Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.3-General-trinomial/129-1.2.3.4-a

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**113**]. This is test number [129].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	97.35 (110)	2.65 (3)
Rubi	96.46 (109)	3.54 (4)
Maple	78.76 (89)	21.24 (24)
Fricas	69.91 (79)	30.09 (34)
Mupad	68.14 (77)	31.86 (36)
Giac	64.60 (73)	35.40 (40)
Maxima	46.02 (52)	53.98 (61)
Sympy	45.13 (51)	54.87 (62)
Reduce	43.36 (49)	56.64 (64)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

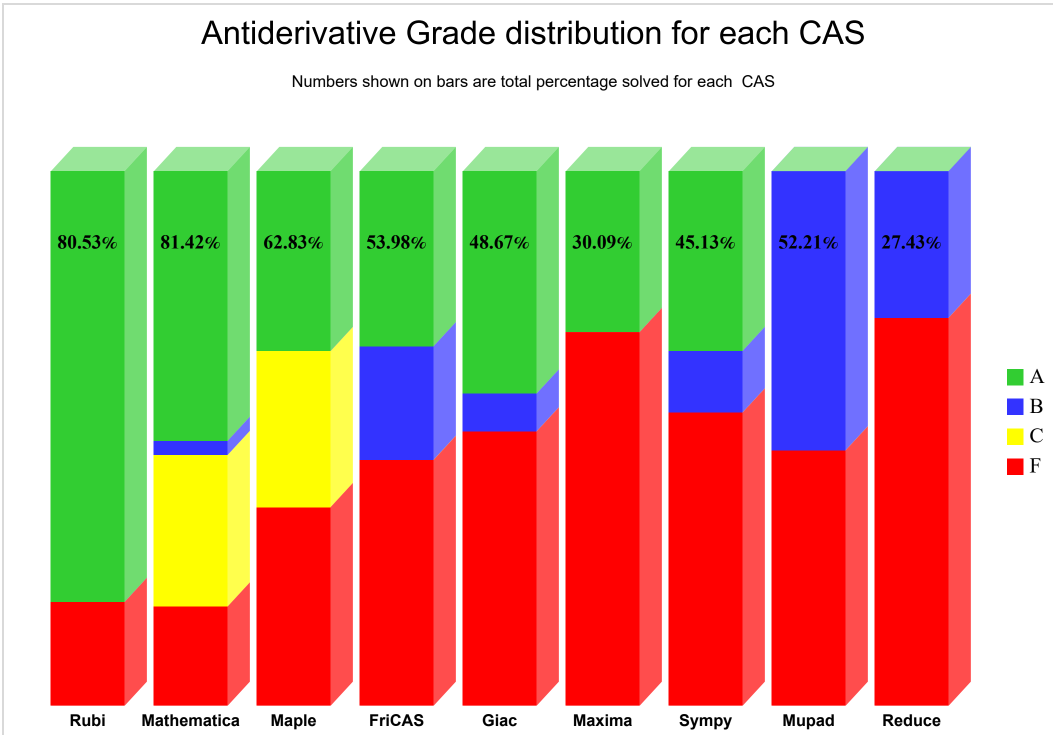
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

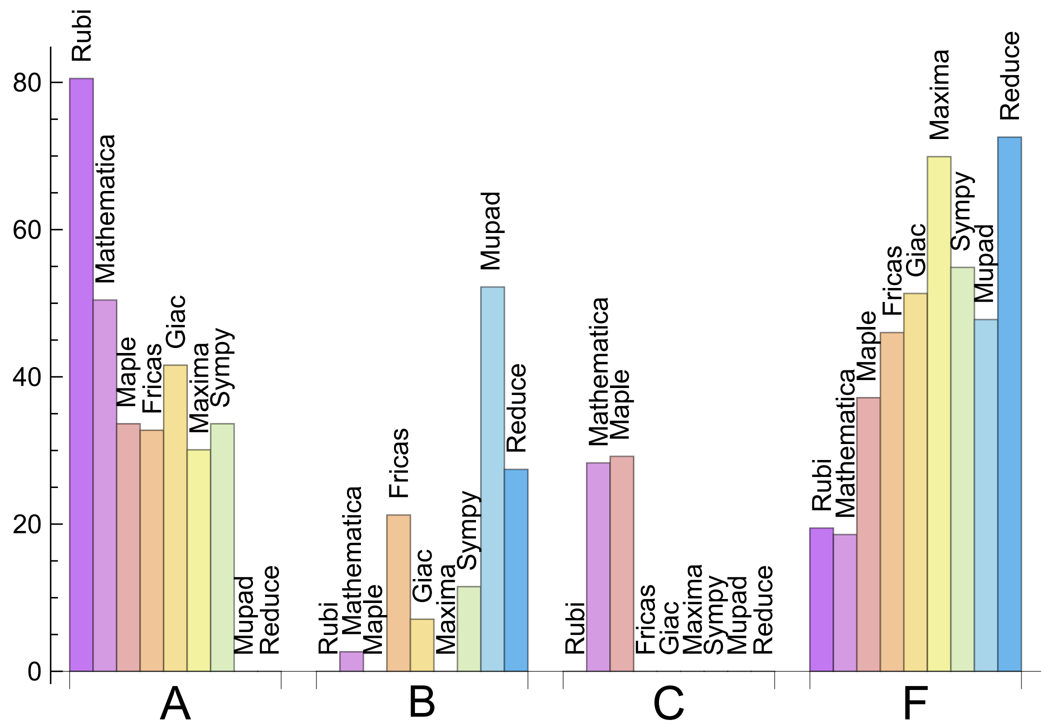
System	% A grade	% B grade	% C grade	% F grade
Rubi	80.531	0.000	0.000	19.469
Mathematica	50.442	2.655	28.319	18.584
Giac	41.593	7.080	0.000	51.327
Maple	33.628	0.000	29.204	37.168
Sympy	33.628	11.504	0.000	54.867
Fricas	32.743	21.239	0.000	46.018
Maxima	30.088	0.000	0.000	69.912
Mupad	0.000	52.212	0.000	47.788
Reduce	0.000	27.434	0.000	72.566

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	3	100.00	0.00	0.00
Rubi	4	100.00	0.00	0.00
Maple	24	100.00	0.00	0.00
Fricas	34	70.59	29.41	0.00
Mupad	36	0.00	100.00	0.00
Giac	40	97.50	2.50	0.00
Sympy	62	0.00	100.00	0.00
Maxima	61	72.13	0.00	27.87
Reduce	64	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.10
Maple	0.18
Giac	0.22
Reduce	0.34
Rubi	0.55
Mathematica	1.40
Sympy	1.80
Fricas	2.10
Mupad	24.88

