2*n))^(3/2)/b^2/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.57

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^{2n}\sqrt{(a+bx^n)^2}(3a+2bx^n)}{6n(a+bx^n)}$$

input
```
Integrate[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(x^(2*n)*Sqrt[(a + b*x^n)^2]*(3*a + 2*b*x^n))/(6*n*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

$$\Big\downarrow \ 1693$$

$$\frac{\int x^n\sqrt{2abx^n+b^2x^{2n}+a^2}dx^n}{n}$$

$$\Big\downarrow \ 1100$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{3b^2}-\frac{a\int \sqrt{2abx^n+b^2x^{2n}+a^2}dx^n}{b}}{n}$$

$$\Big\downarrow \ 1079$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{3b^2}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\int (b^2x^n+ab)\,dx^n}{b^2(a+bx^n)}}{n}$$

$$\Big\downarrow \ 17$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{3b^2}-\frac{a(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}{2b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(-1/2*(a*(a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/b^2 + (a^2 + 2*a
*b*x^n + b^2*x^(2*n))^(3/2)/(3*b^2))/n
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b\,x^{3n}}{3\left(a+b\,x^{n}\right)n}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a\,x^{2n}}{2\left(a+b\,x^{n}\right)n}$	64

input
```
int(x^(-1+2*n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/n*(x^n)^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b
*x^n)*a/n*(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.29

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="frica
s")
```

output
```
1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

Sympy [F]

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^{2n-1}\sqrt{(a+bx^n)^2}\,dx$$

input
```
integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**(2*n - 1)*sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.29

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxim
a")
```

output
```
1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

Giac [F]

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int \sqrt{b^2x^{2\,n}+2\,abx^n+a^2}x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^{2\,n-1}\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

input
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.26

$$\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^{2n}(2x^nb+3a)}{6n}$$

input
```
int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
(x**(2*n)*(2*x***n*b + 3*a))/(6*n)
```

3.161 $\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 80

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{b^2n} - \frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/b^2/n-a*(a+b*x^n)*ln(a+b*x^n)/b^2/n/(a^2
+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{(a+bx^n)(bx^n-a\log(bn(a+bx^n)))}{b^2n\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
((a + b*x^n)*(b*x^n - a*Log[b*n*(a + b*x^n)]))/(b^2*n*Sqrt[(a + b*x^n)^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1693, 1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int \frac{x^n}{\sqrt{2abx^n+b^2x^{2n}+a^2}}dx^n}{n}$$

$$\Big\downarrow \text{1100}$$

$$\frac{\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{b^2}-\frac{a\int \frac{1}{\sqrt{2abx^n+b^2x^{2n}+a^2}}dx^n}{b}}{n}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{b^2}-\frac{a(a+bx^n)\int \frac{1}{b^2x^n+ab}dx^n}{\sqrt{a^2+2abx^n+b^2x^{2n}}}}{n}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{b^2}-\frac{a(a+bx^n)\log(a+bx^n)}{b^2\sqrt{a^2+2abx^n+b^2x^{2n}}}}{n}$$

input
```
Int[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/b^2 - (a*(a + b*x^n)*Log[a + b*x^n])/
(b^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))/n
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,x^{n}}{(a+b\,x^{n})bn}-\frac{\sqrt{(a+b\,x^{n})^{2}}\,a\ln(x^{n}+\frac{a}{b})}{(a+b\,x^{n})b^{2}n}$	71

input
```
int(x^(-1+2*n)/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)/b/n*x^n-((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/b^2/
n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.30

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{bx^n-a\log\left(bx^n+a\right)}{b^2n}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="frica
s")
```

output
```
(b*x^n - a*log(b*x^n + a))/(b^2*n)
```

Sympy [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{x^{2n-1}}{\sqrt{\left(a+bx^n\right)^2}}\,dx$$

input
```
integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**(2*n - 1)/sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.40

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{x^n}{bn}-\frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2n}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxim
a")
```

output
```
x^n/(b*n) - a*log((b*x^n + a)/b)/(b^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(x^(2*n - 1)/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

input
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

output
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.30

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{x^nb-\log(x^nb+a)\,a}{b^2n}$$

input
```
int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
(x**n*b - log(x**n*b + a)*a)/(b**2*n)
```

3.162 $\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 48

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{x^{2n}}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
1/2*x^(2*n)/a/n/(a+b*x^n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx=\frac{\left(-a-2bx^n\right)\left(a+bx^n\right)}{2b^2n\left(\left(a+bx^n\right)^2\right)^{3/2}}$$

input
```
Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
((-a - 2*b*x^n)*(a + b*x^n))/(2*b^2*n*((a + b*x^n)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1693, 1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int \frac{x^n}{\left(2abx^n+b^2x^{2n}+a^2\right)^{3/2}}dx^n}{n}$$

$$\Big\downarrow \text{1100}$$

$$\frac{-\frac{a\int \frac{1}{\left(2abx^n+b^2x^{2n}+a^2\right)^{3/2}}dx^n}{b}-\frac{1}{b^2\sqrt{a^2+2abx^n+b^2x^{2n}}}}{n}$$

$$\Big\downarrow \text{1078}$$

$$\frac{\frac{a}{2b^2(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{b^2\sqrt{a^2+2abx^n+b^2x^{2n}}}}{n}$$

input
```
Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(-(1/(b^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])) + a/(2*b^2*(a + b*x^n)*Sqr
t[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))/n
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,\left(2b\,x^{n}+a\right)}{2\left(a+b\,x^{n}\right)^{3} b^{2} n}$	37

input
```
int(x^(-1+2*n)/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^3*(2*b*x^n+a)/b^2/n
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{2\,bx^n+a}{2\,(b^4nx^{2\,n}+2\,ab^3nx^n+a^2b^2n)}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="frica
s")
```

output
```
-1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

Sympy [F]

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{x^{2n-1}}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x**(2*n - 1)/((a + b*x**n)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{2\,bx^n+a}{2\,(b^4nx^{2\,n}+2\,ab^3nx^n+a^2b^2n)}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxim
a")
```

output
```
-1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{x^{2\,n-1}}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac"
)
```

output
```
integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{x^{2\,n-1}}{(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)^{3/2}}\,dx$$

input
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

output
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \frac{x^{2n}}{2an\,(x^{2n}b^2+2x^nab+a^2)}$$

input
```
int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
x**(2*n)/(2*a*n*(x**(2*n)*b**2 + 2*x**n*a*b + a**2))
```

3.163 $\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 79

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = -\frac{1}{3b^2n\,(a^2+2abx^n+b^2x^{2n})^{3/2}} + \frac{a}{4b^2n\,(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-1/3/b^2/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)+1/4*a/b^2/n/(a+b*x^n)^3/(a^2+
2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.51

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = \frac{(-a-4bx^n)\,(a+bx^n)}{12b^2n\,\left((a+bx^n)^2\right)^{5/2}}$$

input
```
Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
((-a - 4*b*x^n)*(a + b*x^n))/(12*b^2*n*((a + b*x^n)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1693, 1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int \frac{x^n}{(2abx^n+b^2x^{2n}+a^2)^{5/2}}dx^n}{n}$$

$$\Big\downarrow \text{1100}$$

$$\frac{-\frac{a\int \frac{1}{\left(2abx^n+b^2x^{2n}+a^2\right)^{5/2}}dx^n}{b}-\frac{1}{3b^2(a^2+2abx^n+b^2x^{2n})^{3/2}}}{n}$$

$$\Big\downarrow \text{1078}$$

$$\frac{\frac{a}{4b^2(a+bx^n)(a^2+2abx^n+b^2x^{2n})^{3/2}}-\frac{1}{3b^2(a^2+2abx^n+b^2x^{2n})^{3/2}}}{n}$$

input
```
Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
(-1/3*1/(b^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)) + a/(4*b^2*(a + b*x^n)
*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)))/n
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

method	result	size
risch	$-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,\left(4 b\,x^{n}+a\right)}{12 \left(a+b\,x^{n}\right)^{5} b^{2} n}$	37

input
```
int(x^(-1+2*n)/(a^2+2*x^n*a*b+b^2*x^(2*n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^5*(4*b*x^n+a)/b^2/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = -\frac{4\,bx^n+a}{12\,(b^6nx^{4\,n}+4\,ab^5nx^{3\,n}+6\,a^2b^4nx^{2\,n}+4\,a^3b^3nx^n+a^4b^2n)}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="frica
s")
```

output
```
-1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*
n) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = -\frac{4\,bx^n+a}{12\,(b^6nx^{4\,n}+4\,ab^5nx^{3\,n}+6\,a^2b^4nx^{2\,n}+4\,a^3b^3nx^n+a^4b^2n)}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxim
a")
```

output
```
-1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*
n) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx=\int \frac{x^{2\,n-1}}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac"
)
```

output
```
integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx=\int \frac{x^{2\,n-1}}{(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)^{5/2}}\,dx$$

input
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
```

output
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx = \frac{-4x^n b-a}{12b^2n\,(x^{4n}b^4+4x^{3n}a\,b^3+6x^{2n}a^2b^2+4x^na^3b+a^4)}$$

input
```
int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

output
```
( - 4*x**n*b - a)/(12*b**2*n*(x**(4*n)*b**4 + 4*x**(3*n)*a*b**3 + 6*x**(2*
n)*a**2*b**2 + 4*x**n*a**3*b + a**4))
```

3.164 $\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 79

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx = -\frac{1}{5b^2n\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}} + \frac{a}{6b^2n\left(a+bx^n\right)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-1/5/b^2/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2)+1/6*a/b^2/n/(a+b*x^n)^5/(a^2+
2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.51

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx = \frac{\left(-a-6bx^n\right)\left(a+bx^n\right)}{30b^2n\left(\left(a+bx^n\right)^2\right)^{7/2}}$$

input
```
Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

output
```
((-a - 6*b*x^n)*(a + b*x^n))/(30*b^2*n*((a + b*x^n)^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1693, 1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int \frac{x^n}{(2abx^n+b^2x^{2n}+a^2)^{7/2}}dx^n}{n}$$

$$\Big\downarrow \text{1100}$$

$$\frac{-\frac{a\int \frac{1}{\left(2abx^n+b^2x^{2n}+a^2\right)^{7/2}}dx^n}{b}-\frac{1}{5b^2(a^2+2abx^n+b^2x^{2n})^{5/2}}}{n}$$

$$\Big\downarrow \text{1078}$$

$$\frac{\frac{a}{6b^2(a+bx^n)(a^2+2abx^n+b^2x^{2n})^{5/2}}-\frac{1}{5b^2(a^2+2abx^n+b^2x^{2n})^{5/2}}}{n}$$

input
```
Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

output
```
(-1/5*1/(b^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2)) + a/(6*b^2*(a + b*x^n)
*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2)))/n
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

method	result	size
risch	$-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,\left(6b\,x^{n}+a\right)}{30\left(a+b\,x^{n}\right)^{7} b^{2} n}$	37

input
```
int(x^(-1+2*n)/(a^2+2*x^n*a*b+b^2*x^(2*n))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^7*(6*b*x^n+a)/b^2/n
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = -\frac{6\,bx^n+a}{30\,(b^8nx^{6\,n}+6\,ab^7nx^{5\,n}+15\,a^2b^6nx^{4\,n}+20\,a^3b^5nx^{3\,n}+15\,a^4b^4nx^{2\,n}+6\,a^5b^3nx^n+a^6b^2n)}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="frica
s")
```

output
```
-1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4
*n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*
b^2*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = -\frac{6\,bx^n+a}{30\,(b^8nx^{6\,n}+6\,ab^7nx^{5\,n}+15\,a^2b^6nx^{4\,n}+20\,a^3b^5nx^{3\,n}+15\,a^4b^4nx^{2\,n}+6\,a^5b^3nx^n+a^6b^2n)}$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="maxim
a")
```

output
```
-1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4
*n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*
b^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx=\int \frac{x^{2\,n-1}}{\left(b^2x^{2\,n}+2\,abx^n+a^2\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="giac"
)
```

output
```
integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx=\int \frac{x^{2\,n-1}}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{7/2}}\,dx$$

input
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2),x)
```

output
```
int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}}\,dx=\frac{-6x^nb-a}{30b^2n\left(x^{6n}b^6+6x^{5n}a\,b^5+15x^{4n}a^2b^4+20x^{3n}a^3b^3+15x^{2n}a^4b^2+6x^na^5b+\right.}$$

input
```
int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x)
```

output
```
( - 6*x**n*b - a)/(30*b**2*n*(x**(6*n)*b**6 + 6*x**(5*n)*a*b**5 + 15*x**(4
*n)*a**2*b**4 + 20*x**(3*n)*a**3*b**3 + 15*x**(2*n)*a**4*b**2 + 6*x**n*a**
5*b + a**6))
```

3.165 $\int x^{-1-n}(a^2+2abx^n+b^2x^{2n})^{5/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 303

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx = -\frac{a^5x^{-n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(a+bx^n)}+\frac{10a^3b^3x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{5a^2b^4x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{5ab^5x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\,(ab+b^2x^n)}+\frac{b^6x^{4n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\,(ab+b^2x^n)}+\frac{5a^4b^2\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{ab+b^2x^n}$$

output

```
-a^5*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(x^n)/(a+b*x^n)+10*a^3*b^3*x^n*(a
^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+5*a^2*b^4*x^(2*n)*(a^2+2*a
*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+5/3*a*b^5*x^(3*n)*(a^2+2*a*b*x^n
+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+1/4*b^6*x^(4*n)*(a^2+2*a*b*x^n+b^2*x^(
2*n))^(1/2)/n/(a*b+b^2*x^n)+5*a^4*b^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)*ln
(x)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.33

$$\int x^{-1-n}(a^2+2abx^n+b^2x^{2n})^{5/2}\,dx=\frac{x^{-n}\sqrt{(a+bx^n)^2}(-12a^5+120a^3b^2x^{2n}+60a^2b^3x^{3n}+20ab^4x^{4n}+3b^5x^{5n}+60a^4bx^n\log(x}{12n(a+bx^n)}$$

input
```
Integrate[x^(-1 - n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(-12*a^5 + 120*a^3*b^2*x^(2*n) + 60*a^2*b^3*x^(3*n) +
 20*a*b^4*x^(4*n) + 3*b^5*x^(5*n) + 60*a^4*b*x^n*Log[x^n]))/(12*n*x^n*(a +
 b*x^n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.38, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1384, 798, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}(a^2+2abx^n+b^2x^{2n})^{5/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-n-1}(b^2x^n+ab)^5\,dx}{ab^5+b^6x^n}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int b^5x^{-2n}(bx^n+a)^5\,dx^n}{n(ab^5+b^6x^n)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^5\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-2n}(bx^n+a)^5\,dx^n}{n\,(ab^5+b^6x^n)}$$

$$\Big\downarrow \ 49$$

$$\frac{b^5\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(a^5x^{-2n}+5a^4bx^{-n}+10a^2b^3x^n+5ab^4x^{2n}+b^5x^{3n}+10a^3b^2\right)dx^n}{n\,(ab^5+b^6x^n)}$$

$$\Big\downarrow \ 2009$$

$$\frac{b^5\sqrt{a^2+2abx^n+b^2x^{2n}}\left(-a^5x^{-n}+5a^4b\log(x^n)+10a^3b^2x^n+5a^2b^3x^{2n}+\frac{5}{3}ab^4x^{3n}+\frac{1}{4}b^5x^{4n}\right)}{n\,(ab^5+b^6x^n)}$$

input
```
Int[x^(-1 - n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
(b^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(-(a^5/x^n) + 10*a^3*b^2*x^n + 5*
a^2*b^3*x^(2*n) + (5*a*b^4*x^(3*n))/3 + (b^5*x^(4*n))/4 + 5*a^4*b*Log[x^n]
))/(n*(a*b^5 + b^6*x^n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.66

method	result
risch	$\frac{5\sqrt{(a+b\,x^{n})^{2}}\,a^{4} b \ln(x)}{a+b\,x^{n}}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{5} x^{4n}}{4(a+b\,x^{n})n}+\frac{5\sqrt{(a+b\,x^{n})^{2}}\,a\,b^{4} x^{3n}}{3(a+b\,x^{n})n}+\frac{5\sqrt{(a+b\,x^{n})^{2}}\,a^{2} b^{3} x^{2n}}{(a+b\,x^{n})n}+\frac{10\sqrt{(a+b\,x^{n})^{2}}\,a^{3} b^{2} x^{n}}{(a+b\,x^{n})n}-$

input
```
int(x^(-1-n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
5*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^4*b*ln(x)+1/4*((a+b*x^n)^2)^(1/2)/(a+b*x
^n)*b^5/n*(x^n)^4+5/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^4/n*(x^n)^3+5*((a+
b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b^3/n*(x^n)^2+10*((a+b*x^n)^2)^(1/2)/(a+b*x^
n)*a^3*b^2/n*x^n-((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^5/n/(x^n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.25

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{60\,a^4bnx^n\log\left(x\right)+3\,b^5x^{5\,n}+20\,ab^4x^{4\,n}+60\,a^2b^3x^{3\,n}+120\,a^3b^2x^{2\,n}-12\,a^5}{12\,nx^n}$$

input
```
integrate(x^(-1-n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="fricas"
)
```

output
```
1/12*(60*a^4*b*n*x^n*log(x) + 3*b^5*x^(5*n) + 20*a*b^4*x^(4*n) + 60*a^2*b^
3*x^(3*n) + 120*a^3*b^2*x^(2*n) - 12*a^5)/(n*x^n)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1-n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.24

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=5\,a^4b\log\left(x\right)+\frac{3\,b^5x^{5\,n}+20\,ab^4x^{4\,n}+60\,a^2b^3x^{3\,n}+120\,a^3b^2x^{2\,n}-12\,a^5}{12\,nx^n}$$

input
```
integrate(x^(-1-n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxima"
)
```

output
```
5*a^4*b*log(x) + 1/12*(3*b^5*x^(5*n) + 20*a*b^4*x^(4*n) + 60*a^2*b^3*x^(3*
n) + 120*a^3*b^2*x^(2*n) - 12*a^5)/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.41

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{60\,a^4bnx^n\log\left(x\right)\operatorname{sgn}(bx^n+a)+3\,b^5x^{5\,n}\operatorname{sgn}(bx^n+a)+20\,ab^4x^{4\,n}\operatorname{sgn}(bx^n+a)+60\,a^2b^3x}{12\,nx^n}$$

input
```
integrate(x^(-1-n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac")
```

output
```
1/12*(60*a^4*b*n*x^n*log(x)*sgn(b*x^n + a) + 3*b^5*x^(5*n)*sgn(b*x^n + a)
+ 20*a*b^4*x^(4*n)*sgn(b*x^n + a) + 60*a^2*b^3*x^(3*n)*sgn(b*x^n + a) + 12
0*a^3*b^2*x^(2*n)*sgn(b*x^n + a) - 12*a^5*sgn(b*x^n + a))/(n*x^n)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\int\frac{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{5/2}}{x^{n+1}}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2)/x^(n + 1),x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2)/x^(n + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.25

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{3x^{5n}b^5+20x^{4n}a\,b^4+60x^{3n}a^2b^3+120x^{2n}a^3b^2+60x^n\log(x)\,a^4bn-12a^5}{12x^nn}$$

input
```
int(x^(-1-n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

output
```
(3*x**(5*n)*b**5 + 20*x**(4*n)*a*b**4 + 60*x**(3*n)*a**2*b**3 + 120*x**(2*
n)*a**3*b**2 + 60*x**n*log(x)*a**4*b*n - 12*a**5)/(12*x**n*n)
```

3.166 $\int x^{-1-n}(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 195

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx = -\frac{a^3x^{-n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(a+bx^n)}+\frac{3ab^3x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{b^4x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(ab+b^2x^n)}+\frac{3a^2b^2\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{ab+b^2x^n}$$

output

```
-a^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(x^n)/(a+b*x^n)+3*a*b^3*x^n*(a^2+
2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+1/2*b^4*x^(2*n)*(a^2+2*a*b*x^
n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+3*a^2*b^2*(a^2+2*a*b*x^n+b^2*x^(2*n))
^(1/2)*ln(x)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.37

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{x^{-n}\sqrt{(a+bx^n)^2}\left(-2a^3+6ab^2x^{2n}+b^3x^{3n}+6a^2bx^n\log(x^n)\right)}{2n(a+bx^n)}$$

input
```
Integrate[x^(-1 - n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(-2*a^3 + 6*a*b^2*x^(2*n) + b^3*x^(3*n) + 6*a^2*b*x^n
*Log[x^n]))/(2*n*x^n*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1384, 798, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-n-1}\left(b^2x^n+ab\right)^3dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int b^3x^{-2n}(bx^n+a)^3dx^n}{n\left(ab^3+b^4x^n\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^3\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-2n}(bx^n+a)^3dx^n}{n\left(ab^3+b^4x^n\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{b^3\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(a^3x^{-2n}+3a^2bx^{-n}+b^3x^n+3ab^2\right)dx^n}{n\left(ab^3+b^4x^n\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3\sqrt{a^2+2abx^n+b^2x^{2n}}\left(-a^3x^{-n}+3a^2b\log\left(x^n\right)+3ab^2x^n+\frac{1}{2}b^3x^{2n}\right)}{n\left(ab^3+b^4x^n\right)}$$

input
```
Int[x^(-1 - n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(b^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(-(a^3/x^n) + 3*a*b^2*x^n + (b^3*
x^(2*n))/2 + 3*a^2*b*Log[x^n]))/(n*(a*b^3 + b^4*x^n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{3\sqrt{(a+b\,x^{n})^{2}}\,b\,a^{2}\ln(x)}{a+b\,x^{n}}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{3}x^{2n}}{2(a+b\,x^{n})n}+\frac{3\sqrt{(a+b\,x^{n})^{2}}\,b^{2}a\,x^{n}}{(a+b\,x^{n})n}-\frac{\sqrt{(a+b\,x^{n})^{2}}\,a^{3}x^{-n}}{(a+b\,x^{n})n}$	128

input
```
int(x^(-1-n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b*a^2*ln(x)+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x
^n)*b^3/n*(x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2*a/n*x^n-((a+b*x^n)^2
)^(1/2)/(a+b*x^n)*a^3/n/(x^n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.25

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{6\,a^2bnx^n\log\left(x\right)+b^3x^{3\,n}+6\,ab^2x^{2\,n}-2\,a^3}{2\,nx^n}$$

input
```
integrate(x^(-1-n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas"
)
```

output
```
1/2*(6*a^2*b*n*x^n*log(x) + b^3*x^(3*n) + 6*a*b^2*x^(2*n) - 2*a^3)/(n*x^n)
```

Sympy [F]

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int x^{-n-1}\left((a+bx^n)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**(-1-n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x**(-n - 1)*((a + b*x**n)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.23

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=3\,a^2b\log\left(x\right)+\frac{b^3x^{3\,n}+6\,ab^2x^{2\,n}-2\,a^3}{2\,nx^n}$$

input
```
integrate(x^(-1-n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima"
)
```

output
```
3*a^2*b*log(x) + 1/2*(b^3*x^(3*n) + 6*a*b^2*x^(2*n) - 2*a^3)/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.41

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{6\,a^2bnx^n\log\left(x\right)\mathrm{sgn}(bx^n+a)+b^3x^{3\,n}\mathrm{sgn}(bx^n+a)+6\,ab^2x^{2\,n}\mathrm{sgn}(bx^n+a)-2\,a^3\mathrm{sgn}(bx^n$$
$$2\,nx^n$$

input
```
integrate(x^(-1-n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
1/2*(6*a^2*b*n*x^n*log(x)*sgn(b*x^n + a) + b^3*x^(3*n)*sgn(b*x^n + a) + 6*
a*b^2*x^(2*n)*sgn(b*x^n + a) - 2*a^3*sgn(b*x^n + a))/(n*x^n)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int \frac{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}{x^{n+1}}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^(n + 1),x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^(n + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.25

$$\int x^{-1-n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{x^{3n}b^3+6x^{2n}a\,b^2+6x^n\log(x)\,a^2bn-2a^3}{2x^nn}$$

input
```
int(x^(-1-n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
(x**(3*n)*b**3 + 6*x**(2*n)*a*b**2 + 6*x**n*log(x)*a**2*b*n - 2*a**3)/(2*x
**n*n)
```

3.167 $\int x^{-1-n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 82

$$\int x^{-1-n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=-\frac{ax^{-n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{n(a+bx^n)}+\frac{b\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{a+bx^n}$$

output
```
-a*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(x^n)/(a+b*x^n)+b*(a^2+2*a*b*x^n+b^
2*x^(2*n))^(1/2)*ln(x)/(a+b*x^n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int x^{-1-n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^{-n}\sqrt{(a+bx^n)^2}(-a+bx^n\log(x^n))}{n(a+bx^n)}$$

input
```
Integrate[x^(-1 - n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(-a + b*x^n*Log[x^n]))/(n*x^n*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-n-1}\left(b^2x^n+ab\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \ 802$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(abx^{-n-1}+\frac{b^2}{x}\right)dx}{ab+b^2x^n}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\left(b^2\log(x)-\frac{abx^{-n}}{n}\right)}{ab+b^2x^n}$$

input
```
Int[x^(-1 - n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(-((a*b)/(n*x^n)) + b^2*Log[x]))/(a*b
 + b^2*x^n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,b\ln(x)}{a+b\,x^{n}}-\frac{\sqrt{(a+b\,x^{n})^{2}}\,a\,x^{-n}}{(a+b\,x^{n})n}$	57

input
```
int(x^(-1-n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b*ln(x)-((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/n/(x
^n)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.26

$$\int x^{-1-n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{bnx^n\log{(x)}-a}{nx^n}$$

input
```
integrate(x^(-1-n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas"
)
```

output
```
(b*n*x^n*log(x) - a)/(n*x^n)
```

Sympy [F]

$$\int x^{-1-n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^{-n-1}\sqrt{(a+bx^n)^2}\,dx$$

input
```
integrate(x**(-1-n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**(-n - 1)*sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.20

$$\int x^{-1-n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=b\log(x)-\frac{a}{nx^n}$$

input
```
integrate(x^(-1-n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima"
)
```

output
```
b*log(x) - a/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.40

$$\int x^{-1-n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=b\log(|x|)\operatorname{sgn}(bx^n+a)-\frac{a\operatorname{sgn}(bx^n+a)}{nx^n}$$

input
```
integrate(x^(-1-n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
b*log(abs(x))*sgn(b*x^n + a) - a*sgn(b*x^n + a)/(n*x^n)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int \frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}{x^{n+1}}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^(n + 1),x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^(n + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.26

$$\int x^{-1-n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^n\log(x)\,bn-a}{x^n n}$$

input
```
int(x^(-1-n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
(x**n*log(x)*b*n - a)/(x**n*n)
```

3.168 $\int \frac{x^{-1-n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 137

$$\int \frac{x^{-1-n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}} \, dx = -\frac{x^{-n}(a+bx^n)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{(ab+b^2x^n)\log(x)}{a^2\sqrt{a^2+2abx^n+b^2x^{2n}}} + \frac{(ab+b^2x^n)\log(a+bx^n)}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-(a+b*x^n)/a/n/(x^n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-(a*b+b^2*x^n)*ln(x)
/a^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)+(a*b+b^2*x^n)*ln(a+b*x^n)/a^2/n/(a^
2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.42

$$\int \frac{x^{-1-n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}} \, dx = -\frac{x^{-n}(a+bx^n)\left(a+bx^n\log(x^n)-bx^n\log(a+bx^n)\right)}{a^2n\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[x^(-1 - n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
-(((a + b*x^n)*(a + b*x^n*Log[x^n] - b*x^n*Log[a + b*x^n]))/(a^2*n*x^n*Sqr
t[(a + b*x^n)^2]))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.55, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1384, 798, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{(ab+b^2x^n)\int \frac{x^{-n-1}}{b^2x^n+ab}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{798}$$

$$\frac{(ab+b^2x^n)\int \frac{x^{-2n}}{b(bx^n+a)}dx^n}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ab+b^2x^n)\int \frac{x^{-2n}}{bx^n+a}dx^n}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(ab+b^2x^n)\int \left(\frac{x^{-2n}}{a}-\frac{bx^{-n}}{a^2}+\frac{b^2}{a^2(bx^n+a)}\right)dx^n}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ab+b^2x^n)\left(-\frac{b\log(x^n)}{a^2}+\frac{b\log(a+bx^n)}{a^2}-\frac{x^{-n}}{a}\right)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x^(-1 - n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
((a*b + b^2*x^n)*(-(1/(a*x^n)) - (b*Log[x^n])/a^2 + (b*Log[a + b*x^n])/a^2
))/(b*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,x^{-n}}{\left(a+b\,x^{n}\right)a n}-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b\ln\left(x\right)}{\left(a+b\,x^{n}\right)a^{2}}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b\ln\left(x^{n}+\frac{a}{b}\right)}{\left(a+b\,x^{n}\right)a^{2} n}$	101

input
```
int(x^(-1-n)/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a+b*x^n)^2)^(1/2)/(a+b*x^n)/a/n/(x^n)-((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b*l
n(x)/a^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/a^2/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.27

$$\int \frac{x^{-1-n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=-\frac{bnx^n\log{(x)}-bx^n\log{(bx^n+a)}+a}{a^2nx^n}$$

input
```
integrate(x^(-1-n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas"
)
```

output
```
-(b*n*x^n*log(x) - b*x^n*log(b*x^n + a) + a)/(a^2*n*x^n)
```

Sympy [F]

$$\int \frac{x^{-1-n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{x^{-n-1}}{\sqrt{(a+bx^n)^2}}\,dx$$

input
```
integrate(x**(-1-n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**(-n - 1)/sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.31

$$\int \frac{x^{-1-n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{b\log(x)}{a^2}+\frac{b\log\left(\frac{bx^n+a}{b}\right)}{a^2n}-\frac{1}{anx^n}$$

input
```
integrate(x^(-1-n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima"
)
```

output
```
-b*log(x)/a^2 + b*log((b*x^n + a)/b)/(a^2*n) - 1/(a*n*x^n)
```

Giac [F]

$$\int \frac{x^{-1-n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^{-n-1}}{\sqrt{b^2x^{2n}+2\,abx^n+a^2}}\,dx$$

input
```
integrate(x^(-1-n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{1}{x^{n+1}\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

input
```
int(1/(x^(n + 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
```

output
```
int(1/(x^(n + 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.28

$$\int \frac{x^{-1-n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{x^n \log(x^n b+a)\,b-x^n \log(x)\,bn-a}{x^n a^2 n}$$

input
```
int(x^(-1-n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
(x**n*log(x**n*b + a)*b - x**n*log(x)*b*n - a)/(x**n*a**2*n)
```

3.169 $\int \frac{x^{-1-n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 244

$$\int \frac{x^{-1-n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = -\frac{x^{-n}(a+bx^n)}{a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{ab+b^2x^n}{2a^2n\left(a+bx^n\right)^2\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{2(ab+b^2x^n)}{a^3n\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{3(ab+b^2x^n)\log(x)}{a^4\sqrt{a^2+2abx^n+b^2x^{2n}}} + \frac{3(ab+b^2x^n)\log\left(a+bx^n\right)}{a^4n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-(a+b*x^n)/a^3/n/(x^n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-1/2*(a*b+b^2*x^n)
/a^2/n/(a+b*x^n)^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-2*(a*b+b^2*x^n)/a^3/n
/(a+b*x^n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-3*(a*b+b^2*x^n)*ln(x)/a^4/(a^
2+2*a*b*x^n+b^2*x^(2*n))^(1/2)+3*(a*b+b^2*x^n)*ln(a+b*x^n)/a^4/n/(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.43

$$\int \frac{x^{-1-n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \frac{x^{-n}\left(-a(2a^2+9abx^n+6b^2x^{2n})-6bx^n(a+bx^n)^2\log(x^n)+6bx^n(a+bx\right.}{2a^4n\,(a+bx^n)\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[x^(-1 - n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(-(a*(2*a^2 + 9*a*b*x^n + 6*b^2*x^(2*n))) - 6*b*x^n*(a + b*x^n)^2*Log[x^n]
 + 6*b*x^n*(a + b*x^n)^2*Log[a + b*x^n])/(2*a^4*n*x^n*(a + b*x^n)*Sqrt[(a
+ b*x^n)^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1384, 798, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{(ab^3+b^4x^n)\int \frac{x^{-n-1}}{(b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \; 798$$

$$\frac{(ab^3+b^4x^n)\int \frac{x^{-2n}}{b^3(bx^n+a)^3}dx^n}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \; 27$$

$$\frac{(ab^3+b^4x^n)\int \frac{x^{-2n}}{(bx^n+a)^3}dx^n}{b^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(ab^3+b^4x^n)\int\left(\frac{x^{-2n}}{a^3}-\frac{3bx^{-n}}{a^4}+\frac{3b^2}{a^4(bx^n+a)}+\frac{2b^2}{a^3(bx^n+a)^2}+\frac{b^2}{a^2(bx^n+a)^3}\right)dx^n}{b^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ab^3+b^4x^n)\left(-\frac{3b\log(x^n)}{a^4}+\frac{3b\log(a+bx^n)}{a^4}-\frac{2b}{a^3(a+bx^n)}-\frac{x^{-n}}{a^3}-\frac{b}{2a^2(a+bx^n)^2}\right)}{b^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x^(-1 - n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
((a*b^3 + b^4*x^n)*(-(1/(a^3*x^n)) - b/(2*a^2*(a + b*x^n)^2) - (2*b)/(a^3*
(a + b*x^n)) - (3*b*Log[x^n])/a^4 + (3*b*Log[a + b*x^n])/a^4))/(b^3*n*Sqrt
[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.58

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^{n})^{2}}\,x^{-n}}{(a+b\,x^{n})a^{3}n}-\frac{3\sqrt{(a+b\,x^{n})^{2}}\,b\ln(x)}{(a+b\,x^{n})a^{4}}-\frac{\sqrt{(a+b\,x^{n})^{2}}\,b(4b\,x^{n}+5a)}{2(a+b\,x^{n})^{3}a^{3}n}+\frac{3\sqrt{(a+b\,x^{n})^{2}}\,b\ln(x^{n}+\frac{a}{b})}{(a+b\,x^{n})a^{4}n}$	141

input
```
int(x^(-1-n)/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-((a+b*x^n)^2)^(1/2)/(a+b*x^n)/a^3/n/(x^n)-3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)
*b*ln(x)/a^4-1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^3*b*(4*b*x^n+5*a)/a^3/n+3*(
(a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/a^4/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.57

$$\int \frac{x^{-1-n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{6\,b^3nx^{3\,n}\log\left(x\right)+2\,a^3+6\,(2\,ab^2n\log\left(x\right)+ab^2)x^{2\,n}+3\,(2\,a^2bn\log\left(x\right)+3\,a^2b)x^n-6\,(b^3x^{3\,n}+2\,ab^2x^{2\,n}}{2\,(a^4b^2nx^{3\,n}+2\,a^5bnx^{2\,n}+a^6nx^n)}$$

input
```
integrate(x^(-1-n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas"
)
```

output
```
-1/2*(6*b^3*n*x^(3*n)*log(x) + 2*a^3 + 6*(2*a*b^2*n*log(x) + a*b^2)*x^(2*n
) + 3*(2*a^2*b*n*log(x) + 3*a^2*b)*x^n - 6*(b^3*x^(3*n) + 2*a*b^2*x^(2*n)
+ a^2*b*x^n)*log(b*x^n + a))/(a^4*b^2*n*x^(3*n) + 2*a^5*b*n*x^(2*n) + a^6*
n*x^n)
```

Sympy [F]

$$\int \frac{x^{-1-n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x^{-n-1}}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(-1-n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Integral(x**(-n - 1)/((a + b*x**n)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.37

$$\int \frac{x^{-1-n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = -\frac{6\,b^2x^{2\,n}+9\,abx^n+2\,a^2}{2\,(a^3b^2nx^{3\,n}+2\,a^4bnx^{2\,n}+a^5nx^n)}-\frac{3\,b\log(x)}{a^4}+\frac{3\,b\log\left(\frac{bx^n+a}{b}\right)}{a^4n}$$

input
```
integrate(x^(-1-n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima"
)
```

output
```
-1/2*(6*b^2*x^(2*n) + 9*a*b*x^n + 2*a^2)/(a^3*b^2*n*x^(3*n) + 2*a^4*b*n*x^
(2*n) + a^5*n*x^n) - 3*b*log(x)/a^4 + 3*b*log((b*x^n + a)/b)/(a^4*n)
```

Giac [F]

$$\int \frac{x^{-1-n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{x^{-n-1}}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1-n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \int \frac{1}{x^{n+1}\,(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n)^{3/2}}\,dx$$

input
```
int(1/(x^(n + 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
```

output
```
int(1/(x^(n + 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.63

$$\int \frac{x^{-1-n}}{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}\,dx = \frac{6x^{3n}\log(x^nb + a)\,b^3 - 6x^{3n}\log(x)\,b^3n + 3x^{3n}b^3 + 12x^{2n}\log(x^nb + a)\,a\,b^2 -}{2x^na^4n\,(x^{2n}b^2}$$

input
```
int(x^(-1-n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
(6*x**(3*n)*log(x**n*b + a)*b**3 - 6*x**(3*n)*log(x)*b**3*n + 3*x**(3*n)*b
**3 + 12*x**(2*n)*log(x**n*b + a)*a*b**2 - 12*x**(2*n)*log(x)*a*b**2*n + 6
*x**n*log(x**n*b + a)*a**2*b - 6*x**n*log(x)*a**2*b*n - 6*x**n*a**2*b - 2*
a**3)/(2*x**n*a**4*n*(x**(2*n)*b**2 + 2*x**n*a*b + a**2))
```

3.170 $\int x^{-1-2n}(a^2+2abx^n+b^2x^{2n})^{5/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 305

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx = -\frac{a^5x^{-2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(a+bx^n)}-\frac{5a^4b^2x^{-n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{10a^2b^4x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{5ab^5x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(ab+b^2x^n)}+\frac{b^6x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\,(ab+b^2x^n)}+\frac{10a^3b^3\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{ab+b^2x^n}$$

output
```
-1/2*a^5*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(x^(2*n))/(a+b*x^n)-5*a^4*b^2
*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(x^n)/(a*b+b^2*x^n)+10*a^2*b^4*x^n*(a
^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+5/2*a*b^5*x^(2*n)*(a^2+2*a
*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+1/3*b^6*x^(3*n)*(a^2+2*a*b*x^n+b
^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+10*a^3*b^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(
1/2)*ln(x)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.33

$$\int x^{-1-2n}(a^2+2abx^n+b^2x^{2n})^{5/2}\,dx = \frac{x^{-2n}\sqrt{(a+bx^n)^2}(-3a^5-30a^4bx^n+60a^2b^3x^{3n}+15ab^4x^{4n}+2b^5x^{5n}+60a^3b^2x^{2n}\log(x^n)}{6n(a+bx^n)}$$

input
```
Integrate[x^(-1 - 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
(Sqrt[(a + b*x^n)^2]*(-3*a^5 - 30*a^4*b*x^n + 60*a^2*b^3*x^(3*n) + 15*a*b^
4*x^(4*n) + 2*b^5*x^(5*n) + 60*a^3*b^2*x^(2*n)*Log[x^n]))/(6*n*x^(2*n)*(a
+ b*x^n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1384, 798, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}(a^2+2abx^n+b^2x^{2n})^{5/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-2n-1}(b^2x^n+ab)^5\,dx}{ab^5+b^6x^n}$$

$$\Big\downarrow \text{798}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int b^5x^{-3n}(bx^n+a)^5\,dx^n}{n(ab^5+b^6x^n)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^5\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-3n}(bx^n+a)^5\,dx^n}{n\,(ab^5+b^6x^n)}$$

$$\Big\downarrow \text{49}$$

$$\frac{b^5\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(a^5x^{-3n}+5a^4bx^{-2n}+10a^3b^2x^{-n}+5ab^4x^n+b^5x^{2n}+10a^2b^3\right)dx^n}{n\,(ab^5+b^6x^n)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^5\sqrt{a^2+2abx^n+b^2x^{2n}}\left(-\frac{1}{2}a^5x^{-2n}-5a^4bx^{-n}+10a^3b^2\log(x^n)+10a^2b^3x^n+\frac{5}{2}ab^4x^{2n}+\frac{1}{3}b^5x^{3n}\right)}{n\,(ab^5+b^6x^n)}$$

input
```
Int[x^(-1 - 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

output
```
(b^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(-1/2*a^5/x^(2*n) - (5*a^4*b)/x^n
 + 10*a^2*b^3*x^n + (5*a*b^4*x^(2*n))/2 + (b^5*x^(3*n))/3 + 10*a^3*b^2*Log
[x^n]))/(n*(a*b^5 + b^6*x^n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.66

method	result
risch	$\frac{10\sqrt{(a+b\,x^{n})^{2}}\,a^{3}b^{2}\ln(x)}{a+b\,x^{n}}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{5}x^{3n}}{3(a+b\,x^{n})n}+\frac{5\sqrt{(a+b\,x^{n})^{2}}\,a\,b^{4}x^{2n}}{2(a+b\,x^{n})n}+\frac{10\sqrt{(a+b\,x^{n})^{2}}\,a^{2}b^{3}x^{n}}{(a+b\,x^{n})n}-\frac{5\sqrt{(a+b\,x^{n})^{2}}\,a^{4}b\,x^{-n}}{(a+b\,x^{n})n}$

input
```
int(x^(-1-2*n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
10*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*b^2*ln(x)+1/3*((a+b*x^n)^2)^(1/2)/(a+
b*x^n)*b^5/n*(x^n)^3+5/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^4/n*(x^n)^2+10*
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b^3/n*x^n-5*((a+b*x^n)^2)^(1/2)/(a+b*x^n
)*a^4*b/n/(x^n)-1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^5/n/(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.25

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{60\,a^3b^2nx^{2\,n}\log\left(x\right)+2\,b^5x^{5\,n}+15\,ab^4x^{4\,n}+60\,a^2b^3x^{3\,n}-30\,a^4bx^n-3\,a^5}{6\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="frica
s")
```

output
```
1/6*(60*a^3*b^2*n*x^(2*n)*log(x) + 2*b^5*x^(5*n) + 15*a*b^4*x^(4*n) + 60*a
^2*b^3*x^(3*n) - 30*a^4*b*x^n - 3*a^5)/(n*x^(2*n))
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1-2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.24

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=10\,a^3b^2\log\left(x\right)+\frac{2\,b^5x^{5\,n}+15\,ab^4x^{4\,n}+60\,a^2b^3x^{3\,n}-30\,a^4bx^n-3\,a^5}{6\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxim
a")
```

output
```
10*a^3*b^2*log(x) + 1/6*(2*b^5*x^(5*n) + 15*a*b^4*x^(4*n) + 60*a^2*b^3*x^(
3*n) - 30*a^4*b*x^n - 3*a^5)/(n*x^(2*n))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.41

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{60\,a^3b^2nx^{2\,n}\log\left(x\right)\mathrm{sgn}(bx^n+a)+2\,b^5x^{5\,n}\mathrm{sgn}(bx^n+a)+15\,ab^4x^{4\,n}\mathrm{sgn}(bx^n+a)+60\,a^2b}{6\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac"
)
```

output
```
1/6*(60*a^3*b^2*n*x^(2*n)*log(x)*sgn(b*x^n + a) + 2*b^5*x^(5*n)*sgn(b*x^n
+ a) + 15*a*b^4*x^(4*n)*sgn(b*x^n + a) + 60*a^2*b^3*x^(3*n)*sgn(b*x^n + a)
 - 30*a^4*b*x^n*sgn(b*x^n + a) - 3*a^5*sgn(b*x^n + a))/(n*x^(2*n))
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\int \frac{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{5/2}}{x^{2\,n+1}}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2)/x^(2*n + 1),x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2)/x^(2*n + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.25

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx=\frac{2x^{5n}b^5+15x^{4n}a\,b^4+60x^{3n}a^2b^3+60x^{2n}\log(x)\,a^3b^2n-30x^na^4b-3a^5}{6x^{2n}n}$$

input
```
int(x^(-1-2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x)
```

output
```
(2*x**(5*n)*b**5 + 15*x**(4*n)*a*b**4 + 60*x**(3*n)*a**2*b**3 + 60*x**(2*n
)*log(x)*a**3*b**2*n - 30*x**n*a**4*b - 3*a**5)/(6*x**(2*n)*n)
```

3.171 $\int x^{-1-2n}(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 194

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx = -\frac{a^3x^{-2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(a+bx^n)}-\frac{3a^2b^2x^{-n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{b^4x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{3ab^3\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{ab+b^2x^n}$$

output
```
-1/2*a^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(x^(2*n))/(a+b*x^n)-3*a^2*b^2
*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(x^n)/(a*b+b^2*x^n)+b^4*x^n*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+3*a*b^3*(a^2+2*a*b*x^n+b^2*x^(2*n
))^(1/2)*ln(x)/(a*b+b^2*x^n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.37

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx = -\frac{x^{-2n}\sqrt{(a+bx^n)^2}\left(a^3+6a^2bx^n-2b^3x^{3n}-6ab^2x^{2n}\log\left(x^n\right)\right)}{2n\left(a+bx^n\right)}$$

input
```
Integrate[x^(-1 - 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
-1/2*(Sqrt[(a + b*x^n)^2]*(a^3 + 6*a^2*b*x^n - 2*b^3*x^(3*n) - 6*a*b^2*x^(
2*n)*Log[x^n]))/(n*x^(2*n)*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1384, 798, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-2n-1}\left(b^2x^n+ab\right)^3dx}{ab^3+b^4x^n}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int b^3x^{-3n}\left(bx^n+a\right)^3dx^n}{n\left(ab^3+b^4x^n\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b^3\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-3n}\left(bx^n+a\right)^3dx^n}{n\left(ab^3+b^4x^n\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{b^3\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(a^3x^{-3n}+3a^2bx^{-2n}+3ab^2x^{-n}+b^3\right)dx^n}{n\left(ab^3+b^4x^n\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3\sqrt{a^2+2abx^n+b^2x^{2n}}\left(-\frac{1}{2}a^3x^{-2n}-3a^2bx^{-n}+3ab^2\log\left(x^n\right)+b^3x^n\right)}{n\left(ab^3+b^4x^n\right)}$$

input
```
Int[x^(-1 - 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(b^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*(-1/2*a^3/x^(2*n) - (3*a^2*b)/x^n
 + b^3*x^n + 3*a*b^2*Log[x^n]))/(n*(a*b^3 + b^4*x^n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{3\sqrt{(a+b\,x^{n})^{2}}\,b^{2} a \ln(x)}{a+b\,x^{n}}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{3} x^{n}}{(a+b\,x^{n}) n}-\frac{3\sqrt{(a+b\,x^{n})^{2}}\,b\,a^{2} x^{-n}}{(a+b\,x^{n}) n}-\frac{\sqrt{(a+b\,x^{n})^{2}}\,a^{3} x^{-2n}}{2(a+b\,x^{n}) n}$	127

input
```
int(x^(-1-2*n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2*a*ln(x)+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*
b^3/n*x^n-3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b*a^2/n/(x^n)-1/2*((a+b*x^n)^2)^
(1/2)/(a+b*x^n)*a^3/n/(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.26

$$\int x^{-1-2n} \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{6\,ab^2nx^{2\,n}\log\left(x\right)+2\,b^3x^{3\,n}-6\,a^2bx^n-a^3}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="frica
s")
```

output
```
1/2*(6*a*b^2*n*x^(2*n)*log(x) + 2*b^3*x^(3*n) - 6*a^2*b*x^n - a^3)/(n*x^(2
*n))
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x**(-1-2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.24

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=3\,ab^2\log\left(x\right)+\frac{2\,b^3x^{3\,n}-6\,a^2bx^n-a^3}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxim
a")
```

output
```
3*a*b^2*log(x) + 1/2*(2*b^3*x^(3*n) - 6*a^2*b*x^n - a^3)/(n*x^(2*n))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.43

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{6\,ab^2nx^{2\,n}\log\left(x\right)\mathrm{sgn}(bx^n+a)+2\,b^3x^{3\,n}\mathrm{sgn}(bx^n+a)-6\,a^2bx^n\mathrm{sgn}(bx^n+a)-a^3\mathrm{sgn}(bx^n}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac"
)
```

output
```
1/2*(6*a*b^2*n*x^(2*n)*log(x)*sgn(b*x^n + a) + 2*b^3*x^(3*n)*sgn(b*x^n + a
) - 6*a^2*b*x^n*sgn(b*x^n + a) - a^3*sgn(b*x^n + a))/(n*x^(2*n))
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\int\frac{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}{x^{2\,n+1}}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^(2*n + 1),x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^(2*n + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.26

$$\int x^{-1-2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{2x^{3n}b^3+6x^{2n}\log(x)\,a\,b^2n-6x^na^2b-a^3}{2x^{2n}n}$$

input
```
int(x^(-1-2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
(2*x**(3*n)*b**3 + 6*x**(2*n)*log(x)*a*b**2*n - 6*x**n*a**2*b - a**3)/(2*x
**(2*n)*n)
```

3.172 $\int x^{-1-2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 46

$$\int x^{-1-2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=-\frac{x^{-2n}(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}{2an}$$

output
```
-1/2*(a+b*x^n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/a/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^{-1-2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{x^{-2n}(-a-2bx^n)\sqrt{(a+bx^n)^2}}{2n\,(a+bx^n)}$$

input
```
Integrate[x^(-1 - 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
((-a - 2*b*x^n)*Sqrt[(a + b*x^n)^2])/(2*n*x^(2*n)*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1693, 1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int x^{-3n}\sqrt{2abx^n+b^2x^{2n}+a^2}dx^n}{n}$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int bx^{-3n}(bx^n+a)\,dx^n}{bn\,(a+bx^n)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-3n}(bx^n+a)\,dx^n}{n\,(a+bx^n)}$$

$$\Big\downarrow \text{48}$$

$$-\frac{x^{-2n}(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}{2an}$$

input
```
Int[x^(-1 - 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
-1/2*((a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(a*n*x^(2*n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

method	result	size
risch	$-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,b\,x^{-n}}{\left(a+b\,x^{n}\right)n}-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,a\,x^{-2n}}{2\left(a+b\,x^{n}\right)n}$	64

input
```
int(x^(-1-2*n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/n/(x^n)-1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)
*a/n/(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.43

$$\int x^{-1-2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=-\frac{2\,bx^n+a}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="frica
s")
```

output
```
-1/2*(2*b*x^n + a)/(n*x^(2*n))
```

Sympy [F]

$$\int x^{-1-2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int x^{-2n-1}\sqrt{(a+bx^n)^2}\,dx$$

input
```
integrate(x**(-1-2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**(-2*n - 1)*sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.43

$$\int x^{-1-2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=-\frac{2\,bx^n+a}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxim
a")
```

output
```
-1/2*(2*b*x^n + a)/(n*x^(2*n))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int x^{-1-2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=-\frac{2\,bx^n\mathrm{sgn}(bx^n+a)+a\mathrm{sgn}(bx^n+a)}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac"
)
```

output
```
-1/2*(2*b*x^n*sgn(b*x^n + a) + a*sgn(b*x^n + a))/(n*x^(2*n))
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\int\frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}{x^{2\,n+1}}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^(2*n + 1),x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^(2*n + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int x^{-1-2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx=\frac{-2x^nb-a}{2x^{2n}n}$$

input
```
int(x^(-1-2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
( - 2*x**n*b - a)/(2*x**(2*n)*n)
```

3.173 $\int \frac{x^{-1-2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 190

$$\int \frac{x^{-1-2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{x^{-2n}(a+bx^n)}{2an\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{x^{-n}(ab+b^2x^n)}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}} + \frac{(ab^2+b^3x^n)\log(x)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(ab^2+b^3x^n)\log(a+bx^n)}{a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

output
```
-1/2*(a+b*x^n)/a/n/(x^(2*n))/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)+(a*b+b^2*x^
n)/a^2/n/(x^n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)+(a*b^2+b^3*x^n)*ln(x)/a^3
/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-(a*b^2+b^3*x^n)*ln(a+b*x^n)/a^3/n/(a^2+
2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

$$\int \frac{x^{-1-2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = -\frac{x^{-2n}(a+bx^n)\left(a(a-2bx^n)-2b^2x^{2n}\log\left(x^n\right)+2b^2x^{2n}\log\left(a+bx^n\right)\right)}{2a^3n\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[x^(-1 - 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
-1/2*((a + b*x^n)*(a*(a - 2*b*x^n) - 2*b^2*x^(2*n)*Log[x^n] + 2*b^2*x^(2*n
)*Log[a + b*x^n]))/(a^3*n*x^(2*n)*Sqrt[(a + b*x^n)^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1384, 798, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2n-1}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\left(ab+b^2x^n\right)\int \frac{x^{-2n-1}}{b^2x^n+ab}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\left(ab+b^2x^n\right)\int \frac{x^{-3n}}{b(bx^n+a)}dx^n}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(ab+b^2x^n\right)\int \frac{x^{-3n}}{bx^n+a}dx^n}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\left(ab+b^2x^n\right)\int \left(\frac{x^{-3n}}{a}-\frac{bx^{-2n}}{a^2}+\frac{b^2x^{-n}}{a^3}-\frac{b^3}{a^3(bx^n+a)}\right)dx^n}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(ab+b^2x^n\right)\left(\frac{b^2\log(x^n)}{a^3}-\frac{b^2\log(a+bx^n)}{a^3}+\frac{bx^{-n}}{a^2}-\frac{x^{-2n}}{2a}\right)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x^(-1 - 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

output
```
((a*b + b^2*x^n)*(-1/2*1/(a*x^(2*n)) + b/(a^2*x^n) + (b^2*Log[x^n])/a^3 -
(b^2*Log[a + b*x^n])/a^3))/(b*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,b\,x^{-n}}{(a+b\,x^{n})a^{2}n}-\frac{\sqrt{(a+b\,x^{n})^{2}}\,x^{-2n}}{2(a+b\,x^{n})an}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{2}\ln(x)}{(a+b\,x^{n})a^{3}}-\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{2}\ln\left(x^{n}+\frac{a}{b}\right)}{(a+b\,x^{n})a^{3}n}$	138

input
```
int(x^(-1-2*n)/(a^2+2*x^n*a*b+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/a^2/n/(x^n)-1/2*((a+b*x^n)^2)^(1/2)/(a+b*x
^n)/a/n/(x^n)^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2/a^3*ln(x)-((a+b*x^n)^2)^
(1/2)/(a+b*x^n)*b^2/a^3/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.31

$$\int \frac{x^{-1-2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{2\,b^2nx^{2\,n}\log\left(x\right)-2\,b^2x^{2\,n}\log\left(bx^n+a\right)+2\,abx^n-a^2}{2\,a^3nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="frica
s")
```

output
```
1/2*(2*b^2*n*x^(2*n)*log(x) - 2*b^2*x^(2*n)*log(b*x^n + a) + 2*a*b*x^n - a
^2)/(a^3*n*x^(2*n))
```

Sympy [F]

$$\int \frac{x^{-1-2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{x^{-2n-1}}{\sqrt{(a+bx^n)^2}}\,dx$$

input
```
integrate(x**(-1-2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

output
```
Integral(x**(-2*n - 1)/sqrt((a + b*x**n)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.31

$$\int \frac{x^{-1-2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{b^2\log{(x)}}{a^3} - \frac{b^2\log{\left(\frac{bx^n+a}{b}\right)}}{a^3n} + \frac{2\,bx^n-a}{2\,a^2nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxim
a")
```

output
```
b^2*log(x)/a^3 - b^2*log((b*x^n + a)/b)/(a^3*n) + 1/2*(2*b*x^n - a)/(a^2*n
*x^(2*n))
```

Giac [F]

$$\int \frac{x^{-1-2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \int \frac{x^{-2\,n-1}}{\sqrt{b^2x^{2\,n}+2\,abx^n+a^2}}\,dx$$

input
```
integrate(x^(-1-2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(x^(-2*n - 1)/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\int \frac{1}{x^{2\,n+1}\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

input
```
int(1/(x^(2*n + 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
```

output
```
int(1/(x^(2*n + 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.31

$$\int \frac{x^{-1-2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx=\frac{-2x^{2n}\log(x^nb+a)\,b^2+2x^{2n}\log(x)\,b^2n+2x^nab-a^2}{2x^{2n}a^3n}$$

input
```
int(x^(-1-2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

output
```
( - 2*x**(2*n)*log(x**n*b + a)*b**2 + 2*x**(2*n)*log(x)*b**2*n + 2*x**n*a*
b - a**2)/(2*x**(2*n)*a**3*n)
```

3.174 $\int \frac{x^{-1-2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 302

$$\begin{aligned}\int \frac{x^{-1-2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx &= -\frac{x^{-2n}(a+bx^n)}{2a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}} \\ &+ \frac{3x^{-n}(ab+b^2x^n)}{a^4n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{ab^2+b^3x^n}{2a^3n\,(a+bx^n)^2\sqrt{a^2+2abx^n+b^2x^{2n}}} \\ &+ \frac{3(ab^2+b^3x^n)}{a^4n\,(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}} \\ &+ \frac{6(ab^2+b^3x^n)\log(x)}{a^5\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{6(ab^2+b^3x^n)\log(a+bx^n)}{a^5n\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

output
```
-1/2*(a+b*x^n)/a^3/n/(x^(2*n))/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)+3*(a*b+b^
2*x^n)/a^4/n/(x^n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)+1/2*(a*b^2+b^3*x^n)/a
^3/n/(a+b*x^n)^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)+3*(a*b^2+b^3*x^n)/a^4/n
/(a+b*x^n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)+6*(a*b^2+b^3*x^n)*ln(x)/a^5/(
a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-6*(a*b^2+b^3*x^n)*ln(a+b*x^n)/a^5/n/(a^2+
2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.42

$$\int \frac{x^{-1-2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \frac{x^{-2n}\left(a(-a^3+4a^2bx^n+18ab^2x^{2n}+12b^3x^{3n})+12b^2x^{2n}(a+bx^n)^2\log(x^n\right.}{2a^5n\,(a+bx^n)\sqrt{(a+bx^n)^2}}$$

input
```
Integrate[x^(-1 - 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
(a*(-a^3 + 4*a^2*b*x^n + 18*a*b^2*x^(2*n) + 12*b^3*x^(3*n)) + 12*b^2*x^(2*
n)*(a + b*x^n)^2*Log[x^n] - 12*b^2*x^(2*n)*(a + b*x^n)^2*Log[a + b*x^n])/(
2*a^5*n*x^(2*n)*(a + b*x^n)*Sqrt[(a + b*x^n)^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.44, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1384, 798, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2n-1}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{(ab^3+b^4x^n)\int \frac{x^{-2n-1}}{(b^2x^n+ab)^3}dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \ 798$$

$$\frac{(ab^3+b^4x^n)\int \frac{x^{-3n}}{b^3(bx^n+a)^3}dx^n}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \ 27$$

$$\frac{(ab^3+b^4x^n)\int \frac{x^{-3n}}{(bx^n+a)^3}dx^n}{b^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(ab^3+b^4x^n)\int\left(\frac{x^{-3n}}{a^3}-\frac{3bx^{-2n}}{a^4}+\frac{6b^2x^{-n}}{a^5}-\frac{6b^3}{a^5(bx^n+a)}-\frac{3b^3}{a^4(bx^n+a)^2}-\frac{b^3}{a^3(bx^n+a)^3}\right)dx^n}{b^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ab^3+b^4x^n)\left(\frac{6b^2\log(x^n)}{a^5}-\frac{6b^2\log(a+bx^n)}{a^5}+\frac{3b^2}{a^4(a+bx^n)}+\frac{3bx^{-n}}{a^4}+\frac{b^2}{2a^3(a+bx^n)^2}-\frac{x^{-2n}}{2a^3}\right)}{b^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

input
```
Int[x^(-1 - 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

output
```
((a*b^3 + b^4*x^n)*(-1/2*1/(a^3*x^(2*n)) + (3*b)/(a^4*x^n) + b^2/(2*a^3*(a
 + b*x^n)^2) + (3*b^2)/(a^4*(a + b*x^n)) + (6*b^2*Log[x^n])/a^5 - (6*b^2*L
og[a + b*x^n])/a^5))/(b^3*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.60

method	result
risch	$\frac{3\sqrt{(a+b\,x^{n})^{2}}\,b\,x^{-n}}{(a+b\,x^{n})a^{4}n}-\frac{\sqrt{(a+b\,x^{n})^{2}}\,x^{-2n}}{2(a+b\,x^{n})a^{3}n}+\frac{6\sqrt{(a+b\,x^{n})^{2}}\,b^{2}\ln(x)}{(a+b\,x^{n})a^{5}}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,b^{2}(6b\,x^{n}+7a)}{2(a+b\,x^{n})^{3}a^{4}n}-\frac{6\sqrt{(a+b\,x^{n})^{2}}\,b^{2}\ln(x^{n}+}{(a+b\,x^{n})n\,a^{5}}$

input
```
int(x^(-1-2*n)/(a^2+2*x^n*a*b+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/a^4/n/(x^n)-1/2*((a+b*x^n)^2)^(1/2)/(a+b
*x^n)/a^3/n/(x^n)^2+6*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2/a^5*ln(x)+1/2*((a+
b*x^n)^2)^(1/2)/(a+b*x^n)^3*b^2*(6*b*x^n+7*a)/a^4/n-6*((a+b*x^n)^2)^(1/2)/
(a+b*x^n)*b^2/n/a^5*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.53

$$\int \frac{x^{-1-2n}}{\left(a^{2}+2abx^{n}+b^{2}x^{2n}\right)^{3/2}}\,dx=\frac{12\,b^{4}nx^{4\,n}\log\left(x\right)+4\,a^{3}bx^{n}-a^{4}+12\left(2\,ab^{3}n\log\left(x\right)+ab^{3}\right)x^{3\,n}+6\left(2\,a^{2}b^{2}\right.}{2\left(a^{5}b^{2}nx^{4\,n}+2\,a^{6}b\right.}$$

input
```
integrate(x^(-1-2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="frica
s")
```

output
```
1/2*(12*b^4*n*x^(4*n)*log(x) + 4*a^3*b*x^n - a^4 + 12*(2*a*b^3*n*log(x) +
a*b^3)*x^(3*n) + 6*(2*a^2*b^2*n*log(x) + 3*a^2*b^2)*x^(2*n) - 12*(b^4*x^(4
*n) + 2*a*b^3*x^(3*n) + a^2*b^2*x^(2*n))*log(b*x^n + a))/(a^5*b^2*n*x^(4*n
) + 2*a^6*b*n*x^(3*n) + a^7*n*x^(2*n))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.36

$$\int \frac{x^{-1-2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \frac{12\,b^3x^{3\,n}+18\,ab^2x^{2\,n}+4\,a^2bx^n-a^3}{2\,(a^4b^2nx^{4\,n}+2\,a^5bnx^{3\,n}+a^6nx^{2\,n})} + \frac{6\,b^2\log(x)}{a^5} - \frac{6\,b^2\log\left(\frac{bx^n+a}{b}\right)}{a^5n}$$

input
```
integrate(x^(-1-2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxim
a")
```

output
```
1/2*(12*b^3*x^(3*n) + 18*a*b^2*x^(2*n) + 4*a^2*b*x^n - a^3)/(a^4*b^2*n*x^(
4*n) + 2*a^5*b*n*x^(3*n) + a^6*n*x^(2*n)) + 6*b^2*log(x)/a^5 - 6*b^2*log((
b*x^n + a)/b)/(a^5*n)
```

Giac [F]

$$\int \frac{x^{-1-2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \int \frac{x^{-2\,n-1}}{(b^2x^{2\,n}+2\,abx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1-2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac"
)
```

output
```
integrate(x^(-2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\int \frac{1}{x^{2\,n+1}\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

input
```
int(1/(x^(2*n + 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
```

output
```
int(1/(x^(2*n + 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.58

$$\int \frac{x^{-1-2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx=\frac{-12x^{4n}\log(x^nb+a)\,b^4+12x^{4n}\log(x)\,b^4n-6x^{4n}b^4-24x^{3n}\log(x^nb+a)\,a}{2x^2}$$

input
```
int(x^(-1-2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

output
```
( - 12*x**(4*n)*log(x**n*b + a)*b**4 + 12*x**(4*n)*log(x)*b**4*n - 6*x**(4
*n)*b**4 - 24*x**(3*n)*log(x**n*b + a)*a*b**3 + 24*x**(3*n)*log(x)*a*b**3*
n - 12*x**(2*n)*log(x**n*b + a)*a**2*b**2 + 12*x**(2*n)*log(x)*a**2*b**2*n
 + 12*x**(2*n)*a**2*b**2 + 4*x**n*a**3*b - a**4)/(2*x**(2*n)*a**5*n*(x**(2
*n)*b**2 + 2*x**n*a*b + a**2))
```

3.175 $\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx$

Optimal result

Integrand size = 35, antiderivative size = 129

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx = -\frac{(dx)^{-2n(1+p)}\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2dn(1+p)} + \frac{bx^n(dx)^{-2n(1+p)}\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2adn\left(1+3p+2p^2\right)}$$

output
```
-1/2*(1+b*x^n/a)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/d/n/(p+1)/((d*x)^(2*n*(p+1)
))+1/2*b*x^n*(1+b*x^n/a)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/a/d/n/(2*p^2+3*p+1)
/((d*x)^(2*n*(p+1)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx = -\frac{x(dx)^{-1-2n(1+p)}\left((a+bx^n)^2\right)^p\left(1+\frac{bx^n}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(-2p,-2(1+p),1-2(1+p),-\frac{bx^n}{a}\right)}{2n(1+p)}$$

input
```
Integrate[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

output
```
-1/2*(x*(d*x)^(-1 - 2*n*(1 + p))*((a + b*x^n)^2)^p*Hypergeometric2F1[-2*p,
 -2*(1 + p), 1 - 2*(1 + p), -((b*x^n)/a)])/(n*(1 + p)*(1 + (b*x^n)/a)^(2*p
))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1385, 805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{-2n(p+1)-1}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{ 1385}$$

$$\left(\frac{bx^n}{a}+1\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\int (dx)^{-2n(p+1)-1}\left(\frac{bx^n}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{ 805}$$

$$\left(\frac{bx^n}{a}+1\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\left(-\frac{\int (dx)^{-2n(p+1)-1}\left(\frac{bx^n}{a}+1\right)^{2p+1}dx}{2p+1}-\frac{(dx)^{-2n(p+1)}\left(\frac{bx^n}{a}+1\right)^{2p+1}}{dn(2p+1)}\right)$$

$$\Big\downarrow \text{ 796}$$

$$\left(\frac{bx^n}{a}+1\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\left(\frac{(dx)^{-2n(p+1)}\left(\frac{bx^n}{a}+1\right)^{2(p+1)}}{2dn(p+1)(2p+1)}-\frac{(dx)^{-2n(p+1)}\left(\frac{bx^n}{a}+1\right)^{2p+1}}{dn(2p+1)}\right)$$

input
```
Int[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

output
```
((a^2 + 2*a*b*x^n + b^2*x^(2*n))^p*((1 + (b*x^n)/a)^(2*(1 + p))/(2*d*n*(1
+ p)*(1 + 2*p)*(d*x)^(2*n*(1 + p))) - (1 + (b*x^n)/a)^(1 + 2*p)/(d*n*(1 +
2*p)*(d*x)^(2*n*(1 + p)))))/(1 + (b*x^n)/a)^(2*p)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int (dx)^{-1-2n(p+1)} \left(a^2 + 2x^n ab + b^2x^{2n}\right)^p dx$$

input
```
int((d*x)^(-1-2*n*(p+1))*(a^2+2*x^n*a*b+b^2*x^(2*n))^p,x)
```

output
```
int((d*x)^(-1-2*n*(p+1))*(a^2+2*x^n*a*b+b^2*x^(2*n))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.28

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx = -\frac{\left(2\,abpxx^n e^{\left(-(2\,np+2\,n+1)\log(d)-(2\,np+2\,n+1)\log(x)\right)} - b^2xx^{2\,n}e^{\left(-(2\,np+2\,n+1)\log(d)-(2\,np+2\,n+1)\log(x)\right)} + (2\,a^2p+\right.}{2\left(2\,a^2np^2+3\,a^2np+a^2n\right)}$$

input
```
integrate((d*x)^(-1-2*n*(p+1))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm=
"fricas")
```

output
```
-1/2*(2*a*b*p*x*x^n*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x
)) - b^2*x*x^(2*n)*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x)
) + (2*a^2*p + a^2)*x*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log
(x)))*(b^2*x^(2*n) + 2*a*b*x^n + a^2)^p/(2*a^2*n*p^2 + 3*a^2*n*p + a^2*n)
```

Sympy [F]

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx = \int (dx)^{-2n(p+1)-1}\left((a+bx^n)^2\right)^p \, dx$$

input
```
integrate((d*x)**(-1-2*n*(p+1))*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

output
```
Integral((d*x)**(-2*n*(p + 1) - 1)*((a + b*x**n)**2)**p, x)
```

Maxima [F]

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx = \int \left(b^2x^{2\,n}+2\,abx^n+a^2\right)^p (dx)^{-2\,n(p+1)-1} \, dx$$

input
```
integrate((d*x)^(-1-2*n*(p+1))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm=
"maxima")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

Giac [F]

$$\int (dx)^{-1-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\int \left(b^2x^{2n}+2\,abx^n+a^2\right)^p(dx)^{-2\,n(p+1)-1}\,dx$$

input
```
integrate((d*x)^(-1-2*n*(p+1))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm=
"giac")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{-1-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\int \frac{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^p}{(d\,x)^{2\,n\,(p+1)+1}}\,dx$$

input
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1),x)
```

output
```
int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.76

$$\int (dx)^{-1-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{\left(x^{2n}b^2+2x^nab+a^2\right)^p\left(x^{2n}b^2-2x^nabp-2a^2p-a^2\right)}{2x^{2np+2n}d^{2np+2n}a^2dn\left(2p^2+3p+1\right)}$$

input
```
int((d*x)^(-1-2*n*(p+1))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

output
```
((x**(2*n)*b**2 + 2*x**n*a*b + a**2)**p*(x**(2*n)*b**2 - 2*x**n*a*b*p - 2*
a**2*p - a**2))/(2*x**(2*n*p + 2*n)*d**(2*n*p + 2*n)*a**2*d*n*(2*p**2 + 3*
p + 1))
```

3.176 $\int x^{-1+n}(a^2+2abx^n+b^2x^{2n})^p\,dx$

Optimal result

Integrand size = 28, antiderivative size = 43

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{bn(1+2p)}$$

output
```
(a+b*x^n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/b/n/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{(a+bx^n)\left((a+bx^n)^2\right)^p}{bn(1+2p)}$$

input
```
Integrate[x^(-1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

output
```
((a + b*x^n)*((a + b*x^n)^2)^p)/(b*n*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1690, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\left(a^2+2abx^n+b^2x^{2n}\right)^p dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{\int \left(2abx^n+b^2x^{2n}+a^2\right)^p dx^n}{n}$$

$$\Big\downarrow \text{1079}$$

$$\frac{\left(ab+b^2x^n\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\int \left(b^2x^n+ab\right)^{2p}dx^n}{n}$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(ab+b^2x^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(2p+1)}$$

input
```
Int[x^(-1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

output
```
((a*b + b^2*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(b^2*n*(1 + 2*p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

method	result	size
risch	$\frac{(a+b\,x^{n})(a+b\,x^{n})^{2p}\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}\left(i(a+b\,x^{n})^{2}\right)p\left(-\mathrm{csgn}\left(i(a+b\,x^{n})^{2}\right)+\mathrm{csgn}\left(i(a+b\,x^{n})\right)\right)^{2}}{2}}}{b(1+2p)n}$	81

input
```
int(x^(-1+n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^p,x,method=_RETURNVERBOSE)
```

output
```
(a+b*x^n)/b/(1+2*p)/n*(a+b*x^n)^(2*p)*exp(-1/2*I*Pi*csgn(I*(a+b*x^n)^2)*p*
(-csgn(I*(a+b*x^n)^2)+csgn(I*(a+b*x^n)))^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{(bx^n+a)(b^2x^{2\,n}+2\,abx^n+a^2)^p}{2\,bnp+bn}$$

input
```
integrate(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="fricas")
```

output
```
(b*x^n + a)*(b^2*x^(2*n) + 2*a*b*x^n + a^2)^p/(2*b*n*p + b*n)
```

Sympy [F]

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx$$
$$=\begin{cases} \frac{\log{(x)}}{\sqrt{a^2}} & \text{for } b=0\wedge n=0\wedge p=-\frac{1}{2} \\ \frac{xx^{n-1}(a^2)^p}{n} & \text{for } b=0 \\ (a^2+2ab+b^2)^p\log{(x)} & \text{for } n=0 \\ \int\frac{x^{n-1}}{\sqrt{(a+bx^n)^2}}\,dx & \text{for } p=-\frac{1}{2} \\ \frac{a(a^2+2abx^n+b^2x^{2n})^p}{2bnp+bn}+\frac{bx^n(a^2+2abx^n+b^2x^{2n})^p}{2bnp+bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

output
```
Piecewise((log(x)/sqrt(a**2), Eq(b, 0) & Eq(n, 0) & Eq(p, -1/2)), (x*x**(n
 - 1)*(a**2)**p/n, Eq(b, 0)), ((a**2 + 2*a*b + b**2)**p*log(x), Eq(n, 0)),
 (Integral(x**(n - 1)/sqrt((a + b*x**n)**2), x), Eq(p, -1/2)), (a*(a**2 +
2*a*b*x**n + b**2*x**(2*n))**p/(2*b*n*p + b*n) + b*x**n*(a**2 + 2*a*b*x**n
 + b**2*x**(2*n))**p/(2*b*n*p + b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{(bx^n+a)(bx^n+a)^{2p}}{bn(2p+1)}$$

input
```
integrate(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="maxima")
```

output
```
(b*x^n + a)*(b*x^n + a)^(2*p)/(b*n*(2*p + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{(b^2x^{2\,n}+2\,abx^n+a^2)^pbx^n+(b^2x^{2\,n}+2\,abx^n+a^2)^pa}{(2\,bp+b)n}$$

input
```
integrate(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="giac")
```

output
```
((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*b*x^n + (b^2*x^(2*n) + 2*a*b*x^n + a^2)
^p*a)/((2*b*p + b)*n)
```

Mupad [B] (verification not implemented)

Time = 10.64 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{(a+b\,x^n)\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^p}{b\,n\,\left(2\,p+1\right)}$$

input
```
int(x^(n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p,x)
```

output
```
((a + b*x^n)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p)/(b*n*(2*p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^{-1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{(x^{2n}b^2+2x^nab+a^2)^p\,(x^nb+a)}{bn\,(2p+1)}$$

input
```
int(x^(-1+n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

output
```
((x**(2*n)*b**2 + 2*x**n*a*b + a**2)**p*(x**n*b + a))/(b*n*(2*p + 1))
```

3.177 $\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 85

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=-\frac{a(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(1+2p)}+\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{1+p}}{2b^2n(1+p)}$$

output
```
-a*(a+b*x^n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/b^2/n/(1+2*p)+1/2*(a^2+2*a*b*x^
n+b^2*x^(2*n))^(p+1)/b^2/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{(a+bx^n)\left((a+bx^n)^2\right)^p(-a+b(1+2p)x^n)}{2b^2n(1+p)(1+2p)}$$

input
```
Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

output
```
((a + b*x^n)*((a + b*x^n)^2)^p*(-a + b*(1 + 2*p)*x^n))/(2*b^2*n*(1 + p)*(1
 + 2*p))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1693, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx$$

$$\Big\downarrow \ 1693$$

$$\frac{\int x^n\left(2abx^n+b^2x^{2n}+a^2\right)^p dx^n}{n}$$

$$\Big\downarrow \ 1100$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{p+1}}{2b^2(p+1)}-\frac{a\int \left(2abx^n+b^2x^{2n}+a^2\right)^p dx^n}{b}}{n}$$

$$\Big\downarrow \ 1079$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{p+1}}{2b^2(p+1)}-\frac{a\left(ab+b^2x^n\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\int \left(b^2x^n+ab\right)^{2p}dx^n}{b}}{n}$$

$$\Big\downarrow \ 17$$

$$\frac{\frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{p+1}}{2b^2(p+1)}-\frac{a\left(ab+b^2x^n\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^3(2p+1)}}{n}$$

input
```
Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

output
```
(-((a*(a*b + b^2*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(b^3*(1 + 2*p)))
+ (a^2 + 2*a*b*x^n + b^2*x^(2*n))^(1 + p)/(2*b^2*(1 + p)))/n
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.33

method	result	size
risch	$-\frac{\left(-2 b^{2} p \,x^{2 n}-2 a p \,x^{n} b -b^{2} x^{2 n}+a^{2}\right) \left(a +b \,x^{n}\right)^{2 p} {\mathrm e}^{-\frac{i \pi \,\operatorname{csgn}\left(i \left(a +b \,x^{n}\right)^{2}\right) p \left(-\operatorname{csgn}\left(i \left(a +b \,x^{n}\right)^{2}\right)+\operatorname{csgn}\left(i \left(a +b \,x^{n}\right)\right)\right)^{2}}{2}}}{2 \left(1+2 p \right) \left(p +1\right) b^{2} n}$	113

input
```
int(x^(-1+2*n)*(a^2+2*x^n*a*b+b^2*x^(2*n))^p,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*b^2*p*(x^n)^2-2*a*p*x^n*b-(x^n)^2*b^2+a^2)/(1+2*p)/(p+1)/b^2/n*(a
+b*x^n)^(2*p)*exp(-1/2*I*Pi*csgn(I*(a+b*x^n)^2)*p*(-csgn(I*(a+b*x^n)^2)+cs
gn(I*(a+b*x^n)))^2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx = \frac{(2\,abpx^n-a^2+(2\,b^2p+b^2)x^{2\,n})(b^2x^{2\,n}+2\,abx^n+a^2)^p}{2\,(2\,b^2np^2+3\,b^2np+b^2n)}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*p*x^n - a^2 + (2*b^2*p + b^2)*x^(2*n))*(b^2*x^(2*n) + 2*a*b*x^n
 + a^2)^p/(2*b^2*n*p^2 + 3*b^2*n*p + b^2*n)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx = \frac{(b^2(2\,p+1)x^{2\,n}+2\,abpx^n-a^2)(bx^n+a)^{2\,p}}{2\,(2\,p^2+3\,p+1)b^2n}$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="maxima")
```

output
```
1/2*(b^2*(2*p + 1)*x^(2*n) + 2*a*b*p*x^n - a^2)*(b*x^n + a)^(2*p)/((2*p^2
+ 3*p + 1)*b^2*n)
```

Giac [F]

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\int \left(b^2x^{2\,n}+2\,abx^n+a^2\right)^p x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\int x^{2\,n-1}\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^p\,dx$$

input
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p,x)
```

output
```
int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\frac{\left(x^{2n}b^2+2x^nab+a^2\right)^p\left(2x^{2n}b^2p+x^{2n}b^2+2x^nabp-a^2\right)}{2b^2n\left(2p^2+3p+1\right)}$$

input
```
int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

output
```
((x**(2*n)*b**2 + 2*x**n*a*b + a**2)**p*(2*x**(2*n)*b**2*p + x**(2*n)*b**2
 + 2*x**n*a*b*p - a**2))/(2*b**2*n*(2*p**2 + 3*p + 1))
```

3.178 $\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,n}+\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}$$

output
```
-b*x^n/c^2/n+1/2*x^(2*n)/c/n+b*(-3*a*c+b^2)*arctanh((b+2*c*x^n)/(-4*a*c+b^
2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)/n+1/2*(-a*c+b^2)*ln(a+b*x^n+c*x^(2*n))/c^
3/n
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \frac{cx^n(-2b+cx^n)-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log\left(a+x^n(b+cx^n)\right)}{2c^3n}$$

input
```
Integrate[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(c*x^n*(-2*b + c*x^n) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2
+ 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x^n*(b + c*x^n)])/(2*c
^3*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4n-1}}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \; 1693$$

$$\frac{\int \frac{x^{3n}}{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \; 1143$$

$$\frac{\int \left(\frac{x^n}{c}+\frac{(b^2-ac)x^n+ab}{c^2(bx^n+cx^{2n}+a)}-\frac{b}{c^2}\right)dx^n}{n}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{b(b^2-3ac)\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2c^3}-\frac{bx^n}{c^2}+\frac{x^{2n}}{2c}}{n}$$

input
```
Int[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-((b*x^n)/c^2) + x^(2*n)/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)/S
qrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^n +
c*x^(2*n)])/(2*c^3))/n
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(103) = 206$.

Time = 0.21 (sec) , antiderivative size = 973, normalized size of antiderivative = 8.77

method	result
risch	$-\frac{\ln(x)a}{c^2}+\frac{\ln(x)b^2}{c^3}+\frac{x^{2n}}{2cn}-\frac{b\,x^n}{c^2n}+\frac{4n^2\ln(x)a^2c^2}{4a\,c^4n^2-b^2c^3n^2}-\frac{5n^2\ln(x)a\,b^2c}{4a\,c^4n^2-b^2c^3n^2}+\frac{n^2\ln(x)b^4}{4a\,c^4n^2-b^2c^3n^2}-\frac{2\ln\left(x^n+\frac{3a\,b^2c-b^4+\sqrt{-}}{}\right.}{}$

input
```
int(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*ln(x)*a+1/c^3*ln(x)*b^2+1/2/c/n*(x^n)^2-b*x^n/c^2/n+4/(4*a*c^4*n^2-
b^2*c^3*n^2)*n^2*ln(x)*a^2*c^2-5/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*a*b^2
*c+1/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*b^4-2/c/(4*a*c-b^2)/n*ln(x^n+1/2*
(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/
(3*a*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*
b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*a*b^2-1/2/c
^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2
-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4+1/2/c^3/(4*a*c-b^2)/n*ln(x^n+
1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/
c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)-2/c
/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-
10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n-1
/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/
c/b/(3*a*c-b^2))*a*b^2-1/2/c^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-
36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4-
1/2/c^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b
^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4
*c^2-10*a*b^6*c+b^8)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.18

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx$$
$$= \left[-\frac{(b^3-3\,abc)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)x^n-\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)-(b^2c^2-4\,ac^3)x^{2\,n}+2\,(b^3c-}{2\,(b^2c^3-4\,ac^4)n}\right.$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c
+ 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*
x^n + a)) - (b^2*c^2 - 4*a*c^3)*x^(2*n) + 2*(b^3*c - 4*a*b*c^2)*x^n - (b^4
 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^3 - 4*a*c^4)
*n), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*
c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^2*c^2 - 4*a*c^3)*x^(2
*n) - 2*(b^3*c - 4*a*b*c^2)*x^n + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2
*n) + b*x^n + a))/((b^2*c^3 - 4*a*c^4)*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+4*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{4\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
(b^2 - a*c)*log(x)/c^3 + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n) + integrate(-(a
*b^2 - a^2*c + (b^3 - 2*a*b*c)*x^n)/(c^4*x*x^(2*n) + b*c^3*x*x^n + a*c^3*x
), x)
```

Giac [F]

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{4\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{4\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^(4*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(4*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.86

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \frac{6\sqrt{4ac-b^2}\,atan\left(\frac{2x^nc+b}{\sqrt{4ac-b^2}}\right)abc-2\sqrt{4ac-b^2}\,atan\left(\frac{2x^nc+b}{\sqrt{4ac-b^2}}\right)b^3+4x^{2n}a\,c^3-x^{2n}b^2c^2-8x^nab\,c^2+2x^nb^3}{2c^3n\,(4ac-b^2)}$$

input
```
int(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
(6*sqrt(4*a*c - b**2)*atan((2*x**n*c + b)/sqrt(4*a*c - b**2))*a*b*c - 2*sq
rt(4*a*c - b**2)*atan((2*x**n*c + b)/sqrt(4*a*c - b**2))*b**3 + 4*x**(2*n)
*a*c**3 - x**(2*n)*b**2*c**2 - 8*x**n*a*b*c**2 + 2*x**n*b**3*c - 4*log(x**
(2*n)*c + x**n*b + a)*a**2*c**2 + 5*log(x**(2*n)*c + x**n*b + a)*a*b**2*c
- log(x**(2*n)*c + x**n*b + a)*b**4)/(2*c**3*n*(4*a*c - b**2))
```

3.179 $\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \frac{x^n}{cn} - \frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}n} - \frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}$$

output
```
x^n/c/n-(-2*a*c+b^2)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b
^2)^(1/2)/n-1/2*b*ln(a+b*x^n+c*x^(2*n))/c^2/n
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \frac{2cx^n + \frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - b\log\left(a+x^n(b+cx^n)\right)}{2c^2n}$$

input
```
Integrate[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*c*x^n + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] - b*Log[a + x^n*(b + c*x^n)])/(2*c^2*n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{a+bx^n+cx^{2n}}\,dx$$
$$\Big\downarrow \ 1693$$
$$\frac{\int \frac{x^{2n}}{bx^n+cx^{2n}+a}dx^n}{n}$$
$$\Big\downarrow \ 1143$$
$$\frac{\int \left(\frac{1}{c}-\frac{bx^n+a}{c(bx^n+cx^{2n}+a)}\right)dx^n}{n}$$
$$\Big\downarrow \ 2009$$
$$\frac{-\frac{(b^2-2ac)\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2}+\frac{x^n}{c}}{n}$$

input
```
Int[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(x^n/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqr
t[b^2 - 4*a*c]) - (b*Log[a + b*x^n + c*x^(2*n)])/(2*c^2))/n
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(81) = 162$.

Time = 0.18 (sec) , antiderivative size = 664, normalized size of antiderivative = 7.63

method	result
risch	$-\frac{b\ln(x)}{c^2}+\frac{x^n}{cn}+\frac{4n^2\ln(x)abc}{4a\,c^3n^2-b^2c^2n^2}-\frac{n^2\ln(x)b^3}{4a\,c^3n^2-b^2c^2n^2}-\frac{2\ln\left(x^n-\frac{-2abc+b^3+\sqrt{-16a^3c^3+20a^2b^2c^2-8a\,b^4c+b^6}}{2c\left(2ac-b^2\right)}\right)ab}{c(4ac-b^2)n}+\frac{\ln\left(x^n\right.}{}$

input
```
int(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-b/c^2*ln(x)+x^n/c/n+4/(4*a*c^3*n^2-b^2*c^2*n^2)*n^2*ln(x)*a*b*c-1/(4*a*c^
3*n^2-b^2*c^2*n^2)*n^2*ln(x)*b^3-2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^
3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*a*b+1/2
/c^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*
a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3+1/2/c^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-
2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2
))*(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)-2/c/(4*a*c-b^2)/n*ln(x
^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2
*a*c-b^2))*a*b+1/2/c^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+
20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3-1/2/c^2/(4*a*c-b^2
)/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/
2))/c/(2*a*c-b^2))*(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.28

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx$$
$$= \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)x^n+\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)-2\,(b^2c-4\,ac^2)x^n+(b^3-4\,ab}{2\,(b^2c^2-4\,ac^3)n}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}cx^n+\sqrt{-b^2+4\,ac}b}{b^2-4\,ac}\right)-2\,(b^2c-4\,ac^2)x^n+(b^3-4\,abc)\log\left(cx\right.}{2\,(b^2c^2-4\,ac^3)n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c +
2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^
n + a)) - 2*(b^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n
+ a))/((b^2*c^2 - 4*a*c^3)*n), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*ar
ctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) -
 2*(b^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a))/((b
^2*c^2 - 4*a*c^3)*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{3\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-b*log(x)/c^2 + x^n/(c*n) - integrate(-(a*b + (b^2 - a*c)*x^n)/(c^3*x*x^(2
*n) + b*c^2*x*x^n + a*c^2*x), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{3\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{3\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.78

$$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx$$
$$=\frac{-4\sqrt{4ac-b^2}\,atan\left(\frac{2x^nc+b}{\sqrt{4ac-b^2}}\right)ac+2\sqrt{4ac-b^2}\,atan\left(\frac{2x^nc+b}{\sqrt{4ac-b^2}}\right)b^2+8x^nac^2-2x^nb^2c-4\log(x^{2n}c+x^nb -}{2c^2n\,(4ac-b^2)}$$

input
```
int(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((2*x**n*c + b)/sqrt(4*a*c - b**2))*a*c + 2*s
qrt(4*a*c - b**2)*atan((2*x**n*c + b)/sqrt(4*a*c - b**2))*b**2 + 8*x**n*a*
c**2 - 2*x**n*b**2*c - 4*log(x**(2*n)*c + x**n*b + a)*a*b*c + log(x**(2*n)
*c + x**n*b + a)*b**3)/(2*c**2*n*(4*a*c - b**2))
```

3.180 $\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \frac{b\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,n}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

output
```
b*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)/n+1/2*ln(a+
b*x^n+c*x^(2*n))/c/n
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \frac{-\frac{2b\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+x^n(b+cx^n)\right)}{2cn}$$

input
```
Integrate[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[
a + x^n*(b + c*x^n)])/(2*c*n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1693, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int \frac{x^n}{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\int \frac{2cx^n+b}{bx^n+cx^{2n}+a}dx^n}{2c}-\frac{b\int \frac{1}{bx^n+cx^{2n}+a}dx^n}{2c}}{n}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{b\int \frac{1}{-x^{2n}+b^2-4ac}d(2cx^n+b)}{c}+\frac{\int \frac{2cx^n+b}{bx^n+cx^{2n}+a}dx^n}{2c}}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\int \frac{2cx^n+b}{bx^n+cx^{2n}+a}dx^n}{2c}+\frac{b\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}}{n}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{b\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2c}}{n}$$

input
```
Int[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
((b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^n + c*x^(2*n)]/(2*c))/n
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(62) = 124$.

Time = 0.16 (sec) , antiderivative size = 402, normalized size of antiderivative = 5.91

method	result
risch	$\frac{\ln(x)}{c} - \frac{4n^2\ln(x)ac}{4a\,c^2n^2-b^2c\,n^2} + \frac{n^2\ln(x)b^2}{4a\,c^2n^2-b^2c\,n^2} + \frac{2\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)a}{(4ac-b^2)n} - \frac{\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)b^2}{2c(4ac-b^2)n} + \frac{\ln\left(x^n-\right.}{}$

input
```
int(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/c*ln(x)-4/(4*a*c^2*n^2-b^2*c*n^2)*n^2*ln(x)*a*c+1/(4*a*c^2*n^2-b^2*c*n^2
)*n^2*ln(x)*b^2+2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b
/c)*a-1/2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^
2+1/2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*
b^2*c+b^4)^(1/2)+2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b
/c)*a-1/2/c/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2
-1/2/c/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^
2*c+b^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.40

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)x^n+\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)+(b^2-4\,ac)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,(b^2c-4\,ac^2)n}, \frac{2\sqrt{-b^2}}{}\right.$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt
(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + (b^
2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a*c^2)*n), 1/2*(2*sqrt(
-b^2 + 4*a*c)*b*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b
)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a
*c^2)*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{2n-1}}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
log(x)/c - integrate((b*x^n + a)/(c^2*x*x^(2*n) + b*c*x*x^n + a*c*x), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{2n-1}}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{2\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^(2*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(2*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.43

$$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx = \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2x^n c+b}{\sqrt{4ac-b^2}}\right)b+4\log(x^{2n}c+x^n b+a)\,ac-\log(x^{2n}c+x^n b+a)\,b^2}{2cn\,(4ac-b^2)}$$

input
```
int(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((2*x**n*c + b)/sqrt(4*a*c - b**2))*b + 4*log
(x**(2*n)*c + x**n*b + a)*a*c - log(x**(2*n)*c + x**n*b + a)*b**2)/(2*c*n*
(4*a*c - b**2))
```

3.181 $\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,n}$$

output
```
-2*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx=\frac{2\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\,n}$$

input
```
Integrate[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*n)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1690, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{a+bx^n+cx^{2n}}dx$$
$$\Big\downarrow \text{ 1690}$$
$$\frac{\int \frac{1}{bx^n+cx^{2n}+a}dx^n}{n}$$
$$\Big\downarrow \text{ 1083}$$
$$-\frac{2\int \frac{1}{-x^{2n}+b^2-4ac}d(2cx^n+b)}{n}$$
$$\Big\downarrow \text{ 219}$$
$$-\frac{2\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

input
```
Int[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*n)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(35) = 70$.

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.90

method	result	size
risch	$-\frac{\ln\left(x^{n}+\frac{b^{2}-4ac+b\sqrt{-4ac+b^{2}}}{2c\sqrt{-4ac+b^{2}}}\right)}{\sqrt{-4ac+b^{2}}\, n}+\frac{\ln\left(x^{n}+\frac{b\sqrt{-4ac+b^{2}}+4ac-b^{2}}{2c\sqrt{-4ac+b^{2}}}\right)}{\sqrt{-4ac+b^{2}}\, n}$	113

input
```
int(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))/c/(-4*
a*c+b^2)^(1/2))+1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*(b*(-4*a*c+b^2)^(1/2)+4*
a*c-b^2)/c/(-4*a*c+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 4.08

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)x^n-\sqrt{b^2-4\,ac}b}{cx^{2\,n}+bx^n+a}\right)}{\sqrt{b^2-4\,ac}n}, -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}cx^n+\sqrt{-b^2+4\,ac}b}{b^2-4\,ac}\right)}{(b^2-4\,ac)n}\right]$$

input
```
integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sq
rt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a))/(sqrt(b^2 - 4*a*c)*n), -2*sqrt
(-b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)
/(b^2 - 4*a*c))/((b^2 - 4*a*c)*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(x^(n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx=\frac{2\,\arctan\left(\frac{2\,cx^n+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}n}$$

input
```
integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
2*arctan((2*c*x^n + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*n)
```

Mupad [B] (verification not implemented)

Time = 11.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx=\frac{2\,\mathrm{atan}\left(\frac{b+2\,c\,x^n}{\sqrt{4\,a\,c-b^2}}\right)}{n\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^(n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
(2*atan((b + 2*c*x^n)/(4*a*c - b^2)^(1/2)))/(n*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \frac{2\sqrt{4ac-b^2}\,atan\left(\frac{2x^n c+b}{\sqrt{4ac-b^2}}\right)}{n\,(4ac-b^2)}$$

input
```
int(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
(2*sqrt(4*a*c - b**2)*atan((2*x**n*c + b)/sqrt(4*a*c - b**2)))/(n*(4*a*c -
 b**2))
```

3.182 $\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx = -\frac{x^{-n}}{an} - \frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}n} - \frac{b\log(x)}{a^2} + \frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}$$

output
```
-1/a/n/(x^n)-(-2*a*c+b^2)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/a^2/(-4*
a*c+b^2)^(1/2)/n-b*ln(x)/a^2+1/2*b*ln(a+b*x^n+c*x^(2*n))/a^2/n
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx = \frac{-2ax^{-n} + \frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 2b\log(x^n) + b\log(a+x^n(b+cx^n))}{2a^2n}$$

input
```
Integrate[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
((-2*a)/x^n + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/S
qrt[-b^2 + 4*a*c] - 2*b*Log[x^n] + b*Log[a + x^n*(b + c*x^n)])/(2*a^2*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{a+bx^n+cx^{2n}}dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{\int \frac{x^{-2n}}{bx^n+cx^{2n}+a}dx^n}{n}$$
$$\Big\downarrow \text{1145}$$
$$\frac{\frac{\int -\frac{x^{-n}(cx^n+b)}{bx^n+cx^{2n}+a}dx^n}{a}-\frac{x^{-n}}{a}}{n}$$
$$\Big\downarrow \text{25}$$
$$\frac{-\frac{\int \frac{x^{-n}(cx^n+b)}{bx^n+cx^{2n}+a}dx^n}{a}-\frac{x^{-n}}{a}}{n}$$
$$\Big\downarrow \text{1200}$$
$$\frac{-\frac{\int \left(\frac{bx^{-n}}{a}+\frac{-bcx^n-b^2+ac}{a\left(bx^n+cx^{2n}+a\right)}\right)dx^n}{a}-\frac{x^{-n}}{a}}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{\frac{\left(b^2-2ac\right)\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a}+\frac{b\log(x^n)}{a}}{a}-\frac{x^{-n}}{a}}{n}$$

input
```
Int[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-(1/(a*x^n)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/
(a*Sqrt[b^2 - 4*a*c]) + (b*Log[x^n])/a - (b*Log[a + b*x^n + c*x^(2*n)])/(2
*a))/a)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. 2(92) = 184.

Time = 0.16 (sec) , antiderivative size = 658, normalized size of antiderivative = 6.71

method	result
risch	$-\frac{x^{-n}}{an}-\frac{4n^{2}\ln\left(x\right)abc}{4a^{3}c\,n^{2}-a^{2}b^{2}n^{2}}+\frac{n^{2}\ln\left(x\right)b^{3}}{4a^{3}c\,n^{2}-a^{2}b^{2}n^{2}}+\frac{2\ln\left(x^{n}-\frac{-2abc+b^{3}+\sqrt{-16a^{3}c^{3}+20a^{2}b^{2}c^{2}-8a\,b^{4}c+b^{6}}}{2c\left(2ac-b^{2}\right)}\right)bc}{a\left(4ac-b^{2}\right)n}-\frac{\ln\left(x^{n}-\frac{-2abc+\cdots}{}\right)}{}$

input
```
int(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-1/a/n/(x^n)-4/(4*a^3*c*n^2-a^2*b^2*n^2)*n^2*ln(x)*a*b*c+1/(4*a^3*c*n^2-a^
2*b^2*n^2)*n^2*ln(x)*b^3+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a
^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b*c-1/2/a^2/(4*
a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+
b^6)^(1/2))/c/(2*a*c-b^2))*b^3+1/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+
b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-16*
a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(
2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2
))*b*c-1/2/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b
^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3-1/2/a^2/(4*a*c-b^2)/n*ln(x
^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2
*a*c-b^2))*(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.40

$$\int \frac{x^{-1-n}}{a+bx^{n}+cx^{2n}}\,dx$$

$$= \left[-\frac{2\,(b^{3}-4\,abc)nx^{n}\log\left(x\right)+(b^{2}-2\,ac)\sqrt{b^{2}-4\,ac}x^{n}\log\left(\frac{2\,c^{2}x^{2\,n}+b^{2}-2\,ac+2\,\left(bc+\sqrt{b^{2}-4\,ac}c\right)x^{n}+\sqrt{b^{2}-4\,ac}b}{cx^{2\,n}+bx^{n}+a}\right)+\cdots}{2\,(a^{2}b^{2}-4\,a^{3}c)nx^{n}}\right.$$

$$-\frac{2\,(b^{3}-4\,abc)nx^{n}\log\left(x\right)+2\,(b^{2}-2\,ac)\sqrt{-b^{2}+4\,ac}x^{n}\arctan\left(-\frac{2\,\sqrt{-b^{2}+4\,ac}cx^{n}+\sqrt{-b^{2}+4\,ac}b}{b^{2}-4\,ac}\right)+2\,ab^{2}-\cdots}{2\,(a^{2}b^{2}-4\,a^{3}c)nx^{n}}$$

input
```
integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[-1/2*(2*(b^3 - 4*a*b*c)*n*x^n*log(x) + (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^
n*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + s
qrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + 2*a*b^2 - 8*a^2*c - (b^3 -
4*a*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a^2*b^2 - 4*a^3*c)*n*x^n), -1/2
*(2*(b^3 - 4*a*b*c)*n*x^n*log(x) + 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x^n*
arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c))
 + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a
^2*b^2 - 4*a^3*c)*n*x^n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-1/(a*n*x^n) - integrate((c*x^n + b)/(a*c*x*x^(2*n) + a*b*x*x^n + a^2*x),
x)
```

Giac [F]

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{x^{n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.93

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx$$
$$=\frac{-4x^n\sqrt{4ac-b^2}\,atan\left(\frac{2x^nc+b}{\sqrt{4ac-b^2}}\right)ac+2x^n\sqrt{4ac-b^2}\,atan\left(\frac{2x^nc+b}{\sqrt{4ac-b^2}}\right)b^2+4x^n\log\left(x^{2n}c+x^nb+a\right)abc-x^n}{2x^na^2n\left(4ac-b^2\right)}$$

input
```
int(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
( - 4*x***n*sqrt(4*a*c - b**2)*atan((2*x***n*c + b)/sqrt(4*a*c - b**2))*a*c
+ 2*x***n*sqrt(4*a*c - b**2)*atan((2*x***n*c + b)/sqrt(4*a*c - b**2))*b**2 +
 4*x***n*log(x**(2*n)*c + x***n*b + a)*a*b*c - x***n*log(x**(2*n)*c + x***n*b
+ a)*b**3 - 8*x***n*log(x)*a*b*c*n + 2*x***n*log(x)*b**3*n - 8*a**2*c + 2*a*
b**2)/(2*x***n*a**2*n*(4*a*c - b**2))
```

3.183 $\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\,n}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}$$

output
```
-1/2/a/n/(x^(2*n))+b/a^2/n/(x^n)+b*(-3*a*c+b^2)*arctanh((b+2*c*x^n)/(-4*a*
c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)/n+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)
*ln(a+b*x^n+c*x^(2*n))/a^3/n
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \frac{ax^{-2n}(-a+2bx^n)-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2(b^2-ac)\log(x^n)-(b^2-ac)\log\left(a+x^n(b+cx^n)\right)}{2a^3n}$$

input
```
Integrate[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
((a*(-a + 2*b*x^n))/x^(2*n) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt
[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x^n] - (b^2 - a*c)
*Log[a + x^n*(b + c*x^n)])/(2*a^3*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2n-1}}{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{\int \frac{x^{-3n}}{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \text{ 1145}$$

$$\frac{\frac{\int -\frac{x^{-2n}(cx^n+b)}{bx^n+cx^{2n}+a}dx^n}{a}-\frac{x^{-2n}}{2a}}{n}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-\frac{\int \frac{x^{-2n}(cx^n+b)}{bx^n+cx^{2n}+a}dx^n}{a}-\frac{x^{-2n}}{2a}}{n}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{-\frac{\int \left(\frac{bx^{-2n}}{a}+\frac{(ac-b^2)x^{-n}}{a^2}+\frac{c(b^2-ac)x^n+b(b^2-2ac)}{a^2(bx^n+cx^{2n}+a)}\right)dx^n}{a}-\frac{x^{-2n}}{2a}}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{-\frac{b(b^2-3ac)\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{(b^2-ac)\log(x^n)}{a^2}+\frac{(b^2-ac)\log(a+bx^n+cx^{2n})}{2a^2}-\frac{bx^{-n}}{a}}{a}-\frac{x^{-2n}}{2a}}{n}$$

input
```
Int[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-1/2*1/(a*x^(2*n)) - (-(b/(a*x^n)) - (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^
n)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - a*c)*Log[x^n])/a^
2 + ((b^2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^2))/a)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(120) = 240$.

Time = 0.18 (sec) , antiderivative size = 958, normalized size of antiderivative = 7.60

method	result
risch	$\frac{b\,x^{-n}}{a^2 n} - \frac{x^{-2n}}{2an} - \frac{4n^2\ln(x)a^2c^2}{4a^4c\,n^2-a^3b^2n^2} + \frac{5n^2\ln(x)a\,b^2c}{4a^4c\,n^2-a^3b^2n^2} - \frac{n^2\ln(x)b^4}{4a^4c\,n^2-a^3b^2n^2} + \frac{2\ln\left(x^n+\frac{3a\,b^2c-b^4+\sqrt{-36a^3b^2c^3+33a^2b^4c^2-1}}{2cb\left(3ac-b^2\right)}\right.}{a(4ac-b^2)n}$

input
```
int(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
b/a^2/n/(x^n)-1/2/a/n/(x^n)^2-4/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a^2*c^
2+5/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a*b^2*c-1/(4*a^4*c*n^2-a^3*b^2*n^2
)*n^2*ln(x)*b^4+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c
^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*c^2-5/2/a^2/(4*a
*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b
^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^2*c+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(
3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(
3*a*c-b^2))*b^4+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b
^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2
*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-3
*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3
*a*c-b^2))*c^2-5/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b
^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^2*c+1/2/a^
3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2
-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4-1/2/a^3/(4*a*c-b^2)/n*ln(x^n-
1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))
/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.40

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx$$
$$= \left[-\frac{a^2b^2-4\,a^3c-2\,(b^4-5\,ab^2c+4\,a^2c^2)nx^{2\,n}\log\left(x\right)+(b^3-3\,abc)\sqrt{b^2-4\,ac}x^{2\,n}\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,(}{cx}\right.}{2\,(a^3b^2-4\,a^4}\right.$$
$$-\frac{a^2b^2-4\,a^3c-2\,(b^4-5\,ab^2c+4\,a^2c^2)nx^{2\,n}\log\left(x\right)-2\,(b^3-3\,abc)\sqrt{-b^2+4\,ac}x^{2\,n}\arctan\left(-\frac{2\,\sqrt{-b^2+4}}{}\right.}{2\,(a^3b^2-4\,a^4c)nx}$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[-1/2*(a^2*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x
) + (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^(2*n)*log((2*c^2*x^(2*n) + b^2 - 2
*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n)
 + b*x^n + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*n)*log(c*x^(2*n) + b*x
^n + a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b^2 - 4*a^4*c)*n*x^(2*n)), -1/2
*(a^2*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x) - 2
*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^(2*n)*arctan(-(2*sqrt(-b^2 + 4*a*c)*
c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^
2)*x^(2*n)*log(c*x^(2*n) + b*x^n + a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b
^2 - 4*a^4*c)*n*x^(2*n))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-2*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-2\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
1/2*(2*b*x^n - a)/(a^2*n*x^(2*n)) + integrate((b*c*x^n + b^2 - a*c)/(a^2*c
*x*x^(2*n) + a^2*b*x*x^n + a^3*x), x)
```

Giac [F]

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-2\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{2\,n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.15

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \frac{6x^{2n}\sqrt{4ac-b^2}\,atan\left(\frac{2x^n c+b}{\sqrt{4ac-b^2}}\right)abc-2x^{2n}\sqrt{4ac-b^2}\,atan\left(\frac{2x^n c+b}{\sqrt{4ac-b^2}}\right)b^3+4x^{2n}\log(x^{2n}c+x^n b+a)\,a^2c^2-$$

input
```
int(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
(6*x**(2*n)*sqrt(4*a*c - b**2)*atan((2*x**n*c + b)/sqrt(4*a*c - b**2))*a*b
*c - 2*x**(2*n)*sqrt(4*a*c - b**2)*atan((2*x**n*c + b)/sqrt(4*a*c - b**2))
*b**3 + 4*x**(2*n)*log(x**(2*n)*c + x**n*b + a)*a**2*c**2 - 5*x**(2*n)*log
(x**(2*n)*c + x**n*b + a)*a*b**2*c + x**(2*n)*log(x**(2*n)*c + x**n*b + a)
*b**4 - 8*x**(2*n)*log(x)*a**2*c**2*n + 10*x**(2*n)*log(x)*a*b**2*c*n - 2*
x**(2*n)*log(x)*b**4*n + 8*x**n*a**2*b*c - 2*x**n*a*b**3 - 4*a**3*c + a**2
*b**2)/(2*x**(2*n)*a**3*n*(4*a*c - b**2))
```

3.184 $\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 164

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = -\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{(b^2-ac)\,x^{-n}}{a^3n} - \frac{(b^4-4ab^2c+2a^2c^2)\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}n} - \frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}$$

output
```
-1/3/a/n/(x^(3*n))+1/2*b/a^2/n/(x^(2*n))-(-a*c+b^2)/a^3/n/(x^n)-(2*a^2*c^2
-4*a*b^2*c+b^4)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(
1/2)/n-b*(-2*a*c+b^2)*ln(x)/a^4+1/2*b*(-2*a*c+b^2)*ln(a+b*x^n+c*x^(2*n))/a
^4/n
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.90

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx$$
$$= \frac{ax^{-3n}(-2a^2-6b^2x^{2n}+3ax^n(b+2cx^n)) + \frac{6(b^4-4ab^2c+2a^2c^2)\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 6(b^3-2abc)\log(x^n) + 3(b}{6a^4n}$$

input
```
Integrate[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
((a*(-2*a^2 - 6*b^2*x^(2*n) + 3*a*x^n*(b + 2*c*x^n)))/x^(3*n) + (6*(b^4 -
4*a*b^2*c + 2*a^2*c^2)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2
 + 4*a*c] - 6*(b^3 - 2*a*b*c)*Log[x^n] + 3*(b^3 - 2*a*b*c)*Log[a + x^n*(b
+ c*x^n)])/(6*a^4*n)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1693, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-3n-1}}{a+bx^n+cx^{2n}}\,dx$$
$$\Big\downarrow \text{1693}$$
$$\frac{\int \frac{x^{-4n}}{bx^n+cx^{2n}+a}dx^n}{n}$$
$$\Big\downarrow \text{1145}$$
$$\frac{\frac{\int -\frac{x^{-3n}(cx^n+b)}{bx^n+cx^{2n}+a}dx^n}{a} - \frac{x^{-3n}}{3a}}{n}$$
$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{x^{-3n}(cx^n+b)}{bx^n+cx^{2n}+a}dx^n}{a}-\frac{x^{-3n}}{3a}}{n}$$

$$\Big\downarrow \ 1200$$

$$\frac{-\frac{\int \left(\frac{bx^{-3n}}{a}+\frac{(ac-b^2)x^{-2n}}{a^2}+\frac{(b^3-2abc)x^{-n}}{a^3}+\frac{-bc(b^2-2ac)x^n-b^4-a^2c^2+3ab^2c}{a^3(bx^n+cx^{2n}+a)}\right)dx^n}{a}-\frac{x^{-3n}}{3a}}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{-\frac{b(b^2-2ac)\log(x^n)}{a^3}-\frac{b(b^2-2ac)\log(a+bx^n+cx^{2n})}{2a^3}+\frac{x^{-n}(b^2-ac)}{a^2}+\frac{(2a^2c^2-4ab^2c+b^4)\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-\frac{bx^{-2n}}{2a}}{a}-\frac{x^{-3n}}{3a}}{n}$$

input
```
Int[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-1/3*1/(a*x^(3*n)) - (-1/2*b/(a*x^(2*n)) + (b^2 - a*c)/(a^2*x^n) + ((b^4
- 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a^3*Sq
rt[b^2 - 4*a*c]) + (b*(b^2 - 2*a*c)*Log[x^n])/a^3 - (b*(b^2 - 2*a*c)*Log[a
 + b*x^n + c*x^(2*n)])/(2*a^3))/a)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. $2(158) = 316$.

Time = 0.22 (sec) , antiderivative size = 1300, normalized size of antiderivative = 7.93

method	result	size
risch	Expression too large to display	1300

input
```
int(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/a^2/n/(x^n)*c-1/a^3/n/(x^n)*b^2+1/2*b/a^2/n/(x^n)^2-1/3/a/n/(x^n)^3+8/(4
*a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*a^2*b*c^2-6/(4*a^5*c*n^2-a^4*b^2*n^2)*n^
2*ln(x)*a*b^3*c+1/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*b^5-4/a^2/(4*a*c-b^2
)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a
^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b
^4))*b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*
a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2
))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-b^2)/n*ln(x^n+1/2*(2*
a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*
b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^5+1/2/a^4/(
4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2
*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*
a*b^2*c+b^4))*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12
*a*b^8*c+b^10)^(1/2)-4/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*
c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c
+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^
n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c
^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^3
*c-1/2/a^4/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5*c
^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))...
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 522, normalized size of antiderivative = 3.18

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx$$

$$= \left[-\frac{2\,a^3b^2-8\,a^4c+6\,(b^5-6\,ab^3c+8\,a^2bc^2)nx^{3\,n}\log{(x)}-3\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{b^2-4\,ac}x^{3\,n}\log\left(\frac{2\,c}{\ldots}\right.}{\ldots}\right.$$

$$-\frac{2\,a^3b^2-8\,a^4c+6\,(b^5-6\,ab^3c+8\,a^2bc^2)nx^{3\,n}\log{(x)}+6\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{-b^2+4\,ac}x^{3\,n}\arctan\ldots}{\ldots}$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[-1/6*(2*a^3*b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*l
og(x) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^(3*n)*log((2*c
^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 -
4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^(
3*n)*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^(2
*n) - 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4*a^5*c)*n*x^(3*n)), -1/6*(
2*a^3*b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*log(x) +
 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^(3*n)*arctan(-(2*sqr
t(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - 3*(b^5 - 6*
a*b^3*c + 8*a^2*b*c^2)*x^(3*n)*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a
^2*b^2*c + 4*a^3*c^2)*x^(2*n) - 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4
*a^5*c)*n*x^(3*n))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-3*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-3\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
1/6*(3*a*b*x^n - 2*a^2 - 6*(b^2 - a*c)*x^(2*n))/(a^3*n*x^(3*n)) + integrat
e(-(b^3 - 2*a*b*c + (b^2*c - a*c^2)*x^n)/(a^3*c*x*x^(2*n) + a^3*b*x*x^n +
a^4*x), x)
```

Giac [F]

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{-3\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{x^{3\,n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.22

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx$$
$$=\frac{12x^{3n}\sqrt{4ac-b^2}\,atan\left(\frac{2x^nc+b}{\sqrt{4ac-b^2}}\right)a^2c^2-24x^{3n}\sqrt{4ac-b^2}\,atan\left(\frac{2x^nc+b}{\sqrt{4ac-b^2}}\right)a\,b^2c+6x^{3n}\sqrt{4ac-b^2}\,atan\left(\frac{2x^nc}{\sqrt{4ac}}\right.}{}$$

input
```
int(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
(12*x**(3*n)*sqrt(4*a*c - b**2)*atan((2*x**n*c + b)/sqrt(4*a*c - b**2))*a*
*2*c**2 - 24*x**(3*n)*sqrt(4*a*c - b**2)*atan((2*x**n*c + b)/sqrt(4*a*c -
b**2))*a*b**2*c + 6*x**(3*n)*sqrt(4*a*c - b**2)*atan((2*x**n*c + b)/sqrt(4
*a*c - b**2))*b**4 - 24*x**(3*n)*log(x**(2*n)*c + x**n*b + a)*a**2*b*c**2
+ 18*x**(3*n)*log(x**(2*n)*c + x**n*b + a)*a*b**3*c - 3*x**(3*n)*log(x**(2
*n)*c + x**n*b + a)*b**5 + 48*x**(3*n)*log(x)*a**2*b*c**2*n - 36*x**(3*n)*
log(x)*a*b**3*c*n + 6*x**(3*n)*log(x)*b**5*n + 24*x**(2*n)*a**3*c**2 - 30*
x**(2*n)*a**2*b**2*c + 6*x**(2*n)*a*b**4 + 12*x**n*a**3*b*c - 3*x**n*a**2*
b**3 - 8*a**4*c + 2*a**3*b**2)/(6*x**(3*n)*a**4*n*(4*a*c - b**2))
```

3.185 $\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 353

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \frac{2\,2^{3/4}c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2\,2^{3/4}c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n} + \frac{2\,2^{3/4}c^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2\,2^{3/4}c^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n}$$

output
```
2*2^(3/4)*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/4*n)/(-b-(-4*a*c+b^2)^(1/2))
^(1/4))/(-4*a*c+b^2)^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/n-2*2^(3/4)*c^(3/
4)*arctan(2^(1/4)*c^(1/4)*x^(1/4*n)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/(-4*a*c
+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)/n+2*2^(3/4)*c^(3/4)*arctanh(2^(1
/4)*c^(1/4)*x^(1/4*n)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-
b-(-4*a*c+b^2)^(1/2))^(3/4)/n-2*2^(3/4)*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^
(1/4*n)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)
^(1/2))^(3/4)/n
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$$
$$= \frac{2\,2^{3/4}c^{3/4}\left(-\frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}-\frac{\sqrt[4]{-b-\sqrt{\ldots}}\ldots}{\ldots}\right.}{n}$$

input
```
Integrate[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*2^(3/4)*c^(3/4)*(-(((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1
/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(b^2 - 4*a*c + b*Sqrt[b^2 -
4*a*c])) - ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)
]/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((-b - Sqrt[b^2 - 4
*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1
/4)])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]) - ArcTanh[(2^(1/4)*c^(1/4)*x^(n/
4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*
a*c])^(3/4))))/n
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1717, 1685, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{4}-1}}{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{ 1717}$$

$$\frac{4\int \frac{1}{bx^n+cx^{2n}+a}dx^{n/4}}{n}$$

$$\Big\downarrow \text{ 1685}$$

$$\frac{4\left(\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^{n/4}}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^{n/4}}{\sqrt{b^2-4ac}}\right)}{n}$$

$$\Big\downarrow \text{ 756}$$

$$\frac{4\left(\frac{c\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^{n/2}}dx^{n/4}}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^{n/2}+\sqrt{\sqrt{b^2-4ac}-b}}dx^{n/4}}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^{n/2}}dx^{n/4}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^{n/2}+\sqrt{-b-\cdots}}}{\sqrt{-\sqrt{b^2-\cdots}}}\cdots\right)}{\sqrt{b^2-4ac}}\cdots\right)}{n}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{4\left(\frac{c\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\sqrt{2}\sqrt{c}x^{n/2}}}dx^{n/4}}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x^{n/2}}}dx^{n/4}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{\cdots}}{\sqrt[4]{-\sqrt{b^2\cdots}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2\cdots}\right.}\cdots\right)}{\sqrt{b^2-4ac}}\cdots\right)}{n}$$

$\downarrow$ 221

$$\frac{4\left(\frac{c\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\cdots}{\sqrt[4]{\cdots}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\cdots}\right)}{\sqrt{b^2-4ac}}\cdots\right)}{n}$$

input
```
Int[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(4*(-((c*(-(ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4
)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^
(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt
[b^2 - 4*a*c])^(3/4))))/Sqrt[b^2 - 4*a*c]) + (c*(-(ArcTan[(2^(1/4)*c^(1/4)
*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2
- 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a
*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/Sqrt[b^2 -
4*a*c]))/n
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

rule 1717
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Simp[1/(m + 1)   Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[
2*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !
IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.79

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left((256a^7c^4n^8-256a^6b^2c^3n^8+96a^5b^4c^2n^8-16a^4b^6c\,n^8+a^3b^8n^8)\_Z^8+(-48a^3b\,c^3n^4+40a^2b^3c^2n^4-11a\,b^5c\,n^4+b^7n^4)\_\right.}$

input
```
int(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(x^(1/4*n)+(16/(a*c^2-b^2*c)*n^5*b*a^5*c^2-8/(a*c^2-b^2*c)*n^5*b^
3*a^4*c+1/(a*c^2-b^2*c)*n^5*b^5*a^3)*_R^5+(2/(a*c^2-b^2*c)*n*a^2*c^2-4/(a*
c^2-b^2*c)*n*b^2*a*c+1/(a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((256*a^7*c^4*n^8
-256*a^6*b^2*c^3*n^8+96*a^5*b^4*c^2*n^8-16*a^4*b^6*c*n^8+a^3*b^8*n^8)*_Z^8
+(-48*a^3*b*c^3*n^4+40*a^2*b^3*c^2*n^4-11*a*b^5*c*n^4+b^7*n^4)*_Z^4+c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3481 vs. $2(273) = 546$.

Time = 0.31 (sec) , antiderivative size = 3481, normalized size of antiderivative = 9.86

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{n}{4}-1}}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**(-1+1/4*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(n/4 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{n}{4}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^(n/4 - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(n/4 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{n}{4}}}{x^{2n}cx+x^nbx+ax}dx$$

input
```
int(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(x**(n/4)/(x**(2*n)*c*x + x**n*b*x + a*x),x)
```

3.186 $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 610

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = -\frac{2^{2/3}\sqrt{3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{2^{2/3}\sqrt{3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{c^{2/3}\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{c^{2/3}\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x^{n/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

output
```
-2^(2/3)*3^(1/2)*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x^(1/3*n)/(b-(-4*
a*c+b^2)^(1/2))^(1/3))*3^(1/2))/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(2/3)/n+2^(2/3)*3^(1/2)*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x^(1/3*n)/
(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(2/3)/n+2^(2/3)*c^(2/3)*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(
1/3)*x^(1/3*n))/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)/n-2^(2/3)*
c^(2/3)*ln((b+(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x^(1/3*n))/(-4*a*c
+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)/n-1/2*c^(2/3)*ln((b-(-4*a*c+b^2)^
(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x^(1/3*n)+2^(2/3
)*c^(2/3)*x^(2/3*n))*2^(2/3)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/
3)/n+1/2*c^(2/3)*ln((b+(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b+(-4*a*
c+b^2)^(1/2))^(1/3)*x^(1/3*n)+2^(2/3)*c^(2/3)*x^(2/3*n))*2^(2/3)/(-4*a*c+b
^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)/n
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$$

$$= \frac{c^{2/3}\left(-2\sqrt{3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)+2\sqrt{3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}\arctan\left(\frac{1-\frac{}{\sqrt[3]{}}}{}\right.}{}$$

input
```
Integrate[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(c^(2/3)*(-2*Sqrt[3]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*
c^(1/3)*x^(n/3))/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]] + 2*Sqrt[3]*(b -
Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b + Sqrt
[b^2 - 4*a*c])^(1/3))/Sqrt[3]] + 2*(b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b -
Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] - 2*(b - Sqrt[b^2 - 4*
a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] -
 (b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)
*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/
3)] + (b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((
2*n)/3)]))/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*(b + S
qrt[b^2 - 4*a*c])^(2/3)*n)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 533, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1717, 1685, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{3}-1}}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1717}$$

$$\frac{3\int \frac{1}{bx^n+cx^{2n}+a}dx^{n/3}}{n}$$

$$\Big\downarrow \text{1685}$$

$$\frac{3\left(\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^{n/3}}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^{n/3}}{\sqrt{b^2-4ac}}\right)}{n}$$

$$\Big\downarrow \text{750}$$

$$3\left(\frac{c\left(\frac{2^{2/3}\int\frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x^{n/3}\right)}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int\frac{1}{\sqrt[3]{c}x^{n/3}+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}dx^{n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}\right.$$

$\downarrow$ 16

$$3\left(\frac{c\left(\frac{2^{2/3}\int\frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x^{n/3}\right)}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{\sqrt{b^2-4ac}}\right.$$

$\downarrow$ 27

$$3\left(\frac{c\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x^{n/3}}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\sqrt{b^2-4ac}}\right.$$

$\downarrow$ 1142

$$3\left(\frac{c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}}{2\sqrt[3]{2}}-\frac{\int-\frac{\sqrt[3]{c}\left(\cdots\right.}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\cdots}}\cdots}{\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\right.}{\sqrt{b^2-4ac}\cdots}\right.\cdots$$

$\downarrow$ 25

$$3\left(\frac{c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\,x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}}{2\sqrt[3]{2}}+\int\frac{\sqrt[3]{c}\left(2\right.}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{}}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\sqrt{b^2-4ac}}\right.$$

$\downarrow$ 27

$$3\left(\frac{c\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{2^{\cdots}}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\cdots}}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\cdots\right.}{\sqrt{b^2-4ac}\cdots}\cdots\right.$$

↓ 1082

$$3\left(\frac{c\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x^{n/3}}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}+\frac{3\int\frac{1}{-x^{2n/3}-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\sqrt{b^2-4ac}}\right.$$

$$\Big\downarrow 217$$

$$3\left(\frac{c\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x^{n/3}}{-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{n/3}+2c^{2/3}x^{2n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\cdots\right)}{\sqrt{b^2-4ac}}\cdots\right.$$

↓ 1103

$$3\left(\frac{c\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{4\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\cdots}{\cdots}\right.}{\sqrt{b^2-4ac}}\cdots\right.$$

input
```
Int[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(3*((c*((2^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/
3)])/(3*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]
*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sq
rt[3]])/c^(1/3) - Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b -
 Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)]/(4*c^(1/3
))))/(3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/Sqrt[b^2 - 4*a*c] - (c*((2^(2/3)*
Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)])/(3*c^(1/3)*(
b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2
^(1/3)*c^(1/3)*x^(n/3))/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) -
 Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c
])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)]/(4*c^(1/3))))/(3*(b + Sqrt
[b^2 - 4*a*c])^(2/3))))/Sqrt[b^2 - 4*a*c]))/n
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

rule 1717
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Simp[1/(m + 1)   Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[
2*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !
IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.42 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.43

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(\left(64a^5c^3n^6-48a^4b^2c^2n^6+12a^3b^4c\,n^6-a^2b^6n^6\right)\_Z^6+\left(16a^2b\,c^2n^3-8a\,b^3c\,n^3+b^5n^3\right)\_Z^3+c^2\right)} \_R\ln\left(x^{\frac{n}{3}}+\left(-\right.\right.$

input
```
int(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(x^(1/3*n)+(-16/(2*a*c^2-b^2*c)*n^4*b*a^4*c^2+8/(2*a*c^2-b^2*c)*n
^4*b^3*a^3*c-1/(2*a*c^2-b^2*c)*n^4*b^5*a^2)*_R^4+(4/(2*a*c^2-b^2*c)*n*a^2*
c^2-5/(2*a*c^2-b^2*c)*n*b^2*a*c+1/(2*a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((64
*a^5*c^3*n^6-48*a^4*b^2*c^2*n^6+12*a^3*b^4*c*n^6-a^2*b^6*n^6)*_Z^6+(16*a^2
*b*c^2*n^3-8*a*b^3*c*n^3+b^5*n^3)*_Z^3+c^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2461 vs. $2(465) = 930$.

Time = 0.15 (sec) , antiderivative size = 2461, normalized size of antiderivative = 4.03

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
-1/2*(1/2)^(1/3)*(sqrt(-3) + 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*
b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)
*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-(4*(b^2*c - 2*a*c^2)*x*x
^(1/3*n - 1) + (1/2)^(1/3)*(sqrt(-3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2)*n + (b^
4 - 6*a*b^2*c + 8*a^2*c^2)*n - (sqrt(-3)*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b
*c^2)*n^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4)*sqrt((b^4 - 4*a*b^
2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n
^6)))*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b
^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*
a^3*c)*n^3))^(1/3))/x) + 1/2*(1/2)^(1/3)*(sqrt(-3) - 1)*(((a^2*b^2 - 4*a^3
*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a
^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-
(4*(b^2*c - 2*a*c^2)*x*x^(1/3*n - 1) - (1/2)^(1/3)*(sqrt(-3)*(b^4 - 6*a*b^
2*c + 8*a^2*c^2)*n - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n - (sqrt(-3)*(a^2*b^5
- 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4 - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)
*n^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6
*b^2*c^2 - 64*a^7*c^3)*n^6)))*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^
2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n
^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3))/x) - 1/2*(1/2)^(1/3)*(sqrt(-3)
 + 1)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a...
```

Sympy [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{a + bx^n + cx^{2n}}\,dx = \int \frac{x^{\frac{n}{3}-1}}{a + bx^n + cx^{2n}}\,dx$$

input
```
integrate(x**(-1+1/3*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(n/3 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{1}{3}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{1}{3}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{n}{3}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^(n/3 - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(n/3 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{\frac{n}{3}}}{x^{2n}cx+x^nbx+ax}dx$$

input
```
int(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(x**(n/3)/(x**(2*n)*c*x + x**n*b*x + a*x),x)
```

3.187 $\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 169

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}n}-\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}n}$$

output
```
2*2^(1/2)*c^(1/2)*arctan(2^(1/2)*c^(1/2)*x^(1/2*n)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/n-2*2^(1/2)*c^(1/2)
*arctan(2^(1/2)*c^(1/2)*x^(1/2*n)/(b+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^
2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\frac{2\sqrt{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,n}$$

input
```
Integrate[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4
*a*c]]]/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqr
t[b + Sqrt[b^2 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[b^2 - 4*a*c]
*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1717, 1406, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{2}-1}}{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{1717}$$

$$\frac{2\int \frac{1}{bx^n+cx^{2n}+a}dx^{n/2}}{n}$$

$$\Big\downarrow \text{1406}$$

$$\frac{2\left(\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^{n/2}}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^{n/2}}{\sqrt{b^2-4ac}}\right)}{n}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n}$$

input
```
Int[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*((Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[
c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/n
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1717
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Simp[1/(m + 1)   Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[
2*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !
IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.67

method	result
risch	$\sum_{\_R=\operatorname{RootOf}\left(\left(16a^3c^2n^4-8a^2b^2c\,n^4+a\,b^4n^4\right)\_Z^4+\left(-4abc\,n^2+b^3n^2\right)\_Z^2+c\right)} \_R\ln\left(x^{\frac{n}{2}}+\left(4n^3b\,a^2-\frac{n^3b^3a}{c}\right)\_R^3+\right.$

input
```
int(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(x^(1/2*n)+(4*n^3*b*a^2-1/c*n^3*b^3*a)*_R^3+(2*a*n-1/c*n*b^2)*_R)
,_R=RootOf((16*a^3*c^2*n^4-8*a^2*b^2*c*n^4+a*b^4*n^4)*_Z^4+(-4*a*b*c*n^2+b
^3*n^2)*_Z^2+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 801 vs. $2(129) = 258$.

Time = 0.09 (sec) , antiderivative size = 801, normalized size of antiderivative = 4.74

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4))
 + b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) + sqrt(2)*((a*b^3
- 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 - 4*a*c)*n)*sqrt
(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) + b)/((a*b^2 -
4*a^2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2
*b^2 - 4*a^3*c)*n^4)) + b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n -
1) - sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) -
(b^2 - 4*a*c)*n)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*
n^4)) + b)/((a*b^2 - 4*a^2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(((a*b^2 - 4*a^2
*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2))*lo
g((4*c*x*x^(1/2*n - 1) + sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2
 - 4*a^3*c)*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((
a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))/x) + 1/2*sqrt(2)*s
qrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2
- 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) - sqrt(2)*((a*b^3 - 4*a^2*b*c)*n
^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a
^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))
/x)
```

Sympy [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{n}{2}-1}}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**(-1+1/2*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(n/2 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. $2(129) = 258$.

Time = 0.43 (sec) , antiderivative size = 1037, normalized size of antiderivative = 6.14

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\text{Too large to display}$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
1/2*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^
2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*sqrt(x^n)/sqrt((b + sqrt(b^2 - 4*a*c)
)/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2
*c^2 - 4*a^2*c^3)*abs(c)) + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 +
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqr...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{n}{2}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx=\int \frac{x^{\frac{n}{2}}}{x^{2n}cx+x^nbx+ax}dx$$

input
```
int(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(x**(n/2)/(x**(2*n)*c*x + x**n*b*x + a*x),x)
```

3.188 $\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 205

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = -\frac{2x^{-n/2}}{an}+\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b-\sqrt{b^2-4ac}}n}+\frac{\sqrt{2}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b+\sqrt{b^2-4ac}}n}$$

output

```
-2/a/n/(x^(1/2*n))+2^(1/2)*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1
/2)*a^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(x^(1/2*n)))/a^(3/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2)/n+2^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1
/2)*a^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(x^(1/2*n)))/a^(3/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.62

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \frac{4cx^{-n/2}\left(\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\right)}{n}$$

input
```
Integrate[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(4*c*(Hypergeometric2F1[-1/2, 1, 1/2, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/
(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/2, 1, 1/2, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x
^(n/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1717, 1679, 1442, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{2}-1}}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{ 1717}$$

$$-\frac{2\int \frac{1}{bx^n+cx^{2n}+a}dx^{-n/2}}{n}$$

$$\Big\downarrow \text{ 1679}$$

$$-\frac{2\int \frac{x^{-2n}}{ax^{-2n}+bx^{-n}+c}dx^{-n/2}}{n}$$

$$\Big\downarrow \textbf{1442}$$

$$-\frac{2\left(\frac{x^{-n/2}}{a}-\frac{\int \frac{bx^{-n}+c}{ax^{-2n}+bx^{-n}+c}dx^{-n/2}}{a}\right)}{n}$$

$$\Big\downarrow \textbf{1480}$$

$$-\frac{2\left(\frac{x^{-n/2}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{ax^{-n}+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^{-n/2}+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{ax^{-n}+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^{-n/2}}{a}\right)}{n}$$

$$\Big\downarrow \textbf{218}$$

$$-\frac{2\left(\frac{x^{-n/2}}{a}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{a}\sqrt{\sqrt{b^2-4ac}+b}}}{a}\right)}{n}$$

input

```
Int[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

output

```
(-2*(1/(a*x^(n/2)) - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[a])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(Sqrt[2]*Sqrt[a]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[a])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(Sqrt[2]*Sqrt[a]*S
qrt[b + Sqrt[b^2 - 4*a*c]]))/a))/n
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 1717
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Simp[1/(m + 1)   Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[
2*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !
IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{2x^{-\frac{n}{2}}}{an}+\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^5c^2n^4-8a^4b^2c\,n^4+a^3b^4n^4\right)\_Z^4+\left(12a^2b\,c^2n^2-7a\,b^3c\,n^2+b^5n^2\right)\_Z^2+c^3\right)}\_R\ln\left(x^{\frac{n}{2}}+\left(-\right.\right.\right.$

input
```
int(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-2/a/n/(x^(1/2*n))+sum(_R*ln(x^(1/2*n)+(-8/(a*c^3-b^2*c^2)*n^3*a^5*c^2+6/(
a*c^3-b^2*c^2)*n^3*b^2*a^4*c-1/(a*c^3-b^2*c^2)*n^3*b^4*a^3)*_R^3+(-5/(a*c^
3-b^2*c^2)*n*b*a^2*c^2+5/(a*c^3-b^2*c^2)*n*b^3*a*c-1/(a*c^3-b^2*c^2)*n*b^5
)*_R),_R=RootOf((16*a^5*c^2*n^4-8*a^4*b^2*c*n^4+a^3*b^4*n^4)*_Z^4+(12*a^2*
b*c^2*n^2-7*a*b^3*c*n^2+b^5*n^2)*_Z^2+c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. 2(169) = 338.

Time = 0.10 (sec) , antiderivative size = 1229, normalized size of antiderivative = 6.00

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^
2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) + sqrt(2)*((a^3*b^3 - 4*a^4*b
*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - (b^4
 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 -
 4*a^4*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*
b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) - sqrt(2)*((
a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^
7*c)*n^4)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n
^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a
*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*
c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 +
 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n -
 1) + sqrt(2)*((a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
((a^6*b^2 - 4*a^7*c)*n^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(((a^3*b
^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^
4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) + sqrt(2)*a*n*sqrt(((a
^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^...
```

Sympy [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{n}{2}-1}}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**(-1-1/2*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(-n/2 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-2/(a*n*x^(1/2*n)) - integrate((c*x^(3/2*n) + b*x^(1/2*n))/(a*c*x*x^(2*n)
+ a*b*x*x^n + a^2*x), x)
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{2}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{2}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(n/2 + 1)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n/2 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{\frac{5n}{2}}cx+x^{\frac{3n}{2}}bx+x^{\frac{n}{2}}ax}dx$$

input
```
int(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(x**((5*n)/2)*c*x + x**((3*n)/2)*b*x + x**(n/2)*a*x),x)
```

3.189 $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 699

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = -\frac{3x^{-n/3}}{an} - \frac{\sqrt{3}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$
$$-\frac{\sqrt{3}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$
$$+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$
$$+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$
$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}\right)}{2\sqrt[3]{2}a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}$$
$$-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b+\sqrt{b^2-4ac}}x^{-n/3}\right)}{2\sqrt[3]{2}a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

output
```
-3/a/n/(x^(1/3*n))-1/2*3^(1/2)*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(
1/3*(1-2*2^(1/3)*a^(1/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)/(x^(1/3*n)))*3^(1/2)
)*2^(2/3)/a^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)/n-1/2*3^(1/2)*(b+(-2*a*c+b^
2)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/3)*a^(1/3)/(b+(-4*a*c+b^2)^(1/
2))^(1/3)/(x^(1/3*n)))*3^(1/2))*2^(2/3)/a^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/
3)/n+1/2*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(1/
3)+2^(1/3)*a^(1/3)/(x^(1/3*n)))*2^(2/3)/a^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/
3)/n+1/2*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(1/
3)+2^(1/3)*a^(1/3)/(x^(1/3*n)))*2^(2/3)/a^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/
3)/n-1/4*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(2/
3)+2^(2/3)*a^(2/3)/(x^(2/3*n))-2^(1/3)*a^(1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3
)/(x^(1/3*n)))*2^(2/3)/a^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)/n-1/4*(b+(-2*a
*c+b^2)/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(2/3)+2^(2/3)*a^(2/3
)/(x^(2/3*n))-2^(1/3)*a^(1/3)*(b+(-4*a*c+b^2)^(1/2))^(1/3)/(x^(1/3*n)))*2^
(2/3)/a^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.18

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \frac{6cx^{-n/3}\left(\frac{\text{Hypergeometric2F1}\left(-\frac{1}{3},1,\frac{2}{3},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{\text{Hypergeometric2F1}\left(-\frac{1}{3},1,\frac{2}{3},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\right)}{n}$$

input
```
Integrate[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(6*c*(Hypergeometric2F1[-1/3, 1, 2/3, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/
(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/3, 1, 2/3, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x
^(n/3))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.82, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1717, 1679, 1703, 1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{3}-1}}{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{1717}$$

$$-\frac{3\int \frac{1}{bx^n+cx^{2n}+a}dx^{-n/3}}{n}$$

$$\Big\downarrow \text{1679}$$

$$-\frac{3\int \frac{x^{-2n}}{ax^{-2n}+bx^{-n}+c}dx^{-n/3}}{n}$$

$$\Big\downarrow \text{1703}$$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\int \frac{bx^{-n}+c}{ax^{-2n}+bx^{-n}+c}dx^{-n/3}}{a}\right)}{n}$$

$$\Big\downarrow \text{1752}$$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{ax^{-n}+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^{-n/3}+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{ax^{-n}+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^{-n/3}}{a}\right)}{n}$$

$$\Big\downarrow \text{750}$$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int\frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{a}x^{-n/3}\right)}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{-n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int\frac{}{\sqrt[3]{a}x^{-n/}}}{}\right.}{}\right.}{}$$

$\downarrow$ 16

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int\frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{a}x^{-n/3}\right)}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{-n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-}\right.}{3\sqrt[3]{}}\right.}{}\right.}{}$$

$\downarrow$ 27

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{a}x^{-n/3}}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{-n/3}}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b \cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 1142

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b-\sqrt{b^2-4ac}}\right)}{3\sqrt[3]{a}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2\,2^{2/3}\left(3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{\cdots}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{\cdots}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$\downarrow$ 25

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b-\sqrt{b^2-4ac}}\right)}{3\sqrt[3]{a}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{}}}{}\right.}{}\right.}{}\right.}{}$$

$\downarrow$ 27

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/}}}{2\sqrt[3]{2}}\right.}{3\left(b-\sqrt{b^2-4ac}\right.}\right.}{}\right.}{}$$

↓ 1082

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{a}x^{-n/3}}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{-n/3}+\frac{3\int\frac{1}{-x^{-2n/}}}{}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}\right.}{}$$

↓ 217

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{a}x^{-n/3}}{2a^{2/3}x^{-2n/3}-2^{2/3}\sqrt[3]{a}\sqrt[3]{b-\sqrt{b^2-4ac}}x^{-n/3}+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx^{-n/3}-\frac{\sqrt{3}\arctan\left(\cdots\right)}{\cdots}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

↓ 1103

$$-3\left(\frac{x^{-n/3}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(-\frac{\log\left(2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}\right)}{4\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\cdots}{\sqrt[3]{\cdots}}\right)}{\cdots}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\cdots}\cdots\right)$$

input
```
Int[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-3*(1/(a*x^(n/3)) - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log
[(b - Sqrt[b^2 - 4*a*c])^(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(3*a^(1/3)*(b
 - Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^
(1/3)*a^(1/3))/((b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3)))/Sqrt[3]])/a^(1/3)
- Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(
1/3)*a^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3))/x^(n/3)]/(4*a^(1/3))))/(3*(b -
 Sqrt[b^2 - 4*a*c])^(2/3))))/2 + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*((
2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(3
*a^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan
[(1 - (2*2^(1/3)*a^(1/3))/((b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3)))/Sqrt[3]
])/a^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n
)/3) - (2^(1/3)*a^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))/x^(n/3)]/(4*a^(1/3)
)))/(3*(b + Sqrt[b^2 - 4*a*c])^(2/3))))/2)/a))/n
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1717
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Simp[1/(m + 1)   Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[
2*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !
IntegerQ[n]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 534, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{3x^{-\frac{n}{3}}}{an}+\left(\sum_{\_R=\text{RootOf}\left(\left(64a^7c^3n^6-48a^6b^2c^2n^6+12a^5b^4c\,n^6-a^4b^6n^6\right)\_Z^6+\left(-32a^3b\,c^3n^3+32a^2b^3c^2n^3-10a\,b^5c\,n^3+b^7n^3\right)\_\right.}\right.$

input
```
int(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-3/a/n/(x^(1/3*n))+sum(_R*ln(x^(1/3*n)+(-64/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3
)*n^5*a^8*c^4+112/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^2*a^7*c^3-60/(2*a^
2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^4*a^6*c^2+13/(2*a^2*c^5-4*a*b^2*c^4+b^4*c
^3)*n^5*b^6*a^5*c-1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^8*a^4)*_R^5+(28/
(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b*a^4*c^4-63/(2*a^2*c^5-4*a*b^2*c^4+b^
4*c^3)*n^2*b^3*a^3*c^3+42/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^5*a^2*c^2-
11/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^7*a*c+1/(2*a^2*c^5-4*a*b^2*c^4+b^
4*c^3)*n^2*b^9)*_R^2),_R=RootOf((64*a^7*c^3*n^6-48*a^6*b^2*c^2*n^6+12*a^5*
b^4*c*n^6-a^4*b^6*n^6)*_Z^6+(-32*a^3*b*c^3*n^3+32*a^2*b^3*c^2*n^3-10*a*b^5
*c*n^3+b^7*n^3)*_Z^3+c^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3155 vs. 2(567) = 1134.

Time = 0.17 (sec) , antiderivative size = 3155, normalized size of antiderivative = 4.51

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{n}{3}-1}}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**(-1-1/3*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(-n/3 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-3/(a*n*x^(1/3*n)) - integrate((c*x^(5/3*n) + b*x^(2/3*n))/(a*c*x*x^(2*n)
+ a*b*x*x^n + a^2*x), x)
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{3}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{3}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{\frac{7n}{3}}cx+x^{\frac{4n}{3}}bx+x^{\frac{n}{3}}ax}dx$$

input
```
int(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(x**((7*n)/3)*c*x + x**((4*n)/3)*b*x + x**(n/3)*a*x),x)
```

3.190 $\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 414

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = -\frac{4x^{-n/4}}{an} - \frac{2^{3/4}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2^{3/4}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n}$$

output
```
-4/a/n/(x^(1/4*n))-2^(3/4)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1
/4)*a^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(x^(1/4*n)))/a^(5/4)/(-b-(-4*a*c
+b^2)^(1/2))^(3/4)/n-2^(3/4)*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^
(1/4)*a^(1/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)/(x^(1/4*n)))/a^(5/4)/(-b+(-4*a
*c+b^2)^(1/2))^(3/4)/n-2^(3/4)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh
(2^(1/4)*a^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(x^(1/4*n)))/a^(5/4)/(-b-(-
4*a*c+b^2)^(1/2))^(3/4)/n-2^(3/4)*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arct
anh(2^(1/4)*a^(1/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)/(x^(1/4*n)))/a^(5/4)/(-b
+(-4*a*c+b^2)^(1/2))^(3/4)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.31

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$$
$$= \frac{8cx^{-n/4}\left(\frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}\right)}{n}$$

input
```
Integrate[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(8*c*(Hypergeometric2F1[-1/4, 1, 3/4, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/
(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/4, 1, 3/4, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x
^(n/4))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1717, 1679, 1703, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{4}-1}}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{ 1717}$$

$$-\frac{4\int \frac{1}{bx^n+cx^{2n}+a}dx^{-n/4}}{n}$$

$$\Big\downarrow \text{ 1679}$$

$$-\frac{4\int \frac{x^{-2n}}{ax^{-2n}+bx^{-n}+c}dx^{-n/4}}{n}$$

$$\Big\downarrow \text{ 1703}$$

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{\int \frac{bx^{-n}+c}{ax^{-2n}+bx^{-n}+c}dx^{-n/4}}{a}\right)}{n}$$

$$\Big\downarrow \text{ 1752}$$

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{ax^{-n}+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^{-n/4}+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{ax^{-n}+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^{-n/4}}{a}\right)}{n}$$

$$\Big\downarrow \text{ 756}$$

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{a}x^{-n/2}}dx^{-n/4}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{a}x^{-n/2}+\sqrt{-b-\sqrt{b^2-4ac}}}dx^{-n/4}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\cdots\right)}{a}\right)}{n}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{a}x^{-n/2}}}dx^{-n/4}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{a}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\cdots}}}{\cdots}\right.}{a}\right.}{n}$$

$$\Big\downarrow \; 221$$

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{a}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{a}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\operatorname{arcta}\cdots}{\cdots}\right.}{a}\right.}{n}$$

input
```
Int[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-4*(1/(a*x^(n/4)) - (((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^
(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))]/(2^(1/4)*a^(1/4)*
(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*a^(1/4))/((-b - Sqrt[b
^2 - 4*a*c])^(1/4)*x^(n/4))]/(2^(1/4)*a^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/
4))))/2 + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*a^(1/4
))/((-b + Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))]/(2^(1/4)*a^(1/4)*(-b + Sqrt[b
^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])
^(1/4)*x^(n/4))]/(2^(1/4)*a^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2)/a))
/n
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1717
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Simp[1/(m + 1)   Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[
2*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !
IntegerQ[n]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.95 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{4x^{-\frac{n}{4}}}{an}+\left(\sum_{\_R=\text{RootOf}\left(\left(256a^{9}c^{4}n^{8}-256a^{8}b^{2}c^{3}n^{8}+96a^{7}b^{4}c^{2}n^{8}-16a^{6}b^{6}c\,n^{8}+a^{5}b^{8}n^{8}\right)\_Z^{8}+\left(80a^{4}b\,c^{4}n^{4}-120a^{3}b^{3}c^{3}n^{4}+61c\right.\right.}\right.$

input
```
int(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
-4/a/n/(x^(1/4*n))+sum(_R*ln(x^(1/4*n)+(-128/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)
*n^7*a^10*c^5+352/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^2*a^9*c^4-280/(a^2*c
^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^4*a^8*c^3+98/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n
^7*b^6*a^7*c^2-16/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^8*a^6*c+1/(a^2*c^6-3
*a*b^2*c^5+b^4*c^4)*n^7*b^10*a^5)*_R^7+(-36/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*
n^3*b*a^5*c^5+129/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^3*a^4*c^4-138/(a^2*c
^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^5*a^3*c^3+63/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n
^3*b^7*a^2*c^2-13/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^9*a*c+1/(a^2*c^6-3*a
*b^2*c^5+b^4*c^4)*n^3*b^11)*_R^3),_R=RootOf((256*a^9*c^4*n^8-256*a^8*b^2*c
^3*n^8+96*a^7*b^4*c^2*n^8-16*a^6*b^6*c*n^8+a^5*b^8*n^8)*_Z^8+(80*a^4*b*c^4
*n^4-120*a^3*b^3*c^3*n^4+61*a^2*b^5*c^2*n^4-13*a*b^7*c*n^4+b^9*n^4)*_Z^4+c
^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4375 vs. $2(342) = 684$.

Time = 0.26 (sec) , antiderivative size = 4375, normalized size of antiderivative = 10.57

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{n}{4}-1}}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**(-1-1/4*n)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**(-n/4 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-4/(a*n*x^(1/4*n)) - integrate((c*x^(7/4*n) + b*x^(3/4*n))/(a*c*x*x^(2*n)
+ a*b*x*x^n + a^2*x), x)
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^{-\frac{1}{4}\,n-1}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{\frac{n}{4}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \int \frac{1}{x^{\frac{9n}{4}}cx+x^{\frac{5n}{4}}bx+x^{\frac{n}{4}}ax}dx$$

input
```
int(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(x**((9*n)/4)*c*x + x**((5*n)/4)*b*x + x**(n/4)*a*x),x)
```

3.191 $\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx = -\frac{2cx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)} - \frac{2cx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)}$$

output
```
-2*c*x^3*hypergeom([1, 3/n],[(3+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(3*
b^2-12*a*c-3*b*(-4*a*c+b^2)^(1/2))-2*c*x^3*hypergeom([1, 3/n],[(3+n)/n],-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(3*b^2-12*a*c+3*b*(-4*a*c+b^2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.89

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx = -\frac{2}{3}cx^3\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-3/n}\text{Hypergeometric2F1}\left(-\frac{3}{n},-\frac{3}{n},\frac{-3+n}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} + \frac{1-8^{-1/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-3/n}\text{Hypergeometric2F1}\left(-\frac{3}{n},-\frac{3}{n},\frac{-3+n}{n},\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

input
```
Integrate[x^2/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-2*c*x^3*((1 - Hypergeometric2F1[-3/n, -3/n, (-3 + n)/n, (b - Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 - 4*a
*c])/c + x^n))^(3/n))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hypergeom
etric2F1[-3/n, -3/n, (-3 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^n)]/(8^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(3
/n)))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1719}$$

$$\frac{2c\int \frac{x^2}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x^2}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2cx^3 \text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^3 \text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input

```
Int[x^2/(a + b*x^n + c*x^(2*n)),x]
```

output

```
(2*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])) - (2*c*x^3*Hypergeo
metric2F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[
b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))
```

Defintions of rubi rules used

rule 888

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{x^2}{a+b\,x^n+c\,x^{2n}}dx$$

input
```
int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^2}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(x^2/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^2}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**2/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x**2/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^2}{a + bx^n + cx^{2n}} \, dx = \int \frac{x^2}{cx^{2n} + bx^n + a} \, dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^2}{a + bx^n + cx^{2n}} \, dx = \int \frac{x^2}{cx^{2n} + bx^n + a} \, dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + bx^n + cx^{2n}} \, dx = \int \frac{x^2}{a + b\,x^n + c\,x^{2\,n}} \, dx$$

input
```
int(x^2/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x^2/(a + b*x^n + c*x^(2*n)), x)
```

Reduce [F]

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^2}{x^{2n}c+x^n b+a}dx$$

input
```
int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(x**2/(x**(2*n)*c + x**n*b + a),x)
```

3.192 $\int \frac{x}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 136

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx = -\frac{cx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{cx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

output
```
-c*x^2*hypergeom([1, 2/n],[(2+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-
4*a*c-b*(-4*a*c+b^2)^(1/2))-c*x^2*hypergeom([1, 2/n],[(2+n)/n],-2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))/(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.93

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx = -cx^2\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-2/n}\text{Hypergeometric2F1}\left(-\frac{2}{n},-\frac{2}{n},\frac{-2+n}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} + \frac{1-4^{-1/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-2/n}\text{Hypergeometric2F1}\left(-\frac{2}{n},-\frac{2}{n},\frac{-2+n}{n},\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

input
```
Integrate[x/(a + b*x^n + c*x^(2*n)),x]
```

output
```
-(c*x^2*((1 - Hypergeometric2F1[-2/n, -2/n, (-2 + n)/n, (b - Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c
])/c + x^n))^(2/n))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hypergeomet
ric2F1[-2/n, -2/n, (-2 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^n)]/(4^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(2/n
)))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \; 1719$$

$$\frac{2c\int \frac{x}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{888}$$

$$\frac{cx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{cx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[x/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])) - (c*x^2*Hypergeometric
2F1[1, 2/n, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*
a*c]*(b + Sqrt[b^2 - 4*a*c]))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{x}{a+b\,x^{n}+c\,x^{2n}}dx$$

input
```
int(x/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(x/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx=\int \frac{x}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(x/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx=\int \frac{x}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(x/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx = \int \frac{x}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx = \int \frac{x}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx = \int \frac{x}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(x/(a + b*x^n + c*x^(2*n)), x)
```

Reduce [F]

$$\int \frac{x}{a + bx^n + cx^{2n}}\,dx = \int \frac{x}{x^{2n}c + x^n b + a}dx$$

input
```
int(x/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(x/(x**(2*n)*c + x**n*b + a),x)
```

3.193 $\int \frac{1}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 124

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx = -\frac{2cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

output
```
-2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*
a*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4
*a*c+b^2)^(1/2)))/(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 261 vs. $2(124) = 248$.

Time = 0.57 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.10

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx = -2cx\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-1/n}\text{Hypergeometric2F1}\left(-\frac{1}{n},-\frac{1}{n},\frac{-1+n}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}\right.$$
$$\left.+\frac{1-2^{-1/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}\text{Hypergeometric2F1}\left(-\frac{1}{n},-\frac{1}{n},\frac{-1+n}{n},\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(-1),x]
```

output
```
-2*c*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 -
4*a*c])/c + x^n))^n^(-1))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hyper
geometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)]/(2^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n))^n^(-1)))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1685, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1685}$$

$$\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{778}$$

$$\frac{2cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(-1),x]
```

output
```
(2*c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])) - (2*c*x*Hypergeome
tric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[
b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{1}{a+b\,x^{n}+c\,x^{2n}}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(1/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(1/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(1/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(1/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(1/(a + b*x^n + c*x^(2*n)), x)
```

Reduce [F]

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{x^{2n}c+x^nb+a}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(x**(2*n)*c + x**n*b + a),x)
```

3.194 $\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 74

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx = \frac{b\operatorname{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}n}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}$$

output
```
b*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)/n+ln(x)/a-1
/2*ln(a+b*x^n+c*x^(2*n))/a/n
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx = -\frac{\frac{2b\arctan\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2\log(x^n)+\log(a+x^n(b+cx^n))}{2an}$$

input
```
Integrate[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

output
```
-1/2*((2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] -
2*Log[x^n] + Log[a + x^n*(b + c*x^n)])/(a*n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1693, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^n+cx^{2n})}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int \frac{x^{-n}}{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \text{1144}$$

$$\frac{\frac{\int -\frac{cx^n+b}{bx^n+cx^{2n}+a}dx^n}{a}+\frac{\log(x^n)}{a}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{\int \frac{cx^n+b}{bx^n+cx^{2n}+a}dx^n}{a}}{n}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{\frac{1}{2}b\int \frac{1}{bx^n+cx^{2n}+a}dx^n+\frac{1}{2}\int \frac{2cx^n+b}{bx^n+cx^{2n}+a}dx^n}{a}}{n}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{\frac{1}{2}\int \frac{2cx^n+b}{bx^n+cx^{2n}+a}dx^n-b\int \frac{1}{-x^{2n}+b^2-4ac}d(2cx^n+b)}{a}}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{\frac{1}{2}\int \frac{2cx^n+b}{bx^n+cx^{2n}+a}dx^n-\frac{b\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{n}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\log(x^n)}{a} - \frac{\frac{1}{2}\log(a+bx^n+cx^{2n}) - \frac{b\,\text{arctanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{n}$$

input
```
Int[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(Log[x^n]/a - (-((b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4
*a*c]) + Log[a + b*x^n + c*x^(2*n)]/2)/a)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(68) = 136$.

Time = 0.12 (sec) , antiderivative size = 397, normalized size of antiderivative = 5.36

method	result
risch	$\frac{4n^2\ln(x)ac}{4a^2cn^2-a\,b^2n^2}-\frac{n^2\ln(x)b^2}{4a^2cn^2-a\,b^2n^2}-\frac{2\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)c}{(4ac-b^2)n}+\frac{\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)b^2}{2a(4ac-b^2)n}+\frac{\ln\left(x^n-\frac{-b^2+\sqrt{-4}}{2b}\right.}{2a(}$

input
```
int(1/x/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
4/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x)*a*c-1/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x
)*b^2-2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/
a/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2+1/2/a/(4
*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)
^(1/2)-2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/
a/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/a/(4*
a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.50

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx$$

$$= \left[\frac{2(b^2-4ac)n\log(x)+\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2x^{2n}+b^2-2ac+2\left(bc+\sqrt{b^2-4ac}\,c\right)x^n+\sqrt{b^2-4ac}\,b}{cx^{2n}+bx^n+a}\right)-(b^2-4ac)\log(cx}{2(ab^2-4a^2c)n}\right.$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
[1/2*(2*(b^2 - 4*a*c)*n*log(x) + sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) +
b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*
x^(2*n) + b*x^n + a)) - (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2
- 4*a^2*c)*n), 1/2*(2*(b^2 - 4*a*c)*n*log(x) + 2*sqrt(-b^2 + 4*a*c)*b*arct
an(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - (
b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4*a^2*c)*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x(a+bx^n+cx^{2n})}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)}\,dx=\int \frac{1}{(cx^{2n}+bx^n+a)x}\,dx$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)}\,dx=\int \frac{1}{(cx^{2n}+bx^n+a)x}\,dx$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 10.93 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.03

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)}\,dx = \frac{\ln\left(-\frac{1}{c\,x}-\frac{(2\,a\,n+b\,n\,x^n)\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{2\,c\,x\,(a\,b^2\,n-4\,a^2\,c\,n)}\right)\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{2\,\left(a\,b^2\,n-4\,a^2\,c\,n\right)} - \frac{\ln\left(\frac{(2\,a\,n+b\,n\,x^n)\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2\right)}{2\,c\,x\,(a\,b^2\,n-4\,a^2\,c\,n)}-\frac{1}{c\,x}\right)\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2\right)}{2\,\left(a\,b^2\,n-4\,a^2\,c\,n\right)} + \frac{\ln\left(x\right)\,\left(n-1\right)}{a\,n}$$

input
```
int(1/(x*(a + b*x^n + c*x^(2*n))),x)
```

output
```
(log(- 1/(c*x) - ((2*a*n + b*n*x^n)*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2))
/(2*c*x*(a*b^2*n - 4*a^2*c*n)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2))/(2*
(a*b^2*n - 4*a^2*c*n)) - (log(((2*a*n + b*n*x^n)*(b*(b^2 - 4*a*c)^(1/2) -
4*a*c + b^2))/(2*c*x*(a*b^2*n - 4*a^2*c*n)) - 1/(c*x))*(b*(b^2 - 4*a*c)^(1
/2) - 4*a*c + b^2))/(2*(a*b^2*n - 4*a^2*c*n)) + (log(x)*(n - 1))/(a*n)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)}\,dx$$
$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2x^nc+b}{\sqrt{4ac-b^2}}\right)b-4\log(x^{2n}c+x^nb+a)\,ac+\log(x^{2n}c+x^nb+a)\,b^2+8\log(x)\,acn-2\log}{2an\left(4ac-b^2\right)}$$

input
```
int(1/x/(a+b*x^n+c*x^(2*n)),x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((2*x**n*c + b)/sqrt(4*a*c - b**2))*b - 4*log
(x**(2*n)*c + x**n*b + a)*a*c + log(x**(2*n)*c + x**n*b + a)*b**2 + 8*log(
x)*a*c*n - 2*log(x)*b**2*n)/(2*a*n*(4*a*c - b**2))
```

3.195 $\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx = \frac{2c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x} + \frac{2c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x}$$

output
```
2*c*hypergeom([1, -1/n],[-(1-n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4
*a*c-b*(-4*a*c+b^2)^(1/2))/x+2*c*hypergeom([1, -1/n],[-(1-n)/n],-2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))/(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)/x
```

Mathematica [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx = \frac{2^{1+\frac{1}{n}}c\left(\frac{\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(1+\frac{1}{n},1+\frac{1}{n},2+\frac{1}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{-b+\sqrt{b^2-4ac}-2cx^n} + \frac{x^{-n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{1+\frac{1}{n}}\text{Hypergeomet}}{c}\right.}{\sqrt{b^2-4ac}(1+n)x}$$

input
```
Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(2^(1 + n^(-1))*c*((((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)*Hyp
ergeometric2F1[1 + n^(-1), 1 + n^(-1), 2 + n^(-1), (b - Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n) +
(((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(1 + n^(-1))*Hypergeometric2F
1[1 + n^(-1), 1 + n^(-1), 2 + n^(-1), (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)])/(c*x^n)))/(Sqrt[b^2 - 4*a*c]*(1 + n)*x)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^n+cx^{2n})} \, dx$$

$$\Big\downarrow \text{1719}$$

$$\frac{2c\int \frac{1}{x^2\left(2cx^n+b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{1}{x^2\left(2cx^n+b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}$$

input
```
Int[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(-2*c*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*x) + (2*c*Hypergeo
metric2F1[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(
Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*x)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{1}{x^2 (a+b\,x^n+c\,x^{2n})}dx$$

input
```
int(1/x^2/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/x^2/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^n+cx^{2n})}\,dx=\int \frac{1}{(cx^{2\,n}+bx^n+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(1/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})} \, dx = \int \frac{1}{x^2 \left(a + b\,x^n + c\,x^{2\,n}\right)} \, dx$$

input
```
int(1/(x^2*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^2*(a + b*x^n + c*x^(2*n))), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})} \, dx = \int \frac{1}{x^{2n} c\,x^2 + x^n b\,x^2 + a\,x^2} dx$$

input
```
int(1/x^2/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(x**(2*n)*c*x**2 + x**n*b*x**2 + a*x**2),x)
```

3.196 $\int \frac{1}{x^3(a+bx^n+cx^{2n})}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{1}{x^3(a+bx^n+cx^{2n})}\,dx = \frac{c\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2} + \frac{c\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x^2}$$

output
```
c*hypergeom([1, -2/n],[-(2-n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*a
*c-b*(-4*a*c+b^2)^(1/2))/x^2+c*hypergeom([1, -2/n],[-(2-n)/n],-2*c*x^n/(b+
(-4*a*c+b^2)^(1/2)))/(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)/x^2
```

Mathematica [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.84

$$\int \frac{1}{x^3(a+bx^n+cx^{2n})}\,dx = \frac{2^{\frac{2+n}{n}}c\left(\frac{\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{2/n}\text{Hypergeometric2F1}\left(\frac{2+n}{n},\frac{2+n}{n},2+\frac{2}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{-b+\sqrt{b^2-4ac}-2cx^n} + \frac{x^{-n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{\frac{2+n}{n}}\text{Hypergeomet}}{c}\right.}{\sqrt{b^2-4ac}(2+n)x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(2^((2 + n)/n)*c*((((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(2/n)*Hyper
geometric2F1[(2 + n)/n, (2 + n)/n, 2 + 2/n, (b - Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c] + 2*c*x^n)])/(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n) + (((c*x^
n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((2 + n)/n)*Hypergeometric2F1[(2 + n
)/n, (2 + n)/n, 2 + 2/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] +
2*c*x^n)])/(c*x^n)))/(Sqrt[b^2 - 4*a*c]*(2 + n)*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})} dx$$

$$\Big\downarrow \text{1719}$$

$$\frac{2c \int \frac{1}{x^3 \left(2cx^n + b - \sqrt{b^2 - 4ac}\right)} dx}{\sqrt{b^2 - 4ac}} - \frac{2c \int \frac{1}{x^3 \left(2cx^n + b + \sqrt{b^2 - 4ac}\right)} dx}{\sqrt{b^2 - 4ac}}$$

$$\Big\downarrow \text{888}$$

$$\frac{c \,\text{Hypergeometric2F1}\left(1, -\frac{2}{n}, -\frac{2-n}{n}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x^2 \sqrt{b^2 - 4ac} \left(\sqrt{b^2 - 4ac} + b\right)} - \frac{c \,\text{Hypergeometric2F1}\left(1, -\frac{2}{n}, -\frac{2-n}{n}, -\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x^2 \sqrt{b^2 - 4ac} \left(b - \sqrt{b^2 - 4ac}\right)}$$

input
```
Int[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

output
```
-((c*Hypergeometric2F1[1, -2/n, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*x^2)) + (c*Hypergeomet
ric2F1[1, -2/n, -((2 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b
^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{1}{x^3 \left(a+b\,x^n+c\,x^{2n}\right)}dx$$

input
```
int(1/x^3/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/x^3/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{1}{x^3 \left(a+bx^n+cx^{2n}\right)}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(1/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\,(a+bx^n+cx^{2n})}\,dx=\int \frac{1}{x^3\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/(x^3*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/(x^3*(a + b*x^n + c*x^(2*n))), x)
```

Reduce [F]

$$\int \frac{1}{x^3\,(a+bx^n+cx^{2n})}\,dx=\int \frac{1}{x^{2n}c\,x^3+x^n b\,x^3+a\,x^3}dx$$

input
```
int(1/x^3/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(x**(2*n)*c*x**3 + x**n*b*x**3 + a*x**3),x)
```

3.197 $\int \sqrt{dx}(a+bx^n+cx^{2n})\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx=\frac{2a(dx)^{3/2}}{3d}+\frac{2bx^n(dx)^{3/2}}{d(3+2n)}+\frac{2cx^{2n}(dx)^{3/2}}{d(3+4n)}$$

output
```
2/3*a*(d*x)^(3/2)/d+2*b*x^n*(d*x)^(3/2)/d/(3+2*n)+2*c*x^(2*n)*(d*x)^(3/2)/
d/(3+4*n)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.64

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx=\frac{2}{3}x\sqrt{dx}\left(a+3x^n\left(\frac{b}{3+2n}+\frac{cx^n}{3+4n}\right)\right)$$

input
```
Integrate[Sqrt[d*x]*(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*x*Sqrt[d*x]*(a + 3*x^n*(b/(3 + 2*n) + (c*x^n)/(3 + 4*n))))/3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx$$

$$\Big\downarrow \text{ 1691}$$

$$\int \left(a\sqrt{dx}+b\sqrt{dx}x^n+c\sqrt{dx}x^{2n}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a(dx)^{3/2}}{3d}+\frac{2b\sqrt{dx}x^{n+1}}{2n+3}+\frac{2c\sqrt{dx}x^{2n+1}}{4n+3}$$

input
```
Int[Sqrt[d*x]*(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*b*x^(1 + n)*Sqrt[d*x])/(3 + 2*n) + (2*c*x^(1 + 2*n)*Sqrt[d*x])/(3 + 4*n
) + (2*a*(d*x)^(3/2))/(3*d)
```

Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(56) = 112.

Time = 0.10 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.88

method	result
orering	$\frac{2x(8n^2+24n+13)\sqrt{dx}\,(a+b\,x^n+c\,x^{2n})}{3(3+4n)(3+2n)} - \frac{4x^2(1+2n)\left(\frac{(a+b\,x^n+c\,x^{2n})d}{2\sqrt{dx}}+\sqrt{dx}\left(\frac{b\,x^n n}{x}+\frac{2c\,x^{2n} n}{x}\right)\right)}{(3+4n)(3+2n)} + \frac{8x^3\left(-\frac{(a+b\,x^n+c\,x^{2n})d}{4(dx)^{\frac{3}{2}}}\right.}{}$

input
```
int((d*x)^(1/2)*(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
2/3*x*(8*n^2+24*n+13)/(3+4*n)/(3+2*n)*(d*x)^(1/2)*(a+b*x^n+c*x^(2*n))-4*x^
2*(1+2*n)/(3+4*n)/(3+2*n)*(1/2/(d*x)^(1/2)*(a+b*x^n+c*x^(2*n))*d+(d*x)^(1/
2)*(b*x^n*n/x+2*c*x^(2*n)*n/x))+8/3/(8*n^2+18*n+9)*x^3*(-1/4/(d*x)^(3/2)*(
a+b*x^n+c*x^(2*n))*d^2+1/(d*x)^(1/2)*(b*x^n*n/x+2*c*x^(2*n)*n/x)*d+(d*x)^(
1/2)*(b*x^n*n^2/x^2-b*x^n*n/x^2+4*c*x^(2*n)*n^2/x^2-2*c*x^(2*n)*n/x^2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx = \frac{2\left(3\left(2\,cn+3\,c\right)\sqrt{dx}xx^{2\,n}+3\left(4\,bn+3\,b\right)\sqrt{dx}xx^{n}+\left(8\,an^2+18\,an+9\,a\right)\sqrt{dx}x\right)}{3\left(8\,n^2+18\,n+9\right)}$$

input
```
integrate((d*x)^(1/2)*(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
2/3*(3*(2*c*n + 3*c)*sqrt(d*x)*x*x^(2*n) + 3*(4*b*n + 3*b)*sqrt(d*x)*x*x^n
 + (8*a*n^2 + 18*a*n + 9*a)*sqrt(d*x)*x)/(8*n^2 + 18*n + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(54) = 108$.

Time = 147.92 (sec) , antiderivative size = 284, normalized size of antiderivative = 4.44

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx$$
$$= \begin{cases} a\left(\begin{cases} \frac{2(dx)^{\frac{3}{2}}}{3d} & \text{for } d \neq 0 \\ 0 & \text{otherwise} \end{cases}\right) + \frac{b\sqrt{dx}\log(x)}{\sqrt{x}} + c\left(\begin{cases} -\frac{2d^2}{3(dx)^{\frac{3}{2}}} & \text{for } d \neq 0 \\ 0 & \text{otherwise} \end{cases}\right) & \text{for } \\ a\left(\begin{cases} \frac{2(dx)^{\frac{3}{2}}}{3d} & \text{for } d \neq 0 \\ 0 & \text{otherwise} \end{cases}\right) + \frac{4b\sqrt[4]{x}\sqrt{dx}}{3} + \frac{c\sqrt{dx}\log(x)}{\sqrt{x}} & \text{for } \\ \frac{16an^2x\sqrt{dx}}{24n^2+54n+27} + \frac{36anx\sqrt{dx}}{24n^2+54n+27} + \frac{18ax\sqrt{dx}}{24n^2+54n+27} + \frac{24bnxx^n\sqrt{dx}}{24n^2+54n+27} + \frac{18bxx^n\sqrt{dx}}{24n^2+54n+27} + \frac{12cnxx^{2n}\sqrt{dx}}{24n^2+54n+27} + \frac{18cxx^{2n}\sqrt{dx}}{24n^2+54n+27} & \text{othe} \end{cases}$$

input
```
integrate((d*x)**(1/2)*(a+b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((a*Piecewise((2*(d*x)**(3/2)/(3*d), Ne(d, 0)), (0, True)) + b*sq
rt(d*x)*log(x)/sqrt(x) + c*Piecewise((-2*d**2/(3*(d*x)**(3/2)), Ne(d, 0)),
 (0, True)), Eq(n, -3/2)), (a*Piecewise((2*(d*x)**(3/2)/(3*d), Ne(d, 0)),
(0, True)) + 4*b*x**(1/4)*sqrt(d*x)/3 + c*sqrt(d*x)*log(x)/sqrt(x), Eq(n,
-3/4)), (16*a*n**2*x*sqrt(d*x)/(24*n**2 + 54*n + 27) + 36*a*n*x*sqrt(d*x)/
(24*n**2 + 54*n + 27) + 18*a*x*sqrt(d*x)/(24*n**2 + 54*n + 27) + 24*b*n*x*
x**n*sqrt(d*x)/(24*n**2 + 54*n + 27) + 18*b*x*x**n*sqrt(d*x)/(24*n**2 + 54
*n + 27) + 12*c*n*x*x**(2*n)*sqrt(d*x)/(24*n**2 + 54*n + 27) + 18*c*x*x**(
2*n)*sqrt(d*x)/(24*n**2 + 54*n + 27), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx = \frac{2\,c\sqrt{d}x^{\frac{3}{2}}x^{2\,n}}{4\,n+3} + \frac{2\,b\sqrt{d}x^{\frac{3}{2}}x^{n}}{2\,n+3} + \frac{2\,(dx)^{\frac{3}{2}}\,a}{3\,d}$$

input
```
integrate((d*x)^(1/2)*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
2*c*sqrt(d)*x^(3/2)*x^(2*n)/(4*n + 3) + 2*b*sqrt(d)*x^(3/2)*x^n/(2*n + 3)
+ 2/3*(d*x)^(3/2)*a/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx=\frac{2}{3}\sqrt{dx}ax+\frac{2\sqrt{dx}cxx^{2n}}{4n+3}+\frac{2\sqrt{dx}bxx^n}{2n+3}$$

input
```
integrate((d*x)^(1/2)*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
2/3*sqrt(d*x)*a*x + 2*sqrt(d*x)*c*x*x^(2*n)/(4*n + 3) + 2*sqrt(d*x)*b*x*x^
n/(2*n + 3)
```

Mupad [B] (verification not implemented)

Time = 10.80 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx=\frac{2\,a\,x\,\sqrt{d\,x}}{3}+\frac{2\,c\,x\,x^{2\,n}\,\sqrt{d\,x}}{4\,n+3}+\frac{2\,b\,x\,x^n\,\sqrt{d\,x}}{2\,n+3}$$

input
```
int((d*x)^(1/2)*(a + b*x^n + c*x^(2*n)),x)
```

output
```
(2*a*x*(d*x)^(1/2))/3 + (2*c*x*x^(2*n)*(d*x)^(1/2))/(4*n + 3) + (2*b*x*x^n
*(d*x)^(1/2))/(2*n + 3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x(6x^{2n}cn+9x^{2n}c+12x^nbn+9x^nb+8a\,n^2+18an+9a)}{24n^2+54n+27}$$

input
```
int((d*x)^(1/2)*(a+b*x^n+c*x^(2*n)),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x*(6*x**(2*n)*c*n + 9*x**(2*n)*c + 12*x**n*b*n + 9*x**n
*b + 8*a*n**2 + 18*a*n + 9*a))/(3*(8*n**2 + 18*n + 9))
```

3.198 $\int \sqrt{dx}(a+bx^n+cx^{2n})\ dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\ dx=\frac{2a(dx)^{3/2}}{3d}+\frac{2bx^n(dx)^{3/2}}{d(3+2n)}+\frac{2cx^{2n}(dx)^{3/2}}{d(3+4n)}$$

output
```
2/3*a*(d*x)^(3/2)/d+2*b*x^n*(d*x)^(3/2)/d/(3+2*n)+2*c*x^(2*n)*(d*x)^(3/2)/
d/(3+4*n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.64

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\ dx=\frac{2}{3}x\sqrt{dx}\left(a+3x^n\left(\frac{b}{3+2n}+\frac{cx^n}{3+4n}\right)\right)$$

input
```
Integrate[Sqrt[d*x]*(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*x*Sqrt[d*x]*(a + 3*x^n*(b/(3 + 2*n) + (c*x^n)/(3 + 4*n))))/3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx$$

$$\Big\downarrow \ 1691$$

$$\int \left(a\sqrt{dx}+b\sqrt{dx}x^n+c\sqrt{dx}x^{2n}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2a(dx)^{3/2}}{3d}+\frac{2b\sqrt{dx}x^{n+1}}{2n+3}+\frac{2c\sqrt{dx}x^{2n+1}}{4n+3}$$

input
```
Int[Sqrt[d*x]*(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*b*x^(1 + n)*Sqrt[d*x])/(3 + 2*n) + (2*c*x^(1 + 2*n)*Sqrt[d*x])/(3 + 4*n
) + (2*a*(d*x)^(3/2))/(3*d)
```

Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(56) = 112$.

Time = 0.07 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.88

method	result
orering	$\frac{2x(8n^2+24n+13)\sqrt{dx}\,(a+b\,x^n+c\,x^{2n})}{3(3+4n)(3+2n)} - \frac{4x^2(1+2n)\left(\frac{(a+b\,x^n+c\,x^{2n})d}{2\sqrt{dx}}+\sqrt{dx}\left(\frac{b\,x^n n}{x}+\frac{2c\,x^{2n} n}{x}\right)\right)}{(3+4n)(3+2n)} + \frac{8x^3\left(-\frac{(a+b\,x^n+c\,x^{2n})d}{4(dx)^{\frac{3}{2}}}\right.}{}$

input
```
int((d*x)^(1/2)*(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
2/3*x*(8*n^2+24*n+13)/(3+4*n)/(3+2*n)*(d*x)^(1/2)*(a+b*x^n+c*x^(2*n))-4*x^
2*(1+2*n)/(3+4*n)/(3+2*n)*(1/2/(d*x)^(1/2)*(a+b*x^n+c*x^(2*n))*d+(d*x)^(1/
2)*(b*x^n*n/x+2*c*x^(2*n)*n/x))+8/3/(8*n^2+18*n+9)*x^3*(-1/4/(d*x)^(3/2)*(
a+b*x^n+c*x^(2*n))*d^2+1/(d*x)^(1/2)*(b*x^n*n/x+2*c*x^(2*n)*n/x)*d+(d*x)^(
1/2)*(b*x^n*n^2/x^2-b*x^n*n/x^2+4*c*x^(2*n)*n^2/x^2-2*c*x^(2*n)*n/x^2))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx = \frac{2\left(3\,(2\,cn+3\,c)\sqrt{dx}xx^{2\,n}+3\,(4\,bn+3\,b)\sqrt{dx}xx^{n}+(8\,an^2+18\,an+9\,a)\sqrt{dx}x\right)}{3\,(8\,n^2+18\,n+9)}$$

input
```
integrate((d*x)^(1/2)*(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
2/3*(3*(2*c*n + 3*c)*sqrt(d*x)*x*x^(2*n) + 3*(4*b*n + 3*b)*sqrt(d*x)*x*x^n
 + (8*a*n^2 + 18*a*n + 9*a)*sqrt(d*x)*x)/(8*n^2 + 18*n + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(54) = 108$.

Time = 140.26 (sec) , antiderivative size = 284, normalized size of antiderivative = 4.44

$$\int \sqrt{dx}(a+bx^n+cx^{2n})\,dx$$
$$= \begin{cases} a\left(\begin{cases}\frac{2(dx)^{\frac{3}{2}}}{3d} & \text{for } d\neq 0\\ 0 & \text{otherwise}\end{cases}\right)+\frac{b\sqrt{dx}\log(x)}{\sqrt{x}}+c\left(\begin{cases}-\frac{2d^2}{3(dx)^{\frac{3}{2}}} & \text{for } d\neq 0\\ 0 & \text{otherwise}\end{cases}\right) & \text{for } n = -\frac{3}{2} \\ a\left(\begin{cases}\frac{2(dx)^{\frac{3}{2}}}{3d} & \text{for } d\neq 0\\ 0 & \text{otherwise}\end{cases}\right)+\frac{4b\sqrt[4]{x}\sqrt{dx}}{3}+\frac{c\sqrt{dx}\log(x)}{\sqrt{x}} & \text{for } n = -\frac{3}{4} \\ \frac{16an^2x\sqrt{dx}}{24n^2+54n+27}+\frac{36anx\sqrt{dx}}{24n^2+54n+27}+\frac{18ax\sqrt{dx}}{24n^2+54n+27}+\frac{24bnxx^n\sqrt{dx}}{24n^2+54n+27}+\frac{18bxx^n\sqrt{dx}}{24n^2+54n+27}+\frac{12cnxx^{2n}\sqrt{dx}}{24n^2+54n+27}+\frac{18cxx^{2n}\sqrt{dx}}{24n^2+54n+27} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x)**(1/2)*(a+b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise((a*Piecewise((2*(d*x)**(3/2)/(3*d), Ne(d, 0)), (0, True)) + b*sq
rt(d*x)*log(x)/sqrt(x) + c*Piecewise((-2*d**2/(3*(d*x)**(3/2)), Ne(d, 0)),
 (0, True)), Eq(n, -3/2)), (a*Piecewise((2*(d*x)**(3/2)/(3*d), Ne(d, 0)),
(0, True)) + 4*b*x**(1/4)*sqrt(d*x)/3 + c*sqrt(d*x)*log(x)/sqrt(x), Eq(n,
-3/4)), (16*a*n**2*x*sqrt(d*x)/(24*n**2 + 54*n + 27) + 36*a*n*x*sqrt(d*x)/
(24*n**2 + 54*n + 27) + 18*a*x*sqrt(d*x)/(24*n**2 + 54*n + 27) + 24*b*n*x*
x**n*sqrt(d*x)/(24*n**2 + 54*n + 27) + 18*b*x*x**n*sqrt(d*x)/(24*n**2 + 54
*n + 27) + 12*c*n*x*x**(2*n)*sqrt(d*x)/(24*n**2 + 54*n + 27) + 18*c*x*x**(
2*n)*sqrt(d*x)/(24*n**2 + 54*n + 27), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \sqrt{dx}(a+bx^n+cx^{2n})\,dx = \frac{2\,c\sqrt{d}x^{\frac{3}{2}}x^{2\,n}}{4\,n+3}+\frac{2\,b\sqrt{d}x^{\frac{3}{2}}x^{n}}{2\,n+3}+\frac{2\,(dx)^{\frac{3}{2}}\,a}{3\,d}$$

input
```
integrate((d*x)^(1/2)*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
2*c*sqrt(d)*x^(3/2)*x^(2*n)/(4*n + 3) + 2*b*sqrt(d)*x^(3/2)*x^n/(2*n + 3)
+ 2/3*(d*x)^(3/2)*a/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx=\frac{2}{3}\sqrt{dx}ax+\frac{2\sqrt{dx}cxx^{2n}}{4n+3}+\frac{2\sqrt{dx}bxx^n}{2n+3}$$

input
```
integrate((d*x)^(1/2)*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
2/3*sqrt(d*x)*a*x + 2*sqrt(d*x)*c*x*x^(2*n)/(4*n + 3) + 2*sqrt(d*x)*b*x*x^
n/(2*n + 3)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx=\frac{2\,a\,x\,\sqrt{d\,x}}{3}+\frac{2\,c\,x\,x^{2\,n}\,\sqrt{d\,x}}{4\,n+3}+\frac{2\,b\,x\,x^n\,\sqrt{d\,x}}{2\,n+3}$$

input
```
int((d*x)^(1/2)*(a + b*x^n + c*x^(2*n)),x)
```

output
```
(2*a*x*(d*x)^(1/2))/3 + (2*c*x*x^(2*n)*(d*x)^(1/2))/(4*n + 3) + (2*b*x*x^n
*(d*x)^(1/2))/(2*n + 3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \sqrt{dx}\left(a+bx^n+cx^{2n}\right)\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x(6x^{2n}cn+9x^{2n}c+12x^nbn+9x^nb+8a\,n^2+18an+9a)}{24n^2+54n+27}$$

input
```
int((d*x)^(1/2)*(a+b*x^n+c*x^(2*n)),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x*(6*x**(2*n)*c*n + 9*x**(2*n)*c + 12*x**n*b*n + 9*x**n
*b + 8*a*n**2 + 18*a*n + 9*a))/(3*(8*n**2 + 18*n + 9))
```

3.199 $\int \frac{a+bx^n+cx^{2n}}{\sqrt{dx}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{a+bx^n+cx^{2n}}{\sqrt{dx}} \, dx = \frac{2a\sqrt{dx}}{d} + \frac{2bx^n\sqrt{dx}}{d(1+2n)} + \frac{2cx^{2n}\sqrt{dx}}{d(1+4n)}$$

output
```
2*a*(d*x)^(1/2)/d+2*b*x^n*(d*x)^(1/2)/d/(1+2*n)+2*c*x^(2*n)*(d*x)^(1/2)/d/
(1+4*n)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^n+cx^{2n}}{\sqrt{dx}} \, dx = \frac{2x(a(1+6n+8n^2)+x^n(b+4bn+c(1+2n)x^n))}{(1+6n+8n^2)\sqrt{dx}}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))/Sqrt[d*x],x]
```

output
```
(2*x*(a*(1 + 6*n + 8*n^2) + x^n*(b + 4*b*n + c*(1 + 2*n)*x^n)))/((1 + 6*n
+ 8*n^2)*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n+cx^{2n}}{\sqrt{dx}}\,dx$$

$$\Big\downarrow \text{1691}$$

$$\int \left(\frac{a}{\sqrt{dx}}+\frac{bx^n}{\sqrt{dx}}+\frac{cx^{2n}}{\sqrt{dx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a\sqrt{dx}}{d}+\frac{2bx^{n+1}}{(2n+1)\sqrt{dx}}+\frac{2cx^{2n+1}}{(4n+1)\sqrt{dx}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))/Sqrt[d*x],x]
```

output
```
(2*b*x^(1 + n))/((1 + 2*n)*Sqrt[d*x]) + (2*c*x^(1 + 2*n))/((1 + 4*n)*Sqrt[
d*x]) + (2*a*Sqrt[d*x])/d
```

Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{2\sqrt{dx}\,a+\frac{2b\sqrt{dx}\,\mathrm{e}^{n\ln(x)}}{1+2n}+\frac{2c\sqrt{dx}\,\mathrm{e}^{2n\ln(x)}}{1+4n}}{d}$
default	$\frac{2\sqrt{dx}\,a+\frac{2b\sqrt{dx}\,\mathrm{e}^{n\ln(x)}}{1+2n}+\frac{2c\sqrt{dx}\,\mathrm{e}^{2n\ln(x)}}{1+4n}}{d}$
orering	$\frac{2x(8n^2+1)(a+b\,x^n+c\,x^{2n})}{(1+4n)(1+2n)\sqrt{dx}}-\frac{12x^2(-1+2n)\left(\frac{\frac{b\,x^n n}{x}+\frac{2c\,x^{2n}n}{x}}{\sqrt{dx}}-\frac{(a+b\,x^n+c\,x^{2n})d}{2(dx)^{\frac{3}{2}}}\right)}{(1+4n)(1+2n)}+\frac{8x^3\left(\frac{\frac{b\,x^n n^2}{x^2}-\frac{b\,x^n n}{x^2}+\frac{4c\,x^{2n}}{x^2}}{\sqrt{dx}}\right.}{}$

input
```
int((a+b*x^n+c*x^(2*n))/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*((d*x)^(1/2)*a+b/(1+2*n)*(d*x)^(1/2)*exp(n*ln(x))+c/(1+4*n)*(d*x)^(1/2
)*exp(2*n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^n+cx^{2n}}{\sqrt{dx}}\,dx = \frac{2\left((2\,cn+c)\sqrt{dx}x^{2\,n}+(4\,bn+b)\sqrt{dx}x^n+(8\,an^2+6\,an+a)\sqrt{dx}\right)}{8\,dn^2+6\,dn+d}$$

input
```
integrate((a+b*x^n+c*x^(2*n))/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2*((2*c*n + c)*sqrt(d*x)*x^(2*n) + (4*b*n + b)*sqrt(d*x)*x^n + (8*a*n^2 +
6*a*n + a)*sqrt(d*x))/(8*d*n^2 + 6*d*n + d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(53) = 106$.

Time = 17.10 (sec) , antiderivative size = 377, normalized size of antiderivative = 6.08

$$\int \frac{a + bx^n + cx^{2n}}{\sqrt{dx}}\, dx$$
$$= \begin{cases} -2a\left(\begin{cases} \tilde{\infty}x & \text{for } d = 0 \\ -\frac{\sqrt{dx}}{d} & \text{otherwise} \end{cases}\right) - \frac{2b\sqrt{x}\log\left(\frac{1}{\sqrt{x}}\right)}{\sqrt{dx}} - \frac{2c}{\sqrt{dx}} \\ -4a\left(\begin{cases} \tilde{\infty}x & \text{for } d = 0 \\ -\frac{\sqrt{dx}}{2d} & \text{otherwise} \end{cases}\right) + \frac{4bx^{\frac{3}{4}}}{\sqrt{dx}} - \frac{4c\sqrt{x}\log\left(\frac{1}{\sqrt[4]{x}}\right)}{\sqrt{dx}} \\ \frac{16an^2x}{8n^2\sqrt{dx}+6n\sqrt{dx}+\sqrt{dx}} + \frac{12anx}{8n^2\sqrt{dx}+6n\sqrt{dx}+\sqrt{dx}} + \frac{2ax}{8n^2\sqrt{dx}+6n\sqrt{dx}+\sqrt{dx}} + \frac{8bnxx^n}{8n^2\sqrt{dx}+6n\sqrt{dx}+\sqrt{dx}} + \frac{2bxx^n}{8n^2\sqrt{dx}+6n\sqrt{dx}+\sqrt{dx}} + \end{cases}$$

input
```
integrate((a+b*x**n+c*x**(2*n))/(d*x)**(1/2),x)
```

output
```
Piecewise((-2*a*Piecewise((zoo*x, Eq(d, 0)), (-sqrt(d*x)/d, True)) - 2*b*s
qrt(x)*log(1/sqrt(x))/sqrt(d*x) - 2*c/sqrt(d*x), Eq(n, -1/2)), (-4*a*Piece
wise((zoo*x, Eq(d, 0)), (-sqrt(d*x)/(2*d), True)) + 4*b*x**(3/4)/sqrt(d*x)
 - 4*c*sqrt(x)*log(x**(-1/4))/sqrt(d*x), Eq(n, -1/4)), (16*a*n**2*x/(8*n**
2*sqrt(d*x) + 6*n*sqrt(d*x) + sqrt(d*x)) + 12*a*n*x/(8*n**2*sqrt(d*x) + 6*
n*sqrt(d*x) + sqrt(d*x)) + 2*a*x/(8*n**2*sqrt(d*x) + 6*n*sqrt(d*x) + sqrt(
d*x)) + 8*b*n*x*x**n/(8*n**2*sqrt(d*x) + 6*n*sqrt(d*x) + sqrt(d*x)) + 2*b*
x*x**n/(8*n**2*sqrt(d*x) + 6*n*sqrt(d*x) + sqrt(d*x)) + 4*c*n*x*x**(2*n)/(
8*n**2*sqrt(d*x) + 6*n*sqrt(d*x) + sqrt(d*x)) + 2*c*x*x**(2*n)/(8*n**2*sqr
t(d*x) + 6*n*sqrt(d*x) + sqrt(d*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{a + bx^n + cx^{2n}}{\sqrt{dx}}\, dx = \frac{2\,c\sqrt{x}x^{2\,n}}{\sqrt{d}(4\,n+1)} + \frac{2\,b\sqrt{x}x^n}{\sqrt{d}(2\,n+1)} + \frac{2\,\sqrt{dx}a}{d}$$

input
```
integrate((a+b*x^n+c*x^(2*n))/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2*c*sqrt(x)*x^(2*n)/(sqrt(d)*(4*n + 1)) + 2*b*sqrt(x)*x^n/(sqrt(d)*(2*n +
1)) + 2*sqrt(d*x)*a/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \frac{a + bx^n + cx^{2n}}{\sqrt{dx}}\,dx = \frac{2\left(\sqrt{dx}a + \frac{\sqrt{dx}cx^{2n}}{4n+1} + \frac{\sqrt{dx}bx^n}{2n+1}\right)}{d}$$

input
```
integrate((a+b*x^n+c*x^(2*n))/(d*x)^(1/2),x, algorithm="giac")
```

output
```
2*(sqrt(d*x)*a + sqrt(d*x)*c*x^(2*n)/(4*n + 1) + sqrt(d*x)*b*x^n/(2*n + 1)
)/d
```

Mupad [B] (verification not implemented)

Time = 10.90 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{a + bx^n + cx^{2n}}{\sqrt{dx}}\,dx = \frac{2\,a\,\sqrt{d\,x}}{d} + \frac{2\,c\,x\,x^{2\,n}}{(4\,n+1)\,\sqrt{d\,x}} + \frac{2\,b\,x\,x^n}{(2\,n+1)\,\sqrt{d\,x}}$$

input
```
int((a + b*x^n + c*x^(2*n))/(d*x)^(1/2),x)
```

output
```
(2*a*(d*x)^(1/2))/d + (2*c*x*x^(2*n))/((4*n + 1)*(d*x)^(1/2)) + (2*b*x*x^n
)/((2*n + 1)*(d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^n+cx^{2n}}{\sqrt{dx}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,(2x^{2n}cn+x^{2n}c+4x^nbn+x^nb+8a\,n^2+6an+a)}{d\,(8n^2+6n+1)}$$

input
```
int((a+b*x^n+c*x^(2*n))/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*(2*x**(2*n)*c*n + x**(2*n)*c + 4*x**n*b*n + x**n*b + 8*
a*n**2 + 6*a*n + a))/(d*(8*n**2 + 6*n + 1))
```

3.200 $\int \frac{a+bx^n+cx^{2n}}{(dx)^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{3/2}} \, dx = -\frac{2a}{d\sqrt{dx}}-\frac{2bx^n}{d(1-2n)\sqrt{dx}}-\frac{2cx^{2n}}{d(1-4n)\sqrt{dx}}$$

output
```
-2*a/d/(d*x)^(1/2)-2*b*x^n/d/(1-2*n)/(d*x)^(1/2)-2*c*x^(2*n)/d/(1-4*n)/(d*
x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{3/2}} \, dx = \frac{2x\left(-a+x^n\left(\frac{b}{-1+2n}+\frac{cx^n}{-1+4n}\right)\right)}{(dx)^{3/2}}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))/(d*x)^(3/2),x]
```

output
```
(2*x*(-a + x^n*(b/(-1 + 2*n) + (c*x^n)/(-1 + 4*n))))/(d*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1691}$$

$$\int \left(\frac{a}{(dx)^{3/2}}+\frac{bx^n}{(dx)^{3/2}}+\frac{cx^{2n}}{(dx)^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a}{d\sqrt{dx}}-\frac{2bx^{n+1}}{(1-2n)(dx)^{3/2}}-\frac{2cx^{2n+1}}{(1-4n)(dx)^{3/2}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))/(d*x)^(3/2),x]
```

output
```
(-2*b*x^(1 + n))/((1 - 2*n)*(d*x)^(3/2)) - (2*c*x^(1 + 2*n))/((1 - 4*n)*(d
*x)^(3/2)) - (2*a)/(d*Sqrt[d*x])
```

Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(56) = 112$.

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 4.02

method	result
orering	$-\frac{2x(8n^2-24n+13)(a+b\,x^n+c\,x^{2n})}{(-1+4n)(-1+2n)(dx)^{\frac{3}{2}}}+\frac{12x^2(-3+2n)\left(\frac{\frac{b\,x^n n}{x}+\frac{2c\,x^{2n}n}{x}}{(dx)^{\frac{3}{2}}}-\frac{3\left(a+b\,x^n+c\,x^{2n}\right)d}{2(dx)^{\frac{5}{2}}}\right)}{(-1+4n)(-1+2n)}-\frac{8x^3\left(\frac{\frac{b\,x^n n^2}{x^2}-\frac{b\,x^n n}{x^2}+\frac{4c\,x^{2n}n^2}{x^2}}{(dx)^{\frac{3}{2}}}\right.}{}$

input
```
int((a+b*x^n+c*x^(2*n))/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x*(8*n^2-24*n+13)/(-1+4*n)/(-1+2*n)*(a+b*x^n+c*x^(2*n))/(d*x)^(3/2)+12*
x^2*(-3+2*n)/(-1+4*n)/(-1+2*n)*((b*x^n*n/x+2*c*x^(2*n)*n/x)/(d*x)^(3/2)-3/
2*(a+b*x^n+c*x^(2*n))/(d*x)^(5/2)*d)-8/(8*n^2-6*n+1)*x^3*((b*x^n*n^2/x^2-b
*x^n*n/x^2+4*c*x^(2*n)*n^2/x^2-2*c*x^(2*n)*n/x^2)/(d*x)^(3/2)-3*(b*x^n*n/x
+2*c*x^(2*n)*n/x)/(d*x)^(5/2)*d+15/4*(a+b*x^n+c*x^(2*n))/(d*x)^(7/2)*d^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{3/2}}\,dx=\frac{2\left((2\,cn-c)\sqrt{dx}x^{2\,n}+(4\,bn-b)\sqrt{dx}x^n-(8\,an^2-6\,an+a)\sqrt{dx}\right)}{(8\,d^2n^2-6\,d^2n+d^2)x}$$

input
```
integrate((a+b*x^n+c*x^(2*n))/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2*((2*c*n - c)*sqrt(d*x)*x^(2*n) + (4*b*n - b)*sqrt(d*x)*x^n - (8*a*n^2 -
6*a*n + a)*sqrt(d*x))/((8*d^2*n^2 - 6*d^2*n + d^2)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(54) = 108$.

Time = 12.21 (sec) , antiderivative size = 376, normalized size of antiderivative = 6.06

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{3/2}}\,dx = \begin{cases} 4a\left(\begin{cases} \tilde{\infty}x & \text{for } d=0 \\ -\frac{1}{2d\sqrt{dx}} & \text{otherwise} \end{cases}\right) - \frac{4bx^{\frac{5}{4}}}{(dx)^{\frac{3}{2}}} + \frac{4cx^{\frac{3}{2}}\log\left(\sqrt[4]{x}\right)}{(dx)^{\frac{3}{2}}} \\ 2a\left(\begin{cases} \tilde{\infty}x & \text{for } d=0 \\ -\frac{1}{d\sqrt{dx}} & \text{otherwise} \end{cases}\right) + \frac{2bx^{\frac{3}{2}}\log\left(\sqrt{x}\right)}{(dx)^{\frac{3}{2}}} + \frac{2cx^2}{(dx)^{\frac{3}{2}}} \\ -\frac{16an^2x}{8n^2(dx)^{\frac{3}{2}}-6n(dx)^{\frac{3}{2}}+(dx)^{\frac{3}{2}}} + \frac{12anx}{8n^2(dx)^{\frac{3}{2}}-6n(dx)^{\frac{3}{2}}+(dx)^{\frac{3}{2}}} - \frac{2ax}{8n^2(dx)^{\frac{3}{2}}-6n(dx)^{\frac{3}{2}}+(dx)^{\frac{3}{2}}} + \frac{}{8n^2(dx)} \end{cases}$$

input
```
integrate((a+b*x**n+c*x**(2*n))/(d*x)**(3/2),x)
```

output
```
Piecewise((4*a*Piecewise((zoo*x, Eq(d, 0)), (-1/(2*d*sqrt(d*x)), True)) -
4*b*x**(5/4)/(d*x)**(3/2) + 4*c*x**(3/2)*log(x**(1/4))/(d*x)**(3/2), Eq(n,
 1/4)), (2*a*Piecewise((zoo*x, Eq(d, 0)), (-1/(d*sqrt(d*x)), True)) + 2*b*
x**(3/2)*log(sqrt(x))/(d*x)**(3/2) + 2*c*x**2/(d*x)**(3/2), Eq(n, 1/2)), (
-16*a*n**2*x/(8*n**2*(d*x)**(3/2) - 6*n*(d*x)**(3/2) + (d*x)**(3/2)) + 12*
a*n*x/(8*n**2*(d*x)**(3/2) - 6*n*(d*x)**(3/2) + (d*x)**(3/2)) - 2*a*x/(8*n
**2*(d*x)**(3/2) - 6*n*(d*x)**(3/2) + (d*x)**(3/2)) + 8*b*n*x*x**n/(8*n**2
*(d*x)**(3/2) - 6*n*(d*x)**(3/2) + (d*x)**(3/2)) - 2*b*x*x**n/(8*n**2*(d*x
)**(3/2) - 6*n*(d*x)**(3/2) + (d*x)**(3/2)) + 4*c*n*x*x**(2*n)/(8*n**2*(d*
x)**(3/2) - 6*n*(d*x)**(3/2) + (d*x)**(3/2)) - 2*c*x*x**(2*n)/(8*n**2*(d*x
)**(3/2) - 6*n*(d*x)**(3/2) + (d*x)**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{3/2}}\,dx = -\frac{2\,a}{\sqrt{dx}d} + \frac{2\,cx^{2\,n}}{d^{\frac{3}{2}}(4\,n-1)\sqrt{x}} + \frac{2\,bx^n}{d^{\frac{3}{2}}(2\,n-1)\sqrt{x}}$$

input
```
integrate((a+b*x^n+c*x^(2*n))/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-2*a/(sqrt(d*x)*d) + 2*c*x^(2*n)/(d^(3/2)*(4*n - 1)*sqrt(x)) + 2*b*x^n/(d^
(3/2)*(2*n - 1)*sqrt(x))
```

Giac [F]

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{3/2}}\,dx=\int \frac{cx^{2\,n}+bx^n+a}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)/(d*x)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.91 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{3/2}}\,dx=\frac{2\,c\,x^{2\,n}}{d\,\left(4\,n-1\right)\,\sqrt{d\,x}}-\frac{2\,a\,\sqrt{d\,x}}{d^2\,x}+\frac{2\,b\,x^n}{d\,\left(2\,n-1\right)\,\sqrt{d\,x}}$$

input
```
int((a + b*x^n + c*x^(2*n))/(d*x)^(3/2),x)
```

output
```
(2*c*x^(2*n))/(d*(4*n - 1)*(d*x)^(1/2)) - (2*a*(d*x)^(1/2))/(d^2*x) + (2*b
*x^n)/(d*(2*n - 1)*(d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{3/2}}\,dx=\frac{2\sqrt{d}\,(2x^{2n}cn-x^{2n}c+4x^n bn-x^n b-8a\,n^2+6an-a)}{\sqrt{x}\,d^2\,(8n^2-6n+1)}$$

input
```
int((a+b*x^n+c*x^(2*n))/(d*x)^(3/2),x)
```

output
```
(2*sqrt(d)*(2*x**(2*n)*c*n - x**(2*n)*c + 4*x**n*b*n - x**n*b - 8*a*n**2 +
 6*a*n - a))/(sqrt(x)*d**2*(8*n**2 - 6*n + 1))
```

3.201 $\int \frac{a+bx^n+cx^{2n}}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{5/2}}\,dx = -\frac{2a}{3d(dx)^{3/2}}-\frac{2bx^n}{d(3-2n)(dx)^{3/2}}-\frac{2cx^{2n}}{d(3-4n)(dx)^{3/2}}$$

output
```
-2/3*a/d/(d*x)^(3/2)-2*b*x^n/d/(3-2*n)/(d*x)^(3/2)-2*c*x^(2*n)/d/(3-4*n)/(
d*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.67

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{5/2}}\,dx = \frac{2x\left(-a+3x^n\left(\frac{b}{-3+2n}+\frac{cx^n}{-3+4n}\right)\right)}{3(dx)^{5/2}}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))/(d*x)^(5/2),x]
```

output
```
(2*x*(-a + 3*x^n*(b/(-3 + 2*n) + (c*x^n)/(-3 + 4*n))))/(3*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{5/2}}dx$$
$$\Big\downarrow \text{1691}$$
$$\int \left(\frac{a}{(dx)^{5/2}}+\frac{bx^n}{(dx)^{5/2}}+\frac{cx^{2n}}{(dx)^{5/2}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2a}{3d(dx)^{3/2}}-\frac{2bx^{n+1}}{(3-2n)(dx)^{5/2}}-\frac{2cx^{2n+1}}{(3-4n)(dx)^{5/2}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))/(d*x)^(5/2),x]
```

output
```
(-2*b*x^(1 + n))/((3 - 2*n)*(d*x)^(5/2)) - (2*c*x^(1 + 2*n))/((3 - 4*n)*(d
*x)^(5/2)) - (2*a)/(3*d*(d*x)^(3/2))
```

Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(56) = 112$.

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.89

method	result
orering	$-\frac{2x(8n^2-48n+49)(a+b\,x^n+c\,x^{2n})}{3(-3+4n)(-3+2n)(dx)^{\frac{5}{2}}}+\frac{4x^2(-5+2n)\left(\frac{\frac{b\,x^n n}{x}+\frac{2c\,x^{2n} n}{x}}{(dx)^{\frac{5}{2}}}-\frac{5\left(a+b\,x^n+c\,x^{2n}\right)d}{2(dx)^{\frac{7}{2}}}\right)}{(-3+4n)(-3+2n)}-\frac{8x^3\left(\frac{\frac{b\,x^n n^2}{x^2}-\frac{b\,x^n n}{x^2}+\frac{4c\,x^{2n} n^2}{x^2}}{(dx)^{\frac{5}{2}}}\right.}{}$

input
```
int((a+b*x^n+c*x^(2*n))/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(8*n^2-48*n+49)/(-3+4*n)/(-3+2*n)*(a+b*x^n+c*x^(2*n))/(d*x)^(5/2)+4
*x^2*(-5+2*n)/(-3+4*n)/(-3+2*n)*((b*x^n*n/x+2*c*x^(2*n)*n/x)/(d*x)^(5/2)-5
/2*(a+b*x^n+c*x^(2*n))/(d*x)^(7/2)*d)-8/3/(8*n^2-18*n+9)*x^3*((b*x^n*n^2/x
^2-b*x^n*n/x^2+4*c*x^(2*n)*n^2/x^2-2*c*x^(2*n)*n/x^2)/(d*x)^(5/2)-5*(b*x^n
*n/x+2*c*x^(2*n)*n/x)/(d*x)^(7/2)*d+35/4*(a+b*x^n+c*x^(2*n))/(d*x)^(9/2)*d
^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.36

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{5/2}}\,dx = \frac{2\left(3\,(2\,cn-3\,c)\sqrt{dx}x^{2\,n}+3\,(4\,bn-3\,b)\sqrt{dx}x^n-(8\,an^2-18\,an+9\,a)\sqrt{dx}\right)}{3\,(8\,d^3n^2-18\,d^3n+9\,d^3)x^2}$$

input
```
integrate((a+b*x^n+c*x^(2*n))/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*(2*c*n - 3*c)*sqrt(d*x)*x^(2*n) + 3*(4*b*n - 3*b)*sqrt(d*x)*x^n - (
8*a*n^2 - 18*a*n + 9*a)*sqrt(d*x))/((8*d^3*n^2 - 18*d^3*n + 9*d^3)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(56) = 112$.