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	121.79	0.99	41.00	1.02
Sympy	138.78	1.13	128.00	0.73
Maple	160.16	0.71	48.00	0.85
Mathematica	205.25	0.91	124.50	1.00
Giac	262.33	1.08	68.00	0.98
Rubi	287.22	1.01	279.00	1.00
Fricas	1313.89	3.43	137.00	1.54
Reduce	2855.88	97.72	286.00	1.18
Mupad	4041.52	7.41	309.00	1.34

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

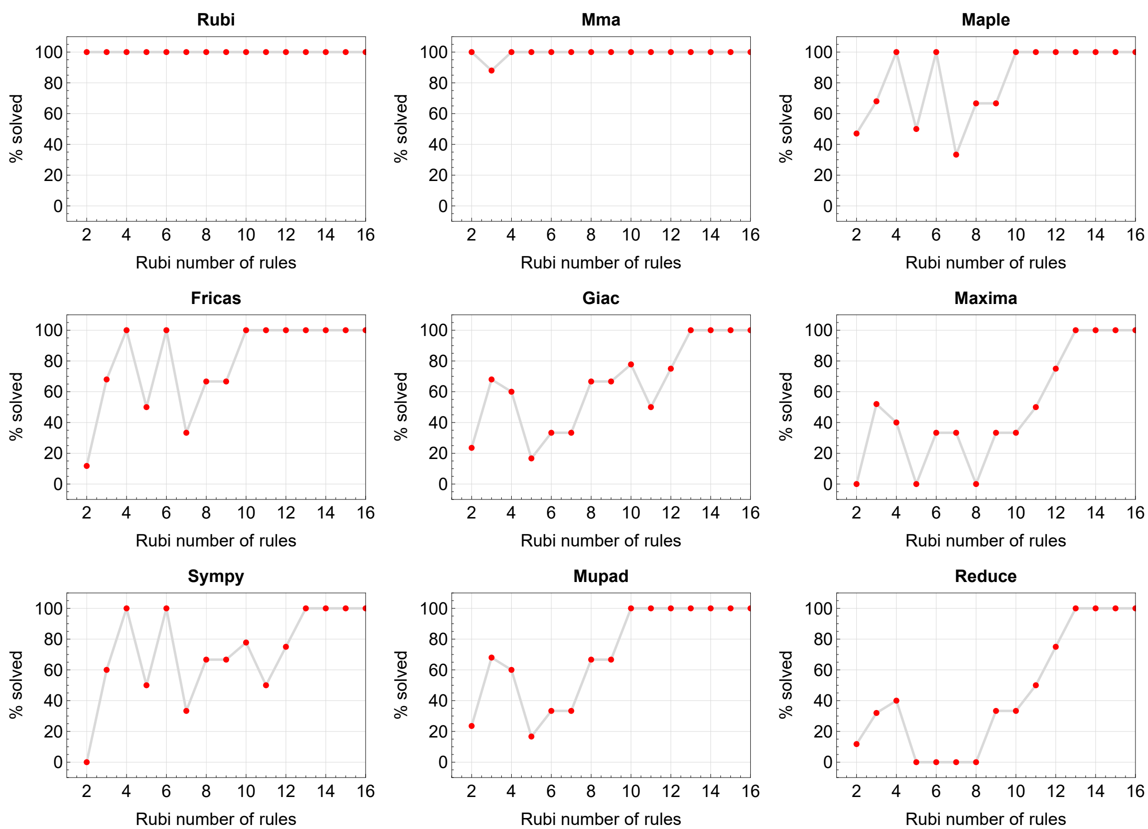


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

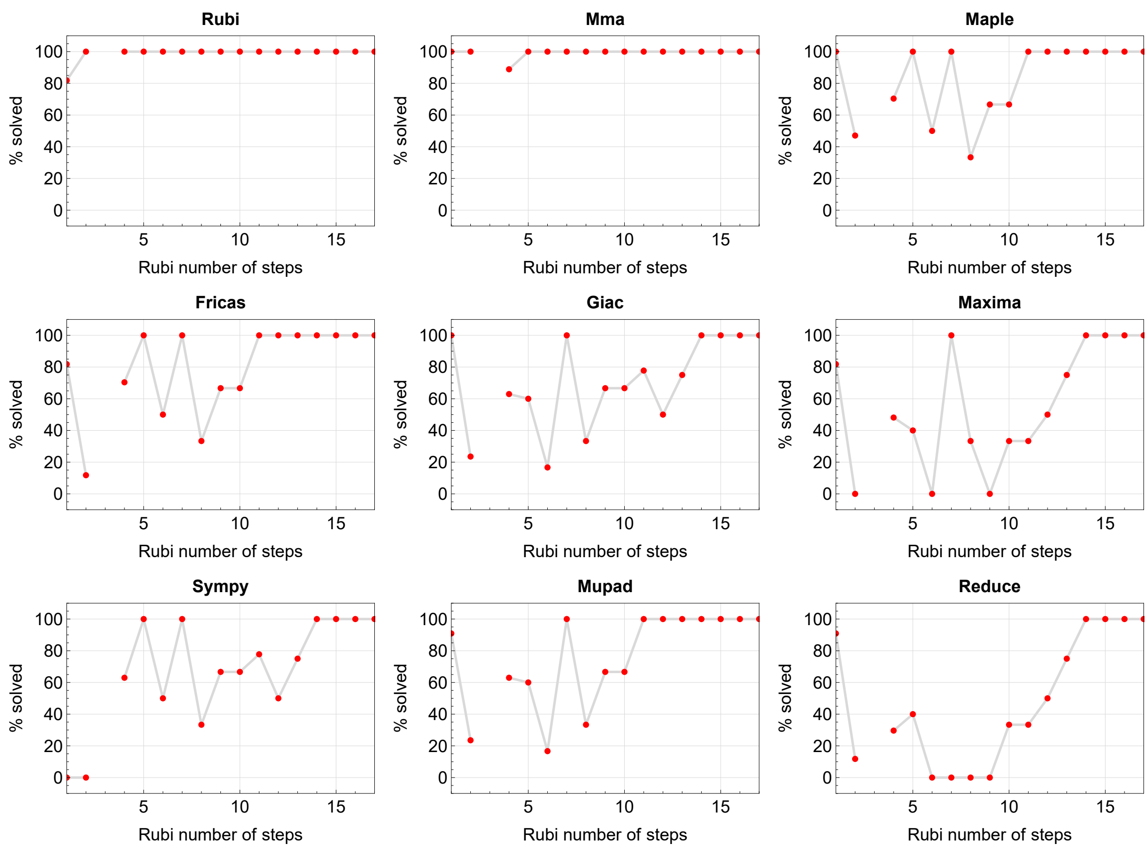


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

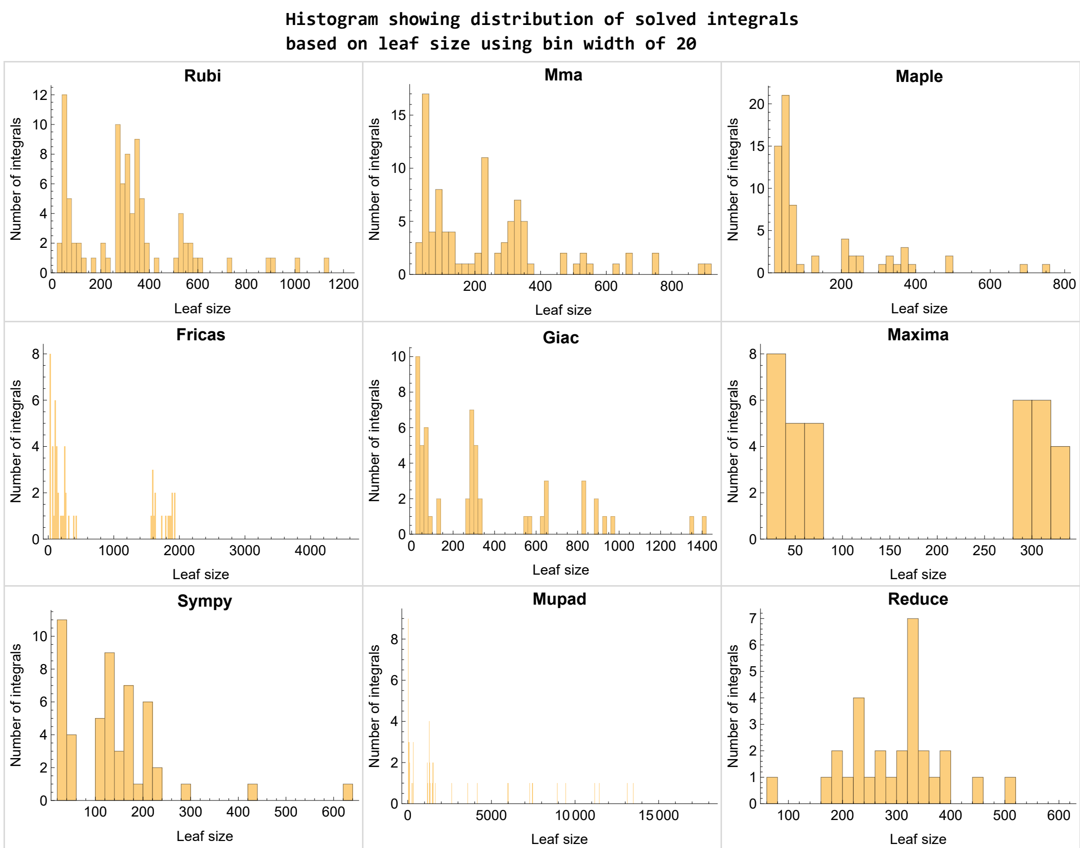


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

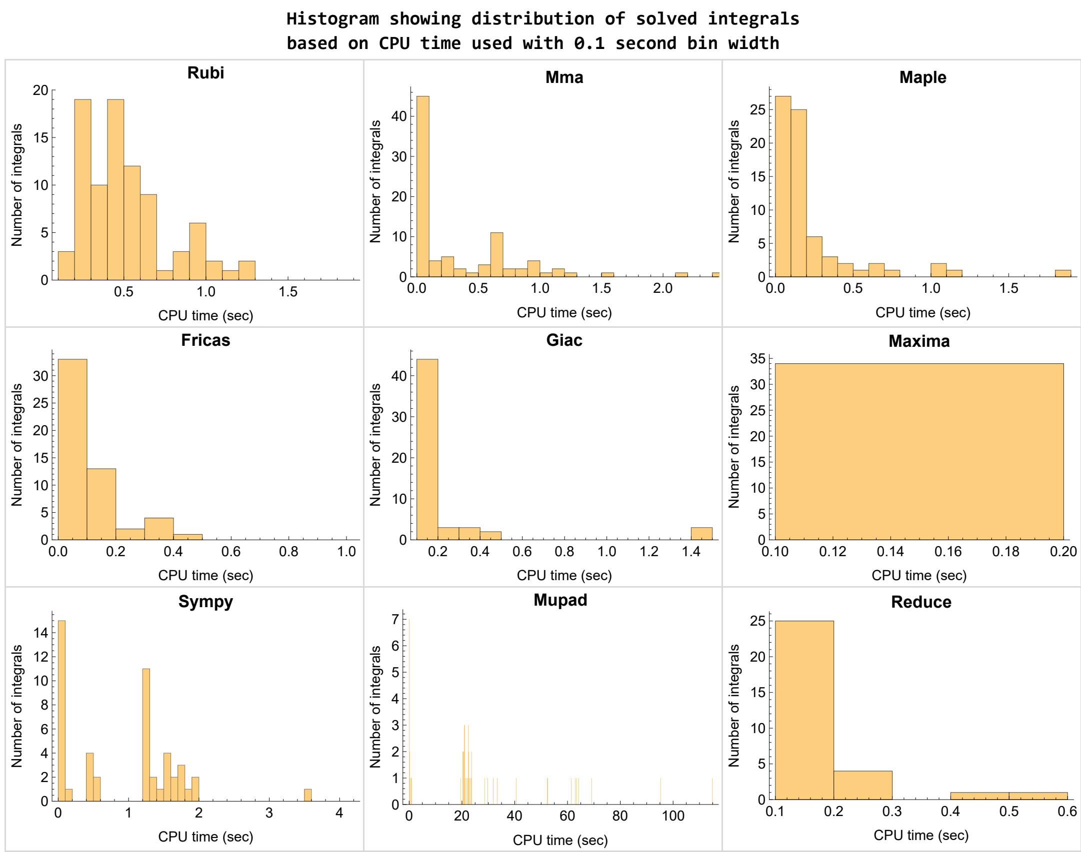


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

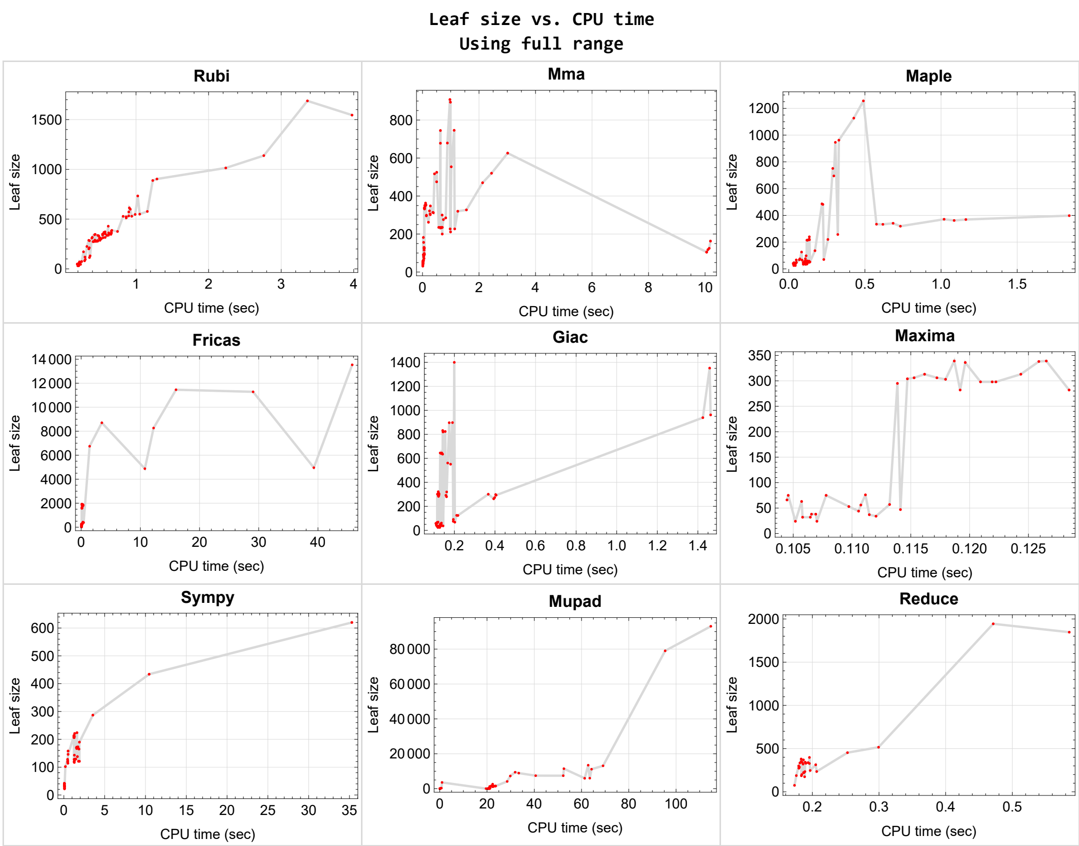


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{91, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108, 109, 110, 111, 112, 113\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 86, 87, 100, 101, 102, 104\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