Time = 161.92 (sec) , antiderivative size = 384, normalized size of antiderivative = 6.00

$$\int \frac{a + bx^n + cx^{2n}}{(dx)^{5/2}}\,dx = \begin{cases} a\left(\begin{cases} -\frac{2}{3d(dx)^{\frac{3}{2}}} & \text{for } d \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}\right) - \frac{4bx^{\frac{7}{4}}}{3(dx)^{\frac{5}{2}}} + \frac{cx^{\frac{5}{2}}\log(x)}{(dx)^{\frac{5}{2}}} \\ a\left(\begin{cases} -\frac{2}{3d(dx)^{\frac{3}{2}}} & \text{for } d \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}\right) + \frac{bx^{\frac{5}{2}}\log(x)}{(dx)^{\frac{5}{2}}} + c\left(\begin{cases} \frac{2(dx)^{\frac{3}{2}}}{3d^4} & \text{for } d \neq 0 \\ \tilde{\infty}x^4 & \text{otherwise} \end{cases}\right) \\ -\frac{16an^2x}{24n^2(dx)^{\frac{5}{2}} - 54n(dx)^{\frac{5}{2}} + 27(dx)^{\frac{5}{2}}} + \frac{36anx}{24n^2(dx)^{\frac{5}{2}} - 54n(dx)^{\frac{5}{2}} + 27(dx)^{\frac{5}{2}}} - \frac{18ax}{24n^2(dx)^{\frac{5}{2}} - 54n(dx)^{\frac{5}{2}} + 27(dx)} \end{cases}$$

input
```
integrate((a+b*x**n+c*x**(2*n))/(d*x)**(5/2),x)
```

output
```
Piecewise((a*Piecewise((-2/(3*d*(d*x)**(3/2)), Ne(d, 0)), (zoo*x, True)) -
 4*b*x**(7/4)/(3*(d*x)**(5/2)) + c*x**(5/2)*log(x)/(d*x)**(5/2), Eq(n, 3/4
)), (a*Piecewise((-2/(3*d*(d*x)**(3/2)), Ne(d, 0)), (zoo*x, True)) + b*x**
(5/2)*log(x)/(d*x)**(5/2) + c*Piecewise((2*(d*x)**(3/2)/(3*d**4), Ne(d, 0)
), (zoo*x**4, True)), Eq(n, 3/2)), (-16*a*n**2*x/(24*n**2*(d*x)**(5/2) - 5
4*n*(d*x)**(5/2) + 27*(d*x)**(5/2)) + 36*a*n*x/(24*n**2*(d*x)**(5/2) - 54*
n*(d*x)**(5/2) + 27*(d*x)**(5/2)) - 18*a*x/(24*n**2*(d*x)**(5/2) - 54*n*(d
*x)**(5/2) + 27*(d*x)**(5/2)) + 24*b*n*x*x**n/(24*n**2*(d*x)**(5/2) - 54*n
*(d*x)**(5/2) + 27*(d*x)**(5/2)) - 18*b*x*x**n/(24*n**2*(d*x)**(5/2) - 54*
n*(d*x)**(5/2) + 27*(d*x)**(5/2)) + 12*c*n*x*x**(2*n)/(24*n**2*(d*x)**(5/2
) - 54*n*(d*x)**(5/2) + 27*(d*x)**(5/2)) - 18*c*x*x**(2*n)/(24*n**2*(d*x)*
*(5/2) - 54*n*(d*x)**(5/2) + 27*(d*x)**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{a + bx^n + cx^{2n}}{(dx)^{5/2}}\,dx = -\frac{2\,a}{3\,(dx)^{\frac{3}{2}}\,d} + \frac{2\,cx^{2\,n}}{d^{\frac{5}{2}}(4\,n-3)x^{\frac{3}{2}}} + \frac{2\,bx^n}{d^{\frac{5}{2}}(2\,n-3)x^{\frac{3}{2}}}$$

input
```
integrate((a+b*x^n+c*x^(2*n))/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*a/((d*x)^(3/2)*d) + 2*c*x^(2*n)/(d^(5/2)*(4*n - 3)*x^(3/2)) + 2*b*x^n
/(d^(5/2)*(2*n - 3)*x^(3/2))
```

Giac [F]

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{5/2}}\,dx=\int \frac{cx^{2\,n}+bx^n+a}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)/(d*x)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.84 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{5/2}}\,dx=\frac{2\,c\,x^{2\,n}}{d^2\,x\,\left(4\,n-3\right)\,\sqrt{d\,x}}-\frac{2\,a\,\sqrt{d\,x}}{3\,d^3\,x^2}+\frac{2\,b\,x^n}{d^2\,x\,\left(2\,n-3\right)\,\sqrt{d\,x}}$$

input
```
int((a + b*x^n + c*x^(2*n))/(d*x)^(5/2),x)
```

output
```
(2*c*x^(2*n))/(d^2*x*(4*n - 3)*(d*x)^(1/2)) - (2*a*(d*x)^(1/2))/(3*d^3*x^2
) + (2*b*x^n)/(d^2*x*(2*n - 3)*(d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{a+bx^n+cx^{2n}}{(dx)^{5/2}}\,dx=\frac{2\sqrt{d}\,(6x^{2n}cn-9x^{2n}c+12x^nbn-9x^nb-8a\,n^2+18an-9a)}{3\sqrt{x}\,d^3x\,(8n^2-18n+9)}$$

input
```
int((a+b*x^n+c*x^(2*n))/(d*x)^(5/2),x)
```

output
```
(2*sqrt(d)*(6*x**(2*n)*c*n - 9*x**(2*n)*c + 12*x**n*b*n - 9*x**n*b - 8*a*n
**2 + 18*a*n - 9*a))/(3*sqrt(x)*d**3*x*(8*n**2 - 18*n + 9))
```

3.202 $\int \frac{(dx)^{3/2}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 162

$$\int \frac{(dx)^{3/2}}{a+bx^n+cx^{2n}}\,dx=-\frac{4c(dx)^{5/2}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2n},1+\frac{5}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{5\left(b^2-4ac-b\sqrt{b^2-4ac}\right)d}-\frac{4c(dx)^{5/2}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2n},1+\frac{5}{2n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{5\left(b^2-4ac+b\sqrt{b^2-4ac}\right)d}$$

output
```
-4/5*c*(d*x)^(5/2)*hypergeom([1, 5/2/n],[1+5/2/n],-2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))/d-4/5*c*(d*x)^(5/2)*hypergeom([1
, 5/2/n],[1+5/2/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(b*(-4*a*c+b^2)^(1/2)-
4*a*c+b^2)/d
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.78

$$\int \frac{(dx)^{3/2}}{a+bx^n+cx^{2n}}\,dx =$$

$$-\frac{4}{5}cx(dx)^{3/2}\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{5}{2}/n}\text{Hypergeometric2F1}\left(-\frac{5}{2n},-\frac{5}{2n},1-\frac{5}{2n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{1-}{}\right.$$

input
```
Integrate[(d*x)^(3/2)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-4*c*x*(d*x)^(3/2)*((1 - Hypergeometric2F1[-5/(2*n), -5/(2*n), 1 - 5/(2*n
), (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(
-b + Sqrt[b^2 - 4*a*c])/c + x^n))^(5/(2*n)))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4
*a*c]) + (1 - Hypergeometric2F1[-5/(2*n), -5/(2*n), 1 - 5/(2*n), (b + Sqrt
[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(2^(5/(2*n))*((c*x^n)/(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(5/(2*n))))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[
b^2 - 4*a*c]))))/5
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{ 1719}$$

$$\frac{2c\int \frac{(dx)^{3/2}}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{(dx)^{3/2}}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{888}$$

$$\frac{4c(dx)^{5/2}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2n},1+\frac{5}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{5d\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{4c(dx)^{5/2}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2n},1+\frac{5}{2n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(d*x)^(3/2)/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(4*c*(d*x)^(5/2)*Hypergeometric2F1[1, 5/(2*n), 1 + 5/(2*n), (-2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])])/(5*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d) - (
4*c*(d*x)^(5/2)*Hypergeometric2F1[1, 5/(2*n), 1 + 5/(2*n), (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/(5*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{(dx)^{\frac{3}{2}}}{a+b\,x^{n}+c\,x^{2n}}dx$$

input
```
int((d*x)^(3/2)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int((d*x)^(3/2)/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{(dx)^{3/2}}{a+bx^n+cx^{2n}}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*d*x/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{3/2}}{a+bx^n+cx^{2n}}\,dx=\text{Timed out}$$

input
```
integrate((d*x)**(3/2)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(dx)^{3/2}}{a+bx^n+cx^{2n}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-2*d^(3/2)*x^(5/2)/(c*(4*n - 5)*x^(2*n) + b*(4*n - 5)*x^n + a*(4*n - 5)) +
 integrate(2*(b*d^(3/2)*n*x^n + 2*a*d^(3/2)*n)*x^(3/2)/(c^2*(4*n - 5)*x^(4
*n) + 2*b*c*(4*n - 5)*x^(3*n) + 2*a*b*(4*n - 5)*x^n + a^2*(4*n - 5) + (b^2
*(4*n - 5) + 2*a*c*(4*n - 5))*x^(2*n)), x)
```

Giac [F]

$$\int \frac{(dx)^{3/2}}{a+bx^n+cx^{2n}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{3/2}}{a+bx^n+cx^{2n}}\,dx = \int \frac{(d\,x)^{3/2}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((d*x)^(3/2)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int((d*x)^(3/2)/(a + b*x^n + c*x^(2*n)), x)
```

Reduce [F]

$$\int \frac{(dx)^{3/2}}{a+bx^n+cx^{2n}}\,dx = \sqrt{d}\left(\int \frac{\sqrt{x}\,x}{x^{2n}c+x^nb+a}dx\right)d$$

input
```
int((d*x)^(3/2)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
sqrt(d)*int((sqrt(x)*x)/(x**(2*n)*c + x**n*b + a),x)*d
```

3.203 $\int \frac{\sqrt{dx}}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 162

$$\int \frac{\sqrt{dx}}{a+bx^n+cx^{2n}}\,dx = -\frac{4c(dx)^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1+\frac{3}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)d} - \frac{4c(dx)^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1+\frac{3}{2n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)d}$$

output
```
-4/3*c*(d*x)^(3/2)*hypergeom([1, 3/2/n],[1+3/2/n],-2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))/d-4/3*c*(d*x)^(3/2)*hypergeom([1
, 3/2/n],[1+3/2/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(b*(-4*a*c+b^2)^(1/2)-
4*a*c+b^2)/d
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{dx}}{a+bx^n+cx^{2n}}\,dx = -\frac{4}{3}cx\sqrt{dx}\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{3}{2}/n}\text{Hypergeometric2F1}\left(-\frac{3}{2n},-\frac{3}{2n},1-\frac{3}{2n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} + \frac{1-2^{-\frac{3}{2}/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-\frac{3}{2}/n}\text{Hypergeometric2F1}\left(-\frac{3}{2n},-\frac{3}{2n},1-\frac{3}{2n},\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

input
```
Integrate[Sqrt[d*x]/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(-4*c*x*Sqrt[d*x]*((1 - Hypergeometric2F1[-3/(2*n), -3/(2*n), 1 - 3/(2*n),
 (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b
 + Sqrt[b^2 - 4*a*c])/c + x^n))^(3/(2*n)))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a
*c]) + (1 - Hypergeometric2F1[-3/(2*n), -3/(2*n), 1 - 3/(2*n), (b + Sqrt[b
^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(2^(3/(2*n))*((c*x^n)/(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(3/(2*n))))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^
2 - 4*a*c]))))/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1719}$$

$$\frac{2c\int \frac{\sqrt{dx}}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{\sqrt{dx}}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{888}$$

$$\frac{4c(dx)^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1+\frac{3}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3d\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{4c(dx)^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1+\frac{3}{2n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[Sqrt[d*x]/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(4*c*(d*x)^(3/2)*Hypergeometric2F1[1, 3/(2*n), 1 + 3/(2*n), (-2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d) - (
4*c*(d*x)^(3/2)*Hypergeometric2F1[1, 3/(2*n), 1 + 3/(2*n), (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/(3*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{\sqrt{dx}}{a+b\,x^{n}+c\,x^{2n}}dx$$

input
```
int((d*x)^(1/2)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int((d*x)^(1/2)/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{\sqrt{dx}}{a+bx^n+cx^{2n}}\,dx = \int \frac{\sqrt{dx}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{a+bx^n+cx^{2n}}\,dx = \int \frac{\sqrt{dx}}{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate((d*x)**(1/2)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(sqrt(d*x)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{\sqrt{dx}}{a+bx^n+cx^{2n}}\,dx=\int \frac{\sqrt{dx}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-2*sqrt(d)*x^(3/2)/(c*(4*n - 3)*x^(2*n) + b*(4*n - 3)*x^n + a*(4*n - 3)) +
 integrate(2*(b*sqrt(d)*n*x^n + 2*a*sqrt(d)*n)*sqrt(x)/(c^2*(4*n - 3)*x^(4
*n) + 2*b*c*(4*n - 3)*x^(3*n) + 2*a*b*(4*n - 3)*x^n + a^2*(4*n - 3) + (b^2
*(4*n - 3) + 2*a*c*(4*n - 3))*x^(2*n)), x)
```

Giac [F]

$$\int \frac{\sqrt{dx}}{a+bx^n+cx^{2n}}\,dx=\int \frac{\sqrt{dx}}{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx}}{a+bx^n+cx^{2n}}\,dx=\int \frac{\sqrt{d\,x}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((d*x)^(1/2)/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int((d*x)^(1/2)/(a + b*x^n + c*x^(2*n)), x)
```

Reduce [F]

$$\int \frac{\sqrt{dx}}{a+bx^n+cx^{2n}}\,dx=\sqrt{d}\left(\int \frac{\sqrt{x}}{x^{2n}c+x^nb+a}dx\right)$$

input
```
int((d*x)^(1/2)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
sqrt(d)*int(sqrt(x)/(x**(2*n)*c + x**n*b + a),x)
```

3.204 $\int \frac{1}{\sqrt{dx}(a+bx^n+cx^{2n})} dx$

Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})}\,dx = -\frac{4c\sqrt{dx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)d} - \frac{4c\sqrt{dx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)d}$$

output
```
-4*c*(d*x)^(1/2)*hypergeom([1, 1/2/n],[1+1/2/n],-2*c*x^n/(b-(-4*a*c+b^2)^(
1/2)))/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))/d-4*c*(d*x)^(1/2)*hypergeom([1, 1/
2/n],[1+1/2/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(b*(-4*a*c+b^2)^(1/2)-4*a*
c+b^2)/d
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})}\,dx = -\frac{4cx\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{2}/n}\text{Hypergeometric2F1}\left(-\frac{1}{2n},-\frac{1}{2n},1-\frac{1}{2n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{1-2^{-\frac{1}{2}/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-}}{}\right)}{\sqrt{dx}}$$

input
```
Integrate[1/(Sqrt[d*x]*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(-4*c*x*((1 - Hypergeometric2F1[-1/2*1/n, -1/2*1/n, 1 - 1/(2*n), (b - Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^
2 - 4*a*c])/c + x^n))^(1/(2*n)))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1
- Hypergeometric2F1[-1/2*1/n, -1/2*1/n, 1 - 1/(2*n), (b + Sqrt[b^2 - 4*a*c
])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(2^(1/(2*n))*((c*x^n)/(b + Sqrt[b^2
- 4*a*c] + 2*c*x^n))^(1/(2*n))))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]
))))/Sqrt[d*x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})}\,dx$$

$$\Big\downarrow \; 1719$$

$$\frac{2c\int \frac{1}{\sqrt{dx}\left(2cx^n+b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}} - \frac{2c\int \frac{1}{\sqrt{dx}\left(2cx^n+b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{888}$$

$$\frac{4c\sqrt{dx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{4c\sqrt{dx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[1/(Sqrt[d*x]*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(4*c*Sqrt[d*x]*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, (-2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d) - (4
*c*Sqrt[d*x]*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{1}{\sqrt{dx}\,(a+b\,x^n+c\,x^{2n})}dx$$

input
```
int(1/(d*x)^(1/2)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(d*x)^(1/2)/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(c*d*x*x^(2*n) + b*d*x*x^n + a*d*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x)**(1/2)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-2*sqrt(x)/(c*sqrt(d)*(4*n - 1)*x^(2*n) + b*sqrt(d)*(4*n - 1)*x^n + a*sqrt
(d)*(4*n - 1)) + integrate(2*(b*n*x^n + 2*a*n)/((c^2*sqrt(d)*(4*n - 1)*x^(
4*n) + 2*b*c*sqrt(d)*(4*n - 1)*x^(3*n) + 2*a*b*sqrt(d)*(4*n - 1)*x^n + a^2
*sqrt(d)*(4*n - 1) + (b^2*sqrt(d)*(4*n - 1) + 2*a*c*sqrt(d)*(4*n - 1))*x^(
2*n))*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*sqrt(d*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{\sqrt{d\,x}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/((d*x)^(1/2)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/((d*x)^(1/2)*(a + b*x^n + c*x^(2*n))), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})}\,dx = \frac{\sqrt{d}\left(\int \frac{1}{x^{2n+\frac{1}{2}}c+x^{n+\frac{1}{2}}b+\sqrt{x}\,a}dx\right)}{d}$$

input
```
int(1/(d*x)^(1/2)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
(sqrt(d)*int(1/(x**((4*n + 1)/2)*c + x**((2*n + 1)/2)*b + sqrt(x)*a),x))/d
```

3.205 $\int \frac{1}{(dx)^{3/2}(a+bx^n+cx^{2n})}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{1}{(dx)^{3/2}(a+bx^n+cx^{2n})}\,dx = \frac{4c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)d\sqrt{dx}} + \frac{4c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)d\sqrt{dx}}$$

output
```
4*c*hypergeom([1, -1/2/n],[1-1/2/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-
4*a*c-b*(-4*a*c+b^2)^(1/2))/d/(d*x)^(1/2)+4*c*hypergeom([1, -1/2/n],[1-1/2
/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)/d/(d
*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.95 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.82

$$\int \frac{1}{(dx)^{3/2}(a+bx^n+cx^{2n})}\,dx = \frac{x\left(-2\sqrt{b^2-4ac}+2^{\frac{1}{2}/n}\left(b+\sqrt{b^2-4ac}\right)\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{\frac{1}{2}/n}\text{Hypergeom}\right.}{}$$

input
```
Integrate[1/((d*x)^(3/2)*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(x*(-2*Sqrt[b^2 - 4*a*c] + 2^(1/(2*n))*(b + Sqrt[b^2 - 4*a*c])*((c*x^n)/(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(1/(2*n))*Hypergeometric2F1[1/(2*n), 1/(2
*n), 1 + 1/(2*n), (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n
)] + 2^(1/(2*n))*(-b + Sqrt[b^2 - 4*a*c])*((c*x^n)/(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^n))^(1/(2*n))*Hypergeometric2F1[1/(2*n), 1/(2*n), 1 + 1/(2*n), (b
+ Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]))/(a*Sqrt[b^2 - 4*
a*c]*(d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^n+cx^{2n}\right)}dx$$

$$\Big\downarrow \text{1719}$$

$$\frac{2c\int \frac{1}{(dx)^{3/2}\left(2cx^n+b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{1}{(dx)^{3/2}\left(2cx^n+b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{888}$$

$$\frac{4c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}-\frac{4c\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}$$

input
```
Int[1/((d*x)^(3/2)*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(-4*c*Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), (-2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*Sqrt[d*x]) + (4*
c*Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*Sqrt[d*x])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{1}{(dx)^{\frac{3}{2}} (a+b\,x^{n}+c\,x^{2n})}dx$$

input
```
int(1/(d*x)^(3/2)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(d*x)^(3/2)/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{1}{(dx)^{3/2} (a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)\,(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(c*d^2*x^2*x^(2*n) + b*d^2*x^2*x^n + a*d^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{3/2}\,(a+bx^n+cx^{2n})}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x)**(3/2)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(dx)^{3/2}\,(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)\,(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-2/((c*d^(3/2)*(4*n + 1)*x^(2*n) + b*d^(3/2)*(4*n + 1)*x^n + a*d^(3/2)*(4*
n + 1))*sqrt(x)) + integrate(2*(b*n*x^n + 2*a*n)/((c^2*d^(3/2)*(4*n + 1)*x
^(4*n) + 2*b*c*d^(3/2)*(4*n + 1)*x^(3*n) + 2*a*b*d^(3/2)*(4*n + 1)*x^n + a
^2*d^(3/2)*(4*n + 1) + (b^2*d^(3/2)*(4*n + 1) + 2*a*c*d^(3/2)*(4*n + 1))*x
^(2*n))*x^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(dx)^{3/2}(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*(d*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{3/2}(a+bx^n+cx^{2n})}\,dx = \int \frac{1}{(d\,x)^{3/2}\,(a+b\,x^n+c\,x^{2\,n})}\,dx$$

input
```
int(1/((d*x)^(3/2)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/((d*x)^(3/2)*(a + b*x^n + c*x^(2*n))), x)
```

Reduce [F]

$$\int \frac{1}{(dx)^{3/2}(a+bx^n+cx^{2n})}\,dx = \frac{\sqrt{d}\left(\int \frac{1}{x^{2n+\frac{1}{2}}cx+x^{n+\frac{1}{2}}bx+\sqrt{x}\,ax}dx\right)}{d^2}$$

input
```
int(1/(d*x)^(3/2)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
(sqrt(d)*int(1/(x**((4*n + 1)/2)*c*x + x**((2*n + 1)/2)*b*x + sqrt(x)*a*x)
,x))/d**2
```

3.206 $\int \frac{1}{(dx)^{5/2}(a+bx^n+cx^{2n})} dx$

Optimal result

Integrand size = 24, antiderivative size = 162

$$\int \frac{1}{(dx)^{5/2}(a+bx^n+cx^{2n})} dx = \frac{4c\,\text{Hypergeometric2F1}\left(1,-\frac{3}{2n},1-\frac{3}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)d(dx)^{3/2}} + \frac{4c\,\text{Hypergeometric2F1}\left(1,-\frac{3}{2n},1-\frac{3}{2n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)d(dx)^{3/2}}$$

output
```
4/3*c*hypergeom([1, -3/2/n],[1-3/2/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^
2-4*a*c-b*(-4*a*c+b^2)^(1/2))/d/(d*x)^(3/2)+4/3*c*hypergeom([1, -3/2/n],[1
-3/2/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)/
d/(d*x)^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.42 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.79

$$\int \frac{1}{(dx)^{5/2}(a+bx^n+cx^{2n})} dx = \frac{x\left(-2\sqrt{b^2-4ac}+2^{\frac{3}{2}/n}\left(b+\sqrt{b^2-4ac}\right)\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{\frac{3}{2}/n}\text{Hypergeor}\right.}{}$$

input
```
Integrate[1/((d*x)^(5/2)*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(x*(-2*Sqrt[b^2 - 4*a*c] + 2^(3/(2*n))*(b + Sqrt[b^2 - 4*a*c])*((c*x^n)/(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(3/(2*n))*Hypergeometric2F1[3/(2*n), 3/(2
*n), 1 + 3/(2*n), (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n
)] + 2^(3/(2*n))*(-b + Sqrt[b^2 - 4*a*c])*((c*x^n)/(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^n))^(3/(2*n))*Hypergeometric2F1[3/(2*n), 3/(2*n), 1 + 3/(2*n), (b
+ Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]))/(3*a*Sqrt[b^2 -
4*a*c]*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{5/2}(a+bx^n+cx^{2n})}dx$$

$$\Big\downarrow \text{ 1719}$$

$$\frac{2c\int \frac{1}{(dx)^{5/2}\left(2cx^n+b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{1}{(dx)^{5/2}\left(2cx^n+b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{4c\,\text{Hypergeometric2F1}\left(1,-\frac{3}{2n},1-\frac{3}{2n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3d(dx)^{3/2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}-\frac{4c\,\text{Hypergeometric2F1}\left(1,-\frac{3}{2n},1-\frac{3}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3d(dx)^{3/2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}$$

input
```
Int[1/((d*x)^(5/2)*(a + b*x^n + c*x^(2*n))),x]
```

output
```
(-4*c*Hypergeometric2F1[1, -3/(2*n), 1 - 3/(2*n), (-2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(d*x)^(3/2)) +
 (4*c*Hypergeometric2F1[1, -3/(2*n), 1 - 3/(2*n), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])/(3*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(d*x)^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{1}{(dx)^{\frac{5}{2}}\left(a+b\,x^{n}+c\,x^{2n}\right)}dx$$

input
```
int(1/(d*x)^(5/2)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(d*x)^(5/2)/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{1}{(dx)^{5/2}\left(a+bx^n+cx^{2n}\right)}\,dx = \int \frac{1}{\left(cx^{2\,n}+bx^{n}+a\right)(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(5/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(c*d^3*x^3*x^(2*n) + b*d^3*x^3*x^n + a*d^3*x^3), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(dx)^{5/2}\left(a+bx^n+cx^{2n}\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(d*x)**(5/2)/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(dx)^{5/2}\left(a+bx^n+cx^{2n}\right)}\,dx = \int \frac{1}{\left(cx^{2\,n}+bx^n+a\right)(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(5/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
-2/((c*d^(5/2)*(4*n + 3)*x^(2*n) + b*d^(5/2)*(4*n + 3)*x^n + a*d^(5/2)*(4*
n + 3))*x^(3/2)) + integrate(2*(b*n*x^n + 2*a*n)/((c^2*d^(5/2)*(4*n + 3)*x
^(4*n) + 2*b*c*d^(5/2)*(4*n + 3)*x^(3*n) + 2*a*b*d^(5/2)*(4*n + 3)*x^n + a
^2*d^(5/2)*(4*n + 3) + (b^2*d^(5/2)*(4*n + 3) + 2*a*c*d^(5/2)*(4*n + 3))*x
^(2*n))*x^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(dx)^{5/2}\left(a+bx^n+cx^{2n}\right)}\,dx=\int \frac{1}{\left(cx^{2n}+bx^n+a\right)(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(5/2)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)*(d*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{5/2}\left(a+bx^n+cx^{2n}\right)}\,dx=\int \frac{1}{(d\,x)^{5/2}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

input
```
int(1/((d*x)^(5/2)*(a + b*x^n + c*x^(2*n))),x)
```

output
```
int(1/((d*x)^(5/2)*(a + b*x^n + c*x^(2*n))), x)
```

Reduce [F]

$$\int \frac{1}{(dx)^{5/2}\left(a+bx^n+cx^{2n}\right)}\,dx=\frac{\sqrt{d}\left(\int \frac{1}{x^{2n+\frac{1}{2}}c\,x^2+x^{n+\frac{1}{2}}b\,x^2+\sqrt{x}\,a\,x^2}dx\right)}{d^3}$$

input
```
int(1/(d*x)^(5/2)/(a+b*x^n+c*x^(2*n)),x)
```

output
```
(sqrt(d)*int(1/(x**((4*n + 1)/2)*c*x**2 + x**((2*n + 1)/2)*b*x**2 + sqrt(x
)*a*x**2),x))/d**3
```

3.207 $\int \frac{(dx)^{3/2}}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 330

$$\int \frac{(dx)^{3/2}}{(a+bx^n+cx^{2n})^2}\,dx = \frac{(dx)^{5/2}(b^2-2ac+bcx^n)}{a(b^2-4ac)\,dn\,(a+bx^n+cx^{2n})}$$
$$-\frac{c\left(4ac(5-4n)-b^2(5-2n)-b\sqrt{b^2-4ac}(5-2n)\right)(dx)^{5/2}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2n},1+\frac{5}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{5a(b^2-4ac)\left(b^2-4ac-b\sqrt{b^2-4ac}\right)dn}$$
$$-\frac{c\left(4ac(5-4n)-b^2(5-2n)+b\sqrt{b^2-4ac}(5-2n)\right)(dx)^{5/2}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2n},1+\frac{5}{2n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{5a(b^2-4ac)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)dn}$$

output
```
(d*x)^(5/2)*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(a+b*x^n+c*x^(2*n))-1/5
*c*(4*a*c*(5-4*n)-b^2*(5-2*n)-b*(-4*a*c+b^2)^(1/2)*(5-2*n))*(d*x)^(5/2)*hy
pergeom([1, 5/2/n],[1+5/2/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^
2)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))/d/n-1/5*c*(4*a*c*(5-4*n)-b^2*(5-2*n)+b
*(-4*a*c+b^2)^(1/2)*(5-2*n))*(d*x)^(5/2)*hypergeom([1, 5/2/n],[1+5/2/n],-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/(b*(-4*a*c+b^2)^(1/2)-4*a*c+
b^2)/d/n
```

Mathematica [A] (warning: unable to verify)

Time = 4.54 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.54

$$\int \frac{(dx)^{3/2}}{(a+bx^n+cx^{2n})^2}\,dx =$$

$$-\frac{cx(dx)^{3/2}\left(\frac{2^{-\frac{5}{2}/n}\left(4ac\sqrt{b^2-4ac}(5-4n)+4abc(5-2n)+b^3(-5+2n)+b^2\sqrt{b^2-4ac}(-5+2n)\right)\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-\frac{5}{2}/n}\text{Hypergeometric2}}{-b^2+4ac+b\sqrt{b^2-4ac}}\right.}{}$$

input
```
Integrate[(d*x)^(3/2)/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
-1/5*(c*x*(d*x)^(3/2)*(((4*a*c*Sqrt[b^2 - 4*a*c]*(5 - 4*n) + 4*a*b*c*(5 -
2*n) + b^3*(-5 + 2*n) + b^2*Sqrt[b^2 - 4*a*c]*(-5 + 2*n))*Hypergeometric2F
1[-5/(2*n), -5/(2*n), 1 - 5/(2*n), (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c] + 2*c*x^n)])/(2^(5/(2*n))*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*((c
*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(5/(2*n))) + ((4*(b^2 - 4*a*c)*(-
8*a^2*c*n + b^2*(-5 + 2*n)*x^n*(b + c*x^n) + a*(2*b^2*n - b*c*(-15 + 8*n)*
x^n - 2*c^2*(-5 + 4*n)*x^(2*n))))/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(a
+ x^n*(b + c*x^n))) + ((b^2*(5 - 2*n) + b*Sqrt[b^2 - 4*a*c]*(-5 + 2*n) + 4
*a*c*(-5 + 4*n))*Hypergeometric2F1[-5/(2*n), -5/(2*n), 1 - 5/(2*n), (b + S
qrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^(5/(2*n))*((c*x^n
)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(5/(2*n))))/(b + Sqrt[b^2 - 4*a*c])))
/(a*(b^2 - 4*a*c)^(3/2)*n)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1720, 27, 1884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{(a+bx^n+cx^{2n})^2}\,dx$$

$$\Big\downarrow \; 1720$$

$$\frac{(dx)^{5/2}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}-\frac{\int -\frac{(dx)^{3/2}\left(-bc(5-2n)x^n+2ac(5-4n)-2b^2\left(\frac{5}{2}-n\right)\right)}{2(bx^n+cx^{2n}+a)}dx}{an\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(dx)^{3/2}\left(-bc(5-2n)x^n+2ac(5-4n)-b^2(5-2n)\right)}{bx^n+cx^{2n}+a}dx}{2an\left(b^2-4ac\right)}+\frac{(dx)^{5/2}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 1884$$

$$\frac{\int \left(\frac{\left(\frac{c\left(2nb^2-5b^2+20ac-16acn\right)}{\sqrt{b^2-4ac}}-bc(5-2n)\right)(dx)^{3/2}}{2cx^n+b-\sqrt{b^2-4ac}}+\frac{\left(-bc(5-2n)-\frac{c\left(2nb^2-5b^2+20ac-16acn\right)}{\sqrt{b^2-4ac}}\right)(dx)^{3/2}}{2cx^n+b+\sqrt{b^2-4ac}}\right)dx}{2an\left(b^2-4ac\right)}+$$
$$\frac{(dx)^{5/2}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{2c(dx)^{5/2}\left(-b(5-2n)\sqrt{b^2-4ac}+4ac(5-4n)-\left(b^2(5-2n)\right)\right)\text{Hypergeometric2F1}\left(1,\frac{5}{2n},1+\frac{5}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{5d\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2c(dx)^{5/2}\left(\frac{4ac(5-4n)-b^2(5-}{\sqrt{b^2-4ac}}\right.}{}}{2an\left(b^2-4ac\right)}$$
$$\frac{(dx)^{5/2}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

input
```
Int[(d*x)^(3/2)/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
((d*x)^(5/2)*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*d*n*(a + b*x^n + c*
x^(2*n))) + ((-2*c*(4*a*c*(5 - 4*n) - b^2*(5 - 2*n) - b*Sqrt[b^2 - 4*a*c]*
(5 - 2*n))*(d*x)^(5/2)*Hypergeometric2F1[1, 5/(2*n), 1 + 5/(2*n), (-2*c*x^
n)/(b - Sqrt[b^2 - 4*a*c])])/(5*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*d) - (
2*c*((4*a*c*(5 - 4*n) - b^2*(5 - 2*n))/Sqrt[b^2 - 4*a*c] + b*(5 - 2*n))*(d
*x)^(5/2)*Hypergeometric2F1[1, 5/(2*n), 1 + 5/(2*n), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])])/(5*(b + Sqrt[b^2 - 4*a*c])*d))/(2*a*(b^2 - 4*a*c)*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1720
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p +
 1) + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*
a*c, 0] && ILtQ[p + 1, 0]
```

rule 1884
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && (
IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(dx)^{\frac{3}{2}}}{(a+b\,x^n+c\,x^{2n})^2}dx$$

input
```
int((d*x)^(3/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int((d*x)^(3/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{(dx)^{3/2}}{(a + bx^n + cx^{2n})^2} \, dx = \int \frac{(dx)^{\frac{3}{2}}}{(cx^{2n} + bx^n + a)^2} \, dx$$

input
```
integrate((d*x)^(3/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*d*x/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b
*c*x^n + a*c)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{3/2}}{(a + bx^n + cx^{2n})^2} \, dx = \text{Timed out}$$

input
```
integrate((d*x)**(3/2)/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(dx)^{3/2}}{(a + bx^n + cx^{2n})^2} \, dx = \int \frac{(dx)^{\frac{3}{2}}}{(cx^{2n} + bx^n + a)^2} \, dx$$

input
```
integrate((d*x)^(3/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
-2*d^(3/2)*x^(5/2)/(c^2*(8*n - 5)*x^(4*n) + 2*b*c*(8*n - 5)*x^(3*n) + 2*a*
b*(8*n - 5)*x^n + a^2*(8*n - 5) + (b^2*(8*n - 5) + 2*a*c*(8*n - 5))*x^(2*n
)) + integrate(4*(b*d^(3/2)*n*x^n + 2*a*d^(3/2)*n)*x^(3/2)/(c^3*(8*n - 5)*
x^(6*n) + 3*b*c^2*(8*n - 5)*x^(5*n) + 3*a^2*b*(8*n - 5)*x^n + a^3*(8*n - 5
) + 3*(b^2*c*(8*n - 5) + a*c^2*(8*n - 5))*x^(4*n) + (b^3*(8*n - 5) + 6*a*b
*c*(8*n - 5))*x^(3*n) + 3*(a*b^2*(8*n - 5) + a^2*c*(8*n - 5))*x^(2*n)), x)
```

Giac [F]

$$\int \frac{(dx)^{3/2}}{(a+bx^n+cx^{2n})^2}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)/(c*x^(2*n) + b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{3/2}}{(a+bx^n+cx^{2n})^2}\,dx=\int \frac{(d\,x)^{3/2}}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

input
```
int((d*x)^(3/2)/(a + b*x^n + c*x^(2*n))^2,x)
```

output
```
int((d*x)^(3/2)/(a + b*x^n + c*x^(2*n))^2, x)
```

Reduce [F]

$$\int \frac{(dx)^{3/2}}{\left(a+bx^n+cx^{2n}\right)^2}\,dx = \sqrt{d}\left(\int \frac{\sqrt{x}\,x}{x^{4n}c^2+2x^{3n}bc+2x^{2n}ac+x^{2n}b^2+2x^nab+a^2}dx\right)d$$

input
```
int((d*x)^(3/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
sqrt(d)*int((sqrt(x)*x)/(x**(4*n)*c**2 + 2*x**(3*n)*b*c + 2*x**(2*n)*a*c +
 x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)*d
```

3.208 $\int \frac{\sqrt{dx}}{(a+bx^n+cx^{2n})^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 330

$$\int \frac{\sqrt{dx}}{(a+bx^n+cx^{2n})^2} dx = \frac{(dx)^{3/2}(b^2-2ac+bcx^n)}{a(b^2-4ac)dn(a+bx^n+cx^{2n})}$$
$$-\frac{c\left(4ac(3-4n)-b^2(3-2n)-b\sqrt{b^2-4ac}(3-2n)\right)(dx)^{3/2}\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1+\frac{3}{2n},-\frac{2c\ldots}{b-\sqrt{b\ldots}}\right.}{3a(b^2-4ac)\left(b^2-4ac-b\sqrt{b^2-4ac}\right)dn}$$
$$-\frac{c\left(4ac(3-4n)-b^2(3-2n)+b\sqrt{b^2-4ac}(3-2n)\right)(dx)^{3/2}\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1+\frac{3}{2n},-\frac{2c\ldots}{b+\sqrt{b\ldots}}\right.}{3a(b^2-4ac)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)dn}$$

output
```
(d*x)^(3/2)*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(a+b*x^n+c*x^(2*n))-1/3
*c*(4*a*c*(3-4*n)-b^2*(3-2*n)-b*(-4*a*c+b^2)^(1/2)*(3-2*n))*(d*x)^(3/2)*hy
pergeom([1, 3/2/n],[1+3/2/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^
2)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))/d/n-1/3*c*(4*a*c*(3-4*n)-b^2*(3-2*n)+b
*(-4*a*c+b^2)^(1/2)*(3-2*n))*(d*x)^(3/2)*hypergeom([1, 3/2/n],[1+3/2/n],-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/(b*(-4*a*c+b^2)^(1/2)-4*a*c+
b^2)/d/n
```

Mathematica [A] (warning: unable to verify)

Time = 4.55 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{dx}}{(a+bx^n+cx^{2n})^2}\,dx =$$

$$-\frac{cx\sqrt{dx}\left(\frac{2^{-\frac{3}{2}/n}\left(4ac\sqrt{b^2-4ac}(3-4n)+4abc(3-2n)+b^3(-3+2n)+b^2\sqrt{b^2-4ac}(-3+2n)\right)\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-\frac{3}{2}/n}\text{Hypergeometric2F}}{-b^2+4ac+b\sqrt{b^2-4ac}}\right.}{}$$

input
```
Integrate[Sqrt[d*x]/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
-1/3*(c*x*Sqrt[d*x]*(((4*a*c*Sqrt[b^2 - 4*a*c]*(3 - 4*n) + 4*a*b*c*(3 - 2*
n) + b^3*(-3 + 2*n) + b^2*Sqrt[b^2 - 4*a*c]*(-3 + 2*n))*Hypergeometric2F1[
-3/(2*n), -3/(2*n), 1 - 3/(2*n), (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^n)])/(2^(3/(2*n))*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*((c*x
^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(3/(2*n))) + ((4*(b^2 - 4*a*c)*(-8*
a^2*c*n + b^2*(-3 + 2*n)*x^n*(b + c*x^n) + a*(2*b^2*n - b*c*(-9 + 8*n)*x^n
 - 2*c^2*(-3 + 4*n)*x^(2*n))))/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(a + x
^n*(b + c*x^n))) + ((b^2*(3 - 2*n) + b*Sqrt[b^2 - 4*a*c]*(-3 + 2*n) + 4*a*
c*(-3 + 4*n))*Hypergeometric2F1[-3/(2*n), -3/(2*n), 1 - 3/(2*n), (b + Sqrt
[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^(3/(2*n))*((c*x^n)/(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(3/(2*n)))))/(b + Sqrt[b^2 - 4*a*c])))/(a
*(b^2 - 4*a*c)^(3/2)*n)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1720, 27, 1884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{\left(a+bx^n+cx^{2n}\right)^2}dx$$

$\big\downarrow$ 1720

$$\frac{(dx)^{3/2}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}-\frac{\int -\frac{\sqrt{dx}\left(-bc(3-2n)x^n+2ac(3-4n)-2b^2\left(\frac{3}{2}-n\right)\right)}{2\left(bx^n+cx^{2n}+a\right)}dx}{an\left(b^2-4ac\right)}$$

$\big\downarrow$ 27

$$\frac{\int \frac{\sqrt{dx}\left(-bc(3-2n)x^n+2ac(3-4n)-b^2(3-2n)\right)}{bx^n+cx^{2n}+a}dx}{2an\left(b^2-4ac\right)}+\frac{(dx)^{3/2}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$\big\downarrow$ 1884

$$\frac{\int \left(\frac{\sqrt{dx}\left(-bc(3-2n)-\frac{c\left(2nb^2-3b^2+12ac-16acn\right)}{\sqrt{b^2-4ac}}\right)}{2cx^n+b+\sqrt{b^2-4ac}}+\frac{\left(\frac{c\left(2nb^2-3b^2+12ac-16acn\right)}{\sqrt{b^2-4ac}}-bc(3-2n)\right)\sqrt{dx}}{2cx^n+b-\sqrt{b^2-4ac}}\right)dx}{2an\left(b^2-4ac\right)}+\frac{(dx)^{3/2}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$\big\downarrow$ 2009

$$\frac{-\frac{2c(dx)^{3/2}\left(-b(3-2n)\sqrt{b^2-4ac}+4ac(3-4n)-\left(b^2(3-2n)\right)\right)\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1+\frac{3}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3d\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2c(dx)^{3/2}\left(\frac{4ac(3-4n)-b^2(3-\ldots}{\sqrt{b^2-4ac}}\ldots\right.}{\ldots}}{2an\left(b^2-4ac\right)}+\frac{(dx)^{3/2}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

input
```
Int[Sqrt[d*x]/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
((d*x)^(3/2)*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*d*n*(a + b*x^n + c*
x^(2*n))) + ((-2*c*(4*a*c*(3 - 4*n) - b^2*(3 - 2*n) - b*Sqrt[b^2 - 4*a*c]*
(3 - 2*n))*(d*x)^(3/2)*Hypergeometric2F1[1, 3/(2*n), 1 + 3/(2*n), (-2*c*x^
n)/(b - Sqrt[b^2 - 4*a*c])])/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*d) - (
2*c*((4*a*c*(3 - 4*n) - b^2*(3 - 2*n))/Sqrt[b^2 - 4*a*c] + b*(3 - 2*n))*(d
*x)^(3/2)*Hypergeometric2F1[1, 3/(2*n), 1 + 3/(2*n), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])])/(3*(b + Sqrt[b^2 - 4*a*c])*d))/(2*a*(b^2 - 4*a*c)*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1720
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p +
 1) + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*
a*c, 0] && ILtQ[p + 1, 0]
```

rule 1884
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && (
IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{\sqrt{dx}}{(a+b\,x^n+c\,x^{2n})^2}dx$$

input
```
int((d*x)^(1/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int((d*x)^(1/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{dx}}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{\sqrt{dx}}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*x
^n + a*c)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx}}{(a+bx^n+cx^{2n})^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(1/2)/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{dx}}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{\sqrt{dx}}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
-2*sqrt(d)*x^(3/2)/(c^2*(8*n - 3)*x^(4*n) + 2*b*c*(8*n - 3)*x^(3*n) + 2*a*
b*(8*n - 3)*x^n + a^2*(8*n - 3) + (b^2*(8*n - 3) + 2*a*c*(8*n - 3))*x^(2*n
)) + integrate(4*(b*sqrt(d)*n*x^n + 2*a*sqrt(d)*n)*sqrt(x)/(c^3*(8*n - 3)*
x^(6*n) + 3*b*c^2*(8*n - 3)*x^(5*n) + 3*a^2*b*(8*n - 3)*x^n + a^3*(8*n - 3
) + 3*(b^2*c*(8*n - 3) + a*c^2*(8*n - 3))*x^(4*n) + (b^3*(8*n - 3) + 6*a*b
*c*(8*n - 3))*x^(3*n) + 3*(a*b^2*(8*n - 3) + a^2*c*(8*n - 3))*x^(2*n)), x)
```

Giac [F]

$$\int \frac{\sqrt{dx}}{(a+bx^n+cx^{2n})^2}\,dx=\int \frac{\sqrt{dx}}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)/(c*x^(2*n) + b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx}}{(a+bx^n+cx^{2n})^2}\,dx=\int \frac{\sqrt{d\,x}}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

input
```
int((d*x)^(1/2)/(a + b*x^n + c*x^(2*n))^2,x)
```

output
```
int((d*x)^(1/2)/(a + b*x^n + c*x^(2*n))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{dx}}{(a+bx^n+cx^{2n})^2}\,dx = \sqrt{d}\left(\int \frac{\sqrt{x}}{x^{4n}c^2+2x^{3n}bc+2x^{2n}ac+x^{2n}b^2+2x^nab+a^2}dx\right)$$

input
```
int((d*x)^(1/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
sqrt(d)*int(sqrt(x)/(x**(4*n)*c**2 + 2*x**(3*n)*b*c + 2*x**(2*n)*a*c + x**
(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.209 $\int \frac{1}{\sqrt{dx}(a+bx^n+cx^{2n})^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 326

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})^2}\,dx = \frac{\sqrt{dx}(b^2-2ac+bcx^n)}{a\,(b^2-4ac)\,dn\,(a+bx^n+cx^{2n})} - \frac{c\big(4ac(1-4n)-b^2(1-2n)-b\sqrt{b^2-4ac}(1-2n)\big)\sqrt{dx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{2c}{b-\sqrt{b}}\right.}{a\,(b^2-4ac)\left(b^2-4ac-b\sqrt{b^2-4ac}\right)dn} - \frac{c\big(4ac(1-4n)-b^2(1-2n)+b\sqrt{b^2-4ac}(1-2n)\big)\sqrt{dx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{2c}{b+\sqrt{b}}\right.}{a\,(b^2-4ac)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)dn}$$

output
```
(d*x)^(1/2)*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(a+b*x^n+c*x^(2*n))-c*(
4*a*c*(1-4*n)-b^2*(1-2*n)-b*(-4*a*c+b^2)^(1/2)*(1-2*n))*(d*x)^(1/2)*hyperg
eom([1, 1/2/n],[1+1/2/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/(
b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))/d/n-c*(4*a*c*(1-4*n)-b^2*(1-2*n)+b*(-4*a*c
+b^2)^(1/2)*(1-2*n))*(d*x)^(1/2)*hypergeom([1, 1/2/n],[1+1/2/n],-2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)/d/n
```

Mathematica [A] (warning: unable to verify)

Time = 4.23 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})^2}\,dx =$$

$$-\frac{cx\left(\frac{2^{-\frac{1}{2}/n}\left(4ac\sqrt{b^2-4ac}(1-4n)+4abc(1-2n)+b^3(-1+2n)+b^2\sqrt{b^2-4ac}(-1+2n)\right)\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-\frac{1}{2}/n}\text{Hypergeometric2F1}\left(-\right.}{-b^2+4ac+b\sqrt{b^2-4ac}}}{}$$

input
```
Integrate[1/(Sqrt[d*x]*(a + b*x^n + c*x^(2*n))^2),x]
```

output
```
-((c*x*(((4*a*c*Sqrt[b^2 - 4*a*c]*(1 - 4*n) + 4*a*b*c*(1 - 2*n) + b^3*(-1
+ 2*n) + b^2*Sqrt[b^2 - 4*a*c]*(-1 + 2*n))*Hypergeometric2F1[-1/2*1/n, -1/
2*1/n, 1 - 1/(2*n), (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x
^n)])/(2^(1/(2*n))*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*((c*x^n)/(b - Sqrt
[b^2 - 4*a*c] + 2*c*x^n))^(1/(2*n))) + ((4*(b^2 - 4*a*c)*(-8*a^2*c*n + b^2
*(-1 + 2*n)*x^n*(b + c*x^n) + a*(2*b^2*n - b*c*(-3 + 8*n)*x^n - 2*c^2*(-1
+ 4*n)*x^(2*n))))/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(a + x^n*(b + c*x^n
))) + ((b^2*(1 - 2*n) + b*Sqrt[b^2 - 4*a*c]*(-1 + 2*n) + 4*a*c*(-1 + 4*n))
*Hypergeometric2F1[-1/2*1/n, -1/2*1/n, 1 - 1/(2*n), (b + Sqrt[b^2 - 4*a*c]
)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^(1/(2*n))*((c*x^n)/(b + Sqrt[b^2
- 4*a*c] + 2*c*x^n))^(1/(2*n))))/(b + Sqrt[b^2 - 4*a*c])))/(a*(b^2 - 4*a*c
)^(3/2)*n*Sqrt[d*x]))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1720, 27, 1884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})^2}\,dx$$

$$\Big\downarrow \; 1720$$

$$\frac{\sqrt{dx}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}-\frac{\int -\frac{-bc(1-2n)x^n+2ac(1-4n)-2b^2\left(\frac{1}{2}-n\right)}{2\sqrt{dx}\left(bx^n+cx^{2n}+a\right)}dx}{an\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-bc(1-2n)x^n+2ac(1-4n)-b^2(1-2n)}{\sqrt{dx}\left(bx^n+cx^{2n}+a\right)}dx}{2an\left(b^2-4ac\right)}+\frac{\sqrt{dx}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 1884$$

$$\frac{\int \left(\frac{-bc(1-2n)-\frac{c\left(2nb^2-b^2+4ac-16acn\right)}{\sqrt{b^2-4ac}}}{\sqrt{dx}\left(2cx^n+b+\sqrt{b^2-4ac}\right)}+\frac{\frac{c\left(2nb^2-b^2+4ac-16acn\right)}{\sqrt{b^2-4ac}}-bc(1-2n)}{\sqrt{dx}\left(2cx^n+b-\sqrt{b^2-4ac}\right)}\right)dx}{2an\left(b^2-4ac\right)}+\frac{\sqrt{dx}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{2c\sqrt{dx}\left(\frac{4ac(1-4n)-b^2(1-2n)}{\sqrt{b^2-4ac}}-b(1-2n)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\sqrt{dx}\left(\frac{4ac(1-4n)-b^2(1-2n)}{\sqrt{b^2-4ac}}-2bn+b\right)\text{Hyperg}}{d\left(\sqrt{b^2-\right.}}{2an\left(b^2-4ac\right)}$$

$$\frac{\sqrt{dx}\left(-2ac+b^2+bcx^n\right)}{adn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

input
```
Int[1/(Sqrt[d*x]*(a + b*x^n + c*x^(2*n))^2),x]
```

output
```
(Sqrt[d*x]*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*d*n*(a + b*x^n + c*x^
(2*n))) + ((2*c*((4*a*c*(1 - 4*n) - b^2*(1 - 2*n))/Sqrt[b^2 - 4*a*c] - b*(
1 - 2*n))*Sqrt[d*x]*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, (-2*c*x^
n)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*d) - (2*c*(b + (4*a*
c*(1 - 4*n) - b^2*(1 - 2*n))/Sqrt[b^2 - 4*a*c] - 2*b*n)*Sqrt[d*x]*Hypergeo
metric2F1[1, 1/(2*n), (2 + n^(-1))/2, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
/((b + Sqrt[b^2 - 4*a*c])*d))/(2*a*(b^2 - 4*a*c)*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1720
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p +
 1) + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*
a*c, 0] && ILtQ[p + 1, 0]
```

rule 1884
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && (
IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{1}{\sqrt{dx}\,(a+b\,x^{n}+c\,x^{2n})^{2}}dx$$

input
```
int(1/(d*x)^(1/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int(1/(d*x)^(1/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})^2}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^2\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(c^2*d*x*x^(4*n) + b^2*d*x*x^(2*n) + 2*a*b*d*x*x^n + a^
2*d*x + 2*(b*c*d*x*x^n + a*c*d*x)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x)**(1/2)/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})^2}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^2\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
-2*sqrt(x)/(c^2*sqrt(d)*(8*n - 1)*x^(4*n) + 2*b*c*sqrt(d)*(8*n - 1)*x^(3*n
) + 2*a*b*sqrt(d)*(8*n - 1)*x^n + a^2*sqrt(d)*(8*n - 1) + (b^2*sqrt(d)*(8*
n - 1) + 2*a*c*sqrt(d)*(8*n - 1))*x^(2*n)) + integrate(4*(b*n*x^n + 2*a*n)
/((c^3*sqrt(d)*(8*n - 1)*x^(6*n) + 3*b*c^2*sqrt(d)*(8*n - 1)*x^(5*n) + 3*a
^2*b*sqrt(d)*(8*n - 1)*x^n + a^3*sqrt(d)*(8*n - 1) + 3*(b^2*c*sqrt(d)*(8*n
 - 1) + a*c^2*sqrt(d)*(8*n - 1))*x^(4*n) + (b^3*sqrt(d)*(8*n - 1) + 6*a*b*
c*sqrt(d)*(8*n - 1))*x^(3*n) + 3*(a*b^2*sqrt(d)*(8*n - 1) + a^2*c*sqrt(d)*
(8*n - 1))*x^(2*n))*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})^2}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^2\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^2*sqrt(d*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})^2}\,dx = \int \frac{1}{\sqrt{d\,x}\,(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

input
```
int(1/((d*x)^(1/2)*(a + b*x^n + c*x^(2*n))^2),x)
```

output
```
int(1/((d*x)^(1/2)*(a + b*x^n + c*x^(2*n))^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx^n+cx^{2n})^2}\,dx = \frac{\sqrt{d}\left(\int \frac{1}{x^{4n+\frac{1}{2}}c^2+2x^{3n+\frac{1}{2}}bc+2x^{2n+\frac{1}{2}}ac+x^{2n+\frac{1}{2}}b^2+2x^{n+\frac{1}{2}}ab+\sqrt{x}\,a^2}dx\right)}{d}$$

input
```
int(1/(d*x)^(1/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
(sqrt(d)*int(1/(x**((8*n + 1)/2)*c**2 + 2*x**((6*n + 1)/2)*b*c + 2*x**((4*
n + 1)/2)*a*c + x**((4*n + 1)/2)*b**2 + 2*x**((2*n + 1)/2)*a*b + sqrt(x)*a
**2),x))/d
```

3.210 $\int \frac{1}{(dx)^{3/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 322

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx = \frac{b^2-2ac+bcx^n}{a\left(b^2-4ac\right)dn\sqrt{dx}\left(a+bx^n+cx^{2n}\right)} + \frac{c\left(b^2(1+2n)+b\sqrt{b^2-4ac}(1+2n)-4ac(1+4n)\right)\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{a\left(b^2-4ac\right)\left(b^2-4ac-b\sqrt{b^2-4ac}\right)dn\sqrt{dx}} + \frac{c\left(b^2(1+2n)-b\sqrt{b^2-4ac}(1+2n)-4ac(1+4n)\right)\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\left(b^2-4ac\right)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)dn\sqrt{dx}}$$

output
```
(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(d*x)^(1/2)/(a+b*x^n+c*x^(2*n))+c*(
b^2*(1+2*n)+b*(-4*a*c+b^2)^(1/2)*(1+2*n)-4*a*c*(1+4*n))*hypergeom([1, -1/2
/n],[1-1/2/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/(b^2-4*a*c-b
*(-4*a*c+b^2)^(1/2))/d/n/(d*x)^(1/2)+c*(b^2*(1+2*n)-b*(-4*a*c+b^2)^(1/2)*(
1+2*n)-4*a*c*(1+4*n))*hypergeom([1, -1/2/n],[1-1/2/n],-2*c*x^n/(b+(-4*a*c+
b^2)^(1/2)))/a/(-4*a*c+b^2)/(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)/d/n/(d*x)^(1/
2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.85 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.55

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx =$$

$$-\frac{x\left(-4b^2+16ac-\frac{2b^2}{n}+\frac{4ac}{n}+\frac{2ab^2}{n(a+x^n(b+cx^n))}-\frac{4a^2c}{n(a+x^n(b+cx^n))}+\frac{2abcx^n}{n(a+x^n(b+cx^n))}+\frac{2^{\frac{1}{2}/n}\left(b^3(1+2n)+b^2\sqrt{b^2-4ac}(1+\right.}{}\right.}{}$$

input
```
Integrate[1/((d*x)^(3/2)*(a + b*x^n + c*x^(2*n))^2),x]
```

output
```
-1/2*(x*(-4*b^2 + 16*a*c - (2*b^2)/n + (4*a*c)/n + (2*a*b^2)/(n*(a + x^n*(
b + c*x^n))) - (4*a^2*c)/(n*(a + x^n*(b + c*x^n))) + (2*a*b*c*x^n)/(n*(a +
 x^n*(b + c*x^n))) + (2^(1/(2*n))*(b^3*(1 + 2*n) + b^2*Sqrt[b^2 - 4*a*c]*(
1 + 2*n) - 4*a*b*c*(1 + 3*n) - 2*a*c*Sqrt[b^2 - 4*a*c]*(1 + 4*n))*((c*x^n)
/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(1/(2*n))*Hypergeometric2F1[1/(2*n), 1
/(2*n), 1 + 1/(2*n), (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)])/(Sqrt[b^2 - 4*a*c]*n) + (2^(1/(2*n))*(-(b^3*(1 + 2*n)) + b^2*Sqrt[b
^2 - 4*a*c]*(1 + 2*n) + 4*a*b*c*(1 + 3*n) - 2*a*c*Sqrt[b^2 - 4*a*c]*(1 + 4
*n))*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(1/(2*n))*Hypergeometric2
F1[1/(2*n), 1/(2*n), 1 + 1/(2*n), (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 -
4*a*c] + 2*c*x^n)])/(Sqrt[b^2 - 4*a*c]*n)))/(a^2*(-b^2 + 4*a*c)*(d*x)^(3/2
))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1720, 27, 1884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx$$

$$\Big\downarrow \; 1720$$

$$\frac{-2ac+b^2+bcx^n}{adn\sqrt{dx}\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}-\frac{\int -\frac{bc(2n+1)x^n+b^2(2n+1)-2ac(4n+1)}{2(dx)^{3/2}\left(bx^n+cx^{2n}+a\right)}dx}{an\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{bc(2n+1)x^n+b^2(2n+1)-2ac(4n+1)}{(dx)^{3/2}\left(bx^n+cx^{2n}+a\right)}dx}{2an\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^n}{adn\sqrt{dx}\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 1884$$

$$\frac{\int \left(\frac{bc(2n+1)-\frac{c\left(2nb^2+b^2-4ac-16acn\right)}{\sqrt{b^2-4ac}}}{(dx)^{3/2}\left(2cx^n+b+\sqrt{b^2-4ac}\right)}+\frac{bc(2n+1)+\frac{c\left(2nb^2+b^2-4ac-16acn\right)}{\sqrt{b^2-4ac}}}{(dx)^{3/2}\left(2cx^n+b-\sqrt{b^2-4ac}\right)}\right)dx}{2an\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^n}{adn\sqrt{dx}\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{2c\left(\frac{b^2(2n+1)-4ac(4n+1)}{\sqrt{b^2-4ac}}+2bn+b\right)\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(-\frac{b^2(2n+1)-4ac(4n+1)}{\sqrt{b^2-4ac}}+2bn+b\right)\text{Hypergeometr}}{d\sqrt{dx}\left(\sqrt{b^2-4ac}+\right.}}{2an\left(b^2-4ac\right)}$$
$$\frac{-2ac+b^2+bcx^n}{adn\sqrt{dx}\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

input

```
Int[1/((d*x)^(3/2)*(a + b*x^n + c*x^(2*n))^2),x]
```

output

```
(b^2 - 2*a*c + b*c*x^n)/(a*(b^2 - 4*a*c)*d*n*Sqrt[d*x]*(a + b*x^n + c*x^(2
*n))) + ((-2*c*(b + 2*b*n + (b^2*(1 + 2*n) - 4*a*c*(1 + 4*n))/Sqrt[b^2 - 4
*a*c])*Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), (-2*c*x^n)/(b - Sqrt[b^
2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*d*Sqrt[d*x]) - (2*c*(b + 2*b*n - (b
^2*(1 + 2*n) - 4*a*c*(1 + 4*n))/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, -1
/2*1/n, 1 - 1/(2*n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^2 -
 4*a*c])*d*Sqrt[d*x]))/(2*a*(b^2 - 4*a*c)*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1720
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p +
 1) + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*
a*c, 0] && ILtQ[p + 1, 0]
```

rule 1884
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && (
IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+b\,x^{n}+c\,x^{2n}\right)^{2}}dx$$

input
```
int(1/(d*x)^(3/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int(1/(d*x)^(3/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx=\int \frac{1}{\left(cx^{2\,n}+bx^n+a\right)^2(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(c^2*d^2*x^2*x^(4*n) + b^2*d^2*x^2*x^(2*n) + 2*a*b*d^2*
x^2*x^n + a^2*d^2*x^2 + 2*(b*c*d^2*x^2*x^n + a*c*d^2*x^2)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*x)**(3/2)/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx=\int \frac{1}{\left(cx^{2\,n}+bx^n+a\right)^2(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
-2/((c^2*d^(3/2)*(8*n + 1)*x^(4*n) + 2*b*c*d^(3/2)*(8*n + 1)*x^(3*n) + 2*a
*b*d^(3/2)*(8*n + 1)*x^n + a^2*d^(3/2)*(8*n + 1) + (b^2*d^(3/2)*(8*n + 1)
+ 2*a*c*d^(3/2)*(8*n + 1))*x^(2*n))*sqrt(x)) + integrate(4*(b*n*x^n + 2*a*
n)/((c^3*d^(3/2)*(8*n + 1)*x^(6*n) + 3*b*c^2*d^(3/2)*(8*n + 1)*x^(5*n) + 3
*a^2*b*d^(3/2)*(8*n + 1)*x^n + a^3*d^(3/2)*(8*n + 1) + 3*(b^2*c*d^(3/2)*(8
*n + 1) + a*c^2*d^(3/2)*(8*n + 1))*x^(4*n) + (b^3*d^(3/2)*(8*n + 1) + 6*a*
b*c*d^(3/2)*(8*n + 1))*x^(3*n) + 3*(a*b^2*d^(3/2)*(8*n + 1) + a^2*c*d^(3/2
)*(8*n + 1))*x^(2*n))*x^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(dx)^{3/2} (a+bx^n+cx^{2n})^2} dx = \int \frac{1}{(cx^{2n}+bx^n+a)^2 (dx)^{\frac{3}{2}}} dx$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^2*(d*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{3/2} (a+bx^n+cx^{2n})^2} dx = \int \frac{1}{(d\,x)^{3/2}\,(a+b\,x^n+c\,x^{2\,n})^2} dx$$

input
```
int(1/((d*x)^(3/2)*(a + b*x^n + c*x^(2*n))^2),x)
```

output
```
int(1/((d*x)^(3/2)*(a + b*x^n + c*x^(2*n))^2), x)
```

Reduce [F]

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx = \frac{\sqrt{d}\left(\int \frac{1}{x^{4n+\frac{1}{2}}c^2x+2x^{3n+\frac{1}{2}}bcx+2x^{2n+\frac{1}{2}}acx+x^{2n+\frac{1}{2}}b^2x+2x^{n+\frac{1}{2}}abx+\sqrt{x}\,a^2x}dx\right)}{d^2}$$

input
```
int(1/(d*x)^(3/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
(sqrt(d)*int(1/(x**((8*n + 1)/2)*c**2*x + 2*x**((6*n + 1)/2)*b*c*x + 2*x**
((4*n + 1)/2)*a*c*x + x**((4*n + 1)/2)*b**2*x + 2*x**((2*n + 1)/2)*a*b*x +
 sqrt(x)*a**2*x),x))/d**2
```

3.211 $\int \frac{1}{(dx)^{5/2}(a+bx^n+cx^{2n})^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 328

$$\int \frac{1}{(dx)^{5/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx = \frac{b^2-2ac+bcx^n}{a\left(b^2-4ac\right)dn(dx)^{3/2}\left(a+bx^n+cx^{2n}\right)}$$
$$+\frac{c\left(b^2(3+2n)+b\sqrt{b^2-4ac}(3+2n)-4ac(3+4n)\right)\text{Hypergeometric2F1}\left(1,-\frac{3}{2n},1-\frac{3}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3a\left(b^2-4ac\right)\left(b^2-4ac-b\sqrt{b^2-4ac}\right)dn(dx)^{3/2}}$$
$$+\frac{c\left(b^2(3+2n)-b\sqrt{b^2-4ac}(3+2n)-4ac(3+4n)\right)\text{Hypergeometric2F1}\left(1,-\frac{3}{2n},1-\frac{3}{2n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\left(b^2-4ac\right)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)dn(dx)^{3/2}}$$

output
```
(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(d*x)^(3/2)/(a+b*x^n+c*x^(2*n))+1/3
*c*(b^2*(3+2*n)+b*(-4*a*c+b^2)^(1/2)*(3+2*n)-4*a*c*(3+4*n))*hypergeom([1,
-3/2/n],[1-3/2/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/(b^2-4*a
*c-b*(-4*a*c+b^2)^(1/2))/d/n/(d*x)^(3/2)+1/3*c*(b^2*(3+2*n)-b*(-4*a*c+b^2)
^(1/2)*(3+2*n)-4*a*c*(3+4*n))*hypergeom([1, -3/2/n],[1-3/2/n],-2*c*x^n/(b+
(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)/d/n/(
d*x)^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.84 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.44

$$\int \frac{1}{(dx)^{5/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx = \frac{x\left(\frac{2\left(8a^2cn-b^2(3+2n)x^n(b+cx^n)+a\left(-2b^2n+bc(9+8n)x^n+2c^2(3+4n)x^{2n}\right)\right)}{a+x^n(b+cx^n)}+\frac{2^{\frac{3}{2}/n}\left(-12abc\right.}{}\right.}{}$$

input
```
Integrate[1/((d*x)^(5/2)*(a + b*x^n + c*x^(2*n))^2),x]
```

output
```
(x*((2*(8*a^2*c*n - b^2*(3 + 2*n)*x^n*(b + c*x^n) + a*(-2*b^2*n + b*c*(9 +
 8*n)*x^n + 2*c^2*(3 + 4*n)*x^(2*n))))/(a + x^n*(b + c*x^n)) + (2^(3/(2*n)
)*(-12*a*b*c*(1 + n) + b^3*(3 + 2*n) + b^2*Sqrt[b^2 - 4*a*c]*(3 + 2*n) - 2
*a*c*Sqrt[b^2 - 4*a*c]*(3 + 4*n))*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
n))^(3/(2*n))*Hypergeometric2F1[3/(2*n), 3/(2*n), 1 + 3/(2*n), (b - Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/Sqrt[b^2 - 4*a*c] + (2^(3
/(2*n))*(12*a*b*c*(1 + n) - b^3*(3 + 2*n) + b^2*Sqrt[b^2 - 4*a*c]*(3 + 2*n
) - 2*a*c*Sqrt[b^2 - 4*a*c]*(3 + 4*n))*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^n))^(3/(2*n))*Hypergeometric2F1[3/(2*n), 3/(2*n), 1 + 3/(2*n), (b + S
qrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/Sqrt[b^2 - 4*a*c]))/
(6*a^2*(b^2 - 4*a*c)*n*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1720, 27, 1884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{5/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx$$

$$\Big\downarrow \; 1720$$

$$\frac{-2ac+b^2+bcx^n}{adn(dx)^{3/2}\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}-\frac{\int -\frac{bc(2n+3)x^n+b^2(2n+3)-2ac(4n+3)}{2(dx)^{5/2}\left(bx^n+cx^{2n}+a\right)}dx}{an\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{bc(2n+3)x^n+b^2(2n+3)-2ac(4n+3)}{(dx)^{5/2}\left(bx^n+cx^{2n}+a\right)}dx}{2an\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^n}{adn(dx)^{3/2}\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 1884$$

$$\frac{\int \left(\frac{bc(2n+3)-\frac{c\left(2nb^2+3b^2-12ac-16acn\right)}{\sqrt{b^2-4ac}}}{(dx)^{5/2}\left(2cx^n+b+\sqrt{b^2-4ac}\right)}+\frac{bc(2n+3)+\frac{c\left(2nb^2+3b^2-12ac-16acn\right)}{\sqrt{b^2-4ac}}}{(dx)^{5/2}\left(2cx^n+b-\sqrt{b^2-4ac}\right)}\right)dx}{2an\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^n}{adn(dx)^{3/2}\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{2c\left(\frac{b^2(2n+3)-4ac(4n+3)}{\sqrt{b^2-4ac}}+b(2n+3)\right)\text{Hypergeometric2F1}\left(1,-\frac{3}{2n},1-\frac{3}{2n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3d(dx)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(b(2n+3)-\frac{b^2(2n+3)-4ac(4n+3)}{\sqrt{b^2-4ac}}\right)\text{Hypergeome}}{3d(dx)^{3/2}\left(\sqrt{b^2-4\right.}}}{2an\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx^n}{adn(dx)^{3/2}\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

input
```
Int[1/((d*x)^(5/2)*(a + b*x^n + c*x^(2*n))^2),x]
```

output
```
(b^2 - 2*a*c + b*c*x^n)/(a*(b^2 - 4*a*c)*d*n*(d*x)^(3/2)*(a + b*x^n + c*x^
(2*n))) + ((-2*c*(b*(3 + 2*n) + (b^2*(3 + 2*n) - 4*a*c*(3 + 4*n))/Sqrt[b^2
 - 4*a*c])*Hypergeometric2F1[1, -3/(2*n), 1 - 3/(2*n), (-2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])])/(3*(b - Sqrt[b^2 - 4*a*c])*d*(d*x)^(3/2)) - (2*c*(b*(3 +
 2*n) - (b^2*(3 + 2*n) - 4*a*c*(3 + 4*n))/Sqrt[b^2 - 4*a*c])*Hypergeometri
c2F1[1, -3/(2*n), 1 - 3/(2*n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*(b
+ Sqrt[b^2 - 4*a*c])*d*(d*x)^(3/2)))/(2*a*(b^2 - 4*a*c)*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1720
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p +
 1) + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*
a*c, 0] && ILtQ[p + 1, 0]
```

rule 1884
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && (
IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{1}{(dx)^{\frac{5}{2}} (a+b\,x^{n}+c\,x^{2n})^{2}}dx$$

input
```
int(1/(d*x)^(5/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int(1/(d*x)^(5/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{1}{(dx)^{5/2} (a + bx^n + cx^{2n})^2} \, dx = \int \frac{1}{(cx^{2\,n} + bx^n + a)^2 (dx)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/(d*x)^(5/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(c^2*d^3*x^3*x^(4*n) + b^2*d^3*x^3*x^(2*n) + 2*a*b*d^3*
x^3*x^n + a^2*d^3*x^3 + 2*(b*c*d^3*x^3*x^n + a*c*d^3*x^3)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{5/2} (a + bx^n + cx^{2n})^2} \, dx = \text{Timed out}$$

input
```
integrate(1/(d*x)**(5/2)/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(dx)^{5/2} (a + bx^n + cx^{2n})^2} \, dx = \int \frac{1}{(cx^{2\,n} + bx^n + a)^2 (dx)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/(d*x)^(5/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
-2/((c^2*d^(5/2)*(8*n + 3)*x^(4*n) + 2*b*c*d^(5/2)*(8*n + 3)*x^(3*n) + 2*a
*b*d^(5/2)*(8*n + 3)*x^n + a^2*d^(5/2)*(8*n + 3) + (b^2*d^(5/2)*(8*n + 3)
+ 2*a*c*d^(5/2)*(8*n + 3))*x^(2*n))*x^(3/2)) + integrate(4*(b*n*x^n + 2*a*
n)/((c^3*d^(5/2)*(8*n + 3)*x^(6*n) + 3*b*c^2*d^(5/2)*(8*n + 3)*x^(5*n) + 3
*a^2*b*d^(5/2)*(8*n + 3)*x^n + a^3*d^(5/2)*(8*n + 3) + 3*(b^2*c*d^(5/2)*(8
*n + 3) + a*c^2*d^(5/2)*(8*n + 3))*x^(4*n) + (b^3*d^(5/2)*(8*n + 3) + 6*a*
b*c*d^(5/2)*(8*n + 3))*x^(3*n) + 3*(a*b^2*d^(5/2)*(8*n + 3) + a^2*c*d^(5/2
)*(8*n + 3))*x^(2*n))*x^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(dx)^{5/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx=\int \frac{1}{\left(cx^{2n}+bx^n+a\right)^2(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(5/2)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^2*(d*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{5/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx=\int \frac{1}{{\left(d\,x\right)}^{5/2}\,{\left(a+b\,x^n+c\,x^{2\,n}\right)}^2}\,dx$$

input
```
int(1/((d*x)^(5/2)*(a + b*x^n + c*x^(2*n))^2),x)
```

output
```
int(1/((d*x)^(5/2)*(a + b*x^n + c*x^(2*n))^2), x)
```

Reduce [F]

$$\int \frac{1}{(dx)^{5/2}\left(a+bx^n+cx^{2n}\right)^2}\,dx = \frac{\sqrt{d}\left(\int \frac{1}{x^{4n+\frac{1}{2}}c^2x^2+2x^{3n+\frac{1}{2}}bc\,x^2+2x^{2n+\frac{1}{2}}ac\,x^2+x^{2n+\frac{1}{2}}b^2x^2+2x^{n+\frac{1}{2}}ab\,x^2+\sqrt{x}\,a^2x^2}dx\right)}{d^3}$$

input
```
int(1/(d*x)^(5/2)/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
(sqrt(d)*int(1/(x**((8*n + 1)/2)*c**2*x**2 + 2*x**((6*n + 1)/2)*b*c*x**2 +
 2*x**((4*n + 1)/2)*a*c*x**2 + x**((4*n + 1)/2)*b**2*x**2 + 2*x**((2*n + 1
)/2)*a*b*x**2 + sqrt(x)*a**2*x**2),x))/d**3
```

3.212 $\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^4\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{4}{n},-\frac{1}{2},-\frac{1}{2},\frac{4+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output

```
1/4*x^4*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(4/n,-1/2,-1/2,(4+n)/n,-2*c*x^n/
(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(148) = 296.

Time = 0.92 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.47

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^4\left(4(4+n)\left(a+x^n(b+cx^n)\right)+an(4+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{4}{n},\frac{1}{2},\frac{1}{2},\frac{4+n}{n},-\ldots\right.\right.}{4(4+\ldots}$$

input
```
Integrate[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^4*(4*(4 + n)*(a + x^n*(b + c*x^n)) + a*n*(4 + n)*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*b*n*x
^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(4 + n)/
n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])]))/(4*(4 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}\int x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{4}{n},-\frac{1}{2},-\frac{1}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -1/2, -1/2, (4 + n)/n, (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^3\sqrt{a+b\,x^n+c\,x^{2n}}dx$$

input
```
int(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx = \int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate(x**3*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(x**3*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx = \int \sqrt{cx^{2n}+bx^n+a}x^3\,dx$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2n}+bx^n+a}x^3\,dx$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx=\int x^3\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^3*(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(x^3*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$$
$$=\frac{2\sqrt{x^{2n}c+x^nb+a}\,x^4-\left(\int \frac{x^{2n}\sqrt{x^{2n}c+x^nb+a}\,x^3}{x^{2n}cn+8x^{2n}c+x^nbn+8x^nb+an+8a}dx\right)c\,n^2-8\left(\int \frac{x^{2n}\sqrt{x^{2n}c+x^nb+a}\,x^3}{x^{2n}cn+8x^{2n}c+x^nbn+8x^nb+an+8a}dx\right)cn}{n+8}$$

input
```
int(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
(2*sqrt(x**(2*n)*c + x**n*b + a)*x**4 - int((x**(2*n)*sqrt(x**(2*n)*c + x*
*n*b + a)*x**3)/(x**(2*n)*c*n + 8*x**(2*n)*c + x**n*b*n + 8*x**n*b + a*n +
 8*a),x)*c*n**2 - 8*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*x**3)/(x**
(2*n)*c*n + 8*x**(2*n)*c + x**n*b*n + 8*x**n*b + a*n + 8*a),x)*c*n + int((
sqrt(x**(2*n)*c + x**n*b + a)*x**3)/(x**(2*n)*c*n + 8*x**(2*n)*c + x**n*b*
n + 8*x**n*b + a*n + 8*a),x)*a*n**2 + 8*int((sqrt(x**(2*n)*c + x**n*b + a)
*x**3)/(x**(2*n)*c*n + 8*x**(2*n)*c + x**n*b*n + 8*x**n*b + a*n + 8*a),x)*
a*n)/(n + 8)
```

3.213 $\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^3\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{3}{n},-\frac{1}{2},-\frac{1}{2},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/3*x^3*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(3/n,-1/2,-1/2,(3+n)/n,-2*c*x^n/
(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 366 vs. 2(148) = 296.

Time = 0.88 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.47

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^3\left(6(3+n)\left(a+x^n(b+cx^n)\right)+2an(3+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{n},\frac{1}{2},\frac{1}{2},\frac{3+n}{n},-\right.\right.}{6(3+}$$

input
```
Integrate[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^3*(6*(3 + n)*(a + x^n*(b + c*x^n)) + 2*a*n*(3 + n)*Sqrt[(b - Sqrt[b^2 -
 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*n
*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 + n
)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])]))/(6*(3 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}\int x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^3\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{3}{n},-\frac{1}{2},-\frac{1}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -1/2, -1/2, (3 + n)/n, (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^2\sqrt{a+b\,x^n+c\,x^{2n}}dx$$

input
```
int(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \sqrt{a + bx^n + cx^{2n}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2 \sqrt{a + bx^n + cx^{2n}}\, dx = \int x^2 \sqrt{a + bx^n + cx^{2n}}\, dx$$

input
```
integrate(x**2*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int x^2 \sqrt{a + bx^n + cx^{2n}}\, dx = \int \sqrt{cx^{2n} + bx^n + a}x^2\, dx$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2n}+bx^n+a}x^2\,dx$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx=\int x^2\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x^2*(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(x^2*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$$
$$=\frac{2\sqrt{x^{2n}c+x^nb+a}\,x^3-\left(\int\frac{x^{2n}\sqrt{x^{2n}c+x^nb+a}\,x^2}{x^{2n}cn+6x^{2n}c+x^nbn+6x^nb+an+6a}dx\right)c\,n^2-6\left(\int\frac{x^{2n}\sqrt{x^{2n}c+x^nb+a}\,x^2}{x^{2n}cn+6x^{2n}c+x^nbn+6x^nb+an+6a}dx\right)cn}{n+6}$$

input
```
int(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
(2*sqrt(x**(2*n)*c + x**n*b + a)*x**3 - int((x**(2*n)*sqrt(x**(2*n)*c + x*
*n*b + a)*x**2)/(x**(2*n)*c*n + 6*x**(2*n)*c + x**n*b*n + 6*x**n*b + a*n +
 6*a),x)*c*n**2 - 6*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*x**2)/(x**
(2*n)*c*n + 6*x**(2*n)*c + x**n*b*n + 6*x**n*b + a*n + 6*a),x)*c*n + int((
sqrt(x**(2*n)*c + x**n*b + a)*x**2)/(x**(2*n)*c*n + 6*x**(2*n)*c + x**n*b*
n + 6*x**n*b + a*n + 6*a),x)*a*n**2 + 6*int((sqrt(x**(2*n)*c + x**n*b + a)
*x**2)/(x**(2*n)*c*n + 6*x**(2*n)*c + x**n*b*n + 6*x**n*b + a*n + 6*a),x)*
a*n)/(n + 6)
```

3.214 $\int x\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 148

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^2\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{2}{n},-\frac{1}{2},-\frac{1}{2},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output

```
1/2*x^2*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(2/n,-1/2,-1/2,(2+n)/n,-2*c*x^n/
(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 364 vs. $2(148) = 296$.

Time = 0.83 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.46

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x^2\left(2(2+n)\left(a+x^n(b+cx^n)\right)+an(2+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{2+n}{n},-\right.\right.}{2(2+n}$$

input
```
Integrate[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^2*(2*(2 + n)*(a + x^n*(b + c*x^n)) + a*n*(2 + n)*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(2 + n)/n,
 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])]))/(2*(2 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^n+cx^{2n}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}\int x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{2}{n},-\frac{1}{2},-\frac{1}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -1/2, -1/2, (2 + n)/n, (-2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x\sqrt{a+b\,x^{n}+c\,x^{2n}}dx$$

input
```
int(x*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(x*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a + bx^n + cx^{2n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{a + bx^n + cx^{2n}}\,dx = \int x\sqrt{a + bx^n + cx^{2n}}\,dx$$

input
```
integrate(x*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(x*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int x\sqrt{a + bx^n + cx^{2n}}\,dx = \int \sqrt{cx^{2n} + bx^n + a}x\,dx$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}x\,dx$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx=\int x\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int(x*(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(x*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{2\sqrt{x^{2n}c+x^nb+a}\,x^2-\left(\int \frac{x^{2n}\sqrt{x^{2n}c+x^nb+a}\,x}{x^{2n}cn+4x^{2n}c+x^nbn+4x^nb+an+4a}dx\right)c\,n^2-4\left(\int \frac{x^{2n}\sqrt{x^{2n}c+x^nb+a}\,x}{x^{2n}cn+4x^{2n}c+x^nbn+4x^nb+an+4a}dx\right)cn}{n+4}$$

input
```
int(x*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
(2*sqrt(x**(2*n)*c + x**n*b + a)*x**2 - int((x**(2*n)*sqrt(x**(2*n)*c + x*
*n*b + a)*x)/(x**(2*n)*c*n + 4*x**(2*n)*c + x**n*b*n + 4*x**n*b + a*n + 4*
a),x)*c*n**2 - 4*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*x)/(x**(2*n)*
c*n + 4*x**(2*n)*c + x**n*b*n + 4*x**n*b + a*n + 4*a),x)*c*n + int((sqrt(x
**(2*n)*c + x**n*b + a)*x)/(x**(2*n)*c*n + 4*x**(2*n)*c + x**n*b*n + 4*x**
n*b + a*n + 4*a),x)*a*n**2 + 4*int((sqrt(x**(2*n)*c + x**n*b + a)*x)/(x**(
2*n)*c*n + 4*x**(2*n)*c + x**n*b*n + 4*x**n*b + a*n + 4*a),x)*a*n)/(n + 4)
```

3.215 $\int \sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 139

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
x*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(1/n,-1/2,-1/2,1+1/n,-2*c*x^n/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(139) = 278.

Time = 0.85 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.53

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x\left(bnx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+2(1+\right.}{2(1+n)^2}$$

input
```
Integrate[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*(b*n*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1
[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 + n)*(a + x^n*(b + c*x^n) + a*n*Sq
rt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2,
1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])))/(2*(1 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \ 1686$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}\int \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \ 936$$

$$\frac{x\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -1/2, -1/2, 1 + n^(-1), (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sqrt{a+b\,x^{n}+c\,x^{2n}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a + bx^n + cx^{2n}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{a + bx^n + cx^{2n}}\, dx = \int \sqrt{a + bx^n + cx^{2n}}\, dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \sqrt{a + bx^n + cx^{2n}}\, dx = \int \sqrt{cx^{2n} + bx^n + a}\, dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{2\sqrt{x^{2n}c+x^nb+a}\,x+\left(\int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{2n}cn+2x^{2n}c+x^nbn+2x^nb+an+2a}dx\right)a\,n^2+2\left(\int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{2n}cn+2x^{2n}c+x^nbn+2x^nb+an+2a}dx\right)an}{n+2}$$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
(2*sqrt(x**(2*n)*c + x**n*b + a)*x + int(sqrt(x**(2*n)*c + x**n*b + a)/(x*
*(2*n)*c*n + 2*x**(2*n)*c + x**n*b*n + 2*x**n*b + a*n + 2*a),x)*a*n**2 + 2
*int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(2*n)*c*n + 2*x**(2*n)*c + x**n*b*n
 + 2*x**n*b + a*n + 2*a),x)*a*n - int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b +
 a))/(x**(2*n)*c*n + 2*x**(2*n)*c + x**n*b*n + 2*x**n*b + a*n + 2*a),x)*c*
n**2 - 2*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a))/(x**(2*n)*c*n + 2*x*
*(2*n)*c + x**n*b*n + 2*x**n*b + a*n + 2*a),x)*c*n)/(n + 2)
```

3.216 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x} dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x} dx = \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n}$$

output
```
(a+b*x^n+c*x^(2*n))^(1/2)/n-a^(1/2)*arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x
^n+c*x^(2*n))^(1/2))/n+1/2*b*arctanh(1/2*(b+2*c*x^n)/c^(1/2)/(a+b*x^n+c*x^
(2*n))^(1/2))/c^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x} dx = \frac{2\sqrt{a+x^n(b+cx^n)} + 4\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}x^n-\sqrt{a+x^n(b+cx^n)}}{\sqrt{a}}\right) - \frac{b\log\left(n\left(b+2cx^n-2\sqrt{c}\sqrt{a+x^n(b+cx^n)}\right)\right)}{\sqrt{c}}}{2n}$$

input
```
Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

output
```
(2*Sqrt[a + x^n*(b + c*x^n)] + 4*Sqrt[a]*ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x
^n*(b + c*x^n)])/Sqrt[a]] - (b*Log[n*(b + 2*c*x^n - 2*Sqrt[c]*Sqrt[a + x^n
*(b + c*x^n)])])/Sqrt[c])/(2*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1693, 1162, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{\int x^{-n}\sqrt{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \text{ 1162}$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}-\frac{1}{2}\int -\frac{x^{-n}(bx^n+2a)}{\sqrt{bx^n+cx^{2n}+a}}dx^n}{n}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{1}{2}\int \frac{x^{-n}(bx^n+2a)}{\sqrt{bx^n+cx^{2n}+a}}dx^n+\sqrt{a+bx^n+cx^{2n}}}{n}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{\frac{1}{2}\left(b\int \frac{1}{\sqrt{bx^n+cx^{2n}+a}}dx^n+2a\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n\right)+\sqrt{a+bx^n+cx^{2n}}}{n}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{\frac{1}{2}\left(2b\int \frac{1}{4c-x^{2n}}d\frac{2cx^n+b}{\sqrt{bx^n+cx^{2n}+a}}+2a\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n\right)+\sqrt{a+bx^n+cx^{2n}}}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(2a\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n+\frac{b\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{c}}\right)+\sqrt{a+bx^n+cx^{2n}}}{n}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{c}}-4a\int \frac{1}{4a-x^{2n}}d\frac{bx^n+2a}{\sqrt{bx^n+cx^{2n}+a}}\right)+\sqrt{a+bx^n+cx^{2n}}}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{c}}-2\sqrt{a}\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)\right)+\sqrt{a+bx^n+cx^{2n}}}{n}$$

input
```
Int[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

output
```
(Sqrt[a + b*x^n + c*x^(2*n)] + (-2*Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a
]*Sqrt[a + b*x^n + c*x^(2*n)])] + (b*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt
[a + b*x^n + c*x^(2*n)])])/Sqrt[c])/2)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05

method	result
risch	$\frac{\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}+c\,{\mathrm e}^{2n\ln\left(x\right)}}}{n}+\frac{b\ln\left(\frac{\frac{b}{2}+{\mathrm e}^{n\ln\left(x\right)}c}{\sqrt{c}}+\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}+c\,{\mathrm e}^{2n\ln\left(x\right)}}\right)}{2n\sqrt{c}}-\frac{\sqrt{a}\,\ln\left(\left(2a+b\,{\mathrm e}^{n\ln\left(x\right)}+2\sqrt{a}\,\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}+c\,{\mathrm e}^{2n}}\right.\right.}{n}$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)+1/2/n*b*ln((1/2*b+exp(n*ln(x
))*c)/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))/c^(1/2)-1/n*a^(1/
2)*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1
/2))/exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 658, normalized size of antiderivative = 5.53

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx$$

$$= \left[\frac{b\sqrt{c}\log\left(-8\,c^2x^{2\,n}-8\,bcx^n-b^2-4\,ac-4\left(2\,c^{\frac{3}{2}}x^n+b\sqrt{c}\right)\sqrt{cx^{2\,n}+bx^n+a}\right)+2\,\sqrt{a}c\log\left(-\frac{8\,abx^n+8}{}\right.}{4\,cn}\right.$$

$$-\frac{b\sqrt{-c}\arctan\left(\frac{\left(2\,\sqrt{-c}cx^n+b\sqrt{-c}\right)\sqrt{cx^{2\,n}+bx^n+a}}{2\,\left(c^2x^{2\,n}+bcx^n+ac\right)}\right)-\sqrt{a}c\log\left(-\frac{8\,abx^n+8\,a^2+\left(b^2+4\,ac\right)x^{2\,n}-4\left(\sqrt{a}bx^n+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n}+bx^{n}}}{x^{2\,n}}\right.}{2\,cn}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/4*(b*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2
)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 2*sqrt(a)*c*log(-(8*a*b*
x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c
*x^(2*n) + b*x^n + a))/x^(2*n)) + 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n),
-1/2*(b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n)
 + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) - sqrt(a)*c*log(-(8*a*b*x^n +
 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2
*n) + b*x^n + a))/x^(2*n)) - 2*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/4*(
4*sqrt(-a)*c*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b
*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) + b*sqrt(c)*log(-8*c^2*x^(2*n) -
8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b
*x^n + a)) + 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/2*(2*sqrt(-a)*c*arc
tan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x
^(2*n) + a*b*x^n + a^2)) - b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqr
t(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*sqrt
(c*x^(2*n) + b*x^n + a)*c)/(c*n)]
```

Sympy [F]

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x}\, dx = \int \frac{\sqrt{a + bx^n + cx^{2n}}}{x}\, dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*x**n + c*x**(2*n))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x}\, dx = \int \frac{\sqrt{cx^{2n} + bx^n + a}}{x}\, dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx=\int \frac{\sqrt{cx^{2n}+bx^n+a}}{x}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx=\int \frac{\sqrt{a+b\,x^n+c\,x^{2\,n}}}{x}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(1/2)/x,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx = \frac{2\sqrt{x^{2n}c+x^nb+a}+\left(\int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{2n}cx+x^nbx+ax}dx\right)an-\left(\int \frac{x^{2n}\sqrt{x^{2n}c+x^nb+a}}{x^{2n}cx+x^nbx+ax}dx\right)cn}{n}$$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2)/x,x)
```

output
```
(2*sqrt(x**(2*n)*c + x**n*b + a) + int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(
2*n)*c*x + x**n*b*x + a*x),x)*a*n - int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b
 + a))/(x**(2*n)*c*x + x**n*b*x + a*x),x)*c*n)/n
```

3.217 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = -\frac{\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{1}{2},-\frac{1}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
-(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(-1/n,-1/2,-1/2,-(1-n)/n,-2*c*x^n/(b-(-
4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/x/(1+2*c*x^n/(b-(-4*a*c
+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(149) = 298.

Time = 0.76 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.45

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \frac{2(-1+n)\left(a+x^n(b+cx^n)\right)-2a(-1+n)n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},\frac{-1+n}{n}\right.}{2(-1+}$$

input
```
Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

output
```
(2*(-1 + n)*(a + x^n*(b + c*x^n)) - 2*a*(-1 + n)*n*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n
*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(-1 +
n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/
(-b + Sqrt[b^2 - 4*a*c])])/(2*(-1 + n)^2*x*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}\int \frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}{x^2}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{1}{2},-\frac{1}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

output
```
-((Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -1/2, -1/2, -((1 - n)/n),
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}+c\,x^{2n}}}{x^{2}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*x**n + c*x**(2*n))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \int \frac{\sqrt{cx^{2n}+bx^n+a}}{x^2}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx=\int \frac{\sqrt{cx^{2\,n}+bx^n+a}}{x^2}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx=\int \frac{\sqrt{a+b\,x^n+c\,x^{2\,n}}}{x^2}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(1/2)/x^2,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \frac{2\sqrt{x^{2n}c+x^nb+a}+\left(\int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{2n}cn\,x^2-2x^{2n}c\,x^2+x^nbn\,x^2-2x^nb\,x^2+an\,x^2-2a\,x^2}dx\right)a\,n^2x-2\left(\int \frac{\sqrt{x^{2n}c+x^n}}{x^{2n}cn\,x^2-2x^{2n}c\,x^2+x^nbn\,x^2-}\right.}{}$$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x)
```

output
```
(2*sqrt(x**(2*n)*c + x**n*b + a) + int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(
2*n)*c*n*x**2 - 2*x**(2*n)*c*x**2 + x**n*b*n*x**2 - 2*x**n*b*x**2 + a*n*x*
*2 - 2*a*x**2),x)*a*n**2*x - 2*int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(2*n)
*c*n*x**2 - 2*x**(2*n)*c*x**2 + x**n*b*n*x**2 - 2*x**n*b*x**2 + a*n*x**2 -
 2*a*x**2),x)*a*n*x - int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a))/(x**(2*
n)*c*n*x**2 - 2*x**(2*n)*c*x**2 + x**n*b*n*x**2 - 2*x**n*b*x**2 + a*n*x**2
 - 2*a*x**2),x)*c*n**2*x + 2*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a))/
(x**(2*n)*c*n*x**2 - 2*x**(2*n)*c*x**2 + x**n*b*n*x**2 - 2*x**n*b*x**2 + a
*n*x**2 - 2*a*x**2),x)*c*n*x)/(x*(n - 2))
```

3.218 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = -\frac{\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{2}{n},-\frac{1}{2},-\frac{1}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
-1/2*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(-2/n,-1/2,-1/2,-(2-n)/n,-2*c*x^n/(
b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/x^2/(1+2*c*x^n/(b-(
-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(151) = 302.

Time = 0.81 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.42

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = \frac{2(-2+n)\left(a+x^n(b+cx^n)\right)-a(-2+n)n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{-2+n}{n},\right.}{2(-2+}$$

input
```
Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

output
```
(2*(-2 + n)*(a + x^n*(b + c*x^n)) - a*(-2 + n)*n*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n*
Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(-2 + n)/n,
 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(2*(-2 + n)^2*x^2*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}\int \frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}{x^3}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(-\frac{2}{n},-\frac{1}{2},-\frac{1}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

output
```
-1/2*(Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -1/2, -1/2, -((2 - n)/n),
 (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
(x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}+c\,x^{2n}}}{x^{3}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x^3} \, dx = \int \frac{\sqrt{a + bx^n + cx^{2n}}}{x^3} \, dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a + b*x**n + c*x**(2*n))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x^3} \, dx = \int \frac{\sqrt{cx^{2n} + bx^n + a}}{x^3} \, dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx=\int \frac{\sqrt{cx^{2n}+bx^n+a}}{x^3}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx=\int \frac{\sqrt{a+b\,x^n+c\,x^{2\,n}}}{x^3}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(1/2)/x^3,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx$$
$$=\frac{2\sqrt{x^{2n}c+x^nb+a}+\left(\int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{2n}cn\,x^3-4x^{2n}c\,x^3+x^nbn\,x^3-4x^nb\,x^3+an\,x^3-4a\,x^3}dx\right)a\,n^2x^2-4\left(\int \frac{\sqrt{x^{2n}c+x}}{x^{2n}cn\,x^3-4x^{2n}c\,x^3+x^nbn\,x^3}\right.}{}$$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x)
```

output
```
(2*sqrt(x**(2*n)*c + x**n*b + a) + int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(
2*n)*c*n*x**3 - 4*x**(2*n)*c*x**3 + x**n*b*n*x**3 - 4*x**n*b*x**3 + a*n*x*
*3 - 4*a*x**3),x)*a*n**2*x**2 - 4*int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(2
*n)*c*n*x**3 - 4*x**(2*n)*c*x**3 + x**n*b*n*x**3 - 4*x**n*b*x**3 + a*n*x**
3 - 4*a*x**3),x)*a*n*x**2 - int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a))/(
x**(2*n)*c*n*x**3 - 4*x**(2*n)*c*x**3 + x**n*b*n*x**3 - 4*x**n*b*x**3 + a*
n*x**3 - 4*a*x**3),x)*c*n**2*x**2 + 4*int((x**(2*n)*sqrt(x**(2*n)*c + x**n
*b + a))/(x**(2*n)*c*n*x**3 - 4*x**(2*n)*c*x**3 + x**n*b*n*x**3 - 4*x**n*b
*x**3 + a*n*x**3 - 4*a*x**3),x)*c*n*x**2)/(x**2*(n - 4))
```

3.219 $\int x^3(a+bx^n+cx^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int x^3\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\frac{ax^4\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{4}{n},-\frac{3}{2},-\frac{3}{2},\frac{4+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/4*a*x^4*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(4/n,-3/2,-3/2,(4+n)/n,-2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^n/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 469 vs. 2(149) = 298.

Time = 1.68 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.15

$$\int x^3\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\frac{x^4\Big(2(4+n)\left(3b^2n^2+32ac(2+3n+n^2)+2bc(32+36n+7n^2)\,x^n+8c^2(8+6n+n^2)\,x^{2n}\right.}{}$$

input
```
Integrate[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^4*(2*(4 + n)*(3*b^2*n^2 + 32*a*c*(2 + 3*n + n^2) + 2*b*c*(32 + 36*n + 7
*n^2)*x^n + 8*c^2*(8 + 6*n + n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 6*a*n^2
*(4 + n)*(b^2 - 2*a*c*(2 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(b^2*(8 + n) - 4*a*c
*(8 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appe
llF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(16*c*(2 + n)*(4 + n)^2*(4 + 3*n)*Sqrt[
a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}} \int x^3 \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2} \left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2} dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{ax^4\sqrt{a+bx^n+cx^{2n}} \operatorname{AppellF1}\left(\frac{4}{n},-\frac{3}{2},-\frac{3}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(a*x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -3/2, -3/2, (4 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*
Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^3 \left(a + b\,x^n + c\,x^{2n}\right)^{\frac{3}{2}} dx$$

input
```
int(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3 \left(a + bx^n + cx^{2n}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3 \left(a + bx^n + cx^{2n}\right)^{3/2} dx = \int x^3 \left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**3*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(x**3*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int x^3 \left(a + bx^n + cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} x^3 dx$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

Giac [F]

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} x^3 \, dx$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int x^3 \left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2} dx$$

input
```
int(x^3*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(x^3*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

Reduce [F]

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \text{too large to display}$$

input
```
int(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output

```
(8*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c*n**2*x**4 + 80*x**(2*n)*sqrt(x
**(2*n)*c + x**n*b + a)*c*n*x**4 + 128*x**(2*n)*sqrt(x**(2*n)*c + x**n*b +
 a)*c*x**4 + 14*x**n*sqrt(x**(2*n)*c + x**n*b + a)*b*n**2*x**4 + 128*x**n*
sqrt(x**(2*n)*c + x**n*b + a)*b*n*x**4 + 128*x**n*sqrt(x**(2*n)*c + x**n*b
 + a)*b*x**4 + 68*sqrt(x**(2*n)*c + x**n*b + a)*a*n**2*x**4 + 176*sqrt(x**
(2*n)*c + x**n*b + a)*a*n*x**4 + 128*sqrt(x**(2*n)*c + x**n*b + a)*a*x**4
- 108*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*x**3)/(3*x**(2*n)*c*n**3
 + 34*x**(2*n)*c*n**2 + 88*x**(2*n)*c*n + 64*x**(2*n)*c + 3*x**n*b*n**3 +
34*x**n*b*n**2 + 88*x**n*b*n + 64*x**n*b + 3*a*n**3 + 34*a*n**2 + 88*a*n +
 64*a),x)*a*c*n**6 - 1512*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*x**3
)/(3*x**(2*n)*c*n**3 + 34*x**(2*n)*c*n**2 + 88*x**(2*n)*c*n + 64*x**(2*n)*
c + 3*x**n*b*n**3 + 34*x**n*b*n**2 + 88*x**n*b*n + 64*x**n*b + 3*a*n**3 +
34*a*n**2 + 88*a*n + 64*a),x)*a*c*n**5 - 6432*int((x**(2*n)*sqrt(x**(2*n)*
c + x**n*b + a)*x**3)/(3*x**(2*n)*c*n**3 + 34*x**(2*n)*c*n**2 + 88*x**(2*n
)*c*n + 64*x**(2*n)*c + 3*x**n*b*n**3 + 34*x**n*b*n**2 + 88*x**n*b*n + 64*
x**n*b + 3*a*n**3 + 34*a*n**2 + 88*a*n + 64*a),x)*a*c*n**4 - 10752*int((x*
*(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*x**3)/(3*x**(2*n)*c*n**3 + 34*x**(2*n
)*c*n**2 + 88*x**(2*n)*c*n + 64*x**(2*n)*c + 3*x**n*b*n**3 + 34*x**n*b*n**
2 + 88*x**n*b*n + 64*x**n*b + 3*a*n**3 + 34*a*n**2 + 88*a*n + 64*a),x)*a*c
*n**3 - 6144*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*x**3)/(3*x**(2...
```

3.220 $\int x^2(a+bx^n+cx^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int x^2(a+bx^n+cx^{2n})^{3/2}\,dx=\frac{ax^3\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{3}{n},-\frac{3}{2},-\frac{3}{2},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output

```
1/3*a*x^3*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(3/n,-3/2,-3/2,(3+n)/n,-2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^n/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 475 vs. 2(149) = 298.

Time = 1.68 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.19

$$\int x^2(a+bx^n+cx^{2n})^{3/2}\,dx=\frac{x^3\left(2(3+n)\left(3b^2n^2+4ac(9+18n+8n^2)+2bc(18+27n+7n^2)x^n+4c^2(9+9n+2n^2)x\right.\right.}{}$$

input
```
Integrate[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^3*(2*(3 + n)*(3*b^2*n^2 + 4*a*c*(9 + 18*n + 8*n^2) + 2*b*c*(18 + 27*n +
 7*n^2)*x^n + 4*c^2*(9 + 9*n + 2*n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) + 2*a
*n^2*(3 + n)*(-3*b^2 + 4*a*c*(3 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(-12*a*c*(2
 + n) + b^2*(6 + n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c])]*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(24*c*(1 + n)*(3 + n)^2*(3 + 2
*n)*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^n+cx^{2n}\right)^{3/2} dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}} \int x^2 \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2} \left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2} dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{ax^3\sqrt{a+bx^n+cx^{2n}} \operatorname{AppellF1}\left(\frac{3}{n},-\frac{3}{2},-\frac{3}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(a*x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -3/2, -3/2, (3 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*
Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^2 \left(a + b\,x^n + c\,x^{2n}\right)^{\frac{3}{2}} dx$$

input
```
int(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \left(a + bx^n + cx^{2n}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2 \left(a + bx^n + cx^{2n}\right)^{3/2} dx = \int x^2 \left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**2*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(x**2*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int x^2 \left(a + bx^n + cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n} + bx^n + a\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int x^2\,\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

input
```
int(x^2*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(x^2*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

Reduce [F]

$$\int x^2\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\text{too large to display}$$

input
```
int(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
(8*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c*n**2*x**3 + 60*x**(2*n)*sqrt(x
**(2*n)*c + x**n*b + a)*c*n*x**3 + 72*x**(2*n)*sqrt(x**(2*n)*c + x**n*b +
a)*c*x**3 + 14*x**n*sqrt(x**(2*n)*c + x**n*b + a)*b*n**2*x**3 + 96*x**n*sq
rt(x**(2*n)*c + x**n*b + a)*b*n*x**3 + 72*x**n*sqrt(x**(2*n)*c + x**n*b +
a)*b*x**3 + 68*sqrt(x**(2*n)*c + x**n*b + a)*a*n**2*x**3 + 132*sqrt(x**(2*
n)*c + x**n*b + a)*a*n*x**3 + 72*sqrt(x**(2*n)*c + x**n*b + a)*a*x**3 - 72
*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*x**2)/(2*x**(2*n)*c*n**3 + 17
*x**(2*n)*c*n**2 + 33*x**(2*n)*c*n + 18*x**(2*n)*c + 2*x**n*b*n**3 + 17*x*
*n*b*n**2 + 33*x**n*b*n + 18*x**n*b + 2*a*n**3 + 17*a*n**2 + 33*a*n + 18*a
),x)*a*c*n**6 - 756*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*x**2)/(2*x
**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 33*x**(2*n)*c*n + 18*x**(2*n)*c + 2*
x**n*b*n**3 + 17*x**n*b*n**2 + 33*x**n*b*n + 18*x**n*b + 2*a*n**3 + 17*a*n
**2 + 33*a*n + 18*a),x)*a*c*n**5 - 2412*int((x**(2*n)*sqrt(x**(2*n)*c + x*
*n*b + a)*x**2)/(2*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 33*x**(2*n)*c*n
+ 18*x**(2*n)*c + 2*x**n*b*n**3 + 17*x**n*b*n**2 + 33*x**n*b*n + 18*x**n*b
 + 2*a*n**3 + 17*a*n**2 + 33*a*n + 18*a),x)*a*c*n**4 - 3024*int((x**(2*n)*
sqrt(x**(2*n)*c + x**n*b + a)*x**2)/(2*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**
2 + 33*x**(2*n)*c*n + 18*x**(2*n)*c + 2*x**n*b*n**3 + 17*x**n*b*n**2 + 33*
x**n*b*n + 18*x**n*b + 2*a*n**3 + 17*a*n**2 + 33*a*n + 18*a),x)*a*c*n**3 -
 1296*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*x**2)/(2*x**(2*n)*c*n...
```

3.221 $\int x(a+bx^n+cx^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 149

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\frac{ax^2\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{2}{n},-\frac{3}{2},-\frac{3}{2},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
1/2*a*x^2*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(2/n,-3/2,-3/2,(2+n)/n,-2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^n/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 471 vs. 2(149) = 298.

Time = 1.66 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.16

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\frac{x^2\Big(2(2+n)\left(3b^2n^2+16ac(1+3n+2n^2)+2bc(8+18n+7n^2)\,x^n+8c^2(2+3n+n^2)\,x^{2n}\right.}{}$$

input
```
Integrate[x*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^2*(2*(2 + n)*(3*b^2*n^2 + 16*a*c*(1 + 3*n + 2*n^2) + 2*b*c*(8 + 18*n +
7*n^2)*x^n + 8*c^2*(2 + 3*n + n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 6*a*n^
2*(2 + n)*(b^2 - 4*a*c*(1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(b^2*(4 + n) - 4*a*
c*(4 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*App
ellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(16*c*(1 + n)*(2 + n)^2*(2 + 3*n)*Sqrt
[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}}\int x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{ax^2\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{2}{n},-\frac{3}{2},-\frac{3}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[x*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(a*x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -3/2, -3/2, (2 + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*
Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x\left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}dx$$

input
```
int(x*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(x*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int x\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(x*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

Giac [F]

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\int x\,\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

input
```
int(x*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(x*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

Reduce [F]

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\text{too large to display}$$

input
```
int(x*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output

```
(8*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c*n**2*x**2 + 40*x**(2*n)*sqrt(x
**(2*n)*c + x**n*b + a)*c*n*x**2 + 32*x**(2*n)*sqrt(x**(2*n)*c + x**n*b +
a)*c*x**2 + 14*x**n*sqrt(x**(2*n)*c + x**n*b + a)*b*n**2*x**2 + 64*x**n*sq
rt(x**(2*n)*c + x**n*b + a)*b*n*x**2 + 32*x**n*sqrt(x**(2*n)*c + x**n*b +
a)*b*x**2 + 68*sqrt(x**(2*n)*c + x**n*b + a)*a*n**2*x**2 + 88*sqrt(x**(2*n
)*c + x**n*b + a)*a*n*x**2 + 32*sqrt(x**(2*n)*c + x**n*b + a)*a*x**2 - 108
*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*x)/(3*x**(2*n)*c*n**3 + 17*x*
*(2*n)*c*n**2 + 22*x**(2*n)*c*n + 8*x**(2*n)*c + 3*x**n*b*n**3 + 17*x**n*b
*n**2 + 22*x**n*b*n + 8*x**n*b + 3*a*n**3 + 17*a*n**2 + 22*a*n + 8*a),x)*a
*c*n**6 - 756*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*x)/(3*x**(2*n)*c
*n**3 + 17*x**(2*n)*c*n**2 + 22*x**(2*n)*c*n + 8*x**(2*n)*c + 3*x**n*b*n**
3 + 17*x**n*b*n**2 + 22*x**n*b*n + 8*x**n*b + 3*a*n**3 + 17*a*n**2 + 22*a*
n + 8*a),x)*a*c*n**5 - 1608*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*x)
/(3*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 22*x**(2*n)*c*n + 8*x**(2*n)*c
+ 3*x**n*b*n**3 + 17*x**n*b*n**2 + 22*x**n*b*n + 8*x**n*b + 3*a*n**3 + 17*
a*n**2 + 22*a*n + 8*a),x)*a*c*n**4 - 1344*int((x**(2*n)*sqrt(x**(2*n)*c +
x**n*b + a)*x)/(3*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 22*x**(2*n)*c*n +
 8*x**(2*n)*c + 3*x**n*b*n**3 + 17*x**n*b*n**2 + 22*x**n*b*n + 8*x**n*b +
3*a*n**3 + 17*a*n**2 + 22*a*n + 8*a),x)*a*c*n**3 - 384*int((x**(2*n)*sqrt(
x**(2*n)*c + x**n*b + a)*x)/(3*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 2...
```

3.222 $\int (a+bx^n+cx^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 140

$$\int (a+bx^n+cx^{2n})^{3/2}\,dx=\frac{ax\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
a*x*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(1/n,-3/2,-3/2,1+1/n,-2*c*x^n/(b-(-4
*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^n/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 466 vs. 2(140) = 280.

Time = 1.53 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.33

$$\int (a+bx^n+cx^{2n})^{3/2}\,dx=\frac{x\left(-3bn^2(b^2(2+n)-4ac(2+3n))\,x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(1+\frac{1}{n},\right.\right.}{}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x*(-3*b*n^2*(b^2*(2 + n) - 4*a*c*(2 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*
(1 + n)*((3*b^2*n^2 + 4*a*c*(1 + 6*n + 8*n^2) + 2*b*c*(2 + 9*n + 7*n^2)*x^
n + 4*c^2*(1 + 3*n + 2*n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 3*a*n^2*(b^2
- 4*a*c*(1 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*A
ppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(8*c*(1 + n)^2*(1 + 2*n)*(1 + 3*n)*
Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2} dx$$

$$\Big\downarrow \text{1686}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}}\int \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{ax\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(a*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -3/2, -3/2, 1 + n^(-1),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(
Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \int \left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \int \left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2} dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(3/2), x)
```

Reduce [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \text{too large to display}$$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
(8*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c*n**2*x + 20*x**(2*n)*sqrt(x**(
2*n)*c + x**n*b + a)*c*n*x + 8*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c*x
+ 14*x**n*sqrt(x**(2*n)*c + x**n*b + a)*b*n**2*x + 32*x**n*sqrt(x**(2*n)*c
 + x**n*b + a)*b*n*x + 8*x**n*sqrt(x**(2*n)*c + x**n*b + a)*b*x + 68*sqrt(
x**(2*n)*c + x**n*b + a)*a*n**2*x + 44*sqrt(x**(2*n)*c + x**n*b + a)*a*n*x
 + 8*sqrt(x**(2*n)*c + x**n*b + a)*a*x + 144*int(sqrt(x**(2*n)*c + x**n*b
+ a)/(6*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 11*x**(2*n)*c*n + 2*x**(2*n
)*c + 6*x**n*b*n**3 + 17*x**n*b*n**2 + 11*x**n*b*n + 2*x**n*b + 6*a*n**3 +
 17*a*n**2 + 11*a*n + 2*a),x)*a**2*n**6 + 408*int(sqrt(x**(2*n)*c + x**n*b
 + a)/(6*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 11*x**(2*n)*c*n + 2*x**(2*
n)*c + 6*x**n*b*n**3 + 17*x**n*b*n**2 + 11*x**n*b*n + 2*x**n*b + 6*a*n**3
+ 17*a*n**2 + 11*a*n + 2*a),x)*a**2*n**5 + 264*int(sqrt(x**(2*n)*c + x**n*
b + a)/(6*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 11*x**(2*n)*c*n + 2*x**(2
*n)*c + 6*x**n*b*n**3 + 17*x**n*b*n**2 + 11*x**n*b*n + 2*x**n*b + 6*a*n**3
 + 17*a*n**2 + 11*a*n + 2*a),x)*a**2*n**4 + 48*int(sqrt(x**(2*n)*c + x**n*
b + a)/(6*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 11*x**(2*n)*c*n + 2*x**(2
*n)*c + 6*x**n*b*n**3 + 17*x**n*b*n**2 + 11*x**n*b*n + 2*x**n*b + 6*a*n**3
 + 17*a*n**2 + 11*a*n + 2*a),x)*a**2*n**3 - 216*int((x**(2*n)*sqrt(x**(2*n
)*c + x**n*b + a))/(6*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 11*x**(2*n)*c
*n + 2*x**(2*n)*c + 6*x**n*b*n**3 + 17*x**n*b*n**2 + 11*x**n*b*n + 2*x*...
```

3.223 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 173

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x} \, dx = \frac{(b^2+8ac+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{(a+bx^n+cx^{2n})^{3/2}}{3n} - \frac{a^{3/2}\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} - \frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n}$$

output
```
1/8*(b^2+8*a*c+2*b*c*x^n)*(a+b*x^n+c*x^(2*n))^(1/2)/c/n+1/3*(a+b*x^n+c*x^(
2*n))^(3/2)/n-a^(3/2)*arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^
(1/2))/n-1/16*b*(-12*a*c+b^2)*arctanh(1/2*(b+2*c*x^n)/c^(1/2)/(a+b*x^n+c*x
^(2*n))^(1/2))/c^(3/2)/n
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx = \frac{2\sqrt{c}\sqrt{a+x^n\left(b+cx^n\right)}(3b^2+14bcx^n+8c(4a+cx^{2n}))+96a^{3/2}c^{3/2}\text{arctanh}\Big(\sqrt{\ldots}}{48\ldots}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x^n*(b + c*x^n)]*(3*b^2 + 14*b*c*x^n + 8*c*(4*a + c*x^
(2*n))) + 96*a^(3/2)*c^(3/2)*ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^n*(b + c*x^
n)])/Sqrt[a]] + 3*(b^3 - 12*a*b*c)*Log[c*n*(b + 2*c*x^n - 2*Sqrt[c]*Sqrt[a
 + x^n*(b + c*x^n)])])/(48*c^(3/2)*n)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1693, 1162, 25, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx$$

$$\Big\downarrow \; 1693$$

$$\frac{\int x^{-n}\left(bx^n+cx^{2n}+a\right)^{3/2}dx^n}{n}$$

$$\Big\downarrow \; 1162$$

$$\frac{\frac{1}{3}\left(a+bx^n+cx^{2n}\right)^{3/2}-\frac{1}{2}\int -x^{-n}(bx^n+2a)\sqrt{bx^n+cx^{2n}+a}dx^n}{n}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\int x^{-n}(bx^n+2a)\sqrt{bx^n+cx^{2n}+a}dx^n+\frac{1}{3}\left(a+bx^n+cx^{2n}\right)^{3/2}}{n}$$

$$\Big\downarrow \; 1231$$

$$\frac{\frac{1}{2}\left(\frac{\left(8ac+b^2+2bcx^n\right)\sqrt{a+bx^n+cx^{2n}}}{4c}-\frac{\int -\frac{x^{-n}\left(16a^2c-b\left(b^2-12ac\right)x^n\right)}{2\sqrt{bx^n+cx^{2n}+a}}dx^n}{4c}\right)+\frac{1}{3}\left(a+bx^n+cx^{2n}\right)^{3/2}}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}\left(\frac{\int \frac{x^{-n}\left(16a^2c-b\left(b^2-12ac\right)x^n\right)}{\sqrt{bx^n+cx^{2n}+a}}dx^n}{8c}+\frac{\sqrt{a+bx^n+cx^{2n}}\left(8ac+b^2+2bcx^n\right)}{4c}\right)+\frac{1}{3}\left(a+bx^n+cx^{2n}\right)^{3/2}}{n}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{1}{2}\left(\frac{16a^2c\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n-b\left(b^2-12ac\right)\int \frac{1}{\sqrt{bx^n+cx^{2n}+a}}dx^n}{8c}+\frac{\sqrt{a+bx^n+cx^{2n}}\left(8ac+b^2+2bcx^n\right)}{4c}\right)+\frac{1}{3}\left(a+bx^n+cx^{2n}\right)^{3/2}}{n}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{1}{2}\left(\frac{16a^2c\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n-2b\left(b^2-12ac\right)\int \frac{1}{4c-x^{2n}}d\frac{2cx^n+b}{\sqrt{bx^n+cx^{2n}+a}}}{8c}+\frac{\sqrt{a+bx^n+cx^{2n}}\left(8ac+b^2+2bcx^n\right)}{4c}\right)+\frac{1}{3}\left(a+bx^n+cx^{2n}\right)^{3/2}}{n}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{2}\left(\frac{16a^2c\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n-\frac{b\left(b^2-12ac\right)\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^n+cx^{2n}}\left(8ac+b^2+2bcx^n\right)}{4c}\right)+\frac{1}{3}\left(a+bx^n+cx^{2n}\right)}{n}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{1}{2}\left(\frac{-32a^2c\int \frac{1}{4a-x^{2n}}d\frac{bx^n+2a}{\sqrt{bx^n+cx^{2n}+a}}-\frac{b\left(b^2-12ac\right)\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^n+cx^{2n}}\left(8ac+b^2+2bcx^n\right)}{4c}\right)+\frac{1}{3}\left(a+bx^n+\right.}{n}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{2}\left(\frac{-16a^{3/2}c\,\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^n+cx^{2n}}(8ac+b^2+2bcx^n)}{4c}\right)+\frac{1}{3}(a+}{n}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

output
```
((a + b*x^n + c*x^(2*n))^(3/2)/3 + (((b^2 + 8*a*c + 2*b*c*x^n)*Sqrt[a + b*
x^n + c*x^(2*n)])/(4*c) + (-16*a^(3/2)*c*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*
Sqrt[a + b*x^n + c*x^(2*n)])] - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^n)/(2
*Sqrt[c]*Sqrt[a + b*x^n + c*x^(2*n)])])/Sqrt[c])/(8*c))/2)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.21

method	result
risch	$\frac{\left(8c^{2}\mathrm{e}^{2n\ln(x)}+14bc\,\mathrm{e}^{n\ln(x)}+32ac+3b^{2}\right)\sqrt{a+b\,\mathrm{e}^{n\ln(x)}+c\,\mathrm{e}^{2n\ln(x)}}}{24cn}-\frac{b^{3}\ln\left(\frac{\frac{b}{2}+\mathrm{e}^{n\ln(x)}c}{\sqrt{c}}+\sqrt{a+b\,\mathrm{e}^{n\ln(x)}+c\,\mathrm{e}^{2n\ln(x)}}\right)}{16c^{\frac{3}{2}}n}+\frac{3ab\ln\left(\right.}{}$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*c^2*exp(n*ln(x))^2+14*b*c*exp(n*ln(x))+32*a*c+3*b^2)*(a+b*exp(n*ln
(x))+c*exp(n*ln(x))^2)^(1/2)/c/n-1/16/c^(3/2)/n*b^3*ln((1/2*b+exp(n*ln(x))
*c)/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))+3/4/c^(1/2)/n*a*b*l
n((1/2*b+exp(n*ln(x))*c)/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)
)-1/n*a^(3/2)*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*l
n(x))^2)^(1/2))/exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 827, normalized size of antiderivative = 4.78

$$\int \frac{\left(a+bx^{n}+cx^{2n}\right)^{3/2}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/96*(48*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*
(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) - 3*(b^3
 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c
^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) +
14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1
/48*(24*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(s
qrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 3*(b^3 -
 12*a*b*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2
*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*(8*c^3*x^(2*n) + 14*b*
c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/96*(
96*sqrt(-a)*a*c^2*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n
) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)
*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(
c))*sqrt(c*x^(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2
*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(48*sqrt(-a)*a*c
^2*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/
(a*c*x^(2*n) + a*b*x^n + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*(2
*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b
*c*x^n + a*c)) + 2*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqr
t(c*x^(2*n) + b*x^n + a))/(c^2*n)]
```

Sympy [F]

$$\int \frac{(a + bx^n + cx^{2n})^{3/2}}{x}\,dx = \int \frac{(a + bx^n + cx^{2n})^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(3/2)/x,x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx = \int \frac{(cx^{2\,n}+bx^n+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx = \int \frac{(cx^{2\,n}+bx^n+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx = \int \frac{(a+b\,x^n+c\,x^{2\,n})^{3/2}}{x}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(3/2)/x,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx = \frac{8x^{2n}\sqrt{x^{2n}c+x^nb+a}\,c+14x^n\sqrt{x^{2n}c+x^nb+a}\,b+68\sqrt{x^{2n}c+x^nb+a}\,a+24}{}$$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2)/x,x)
```

output
```
(8*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c + 14*x**n*sqrt(x**(2*n)*c + x*
*n*b + a)*b + 68*sqrt(x**(2*n)*c + x**n*b + a)*a + 24*int(sqrt(x**(2*n)*c
+ x**n*b + a)/(x**(2*n)*c*x + x**n*b*x + a*x),x)*a**2*n - 36*int((x**(2*n)
*sqrt(x**(2*n)*c + x**n*b + a))/(x**(2*n)*c*x + x**n*b*x + a*x),x)*a*c*n +
 3*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a))/(x**(2*n)*c*x + x**n*b*x +
 a*x),x)*b**2*n)/(24*n)
```

3.224 $\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^2} dx = -\frac{a\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{3}{2},-\frac{3}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
-a*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(-1/n,-3/2,-3/2,-(1-n)/n,-2*c*x^n/(b-
(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/x/(1+2*c*x^n/(b-(-4*a
*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 477 vs. 2(150) = 300.