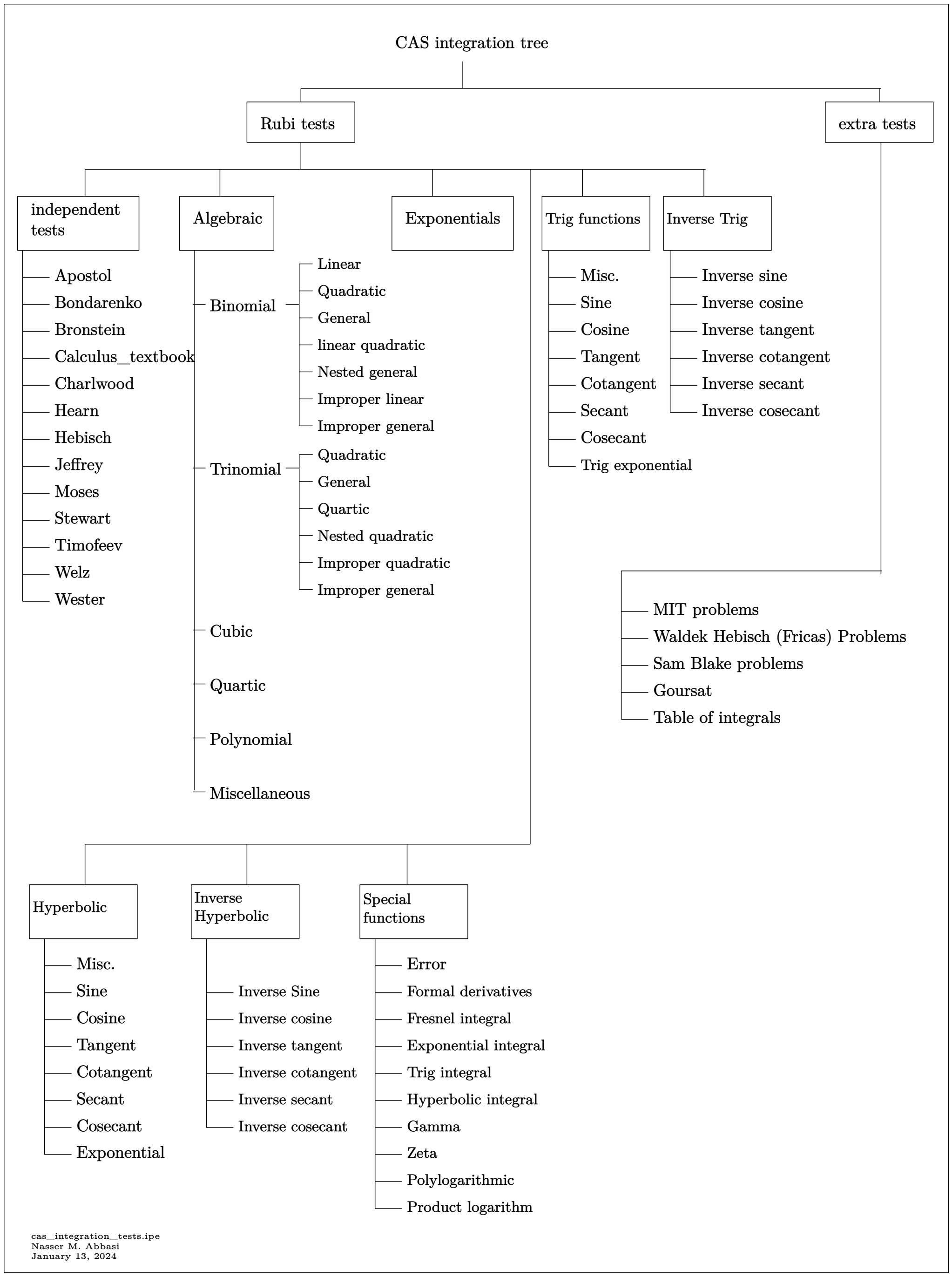
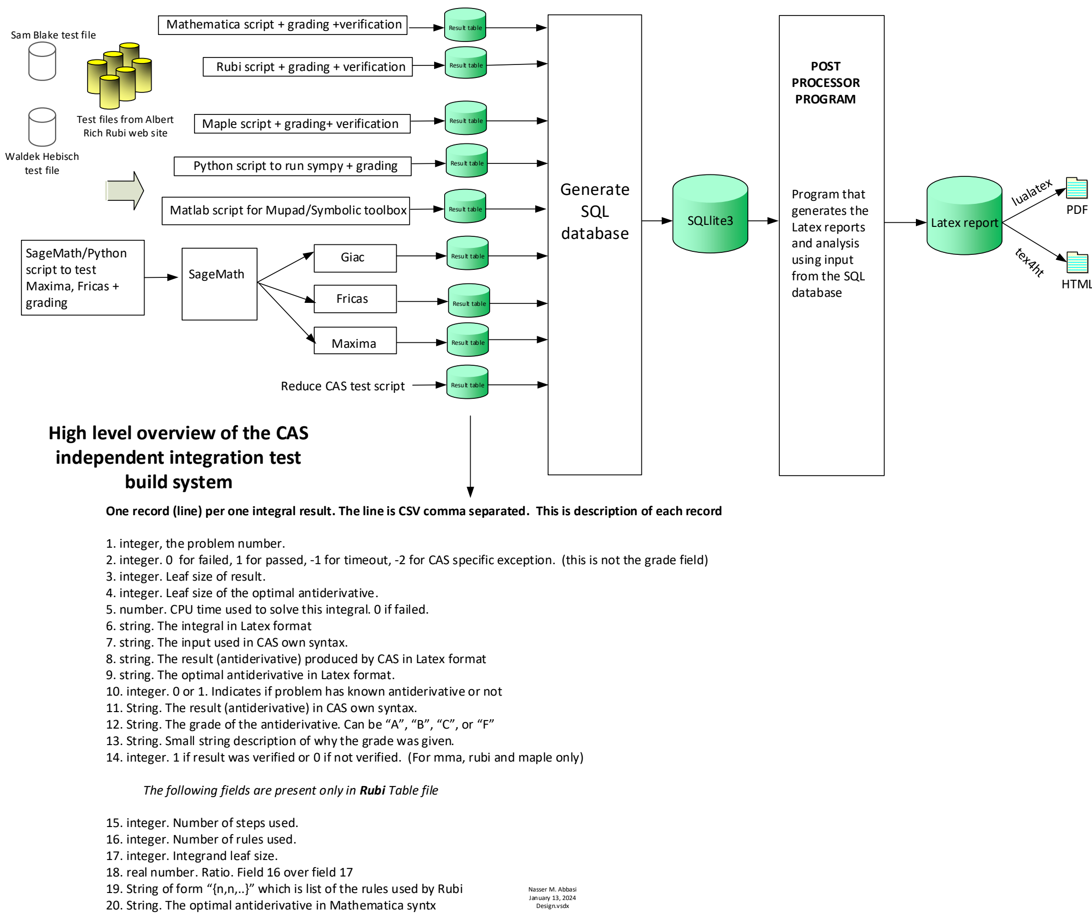


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 100, 101, 102, 103, 104 }

B grade { }

C grade { }

F normal fail { 32, 33, 34, 35 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 47, 48, 49, 59, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 100, 101, 102, 104 }

B grade { 4, 5, 15 }

C grade { 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74 }

F normal fail { 89, 90, 103 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 12, 13, 14, 15, 16, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 47, 48, 49, 50, 51, 59, 60, 61, 66, 67, 68, 69, 70, 71 }

B grade { }

C grade { 6, 7, 8, 9, 10, 11, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 41, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65 }

F normal fail { 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 100, 101, 102, 103, 104 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 12, 13, 14, 15, 16, 27, 36, 37, 38, 39, 40, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 66, 67, 68, 69, 70, 71 }

B grade { 6, 7, 8, 9, 10, 11, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 41, 42, 43, 44, 45, 46 }

C grade { }

F normal fail { 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 100, 101, 102, 103, 104 }

F(-1) timedout fail { 30, 31, 32, 33, 34, 35, 62, 63, 64, 65 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 47, 48, 49, 50, 51, 59, 60 }

B grade { }

C grade { }

F normal fail { 41, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 57, 58, 61, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 100, 101, 102, 103, 104 }

F(-1) timedout fail { }

F(-2) exception fail { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 62, 63, 64, 65 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 47, 48, 49, 50, 51, 59, 60, 61 }

B grade { 34, 52, 53, 54, 55, 56, 57, 58 }

C grade { }

F normal fail { 42, 43, 44, 45, 46, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 100, 101, 102, 103, 104 }

F(-1) timedout fail { 41 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 65 }

C grade { }

F normal fail { }

F(-1) timedout fail { 30, 31, 32, 33, 62, 63, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 100, 101, 102, 103, 104 }

F(-2) exception fail { }

Sympy

A grade { 6, 7, 8, 9, 10, 11, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 66, 67, 68, 69, 70, 71 }

B grade { 1, 2, 3, 4, 5, 12, 13, 14, 15, 16, 36, 37, 38 }

C grade { }

F normal fail { }

F(-1) timedout fail { 28, 29, 30, 31, 32, 33, 34, 35, 39, 40, 41, 42, 43, 44, 45, 46, 62, 63, 64, 65, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33 }

C grade { }

F normal fail { 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 100, 101, 102, 103, 104 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	70	61	60	75	138	146	62	247	95
N.S.	1	0.99	0.86	0.85	1.06	1.94	2.06	0.87	3.48	1.34
time (sec)	N/A	0.221	0.035	0.121	0.105	0.082	0.519	0.115	0.197	0.200