Time = 1.61 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.18

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^2} dx = \frac{2(-1+n)\left(3b^2n^2+4ac(1-6n+8n^2)+2bc(2-9n+7n^2)\,x^n+4c^2(1-3n\right.}{}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```

output
```
(2*(-1 + n)*(3*b^2*n^2 + 4*a*c*(1 - 6*n + 8*n^2) + 2*b*c*(2 - 9*n + 7*n^2)
*x^n + 4*c^2*(1 - 3*n + 2*n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 6*a*(-1 +
n)*n^2*(b^2 + 4*a*c*(-1 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*(4*a*c*(2 - 3*n) +
 b^2*(-2 + n))*n^2*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^
2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
)]*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(8*c*(-1 + n)^2*(-1 + 2*n)*(-
1 + 3*n)*x*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}}\int \frac{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}{x^2}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{1}{n},-\frac{3}{2},-\frac{3}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```

output
```
-((a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -3/2, -3/2, -((1 - n)/n
), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
)/(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(a + b\,x^{n} + c\,x^{2n})^{\frac{3}{2}}}{x^{2}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**2,x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \int \frac{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \int \frac{(a+b\,x^n+c\,x^{2\,n})^{3/2}}{x^2}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(3/2)/x^2,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \text{too large to display}$$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x)
```

output
```
(8*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c*n**2 - 20*x**(2*n)*sqrt(x**(2*
n)*c + x**n*b + a)*c*n + 8*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c + 14*x
**n*sqrt(x**(2*n)*c + x**n*b + a)*b*n**2 - 32*x**n*sqrt(x**(2*n)*c + x**n*
b + a)*b*n + 8*x**n*sqrt(x**(2*n)*c + x**n*b + a)*b + 68*sqrt(x**(2*n)*c +
 x**n*b + a)*a*n**2 - 44*sqrt(x**(2*n)*c + x**n*b + a)*a*n + 8*sqrt(x**(2*
n)*c + x**n*b + a)*a + 144*int(sqrt(x**(2*n)*c + x**n*b + a)/(6*x**(2*n)*c
*n**3*x**2 - 17*x**(2*n)*c*n**2*x**2 + 11*x**(2*n)*c*n*x**2 - 2*x**(2*n)*c
*x**2 + 6*x**n*b*n**3*x**2 - 17*x**n*b*n**2*x**2 + 11*x**n*b*n*x**2 - 2*x*
*n*b*x**2 + 6*a*n**3*x**2 - 17*a*n**2*x**2 + 11*a*n*x**2 - 2*a*x**2),x)*a*
*2*n**6*x - 408*int(sqrt(x**(2*n)*c + x**n*b + a)/(6*x**(2*n)*c*n**3*x**2
- 17*x**(2*n)*c*n**2*x**2 + 11*x**(2*n)*c*n*x**2 - 2*x**(2*n)*c*x**2 + 6*x
**n*b*n**3*x**2 - 17*x**n*b*n**2*x**2 + 11*x**n*b*n*x**2 - 2*x**n*b*x**2 +
 6*a*n**3*x**2 - 17*a*n**2*x**2 + 11*a*n*x**2 - 2*a*x**2),x)*a**2*n**5*x +
 264*int(sqrt(x**(2*n)*c + x**n*b + a)/(6*x**(2*n)*c*n**3*x**2 - 17*x**(2*
n)*c*n**2*x**2 + 11*x**(2*n)*c*n*x**2 - 2*x**(2*n)*c*x**2 + 6*x**n*b*n**3*
x**2 - 17*x**n*b*n**2*x**2 + 11*x**n*b*n*x**2 - 2*x**n*b*x**2 + 6*a*n**3*x
**2 - 17*a*n**2*x**2 + 11*a*n*x**2 - 2*a*x**2),x)*a**2*n**4*x - 48*int(sqr
t(x**(2*n)*c + x**n*b + a)/(6*x**(2*n)*c*n**3*x**2 - 17*x**(2*n)*c*n**2*x*
*2 + 11*x**(2*n)*c*n*x**2 - 2*x**(2*n)*c*x**2 + 6*x**n*b*n**3*x**2 - 17*x*
*n*b*n**2*x**2 + 11*x**n*b*n*x**2 - 2*x**n*b*x**2 + 6*a*n**3*x**2 - 17*...
```

3.225 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3} dx = -\frac{a\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(-\frac{2}{n},-\frac{3}{2},-\frac{3}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
-1/2*a*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(-2/n,-3/2,-3/2,-(2-n)/n,-2*c*x^n
/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/x^2/(1+2*c*x^n/(b
-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 471 vs. 2(152) = 304.

Time = 1.57 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.10

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3} dx = \frac{2(-2+n)\left(3b^2n^2+16ac(1-3n+2n^2)+2bc(8-18n+7n^2)x^n+8c^2(2-3\right.}{}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

output
```
(2*(-2 + n)*(3*b^2*n^2 + 16*a*c*(1 - 3*n + 2*n^2) + 2*b*c*(8 - 18*n + 7*n^
2)*x^n + 8*c^2*(2 - 3*n + n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 6*a*(b^2 +
 4*a*c*(-1 + n))*(-2 + n)*n^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*(4*a*c*(4 - 3*n) + b^2*
(-4 + n))*n^2*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(16*c*(-2 + n)^2*(-1 + n)*(-2 + 3*n)*
x^2*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^n+cx^{2n}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}}\int \frac{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}{x^3}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(-\frac{2}{n},-\frac{3}{2},-\frac{3}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

output
```
-1/2*(a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -3/2, -3/2, -((2 - n)/n
), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
)/(x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{\left(a + b\,x^{n} + c\,x^{2n}\right)^{\frac{3}{2}}}{x^{3}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = \int \frac{(a+bx^n+cx^{2n})^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**3,x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = \int \frac{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx=\int \frac{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx=\int \frac{(a+b\,x^n+c\,x^{2\,n})^{3/2}}{x^3}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(3/2)/x^3,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx=\text{too large to display}$$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x)
```

output

```
(8*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c*n**2 - 40*x**(2*n)*sqrt(x**(2*
n)*c + x**n*b + a)*c*n + 32*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c + 14*
x**n*sqrt(x**(2*n)*c + x**n*b + a)*b*n**2 - 64*x**n*sqrt(x**(2*n)*c + x**n
*b + a)*b*n + 32*x**n*sqrt(x**(2*n)*c + x**n*b + a)*b + 68*sqrt(x**(2*n)*c
 + x**n*b + a)*a*n**2 - 88*sqrt(x**(2*n)*c + x**n*b + a)*a*n + 32*sqrt(x**
(2*n)*c + x**n*b + a)*a + 72*int(sqrt(x**(2*n)*c + x**n*b + a)/(3*x**(2*n)
*c*n**3*x**3 - 17*x**(2*n)*c*n**2*x**3 + 22*x**(2*n)*c*n*x**3 - 8*x**(2*n)
*c*x**3 + 3*x**n*b*n**3*x**3 - 17*x**n*b*n**2*x**3 + 22*x**n*b*n*x**3 - 8*
x**n*b*x**3 + 3*a*n**3*x**3 - 17*a*n**2*x**3 + 22*a*n*x**3 - 8*a*x**3),x)*
a**2*n**6*x**2 - 408*int(sqrt(x**(2*n)*c + x**n*b + a)/(3*x**(2*n)*c*n**3*
x**3 - 17*x**(2*n)*c*n**2*x**3 + 22*x**(2*n)*c*n*x**3 - 8*x**(2*n)*c*x**3
+ 3*x**n*b*n**3*x**3 - 17*x**n*b*n**2*x**3 + 22*x**n*b*n*x**3 - 8*x**n*b*x
**3 + 3*a*n**3*x**3 - 17*a*n**2*x**3 + 22*a*n*x**3 - 8*a*x**3),x)*a**2*n**
5*x**2 + 528*int(sqrt(x**(2*n)*c + x**n*b + a)/(3*x**(2*n)*c*n**3*x**3 - 1
7*x**(2*n)*c*n**2*x**3 + 22*x**(2*n)*c*n*x**3 - 8*x**(2*n)*c*x**3 + 3*x**n
*b*n**3*x**3 - 17*x**n*b*n**2*x**3 + 22*x**n*b*n*x**3 - 8*x**n*b*x**3 + 3*
a*n**3*x**3 - 17*a*n**2*x**3 + 22*a*n*x**3 - 8*a*x**3),x)*a**2*n**4*x**2 -
 192*int(sqrt(x**(2*n)*c + x**n*b + a)/(3*x**(2*n)*c*n**3*x**3 - 17*x**(2*
n)*c*n**2*x**3 + 22*x**(2*n)*c*n*x**3 - 8*x**(2*n)*c*x**3 + 3*x**n*b*n**3*
x**3 - 17*x**n*b*n**2*x**3 + 22*x**n*b*n*x**3 - 8*x**n*b*x**3 + 3*a*n**...
```

3.226 $\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{4}{n},\frac{1}{2},\frac{1}{2},\frac{4+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

output
```
1/4*x^4*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)*AppellF1(4/n,1/2,1/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{4}{n},\frac{1}{2},\frac{1}{2},\frac{4+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+x^n\left(b+cx^n\right)}}$$

input
```
Integrate[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1
/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(4*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{x^3}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{4}{n},\frac{1}{2},\frac{1}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^n + c
*x^(2*n)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x^3}{\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

input
```
int(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate(x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(x**3/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^3}{\sqrt{cx^{2\,n}+bx^n+a}}\,dx$$

input
```
integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^3}{\sqrt{cx^{2\,n}+bx^n+a}}\,dx$$

input
```
integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^3}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(x^3/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(x^3/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{\sqrt{x^{2n}c+x^nb+a}\,x^3}{x^{2n}c+x^nb+a}dx$$

input
```
int(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int((sqrt(x**(2*n)*c + x**n*b + a)*x**3)/(x**(2*n)*c + x**n*b + a),x)
```

3.227 $\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{n},\frac{1}{2},\frac{1}{2},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

output
```
1/3*x^3*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)*AppellF1(3/n,1/2,1/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{n},\frac{1}{2},\frac{1}{2},\frac{3+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+x^n(b+cx^n)}}$$

input
```
Integrate[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1
/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(3*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{x^2}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\text{AppellF1}\left(\frac{3}{n},\frac{1}{2},\frac{1}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a + b*x^n + c
*x^(2*n)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

input
```
int(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a + bx^n + cx^{2n}}}\,dx = \int \frac{x^2}{\sqrt{a + bx^n + cx^{2n}}}\,dx$$

input
```
integrate(x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(x**2/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a + bx^n + cx^{2n}}}\,dx = \int \frac{x^2}{\sqrt{cx^{2n} + bx^n + a}}\,dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a + bx^n + cx^{2n}}}\,dx = \int \frac{x^2}{\sqrt{cx^{2n} + bx^n + a}}\,dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{x^2}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(x^2/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(x^2/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{\sqrt{x^{2n}c+x^nb+a}\,x^2}{x^{2n}c+x^nb+a}dx$$

input
```
int(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int((sqrt(x**(2*n)*c + x**n*b + a)*x**2)/(x**(2*n)*c + x**n*b + a),x)
```

3.228 $\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 148

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

output
```
1/2*x^2*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)*AppellF1(2/n,1/2,1/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{2+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+x^n\left(b+cx^n\right)}}$$

input
```
Integrate[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1
/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(2*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \text{ 1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{x}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\ \text{AppellF1}\left(\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^n + c
*x^(2*n)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x}{\sqrt{a+b\,x^{n}+c\,x^{2 n}}}dx$$

input
```
int(x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate(x/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{x}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{x}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{x}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(x/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(x/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{\sqrt{x^{2n}c+x^nb+a}\,x}{x^{2n}c+x^nb+a}dx$$

input
```
int(x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int((sqrt(x**(2*n)*c + x**n*b + a)*x)/(x**(2*n)*c + x**n*b + a),x)
```

3.229 $\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 139

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

output
```
x*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)*AppellF1(1/n,1/2,1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+x^n\left(b+cx^n\right)}}$$

input
```
Integrate[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1),
1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])])/Sqrt[a + x^n*(b + c*x^n)]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{1686, 936\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \; 1686$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \; 936$$

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^n + c*
x^(2*n)]
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sqrt{a+b\,x^{n}+c\,x^{2n}}}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate(1/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{1}{\sqrt{cx^{2\,n}+bx^n+a}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{1}{\sqrt{cx^{2\,n}+bx^n+a}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{1}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(1/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(1/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{2n}c+x^nb+a}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(2*n)*c + x**n*b + a),x)
```

3.230 $\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

output
```
-arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/a^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}x^n-\sqrt{a+x^n(b+cx^n)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

output
```
(2*ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^n*(b + c*x^n)])/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1693, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n}{n}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{2\int \frac{1}{4a-x^{2n}}d\frac{bx^n+2a}{\sqrt{bx^n+cx^{2n}+a}}}{n}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

input
```
Int[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

output
```
-(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(Sqrt[a]*
n))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{x\sqrt{a+b\,x^{n}+c\,x^{2n}}}dx$$

input
```
int(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.15

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \left[\frac{\log\left(-\frac{8\,abx^{n}+8\,a^{2}+\left(b^{2}+4\,ac\right)x^{2\,n}-4\left(\sqrt{a}bx^{n}+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n}+bx^{n}+a}}{x^{2\,n}}\right)}{2\,\sqrt{a}n}, \frac{\sqrt{-a}\arctan\left(\frac{\left(\sqrt{-a}bx^{n}+2\,\sqrt{-a}a\right)\sqrt{cx^{2\,n}+bx^{n}+a}}{2\,\left(acx^{2\,n}+abx^{n}+a^{2}\right)}\right)}{an}\right]$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n +
2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n))/(sqrt(a)*n), sqrt(-a)*arc
tan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x
^(2*n) + a*b*x^n + a^2))/(a*n)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate(1/x/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*x**n + c*x**(2*n))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x}\,dx$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x}\,dx$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)),x)
```

output
```
int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{2n}cx+x^nbx+ax}dx$$

input
```
int(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(2*n)*c*x + x**n*b*x + a*x),x)
```

3.231 $\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

output
```
-(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)
))^(1/2)*AppellF1(-1/n,1/2,1/2,-(1-n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/x/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},\frac{-1+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+x^n\,(b+cx^n)}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

output
```
-((Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1),
 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + x^n*(b + c*x^n)]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{1721, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \ 1721$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \ 1012$$

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{n},\frac{1}{2},\frac{1}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

output
```
-((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, -((1 - n)/n), (-2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^
n + c*x^(2*n)]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{1}{x^2\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

input
```
int(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**n + c*x**(2*n))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{x^2\,\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(1/(x^2*(a + b*x^n + c*x^(2*n))^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{2n}c\,x^2+x^nb\,x^2+a\,x^2}dx$$

input
```
int(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(2*n)*c*x**2 + x**n*b*x**2 + a*x**2)
,x)
```

3.232 $\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

output
```
-1/2*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(
1/2)))^(1/2)*AppellF1(-2/n,1/2,1/2,-(2-n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/x^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = -\frac{\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},\frac{-2+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+x^n\left(b+cx^n\right)}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

output
```
-1/2*(Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n,
 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
 Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(-\frac{2}{n},\frac{1}{2},\frac{1}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

output
```
-1/2*(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, -((2 - n)/n), (-2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[a + b*
x^n + c*x^(2*n)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{1}{x^3\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

input
```
int(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a + b*x**n + c*x**(2*n))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n}+bx^n+a}\,x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{1}{x^3\,\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(1/(x^3*(a + b*x^n + c*x^(2*n))^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx=\int \frac{\sqrt{x^{2n}c+x^n b+a}}{x^{2n}c\,x^3+x^n b\,x^3+a\,x^3}dx$$

input
```
int(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(2*n)*c*x**3 + x**n*b*x**3 + a*x**3)
,x)
```

3.233 $\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}} dx = \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\text{AppellF1}\left(\frac{4}{n},\frac{3}{2},\frac{3}{2},\frac{4+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2}}\right.}{4a\sqrt{a+bx^n+cx^{2n}}}$$

output
```
1/4*x^4*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)*AppellF1(4/n,3/2,3/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 398 vs. 2(151) = 302.

Time = 1.04 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.64

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}} dx = \frac{x^4\left(-8(4+n)\left(b^2-2ac+bcx^n\right)-\left(b^2(-8+n)-4ac(-4+n)\right)(4+n)\sqrt{\frac{b-\sqrt{}}{b}}\right.}{}$$

input
```
Integrate[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^4*(-8*(4 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-8 + n) - 4*a*c*(-4 + n))
*(4 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/
n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])] + 32*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(4*a*(-b^2 + 4*a*c)*
n*(4 + n)*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{x^3}{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{4}{n},\frac{3}{2},\frac{3}{2},\frac{n+4}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[4/n, 3/2, 3/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^n +
 c*x^(2*n)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x^3}{\left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^3}{(a+bx^n+cx^{2n})^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(x**3/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^3}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^3}{(cx^{2\,n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^3}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

input
```
int(x^3/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(x^3/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{\sqrt{x^{2n}c+x^nb+a}\,x^3}{x^{4n}c^2+2x^{3n}bc+2x^{2n}ac+x^{2n}b^2+2x^nab+a^2}\,dx$$

input
```
int(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int((sqrt(x**(2*n)*c + x**n*b + a)*x**3)/(x**(4*n)*c**2 + 2*x**(3*n)*b*c +
 2*x**(2*n)*a*c + x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.234 $\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{n},\frac{3}{2},\frac{3}{2},\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2 \ldots}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

output
```
1/3*x^3*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)*AppellF1(3/n,3/2,3/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 398 vs. $2(151) = 302$.

Time = 1.04 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.64

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x^3\left(-6(3+n)\left(b^2-2ac+bcx^n\right)-\left(b^2(-6+n)-4ac(-3+n)\right)(3+n)\sqrt{\frac{b-\sqrt{\ldots}}{b\ldots}}\ldots\right.}{\ldots}$$

input
```
Integrate[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^3*(-6*(3 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-6 + n) - 4*a*c*(-3 + n))
*(3 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/
n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])] + 18*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(3*a*(-b^2 + 4*a*c)*
n*(3 + n)*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{x^2}{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{n},\frac{3}{2},\frac{3}{2},\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[3/n, 3/2, 3/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*Sqrt[a + b*x^n +
 c*x^(2*n)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x^2}{(a+b\,x^n+c\,x^{2n})^{\frac{3}{2}}}dx$$

input
```
int(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a + bx^n + cx^{2n})^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{(a + bx^n + cx^{2n})^{3/2}}\, dx = \int \frac{x^2}{(a + bx^n + cx^{2n})^{\frac{3}{2}}}\, dx$$

input
```
integrate(x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(x**2/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a + bx^n + cx^{2n})^{3/2}}\, dx = \int \frac{x^2}{(cx^{2\,n} + bx^n + a)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^2}{(cx^{2\,n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x^2}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

input
```
int(x^2/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(x^2/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{\sqrt{x^{2n}c+x^nb+a}\,x^2}{x^{4n}c^2+2x^{3n}bc+2x^{2n}ac+x^{2n}b^2+2x^nab+a^2}dx$$

input
```
int(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int((sqrt(x**(2*n)*c + x**n*b + a)*x**2)/(x**(4*n)*c**2 + 2*x**(3*n)*b*c +
 2*x**(2*n)*a*c + x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.235 $\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 151

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{n},\frac{3}{2},\frac{3}{2},\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2}}\right.}{2a\sqrt{a+bx^n+cx^{2n}}}$$

output
```
1/2*x^2*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)*AppellF1(2/n,3/2,3/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 398 vs. 2(151) = 302.

Time = 1.01 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.64

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x^2\left(-4(2+n)\left(b^2-2ac+bcx^n\right)-\left(b^2(-4+n)-4ac(-2+n)\right)(2+n)\sqrt{\frac{b-\sqrt{}}{b}}\right.}{}$$

input
```
Integrate[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^2*(-4*(2 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-4 + n) - 4*a*c*(-2 + n))
*(2 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/
n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b
+ Sqrt[b^2 - 4*a*c])] + 8*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(2*a*(-b^2 + 4*a*c)*n
*(2 + n)*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{x}{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{2}{n},\frac{3}{2},\frac{3}{2},\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[2/n, 3/2, 3/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^n +
 c*x^(2*n)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x}{(a + b\,x^n + c\,x^{2n})^{\frac{3}{2}}}dx$$

input
```
int(x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x}{(a+bx^n+cx^{2n})^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(x/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{x}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\int \frac{x}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\int \frac{x}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

input
```
int(x/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(x/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx=\int \frac{\sqrt{x^{2n}c+x^nb+a}\,x}{x^{4n}c^2+2x^{3n}bc+2x^{2n}ac+x^{2n}b^2+2x^nab+a^2}dx$$

input
```
int(x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int((sqrt(x**(2*n)*c + x**n*b + a)*x)/(x**(4*n)*c**2 + 2*x**(3*n)*b*c + 2*
x**(2*n)*a*c + x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.236 $\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 142

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2c\ldots}{b+\sqrt{b\ldots}}\right.}{a\sqrt{a+bx^n+cx^{2n}}}$$

output
```
x*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)*AppellF1(1/n,3/2,3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 384 vs. 2(142) = 284.

Time = 1.02 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.70

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x\left(2bcx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2\ldots}{b+\sqrt{\ldots}}\right.\right.}{\ldots}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

output
```
(x*(2*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c
])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appell
F1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (1 + n)*(2*(b^2 - 2*a*c + b*c*x^n) +
(b^2*(-2 + n) - 4*a*c*(-1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*n*(1
 + n)*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \text{1686}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

output
```
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3/2, 1 + n^(-1), (-2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^n +
 c*x^(2*n)])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{(a+b\,x^{n}+c\,x^{2n})^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(a+bx^n+cx^{2n})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

input
```
int(1/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(1/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{4n}c^2+2x^{3n}bc+2x^{2n}ac+x^{2n}b^2+2x^nab+a^2}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(4*n)*c**2 + 2*x**(3*n)*b*c + 2*x**(
2*n)*a*c + x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.237 $\int \frac{1}{x(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\frac{2(b^2-2ac+bcx^n)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}}-\frac{\operatorname{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

output
```
2*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))^(1/2)-arctanh(1
/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/a^(3/2)/n
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\frac{2\left(-\frac{\sqrt{a}(-b^2+2ac-bcx^n)}{(b^2-4ac)\sqrt{a+x^n(b+cx^n)}}+\operatorname{arctanh}\left(\frac{\sqrt{c}x^n-\sqrt{a+x^n(b+cx^n)}}{\sqrt{a}}\right)\right)}{a^{3/2}n}$$

input
```
Integrate[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

output
```
(2*(-((Sqrt[a]*(-b^2 + 2*a*c - b*c*x^n))/((b^2 - 4*a*c)*Sqrt[a + x^n*(b +
c*x^n)])) + ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^n*(b + c*x^n)])/Sqrt[a]]))/(
a^(3/2)*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1693, 1165, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1693}$$

$$\frac{\int \frac{x^{-n}}{\left(bx^n+cx^{2n}+a\right)^{3/2}}dx^n}{n}$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{\frac{2\left(-2ac+b^2+bcx^n\right)}{a(b^2-4ac)\sqrt{a+bx^n+cx^{2n}}}-\frac{2\int -\frac{\left(b^2-4ac\right)x^{-n}}{2\sqrt{bx^n+cx^{2n}+a}}dx^n}{a(b^2-4ac)}}{n}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{x^{-n}}{\sqrt{bx^n+cx^{2n}+a}}dx^n}{a}+\frac{2\left(-2ac+b^2+bcx^n\right)}{a(b^2-4ac)\sqrt{a+bx^n+cx^{2n}}}}{n}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{\frac{2\left(-2ac+b^2+bcx^n\right)}{a(b^2-4ac)\sqrt{a+bx^n+cx^{2n}}}-\frac{2\int \frac{1}{4a-x^{2n}}d\frac{bx^n+2a}{\sqrt{bx^n+cx^{2n}+a}}}{a}}{n}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{2\left(-2ac+b^2+bcx^n\right)}{a(b^2-4ac)\sqrt{a+bx^n+cx^{2n}}}-\frac{\text{arctanh}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}}}{n}$$

input
```
Int[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^n + c*x^(2*n)])
 - ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/a^(3/2))
/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{x\left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(88) = 176$.

Time = 0.27 (sec) , antiderivative size = 449, normalized size of antiderivative = 4.58

$$\int \frac{1}{x\left(a+bx^{n}+cx^{2n}\right)^{3/2}}\,dx = \left[\frac{\left((b^2c-4\,ac^2)\sqrt{a}x^{2\,n}+(b^3-4\,abc)\sqrt{a}x^{n}+(ab^2-4\,a^2c)\sqrt{a}\right)\log\left(-\frac{8\,abx^{n}+\cdots}{\cdots}\right)}{2\left((a^2b^2c-4\,a^3c^2)nx^{2\,n}+\cdots\right.}\right.$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*sqrt(a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(a)*x^n + (
a*b^2 - 4*a^2*c)*sqrt(a))*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n)
- 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 4*
(a*b*c*x^n + a*b^2 - 2*a^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4
*a^3*c^2)*n*x^(2*n) + (a^2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n)
, (((b^2*c - 4*a*c^2)*sqrt(-a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(-a)*x^n + (a
*b^2 - 4*a^2*c)*sqrt(-a))*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(
c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) + 2*(a*b*c*x^n + a*b
^2 - 2*a^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4*a^3*c^2)*n*x^(2
*n) + (a^2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(1/(x*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \int \frac{1}{\left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \int \frac{1}{\left(cx^{2\,n}+bx^n+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\int \frac{1}{x\,\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x)
```

output
```
int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx=\int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{4n}c^2x+2x^{3n}bcx+2x^{2n}acx+x^{2n}b^2x+2x^nabx+a^2x}dx$$

input
```
int(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(4*n)*c**2*x + 2*x**(3*n)*b*c*x + 2*
x**(2*n)*a*c*x + x**(2*n)*b**2*x + 2*x**n*a*b*x + a**2*x),x)
```

3.238 $\int \frac{1}{x^2(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{n},\frac{3}{2},\frac{3}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

output
```
-(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)
))^(1/2)*AppellF1(-1/n,3/2,3/2,-(1-n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/a/x/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 395 vs. 2(152) = 304.

Time = 1.01 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.60

$$\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{(-1+n)\left(-4ac(1+n)+b^2(2+n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{App}}{}$$

input
```
Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

output
```
((-1 + n)*(-4*a*c*(1 + n) + b^2*(2 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 2*((-1 + n)*(
b^2 - 2*a*c + b*c*x^n) + b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(a*(-b^2 + 4*a*c)*
(-1 + n)*n*x*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+bx^n+cx^{2n}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^2 \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{n},\frac{3}{2},\frac{3}{2},-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

output
```
-((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-n^(-1), 3/2, 3/2, -((1 - n)/n), (-2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*
x^n + c*x^(2*n)]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{1}{x^2\,(a+b\,x^n+c\,x^{2n})^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})^{3/2}} dx = \int \frac{1}{x^2 (a + bx^n + cx^{2n})^{\frac{3}{2}}} dx$$

input
```
integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^n + cx^{2n})^{3/2}} dx = \int \frac{1}{(cx^{2n} + bx^n + a)^{\frac{3}{2}} x^2} dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{x^2 (a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{4n}c^2x^2+2x^{3n}bc\,x^2+2x^{2n}ac\,x^2+x^{2n}b^2x^2+2x^nab\,x^2+a^2x^2}dx$$

input
```
int(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(4*n)*c**2*x**2 + 2*x**(3*n)*b*c*x**
2 + 2*x**(2*n)*a*c*x**2 + x**(2*n)*b**2*x**2 + 2*x**n*a*b*x**2 + a**2*x**2
),x)
```

3.239 $\int \frac{1}{x^3(a+bx^n+cx^{2n})^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{1}{x^3 (a+bx^n+cx^{2n})^{3/2}} \, dx = -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{2}{n},\frac{3}{2},\frac{3}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

output
```
-1/2*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(
1/2)))^(1/2)*AppellF1(-2/n,3/2,3/2,-(2-n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/a/x^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 399 vs. 2(154) = 308.

Time = 1.01 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.59

$$\int \frac{1}{x^3 (a+bx^n+cx^{2n})^{3/2}} \, dx = \frac{(-2+n)\left(-4ac(2+n)+b^2(4+n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{App}}{}$$

input
```
Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

output
```
((-2 + n)*(-4*a*c*(2 + n) + b^2*(4 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*((-2 + n)*(b^2
 - 2*a*c + b*c*x^n) + 2*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(2*a*(-b^2 + 4*a*c)*(-
2 + n)*n*x^2*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^n+cx^{2n})^{3/2}} \, dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{x^3\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{2}{n},\frac{3}{2},\frac{3}{2},-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 3/2, 3/2, -((2 - n)/n), (-2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*x^2*Sqrt[a +
b*x^n + c*x^(2*n)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{1}{x^3 \left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})^{3/2}} \, dx = \int \frac{1}{x^3 (a + bx^n + cx^{2n})^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^n + cx^{2n})^{3/2}} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)^{\frac{3}{2}} x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^n+cx^{2n})^{3/2}} \, dx = \int \frac{1}{(cx^{2n}+bx^n+a)^{\frac{3}{2}} x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^n+cx^{2n})^{3/2}} \, dx = \int \frac{1}{x^3 (a+b\,x^n+c\,x^{2\,n})^{3/2}} \, dx$$

input
```
int(1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x)
```

output
```
int(1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^n+cx^{2n})^{3/2}} \, dx = \int \frac{\sqrt{x^{2n}c+x^n b+a}}{x^{4n}c^2x^3+2x^{3n}bc\,x^3+2x^{2n}ac\,x^3+x^{2n}b^2x^3+2x^n ab\,x^3+a^2x^3} dx$$

input
```
int(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(4*n)*c**2*x**3 + 2*x**(3*n)*b*c*x**
3 + 2*x**(2*n)*a*c*x**3 + x**(2*n)*b**2*x**3 + 2*x**n*a*b*x**3 + a**2*x**3
),x)
```

3.240 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 200

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx = \frac{a^3(dx)^{1+m}}{d(1+m)}+\frac{3a^2bx^n(dx)^{1+m}}{d(1+m+n)}+\frac{3a(b^2+ac)\,x^{2n}(dx)^{1+m}}{d(1+m+2n)} + \frac{b(b^2+6ac)\,x^{3n}(dx)^{1+m}}{d(1+m+3n)}+\frac{3c(b^2+ac)\,x^{4n}(dx)^{1+m}}{d(1+m+4n)} + \frac{3bc^2x^{5n}(dx)^{1+m}}{d(1+m+5n)}+\frac{c^3x^{6n}(dx)^{1+m}}{d(1+m+6n)}$$

output
```
a^3*(d*x)^(1+m)/d/(1+m)+3*a^2*b*x^n*(d*x)^(1+m)/d/(1+m+n)+3*a*(a*c+b^2)*x^
(2*n)*(d*x)^(1+m)/d/(1+m+2*n)+b*(6*a*c+b^2)*x^(3*n)*(d*x)^(1+m)/d/(1+m+3*n
)+3*c*(a*c+b^2)*x^(4*n)*(d*x)^(1+m)/d/(1+m+4*n)+3*b*c^2*x^(5*n)*(d*x)^(1+m
)/d/(1+m+5*n)+c^3*x^(6*n)*(d*x)^(1+m)/d/(1+m+6*n)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.68

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx = x(dx)^m \left(\frac{a^3}{1+m}+\frac{3a^2bx^n}{1+m+n}+\frac{3a(b^2+ac)\,x^{2n}}{1+m+2n} + \frac{b(b^2+6ac)\,x^{3n}}{1+m+3n}+\frac{3c(b^2+ac)\,x^{4n}}{1+m+4n}+\frac{3bc^2x^{5n}}{1+m+5n} + \frac{c^3x^{6n}}{1+m+6n}\right)$$

input

```
Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

output

```
x*(d*x)^m*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(2
*n))/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(3*n))/(1 + m + 3*n) + (3*c*(b^2 +
 a*c)*x^(4*n))/(1 + m + 4*n) + (3*b*c^2*x^(5*n))/(1 + m + 5*n) + (c^3*x^(6
*n))/(1 + m + 6*n))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx$$

$$\Big\downarrow \text{1691}$$

$$\int \left(a^3(dx)^m+3a^2bx^n(dx)^m+3ab^2x^{2n}\left(\frac{ac}{b^2}+1\right)(dx)^m+3b^2cx^{4n}\left(\frac{ac}{b^2}+1\right)(dx)^m+b^3x^{3n}\left(\frac{6ac}{b^2}+1\right)(dx)^m+3$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2bx^{n+1}(dx)^m}{m+n+1}+\frac{3ax^{2n+1}(ac+b^2)(dx)^m}{m+2n+1}+\frac{bx^{3n+1}(6ac+b^2)(dx)^m}{m+3n+1}+\frac{3cx^{4n+1}(ac+b^2)(dx)^m}{m+4n+1}+\frac{3bc^2x^{5n+1}(dx)^m}{m+5n+1}+\frac{c^3x^{6n+1}(dx)^m}{m+6n+1}$$

input
```
Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
(3*a^2*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(1 + 2*n)*(d*
x)^m)/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n)
+ (3*c*(b^2 + a*c)*x^(1 + 4*n)*(d*x)^m)/(1 + m + 4*n) + (3*b*c^2*x^(1 + 5*
n)*(d*x)^m)/(1 + m + 5*n) + (c^3*x^(1 + 6*n)*(d*x)^m)/(1 + m + 6*n) + (a^3
*(d*x)^(1 + m))/(d*(1 + m))
```

Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.96 (sec) , antiderivative size = 3765, normalized size of antiderivative = 18.82

method	result	size
risch	Expression too large to display	3765
parallelrisch	Expression too large to display	5804
orering	Expression too large to display	9909

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x,method=_RETURNVERBOSE)
```

output
```
x*(540*a*b*c*m^4*n*(x^n)^3+2904*a*b*c*m^3*n^2*(x^n)^3+21*a^3*n+36*a*b*c*(x
^n)^3*m+108*a*b*c*(x^n)^3*n+1284*b^2*c*m*n^2*(x^n)^4+120*a*b*c*m^3*(x^n)^3
+1860*a^2*b*m^3*n^2*x^n+921*a*c^2*m^3*n^3*(x^n)^4+1188*a*c^2*m^2*n^4*(x^n)
^4+540*a*c^2*m*n^5*(x^n)^4+51*b^2*c*m^5*n*(x^n)^4+321*b^2*c*m^4*n^2*(x^n)^
4+921*b^2*c*m^3*n^3*(x^n)^4+1188*b^2*c*m^2*n^4*(x^n)^4+6*m*a^3+180*b^3*m^3
*(x^n)^3*n+1740*a^2*b*n^3*x^n+1383*a*b^2*n^3*(x^n)^2+726*b^3*m^2*n^2*(x^n)
^3+1116*b^3*m*n^3*(x^n)^3+726*a*b*c*m^4*n^2*(x^n)^3+36*a*b*c*m^5*(x^n)^3+1
440*a*b*c*n^5*(x^n)^3+510*a*c^2*m^3*n*(x^n)^4+1926*a*c^2*m^2*n^2*(x^n)^4+2
763*a*c^2*m*n^3*(x^n)^4+a^3+548*c^3*m*n^4*(x^n)^6+3*a*c^2*m^6*(x^n)^4+3*b^
2*c*m^6*(x^n)^4+18*b*c^2*m^5*(x^n)^5+432*b*c^2*n^5*(x^n)^5+150*c^3*m^3*n*(
x^n)^6+510*c^3*m^2*n^2*(x^n)^6+675*c^3*m*n^3*(x^n)^6+18*a*c^2*m^5*(x^n)^4+
540*a*c^2*n^5*(x^n)^4+15*c^3*m^5*n*(x^n)^6+85*c^3*m^4*n^2*(x^n)^6+120*c^3*
m*n^5*(x^n)^6+3*b*c^2*m^6*(x^n)^5+75*c^3*m^4*n*(x^n)^6+340*c^3*m^3*n^2*(x^
n)^6+675*c^3*m^2*n^3*(x^n)^6+1116*b^3*m^2*n^3*(x^n)^3+1016*b^3*m*n^4*(x^n)
^3+45*b^2*c*m^4*(x^n)^4+1188*b^2*c*n^4*(x^n)^4+60*b*c^2*m^3*(x^n)^5+780*b*
c^2*n^3*(x^n)^5+75*c^3*m*n*(x^n)^6+3*a^2*b*m^6*x^n+18*a^2*c*m^5*(x^n)^2+10
80*a^2*c*n^5*(x^n)^2+18*a*b^2*m^5*(x^n)^2+225*c^3*m^3*n^3*(x^n)^6+274*c^3*
m^2*n^4*(x^n)^6+18*b^2*c*m^5*(x^n)^4+540*b^2*c*n^5*(x^n)^4+45*b*c^2*m^4*(x
^n)^5+972*b*c^2*n^4*(x^n)^5+150*c^3*m^2*n*(x^n)^6+340*c^3*m*n^2*(x^n)^6+3*
a^2*c*m^6*(x^n)^2+3*a*b^2*m^6*(x^n)^2+45*a*c^2*m^4*(x^n)^4+1188*a*c^2*n...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2303 vs. $2(200) = 400$.

Time = 0.18 (sec) , antiderivative size = 2303, normalized size of antiderivative = 11.52

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
((c^3*m^6 + 6*c^3*m^5 + 15*c^3*m^4 + 20*c^3*m^3 + 120*(c^3*m + c^3)*n^5 +
15*c^3*m^2 + 274*(c^3*m^2 + 2*c^3*m + c^3)*n^4 + 6*c^3*m + 225*(c^3*m^3 +
3*c^3*m^2 + 3*c^3*m + c^3)*n^3 + c^3 + 85*(c^3*m^4 + 4*c^3*m^3 + 6*c^3*m^2
 + 4*c^3*m + c^3)*n^2 + 15*(c^3*m^5 + 5*c^3*m^4 + 10*c^3*m^3 + 10*c^3*m^2
+ 5*c^3*m + c^3)*n)*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 3*(b*c^2*m^6 + 6*b
*c^2*m^5 + 15*b*c^2*m^4 + 20*b*c^2*m^3 + 144*(b*c^2*m + b*c^2)*n^5 + 15*b*
c^2*m^2 + 324*(b*c^2*m^2 + 2*b*c^2*m + b*c^2)*n^4 + 6*b*c^2*m + 260*(b*c^2
*m^3 + 3*b*c^2*m^2 + 3*b*c^2*m + b*c^2)*n^3 + b*c^2 + 95*(b*c^2*m^4 + 4*b*
c^2*m^3 + 6*b*c^2*m^2 + 4*b*c^2*m + b*c^2)*n^2 + 16*(b*c^2*m^5 + 5*b*c^2*m
^4 + 10*b*c^2*m^3 + 10*b*c^2*m^2 + 5*b*c^2*m + b*c^2)*n)*x*x^(5*n)*e^(m*lo
g(d) + m*log(x)) + 3*((b^2*c + a*c^2)*m^6 + 6*(b^2*c + a*c^2)*m^5 + 180*(b
^2*c + a*c^2 + (b^2*c + a*c^2)*m)*n^5 + 15*(b^2*c + a*c^2)*m^4 + 396*(b^2*
c + a*c^2 + (b^2*c + a*c^2)*m^2 + 2*(b^2*c + a*c^2)*m)*n^4 + 20*(b^2*c + a
*c^2)*m^3 + 307*((b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 3*(b^2*c + a*c^2)*m
^2 + 3*(b^2*c + a*c^2)*m)*n^3 + b^2*c + a*c^2 + 15*(b^2*c + a*c^2)*m^2 + 1
07*((b^2*c + a*c^2)*m^4 + 4*(b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 6*(b^2*c
 + a*c^2)*m^2 + 4*(b^2*c + a*c^2)*m)*n^2 + 6*(b^2*c + a*c^2)*m + 17*((b^2*
c + a*c^2)*m^5 + 5*(b^2*c + a*c^2)*m^4 + 10*(b^2*c + a*c^2)*m^3 + b^2*c +
a*c^2 + 10*(b^2*c + a*c^2)*m^2 + 5*(b^2*c + a*c^2)*m)*n)*x*x^(4*n)*e^(m*lo
g(d) + m*log(x)) + ((b^3 + 6*a*b*c)*m^6 + 6*(b^3 + 6*a*b*c)*m^5 + 240*(...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68491 vs. $2(180) = 360$.

Time = 67.56 (sec) , antiderivative size = 68491, normalized size of antiderivative = 342.46

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**3,x)
```

output
```
Piecewise(((a + b + c)**3*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a**3*log(x) +
 3*a**2*b*x**n/n + 3*a**2*c*x**(2*n)/(2*n) + 3*a*b**2*x**(2*n)/(2*n) + 2*a
*b*c*x**(3*n)/n + 3*a*c**2*x**(4*n)/(4*n) + b**3*x**(3*n)/(3*n) + 3*b**2*c
*x**(4*n)/(4*n) + 3*b*c**2*x**(5*n)/(5*n) + c**3*x**(6*n)/(6*n))/d, Eq(m,
-1)), (a**3*Piecewise((0**(-6*n - 1)*x, Eq(d, 0)), (Piecewise((-1/(6*n*(d*
x)**(6*n)), Ne(n, 0)), (log(d*x), True))/d, True)) + 3*a**2*b*Piecewise((-
x*x**n*(d*x)**(-6*n - 1)/(5*n), Ne(n, 0)), (x*x**n*(d*x)**(-6*n - 1)*log(x
), True)) + 3*a**2*c*Piecewise((-x*x**(2*n)*(d*x)**(-6*n - 1)/(4*n), Ne(n,
 0)), (x*x**(2*n)*(d*x)**(-6*n - 1)*log(x), True)) + 3*a*b**2*Piecewise((-
x*x**(2*n)*(d*x)**(-6*n - 1)/(4*n), Ne(n, 0)), (x*x**(2*n)*(d*x)**(-6*n -
1)*log(x), True)) + 6*a*b*c*Piecewise((-x*x**(3*n)*(d*x)**(-6*n - 1)/(3*n)
, Ne(n, 0)), (x*x**(3*n)*(d*x)**(-6*n - 1)*log(x), True)) + 3*a*c**2*Piece
wise((-x*x**(4*n)*(d*x)**(-6*n - 1)/(2*n), Ne(n, 0)), (x*x**(4*n)*(d*x)**(
-6*n - 1)*log(x), True)) + b**3*Piecewise((-x*x**(3*n)*(d*x)**(-6*n - 1)/(
3*n), Ne(n, 0)), (x*x**(3*n)*(d*x)**(-6*n - 1)*log(x), True)) + 3*b**2*c*P
iecewise((-x*x**(4*n)*(d*x)**(-6*n - 1)/(2*n), Ne(n, 0)), (x*x**(4*n)*(d*x
)**(-6*n - 1)*log(x), True)) + 3*b*c**2*Piecewise((-x*x**(5*n)*(d*x)**(-6*
n - 1)/n, Ne(n, 0)), (x*x**(5*n)*(d*x)**(-6*n - 1)*log(x), True)) + c**3*x
*x**(6*n)*(d*x)**(-6*n - 1)*log(x), Eq(m, -6*n - 1)), (a**3*Piecewise((0**
(-5*n - 1)*x, Eq(d, 0)), (Piecewise((-1/(5*n*(d*x)**(5*n)), Ne(n, 0)), ...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.36

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx = \frac{c^3d^mxe^{(m\log(x)+6\,n\log(x))}}{m+6\,n+1} + \frac{3\,bc^2d^mxe^{(m\log(x)+5\,n\log(x))}}{m+5\,n+1} + \frac{3\,b^2cd^mxe^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{3\,ac^2d^mxe^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{b^3d^mxe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{6\,abcd^mxe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{3\,ab^2d^mxe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,a^2cd^mxe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,a^2bd^mxe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(dx)^{m+1}a^3}{d(m+1)}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
c^3*d^m*x*e^(m*log(x) + 6*n*log(x))/(m + 6*n + 1) + 3*b*c^2*d^m*x*e^(m*log
(x) + 5*n*log(x))/(m + 5*n + 1) + 3*b^2*c*d^m*x*e^(m*log(x) + 4*n*log(x))/
(m + 4*n + 1) + 3*a*c^2*d^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + b^
3*d^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 6*a*b*c*d^m*x*e^(m*log(x
) + 3*n*log(x))/(m + 3*n + 1) + 3*a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x))/(m
 + 2*n + 1) + 3*a^2*c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 3*a^
2*b*d^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (d*x)^(m + 1)*a^3/(d*(m +
1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25656 vs. $2(200) = 400$.

Time = 0.30 (sec) , antiderivative size = 25656, normalized size of antiderivative = 128.28

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

output

```
(c^3*m^6*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^5*n*x*x^(6*n)*e^(m*l
og(d) + m*log(x)) + 85*c^3*m^4*n^2*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 225
*c^3*m^3*n^3*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 274*c^3*m^2*n^4*x*x^(6*n)
*e^(m*log(d) + m*log(x)) + 120*c^3*m*n^5*x*x^(6*n)*e^(m*log(d) + m*log(x))
 + 3*b*c^2*m^6*x*x^(5*n)*e^(m*log(d) + m*log(x)) + c^3*m^6*x*x^(5*n)*e^(m*
log(d) + m*log(x)) + 48*b*c^2*m^5*n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 15
*c^3*m^5*n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 285*b*c^2*m^4*n^2*x*x^(5*n)
*e^(m*log(d) + m*log(x)) + 85*c^3*m^4*n^2*x*x^(5*n)*e^(m*log(d) + m*log(x)
) + 780*b*c^2*m^3*n^3*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 225*c^3*m^3*n^3*
x*x^(5*n)*e^(m*log(d) + m*log(x)) + 972*b*c^2*m^2*n^4*x*x^(5*n)*e^(m*log(d
) + m*log(x)) + 274*c^3*m^2*n^4*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 432*b*
c^2*m*n^5*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 120*c^3*m*n^5*x*x^(5*n)*e^(m
*log(d) + m*log(x)) + 3*b^2*c*m^6*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 3*a*
c^2*m^6*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 3*b*c^2*m^6*x*x^(4*n)*e^(m*log
(d) + m*log(x)) + c^3*m^6*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 51*b^2*c*m^5
*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 51*a*c^2*m^5*n*x*x^(4*n)*e^(m*log(d
) + m*log(x)) + 48*b*c^2*m^5*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 15*c^3*
m^5*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 321*b^2*c*m^4*n^2*x*x^(4*n)*e^(m
*log(d) + m*log(x)) + 321*a*c^2*m^4*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x))
+ 285*b*c^2*m^4*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 85*c^3*m^4*n^2*...
```

Mupad [B] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 1734, normalized size of antiderivative = 8.67

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

input

```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^3,x)
```

output
```
(a^3*x*(d*x)^m)/(m + 1) + (c^3*x*x^(6*n)*(d*x)^m*(5*m + 15*n + 60*m*n + 25
5*m*n^2 + 90*m^2*n + 450*m*n^3 + 60*m^3*n + 274*m*n^4 + 15*m^4*n + 10*m^2
+ 10*m^3 + 5*m^4 + m^5 + 85*n^2 + 225*n^3 + 274*n^4 + 120*n^5 + 255*m^2*n^
2 + 225*m^2*n^3 + 85*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210
*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21
*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624
*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 16
24*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*a*x*x^(2*n)*(d*x)^m*(a*c
+ b^2)*(5*m + 19*n + 76*m*n + 411*m*n^2 + 114*m^2*n + 922*m*n^3 + 76*m^3*n
 + 702*m*n^4 + 19*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 137*n^2 + 461*n^
3 + 702*n^4 + 360*n^5 + 411*m^2*n^2 + 461*m^2*n^3 + 137*m^3*n^2 + 1))/(6*m
 + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*
m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m
^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^
2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2
+ 1) + (b*x*x^(3*n)*(d*x)^m*(6*a*c + b^2)*(5*m + 18*n + 72*m*n + 363*m*n^2
 + 108*m^2*n + 744*m*n^3 + 72*m^3*n + 508*m*n^4 + 18*m^4*n + 10*m^2 + 10*m
^3 + 5*m^4 + m^5 + 121*n^2 + 372*n^3 + 508*n^4 + 240*n^5 + 363*m^2*n^2 + 3
72*m^2*n^3 + 121*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2
*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3880, normalized size of antiderivative = 19.40

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x)
```

output

```
(x**m*d**m*x*(x**(6*n)*c**3*m**6 + 15*x**(6*n)*c**3*m**5*n + 6*x**(6*n)*c*
*3*m**5 + 85*x**(6*n)*c**3*m**4*n**2 + 75*x**(6*n)*c**3*m**4*n + 15*x**(6*
n)*c**3*m**4 + 225*x**(6*n)*c**3*m**3*n**3 + 340*x**(6*n)*c**3*m**3*n**2 +
 150*x**(6*n)*c**3*m**3*n + 20*x**(6*n)*c**3*m**3 + 274*x**(6*n)*c**3*m**2
*n**4 + 675*x**(6*n)*c**3*m**2*n**3 + 510*x**(6*n)*c**3*m**2*n**2 + 150*x*
*(6*n)*c**3*m**2*n + 15*x**(6*n)*c**3*m**2 + 120*x**(6*n)*c**3*m*n**5 + 54
8*x**(6*n)*c**3*m*n**4 + 675*x**(6*n)*c**3*m*n**3 + 340*x**(6*n)*c**3*m*n*
*2 + 75*x**(6*n)*c**3*m*n + 6*x**(6*n)*c**3*m + 120*x**(6*n)*c**3*n**5 + 2
74*x**(6*n)*c**3*n**4 + 225*x**(6*n)*c**3*n**3 + 85*x**(6*n)*c**3*n**2 + 1
5*x**(6*n)*c**3*n + x**(6*n)*c**3 + 3*x**(5*n)*b*c**2*m**6 + 48*x**(5*n)*b
*c**2*m**5*n + 18*x**(5*n)*b*c**2*m**5 + 285*x**(5*n)*b*c**2*m**4*n**2 + 2
40*x**(5*n)*b*c**2*m**4*n + 45*x**(5*n)*b*c**2*m**4 + 780*x**(5*n)*b*c**2*
m**3*n**3 + 1140*x**(5*n)*b*c**2*m**3*n**2 + 480*x**(5*n)*b*c**2*m**3*n +
60*x**(5*n)*b*c**2*m**3 + 972*x**(5*n)*b*c**2*m**2*n**4 + 2340*x**(5*n)*b*
c**2*m**2*n**3 + 1710*x**(5*n)*b*c**2*m**2*n**2 + 480*x**(5*n)*b*c**2*m**2
*n + 45*x**(5*n)*b*c**2*m**2 + 432*x**(5*n)*b*c**2*m*n**5 + 1944*x**(5*n)*
b*c**2*m*n**4 + 2340*x**(5*n)*b*c**2*m*n**3 + 1140*x**(5*n)*b*c**2*m*n**2
+ 240*x**(5*n)*b*c**2*m*n + 18*x**(5*n)*b*c**2*m + 432*x**(5*n)*b*c**2*n**
5 + 972*x**(5*n)*b*c**2*n**4 + 780*x**(5*n)*b*c**2*n**3 + 285*x**(5*n)*b*c
**2*n**2 + 48*x**(5*n)*b*c**2*n + 3*x**(5*n)*b*c**2 + 3*x**(4*n)*a*c**2...
```

3.241 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 129

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx = \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2abx^n(dx)^{1+m}}{d(1+m+n)}+\frac{\left(b^2+2ac\right)x^{2n}(dx)^{1+m}}{d(1+m+2n)} +\frac{2bcx^{3n}(dx)^{1+m}}{d(1+m+3n)}+\frac{c^2x^{4n}(dx)^{1+m}}{d(1+m+4n)}$$

output
```
a^2*(d*x)^(1+m)/d/(1+m)+2*a*b*x^n*(d*x)^(1+m)/d/(1+m+n)+(2*a*c+b^2)*x^(2*n
)*(d*x)^(1+m)/d/(1+m+2*n)+2*b*c*x^(3*n)*(d*x)^(1+m)/d/(1+m+3*n)+c^2*x^(4*n
)*(d*x)^(1+m)/d/(1+m+4*n)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.67

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx = x(dx)^m\left(\frac{a^2}{1+m}+\frac{2abx^n}{1+m+n}+\frac{\left(b^2+2ac\right)x^{2n}}{1+m+2n} +\frac{2bcx^{3n}}{1+m+3n}+\frac{c^2x^{4n}}{1+m+4n}\right)$$

input
```
Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^n)/(1 + m + n) + ((b^2 + 2*a*c)*x^(2*n))
/(1 + m + 2*n) + (2*b*c*x^(3*n))/(1 + m + 3*n) + (c^2*x^(4*n))/(1 + m + 4*
n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 dx$$

$$\Big\downarrow \text{1691}$$

$$\int \left(a^2(dx)^m+b^2x^{2n}\left(\frac{2ac}{b^2}+1\right)(dx)^m+2abx^n(dx)^m+2bcx^{3n}(dx)^m+c^2x^{4n}(dx)^m\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{x^{2n+1}\left(2ac+b^2\right)(dx)^m}{m+2n+1}+\frac{2abx^{n+1}(dx)^m}{m+n+1}+\frac{2bcx^{3n+1}(dx)^m}{m+3n+1}+\frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

input
```
Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
(2*a*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + ((b^2 + 2*a*c)*x^(1 + 2*n)*(d*x)^m
)/(1 + m + 2*n) + (2*b*c*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) + (c^2*x^(1 +
4*n)*(d*x)^m)/(1 + m + 4*n) + (a^2*(d*x)^(1 + m))/(d*(1 + m))
```

Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 1032, normalized size of antiderivative = 8.00

method	result	size
risch	Expression too large to display	1032
parallelrisch	Expression too large to display	1566
orering	Expression too large to display	2292

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x,method=_RETURNVERBOSE)
```

output
```
x*(a^2+35*a^2*n^2+38*b^2*m*n^2*(x^n)^2+19*b^2*m^2*n^2*(x^n)^2+12*b^2*m*n^3
*(x^n)^2+8*b*c*m^3*(x^n)^3+16*b*c*n^3*(x^n)^3+18*c^2*m*n*(x^n)^4+2*a*b*m^4
*x^n+8*a*c*m^3*(x^n)^2+24*a*c*n^3*(x^n)^2+24*b^2*m^2*n*(x^n)^2+2*a*c*m^4*(
x^n)^2+8*b^2*m^3*n*(x^n)^2+6*c^2*m^3*n*(x^n)^4+11*c^2*m^2*n^2*(x^n)^4+6*c^
2*m*n^3*(x^n)^4+2*b*c*m^4*(x^n)^3+18*c^2*m^2*n*(x^n)^4+22*c^2*m*n^2*(x^n)^
4+16*a*c*(x^n)^2*n+24*b^2*m*n*(x^n)^2+8*m*b*c*(x^n)^3+14*b*c*(x^n)^3*n+12*
a*b*m^2*x^n+52*a*b*n^2*x^n+8*a*c*(x^n)^2*m+12*b*c*m^2*(x^n)^3+28*b*c*n^2*(
x^n)^3+8*a*b*m^3*x^n+48*a*b*n^3*x^n+12*a*c*m^2*(x^n)^2+38*a*c*n^2*(x^n)^2+
8*x^n*a*b*m+18*x^n*n*a*b+54*a*b*m^2*n*x^n+104*a*b*m*n^2*x^n+48*a*c*m*n*(x^
n)^2+54*a*b*m*n*x^n+16*a*c*m^3*n*(x^n)^2+38*a*c*m^2*n^2*(x^n)^2+24*a*c*m*n
^3*(x^n)^2+42*b*c*m^2*n*(x^n)^3+76*a*c*m*n^2*(x^n)^2+42*b*c*m*n*(x^n)^3+14
*b*c*m^3*n*(x^n)^3+28*b*c*m^2*n^2*(x^n)^3+16*b*c*m*n^3*(x^n)^3+2*(x^n)^2*a
*c+50*a^2*n^3+6*a^2*m^2+56*b*c*m*n^2*(x^n)^3+18*a*b*m^3*n*x^n+52*a*b*m^2*n
^2*x^n+48*a*b*m*n^3*x^n+48*a*c*m^2*n*(x^n)^2+4*b^2*m^3*(x^n)^2+10*a^2*n+4*
a^2*m+(x^n)^2*b^2+24*a^2*n^4+6*b^2*m^2*(x^n)^2+4*b^2*(x^n)^2*m+a^2*m^4+4*a
^2*m^3+6*c^2*m^2*(x^n)^4+(x^n)^4*c^2+11*c^2*n^2*(x^n)^4+12*b^2*n^3*(x^n)^2
+6*c^2*(x^n)^4*n+19*b^2*n^2*(x^n)^2+b^2*m^4*(x^n)^2+8*b^2*(x^n)^2*n+30*a^2
*m^2*n+70*a^2*m*n^2+30*a^2*m*n+35*a^2*m^2*n^2+50*a^2*m*n^3+4*m*c^2*(x^n)^4
+4*c^2*m^3*(x^n)^4+c^2*m^4*(x^n)^4+6*c^2*n^3*(x^n)^4+2*x^n*a*b+10*a^2*m^3*
n+2*(x^n)^3*b*c)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)/(1+m+4*n)*x^m*d^m*ex...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(129) = 258$.

Time = 0.21 (sec) , antiderivative size = 706, normalized size of antiderivative = 5.47

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \,dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
((c^2*m^4 + 4*c^2*m^3 + 6*c^2*m^2 + 6*(c^2*m + c^2)*n^3 + 4*c^2*m + 11*(c^
2*m^2 + 2*c^2*m + c^2)*n^2 + c^2 + 6*(c^2*m^3 + 3*c^2*m^2 + 3*c^2*m + c^2)
*n)*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*(b*c*m^4 + 4*b*c*m^3 + 6*b*c*m^2
 + 8*(b*c*m + b*c)*n^3 + 4*b*c*m + 14*(b*c*m^2 + 2*b*c*m + b*c)*n^2 + b*c
+ 7*(b*c*m^3 + 3*b*c*m^2 + 3*b*c*m + b*c)*n)*x*x^(3*n)*e^(m*log(d) + m*log
(x)) + ((b^2 + 2*a*c)*m^4 + 4*(b^2 + 2*a*c)*m^3 + 12*(b^2 + 2*a*c + (b^2 +
 2*a*c)*m)*n^3 + 6*(b^2 + 2*a*c)*m^2 + 19*((b^2 + 2*a*c)*m^2 + b^2 + 2*a*c
 + 2*(b^2 + 2*a*c)*m)*n^2 + b^2 + 2*a*c + 4*(b^2 + 2*a*c)*m + 8*((b^2 + 2*
a*c)*m^3 + 3*(b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 3*(b^2 + 2*a*c)*m)*n)*x*x^(
2*n)*e^(m*log(d) + m*log(x)) + 2*(a*b*m^4 + 4*a*b*m^3 + 6*a*b*m^2 + 24*(a*
b*m + a*b)*n^3 + 4*a*b*m + 26*(a*b*m^2 + 2*a*b*m + a*b)*n^2 + a*b + 9*(a*b
*m^3 + 3*a*b*m^2 + 3*a*b*m + a*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^2*
m^4 + 24*a^2*n^4 + 4*a^2*m^3 + 6*a^2*m^2 + 50*(a^2*m + a^2)*n^3 + 4*a^2*m
+ 35*(a^2*m^2 + 2*a^2*m + a^2)*n^2 + a^2 + 10*(a^2*m^3 + 3*a^2*m^2 + 3*a^2
*m + a^2)*n)*x*e^(m*log(d) + m*log(x)))/(m^5 + 24*(m + 1)*n^4 + 5*m^4 + 50
*(m^2 + 2*m + 1)*n^3 + 10*m^3 + 35*(m^3 + 3*m^2 + 3*m + 1)*n^2 + 10*m^2 +
10*(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)*n + 5*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12323 vs. $2(114) = 228$.

Time = 16.31 (sec) , antiderivative size = 12323, normalized size of antiderivative = 95.53

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Piecewise(((a + b + c)**2*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a**2*log(x) +
 2*a*b*x**n/n + a*c*x**(2*n)/n + b**2*x**(2*n)/(2*n) + 2*b*c*x**(3*n)/(3*n
) + c**2*x**(4*n)/(4*n))/d, Eq(m, -1)), (a**2*Piecewise((0**(-4*n - 1)*x,
Eq(d, 0)), (Piecewise((-1/(4*n*(d*x)**(4*n)), Ne(n, 0)), (log(d*x), True))
/d, True)) + 2*a*b*Piecewise((-x*x**n*(d*x)**(-4*n - 1)/(3*n), Ne(n, 0)),
(x*x**n*(d*x)**(-4*n - 1)*log(x), True)) + 2*a*c*Piecewise((-x*x**(2*n)*(d
*x)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*x**(2*n)*(d*x)**(-4*n - 1)*log(x), Tr
ue)) + b**2*Piecewise((-x*x**(2*n)*(d*x)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*
x**(2*n)*(d*x)**(-4*n - 1)*log(x), True)) + 2*b*c*Piecewise((-x*x**(3*n)*(
d*x)**(-4*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(d*x)**(-4*n - 1)*log(x), True)
) + c**2*x*x**(4*n)*(d*x)**(-4*n - 1)*log(x), Eq(m, -4*n - 1)), (a**2*Piec
ewise((0**(-3*n - 1)*x, Eq(d, 0)), (Piecewise((-1/(3*n*(d*x)**(3*n)), Ne(n
, 0)), (log(d*x), True))/d, True)) + 2*a*b*Piecewise((-x*x**n*(d*x)**(-3*n
 - 1)/(2*n), Ne(n, 0)), (x*x**n*(d*x)**(-3*n - 1)*log(x), True)) + 2*a*c*P
iecewise((-x*x**(2*n)*(d*x)**(-3*n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(d*x)**(
-3*n - 1)*log(x), True)) + b**2*Piecewise((-x*x**(2*n)*(d*x)**(-3*n - 1)/n
, Ne(n, 0)), (x*x**(2*n)*(d*x)**(-3*n - 1)*log(x), True)) + 2*b*c*x*x**(3*
n)*(d*x)**(-3*n - 1)*log(x) + c**2*Piecewise((x*x**(4*n)*(d*x)**(-3*n - 1)
/n, Ne(n, 0)), (x*x**(4*n)*(d*x)**(-3*n - 1)*log(x), True)), Eq(m, -3*n -
1)), (a**2*Piecewise((0**(-2*n - 1)*x, Eq(d, 0)), (Piecewise((-1/(2*n*(...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.18

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^2 \, dx = \frac{c^2 d^m x e^{(m \log(x) + 4 n \log(x))}}{m + 4n + 1} + \frac{2 bcd^m x e^{(m \log(x) + 3 n \log(x))}}{m + 3n + 1} + \frac{b^2 d^m x e^{(m \log(x) + 2 n \log(x))}}{m + 2n + 1} + \frac{2 acd^m x e^{(m \log(x) + 2 n \log(x))}}{m + 2n + 1} + \frac{2 abd^m x e^{(m \log(x) + n \log(x))}}{m + n + 1} + \frac{(dx)^{m+1} a^2}{d(m + 1)}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
c^2*d^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 2*b*c*d^m*x*e^(m*log(x
) + 3*n*log(x))/(m + 3*n + 1) + b^2*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2
*n + 1) + 2*a*c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*a*b*d^m*
x*e^(m*log(x) + n*log(x))/(m + n + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5454 vs. $2(129) = 258$.

Time = 0.17 (sec) , antiderivative size = 5454, normalized size of antiderivative = 42.28

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
(c^2*m^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(4*n)*e^(m*lo
g(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^
2*m*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*x^(3*n)*e^(m*log(d
) + m*log(x)) + c^2*m^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x
*x^(3*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(3*n)*e^(m*log(d) + m*l
og(x)) + 28*b*c*m^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 11*c^2*m^2*n^2
*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^(3*n)*e^(m*log(d) +
m*log(x)) + 6*c^2*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^(2
*n)*e^(m*log(d) + m*log(x)) + 2*a*c*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x))
 + 2*b*c*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c^2*m^4*x*x^(2*n)*e^(m*log
(d) + m*log(x)) + 8*b^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*a*c*m
^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^(2*n)*e^(m*log(d
) + m*log(x)) + 6*c^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 19*b^2*m^2
*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 38*a*c*m^2*n^2*x*x^(2*n)*e^(m*log
(d) + m*log(x)) + 28*b*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 11*c^
2*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*x^(2*n)*e^(m*
log(d) + m*log(x)) + 24*a*c*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*b
*c*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m*n^3*x*x^(2*n)*e^(m*lo
g(d) + m*log(x)) + 2*a*b*m^4*x*x^n*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^n
*e^(m*log(d) + m*log(x)) + 2*a*c*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 2*...
```

Mupad [B] (verification not implemented)

Time = 10.79 (sec) , antiderivative size = 543, normalized size of antiderivative = 4.21

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx = \frac{a^2 \, x \, (d \, x)^m}{m+1}$$
$$+ \frac{x \, x^{2\,n} \, (d \, x)^m \, (b^2+2 \, a \, c) \, (m^3+8 \, m^2 \, n+3 \, m^2+19 \, m \, n^2+16 \, m \, n+3 \, m+12 \, n^3+19 \, n^2 +}{m^4+10 \, m^3 \, n+4 \, m^3+35 \, m^2 \, n^2+30 \, m^2 \, n+6 \, m^2+50 \, m \, n^3+70 \, m \, n^2+30 \, m \, n+4 \, m+24 \, n^4+50}$$
$$+ \frac{c^2 \, x \, x^{4\,n} \, (d \, x)^m \, (m^3+6 \, m^2 \, n+3 \, m^2+11 \, m \, n^2+12 \, m \, n+3 \, m+6 \, n^3+11 \, n^2+6 \, n}{m^4+10 \, m^3 \, n+4 \, m^3+35 \, m^2 \, n^2+30 \, m^2 \, n+6 \, m^2+50 \, m \, n^3+70 \, m \, n^2+30 \, m \, n+4 \, m+24 \, n^4+50}$$
$$+ \frac{2 \, a \, b \, x \, x^n \, (d \, x)^m \, (m^3+9 \, m^2 \, n+3 \, m^2+26 \, m \, n^2+18 \, m \, n+3 \, m+24 \, n^3+26 \, n^2+9 \, n}{m^4+10 \, m^3 \, n+4 \, m^3+35 \, m^2 \, n^2+30 \, m^2 \, n+6 \, m^2+50 \, m \, n^3+70 \, m \, n^2+30 \, m \, n+4 \, m+24 \, n^4+50}$$
$$+ \frac{2 \, b \, c \, x \, x^{3\,n} \, (d \, x)^m \, (m^3+7 \, m^2 \, n+3 \, m^2+14 \, m \, n^2+14 \, m \, n+3 \, m+8 \, n^3+14 \, n^2+7 \, n}{m^4+10 \, m^3 \, n+4 \, m^3+35 \, m^2 \, n^2+30 \, m^2 \, n+6 \, m^2+50 \, m \, n^3+70 \, m \, n^2+30 \, m \, n+4 \, m+24 \, n^4+50}$$

input
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^2,x)
```

output
```
(a^2*x*(d*x)^m)/(m + 1) + (x*x^(2*n)*(d*x)^m*(2*a*c + b^2)*(3*m + 8*n + 16
*m*n + 19*m*n^2 + 8*m^2*n + 3*m^2 + m^3 + 19*n^2 + 12*n^3 + 1))/(4*m + 10*
n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m
^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (c^2*x*x^(4*n)*(d*x)^m*(
3*m + 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3*m^2 + m^3 + 11*n^2 + 6*n^3 + 1
))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^
2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (2*a*b*x*x^
n*(d*x)^m*(3*m + 9*n + 18*m*n + 26*m*n^2 + 9*m^2*n + 3*m^2 + m^3 + 26*n^2
+ 24*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*
m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) +
 (2*b*c*x*x^(3*n)*(d*x)^m*(3*m + 7*n + 14*m*n + 14*m*n^2 + 7*m^2*n + 3*m^2
 + m^3 + 14*n^2 + 8*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n +
 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35
*m^2*n^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1063, normalized size of antiderivative = 8.24

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 dx = \text{Too large to display}$$

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
(x**m*d**m*x*(x**(4*n)*c**2*m**4 + 6*x**(4*n)*c**2*m**3*n + 4*x**(4*n)*c**
2*m**3 + 11*x**(4*n)*c**2*m**2*n**2 + 18*x**(4*n)*c**2*m**2*n + 6*x**(4*n)
*c**2*m**2 + 6*x**(4*n)*c**2*m*n**3 + 22*x**(4*n)*c**2*m*n**2 + 18*x**(4*n
)*c**2*m*n + 4*x**(4*n)*c**2*m + 6*x**(4*n)*c**2*n**3 + 11*x**(4*n)*c**2*n
**2 + 6*x**(4*n)*c**2*n + x**(4*n)*c**2 + 2*x**(3*n)*b*c*m**4 + 14*x**(3*n
)*b*c*m**3*n + 8*x**(3*n)*b*c*m**3 + 28*x**(3*n)*b*c*m**2*n**2 + 42*x**(3*
n)*b*c*m**2*n + 12*x**(3*n)*b*c*m**2 + 16*x**(3*n)*b*c*m*n**3 + 56*x**(3*n
)*b*c*m*n**2 + 42*x**(3*n)*b*c*m*n + 8*x**(3*n)*b*c*m + 16*x**(3*n)*b*c*n*
*3 + 28*x**(3*n)*b*c*n**2 + 14*x**(3*n)*b*c*n + 2*x**(3*n)*b*c + 2*x**(2*n
)*a*c*m**4 + 16*x**(2*n)*a*c*m**3*n + 8*x**(2*n)*a*c*m**3 + 38*x**(2*n)*a*
c*m**2*n**2 + 48*x**(2*n)*a*c*m**2*n + 12*x**(2*n)*a*c*m**2 + 24*x**(2*n)*
a*c*m*n**3 + 76*x**(2*n)*a*c*m*n**2 + 48*x**(2*n)*a*c*m*n + 8*x**(2*n)*a*c
*m + 24*x**(2*n)*a*c*n**3 + 38*x**(2*n)*a*c*n**2 + 16*x**(2*n)*a*c*n + 2*x
**(2*n)*a*c + x**(2*n)*b**2*m**4 + 8*x**(2*n)*b**2*m**3*n + 4*x**(2*n)*b**
2*m**3 + 19*x**(2*n)*b**2*m**2*n**2 + 24*x**(2*n)*b**2*m**2*n + 6*x**(2*n)
*b**2*m**2 + 12*x**(2*n)*b**2*m*n**3 + 38*x**(2*n)*b**2*m*n**2 + 24*x**(2*
n)*b**2*m*n + 4*x**(2*n)*b**2*m + 12*x**(2*n)*b**2*n**3 + 19*x**(2*n)*b**2
*n**2 + 8*x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*m**4 + 18*x**n*a*b*
m**3*n + 8*x**n*a*b*m**3 + 52*x**n*a*b*m**2*n**2 + 54*x**n*a*b*m**2*n + 12
*x**n*a*b*m**2 + 48*x**n*a*b*m*n**3 + 104*x**n*a*b*m*n**2 + 54*x**n*a*b...
```

3.242 $\int (dx)^m \left(a+bx^n+cx^{2n}\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right) dx = \frac{a(dx)^{1+m}}{d(1+m)}+\frac{bx^n(dx)^{1+m}}{d(1+m+n)}+\frac{cx^{2n}(dx)^{1+m}}{d(1+m+2n)}$$

output
```
a*(d*x)^(1+m)/d/(1+m)+b*x^n*(d*x)^(1+m)/d/(1+m+n)+c*x^(2*n)*(d*x)^(1+m)/d/
(1+m+2*n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.64

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right) dx = x(dx)^m \left(\frac{a}{1+m}+x^n\left(\frac{b}{1+m+n}+\frac{cx^n}{1+m+2n}\right)\right)$$

input
```
Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

output
```
x*(d*x)^m*(a/(1 + m) + x^n*(b/(1 + m + n) + (c*x^n)/(1 + m + 2*n)))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right) dx$$

$$\Big\downarrow \text{1691}$$

$$\int \left(a(dx)^m+bx^n(dx)^m+cx^{2n}(dx)^m\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{bx^{n+1}(dx)^m}{m+n+1}+\frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

input
```
Int[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

output
```
(b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (c*x^(1 + 2*n)*(d*x)^m)/(1 + m + 2*n)
+ (a*(d*x)^(1 + m))/(d*(1 + m))
```

Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.69

method	result
risch	$\frac{x(c\,m^2x^{2n}+cmn\,x^{2n}+x^n b\,m^2+2mb\,x^n n+2x^{2n}cm+c\,x^{2n}n+a\,m^2+3amn+2a\,n^2+2mb\,x^n+2b\,x^n n+c\,x^{2n}+2am+3an+b\,x^n}{(1+m)(1+m+n)(1+m+2n)}$
parallelrisch	$\frac{x(dx)^m a\,m^2+2x(dx)^m a\,n^2+x\,x^n(dx)^m b+x\,x^{2n}(dx)^m c+2x(dx)^m am+3x(dx)^m an+2x\,x^n(dx)^m bmn+x\,x^{2n}(dx)^m cmn+x}{(1+m)(1+m}$
orering	$\frac{x(3m^2+6mn+2n^2+3m+3n+1)(dx)^m(a+b\,x^n+c\,x^{2n})}{(m^2+2mn+2m+2n+1)(1+m+n)}-\frac{3x^2(m+n)\left(\frac{(dx)^m m(a+b\,x^n+c\,x^{2n})}{x}+(dx)^m\left(\frac{b\,x^n n}{x}+\frac{2c\,x^{2n}n}{x}\right)\right)}{(m^2+2mn+2m+2n+1)(1+m+n)}$

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
x*(c*m^2*(x^n)^2+c*m*n*(x^n)^2+x^n*b*m^2+2*m*b*x^n*n+2*(x^n)^2*c*m+c*(x^n)
^2*n+a*m^2+3*a*m*n+2*a*n^2+2*m*b*x^n+2*b*x^n*n+c*(x^n)^2+2*a*m+3*a*n+b*x^n
+a)/(1+m)/(1+m+n)/(1+m+2*n)*x^m*d^m*exp(1/2*I*csgn(I*d*x)*Pi*m*(csgn(I*d*x
)-csgn(I*x))*(-csgn(I*d*x)+csgn(I*d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(64) = 128$.

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.22

$$\int (dx)^m\left(a+bx^n+cx^{2n}\right)\,dx$$
$$=\frac{(cm^2+2\,cm+(cm+c)n+c)xx^{2\,n}e^{(m\log(d)+m\log(x))}+(bm^2+2\,bm+2\,(bm+b)n+b)xx^n e^{(m\log(d)+m\log}}{m^3+2\,(m+1)n^2+3\,m^2+3\,(m^2+2\,m+1)n}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
((c*m^2 + 2*c*m + (c*m + c)*n + c)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + (b*
m^2 + 2*b*m + 2*(b*m + b)*n + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m^2 +
2*a*n^2 + 2*a*m + 3*(a*m + a)*n + a)*x*e^(m*log(d) + m*log(x)))/(m^3 + 2*(
m + 1)*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*n + 3*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(53) = 106$.

Time = 3.37 (sec) , antiderivative size = 1096, normalized size of antiderivative = 17.12

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)\, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(a+b*x**n+c*x**(2*n)),x)
```

output
```
Piecewise(((a + b + c)*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a*log(x) + b*x**
n/n + c*x**(2*n)/(2*n))/d, Eq(m, -1)), (a*Piecewise((0**(-2*n - 1)*x, Eq(d
, 0)), (Piecewise((-1/(2*n*(d*x)**(2*n)), Ne(n, 0)), (log(d*x), True))/d,
True)) + b*Piecewise((-x*x**n*(d*x)**(-2*n - 1)/n, Ne(n, 0)), (x*x**n*(d*x
)**(-2*n - 1)*log(x), True)) + c*x*x**(2*n)*(d*x)**(-2*n - 1)*log(x), Eq(m
, -2*n - 1)), (a*Piecewise((0**(-n - 1)*x, Eq(d, 0)), (Piecewise((-1/(n*(d
*x)**n), Ne(n, 0)), (log(d*x), True))/d, True)) + b*x*x**n*(d*x)**(-n - 1)
*log(x) + c*Piecewise((x*x**(2*n)*(d*x)**(-n - 1)/n, Ne(n, 0)), (x*x**(2*n
)*(d*x)**(-n - 1)*log(x), True)), Eq(m, -n - 1)), (a*m**2*x*(d*x)**m/(m**3
 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 3*a*m*
n*x*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 +
 3*n + 1) + 2*a*m*x*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n
+ 3*m + 2*n**2 + 3*n + 1) + 2*a*n**2*x*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2
+ 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 3*a*n*x*(d*x)**m/(m**3 + 3*
m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + a*x*(d*x)**
m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) +
 b*m**2*x*x**n*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m
 + 2*n**2 + 3*n + 1) + 2*b*m*n*x*x**n*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 +
 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*b*m*x*x**n*(d*x)**m/(m**3
+ 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*b*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right) \, dx = \frac{cd^m x e^{(m \log(x)+2 n \log(x))}}{m+2n+1} + \frac{bd^m x e^{(m \log(x)+n \log(x))}}{m+n+1} + \frac{(dx)^{m+1} a}{d(m+1)}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + b*d^m*x*e^(m*log(x) + n*
log(x))/(m + n + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(64) = 128.

Time = 0.13 (sec) , antiderivative size = 557, normalized size of antiderivative = 8.70

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
(c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c*m*n*x*x^(2*n)*e^(m*log(d) + m
*log(x)) + b*m^2*x*x^n*e^(m*log(d) + m*log(x)) + c*m^2*x*x^n*e^(m*log(d) +
 m*log(x)) + 2*b*m*n*x*x^n*e^(m*log(d) + m*log(x)) + c*m*n*x*x^n*e^(m*log(
d) + m*log(x)) + a*m^2*x*e^(m*log(d) + m*log(x)) + b*m^2*x*e^(m*log(d) + m
*log(x)) + c*m^2*x*e^(m*log(d) + m*log(x)) + 3*a*m*n*x*e^(m*log(d) + m*log
(x)) + 2*b*m*n*x*e^(m*log(d) + m*log(x)) + c*m*n*x*e^(m*log(d) + m*log(x))
 + 2*a*n^2*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*x^(2*n)*e^(m*log(d) + m*log
(x)) + c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*m*x*x^n*e^(m*log(d) + m
*log(x)) + 2*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*n*x*x^n*e^(m*log(d) +
 m*log(x)) + c*n*x*x^n*e^(m*log(d) + m*log(x)) + 2*a*m*x*e^(m*log(d) + m*l
og(x)) + 2*b*m*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*e^(m*log(d) + m*log(x))
 + 3*a*n*x*e^(m*log(d) + m*log(x)) + 2*b*n*x*e^(m*log(d) + m*log(x)) + c*n
*x*e^(m*log(d) + m*log(x)) + c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + b*x*x^n
*e^(m*log(d) + m*log(x)) + c*x*x^n*e^(m*log(d) + m*log(x)) + a*x*e^(m*log(
d) + m*log(x)) + b*x*e^(m*log(d) + m*log(x)) + c*x*e^(m*log(d) + m*log(x))
)/(m^3 + 3*m^2*n + 2*m*n^2 + 3*m^2 + 6*m*n + 2*n^2 + 3*m + 3*n + 1)
```

Mupad [B] (verification not implemented)

Time = 10.62 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)\,dx=(d\,x)^m\,\left(\frac{a\,x}{m+1}+\frac{b\,x\,x^n\,(m+2\,n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}+\frac{c\,x\,x^{2\,n}\,(m+n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}\right)$$

input
```
int((d*x)^m*(a + b*x^n + c*x^(2*n)),x)
```

output
```
(d*x)^m*((a*x)/(m + 1) + (b*x*x^n*(m + 2*n + 1))/(2*m + 3*n + 3*m*n + m^2
+ 2*n^2 + 1) + (c*x*x^(2*n)*(m + n + 1))/(2*m + 3*n + 3*m*n + m^2 + 2*n^2
+ 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.36

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)\,dx$$
$$=\frac{x^m d^m x(x^{2n}c\,m^2+x^{2n}cmn+2x^{2n}cm+x^{2n}cn+x^{2n}c+x^n b\,m^2+2x^n bmn+2x^n bm+2x^n bn+x^n b+a}{m^3+3m^2n+2m\,n^2+3m^2+6mn+2n^2+3m+3n+1}$$

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n)),x)
```

output
```
(x**m*d**m*x*(x**(2*n)*c*m**2 + x**(2*n)*c*m*n + 2*x**(2*n)*c*m + x**(2*n)
*c*n + x**(2*n)*c + x**n*b*m**2 + 2*x**n*b*m*n + 2*x**n*b*m + 2*x**n*b*n +
 x**n*b + a*m**2 + 3*a*m*n + 2*a*m + 2*a*n**2 + 3*a*n + a))/(m**3 + 3*m**2
*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1)
```

3.243 $\int \frac{(dx)^m}{a+bx^n+cx^{2n}} dx$

Optimal result

Integrand size = 22, antiderivative size = 175

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}} dx = \frac{2c(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

output
```
2*c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2
)^(1/2)))/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))/d/(1+m)-2*c*(d*x)^(1+m
)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(-4*
a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/d/(1+m)
```

Mathematica [A] (warning: unable to verify)

Time = 1.27 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.75

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx = -\frac{x(dx)^m\left(\frac{2c\left(1-2^{-\frac{1+m}{n}}\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-\frac{1+m}{n}}\text{Hypergeometric2F1}\left(-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{2c\left(1-2^{-}\right.}{}\right.}{1+m}$$

input
```
Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

output
```
-((x*(d*x)^m*((2*c*(1 - Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 -
(1 + m)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(2^(
(1 + m)/n)*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n))))/(b^2
 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (2*c*(1 - Hypergeometric2F1[-((1 + m)/n)
, -((1 + m)/n), (-1 - m + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^n)]/(2^((1 + m)/n)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)
)^((1 + m)/n))))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))/(1 + m))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1719, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1719}$$

$$\frac{2c\int \frac{(dx)^m}{2cx^n+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{(dx)^m}{2cx^n+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2c(dx)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

output
```
(2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^
n)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*
(1 + m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 -
 4*a*c])*d*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1719
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(d*x)^m/(b - q + 2
*c*x^n), x], x] - Simp[2*(c/q)   Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /;
FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{(dx)^m}{a + b\,x^n + c\,x^{2n}}dx$$

input
```
int((d*x)^m/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int((d*x)^m/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{a + bx^n + cx^{2n}}\,dx = \int \frac{(dx)^m}{cx^{2\,n} + bx^n + a}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{a + bx^n + cx^{2n}}\,dx = \int \frac{(dx)^m}{a + bx^n + cx^{2n}}\,dx$$

input
```
integrate((d*x)**m/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral((d*x)**m/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx=\int \frac{(dx)^m}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx=\int \frac{(dx)^m}{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx=\int \frac{(d\,x)^m}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((d*x)^m/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int((d*x)^m/(a + b*x^n + c*x^(2*n)), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{a + bx^n + cx^{2n}}\, dx = d^m \left(\int \frac{x^m}{x^{2n}c + x^n b + a} dx \right)$$

input
```
int((d*x)^m/(a+b*x^n+c*x^(2*n)),x)
```

output
```
d**m*int(x**m/(x**(2*n)*c + x**n*b + a),x)
```

3.244 $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 328

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)} + \frac{c\left(4ac(1+m-2n)-b^2(1+m-n)-b\sqrt{b^2-4ac}(1+m-n)\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n}\right.}{a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+m)n} - \frac{c\left(4ac(1+m-2n)-b^2(1+m-n)+b\sqrt{b^2-4ac}(1+m-n)\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n}\right.}{a\left(b^2-4ac\right)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d(1+m)n}$$

output
```
(d*x)^(1+m)*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(a+b*x^n+c*x^(2*n))+c*(
4*a*c*(1+m-2*n)-b^2*(1+m-n)-b*(-4*a*c+b^2)^(1/2)*(1+m-n))*(d*x)^(1+m)*hype
rgeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+
b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))/d/(1+m)/n-c*(4*a*c*(1+m-2*n)-b^2*(1+m-n)
+b*(-4*a*c+b^2)^(1/2)*(1+m-n))*(d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)
/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^
(1/2))/d/(1+m)/n
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3515 vs. $2(328) = 656$.

Time = 6.73 (sec) , antiderivative size = 3515, normalized size of antiderivative = 10.72

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
(x*(d*x)^m*(-b^2 + 2*a*c - b*c*x^n))/(a*(-b^2 + 4*a*c)*n*(a + b*x^n + c*x^
(2*n))) - (b*c*x^(1 + n)*(d*x)^m*(x^n)^((1 + m)/n - (1 + m + n)/n)*(-(((x^
n/(-1/2*(-b - Sqrt[b^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hypergeometric2
F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -1/2*(-b - Sqrt[b^2 - 4*a*c]
)/(c*(-1/2*(-b - Sqrt[b^2 - 4*a*c])/c + x^n))])/Sqrt[b^2 - 4*a*c]) + ((x^n
/(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hypergeometric2F
1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -1/2*(-b + Sqrt[b^2 - 4*a*c])
/(c*(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))])/Sqrt[b^2 - 4*a*c]))/(a*(-b^
2 + 4*a*c)*(1 + m)) + (b*c*x^(1 + n)*(d*x)^m*(x^n)^((1 + m)/n - (1 + m + n
)/n)*(-(((x^n/(-1/2*(-b - Sqrt[b^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hyp
ergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -1/2*(-b - Sqrt[
b^2 - 4*a*c])/(c*(-1/2*(-b - Sqrt[b^2 - 4*a*c])/c + x^n))])/Sqrt[b^2 - 4*a
*c]) + ((x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))^(-n^(-1) - m/n)*Hype
rgeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, -1/2*(-b + Sqrt[b
^2 - 4*a*c])/(c*(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))])/Sqrt[b^2 - 4*a*
c]))/(a*(-b^2 + 4*a*c)*(1 + m)*n) + (b*c*m*x^(1 + n)*(d*x)^m*(x^n)^((1 + m
)/n - (1 + m + n)/n)*(-(((x^n/(-1/2*(-b - Sqrt[b^2 - 4*a*c])/c + x^n))^(-n
^(-1) - m/n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n,
-1/2*(-b - Sqrt[b^2 - 4*a*c])/(c*(-1/2*(-b - Sqrt[b^2 - 4*a*c])/c + x^n))]
)/Sqrt[b^2 - 4*a*c]) + ((x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c])/c + x^n))^(...
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1720, 25, 1884, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}dx$$

$$\Big\downarrow \text{ 1720}$$

$$\frac{(dx)^{m+1}(-2ac+b^2+bcx^n)}{adn(b^2-4ac)(a+bx^n+cx^{2n})}-\frac{\int -\frac{(dx)^m(-bc(m-n+1)x^n+2ac(m-2n+1)-b^2(m-n+1))}{bx^n+cx^{2n}+a}dx}{an(b^2-4ac)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(dx)^m(-bc(m-n+1)x^n+2ac(m-2n+1)-b^2(m-n+1))}{bx^n+cx^{2n}+a}dx}{an(b^2-4ac)}+\frac{(dx)^{m+1}(-2ac+b^2+bcx^n)}{adn(b^2-4ac)(a+bx^n+cx^{2n})}$$

$$\Big\downarrow \text{ 1884}$$

$$\frac{\int \left(\frac{\left(-bc(m-n+1)-\frac{c(mb^2-nb^2+b^2-4ac-4acm+8acn)}{\sqrt{b^2-4ac}}\right)(dx)^m}{2cx^n+b-\sqrt{b^2-4ac}}+\frac{\left(\frac{c(mb^2-nb^2+b^2-4ac-4acm+8acn)}{\sqrt{b^2-4ac}}-bc(m-n+1)\right)(dx)^m}{2cx^n+b+\sqrt{b^2-4ac}}\right)dx}{an(b^2-4ac)}+\frac{(dx)^{m+1}(-2ac+b^2+bcx^n)}{adn(b^2-4ac)(a+bx^n+cx^{2n})}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{c(dx)^{m+1}\left(\frac{4ac(m-2n+1)-b^2(m-n+1)}{\sqrt{b^2-4ac}}-b(m-n+1)\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(\frac{4ac(m-2n+1)-b^2(m-}{\sqrt{b^2-4ac}}\right.}{}}{an(b^2-4ac)}$$
$$\frac{(dx)^{m+1}(-2ac+b^2+bcx^n)}{adn(b^2-4ac)(a+bx^n+cx^{2n})}$$

input
```
Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*d*n*(a + b*x^n +
c*x^(2*n))) + ((c*((4*a*c*(1 + m - 2*n) - b^2*(1 + m - n))/Sqrt[b^2 - 4*a*
c] - b*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m +
 n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*d*(1
+ m)) - (c*((4*a*c*(1 + m - 2*n) - b^2*(1 + m - n))/Sqrt[b^2 - 4*a*c] + b*
(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^2 - 4*a*c])*d*(1 + m)))/
(a*(b^2 - 4*a*c)*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1720
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p +
 1) + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*
a*c, 0] && ILtQ[p + 1, 0]
```

rule 1884
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] &&  !RationalQ[n] && (
IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(dx)^m}{(a+b\,x^n+c\,x^{2n})^2}dx$$

input
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*x^n
 + a*c)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
(b*c*d^m*x*e^(m*log(x) + n*log(x)) + (b^2*d^m - 2*a*c*d^m)*x*x^m)/(a^2*b^2
*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*
n)*x^n) + integrate(-(b*c*d^m*(m - n + 1)*e^(m*log(x) + n*log(x)) + (b^2*d
^m*(m - n + 1) - 2*a*c*d^m*(m - 2*n + 1))*x^m)/(a^2*b^2*n - 4*a^3*c*n + (a
*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \int \frac{(d\,x)^m}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

input
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^2,x)
```

output
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^2, x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = d^m \left(\int \frac{x^m}{x^{4n}c^2+2x^{3n}bc+2x^{2n}ac+x^{2n}b^2+2x^nab+a^2}dx\right)$$

input

```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x)
```

output

```
d**m*int(x**m/(x**(4*n)*c**2 + 2*x**(3*n)*b*c + 2*x**(2*n)*a*c + x**(2*n)*
b**2 + 2*x**n*a*b + a**2),x)
```

3.245 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 161

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \frac{a(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1+m}{n},-\frac{3}{2},-\frac{3}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
a*(d*x)^(1+m)*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1((1+m)/n,-3/2,-3/2,(1+m+n)
/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m
)/(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 545 vs. 2(161) = 322.

Time = 3.96 (sec) , antiderivative size = 545, normalized size of antiderivative = 3.39

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \frac{x(dx)^m\left(-6an^2(1+m+n)\left(b^2(1+m)-4ac(1+m+2n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+\cdots}{b+\sqrt{b^2-4a\cdots}}}\right.}{}$$

input
```
Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x*(d*x)^m*(-6*a*n^2*(1 + m + n)*(b^2*(1 + m) - 4*a*c*(1 + m + 2*n))*Sqrt[
(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2,
1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + (1 + m)*(2*(1 + m + n)*(3*b^2*n^2 + 4*a*c*(1 + m^2 + 6
*n + 8*n^2 + m*(2 + 6*n)) + 2*b*c*(2 + 4*m + 2*m^2 + 9*n + 9*m*n + 7*n^2)*
x^n + 4*c^2*(1 + 2*m + m^2 + 3*n + 3*m*n + 2*n^2)*x^(2*n))*(a + x^n*(b + c
*x^n)) - 3*b*n^2*(b^2*(2 + 2*m + n) - 4*a*c*(2 + 2*m + 3*n))*x^n*Sqrt[(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m + n)/n, 1/2,
1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])])))/(8*c*(1 + m)*(1 + m + n)^2*(1 + m + 2*n)*(1 + m + 3
*n)*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}}\int (dx)^m \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{a(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{m+1}{n},-\frac{3}{2},-\frac{3}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(a*(d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -3/2, -3/
2, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int (dx)^m \left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}dx$$

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^{3/2} \, dx = \int (dx)^m \left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral((d*x)**m*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^{3/2} \, dx = \int \left(cx^{2\,n} + bx^n + a\right)^{\frac{3}{2}} (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \int (d\,x)^m \left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

input
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

Reduce [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \text{too large to display}$$

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output

```
(d**m*(8*x**(m + 2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c*m**2*x + 20*x**(m +
2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c*m*n*x + 16*x**(m + 2*n)*sqrt(x**(2*n)
*c + x**n*b + a)*c*m*x + 8*x**(m + 2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c*n*
*2*x + 20*x**(m + 2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c*n*x + 8*x**(m + 2*n
)*sqrt(x**(2*n)*c + x**n*b + a)*c*x + 8*x**(m + n)*sqrt(x**(2*n)*c + x**n*
b + a)*b*m**2*x + 32*x**(m + n)*sqrt(x**(2*n)*c + x**n*b + a)*b*m*n*x + 16
*x**(m + n)*sqrt(x**(2*n)*c + x**n*b + a)*b*m*x + 14*x**(m + n)*sqrt(x**(2
*n)*c + x**n*b + a)*b*n**2*x + 32*x**(m + n)*sqrt(x**(2*n)*c + x**n*b + a)
*b*n*x + 8*x**(m + n)*sqrt(x**(2*n)*c + x**n*b + a)*b*x + 8*x**m*sqrt(x**(
2*n)*c + x**n*b + a)*a*m**2*x + 44*x**m*sqrt(x**(2*n)*c + x**n*b + a)*a*m*
n*x + 16*x**m*sqrt(x**(2*n)*c + x**n*b + a)*a*m*x + 68*x**m*sqrt(x**(2*n)*
c + x**n*b + a)*a*n**2*x + 44*x**m*sqrt(x**(2*n)*c + x**n*b + a)*a*n*x + 8
*x**m*sqrt(x**(2*n)*c + x**n*b + a)*a*x - 48*int((x**(m + 2*n)*sqrt(x**(2*
n)*c + x**n*b + a))/(2*x**(2*n)*c*m**3 + 11*x**(2*n)*c*m**2*n + 6*x**(2*n)
*c*m**2 + 17*x**(2*n)*c*m*n**2 + 22*x**(2*n)*c*m*n + 6*x**(2*n)*c*m + 6*x*
*(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 11*x**(2*n)*c*n + 2*x**(2*n)*c + 2*x*
*n*b*m**3 + 11*x**n*b*m**2*n + 6*x**n*b*m**2 + 17*x**n*b*m*n**2 + 22*x**n*
b*m*n + 6*x**n*b*m + 6*x**n*b*n**3 + 17*x**n*b*n**2 + 11*x**n*b*n + 2*x**n
*b + 2*a*m**3 + 11*a*m**2*n + 6*a*m**2 + 17*a*m*n**2 + 22*a*m*n + 6*a*m +
6*a*n**3 + 17*a*n**2 + 11*a*n + 2*a),x)*a*c*m**4*n**2 - 336*int((x**(m ...
```

3.246 $\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 160

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1+m}{n},-\frac{1}{2},-\frac{1}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
(d*x)^(1+m)*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1((1+m)/n,-1/2,-1/2,(1+m+n)/n
,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/
(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 388 vs. 2(160) = 320.

Time = 1.12 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.42

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x(dx)^m\left(2an(1+m+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},\frac{1}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{}{-b+\cdots}\right.\right.}{}$$

input
```
Integrate[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*(d*x)^m*(2*a*n*(1 + m + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*(2*(1 + m +
 n)*(a + x^n*(b + c*x^n)) + b*n*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[(1 + m + n)/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(2*(1
+ m)*(1 + m + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \ 1721$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}\int (dx)^m\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \ 1012$$

$$\frac{(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{m+1}{n},-\frac{1}{2},-\frac{1}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
((d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -1/2, -1/2,
 (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int (dx)^m \sqrt{a+b\,x^n+c\,x^{2n}}dx$$

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m \sqrt{a + bx^n + cx^{2n}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (dx)^m \sqrt{a + bx^n + cx^{2n}}\, dx = \int (dx)^m \sqrt{a + bx^n + cx^{2n}}\, dx$$

input
```
integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int (dx)^m \sqrt{a + bx^n + cx^{2n}}\, dx = \int \sqrt{cx^{2\,n} + bx^n + a}(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}(dx)^m\;dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx=\int (d\,x)^m\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

Reduce [F]

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{d^m\left(2x^m\sqrt{x^{2n}c+x^nb+a}\,x-2\left(\int \frac{x^{m+2n}\sqrt{x^{2n}c+x^nb+a}}{2x^{2n}cm+x^{2n}cn+2x^{2n}c+2x^nbm+x^nbn+2x^nb+2am+an+2a}dx\right)cmn-\left(\int \frac{}{2x^{2n}cm+x}\right.\right.}{}$$

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
(d**m*(2*x**m*sqrt(x**(2*n)*c + x**n*b + a)*x - 2*int((x**(m + 2*n)*sqrt(x
**(2*n)*c + x**n*b + a))/(2*x**(2*n)*c*m + x**(2*n)*c*n + 2*x**(2*n)*c + 2
*x**n*b*m + x**n*b*n + 2*x**n*b + 2*a*m + a*n + 2*a),x)*c*m*n - int((x**(m
 + 2*n)*sqrt(x**(2*n)*c + x**n*b + a))/(2*x**(2*n)*c*m + x**(2*n)*c*n + 2*
x**(2*n)*c + 2*x**n*b*m + x**n*b*n + 2*x**n*b + 2*a*m + a*n + 2*a),x)*c*n*
*2 - 2*int((x**(m + 2*n)*sqrt(x**(2*n)*c + x**n*b + a))/(2*x**(2*n)*c*m +
x**(2*n)*c*n + 2*x**(2*n)*c + 2*x**n*b*m + x**n*b*n + 2*x**n*b + 2*a*m + a
*n + 2*a),x)*c*n + 2*int((x**m*sqrt(x**(2*n)*c + x**n*b + a))/(2*x**(2*n)*
c*m + x**(2*n)*c*n + 2*x**(2*n)*c + 2*x**n*b*m + x**n*b*n + 2*x**n*b + 2*a
*m + a*n + 2*a),x)*a*m*n + int((x**m*sqrt(x**(2*n)*c + x**n*b + a))/(2*x**
(2*n)*c*m + x**(2*n)*c*n + 2*x**(2*n)*c + 2*x**n*b*m + x**n*b*n + 2*x**n*b
 + 2*a*m + a*n + 2*a),x)*a*n**2 + 2*int((x**m*sqrt(x**(2*n)*c + x**n*b + a
))/(2*x**(2*n)*c*m + x**(2*n)*c*n + 2*x**(2*n)*c + 2*x**n*b*m + x**n*b*n +
 2*x**n*b + 2*a*m + a*n + 2*a),x)*a*n))/(2*m + n + 2)
```

3.247 $\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 160

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},\frac{1}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

output
```
(d*x)^(1+m)*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c
+b^2)^(1/2)))^(1/2)*AppellF1((1+m)/n,1/2,1/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c
+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/(a+b*x^n+c*x^(2*n))^
(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.14

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x(dx)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},\frac{1}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{a+x^n\left(b+cx^n\right)}}$$

input
```
Integrate[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[
(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{(dx)^m}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+1}{n},\frac{1}{2},\frac{1}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n,
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(
d*(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\,x^n+c\,x^{2n}}}dx$$

input
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx$$

input
```
integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^{2n}+bx^n+a}}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{(d\,x)^m}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = d^m\left(\int \frac{x^m\sqrt{x^{2n}c+x^nb+a}}{x^{2n}c+x^nb+a}dx\right)$$

input
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
d**m*int((x**m*sqrt(x**(2*n)*c + x**n*b + a))/(x**(2*n)*c + x**n*b + a),x)
```

3.248 $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{3}{2},\frac{3}{2},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-}}\right.}{ad(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

output
```
(d*x)^(1+m)*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c
+b^2)^(1/2)))^(1/2)*AppellF1((1+m)/n,3/2,3/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c
+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/a/d/(1+m)/(a+b*x^n+c*x^(2*n)
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 428 vs. 2(163) = 326.