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	58	56	48	66	120	114	51	235	114
N.S.	1	0.98	0.95	0.81	1.12	2.03	1.93	0.86	3.98	1.93
time (sec)	N/A	0.219	0.028	0.096	0.104	0.075	0.494	0.130	0.206	21.166

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	49	48	36	53	106	122	38	223	111
N.S.	1	1.02	1.00	0.75	1.10	2.21	2.54	0.79	4.65	2.31
time (sec)	N/A	0.191	0.025	0.097	0.110	0.072	0.483	0.135	0.187	0.232

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	58	156	47	63	118	117	46	234	226
N.S.	1	1.05	2.84	0.85	1.15	2.15	2.13	0.84	4.25	4.11
time (sec)	N/A	0.220	0.040	0.104	0.106	0.082	1.280	0.111	0.189	21.276

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	67	182	59	76	144	129	65	273	135
N.S.	1	1.02	2.76	0.89	1.15	2.18	1.95	0.98	4.14	2.05
time (sec)	N/A	0.233	0.031	0.108	0.111	0.088	1.333	0.114	0.180	0.304

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	287	353	52	339	1785	214	292	380	1273
N.S.	1	0.85	1.05	0.15	1.01	5.30	0.64	0.87	1.13	3.78
time (sec)	N/A	0.504	0.123	0.139	0.127	0.319	1.236	0.158	0.183	22.454

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	303	351	49	336	1596	170	311	338	1267
N.S.	1	0.91	1.05	0.15	1.01	4.79	0.51	0.93	1.02	3.80
time (sec)	N/A	0.522	0.085	0.135	0.120	0.107	1.542	0.122	0.194	22.245

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	280	337	38	313	1874	211	280	370	1498
N.S.	1	0.86	1.03	0.12	0.96	5.75	0.65	0.86	1.13	4.60
time (sec)	N/A	0.447	0.071	0.120	0.116	0.126	1.289	0.161	0.187	23.125

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	279	337	36	313	1613	168	303	328	1293
N.S.	1	0.86	1.04	0.11	0.97	4.99	0.52	0.94	1.02	4.00
time (sec)	N/A	0.473	0.063	0.118	0.124	0.157	1.745	0.125	0.196	21.126

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	285	363	215	339	1845	209	320	352	1487
N.S.	1	0.85	1.08	0.64	1.01	5.51	0.62	0.96	1.05	4.44
time (sec)	N/A	0.474	0.108	0.119	0.119	0.325	1.268	0.162	0.183	22.615

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	307	322	217	338	1572	172	322	398	1149
N.S.	1	0.91	0.96	0.64	1.00	4.66	0.51	0.96	1.18	3.41
time (sec)	N/A	0.497	0.248	0.129	0.126	0.114	1.571	0.120	0.196	21.162

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	69	61	58	56	135	158	58	200	54
N.S.	1	0.99	0.87	0.83	0.80	1.93	2.26	0.83	2.86	0.77
time (sec)	N/A	0.225	0.026	0.118	0.111	0.082	0.514	0.109	0.184	19.579

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	58	48	47	115	119	47	186	44
N.S.	1	0.98	1.00	0.83	0.81	1.98	2.05	0.81	3.21	0.76
time (sec)	N/A	0.212	0.022	0.095	0.114	0.074	0.459	0.135	0.184	20.581

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	48	47	35	34	102	128	34	174	35
N.S.	1	1.02	1.00	0.74	0.72	2.17	2.72	0.72	3.70	0.74
time (sec)	N/A	0.191	0.019	0.095	0.112	0.077	0.454	0.114	0.188	0.099

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	57	115	45	44	113	128	42	188	42
N.S.	1	1.06	2.13	0.83	0.81	2.09	2.37	0.78	3.48	0.78
time (sec)	N/A	0.218	0.057	0.104	0.111	0.077	1.256	0.112	0.176	20.550

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	124	57	57	140	143	61	221	97
N.S.	1	1.03	1.91	0.88	0.88	2.15	2.20	0.94	3.40	1.49
time (sec)	N/A	0.228	0.054	0.104	0.113	0.077	1.288	0.136	0.187	20.994

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	376	296	54	303	1582	173	292	300	1147
N.S.	1	1.17	0.92	0.17	0.94	4.91	0.54	0.91	0.93	3.56
time (sec)	N/A	0.745	0.137	0.130	0.118	0.152	1.721	0.124	0.179	22.537

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	347	346	45	295	1622	167	290	286	1308
N.S.	1	1.12	1.11	0.14	0.95	5.22	0.54	0.93	0.92	4.21
time (sec)	N/A	0.655	0.089	0.122	0.114	0.109	1.573	0.121	0.181	22.700

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	352	334	34	282	1631	165	283	276	1331
N.S.	1	1.15	1.10	0.11	0.92	5.35	0.54	0.93	0.90	4.36
time (sec)	N/A	0.578	0.072	0.109	0.119	0.152	1.849	0.122	0.180	22.145

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	351	306	218	298	1598	168	302	338	1187
N.S.	1	1.11	0.97	0.69	0.94	5.06	0.53	0.96	1.07	3.76
time (sec)	N/A	0.626	0.270	0.136	0.121	0.112	1.573	0.116	0.185	22.388

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	380	314	240	306	1734	190	304	343	1261
N.S.	1	1.16	0.96	0.73	0.94	5.30	0.58	0.93	1.05	3.86
time (sec)	N/A	0.660	0.368	0.135	0.115	0.197	1.909	0.123	0.189	22.329

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	338	299	58	306	1931	224	293	334	1461
N.S.	1	1.12	0.99	0.19	1.02	6.42	0.74	0.97	1.11	4.85
time (sec)	N/A	0.621	0.141	0.133	0.117	0.141	1.605	0.406	0.181	23.448

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	316	349	49	298	1825	211	273	321	1319
N.S.	1	1.09	1.20	0.17	1.03	6.29	0.73	0.94	1.11	4.55
time (sec)	N/A	0.575	0.087	0.122	0.122	0.324	1.223	0.399	0.186	22.524

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	310	332	36	282	1920	207	263	313	1537
N.S.	1	1.11	1.19	0.13	1.01	6.88	0.74	0.94	1.12	5.51
time (sec)	N/A	0.508	0.073	0.118	0.128	0.131	1.254	0.394	0.205	23.644

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	313	302	214	298	1885	206	298	333	1526
N.S.	1	1.09	1.05	0.74	1.03	6.55	0.72	1.03	1.16	5.30
time (sec)	N/A	0.548	0.276	0.123	0.122	0.313	1.249	0.403	0.190	23.636