Time = 2.32 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.63

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x(dx)^m\left(\left(-4ac(1+m-n)+b^2(2+2m-n)\right)(1+m+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\right.}{}$$

input
```
Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x*(d*x)^m*((-4*a*c*(1 + m - n) + b^2*(2 + 2*m - n))*(1 + m + n)*Sqrt[(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2,
 (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])] - 2*(1 + m)*((1 + m + n)*(b^2 - 2*a*c + b*c*x^n) - b*c*(1 +
m)*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 +
 m + n)/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*(1 + m)*n*(1 + m
+ n)*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}dx$$

$$\Big\downarrow \text{1721}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{(dx)^m}{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\text{AppellF1}\left(\frac{m+1}{n},\frac{3}{2},\frac{3}{2},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 3/2, 3/2, (1 + m + n)/n,
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(
a*d*(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(dx)^m}{(a+b\,x^n+c\,x^{2n})^{\frac{3}{2}}}dx$$

input
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(dx)^m}{(a + bx^n + cx^{2n})^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a + bx^n + cx^{2n})^{3/2}} dx = \int \frac{(dx)^m}{(a + bx^n + cx^{2n})^{\frac{3}{2}}} dx$$

input
```
integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral((d*x)**m/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a + bx^n + cx^{2n})^{3/2}} dx = \int \frac{(dx)^m}{(cx^{2n} + bx^n + a)^{\frac{3}{2}}} dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{(d\,x)^m}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

input
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int((d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = d^m\left(\int \frac{x^m\sqrt{x^{2n}c+x^nb+a}}{x^{4n}c^2+2x^{3n}bc+2x^{2n}ac+x^{2n}b^2+2x^nab+a^2}dx\right)$$

input
```
int((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
d**m*int((x**m*sqrt(x**(2*n)*c + x**n*b + a))/(x**(4*n)*c**2 + 2*x**(3*n)*
b*c + 2*x**(2*n)*a*c + x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.249 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 158

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$= \frac{(dx)^{1+m} \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} \left(a+bx^n+cx^{2n}\right)^p \text{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2c}{b-\sqrt{b}}\right.}{d(1+m)}$$

output
```
(d*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*
x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$= \frac{x(dx)^m \left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} \left(a+x^n(b+cx^n)\right)^p \text{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{}{b+}\right.}{1+m}$$

input
```
Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x*(d*x)^m*(a + x^n*(b + c*x^n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)
/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
])/((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*
((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p dx$$

$$\Big\downarrow \text{ 1721}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\int (dx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-\cdots}}\right.$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{(dx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(\frac{m+1}{n},-p,-p,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\cdots\right.}{d(m+1)}$$

input
```
Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
((d*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 +
m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/(d*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int (dx)^m \left(a + b\,x^n + c\,x^{2n}\right)^p dx$$

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^p dx = \int \left(cx^{2\,n} + bx^n + a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(cx^{2\,n}+bx^n+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(cx^{2\,n}+bx^n+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p\,dx=\int (d\,x)^m\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

input
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int((d*x)^m*(a + b*x^n + c*x^(2*n))^p, x)
```

Reduce [F]

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p\,dx = \frac{d^m\left(x^m(x^{2n}c+x^nb+a)^p x-\left(\int \frac{x^{m+2n}(x^{2n}c+x^nb+a)^p}{x^{2n}cm+x^{2n}cnp+x^{2n}c+x^nbm+x^nbnp+x^nb+am+anp+a}dx\right)cmnp-\left(\int \frac{}{x^{2n}cm+x^{2n}cnp}\right.\right.}{}$$

input
```
int((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
(d**m*(x**m*(x**(2*n)*c + x**n*b + a)**p*x - int((x**(m + 2*n)*(x**(2*n)*c
 + x**n*b + a)**p)/(x**(2*n)*c*m + x**(2*n)*c*n*p + x**(2*n)*c + x**n*b*m
+ x**n*b*n*p + x**n*b + a*m + a*n*p + a),x)*c*m*n*p - int((x**(m + 2*n)*(x
**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*m + x**(2*n)*c*n*p + x**(2*n)*c +
x**n*b*m + x**n*b*n*p + x**n*b + a*m + a*n*p + a),x)*c*n**2*p**2 - int((x*
*(m + 2*n)*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*m + x**(2*n)*c*n*p +
x**(2*n)*c + x**n*b*m + x**n*b*n*p + x**n*b + a*m + a*n*p + a),x)*c*n*p +
int((x**m*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*m + x**(2*n)*c*n*p + x
**(2*n)*c + x**n*b*m + x**n*b*n*p + x**n*b + a*m + a*n*p + a),x)*a*m*n*p +
 int((x**m*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*m + x**(2*n)*c*n*p +
x**(2*n)*c + x**n*b*m + x**n*b*n*p + x**n*b + a*m + a*n*p + a),x)*a*n**2*p
**2 + int((x**m*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*m + x**(2*n)*c*n
*p + x**(2*n)*c + x**n*b*m + x**n*b*n*p + x**n*b + a*m + a*n*p + a),x)*a*n
*p))/(m + n*p + 1)
```

3.250 $\int x^3(a + bx^n + cx^{2n})^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 146

$$\int x^3(a+bx^n+cx^{2n})^p\,dx=\frac{1}{4}x^4\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p\operatorname{AppellF1}\left(\frac{4}{n},-p,-p,\frac{4+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/4*x^4*(a+b*x^n+c*x^(2*n))^p*AppellF1(4/n,-p,-p,(4+n)/n,-2*c*x^n/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2
)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.18

$$\int x^3(a+bx^n+cx^{2n})^p\,dx=\frac{1}{4}x^4\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\operatorname{AppellF1}\left(\frac{4}{n},-p,-p,\frac{4+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^3*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x^4*(a + x^n*(b + c*x^n))^p*AppellF1[4/n, -p, -p, (4 + n)/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(4*((b - Sqrt
[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^p \, dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\int x^3\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-4a}}\right.$$

$$\Big\downarrow \text{1012}$$

$$\frac{1}{4}x^4\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(\frac{4}{n},-p,-p,\frac{n+4}{n},-\frac{2c}{b-\sqrt{b^2}}\right.$$

input
```
Int[x^3*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x^4*(a + b*x^n + c*x^(2*n))^p*AppellF1[4/n, -p, -p, (4 + n)/n, (-2*c*x^n)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*(1 + (2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^
p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^{3} \left(a +b \,x^{n}+c \,x^{2 n}\right)^{p}d x$$

input
```
int(x^3*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int(x^3*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int x^3 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int { {\left(c x^{2 \, n} + b x^{n} + a\right)}^{p} x^{3} \,d x }$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b x^n + c x^{2n}\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3 \left(a + b x^n + c x^{2n}\right)^p \, dx = \int \left(c x^{2\,n} + b x^n + a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*x^3, x)
```

Giac [F]

$$\int x^3 \left(a + b x^n + c x^{2n}\right)^p \, dx = \int \left(c x^{2\,n} + b x^n + a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+bx^n+cx^{2n})^p\,dx=\int x^3\left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

input
```
int(x^3*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int(x^3*(a + b*x^n + c*x^(2*n))^p, x)
```

Reduce [F]

$$\int x^3(a+bx^n+cx^{2n})^p\,dx$$
$$=\frac{(x^{2n}c+x^nb+a)^p\,x^4+\left(\int\frac{(x^{2n}c+x^nb+a)^p x^3}{x^{2n}cnp+4x^{2n}c+x^nbnp+4x^nb+anp+4a}dx\right)a\,n^2p^2+4\left(\int\frac{(x^{2n}c+x^nb+a)^p x^3}{x^{2n}cnp+4x^{2n}c+x^nbnp+4x^nb+anp+4a}\right.}{np+}$$

input
```
int(x^3*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
((x**(2*n)*c + x**n*b + a)**p*x**4 + int(((x**(2*n)*c + x**n*b + a)**p*x**
3)/(x**(2*n)*c*n*p + 4*x**(2*n)*c + x**n*b*n*p + 4*x**n*b + a*n*p + 4*a),x
)*a*n**2*p**2 + 4*int(((x**(2*n)*c + x**n*b + a)**p*x**3)/(x**(2*n)*c*n*p
+ 4*x**(2*n)*c + x**n*b*n*p + 4*x**n*b + a*n*p + 4*a),x)*a*n*p - int((x**(
2*n)*(x**(2*n)*c + x**n*b + a)**p*x**3)/(x**(2*n)*c*n*p + 4*x**(2*n)*c + x
**n*b*n*p + 4*x**n*b + a*n*p + 4*a),x)*c*n**2*p**2 - 4*int((x**(2*n)*(x**(
2*n)*c + x**n*b + a)**p*x**3)/(x**(2*n)*c*n*p + 4*x**(2*n)*c + x**n*b*n*p
+ 4*x**n*b + a*n*p + 4*a),x)*c*n*p)/(n*p + 4)
```

3.251 $\int x^2(a+bx^n+cx^{2n})^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 146

$$\int x^2\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{1}{3}x^3\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(\frac{3}{n},-p,-p,\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/3*x^3*(a+b*x^n+c*x^(2*n))^p*AppellF1(3/n,-p,-p,(3+n)/n,-2*c*x^n/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2
)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.18

$$\int x^2\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{1}{3}x^3\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+x^n(b+cx^n)\right)^p\text{AppellF1}\left(\frac{3}{n},-p,-p,\frac{3+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^2*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x^3*(a + x^n*(b + c*x^n))^p*AppellF1[3/n, -p, -p, (3 + n)/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(3*((b - Sqrt
[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^n+cx^{2n}\right)^p \, dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^n+cx^{2n}\right)^p \int x^2 \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p \left(\frac{2cx^n}{b+\sqrt{b^2-4a}}\right.$$

$$\Big\downarrow \text{1012}$$

$$\frac{1}{3}x^3 \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^n+cx^{2n}\right)^p \text{AppellF1}\left(\frac{3}{n},-p,-p,\frac{n+3}{n},-\frac{2c}{b-\sqrt{b^2}}\right.$$

input
```
Int[x^2*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x^3*(a + b*x^n + c*x^(2*n))^p*AppellF1[3/n, -p, -p, (3 + n)/n, (-2*c*x^n)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*(1 + (2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^
p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^2 \left(a + b\,x^n + c\,x^{2n}\right)^p dx$$

input
```
int(x^2*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int(x^2*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int x^2 \left(a + bx^n + cx^{2n}\right)^p \, dx = \int \left(cx^{2\,n} + bx^n + a\right)^p x^2 \, dx$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 (a + bx^n + cx^{2n})^p \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 (a + bx^n + cx^{2n})^p \, dx = \int (cx^{2n} + bx^n + a)^p x^2 \, dx$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*x^2, x)
```

Giac [F]

$$\int x^2 (a + bx^n + cx^{2n})^p \, dx = \int (cx^{2n} + bx^n + a)^p x^2 \, dx$$

input
```
integrate(x^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+bx^n+cx^{2n}\right)^p \, dx=\int x^2\,{\left(a+b\,x^n+c\,x^{2\,n}\right)}^p \,dx$$

input
```
int(x^2*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int(x^2*(a + b*x^n + c*x^(2*n))^p, x)
```

Reduce [F]

$$\int x^2 \left(a+bx^n+cx^{2n}\right)^p \, dx = \frac{(x^{2n}c+x^n b+a)^p x^3+\left(\int \frac{(x^{2n}c+x^n b+a)^p x^2}{x^{2n}cnp+3x^{2n}c+x^n bnp+3x^n b+anp+3a}dx\right) a\,n^2p^2+3\left(\int \frac{(x^{2n}c+x^n b+a)^p x^2}{x^{2n}cnp+3x^{2n}c+x^n bnp+3x^n b+anp+3a}\right.}{np+}$$

input
```
int(x^2*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
((x**(2*n)*c + x**n*b + a)**p*x**3 + int(((x**(2*n)*c + x**n*b + a)**p*x**
2)/(x**(2*n)*c*n*p + 3*x**(2*n)*c + x**n*b*n*p + 3*x**n*b + a*n*p + 3*a),x
)*a*n**2*p**2 + 3*int(((x**(2*n)*c + x**n*b + a)**p*x**2)/(x**(2*n)*c*n*p
+ 3*x**(2*n)*c + x**n*b*n*p + 3*x**n*b + a*n*p + 3*a),x)*a*n*p - int((x**(
2*n)*(x**(2*n)*c + x**n*b + a)**p*x**2)/(x**(2*n)*c*n*p + 3*x**(2*n)*c + x
**n*b*n*p + 3*x**n*b + a*n*p + 3*a),x)*c*n**2*p**2 - 3*int((x**(2*n)*(x**(
2*n)*c + x**n*b + a)**p*x**2)/(x**(2*n)*c*n*p + 3*x**(2*n)*c + x**n*b*n*p
+ 3*x**n*b + a*n*p + 3*a),x)*c*n*p)/(n*p + 3)
```

3.252 $\int x(a+bx^n+cx^{2n})^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 146

$$\int x\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{1}{2}x^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\operatorname{AppellF1}\left(\frac{2}{n},-p,-p,\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/2*x^2*(a+b*x^n+c*x^(2*n))^p*AppellF1(2/n,-p,-p,(2+n)/n,-2*c*x^n/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2
)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.18

$$\int x\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{1}{2}x^2\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+x^n(b+cx^n)\right)^p\operatorname{AppellF1}\left(\frac{2}{n},-p,-p,\frac{2+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x^2*(a + x^n*(b + c*x^n))^p*AppellF1[2/n, -p, -p, (2 + n)/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(2*((b - Sqrt
[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\int x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right.$$

$$\Big\downarrow \text{1012}$$

$$\frac{1}{2}x^2\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(\frac{2}{n},-p,-p,\frac{n+2}{n},-\frac{2c}{b-\sqrt{b^2}}\right.$$

input
```
Int[x*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x^2*(a + b*x^n + c*x^(2*n))^p*AppellF1[2/n, -p, -p, (2 + n)/n, (-2*c*x^n)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*(1 + (2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^
p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x\left(a+b\,x^{n}+c\,x^{2n}\right)^{p}dx$$

input
```
int(x*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int(x*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int x\left(a+bx^{n}+cx^{2n}\right)^{p}\,dx=\int\left(cx^{2\,n}+bx^{n}+a\right)^{p}x\,dx$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+bx^n+cx^{2n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+bx^n+cx^{2n}\right)^p\,dx=\int\left(cx^{2\,n}+bx^n+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+bx^n+cx^{2n}\right)^p\,dx=\int\left(cx^{2\,n}+bx^n+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^n+cx^{2n})^p\,dx=\int x\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

input
```
int(x*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int(x*(a + b*x^n + c*x^(2*n))^p, x)
```

Reduce [F]

$$\int x(a+bx^n+cx^{2n})^p\,dx = \frac{(x^{2n}c+x^nb+a)^p\,x^2+\left(\int\frac{(x^{2n}c+x^nb+a)^p x}{x^{2n}cnp+2x^{2n}c+x^nbnp+2x^nb+anp+2a}dx\right)a\,n^2p^2+2\left(\int\frac{(x^{2n}c+x^nb+a)^p x}{x^{2n}cnp+2x^{2n}c+x^nbnp+2x^nb+anp+2a}\right.}{np+}$$

input
```
int(x*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
((x**(2*n)*c + x**n*b + a)**p*x**2 + int(((x**(2*n)*c + x**n*b + a)**p*x)/
(x**(2*n)*c*n*p + 2*x**(2*n)*c + x**n*b*n*p + 2*x**n*b + a*n*p + 2*a),x)*a
*n**2*p**2 + 2*int(((x**(2*n)*c + x**n*b + a)**p*x)/(x**(2*n)*c*n*p + 2*x*
*(2*n)*c + x**n*b*n*p + 2*x**n*b + a*n*p + 2*a),x)*a*n*p - int((x**(2*n)*(
x**(2*n)*c + x**n*b + a)**p*x)/(x**(2*n)*c*n*p + 2*x**(2*n)*c + x**n*b*n*p
 + 2*x**n*b + a*n*p + 2*a),x)*c*n**2*p**2 - 2*int((x**(2*n)*(x**(2*n)*c +
x**n*b + a)**p*x)/(x**(2*n)*c*n*p + 2*x**(2*n)*c + x**n*b*n*p + 2*x**n*b +
 a*n*p + 2*a),x)*c*n*p)/(n*p + 2)
```

3.253 $\int (a + bx^n + cx^{2n})^p \, dx$

Optimal result

Integrand size = 16, antiderivative size = 137

$$\int (a + bx^n + cx^{2n})^p \, dx = x\left(1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right)^{-p}\left(1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)^{-p}(a + bx^n + cx^{2n})^p \operatorname{AppellF1}\left(\frac{1}{n}, -p, -p, 1 + \frac{1}{n}, -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)$$

output
```
x*(a+b*x^n+c*x^(2*n))^p*AppellF1(1/n,-p,-p,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2))
)^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.20

$$\int (a + bx^n + cx^{2n})^p \, dx = x\left(\frac{b - \sqrt{b^2 - 4ac} + 2cx^n}{b - \sqrt{b^2 - 4ac}}\right)^{-p}\left(\frac{b + \sqrt{b^2 - 4ac} + 2cx^n}{b + \sqrt{b^2 - 4ac}}\right)^{-p}(a + x^n(b + cx^n))^p \operatorname{AppellF1}\left(\frac{1}{n}, -p, -p, 1 + \frac{1}{n}, -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^n}{-b + \sqrt{b^2 - 4ac}}\right)$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x*(a + x^n*(b + c*x^n))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt
[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^n+cx^{2n}\right)^p \, dx$$

$$\Big\downarrow \text{1686}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\int \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right.$$

$$\Big\downarrow \text{936}$$

$$x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-}}\right.$$

input
```
Int[(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x*(a + b*x^n + c*x^(2*n))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^
n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^
p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int (a+b\,x^{n}+c\,x^{2n})^{p}\,dx$$

input
```
int((a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (a+bx^n+cx^{2n})^p\,dx=\int (cx^{2\,n}+bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Timed out}$$

input
```
integrate((a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(cx^{2n}+bx^n+a\right)^p \, dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(cx^{2n}+bx^n+a\right)^p \, dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^n+cx^{2n}\right)^p\,dx=\int \left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^p, x)
```

Reduce [F]

$$\int \left(a+bx^n+cx^{2n}\right)^p\,dx$$
$$=\frac{\left(x^{2n}c+x^nb+a\right)^p x+\left(\int \frac{\left(x^{2n}c+x^nb+a\right)^p}{x^{2n}cnp+x^{2n}c+x^nbnp+x^nb+anp+a}dx\right) a\,n^2p^2+\left(\int \frac{\left(x^{2n}c+x^nb+a\right)^p}{x^{2n}cnp+x^{2n}c+x^nbnp+x^nb+anp+a}dx\right) an}{np+1}$$

input
```
int((a+b*x^n+c*x^(2*n))^p,x)
```

output
```
((x**(2*n)*c + x**n*b + a)**p*x + int((x**(2*n)*c + x**n*b + a)**p/(x**(2*
n)*c*n*p + x**(2*n)*c + x**n*b*n*p + x**n*b + a*n*p + a),x)*a*n**2*p**2 +
int((x**(2*n)*c + x**n*b + a)**p/(x**(2*n)*c*n*p + x**(2*n)*c + x**n*b*n*p
 + x**n*b + a*n*p + a),x)*a*n*p - int((x**(2*n)*(x**(2*n)*c + x**n*b + a)*
*p)/(x**(2*n)*c*n*p + x**(2*n)*c + x**n*b*n*p + x**n*b + a*n*p + a),x)*c*n
**2*p**2 - int((x**(2*n)*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*n*p + x
**(2*n)*c + x**n*b*n*p + x**n*b + a*n*p + a),x)*c*n*p)/(n*p + 1)
```

3.254 $\int \frac{(a+bx^n+cx^{2n})^p}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 167

$$\int \frac{(a+bx^n+cx^{2n})^p}{x} dx = \frac{2^{-1+2p}\left(\frac{x^{-n}\left(b-\sqrt{b^2-4ac}+2cx^n\right)}{c}\right)^{-p}\left(\frac{x^{-n}\left(b+\sqrt{b^2-4ac}+2cx^n\right)}{c}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(-2p,-p,-p,1-\right.}{np}$$

output
```
2^(-1+2*p)*(a+b*x^n+c*x^(2*n))^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*a*c
+b^2)^(1/2))/c/(x^n),-1/2*(b+(-4*a*c+b^2)^(1/2))/c/(x^n))/n/p/(((b-(-4*a*c
+b^2)^(1/2)+2*c*x^n)/c/(x^n))^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^n)/c/(x^n))
^p)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^n+cx^{2n})^p}{x} dx = \frac{2^{-1+2p}\left(\frac{x^{-n}\left(b-\sqrt{b^2-4ac}+2cx^n\right)}{c}\right)^{-p}\left(\frac{x^{-n}\left(b+\sqrt{b^2-4ac}+2cx^n\right)}{c}\right)^{-p}(a+x^n(b+cx^n))^p \operatorname{AppellF1}\left(-2p,-p,-p,1\right.}{np}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^p/x,x]
```

output
```
(2^(-1 + 2*p)*(a + x^n*(b + c*x^n))^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2
*(b + Sqrt[b^2 - 4*a*c])/(c*x^n), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^n)])/(n*
p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(c*x^n))^p*((b + Sqrt[b^2 - 4*a*c] +
2*c*x^n)/(c*x^n))^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1693, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n+cx^{2n})^p}{x}\,dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{\int x^{-n}\left(bx^n+cx^{2n}+a\right)^p dx^n}{n}$$

$$\Big\downarrow \text{1178}$$

$$-\frac{4^p(x^{-n})^{2p}\left(\frac{x^{-n}\left(-\sqrt{b^2-4ac}+b+2cx^n\right)}{c}\right)^{-p}\left(\frac{x^{-n}\left(\sqrt{b^2-4ac}+b+2cx^n\right)}{c}\right)^{-p}(a+bx^n+cx^{2n})^p\int (x^{-n})^{-2p-1}\left(\frac{\left(b-\sqrt{b^2-4\right.}}{2c}\right.}{n}$$

$$\Big\downarrow \text{150}$$

$$\frac{2^{2p-1}\left(\frac{x^{-n}\left(-\sqrt{b^2-4ac}+b+2cx^n\right)}{c}\right)^{-p}\left(\frac{x^{-n}\left(\sqrt{b^2-4ac}+b+2cx^n\right)}{c}\right)^{-p}(a+bx^n+cx^{2n})^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p\right.}{np}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^p/x,x]
```

output
```
(2^(-1 + 2*p)*(a + b*x^n + c*x^(2*n))^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1
/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^n), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^n)])
/(n*p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(c*x^n))^p*((b + Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(c*x^n))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(a+b\,x^n+c\,x^{2n})^p}{x}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^p/x,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^p/x,x)
```

Fricas [F]

$$\int \frac{(a+bx^n+cx^{2n})^p}{x}\,dx=\int \frac{(cx^{2n}+bx^n+a)^p}{x}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/x,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p/x, x)
```

Sympy [F]

$$\int \frac{(a+bx^n+cx^{2n})^p}{x}\,dx=\int \frac{(a+bx^n+cx^{2n})^p}{x}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**p/x,x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**p/x, x)
```

Maxima [F]

$$\int \frac{(a+bx^n+cx^{2n})^p}{x}\,dx=\int \frac{(cx^{2n}+bx^n+a)^p}{x}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/x,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/x, x)
```

Giac [F]

$$\int \frac{(a+bx^n+cx^{2n})^p}{x}\,dx = \int \frac{(cx^{2n}+bx^n+a)^p}{x}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/x,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^p}{x}\,dx = \int \frac{(a+b\,x^n+c\,x^{2\,n})^p}{x}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^p/x,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^p/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^n+cx^{2n})^p}{x}\,dx = \frac{(x^{2n}c+x^nb+a)^p+\left(\int \frac{(x^{2n}c+x^nb+a)^p}{x^{2n}cx+x^nbx+ax}dx\right)anp-\left(\int \frac{x^{2n}(x^{2n}c+x^nb+a)^p}{x^{2n}cx+x^nbx+ax}dx\right)cnp}{np}$$

input
```
int((a+b*x^n+c*x^(2*n))^p/x,x)
```

output
```
((x**(2*n)*c + x**n*b + a)**p + int((x**(2*n)*c + x**n*b + a)**p/(x**(2*n)
*c*x + x**n*b*x + a*x),x)*a*n*p - int((x**(2*n)*(x**(2*n)*c + x**n*b + a)*
*p)/(x**(2*n)*c*x + x**n*b*x + a*x),x)*c*n*p)/(n*p)
```

3.255 $\int \frac{(a+bx^n+cx^{2n})^p}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 147

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^2} dx = -\frac{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \text{AppellF1}\left(-\frac{1}{n},-p,-p,-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\right.}{x}$$

output
```
-(a+b*x^n+c*x^(2*n))^p*AppellF1(-1/n,-p,-p,-(1-n)/n,-2*c*x^n/(b-(-4*a*c+b^
2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/x/((1+2*c*x^n/(b-(-4*a*c+b^2)^(
1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^2} dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p \text{AppellF1}\left(-\frac{1}{n},-p,-p,\frac{-1+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\right.}{x}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^p/x^2,x]
```

output
```
-(((a + x^n*(b + c*x^n))^p*AppellF1[-n^(-1), -p, -p, (-1 + n)/n, (-2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(x*((b - S
qrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^2}dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\int \frac{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^p}{x^2}dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p \text{AppellF1}\left(-\frac{1}{n},-p,-p,-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-}}\right.}{x}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^p/x^2,x]
```

output
```
-(((a + b*x^n + c*x^(2*n))^p*AppellF1[-n^(-1), -p, -p, -((1 - n)/n), (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1
+ (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c]))^p))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(a+b\,x^{n}+c\,x^{2n})^{p}}{x^{2}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^p/x^2,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^{n}+cx^{2n})^{p}}{x^{2}}\,dx=\int \frac{(cx^{2\,n}+bx^{n}+a)^{p}}{x^{2}}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/x^2,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**n+c*x**(2*n))**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^2}\,dx = \int \frac{(cx^{2n}+bx^n+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^2}\,dx = \int \frac{(cx^{2n}+bx^n+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^2}\,dx = \int \frac{(a+b\,x^n+c\,x^{2\,n})^p}{x^2}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^p/x^2,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^p/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^2}\,dx = \frac{(x^{2n}c+x^nb+a)^p+\left(\int \frac{(x^{2n}c+x^nb+a)^p}{x^{2n}cnp\,x^2-x^{2n}c\,x^2+x^nbnp\,x^2-x^nb\,x^2+anp\,x^2-a\,x^2}dx\right)a\,n^2p^2x-\left(\int \frac{(x^{2n}c+x^nb}{x^{2n}cnp\,x^2-x^{2n}c\,x^2+x^nbnp\,x}\right.}{}$$

input
```
int((a+b*x^n+c*x^(2*n))^p/x^2,x)
```

output
```
((x**(2*n)*c + x**n*b + a)**p + int((x**(2*n)*c + x**n*b + a)**p/(x**(2*n)
*c*n*p*x**2 - x**(2*n)*c*x**2 + x**n*b*n*p*x**2 - x**n*b*x**2 + a*n*p*x**2
 - a*x**2),x)*a*n**2*p**2*x - int((x**(2*n)*c + x**n*b + a)**p/(x**(2*n)*c
*n*p*x**2 - x**(2*n)*c*x**2 + x**n*b*n*p*x**2 - x**n*b*x**2 + a*n*p*x**2 -
 a*x**2),x)*a*n*p*x - int((x**(2*n)*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n
)*c*n*p*x**2 - x**(2*n)*c*x**2 + x**n*b*n*p*x**2 - x**n*b*x**2 + a*n*p*x**
2 - a*x**2),x)*c*n**2*p**2*x + int((x**(2*n)*(x**(2*n)*c + x**n*b + a)**p)
/(x**(2*n)*c*n*p*x**2 - x**(2*n)*c*x**2 + x**n*b*n*p*x**2 - x**n*b*x**2 +
a*n*p*x**2 - a*x**2),x)*c*n*p*x)/(x*(n*p - 1))
```

3.256 $\int \frac{(a+bx^n+cx^{2n})^p}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 149

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^3} dx = -\frac{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(-\frac{2}{n},-p,-p,-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\right.}{2x^2}$$

output
```
-1/2*(a+b*x^n+c*x^(2*n))^p*AppellF1(-2/n,-p,-p,-(2-n)/n,-2*c*x^n/(b-(-4*a*
c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/x^2/((1+2*c*x^n/(b-(-4*a*c+
b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^3} dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p \operatorname{AppellF1}\left(-\frac{2}{n},-p,-p,\frac{-2+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\right.}{2x^2}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^p/x^3,x]
```

output
```
-1/2*((a + x^n*(b + c*x^n))^p*AppellF1[-2/n, -p, -p, (-2 + n)/n, (-2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(x^2*((b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 -
4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^3}\,dx$$

$$\Big\downarrow \text{ 1721}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\int\frac{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^p}{x^3}dx$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p\operatorname{AppellF1}\left(-\frac{2}{n},-p,-p,-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^p/x^3,x]
```

output
```
-1/2*((a + b*x^n + c*x^(2*n))^p*AppellF1[-2/n, -p, -p, -((2 - n)/n), (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*(
1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(a+b\,x^{n}+c\,x^{2n})^{p}}{x^{3}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^p/x^3,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^p/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^3}\,dx=\int \frac{(cx^{2\,n}+bx^n+a)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/x^3,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + bx^n + cx^{2n})^p}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**n+c*x**(2*n))**p/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + bx^n + cx^{2n})^p}{x^3}\,dx = \int \frac{(cx^{2\,n} + bx^n + a)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(a + bx^n + cx^{2n})^p}{x^3}\,dx = \int \frac{(cx^{2\,n} + bx^n + a)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/x^3,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^3}\,dx = \int \frac{(a+b\,x^n+c\,x^{2\,n})^p}{x^3}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^p/x^3,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^p/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^n+cx^{2n})^p}{x^3}\,dx$$
$$= \frac{(x^{2n}c+x^nb+a)^p+\left(\int \frac{(x^{2n}c+x^nb+a)^p}{x^{2n}cnp\,x^3-2x^{2n}c\,x^3+x^nbnp\,x^3-2x^nb\,x^3+anp\,x^3-2a\,x^3}dx\right)a\,n^2p^2x^2-2\left(\int \frac{(x^{2}}{x^{2n}cnp\,x^3-2x^{2n}c\,x^3+}\right.}{}$$

input
```
int((a+b*x^n+c*x^(2*n))^p/x^3,x)
```

output
```
((x**(2*n)*c + x**n*b + a)**p + int((x**(2*n)*c + x**n*b + a)**p/(x**(2*n)
*c*n*p*x**3 - 2*x**(2*n)*c*x**3 + x**n*b*n*p*x**3 - 2*x**n*b*x**3 + a*n*p*
x**3 - 2*a*x**3),x)*a*n**2*p**2*x**2 - 2*int((x**(2*n)*c + x**n*b + a)**p/
(x**(2*n)*c*n*p*x**3 - 2*x**(2*n)*c*x**3 + x**n*b*n*p*x**3 - 2*x**n*b*x**3
 + a*n*p*x**3 - 2*a*x**3),x)*a*n*p*x**2 - int((x**(2*n)*(x**(2*n)*c + x**n
*b + a)**p)/(x**(2*n)*c*n*p*x**3 - 2*x**(2*n)*c*x**3 + x**n*b*n*p*x**3 - 2
*x**n*b*x**3 + a*n*p*x**3 - 2*a*x**3),x)*c*n**2*p**2*x**2 + 2*int((x**(2*n
)*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*n*p*x**3 - 2*x**(2*n)*c*x**3 +
 x**n*b*n*p*x**3 - 2*x**n*b*x**3 + a*n*p*x**3 - 2*a*x**3),x)*c*n*p*x**2)/(
x**2*(n*p - 2))
```

3.257 $\int x^{3n}(a+bx^n+cx^{2n})^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int x^{3n}\left(a+bx^n+cx^{2n}\right)^p\,dx$$
$$=\frac{x^{1+3n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\operatorname{AppellF1}\left(3+\frac{1}{n},-p,-p,4+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4}}\right.}{1+3n}$$

output
```
x^(1+3*n)*(a+b*x^n+c*x^(2*n))^p*AppellF1(3+1/n,-p,-p,4+1/n,-2*c*x^n/(b-(-4
*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+3*n)/((1+2*c*x^n/(b-(
-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.18

$$\int x^{3n}\left(a+bx^n+cx^{2n}\right)^p\,dx$$
$$=\frac{x^{1+3n}\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+x^n(b+cx^n)\right)^p\operatorname{AppellF1}\left(3+\frac{1}{n},-p,-p,4+\frac{1}{n},-\frac{2c}{b+\sqrt{b}}\right.}{1+3n}$$

input
```
Integrate[x^(3*n)*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x^(1 + 3*n)*(a + x^n*(b + c*x^n))^p*AppellF1[3 + n^(-1), -p, -p, 4 + n^(-
1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
])/((1 + 3*n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^
p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3n}\left(a+bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\int x^{3n}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-4\right.$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{3n+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\operatorname{AppellF1}\left(3+\frac{1}{n},-p,-p,4+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\right.}{3n+1}$$

input
```
Int[x^(3*n)*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x^(1 + 3*n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[3 + n^(-1), -p, -p, 4 + n^
(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
])])/((1 + 3*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^{3n}\left(a+b\,x^{n}+c\,x^{2n}\right)^{p}dx$$

input
```
int(x^(3*n)*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int(x^(3*n)*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int x^{3n}\left(a+bx^{n}+cx^{2n}\right)^{p}\,dx=\int\left(cx^{2\,n}+bx^{n}+a\right)^{p}x^{3\,n}\,dx$$

input
```
integrate(x^(3*n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p*x^(3*n), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{3n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**(3*n)*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{3n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\int\left(cx^{2n}+bx^n+a\right)^p x^{3n}\,dx$$

input
```
integrate(x^(3*n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*x^(3*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{3n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(3*n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-256,[1,0,7,4,7,5,2,8]%%%}+%%%{-1280,[1,0,7,4,7,4,2,8]%%%}
+%%%{-256
```

Mupad [F(-1)]

Timed out.

$$\int x^{3n}(a+bx^n+cx^{2n})^p\,dx=\int x^{3n}\left(a+b\,x^n+c\,x^{2n}\right)^p\,dx$$

input
```
int(x^(3*n)*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int(x^(3*n)*(a + b*x^n + c*x^(2*n))^p, x)
```

Reduce [F]

$$\int x^{3n}(a+bx^n+cx^{2n})^p\,dx=\text{too large to display}$$

input
```
int(x^(3*n)*(a+b*x^n+c*x^(2*n))^p,x)
```

output

```
(4*x**(3*n)*(x**(2*n)*c + x**n*b + a)**p*b*c**2*n**3*p**3*x + 6*x**(3*n)*(
x**(2*n)*c + x**n*b + a)**p*b*c**2*n**3*p**2*x + 2*x**(3*n)*(x**(2*n)*c +
x**n*b + a)**p*b*c**2*n**3*p*x + 8*x**(3*n)*(x**(2*n)*c + x**n*b + a)**p*b
*c**2*n**2*p**2*x + 9*x**(3*n)*(x**(2*n)*c + x**n*b + a)**p*b*c**2*n**2*p*
x + 2*x**(3*n)*(x**(2*n)*c + x**n*b + a)**p*b*c**2*n**2*x + 5*x**(3*n)*(x*
*(2*n)*c + x**n*b + a)**p*b*c**2*n*p*x + 3*x**(3*n)*(x**(2*n)*c + x**n*b +
 a)**p*b*c**2*n*x + x**(3*n)*(x**(2*n)*c + x**n*b + a)**p*b*c**2*x + 2*x**
(2*n)*(x**(2*n)*c + x**n*b + a)**p*b**2*c*n**3*p**3*x + x**(2*n)*(x**(2*n)
*c + x**n*b + a)**p*b**2*c*n**3*p**2*x + 3*x**(2*n)*(x**(2*n)*c + x**n*b +
 a)**p*b**2*c*n**2*p**2*x + x**(2*n)*(x**(2*n)*c + x**n*b + a)**p*b**2*c*n
**2*p*x + x**(2*n)*(x**(2*n)*c + x**n*b + a)**p*b**2*c*n*p*x + 4*x**n*(x**
(2*n)*c + x**n*b + a)**p*a*b*c*n**3*p**3*x + 4*x**n*(x**(2*n)*c + x**n*b +
 a)**p*a*b*c*n**3*p**2*x + 6*x**n*(x**(2*n)*c + x**n*b + a)**p*a*b*c*n**2*
p**2*x + 4*x**n*(x**(2*n)*c + x**n*b + a)**p*a*b*c*n**2*p*x + 2*x**n*(x**(
2*n)*c + x**n*b + a)**p*a*b*c*n*p*x - x**n*(x**(2*n)*c + x**n*b + a)**p*b*
*3*n**3*p**3*x - 2*x**n*(x**(2*n)*c + x**n*b + a)**p*b**3*n**3*p**2*x - 2*
x**n*(x**(2*n)*c + x**n*b + a)**p*b**3*n**2*p**2*x - 2*x**n*(x**(2*n)*c +
x**n*b + a)**p*b**3*n**2*p*x - x**n*(x**(2*n)*c + x**n*b + a)**p*b**3*n*p*
x - 4*(x**(2*n)*c + x**n*b + a)**p*a**2*c*n**3*p**2*x - 4*(x**(2*n)*c + x*
*n*b + a)**p*a**2*c*n**3*p*x - 4*(x**(2*n)*c + x**n*b + a)**p*a**2*c*n*...
```

3.258 $\int x^{2/n}(a+bx^n+cx^{2n})^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 160

$$\int x^{2/n}(a+bx^n+cx^{2n})^p\,dx = \frac{nx^{\frac{2+n}{n}}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p\operatorname{AppellF1}\left(\frac{2+n}{n^2},-p,-p,\frac{2+n+n^2}{n^2}\right.}{2+n}$$

output
```
n*x^((2+n)/n)*(a+b*x^n+c*x^(2*n))^p*AppellF1((2+n)/n^2,-p,-p,(n^2+n+2)/n^2
,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(2+n)/((
1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p
)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.16

$$\int x^{2/n}(a+bx^n+cx^{2n})^p\,dx = \frac{nx^{\frac{2+n}{n}}\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\operatorname{AppellF1}\left(\frac{2+n}{n^2},-p,-p,1+\right.}{2+n}$$

input
```
Integrate[x^(2/n)*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(n*x^((2 + n)/n)*(a + x^n*(b + c*x^n))^p*AppellF1[(2 + n)/n^2, -p, -p, 1 +
 (2 + n)/n^2, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
 - 4*a*c])])/((2 + n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2/n}\left(a+bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\int x^{2/n}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-\right.$$

$$\Big\downarrow \text{1012}$$

$$\frac{nx^{\frac{n+2}{n}}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(\frac{n+2}{n^2},-p,-p,\frac{n^2+n+2}{n^2},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\right.}{n+2}$$

input
```
Int[x^(2/n)*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(n*x^((2 + n)/n)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(2 + n)/n^2, -p, -p, (
2 + n + n^2)/n^2, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])/((2 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^{\frac{2}{n}} \left(a + b\,x^{n} + c\,x^{2n}\right)^{p} dx$$

input
```
int(x^(2/n)*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int(x^(2/n)*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int x^{2/n} (a + bx^n + cx^{2n})^p \, dx = \int (cx^{2\,n} + bx^n + a)^p x^{\frac{2}{n}} \, dx$$

input
```
integrate(x^(2/n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p*x^(2/n), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{2/n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**(2/n)*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{2/n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^p x^{\frac{2}{n}}\,dx$$

input
```
integrate(x^(2/n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*x^(2/n), x)
```

Giac [F]

$$\int x^{2/n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^p x^{\frac{2}{n}}\,dx$$

input
```
integrate(x^(2/n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*x^(2/n), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{2/n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\int x^{2/n}\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

input
```
int(x^(2/n)*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int(x^(2/n)*(a + b*x^n + c*x^(2*n))^p, x)
```

Reduce [F]

$$\int x^{2/n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{n\left(x^{\frac{2}{n}}\left(x^{2n}c+x^nb+a\right)^p x-\left(\int \frac{x^{\frac{2n^2+2}{n}}\left(x^{2n}c+x^nb+a\right)^p}{x^{2n}c\,n^2p+x^{2n}cn+2x^{2n}c+x^nb\,n^2p+x^nbn+2x^nb+a\,n^2p+an+2a}dx\right)c\,n^3p^2-\right.}{}$$

input
```
int(x^(2/n)*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
(n*(x**(2/n)*(x**(2*n)*c + x**n*b + a)**p*x - int((x**((2*n**2 + 2)/n)*(x*
*(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*n**2*p + x**(2*n)*c*n + 2*x**(2*n)*
c + x**n*b*n**2*p + x**n*b*n + 2*x**n*b + a*n**2*p + a*n + 2*a),x)*c*n**3*
p**2 - int((x**((2*n**2 + 2)/n)*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*
n**2*p + x**(2*n)*c*n + 2*x**(2*n)*c + x**n*b*n**2*p + x**n*b*n + 2*x**n*b
 + a*n**2*p + a*n + 2*a),x)*c*n**2*p - 2*int((x**((2*n**2 + 2)/n)*(x**(2*n
)*c + x**n*b + a)**p)/(x**(2*n)*c*n**2*p + x**(2*n)*c*n + 2*x**(2*n)*c + x
**n*b*n**2*p + x**n*b*n + 2*x**n*b + a*n**2*p + a*n + 2*a),x)*c*n*p + int(
(x**(2/n)*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*n**2*p + x**(2*n)*c*n
+ 2*x**(2*n)*c + x**n*b*n**2*p + x**n*b*n + 2*x**n*b + a*n**2*p + a*n + 2*
a),x)*a*n**3*p**2 + int((x**(2/n)*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*
c*n**2*p + x**(2*n)*c*n + 2*x**(2*n)*c + x**n*b*n**2*p + x**n*b*n + 2*x**n
*b + a*n**2*p + a*n + 2*a),x)*a*n**2*p + 2*int((x**(2/n)*(x**(2*n)*c + x**
n*b + a)**p)/(x**(2*n)*c*n**2*p + x**(2*n)*c*n + 2*x**(2*n)*c + x**n*b*n**
2*p + x**n*b*n + 2*x**n*b + a*n**2*p + a*n + 2*a),x)*a*n*p))/(n**2*p + n +
 2)
```

3.259 $\int x^{\frac{1}{n}}(a+bx^n+cx^{2n})^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 156

$$\int x^{\frac{1}{n}}\left(a+bx^n+cx^{2n}\right)^p\,dx = \frac{nx^{1+\frac{1}{n}}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{AppellF1}\left(\frac{1+n}{n^2},-p,-p,1+\frac{1}{n^2}+\frac{1}{n},-\frac{\cdots}{b-\cdots}\right.}{1+n}$$

output
```
n*x^(1+1/n)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+n)/n^2,-p,-p,1+1/n^2+1/n,-2*
c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+n)/((1+2*
c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.17

$$\int x^{\frac{1}{n}}\left(a+bx^n+cx^{2n}\right)^p\,dx = \frac{nx^{1+\frac{1}{n}}\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+x^n(b+cx^n)\right)^p \operatorname{AppellF1}\left(\frac{1+n}{n^2},-p,-p,1+\frac{1}{n^2}+\frac{1}{n},-\cdots\right.}{1+n}$$

input
```
Integrate[x^n^(-1)*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(n*x^(1 + n^(-1))*(a + x^n*(b + c*x^n))^p*AppellF1[(1 + n)/n^2, -p, -p, 1
+ n^(-2) + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])])/((1 + n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^
2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))
^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{\frac{1}{n}}\left(a+bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\int x^{\frac{1}{n}}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-4}}\right.$$

$$\Big\downarrow \text{1012}$$

$$\frac{nx^{\frac{1}{n}+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(\frac{n+1}{n^2},-p,-p,1+\frac{1}{n}+\frac{1}{n^2},-\frac{2cx^n}{b-\sqrt{b^2-4a}}\right.}{n+1}$$

input
```
Int[x^n^(-1)*(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(n*x^(1 + n^(-1))*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + n)/n^2, -p, -p,
1 + n^(-2) + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])])/((1 + n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1
+ (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^{\frac{1}{n}} \left(a+b\,x^{n}+c\,x^{2n}\right)^{p} dx$$

input
```
int(x^(1/n)*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int(x^(1/n)*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int x^{\frac{1}{n}} \left(a+bx^n+cx^{2n}\right)^p \, dx=\int \left(cx^{2\,n}+bx^n+a\right)^p x^{\left(\frac{1}{n}\right)}\, dx$$

input
```
integrate(x^(1/n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p*x^(1/n), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{\frac{1}{n}}(a+bx^n+cx^{2n})^p\,dx = \text{Timed out}$$

input
```
integrate(x**(1/n)*(a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{\frac{1}{n}}(a+bx^n+cx^{2n})^p\,dx = \int (cx^{2n}+bx^n+a)^p x^{\left(\frac{1}{n}\right)}\,dx$$

input
```
integrate(x^(1/n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*x^(1/n), x)
```

Giac [F]

$$\int x^{\frac{1}{n}}(a+bx^n+cx^{2n})^p\,dx = \int (cx^{2n}+bx^n+a)^p x^{\left(\frac{1}{n}\right)}\,dx$$

input
```
integrate(x^(1/n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p*x^(1/n), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{\frac{1}{n}}\left(a+bx^n+cx^{2n}\right)^p\,dx=\int x^{1/n}\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

input
```
int(x^(1/n)*(a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int(x^(1/n)*(a + b*x^n + c*x^(2*n))^p, x)
```

Reduce [F]

$$\int x^{\frac{1}{n}}\left(a+bx^n+cx^{2n}\right)^p\,dx$$
$$=\frac{n\left(x^{\frac{1}{n}}\left(x^{2n}c+x^nb+a\right)^px-\left(\int\frac{x^{\frac{2n^2+1}{n}}\left(x^{2n}c+x^nb+a\right)^p}{x^{2n}c\,n^2p+x^{2n}cn+x^{2n}c+x^nb\,n^2p+x^nbn+x^nb+a\,n^2p+an+a}dx\right)c\,n^3p^2-\left(\int\frac{}{x^{2n}c\,n^2p+x}\right.\right.}{}$$

input
```
int(x^(1/n)*(a+b*x^n+c*x^(2*n))^p,x)
```

output
```
(n*(x**(1/n)*(x**(2*n)*c + x**n*b + a)**p*x - int((x**((2*n**2 + 1)/n)*(x*
*(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*n**2*p + x**(2*n)*c*n + x**(2*n)*c
+ x**n*b*n**2*p + x**n*b*n + x**n*b + a*n**2*p + a*n + a),x)*c*n**3*p**2 -
 int((x**((2*n**2 + 1)/n)*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*n**2*p
 + x**(2*n)*c*n + x**(2*n)*c + x**n*b*n**2*p + x**n*b*n + x**n*b + a*n**2*
p + a*n + a),x)*c*n**2*p - int((x**((2*n**2 + 1)/n)*(x**(2*n)*c + x**n*b +
 a)**p)/(x**(2*n)*c*n**2*p + x**(2*n)*c*n + x**(2*n)*c + x**n*b*n**2*p + x
**n*b*n + x**n*b + a*n**2*p + a*n + a),x)*c*n*p + int((x**(1/n)*(x**(2*n)*
c + x**n*b + a)**p)/(x**(2*n)*c*n**2*p + x**(2*n)*c*n + x**(2*n)*c + x**n*
b*n**2*p + x**n*b*n + x**n*b + a*n**2*p + a*n + a),x)*a*n**3*p**2 + int((x
**(1/n)*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*n**2*p + x**(2*n)*c*n +
x**(2*n)*c + x**n*b*n**2*p + x**n*b*n + x**n*b + a*n**2*p + a*n + a),x)*a*
n**2*p + int((x**(1/n)*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*n**2*p +
x**(2*n)*c*n + x**(2*n)*c + x**n*b*n**2*p + x**n*b*n + x**n*b + a*n**2*p +
 a*n + a),x)*a*n*p))/(n**2*p + n + 1)
```

3.260 $\int (a+bx^n+cx^{2n})^p \, dx$

Optimal result

Integrand size = 16, antiderivative size = 137

$$\int (a+bx^n+cx^{2n})^p \, dx = x\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)$$

output
```
x*(a+b*x^n+c*x^(2*n))^p*AppellF1(1/n,-p,-p,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2))
)^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.20

$$\int (a+bx^n+cx^{2n})^p \, dx = x\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p \operatorname{AppellF1}\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x*(a + x^n*(b + c*x^n))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt
[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^n+cx^{2n}\right)^p \, dx$$

$$\Big\downarrow \text{1686}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\int \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right.$$

$$\Big\downarrow \text{936}$$

$$x\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(\frac{1}{n},-p,-p,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-}}\right.$$

input
```
Int[(a + b*x^n + c*x^(2*n))^p,x]
```

output
```
(x*(a + b*x^n + c*x^(2*n))^p*AppellF1[n^(-1), -p, -p, 1 + n^(-1), (-2*c*x^
n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*
c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^
p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a+b\,x^{n}+c\,x^{2n}\right)^{p}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^p,x)
```

output
```
int((a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int \left(a+bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^p dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^n+cx^{2n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((a+b*x**n+c*x**(2*n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^p\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^n+cx^{2n}\right)^p\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^p\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^n+cx^{2n}\right)^p\,dx=\int \left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^p,x)
```

output
```
int((a + b*x^n + c*x^(2*n))^p, x)
```

Reduce [F]

$$\int \left(a+bx^n+cx^{2n}\right)^p\,dx$$
$$=\frac{\left(x^{2n}c+x^nb+a\right)^p x+\left(\int \frac{\left(x^{2n}c+x^nb+a\right)^p}{x^{2n}cnp+x^{2n}c+x^nbnp+x^nb+anp+a}dx\right) a\,n^2p^2+\left(\int \frac{\left(x^{2n}c+x^nb+a\right)^p}{x^{2n}cnp+x^{2n}c+x^nbnp+x^nb+anp+a}dx\right) an}{np+1}$$

input
```
int((a+b*x^n+c*x^(2*n))^p,x)
```

output
```
((x**(2*n)*c + x**n*b + a)**p*x + int((x**(2*n)*c + x**n*b + a)**p/(x**(2*
n)*c*n*p + x**(2*n)*c + x**n*b*n*p + x**n*b + a*n*p + a),x)*a*n**2*p**2 +
int((x**(2*n)*c + x**n*b + a)**p/(x**(2*n)*c*n*p + x**(2*n)*c + x**n*b*n*p
 + x**n*b + a*n*p + a),x)*a*n*p - int((x**(2*n)*(x**(2*n)*c + x**n*b + a)*
*p)/(x**(2*n)*c*n*p + x**(2*n)*c + x**n*b*n*p + x**n*b + a*n*p + a),x)*c*n
**2*p**2 - int((x**(2*n)*(x**(2*n)*c + x**n*b + a)**p)/(x**(2*n)*c*n*p + x
**(2*n)*c + x**n*b*n*p + x**n*b + a*n*p + a),x)*c*n*p)/(n*p + 1)
```

3.261 $\int x^{-1/n}(a+bx^n+cx^{2n})^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 174

$$\int x^{-1/n}(a+bx^n+cx^{2n})^p\,dx = -\frac{nx^{-\frac{1-n}{n}}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p\operatorname{AppellF1}\left(-\frac{1-n}{n^2},-p,-p,-\frac{1-n-n^2}{n^2},-\frac{}{b-}\right.}{1-n}$$

output
```
-n*(a+b*x^n+c*x^(2*n))^p*AppellF1(-(1-n)/n^2,-p,-p,-(-n^2-n+1)/n^2,-2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1-n)/(x^((1-n)/
n))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/
2)))^p)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07

$$\int x^{-1/n}(a+bx^n+cx^{2n})^p\,dx = \frac{nx^{\frac{-1+n}{n}}\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\operatorname{AppellF1}\left(\frac{-1+n}{n^2},-p,-p,\right.}{-1+n}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^p/x^n^(-1),x]
```

output
```
(n*x^((-1 + n)/n)*(a + x^n*(b + c*x^n))^p*AppellF1[(-1 + n)/n^2, -p, -p, 1
 + (-1 + n)/n^2, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/((-1 + n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^
p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-1/n}\left(a+bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\int x^{-1/n}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-\ldots}}\right.$$

$$\Big\downarrow \text{1012}$$

$$-\frac{nx^{-\frac{1-n}{n}}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\operatorname{AppellF1}\left(-\frac{1-n}{n^2},-p,-p,-\frac{-n^2-n+1}{n^2},-\frac{2c\ldots}{b-\sqrt{\ldots}}\right.}{1-n}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^p/x^n^(-1),x]
```

output
```
-((n*(a + b*x^n + c*x^(2*n))^p*AppellF1[-((1 - n)/n^2), -p, -p, -((1 - n -
 n^2)/n^2), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/((1 - n)*x^((1 - n)/n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^
p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \left(a+b\,x^{n}+c\,x^{2n}\right)^{p} x^{-\frac{1}{n}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^p/(x^(1/n)),x)
```

output
```
int((a+b*x^n+c*x^(2*n))^p/(x^(1/n)),x)
```

Fricas [F]

$$\int x^{-1/n}\left(a+bx^{n}+cx^{2n}\right)^{p}\,dx=\int \frac{\left(cx^{2\,n}+bx^{n}+a\right)^{p}}{x^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(x^(1/n)),x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p/x^(1/n), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1/n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((a+b*x**n+c*x**(2*n))**p/(x**(1/n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1/n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\int \frac{(cx^{2n}+bx^n+a)^p}{x^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(x^(1/n)),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/x^(1/n), x)
```

Giac [F]

$$\int x^{-1/n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\int \frac{(cx^{2n}+bx^n+a)^p}{x^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(x^(1/n)),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/x^(1/n), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1/n}(a+bx^n+cx^{2n})^p\,dx=\int \frac{(a+b\,x^n+c\,x^{2\,n})^p}{x^{1/n}}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^p/x^(1/n),x)
```

output
```
int((a + b*x^n + c*x^(2*n))^p/x^(1/n), x)
```

Reduce [F]

$$\int x^{-1/n}(a+bx^n+cx^{2n})^p\,dx=\text{Too large to display}$$

input
```
int((a+b*x^n+c*x^(2*n))^p/(x^(1/n)),x)
```

output
```
(n*((x**(2*n)*c + x**n*b + a)**p*x + x**(1/n)*int((x**(2*n)*c + x**n*b + a
)**p/(x**((2*n**2 + 1)/n)*c*n**2*p + x**((2*n**2 + 1)/n)*c*n - x**((2*n**2
 + 1)/n)*c + x**((n**2 + 1)/n)*b*n**2*p + x**((n**2 + 1)/n)*b*n - x**((n**
2 + 1)/n)*b + x**(1/n)*a*n**2*p + x**(1/n)*a*n - x**(1/n)*a),x)*a*n**3*p**
2 + x**(1/n)*int((x**(2*n)*c + x**n*b + a)**p/(x**((2*n**2 + 1)/n)*c*n**2*
p + x**((2*n**2 + 1)/n)*c*n - x**((2*n**2 + 1)/n)*c + x**((n**2 + 1)/n)*b*
n**2*p + x**((n**2 + 1)/n)*b*n - x**((n**2 + 1)/n)*b + x**(1/n)*a*n**2*p +
 x**(1/n)*a*n - x**(1/n)*a),x)*a*n**2*p - x**(1/n)*int((x**(2*n)*c + x**n*
b + a)**p/(x**((2*n**2 + 1)/n)*c*n**2*p + x**((2*n**2 + 1)/n)*c*n - x**((2
*n**2 + 1)/n)*c + x**((n**2 + 1)/n)*b*n**2*p + x**((n**2 + 1)/n)*b*n - x**
((n**2 + 1)/n)*b + x**(1/n)*a*n**2*p + x**(1/n)*a*n - x**(1/n)*a),x)*a*n*p
 - x**(1/n)*int((x**(2*n)*(x**(2*n)*c + x**n*b + a)**p)/(x**((2*n**2 + 1)/
n)*c*n**2*p + x**((2*n**2 + 1)/n)*c*n - x**((2*n**2 + 1)/n)*c + x**((n**2
+ 1)/n)*b*n**2*p + x**((n**2 + 1)/n)*b*n - x**((n**2 + 1)/n)*b + x**(1/n)*
a*n**2*p + x**(1/n)*a*n - x**(1/n)*a),x)*c*n**3*p**2 - x**(1/n)*int((x**(2
*n)*(x**(2*n)*c + x**n*b + a)**p)/(x**((2*n**2 + 1)/n)*c*n**2*p + x**((2*n
**2 + 1)/n)*c*n - x**((2*n**2 + 1)/n)*c + x**((n**2 + 1)/n)*b*n**2*p + x**
((n**2 + 1)/n)*b*n - x**((n**2 + 1)/n)*b + x**(1/n)*a*n**2*p + x**(1/n)*a*
n - x**(1/n)*a),x)*c*n**2*p + x**(1/n)*int((x**(2*n)*(x**(2*n)*c + x**n*b
+ a)**p)/(x**((2*n**2 + 1)/n)*c*n**2*p + x**((2*n**2 + 1)/n)*c*n - x**(...
```

3.262 $\int x^{-2/n}(a+bx^n+cx^{2n})^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 174

$$\int x^{-2/n}(a+bx^n+cx^{2n})^p\,dx = -\frac{nx^{-\frac{2-n}{n}}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p\,\text{AppellF1}\left(-\frac{2-n}{n^2},-p,-p,-\frac{2-n-n^2}{n^2},-\frac{}{b-}\right.}{2-n}$$

output
```
-n*(a+b*x^n+c*x^(2*n))^p*AppellF1(-(2-n)/n^2,-p,-p,-(-n^2-n+2)/n^2,-2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(2-n)/(x^((2-n)/
n))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/
2)))^p)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07

$$\int x^{-2/n}(a+bx^n+cx^{2n})^p\,dx = \frac{nx^{\frac{-2+n}{n}}\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\,\text{AppellF1}\left(\frac{-2+n}{n^2},-p,-p,\right.}{-2+n}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^p/x^(2/n),x]
```

output
```
(n*x^((-2 + n)/n)*(a + x^n*(b + c*x^n))^p*AppellF1[(-2 + n)/n^2, -p, -p, 1
 + (-2 + n)/n^2, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/((-2 + n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^
p)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2/n}\left(a+bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\int x^{-2/n}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-}\right.$$

$$\Big\downarrow \text{1012}$$

$$-\frac{nx^{-\frac{2-n}{n}}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\,\text{AppellF1}\left(-\frac{2-n}{n^2},-p,-p,-\frac{-n^2-n+2}{n^2},-\frac{2c}{b-\sqrt{}}\right.}{2-n}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^p/x^(2/n),x]
```

output
```
-((n*(a + b*x^n + c*x^(2*n))^p*AppellF1[-((2 - n)/n^2), -p, -p, -((2 - n -
 n^2)/n^2), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/((2 - n)*x^((2 - n)/n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^
p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \left(a+b\,x^{n}+c\,x^{2n}\right)^{p} x^{-\frac{2}{n}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^p/(x^(2/n)),x)
```

output
```
int((a+b*x^n+c*x^(2*n))^p/(x^(2/n)),x)
```

Fricas [F]

$$\int x^{-2/n}(a+bx^n+cx^{2n})^p\,dx=\int\frac{(cx^{2\,n}+bx^n+a)^p}{x^{\frac{2}{n}}}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(x^(2/n)),x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p/x^(2/n), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-2/n}(a + bx^n + cx^{2n})^p \, dx = \text{Timed out}$$

input
```
integrate((a+b*x**n+c*x**(2*n))**p/(x**(2/n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-2/n}(a + bx^n + cx^{2n})^p \, dx = \int \frac{(cx^{2n} + bx^n + a)^p}{x^{\frac{2}{n}}} \, dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(x^(2/n)),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/x^(2/n), x)
```

Giac [F]

$$\int x^{-2/n}(a + bx^n + cx^{2n})^p \, dx = \int \frac{(cx^{2n} + bx^n + a)^p}{x^{\frac{2}{n}}} \, dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(x^(2/n)),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/x^(2/n), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-2/n}(a+bx^n+cx^{2n})^p\,dx=\int \frac{(a+b\,x^n+c\,x^{2\,n})^p}{x^{2/n}}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^p/x^(2/n),x)
```

output
```
int((a + b*x^n + c*x^(2*n))^p/x^(2/n), x)
```

Reduce [F]

$$\int x^{-2/n}(a+bx^n+cx^{2n})^p\,dx=\text{Too large to display}$$

input
```
int((a+b*x^n+c*x^(2*n))^p/(x^(2/n)),x)
```

output
```
(n*((x**(2*n)*c + x**n*b + a)**p*x + x**(2/n)*int((x**(2*n)*c + x**n*b + a
)**p/(x**((2*n**2 + 2)/n)*c*n**2*p + x**((2*n**2 + 2)/n)*c*n - 2*x**((2*n*
*2 + 2)/n)*c + x**((n**2 + 2)/n)*b*n**2*p + x**((n**2 + 2)/n)*b*n - 2*x**(
(n**2 + 2)/n)*b + x**(2/n)*a*n**2*p + x**(2/n)*a*n - 2*x**(2/n)*a),x)*a*n*
*3*p**2 + x**(2/n)*int((x**(2*n)*c + x**n*b + a)**p/(x**((2*n**2 + 2)/n)*c
*n**2*p + x**((2*n**2 + 2)/n)*c*n - 2*x**((2*n**2 + 2)/n)*c + x**((n**2 +
2)/n)*b*n**2*p + x**((n**2 + 2)/n)*b*n - 2*x**((n**2 + 2)/n)*b + x**(2/n)*
a*n**2*p + x**(2/n)*a*n - 2*x**(2/n)*a),x)*a*n**2*p - 2*x**(2/n)*int((x**(
2*n)*c + x**n*b + a)**p/(x**((2*n**2 + 2)/n)*c*n**2*p + x**((2*n**2 + 2)/n
)*c*n - 2*x**((2*n**2 + 2)/n)*c + x**((n**2 + 2)/n)*b*n**2*p + x**((n**2 +
 2)/n)*b*n - 2*x**((n**2 + 2)/n)*b + x**(2/n)*a*n**2*p + x**(2/n)*a*n - 2*
x**(2/n)*a),x)*a*n*p - x**(2/n)*int((x**(2*n)*(x**(2*n)*c + x**n*b + a)**p
)/(x**((2*n**2 + 2)/n)*c*n**2*p + x**((2*n**2 + 2)/n)*c*n - 2*x**((2*n**2
+ 2)/n)*c + x**((n**2 + 2)/n)*b*n**2*p + x**((n**2 + 2)/n)*b*n - 2*x**((n*
*2 + 2)/n)*b + x**(2/n)*a*n**2*p + x**(2/n)*a*n - 2*x**(2/n)*a),x)*c*n**3*
p**2 - x**(2/n)*int((x**(2*n)*(x**(2*n)*c + x**n*b + a)**p)/(x**((2*n**2 +
 2)/n)*c*n**2*p + x**((2*n**2 + 2)/n)*c*n - 2*x**((2*n**2 + 2)/n)*c + x**(
(n**2 + 2)/n)*b*n**2*p + x**((n**2 + 2)/n)*b*n - 2*x**((n**2 + 2)/n)*b + x
**(2/n)*a*n**2*p + x**(2/n)*a*n - 2*x**(2/n)*a),x)*c*n**2*p + 2*x**(2/n)*i
nt((x**(2*n)*(x**(2*n)*c + x**n*b + a)**p)/(x**((2*n**2 + 2)/n)*c*n**2*...
```

3.263 $\int x^{-3/n}(a+bx^n+cx^{2n})^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 174

$$\int x^{-3/n}(a+bx^n+cx^{2n})^p\,dx = -\frac{nx^{-\frac{3-n}{n}}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p\operatorname{AppellF1}\left(-\frac{3-n}{n^2},-p,-p,-\frac{3-n-n^2}{n^2},-\frac{}{b-}\right.}{3-n}$$

output
```
-n*(a+b*x^n+c*x^(2*n))^p*AppellF1(-(3-n)/n^2,-p,-p,-(-n^2-n+3)/n^2,-2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(3-n)/(x^((3-n)/
n))/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/
2)))^p)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07

$$\int x^{-3/n}(a+bx^n+cx^{2n})^p\,dx = \frac{nx^{\frac{-3+n}{n}}\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\operatorname{AppellF1}\left(\frac{-3+n}{n^2},-p,-p,\right.}{-3+n}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^p/x^(3/n),x]
```

output
```
(n*x^((-3 + n)/n)*(a + x^n*(b + c*x^n))^p*AppellF1[(-3 + n)/n^2, -p, -p, 1
 + (-3 + n)/n^2, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/((-3 + n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^
p)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3/n}\left(a+bx^n+cx^{2n}\right)^p\,dx$$

$$\Big\downarrow \text{1721}$$

$$\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\int x^{-3/n}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^n}{b+\sqrt{b^2-}\right.$$

$$\Big\downarrow \text{1012}$$

$$-\frac{nx^{-\frac{3-n}{n}}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\text{AppellF1}\left(-\frac{3-n}{n^2},-p,-p,-\frac{-n^2-n+3}{n^2},-\frac{2c}{b-\sqrt{}}\right.}{3-n}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^p/x^(3/n),x]
```

output
```
-((n*(a + b*x^n + c*x^(2*n))^p*AppellF1[-((3 - n)/n^2), -p, -p, -((3 - n -
 n^2)/n^2), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/((3 - n)*x^((3 - n)/n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^
p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \left(a+b\,x^{n}+c\,x^{2n}\right)^{p} x^{-\frac{3}{n}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^p/(x^(3/n)),x)
```

output
```
int((a+b*x^n+c*x^(2*n))^p/(x^(3/n)),x)
```

Fricas [F]

$$\int x^{-3/n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\int \frac{(cx^{2\,n}+bx^n+a)^p}{x^{\frac{3}{n}}}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(x^(3/n)),x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^p/x^(3/n), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-3/n}(a+bx^n+cx^{2n})^p\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**n+c*x**(2*n))**p/(x**(3/n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-3/n}(a+bx^n+cx^{2n})^p\,dx = \int \frac{(cx^{2\,n}+bx^n+a)^p}{x^{\frac{3}{n}}}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(x^(3/n)),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/x^(3/n), x)
```

Giac [F]

$$\int x^{-3/n}(a+bx^n+cx^{2n})^p\,dx = \int \frac{(cx^{2\,n}+bx^n+a)^p}{x^{\frac{3}{n}}}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^p/(x^(3/n)),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^p/x^(3/n), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-3/n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\int \frac{\left(a+b\,x^n+c\,x^{2\,n}\right)^p}{x^{3/n}}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^p/x^(3/n),x)
```

output
```
int((a + b*x^n + c*x^(2*n))^p/x^(3/n), x)
```

Reduce [F]

$$\int x^{-3/n}\left(a+bx^n+cx^{2n}\right)^p\,dx=\text{Too large to display}$$

input
```
int((a+b*x^n+c*x^(2*n))^p/(x^(3/n)),x)
```

output
```
(n*((x**(2*n)*c + x**n*b + a)**p*x + x**(3/n)*int((x**(2*n)*c + x**n*b + a
)**p/(x**((2*n**2 + 3)/n)*c*n**2*p + x**((2*n**2 + 3)/n)*c*n - 3*x**((2*n*
*2 + 3)/n)*c + x**((n**2 + 3)/n)*b*n**2*p + x**((n**2 + 3)/n)*b*n - 3*x**(
(n**2 + 3)/n)*b + x**(3/n)*a*n**2*p + x**(3/n)*a*n - 3*x**(3/n)*a),x)*a*n*
*3*p**2 + x**(3/n)*int((x**(2*n)*c + x**n*b + a)**p/(x**((2*n**2 + 3)/n)*c
*n**2*p + x**((2*n**2 + 3)/n)*c*n - 3*x**((2*n**2 + 3)/n)*c + x**((n**2 +
3)/n)*b*n**2*p + x**((n**2 + 3)/n)*b*n - 3*x**((n**2 + 3)/n)*b + x**(3/n)*
a*n**2*p + x**(3/n)*a*n - 3*x**(3/n)*a),x)*a*n**2*p - 3*x**(3/n)*int((x**(
2*n)*c + x**n*b + a)**p/(x**((2*n**2 + 3)/n)*c*n**2*p + x**((2*n**2 + 3)/n
)*c*n - 3*x**((2*n**2 + 3)/n)*c + x**((n**2 + 3)/n)*b*n**2*p + x**((n**2 +
 3)/n)*b*n - 3*x**((n**2 + 3)/n)*b + x**(3/n)*a*n**2*p + x**(3/n)*a*n - 3*
x**(3/n)*a),x)*a*n*p - x**(3/n)*int((x**(2*n)*(x**(2*n)*c + x**n*b + a)**p
)/(x**((2*n**2 + 3)/n)*c*n**2*p + x**((2*n**2 + 3)/n)*c*n - 3*x**((2*n**2
+ 3)/n)*c + x**((n**2 + 3)/n)*b*n**2*p + x**((n**2 + 3)/n)*b*n - 3*x**((n*
*2 + 3)/n)*b + x**(3/n)*a*n**2*p + x**(3/n)*a*n - 3*x**(3/n)*a),x)*c*n**3*
p**2 - x**(3/n)*int((x**(2*n)*(x**(2*n)*c + x**n*b + a)**p)/(x**((2*n**2 +
 3)/n)*c*n**2*p + x**((2*n**2 + 3)/n)*c*n - 3*x**((2*n**2 + 3)/n)*c + x**(
(n**2 + 3)/n)*b*n**2*p + x**((n**2 + 3)/n)*b*n - 3*x**((n**2 + 3)/n)*b + x
**(3/n)*a*n**2*p + x**(3/n)*a*n - 3*x**(3/n)*a),x)*c*n**2*p + 3*x**(3/n)*i
nt((x**(2*n)*(x**(2*n)*c + x**n*b + a)**p)/(x**((2*n**2 + 3)/n)*c*n**2*...
```

3.264 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$

Optimal result

Integrand size = 31, antiderivative size = 55

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx = \frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

output
```
1/4*a*f^3*(e*x+d)^4/e+1/6*b*f^3*(e*x+d)^6/e+1/8*c*f^3*(e*x+d)^8/e
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(55) = 110.

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.80

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx = f^3\left(d^3\left(a+bd^2+cd^4\right)x+\frac{1}{2}d^2\left(3a+5bd^2+7cd^4\right)ex^2+\frac{1}{3}d\left(3a+10bd^2+21cd^4\right)e^2x^3 + \frac{1}{4}\left(a+10bd^2+35cd^4\right)e^3x^4+d\left(b+7cd^2\right)e^4x^5+\frac{1}{6}\left(b+21cd^2\right)e^5x^6+cde^6x^7+\frac{1}{8}ce^7x^8\right)$$

input
```
Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
f^3*(d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 +
 (d*(3*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^
3*x^4)/4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*
x^7 + (c*e^7*x^8)/8)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1462, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f^3 \int (d+ex)^3 \left(c(d+ex)^4+b(d+ex)^2+a\right) d(d+ex)}{e}$$

$$\Big\downarrow \text{1433}$$

$$\frac{f^3 \int \left(c(d+ex)^7+b(d+ex)^5+a(d+ex)^3\right) d(d+ex)}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^3 \left(\frac{1}{4}a(d+ex)^4+\frac{1}{6}b(d+ex)^6+\frac{1}{8}c(d+ex)^8\right)}{e}$$

input
```
Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f^3*((a*(d + e*x)^4)/4 + (b*(d + e*x)^6)/6 + (c*(d + e*x)^8)/8))/e
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(49) = 98$.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.25

method	result
gosper	$\frac{f^{3} x \left(3 c \,e^{7} x^{7}+24 c d \,e^{6} x^{6}+84 x^{5} c \,d^{2} e^{5}+168 c \,d^{3} e^{4} x^{4}+4 x^{5} b \,e^{5}+210 x^{3} c \,d^{4} e^{3}+24 b d \,e^{4} x^{4}+168 x^{2} c \,d^{5} e^{2}+60 x^{3} b \,d^{2} e^{3}+84 x c \,d^{6} e+\right.}{24}$
norman	$\left(\frac{7}{2} d^{2} f^{3} e^{5} c +\frac{1}{6} b \,e^{5} f^{3}\right) x^{6}+\left(7 c \,d^{5} e^{2} f^{3}+\frac{10}{3} b \,d^{3} e^{2} f^{3}+a d \,e^{2} f^{3}\right) x^{3}+\left(\frac{7}{2} c \,d^{6} e \,f^{3}+\frac{5}{2} b \,d^{4} e \,f^{3}\right.$
risch	$\frac{1}{8} e^{7} f^{3} c \,x^{8}+d \,f^{3} e^{6} c \,x^{7}+\frac{7}{2} f^{3} x^{6} c \,d^{2} e^{5}+\frac{1}{6} f^{3} x^{6} b \,e^{5}+7 f^{3} c \,d^{3} e^{4} x^{5}+f^{3} b d \,e^{4} x^{5}+\frac{35}{4} f^{3} x^{4} c \,d^{4} e$
parallelrisch	$\frac{1}{8} e^{7} f^{3} c \,x^{8}+d \,f^{3} e^{6} c \,x^{7}+\frac{7}{2} f^{3} x^{6} c \,d^{2} e^{5}+\frac{1}{6} f^{3} x^{6} b \,e^{5}+7 f^{3} c \,d^{3} e^{4} x^{5}+f^{3} b d \,e^{4} x^{5}+\frac{35}{4} f^{3} x^{4} c \,d^{4} e$
orering	$\frac{x \left(3 c \,e^{7} x^{7}+24 c d \,e^{6} x^{6}+84 x^{5} c \,d^{2} e^{5}+168 c \,d^{3} e^{4} x^{4}+4 x^{5} b \,e^{5}+210 x^{3} c \,d^{4} e^{3}+24 b d \,e^{4} x^{4}+168 x^{2} c \,d^{5} e^{2}+60 x^{3} b \,d^{2} e^{3}+84 x c \,d^{6} e+80\right.}{24 (c \,e^{4} x^{4}+4 c d \,e^{3} x^{3}+6 c \,d^{2} e^{2} x^{2}+4 c \,d^{3} e x+b \,e}$
default	$\frac{e^{7} f^{3} c \,x^{8}}{8}+d \,f^{3} e^{6} c \,x^{7}+\frac{\left(15 d^{2} f^{3} e^{5} c +e^{3} f^{3} \left(6 c \,d^{2} e^{2}+b \,e^{2}\right)\right) x^{6}}{6}+\frac{\left(13 d^{3} f^{3} c \,e^{4}+3 d \,f^{3} e^{2} \left(6 c \,d^{2} e^{2}+b \,e^{2}\right)+e^{3} f^{3} \left(4 c \,d^{3} e +\right.\right.}{5}$

input
```
int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/24*f^3*x*(3*c*e^7*x^7+24*c*d*e^6*x^6+84*c*d^2*e^5*x^5+168*c*d^3*e^4*x^4+
4*b*e^5*x^5+210*c*d^4*e^3*x^3+24*b*d*e^4*x^4+168*c*d^5*e^2*x^2+60*b*d^2*e^
3*x^3+84*c*d^6*e*x+80*b*d^3*e^2*x^2+24*c*d^7+6*a*e^3*x^3+60*b*d^4*e*x+24*a
*d*e^2*x^2+24*b*d^5+36*a*d^2*e*x+24*a*d^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(49) = 98$.

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.02

$$\int (df + efx)^3 \left(a + b(d + ex)^2 + c(d + ex)^4\right) dx = \frac{1}{8} ce^7 f^3 x^8 + cde^6 f^3 x^7 + \frac{1}{6} \left(21\, cd^2 + b\right) e^5 f^3 x^6 + \left(7\, cd^3 + bd\right) e^4 f^3 x^5 + \frac{1}{4} \left(35\, cd^4 + 10\, bd^2 + a\right) e^3 f^3 x^4 + \frac{1}{3} \left(21\, cd^5 + 10\, bd^3 + 3\, ad\right) e^2 f^3 x^3 + \frac{1}{2} \left(7\, cd^6 + 5\, bd^4 + 3\, ad^2\right) ef^3 x^2 + \left(cd^7 + bd^5 + ad^3\right) f^3 x$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
1/8*c*e^7*f^3*x^8 + c*d*e^6*f^3*x^7 + 1/6*(21*c*d^2 + b)*e^5*f^3*x^6 + (7*
c*d^3 + b*d)*e^4*f^3*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*f^3*x^4 + 1/3
*(21*c*d^5 + 10*b*d^3 + 3*a*d)*e^2*f^3*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*
d^2)*e*f^3*x^2 + (c*d^7 + b*d^5 + a*d^3)*f^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(44) = 88$.

Time = 0.04 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.36

$$\int (df + efx)^3 \left(a + b(d + ex)^2 + c(d + ex)^4\right) dx = cde^6 f^3 x^7 + \frac{ce^7 f^3 x^8}{8} + x^6 \left(\frac{be^5 f^3}{6} + \frac{7cd^2 e^5 f^3}{2}\right) + x^5 \left(bde^4 f^3 + 7cd^3 e^4 f^3\right) + x^4 \left(\frac{ae^3 f^3}{4} + \frac{5bd^2 e^3 f^3}{2} + \frac{35cd^4 e^3 f^3}{4}\right) + x^3 \left(ade^2 f^3 + \frac{10bd^3 e^2 f^3}{3} + 7cd^5 e^2 f^3\right) + x^2 \cdot \left(\frac{3ad^2 ef^3}{2} + \frac{5bd^4 ef^3}{2} + \frac{7cd^6 ef^3}{2}\right) + x \left(ad^3 f^3 + bd^5 f^3 + cd^7 f^3\right)$$

input
```
integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
c*d*e**6*f**3*x**7 + c*e**7*f**3*x**8/8 + x**6*(b*e**5*f**3/6 + 7*c*d**2*e
**5*f**3/2) + x**5*(b*d*e**4*f**3 + 7*c*d**3*e**4*f**3) + x**4*(a*e**3*f**
3/4 + 5*b*d**2*e**3*f**3/2 + 35*c*d**4*e**3*f**3/4) + x**3*(a*d*e**2*f**3
+ 10*b*d**3*e**2*f**3/3 + 7*c*d**5*e**2*f**3) + x**2*(3*a*d**2*e*f**3/2 +
5*b*d**4*e*f**3/2 + 7*c*d**6*e*f**3/2) + x*(a*d**3*f**3 + b*d**5*f**3 + c*
d**7*f**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(49) = 98$.

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.02

$$\begin{aligned}\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx &= \frac{1}{8}\,ce^7f^3x^8+cde^6f^3x^7+\frac{1}{6}\left(21\,cd^2+b\right)e^5f^3x^6+\left(7\,cd^3+bd\right)e^4f^3x^5\\&\quad+\frac{1}{4}\left(35\,cd^4+10\,bd^2+a\right)e^3f^3x^4+\frac{1}{3}\left(21\,cd^5+10\,bd^3+3\,ad\right)e^2f^3x^3\\&\quad+\frac{1}{2}\left(7\,cd^6+5\,bd^4+3\,ad^2\right)ef^3x^2+\left(cd^7+bd^5+ad^3\right)f^3x\end{aligned}$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
1/8*c*e^7*f^3*x^8 + c*d*e^6*f^3*x^7 + 1/6*(21*c*d^2 + b)*e^5*f^3*x^6 + (7*
c*d^3 + b*d)*e^4*f^3*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*f^3*x^4 + 1/3
*(21*c*d^5 + 10*b*d^3 + 3*a*d)*e^2*f^3*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*
d^2)*e*f^3*x^2 + (c*d^7 + b*d^5 + a*d^3)*f^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(49) = 98$.

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.71

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx$$
$$=\frac{1}{2}\left(efx^2+2\,dfx\right)cd^6f^2+\frac{1}{2}\left(efx^2+2\,dfx\right)bd^4f^2+\frac{1}{2}\left(efx^2+2\,dfx\right)ad^2f^2$$
$$+\frac{18\left(efx^2+2\,dfx\right)^2cd^4ef^2+12\left(efx^2+2\,dfx\right)^3cd^2e^2f+3\left(efx^2+2\,dfx\right)^4ce^3+12\left(efx^2+2\,dfx\right)^2bd^2}{24\,f}$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
1/2*(e*f*x^2 + 2*d*f*x)*c*d^6*f^2 + 1/2*(e*f*x^2 + 2*d*f*x)*b*d^4*f^2 + 1/
2*(e*f*x^2 + 2*d*f*x)*a*d^2*f^2 + 1/24*(18*(e*f*x^2 + 2*d*f*x)^2*c*d^4*e*f
^2 + 12*(e*f*x^2 + 2*d*f*x)^3*c*d^2*e^2*f + 3*(e*f*x^2 + 2*d*f*x)^4*c*e^3
+ 12*(e*f*x^2 + 2*d*f*x)^2*b*d^2*e*f^2 + 4*(e*f*x^2 + 2*d*f*x)^3*b*e^2*f +
 6*(e*f*x^2 + 2*d*f*x)^2*a*e*f^2)/f
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.98

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx$$
$$=\frac{e^5\,f^3\,x^6\,\left(21\,c\,d^2+b\right)}{6}+\frac{c\,e^7\,f^3\,x^8}{8}+d^3\,f^3\,x\,\left(c\,d^4+b\,d^2+a\right)$$
$$+\frac{e^3\,f^3\,x^4\,\left(35\,c\,d^4+10\,b\,d^2+a\right)}{4}+\frac{d^2\,e\,f^3\,x^2\,\left(7\,c\,d^4+5\,b\,d^2+3\,a\right)}{2}$$
$$+\frac{d\,e^2\,f^3\,x^3\,\left(21\,c\,d^4+10\,b\,d^2+3\,a\right)}{3}+d\,e^4\,f^3\,x^5\,\left(7\,c\,d^2+b\right)+c\,d\,e^6\,f^3\,x^7$$

input
```
int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
(e^5*f^3*x^6*(b + 21*c*d^2))/6 + (c*e^7*f^3*x^8)/8 + d^3*f^3*x*(a + b*d^2
+ c*d^4) + (e^3*f^3*x^4*(a + 10*b*d^2 + 35*c*d^4))/4 + (d^2*e*f^3*x^2*(3*a
 + 5*b*d^2 + 7*c*d^4))/2 + (d*e^2*f^3*x^3*(3*a + 10*b*d^2 + 21*c*d^4))/3 +
 d*e^4*f^3*x^5*(b + 7*c*d^2) + c*d*e^6*f^3*x^7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.24

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)\,dx$$
$$= \frac{f^3x(3c\,e^7x^7+24cd\,e^6x^6+84c\,d^2e^5x^5+168c\,d^3e^4x^4+4b\,e^5x^5+210c\,d^4e^3x^3+24bd\,e^4x^4+168c\,d^5e^2x^2 -}{}$$

input
```
int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

output
```
(f**3*x*(24*a*d**3 + 36*a*d**2*e*x + 24*a*d*e**2*x**2 + 6*a*e**3*x**3 + 24
*b*d**5 + 60*b*d**4*e*x + 80*b*d**3*e**2*x**2 + 60*b*d**2*e**3*x**3 + 24*b
*d*e**4*x**4 + 4*b*e**5*x**5 + 24*c*d**7 + 84*c*d**6*e*x + 168*c*d**5*e**2
*x**2 + 210*c*d**4*e**3*x**3 + 168*c*d**3*e**4*x**4 + 84*c*d**2*e**5*x**5
+ 24*c*d*e**6*x**6 + 3*c*e**7*x**7))/24
```

3.265 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

Optimal result

Integrand size = 33, antiderivative size = 104

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx = \frac{a^2f^3(d+ex)^4}{4e}+\frac{abf^3(d+ex)^6}{3e}+\frac{(b^2+2ac)\,f^3(d+ex)^8}{8e}+\frac{bcf^3(d+ex)^{10}}{5e}+\frac{c^2f^3(d+ex)^{12}}{12e}$$

output
```
1/4*a^2*f^3*(e*x+d)^4/e+1/3*a*b*f^3*(e*x+d)^6/e+1/8*(2*a*c+b^2)*f^3*(e*x+d
)^8/e+1/5*b*c*f^3*(e*x+d)^10/e+1/12*c^2*f^3*(e*x+d)^12/e
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 405 vs. $2(104) = 208$.

Time = 0.13 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.89

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$$
$$= f^3 \left(d^3 \left(a+bd^2+cd^4\right)^2 x + \frac{1}{2} d^2 \left(3a^2+10abd^2+7b^2d^4+14acd^4+18bcd^6+11c^2d^8\right) ex^2 + \frac{1}{3} d \left(3a^2+20abd^2+21b^2d^4+42acd^4+72bcd^6+55c^2d^8\right) e^2x^3 + \frac{1}{4} \left(a^2+20abd^2+35b^2d^4+70acd^4+168bcd^6+165c^2d^8\right) e^3x^4 + \frac{1}{5} d \left(10ab+35b^2d^2+70acd^2+252bcd^4+330c^2d^6\right) e^4x^5 + \frac{1}{6} \left(2ab+21b^2d^2+42acd^2+252bcd^4+462c^2d^6\right) e^5x^6 + d\left(b^2+2ac+24bcd^2+66c^2d^4\right) e^6x^7 + \frac{1}{8} \left(b^2+2ac+72bcd^2+330c^2d^4\right) e^7x^8 + \frac{1}{3} cd \left(6b+55cd^2\right) e^8x^9 + \frac{1}{10} c \left(2b+55cd^2\right) e^9x^{10} + c^2de^{10}x^{11} + \frac{1}{12} c^2e^{11}x^{12}\right)$$

input
```
Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
f^3*(d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 +
14*a*c*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 +
21*b^2*d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e^2*x^3)/3 + ((a^2 + 20
*a*b*d^2 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4
 + (d*(10*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x
^5)/5 + ((2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5
*x^6)/6 + d*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*
c + 72*b*c*d^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9)/
3 + (c*(2*b + 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/1
2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1462, 1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx$$

$$\Big\downarrow \text{ 1462}$$

$$\frac{f^3 \int (d+ex)^3 \left(c(d+ex)^4+b(d+ex)^2+a\right)^2 d(d+ex)}{e}$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{f^3 \int (d+ex)^2 \left(c(d+ex)^4+b(d+ex)^2+a\right)^2 d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{ 1140}$$

$$\frac{f^3 \int \left(c^2(d+ex)^{10}+2bc(d+ex)^8+\left(b^2+2ac\right)(d+ex)^6+2ab(d+ex)^4+a^2(d+ex)^2\right) d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f^3 \left(\frac{1}{2}a^2(d+ex)^4+\frac{1}{4}\left(2ac+b^2\right)(d+ex)^8+\frac{2}{3}ab(d+ex)^6+\frac{2}{5}bc(d+ex)^{10}+\frac{1}{6}c^2(d+ex)^{12}\right)}{2e}$$

input
```
Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f^3*((a^2*(d + e*x)^4)/2 + (2*a*b*(d + e*x)^6)/3 + ((b^2 + 2*a*c)*(d + e*
x)^8)/4 + (2*b*c*(d + e*x)^10)/5 + (c^2*(d + e*x)^12)/6))/(2*e)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(94) = 188$.

Time = 0.12 (sec) , antiderivative size = 566, normalized size of antiderivative = 5.44

method	result
gosper	$\frac{f^3x(10c^2e^{11}x^{11}+120c^2d\,e^{10}x^{10}+660x^9c^2d^2e^9+2200x^8c^2d^3e^8+24x^9bc\,e^9+4950x^7c^2d^4e^7+240x^8bcd\,e^8+7920c^2d^5e^6x^6+10}{}$
norman	$\left(\frac{11}{2}d^2f^3e^9c^2+\frac{1}{5}bc\,e^9f^3\right)x^{10}+\left(\frac{55}{3}d^3f^3c^2e^8+2bcd\,e^8f^3\right)x^9+\left(\frac{165}{4}d^4f^3c^2e^7+9bc\,d^2e^7f^3+\right.$
orering	$\frac{x(10c^2e^{11}x^{11}+120c^2d\,e^{10}x^{10}+660x^9c^2d^2e^9+2200x^8c^2d^3e^8+24x^9bc\,e^9+4950x^7c^2d^4e^7+240x^8bcd\,e^8+7920c^2d^5e^6x^6+1080}{}$
risch	$42f^3x^6bc\,d^4e^5+7f^3x^6ac\,d^2e^5+\frac{252}{5}f^3x^5bc\,d^5e^4+14f^3x^5ac\,d^3e^4+2f^3x^5abd\,e^4+42f^3x^4bc$
parallelrisch	$42f^3x^6bc\,d^4e^5+7f^3x^6ac\,d^2e^5+\frac{252}{5}f^3x^5bc\,d^5e^4+14f^3x^5ac\,d^3e^4+2f^3x^5abd\,e^4+42f^3x^4bc$
default	Expression too large to display

input
```
int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/120*f^3*x*(10*c^2*e^11*x^11+120*c^2*d*e^10*x^10+660*c^2*d^2*e^9*x^9+2200
*c^2*d^3*e^8*x^8+24*b*c*e^9*x^9+4950*c^2*d^4*e^7*x^7+240*b*c*d*e^8*x^8+792
0*c^2*d^5*e^6*x^6+1080*b*c*d^2*e^7*x^7+9240*c^2*d^6*e^5*x^5+2880*b*c*d^3*e
^6*x^6+7920*c^2*d^7*e^4*x^4+30*a*c*e^7*x^7+15*b^2*e^7*x^7+5040*b*c*d^4*e^5
*x^5+4950*c^2*d^8*e^3*x^3+240*a*c*d*e^6*x^6+120*b^2*d*e^6*x^6+6048*b*c*d^5
*e^4*x^4+2200*c^2*d^9*e^2*x^2+840*a*c*d^2*e^5*x^5+420*b^2*d^2*e^5*x^5+5040
*b*c*d^6*e^3*x^3+660*c^2*d^10*e*x+1680*a*c*d^3*e^4*x^4+840*b^2*d^3*e^4*x^4
+2880*b*c*d^7*e^2*x^2+120*c^2*d^11+40*a*b*e^5*x^5+2100*a*c*d^4*e^3*x^3+105
0*b^2*d^4*e^3*x^3+1080*b*c*d^8*e*x+240*a*b*d*e^4*x^4+1680*a*c*d^5*e^2*x^2+
840*b^2*d^5*e^2*x^2+240*b*c*d^9+600*a*b*d^2*e^3*x^3+840*a*c*d^6*e*x+420*b^
2*d^6*e*x+800*a*b*d^3*e^2*x^2+240*a*c*d^7+120*b^2*d^7+30*a^2*e^3*x^3+600*a
*b*d^4*e*x+120*a^2*d*e^2*x^2+240*a*b*d^5+180*a^2*d^2*e*x+120*a^2*d^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(94) = 188$.

Time = 0.17 (sec) , antiderivative size = 439, normalized size of antiderivative = 4.22

$$\begin{aligned}
&\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx \\
&= \frac{1}{12}c^2e^{11}f^3x^{12}+c^2de^{10}f^3x^{11}+\frac{1}{10}\left(55\,c^2d^2+2\,bc\right)e^9f^3x^{10} \\
&\quad+\frac{1}{3}\left(55\,c^2d^3+6\,bcd\right)e^8f^3x^9+\frac{1}{8}\left(330\,c^2d^4+72\,bcd^2+b^2+2\,ac\right)e^7f^3x^8 \\
&\quad+\left(66\,c^2d^5+24\,bcd^3+\left(b^2+2\,ac\right)d\right)e^6f^3x^7 \\
&\quad+\frac{1}{6}\left(462\,c^2d^6+252\,bcd^4+21\left(b^2+2\,ac\right)d^2+2\,ab\right)e^5f^3x^6 \\
&\quad+\frac{1}{5}\left(330\,c^2d^7+252\,bcd^5+35\left(b^2+2\,ac\right)d^3+10\,abd\right)e^4f^3x^5 \\
&\quad+\frac{1}{4}\left(165\,c^2d^8+168\,bcd^6+35\left(b^2+2\,ac\right)d^4+20\,abd^2+a^2\right)e^3f^3x^4 \\
&\quad+\frac{1}{3}\left(55\,c^2d^9+72\,bcd^7+21\left(b^2+2\,ac\right)d^5+20\,abd^3+3\,a^2d\right)e^2f^3x^3 \\
&\quad+\frac{1}{2}\left(11\,c^2d^{10}+18\,bcd^8+7\left(b^2+2\,ac\right)d^6+10\,abd^4+3\,a^2d^2\right)ef^3x^2 \\
&\quad+\left(c^2d^{11}+2\,bcd^9+\left(b^2+2\,ac\right)d^7+2\,abd^5+a^2d^3\right)f^3x
\end{aligned}$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas
")
```

output
```
1/12*c^2*e^11*f^3*x^12 + c^2*d*e^10*f^3*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e
^9*f^3*x^10 + 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*f^3*x^9 + 1/8*(330*c^2*d^4 +
72*b*c*d^2 + b^2 + 2*a*c)*e^7*f^3*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 +
2*a*c)*d)*e^6*f^3*x^7 + 1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*
d^2 + 2*a*b)*e^5*f^3*x^6 + 1/5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*
c)*d^3 + 10*a*b*d)*e^4*f^3*x^5 + 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2
+ 2*a*c)*d^4 + 20*a*b*d^2 + a^2)*e^3*f^3*x^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^
7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a^2*d)*e^2*f^3*x^3 + 1/2*(11*c^2
*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*b*d^4 + 3*a^2*d^2)*e*f^3*x
^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*f^3*
x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(88) = 176$.

Time = 0.07 (sec) , antiderivative size = 722, normalized size of antiderivative = 6.94

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx$$
$$= c^2de^{10}f^3x^{11}+\frac{c^2e^{11}f^3x^{12}}{12}+x^{10}\left(\frac{bce^9f^3}{5}+\frac{11c^2d^2e^9f^3}{2}\right)+x^9 \cdot\left(2bcde^8f^3+\frac{55c^2d^3e^8f^3}{3}\right)+x^8\left(\frac{ace^7f^3}{4}+\frac{b^2e^7f^3}{8}+9bcd^2e^7f^3+\frac{165c^2d^4e^7f^3}{4}\right) + x^7\cdot\left(2acde^6f^3+b^2de^6f^3+24bcd^3e^6f^3+66c^2d^5e^6f^3\right) + x^6\left(\frac{abe^5f^3}{3}+7acd^2e^5f^3+\frac{7b^2d^2e^5f^3}{2}+42bcd^4e^5f^3+77c^2d^6e^5f^3\right)+x^5 \cdot\left(2abde^4f^3+14acd^3e^4f^3+7b^2d^3e^4f^3+\frac{252bcd^5e^4f^3}{5}+66c^2d^7e^4f^3\right) + x^4\left(\frac{a^2e^3f^3}{4}+5abd^2e^3f^3+\frac{35acd^4e^3f^3}{2}+\frac{35b^2d^4e^3f^3}{4}+42bcd^6e^3f^3+\frac{165c^2d^8e^3f^3}{4}\right) + x^3\left(a^2de^2f^3+\frac{20abd^3e^2f^3}{3}+14acd^5e^2f^3+7b^2d^5e^2f^3+24bcd^7e^2f^3+\frac{55c^2d^9e^2f^3}{3}\right) + x^2\cdot\left(\frac{3a^2d^2ef^3}{2}+5abd^4ef^3+7acd^6ef^3+\frac{7b^2d^6ef^3}{2}+9bcd^8ef^3+\frac{11c^2d^{10}ef^3}{2}\right) + x\left(a^2d^3f^3+2abd^5f^3+2acd^7f^3+b^2d^7f^3+2bcd^9f^3+c^2d^{11}f^3\right)$$

input
```
integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
c**2*d*e**10*f**3*x**11 + c**2*e**11*f**3*x**12/12 + x**10*(b*c*e**9*f**3/
5 + 11*c**2*d**2*e**9*f**3/2) + x**9*(2*b*c*d*e**8*f**3 + 55*c**2*d**3*e**
8*f**3/3) + x**8*(a*c*e**7*f**3/4 + b**2*e**7*f**3/8 + 9*b*c*d**2*e**7*f**
3 + 165*c**2*d**4*e**7*f**3/4) + x**7*(2*a*c*d*e**6*f**3 + b**2*d*e**6*f**
3 + 24*b*c*d**3*e**6*f**3 + 66*c**2*d**5*e**6*f**3) + x**6*(a*b*e**5*f**3/
3 + 7*a*c*d**2*e**5*f**3 + 7*b**2*d**2*e**5*f**3/2 + 42*b*c*d**4*e**5*f**3
 + 77*c**2*d**6*e**5*f**3) + x**5*(2*a*b*d*e**4*f**3 + 14*a*c*d**3*e**4*f*
*3 + 7*b**2*d**3*e**4*f**3 + 252*b*c*d**5*e**4*f**3/5 + 66*c**2*d**7*e**4*
f**3) + x**4*(a**2*e**3*f**3/4 + 5*a*b*d**2*e**3*f**3 + 35*a*c*d**4*e**3*f
**3/2 + 35*b**2*d**4*e**3*f**3/4 + 42*b*c*d**6*e**3*f**3 + 165*c**2*d**8*e
**3*f**3/4) + x**3*(a**2*d*e**2*f**3 + 20*a*b*d**3*e**2*f**3/3 + 14*a*c*d*
*5*e**2*f**3 + 7*b**2*d**5*e**2*f**3 + 24*b*c*d**7*e**2*f**3 + 55*c**2*d**
9*e**2*f**3/3) + x**2*(3*a**2*d**2*e*f**3/2 + 5*a*b*d**4*e*f**3 + 7*a*c*d*
*6*e*f**3 + 7*b**2*d**6*e*f**3/2 + 9*b*c*d**8*e*f**3 + 11*c**2*d**10*e*f**
3/2) + x*(a**2*d**3*f**3 + 2*a*b*d**5*f**3 + 2*a*c*d**7*f**3 + b**2*d**7*f
**3 + 2*b*c*d**9*f**3 + c**2*d**11*f**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(94) = 188$.

Time = 0.04 (sec) , antiderivative size = 439, normalized size of antiderivative = 4.22

$$\begin{aligned}
&\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx \\
&= \frac{1}{12}c^2e^{11}f^3x^{12} + c^2de^{10}f^3x^{11} + \frac{1}{10}\left(55\,c^2d^2+2\,bc\right)e^9f^3x^{10} \\
&\quad + \frac{1}{3}\left(55\,c^2d^3+6\,bcd\right)e^8f^3x^9 + \frac{1}{8}\left(330\,c^2d^4+72\,bcd^2+b^2+2\,ac\right)e^7f^3x^8 \\
&\quad + \left(66\,c^2d^5+24\,bcd^3+\left(b^2+2\,ac\right)d\right)e^6f^3x^7 \\
&\quad + \frac{1}{6}\left(462\,c^2d^6+252\,bcd^4+21\left(b^2+2\,ac\right)d^2+2\,ab\right)e^5f^3x^6 \\
&\quad + \frac{1}{5}\left(330\,c^2d^7+252\,bcd^5+35\left(b^2+2\,ac\right)d^3+10\,abd\right)e^4f^3x^5 \\
&\quad + \frac{1}{4}\left(165\,c^2d^8+168\,bcd^6+35\left(b^2+2\,ac\right)d^4+20\,abd^2+a^2\right)e^3f^3x^4 \\
&\quad + \frac{1}{3}\left(55\,c^2d^9+72\,bcd^7+21\left(b^2+2\,ac\right)d^5+20\,abd^3+3\,a^2d\right)e^2f^3x^3 \\
&\quad + \frac{1}{2}\left(11\,c^2d^{10}+18\,bcd^8+7\left(b^2+2\,ac\right)d^6+10\,abd^4+3\,a^2d^2\right)ef^3x^2 \\
&\quad + \left(c^2d^{11}+2\,bcd^9+\left(b^2+2\,ac\right)d^7+2\,abd^5+a^2d^3\right)f^3x
\end{aligned}$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima
")
```

output
```
1/12*c^2*e^11*f^3*x^12 + c^2*d*e^10*f^3*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e
^9*f^3*x^10 + 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*f^3*x^9 + 1/8*(330*c^2*d^4 +
72*b*c*d^2 + b^2 + 2*a*c)*e^7*f^3*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 +
2*a*c)*d)*e^6*f^3*x^7 + 1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*
d^2 + 2*a*b)*e^5*f^3*x^6 + 1/5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*
c)*d^3 + 10*a*b*d)*e^4*f^3*x^5 + 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2
+ 2*a*c)*d^4 + 20*a*b*d^2 + a^2)*e^3*f^3*x^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^
7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a^2*d)*e^2*f^3*x^3 + 1/2*(11*c^2
*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*b*d^4 + 3*a^2*d^2)*e*f^3*x
^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*f^3*
x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(94) = 188$.

Time = 0.12 (sec) , antiderivative size = 597, normalized size of antiderivative = 5.74

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx$$
$$= \frac{1}{2}\left(efx^2+2\,dfx\right)c^2d^{10}f^2 + \left(efx^2+2\,dfx\right)bcd^8f^2 + \frac{1}{2}\left(efx^2+2\,dfx\right)b^2d^6f^2 + \left(efx^2+2\,dfx\right)acd^6f^2 + \left(efx^2+2\,dfx\right)abd^4f^2 + \frac{1}{2}\left(efx^2+2\,dfx\right)a^2d^2f^2 + \frac{150\left(efx^2+2\,dfx\right)^2c^2d^8ef^4 + 200\left(efx^2+2\,dfx\right)^3c^2d^6e^2f^3 + 150\left(efx^2+2\,dfx\right)^4c^2d^4e^3f^2 + 240\,(efx$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
1/2*(e*f*x^2 + 2*d*f*x)*c^2*d^10*f^2 + (e*f*x^2 + 2*d*f*x)*b*c*d^8*f^2 + 1
/2*(e*f*x^2 + 2*d*f*x)*b^2*d^6*f^2 + (e*f*x^2 + 2*d*f*x)*a*c*d^6*f^2 + (e*
f*x^2 + 2*d*f*x)*a*b*d^4*f^2 + 1/2*(e*f*x^2 + 2*d*f*x)*a^2*d^2*f^2 + 1/120
*(150*(e*f*x^2 + 2*d*f*x)^2*c^2*d^8*e*f^4 + 200*(e*f*x^2 + 2*d*f*x)^3*c^2*
d^6*e^2*f^3 + 150*(e*f*x^2 + 2*d*f*x)^4*c^2*d^4*e^3*f^2 + 240*(e*f*x^2 + 2
*d*f*x)^2*b*c*d^6*e*f^4 + 60*(e*f*x^2 + 2*d*f*x)^5*c^2*d^2*e^4*f + 240*(e*
f*x^2 + 2*d*f*x)^3*b*c*d^4*e^2*f^3 + 10*(e*f*x^2 + 2*d*f*x)^6*c^2*e^5 + 12
0*(e*f*x^2 + 2*d*f*x)^4*b*c*d^2*e^3*f^2 + 90*(e*f*x^2 + 2*d*f*x)^2*b^2*d^4
*e*f^4 + 180*(e*f*x^2 + 2*d*f*x)^2*a*c*d^4*e*f^4 + 24*(e*f*x^2 + 2*d*f*x)^
5*b*c*e^4*f + 60*(e*f*x^2 + 2*d*f*x)^3*b^2*d^2*e^2*f^3 + 120*(e*f*x^2 + 2*
d*f*x)^3*a*c*d^2*e^2*f^3 + 15*(e*f*x^2 + 2*d*f*x)^4*b^2*e^3*f^2 + 30*(e*f*
x^2 + 2*d*f*x)^4*a*c*e^3*f^2 + 120*(e*f*x^2 + 2*d*f*x)^2*a*b*d^2*e*f^4 + 4
0*(e*f*x^2 + 2*d*f*x)^3*a*b*e^2*f^3 + 30*(e*f*x^2 + 2*d*f*x)^2*a^2*e*f^4)/
f^3
```

Mupad [B] (verification not implemented)

Time = 10.64 (sec) , antiderivative size = 419, normalized size of antiderivative = 4.03

$$\begin{aligned}
&\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2\,dx\\
&=\frac{e^3\,f^3\,x^4\,(a^2+20\,a\,b\,d^2+70\,a\,c\,d^4+35\,b^2\,d^4+168\,b\,c\,d^6+165\,c^2\,d^8)}{4}+\frac{c^2\,e^{11}\,f^3\,x^{12}}{12}\\
&+d^3\,f^3\,x\,\left(c\,d^4+b\,d^2+a\right)^2+\frac{e^7\,f^3\,x^8\,(b^2+72\,b\,c\,d^2+330\,c^2\,d^4+2\,a\,c)}{8}\\
&+\frac{e^5\,f^3\,x^6\,(21\,b^2\,d^2+252\,b\,c\,d^4+2\,a\,b+462\,c^2\,d^6+42\,a\,c\,d^2)}{6}\\
&+\frac{d^2\,e\,f^3\,x^2\,(3\,a^2+10\,a\,b\,d^2+14\,a\,c\,d^4+7\,b^2\,d^4+18\,b\,c\,d^6+11\,c^2\,d^8)}{2}\\
&+\frac{d\,e^2\,f^3\,x^3\,(3\,a^2+20\,a\,b\,d^2+42\,a\,c\,d^4+21\,b^2\,d^4+72\,b\,c\,d^6+55\,c^2\,d^8)}{3}\\
&+d\,e^6\,f^3\,x^7\,\left(b^2+24\,b\,c\,d^2+66\,c^2\,d^4+2\,a\,c\right)\\
&+\frac{d\,e^4\,f^3\,x^5\,(35\,b^2\,d^2+252\,b\,c\,d^4+10\,a\,b+330\,c^2\,d^6+70\,a\,c\,d^2)}{5}\\
&+\frac{c\,e^9\,f^3\,x^{10}\,(55\,c\,d^2+2\,b)}{10}+c^2\,d\,e^{10}\,f^3\,x^{11}+\frac{c\,d\,e^8\,f^3\,x^9\,(55\,c\,d^2+6\,b)}{3}
\end{aligned}$$

input
```
int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
(e^3*f^3*x^4*(a^2 + 35*b^2*d^4 + 165*c^2*d^8 + 20*a*b*d^2 + 70*a*c*d^4 + 1
68*b*c*d^6))/4 + (c^2*e^11*f^3*x^12)/12 + d^3*f^3*x*(a + b*d^2 + c*d^4)^2
+ (e^7*f^3*x^8*(2*a*c + b^2 + 330*c^2*d^4 + 72*b*c*d^2))/8 + (e^5*f^3*x^6*
(2*a*b + 21*b^2*d^2 + 462*c^2*d^6 + 42*a*c*d^2 + 252*b*c*d^4))/6 + (d^2*e*
f^3*x^2*(3*a^2 + 7*b^2*d^4 + 11*c^2*d^8 + 10*a*b*d^2 + 14*a*c*d^4 + 18*b*c
*d^6))/2 + (d*e^2*f^3*x^3*(3*a^2 + 21*b^2*d^4 + 55*c^2*d^8 + 20*a*b*d^2 +
42*a*c*d^4 + 72*b*c*d^6))/3 + d*e^6*f^3*x^7*(2*a*c + b^2 + 66*c^2*d^4 + 24
*b*c*d^2) + (d*e^4*f^3*x^5*(10*a*b + 35*b^2*d^2 + 330*c^2*d^6 + 70*a*c*d^2
 + 252*b*c*d^4))/5 + (c*e^9*f^3*x^10*(2*b + 55*c*d^2))/10 + c^2*d*e^10*f^3
*x^11 + (c*d*e^8*f^3*x^9*(6*b + 55*c*d^2))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 565, normalized size of antiderivative = 5.43

$$\int (df + efx)^3 \left(a + b(d+ex)^2 + c(d+ex)^4\right)^2 \, dx$$
$$= \frac{f^3x(10c^2e^{11}x^{11} + 120c^2d\,e^{10}x^{10} + 660c^2d^2e^9x^9 + 2200c^2d^3e^8x^8 + 24bc\,e^9x^9 + 4950c^2d^4e^7x^7 + 240bcd\,e^8x}{}$$

input
```
int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

output
```
(f**3*x*(120*a**2*d**3 + 180*a**2*d**2*e*x + 120*a**2*d*e**2*x**2 + 30*a**
2*e**3*x**3 + 240*a*b*d**5 + 600*a*b*d**4*e*x + 800*a*b*d**3*e**2*x**2 + 6
00*a*b*d**2*e**3*x**3 + 240*a*b*d*e**4*x**4 + 40*a*b*e**5*x**5 + 240*a*c*d
**7 + 840*a*c*d**6*e*x + 1680*a*c*d**5*e**2*x**2 + 2100*a*c*d**4*e**3*x**3
 + 1680*a*c*d**3*e**4*x**4 + 840*a*c*d**2*e**5*x**5 + 240*a*c*d*e**6*x**6
+ 30*a*c*e**7*x**7 + 120*b**2*d**7 + 420*b**2*d**6*e*x + 840*b**2*d**5*e**
2*x**2 + 1050*b**2*d**4*e**3*x**3 + 840*b**2*d**3*e**4*x**4 + 420*b**2*d**
2*e**5*x**5 + 120*b**2*d*e**6*x**6 + 15*b**2*e**7*x**7 + 240*b*c*d**9 + 10
80*b*c*d**8*e*x + 2880*b*c*d**7*e**2*x**2 + 5040*b*c*d**6*e**3*x**3 + 6048
*b*c*d**5*e**4*x**4 + 5040*b*c*d**4*e**5*x**5 + 2880*b*c*d**3*e**6*x**6 +
1080*b*c*d**2*e**7*x**7 + 240*b*c*d*e**8*x**8 + 24*b*c*e**9*x**9 + 120*c**
2*d**11 + 660*c**2*d**10*e*x + 2200*c**2*d**9*e**2*x**2 + 4950*c**2*d**8*e
**3*x**3 + 7920*c**2*d**7*e**4*x**4 + 9240*c**2*d**6*e**5*x**5 + 7920*c**2
*d**5*e**6*x**6 + 4950*c**2*d**4*e**7*x**7 + 2200*c**2*d**3*e**8*x**8 + 66
0*c**2*d**2*e**9*x**9 + 120*c**2*d*e**10*x**10 + 10*c**2*e**11*x**11))/120
```

3.266 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx$

Optimal result

Integrand size = 33, antiderivative size = 159

$$\begin{aligned}\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx &= \frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{3a(b^2+ac)\,f^3(d+ex)^8}{8e} \\ &+\frac{b(b^2+6ac)\,f^3(d+ex)^{10}}{10e}+\frac{c(b^2+ac)\,f^3(d+ex)^{12}}{4e} \\ &+\frac{3bc^2f^3(d+ex)^{14}}{14e}+\frac{c^3f^3(d+ex)^{16}}{16e}\end{aligned}$$

output
```
1/4*a^3*f^3*(e*x+d)^4/e+1/2*a^2*b*f^3*(e*x+d)^6/e+3/8*a*(a*c+b^2)*f^3*(e*x
+d)^8/e+1/10*b*(6*a*c+b^2)*f^3*(e*x+d)^10/e+1/4*c*(a*c+b^2)*f^3*(e*x+d)^12
/e+3/14*b*c^2*f^3*(e*x+d)^14/e+1/16*c^3*f^3*(e*x+d)^16/e
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 801 vs. 2(159) = 318.

Time = 0.33 (sec) , antiderivative size = 801, normalized size of antiderivative = 5.04

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx$$
$$=f^3\Bigg(d^3\left(a+bd^2+cd^4\right)^3x+\frac{3}{2}d^2\left(a+bd^2+cd^4\right)^2\left(a+3bd^2+5cd^4\right)ex^2+d\left(a^3+10a^2bd^2\right.$$
$$\left.+21ab^2d^4+21a^2cd^4+12b^3d^6+72abcd^6+55b^2cd^8+55ac^2d^8+78bc^2d^{10}+35c^3d^{12}\right)e^2x^3$$
$$+\frac{1}{4}\left(a^3+30a^2bd^2+105ab^2d^4+105a^2cd^4+84b^3d^6+504abcd^6+495b^2cd^8+495ac^2d^8\right.$$
$$\left.+858bc^2d^{10}+455c^3d^{12}\right)e^3x^4+\frac{3}{5}d\left(5a^2b+35ab^2d^2+35a^2cd^2+42b^3d^4+252abcd^4\right.$$
$$\left.+330b^2cd^6+330ac^2d^6+715bc^2d^8+455c^3d^{10}\right)e^4x^5+\frac{1}{2}\left(a^2b+21ab^2d^2+21a^2cd^2\right.$$
$$\left.+42b^3d^4+252abcd^4+462b^2cd^6+462ac^2d^6+1287bc^2d^8+1001c^3d^{10}\right)e^5x^6+\frac{1}{7}d\left(21ab^2\right.$$
$$\left.+21a^2c+84b^3d^2+504abcd^2+1386b^2cd^4+1386ac^2d^4+5148bc^2d^6+5005c^3d^8\right)e^6x^7$$
$$+\frac{3}{8}\left(ab^2+a^2c+12b^3d^2+72abcd^2+330b^2cd^4+330ac^2d^4+1716bc^2d^6+2145c^3d^8\right)e^7x^8$$
$$+d\left(b^3+6abc+55b^2cd^2+55ac^2d^2+429bc^2d^4+715c^3d^6\right)e^8x^9$$
$$+\frac{1}{10}\left(b^3+6abc+165b^2cd^2+165ac^2d^2+2145bc^2d^4+5005c^3d^6\right)e^9x^{10}$$
$$+3cd\left(b^2+ac+26bcd^2+91c^2d^4\right)e^{10}x^{11}+\frac{1}{4}c\left(b^2+ac+78bcd^2+455c^2d^4\right)e^{11}x^{12}$$
$$+c^2d\left(3b+35cd^2\right)e^{12}x^{13}+\frac{3}{14}c^2\left(b+35cd^2\right)e^{13}x^{14}+c^3de^{14}x^{15}+\frac{1}{16}c^3e^{15}x^{16}\Bigg)$$

input

```
Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
f^3*(d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d
^2 + 5*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d
^4 + 12*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^
10 + 35*c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2
*c*d^4 + 84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*
b*c^2*d^10 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*
a^2*c*d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 7
15*b*c^2*d^8 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*
c*d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*
b*c^2*d^8 + 1001*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d
^2 + 504*a*b*c*d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 50
05*c^3*d^8)*e^6*x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 3
30*b^2*c*d^4 + 330*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 +
 d*(b^3 + 6*a*b*c + 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*
d^6)*e^8*x^9 + ((b^3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^
2*d^4 + 5005*c^3*d^6)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^
2*d^4)*e^10*x^11 + (c*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4
+ c^2*d*(3*b + 35*c*d^2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 +
 c^3*d*e^14*x^15 + (c^3*e^15*x^16)/16)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1462, 1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx$$

$$\Big\downarrow \text{ 1462}$$

$$\frac{f^3 \int (d+ex)^3 \left(c(d+ex)^4+b(d+ex)^2+a\right)^3 d(d+ex)}{e}$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{f^3 \int (d+ex)^2 \left(c(d+ex)^4+b(d+ex)^2+a\right)^3 d(d+ex)^2}{2e}$$

$\big\downarrow$ 1140

$$\frac{f^3\int\left(c^3(d+ex)^{14}+3bc^2(d+ex)^{12}+3c\left(b^2+ac\right)(d+ex)^{10}+b\left(b^2+6ac\right)(d+ex)^8+3a\left(b^2+ac\right)(d+ex)^6+\right.}{2e}$$

$\big\downarrow$ 2009

$$\frac{f^3\left(\frac{1}{2}a^3(d+ex)^4+a^2b(d+ex)^6+\frac{1}{2}c\left(ac+b^2\right)(d+ex)^{12}+\frac{1}{5}b\left(6ac+b^2\right)(d+ex)^{10}+\frac{3}{4}a\left(ac+b^2\right)(d+ex)^8+\right.}{2e}$$

input
```
Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f^3*((a^3*(d + e*x)^4)/2 + a^2*b*(d + e*x)^6 + (3*a*(b^2 + a*c)*(d + e*x)
^8)/4 + (b*(b^2 + 6*a*c)*(d + e*x)^10)/5 + (c*(b^2 + a*c)*(d + e*x)^12)/2
+ (3*b*c^2*(d + e*x)^14)/7 + (c^3*(d + e*x)^16)/8))/(2*e)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1317 vs. $2(145) = 290$.

Time = 0.15 (sec) , antiderivative size = 1318, normalized size of antiderivative = 8.29

method	result	size
gosper	Expression too large to display	1318
orering	Expression too large to display	1420
norman	Expression too large to display	1430
risch	Expression too large to display	1636
parallelrisch	Expression too large to display	1636
default	Expression too large to display	7697

input
```
int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/560*f^3*x*(35*c^3*e^15*x^15+560*c^3*d*e^14*x^14+4200*c^3*d^2*e^13*x^13+1
9600*c^3*d^3*e^12*x^12+120*b*c^2*e^13*x^13+63700*c^3*d^4*e^11*x^11+1680*b*
c^2*d*e^12*x^12+152880*c^3*d^5*e^10*x^10+10920*b*c^2*d^2*e^11*x^11+280280*
c^3*d^6*e^9*x^9+43680*b*c^2*d^3*e^10*x^10+400400*c^3*d^7*e^8*x^8+140*a*c^2
*e^11*x^11+140*b^2*c*e^11*x^11+120120*b*c^2*d^4*e^9*x^9+450450*c^3*d^8*e^7
*x^7+1680*a*c^2*d*e^10*x^10+1680*b^2*c*d*e^10*x^10+240240*b*c^2*d^5*e^8*x^
8+400400*c^3*d^9*e^6*x^6+9240*a*c^2*d^2*e^9*x^9+9240*b^2*c*d^2*e^9*x^9+360
360*b*c^2*d^6*e^7*x^7+280280*c^3*d^10*e^5*x^5+30800*a*c^2*d^3*e^8*x^8+3080
0*b^2*c*d^3*e^8*x^8+411840*b*c^2*d^7*e^6*x^6+152880*c^3*d^11*e^4*x^4+336*a
*b*c*e^9*x^9+69300*a*c^2*d^4*e^7*x^7+56*b^3*e^9*x^9+69300*b^2*c*d^4*e^7*x^
7+360360*b*c^2*d^8*e^5*x^5+63700*c^3*d^12*e^3*x^3+3360*a*b*c*d*e^8*x^8+110
880*a*c^2*d^5*e^6*x^6+560*b^3*d*e^8*x^8+110880*b^2*c*d^5*e^6*x^6+240240*b*
c^2*d^9*e^4*x^4+19600*c^3*d^13*e^2*x^2+15120*a*b*c*d^2*e^7*x^7+129360*a*c^
2*d^6*e^5*x^5+2520*b^3*d^2*e^7*x^7+129360*b^2*c*d^6*e^5*x^5+120120*b*c^2*d
^10*e^3*x^3+4200*c^3*d^14*e*x+40320*a*b*c*d^3*e^6*x^6+110880*a*c^2*d^7*e^4
*x^4+6720*b^3*d^3*e^6*x^6+110880*b^2*c*d^7*e^4*x^4+43680*b*c^2*d^11*e^2*x^
2+560*c^3*d^15+210*a^2*c*e^7*x^7+210*a*b^2*e^7*x^7+70560*a*b*c*d^4*e^5*x^5
+69300*a*c^2*d^8*e^3*x^3+11760*b^3*d^4*e^5*x^5+69300*b^2*c*d^8*e^3*x^3+109
20*b*c^2*d^12*e*x+1680*a^2*c*d*e^6*x^6+1680*a*b^2*d*e^6*x^6+84672*a*b*c*d^
5*e^4*x^4+30800*a*c^2*d^9*e^2*x^2+14112*b^3*d^5*e^4*x^4+30800*b^2*c*d^9...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(145) = 290$.

Time = 0.07 (sec) , antiderivative size = 920, normalized size of antiderivative = 5.79

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas
")
```

output
```
1/16*c^3*e^15*f^3*x^16 + c^3*d*e^14*f^3*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e
^13*f^3*x^14 + (35*c^3*d^3 + 3*b*c^2*d)*e^12*f^3*x^13 + 1/4*(455*c^3*d^4 +
 78*b*c^2*d^2 + b^2*c + a*c^2)*e^11*f^3*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^
3 + (b^2*c + a*c^2)*d)*e^10*f^3*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4
 + b^3 + 6*a*b*c + 165*(b^2*c + a*c^2)*d^2)*e^9*f^3*x^10 + (715*c^3*d^7 +
429*b*c^2*d^5 + 55*(b^2*c + a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*f^3*x^9 +
3/8*(2145*c^3*d^8 + 1716*b*c^2*d^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2
*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7*f^3*x^8 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*
d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*
c)*d)*e^6*f^3*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c^2*d^8 + 462*(b^2*c + a*c
^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b^2 + a^2*c)*d^2)*e^5*f^3
*x^6 + 3/5*(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c + a*c^2)*d^7 + 42*(b
^3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*e^4*f^3*x^5 + 1/4*
(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84*(b^3 + 6*a*b
*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*f^3*x^4 + (35*
c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b*c)*d^7
 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*f^3*x^3 + 3/2*(5*c^3
*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 +
5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d^6 + a^3*d^2)*e*f^3*x^2 + (c^3*d^15 + 3*b
*c^2*d^13 + 3*(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1654 vs. $2(141) = 282$.

Time = 0.14 (sec) , antiderivative size = 1654, normalized size of antiderivative = 10.40

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx=\text{Too large to display}$$

input
```
integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
c**3*d*e**14*f**3*x**15 + c**3*e**15*f**3*x**16/16 + x**14*(3*b*c**2*e**13
*f**3/14 + 15*c**3*d**2*e**13*f**3/2) + x**13*(3*b*c**2*d*e**12*f**3 + 35*
c**3*d**3*e**12*f**3) + x**12*(a*c**2*e**11*f**3/4 + b**2*c*e**11*f**3/4 +
 39*b*c**2*d**2*e**11*f**3/2 + 455*c**3*d**4*e**11*f**3/4) + x**11*(3*a*c*
*2*d*e**10*f**3 + 3*b**2*c*d*e**10*f**3 + 78*b*c**2*d**3*e**10*f**3 + 273*
c**3*d**5*e**10*f**3) + x**10*(3*a*b*c*e**9*f**3/5 + 33*a*c**2*d**2*e**9*f
**3/2 + b**3*e**9*f**3/10 + 33*b**2*c*d**2*e**9*f**3/2 + 429*b*c**2*d**4*e
**9*f**3/2 + 1001*c**3*d**6*e**9*f**3/2) + x**9*(6*a*b*c*d*e**8*f**3 + 55*
a*c**2*d**3*e**8*f**3 + b**3*d*e**8*f**3 + 55*b**2*c*d**3*e**8*f**3 + 429*
b*c**2*d**5*e**8*f**3 + 715*c**3*d**7*e**8*f**3) + x**8*(3*a**2*c*e**7*f**
3/8 + 3*a*b**2*e**7*f**3/8 + 27*a*b*c*d**2*e**7*f**3 + 495*a*c**2*d**4*e**
7*f**3/4 + 9*b**3*d**2*e**7*f**3/2 + 495*b**2*c*d**4*e**7*f**3/4 + 1287*b*
c**2*d**6*e**7*f**3/2 + 6435*c**3*d**8*e**7*f**3/8) + x**7*(3*a**2*c*d*e**
6*f**3 + 3*a*b**2*d*e**6*f**3 + 72*a*b*c*d**3*e**6*f**3 + 198*a*c**2*d**5*
e**6*f**3 + 12*b**3*d**3*e**6*f**3 + 198*b**2*c*d**5*e**6*f**3 + 5148*b*c*
*2*d**7*e**6*f**3/7 + 715*c**3*d**9*e**6*f**3) + x**6*(a**2*b*e**5*f**3/2
+ 21*a**2*c*d**2*e**5*f**3/2 + 21*a*b**2*d**2*e**5*f**3/2 + 126*a*b*c*d**4
*e**5*f**3 + 231*a*c**2*d**6*e**5*f**3 + 21*b**3*d**4*e**5*f**3 + 231*b**2
*c*d**6*e**5*f**3 + 1287*b*c**2*d**8*e**5*f**3/2 + 1001*c**3*d**10*e**5*f*
*3/2) + x**5*(3*a**2*b*d*e**4*f**3 + 21*a**2*c*d**3*e**4*f**3 + 21*a*b*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(145) = 290$.

Time = 0.04 (sec) , antiderivative size = 920, normalized size of antiderivative = 5.79

$$\int (df + efx)^3 \left(a + b(d + ex)^2 + c(d + ex)^4\right)^3 dx = \text{Too large to display}$$

input

```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima
")
```

output

```
1/16*c^3*e^15*f^3*x^16 + c^3*d*e^14*f^3*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e
^13*f^3*x^14 + (35*c^3*d^3 + 3*b*c^2*d)*e^12*f^3*x^13 + 1/4*(455*c^3*d^4 +
 78*b*c^2*d^2 + b^2*c + a*c^2)*e^11*f^3*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^
3 + (b^2*c + a*c^2)*d)*e^10*f^3*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4
 + b^3 + 6*a*b*c + 165*(b^2*c + a*c^2)*d^2)*e^9*f^3*x^10 + (715*c^3*d^7 +
429*b*c^2*d^5 + 55*(b^2*c + a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*f^3*x^9 +
3/8*(2145*c^3*d^8 + 1716*b*c^2*d^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2
*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7*f^3*x^8 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*
d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*
c)*d)*e^6*f^3*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c^2*d^8 + 462*(b^2*c + a*c
^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b^2 + a^2*c)*d^2)*e^5*f^3
*x^6 + 3/5*(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c + a*c^2)*d^7 + 42*(b
^3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*e^4*f^3*x^5 + 1/4*
(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84*(b^3 + 6*a*b
*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*f^3*x^4 + (35*
c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b*c)*d^7
 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*f^3*x^3 + 3/2*(5*c^3
*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 +
5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d^6 + a^3*d^2)*e*f^3*x^2 + (c^3*d^15 + 3*b
*c^2*d^13 + 3*(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1330 vs. $2(145) = 290$.

Time = 0.16 (sec) , antiderivative size = 1330, normalized size of antiderivative = 8.36

$$\int (df + efx)^3 \left(a + b(d + ex)^2 + c(d + ex)^4\right)^3 dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
1/2*(e*f*x^2 + 2*d*f*x)*c^3*d^14*f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*b*c^2*d^12*
f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*b^2*c*d^10*f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*a*c
^2*d^10*f^2 + 1/2*(e*f*x^2 + 2*d*f*x)*b^3*d^8*f^2 + 3*(e*f*x^2 + 2*d*f*x)*
a*b*c*d^8*f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*a*b^2*d^6*f^2 + 3/2*(e*f*x^2 + 2*d
*f*x)*a^2*c*d^6*f^2 + 3/2*(e*f*x^2 + 2*d*f*x)*a^2*b*d^4*f^2 + 1/2*(e*f*x^2
 + 2*d*f*x)*a^3*d^2*f^2 + 1/560*(980*(e*f*x^2 + 2*d*f*x)^2*c^3*d^12*e*f^6
+ 1960*(e*f*x^2 + 2*d*f*x)^3*c^3*d^10*e^2*f^5 + 2450*(e*f*x^2 + 2*d*f*x)^4
*c^3*d^8*e^3*f^4 + 2520*(e*f*x^2 + 2*d*f*x)^2*b*c^2*d^10*e*f^6 + 1960*(e*f
*x^2 + 2*d*f*x)^5*c^3*d^6*e^4*f^3 + 4200*(e*f*x^2 + 2*d*f*x)^3*b*c^2*d^8*e
^2*f^5 + 980*(e*f*x^2 + 2*d*f*x)^6*c^3*d^4*e^5*f^2 + 4200*(e*f*x^2 + 2*d*f
*x)^4*b*c^2*d^6*e^3*f^4 + 2100*(e*f*x^2 + 2*d*f*x)^2*b^2*c*d^8*e*f^6 + 210
0*(e*f*x^2 + 2*d*f*x)^2*a*c^2*d^8*e*f^6 + 280*(e*f*x^2 + 2*d*f*x)^7*c^3*d^
2*e^6*f + 2520*(e*f*x^2 + 2*d*f*x)^5*b*c^2*d^4*e^4*f^3 + 2800*(e*f*x^2 + 2
*d*f*x)^3*b^2*c*d^6*e^2*f^5 + 2800*(e*f*x^2 + 2*d*f*x)^3*a*c^2*d^6*e^2*f^5
 + 35*(e*f*x^2 + 2*d*f*x)^8*c^3*e^7 + 840*(e*f*x^2 + 2*d*f*x)^6*b*c^2*d^2*
e^5*f^2 + 2100*(e*f*x^2 + 2*d*f*x)^4*b^2*c*d^4*e^3*f^4 + 2100*(e*f*x^2 + 2
*d*f*x)^4*a*c^2*d^4*e^3*f^4 + 560*(e*f*x^2 + 2*d*f*x)^2*b^3*d^6*e*f^6 + 33
60*(e*f*x^2 + 2*d*f*x)^2*a*b*c*d^6*e*f^6 + 120*(e*f*x^2 + 2*d*f*x)^7*b*c^2
*e^6*f + 840*(e*f*x^2 + 2*d*f*x)^5*b^2*c*d^2*e^4*f^3 + 840*(e*f*x^2 + 2*d*
f*x)^5*a*c^2*d^2*e^4*f^3 + 560*(e*f*x^2 + 2*d*f*x)^3*b^3*d^4*e^2*f^5 + ...
```

Mupad [B] (verification not implemented)

Time = 10.93 (sec) , antiderivative size = 825, normalized size of antiderivative = 5.19

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$$
$$= \frac{3 e^7 f^3 x^8 (a^2 c + a b^2 + 72 a b c d^2 + 330 a c^2 d^4 + 12 b^3 d^2 + 330 b^2 c d^4 + 1716 b c^2 d^6 + 2145 c^3 d^8)}{8}$$
$$+ \frac{e^5 f^3 x^6 (a^2 b + 21 a^2 c d^2 + 21 a b^2 d^2 + 252 a b c d^4 + 462 a c^2 d^6 + 42 b^3 d^4 + 462 b^2 c d^6 + 1287 b c^2 d^8 +}{2}$$
$$+ \frac{e^9 f^3 x^{10} (b^3 + 165 b^2 c d^2 + 2145 b c^2 d^4 + 6 a b c + 5005 c^3 d^6 + 165 a c^2 d^2)}{10}$$
$$+ \frac{c^3 e^{15} f^3 x^{16}}{16} + d^3 f^3 x \left(c d^4 + b d^2 + a\right)^3$$
$$+ \frac{e^3 f^3 x^4 (a^3 + 30 a^2 b d^2 + 105 a^2 c d^4 + 105 a b^2 d^4 + 504 a b c d^6 + 495 a c^2 d^8 + 84 b^3 d^6 + 495 b^2 c d^8 +}{4}$$
$$+ \frac{c e^{11} f^3 x^{12} (b^2 + 78 b c d^2 + 455 c^2 d^4 + a c)}{4}$$
$$+ \frac{d e^6 f^3 x^7 (21 a^2 c + 21 a b^2 + 504 a b c d^2 + 1386 a c^2 d^4 + 84 b^3 d^2 + 1386 b^2 c d^4 + 5148 b c^2 d^6 + 5005 c}{7}$$
$$+ \frac{3 d e^4 f^3 x^5 (5 a^2 b + 35 a^2 c d^2 + 35 a b^2 d^2 + 252 a b c d^4 + 330 a c^2 d^6 + 42 b^3 d^4 + 330 b^2 c d^6 + 715 b c^2}{5}$$
$$+ d e^8 f^3 x^9 \left(b^3 + 55 b^2 c d^2 + 429 b c^2 d^4 + 6 a b c + 715 c^3 d^6 + 55 a c^2 d^2\right)$$
$$+ \frac{3 c^2 e^{13} f^3 x^{14} (35 c d^2 + b)}{14} + c^3 d e^{14} f^3 x^{15} + d e^2 f^3 x^3 \left(a^3 + 10 a^2 b d^2 + 21 a^2 c d^4 \right.$$
$$\left. + 21 a b^2 d^4 + 72 a b c d^6 + 55 a c^2 d^8 + 12 b^3 d^6 + 55 b^2 c d^8 + 78 b c^2 d^{10} + 35 c^3 d^{12}\right)$$
$$+ \frac{3 d^2 e f^3 x^2 (c d^4 + b d^2 + a)^2 (5 c d^4 + 3 b d^2 + a)}{2}$$
$$+ c^2 d e^{12} f^3 x^{13} \left(35 c d^2 + 3 b\right) + 3 c d e^{10} f^3 x^{11} \left(b^2 + 26 b c d^2 + 91 c^2 d^4 + a c\right)$$

input
```
int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
(3*e^7*f^3*x^8*(a*b^2 + a^2*c + 12*b^3*d^2 + 2145*c^3*d^8 + 330*a*c^2*d^4
+ 330*b^2*c*d^4 + 1716*b*c^2*d^6 + 72*a*b*c*d^2))/8 + (e^5*f^3*x^6*(a^2*b
+ 42*b^3*d^4 + 1001*c^3*d^10 + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 462*a*c^2*d^6
 + 462*b^2*c*d^6 + 1287*b*c^2*d^8 + 252*a*b*c*d^4))/2 + (e^9*f^3*x^10*(b^3
 + 5005*c^3*d^6 + 165*a*c^2*d^2 + 165*b^2*c*d^2 + 2145*b*c^2*d^4 + 6*a*b*c
))/10 + (c^3*e^15*f^3*x^16)/16 + d^3*f^3*x*(a + b*d^2 + c*d^4)^3 + (e^3*f^
3*x^4*(a^3 + 84*b^3*d^6 + 455*c^3*d^12 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 10
5*a^2*c*d^4 + 495*a*c^2*d^8 + 495*b^2*c*d^8 + 858*b*c^2*d^10 + 504*a*b*c*d
^6))/4 + (c*e^11*f^3*x^12*(a*c + b^2 + 455*c^2*d^4 + 78*b*c*d^2))/4 + (d*e
^6*f^3*x^7*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 5005*c^3*d^8 + 1386*a*c^2*d
^4 + 1386*b^2*c*d^4 + 5148*b*c^2*d^6 + 504*a*b*c*d^2))/7 + (3*d*e^4*f^3*x^
5*(5*a^2*b + 42*b^3*d^4 + 455*c^3*d^10 + 35*a*b^2*d^2 + 35*a^2*c*d^2 + 330
*a*c^2*d^6 + 330*b^2*c*d^6 + 715*b*c^2*d^8 + 252*a*b*c*d^4))/5 + d*e^8*f^3
*x^9*(b^3 + 715*c^3*d^6 + 55*a*c^2*d^2 + 55*b^2*c*d^2 + 429*b*c^2*d^4 + 6*
a*b*c) + (3*c^2*e^13*f^3*x^14*(b + 35*c*d^2))/14 + c^3*d*e^14*f^3*x^15 + d
*e^2*f^3*x^3*(a^3 + 12*b^3*d^6 + 35*c^3*d^12 + 10*a^2*b*d^2 + 21*a*b^2*d^4
 + 21*a^2*c*d^4 + 55*a*c^2*d^8 + 55*b^2*c*d^8 + 78*b*c^2*d^10 + 72*a*b*c*d
^6) + (3*d^2*e*f^3*x^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4))/2 +
c^2*d*e^12*f^3*x^13*(3*b + 35*c*d^2) + 3*c*d*e^10*f^3*x^11*(a*c + b^2 + 91
*c^2*d^4 + 26*b*c*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1317, normalized size of antiderivative = 8.28

$$\int (df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3\,dx = \text{Too large to display}$$

input
```
int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

output
```
(f**3*x*(560*a**3*d**3 + 840*a**3*d**2*e*x + 560*a**3*d*e**2*x**2 + 140*a*
*3*e**3*x**3 + 1680*a**2*b*d**5 + 4200*a**2*b*d**4*e*x + 5600*a**2*b*d**3*
e**2*x**2 + 4200*a**2*b*d**2*e**3*x**3 + 1680*a**2*b*d*e**4*x**4 + 280*a**
2*b*e**5*x**5 + 1680*a**2*c*d**7 + 5880*a**2*c*d**6*e*x + 11760*a**2*c*d**
5*e**2*x**2 + 14700*a**2*c*d**4*e**3*x**3 + 11760*a**2*c*d**3*e**4*x**4 +
5880*a**2*c*d**2*e**5*x**5 + 1680*a**2*c*d*e**6*x**6 + 210*a**2*c*e**7*x**
7 + 1680*a*b**2*d**7 + 5880*a*b**2*d**6*e*x + 11760*a*b**2*d**5*e**2*x**2
+ 14700*a*b**2*d**4*e**3*x**3 + 11760*a*b**2*d**3*e**4*x**4 + 5880*a*b**2*
d**2*e**5*x**5 + 1680*a*b**2*d*e**6*x**6 + 210*a*b**2*e**7*x**7 + 3360*a*b
*c*d**9 + 15120*a*b*c*d**8*e*x + 40320*a*b*c*d**7*e**2*x**2 + 70560*a*b*c*
d**6*e**3*x**3 + 84672*a*b*c*d**5*e**4*x**4 + 70560*a*b*c*d**4*e**5*x**5 +
 40320*a*b*c*d**3*e**6*x**6 + 15120*a*b*c*d**2*e**7*x**7 + 3360*a*b*c*d*e*
*8*x**8 + 336*a*b*c*e**9*x**9 + 1680*a*c**2*d**11 + 9240*a*c**2*d**10*e*x
+ 30800*a*c**2*d**9*e**2*x**2 + 69300*a*c**2*d**8*e**3*x**3 + 110880*a*c**
2*d**7*e**4*x**4 + 129360*a*c**2*d**6*e**5*x**5 + 110880*a*c**2*d**5*e**6*
x**6 + 69300*a*c**2*d**4*e**7*x**7 + 30800*a*c**2*d**3*e**8*x**8 + 9240*a*
c**2*d**2*e**9*x**9 + 1680*a*c**2*d*e**10*x**10 + 140*a*c**2*e**11*x**11 +
 560*b**3*d**9 + 2520*b**3*d**8*e*x + 6720*b**3*d**7*e**2*x**2 + 11760*b**
3*d**6*e**3*x**3 + 14112*b**3*d**5*e**4*x**4 + 11760*b**3*d**4*e**5*x**5 +
 6720*b**3*d**3*e**6*x**6 + 2520*b**3*d**2*e**7*x**7 + 560*b**3*d*e**8*...
```

3.267 $\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 202

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^4x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

output
```
f^4*x/c-1/2*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*f^4*arctan(2^(1/2)*c^(1/2)
*(e*x+d)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b-(-4*a*c+b^2)^(1/
2))^(1/2)/e-1/2*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*f^4*arctan(2^(1/2)*c^(
1/2)*(e*x+d)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.10

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^4\left(2\sqrt{c}(d+ex)-\frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2c^{3/2}e}$$

input
```
Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f^4*(2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(2*c^(3/2)*e)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1462, 1442, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f^4\int \frac{(d+ex)^4}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{e}$$

$$\Big\downarrow \text{1442}$$

$$\frac{f^4\left(\frac{d+ex}{c}-\frac{\int \frac{b(d+ex)^2+a}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{c}\right)}{e}$$

$$\Big\downarrow \; 1480$$

$$\frac{f^4\left(\frac{d+ex}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{c}\right)}{e}$$

$$\Big\downarrow \; 218$$

$$\frac{f^4\left(\frac{d+ex}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}\right)}{e}$$

input

```
Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output

```
(f^4*((d + e*x)/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]]))/c))/e
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.80

method	result
default	$f^4 \left(\frac{x}{c} + \frac{\sum_{\_R=\operatorname{RootOf}\left(c e^4 \_Z^4+4 c d e^3 \_Z^3+\left(6 c d^2 e^2+b e^2\right) \_Z^2+\left(4 c d^3 e+2 b d e\right) \_Z+c d^4+b d^2+a\right)} \frac{\left(-\_R^2 b e^2-2 \_R b d e-b d^2- \cdots\right.}{2 c e^3 \_R^3+6 c d e^2 \_R^2+6 c d^2 e \_ \cdots}}{2 c e} \right.$
risch	$\frac{f^4 x}{c} + \frac{f^4 \left(\sum_{\_R=\operatorname{RootOf}\left(c e^4 \_Z^4+4 c d e^3 \_Z^3+\left(6 c d^2 e^2+b e^2\right) \_Z^2+\left(4 c d^3 e+2 b d e\right) \_Z+c d^4+b d^2+a\right)} \frac{\left(-\_R^2 b e^2-2 \_R b d e-b d^2 \cdots\right.}{2 c e^3 \_R^3+6 c d e^2 \_R^2+6 c d^2 e \_ \cdots} \right.}{2 c e}$

input
```
int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
f^4*(x/c+1/2/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*_R^2
*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*
c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1346 vs. $2(166) = 332$.

Time = 0.09 (sec) , antiderivative size = 1346, normalized size of antiderivative = 6.66

$$\int \frac{(df + efx)^4}{a + b(d + ex)^2 + c(d + ex)^4} \, dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
1/2*(2*f^4*x - sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((
b^2*c^3 - 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*
c)*d*f^12 + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*
e^3)*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((
b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2
))) + sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 -
 4*a*c^4)*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12
 - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt
(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 -
 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) - sqr
t(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^1
6/((b^2*c^6 - 4*a*c^7)*e^4))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)
*e^2))*log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 + sqrt(1
/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)*f^16/((b^2*c^6 - 4*a*c^7)*e^4))*(b^3*c^3 - 4*a*b*c^4)*e^3)*sqrt(-((b^3 -
 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c...
```

Sympy [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.08

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$=\operatorname{RootSum}\left(t^4\cdot\left(256a^2c^5e^4-128ab^2c^4e^4+16b^4c^3e^4\right)+t^2\cdot\left(48a^2bc^2e^2f^8-28ab^3ce^2f^8+4b^5e^2f^8\right)+a^3f\right.$$
$$+\frac{f^4x}{c}$$

input
```
integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e*
*4) + _t**2*(48*a**2*b*c**2*e**2*f**8 - 28*a*b**3*c*e**2*f**8 + 4*b**5*e**
2*f**8) + a**3*f**16, Lambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t
**3*b**3*c**3*e**3 - 4*_t*a**2*c**2*e*f**8 + 8*_t*a*b**2*c*e*f**8 - 2*_t*b
**4*e*f**8 + a**2*c*d*f**12 - a*b**2*d*f**12)/(a**2*c*e*f**12 - a*b**2*e*f
**12)))) + f**4*x/c
```

Maxima [F]

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\int \frac{(efx+df)^4}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
f^4*x/c - f^4*integrate((b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)/(c*e^4*x^4 + 4
*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e
*x + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1366 vs. $2(166) = 332$.

Time = 0.13 (sec) , antiderivative size = 1366, normalized size of antiderivative = 6.76

$$\int \frac{(df + efx)^4}{a + b(d + ex)^2 + c(d + ex)^4} \, dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
f^4*x/c + 1/2*((b*e^6*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) + d/e)^2 - 2*b*d*e^5*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e) + b*d^2*e^4*f^4 + a*e^4*f^4)*log(x + sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e -
b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) + d/e)) - (b*e^6*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*b*d*e^5*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b*d^2*e^4*f^4 + a*e^4*f^4)*log(x - sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^
3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c
*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) - d/e)) + (b*e^6*f^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*b*d*e^5*f^4*(sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + b*d^2*e^4*f^4 + a*e^4*f^4)*l
og(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*
c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 -
 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + ...
```

Mupad [B] (verification not implemented)

Time = 11.71 (sec) , antiderivative size = 4605, normalized size of antiderivative = 22.80

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \text{Too large to display}$$

input

```
int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output

```
atan((((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c
+ ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^13 - 3
2*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f
^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 -
 a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*
b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 +
2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*
1i + ((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c -
 ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32
*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^
8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 -
a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b
^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2
*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a...
```

Reduce [F]

$$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$= \frac{f^4\left(-\left(\int \frac{x^2}{c e^4x^4+4cd e^3x^3+6c d^2e^2x^2+4c d^3ex+b e^2x^2+c d^4+2bdex+b d^2+a}dx\right) b e^2-2\left(\int \frac{x}{c e^4x^4+4cd e^3x^3+6c d^2e^2x^2+4c d^3ex+b e\cdots}\right.\right.}{}$$

input
```
int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

output
```
(f**4*( - int(x**2/(a + b*d**2 + 2*b*d*e*x + b*e**2*x**2 + c*d**4 + 4*c*d*
*3*e*x + 6*c*d**2*e**2*x**2 + 4*c*d*e**3*x**3 + c*e**4*x**4),x)*b*e**2 - 2
*int(x/(a + b*d**2 + 2*b*d*e*x + b*e**2*x**2 + c*d**4 + 4*c*d**3*e*x + 6*c
*d**2*e**2*x**2 + 4*c*d*e**3*x**3 + c*e**4*x**4),x)*b*d*e - int(1/(a + b*d
**2 + 2*b*d*e*x + b*e**2*x**2 + c*d**4 + 4*c*d**3*e*x + 6*c*d**2*e**2*x**2
 + 4*c*d*e**3*x**3 + c*e**4*x**4),x)*a - int(1/(a + b*d**2 + 2*b*d*e*x + b
*e**2*x**2 + c*d**4 + 4*c*d**3*e*x + 6*c*d**2*e**2*x**2 + 4*c*d*e**3*x**3
+ c*e**4*x**4),x)*b*d**2 + x))/c
```

3.268 $\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4} \, dx$

Optimal result

Integrand size = 33, antiderivative size = 87

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4} \, dx = \frac{bf^3 \text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}e} + \frac{f^3 \log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

output
```
1/2*b*f^3*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/
2)/e+1/4*f^3*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/c/e
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4} \, dx = \frac{f^3\left(-\frac{2b\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log\left(a+b(d+ex)^2+c(d+ex)^4\right)\right)}{4ce}$$

input
```
Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f^3*((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
4*a*c] + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]))/(4*c*e)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1462, 1434, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f^3\int \frac{(d+ex)^3}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{e}$$

$$\Big\downarrow \text{1434}$$

$$\frac{f^3\int \frac{(d+ex)^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1142}$$

$$\frac{f^3\left(\frac{\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2c}-\frac{b\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2c}\right)}{2e}$$

$$\Big\downarrow \text{1083}$$

$$\frac{f^3\left(\frac{b\int \frac{1}{-(d+ex)^4+b^2-4ac}d(2c(d+ex)^2+b)}{c}+\frac{\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2c}\right)}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{f^3\left(\frac{\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2c}+\frac{b\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)}{2e}$$

$$\Big\downarrow \text{1103}$$

$$\frac{f^3\left(\frac{b\,\mathrm{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2c}\right)}{2e}$$

input
```
Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f^3*((b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4
*a*c]) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(2*c)))/(2*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.77

method	result
default	$\frac{f^3\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2bde\right)\_Z+c\,d^4+b\,d^2+a\right)}\frac{\left(\_R^3e^3+3\_R^2d\,e^2+3\_R\,d^2e+d^3\right)\ln\ldots}{2c\,e^3\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2c\,d^3\ldots}\right.}{2e}$
risch	Expression too large to display

input
```
int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*f^3/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*
c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c
*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a)
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 446, normalized size of antiderivative = 5.13

$$\int\frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}bf^3\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac+(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\ldots}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right.}{4\,(b^2c-\ldots}\right.$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2 - 4*a*c)*b*f^3*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2
*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x
 + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c)
)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(
2*c*d^3 + b*d)*e*x + a)) + (b^2 - 4*a*c)*f^3*log(c*e^4*x^4 + 4*c*d*e^3*x^3
 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b
^2*c - 4*a*c^2)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*f^3*arctan(-(2*c*e^2*x^2 +
 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c
)*f^3*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^
2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c - 4*a*c^2)*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(75) = 150$.

Time = 1.02 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.82

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)} + \frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+bd^2f^3}{be^2f^3}\right) + \left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)} + \frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+bd^2f^3}{be^2f^3}\right)$$

input
```
integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*
d*x/e + x**2 + (-8*a*c*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2
)) + f**3/(4*c*e)) + 2*a*f**3 + 2*b**2*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c
*e*(4*a*c - b**2)) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3)) + (b*f**3
*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d*x/e +
x**2 + (-8*a*c*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3
/(4*c*e)) + 2*a*f**3 + 2*b**2*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c
- b**2)) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3))
```

Maxima [F]

$$\int \frac{(df + efx)^3}{a + b(d + ex)^2 + c(d + ex)^4}\,dx = \int \frac{(efx + df)^3}{(ex + d)^4c + (ex + d)^2b + a}\,dx$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.82

$$\int \frac{(df + efx)^3}{a + b(d + ex)^2 + c(d + ex)^4}\,dx = -\frac{bf^3 \arctan\left(\frac{2\,cd^2f + 2\,(efx^2 + 2\,dfx)ce + bf}{\sqrt{-b^2 + 4\,ac}f}\right)}{2\,\sqrt{-b^2 + 4\,ac}ce} + \frac{f^3 \log\left(cd^4f^2 + 2\,(efx^2 + 2\,dfx)cd^2ef + (efx^2 + 2\,dfx)^2ce^2 + bd^2f^2 + (efx^2 + 2\,dfx)bef + af^2\right)}{4\,ce}$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
-1/2*b*f^3*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2
 + 4*a*c)*f))/(sqrt(-b^2 + 4*a*c)*c*e) + 1/4*f^3*log(c*d^4*f^2 + 2*(e*f*x^
2 + 2*d*f*x)*c*d^2*e*f + (e*f*x^2 + 2*d*f*x)^2*c*e^2 + b*d^2*f^2 + (e*f*x^
2 + 2*d*f*x)*b*e*f + a*f^2)/(c*e)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.30

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$= \frac{4\,a\,c\,e\,f^3\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}$$
$$-\frac{b\,f^3\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,d^2}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,e^2\,x^2}{\sqrt{4\,a\,c-b^2}}+\frac{4\,c\,d\,e\,x}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,e\,\sqrt{4\,a\,c-b^2}}$$
$$-\frac{b^2\,e\,f^3\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}$$

input
```
int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
(4*a*c*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6
*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2
) - (b*f^3*atan(b/(4*a*c - b^2)^(1/2) + (2*c*d^2)/(4*a*c - b^2)^(1/2) + (2
*c*e^2*x^2)/(4*a*c - b^2)^(1/2) + (4*c*d*e*x)/(4*a*c - b^2)^(1/2)))/(2*c*e
*(4*a*c - b^2)^(1/2)) - (b^2*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e
^4*x^4 + 2*b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a
*c^2*e^2 - 4*b^2*c*e^2)
```

Reduce [F]

$$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$= \frac{f^3\left(-2\left(\int \frac{x}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2+a}dx\right)b\,e^2-2\left(\int \frac{1}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e\right.\right.}{}$$

input
```
int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

output

```
(f**3*( - 2*int(x/(a + b*d**2 + 2*b*d*e*x + b*e**2*x**2 + c*d**4 + 4*c*d**
3*e*x + 6*c*d**2*e**2*x**2 + 4*c*d*e**3*x**3 + c*e**4*x**4),x)*b*e**2 - 2*
int(1/(a + b*d**2 + 2*b*d*e*x + b*e**2*x**2 + c*d**4 + 4*c*d**3*e*x + 6*c*
d**2*e**2*x**2 + 4*c*d*e**3*x**3 + c*e**4*x**4),x)*b*d*e + log(a + b*d**2
+ 2*b*d*e*x + b*e**2*x**2 + c*d**4 + 4*c*d**3*e*x + 6*c*d**2*e**2*x**2 + 4
*c*d*e**3*x**3 + c*e**4*x**4)))/(4*c*e)
```

3.269 $\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4} dx$

Optimal result

Integrand size = 33, antiderivative size = 170

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4} dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e} + \frac{\sqrt{b+\sqrt{b^2-4ac}}f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}e}$$

output
```
-1/2*(b-(-4*a*c+b^2)^(1/2))^(1/2)*f^2*arctan(2^(1/2)*c^(1/2)*(e*x+d)/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)/e+1/2*(b+(-4*a
*c+b^2)^(1/2))^(1/2)*f^2*arctan(2^(1/2)*c^(1/2)*(e*x+d)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.05

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4} dx = \frac{f^2\left(\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}e}$$

input
```
Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f^2*((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a
*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]))/(Sq
rt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1462, 1450, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f^2\int \frac{(d+ex)^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{e}$$

$$\Big\downarrow \text{1450}$$

$$\frac{f^2\left(\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)\right)}{e}$$

$$\Big\downarrow \text{218}$$

$$\frac{f^2\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{e}$$

input
```
Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f^2*(((1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((
1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/e
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

method	result
default	$\frac{f^2\left(\sum_{\_R=\operatorname{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2bde\right)\_Z+c\,d^4+b\,d^2+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_\right.}{2c\,e^3\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2c\,d^3}\right.}{2e}$
risch	$\frac{f^2\left(\sum_{\_R=\operatorname{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2bde\right)\_Z+c\,d^4+b\,d^2+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_\right.}{2c\,e^3\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2c\,d^3}\right.}{2e}$

input
```
int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*f^2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*
c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(135) = 270$.

Time = 0.12 (sec) , antiderivative size = 799, normalized size of antiderivative = 4.70

$$\int \frac{(df + efx)^2}{a + b(d + ex)^2 + c(d + ex)^4} \, dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^
3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 + sqrt(1/2)*(b^
2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f^4 + (b^2
*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*
e^2))) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2
 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 - sqrt
(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*sqrt(-(b*f
^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c -
4*a*c^2)*e^2))) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/
((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f
^6 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^3*s
qrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/(
(b^2*c - 4*a*c^2)*e^2))) + 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*
sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^
6*x + d*f^6 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^
4))*e^3*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4
))*e^2)/((b^2*c - 4*a*c^2)*e^2)))
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$$
$$=\text{RootSum}\left(t^4\cdot\left(256a^2c^3e^4-128ab^2c^2e^4+16b^4ce^4\right)+t^2\left(-16abce^2f^4+4b^3e^2f^4\right)+af^8,\left(t\mapsto t\log\left(x-\right.\right.\right.$$

input
```
integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4)
 + _t**2*(-16*a*b*c*e**2*f**4 + 4*b**3*e**2*f**4) + a*f**8, Lambda(_t, _t*
log(x + (64*_t**3*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e*f**4 + d*f
**6)/(e*f**6))))
```

Maxima [F]

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\int \frac{(efx+df)^2}{(ex+d)^4c+(ex+d)^2b+a}\,dx$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1443 vs. 2(135) = 270.

Time = 0.14 (sec) , antiderivative size = 1443, normalized size of antiderivative = 8.49

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
-1/2*(e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e)^2 - 2*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) + d/e) + d^2*f^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*
c)*e^2)/(c*e^4)) + d/e)^2 + 6*c*d^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e) - 2*c*d^3*e + b*e^2*(sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) - b*d*e) + 1/2*(e^2*f^2*(sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*d*e*f^2*(sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + d^2*f^2)*l
og(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*
c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 +
 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e
)^2 + 6*c*d^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) - d/e) + 2*c*d^3*e + b*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) - d/e) + b*d*e) - 1/2*(e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 -
 sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + d^2*f^2)*log(x + sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/...
```

Mupad [B] (verification not implemented)

Time = 11.30 (sec) , antiderivative size = 683, normalized size of antiderivative = 4.02

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx =$$

$$-2\,\mathrm{atanh}\left(\frac{\sqrt{-\frac{b^3\,f^4+f^4\,\sqrt{-{(4\,a\,c-b^2)}^3}-4\,a\,b\,c\,f^4}{8\,(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2)}}\,\left(x\,(4\,a\,c^2\,e^{12}\,f^4-2\,b^2\,c\,e^{12}\,f^4)+\frac{(x\,(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14})+8\,b^3\,c^2}{8\,(16}\right.}{a\,c\,e^{10}\,f^6}\right.$$

$$-2\,\mathrm{atanh}\left(\frac{\sqrt{\frac{f^4\,\sqrt{-{(4\,a\,c-b^2)}^3}-b^3\,f^4+4\,a\,b\,c\,f^4}{8\,(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2)}}\,\left(x\,(4\,a\,c^2\,e^{12}\,f^4-2\,b^2\,c\,e^{12}\,f^4)-\frac{(x\,(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14})+8\,b^3\,c^2\,d\,e}{8\,(16\,a^2}\right.}{a\,c\,e^{10}\,f^6}\right.$$

input
```
int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
- 2*atanh(((-(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c*f^4)/(8*(b^
4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12*f^4 -
 2*b^2*c*e^12*f^4) + ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*
e^13 - 32*a*b*c^3*d*e^13)*(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*
c*f^4))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^1
1*f^4 - 2*b^2*c*d*e^11*f^4))/(a*c*e^10*f^6))*(-(b^3*f^4 + f^4*(-(4*a*c - b
^2)^3)^(1/2) - 4*a*b*c*f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e
^2)))^(1/2) - 2*atanh((((f^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*f^4 + 4*a*b*c*
f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2
*e^12*f^4 - 2*b^2*c*e^12*f^4) - ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8
*b^3*c^2*d*e^13 - 32*a*b*c^3*d*e^13)*(f^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*f
^4 + 4*a*b*c*f^4))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*
a*c^2*d*e^11*f^4 - 2*b^2*c*d*e^11*f^4))/(a*c*e^10*f^6))*((f^4*(-(4*a*c - b
^2)^3)^(1/2) - b^3*f^4 + 4*a*b*c*f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a
*b^2*c^2*e^2)))^(1/2)
```

Reduce [F]

$$\int \frac{(df + efx)^2}{a + b(d + ex)^2 + c(d + ex)^4} \, dx$$
$$= f^2 \left(\left(\int \frac{x^2}{c\,e^4x^4 + 4cd\,e^3x^3 + 6c\,d^2e^2x^2 + 4c\,d^3ex + b\,e^2x^2 + c\,d^4 + 2bdex + b\,d^2 + a} dx \right) e^2 + 2 \left(\int \frac{x}{c\,e^4x^4 + 4cd\,e^3x^3 + 6c\,d^2e^2x^2 + 4c\,d^3ex + b\,e^2x^2 + c\,d^4 + 2bdex + b\,d^2 + a} dx \right) de + \left(\int \frac{1}{c\,e^4x^4 + 4cd\,e^3x^3 + 6c\,d^2e^2x^2 + 4c\,d^3ex + b\,e^2x^2 + c\,d^4 + 2bdex + b\,d^2 + a} dx \right) d^2 \right)$$

input
```
int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

output
```
f**2*(int(x**2/(a + b*d**2 + 2*b*d*e*x + b*e**2*x**2 + c*d**4 + 4*c*d**3*e
*x + 6*c*d**2*e**2*x**2 + 4*c*d*e**3*x**3 + c*e**4*x**4),x)*e**2 + 2*int(x
/(a + b*d**2 + 2*b*d*e*x + b*e**2*x**2 + c*d**4 + 4*c*d**3*e*x + 6*c*d**2*
e**2*x**2 + 4*c*d*e**3*x**3 + c*e**4*x**4),x)*d*e + int(1/(a + b*d**2 + 2*
b*d*e*x + b*e**2*x**2 + c*d**4 + 4*c*d**3*e*x + 6*c*d**2*e**2*x**2 + 4*c*d
*e**3*x**3 + c*e**4*x**4),x)*d**2)
```

3.270 $\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 44

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx=-\frac{f\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,e}$$

output
```
-f*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx=\frac{f\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\,e}$$

input
```
Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
(f*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*e
)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1462, 1432, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}dx$$

$$\Big\downarrow \ 1462$$

$$\frac{f\int \frac{d+ex}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{e}$$

$$\Big\downarrow \ 1432$$

$$\frac{f\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{2e}$$

$$\Big\downarrow \ 1083$$

$$-\frac{f\int \frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{e}$$

$$\Big\downarrow \ 219$$

$$-\frac{f\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

input
```
Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

output
```
-((f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*
e))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.95

method	result
default	$\frac{f\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2bde\right)\_Z+c\,d^4+b\,d^2+a\right)}\frac{\left(\_Re+d\right)\ln\left(x-\_R\right)}{2c\,e^3\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2c\,d^3\cdots}\right.}{2e}$
risch	$-\frac{f\ln\left(\left(\sqrt{-4ac+b^2}\,e^2-b\,e^2\right)x^2+\left(2\sqrt{-4ac+b^2}\,de-2bde\right)x+\sqrt{-4ac+b^2}\,d^2-b\,d^2-2a\right)}{2\sqrt{-4ac+b^2}\,e}+\frac{f\ln\left(\left(\sqrt{-4ac+b^2}\,e^2+b\,e^2\right)x^2+\left(2\sqrt{-\cdots}\right.\right.}{2\sqrt{\cdots}}$

input
```
int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*f/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*
b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*
_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 274, normalized size of antiderivative = 6.23

$$\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4}\,dx = \left[\frac{f\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac-(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{b^2-4\,ac}}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right)}{2\,\sqrt{b^2-4\,ac}e}, -\frac{\sqrt{-b^2+4\,ac}f\arctan\left(-\frac{(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{(b^2-4\,ac)e}\right]$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
[1/2*f*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b
*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e
^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^
3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)
)/(sqrt(b^2 - 4*a*c)*e), -sqrt(-b^2 + 4*a*c)*f*arctan(-(2*c*e^2*x^2 + 4*c*
d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/((b^2 - 4*a*c)*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(41) = 82$.

Time = 0.61 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.30

$$\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4}\,dx = -\frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e} + x^2 + \frac{-4acf\sqrt{-\frac{1}{4ac-b^2}} + b^2f\sqrt{-\frac{1}{4ac-b^2}} + bf + 2cd^2f}{2ce^2f}\right)}{2e} + \frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e} + x^2 + \frac{4acf\sqrt{-\frac{1}{4ac-b^2}} - b^2f\sqrt{-\frac{1}{4ac-b^2}} + bf + 2cd^2f}{2ce^2f}\right)}{2e}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
-f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*f*sqrt(-1/(4*a*c -
 b**2)) + b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e**2*f))
/(2*e) + f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*f*sqrt(-1/(
4*a*c - b**2)) - b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e
**2*f))/(2*e)
```

Maxima [F]

$$\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4} \, dx = \int \frac{efx + df}{(ex + d)^4 c + (ex + d)^2 b + a} \, dx$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4} \, dx = \frac{f \arctan\left(\frac{2\,cd^2f + 2\,(efx^2 + 2\,dfx)ce + bf}{\sqrt{-b^2 + 4\,ac}f}\right)}{\sqrt{-b^2 + 4\,ac}e}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
f*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*a*c)
*f))/(sqrt(-b^2 + 4*a*c)*e)
```

Mupad [B] (verification not implemented)

Time = 10.89 (sec) , antiderivative size = 477, normalized size of antiderivative = 10.84

$$\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4}\,dx$$
$$= \frac{f \operatorname{atan}\left(\frac{\frac{f\left(4c^2d^2e^7f+4c^2e^9fx^2-\frac{f(8bc^2d^2e^8+16bc^2de^9x+8bc^2e^{10}x^2+16ac^2e^8)}{2e\sqrt{b^2-4ac}}+8c^2de^8fx\right)1i}{2e\sqrt{b^2-4ac}}+\frac{f\left(4c^2d^2e^7f+4c^2e^9fx^2+\frac{f(8bc^2d^2\cdots}{\cdots}\right.}{\cdots}}{\frac{f\left(4c^2d^2e^7f+4c^2e^9fx^2-\frac{f(8bc^2d^2e^8+16bc^2de^9x+8bc^2e^{10}x^2+16ac^2e^8)}{2e\sqrt{b^2-4ac}}+8c^2de^8fx\right)}{2e\sqrt{b^2-4ac}}-\frac{f\left(4c^2d^2e^7f+4c^2e^9fx^2+\frac{f(8bc^2d^2\cdots}{\cdots}\right.}{\cdots}}\right.}{e\sqrt{b^2-4ac}}$$

input
```
int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

output
```
(f*atan(((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^2*e^8 + 8*b*c^
2*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)
) + 8*c^2*d*e^8*f*x)*1i)/(2*e*(b^2 - 4*a*c)^(1/2)) + (f*(4*c^2*d^2*e^7*f +
 4*c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2 +
 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x)*1i)/(2*e*
(b^2 - 4*a*c)^(1/2)))/((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^
2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2
- 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x))/(2*e*(b^2 - 4*a*c)^(1/2)) - (f*(4*c^2*
d^2*e^7*f + 4*c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2
*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x
))/(2*e*(b^2 - 4*a*c)^(1/2))))*1i)/(e*(b^2 - 4*a*c)^(1/2))
```

Reduce [F]

$$\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4}\,dx$$
$$= f\left(\left(\int \frac{x}{c\,e^4x^4 + 4cd\,e^3x^3 + 6c\,d^2e^2x^2 + 4c\,d^3ex + b\,e^2x^2 + c\,d^4 + 2bdex + b\,d^2 + a}dx\right)e + \left(\int \frac{1}{c\,e^4x^4 + 4cd\,e^3x^3 + 6c\,d^2e^2x^2 + 4c\,d^3ex + b\,e^2x^2 + c\,d^4 + 2bdex + b\,d^2 + a}dx\right)d\right)$$

input
```
int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

output
```
f*(int(x/(a + b*d**2 + 2*b*d*e*x + b*e**2*x**2 + c*d**4 + 4*c*d**3*e*x + 6
*c*d**2*e**2*x**2 + 4*c*d*e**3*x**3 + c*e**4*x**4),x)*e + int(1/(a + b*d**
2 + 2*b*d*e*x + b*e**2*x**2 + c*d**4 + 4*c*d**3*e*x + 6*c*d**2*e**2*x**2 +
 4*c*d*e**3*x**3 + c*e**4*x**4),x)*d)
```

3.271 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 103

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{b\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}ef}+\frac{\log(d+ex)}{aef}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}$$

output
```
1/2*b*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)/e
/f+ln(e*x+d)/a/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a/e/f
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.27

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{4\sqrt{b^2-4ac}\log(d+ex)-\left(b+\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(b\right.}{4a\sqrt{b^2-4ac}ef}$$

input
```
Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b
^2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2
- 4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e*f)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1462, 1434, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)(c(d+ex)^4+b(d+ex)^2+a)}d(d+ex)}{ef}$$

$$\Big\downarrow \text{1434}$$

$$\frac{\int \frac{1}{(d+ex)^2(c(d+ex)^4+b(d+ex)^2+a)}d(d+ex)^2}{2ef}$$

$$\Big\downarrow \text{1144}$$

$$\frac{\frac{\int -\frac{c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{a}+\frac{\log\left((d+ex)^2\right)}{a}}{2ef}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\log\left((d+ex)^2\right)}{a}-\frac{\int \frac{c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{a}}{2ef}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\log\left((d+ex)^2\right)}{a}-\frac{\frac{1}{2}b\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2+\frac{1}{2}\int \frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{a}}{2ef}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{\log((d+ex)^2)}{a}-\frac{\frac{1}{2}\int\frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2-b\int\frac{1}{-(d+ex)^4+b^2-4ac}d(2c(d+ex)^2+b)}{a}}{2ef}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\log((d+ex)^2)}{a}-\frac{\frac{1}{2}\int\frac{2c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2-\frac{b\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{2ef}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\log((d+ex)^2)}{a}-\frac{\frac{1}{2}\log(a+b(d+ex)^2+c(d+ex)^4)-\frac{b\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{2ef}$$

input
```
Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(Log[(d + e*x)^2]/a - (-((b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c
]])/Sqrt[b^2 - 4*a*c]) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/2)/a)/(2*e
*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.68

method	result
risch	$\frac{\ln(ex+d)}{aef}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4a^2f^2e^2c-b^2af^2e^2\right)\_Z^2+\left(4acef-b^2ef\right)\_Z+c\right)}\_R\ln\left(\left(\left(10ace^3f-3b^2e^3f\right)\_R+5ce^2\right)x^2+\left((20ac\right.\right.\right.}{2}$
default	$\frac{\frac{\sum_{\_R=\operatorname{RootOf}\left(ce^4\_Z^4+4cde^3\_Z^3+\left(6cd^2e^2+be^2\right)\_Z^2+\left(4cd^3e+2bde\right)\_Z+cd^4+bd^2+a\right)}\frac{\left(-ce^3\_R^3-3cde^2\_R^2+e\left(-3cd^2-b\right)\_R-\right.}{2ce^3\_R^3+6cde^2\_R^2+6cd^2e\_R+2c}}{2ae}}{f}$

input
```
int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
ln(e*x+d)/a/e/f+1/2*sum(_R*ln(((10*a*c*e^3*f-3*b^2*e^3*f)*_R+5*c*e^2)*x^2+
((20*a*c*d*e^2*f-6*b^2*d*e^2*f)*_R+10*c*d*e)*x+(10*a*c*d^2*e*f-3*b^2*d^2*e
*f-a*b*e*f)*_R+5*c*d^2+2*b),_R=RootOf((4*a^2*c*e^2*f^2-a*b^2*e^2*f^2)*_Z^2
+(4*a*c*e*f-b^2*e*f)*_Z+c))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.60

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2e^4x^4+8\,c^2de^3x^3+2\,c^2d^4+2\,(6\,c^2d^2+bc)e^2x^2+2\,bcd^2+4\,(2\,c^2d^3+bcd)ex+b^2-2\,ac+(2\,ce^2x^2+4\,cdex+2\,cd^2+b)\sqrt{b}}{ce^4x^4+4\,cde^3x^3+cd^4+(6\,cd^2+b)e^2x^2+bd^2+2\,(2\,cd^3+bd)ex+a}\right.}{}\right.$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4
 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b
^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c
*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*
d^3 + b*d)*e*x + a)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4
*a*c)*log(e*x + d))/((a*b^2 - 4*a^2*c)*e*f), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*a
rctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 +
b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(e*x
+ d))/((a*b^2 - 4*a^2*c)*e*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(83) = 166$.

Time = 4.15 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.38

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4aef\,(4ac-b^2)} - \frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)+2ab^2ef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)-2ac+b^2 \ldots}{bce^2}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4aef\,(4ac-b^2)} - \frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)+2ab^2ef\left(\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)-2ac+b^2+bc \ldots}{bce^2}\right) + \frac{\log\left(\frac{d}{e}+x\right)}{aef}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/
e + x**2 + (-8*a**2*c*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2))
 - 1/(4*a*e*f)) + 2*a*b**2*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b
**2)) - 1/(4*a*e*f)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a
*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/e + x**2 + (-
8*a**2*c*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)
) + 2*a*b**2*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*
e*f)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + log(d/e + x)/(a*e*f)
```

Maxima [F]

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(efx+df)}\,dx$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

output
```
-integrate((c*e^3*x^3 + 3*c*d*e^2*x^2 + c*d^3 + (3*c*d^2 + b)*e*x + b*d)/(
c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c
*d^3 + b*d)*e*x + a), x)/(a*f) + log(e*x + d)/(a*e*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(95) = 190$.

Time = 0.15 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.83

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = -\frac{\log\left(|ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a|\right)}{4\,aef} + \frac{\log\left(|ex+d|\right)}{aef} - \frac{\frac{abce^3f\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bdex+2\,\sqrt{b^2-4\,ac}dex+bd^2+\sqrt{b^2-4\,ac}d^2+2\,a\right|\right)}{\sqrt{b^2-4\,ac}} - \frac{abce^3f\log\left(\left|-be^2x^2+\sqrt{b^2-4\,ac}e^2x^2-2\,bdex+2\,\sqrt{}\right.\right.}{\sqrt{b^2-4\,ac}}}{4\,a^2ce^4f^2}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
-1/4*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c
*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a*e*f) + log(abs(e*x + d))/(a*
e*f) - 1/4*(a*b*c*e^3*f*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*
b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)
)/sqrt(b^2 - 4*a*c) - a*b*c*e^3*f*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e
^2*x^2 - 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)
*d^2 - 2*a))/sqrt(b^2 - 4*a*c))/(a^2*c*e^4*f^2)
```

Mupad [B] (verification not implemented)

Time = 11.96 (sec) , antiderivative size = 2520, normalized size of antiderivative = 24.47

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

output
```
log(d + e*x)/(a*e*f) - (log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*
b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3)*(2*b^2*e*f - 8*a*
c*e*f))/(2*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) - (b*atan((16*a^3*f^3*x*(
4*a*c - b^2)^(3/2)*(((3*b^3 - 8*a*b*c)*((b^2*((2*(2*b^2*e*f - 8*a*c*e*f)*(
6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*
e^2*f^2)) + (20*b*c^3*d*e^17)/f))/(16*a^2*e^2*f^2*(4*a*c - b^2)) - ((2*b^2
*e*f - 8*a*c*e*f)^2*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*d*e^18*f - 20*a
*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) + (20*b*c^3*d*e
^17)/f))/(4*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*f - 8*
a*c*e*f)*(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(4*a^2*e^2*f^3*(4*a*b
^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2
)) - (((b*(2*b^2*e*f - 8*a*c*e*f)*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*d
*e^18*f - 20*a*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) +
 (20*b*c^3*d*e^17)/f))/(4*a*e*f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*
c - b^2)^(1/2)) - (b^3*(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(16*a^3
*e^3*f^4*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f)^2*(6*b^3*c^2*d*
e^18*f - 20*a*b*c^3*d*e^18*f))/(4*a*e*f^2*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*
f^2)^2*(4*a*c - b^2)^(1/2)))*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c))/(8*a^3*c^2
*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2))))/(b^2*c^2*e^14) + (2*f^3*(3*b^3 -
8*a*b*c)*(4*a*c - b^2)^(3/2)*((b^2*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^2*e^...
```

Reduce [F]

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$=\frac{\int \frac{1}{c\,e^5x^5+5cd\,e^4x^4+10c\,d^2e^3x^3+10c\,d^3e^2x^2+b\,e^3x^3+5c\,d^4ex+3bd\,e^2x^2+c\,d^5+3b\,d^2ex+b\,d^3+aex+ad}dx}{f}$$

input
```
int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

output
```
int(1/(a*d + a*e*x + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x**3
 + c*d**5 + 5*c*d**4*e*x + 10*c*d**3*e**2*x**2 + 10*c*d**2*e**3*x**3 + 5*c
*d*e**4*x**4 + c*e**5*x**5),x)/f
```

3.272 $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 204

$$\begin{aligned}\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx &= -\frac{1}{aef^2(d+ex)} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}ef^2}\\ &\quad - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}ef^2}\end{aligned}$$

output

```
-1/a/e/f^2/(e*x+d)-1/2*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(
1/2)*(e*x+d)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(b-(-4*a*c+b^2)^(1/2)
)^(1/2)/e/f^2-1/2*c^(1/2)*(1-b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*
(e*x+d)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(b+(-4*a*c+b^2)^(1/2))^(1/
2)/e/f^2
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.02

$$\int \frac{1}{(df+efx)^2 (a+b(d+ex)^2+c(d+ex)^4)} \, dx = -\frac{\frac{2}{d+ex}+\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2aef^2}$$

input
```
Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
-1/2*(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a*e*f^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1462, 1443, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^2 (a+b(d+ex)^2+c(d+ex)^4)} \, dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^2(c(d+ex)^4+b(d+ex)^2+a)}d(d+ex)}{ef^2}$$

$$\Big\downarrow \text{1443}$$

$$\frac{\frac{\int -\frac{c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{1}{a(d+ex)}}{ef^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{c(d+ex)^2+b}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{1}{a(d+ex)}}{ef^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{a}-\frac{1}{a(d+ex)}}{ef^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{a(d+ex)}}{ef^2}$$

input
```
Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(-(1/(a*(d + e*x))) - ((Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*
c]]))/a)/(e*f^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.84

method	result
default	$\frac{-\frac{1}{ae(ex+d)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\left(4c\,d^{3}e+2bde\right)\_Z+c\,d^{4}+b\,d^{2}+a\right)}\frac{\left(-\_R^{2}c\,e^{2}-2\_Rcde-c\,d^{2}\right.}{2c\,e^{3}\_R^{3}+6cd\,e^{2}\_R^{2}+6c\,d^{2}e\_}}{2ae}}{f^{2}}$
risch	$-\frac{1}{ae\,f^{2}(ex+d)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16f^{8}e^{4}c^{2}a^{5}-8b^{2}f^{8}e^{4}c\,a^{4}+b^{4}f^{8}e^{4}a^{3}\right)\_Z^{4}+\left(12a^{2}b\,c^{2}e^{2}f^{4}-7a\,b^{3}c\,e^{2}f^{4}+b^{5}e^{2}f^{4}\right)\_Z^{2}+c^{3}\right)}\_R\ln\left(\right.\right.}{}$

input
```
int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/f^2*(-1/a/e/(e*x+d)+1/2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3
*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(
c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+
c*d^4+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(167) = 334$.

Time = 0.10 (sec) , antiderivative size = 1477, normalized size of antiderivative = 7.24

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas
")
```

output
```
1/2*(sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^
4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 -
3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(
b^2*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^
6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - (b^5 -
 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(-((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*a*b*c)
/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) - sqrt(1/2)*(a*e^2*f^2*x + a*d*e*f^2)*sqr
t(-((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2
 - 4*a^7*c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))*log(
-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6
*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2
 - 4*a^7*c)*e^4*f^8)) - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*sqrt(-((a^3
*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7
*c)*e^4*f^8)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) - sqrt(1/2)
*(a*e^2*f^2*x + a*d*e*f^2)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*f^4*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*
b^2 - 4*a^4*c)*e^2*f^4))*log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3
)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) + (b^5 - 5*a*b^3*c +...
```

Sympy [A] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.26

$$\int \frac{1}{(df+efx)^2 (a+b(d+ex)^2+c(d+ex)^4)} dx$$
$$= \text{RootSum}\left(t^4 \cdot (256a^5c^2e^4f^8 - 128a^4b^2ce^4f^8 + 16a^3b^4e^4f^8) + t^2 \cdot (48a^2bc^2e^2f^4 - 28ab^3ce^2f^4 + 4b^5e^2f^4\right.$$
$$- \frac{1}{adef^2 + ae^2f^2x}$$

input
```
integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
RootSum(_t**4*(256*a**5*c**2*e**4*f**8 - 128*a**4*b**2*c*e**4*f**8 + 16*a*
*3*b**4*e**4*f**8) + _t**2*(48*a**2*b*c**2*e**2*f**4 - 28*a*b**3*c*e**2*f*
*4 + 4*b**5*e**2*f**4) + c**3, Lambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*
e**3*f**6 + 48*_t**3*a**4*b**2*c*e**3*f**6 - 8*_t**3*a**3*b**4*e**3*f**6 -
 10*_t*a**2*b*c**2*e*f**2 + 10*_t*a*b**3*c*e*f**2 - 2*_t*b**5*e*f**2 + a*c
**3*d - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e*f**2 + a*e**2*
f**2*x)
```

Maxima [F]

$$\int \frac{1}{(df+efx)^2 (a+b(d+ex)^2+c(d+ex)^4)} dx$$
$$= \int \frac{1}{\left((ex+d)^4c + (ex+d)^2b + a\right)(efx+df)^2} dx$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima
")
```

output
```
-1/(a*e^2*f^2*x + a*d*e*f^2) - integrate((c*e^2*x^2 + 2*c*d*e*x + c*d^2 +
b)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*
(2*c*d^3 + b*d)*e*x + a), x)/(a*f^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error index.cc index_gcd Error: Bad
 Argument ValueError index.cc index_gcd Error: Bad Argument ValueDone
```

Mupad [B] (verification not implemented)

Time = 12.15 (sec) , antiderivative size = 4339, normalized size of antiderivative = 21.27

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

output
```
- atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
 a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 -
8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f
^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^
3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*
b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^
3*c^2*e^12*f^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d
*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6)*1i + (-(b^5 + b^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*
b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*
c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32
*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^
10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^
4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^8)
*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^...
```

Reduce [F]

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$= \frac{\int \frac{1}{c e^6x^6+6cd e^5x^5+15c d^2e^4x^4+20c d^3e^3x^3+b e^4x^4+15c d^4e^2x^2+4bd e^3x^3+6c d^5ex+6b d^2e^2x^2+c d^6+4b d^3ex+a e^2x^2+b d^4+2adex+a d^2}dx}{f^2}$$

input
```
int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

output
```
int(1/(a*d**2 + 2*a*d*e*x + a*e**2*x**2 + b*d**4 + 4*b*d**3*e*x + 6*b*d**2
*e**2*x**2 + 4*b*d*e**3*x**3 + b*e**4*x**4 + c*d**6 + 6*c*d**5*e*x + 15*c*
d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 15*c*d**2*e**4*x**4 + 6*c*d*e**5*x*
*5 + c*e**6*x**6),x)/f**2
```

3.273 $\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 133

$$\begin{aligned}\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx &= -\frac{1}{2aef^3(d+ex)^2}-\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}ef^3}\\&\quad-\frac{b\log(d+ex)}{a^2ef^3}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}\end{aligned}$$

output
```
-1/2/a/e/f^3/(e*x+d)^2-1/2*(-2*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+
b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/e/f^3-b*ln(e*x+d)/a^2/e/f^3+1/4*b*ln(a+
b*(e*x+d)^2+c*(e*x+d)^4)/a^2/e/f^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.18

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{-\frac{2a}{(d+ex)^2}-4b\log(d+ex)+\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c\right.}{\sqrt{b^2-4ac}}}{4a^2ef^3}$$

input
```
Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*
c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b
^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x
)^2])/Sqrt[b^2 - 4*a*c])/(4*a^2*e*f^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1462, 1434, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^3(c(d+ex)^4+b(d+ex)^2+a)}d(d+ex)}{ef^3}$$

$$\Big\downarrow \text{1434}$$

$$\frac{\int \frac{1}{(d+ex)^4(c(d+ex)^4+b(d+ex)^2+a)}d(d+ex)^2}{2ef^3}$$

$$\Big\downarrow \text{1145}$$

$$\frac{\frac{\int -\frac{c(d+ex)^2+b}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a}-\frac{1}{a(d+ex)^2}}{2ef^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{c(d+ex)^2+b}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a}-\frac{1}{a(d+ex)^2}}{2ef^3}$$

$$\Big\downarrow \text{1200}$$

$$\frac{-\frac{\int \left(\frac{b}{a(d+ex)^2}+\frac{-b^2-c(d+ex)^2b+ac}{a\left(c(d+ex)^4+b(d+ex)^2+a\right)}\right)d(d+ex)^2}{a}-\frac{1}{a(d+ex)^2}}{2ef^3}$$

$\big\downarrow$ 2009

$$\frac{-\frac{\frac{(b^2-2ac)\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a}+\frac{b\log\left((d+ex)^2\right)}{a}}{a}-\frac{1}{a(d+ex)^2}}{2ef^3}$$

input
```
Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(-(1/(a*(d + e*x)^2)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt
[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + (b*Log[(d + e*x)^2])/a - (b*Log[a
+ b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a))/a)/(2*e*f^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.63

method	result
default	$\frac{\frac{\sum_{\_R=\text{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2bde\right)\_Z+c\,d^4+b\,d^2+a\right)}\frac{\left(bc\,e^3\_R^3+3bcd\,e^2\_R^2+e\left(3bc\,d^2-ac+b^2\right)\_ \right.}{2c\,e^3\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_}}{2a^2e}}{f^3}$
risch	$-\frac{1}{2ae\,f^3(ex+d)^2}-\frac{b\ln(ex+d)}{a^2e\,f^3}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^3c\,e^2f^6-a^2b^2e^2f^6\right)\_Z^2+\left(-4abce\,f^3+b^3e\,f^3\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(10a^3c\,e^4f^6\right.\right.\right.\right.}{}$

input
```
int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/f^3*(1/2/a^2/e*sum((b*c*e^3*_R^3+3*b*c*d*e^2*_R^2+e*(3*b*c*d^2-a*c+b^2)*
_R+d^3*b*c-a*c*d+d*b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+
_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^
2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))-1/2/a/e/(e*x+d)^2-1/a^2*b/e
*ln(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(123) = 246$.

Time = 0.18 (sec) , antiderivative size = 828, normalized size of antiderivative = 6.23

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas
")
```

output
```
[-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x
+ (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^
3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b
*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2
 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*
a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
+ (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4
*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x +
 d))/((a^2*b^2 - 4*a^3*c)*e^3*f^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x
+ (a^2*b^2 - 4*a^3*c)*d^2*e*f^3), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*
c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)
*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
 4*a*c)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a
*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
 b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3
- 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)
*e^3*f^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x + (a^2*b^2 - 4*a^3*c)*d^2
*e*f^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(119) = 238$.

Time = 112.04 (sec) , antiderivative size = 532, normalized size of antiderivative = 4.00

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \left(\frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2ef^3\cdot(4ac-b^2)}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^3cef^3\left(\frac{b}{4a^2ef^3}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2ef^3\cdot(4ac-b^2)}\right)+2a^2b^2ef^3\left(\frac{b}{4a^2ef^3}\right.}{2ac^2e^2-b}\right.$$

$$+\left(\frac{b}{4a^2ef^3} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2ef^3\cdot(4ac-b^2)}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^3cef^3\left(\frac{b}{4a^2ef^3}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2ef^3\cdot(4ac-b^2)}\right)+2a^2b^2ef^3\left(\frac{b}{4a^2ef^3}\right.}{2ac^2e^2-b}\right.$$

$$-\frac{1}{2ad^2ef^3+4ade^2f^3x+2ae^3f^3x^2}-\frac{b\log\left(\frac{d}{e}+x\right)}{a^2ef^3}$$

input
```
integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
(b/(4*a**2*e*f**3) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*
a*c - b**2)))*log(2*d*x/e + x**2 + (-8*a**3*c*e*f**3*(b/(4*a**2*e*f**3) -
sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2))) + 2*a**
2*b**2*e*f**3*(b/(4*a**2*e*f**3) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a
**2*e*f**3*(4*a*c - b**2))) + 3*a*b*c + 2*a*c**2*d**2 - b**3 - b**2*c*d**2
)/(2*a*c**2*e**2 - b**2*c*e**2)) + (b/(4*a**2*e*f**3) + sqrt(-4*a*c + b**2
)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2)))*log(2*d*x/e + x**2 + (-8*
a**3*c*e*f**3*(b/(4*a**2*e*f**3) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a
**2*e*f**3*(4*a*c - b**2))) + 2*a**2*b**2*e*f**3*(b/(4*a**2*e*f**3) + sqrt
(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2))) + 3*a*b*c +
 2*a*c**2*d**2 - b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) - 1/(2
*a*d**2*e*f**3 + 4*a*d*e**2*f**3*x + 2*a*e**3*f**3*x**2) - b*log(d/e + x)/
(a**2*e*f**3)
```

Maxima [F]

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(efx+df)^3}\,dx$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima
")
```

output
```
-1/2/(a*e^3*f^3*x^2 + 2*a*d*e^2*f^3*x + a*d^2*e*f^3) + integrate((b*c*e^3*
x^3 + 3*b*c*d*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 - a*c)*e*x + (b^2 - a*c
)*d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 +
2*(2*c*d^3 + b*d)*e*x + a), x)/(a^2*f^3) - b*log(e*x + d)/(a^2*e*f^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(123) = 246.

Time = 0.15 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.68

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{b\log\left(|ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a|\right)}{4\,a^2ef^3} - \frac{b\log\left(|ex+d|\right)}{a^2ef^3} - \frac{1}{2\,(ex+d)^2aef^3} + \frac{\frac{\left(a^2b^2ce^3f^3-2\,a^3c^2e^3f^3\right)\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bdex+2\,\sqrt{b^2-4\,ac}dex+bd^2+\sqrt{b^2-4\,ac}d^2+2\,a\right|\right)}{\sqrt{b^2-4\,ac}} - \frac{\left(a^2b^2ce^3f^3-2\,a^3c^2e^3f^3\right)\log\left(\left|\right.\right.}{}}{4\,a^4ce^4f^6}$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
1/4*b*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x +
c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a^2*e*f^3) - b*log(abs(e*x +
d))/(a^2*e*f^3) - 1/2/((e*x + d)^2*a*e*f^3) + 1/4*((a^2*b^2*c*e^3*f^3 - 2*
a^3*c^2*e^3*f^3)*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x
 + 2*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a))/sqrt(
b^2 - 4*a*c) - (a^2*b^2*c*e^3*f^3 - 2*a^3*c^2*e^3*f^3)*log(abs(-b*e^2*x^2
+ sqrt(b^2 - 4*a*c)*e^2*x^2 - 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^
2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a))/sqrt(b^2 - 4*a*c))/(a^4*c*e^4*f^6)
```

Mupad [B] (verification not implemented)

Time = 14.10 (sec) , antiderivative size = 5947, normalized size of antiderivative = 44.71

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

output
```
(atan((16*a^6*f^9*x*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((2*b^3*e*f^3 - 8*a
*b*c*e*f^3)*((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6))/(a^3*f
^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3
- 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(2*(1
6*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (12*b*c^4*d*e^16)/(a^2*f^6))*(2*b^
3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (2*
c^5*d*e^15)/(a^3*f^9) - (((((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^
17*f^6))/(a^3*f^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9
)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^
2*f^6)))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3
*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*
a*c - b^2))/(4*a^5*e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^
2*e^2*f^6)))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((40*a^4*b
*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)
*(2*a*c - b^2)^2)/(16*a^7*e^2*f^15*(4*a*c - b^2)*(16*a^3*c*e^2*f^6 - 4*a^2
*b^2*e^2*f^6))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((3*b^5 + 13
*a^2*b*c^2 - 15*a*b^3*c)*((((((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*
e^17*f^6))/(a^3*f^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f
^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*
e^2*f^6)))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((40*a^4*...
```

Reduce [F]

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$= \frac{\int \frac{1}{c\,e^7x^7+7cd\,e^6x^6+21c\,d^2e^5x^5+35c\,d^3e^4x^4+b\,e^5x^5+35c\,d^4e^3x^3+5bd\,e^4x^4+21c\,d^5e^2x^2+10b\,d^2e^3x^3+7c\,d^6ex+10b\,d^3e^2x^2+c\,d^7+a\,e^3x^3+5b\,d}{f^3}$$

input
```
int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

output
```
int(1/(a*d**3 + 3*a*d**2*e*x + 3*a*d*e**2*x**2 + a*e**3*x**3 + b*d**5 + 5*
b*d**4*e*x + 10*b*d**3*e**2*x**2 + 10*b*d**2*e**3*x**3 + 5*b*d*e**4*x**4 +
 b*e**5*x**5 + c*d**7 + 7*c*d**6*e*x + 21*c*d**5*e**2*x**2 + 35*c*d**4*e**
3*x**3 + 35*c*d**3*e**4*x**4 + 21*c*d**2*e**5*x**5 + 7*c*d*e**6*x**6 + c*e
**7*x**7),x)/f**3
```

3.274 $\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 236

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = -\frac{1}{3aef^4(d+ex)^3}+\frac{b}{a^2ef^4(d+ex)}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}ef^4}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}ef^4}$$

output
```
-1/3/a/e/f^4/(e*x+d)^3+b/a^2/e/f^4/(e*x+d)+1/2*c^(1/2)*(b+(-2*a*c+b^2)/(-4
*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*(e*x+d)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*2^(1/2)/a^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)/e/f^4+1/2*c^(1/2)*(b-(-2*a*c+
b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*(e*x+d)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*2^(1/2)/a^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/e/f^4
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.01

$$\int \frac{1}{(df+efx)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)} dx$$
$$= \frac{-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}+\frac{3\sqrt{2}\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{6a^2ef^4}$$

input
```
Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c +
b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*S
qrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e
*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 -
4*a*c]]))/(6*a^2*e*f^4)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1462, 1443, 27, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)} dx$$

$$\Big\downarrow \text{ 1462}$$

$$\frac{\int \frac{1}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{ef^4}$$

$$\Big\downarrow \text{ 1443}$$

$$\frac{\frac{\int -\frac{3\left(c(d+ex)^2+b\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{3a}-\frac{1}{3a(d+ex)^3}}{ef^4}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{c(d+ex)^2+b}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{a}-\frac{1}{3a(d+ex)^3}}{ef^4}$$

$$\Big\downarrow \; 1604$$

$$\frac{-\frac{-\frac{\int \frac{b^2+c(d+ex)^2b-ac}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{b}{a(d+ex)}}{a}-\frac{1}{3a(d+ex)^3}}{ef^4}$$

$$\Big\downarrow \; 1480$$

$$\frac{-\frac{-\frac{\frac{1}{2}c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{a}-\frac{b}{a(d+ex)}}{a}-\frac{1}{3a(d+ex)^3}}{ef^4}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{-\frac{\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{b}{a(d+ex)}}{a}-\frac{1}{3a(d+ex)^3}}{ef^4}$$

input
```
Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

output
```
(-1/3*1/(a*(d + e*x)^3) - (-(b/(a*(d + e*x))) - ((Sqrt[c]*(b + (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b - (b^2 -
2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a)/a)/(e*f^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.81

method	result
default	$\frac{\frac{\sum_{\_R=\text{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2bde\right)\_Z+c\,d^4+b\,d^2+a\right)}\frac{\left(\_R^2bc\,e^2+2\_Rbcde+bc\,d^2-ac+b^2\right)\ln\left(x-\right.}{2c\,e^3\_R^3+6cd\,e^2\_R^2+6c\,d^2e\_R+2c\,d^3+eb}}{2a^2e}}{f^4}$
risch	$\frac{\frac{eb\,x^2}{a^2}+\frac{2bdx}{a^2}-\frac{-3b\,d^2+a}{3e\,a^2}}{f^4(ex+d)^3}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(16f^{16}e^4c^2a^7-8a^6b^2c\,e^4f^{16}+a^5b^4e^4f^{16}\right)\_Z^4+\left(-20b\,f^8e^2c^3a^3+25b^3f^8e^2c^2a^2-9b^5f^8e^2ca+\right.\right.}\right.}{}$

input
```
int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

output
```
1/f^4*(1/2/a^2/e*sum((_R^2*b*c*e^2+2*_R*b*c*d*e+b*c*d^2-a*c+b^2)/(2*_R^3*c
*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*
e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*
d^4+b*d^2+a))-1/3/a/e/(e*x+d)^3+1/a^2*b/e/(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2212 vs. 2(200) = 400.

Time = 0.17 (sec) , antiderivative size = 2212, normalized size of antiderivative = 9.37

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas
")
```

output
```
1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3
*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2
- 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
+ a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c
^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)
*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6
*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6
*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - (b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(-((a^5*b^2 -
 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^
2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) - 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*
d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2 - 4*a
^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/(
(a^5*b^2 - 4*a^6*c)*e^2*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x
+ 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*
c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4*f^16)) - (b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*sqrt(-((a^5*b^2 - 4...
```

Sympy [A] (verification not implemented)

Time = 7.84 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.74

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$= \frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3ef^4+9a^2d^2e^2f^4x+9a^2de^3f^4x^2+3a^2e^4f^4x^3}$$
$$+\mathrm{RootSum}\Big(t^4\cdot\left(256a^7c^2e^4f^{16}-128a^6b^2ce^4f^{16}+16a^5b^4e^4f^{16}\right)+t^2\left(-80a^3bc^3e^2f^8+100a^2b^3c^2e^2f^8-\right.$$

input
```
integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

output
```
(-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e*f**4 + 9*a**2*d
**2*e**2*f**4*x + 9*a**2*d*e**3*f**4*x**2 + 3*a**2*e**4*f**4*x**3) + RootS
um(_t**4*(256*a**7*c**2*e**4*f**16 - 128*a**6*b**2*c*e**4*f**16 + 16*a**5*
b**4*e**4*f**16) + _t**2*(-80*a**3*b*c**3*e**2*f**8 + 100*a**2*b**3*c**2*e
**2*f**8 - 36*a*b**5*c*e**2*f**8 + 4*b**7*e**2*f**8) + c**5, Lambda(_t, _t
*log(x + (-96*_t**3*a**7*b*c**2*e**3*f**12 + 56*_t**3*a**6*b**3*c*e**3*f**
12 - 8*_t**3*a**5*b**5*e**3*f**12 - 4*_t*a**4*c**4*e*f**4 + 32*_t*a**3*b**
2*c**3*e*f**4 - 40*_t*a**2*b**4*c**2*e*f**4 + 16*_t*a*b**6*c*e*f**4 - 2*_t
*b**8*e*f**4 + a**2*c**5*d - 3*a*b**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e -
 3*a*b**2*c**4*e + b**4*c**3*e))))
```

Maxima [F]

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)(efx+df)^4}\,dx$$

input
```
integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima
")
```

output
```
1/3*(3*b*e^2*x^2 + 6*b*d*e*x + 3*b*d^2 - a)/(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3
*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4) + integrate((b*c*e^2*x^2 +
 2*b*c*d*e*x + b*c*d^2 + b^2 - a*c)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (
6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a^2*f^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1353 vs. $2(200) = 400$.

Time = 0.14 (sec) , antiderivative size = 1353, normalized size of antiderivative = 5.73

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

output
```
-1/2*((b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
 d/e)^2 - 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^
4)) + d/e) + b*c*d^2 + b^2 - a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^
2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
+ b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e))
 - (b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/
e)^2 + 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 - d/e) + b*c*d^2 + b^2 - a*c)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b
*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) +
(b*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^
2 - 2*b*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e) + b*c*d^2 + b^2 - a*c)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b...
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 5771, normalized size of antiderivative = 24.45

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

output
```
((2*b*d*x)/a^2 - (a - 3*b*d^2)/(3*a^2*e) + (b*e*x^2)/a^2)/(d^3*f^4 + e^3*f
^4*x^3 + 3*d*e^2*f^4*x^2 + 3*d^2*e*f^4*x) - atan((((b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^
8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2
)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e
^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*
b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10
*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20
+ 8*a^10*b^3*c^2*d*e^13*f^20) - 16*a^10*c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12
*f^16 + 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3
*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*
c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12)*1i + ((b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^
(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^
8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - ...
```

Reduce [F]

$$\int \frac{1}{(df+efx)^4 (a+b(d+ex)^2+c(d+ex)^4)}\,dx$$
$$= \frac{\int \frac{1}{c e^8x^8+8cd e^7x^7+28c d^2e^6x^6+56c d^3e^5x^5+b e^6x^6+70c d^4e^4x^4+6bd e^5x^5+56c d^5e^3x^3+15b d^2e^4x^4+28c d^6e^2x^2+20b d^3e^3x^3+8c d^7ex+a e^4x}}{f^4}$$

input
```
int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

output
```
int(1/(a*d**4 + 4*a*d**3*e*x + 6*a*d**2*e**2*x**2 + 4*a*d*e**3*x**3 + a*e*
*4*x**4 + b*d**6 + 6*b*d**5*e*x + 15*b*d**4*e**2*x**2 + 20*b*d**3*e**3*x**
3 + 15*b*d**2*e**4*x**4 + 6*b*d*e**5*x**5 + b*e**6*x**6 + c*d**8 + 8*c*d**
7*e*x + 28*c*d**6*e**2*x**2 + 56*c*d**5*e**3*x**3 + 70*c*d**4*e**4*x**4 +
56*c*d**3*e**5*x**5 + 28*c*d**2*e**6*x**6 + 8*c*d*e**7*x**7 + c*e**8*x**8)
,x)/f**4
```

3.275 $\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 279

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}e}+\frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e}$$

output
```
1/2*f^4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^
4)+1/4*(b-(4*a*c+b^2)/(-4*a*c+b^2)^(1/2))*f^4*arctan(2^(1/2)*c^(1/2)*(e*x+
d)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2)/e+1/4*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))*f^4*arctan(2^(1/2)
*c^(1/2)*(e*x+d)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2
)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= \frac{f^4\left(-\frac{2(-2a(d+ex)-b(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\dots}{\sqrt{b}\dots}\right.}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4\dots}}}\right.}{4e}$$

input
```
Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x
)^2 + c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2
 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(4*e)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1462, 1440, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$\Big\downarrow \text{1462}$$
$$\frac{f^4\int \frac{(d+ex)^4}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{e}$$
$$\Big\downarrow \text{1440}$$

$$\frac{f^4\left(\frac{(d+ex)(2a+b(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int \frac{2a-b(d+ex)^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2(b^2-4ac)}\right)}{e}$$

$\Big\downarrow$ 1480

$$\frac{f^4\left(\frac{(d+ex)(2a+b(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{-\frac{1}{2}\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)-\frac{1}{2}\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{2(b^2-4ac)}\right)}{e}$$

$\Big\downarrow$ 218

$$\frac{f^4\left(\frac{(d+ex)(2a+b(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}\right)}{e}$$

input
```
Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f^4*(((d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)) - (-(((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sq
rt[b - Sqrt[b^2 - 4*a*c]])) - ((b + (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[
c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c))))/e
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.17

method	result
default	$f^4\left(\frac{-\frac{b\,e^2x^3}{2(4ac-b^2)}-\frac{3dbe\,x^2}{2(4ac-b^2)}-\frac{(3b\,d^2+2a)x}{2(4ac-b^2)}-\frac{d(b\,d^2+2a)}{2e(4ac-b^2)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2}}{}\right.$
risch	$\frac{-\frac{b\,e^2f^4x^3}{2(4ac-b^2)}-\frac{3dbe\,f^4x^2}{2(4ac-b^2)}-\frac{f^4(3b\,d^2+2a)x}{2(4ac-b^2)}-\frac{d\,f^4(b\,d^2+2a)}{2e(4ac-b^2)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2+a}+\frac{f^4\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\right.}\right.}{}$

input
```
int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
f^4*((-1/2*b*e^2/(4*a*c-b^2)*x^3-3/2*d*b*e/(4*a*c-b^2)*x^2-1/2*(3*b*d^2+2*
a)/(4*a*c-b^2)*x-1/2*d/e*(b*d^2+2*a)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c
-b^2)/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2*_R^3*c*e^3+6*_R^2*c*d*e^
2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3
*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2578 vs. $2(235) = 470$.

Time = 0.15 (sec) , antiderivative size = 2578, normalized size of antiderivative = 9.24

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas
")
```

output
```
1/4*(2*b*e^3*f^4*x^3 + 6*b*d*e^2*f^4*x^2 + 2*(3*b*d^2 + 2*a)*e*f^4*x + 2*(
b*d^3 + 2*a*d)*f^4 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a
*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2
*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4
 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 +
 sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(
b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^
4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((3*b^2 + 4*a*c)*e*f^12*x +
(3*b^2 + 4*a*c)*d*f^12 + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 +
 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))
*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 +
 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/(
(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((
b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c
+ 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4
*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(252) = 504.

Time = 5.35 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.30

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$

$$= \frac{-2adf^4-bd^3f^4-3bde^2f^4x^2-b\cdots}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2c\cdots}$$

$$+\mathrm{RootSum}\Big(t^4\cdot\big(1048576a^6c^7e^4-1572864a^5b^2c^6e^4+983040a^4b^4c^5e^4-327680a^3b^6c^4e^4+61440a^2b^8c^3\cdots$$

input
```
integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
(-2*a*d*f**4 - b*d**3*f**4 - 3*b*d*e**2*f**4*x**2 - b*e**3*f**4*x**3 + x*(
-2*a*e*f**4 - 3*b*d**2*e*f**4))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e
+ 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5
- 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b
*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16
*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
 + RootSum(_t**4*(1048576*a**6*c**7*e**4 - 1572864*a**5*b**2*c**6*e**4 + 9
83040*a**4*b**4*c**5*e**4 - 327680*a**3*b**6*c**4*e**4 + 61440*a**2*b**8*c
**3*e**4 - 6144*a*b**10*c**2*e**4 + 256*b**12*c*e**4) + _t**2*(-12288*a**4
*b*c**4*e**2*f**8 + 8192*a**3*b**3*c**3*e**2*f**8 - 1536*a**2*b**5*c**2*e*
*2*f**8 + 16*b**9*e**2*f**8) + 16*a**3*c**2*f**16 + 24*a**2*b**2*c*f**16 +
 9*a*b**4*f**16, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3 - 122
88*_t**3*a**2*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t**3*b**
7*c*e**3 + 64*_t*a**2*c**2*e*f**8 - 128*_t*a*b**2*c*e*f**8 - 4*_t*b**4*e*f
**8 + 4*a*c*d*f**12 + 3*b**2*d*f**12)/(4*a*c*e*f**12 + 3*b**2*e*f**12)))
```

Maxima [F]

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{(efx+df)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima
")
```

output
```
-1/2*f^4*integrate(-(b*e^2*x^2 + 2*b*d*e*x + b*d^2 - 2*a)/((b^2*c - 4*a*c^
2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3
- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/
2*(b*e^3*f^4*x^3 + 3*b*d*e^2*f^4*x^2 + (3*b*d^2 + 2*a)*e*f^4*x + (b*d^3 +
2*a*d)*f^4)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (
b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*
d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*d^2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1474 vs. $2(235) = 470$.

Time = 0.13 (sec) , antiderivative size = 1474, normalized size of antiderivative = 5.28

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
-1/4*((b*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 + d/e)^2 - 2*b*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(
c*e^4)) + d/e) + b*d^2*f^4 - 2*a*f^4)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*
e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d
/e)) - (b*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) - d/e)^2 + 2*b*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/
(c*e^4)) - d/e) + b*d^2*f^4 - 2*a*f^4)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2
*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) -
d/e)) + (b*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) + d/e)^2 - 2*b*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) + d/e) + b*d^2*f^4 - 2*a*f^4)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^
2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c...
```

Mupad [B] (verification not implemented)

Time = 12.71 (sec) , antiderivative size = 8025, normalized size of antiderivative = 28.76

$$\int \frac{(df + efx)^4}{(a + b(d + ex)^2 + c(d + ex)^4)^2} \, dx = \text{Too large to display}$$

input
```
int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
atan(((((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c
^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*
b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*
a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^
4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 +
4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*
c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(-(b^9*f^8
 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 +
 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e
^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6
144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^
8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)) + (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/
(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1
/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b
^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1
280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1...
```

Reduce [F]

$$\int \frac{(df + efx)^4}{(a + b(d + ex)^2 + c(d + ex)^4)^2} \, dx = \text{too large to display}$$

input
```
int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

output
```
(f**4*(2*int(x**3/(a**2 + 2*a*b*d**2 + 4*a*b*d*e*x + 2*a*b*e**2*x**2 + 2*a
*c*d**4 + 8*a*c*d**3*e*x + 12*a*c*d**2*e**2*x**2 + 8*a*c*d*e**3*x**3 + 2*a
*c*e**4*x**4 + b**2*d**4 + 4*b**2*d**3*e*x + 6*b**2*d**2*e**2*x**2 + 4*b**
2*d*e**3*x**3 + b**2*e**4*x**4 + 2*b*c*d**6 + 12*b*c*d**5*e*x + 30*b*c*d**
4*e**2*x**2 + 40*b*c*d**3*e**3*x**3 + 30*b*c*d**2*e**4*x**4 + 12*b*c*d*e**
5*x**5 + 2*b*c*e**6*x**6 + c**2*d**8 + 8*c**2*d**7*e*x + 28*c**2*d**6*e**2
*x**2 + 56*c**2*d**5*e**3*x**3 + 70*c**2*d**4*e**4*x**4 + 56*c**2*d**3*e**
5*x**5 + 28*c**2*d**2*e**6*x**6 + 8*c**2*d*e**7*x**7 + c**2*e**8*x**8),x)*
a*b**2*c*e**4 + 4*int(x**3/(a**2 + 2*a*b*d**2 + 4*a*b*d*e*x + 2*a*b*e**2*x
**2 + 2*a*c*d**4 + 8*a*c*d**3*e*x + 12*a*c*d**2*e**2*x**2 + 8*a*c*d*e**3*x
**3 + 2*a*c*e**4*x**4 + b**2*d**4 + 4*b**2*d**3*e*x + 6*b**2*d**2*e**2*x**
2 + 4*b**2*d*e**3*x**3 + b**2*e**4*x**4 + 2*b*c*d**6 + 12*b*c*d**5*e*x + 3
0*b*c*d**4*e**2*x**2 + 40*b*c*d**3*e**3*x**3 + 30*b*c*d**2*e**4*x**4 + 12*
b*c*d*e**5*x**5 + 2*b*c*e**6*x**6 + c**2*d**8 + 8*c**2*d**7*e*x + 28*c**2*
d**6*e**2*x**2 + 56*c**2*d**5*e**3*x**3 + 70*c**2*d**4*e**4*x**4 + 56*c**2
*d**3*e**5*x**5 + 28*c**2*d**2*e**6*x**6 + 8*c**2*d*e**7*x**7 + c**2*e**8*
x**8),x)*a*b*c**2*d**2*e**4 + 2*int(x**3/(a**2 + 2*a*b*d**2 + 4*a*b*d*e*x
+ 2*a*b*e**2*x**2 + 2*a*c*d**4 + 8*a*c*d**3*e*x + 12*a*c*d**2*e**2*x**2 +
8*a*c*d*e**3*x**3 + 2*a*c*e**4*x**4 + b**2*d**4 + 4*b**2*d**3*e*x + 6*b**2
*d**2*e**2*x**2 + 4*b**2*d*e**3*x**3 + b**2*e**4*x**4 + 2*b*c*d**6 + 12...
```

3.276 $\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 103

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^3\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{bf^3\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

output
```
1/2*f^3*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-b*f^3
*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^3\left(\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} - \frac{2b\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}\right)}{2e}$$

input
```
Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f^3*((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*
x)^2))) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4
*a*c)^(3/2)))/(2*e)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1462, 1434, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}dx$$

$$\Big\downarrow \ 1462$$

$$\frac{f^3\int \frac{(d+ex)^3}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{e}$$

$$\Big\downarrow \ 1434$$

$$\frac{f^3\int \frac{(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2e}$$

$$\Big\downarrow \ 1159$$

$$\frac{f^3\left(\frac{b\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{b^2-4ac}+\frac{2a+b(d+ex)^2}{(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{2e}$$

$$\Big\downarrow \ 1083$$

$$\frac{f^3\left(\frac{2a+b(d+ex)^2}{(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{2b\int \frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{b^2-4ac}\right)}{2e}$$

$$\Big\downarrow \ 219$$

$$\frac{f^3\left(\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{2b\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\right)}{2e}$$

input
```
Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f^3*((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)) - (2*b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*
c)^(3/2)))/(2*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.72

method	result
default	$f^3\left(\frac{-\frac{be\,x^2}{2(4ac-b^2)}-\frac{bdx}{4ac-b^2}-\frac{b\,d^2+2a}{2e(4ac-b^2)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2+a}+\frac{b\left(\sum_{\_R=\text{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_\right.}\right.}{}\right.$
risch	$\frac{-\frac{be\,f^3x^2}{2(4ac-b^2)}-\frac{bd\,f^3x}{4ac-b^2}-\frac{f^3\left(b\,d^2+2a\right)}{2e(4ac-b^2)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2+a}+\frac{f^3b\ln\left(\left(\left(-\left(-4ac+b^2\right)^{\frac{3}{2}}e^2+4abc\,e^2-b^3e^2\right)x^2+\left(-2(-4a\right.\right.\right.}{}$

input
```
int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
f^3*((-1/2*b*e/(4*a*c-b^2)*x^2-b*d/(4*a*c-b^2)*x-1/2/e*(b*d^2+2*a)/(4*a*c-
b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)+1/2/(4*a*c-b^2)*b/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2
*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*
c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(97) = 194$.

Time = 0.13 (sec) , antiderivative size = 1077, normalized size of antiderivative = 10.46

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas
")
```

output
```
[1/2*((b^3 - 4*a*b*c)*e^2*f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2
 - 8*a^2*c + (b^3 - 4*a*b*c)*d^2)*f^3 - (b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3
*x^3 + (6*b*c*d^2 + b^2)*e^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*
c*d^4 + b^2*d^2 + a*b)*f^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d
*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*
d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*s
qrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x
^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*
b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 +
 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), 1/2*((b^3 -
4*a*b*c)*e^2*f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2 - 8*a^2*c +
(b^3 - 4*a*b*c)*d^2)*f^3 - 2*(b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3*x^3 + (6*b
*c*d^2 + b^2)*e^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*c*d^4 + b^2
*d^2 + a*b)*f^3)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c
*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8
*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(88) = 176$.

Time = 2.70 (sec) , antiderivative size = 556, normalized size of antiderivative = 5.40

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$

$$= \frac{bf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3cf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e}$$
$$-\frac{bf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2bc^2f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3cf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e}$$
$$+\frac{-2af^3-bd^2f^3-}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b}$$

input
```
integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**2*f*
*3*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**3*c*f**3*sqrt(-1/(4*a*c - b**2)**3)
 - b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3 + 2*b*c*d**2*f**3)/(2*
b*c*e**2*f**3))/(2*e) - b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x*
*2 + (16*a**2*b*c**2*f**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*f**3*sqr
t(-1/(4*a*c - b**2)**3) + b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3
 + 2*b*c*d**2*f**3)/(2*b*c*e**2*f**3))/(2*e) + (-2*a*f**3 - b*d**2*f**3 -
2*b*d*e*f**3*x - b*e**2*f**3*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2
*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e*
*5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*
a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*
(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**
2))
```

Maxima [F]

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\int \frac{(efx+df)^3}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima
")
```

output
```
-b*f^3*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^
2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2
)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a
*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^2*f^3*x^2 + 2*b*d*e*f^3
*x + (b*d^2 + 2*a)*f^3)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d
*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c
 - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^
2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(97) = 194$.

Time = 0.15 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.96

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\frac{bf^3\arctan\left(\frac{2\,cd^2f+2\,(efx^2+2\,dfx)ce+bf}{\sqrt{-b^2+4\,ac}f}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}e}+\frac{bd^2f^5+(efx^2+2\,dfx)bef^4+2\,af^5}{2\,\left(cd^4f^2+2\,(efx^2+2\,dfx)cd^2ef+(efx^2+2\,dfx)^2ce^2+bd^2f^2+(efx^2+2\,dfx)bef+af^2\right)(b^2e-4\,a}$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
b*f^3*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*
a*c)*f))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*e) + 1/2*(b*d^2*f^5 + (e*f*x^2
+ 2*d*f*x)*b*e*f^4 + 2*a*f^5)/((c*d^4*f^2 + 2*(e*f*x^2 + 2*d*f*x)*c*d^2*e*
f + (e*f*x^2 + 2*d*f*x)^2*c*e^2 + b*d^2*f^2 + (e*f*x^2 + 2*d*f*x)*b*e*f +
a*f^2)*(b^2*e - 4*a*c*e))
```

Mupad [B] (verification not implemented)

Time = 11.14 (sec) , antiderivative size = 460, normalized size of antiderivative = 4.47

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$

$$= \frac{b\,f^3\,\mathrm{atan}\left(\frac{(4\,a\,c-b^2)^4\left(x\left(\frac{b^3\,f^6\,(2\,b^3\,c^2\,d\,e^9-8\,a\,b\,c^3\,d\,e^9)}{a\,e^2\,(4\,a\,c-b^2)^{11/2}}-\frac{2\,b^2\,c^2\,d\,e^7\,f^6}{a\,(4\,a\,c-b^2)^{7/2}}\right)+x^2\left(\frac{b^3\,f^6\,(2\,b^3\,c^2\,e^{10}-8\,a\,b\,c^3\,e^{10})}{2\,a\,e^2\,(4\,a\,c-b^2)^{11/2}}-\frac{b^2\,c^2\,e^8\,f^6}{a\,(4\,a\,c-b^2)^{7/2}}\right)-\frac{b^3\,f^6 \cdots}{\cdots}\right)}{2\,b^2\,c^2\,e^6\,f^6}\right)}{e\,(4\,a\,c-b^2)^{3/2}}$$

$$-\frac{\frac{f^3\,(b\,d^2+2\,a)}{2\,e\,(4\,a\,c-b^2)}+\frac{b\,d\,f^3\,x}{4\,a\,c-b^2}+\frac{b\,e\,f^3\,x^2}{2\,(4\,a\,c-b^2)}}{a+x^2\,(6\,c\,d^2\,e^2+b\,e^2)+b\,d^2+c\,d^4+x\,(4\,c\,e\,d^3+2\,b\,e\,d)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3}$$

input
```
int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
(b*f^3*atan(((4*a*c - b^2)^4*(x*((b^3*f^6*(2*b^3*c^2*d*e^9 - 8*a*b*c^3*d*e
^9))/(a*e^2*(4*a*c - b^2)^(11/2)) - (2*b^2*c^2*d*e^7*f^6)/(a*(4*a*c - b^2)
^(7/2))) + x^2*((b^3*f^6*(2*b^3*c^2*e^10 - 8*a*b*c^3*e^10))/(2*a*e^2*(4*a*
c - b^2)^(11/2)) - (b^2*c^2*e^8*f^6)/(a*(4*a*c - b^2)^(7/2))) - (b^3*f^6*(
16*a^2*c^3*e^8 - 4*a*b^2*c^2*e^8 - 2*b^3*c^2*d^2*e^8 + 8*a*b*c^3*d^2*e^8))
/(2*a*e^2*(4*a*c - b^2)^(11/2)) - (b^2*c^2*d^2*e^6*f^6)/(a*(4*a*c - b^2)^(
7/2))))/(2*b^2*c^2*e^6*f^6)))/(e*(4*a*c - b^2)^(3/2)) - ((f^3*(2*a + b*d^2
))/(2*e*(4*a*c - b^2)) + (b*d*f^3*x)/(4*a*c - b^2) + (b*e*f^3*x^2)/(2*(4*a
*c - b^2)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e +
4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

Reduce [F]

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{too large to display}$$

input
```
int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

output

```
(f**3*(20*int(x**3/(a**3 + 3*a**2*b*d**2 + 4*a**2*b*d*e*x + 2*a**2*b*e**2*
x**2 + 3*a**2*c*d**4 + 8*a**2*c*d**3*e*x + 12*a**2*c*d**2*e**2*x**2 + 8*a*
*2*c*d*e**3*x**3 + 2*a**2*c*e**4*x**4 + 3*a*b**2*d**4 + 8*a*b**2*d**3*e*x
+ 8*a*b**2*d**2*e**2*x**2 + 4*a*b**2*d*e**3*x**3 + a*b**2*e**4*x**4 + 6*a*
b*c*d**6 + 24*a*b*c*d**5*e*x + 44*a*b*c*d**4*e**2*x**2 + 48*a*b*c*d**3*e**
3*x**3 + 32*a*b*c*d**2*e**4*x**4 + 12*a*b*c*d*e**5*x**5 + 2*a*b*c*e**6*x**
6 + 3*a*c**2*d**8 + 16*a*c**2*d**7*e*x + 40*a*c**2*d**6*e**2*x**2 + 64*a*c
**2*d**5*e**3*x**3 + 72*a*c**2*d**4*e**4*x**4 + 56*a*c**2*d**3*e**5*x**5 +
 28*a*c**2*d**2*e**6*x**6 + 8*a*c**2*d*e**7*x**7 + a*c**2*e**8*x**8 + b**3
*d**6 + 4*b**3*d**5*e*x + 6*b**3*d**4*e**2*x**2 + 4*b**3*d**3*e**3*x**3 +
b**3*d**2*e**4*x**4 + 3*b**2*c*d**8 + 16*b**2*c*d**7*e*x + 36*b**2*c*d**6*
e**2*x**2 + 44*b**2*c*d**5*e**3*x**3 + 31*b**2*c*d**4*e**4*x**4 + 12*b**2*
c*d**3*e**5*x**5 + 2*b**2*c*d**2*e**6*x**6 + 3*b*c**2*d**10 + 20*b*c**2*d*
*9*e*x + 58*b*c**2*d**8*e**2*x**2 + 96*b*c**2*d**7*e**3*x**3 + 100*b*c**2*
d**6*e**4*x**4 + 68*b*c**2*d**5*e**5*x**5 + 30*b*c**2*d**4*e**6*x**6 + 8*b
*c**2*d**3*e**7*x**7 + b*c**2*d**2*e**8*x**8 + c**3*d**12 + 8*c**3*d**11*e
*x + 28*c**3*d**10*e**2*x**2 + 56*c**3*d**9*e**3*x**3 + 70*c**3*d**8*e**4*
x**4 + 56*c**3*d**7*e**5*x**5 + 28*c**3*d**6*e**6*x**6 + 8*c**3*d**5*e**7*
x**7 + c**3*d**4*e**8*x**8),x)*a**2*b**4*c*e**4 + 232*int(x**3/(a**3 + 3*a
**2*b*d**2 + 4*a**2*b*d*e*x + 2*a**2*b*e**2*x**2 + 3*a**2*c*d**4 + 8*a*...
```

3.277 $\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 263

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

output
```
-1/2*f^2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)
^4)+1/2*c^(1/2)*(2*b-(-4*a*c+b^2)^(1/2))*f^2*arctan(2^(1/2)*c^(1/2)*(e*x+d
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2)/e-1/2*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))*f^2*arctan(2^(1/2)*c^
(1/2)*(e*x+d)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(-4*a*c+b^2)^(3/2)/(b+
(-4*a*c+b^2)^(1/2))^(1/2)/e
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.95

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx =$$

$$-\frac{f^2\left(\frac{b(d+ex)+2c(d+ex)^3}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+}{\sqrt{b+\sqrt{b^2}}}\right.}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{2e}$$

input
```
Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
-1/2*(f^2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^
2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/e
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1462, 1439, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f^2\int \frac{(d+ex)^2}{(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)}{e}$$

$$\Big\downarrow \text{1439}$$

$$\frac{f^2\left(\frac{\int \frac{b-2c(d+ex)^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2(b^2-4ac)}-\frac{(d+ex)(b+2c(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}\right)}{e}$$

$\Big\downarrow$ 1480

$$\frac{f^2\left(\frac{-c\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)-c\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{2(b^2-4ac)}-\frac{(d+ex)(b+2c(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}\right)}{e}$$

$\Big\downarrow$ 218

$$\frac{f^2\left(\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-\frac{(d+ex)(b+2c(d+ex)^2)}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}\right)}{e}$$

input
```
Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f^2*(-1/2*((d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + b*(d + e*
x)^2 + c*(d + e*x)^4)) + (-((Sqrt[2]*Sqrt[c]*(1 - (2*b)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*(1 + (2*b)/Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt
[b^2 - 4*a*c]])/(2*(b^2 - 4*a*c))))/e
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1439
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
1)*(b^2 - 4*a*c))), x] - Simp[d^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.23

method	result
default	$f^{2}\left(\frac{\frac{c\,e^{2}x^{3}}{4ac-b^{2}}+\frac{3dce\,x^{2}}{4ac-b^{2}}+\frac{\left(6c\,d^{2}+b\right)x}{8ac-2b^{2}}+\frac{d\left(2c\,d^{2}+b\right)}{2e\left(4ac-b^{2}\right)}}{c\,e^{4}x^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+c\,d^{4}+2bdex+b\,d^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}\right.}}{}\right.$
risch	$\frac{\frac{c\,e^{2}f^{2}x^{3}}{4ac-b^{2}}+\frac{3dce\,f^{2}x^{2}}{4ac-b^{2}}+\frac{f^{2}\left(6c\,d^{2}+b\right)x}{8ac-2b^{2}}+\frac{d\,f^{2}\left(2c\,d^{2}+b\right)}{2e\left(4ac-b^{2}\right)}}{c\,e^{4}x^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+c\,d^{4}+2bdex+b\,d^{2}+a}+\frac{f^{2}\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z^{2}+\right.}\right.}{}$

input
```
int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
f^2*((c*e^2/(4*a*c-b^2)*x^3+3*d*c*e/(4*a*c-b^2)*x^2+1/2*(6*c*d^2+b)/(4*a*c
-b^2)*x+1/2*d/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*
e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*s
um((2*_R^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c
*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6
*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. $2(222) = 444$.