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	336	312	222	304	1893	221	301	330	1274
N.S.	1	1.12	1.04	0.74	1.02	6.33	0.74	1.01	1.10	4.26
time (sec)	N/A	0.600	0.385	0.134	0.115	0.118	1.321	0.367	0.190	22.401

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	119	91	76	75	75	102	69	75	91
N.S.	1	1.20	0.92	0.77	0.76	0.76	1.03	0.70	0.76	0.92
time (sec)	N/A	0.297	0.028	0.109	0.108	0.077	0.185	0.119	0.173	0.195

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	599	525	486	0	4880	0	551	453	5967
N.S.	1	1.12	0.98	0.91	0.00	9.16	0.00	1.03	0.85	11.20
time (sec)	N/A	0.922	0.503	0.218	0.000	10.764	0.000	0.181	0.253	61.334

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	547	613	517	482	0	4962	0	561	516	6015
N.S.	1	1.12	0.95	0.88	0.00	9.07	0.00	1.03	0.94	11.00
time (sec)	N/A	0.908	0.425	0.224	0.000	39.300	0.000	0.167	0.299	63.489

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	654	888	678	751	0	0	0	898	24	0
N.S.	1	1.36	1.04	1.15	0.00	0.00	0.00	1.37	0.04	0.00
time (sec)	N/A	1.231	0.634	0.289	0.000	0.000	0.000	0.191	200.017	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	903	679	695	0	0	0	897	24	0
N.S.	1	1.37	1.03	1.05	0.00	0.00	0.00	1.36	0.04	0.00
time (sec)	N/A	1.289	0.881	0.296	0.000	0.000	0.000	0.175	200.021	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	F(-1)	F(-1)	A	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	668	0	745	946	0	0	0	962	1847	0
N.S.	1	0.00	1.12	1.42	0.00	0.00	0.00	1.44	2.76	0.00
time (sec)	N/A	0.000	0.635	0.306	0.000	0.000	0.000	1.463	0.585	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	F(-1)	F(-1)	A	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	684	0	746	962	0	0	0	939	1944	0
N.S.	1	0.00	1.09	1.41	0.00	0.00	0.00	1.37	2.84	0.00
time (sec)	N/A	0.000	1.120	0.329	0.000	0.000	0.000	1.424	0.471	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	F(-1)	F(-1)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	824	0	894	1256	0	0	0	1400	24	8933
N.S.	1	0.00	1.08	1.52	0.00	0.00	0.00	1.70	0.03	10.84
time (sec)	N/A	0.000	0.989	0.490	0.000	0.000	0.000	0.200	200.016	33.446

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	F(-1)	F(-1)	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	831	0	908	1127	0	0	0	1352	24	9445
N.S.	1	0.00	1.09	1.36	0.00	0.00	0.00	1.63	0.03	11.37
time (sec)	N/A	0.000	0.971	0.427	0.000	0.000	0.000	1.458	200.023	31.906

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	131	126	136	0	430	620	125	191	3586
N.S.	1	0.99	0.95	1.03	0.00	3.26	4.70	0.95	1.45	27.17
time (sec)	N/A	0.368	0.067	0.172	0.000	0.296	35.292	0.211	0.326	0.969

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	95	93	98	0	305	434	91	122	2624
N.S.	1	0.98	0.96	1.01	0.00	3.14	4.47	0.94	1.26	27.05
time (sec)	N/A	0.302	0.070	0.114	0.000	0.148	10.452	0.195	0.235	22.387

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	74	71	66	0	216	287	68	69	1632
N.S.	1	1.03	0.99	0.92	0.00	3.00	3.99	0.94	0.96	22.67
time (sec)	N/A	0.251	0.053	0.087	0.000	0.098	3.551	0.202	0.243	21.825

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	81	80	75	0	240	0	75	71	4149
N.S.	1	1.04	1.03	0.96	0.00	3.08	0.00	0.96	0.91	53.19
time (sec)	N/A	0.292	0.039	0.073	0.000	0.202	0.000	0.195	0.247	28.536

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	114	130	126	0	385	0	124	388	7282
N.S.	1	1.02	1.16	1.12	0.00	3.44	0.00	1.11	3.46	65.02
time (sec)	N/A	0.359	0.054	0.084	0.000	0.424	0.000	0.217	0.259	29.853

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	723	577	88	70	0	13535	0	0	81	13112
N.S.	1	0.80	0.12	0.10	0.00	18.72	0.00	0.00	0.11	18.14
time (sec)	N/A	1.156	0.049	0.230	0.000	45.781	0.000	0.000	0.255	69.105

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	550	88	67	0	8705	0	0	75	11453
N.S.	1	0.77	0.12	0.09	0.00	12.12	0.00	0.00	0.10	15.95
time (sec)	N/A	1.051	0.048	0.049	0.000	3.498	0.000	0.000	0.259	52.545

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	532	59	49	0	8268	0	0	43	7457
N.S.	1	0.84	0.09	0.08	0.00	13.04	0.00	0.00	0.07	11.76
time (sec)	N/A	0.900	0.030	0.050	0.000	12.238	0.000	0.000	0.258	52.263

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	512	61	47	0	6748	0	0	41	7469
N.S.	1	0.81	0.10	0.07	0.00	10.64	0.00	0.00	0.06	11.78
time (sec)	N/A	0.865	0.030	0.050	0.000	1.438	0.000	0.000	0.251	40.593

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	547	85	71	0	11285	0	0	81	11174
N.S.	1	0.84	0.13	0.11	0.00	17.28	0.00	0.00	0.12	17.11
time (sec)	N/A	0.989	0.045	0.072	0.000	29.042	0.000	0.000	0.252	64.235

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	655	529	89	68	0	11459	0	0	85	13466
N.S.	1	0.81	0.14	0.10	0.00	17.49	0.00	0.00	0.13	20.56
time (sec)	N/A	0.941	0.045	0.069	0.000	16.023	0.000	0.000	0.235	62.819

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	46	38	37	37	42	37	39	39
N.S.	1	1.04	1.00	0.83	0.80	0.80	0.91	0.80	0.85	0.85
time (sec)	N/A	0.224	0.022	0.036	0.111	0.061	0.056	0.144	0.386	0.064

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	24	24	32	24	24	26
N.S.	1	1.00	1.00	0.81	0.77	0.77	1.03	0.77	0.77	0.84
time (sec)	N/A	0.198	0.012	0.033	0.107	0.061	0.055	0.128	0.189	0.038