Time = 0.27 (sec) , antiderivative size = 2600, normalized size of antiderivative = 9.89

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas
")
```

output
```
-1/4*(4*c*e^3*f^2*x^3 + 12*c*d*e^2*f^2*x^2 + 2*(6*c*d^2 + b)*e*f^2*x + 2*(
2*c*d^3 + b*d)*f^2 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a
*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2
*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4
 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 +
 (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b
^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*f^6*x
+ (3*b^2*c + 4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c
^2)*e*f^4 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8
/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^3)*sqrt(-
((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*
e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1
/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*
a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b
^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48
*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(240) = 480$.

Time = 11.83 (sec) , antiderivative size = 646, normalized size of antiderivative = 2.46

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \frac{bdf^2+2cd^3f^2+6cde^2f^2x^2+2\ldots}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2c\ldots}$$
$$+\mathrm{RootSum}\Big(t^4\cdot\big(1048576a^7c^6e^4-1572864a^6b^2c^5e^4+983040a^5b^4c^4e^4-327680a^4b^6c^3e^4+61440a^3b^8c^2\ldots$$

input
```
integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
(b*d*f**2 + 2*c*d**3*f**2 + 6*c*d*e**2*f**2*x**2 + 2*c*e**3*f**2*x**3 + x*
(b*e*f**2 + 6*c*d**2*e*f**2))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e +
8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 -
2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c
*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a
*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) +
 RootSum(_t**4*(1048576*a**7*c**6*e**4 - 1572864*a**6*b**2*c**5*e**4 + 983
040*a**5*b**4*c**4*e**4 - 327680*a**4*b**6*c**3*e**4 + 61440*a**3*b**8*c**
2*e**4 - 6144*a**2*b**10*c*e**4 + 256*a*b**12*e**4) + _t**2*(-12288*a**4*b
*c**4*e**2*f**4 + 8192*a**3*b**3*c**3*e**2*f**4 - 1536*a**2*b**5*c**2*e**2
*f**4 + 16*b**9*e**2*f**4) + 16*a**2*c**3*f**8 + 24*a*b**2*c**2*f**8 + 9*b
**4*c*f**8, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4*e**3 - 8192*_t**3
*a**4*b**2*c**3*e**3 + 512*_t**3*a**2*b**6*c*e**3 - 64*_t**3*a*b**8*e**3 -
 128*_t*a**2*b*c**2*e*f**4 - 16*_t*a*b**3*c*e*f**4 - 4*_t*b**5*e*f**4 + 4*
a*c**2*d*f**6 + 3*b**2*c*d*f**6)/(4*a*c**2*e*f**6 + 3*b**2*c*e*f**6))))
```

Maxima [F]

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \int \frac{(efx+df)^2}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima
")
```

output
```
1/2*f^2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*c - 4*a*c
^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3
 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4
*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1
/2*(2*c*e^3*f^2*x^3 + 6*c*d*e^2*f^2*x^2 + (6*c*d^2 + b)*e*f^2*x + (2*c*d^3
 + b*d)*f^2)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 +
(b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)
*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c
 + (b^3 - 4*a*b*c)*d^2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1482 vs. $2(222) = 444$.

Time = 0.14 (sec) , antiderivative size = 1482, normalized size of antiderivative = 5.63

$$\int \frac{(df + efx)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^2}\, dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
1/4*((2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)
) + d/e)^2 - 4*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/
(c*e^4)) + d/e) + 2*c*d^2*f^2 - b*f^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2
*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) +
d/e)) - (2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) - d/e)^2 + 4*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) - d/e) + 2*c*d^2*f^2 - b*f^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*
e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*
d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 - d/e)) + (2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(
c*e^4)) + d/e)^2 - 4*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)
*e^2)/(c*e^4)) + d/e) + 2*c*d^2*f^2 - b*f^2)*log(x + sqrt(1/2)*sqrt(-(b*e^
2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^
2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e +...
```

Mupad [B] (verification not implemented)

Time = 12.64 (sec) , antiderivative size = 7835, normalized size of antiderivative = 29.79

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
((x*(b*f^2 + 6*c*d^2*f^2))/(2*(4*a*c - b^2)) + (2*c*d^3*f^2 + b*d*f^2)/(2*
e*(4*a*c - b^2)) + (c*e^2*f^2*x^3)/(4*a*c - b^2) + (3*c*d*e*f^2*x^2)/(4*a*
c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*
c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + atan(((((f^4*(-(4*a*c - b^2)^9)^(1
/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3
*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8
*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*
e^2))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b
*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*
c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 +
 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 -
 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2
048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) +
 (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3
*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c - b^2)^9)^(1/2
))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c
^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c
^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^
2))^(1/2) + (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e
^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - ...
```

Reduce [F]

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{too large to display}$$

input
```
int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

output
```
(f**2*( - 4*int(x**3/(a**2*b + 2*a**2*c*d**2 + 2*a*b**2*d**2 + 4*a*b**2*d*
e*x + 2*a*b**2*e**2*x**2 + 6*a*b*c*d**4 + 16*a*b*c*d**3*e*x + 16*a*b*c*d**
2*e**2*x**2 + 8*a*b*c*d*e**3*x**3 + 2*a*b*c*e**4*x**4 + 4*a*c**2*d**6 + 16
*a*c**2*d**5*e*x + 24*a*c**2*d**4*e**2*x**2 + 16*a*c**2*d**3*e**3*x**3 + 4
*a*c**2*d**2*e**4*x**4 + b**3*d**4 + 4*b**3*d**3*e*x + 6*b**3*d**2*e**2*x*
*2 + 4*b**3*d*e**3*x**3 + b**3*e**4*x**4 + 4*b**2*c*d**6 + 20*b**2*c*d**5*
e*x + 42*b**2*c*d**4*e**2*x**2 + 48*b**2*c*d**3*e**3*x**3 + 32*b**2*c*d**2
*e**4*x**4 + 12*b**2*c*d*e**5*x**5 + 2*b**2*c*e**6*x**6 + 5*b*c**2*d**8 +
32*b*c**2*d**7*e*x + 88*b*c**2*d**6*e**2*x**2 + 136*b*c**2*d**5*e**3*x**3
+ 130*b*c**2*d**4*e**4*x**4 + 80*b*c**2*d**3*e**5*x**5 + 32*b*c**2*d**2*e*
*6*x**6 + 8*b*c**2*d*e**7*x**7 + b*c**2*e**8*x**8 + 2*c**3*d**10 + 16*c**3
*d**9*e*x + 56*c**3*d**8*e**2*x**2 + 112*c**3*d**7*e**3*x**3 + 140*c**3*d*
*6*e**4*x**4 + 112*c**3*d**5*e**5*x**5 + 56*c**3*d**4*e**6*x**6 + 16*c**3*
d**3*e**7*x**7 + 2*c**3*d**2*e**8*x**8),x)*a*b*c*d*e**4 - 8*int(x**3/(a**2
*b + 2*a**2*c*d**2 + 2*a*b**2*d**2 + 4*a*b**2*d*e*x + 2*a*b**2*e**2*x**2 +
 6*a*b*c*d**4 + 16*a*b*c*d**3*e*x + 16*a*b*c*d**2*e**2*x**2 + 8*a*b*c*d*e*
*3*x**3 + 2*a*b*c*e**4*x**4 + 4*a*c**2*d**6 + 16*a*c**2*d**5*e*x + 24*a*c*
*2*d**4*e**2*x**2 + 16*a*c**2*d**3*e**3*x**3 + 4*a*c**2*d**2*e**4*x**4 + b
**3*d**4 + 4*b**3*d**3*e*x + 6*b**3*d**2*e**2*x**2 + 4*b**3*d*e**3*x**3 +
b**3*e**4*x**4 + 4*b**2*c*d**6 + 20*b**2*c*d**5*e*x + 42*b**2*c*d**4*e*...
```

3.278 $\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 98

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=-\frac{f(b+2c(d+ex)^2)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{2cf\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

output
```
-1/2*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+2*c*f*
arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=-\frac{f\left(\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}+\frac{4c\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}\right)}{2\left(b^2-4ac\right)e}$$

input
```
Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
-1/2*(f*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*
ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]))/((b
^2 - 4*a*c)*e)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1462, 1432, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^2}dx$$

$$\Big\downarrow \text{ 1462}$$

$$\frac{f\int \frac{d+ex}{(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)}{e}$$

$$\Big\downarrow \text{ 1432}$$

$$\frac{f\int \frac{1}{(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{ 1086}$$

$$\frac{f\left(-\frac{2c\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{b^2-4ac}-\frac{b+2c(d+ex)^2}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}\right)}{2e}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{f\left(\frac{4c\int \frac{1}{-(d+ex)^4+b^2-4ac}d(2c(d+ex)^2+b)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}\right)}{2e}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{f\left(\frac{4c\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2c(d+ex)^2}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}\right)}{2e}$$

input
```
Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

output
```
(f*(-((b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x
)^4))) + (4*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a
*c)^(3/2)))/(2*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.78

method	result
default	$f\left(\frac{\frac{c e\,x^{2}}{4ac-b^{2}}+\frac{2cdx}{4ac-b^{2}}+\frac{2c\,d^{2}+b}{2e\left(4ac-b^{2}\right)}}{c\,e^{4}x^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+c\,d^{4}+2bdex+b\,d^{2}+a}+\frac{c\left(\sum_{\_R=\mathrm{RootOf}\left(c\,e^{4}\_Z^{4}+4cd\,e^{3}\_Z^{3}+\left(6c\,d^{2}e^{2}+b\,e^{2}\right)\_Z \cdots\right.}\cdots\right.}{\cdots}\right.$
risch	$\frac{\frac{cef\,x^{2}}{4ac-b^{2}}+\frac{2cdfx}{4ac-b^{2}}+\frac{f\left(2c\,d^{2}+b\right)}{2e\left(4ac-b^{2}\right)}}{c\,e^{4}x^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+c\,d^{4}+2bdex+b\,d^{2}+a}+\frac{fc\ln\left(\left(\left(\left(-4ac+b^{2}\right)^{\frac{3}{2}}e^{2}+4abc\,e^{2}-b^{3}e^{2}\right)x^{2}+\left(2\left(-4ac+b^{2} \cdots\right.\right.\right.\right.}{\cdots}$

input
```
int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
f*((c*e/(4*a*c-b^2)*x^2+2*c*d/(4*a*c-b^2)*x+1/2/e*(2*c*d^2+b)/(4*a*c-b^2))
/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)+c/(4*a*c-b^2)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6
*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z
^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(92) = 184.

Time = 0.11 (sec) , antiderivative size = 1066, normalized size of antiderivative = 10.88

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(b^2*c - 4*a*c^2)*e^2*f*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x + 2*(c^
2*e^4*f*x^4 + 4*c^2*d*e^3*f*x^3 + (6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d
^3 + b*c*d)*e*f*x + (c^2*d^4 + b*c*d^2 + a*c)*f)*sqrt(b^2 - 4*a*c)*log((2*
c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 +
2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d
*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4
+ (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) + (b^3 - 4*a
*b*c + 2*(b^2*c - 4*a*c^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5
*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c +
 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d
)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*
a*c^2)*e^2*f*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x - 4*(c^2*e^4*f*x^4 + 4*c^2*
d*e^3*f*x^3 + (6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d^3 + b*c*d)*e*f*x +
(c^2*d^4 + b*c*d^2 + a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c
*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c +
 2*(b^2*c - 4*a*c^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 +
 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(87) = 174$.

Time = 2.60 (sec) , antiderivative size = 525, normalized size of antiderivative = 5.36

$$\int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^2}\, dx =$$

$$-\frac{cf\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e} + x^2 + \frac{-16a^2c^3f\sqrt{-\frac{1}{(4ac-b^2)^3}} + 8ab^2c^2f\sqrt{-\frac{1}{(4ac-b^2)^3}} - b^4cf\sqrt{-\frac{1}{(4ac-b^2)^3}} + bcf + 2c^2d^2f}{2c^2e^2f}\right)}{e}$$

$$+\frac{cf\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e} + x^2 + \frac{16a^2c^3f\sqrt{-\frac{1}{(4ac-b^2)^3}} - 8ab^2c^2f\sqrt{-\frac{1}{(4ac-b^2)^3}} + b^4cf\sqrt{-\frac{1}{(4ac-b^2)^3}} + bcf + 2c^2d^2f}{2c^2e^2f}\right)}{e}$$

$$+\frac{bf + 2cd^2f + 4}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\cdot(8ac^2e^5 - 2b^2ce^5) + x^3\cdot(32ac^2de^4 - 8b}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
-c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3*f*sqrt
(-1/(4*a*c - b**2)**3) + 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) - b**4
*c*f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/
e + c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*c**3*f*sq
rt(-1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) + b*
*4*c*f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f)
)/e + (b*f + 2*c*d**2*f + 4*c*d*e*f*x + 2*c*e**2*f*x**2)/(8*a**2*c*e - 2*a
*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4
*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**
2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*
b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e*
*2 - 8*b**2*c*d**3*e**2))
```

Maxima [F]

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx=\int \frac{efx+df}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2}\,dx$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

output
```
2*c*f*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2
)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)
*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*
c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^2*f*x^2 + 4*c*d*e*f*x
+ (2*c*d^2 + b)*f)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*
x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*
a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4
*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(92) = 184$.

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.06

$$\int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^2}\,dx = -\frac{2\,cf\arctan\left(\frac{2\,cd^2f + 2\,(efx^2 + 2\,dfx)ce + bf}{\sqrt{-b^2 + 4\,acf}}\right)}{(b^2 - 4\,ac)\sqrt{-b^2 + 4\,ace}} - \frac{2\,cd^2f^3 + 2\,(efx^2 + 2\,dfx)cef^2 + bf^3}{2\left(cd^4f^2 + 2\,(efx^2 + 2\,dfx)cd^2ef + (efx^2 + 2\,dfx)^2ce^2 + bd^2f^2 + (efx^2 + 2\,dfx)bef + af^2\right)(b^2e - 4\,a}$$

input

```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output

```
-2*c*f*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4
*a*c)*f))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*e) - 1/2*(2*c*d^2*f^3 + 2*(e*f
*x^2 + 2*d*f*x)*c*e*f^2 + b*f^3)/((c*d^4*f^2 + 2*(e*f*x^2 + 2*d*f*x)*c*d^2
*e*f + (e*f*x^2 + 2*d*f*x)^2*c*e^2 + b*d^2*f^2 + (e*f*x^2 + 2*d*f*x)*b*e*f
 + a*f^2)*(b^2*e - 4*a*c*e))
```

Mupad [B] (verification not implemented)

Time = 11.01 (sec) , antiderivative size = 442, normalized size of antiderivative = 4.51

$$\int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^2}\,dx$$
$$= \frac{\frac{f\,(2\,c\,d^2+b)}{2\,e\,(4\,a\,c-b^2)} + \frac{2\,c\,d\,f\,x}{4\,a\,c-b^2} + \frac{c\,e\,f\,x^2}{4\,a\,c-b^2}}{a + x^2\,\left(6\,c\,d^2\,e^2 + b\,e^2\right) + b\,d^2 + c\,d^4 + x\,\left(4\,c\,e\,d^3 + 2\,b\,e\,d\right) + c\,e^4\,x^4 + 4\,c\,d\,e^3\,x^3}$$
$$+ \frac{2\,c\,f\,\mathrm{atan}\left(\frac{{\left(4\,a\,c-b^2\right)}^4\,\left(x\,\left(\frac{8\,c^4\,d\,e^7\,f^2}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}} - \frac{8\,b\,c^2\,f^2\,\left(b^3\,c^2\,d\,e^9-4\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}\right) + x^2\,\left(\frac{4\,c^4\,e^8\,f^2}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}} - \frac{4\,b\,c^2\,f^2\,\left(b^3\,c^2\,e^{10}-4\,a\,b\,c^3\,e^{10}\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}\right)}{8\,c^4\,e^6\,f^2}}{e\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input

```
int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

output
```
((f*(b + 2*c*d^2))/(2*e*(4*a*c - b^2)) + (2*c*d*f*x)/(4*a*c - b^2) + (c*e*
f*x^2)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(
2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + (2*c*f*atan(((4*a*c -
b^2)^4*(x*((8*c^4*d*e^7*f^2)/(a*(4*a*c - b^2)^(7/2)) - (8*b*c^2*f^2*(b^3*c
^2*d*e^9 - 4*a*b*c^3*d*e^9))/(a*e^2*(4*a*c - b^2)^(11/2))) + x^2*((4*c^4*e
^8*f^2)/(a*(4*a*c - b^2)^(7/2)) - (4*b*c^2*f^2*(b^3*c^2*e^10 - 4*a*b*c^3*e
^10))/(a*e^2*(4*a*c - b^2)^(11/2))) + (4*c^4*d^2*e^6*f^2)/(a*(4*a*c - b^2)
^(7/2)) + (4*b*c^2*f^2*(8*a^2*c^3*e^8 - 2*a*b^2*c^2*e^8 - b^3*c^2*d^2*e^8
+ 4*a*b*c^3*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(11/2))))/(8*c^4*e^6*f^2)))/(e*
(4*a*c - b^2)^(3/2))
```

Reduce [F]

$$\int \frac{df + efx}{\left(a + b(d + ex)^2 + c(d + ex)^4\right)^2}\,dx = \text{too large to display}$$

input
```
int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

output

```
(f*( - 4*int(x**3/(a**2*b + 2*a**2*c*d**2 + 2*a*b**2*d**2 + 4*a*b**2*d*e*x
 + 2*a*b**2*e**2*x**2 + 6*a*b*c*d**4 + 16*a*b*c*d**3*e*x + 16*a*b*c*d**2*e
**2*x**2 + 8*a*b*c*d*e**3*x**3 + 2*a*b*c*e**4*x**4 + 4*a*c**2*d**6 + 16*a*
c**2*d**5*e*x + 24*a*c**2*d**4*e**2*x**2 + 16*a*c**2*d**3*e**3*x**3 + 4*a*
c**2*d**2*e**4*x**4 + b**3*d**4 + 4*b**3*d**3*e*x + 6*b**3*d**2*e**2*x**2
+ 4*b**3*d*e**3*x**3 + b**3*e**4*x**4 + 4*b**2*c*d**6 + 20*b**2*c*d**5*e*x
 + 42*b**2*c*d**4*e**2*x**2 + 48*b**2*c*d**3*e**3*x**3 + 32*b**2*c*d**2*e*
*4*x**4 + 12*b**2*c*d*e**5*x**5 + 2*b**2*c*e**6*x**6 + 5*b*c**2*d**8 + 32*
b*c**2*d**7*e*x + 88*b*c**2*d**6*e**2*x**2 + 136*b*c**2*d**5*e**3*x**3 + 1
30*b*c**2*d**4*e**4*x**4 + 80*b*c**2*d**3*e**5*x**5 + 32*b*c**2*d**2*e**6*
x**6 + 8*b*c**2*d*e**7*x**7 + b*c**2*e**8*x**8 + 2*c**3*d**10 + 16*c**3*d*
*9*e*x + 56*c**3*d**8*e**2*x**2 + 112*c**3*d**7*e**3*x**3 + 140*c**3*d**6*
e**4*x**4 + 112*c**3*d**5*e**5*x**5 + 56*c**3*d**4*e**6*x**6 + 16*c**3*d**
3*e**7*x**7 + 2*c**3*d**2*e**8*x**8),x)*a*b*c*e**4 - 8*int(x**3/(a**2*b +
2*a**2*c*d**2 + 2*a*b**2*d**2 + 4*a*b**2*d*e*x + 2*a*b**2*e**2*x**2 + 6*a*
b*c*d**4 + 16*a*b*c*d**3*e*x + 16*a*b*c*d**2*e**2*x**2 + 8*a*b*c*d*e**3*x*
*3 + 2*a*b*c*e**4*x**4 + 4*a*c**2*d**6 + 16*a*c**2*d**5*e*x + 24*a*c**2*d*
*4*e**2*x**2 + 16*a*c**2*d**3*e**3*x**3 + 4*a*c**2*d**2*e**4*x**4 + b**3*d
**4 + 4*b**3*d**3*e*x + 6*b**3*d**2*e**2*x**2 + 4*b**3*d*e**3*x**3 + b**3*
e**4*x**4 + 4*b**2*c*d**6 + 20*b**2*c*d**5*e*x + 42*b**2*c*d**4*e**2*x*...
```

3.279 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 174

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b(b^2-6ac)\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}ef}+\frac{\log(d+ex)}{a^2ef}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef}$$

output
```
1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^
4)+1/2*b*(-6*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^2/(-
4*a*c+b^2)^(3/2)/e/f+ln(e*x+d)/a^2/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a
^2/e/f
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.37

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= \frac{\frac{2a(b^2-2ac+bc(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}+4\log(d+ex)-\frac{\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}+\frac{(b^3\cdots}{\cdots}}{4a^2ef}$$

input
```
Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b
+ c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a
*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2
])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*S
qrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a
*c)^(3/2))/(4*a^2*e*f)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1462, 1434, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$\Big\downarrow \; 1462$$
$$\frac{\int \frac{1}{(d+ex)\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{ef}$$
$$\Big\downarrow \; 1434$$
$$\frac{\int \frac{1}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2ef}$$

$$\Big\downarrow \ 1165$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int -\frac{b^2+c(d+ex)^2b-4ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a(b^2-4ac)}}{2ef}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{b^2+c(d+ex)^2b-4ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{2ef}$$

$$\Big\downarrow \ 1200$$

$$\frac{\frac{\int \left(\frac{b^2-4ac}{a(d+ex)^2}+\frac{-c\left(b^2-4ac\right)(d+ex)^2-b\left(b^2-5ac\right)}{a\left(c(d+ex)^4+b(d+ex)^2+a\right)}\right)d(d+ex)^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{2ef}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\frac{b\left(b^2-6ac\right)\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\left(b^2-4ac\right)\log\left((d+ex)^2\right)}{a}-\frac{\left(b^2-4ac\right)\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a}}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}}{2ef}$$

input
```
Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(a*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(
d + e*x)^4)) + ((b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 -
4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*Log[(d + e*x)^2])/a - ((b^
2 - 4*a*c)*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a))/(a*(b^2 - 4*a*c)
))/(2*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.32

method	result
default	$-\frac{\frac{\frac{abce\,x^2}{8ac-2b^2}+\frac{bcdax}{4ac-b^2}-\frac{a\left(-bc\,d^2+2ac-b^2\right)}{2e\left(4ac-b^2\right)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2bd\right.\right.}}{}}{}$
risch	$\frac{-\frac{bce\,x^2}{2a\left(4ac-b^2\right)}-\frac{cbdx}{a\left(4ac-b^2\right)}+\frac{-bc\,d^2+2ac-b^2}{2ea\left(4ac-b^2\right)}}{f(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2+a)}+\frac{\ln(ex+d)}{a^2ef}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^5c^3e^2f^2-48a^4b^2c^2e^2f^2\right.\right.}\right.}{}$

input
```
int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^2*((1/2*a*b*c*e/(4*a*c-b^2)*x^2+b*c*d*a/(4*a*c-b^2)*x-1/2*a/e*(-
b*c*d^2+2*a*c-b^2)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/2/(4*a*c-b^2)/e*sum((c*e^3
*(4*a*c-b^2)*_R^3+3*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*c^2*d^2-3*b^2*c*d^2+5
*a*b*c-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d-b^3*d)/(2*_R^3*c*e^3+6*_R^2
*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*
c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a
)))+1/a^2/e*ln(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1178 vs. 2(164) = 328.

Time = 0.44 (sec) , antiderivative size = 2486, normalized size of antiderivative = 14.29

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas
")
```

output
```
[1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*e^2*
x^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + ((
b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + (b^3*c - 6*
a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2*x^2 + a*b
^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b
^4 - 6*a*b^2*c)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3
*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3
+ b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(
b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 +
 b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)
*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b
^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 +
(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*
e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x +
a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)}\,dx$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima
")
```

output
```
1/2*(b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 2*a*c)/((a*b^2*c - 4*a^2*
c^2)*e^5*f*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*f*x^3 + (a*b^3 - 4*a^2*b*c
+ 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*f*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3
+ (a*b^3 - 4*a^2*b*c)*d)*e^2*f*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 -
4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e*f) - integrate(((b^2*c - 4*a*c^2)*e^3
*x^3 + 3*(b^2*c - 4*a*c^2)*d*e^2*x^2 + (b^2*c - 4*a*c^2)*d^3 + (b^3 - 5*a*
b*c + 3*(b^2*c - 4*a*c^2)*d^2)*e*x + (b^3 - 5*a*b*c)*d)/((b^2*c - 4*a*c^2)
*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 -
4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/(a^2*f
) + log(e*x + d)/(a^2*e*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. 2(164) = 328.

Time = 0.17 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.81

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = -\frac{(a^2b^3ce^3f-6\,a^3bc^2e^3f)\sqrt{b^2-4\,ac}\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bdex+2\,\sqrt{b^2-4\,ac}dex+bd^2+\cdots\right.\right.}{\cdots} - \frac{\log\left(\left|ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a\right|\right)}{4\,a^2ef} + \frac{\log\left(\left|ex+d\right|\right)}{a^2ef} + \frac{abce^2x^2+2\,abcdex+abcd^2+ab^2-2\,a^2c}{2\,(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a)(b^2-4\,ac)a^2ef}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

output
```
-1/4*((a^2*b^3*c*e^3*f - 6*a^3*b*c^2*e^3*f)*sqrt(b^2 - 4*a*c)*log(abs(b*e^
2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x
+ b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^2*b^3*c*e^3*f - 6*a^3*b*c^2*e
^3*f)*sqrt(b^2 - 4*a*c)*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 - 2
*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a
)))/(a^4*b^4*c*e^4*f^2 - 8*a^5*b^2*c^2*e^4*f^2 + 16*a^6*c^3*e^4*f^2) - 1/4
*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4
 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a^2*e*f) + log(abs(e*x + d))/(a^2*
e*f) + 1/2*(a*b*c*e^2*x^2 + 2*a*b*c*d*e*x + a*b*c*d^2 + a*b^2 - 2*a^2*c)/(
(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2
*x^2 + 2*b*d*e*x + b*d^2 + a)*(b^2 - 4*a*c)*a^2*e*f)
```

Mupad [B] (verification not implemented)

Time = 17.15 (sec) , antiderivative size = 13434, normalized size of antiderivative = 77.21

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

output
```
((b^2 - 2*a*c + b*c*d^2)/(2*e*(a*b^2 - 4*a^2*c)) + (b*c*e*x^2)/(2*(a*b^2 -
 4*a^2*c)) + (b*c*d*x)/(a*b^2 - 4*a^2*c))/(a*f + x^2*(b*e^2*f + 6*c*d^2*e^
2*f) + x*(4*c*d^3*e*f + 2*b*d*e*f) + b*d^2*f + c*d^4*f + c*e^4*f*x^4 + 4*c
*d*e^3*f*x^3) - (log((((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*
c - b^2)^3))^(1/2) - 1)*(((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4
*a*c - b^2)^3))^(1/2) - 1)*((2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^
2 - 10*a*b*c))/(a*f*(4*a*c - b^2)) + (b*c^2*e^16*(a^2*e*f*(-(b^2*(6*a*c -
b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2
*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^2
*f) - (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*f*(4*a*c - b^2)) - (4*b*c^3*d*e
^17*x*(10*a*c - b^2))/(a*f*(4*a*c - b^2))))/(4*a^2*e*f) - (b*c^3*e^15*(4*b
^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*f^2*(4*a*c - b^2)^2) + (
2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2) + (4*b*c^4*d*
e^16*x*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2)))/(4*a^2*e*f) + (b^3*c^
5*e^16*x^2)/(a^3*f^3*(4*a*c - b^2)^3) + (b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d
^2))/(a^3*f^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*f^3*(4*a*c - b^
2)^3))*((b^3*c^5*e^16*x^2)/(a^3*f^3*(4*a*c - b^2)^3) - ((a^2*e*f*(-(b^2*(6
*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*(((a^2*e*f*(-(b^2
*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*((b*c^2*e^16*(
a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + ...
```

Reduce [F]

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= \int \frac{}{c^2e^9x^9+9c^2d\,e^8x^8+36c^2d^2e^7x^7+84c^2d^3e^6x^6+2bc\,e^7x^7+126c^2d^4e^5x^5+14bcd\,e^6x^6+126c^2d^5e^4x^4+42bc\,d^2e^5x^5+84c^2d^6e^3x^3+70bc\,d^3e^4x^4+}$$

input
```
int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

output
```
int(1/(a**2*d + a**2*e*x + 2*a*b*d**3 + 6*a*b*d**2*e*x + 6*a*b*d*e**2*x**2
 + 2*a*b*e**3*x**3 + 2*a*c*d**5 + 10*a*c*d**4*e*x + 20*a*c*d**3*e**2*x**2
+ 20*a*c*d**2*e**3*x**3 + 10*a*c*d*e**4*x**4 + 2*a*c*e**5*x**5 + b**2*d**5
 + 5*b**2*d**4*e*x + 10*b**2*d**3*e**2*x**2 + 10*b**2*d**2*e**3*x**3 + 5*b
**2*d*e**4*x**4 + b**2*e**5*x**5 + 2*b*c*d**7 + 14*b*c*d**6*e*x + 42*b*c*d
**5*e**2*x**2 + 70*b*c*d**4*e**3*x**3 + 70*b*c*d**3*e**4*x**4 + 42*b*c*d**
2*e**5*x**5 + 14*b*c*d*e**6*x**6 + 2*b*c*e**7*x**7 + c**2*d**9 + 9*c**2*d*
*8*e*x + 36*c**2*d**7*e**2*x**2 + 84*c**2*d**6*e**3*x**3 + 126*c**2*d**5*e
**4*x**4 + 126*c**2*d**4*e**5*x**5 + 84*c**2*d**3*e**6*x**6 + 36*c**2*d**2
*e**7*x**7 + 9*c**2*d*e**8*x**8 + c**2*e**9*x**9),x)/f
```

3.280 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 360

$$\begin{aligned}\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx &= -\frac{3b^2-10ac}{2a^2(b^2-4ac)\,ef^2(d+ex)} \\ &\quad+\frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\ &\quad-\frac{\sqrt{c}\left(3b^3-16abc+(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,ef^2} \\ &\quad+\frac{\sqrt{c}\left(3b^3-16abc-(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,ef^2}\end{aligned}$$

output

```
-1/2*(-10*a*c+3*b^2)/a^2/(-4*a*c+b^2)/e/f^2/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*
x+d)^2)/a/(-4*a*c+b^2)/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-1/4*c^(1/
2)*(3*b^3-16*a*b*c+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1
/2)*(e*x+d)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(3/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)/e/f^2+1/4*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b
^2)*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*(e*x+d)/(b+(-4*a*c+b^2)^(1/
2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/e/f
^2
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.95

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= \frac{-\frac{4}{d+ex}+\frac{2(d+ex)\left(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2\right)}{(-b^2+4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+}{4a^2ef^2}$$

input
```
Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
(-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*
(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt
[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*
b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e*f^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1462, 1441, 25, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$\Big\downarrow \text{1462}$$
$$\frac{\int \frac{1}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{ef^2}$$

$$\Big\downarrow \; 1441$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int -\frac{3b^2+3c(d+ex)^2b-10ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a(b^2-4ac)}}{ef^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{3b^2+3c(d+ex)^2b-10ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}}{ef^2}$$

$$\Big\downarrow \; 1604$$

$$\frac{\frac{-\frac{\int \frac{c\left(3b^2-10ac\right)(d+ex)^2+b\left(3b^2-13ac\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{3b^2-10ac}{a(d+ex)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}}{ef^2}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{-\frac{\frac{1}{2}c\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)-\frac{c\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+}{2\sqrt{b^2-4ac}}}{a}}{2a(b^2-4ac)}}{ef^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{-\frac{\frac{\sqrt{c}\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{3b^2-10ac}{a(d+ex)}}{2a(b^2-4ac)}+\frac{}{2a}}{ef^2}$$

input
```
Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(d + e*x)*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) + (-((3*b^2 - 10*a*c)/(a*(d + e*x))) - ((Sqrt[c]*
(3*b^2 - 10*a*c + (3*b^3)/Sqrt[b^2 - 4*a*c] - (16*a*b*c)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*
a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a)
/(2*a*(b^2 - 4*a*c)))/(e*f^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.24

method	result
default	$-\frac{\frac{\frac{c\,e^2\left(2ac-b^2\right)x^3}{8ac-2b^2}+\frac{3dce\left(2ac-b^2\right)x^2}{2\left(4ac-b^2\right)}+\frac{\left(6a\,c^2d^2-3b^2c\,d^2+3abc-b^3\right)x}{8ac-2b^2}+\frac{d\left(2a\,c^2d^2-b^2c\,d^2+3abc-b^3\right)}{2e\left(4ac-b^2\right)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2+a}+\frac{\_R=\text{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\right.}{}}{}$
risch	Expression too large to display

input
```
int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f^2*(-1/a^2*((1/2*c*e^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3+3/2*d*c*e*(2*a*c-b^2
)/(4*a*c-b^2)*x^2+1/2*(6*a*c^2*d^2-3*b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2)*x+
1/2*d/e*(2*a*c^2*d^2-b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*
e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4
/(4*a*c-b^2)/e*sum((c*e^2*(10*a*c-3*b^2)*_R^2+2*c*d*e*(10*a*c-3*b^2)*_R+10
*a*c^2*d^2-3*b^2*c*d^2+13*a*b*c-3*b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c
*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6
*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a)))-1/a^2/e/(e*
x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4520 vs. $2(312) = 624$.

Time = 0.28 (sec) , antiderivative size = 4520, normalized size of antiderivative = 12.56

$$\int \frac{1}{(df+efx)^2 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fric
as")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^2 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)^2 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)^2}\,dx$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxi
ma")
```

output
```
-1/2*((3*b^2*c - 10*a*c^2)*e^4*x^4 + 4*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + (3
*b^2*c - 10*a*c^2)*d^4 + (3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^2)*e
^2*x^2 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*d^2 + 2*(2*(3*b^2*c - 10*a
*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x)/((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^
5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a
^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 +
 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*
c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c
- 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*f^
2) - 1/2*integrate(((3*b^2*c - 10*a*c^2)*e^2*x^2 + 2*(3*b^2*c - 10*a*c^2)*
d*e*x + 3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*d^2)/((b^2*c - 4*a*c^2)*e^
4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c
)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/(a^2*f^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(312) = 624$.

Time = 0.16 (sec) , antiderivative size = 1031, normalized size of antiderivative = 2.86

$$\int \frac{1}{(df + efx)^2 (a + b(d + ex)^2 + c(d + ex)^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac
")
```

output
```
-1/2*(b^2*c/((e*f*x + d*f)*e*f) - 2*a*c^2/((e*f*x + d*f)*e*f) + b^3*f/((e*
f*x + d*f)^3*e) - 3*a*b*c*f/((e*f*x + d*f)^3*e))/((a^2*b^2 - 4*a^3*c)*(c +
 b*f^2/(e*f*x + d*f)^2 + a*f^4/(e*f*x + d*f)^4)) - 1/((e*f*x + d*f)*a^2*e*
f) + 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*sqrt
(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*e^4*f^8 + 2*(3*a^3*b^2*c - 10*a^4*c^2)*sqr
t(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*e^2*f^4*abs(a^2*b^2*e^2
*f^4 - 4*a^3*c*e^2*f^4) - (a^2*b^2*e^2*f^4 - 4*a^3*c*e^2*f^4)^2*(3*b^3 - 1
3*a*b*c)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*f*x +
 d*f)*e*f*sqrt((a^2*b^3*e^2*f^4 - 4*a^3*b*c*e^2*f^4 + sqrt((a^2*b^3*e^2*f^
4 - 4*a^3*b*c*e^2*f^4)^2 - 4*(a^3*b^2*e^4*f^8 - 4*a^4*c*e^4*f^8)*(a^2*b^2*
c - 4*a^3*c^2)))/(a^3*b^2*e^4*f^8 - 4*a^4*c*e^4*f^8))))/((a^5*b^2*c - 4*a^
6*c^2)*sqrt(b^2 - 4*a*c)*e^3*f^6*abs(a^2*b^2*e^2*f^4 - 4*a^3*c*e^2*f^4)*ab
s(a)) - 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*s
qrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*e^4*f^8 - 2*(3*a^3*b^2*c - 10*a^4*c^2)*
sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*e^2*f^4*abs(a^2*b^2*
e^2*f^4 - 4*a^3*c*e^2*f^4) - (a^2*b^2*e^2*f^4 - 4*a^3*c*e^2*f^4)^2*(3*b^3
- 13*a*b*c)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)/((e*f*
x + d*f)*e*f*sqrt((a^2*b^3*e^2*f^4 - 4*a^3*b*c*e^2*f^4 - sqrt((a^2*b^3*e^2
*f^4 - 4*a^3*b*c*e^2*f^4)^2 - 4*(a^3*b^2*e^4*f^8 - 4*a^4*c*e^4*f^8)*(a^2*b
^2*c - 4*a^3*c^2)))/(a^3*b^2*e^4*f^8 - 4*a^4*c*e^4*f^8))))/((a^5*b^2*c ...
```

Mupad [B] (verification not implemented)

Time = 14.37 (sec) , antiderivative size = 12008, normalized size of antiderivative = 33.36

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

output
```
- atan(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 207
7*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5
- 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*
b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 61
44*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^
4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^
3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c -
b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a
^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4
- 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/
2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^
5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^
11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a
^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 384
0*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)
))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 +
61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14
*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^...
```

Reduce [F]

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$
$$= \int \frac{}{c^2e^{10}x^{10}+10c^2d\,e^9x^9+45c^2d^2e^8x^8+120c^2d^3e^7x^7+2bc\,e^8x^8+210c^2d^4e^6x^6+16bcd\,e^7x^7+252c^2d^5e^5x^5+56bc\,d^2e^6x^6+210c^2d^6e^4x^4+112bc\,d^3}$$

input
```
int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

output
```
int(1/(a**2*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2 + 2*a*b*d**4 + 8*a*b*d**3
*e*x + 12*a*b*d**2*e**2*x**2 + 8*a*b*d*e**3*x**3 + 2*a*b*e**4*x**4 + 2*a*c
*d**6 + 12*a*c*d**5*e*x + 30*a*c*d**4*e**2*x**2 + 40*a*c*d**3*e**3*x**3 +
30*a*c*d**2*e**4*x**4 + 12*a*c*d*e**5*x**5 + 2*a*c*e**6*x**6 + b**2*d**6 +
 6*b**2*d**5*e*x + 15*b**2*d**4*e**2*x**2 + 20*b**2*d**3*e**3*x**3 + 15*b*
*2*d**2*e**4*x**4 + 6*b**2*d*e**5*x**5 + b**2*e**6*x**6 + 2*b*c*d**8 + 16*
b*c*d**7*e*x + 56*b*c*d**6*e**2*x**2 + 112*b*c*d**5*e**3*x**3 + 140*b*c*d*
*4*e**4*x**4 + 112*b*c*d**3*e**5*x**5 + 56*b*c*d**2*e**6*x**6 + 16*b*c*d*e
**7*x**7 + 2*b*c*e**8*x**8 + c**2*d**10 + 10*c**2*d**9*e*x + 45*c**2*d**8*
e**2*x**2 + 120*c**2*d**7*e**3*x**3 + 210*c**2*d**6*e**4*x**4 + 252*c**2*d
**5*e**5*x**5 + 210*c**2*d**4*e**6*x**6 + 120*c**2*d**3*e**7*x**7 + 45*c**
2*d**2*e**8*x**8 + 10*c**2*d*e**9*x**9 + c**2*e**10*x**10),x)/f**2
```

3.281 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 228

$$\begin{aligned}\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx &= -\frac{b^2-3ac}{a^2(b^2-4ac)\,ef^3(d+ex)^2} \\ &\quad + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\ &\quad - \frac{(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}\,ef^3} \\ &\quad - \frac{2b\log(d+ex)}{a^3ef^3} + \frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}\end{aligned}$$

output
```
-(-3*a*c+b^2)/a^2/(-4*a*c+b^2)/e/f^3/(e*x+d)^2+1/2*(b^2-2*a*c+b*c*(e*x+d)^
2)/a/(-4*a*c+b^2)/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-(6*a^2*c^2-6
*a*b^2*c+b^4)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^
2)^(3/2)/e/f^3-2*b*ln(e*x+d)/a^3/e/f^3+1/2*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)
/a^3/e/f^3
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.26

$$\int \frac{1}{(df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx$$
$$= \frac{-\frac{a}{(d+ex)^2}+\frac{a\left(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2\right)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)}-4b\log(d+ex)+\frac{\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{\right.}{(b^2-4ac)^{3/2}}}{2a^3ef^3}$$

input
```
Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
(-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e
*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d +
 e*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sq
rt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*
c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*
c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 -
4*a*c)^(3/2))/(2*a^3*e*f^3)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1462, 1434, 1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx$$
$$\Big\downarrow \text{1462}$$
$$\frac{\int \frac{1}{(d+ex)^3\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{ef^3}$$
$$\Big\downarrow \text{1434}$$

$$\frac{\int \frac{1}{(d+ex)^4(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)^2}{2ef^3}$$

$$\Big\downarrow \text{1165}$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int -\frac{2\left(b^2+c(d+ex)^2b-3ac\right)}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a(b^2-4ac)}}{2ef^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{b^2+c(d+ex)^2b-3ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}}{2ef^3}$$

$$\Big\downarrow \text{1200}$$

$$\frac{\frac{2\int \left(\frac{b^2-3ac}{a(d+ex)^4}+\frac{b^4-5acb^2+c\left(b^2-4ac\right)(d+ex)^2b+3a^2c^2}{a^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}+\frac{4abc-b^3}{a^2(d+ex)^2}\right)d(d+ex)^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}}{2ef^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2\left(-\frac{(6a^2c^2-6ab^2c+b^4)\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\left(b^2-4ac\right)\log\left((d+ex)^2\right)}{a^2}+\frac{b\left(b^2-4ac\right)\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^2}-\frac{b^2-3ac}{a(d+ex)^2}\right)}{a(b^2-4ac)}+\frac{-}{a(b^2-4ac)(d}}{2ef^3}$$

input
```
Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(a*(b^2 - 4*a*c)*(d + e*x)^2*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)) + (2*(-((b^2 - 3*a*c)/(a*(d + e*x)^2)) - ((b^4 -
6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(
a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)*Log[(d + e*x)^2])/a^2 + (b*(b^2
- 4*a*c)*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^2)))/(a*(b^2 - 4*a*c
)))/(2*e*f^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.04

method	result
default	$-\frac{\frac{\frac{ace\left(2ac-b^2\right)x^2}{8ac-2b^2}+\frac{cda\left(2ac-b^2\right)x}{4ac-b^2}+\frac{a\left(2a\,c^2d^2-b^2c\,d^2+3abc-b^3\right)}{2e\left(4ac-b^2\right)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,e^4\_Z^4+4cd\,e^3\_Z^3+\left(6c\,d^2e^2+b\,e^2\right)\_Z^2+\left(4c\,d^3e+2bd\right.\right.}}{}}{}$
risch	Expression too large to display

input
```
int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f^3*(-1/a^3*((1/2*a*c*e*(2*a*c-b^2)/(4*a*c-b^2)*x^2+c*d*a*(2*a*c-b^2)/(4
*a*c-b^2)*x+1/2*a/e*(2*a*c^2*d^2-b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2))/(c*e^
4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)+1/(4*a*c-b^2)/e*sum((b*c*e^3*(-4*a*c+b^2)*_R^3+3*b*c*d*e^2*(-4*a*
c+b^2)*_R^2+e*(-12*a*b*c^2*d^2+3*b^3*c*d^2+3*a^2*c^2-5*a*b^2*c+b^4)*_R-4*a
*b*c^2*d^3+b^3*c*d^3+3*a^2*c^2*d-5*a*b^2*c*d+b^4*d)/(2*_R^3*c*e^3+6*_R^2*c
*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*
d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))
)-1/2/a^2/e/(e*x+d)^2-2/a^3*b/e*ln(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2237 vs. 2(220) = 440.

Time = 1.52 (sec) , antiderivative size = 4604, normalized size of antiderivative = 20.19

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fric
as")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)^3}\,dx$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxi
ma")
```

output
```
-1/2*(2*(b^2*c - 3*a*c^2)*e^4*x^4 + 8*(b^2*c - 3*a*c^2)*d*e^3*x^3 + 2*(b^2
*c - 3*a*c^2)*d^4 + (2*b^3 - 7*a*b*c + 12*(b^2*c - 3*a*c^2)*d^2)*e^2*x^2 +
 a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*d^2 + 2*(4*(b^2*c - 3*a*c^2)*d^3 + (2
*b^3 - 7*a*b*c)*d)*e*x)/((a^2*b^2*c - 4*a^3*c^2)*e^7*f^3*x^6 + 6*(a^2*b^2*
c - 4*a^3*c^2)*d*e^6*f^3*x^5 + (a^2*b^3 - 4*a^3*b*c + 15*(a^2*b^2*c - 4*a^
3*c^2)*d^2)*e^5*f^3*x^4 + 4*(5*(a^2*b^2*c - 4*a^3*c^2)*d^3 + (a^2*b^3 - 4*
a^3*b*c)*d)*e^4*f^3*x^3 + (a^3*b^2 - 4*a^4*c + 15*(a^2*b^2*c - 4*a^3*c^2)*
d^4 + 6*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^3*f^3*x^2 + 2*(3*(a^2*b^2*c - 4*a^3*c
^2)*d^5 + 2*(a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e^2*f^3*x +
 ((a^2*b^2*c - 4*a^3*c^2)*d^6 + (a^2*b^3 - 4*a^3*b*c)*d^4 + (a^3*b^2 - 4*a
^4*c)*d^2)*e*f^3) + 2*integrate(((b^3*c - 4*a*b*c^2)*e^3*x^3 + 3*(b^3*c -
4*a*b*c^2)*d*e^2*x^2 + (b^3*c - 4*a*b*c^2)*d^3 + (b^4 - 5*a*b^2*c + 3*a^2*
c^2 + 3*(b^3*c - 4*a*b*c^2)*d^2)*e*x + (b^4 - 5*a*b^2*c + 3*a^2*c^2)*d)/((
b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^
2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2
*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)
*e*x), x)/(a^3*f^3) - 2*b*log(e*x + d)/(a^3*e*f^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(220) = 440$.

Time = 0.17 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.09

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{(a^3b^4ce^3f^3-6\,a^4b^2c^2e^3f^3+6\,a^5c^3e^3f^3)\sqrt{b^2-4\,ac}\log\left(\left|be^2x^2+\sqrt{b^2-4\,ac}e^2x^2+2\,bdex+2\,\sqrt{b^2-4\,ac}\right.\right.}{} + \frac{b\log\left(\left|ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd^4+be^2x^2+2\,bdex+bd^2+a\right|\right)}{2\,a^3ef^3} - \frac{2\,b\log\left(\left|ex+d\right|\right)}{a^3ef^3} - \frac{2\,ab^2cd^4-6\,a^2c^2d^4+2\,ab^3d^2-7\,a^2bcd^2+2\,(ab^2ce^4-3\,a^2c^2e^4)x^4+a^2b^2-4\,a^3c+8\,(ab^2cde^3-3\,a^2c^2}{2\,(ce^4x^4+4\,cde^3x^3+6\,cd^2e^2x^2+4\,cd^3ex+cd}$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac
")
```

output
```
1/2*((a^3*b^4*c*e^3*f^3 - 6*a^4*b^2*c^2*e^3*f^3 + 6*a^5*c^3*e^3*f^3)*sqrt(
b^2 - 4*a*c)*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x + 2
*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^3*b^
4*c*e^3*f^3 - 6*a^4*b^2*c^2*e^3*f^3 + 6*a^5*c^3*e^3*f^3)*sqrt(b^2 - 4*a*c)
*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 - 2*b*d*e*x + 2*sqrt(b^2 -
 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(a^6*b^4*c*e^4*f^6
- 8*a^7*b^2*c^2*e^4*f^6 + 16*a^8*c^3*e^4*f^6) + 1/2*b*log(abs(c*e^4*x^4 +
4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d^4 + b*e^2*x^2 + 2*b*d*
e*x + b*d^2 + a))/(a^3*e*f^3) - 2*b*log(abs(e*x + d))/(a^3*e*f^3) - 1/2*(2
*a*b^2*c*d^4 - 6*a^2*c^2*d^4 + 2*a*b^3*d^2 - 7*a^2*b*c*d^2 + 2*(a*b^2*c*e^
4 - 3*a^2*c^2*e^4)*x^4 + a^2*b^2 - 4*a^3*c + 8*(a*b^2*c*d*e^3 - 3*a^2*c^2*
d*e^3)*x^3 + (12*a*b^2*c*d^2*e^2 - 36*a^2*c^2*d^2*e^2 + 2*a*b^3*e^2 - 7*a^
2*b*c*e^2)*x^2 + 2*(4*a*b^2*c*d^3*e - 12*a^2*c^2*d^3*e + 2*a*b^3*d*e - 7*a
^2*b*c*d*e)*x)/((c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x
 + c*d^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a)*(b^2 - 4*a*c)*(e*x + d)^2*a^
3*e*f^3)
```

Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 14830, normalized size of antiderivative = 65.04

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

output
```
((x*(2*b^3*d - 12*a*c^2*d^3 + 4*b^2*c*d^3 - 7*a*b*c*d))/(4*a^3*c - a^2*b^2
) - (x^4*(3*a*c^2*e^3 - b^2*c*e^3))/(4*a^3*c - a^2*b^2) - (4*x^3*(3*a*c^2*
d*e^2 - b^2*c*d*e^2))/(4*a^3*c - a^2*b^2) + (a*b^2 - 4*a^2*c + 2*b^3*d^2 -
 6*a*c^2*d^4 + 2*b^2*c*d^4 - 7*a*b*c*d^2)/(2*e*(4*a^3*c - a^2*b^2)) + (x^2
*(2*b^3*e - 36*a*c^2*d^2*e + 12*b^2*c*d^2*e - 7*a*b*c*e))/(2*(4*a^3*c - a^
2*b^2)))/(x^3*(20*c*d^3*e^3*f^3 + 4*b*d*e^3*f^3) + x*(2*a*d*e*f^3 + 4*b*d^
3*e*f^3 + 6*c*d^5*e*f^3) + x^4*(b*e^4*f^3 + 15*c*d^2*e^4*f^3) + x^2*(a*e^2
*f^3 + 6*b*d^2*e^2*f^3 + 15*c*d^4*e^2*f^3) + a*d^2*f^3 + b*d^4*f^3 + c*d^6
*f^3 + c*e^6*f^3*x^6 + 6*c*d*e^5*f^3*x^5) + (log((((b + a^3*e*f^3*(-(b^4 +
 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(((b + a^3
*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1
/2))*((4*c^2*e^16*(2*b^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a
*b^3*c + 2*a*b^2*c^2*d^2))/(a^2*f^3*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4
- 30*a^2*c^2 + 2*a*b^2*c))/(a^2*f^3*(4*a*c - b^2)) - (2*b*c^2*e^16*(b + a^
3*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(
1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c
*e^2*x^2 - 20*a*c*d*e*x))/(a^3*f^3) + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 +
2*a*b^2*c))/(a^2*f^3*(4*a*c - b^2))))/(2*a^3*e*f^3) - (4*c^3*e^15*(3*a*c -
 b^2)*(4*b^4 + 3*a^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2*c - 23*a*b*c^2*d^2))/(a^
4*f^6*(4*a*c - b^2)^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b...
```

Reduce [F]

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx$$

$$= \int \frac{}{c^2e^{11}x^{11}+11c^2d\,e^{10}x^{10}+55c^2d^2e^9x^9+165c^2d^3e^8x^8+2bc\,e^9x^9+330c^2d^4e^7x^7+18bcd\,e^8x^8+462c^2d^5e^6x^6+72bc\,d^2e^7x^7+462c^2d^6e^5x^5+168bc}$$

input
```
int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

output
```
int(1/(a**2*d**3 + 3*a**2*d**2*e*x + 3*a**2*d*e**2*x**2 + a**2*e**3*x**3 +
 2*a*b*d**5 + 10*a*b*d**4*e*x + 20*a*b*d**3*e**2*x**2 + 20*a*b*d**2*e**3*x
**3 + 10*a*b*d*e**4*x**4 + 2*a*b*e**5*x**5 + 2*a*c*d**7 + 14*a*c*d**6*e*x
+ 42*a*c*d**5*e**2*x**2 + 70*a*c*d**4*e**3*x**3 + 70*a*c*d**3*e**4*x**4 +
42*a*c*d**2*e**5*x**5 + 14*a*c*d*e**6*x**6 + 2*a*c*e**7*x**7 + b**2*d**7 +
 7*b**2*d**6*e*x + 21*b**2*d**5*e**2*x**2 + 35*b**2*d**4*e**3*x**3 + 35*b*
*2*d**3*e**4*x**4 + 21*b**2*d**2*e**5*x**5 + 7*b**2*d*e**6*x**6 + b**2*e**
7*x**7 + 2*b*c*d**9 + 18*b*c*d**8*e*x + 72*b*c*d**7*e**2*x**2 + 168*b*c*d*
*6*e**3*x**3 + 252*b*c*d**5*e**4*x**4 + 252*b*c*d**4*e**5*x**5 + 168*b*c*d
**3*e**6*x**6 + 72*b*c*d**2*e**7*x**7 + 18*b*c*d*e**8*x**8 + 2*b*c*e**9*x*
*9 + c**2*d**11 + 11*c**2*d**10*e*x + 55*c**2*d**9*e**2*x**2 + 165*c**2*d*
*8*e**3*x**3 + 330*c**2*d**7*e**4*x**4 + 462*c**2*d**6*e**5*x**5 + 462*c**
2*d**5*e**6*x**6 + 330*c**2*d**4*e**7*x**7 + 165*c**2*d**3*e**8*x**8 + 55*
c**2*d**2*e**9*x**9 + 11*c**2*d*e**10*x**10 + c**2*e**11*x**11),x)/f**3
```

3.282 $\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 423

$$\begin{aligned}
&\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx \\
&= -\frac{5b^2-14ac}{6a^2(b^2-4ac)\,ef^4(d+ex)^3}+\frac{b(5b^2-19ac)}{2a^3(b^2-4ac)\,ef^4(d+ex)} \\
&\quad+\frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)\,ef^4(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\
&\quad+\frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,ef^4} \\
&\quad-\frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2-b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,ef^4}
\end{aligned}$$

output
```
-1/6*(-14*a*c+5*b^2)/a^2/(-4*a*c+b^2)/e/f^4/(e*x+d)^3+1/2*b*(-19*a*c+5*b^2
)/a^3/(-4*a*c+b^2)/e/f^4/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b
^2)/e/f^4/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/4*c^(1/2)*(5*b^4-29*a*b^
2*c+28*c^2*a^2+b*(-19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2
)*(e*x+d)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^3/(-4*a*c+b^2)^(3/2)/(b-
(-4*a*c+b^2)^(1/2))^(1/2)/e/f^4-1/4*c^(1/2)*(5*b^4-29*a*b^2*c+28*c^2*a^2-b
*(-19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*(e*x+d)/(b+(-4
*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^3/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)/e/f^4
```

Mathematica [A] (verified)

Time = 3.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.91

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$

$$= \frac{-\frac{4a}{(d+ex)^3}+\frac{24b}{d+ex}+\frac{6(d+ex)\left(b^4-4ab^2c+2a^2c^2+b^3c(d+ex)^2-3abc^2(d+ex)^2\right)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}+\frac{3\sqrt{2}\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2\cdots}\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4\cdots}}}}{12a^3ef^4}$$

input
```
Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output
```
((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2
*a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a +
 (d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*
c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*
b^2*c - 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e*f^4)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1462, 1441, 25, 1604, 27, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}dx$$

$$\Big\downarrow \; 1462$$

$$\frac{\int \frac{1}{(d+ex)^4(c(d+ex)^4+b(d+ex)^2+a)^2}d(d+ex)}{ef^4}$$

$$\Big\downarrow \; 1441$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}-\frac{\int -\frac{5b^2+5c(d+ex)^2b-14ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a(b^2-4ac)}}{ef^4}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{5b^2+5c(d+ex)^2b-14ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}}{ef^4}$$

$$\Big\downarrow \; 1604$$

$$\frac{\frac{-\frac{\int \frac{3\left(c\left(5b^2-14ac\right)(d+ex)^2+b\left(5b^2-19ac\right)\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{3a}-\frac{5b^2-14ac}{3a(d+ex)^3}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}}{ef^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{\int \frac{c\left(5b^2-14ac\right)(d+ex)^2+b\left(5b^2-19ac\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{a}-\frac{5b^2-14ac}{3a(d+ex)^3}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}}{ef^4}$$

$$\Big\downarrow \; 1604$$

$$\frac{-\frac{-\frac{\int \frac{5b^4-24acb^2+c\left(5b^2-19ac\right)(d+ex)^2b+14a^2c^2}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{b\left(5b^2-19ac\right)}{a(d+ex)}}{a}-\frac{5b^2-14ac}{3a(d+ex)^3}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)}}{ef^4}$$

$\Big\downarrow$ 1480

$$-\frac{-\frac{\frac{c\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)}{2\sqrt{b^2-4ac}}-\frac{c\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}}}{2\sqrt{b^2-4ac}}}{a}}{\frac{a}{\frac{2a(b^2-4ac)}{ef^4}}}$$

$\Big\downarrow$ 218

$$-\frac{-\frac{\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}}{\frac{a}{\frac{2a(b^2-4ac)}{ef^4}}}$$

input

```
Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

output

```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(d + e*x)^3*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)) + (-1/3*(5*b^2 - 14*a*c)/(a*(d + e*x)^3) - (-((
b*(5*b^2 - 19*a*c))/(a*(d + e*x))) - ((Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^
2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
 e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2
- 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]
))/a)/a)/(2*a*(b^2 - 4*a*c)))/(e*f^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.17

method	result
default	$-\frac{-\frac{b c e^2 (3ac-b^2) x^3}{2(4ac-b^2)}-\frac{3dbce(3ac-b^2)x^2}{2(4ac-b^2)}+\frac{(-9abc^2d^2+3b^3cd^2+2a^2c^2-4ab^2c+b^4)x}{8ac-2b^2}+\frac{d(-3abc^2d^2+b^3cd^2+2a^2c^2-4ab^2c+b^4)}{2e(4ac-b^2)}}{ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+cd^4+2bdex+bd^2+a}+\frac{\_R=\text{Roo}}{}$
risch	Expression too large to display

input
```
int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f^4*(-1/a^3*((-1/2*b*c*e^2*(3*a*c-b^2)/(4*a*c-b^2)*x^3-3/2*d*b*c*e*(3*a*
c-b^2)/(4*a*c-b^2)*x^2+1/2*(-9*a*b*c^2*d^2+3*b^3*c*d^2+2*a^2*c^2-4*a*b^2*c
+b^4)/(4*a*c-b^2)*x+1/2*d/e*(-3*a*b*c^2*d^2+b^3*c*d^2+2*a^2*c^2-4*a*b^2*c+
b^4)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e
^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((b*c*e^2*(-19*a*c+5*
b^2)*_R^2+2*b*d*e*c*(-19*a*c+5*b^2)*_R-19*a*b*c^2*d^2+5*b^3*c*d^2+14*a^2*c
^2-24*a*b^2*c+5*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*
b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*
_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a)))-1/3/a^2/e/(e*x+d)^3+2/a^3*b/e
/(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5954 vs. 2(373) = 746.

Time = 0.32 (sec) , antiderivative size = 5954, normalized size of antiderivative = 14.08

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fric
as")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\, dx = \text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\, dx$$
$$= \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^2(efx+df)^4}\, dx$$

input
```
integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxi
ma")
```

output
```
1/6*(3*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 18*(5*b^3*c - 19*a*b*c^2)*d*e^5*x^
5 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19*a*b*c^2)*d^2)*e^4
*x^4 + 3*(5*b^3*c - 19*a*b*c^2)*d^6 + 4*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (
15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d)*e^3*x^3 + (15*b^4 - 62*a*b^2*c + 14*a
^2*c^2)*d^4 + (45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6*(
15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^2)*e^2*x^2 - 2*a^2*b^2 + 8*a^3*c + 10*
(a*b^3 - 4*a^2*b*c)*d^2 + 2*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 62
*a*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x)/((a^3*b^2*c -
4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^
3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^
2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4
*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^
4*f^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3
 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 +
 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^
3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)
*d^3)*e*f^4) + 1/2*integrate(((5*b^3*c - 19*a*b*c^2)*e^2*x^2 + 5*b^4 - 24*
a*b^2*c + 14*a^2*c^2 + 2*(5*b^3*c - 19*a*b*c^2)*d*e*x + (5*b^3*c - 19*a*b*
c^2)*d^2)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^
2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2137 vs. $2(373) = 746$.

Time = 0.15 (sec) , antiderivative size = 2137, normalized size of antiderivative = 5.05

$$\int \frac{1}{(df+efx)^4\,(a+b(d+ex)^2+c(d+ex)^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac
")
```

output
```
-1/4*((5*b^3*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4
)) + d/e)^2 - 19*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) + d/e)^2 - 10*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)/(c*e^4)) + d/e) + 38*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4
 - 24*a*b^2*c + 14*a^2*c^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^
2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)) - (5*
b^3*c*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)
^2 - 19*a*b*c^2*e^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^
4)) - d/e)^2 + 10*b^3*c*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)/(c*e^4)) - d/e) - 38*a*b*c^2*d*e*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)/(c*e^4)) - d/e) + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b
^2*c + 14*a^2*c^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (5*b^3*c*...
```

Mupad [B] (verification not implemented)

Time = 17.01 (sec) , antiderivative size = 13781, normalized size of antiderivative = 32.58

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

output
```
atan(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 636
6*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c
^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13
*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2
)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^
2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a
^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*
(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3
*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 +
49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*
a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*
e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*
c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a
^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) -
 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*
c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)
^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*
a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^
2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^
4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1...
```

Reduce [F]

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$$
$$= \frac{\int}{c^2e^{12}x^{12}+12c^2d\,e^{11}x^{11}+66c^2d^2e^{10}x^{10}+220c^2d^3e^9x^9+2bc\,e^{10}x^{10}+495c^2d^4e^8x^8+20bcd\,e^9x^9+792c^2d^5e^7x^7+90bc\,d^2e^8x^8+924c^2d^6e^6x^6+24}$$

input
```
int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)
```

output
```
int(1/(a**2*d**4 + 4*a**2*d**3*e*x + 6*a**2*d**2*e**2*x**2 + 4*a**2*d*e**3
*x**3 + a**2*e**4*x**4 + 2*a*b*d**6 + 12*a*b*d**5*e*x + 30*a*b*d**4*e**2*x
**2 + 40*a*b*d**3*e**3*x**3 + 30*a*b*d**2*e**4*x**4 + 12*a*b*d*e**5*x**5 +
 2*a*b*e**6*x**6 + 2*a*c*d**8 + 16*a*c*d**7*e*x + 56*a*c*d**6*e**2*x**2 +
112*a*c*d**5*e**3*x**3 + 140*a*c*d**4*e**4*x**4 + 112*a*c*d**3*e**5*x**5 +
 56*a*c*d**2*e**6*x**6 + 16*a*c*d*e**7*x**7 + 2*a*c*e**8*x**8 + b**2*d**8
+ 8*b**2*d**7*e*x + 28*b**2*d**6*e**2*x**2 + 56*b**2*d**5*e**3*x**3 + 70*b
**2*d**4*e**4*x**4 + 56*b**2*d**3*e**5*x**5 + 28*b**2*d**2*e**6*x**6 + 8*b
**2*d*e**7*x**7 + b**2*e**8*x**8 + 2*b*c*d**10 + 20*b*c*d**9*e*x + 90*b*c*
d**8*e**2*x**2 + 240*b*c*d**7*e**3*x**3 + 420*b*c*d**6*e**4*x**4 + 504*b*c
*d**5*e**5*x**5 + 420*b*c*d**4*e**6*x**6 + 240*b*c*d**3*e**7*x**7 + 90*b*c
*d**2*e**8*x**8 + 20*b*c*d*e**9*x**9 + 2*b*c*e**10*x**10 + c**2*d**12 + 12
*c**2*d**11*e*x + 66*c**2*d**10*e**2*x**2 + 220*c**2*d**9*e**3*x**3 + 495*
c**2*d**8*e**4*x**4 + 792*c**2*d**7*e**5*x**5 + 924*c**2*d**6*e**6*x**6 +
792*c**2*d**5*e**7*x**7 + 495*c**2*d**4*e**8*x**8 + 220*c**2*d**3*e**9*x**
9 + 66*c**2*d**2*e**10*x**10 + 12*c**2*d*e**11*x**11 + c**2*e**12*x**12),x
)/f**4
```

3.283 $\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 353

$$\begin{aligned}\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx &= \frac{f^4(d+ex)\,(2a+b(d+ex)^2)}{4\,(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)^2} \\ &\quad - \frac{f^4(d+ex)\,(7b^2-4ac+12bc(d+ex)^2)}{8\,(b^2-4ac)^2\,e\,(a+b(d+ex)^2+c(d+ex)^4)} \\ &\quad + \frac{3\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\,(b^2-4ac)^{5/2}\,\sqrt{b-\sqrt{b^2-4ac}}e} \\ &\quad - \frac{3\sqrt{c}\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)f^4\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\,(b^2-4ac)^{5/2}\,\sqrt{b+\sqrt{b^2-4ac}}e}\end{aligned}$$

output

```
1/4*f^4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^
4)^2-1/8*f^4*(e*x+d)*(7*b^2-4*a*c+12*b*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*
(e*x+d)^2+c*(e*x+d)^4)+3/8*c^(1/2)*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))*f^
4*arctan(2^(1/2)*c^(1/2)*(e*x+d)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(-4
*a*c+b^2)^(5/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/e-3/8*c^(1/2)*(3*b^2+4*a*c+2*
b*(-4*a*c+b^2)^(1/2))*f^4*arctan(2^(1/2)*c^(1/2)*(e*x+d)/(b+(-4*a*c+b^2)^(
1/2))^(1/2))*2^(1/2)/(-4*a*c+b^2)^(5/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/e
```

Mathematica [A] (verified)

Time = 4.37 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.94

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$= \frac{f^4\left(-\frac{2(-2a(d+ex)-b(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{(d+ex)(-7b^2+4ac-12bc(d+ex)^2)}{(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)}+\frac{3\sqrt{2}\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+e\ldots}{\sqrt{b-\sqrt{b^2-4\ldots}}}\right.}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.}{8e}$$

input
```
Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x
)^2 + c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2)
)/((b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[
c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*
x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*
c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(8*e)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1462, 1440, 1492, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$

$$\Big\downarrow \ 1462$$

$$\frac{f^4\int \frac{(d+ex)^4}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)}{e}$$

$$\Big\downarrow \; 1440$$

$$\frac{f^4\left(\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\int \frac{2a-5b(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4(b^2-4ac)}\right)}{e}$$

$$\Big\downarrow \; 1492$$

$$\frac{f^4\left(\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{2\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int \frac{3a\left(b^2-4c(d+ex)^2b+4ac\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2a\left(b^2-4ac\right)}}{4(b^2-4ac)}\right)}{e}$$

$$\Big\downarrow \; 27$$

$$\frac{f^4\left(\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{2\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\int \frac{b^2-4c(d+ex)^2b+4ac}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2\left(b^2-4ac\right)}}{4(b^2-4ac)}\right)}{e}$$

$$\Big\downarrow \; 1480$$

$$\frac{f^4\left(\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{2\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(-c\left(2b-\frac{4ac+3b^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)-c\left(\cdots\right.\right.}{2\left(b^2-4ac\right)\cdots}}{4(b^2-4ac)}\cdots\right.}{e}$$

$$\Big\downarrow \; 218$$

$$\frac{f^4\left(\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{2\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\left(-\frac{\sqrt{2}\sqrt{c}\left(2b-\frac{4ac+3b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{4a\cdots}{\sqrt{b\cdots}}\cdots\right.}{\cdots}\right.}{2\left(b^2-4ac\right)\cdots}}{4(b^2-4ac)}\cdots\right.}{e}$$

input
```
Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f^4*(((d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)^2) - (((d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))/
(2*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*(-((Sqrt[2]*Sqr
t[c]*(2*b - (3*b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
+ e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt
[2]*Sqrt[c]*(2*b + (3*b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]]))
/(2*(b^2 - 4*a*c)))/(4*(b^2 - 4*a*c))))/e
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 708, normalized size of antiderivative = 2.01

method	result
default	$f^4 \left(\frac{-\frac{3c^2e^6b\,x^7}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{21c^2d\,e^5b\,x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(-252bc\,d^2+4ac-19b^2\right)c\,e^4x^5}{128a^2c^2-64a\,b^2c+8b^4}+\frac{5cd\,e^3\left(-84bc\,d^2+4ac-19b^2\right)x^4}{8\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{e^2\left(420b\,c^2d^4-4\cdots\right.}{8\left(\cdots\right.}}{\cdots}\right.$
risch	$\frac{-\frac{3c^2e^6b\,f^4x^7}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{21c^2d\,e^5b\,f^4x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(-252bc\,d^2+4ac-19b^2\right)c\,e^4f^4x^5}{128a^2c^2-64a\,b^2c+8b^4}+\frac{5cd\,e^3f^4\left(-84bc\,d^2+4ac-19b^2\right)x^4}{8\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{e^2f^4\left(420b\,c^2d^4-\cdots\right.}{8\left(\cdots\right.}}{\cdots}$

input
```
int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
f^4*((-3/2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-21/2*c^2*d*e^5*b/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^6+1/8*(-252*b*c*d^2+4*a*c-19*b^2)*c*e^4/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^5+5/8*c*d*e^3*(-84*b*c*d^2+4*a*c-19*b^2)/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^4-1/8*e^2*(420*b*c^2*d^4-40*a*c^2*d^2+190*b^2*c*d^2+16*a*b*
c+5*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(252*b*c^2*d^4-40*a*c^2*d^
2+190*b^2*c*d^2+48*a*b*c+15*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(84*b*
c^2*d^6-20*a*c^2*d^4+95*b^2*c*d^4+48*a*b*c*d^2+15*b^3*d^2+12*a^2*c+3*a*b^2
)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/8*d/e*(12*b*c^2*d^6-4*a*c^2*d^4+19*b^2*c*
d^4+16*a*b*c*d^2+5*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)^2+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-4*_R^2*b*c*e^2-8*_R
*b*c*d*e-4*b*c*d^2+4*a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*
c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^
2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6770 vs. $2(305) = 610$.

Time = 0.37 (sec) , antiderivative size = 6770, normalized size of antiderivative = 19.18

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2237 vs. $2(328) = 656$.

Time = 140.36 (sec) , antiderivative size = 2237, normalized size of antiderivative = 6.34

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
(-12*a**2*c*d*f**4 - 3*a*b**2*d*f**4 - 16*a*b*c*d**3*f**4 + 4*a*c**2*d**5*
f**4 - 5*b**3*d**3*f**4 - 19*b**2*c*d**5*f**4 - 12*b*c**2*d**7*f**4 - 84*b
*c**2*d*e**6*f**4*x**6 - 12*b*c**2*e**7*f**4*x**7 + x**5*(4*a*c**2*e**5*f*
*4 - 19*b**2*c*e**5*f**4 - 252*b*c**2*d**2*e**5*f**4) + x**4*(20*a*c**2*d*
e**4*f**4 - 95*b**2*c*d*e**4*f**4 - 420*b*c**2*d**3*e**4*f**4) + x**3*(-16
*a*b*c*e**3*f**4 + 40*a*c**2*d**2*e**3*f**4 - 5*b**3*e**3*f**4 - 190*b**2*
c*d**2*e**3*f**4 - 420*b*c**2*d**4*e**3*f**4) + x**2*(-48*a*b*c*d*e**2*f**
4 + 40*a*c**2*d**3*e**2*f**4 - 15*b**3*d*e**2*f**4 - 190*b**2*c*d**3*e**2*
f**4 - 252*b*c**2*d**5*e**2*f**4) + x*(-12*a**2*c*e*f**4 - 3*a*b**2*e*f**4
 - 48*a*b*c*d**2*e*f**4 + 20*a*c**2*d**4*e*f**4 - 15*b**3*d**2*e*f**4 - 95
*b**2*c*d**4*e*f**4 - 84*b*c**2*d**6*e*f**4))/(128*a**4*c**2*e - 64*a**3*b
**2*c*e + 256*a**3*b*c**2*d**2*e + 256*a**3*c**3*d**4*e + 8*a**2*b**4*e -
128*a**2*b**3*c*d**2*e + 256*a**2*b*c**3*d**6*e + 128*a**2*c**4*d**8*e + 1
6*a*b**5*d**2*e - 48*a*b**4*c*d**4*e - 128*a*b**3*c**2*d**6*e - 64*a*b**2*
c**3*d**8*e + 8*b**6*d**4*e + 16*b**5*c*d**6*e + 8*b**4*c**2*d**8*e + x**8
*(128*a**2*c**4*e**9 - 64*a*b**2*c**3*e**9 + 8*b**4*c**2*e**9) + x**7*(102
4*a**2*c**4*d*e**8 - 512*a*b**2*c**3*d*e**8 + 64*b**4*c**2*d*e**8) + x**6*
(256*a**2*b*c**3*e**7 + 3584*a**2*c**4*d**2*e**7 - 128*a*b**3*c**2*e**7 -
1792*a*b**2*c**3*d**2*e**7 + 16*b**5*c*e**7 + 224*b**4*c**2*d**2*e**7) + x
**5*(1536*a**2*b*c**3*d*e**6 + 7168*a**2*c**4*d**3*e**6 - 768*a*b**3*c*...
```

Maxima [F]

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\int \frac{(efx+df)^4}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima
")
```

output
```
-3/8*f^4*integrate((4*b*c*e^2*x^2 + 8*b*c*d*e*x + 4*b*c*d^2 - b^2 - 4*a*c)
/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2
*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c + 16*a^2*c^2) - 1/8*(12*b*c^2*
e^7*f^4*x^7 + 84*b*c^2*d*e^6*f^4*x^6 + (252*b*c^2*d^2 + 19*b^2*c - 4*a*c^2
)*e^5*f^4*x^5 + 5*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2)*d)*e^4*f^4*x^4 + (4
20*b*c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^2)*d^2)*e^3*f^4*x^3
 + (252*b*c^2*d^5 + 10*(19*b^2*c - 4*a*c^2)*d^3 + 3*(5*b^3 + 16*a*b*c)*d)*
e^2*f^4*x^2 + (84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*d^4 + 3*a*b^2 + 12*a^
2*c + 3*(5*b^3 + 16*a*b*c)*d^2)*e*f^4*x + (12*b*c^2*d^7 + (19*b^2*c - 4*a*
c^2)*d^5 + (5*b^3 + 16*a*b*c)*d^3 + 3*(a*b^2 + 4*a^2*c)*d)*f^4)/((b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b
^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*
b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6
- 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1958 vs. $2(305) = 610$.

Time = 0.18 (sec) , antiderivative size = 1958, normalized size of antiderivative = 5.55

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
3/16*((4*b*c*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) + d/e)^2 - 8*b*c*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)/(c*e^4)) + d/e) + 4*b*c*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)*log(x + sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(s
qrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3
*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)) - (4*b*c*e^2*f^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 8*b*c*d*e*f^4*(sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 4*b*c*d^2*f^4 - b^2*f^4 - 4*
a*c*f^4)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
+ d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
- d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e
^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^2)*(sqrt(1/2)*sqrt(
-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + (4*b*c*e^2*f^4*(sqrt(1
/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 8*b*c*d*e*f^
4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + 4*b*c
*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)*log(x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 - s...
```

Mupad [B] (verification not implemented)

Time = 15.58 (sec) , antiderivative size = 13840, normalized size of antiderivative = 39.21

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
atan(((-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8
 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1
024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^
20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2
- 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2
 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*
e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a
*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13
- 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b
^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 2
40*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 -
24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b
*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b
^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
16*a*b^6*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b
*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4
*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(5
12*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*
c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10
*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a...
```

Reduce [F]

$$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{too large to display}$$

input
```
int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

output

```
(f**4*( - 12*int(x**5/(a**3*b**2 + 12*a**3*b*c*d**2 + 36*a**3*c**2*d**4 +
3*a**2*b**3*d**2 + 6*a**2*b**3*d*e*x + 3*a**2*b**3*e**2*x**2 + 39*a**2*b**
2*c*d**4 + 84*a**2*b**2*c*d**3*e*x + 54*a**2*b**2*c*d**2*e**2*x**2 + 12*a*
*2*b**2*c*d*e**3*x**3 + 3*a**2*b**2*c*e**4*x**4 + 144*a**2*b*c**2*d**6 + 3
60*a**2*b*c**2*d**5*e*x + 324*a**2*b*c**2*d**4*e**2*x**2 + 144*a**2*b*c**2
*d**3*e**3*x**3 + 36*a**2*b*c**2*d**2*e**4*x**4 + 108*a**2*c**3*d**8 + 432
*a**2*c**3*d**7*e*x + 648*a**2*c**3*d**6*e**2*x**2 + 432*a**2*c**3*d**5*e*
*3*x**3 + 108*a**2*c**3*d**4*e**4*x**4 + 3*a*b**4*d**4 + 12*a*b**4*d**3*e*
x + 18*a*b**4*d**2*e**2*x**2 + 12*a*b**4*d*e**3*x**3 + 3*a*b**4*e**4*x**4
+ 42*a*b**3*c*d**6 + 180*a*b**3*c*d**5*e*x + 306*a*b**3*c*d**4*e**2*x**2 +
 264*a*b**3*c*d**3*e**3*x**3 + 126*a*b**3*c*d**2*e**4*x**4 + 36*a*b**3*c*d
*e**5*x**5 + 6*a*b**3*c*e**6*x**6 + 183*a*b**2*c**2*d**8 + 888*a*b**2*c**2
*d**7*e*x + 1812*a*b**2*c**2*d**6*e**2*x**2 + 2040*a*b**2*c**2*d**5*e**3*x
**3 + 1398*a*b**2*c**2*d**4*e**4*x**4 + 600*a*b**2*c**2*d**3*e**5*x**5 + 1
56*a*b**2*c**2*d**2*e**6*x**6 + 24*a*b**2*c**2*d*e**7*x**7 + 3*a*b**2*c**2
*e**8*x**8 + 252*a*b*c**3*d**10 + 1584*a*b*c**3*d**9*e*x + 4248*a*b*c**3*d
**8*e**2*x**2 + 6336*a*b*c**3*d**7*e**3*x**3 + 5760*a*b*c**3*d**6*e**4*x**
4 + 3312*a*b*c**3*d**5*e**5*x**5 + 1224*a*b*c**3*d**4*e**6*x**6 + 288*a*b*
c**3*d**3*e**7*x**7 + 36*a*b*c**3*d**2*e**8*x**8 + 108*a*c**4*d**12 + 864*
a*c**4*d**11*e*x + 3024*a*c**4*d**10*e**2*x**2 + 6048*a*c**4*d**9*e**3*...
```

3.284 $\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 159

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^3(2a+b(d+ex)^2)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3(b+2c(d+ex)^2)}{4\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{3bcf^3\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}e}$$

output
```
1/4*f^3*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2-3/4
*b*f^3*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+3*b*
c*f^3*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$=\frac{f^3\left(-\frac{3b\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}+\frac{(b^2-4ac)\left(2a+b(d+ex)^2\right)}{\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)^2}-\frac{12bc\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}\right)}{4\left(b^2-4ac\right)^2e}$$

input
```
Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f^3*((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (
(b^2 - 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))
^2 - (12*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
 4*a*c]))/(4*(b^2 - 4*a*c)^2*e)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1462, 1434, 1159, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$\Big\downarrow \text{1462}$$
$$\frac{f^3\int \frac{(d+ex)^3}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)}{e}$$
$$\Big\downarrow \text{1434}$$
$$\frac{f^3\int \frac{(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)^2}{2e}$$

$$\Big\downarrow \text{1159}$$

$$\frac{f^3\left(\frac{3b\int \frac{1}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2(b^2-4ac)}+\frac{2a+b(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

$$\Big\downarrow \text{1086}$$

$$\frac{f^3\left(\frac{3b\left(-\frac{2c\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{b^2-4ac}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{2(b^2-4ac)}+\frac{2a+b(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

$$\Big\downarrow \text{1083}$$

$$\frac{f^3\left(\frac{3b\left(\frac{4c\int \frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{2(b^2-4ac)}+\frac{2a+b(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{f^3\left(\frac{3b\left(\frac{4c\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{2(b^2-4ac)}+\frac{2a+b(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

input
```
Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f^3*((2*a + b*(d + e*x)^2)/(2*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e
*x)^4)^2) + (3*b*(-((b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4))) + (4*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c
]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c))))/(2*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.45

method	result
default	$f^3 \left(\frac{-\frac{3c^2e^5b\,x^6}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{9e^4db\,c^2x^5}{16a^2c^2-8a\,b^2c+b^4}-\frac{9bc\,e^3(10c\,d^2+b)x^4}{4(16a^2c^2-8a\,b^2c+b^4)}-\frac{3cd\,e^2b(10c\,d^2+3b)x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{be(45c^2d^4+27bc\,d^2+5ac+b^2)x^2}{2(16a^2c^2-8a\,b^2c+b^4)}-db}{(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b} \right.$
risch	$\frac{-\frac{3c^2e^5b\,f^3x^6}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{9f^3e^4db\,c^2x^5}{16a^2c^2-8a\,b^2c+b^4}-\frac{9bc\,e^3f^3(10c\,d^2+b)x^4}{4(16a^2c^2-8a\,b^2c+b^4)}-\frac{3cd\,e^2b\,f^3(10c\,d^2+3b)x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{be\,f^3(45c^2d^4+27bc\,d^2+5ac+b^2)x^2}{2(16a^2c^2-8a\,b^2c+b^4)}-db}{(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b}$

input
```
int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
f^3*((-3/2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*e^4*d*b*c^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^5-9/4*b*c*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^4-3*c*d*e^2*b*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*b*e*(45
*c^2*d^4+27*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-d*b*(9*c^2*d
^4+9*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/4/e*(6*b*c^2*d^6+9*
b^2*c*d^4+10*a*b*c*d^2+2*b^3*d^2+8*a^2*c+a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)
)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b
*d*e*x+b*d^2+a)^2+3/2*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-_R*e-d)/(2*_R
^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootO
f(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_
Z+c*d^4+b*d^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1856 vs. 2(151) = 302.

Time = 0.26 (sec) , antiderivative size = 3843, normalized size of antiderivative = 24.17

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1794 vs. $2(144) = 288$.

Time = 7.54 (sec) , antiderivative size = 1794, normalized size of antiderivative = 11.28

$$\int \frac{(df + efx)^3}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
3*b*c*f**3*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*b*c*
*4*f**3*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*f**3*sqrt(-1/(4*a*
c - b**2)**5) - 36*a*b**5*c**2*f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*
f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c*f**3 + 6*b*c**2*d**2*f**3)/(6*b
*c**2*e**2*f**3))/(2*e) - 3*b*c*f**3*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/
e + x**2 + (192*a**3*b*c**4*f**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**
3*c**3*f**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**5*c**2*f**3*sqrt(-1/(4*a*
c - b**2)**5) - 3*b**7*c*f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c*f**3 +
 6*b*c**2*d**2*f**3)/(6*b*c**2*e**2*f**3))/(2*e) + (-8*a**2*c*f**3 - a*b**
2*f**3 - 10*a*b*c*d**2*f**3 - 2*b**3*d**2*f**3 - 9*b**2*c*d**4*f**3 - 6*b*
c**2*d**6*f**3 - 36*b*c**2*d*e**5*f**3*x**5 - 6*b*c**2*e**6*f**3*x**6 + x*
*4*(-9*b**2*c*e**4*f**3 - 90*b*c**2*d**2*e**4*f**3) + x**3*(-36*b**2*c*d*e
**3*f**3 - 120*b*c**2*d**3*e**3*f**3) + x**2*(-10*a*b*c*e**2*f**3 - 2*b**3
*e**2*f**3 - 54*b**2*c*d**2*e**2*f**3 - 90*b*c**2*d**4*e**2*f**3) + x*(-20
*a*b*c*d*e*f**3 - 4*b**3*d*e*f**3 - 36*b**2*c*d**3*e*f**3 - 36*b*c**2*d**5
*e*f**3))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**2*d**2*e + 12
8*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 128*a**2*b*c*
*3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d**4*e - 6
4*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b**5*c*d*
*6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3*e*...
```

Maxima [F]

$$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx=\int \frac{(efx+df)^3}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima
")
```

output
```
-3*b*c*f^3*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d
^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c
+ 16*a^2*c^2) - 1/4*(6*b*c^2*e^6*f^3*x^6 + 36*b*c^2*d*e^5*f^3*x^5 + 9*(10*
b*c^2*d^2 + b^2*c)*e^4*f^3*x^4 + 12*(10*b*c^2*d^3 + 3*b^2*c*d)*e^3*f^3*x^3
 + 2*(45*b*c^2*d^4 + 27*b^2*c*d^2 + b^3 + 5*a*b*c)*e^2*f^3*x^2 + 4*(9*b*c^
2*d^5 + 9*b^2*c*d^3 + (b^3 + 5*a*b*c)*d)*e*f^3*x + (6*b*c^2*d^6 + 9*b^2*c*
d^4 + a*b^2 + 8*a^2*c + 2*(b^3 + 5*a*b*c)*d^2)*f^3)/((b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7
 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a
^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c
+ 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6
+ a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^
5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c
^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(151) = 302$.

Time = 0.17 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.71

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = -\frac{3\,bcf^3\arctan\left(\frac{2\,cd^2f+2\,(efx^2+2\,dfx)ce+bf}{\sqrt{-b^2+4\,acf}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ace}} - \frac{6\,bc^2d^6f^7+18\,(efx^2+2\,dfx)bc^2d^4ef^6+18\,(efx^2+2\,dfx)^2bc^2d^2e^2f^5+9\,b^2cd^4f^7+6\,(efx^2+2\,dfx)^3}{4\,\left(cd^4f^2+2\,(efx^2+2\,dfx)cd^2\right.}$$

input
```
integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
-3*b*c*f^3*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2
 + 4*a*c)*f))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)*e) - 1/4*
(6*b*c^2*d^6*f^7 + 18*(e*f*x^2 + 2*d*f*x)*b*c^2*d^4*e*f^6 + 18*(e*f*x^2 +
2*d*f*x)^2*b*c^2*d^2*e^2*f^5 + 9*b^2*c*d^4*f^7 + 6*(e*f*x^2 + 2*d*f*x)^3*b
*c^2*e^3*f^4 + 18*(e*f*x^2 + 2*d*f*x)*b^2*c*d^2*e*f^6 + 9*(e*f*x^2 + 2*d*f
*x)^2*b^2*c*e^2*f^5 + 2*b^3*d^2*f^7 + 10*a*b*c*d^2*f^7 + 2*(e*f*x^2 + 2*d*
f*x)*b^3*e*f^6 + 10*(e*f*x^2 + 2*d*f*x)*a*b*c*e*f^6 + a*b^2*f^7 + 8*a^2*c*
f^7)/((c*d^4*f^2 + 2*(e*f*x^2 + 2*d*f*x)*c*d^2*e*f + (e*f*x^2 + 2*d*f*x)^2
*c*e^2 + b*d^2*f^2 + (e*f*x^2 + 2*d*f*x)*b*e*f + a*f^2)^2*(b^4*e - 8*a*b^2
*c*e + 16*a^2*c^2*e))
```

Mupad [B] (verification not implemented)

Time = 12.22 (sec) , antiderivative size = 1267, normalized size of antiderivative = 7.97

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
- ((a*b^2*f^3 + 8*a^2*c*f^3 + 2*b^3*d^2*f^3 + 9*b^2*c*d^4*f^3 + 6*b*c^2*d^
6*f^3 + 10*a*b*c*d^2*f^3)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3
*e*f^3 + 27*b^2*c*d^2*e*f^3 + 45*b*c^2*d^4*e*f^3 + 5*a*b*c*e*f^3))/(2*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*x^4*(b^2*c*e^3*f^3 + 10*b*c^2*d^2*e^3*f^3
))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*d*x^3*(3*b^2*c*e^2*f^3 + 10*b*c
^2*d^2*e^2*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (d*x*(b^3*f^3 + 9*b^2*c*
d^2*f^3 + 9*b*c^2*d^4*f^3 + 5*a*b*c*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) +
 (3*b*c^2*e^5*f^3*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*d*e^4
*f^3*x^5)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6
*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6
+ 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e +
 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3
*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^
4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 +
2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) - (3*b*c*f^3*atan(((b
^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^
5)*(x^2*((9*b^2*c^4*e^8*f^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (9*b^3*c^2*f^6*(2*b^5*c^2*e^10 - 16*a*b^3*c^3*e^10 + 32*a^2*b*
c^4*e^10))/(2*a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))
+ x*((9*b^3*c^2*f^6*(2*b^5*c^2*d*e^9 - 16*a*b^3*c^3*d*e^9 + 32*a^2*b*c^...
```

Reduce [F]

$$\int \frac{(df + efx)^3}{(a + b(d + ex)^2 + c(d + ex)^4)^3}\,dx = \text{too large to display}$$

input
```
int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

output

```
(f**3*(4*int(x**3/(a**4*b + 2*a**4*c*d**2 + 4*a**3*b**2*d**2 + 6*a**3*b**2
*d*e*x + 3*a**3*b**2*e**2*x**2 + 12*a**3*b*c*d**4 + 24*a**3*b*c*d**3*e*x +
 24*a**3*b*c*d**2*e**2*x**2 + 12*a**3*b*c*d*e**3*x**3 + 3*a**3*b*c*e**4*x*
*4 + 8*a**3*c**2*d**6 + 24*a**3*c**2*d**5*e*x + 36*a**3*c**2*d**4*e**2*x**
2 + 24*a**3*c**2*d**3*e**3*x**3 + 6*a**3*c**2*d**2*e**4*x**4 + 6*a**2*b**3
*d**4 + 18*a**2*b**3*d**3*e*x + 21*a**2*b**3*d**2*e**2*x**2 + 12*a**2*b**3
*d*e**3*x**3 + 3*a**2*b**3*e**4*x**4 + 24*a**2*b**2*c*d**6 + 90*a**2*b**2*
c*d**5*e*x + 153*a**2*b**2*c*d**4*e**2*x**2 + 156*a**2*b**2*c*d**3*e**3*x*
*3 + 99*a**2*b**2*c*d**2*e**4*x**4 + 36*a**2*b**2*c*d*e**5*x**5 + 6*a**2*b
**2*c*e**6*x**6 + 30*a**2*b*c**2*d**8 + 144*a**2*b*c**2*d**7*e*x + 324*a**
2*b*c**2*d**6*e**2*x**2 + 444*a**2*b*c**2*d**5*e**3*x**3 + 399*a**2*b*c**2
*d**4*e**4*x**4 + 240*a**2*b*c**2*d**3*e**5*x**5 + 96*a**2*b*c**2*d**2*e**
6*x**6 + 24*a**2*b*c**2*d*e**7*x**7 + 3*a**2*b*c**2*e**8*x**8 + 12*a**2*c*
*3*d**10 + 72*a**2*c**3*d**9*e*x + 204*a**2*c**3*d**8*e**2*x**2 + 360*a**2
*c**3*d**7*e**3*x**3 + 426*a**2*c**3*d**6*e**4*x**4 + 336*a**2*c**3*d**5*e
**5*x**5 + 168*a**2*c**3*d**4*e**6*x**6 + 48*a**2*c**3*d**3*e**7*x**7 + 6*
a**2*c**3*d**2*e**8*x**8 + 4*a*b**4*d**6 + 18*a*b**4*d**5*e*x + 33*a*b**4*
d**4*e**2*x**2 + 32*a*b**4*d**3*e**3*x**3 + 18*a*b**4*d**2*e**4*x**4 + 6*a
*b**4*d*e**5*x**5 + a*b**4*e**6*x**6 + 20*a*b**3*c*d**8 + 108*a*b**3*c*d**
7*e*x + 258*a*b**3*c*d**6*e**2*x**2 + 364*a*b**3*c*d**5*e**3*x**3 + 339...
```

3.285 $\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3} \, dx$

Optimal result

Integrand size = 33, antiderivative size = 375

$$\begin{aligned}\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3} \, dx &= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \\ &\quad+\frac{f^2(d+ex)\left(b(b^2+8ac)+c\left(b^2+20ac\right)(d+ex)^2\right)}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\ &\quad+\frac{\sqrt{c}\left(b^2+20ac+\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}e} \\ &\quad+\frac{\sqrt{c}\left(b^2+20ac-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)f^2\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}e}\end{aligned}$$

output
```
-1/4*f^2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)
^4)^2+1/8*f^2*(e*x+d)*(b*(8*a*c+b^2)+c*(20*a*c+b^2)*(e*x+d)^2)/a/(-4*a*c+b
^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/16*c^(1/2)*(b^2+20*a*c+b*(-52*a*c+b^
2)/(-4*a*c+b^2)^(1/2))*f^2*arctan(2^(1/2)*c^(1/2)*(e*x+d)/(b-(-4*a*c+b^2)^
(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)/e+1/16
*c^(1/2)*(b^2+20*a*c-b*(-52*a*c+b^2)/(-4*a*c+b^2)^(1/2))*f^2*arctan(2^(1/2
)*c^(1/2)*(e*x+d)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^2/(
b+(-4*a*c+b^2)^(1/2))^(1/2)/e
```

Mathematica [A] (verified)

Time = 5.01 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.03

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$$

$$= \frac{f^2\left(-\frac{4(b(d+ex)+2c(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{2(d+ex)(b^3+8abc+b^2c(d+ex)^2+20ac^2(d+ex)^2)}{a(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b}\right.}{a(b^2-4ac)^{5/2}\sqrt{b-}}\right.}{16e}$$

input
```
Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f^2*((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 +
 20*a*c^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)) + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*S
qrt[c]*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2
 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(16*e)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1462, 1439, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f^2\int \frac{(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)}{e}$$

$$\Big\downarrow \text{1439}$$

$$\frac{f^2\left(\frac{\int \frac{b-10c(d+ex)^2}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4(b^2-4ac)}-\frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{e}$$

$$\Big\downarrow \text{1492}$$

$$\frac{f^2\left(\frac{\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{2a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int -\frac{c\left(b^2+20ac\right)(d+ex)^2+b\left(b^2-16ac\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2a\left(b^2-4ac\right)}}{4(b^2-4ac)}-\frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{e}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\left(\frac{\frac{\int \frac{c\left(b^2+20ac\right)(d+ex)^2+b\left(b^2-16ac\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{2a\left(b^2-4ac\right)}+\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{2a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{4(b^2-4ac)}-\frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{e}$$

$$\Big\downarrow \text{1480}$$

$$f^2\left(\frac{\frac{\frac{1}{2}c\left(-\frac{52abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\int\frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(\frac{52abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\int\frac{1}{c(d+ex)^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d(d+ex)}{2a\left(b^2-4ac\right)}}{4(b^2-4ac)}\right.$$

$$e$$

$\downarrow$ 218

$$f^2\left(\frac{\frac{\frac{\sqrt{c}\left(-\frac{52abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(\frac{52abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{(d+ex)\left(c\left(20ac+\right.\right.}{2a\left(b^2-4ac\right)\left(a\right.}}{4(b^2-4ac)}\right.$$

$$e$$

input
```
Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f^2*(-1/4*((d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + b*(d + e*
x)^2 + c*(d + e*x)^4)^2) + (((d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)
*(d + e*x)^2))/(2*a*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (
(Sqrt[c]*(b^2 + 20*a*c + b^3/Sqrt[b^2 - 4*a*c] - (52*a*b*c)/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b^2 + 20*a*c - b^3/Sqrt[b^2
- 4*a*c] + (52*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x)
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*
a*(b^2 - 4*a*c)))/(4*(b^2 - 4*a*c))))/e
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1439
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
1)*(b^2 - 4*a*c))), x] - Simp[d^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 889, normalized size of antiderivative = 2.37

method	result
default	$f^2\left(\frac{c^2e^6(20ac+b^2)x^7}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{7c^2d\,e^5(20ac+b^2)x^6}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{(420a\,c^2d^2+21b^2c\,d^2+28abc+2b^3)c\,e^4x^5}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{5cd\,e^3(140a\,c^2d^2+7b^2c\,d^2+28abc+2b^3)}{8(16a^2c^2-8a\,b^2c+b^4)a}\right.$
risch	$\frac{c^2e^6f^2(20ac+b^2)x^7}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{7c^2d\,e^5f^2(20ac+b^2)x^6}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{(420a\,c^2d^2+21b^2c\,d^2+28abc+2b^3)c\,e^4f^2x^5}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{5cd\,e^3f^2(140a\,c^2d^2+7b^2c\,d^2+28abc+2b^3)x}{8(16a^2c^2-8a\,b^2c+b^4)a}$

input

```
int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output

```
f^2*((1/8*c^2*e^6*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7+7/8*c^2*d*
e^5*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6+1/8*(420*a*c^2*d^2+21*b^
2*c*d^2+28*a*b*c+2*b^3)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5+5/8*c*d*e^3
*(140*a*c^2*d^2+7*b^2*c*d^2+28*a*b*c+2*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x
^4+1/8*e^2*(700*a*c^3*d^4+35*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+36*a
^2*c^2+5*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3+1/8*d*e*(420*a*c^3*
d^4+21*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+108*a^2*c^2+15*a*b^2*c+3*b
^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^2+1/8*(140*a*c^3*d^6+7*b^2*c^2*d^6+140*
a*b*c^2*d^4+10*b^3*c*d^4+108*a^2*c^2*d^2+15*a*b^2*c*d^2+3*b^4*d^2+16*a^2*b
*c-a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x+1/8*d/e*(20*a*c^3*d^6+b^2*c^2*d^6
+28*a*b*c^2*d^4+2*b^3*c*d^4+36*a^2*c^2*d^2+5*a*b^2*c*d^2+b^4*d^2+16*a^2*b*
c-a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/16/(16*a^2*c^2-8*
a*b^2*c+b^4)/a/e*sum((c*e^2*(20*a*c+b^2)*_R^2+2*c*d*e*(20*a*c+b^2)*_R+20*a
*c^2*d^2+b^2*c*d^2-16*a*b*c+b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e
+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z^3+(6*c*d^2
*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7838 vs. $2(331) = 662$.

Time = 0.54 (sec) , antiderivative size = 7838, normalized size of antiderivative = 20.90

$$\int \frac{(df + efx)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(df + efx)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^3} \, dx = \text{Timed out}$$

input
```
integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(df + efx)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^3} \, dx = \int \frac{(efx + df)^2}{\left((ex + d)^4 c + (ex + d)^2 b + a\right)^3} \, dx$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima
")
```

output
```
1/8*f^2*integrate(((b^2*c + 20*a*c^2)*e^2*x^2 + 2*(b^2*c + 20*a*c^2)*d*e*x
 + b^3 - 16*a*b*c + (b^2*c + 20*a*c^2)*d^2)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c
*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a*b
^4 - 8*a^2*b^2*c + 16*a^3*c^2) + 1/8*((b^2*c^2 + 20*a*c^3)*e^7*f^2*x^7 + 7
*(b^2*c^2 + 20*a*c^3)*d*e^6*f^2*x^6 + (2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2
+ 20*a*c^3)*d^2)*e^5*f^2*x^5 + 5*(7*(b^2*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c +
14*a*b*c^2)*d)*e^4*f^2*x^4 + (35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*
c + 36*a^2*c^2 + 20*(b^3*c + 14*a*b*c^2)*d^2)*e^3*f^2*x^3 + (21*(b^2*c^2 +
 20*a*c^3)*d^5 + 20*(b^3*c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*c + 36*a^2
*c^2)*d)*e^2*f^2*x^2 + (7*(b^2*c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^
2)*d^4 - a*b^3 + 16*a^2*b*c + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*f^2*
x + ((b^2*c^2 + 20*a*c^3)*d^7 + 2*(b^3*c + 14*a*b*c^2)*d^5 + (b^4 + 5*a*b^
2*c + 36*a^2*c^2)*d^3 - (a*b^3 - 16*a^2*b*c)*d)*f^2)/((a*b^4*c^2 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*
d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*
a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3
+ 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5
+ (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a
^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4
*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2679 vs. $2(331) = 662$.

Time = 0.18 (sec) , antiderivative size = 2679, normalized size of antiderivative = 7.14

$$\int \frac{(df + efx)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
-1/16*((b^2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*
e^4)) + d/e)^2 + 20*a*c^2*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)/(c*e^4)) + d/e)^2 - 2*b^2*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) - 40*a*c^2*d*e*f^2*(sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e) + b^2*c*d^2*f^2 + 20*a*c^2*d
^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)*log(x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) + d/e)^3 - 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e)^2 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 +
 b*e^2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) + d/e))
- (b^2*c*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4))
 - d/e)^2 + 20*a*c^2*e^2*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)/(c*e^4)) - d/e)^2 + 2*b^2*c*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) - d/e) + 40*a*c^2*d*e*f^2*(sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e) + b^2*c*d^2*f^2 + 20*a*c^2*d^2*f^
2 + b^3*f^2 - 16*a*b*c*f^2)*log(x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) + d/e)/(2*c*e^4*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)/(c*e^4)) - d/e)^3 + 6*c*d*e^3*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
 - 4*a*c)*e^2)/(c*e^4)) - d/e)^2 + 2*c*d^3*e + b*d*e + (6*c*d^2*e^2 + b*e^
2)*(sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)/(c*e^4)) - d/e)) + ...
```

Mupad [B] (verification not implemented)

Time = 15.13 (sec) , antiderivative size = 16025, normalized size of antiderivative = 42.73

$$\int \frac{(df + efx)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^3} \, dx = \text{Too large to display}$$

input
```
int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
atan((((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^
3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^
13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512
*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a
^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x
*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 -
40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14
))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^
2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c
^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^
4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c
^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3
*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e
^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*
e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4
*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9*c^4*e^12*f^
2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5
*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2
 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10
*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b...
```

Reduce [F]

$$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{too large to display}$$

input
```
int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

output

```
(f**2*( - 28*int(x**4/(a**3*b + 2*a**3*c*d**2 + 3*a**2*b**2*d**2 + 6*a**2*
b**2*d*e*x + 3*a**2*b**2*e**2*x**2 + 9*a**2*b*c*d**4 + 24*a**2*b*c*d**3*e*
x + 24*a**2*b*c*d**2*e**2*x**2 + 12*a**2*b*c*d*e**3*x**3 + 3*a**2*b*c*e**4
*x**4 + 6*a**2*c**2*d**6 + 24*a**2*c**2*d**5*e*x + 36*a**2*c**2*d**4*e**2*
x**2 + 24*a**2*c**2*d**3*e**3*x**3 + 6*a**2*c**2*d**2*e**4*x**4 + 3*a*b**3
*d**4 + 12*a*b**3*d**3*e*x + 18*a*b**3*d**2*e**2*x**2 + 12*a*b**3*d*e**3*x
**3 + 3*a*b**3*e**4*x**4 + 12*a*b**2*c*d**6 + 60*a*b**2*c*d**5*e*x + 126*a
*b**2*c*d**4*e**2*x**2 + 144*a*b**2*c*d**3*e**3*x**3 + 96*a*b**2*c*d**2*e*
*4*x**4 + 36*a*b**2*c*d*e**5*x**5 + 6*a*b**2*c*e**6*x**6 + 15*a*b*c**2*d**
8 + 96*a*b*c**2*d**7*e*x + 264*a*b*c**2*d**6*e**2*x**2 + 408*a*b*c**2*d**5
*e**3*x**3 + 390*a*b*c**2*d**4*e**4*x**4 + 240*a*b*c**2*d**3*e**5*x**5 + 9
6*a*b*c**2*d**2*e**6*x**6 + 24*a*b*c**2*d*e**7*x**7 + 3*a*b*c**2*e**8*x**8
 + 6*a*c**3*d**10 + 48*a*c**3*d**9*e*x + 168*a*c**3*d**8*e**2*x**2 + 336*a
*c**3*d**7*e**3*x**3 + 420*a*c**3*d**6*e**4*x**4 + 336*a*c**3*d**5*e**5*x*
*5 + 168*a*c**3*d**4*e**6*x**6 + 48*a*c**3*d**3*e**7*x**7 + 6*a*c**3*d**2*
e**8*x**8 + b**4*d**6 + 6*b**4*d**5*e*x + 15*b**4*d**4*e**2*x**2 + 20*b**4
*d**3*e**3*x**3 + 15*b**4*d**2*e**4*x**4 + 6*b**4*d*e**5*x**5 + b**4*e**6*
x**6 + 5*b**3*c*d**8 + 36*b**3*c*d**7*e*x + 114*b**3*c*d**6*e**2*x**2 + 20
8*b**3*c*d**5*e**3*x**3 + 240*b**3*c*d**4*e**4*x**4 + 180*b**3*c*d**3*e**5
*x**5 + 86*b**3*c*d**2*e**6*x**6 + 24*b**3*c*d*e**7*x**7 + 3*b**3*c*e**...
```

3.286 $\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 153

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = -\frac{f(b+2c(d+ex)^2)}{4(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3cf(b+2c(d+ex)^2)}{2(b^2-4ac)^2\,e\,(a+b(d+ex)^2+c(d+ex)^4)} - \frac{6c^2 f\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}\,e}$$

output
```
-1/4*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+3/2*
c*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-6*c^2*f
*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \frac{f\left(\frac{(b^2-4ac)(-b-2c(d+ex)^2)}{(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{6c(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4}+\frac{24c^2\arctan\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}\right)}{4(b^2-4ac)^2e}$$

input
```
Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f*(((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x
)^4)^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) +
 (24*c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a
*c]))/(4*(b^2 - 4*a*c)^2*e)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1462, 1432, 1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{f\int \frac{d+ex}{(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)}{e}$$

$$\Big\downarrow \text{1432}$$

$$\frac{f\int \frac{1}{(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)^2}{2e}$$

$$\Big\downarrow \ 1086$$

$$\frac{f\left(-\frac{3c\int \frac{1}{\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{b^2-4ac}-\frac{b+2c(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

$$\Big\downarrow \ 1086$$

$$\frac{f\left(-\frac{3c\left(-\frac{2c\int \frac{1}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)^2}{b^2-4ac}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

$$\Big\downarrow \ 1083$$

$$\frac{f\left(-\frac{3c\left(\frac{4c\int \frac{1}{-(d+ex)^4+b^2-4ac}d\left(2c(d+ex)^2+b\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

$$\Big\downarrow \ 219$$

$$\frac{f\left(-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2c(d+ex)^2}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\right)}{b^2-4ac}-\frac{b+2c(d+ex)^2}{2(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}\right)}{2e}$$

input
```
Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

output
```
(f*(-1/2*(b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)^2) - (3*c*(-((b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4))) + (4*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*
c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c)))/(2*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 543, normalized size of antiderivative = 3.55

method	result
default	$f\left(\frac{\frac{3c^3e^5x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18e^4d\,c^3x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6c^2d\,e^2\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{ce\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2cd\left(9c^2d\right.}{16a}}{\left(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2\right.}\right.$
risch	$\frac{\frac{3c^3e^5f\,x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18f\,e^4d\,c^3x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3f\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6c^2d\,e^2f\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{cef\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2cdf\left(9c^2d^4\right.}{16a^2c}}{\left(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+c\,d^4+2bdex+b\,d^2+\right.}$

input
```
int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
f*((3*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*e^4*d*c^3/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^5+9/2*c^2*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+6*c
^2*d*e^2*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+c*e*(45*c^2*d^4+27*
b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+2*c*d*(9*c^2*d^4+9*b*c*d
^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/4/e*(12*c^3*d^6+18*b*c^2*d^4+
20*a*c^2*d^2+4*b^2*c*d^2+10*a*b*c-b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*
x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)^2+3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_
R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4
+4*c*d*e^3*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^
2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1808 vs. $2(145) = 290$.

Time = 0.48 (sec) , antiderivative size = 3748, normalized size of antiderivative = 24.50

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1707 vs. 2(139) = 278.

Time = 6.90 (sec) , antiderivative size = 1707, normalized size of antiderivative = 11.16

$$\int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
-3*c**2*f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*c**5*
f*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)
**5) - 36*a*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*f*sqrt(-1
/(4*a*c - b**2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + 3*c
**2*f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*c**5*f*sqr
t(-1/(4*a*c - b**2)**5) - 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)**5)
+ 36*a*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*f*sqrt(-1/(4*a
*c - b**2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + (10*a*b*
c*f + 20*a*c**2*d**2*f - b**3*f + 4*b**2*c*d**2*f + 18*b*c**2*d**4*f + 12*
c**3*d**6*f + 72*c**3*d*e**5*f*x**5 + 12*c**3*e**6*f*x**6 + x**4*(18*b*c**
2*e**4*f + 180*c**3*d**2*e**4*f) + x**3*(72*b*c**2*d*e**3*f + 240*c**3*d**
3*e**3*f) + x**2*(20*a*c**2*e**2*f + 4*b**2*c*e**2*f + 108*b*c**2*d**2*e**
2*f + 180*c**3*d**4*e**2*f) + x*(40*a*c**2*d*e*f + 8*b**2*c*d*e*f + 72*b*c
**2*d**3*e*f + 72*c**3*d**5*e*f))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128
*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*
c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e
- 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b
**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**
9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 -
 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*...
```

Maxima [F]

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\int \frac{efx+df}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3}\,dx$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

output
```
6*c^2*f*integrate((e*x + d)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2
+ b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(b^4 - 8*a*b^2*c + 1
6*a^2*c^2) + 1/4*(12*c^3*e^6*f*x^6 + 72*c^3*d*e^5*f*x^5 + 18*(10*c^3*d^2 +
 b*c^2)*e^4*f*x^4 + 24*(10*c^3*d^3 + 3*b*c^2*d)*e^3*f*x^3 + 4*(45*c^3*d^4
+ 27*b*c^2*d^2 + b^2*c + 5*a*c^2)*e^2*f*x^2 + 8*(9*c^3*d^5 + 9*b*c^2*d^3 +
 (b^2*c + 5*a*c^2)*d)*e*f*x + (12*c^3*d^6 + 18*b*c^2*d^4 - b^3 + 10*a*b*c
+ 4*(b^2*c + 5*a*c^2)*d^2)*f)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^
8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6
+ 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d
^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x
^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*
a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c +
 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(145) = 290$.

Time = 0.17 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.80

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\frac{6\,c^2f\arctan\left(\frac{2\,cd^2f+2\,(efx^2+2\,dfx)ce+bf}{\sqrt{-b^2+4\,acf}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ace}}$$
$$+\frac{12\,c^3d^6f^5+36\,(efx^2+2\,dfx)c^3d^4ef^4+36\,(efx^2+2\,dfx)^2c^3d^2e^2f^3+18\,bc^2d^4f^5+12\,(efx^2+2\,dfx)^3}{4\,\left(cd^4f^2+2\,(efx^2+2\,dfx)c\right.}$$

input
```
integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
6*c^2*f*arctan((2*c*d^2*f + 2*(e*f*x^2 + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 +
4*a*c)*f))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)*e) + 1/4*(12
*c^3*d^6*f^5 + 36*(e*f*x^2 + 2*d*f*x)*c^3*d^4*e*f^4 + 36*(e*f*x^2 + 2*d*f*
x)^2*c^3*d^2*e^2*f^3 + 18*b*c^2*d^4*f^5 + 12*(e*f*x^2 + 2*d*f*x)^3*c^3*e^3
*f^2 + 36*(e*f*x^2 + 2*d*f*x)*b*c^2*d^2*e*f^4 + 18*(e*f*x^2 + 2*d*f*x)^2*b
*c^2*e^2*f^3 + 4*b^2*c*d^2*f^5 + 20*a*c^2*d^2*f^5 + 4*(e*f*x^2 + 2*d*f*x)*
b^2*c*e*f^4 + 20*(e*f*x^2 + 2*d*f*x)*a*c^2*e*f^4 - b^3*f^5 + 10*a*b*c*f^5)
/((c*d^4*f^2 + 2*(e*f*x^2 + 2*d*f*x)*c*d^2*e*f + (e*f*x^2 + 2*d*f*x)^2*c*e
^2 + b*d^2*f^2 + (e*f*x^2 + 2*d*f*x)*b*e*f + a*f^2)^2*(b^4*e - 8*a*b^2*c*e
 + 16*a^2*c^2*e))
```

Mupad [B] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 1199, normalized size of antiderivative = 7.84

$$\int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^3} \, dx = \text{Too large to display}$$

input
```
int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

output
```
((x^2*(5*a*c^2*e*f + b^2*c*e*f + 45*c^3*d^4*e*f + 27*b*c^2*d^2*e*f))/(b^4
+ 16*a^2*c^2 - 8*a*b^2*c) + (12*c^3*d^6*f - b^3*f + 20*a*c^2*d^2*f + 4*b^2
*c*d^2*f + 18*b*c^2*d^4*f + 10*a*b*c*f)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (9*x^4*(10*c^3*d^2*e^3*f + b*c^2*e^3*f))/(2*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)) + (2*d*x*(9*c^3*d^4*f + 5*a*c^2*f + b^2*c*f + 9*b*c^2*d^2*f))/(b^4
+ 16*a^2*c^2 - 8*a*b^2*c) + (6*d*x^3*(10*c^3*d^2*e^2*f + 3*b*c^2*e^2*f))/(
b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*c^3*e^5*f*x^6)/(b^4 + 16*a^2*c^2 - 8*a*
b^2*c) + (18*c^3*d*e^4*f*x^5)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*
d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) +
x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^
3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*
c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2
*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*
d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) +
 (6*c^2*f*atan(((b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^
2*c*(4*a*c - b^2)^5)*(x^2*((36*c^6*e^8*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (36*b*c^4*f^2*(b^5*c^2*e^10 - 8*a*b^3*c^3*e^10
+ 16*a^2*b*c^4*e^10))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c))) + x*((72*c^6*d*e^7*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)) + (72*b*c^4*f^2*(b^5*c^2*d*e^9 - 8*a*b^3*c^3*d*e^9 + 16*a...
```

Reduce [F]

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{too large to display}$$

input
```
int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

output
```
(f*(24*int(x**5/(a**3*b**2 + 8*a**3*b*c*d**2 + 12*a**3*c**2*d**4 + 3*a**2*
b**3*d**2 + 6*a**2*b**3*d*e*x + 3*a**2*b**3*e**2*x**2 + 27*a**2*b**2*c*d**
4 + 60*a**2*b**2*c*d**3*e*x + 42*a**2*b**2*c*d**2*e**2*x**2 + 12*a**2*b**2
*c*d*e**3*x**3 + 3*a**2*b**2*c*e**4*x**4 + 60*a**2*b*c**2*d**6 + 168*a**2*
b*c**2*d**5*e*x + 180*a**2*b*c**2*d**4*e**2*x**2 + 96*a**2*b*c**2*d**3*e**
3*x**3 + 24*a**2*b*c**2*d**2*e**4*x**4 + 36*a**2*c**3*d**8 + 144*a**2*c**3
*d**7*e*x + 216*a**2*c**3*d**6*e**2*x**2 + 144*a**2*c**3*d**5*e**3*x**3 +
36*a**2*c**3*d**4*e**4*x**4 + 3*a*b**4*d**4 + 12*a*b**4*d**3*e*x + 18*a*b*
*4*d**2*e**2*x**2 + 12*a*b**4*d*e**3*x**3 + 3*a*b**4*e**4*x**4 + 30*a*b**3
*c*d**6 + 132*a*b**3*c*d**5*e*x + 234*a*b**3*c*d**4*e**2*x**2 + 216*a*b**3
*c*d**3*e**3*x**3 + 114*a*b**3*c*d**2*e**4*x**4 + 36*a*b**3*c*d*e**5*x**5
+ 6*a*b**3*c*e**6*x**6 + 87*a*b**2*c**2*d**8 + 456*a*b**2*c**2*d**7*e*x +
1020*a*b**2*c**2*d**6*e**2*x**2 + 1272*a*b**2*c**2*d**5*e**3*x**3 + 966*a*
b**2*c**2*d**4*e**4*x**4 + 456*a*b**2*c**2*d**3*e**5*x**5 + 132*a*b**2*c**
2*d**2*e**6*x**6 + 24*a*b**2*c**2*d*e**7*x**7 + 3*a*b**2*c**2*e**8*x**8 +
96*a*b*c**3*d**10 + 624*a*b*c**3*d**9*e*x + 1752*a*b*c**3*d**8*e**2*x**2 +
 2784*a*b*c**3*d**7*e**3*x**3 + 2760*a*b*c**3*d**6*e**4*x**4 + 1776*a*b*c*
*3*d**5*e**5*x**5 + 744*a*b*c**3*d**4*e**6*x**6 + 192*a*b*c**3*d**3*e**7*x
**7 + 24*a*b*c**3*d**2*e**8*x**8 + 36*a*c**4*d**12 + 288*a*c**4*d**11*e*x
+ 1008*a*c**4*d**10*e**2*x**2 + 2016*a*c**4*d**9*e**3*x**3 + 2520*a*c**...
```

3.287 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 270

$$\begin{aligned}\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx &= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)\,ef\,(a+b(d+ex)^2+c(d+ex)^4)^2}\\ &\quad+\frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,(d+ex)^2}{4a^2\,(b^2-4ac)^2\,ef\,(a+b(d+ex)^2+c(d+ex)^4)}\\ &\quad+\frac{b(b^4-10ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3\,(b^2-4ac)^{5/2}\,ef}\\ &\quad+\frac{\log(d+ex)}{a^3ef}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3ef}\end{aligned}$$

output
```
1/4*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^
4)^2+1/4*(2*b^4-15*a*b^2*c+16*c^2*a^2+2*b*c*(-7*a*c+b^2)*(e*x+d)^2)/a^2/(-
4*a*c+b^2)^2/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/2*b*(30*a^2*c^2-10*a*b^2*c+
b^4)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(5/2)/
e/f+ln(e*x+d)/a^3/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e/f
```

Mathematica [A] (verified)

Time = 4.28 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.46

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$= \frac{\frac{a^2(-b^2+2ac-bc(d+ex)^2)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{a(2b^4-15ab^2c+16a^2c^2+2b^3c(d+ex)^2-14abc^2(d+ex)^2)}{(b^2-4ac)^2(a+(d+ex)^2(b+c(d+ex)^2))}+4\log(d+ex)-\frac{(b^5-10ab^3c+30a\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
((a^2*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*(d +
e*x)^2 - 14*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c
*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4
*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4
*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2) +
 ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqr
t[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] +
 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3*e*f)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1462, 1434, 1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$\Big\downarrow \text{1462}$$
$$\frac{\int \frac{1}{(d+ex)(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)}{ef}$$

$$\Big\downarrow \; 1434$$

$$\frac{\int \frac{1}{(d+ex)^2(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)^2}{2ef}$$

$$\Big\downarrow \; 1165$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\int -\frac{3bc(d+ex)^2+2\left(b^2-4ac\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2a(b^2-4ac)}}{2ef}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{3bc(d+ex)^2+2\left(b^2-4ac\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2ef}$$

$$\Big\downarrow \; 1235$$

$$\frac{\frac{\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int -\frac{2\left(\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)(d+ex)^2\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a\left(b^2-4ac\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2ef}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{2\int \frac{\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)(d+ex)^2}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a\left(b^2-4ac\right)}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}}{2ef}$$

$$\Big\downarrow \; 1200$$

$$\frac{\frac{\frac{2\int \left(\frac{\left(4ac-b^2\right)^2}{a(d+ex)^2}+\frac{-c\left(b^2-4ac\right)^2(d+ex)^2-b\left(b^4-9acb^2+23a^2c^2\right)}{a\left(c(d+ex)^4+b(d+ex)^2+a\right)}\right)d(d+ex)^2}{a\left(b^2-4ac\right)}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{a\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex\cdots}{2a(b^2-4ac)(a+b(d+ex)^2+\cdots}}{2ef}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{2\left(\frac{b(30a^2c^2-10ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{(b^2-4ac)^2\log((d+ex)^2)}{a}-\frac{(b^2-4ac)^2\log(a+b(d+ex)^2+c(d+ex)^4)}{2a}\right)}{a(b^2-4ac)}+\frac{16a^2c^2+2bc(b^2-7ac)(d+ex}{a(b^2-4ac)(a+b(d+ex)^2}}{2a(b^2-4ac)}}{2ef}$$

input
```
Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + ((2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)
*(d + e*x)^2)/(a*(b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (2*(
(b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)^2*Log[(d + e*x)^2])/a -
((b^2 - 4*a*c)^2*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a)))/(a*(b^2 -
 4*a*c)))/(2*a*(b^2 - 4*a*c)))/(2*e*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 970, normalized size of antiderivative = 3.59

method	result
default	$-\frac{\frac{c^2 e^5 (7ac-b^2) ab\, x^6}{32a^2c^2-16a\,b^2c+2b^4}+\frac{3(7ac-b^2) ab\, c^2 d\, e^4 x^5}{16a^2c^2-8a\,b^2c+b^4}-\frac{e^3 ac(-210ab\,c^2d^2+30b^3c\,d^2+16a^2c^2-29a\,b^2c+4b^4)x^4}{4(16a^2c^2-8a\,b^2c+b^4)}-\frac{cd\,e^2a(-70ab\,c^2d^2+10b^3c\,d^2+16a \cdots}{16a^2c^2-8a\,b^2c \cdots}}{\cdots}$
risch	Expression too large to display

input
```
int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3*((1/2*c^2*e^5*(7*a*c-b^2)*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3
*(7*a*c-b^2)*a*b*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/4*e^3*a*c*(-21
0*a*b*c^2*d^2+30*b^3*c*d^2+16*a^2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^4-c*d*e^2*a*(-70*a*b*c^2*d^2+10*b^3*c*d^2+16*a^2*c^2-29*a*b^2*c
+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*e*a*(105*a*b*c^3*d^4-15*b^3*c^2
*d^4-48*a^2*c^3*d^2+87*a*b^2*c^2*d^2-12*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-b^5)
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+d*a*(21*a*b*c^3*d^4-3*b^3*c^2*d^4-16*a^2*c
^3*d^2+29*a*b^2*c^2*d^2-4*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-b^5)/(16*a^2*c^2-8
*a*b^2*c+b^4)*x-1/4/e*a*(-14*a*b*c^3*d^6+2*b^3*c^2*d^6+16*a^2*c^3*d^4-29*a
*b^2*c^2*d^4+4*b^4*c*d^4-2*a^2*b*c^2*d^2-12*a*b^3*c*d^2+2*b^5*d^2+24*a^3*c
^2-21*a^2*b^2*c+3*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*
x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/2/(
16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((c*e^3*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*c
*d*e^2*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(48*a^2*c^3*d^2-24*a*b^2*c^2*d^2+
3*b^4*c*d^2+23*a^2*b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*a*b^2*c^2*d^3+
b^4*c*d^3+23*a^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6
*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*d*e^3*_Z
^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+c*d^4+b*d^2+a))+1/a^3/
e*ln(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4898 vs. $2(258) = 516$.

Time = 1.75 (sec) , antiderivative size = 9926, normalized size of antiderivative = 36.76

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3(efx+df)}\,dx$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima
")
```

output
```
1/4*(2*(b^3*c^2 - 7*a*b*c^3)*e^6*x^6 + 12*(b^3*c^2 - 7*a*b*c^3)*d*e^5*x^5
+ (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3 + 30*(b^3*c^2 - 7*a*b*c^3)*d^2)*e^4
*x^4 + 2*(b^3*c^2 - 7*a*b*c^3)*d^6 + 4*(10*(b^3*c^2 - 7*a*b*c^3)*d^3 + (4*
b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d)*e^3*x^3 + 3*a*b^4 - 21*a^2*b^2*c + 2
4*a^3*c^2 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^4 + 2*(b^5 - 6*a*b^3*c
 - a^2*b*c^2 + 15*(b^3*c^2 - 7*a*b*c^3)*d^4 + 3*(4*b^4*c - 29*a*b^2*c^2 +
16*a^2*c^3)*d^2)*e^2*x^2 + 2*(b^5 - 6*a*b^3*c - a^2*b*c^2)*d^2 + 4*(3*(b^3
*c^2 - 7*a*b*c^3)*d^5 + (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 -
 6*a*b^3*c - a^2*b*c^2)*d)*e*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
)*e^9*f*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*f*x^7 + 2
*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c
^3 + 16*a^4*c^4)*d^2)*e^7*f*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*
a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*f*x^5 +
 (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 1
6*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*f*
x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c
 - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3
)*d)*e^4*f*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^
4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*f*x^2 + ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1077 vs. $2(258) = 516$.

Time = 0.22 (sec) , antiderivative size = 1077, normalized size of antiderivative = 3.99

$$\int \frac{1}{(df + efx)\left(a + b(d + ex)^2 + c(d + ex)^4\right)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

output
```
-1/4*((a^3*b^7*c*e^3*f - 14*a^4*b^5*c^2*e^3*f + 70*a^5*b^3*c^3*e^3*f - 120
*a^6*b*c^4*e^3*f)*sqrt(b^2 - 4*a*c)*log(abs(b*e^2*x^2 + sqrt(b^2 - 4*a*c)*
e^2*x^2 + 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x + b*d^2 + sqrt(b^2 - 4*a*c
)*d^2 + 2*a)) - (a^3*b^7*c*e^3*f - 14*a^4*b^5*c^2*e^3*f + 70*a^5*b^3*c^3*e
^3*f - 120*a^6*b*c^4*e^3*f)*sqrt(b^2 - 4*a*c)*log(abs(-b*e^2*x^2 + sqrt(b^
2 - 4*a*c)*e^2*x^2 - 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(
b^2 - 4*a*c)*d^2 - 2*a)))/(a^6*b^8*c*e^4*f^2 - 16*a^7*b^6*c^2*e^4*f^2 + 96
*a^8*b^4*c^3*e^4*f^2 - 256*a^9*b^2*c^4*e^4*f^2 + 256*a^10*c^5*e^4*f^2) - 1
/4*log(abs(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + c*d
^4 + b*e^2*x^2 + 2*b*d*e*x + b*d^2 + a))/(a^3*e*f) + log(abs(e*x + d))/(a^
3*e*f) + 1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3*d^6 + 4*a*b^4*c*d^4 - 29*a^2*
b^2*c^2*d^4 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 - 12*a^2*b^3*c*d^2 - 2*a^3*b*c^
2*d^2 + 2*(a*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^6 + 3*a^2*b^4 - 21*a^3*b^2*c
 + 24*a^4*c^2 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c^3*d*e^5)*x^5 + (30*a*b^3*c
^2*d^2*e^4 - 210*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e^4 - 29*a^2*b^2*c^2*e^4 +
16*a^3*c^3*e^4)*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 70*a^2*b*c^3*d^3*e^3 + 4*a
*b^4*c*d*e^3 - 29*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d*e^3)*x^3 + 2*(15*a*b^3*
c^2*d^4*e^2 - 105*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^2*e^2 - 87*a^2*b^2*c^2*
d^2*e^2 + 48*a^3*c^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3*c*e^2 - a^3*b*c^2*e^2
)*x^2 + 4*(3*a*b^3*c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4*a*b^4*c*d^3*e - 2...
```

Mupad [B] (verification not implemented)

Time = 23.77 (sec) , antiderivative size = 22621, normalized size of antiderivative = 83.78

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

output
```
((x^2*(b^5*e + 48*a^2*c^3*d^2*e + 15*b^3*c^2*d^4*e - 6*a*b^3*c*e - a^2*b*c
^2*e + 12*b^4*c*d^2*e - 105*a*b*c^3*d^4*e - 87*a*b^2*c^2*d^2*e))/(2*(a^2*b
^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^4*(4*b^4*c*e^3 + 16*a^2*c^3*e^3 - 29*
a*b^2*c^2*e^3 + 30*b^3*c^2*d^2*e^3 - 210*a*b*c^3*d^2*e^3))/(4*(a^2*b^4 + 1
6*a^4*c^2 - 8*a^3*b^2*c)) + (x^3*(16*a^2*c^3*d*e^2 + 10*b^3*c^2*d^3*e^2 +
4*b^4*c*d*e^2 - 29*a*b^2*c^2*d*e^2 - 70*a*b*c^3*d^3*e^2))/(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c) + (3*x^5*(b^3*c^2*d*e^4 - 7*a*b*c^3*d*e^4))/(a^2*b^4
+ 16*a^4*c^2 - 8*a^3*b^2*c) + (x^6*(b^3*c^2*e^5 - 7*a*b*c^3*e^5))/(2*(a^2*
b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(b^5*d + 4*b^4*c*d^3 + 16*a^2*c^3*d^
3 + 3*b^3*c^2*d^5 - 29*a*b^2*c^2*d^3 - 6*a*b^3*c*d - a^2*b*c^2*d - 21*a*b*
c^3*d^5))/(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*a*b^4 + 24*a^3*c^2 + 2
*b^5*d^2 - 21*a^2*b^2*c + 4*b^4*c*d^4 + 16*a^2*c^3*d^4 + 2*b^3*c^2*d^6 - 2
*a^2*b*c^2*d^2 - 29*a*b^2*c^2*d^4 - 12*a*b^3*c*d^2 - 14*a*b*c^3*d^6)/(4*e*
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(56*c^2*d^5*e^3*f + 4*b^2*d*e^
3*f + 40*b*c*d^3*e^3*f + 8*a*c*d*e^3*f) + x^2*(6*b^2*d^2*e^2*f + 28*c^2*d^
6*e^2*f + 2*a*b*e^2*f + 12*a*c*d^2*e^2*f + 30*b*c*d^4*e^2*f) + x*(4*b^2*d^
3*e*f + 8*c^2*d^7*e*f + 4*a*b*d*e*f + 8*a*c*d^3*e*f + 12*b*c*d^5*e*f) + x^
4*(b^2*e^4*f + 70*c^2*d^4*e^4*f + 2*a*c*e^4*f + 30*b*c*d^2*e^4*f) + x^5*(5
6*c^2*d^3*e^5*f + 12*b*c*d*e^5*f) + a^2*f + x^6*(28*c^2*d^2*e^6*f + 2*b*c*
e^6*f) + b^2*d^4*f + c^2*d^8*f + c^2*e^8*f*x^8 + 2*a*b*d^2*f + 2*a*c*d^...
```

Reduce [F]

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

output

```
int(1/(a**3*d + a**3*e*x + 3*a**2*b*d**3 + 9*a**2*b*d**2*e*x + 9*a**2*b*d*
e**2*x**2 + 3*a**2*b*e**3*x**3 + 3*a**2*c*d**5 + 15*a**2*c*d**4*e*x + 30*a
**2*c*d**3*e**2*x**2 + 30*a**2*c*d**2*e**3*x**3 + 15*a**2*c*d*e**4*x**4 +
3*a**2*c*e**5*x**5 + 3*a*b**2*d**5 + 15*a*b**2*d**4*e*x + 30*a*b**2*d**3*e
**2*x**2 + 30*a*b**2*d**2*e**3*x**3 + 15*a*b**2*d*e**4*x**4 + 3*a*b**2*e**
5*x**5 + 6*a*b*c*d**7 + 42*a*b*c*d**6*e*x + 126*a*b*c*d**5*e**2*x**2 + 210
*a*b*c*d**4*e**3*x**3 + 210*a*b*c*d**3*e**4*x**4 + 126*a*b*c*d**2*e**5*x**
5 + 42*a*b*c*d*e**6*x**6 + 6*a*b*c*e**7*x**7 + 3*a*c**2*d**9 + 27*a*c**2*d
**8*e*x + 108*a*c**2*d**7*e**2*x**2 + 252*a*c**2*d**6*e**3*x**3 + 378*a*c*
*2*d**5*e**4*x**4 + 378*a*c**2*d**4*e**5*x**5 + 252*a*c**2*d**3*e**6*x**6
+ 108*a*c**2*d**2*e**7*x**7 + 27*a*c**2*d*e**8*x**8 + 3*a*c**2*e**9*x**9 +
 b**3*d**7 + 7*b**3*d**6*e*x + 21*b**3*d**5*e**2*x**2 + 35*b**3*d**4*e**3*
x**3 + 35*b**3*d**3*e**4*x**4 + 21*b**3*d**2*e**5*x**5 + 7*b**3*d*e**6*x**
6 + b**3*e**7*x**7 + 3*b**2*c*d**9 + 27*b**2*c*d**8*e*x + 108*b**2*c*d**7*
e**2*x**2 + 252*b**2*c*d**6*e**3*x**3 + 378*b**2*c*d**5*e**4*x**4 + 378*b*
*2*c*d**4*e**5*x**5 + 252*b**2*c*d**3*e**6*x**6 + 108*b**2*c*d**2*e**7*x**
7 + 27*b**2*c*d*e**8*x**8 + 3*b**2*c*e**9*x**9 + 3*b*c**2*d**11 + 33*b*c**
2*d**10*e*x + 165*b*c**2*d**9*e**2*x**2 + 495*b*c**2*d**8*e**3*x**3 + 990*
b*c**2*d**7*e**4*x**4 + 1386*b*c**2*d**6*e**5*x**5 + 1386*b*c**2*d**5*e**6
*x**6 + 990*b*c**2*d**4*e**7*x**7 + 495*b*c**2*d**3*e**8*x**8 + 165*b*c...
```

3.288 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 499

$$\begin{aligned}
&\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx \\
&= -\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2ef^2(d+ex)} \\
&\quad + \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \\
&\quad + \frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)(d+ex)^2}{8a^2\left(b^2-4ac\right)^2ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\
&\quad - \frac{3\sqrt{c}\left((5b^2-12ac)\left(b^2-5ac\right)+\frac{b(5b^4-47ab^2c+124a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}\,ef^2} \\
&\quad - \frac{3\sqrt{c}\left((5b^2-12ac)\left(b^2-5ac\right)-\frac{5b^5-47ab^3c+124a^2bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}\,ef^2}
\end{aligned}$$

output
```
-3/8*(-12*a*c+5*b^2)*(-5*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/f^2/(e*x+d)+1/4*(b^
2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+
d)^4)^2+1/8*(5*b^4-35*a*b^2*c+36*c^2*a^2+b*c*(-32*a*c+5*b^2)*(e*x+d)^2)/a^
2/(-4*a*c+b^2)^2/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/16*c^(1/2)*((
-12*a*c+5*b^2)*(-5*a*c+b^2)+b*(124*a^2*c^2-47*a*b^2*c+5*b^4)/(-4*a*c+b^2)^
(1/2))*arctan(2^(1/2)*c^(1/2)*(e*x+d)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2
)/a^3/(-4*a*c+b^2)^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)/e/f^2-3/16*c^(1/2)*((-12
*a*c+5*b^2)*(-5*a*c+b^2)-(124*a^2*b*c^2-47*a*b^3*c+5*b^5)/(-4*a*c+b^2)^(1/
2))*arctan(2^(1/2)*c^(1/2)*(e*x+d)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a
^3/(-4*a*c+b^2)^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/e/f^2
```

Mathematica [A] (verified)

Time = 6.26 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.15

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$= -\frac{1}{a^3ef^2(d+ex)} + \frac{b^3(d+ex)-3abc(d+ex)+b^2c(d+ex)^3-2ac^2(d+ex)^3}{4a^2\left(-b^2+4ac\right)ef^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$
$$+ \frac{-7b^5(d+ex)+52ab^3c(d+ex)-84a^2bc^2(d+ex)-7b^4c(d+ex)^3+47ab^2c^2(d+ex)^3-52a^2c^3(d+ \cdots}{8a^3\left(-b^2+4ac\right)^2ef^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$
$$- \frac{3\sqrt{c}\left(5b^5-47ab^3c+124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(\cdots}{\sqrt{b-\sqrt{\cdots}}}\right.}{8\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}ef^2}$$
$$- \frac{3\sqrt{c}\left(-5b^5+47ab^3c-124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\cdots}{\sqrt{b+\cdots}}\right.}{8\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}ef^2}$$

input
```
Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
-(1/(a^3*e*f^2*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d
 + e*x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*f^2*(a + b*(d + e
*x)^2 + c*(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*
a^2*b*c^2*(d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*
a^2*c^3*(d + e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*e*f^2*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)) - (3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqr
t[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt
[c]*(-5*b^5 + 47*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*
b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c
)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 474, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1462, 1441, 25, 1600, 27, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^3}d(d+ex)}{ef^2}$$

$$\Big\downarrow \text{1441}$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{4a(b^2-4ac)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{\int -\frac{5b^2+7c(d+ex)^2b-18ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4a(b^2-4ac)}}{ef^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{5b^2+7c(d+ex)^2b-18ac}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)}{4a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{4a(b^2-4ac)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}}{ef^2}$$

$\Big\downarrow$ 1600

$$\frac{\frac{\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{2a\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int -\frac{3\left(bc\left(5b^2-32ac\right)(d+ex)^2+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a\left(b^2-4ac\right)}}{4a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{4a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+\cdots}}{ef^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{bc\left(5b^2-32ac\right)(d+ex)^2+\left(5b^2-12ac\right)\left(b^2-5ac\right)}{(d+ex)^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)}{2a\left(b^2-4ac\right)}+\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{2a\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{4a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{4a(b^2-4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)\cdots}}{ef^2}$$

$\Big\downarrow$ 1604

$$\frac{\frac{\frac{3\left(-\frac{\int \frac{c\left(5b^2-12ac\right)\left(b^2-5ac\right)(d+ex)^2+b\left(5b^4-42acb^2+92a^2c^2\right)}{c(d+ex)^4+b(d+ex)^2+a}d(d+ex)}{a}-\frac{\left(5b^2-12ac\right)\left(b^2-5ac\right)}{a(d+ex)}\right)}{2a\left(b^2-4ac\right)}+\frac{36a^2c^2+bc\left(5b^2-32ac\right)(d+ex)^2-35ab^2c+5b^4}{2a\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{4a(b^2-4ac)}+\frac{\cdots}{4a(b\cdots}}{ef^2}$$

$\Big\downarrow$ 1480

$$\frac{\frac{3\left(-\frac{\frac{1}{2}c\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\int \frac{1}{c(d+ex)^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d(d+ex)+\frac{1}{2}c\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)-\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}\right)\cdots}{a}\cdots\right.}{2a\left(b^2-4ac\right)}\cdots}{4a(b^2-4ac)}\cdots$$

$\Big\downarrow$ 218

$$\frac{\frac{3\left(-\frac{\frac{\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(\left(5b^2-12ac\right)\left(b^2-5ac\right)-\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\cdots}{\sqrt{\cdots}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}\cdots\right.}{2a\left(b^2-4ac\right)}\cdots}{4a(b^2-4ac)}\cdots}{ef^2}$$

input
```
Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*(d + e*x)*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) + ((5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^
2 - 32*a*c)*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(d + e*x)*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)) + (3*(-(((5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(a*(d + e*x))) -
 ((Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*b^2*c + 124*
a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*((5*b
^2 - 12*a*c)*(b^2 - 5*a*c) - (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a))/(2*a*(b^2 - 4*a*c)))/(4*a*(
b^2 - 4*a*c)))/(e*f^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 1201, normalized size of antiderivative = 2.41

method	result	size
default	Expression too large to display	1201
risch	Expression too large to display	2710

input
```
int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f^2*(-1/a^3*((1/8*c^2*e^6*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^7+7/8*c^2*d*e^5*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^6+1/8*(1092*a^2*c^3*d^2-987*a*b^2*c^2*d^2+147*b^4*c*d^2+136*
a^2*b*c^2-99*a*b^3*c+14*b^5)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+5/8*c*d*
e^3*(364*a^2*c^3*d^2-329*a*b^2*c^2*d^2+49*b^4*c*d^2+136*a^2*b*c^2-99*a*b^3
*c+14*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+1/8*e^2*(1820*a^2*c^4*d^4-1645*a
*b^2*c^3*d^4+245*b^4*c^2*d^4+1360*a^2*b*c^3*d^2-990*a*b^3*c^2*d^2+140*b^5*
c*d^2+68*a^3*c^3+25*a^2*b^2*c^2-43*a*b^4*c+7*b^6)/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3+1/8*d*e*(1092*a^2*c^4*d^4-987*a*b^2*c^3*d^4+147*b^4*c^2*d^4+1360*a^
2*b*c^3*d^2-990*a*b^3*c^2*d^2+140*b^5*c*d^2+204*a^3*c^3+75*a^2*b^2*c^2-129
*a*b^4*c+21*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/8*(364*a^2*c^4*d^6-329*a
*b^2*c^3*d^6+49*b^4*c^2*d^6+680*a^2*b*c^3*d^4-495*a*b^3*c^2*d^4+70*b^5*c*d
^4+204*a^3*c^3*d^2+75*a^2*b^2*c^2*d^2-129*a*b^4*c*d^2+21*b^6*d^2+108*a^3*b
*c^2-66*a^2*b^3*c+9*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/8*d/e*(52*a^2*c^
4*d^6-47*a*b^2*c^3*d^6+7*b^4*c^2*d^6+136*a^2*b*c^3*d^4-99*a*b^3*c^2*d^4+14
*b^5*c*d^4+68*a^3*c^3*d^2+25*a^2*b^2*c^2*d^2-43*a*b^4*c*d^2+7*b^6*d^2+108*
a^3*b*c^2-66*a^2*b^3*c+9*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c
*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^
2+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((c*e^2*(60*a^2*c^2-37*a*b^2*c+5*b^
4)*_R^2+2*c*d*e*(60*a^2*c^2-37*a*b^2*c+5*b^4)*_R+60*a^2*c^3*d^2-37*a*b^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10518 vs. $2(453) = 906$.

Time = 1.36 (sec) , antiderivative size = 10518, normalized size of antiderivative = 21.08

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fric
as")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3(efx+df)^2}\,dx$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxi
ma")
```

output
```
-1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 24*(5*b^4*c^2 -
37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2
*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^2)*e^6*x^6 + 6*(28*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 3
92*a^2*b*c^3)*d)*e^5*x^5 + 3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 +
 (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37
*a*b^2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^
3)*d^2)*e^4*x^4 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 + 4*(42*(
5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c - 227*a*b^3*c^2 +
 392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)
*d)*e^3*x^3 + 8*a^2*b^4 - 64*a^3*b^2*c + 128*a^4*c^2 + (15*b^6 - 91*a*b^4*
c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^4 + (84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60
*a^2*c^4)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^2)*e^2*x^2 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2
)*d^2 + 2*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x)
/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a^3*b^4*c^2
 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1704 vs. $2(453) = 906$.

Time = 0.18 (sec) , antiderivative size = 1704, normalized size of antiderivative = 3.41

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac
")
```

output
```
-1/8*(7*b^4*c^2/((e*f*x + d*f)*e*f) - 47*a*b^2*c^3/((e*f*x + d*f)*e*f) + 5
2*a^2*c^4/((e*f*x + d*f)*e*f) + 14*b^5*c*f/((e*f*x + d*f)^3*e) - 99*a*b^3*
c^2*f/((e*f*x + d*f)^3*e) + 136*a^2*b*c^3*f/((e*f*x + d*f)^3*e) + 7*b^6*f^
3/((e*f*x + d*f)^5*e) - 43*a*b^4*c*f^3/((e*f*x + d*f)^5*e) + 25*a^2*b^2*c^
2*f^3/((e*f*x + d*f)^5*e) + 68*a^3*c^3*f^3/((e*f*x + d*f)^5*e) + 9*a*b^5*f
^5/((e*f*x + d*f)^7*e) - 66*a^2*b^3*c*f^5/((e*f*x + d*f)^7*e) + 108*a^3*b*
c^2*f^5/((e*f*x + d*f)^7*e))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c + b*
f^2/(e*f*x + d*f)^2 + a*f^4/(e*f*x + d*f)^4)^2) - 1/((e*f*x + d*f)*a^3*e*f
) + 3/64*((5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^9*c^2 - 4928*a^9*b^7*c
^3 + 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a^12*b*c^6)*sqrt(2*a*b
 + 2*sqrt(b^2 - 4*a*c)*a)*e^4*f^8 + 2*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*
a^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 -
4*a*c)*e^2*f^4*abs(a^3*b^4*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4 + 16*a^5*c^2*e^2*
f^4) - (a^3*b^4*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4 + 16*a^5*c^2*e^2*f^4)^2*(5*b
^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arcta
n(2*sqrt(1/2)/((e*f*x + d*f)*e*f*sqrt((a^3*b^5*e^2*f^4 - 8*a^4*b^3*c*e^2*f
^4 + 16*a^5*b*c^2*e^2*f^4 + sqrt((a^3*b^5*e^2*f^4 - 8*a^4*b^3*c*e^2*f^4 +
16*a^5*b*c^2*e^2*f^4)^2 - 4*(a^4*b^4*e^4*f^8 - 8*a^5*b^2*c*e^4*f^8 + 16*a^
6*c^2*e^4*f^8)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4*b^4*e^4*f^8
 - 8*a^5*b^2*c*e^4*f^8 + 16*a^6*c^2*e^4*f^8))))/((a^7*b^6*c - 12*a^8*b^...
```

Mupad [B] (verification not implemented)

Time = 20.64 (sec) , antiderivative size = 20580, normalized size of antiderivative = 41.24

$$\int \frac{1}{(df+efx)^2 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3} \, dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

output
```
- atan(((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b
*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 -
6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684
160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(
1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^
4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10
*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a
^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*
e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 26
21440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(2717908992
00*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c
^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*
e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*
e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^1
0*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4
*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 - 2
5*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^
2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19
905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 5203968
0*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - ...
```

Reduce [F]

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

output

```
int(1/(a**3*d**2 + 2*a**3*d*e*x + a**3*e**2*x**2 + 3*a**2*b*d**4 + 12*a**2
*b*d**3*e*x + 18*a**2*b*d**2*e**2*x**2 + 12*a**2*b*d*e**3*x**3 + 3*a**2*b*
e**4*x**4 + 3*a**2*c*d**6 + 18*a**2*c*d**5*e*x + 45*a**2*c*d**4*e**2*x**2
+ 60*a**2*c*d**3*e**3*x**3 + 45*a**2*c*d**2*e**4*x**4 + 18*a**2*c*d*e**5*x
**5 + 3*a**2*c*e**6*x**6 + 3*a*b**2*d**6 + 18*a*b**2*d**5*e*x + 45*a*b**2*
d**4*e**2*x**2 + 60*a*b**2*d**3*e**3*x**3 + 45*a*b**2*d**2*e**4*x**4 + 18*
a*b**2*d*e**5*x**5 + 3*a*b**2*e**6*x**6 + 6*a*b*c*d**8 + 48*a*b*c*d**7*e*x
 + 168*a*b*c*d**6*e**2*x**2 + 336*a*b*c*d**5*e**3*x**3 + 420*a*b*c*d**4*e*
*4*x**4 + 336*a*b*c*d**3*e**5*x**5 + 168*a*b*c*d**2*e**6*x**6 + 48*a*b*c*d
*e**7*x**7 + 6*a*b*c*e**8*x**8 + 3*a*c**2*d**10 + 30*a*c**2*d**9*e*x + 135
*a*c**2*d**8*e**2*x**2 + 360*a*c**2*d**7*e**3*x**3 + 630*a*c**2*d**6*e**4*
x**4 + 756*a*c**2*d**5*e**5*x**5 + 630*a*c**2*d**4*e**6*x**6 + 360*a*c**2*
d**3*e**7*x**7 + 135*a*c**2*d**2*e**8*x**8 + 30*a*c**2*d*e**9*x**9 + 3*a*c
**2*e**10*x**10 + b**3*d**8 + 8*b**3*d**7*e*x + 28*b**3*d**6*e**2*x**2 + 5
6*b**3*d**5*e**3*x**3 + 70*b**3*d**4*e**4*x**4 + 56*b**3*d**3*e**5*x**5 +
28*b**3*d**2*e**6*x**6 + 8*b**3*d*e**7*x**7 + b**3*e**8*x**8 + 3*b**2*c*d*
*10 + 30*b**2*c*d**9*e*x + 135*b**2*c*d**8*e**2*x**2 + 360*b**2*c*d**7*e**
3*x**3 + 630*b**2*c*d**6*e**4*x**4 + 756*b**2*c*d**5*e**5*x**5 + 630*b**2*
c*d**4*e**6*x**6 + 360*b**2*c*d**3*e**7*x**7 + 135*b**2*c*d**2*e**8*x**8 +
 30*b**2*c*d*e**9*x**9 + 3*b**2*c*e**10*x**10 + 3*b*c**2*d**12 + 36*b*c...
```

3.289 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 343

$$\begin{aligned}
&\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx \\
&= -\frac{3(b^2-5ac)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2 ef^3(d+ex)^2} \\
&\quad+\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \\
&\quad+\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\left(d+ex\right)^2}{4a^2\left(b^2-4ac\right)^2 ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} \\
&\quad-\frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\operatorname{arctanh}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}ef^3} \\
&\quad-\frac{3b\log(d+ex)}{a^4ef^3}+\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4ef^3}
\end{aligned}$$

output
```
-3/2*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/f^3/(e*x+d)^2+1/4*(b^2
-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x
+d)^4)^2+1/4*(3*b^4-20*a*b^2*c+20*c^2*a^2+3*b*c*(-6*a*c+b^2)*(e*x+d)^2)/a^
2/(-4*a*c+b^2)^2/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/2*(-20*a^3*
c^3+30*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^
(1/2))/a^4/(-4*a*c+b^2)^(5/2)/e/f^3-3*b*ln(e*x+d)/a^4/e/f^3+3/4*b*ln(a+b*(
e*x+d)^2+c*(e*x+d)^4)/a^4/e/f^3
```

Mathematica [A] (verified)

Time = 6.20 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.48

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$$
$$= -\frac{1}{2a^3ef^3(d+ex)^2}+\frac{b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2}{4a^2\left(-b^2+4ac\right)ef^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$
$$+\frac{-4b^5+29ab^3c-46a^2bc^2-4b^4c(d+ex)^2+26ab^2c^2(d+ex)^2-28a^2c^3(d+ex)^2}{4a^3\left(-b^2+4ac\right)^2ef^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$
$$-\frac{3b\log(d+ex)}{a^4ef^3}$$
$$+\frac{3\left(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+16a^2bc^2\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{\right.}}{4a^4\left(b^2-4ac\right)^{5/2}ef^3}$$
$$+\frac{3\left(-b^6+10ab^4c-30a^2b^2c^2+20a^3c^3+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+16a^2bc^2\sqrt{b^2-4ac}\right)\log\left(b+\right.}{4a^4\left(b^2-4ac\right)^{5/2}ef^3}$$

input
```
Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
-1/2*1/(a^3*e*f^3*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*
c^2*(d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*f^3*(a + b*(d + e*x)^2 + c*(d + e
*x)^4)^2) + (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c*(d + e*x)^2 + 26
*a*b^2*c^2*(d + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e
*f^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e*f^3)
 + (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a
*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b
- Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3)
+ (3*(-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a
*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b
+ Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1462, 1434, 1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}dx$$

$$\Big\downarrow \text{1462}$$

$$\frac{\int \frac{1}{(d+ex)^3(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)}{ef^3}$$

$$\Big\downarrow \text{1434}$$

$$\frac{\int \frac{1}{(d+ex)^4(c(d+ex)^4+b(d+ex)^2+a)^3}d(d+ex)^2}{2ef^3}$$

$$\Big\downarrow \text{1165}$$

$$\frac{\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{\int -\frac{3b^2+4c(d+ex)^2b-10ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2a(b^2-4ac)}}{2ef^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3b^2+4c(d+ex)^2b-10ac}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)^2}d(d+ex)^2}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}}{2ef^3}$$

$\Big\downarrow$ 1235

$$\frac{\frac{\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{a\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\int -\frac{6\left(bc\left(b^2-6ac\right)(d+ex)^2+\left(b^2-5ac\right)\left(b^2-2ac\right)\right)}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a\left(b^2-4ac\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+e$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{6\int \frac{bc\left(b^2-6ac\right)(d+ex)^2+\left(b^2-5ac\right)\left(b^2-2ac\right)}{(d+ex)^4\left(c(d+ex)^4+b(d+ex)^2+a\right)}d(d+ex)^2}{a\left(b^2-4ac\right)}+\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{a\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bc(d+ex)^2}{2a(b^2-4ac)(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4}$$

$\Big\downarrow$ 1200

$$\frac{\frac{\frac{6\int \left(-\frac{b\left(4ac-b^2\right)^2}{a^2(d+ex)^2}+\frac{b^6-9acb^4+23a^2c^2b^2+c\left(b^2-4ac\right)^2(d+ex)^2b-10a^3c^3}{a^2\left(c(d+ex)^4+b(d+ex)^2+a\right)}+\frac{\left(b^2-5ac\right)\left(b^2-2ac\right)}{a(d+ex)^4}\right)d(d+ex)^2}{a\left(b^2-4ac\right)}+\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{a\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}}{2a(b^2-4ac)}}{2ef^3}$$

$\Big\downarrow$ 2009

$$\frac{\frac{\frac{20a^2c^2+3bc\left(b^2-6ac\right)(d+ex)^2-20ab^2c+3b^4}{a\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{6\left(-\frac{b\left(b^2-4ac\right)^2\log\left((d+ex)^2\right)}{a^2}+\frac{b\left(b^2-4ac\right)^2\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^2}-\frac{\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b\right.}{a^2\sqrt{b^2}}\right.}{a\left(b^2-4ac\right)}}{2a(b^2-4ac)}}{2ef^3}$$

input
```
Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

output
```
((b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*(d + e*x)^2*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)^2) + ((3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(
b^2 - 6*a*c)*(d + e*x)^2)/(a*(b^2 - 4*a*c)*(d + e*x)^2*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (6*(-(((b^2 - 5*a*c)*(b^2 - 2*a*c))/(a*(d + e*x)^2)) -
 ((b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*(d + e
*x)^2)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)^2*Lo
g[(d + e*x)^2])/a^2 + (b*(b^2 - 4*a*c)^2*Log[a + b*(d + e*x)^2 + c*(d + e*
x)^4])/(2*a^2)))/(a*(b^2 - 4*a*c)))/(2*a*(b^2 - 4*a*c)))/(2*e*f^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1462
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Si
mp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p
, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.48 (sec) , antiderivative size = 1145, normalized size of antiderivative = 3.34

method	result	size
default	Expression too large to display	1145
risch	Expression too large to display	2364

input
```
int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f^3*(-1/a^4*((1/2*c^2*e^5*(14*a^2*c^2-13*a*b^2*c+2*b^4)*a/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^6+3*(14*a^2*c^2-13*a*b^2*c+2*b^4)*a*c^2*d*e^4/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^5+1/4*e^3*a*c*(420*a^2*c^3*d^2-390*a*b^2*c^2*d^2+60*b^4*c*
d^2+74*a^2*b*c^2-55*a*b^3*c+8*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+c*d*e^2*
a*(140*a^2*c^3*d^2-130*a*b^2*c^2*d^2+20*b^4*c*d^2+74*a^2*b*c^2-55*a*b^3*c+
8*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*e*a*(210*a^2*c^4*d^4-195*a*b^2*c
^3*d^4+30*b^4*c^2*d^4+222*a^2*b*c^3*d^2-165*a*b^3*c^2*d^2+24*b^5*c*d^2+18*
a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+d*a
*(42*a^2*c^4*d^4-39*a*b^2*c^3*d^4+6*b^4*c^2*d^4+74*a^2*b*c^3*d^2-55*a*b^3*
c^2*d^2+8*b^5*c*d^2+18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x+1/4/e*a*(28*a^2*c^4*d^6-26*a*b^2*c^3*d^6+4*b^4*c^2*d^6+7
4*a^2*b*c^3*d^4-55*a*b^3*c^2*d^4+8*b^5*c*d^4+36*a^3*c^3*d^2+14*a^2*b^2*c^2
*d^2-24*a*b^4*c*d^2+4*b^6*d^2+58*a^3*b*c^2-36*a^2*b^3*c+5*a*b^5)/(16*a^2*c
^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*
e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((b
*c*e^3*(-16*a^2*c^2+8*a*b^2*c-b^4)*_R^3+3*b*c*d*e^2*(-16*a^2*c^2+8*a*b^2*c
-b^4)*_R^2+e*(-48*a^2*b*c^3*d^2+24*a*b^3*c^2*d^2-3*b^5*c*d^2+10*a^3*c^3-23
*a^2*b^2*c^2+9*a*b^4*c-b^6)*_R-16*a^2*b*c^3*d^3+8*a*b^3*c^2*d^3-b^5*c*d^3+
10*a^3*c^3*d-23*a^2*b^2*c^2*d+9*a*b^4*c*d-b^6*d)/(2*_R^3*c*e^3+6*_R^2*c*d*
e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(c*e^4*_Z^4+4*c*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7550 vs. $2(329) = 658$.

Time = 5.48 (sec) , antiderivative size = 15231, normalized size of antiderivative = 44.41

$$\int \frac{1}{(df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fric
as")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \int \frac{1}{\left((ex+d)^4c+(ex+d)^2b+a\right)^3(efx+df)^3}\,dx$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxi
ma")
```

output
```
-1/4*(6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*e^8*x^8 + 48*(b^4*c^2 - 7*a*b
^2*c^3 + 10*a^2*c^4)*d*e^7*x^7 + 3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3
+ 56*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^2)*e^6*x^6 + 6*(56*(b^4*c^2 -
7*a*b^2*c^3 + 10*a^2*c^4)*d^3 + 3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*
d)*e^5*x^5 + 6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^8 + (6*b^6 - 36*a*b^
4*c + 14*a^2*b^2*c^2 + 100*a^3*c^3 + 420*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c
^4)*d^4 + 45*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^2)*e^4*x^4 + 3*(4*b
^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^6 + 4*(84*(b^4*c^2 - 7*a*b^2*c^3 + 1
0*a^2*c^4)*d^5 + 15*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^3 + 2*(3*b^6
 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d)*e^3*x^3 + 2*a^2*b^4 - 16*a^
3*b^2*c + 32*a^4*c^2 + 2*(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)
*d^4 + (168*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^6 + 9*a*b^5 - 68*a^2*b^
3*c + 122*a^3*b*c^2 + 45*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^4 + 12*
(3*b^6 - 18*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^2)*e^2*x^2 + (9*a*b^5
- 68*a^2*b^3*c + 122*a^3*b*c^2)*d^2 + 2*(24*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^
2*c^4)*d^7 + 9*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^5 + 4*(3*b^6 - 18
*a*b^4*c + 7*a^2*b^2*c^2 + 50*a^3*c^3)*d^3 + (9*a*b^5 - 68*a^2*b^3*c + 122
*a^3*b*c^2)*d)*e*x)/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^11*f^3*x
^10 + 10*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^10*f^3*x^9 + (2*a^
3*b^5*c - 16*a^4*b^3*c^2 + 32*a^5*b*c^3 + 45*(a^3*b^4*c^2 - 8*a^4*b^2*c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1791 vs. $2(329) = 658$.

Time = 0.20 (sec) , antiderivative size = 1791, normalized size of antiderivative = 5.22

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac
")
```

output
```
3/4*((a^4*b^8*c*e^3*f^3 - 14*a^5*b^6*c^2*e^3*f^3 + 70*a^6*b^4*c^3*e^3*f^3
- 140*a^7*b^2*c^4*e^3*f^3 + 80*a^8*c^5*e^3*f^3)*sqrt(b^2 - 4*a*c)*log(abs(
b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^2*x^2 + 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*
e*x + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^4*b^8*c*e^3*f^3 - 14*a^5*
b^6*c^2*e^3*f^3 + 70*a^6*b^4*c^3*e^3*f^3 - 140*a^7*b^2*c^4*e^3*f^3 + 80*a^
8*c^5*e^3*f^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*e^2*x^2 + sqrt(b^2 - 4*a*c)*e^
2*x^2 - 2*b*d*e*x + 2*sqrt(b^2 - 4*a*c)*d*e*x - b*d^2 + sqrt(b^2 - 4*a*c)*
d^2 - 2*a)))/(a^8*b^8*c*e^4*f^6 - 16*a^9*b^6*c^2*e^4*f^6 + 96*a^10*b^4*c^3
*e^4*f^6 - 256*a^11*b^2*c^4*e^4*f^6 + 256*a^12*c^5*e^4*f^6) - 1/4*(6*b^4*c
^2*e^8*x^8 - 42*a*b^2*c^3*e^8*x^8 + 60*a^2*c^4*e^8*x^8 + 48*b^4*c^2*d*e^7*
x^7 - 336*a*b^2*c^3*d*e^7*x^7 + 480*a^2*c^4*d*e^7*x^7 + 168*b^4*c^2*d^2*e^
6*x^6 - 1176*a*b^2*c^3*d^2*e^6*x^6 + 1680*a^2*c^4*d^2*e^6*x^6 + 336*b^4*c^
2*d^3*e^5*x^5 - 2352*a*b^2*c^3*d^3*e^5*x^5 + 3360*a^2*c^4*d^3*e^5*x^5 + 42
0*b^4*c^2*d^4*e^4*x^4 - 2940*a*b^2*c^3*d^4*e^4*x^4 + 4200*a^2*c^4*d^4*e^4*
x^4 + 12*b^5*c*e^6*x^6 - 87*a*b^3*c^2*e^6*x^6 + 138*a^2*b*c^3*e^6*x^6 + 33
6*b^4*c^2*d^5*e^3*x^3 - 2352*a*b^2*c^3*d^5*e^3*x^3 + 3360*a^2*c^4*d^5*e^3*
x^3 + 72*b^5*c*d*e^5*x^5 - 522*a*b^3*c^2*d*e^5*x^5 + 828*a^2*b*c^3*d*e^5*x
^5 + 168*b^4*c^2*d^6*e^2*x^2 - 1176*a*b^2*c^3*d^6*e^2*x^2 + 1680*a^2*c^4*d
^6*e^2*x^2 + 180*b^5*c*d^2*e^4*x^4 - 1305*a*b^3*c^2*d^2*e^4*x^4 + 2070*a^2
*b*c^3*d^2*e^4*x^4 + 48*b^4*c^2*d^7*e*x - 336*a*b^2*c^3*d^7*e*x + 480*a...
```

Mupad [B] (verification not implemented)

Time = 27.34 (sec) , antiderivative size = 25334, normalized size of antiderivative = 73.86

$$\int \frac{1}{(df+efx)^3\,(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

output
```
(log(((27*c^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c -
 b^2)^6) - ((3*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a
*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2
*c^2 - 7*a*b^2*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*
a*b^4*c - 47*a*b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*f^6*(4*a*c - b^2)^4)
- ((3*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2
/(a^8*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3
+ b^6*c*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*
d^2 - 30*a^2*b^2*c^3*d^2))/(a^3*f^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b -
3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*
f^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2
 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^4*f^3) + (6*c^3*e^18*x
^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2
)^2) + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/
(a^3*f^3*(4*a*c - b^2)^2)))/(4*a^4*e*f^3) + (9*b*c^4*e^17*x^2*(6*b^8 + 900
*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a
*c - b^2)^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 -
 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4)))/(4*a^4*e*f^3)
 + (27*c^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4
*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*f^9*(4*a*c...
```

Reduce [F]

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)
```

output

```
int(1/(a**3*d**3 + 3*a**3*d**2*e*x + 3*a**3*d*e**2*x**2 + a**3*e**3*x**3 +
 3*a**2*b*d**5 + 15*a**2*b*d**4*e*x + 30*a**2*b*d**3*e**2*x**2 + 30*a**2*b
*d**2*e**3*x**3 + 15*a**2*b*d*e**4*x**4 + 3*a**2*b*e**5*x**5 + 3*a**2*c*d*
*7 + 21*a**2*c*d**6*e*x + 63*a**2*c*d**5*e**2*x**2 + 105*a**2*c*d**4*e**3*
x**3 + 105*a**2*c*d**3*e**4*x**4 + 63*a**2*c*d**2*e**5*x**5 + 21*a**2*c*d*
e**6*x**6 + 3*a**2*c*e**7*x**7 + 3*a*b**2*d**7 + 21*a*b**2*d**6*e*x + 63*a
*b**2*d**5*e**2*x**2 + 105*a*b**2*d**4*e**3*x**3 + 105*a*b**2*d**3*e**4*x*
*4 + 63*a*b**2*d**2*e**5*x**5 + 21*a*b**2*d*e**6*x**6 + 3*a*b**2*e**7*x**7
 + 6*a*b*c*d**9 + 54*a*b*c*d**8*e*x + 216*a*b*c*d**7*e**2*x**2 + 504*a*b*c
*d**6*e**3*x**3 + 756*a*b*c*d**5*e**4*x**4 + 756*a*b*c*d**4*e**5*x**5 + 50
4*a*b*c*d**3*e**6*x**6 + 216*a*b*c*d**2*e**7*x**7 + 54*a*b*c*d*e**8*x**8 +
 6*a*b*c*e**9*x**9 + 3*a*c**2*d**11 + 33*a*c**2*d**10*e*x + 165*a*c**2*d**
9*e**2*x**2 + 495*a*c**2*d**8*e**3*x**3 + 990*a*c**2*d**7*e**4*x**4 + 1386
*a*c**2*d**6*e**5*x**5 + 1386*a*c**2*d**5*e**6*x**6 + 990*a*c**2*d**4*e**7
*x**7 + 495*a*c**2*d**3*e**8*x**8 + 165*a*c**2*d**2*e**9*x**9 + 33*a*c**2*
d*e**10*x**10 + 3*a*c**2*e**11*x**11 + b**3*d**9 + 9*b**3*d**8*e*x + 36*b*
*3*d**7*e**2*x**2 + 84*b**3*d**6*e**3*x**3 + 126*b**3*d**5*e**4*x**4 + 126
*b**3*d**4*e**5*x**5 + 84*b**3*d**3*e**6*x**6 + 36*b**3*d**2*e**7*x**7 + 9
*b**3*d*e**8*x**8 + b**3*e**9*x**9 + 3*b**2*c*d**11 + 33*b**2*c*d**10*e*x
+ 165*b**2*c*d**9*e**2*x**2 + 495*b**2*c*d**8*e**3*x**3 + 990*b**2*c*d*...
```

3.290 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) \, dx = \frac{1}{21}(2+3x)^7+\frac{1}{42}(2+3x)^{14}+\frac{1}{63}(2+3x)^{21}$$

output
```
1/21*(2+3*x)^7+1/42*(2+3*x)^14+1/63*(2+3*x)^21
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) \, dx = \frac{1}{21}(2+3x)^7+\frac{1}{42}(2+3x)^{14}+\frac{1}{63}(2+3x)^{21}$$

input
```
Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

output
```
(2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1725, 1690, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x+2)^6 \left((3x+2)^{14}+(3x+2)^7+1\right) dx$$

$$\Big\downarrow \text{1725}$$

$$\frac{1}{3} \int (3x+2)^6 \left((3x+2)^{14}+(3x+2)^7+1\right) d(3x+2)$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{21} \int \left((3x+2)^{14}+3x+3\right) d(3x+2)^7$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{21} \left((3x+2)^7+\frac{1}{3}(3x+2)^3+\frac{1}{2}(3x+2)^2\right)$$

input
```
Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

output
```
((2 + 3*x)^2/2 + (2 + 3*x)^3/3 + (2 + 3*x)^7)/21
```

Defintions of rubi rules used

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 1725
```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n + c*x^
(2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] &&
LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(28) = 56.

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06

method	result
gosper	$\frac{x(2324522934x^{20}+32543321076x^{19}+216955473840x^{18}+916034222880x^{17}+2748102668640x^{16}+6229032715584x^{15}+11073}{}$
default	$1056832x + 15808800x^2 + 149902032x^3 + 1010576952x^4 + 5149786572x^5 + 20588764518$
norman	$1056832x + 15808800x^2 + 149902032x^3 + 1010576952x^4 + 5149786572x^5 + 20588764518$
risch	$1056832x + 15808800x^2 + 149902032x^3 + 1010576952x^4 + 5149786572x^5 + 20588764518$
parallelrisch	$1056832x + 15808800x^2 + 149902032x^3 + 1010576952x^4 + 5149786572x^5 + 20588764518$
orering	$\frac{x(2324522934x^{20}+32543321076x^{19}+216955473840x^{18}+916034222880x^{17}+2748102668640x^{16}+6229032715584x^{15}+11073}{1}$

input
```
int((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x,method=_RETURNVERBOSE)
```

output
```
1/14*x*(2324522934*x^20+32543321076*x^19+216955473840*x^18+916034222880*x^
17+2748102668640*x^16+6229032715584*x^15+11073835938816*x^14+1581976722120
3*x^13+18456408111708*x^12+17772887593188*x^11+14218430440032*x^10+9479154
235824*x^9+5266441986624*x^8+2430891860544*x^7+926214166962*x^6+2882427032
52*x^5+72097012008*x^4+14148077328*x^3+2098628448*x^2+221323200*x+14795648
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(28) = 56$.

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06

$$\int (2+3x)^6\left(1+(2+3x)^7+(2+3x)^{14}\right)\,dx = \frac{1162261467}{7}\,x^{21} + 2324522934\,x^{20} + 15496819560\,x^{19} + 65431015920\,x^{18} + 196293047760\,x^{17} + 444930908256\,x^{16} + 790988281344\,x^{15} + \frac{15819767221203}{14}\,x^{14} + 1318314865122\,x^{13} + 1269491970942\,x^{12} + 1015602174288\,x^{11} + 677082445416\,x^{10} + 376174427616\,x^{9} + 173635132896\,x^{8} + 66158154783\,x^{7} + 20588764518\,x^{6} + 5149786572\,x^{5} + 1010576952\,x^{4} + 149902032\,x^{3} + 15808800\,x^{2} + 1056832\,x$$

input
```
integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="fricas")
```

output
```
1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18
+ 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 158197672212
03/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11
+ 677082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^
7 + 20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 15
808800*x^2 + 1056832*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(24) = 48$.

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.15

$$\int (2+3x)^6\left(1+(2+3x)^7+(2+3x)^{14}\right)\,dx = \frac{1162261467x^{21}}{7} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + 790988281344x^{15} + \frac{15819767221203x^{14}}{14} + 1318314865122x^{13} + 1269491970942x^{12} + 1015602174288x^{11} + 677082445416x^{10} + 376174427616x^{9} + 173635132896x^{8} + 66158154783x^{7} + 20588764518x^{6} + 5149786572x^{5} + 1010576952x^{4} + 149902032x^{3} + 15808800x^{2} + 1056832x$$

input
```
integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14),x)
```

output
```
1162261467*x**21/7 + 2324522934*x**20 + 15496819560*x**19 + 65431015920*x*
*18 + 196293047760*x**17 + 444930908256*x**16 + 790988281344*x**15 + 15819
767221203*x**14/14 + 1318314865122*x**13 + 1269491970942*x**12 + 101560217
4288*x**11 + 677082445416*x**10 + 376174427616*x**9 + 173635132896*x**8 +
66158154783*x**7 + 20588764518*x**6 + 5149786572*x**5 + 1010576952*x**4 +
149902032*x**3 + 15808800*x**2 + 1056832*x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(28) = 56$.

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) dx = \frac{1162261467}{7}\,x^{21} + 2324522934\,x^{20} + 15496819560\,x^{19} + 65431015920\,x^{18} + 196293047760\,x^{17} + 444930908256\,x^{16} + 790988281344\,x^{15} + \frac{15819767221203}{14}\,x^{14} + 1318314865122\,x^{13} + 1269491970942\,x^{12} + 1015602174288\,x^{11} + 677082445416\,x^{10} + 376174427616\,x^{9} + 173635132896\,x^{8} + 66158154783\,x^{7} + 20588764518\,x^{6} + 5149786572\,x^{5} + 1010576952\,x^{4} + 149902032\,x^{3} + 15808800\,x^{2} + 1056832\,x$$

input
```
integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="maxima")
```

output
```
1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18
+ 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 158197672212
03/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11
+ 677082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^
7 + 20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 15
808800*x^2 + 1056832*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) dx = \frac{1}{63}(3x+2)^{21} + \frac{1}{42}(3x+2)^{14} + \frac{1}{21}(3x+2)^7$$

input
```
integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="giac")
```

output
```
1/63*(3*x + 2)^21 + 1/42*(3*x + 2)^14 + 1/21*(3*x + 2)^7
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) dx = \frac{(3x+2)^7 \left(3(3x+2)^7 + 2(3x+2)^{14} + 6\right)}{126}$$

input
```
int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1),x)
```

output
```
((3*x + 2)^7*(3*(3*x + 2)^7 + 2*(3*x + 2)^14 + 6))/126
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.03

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right) dx = \frac{x(2324522934x^{20} + 32543321076x^{19} + 216955473840x^{18} + 916034222880x^{17} + 2748102668640x^{16} + 62\cdots}{}$$

input
```
int((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x)
```

output
```
(x*(2324522934*x**20 + 32543321076*x**19 + 216955473840*x**18 + 9160342228
80*x**17 + 2748102668640*x**16 + 6229032715584*x**15 + 11073835938816*x**1
4 + 15819767221203*x**13 + 18456408111708*x**12 + 17772887593188*x**11 + 1
4218430440032*x**10 + 9479154235824*x**9 + 5266441986624*x**8 + 2430891860
544*x**7 + 926214166962*x**6 + 288242703252*x**5 + 72097012008*x**4 + 1414
8077328*x**3 + 2098628448*x**2 + 221323200*x + 14795648))/14
```

3.291 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 56

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx = \frac{1}{21}(2+3x)^7+\frac{1}{21}(2+3x)^{14}+\frac{1}{21}(2+3x)^{21}+\frac{1}{42}(2+3x)^{28}+\frac{1}{105}(2+3x)^{35}$$

output
```
1/21*(2+3*x)^7+1/21*(2+3*x)^14+1/21*(2+3*x)^21+1/42*(2+3*x)^28+1/105*(2+3*
x)^35
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 188 vs. 2(56) = 112.

Time = 0.02 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.36

$$\begin{aligned}
&\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx \\
&= 17451466816x + 443569828128x^2 + 7299544818384x^3 + 87406679578680x^4 \\
&\quad + \frac{4057390785756924x^5}{5} + 6077684727888102x^6 + 37727143432895007x^7 \\
&\quad + 197897276851452864x^8 + 889942562270387136x^9 + \frac{17344958593049772048x^{10}}{5} \\
&\quad + 11821487501620716192x^{11} + 35454069480572048124x^{12} \\
&\quad + 94069263918929616324x^{13} + 221699757548270194389x^{14} \\
&\quad + 465517091041681015296x^{15} + 872775774067455498528x^{16} \\
&\quad + 1463104032160519033200x^{17} + 2194577166014752240080x^{18} \\
&\quad + 2945285062308448290360x^{19} + 3534290697929473864098x^{20} \\
&\quad + \frac{26506949038858918036881x^{21}}{7} + 3614565944605222108800x^{22} \\
&\quad + 3064515076512846852480x^{23} + 2298383223254096766840x^{24} \\
&\quad + \frac{7584660010542711771792x^{25}}{5} + 875152864622814086340x^{26} \\
&\quad + 437576396725285446564x^{27} + \frac{2625458326972530284475x^{28}}{14} \\
&\quad + 67899784121041365504x^{29} + \frac{101849676181562048256x^{30}}{5} \\
&\quad + 4928210137817518464x^{31} + 924039400840784712x^{32} \\
&\quad + 126005372841925188x^{33} + 11118121133111046x^{34} + \frac{16677181699666569x^{35}}{35}
\end{aligned}$$

input
```
Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

output

```
17451466816*x + 443569828128*x^2 + 7299544818384*x^3 + 87406679578680*x^4
+ (4057390785756924*x^5)/5 + 6077684727888102*x^6 + 37727143432895007*x^7
+ 197897276851452864*x^8 + 889942562270387136*x^9 + (17344958593049772048*
x^10)/5 + 11821487501620716192*x^11 + 35454069480572048124*x^12 + 94069263
918929616324*x^13 + 221699757548270194389*x^14 + 465517091041681015296*x^1
5 + 872775774067455498528*x^16 + 1463104032160519033200*x^17 + 21945771660
14752240080*x^18 + 2945285062308448290360*x^19 + 3534290697929473864098*x^
20 + (26506949038858918036881*x^21)/7 + 3614565944605222108800*x^22 + 3064
515076512846852480*x^23 + 2298383223254096766840*x^24 + (7584660010542711 7
71792*x^25)/5 + 875152864622814086340*x^26 + 437576396725285446564*x^27 +
(2625458326972530284475*x^28)/14 + 67899784121041365504*x^29 + (1018496761
81562048256*x^30)/5 + 4928210137817518464*x^31 + 924039400840784712*x^32 +
 126005372841925188*x^33 + 11118121133111046*x^34 + (166771816996665 69*x^3
5)/35
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1725, 1690, 1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x+2)^6 \left((3x+2)^{14}+(3x+2)^7+1\right)^2 dx$$

$$\Big\downarrow \text{1725}$$

$$\frac{1}{3} \int (3x+2)^6 \left((3x+2)^{14}+(3x+2)^7+1\right)^2 d(3x+2)$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{21} \int \left((3x+2)^{14}+3x+3\right)^2 d(3x+2)^7$$

$$\Big\downarrow \text{1085}$$

$$\frac{1}{21} \int \left((3x+2)^{28}+2(3x+2)^{21}+3(3x+2)^{14}+2(3x+2)^7+1\right) d(3x+2)^7$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{21}\left((3x+2)^7+\frac{1}{5}(3x+2)^5+\frac{1}{2}(3x+2)^4+(3x+2)^3+(3x+2)^2\right)$$

input
```
Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

output
```
((2 + 3*x)^2 + (2 + 3*x)^3 + (2 + 3*x)^4/2 + (2 + 3*x)^5/5 + (2 + 3*x)^7)/
21
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 1725
```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n + c*x^
(2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] &&
LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(46) = 92$.

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.11

method	result
gosper	$\frac{x(33354363399333138x^{34}+778268479317773220x^{33}+8820376098934763160x^{32}+64682758058854929840x^{31}+34497470964$
default	$17451466816x + \frac{7584660010542711771792}{5}x^{25} + 35454069480572048124x^{12} + 221699757548270$
norman	$17451466816x + \frac{7584660010542711771792}{5}x^{25} + 35454069480572048124x^{12} + 221699757548270$
risch	$17451466816x + \frac{7584660010542711771792}{5}x^{25} + 35454069480572048124x^{12} + 221699757548270$
parallelrisch	$17451466816x + \frac{7584660010542711771792}{5}x^{25} + 35454069480572048124x^{12} + 221699757548270$
orering	$\frac{x(33354363399333138x^{34}+778268479317773220x^{33}+8820376098934763160x^{32}+64682758058854929840x^{31}+34497470964$

input

```
int((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x,method=_RETURNVERBOSE)
```

output

```
1/70*x*(33354363399333138*x^34+778268479317773220*x^33+8820376098934763160
*x^32+64682758058854929840*x^31+344974709647226292480*x^30+142589546654186
8675584*x^29+4752984888472895585280*x^28+13127291634862651422375*x^27+3063
0347770769981259480*x^26+61260700523596986043800*x^25+10618524014759796480
5088*x^24+160886825627786773678800*x^23+214516055355899279673600*x^22+2530
19616122365547616000*x^21+265069490388589180368810*x^20+247400348855063170
486860*x^19+206169954361591380325200*x^18+153620401621032656805600*x^17+10
2417282251236332324000*x^16+61094304184721884896960*x^15+32586196372917671
070720*x^14+15518983028378913607230*x^13+6584848474325073142680*x^12+24817
84863640043368680*x^11+827504125113450133440*x^10+242829420302696808672*x^
9+62295979358927099520*x^8+13852809379601700480*x^7+2640900040302650490*x^
6+425437930952167140*x^5+56803471000596936*x^4+6118467570507600*x^3+510968
137286880*x^2+31049887968960*x+1221602677120)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(46) = 92$.

Time = 0.06 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.11

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx$$
$$= \frac{16677181699666569}{35} x^{35} + 11118121133111046 x^{34} + 126005372841925188 x^{33} + 924039400840784712 x^{32} + 4928210137817518464 x^{31} + \frac{101849676181562048256}{5} x^{30} + 67899784121041365504 x^{29} + \frac{2625458326972530284475}{14} x^{28} + 437576396725285446564 x^{27} + 875152864622814086340 x^{26} + \frac{7584660010542711771792}{5} x^{25} + 2298383223254096766840 x^{24} + 3064515076512846852480 x^{23} + 3614565944605222108800 x^{22} + \frac{26506949038858918036881}{7} x^{21} + 3534290697929473864098 x^{20} + 2945285062308448290360 x^{19} + 2194577166014752240080 x^{18} + 1463104032160519033200 x^{17} + 872775774067455498528 x^{16} + 465517091041681015296 x^{15} + 221699757548270194389 x^{14} + 94069263918929616324 x^{13} + 35454069480572048124 x^{12} + 11821487501620716192 x^{11} + \frac{17344958593049772048}{5} x^{10} + 889942562270387136 x^{9} + 197897276851452864 x^{8} + 37727143432895007 x^{7} + 6077684727888102 x^{6} + \frac{4057390785756924}{5} x^{5} + 87406679578680 x^{4} + 7299544818384 x^{3} + 443569828128 x^{2} + 17451466816 x$$

input
```
integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="fricas")
```

output

```
16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^
33 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 1018496761815620
48256/5*x^30 + 67899784121041365504*x^29 + 2625458326972530284475/14*x^28
+ 437576396725285446564*x^27 + 875152864622814086340*x^26 + 75846600105427
11771792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^2
3 + 3614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290
697929473864098*x^20 + 2945285062308448290360*x^19 + 219457716601475224008
0*x^18 + 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 465517
091041681015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x
^13 + 35454069480572048124*x^12 + 11821487501620716192*x^11 + 173449585930
49772048/5*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 377271
43432895007*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679
578680*x^4 + 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(41) = 82$.

Time = 0.06 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.34

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx$$
$$= \frac{16677181699666569x^{35}}{35} + 11118121133111046x^{34} + 126005372841925188x^{33} + 924039400840784712x^{32} + 4928210137817518464x^{31} + \frac{101849676181562048256x^{30}}{5} + 67899784121041365504x^{29} + \frac{2625458326972530284475x^{28}}{14} + 437576396725285446564x^{27} + 875152864622814086340x^{26} + \frac{7584660010542711771792x^{25}}{5} + 2298383223254096766840x^{24} + 3064515076512846852480x^{23} + 3614565944605222108800x^{22} + \frac{26506949038858918036881x^{21}}{7} + 3534290697929473864098x^{20} + 2945285062308448290360x^{19} + 2194577166014752240080x^{18} + 1463104032160519033200x^{17} + 872775774067455498528x^{16} + 465517091041681015296x^{15} + 221699757548270194389x^{14} + 94069263918929616324x^{13} + 35454069480572048124x^{12} + 11821487501620716192x^{11} + \frac{17344958593049772048x^{10}}{5} + 889942562270387136x^{9} + 197897276851452864x^{8} + 37727143432895007x^{7} + 6077684727888102x^{6} + \frac{4057390785756924x^{5}}{5} + 87406679578680x^{4} + 7299544818384x^{3} + 443569828128x^{2} + 17451466816x$$

input
```
integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14)**2,x)
```

output
```
16677181699666569*x**35/35 + 1111812113311046*x**34 + 126005372841925188*
x**33 + 924039400840784712*x**32 + 4928210137817518464*x**31 + 10184967618
1562048256*x**30/5 + 67899784121041365504*x**29 + 262545832697253028475*x
**28/14 + 437576396725285446564*x**27 + 875152864622814086340*x**26 + 7584
660010542711771792*x**25/5 + 2298383223254096766840*x**24 + 30645150765128
46852480*x**23 + 3614565944605222108800*x**22 + 26506949038858918036881*x*
*21/7 + 3534290697929473864098*x**20 + 2945285062308448290360*x**19 + 2194
577166014752240080*x**18 + 1463104032160519033200*x**17 + 8727757740674554
98528*x**16 + 465517091041681015296*x**15 + 221699757548270194389*x**14 +
94069263918929616324*x**13 + 35454069480572048124*x**12 + 1182148750162071
6192*x**11 + 17344958593049772048*x**10/5 + 889942562270387136*x**9 + 1978
97276851452864*x**8 + 37727143432895007*x**7 + 6077684727888102*x**6 + 405
7390785756924*x**5/5 + 87406679578680*x**4 + 7299544818384*x**3 + 44356982
8128*x**2 + 17451466816*x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(46) = 92$.

Time = 0.03 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.11

$$
\begin{aligned}
&\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx \\
&= \frac{16677181699666569}{35} x^{35} + 1111812113311046 x^{34} + 12600537284192518 8 x^{33} \\
&\quad + 924039400840784712 x^{32} + 4928210137817518464 x^{31} \\
&\quad + \frac{101849676181562048256}{5} x^{30} + 67899784121041365504 x^{29} \\
&\quad + \frac{2625458326972530284475}{14} x^{28} + 437576396725285446564 x^{27} \\
&\quad + 875152864622814086340 x^{26} + \frac{7584660010542711771792}{5} x^{25} \\
&\quad + 2298383223254096766840 x^{24} + 3064515076512846852480 x^{23} \\
&\quad + 3614565944605222108800 x^{22} + \frac{26506949038858918036881}{7} x^{21} \\
&\quad + 3534290697929473864098 x^{20} + 2945285062308448290360 x^{19} \\
&\quad + 2194577166014752240080 x^{18} + 1463104032160519033200 x^{17} \\
&\quad + 872775774067455498528 x^{16} + 465517091041681015296 x^{15} \\
&\quad + 221699757548270194389 x^{14} + 94069263918929616324 x^{13} \\
&\quad + 35454069480572048124 x^{12} + 11821487501620716192 x^{11} \\
&\quad + \frac{17344958593049772048}{5} x^{10} + 889942562270387136 x^9 + 197897276851452864 x^8 \\
&\quad + 37727143432895007 x^7 + 6077684727888102 x^6 + \frac{4057390785756924}{5} x^5 \\
&\quad + 87406679578680 x^4 + 7299544818384 x^3 + 443569828128 x^2 + 17451466816 x
\end{aligned}
$$

input
```
integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="maxima")
```

output
```
16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^
33 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 1018496761815620
48256/5*x^30 + 67899784121041365504*x^29 + 2625458326972530284475/14*x^28
+ 437576396725285446564*x^27 + 875152864622814086340*x^26 + 75846600105427
11771792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^2
3 + 3614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290
697929473864098*x^20 + 2945285062308448290360*x^19 + 219457716601475224008
0*x^18 + 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 465517
091041681015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x
^13 + 35454069480572048124*x^12 + 11821487501620716192*x^11 + 173449585930
49772048/5*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 377271
43432895007*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679
578680*x^4 + 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int (2+3x)^6\left(1+(2+3x)^7+(2+3x)^{14}\right)^2\,dx = \frac{1}{105}\,(3\,x+2)^{35}+\frac{1}{42}\,(3\,x+2)^{28}+\frac{1}{21}\,(3\,x+2)^{21}+\frac{1}{21}\,(3\,x+2)^{14}+\frac{1}{21}\,(3\,x+2)^{7}$$

input
```
integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="giac")
```

output
```
1/105*(3*x + 2)^35 + 1/42*(3*x + 2)^28 + 1/21*(3*x + 2)^21 + 1/21*(3*x + 2
)^14 + 1/21*(3*x + 2)^7
```

Mupad [B] (verification not implemented)

Time = 10.67 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int (2+3x)^6\left(1+(2+3x)^7+(2+3x)^{14}\right)^2\,dx = \frac{(3\,x+2)^7}{21}+\frac{(3\,x+2)^{14}}{21}+\frac{(3\,x+2)^{21}}{21}+\frac{(3\,x+2)^{28}}{42}+\frac{(3\,x+2)^{35}}{105}$$

input
```
int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1)^2,x)
```

output
```
(3*x + 2)^7/21 + (3*x + 2)^14/21 + (3*x + 2)^21/21 + (3*x + 2)^28/42 + (3*
x + 2)^35/105
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.09

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx = \frac{x(33354363399333138x^{34}+778268479317773220x^{33}+8820376098934763160x^{32}+64682758058854929\cdots}{}$$

input
```
int((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x)
```

output
```
(x*(33354363399333138*x**34 + 778268479317773220*x**33 + 88203760989347631
60*x**32 + 64682758058854929840*x**31 + 344974709647226292480*x**30 + 1425
895466541868675584*x**29 + 4752984888472895585280*x**28 + 1312729163486265
1422375*x**27 + 30630347770769981259480*x**26 + 61260700523596986043800*x*
*25 + 106185240147597964805088*x**24 + 160886825627786773678800*x**23 + 21
4516055355899279673600*x**22 + 253019616122365547616000*x**21 + 2650694903
88589180368810*x**20 + 247400348855063170486860*x**19 + 206169954361591380
325200*x**18 + 153620401621032656805600*x**17 + 102417282251236332324000*x
**16 + 61094304184721884896960*x**15 + 32586196372917671070720*x**14 + 155
18983028378913607230*x**13 + 6584848474325073142680*x**12 + 24817848636400
43368680*x**11 + 827504125113450133440*x**10 + 242829420302696808672*x**9
+ 62295979358927099520*x**8 + 13852809379601700480*x**7 + 2640900040302650
490*x**6 + 425437930952167140*x**5 + 56803471000596936*x**4 + 611846757050
7600*x**3 + 510968137286880*x**2 + 31049887968960*x + 1221602677120))/70
```

3.292 $\int (d(s+tx))^m \left(a+b(s+tx)^n+c(s+tx)^{2n}\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 93

$$\int (d(s+tx))^m \left(a+b(s+tx)^n+c(s+tx)^{2n}\right) dx = \frac{a(d(s+tx))^{1+m}}{d(1+m)t}+\frac{b(s+tx)^n(d(s+tx))^{1+m}}{d(1+m+n)t}+\frac{c(s+tx)^{2n}(d(s+tx))^{1+m}}{d(1+m+2n)t}$$

output
```
a*(d*(t*x+s))^(1+m)/d/(1+m)/t+b*(t*x+s)^n*(d*(t*x+s))^(1+m)/d/(1+m+n)/t+c*
(t*x+s)^(2*n)*(d*(t*x+s))^(1+m)/d/(1+m+2*n)/t
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int (d(s+tx))^m \left(a+b(s+tx)^n+c(s+tx)^{2n}\right) dx = \frac{(s+tx)(d(s+tx))^m \left(\frac{a}{1+m}+(s+tx)^n \left(\frac{b}{1+m+n}+\frac{c(s+tx)^n}{1+m+2n}\right)\right)}{t}$$

input
```
Integrate[(d*(s + t*x))^m*(a + b*(s + t*x)^n + c*(s + t*x)^(2*n)),x]
```

output
```
((s + t*x)*(d*(s + t*x))^m*(a/(1 + m) + (s + t*x)^n*(b/(1 + m + n) + (c*(s
 + t*x)^n)/(1 + m + 2*n))))/t
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1725, 1691, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d(s+tx))^m \left(a+b(s+tx)^n+c(s+tx)^{2n}\right) dx$$

$$\Big\downarrow \text{ 1725}$$

$$\frac{(s+tx)^{-m}(d(s+tx))^m \int (s+tx)^m \left(b(s+tx)^n+c(s+tx)^{2n}+a\right) d(s+tx)}{t}$$

$$\Big\downarrow \text{ 1691}$$

$$\frac{(s+tx)^{-m}(d(s+tx))^m \int \left(a(s+tx)^m+b(s+tx)^{m+n}+c(s+tx)^{m+2n}\right) d(s+tx)}{t}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(s+tx)^{-m}(d(s+tx))^m \left(\frac{a(s+tx)^{m+1}}{m+1}+\frac{b(s+tx)^{m+n+1}}{m+n+1}+\frac{c(s+tx)^{m+2n+1}}{m+2n+1}\right)}{t}$$

input
```
Int[(d*(s + t*x))^m*(a + b*(s + t*x)^n + c*(s + t*x)^(2*n)),x]
```

output
```
((d*(s + t*x))^m*((a*(s + t*x)^(1 + m))/(1 + m) + (b*(s + t*x)^(1 + m + n)
)/(1 + m + n) + (c*(s + t*x)^(1 + m + 2*n))/(1 + m + 2*n)))/(t*(s + t*x)^m
)
```

Defintions of rubi rules used

rule 1691
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

rule 1725
```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n + c*x^
(2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] &&
LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.59

method	result
risch	$\frac{\left(2mtax+2bms(tx+s)^n+2bns(tx+s)^n+btx(tx+s)^n+b\,m^2s(tx+s)^n+3antx+2a\,n^2tx+3amns+a\,m^2tx+as+b\,m^2tx(tx+s)^n\right.}{}$
orering	$\frac{(tx+s)\left(3m^2+6mn+2n^2+3m+3n+1\right)(d(tx+s))^m\left(a+b(tx+s)^n+c(tx+s)^{2n}\right)}{(m^2+2mn+2m+2n+1)(1+m+n)t}-\frac{3(tx+s)^2(m+n)\left(\frac{(d(tx+s))^m mt\left(a+b(tx+\right.}{tx+s}\right.}{(m^3+3m^2n+2m}$
parallelrisch	$\frac{x(d(tx+s))^m a\,m^2st+2x(d(tx+s))^m a\,n^2st+2(tx+s)^n(d(tx+s))^m bmn\,s^2+(tx+s)^{2n}(d(tx+s))^m cmn\,s^2+x(tx+s)^n(d(tx+s)}{}$

input
```
int((d*(t*x+s))^m*(a+b*(t*x+s)^n+c*(t*x+s)^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
(2*c*m*t*x*((t*x+s)^n)^2+c*n*t*x*((t*x+s)^n)^2+2*m*t*a*x+2*b*m*s*(t*x+s)^n
+2*b*n*s*(t*x+s)^n+b*t*x*(t*x+s)^n+c*m^2*s*((t*x+s)^n)^2+b*m^2*s*(t*x+s)^n
+2*c*m*s*((t*x+s)^n)^2+c*n*s*((t*x+s)^n)^2+c*t*x*((t*x+s)^n)^2+3*a*n*t*x+2
*a*n^2*t*x+3*a*m*n*s+a*m^2*t*x+a*s+b*m^2*t*x*(t*x+s)^n+c*m*n*s*((t*x+s)^n)
^2+x*a*t+3*a*m*n*t*x+2*b*m*n*s*(t*x+s)^n+2*b*m*t*x*(t*x+s)^n+2*b*n*t*x*(t*
x+s)^n+c*m^2*t*x*((t*x+s)^n)^2+2*a*m*s+b*s*(t*x+s)^n+a*m^2*s+2*a*n^2*s+3*a
*n*s+c*s*((t*x+s)^n)^2+2*b*m*n*t*x*(t*x+s)^n+c*m*n*t*x*((t*x+s)^n)^2)/(1+m
)/t/(1+m+n)/(1+m+2*n)*d^m*(t*x+s)^m*exp(-1/2*I*Pi*csgn(I*d*(t*x+s))*m*(-cs
gn(I*d*(t*x+s))+csgn(I*(t*x+s)))*(-csgn(I*d*(t*x+s))+csgn(I*d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(93) = 186$.

Time = 0.08 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.60

$$\int (d(s+tx))^m \left(a+b(s+tx)^n+c(s+tx)^{2n}\right)\,dx$$
$$= \frac{((cm^2+2\,cm+(cm+c)n+c)tx+(cm^2+2\,cm+(cm+c)n+c)s)(tx+s)^{2\,n}e^{(m\log(tx+s)+m\log(d))}+((\ldots}{\ldots}$$

input
```
integrate((d*(t*x+s))^m*(a+b*(t*x+s)^n+c*(t*x+s)^(2*n)),x, algorithm="fric
as")
```

output
```
(((c*m^2 + 2*c*m + (c*m + c)*n + c)*t*x + (c*m^2 + 2*c*m + (c*m + c)*n + c
)*s)*(t*x + s)^(2*n)*e^(m*log(t*x + s) + m*log(d)) + ((b*m^2 + 2*b*m + 2*(
b*m + b)*n + b)*t*x + (b*m^2 + 2*b*m + 2*(b*m + b)*n + b)*s)*(t*x + s)^n*e
^(m*log(t*x + s) + m*log(d)) + ((a*m^2 + 2*a*n^2 + 2*a*m + 3*(a*m + a)*n +
 a)*t*x + (a*m^2 + 2*a*n^2 + 2*a*m + 3*(a*m + a)*n + a)*s)*e^(m*log(t*x +
s) + m*log(d)))/((m^3 + 2*(m + 1)*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*n + 3*m
+ 1)*t)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2853 vs. $2(75) = 150$.

Time = 16.73 (sec) , antiderivative size = 2853, normalized size of antiderivative = 30.68

$$\int (d(s+tx))^m \left(a+b(s+tx)^n+c(s+tx)^{2n}\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*(t*x+s))**m*(a+b*(t*x+s)**n+c*(t*x+s)**(2*n)),x)
```

output
```
Piecewise((x*(a + b + c)/(d*s), Eq(m, -1) & Eq(n, 0) & Eq(t, 0)), ((a*log(
s/t + x)/t + b*(s + t*x)**n/(n*t) + c*(s + t*x)**(2*n)/(2*n*t))/d, Eq(m, -
1)), (Piecewise(((a*Piecewise((0**(-2*n - 1)*(s + t*x), Eq(d, 0)), (Piecew
ise((-1/(2*n*(d*(s + t*x))**(2*n)), Ne(n, 0)), (log(d*(s + t*x)), True))/d
, True)) + b*Piecewise((-(d*(s + t*x))**(-2*n - 1)*(s + t*x)*(s + t*x)**n/
n, Ne(n, 0)), ((d*(s + t*x))**(-2*n - 1)*(s + t*x)*(s + t*x)**n*log(s + t*
x), True)) + c*(d*(s + t*x))**(-2*n - 1)*(s + t*x)*(s + t*x)**(2*n)*log(s
+ t*x))/t, Ne(t, 0)), (x*(d*s)**(-2*n - 1)*(a + b*s**n + c*s**(2*n)), True
)), Eq(m, -2*n - 1)), (Piecewise(((a*Piecewise((0**(-n - 1)*(s + t*x), Eq(
d, 0)), (Piecewise((-1/(n*(d*(s + t*x))**n), Ne(n, 0)), (log(d*(s + t*x)),
 True))/d, True)) + b*(d*(s + t*x))**(-n - 1)*(s + t*x)*(s + t*x)**n*log(s
 + t*x) + c*Piecewise(((d*(s + t*x))**(-n - 1)*(s + t*x)*(s + t*x)**(2*n)/
n, Ne(n, 0)), ((d*(s + t*x))**(-n - 1)*(s + t*x)*(s + t*x)**(2*n)*log(s +
t*x), True)))/t, Ne(t, 0)), (x*(d*s)**(-n - 1)*(a + b*s**n + c*s**(2*n)),
True)), Eq(m, -n - 1)), (x*(d*s)**m*(a + b*s**n + c*s**(2*n)), Eq(t, 0)),
(a*m**2*s*(d*s + d*t*x)**m/(m**3*t + 3*m**2*n*t + 3*m**2*t + 2*m*n**2*t +
6*m*n*t + 3*m*t + 2*n**2*t + 3*n*t + t) + a*m**2*t*x*(d*s + d*t*x)**m/(m**
3*t + 3*m**2*n*t + 3*m**2*t + 2*m*n**2*t + 6*m*n*t + 3*m*t + 2*n**2*t + 3*
n*t + t) + 3*a*m*n*s*(d*s + d*t*x)**m/(m**3*t + 3*m**2*n*t + 3*m**2*t + 2*
m*n**2*t + 6*m*n*t + 3*m*t + 2*n**2*t + 3*n*t + t) + 3*a*m*n*t*x*(d*s +...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int (d(s+tx))^m \left(a+b(s+tx)^n+c(s+tx)^{2n}\right) dx = \frac{(d^m tx + d^m s)ce^{(m\log(tx+s)+2\,n\log(tx+s))}}{mt+2\,nt+t} + \frac{(d^m tx + d^m s)be^{(m\log(tx+s)+n\log(tx+s))}}{mt+nt+t} + \frac{(dtx+ds)^{m+1}a}{d(m+1)t}$$

input
```
integrate((d*(t*x+s))^m*(a+b*(t*x+s)^n+c*(t*x+s)^(2*n)),x, algorithm="maxi
ma")
```

output
```
(d^m*t*x + d^m*s)*c*e^(m*log(t*x + s) + 2*n*log(t*x + s))/(m*t + 2*n*t + t
) + (d^m*t*x + d^m*s)*b*e^(m*log(t*x + s) + n*log(t*x + s))/(m*t + n*t + t
) + (d*t*x + d*s)^(m + 1)*a/(d*(m + 1)*t)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1475 vs. 2(93) = 186.

Time = 0.13 (sec) , antiderivative size = 1475, normalized size of antiderivative = 15.86

$$\int (d(s+tx))^m \left(a+b(s+tx)^n+c(s+tx)^{2n}\right) dx = \text{Too large to display}$$

input
```
integrate((d*(t*x+s))^m*(a+b*(t*x+s)^n+c*(t*x+s)^(2*n)),x, algorithm="giac
")
```

output
```
((t*x + s)^n*b*m^2*t*x*e^(m*log(t*x + s) + m*log(d)) + (t*x + s)^(2*n)*c*m
^2*t*x*e^(m*log(t*x + s) + m*log(d)) + (t*x + s)^n*c*m^2*t*x*e^(m*log(t*x
+ s) + m*log(d)) + 2*(t*x + s)^n*b*m*n*t*x*e^(m*log(t*x + s) + m*log(d)) +
 (t*x + s)^(2*n)*c*m*n*t*x*e^(m*log(t*x + s) + m*log(d)) + (t*x + s)^n*c*m
*n*t*x*e^(m*log(t*x + s) + m*log(d)) + (t*x + s)^n*b*m^2*s*e^(m*log(t*x +
s) + m*log(d)) + (t*x + s)^(2*n)*c*m^2*s*e^(m*log(t*x + s) + m*log(d)) + (
t*x + s)^n*c*m^2*s*e^(m*log(t*x + s) + m*log(d)) + 2*(t*x + s)^n*b*m*n*s*e
^(m*log(t*x + s) + m*log(d)) + (t*x + s)^(2*n)*c*m*n*s*e^(m*log(t*x + s) +
 m*log(d)) + (t*x + s)^n*c*m*n*s*e^(m*log(t*x + s) + m*log(d)) + 2*(t*x +
s)^n*b*m*t*x*e^(m*log(t*x + s) + m*log(d)) + 2*(t*x + s)^(2*n)*c*m*t*x*e^(
m*log(t*x + s) + m*log(d)) + 2*(t*x + s)^n*c*m*t*x*e^(m*log(t*x + s) + m*l
og(d)) + a*m^2*t*x*e^(m*log(t*x + s) + m*log(d)) + b*m^2*t*x*e^(m*log(t*x
+ s) + m*log(d)) + c*m^2*t*x*e^(m*log(t*x + s) + m*log(d)) + 2*(t*x + s)^n
*b*n*t*x*e^(m*log(t*x + s) + m*log(d)) + (t*x + s)^(2*n)*c*n*t*x*e^(m*log(
t*x + s) + m*log(d)) + (t*x + s)^n*c*n*t*x*e^(m*log(t*x + s) + m*log(d)) +
 3*a*m*n*t*x*e^(m*log(t*x + s) + m*log(d)) + 2*b*m*n*t*x*e^(m*log(t*x + s)
 + m*log(d)) + c*m*n*t*x*e^(m*log(t*x + s) + m*log(d)) + 2*a*n^2*t*x*e^(m*
log(t*x + s) + m*log(d)) + 2*(t*x + s)^n*b*m*s*e^(m*log(t*x + s) + m*log(d
)) + 2*(t*x + s)^(2*n)*c*m*s*e^(m*log(t*x + s) + m*log(d)) + 2*(t*x + s)^n
*c*m*s*e^(m*log(t*x + s) + m*log(d)) + a*m^2*s*e^(m*log(t*x + s) + m*lo...
```

Mupad [B] (verification not implemented)

Time = 11.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.02

$$\int (d(s+tx))^m \left(a+b(s+tx)^n+c(s+tx)^{2n}\right)\,dx = (d\,(s+t\,x))^m\left(\frac{a\,x}{m+1}+\frac{a\,s}{t\,(m+1)}+\frac{b\,x\,(s+t\,x)^n\,(m+2\,n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}+\frac{c\,x\,(s+t\,x)^{2\,n}\,(m+n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}+\frac{b\,s\,(s+t\,x)^n\,(m+2\,n+1)}{t\,(m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1)}+\frac{c\,s\,(s+t\,x)^{2\,n}\,(m+n+1)}{t\,(m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1)}\right)$$

input
```
int((d*(s + t*x))^m*(a + c*(s + t*x)^(2*n) + b*(s + t*x)^n),x)
```

output
```
(d*(s + t*x))^m*((a*x)/(m + 1) + (a*s)/(t*(m + 1)) + (b*x*(s + t*x)^n*(m +
 2*n + 1))/(2*m + 3*n + 3*m*n + m^2 + 2*n^2 + 1) + (c*x*(s + t*x)^(2*n)*(m
 + n + 1))/(2*m + 3*n + 3*m*n + m^2 + 2*n^2 + 1) + (b*s*(s + t*x)^n*(m + 2
*n + 1))/(t*(2*m + 3*n + 3*m*n + m^2 + 2*n^2 + 1)) + (c*s*(s + t*x)^(2*n)*
(m + n + 1))/(t*(2*m + 3*n + 3*m*n + m^2 + 2*n^2 + 1)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.19

$$\int (d(s+tx))^m \left(a+b(s+tx)^n+c(s+tx)^{2n}\right)\,dx$$
$$= \frac{d^m(tx+s)^m \left(2(tx+s)^n bms+2(tx+s)^n bns+3amns+2amtx+2a\,n^2tx+3antx+a\,m^2tx+atx+\right.}{}$$

input
```
int((d*(t*x+s))^m*(a+b*(t*x+s)^n+c*(t*x+s)^(2*n)),x)
```

output
```
(d**m*(s + t*x)**m*((s + t*x)**(2*n)*c*m**2*s + (s + t*x)**(2*n)*c*m**2*t*
x + (s + t*x)**(2*n)*c*m*n*s + (s + t*x)**(2*n)*c*m*n*t*x + 2*(s + t*x)**(
2*n)*c*m*s + 2*(s + t*x)**(2*n)*c*m*t*x + (s + t*x)**(2*n)*c*n*s + (s + t*
x)**(2*n)*c*n*t*x + (s + t*x)**(2*n)*c*s + (s + t*x)**(2*n)*c*t*x + (s + t
*x)**n*b*m**2*s + (s + t*x)**n*b*m**2*t*x + 2*(s + t*x)**n*b*m*n*s + 2*(s
+ t*x)**n*b*m*n*t*x + 2*(s + t*x)**n*b*m*s + 2*(s + t*x)**n*b*m*t*x + 2*(s
 + t*x)**n*b*n*s + 2*(s + t*x)**n*b*n*t*x + (s + t*x)**n*b*s + (s + t*x)**
n*b*t*x + a*m**2*s + a*m**2*t*x + 3*a*m*n*s + 3*a*m*n*t*x + 2*a*m*s + 2*a*
m*t*x + 2*a*n**2*s + 2*a*n**2*t*x + 3*a*n*s + 3*a*n*t*x + a*s + a*t*x))/(t
*(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1))
```

3.293 $\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 340

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = -\frac{d(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}} + \frac{(d+ex)^2\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

output
```
-d*(e*x+d)*(1+2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^3
/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*AppellF1(1/3,1/2,1/2,4/3,-2*c*(e*x+d)^3/(b-
(-4*a*c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))/e^2/(a+b*(e*x+d
)^3+c*(e*x+d)^6)^(1/2)+1/2*(e*x+d)^2*(1+2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2
)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*AppellF1(2/3,1/2,
1/2,5/3,-2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^
2)^(1/2)))/e^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)
```

Mathematica [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx=\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

input
```
Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

output
```
Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1726, 25, 2322, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

$$\Big\downarrow \text{1726}$$

$$\frac{\int \frac{ex}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}d(d+ex)}{e^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{ex}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}d(d+ex)}{e^2}$$

$$\Big\downarrow \text{2322}$$

$$-\frac{\int \left(\frac{d}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}-\frac{d+ex}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}\right)d(d+ex)}{e^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,\text{Ap}\cdots}{\sqrt{a+b(d+ex}\cdots}}{e^2}$$

input
```
Int[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

output
```
(-((d*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3,
 (-2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[
b^2 - 4*a*c])])/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) + ((d + e*x)^2*Sq
rt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3
)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(2
*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]))/e^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1726
```
Int[((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[v, x, 1]^(m + 1)   Subst[Int[SimplifyIntegrand[(x
- Coefficient[v, x, 0])^m*(a + b*x^n + c*x^(2*n))^p, x], x], x, v], x] /; F
reeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[v, x] && IntegerQ[m] &&
 NeQ[v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2322
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Maple [F]

$$\int \frac{x}{\sqrt{a+b\left(ex+d\right)^{3}+c\left(ex+d\right)^{6}}}dx$$

input
```
int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

output
```
int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx=\int \frac{x}{\sqrt{\left(ex+d\right)^{6}c+\left(ex+d\right)^{3}b+a}}\,dx$$

input
```
integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="fricas")
```

output
```
integral(x/sqrt(c*e^6*x^6 + 6*c*d*e^5*x^5 + 15*c*d^2*e^4*x^4 + c*d^6 + (20
*c*d^3 + b)*e^3*x^3 + 3*(5*c*d^4 + b*d)*e^2*x^2 + b*d^3 + 3*(2*c*d^5 + b*d
^2)*e*x + a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$
$$=\int \frac{x}{\sqrt{a+bd^3+3bd^2ex+3bde^2x^2+be^3x^3+cd^6+6cd^5ex+15cd^4e^2x^2+20cd^3e^3x^3+15cd^2e^4x^4+6cde^5}}$$

input
```
integrate(x/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x**3
+ c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 15*c
*d**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

input
```
integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

input
```
integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x}{\sqrt{a+b\,{\left(d+e\,x\right)}^3+c\,{\left(d+e\,x\right)}^6}}\,dx$$

input
```
int(x/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2),x)
```

output
```
int(x/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$
$$= \int \frac{\sqrt{c\,e^6x^6+6cd\,e^5x^5+15c\,d^2e^4x^4+20c\,d^3e^3x^3+15c\,d^4e^2x^2+6c\,d^5ex+b\,e^3x^3+c\,d^6+3bd\,e^2x^2+3b\,c}}{c\,e^6x^6+6cd\,e^5x^5+15c\,d^2e^4x^4+20c\,d^3e^3x^3+15c\,d^4e^2x^2+6c\,d^5ex+b\,e^3x^3+c\,d^6+3bd\,e^2x^2+3b\,c}$$

input
```
int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

output
```
int((sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x**3 + c*d*
*6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 15*c*d**2*
e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6)*x)/(a + b*d**3 + 3*b*d**2*e*x +
 3*b*d*e**2*x**2 + b*e**3*x**3 + c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x*
*2 + 20*c*d**3*e**3*x**3 + 15*c*d**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*
x**6),x)
```

3.294 $\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 398

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}} \, dx = \frac{d^2(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} - \frac{d(d+ex)^2\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}} + \frac{\operatorname{arctanh}\left(\frac{b+2c(d+ex)^3}{2\sqrt{c}\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\right)}{3\sqrt{c}e^3}$$

output
```
d^2*(e*x+d)*(1+2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^
3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*AppellF1(1/3,1/2,1/2,4/3,-2*c*(e*x+d)^3/(b
-(-4*a*c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))/e^3/(a+b*(e*x+
d)^3+c*(e*x+d)^6)^(1/2)-d*(e*x+d)^2*(1+2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)
))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*AppellF1(2/3,1/2,1
/2,5/3,-2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2
)^(1/2)))/e^3/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)+1/3*arctanh(1/2*(b+2*c*(e*
x+d)^3)/c^(1/2)/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2))/c^(1/2)/e^3
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx=\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

input
```
Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

output
```
Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.107$, Rules used = {1726, 2322, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

$$\Big\downarrow \text{1726}$$

$$\frac{\int \frac{e^2x^2}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}d(d+ex)}{e^3}$$

$$\Big\downarrow \text{2322}$$

$$\frac{\int \left(\frac{d^2}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}-\frac{2(d+ex)d}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}+\frac{(d+ex)^2}{\sqrt{c(d+ex)^6+b(d+ex)^3+a}}\right)d(d+ex)}{e^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{d^2(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,\text{A}}{\sqrt{a+b(d+e}}}{e^3}$$

input
```
Int[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

output
```
((d^2*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3,
 (-2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[
b^2 - 4*a*c])])/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6] - (d*(d + e*x)^2*S
qrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^
3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/S
qrt[a + b*(d + e*x)^3 + c*(d + e*x)^6] + ArcTanh[(b + 2*c*(d + e*x)^3)/(2*
Sqrt[c]*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])]/(3*Sqrt[c]))/e^3
```

Defintions of rubi rules used

rule 1726
```
Int[((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[v, x, 1]^(m + 1)   Subst[Int[SimplifyIntegrand[(x
- Coefficient[v, x, 0])^m*(a + b*x^n + c*x^(2*n))^p, x], x], x, v], x] /; F
reeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[v, x] && IntegerQ[m] &&
 NeQ[v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2322
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{a+b\,(ex+d)^3+c\,(ex+d)^6}}dx$$

input
```
int(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

output
```
int(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x^2}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

input
```
integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2/sqrt(c*e^6*x^6 + 6*c*d*e^5*x^5 + 15*c*d^2*e^4*x^4 + c*d^6 + (
20*c*d^3 + b)*e^3*x^3 + 3*(5*c*d^4 + b*d)*e^2*x^2 + b*d^3 + 3*(2*c*d^5 + b
*d^2)*e*x + a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$
$$= \int \frac{x^2}{\sqrt{a+bd^3+3bd^2ex+3bde^2x^2+be^3x^3+cd^6+6cd^5ex+15cd^4e^2x^2+20cd^3e^3x^3+15cd^2e^4x^4+6cde^5}}$$

input
```
integrate(x**2/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x*
*3 + c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 1
5*c*d**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \int \frac{x^2}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

input
```
integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx=\int \frac{x^2}{\sqrt{(ex+d)^6c+(ex+d)^3b+a}}\,dx$$

input
```
integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt((e*x + d)^6*c + (e*x + d)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx=\int \frac{x^2}{\sqrt{a+b\,(d+e\,x)^3+c\,(d+e\,x)^6}}\,dx$$

input
```
int(x^2/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2),x)
```

output
```
int(x^2/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$
$$=\int \frac{\sqrt{c\,e^6x^6+6cd\,e^5x^5+15c\,d^2e^4x^4+20c\,d^3e^3x^3+15c\,d^4e^2x^2+6c\,d^5ex+b\,e^3x^3+c\,d^6+3bd\,e^2x^2+3b}}{c\,e^6x^6+6cd\,e^5x^5+15c\,d^2e^4x^4+20c\,d^3e^3x^3+15c\,d^4e^2x^2+6c\,d^5ex+b\,e^3x^3+c\,d^6+3bd\,e^2x^2+3b}$$

input
```
int(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

output
```
int((sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x**3 + c*d*
*6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 15*c*d**2*
e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6)*x**2)/(a + b*d**3 + 3*b*d**2*e*
x + 3*b*d*e**2*x**2 + b*e**3*x**3 + c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2
*x**2 + 20*c*d**3*e**3*x**3 + 15*c*d**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e*
*6*x**6),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file