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	40	39	33	32	32	37	32	34	34
N.S.	1	1.03	1.00	0.85	0.82	0.82	0.95	0.82	0.87	0.87
time (sec)	N/A	0.215	0.012	0.031	0.106	0.061	0.058	0.121	0.185	0.052

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	44	44	33	38	34	41	35	34	36
N.S.	1	1.07	1.07	0.80	0.93	0.83	1.00	0.85	0.83	0.88
time (sec)	N/A	0.219	0.017	0.039	0.107	0.061	0.066	0.118	0.194	20.356

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	45	25	24	28	36	24	50	26
N.S.	1	1.00	1.45	0.81	0.77	0.90	1.16	0.77	1.61	0.84
time (sec)	N/A	0.210	0.017	0.043	0.105	0.060	0.058	0.119	0.185	20.339

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	356	47	46	0	222	31	645	24	332
N.S.	1	0.85	0.11	0.11	0.00	0.53	0.07	1.54	0.06	0.79
time (sec)	N/A	0.653	0.015	0.034	0.000	0.068	0.081	0.130	0.180	0.707

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	362	48	44	0	266	32	820	22	309
N.S.	1	0.95	0.13	0.12	0.00	0.70	0.08	2.15	0.06	0.81
time (sec)	N/A	0.624	0.017	0.033	0.000	0.068	0.080	0.144	0.190	20.755

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	340	46	41	0	256	24	635	18	330
N.S.	1	0.90	0.12	0.11	0.00	0.68	0.06	1.68	0.05	0.87
time (sec)	N/A	0.541	0.016	0.029	0.000	0.067	0.085	0.142	0.184	20.858

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	367	55	44	0	199	22	824	37	281
N.S.	1	0.89	0.13	0.11	0.00	0.48	0.05	2.00	0.09	0.68
time (sec)	N/A	0.558	0.015	0.033	0.000	0.070	0.077	0.155	0.188	20.866

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	349	57	44	0	253	26	640	35	319
N.S.	1	0.85	0.14	0.11	0.00	0.62	0.06	1.56	0.09	0.78
time (sec)	N/A	0.538	0.014	0.035	0.000	0.066	0.076	0.135	0.200	20.957

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	372	47	40	0	279	31	832	27	313
N.S.	1	0.89	0.11	0.10	0.00	0.67	0.07	2.00	0.06	0.75
time (sec)	N/A	0.573	0.017	0.041	0.000	0.067	0.087	0.142	0.171	20.761

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	356	47	38	0	255	32	645	30	332
N.S.	1	0.85	0.11	0.09	0.00	0.61	0.08	1.54	0.07	0.79
time (sec)	N/A	0.555	0.014	0.044	0.000	0.069	0.082	0.138	0.181	20.878

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	41	37	33	32	32	37	32	34	34
N.S.	1	1.14	1.03	0.92	0.89	0.89	1.03	0.89	0.94	0.94
time (sec)	N/A	0.216	0.011	0.033	0.106	0.061	0.054	0.124	0.182	0.053

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	45	55	33	38	34	41	35	34	36
N.S.	1	1.15	1.41	0.85	0.97	0.87	1.05	0.90	0.87	0.92
time (sec)	N/A	0.219	0.018	0.040	0.107	0.060	0.065	0.124	0.198	0.061

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	45	55	33	0	34	41	35	34	36
N.S.	1	1.15	1.41	0.85	0.00	0.87	1.05	0.90	0.87	0.92
time (sec)	N/A	0.236	0.012	0.036	0.000	0.063	0.063	0.117	0.220	0.042

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1014	1014	262	220	0	0	0	0	41	0
N.S.	1	1.00	0.26	0.22	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.237	0.211	0.257	0.000	0.000	0.000	0.000	0.219	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1138	1138	348	257	0	0	0	0	41	0
N.S.	1	1.00	0.31	0.23	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.761	0.279	0.322	0.000	0.000	0.000	0.000	0.237	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1378	1545	475	319	0	0	0	0	77	79010
N.S.	1	1.12	0.34	0.23	0.00	0.00	0.00	0.00	0.06	57.34
time (sec)	N/A	3.976	0.501	0.733	0.000	0.000	0.000	0.000	0.270	95.387

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1520	1688	554	371	0	0	0	0	77	93066
N.S.	1	1.11	0.36	0.24	0.00	0.00	0.00	0.00	0.05	61.23
time (sec)	N/A	3.362	1.019	1.020	0.000	0.000	0.000	0.000	0.236	114.714

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	347	163	398	0	137	121	0	240	0
N.S.	1	0.96	0.45	1.11	0.00	0.38	0.34	0.00	0.67	0.00
time (sec)	N/A	0.437	10.192	1.842	0.000	0.078	1.907	0.000	0.312	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	317	125	362	0	104	121	0	180	0
N.S.	1	1.00	0.39	1.14	0.00	0.33	0.38	0.00	0.57	0.00
time (sec)	N/A	0.398	10.148	1.086	0.000	0.075	1.789	0.000	0.264	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	287	98	333	0	72	119	0	124	0
N.S.	1	1.03	0.35	1.20	0.00	0.26	0.43	0.00	0.45	0.00
time (sec)	N/A	0.349	0.061	0.618	0.000	0.079	1.253	0.000	0.262	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	283	118	334	0	72	122	0	146	0
N.S.	1	1.03	0.43	1.22	0.00	0.26	0.45	0.00	0.53	0.00
time (sec)	N/A	0.359	10.102	0.576	0.000	0.083	1.293	0.000	0.259	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	309	105	341	0	79	129	0	146	0
N.S.	1	1.09	0.37	1.20	0.00	0.28	0.45	0.00	0.51	0.00
time (sec)	N/A	0.392	10.057	0.685	0.000	0.069	1.460	0.000	0.318	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	343	105	369	0	99	138	0	146	0
N.S.	1	1.07	0.33	1.15	0.00	0.31	0.43	0.00	0.45	0.00
time (sec)	N/A	0.429	10.056	1.163	0.000	0.076	1.660	0.000	0.331	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	300	311	227	0	0	0	0	0	0	0
N.S.	1	1.04	0.76	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.498	1.140	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	225	227	0	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	0.978	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	172	285	0	0	0	0	0	921	0
N.S.	1	1.01	1.68	0.00	0.00	0.00	0.00	0.00	5.42	0.00
time (sec)	N/A	0.277	0.822	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	289	276	0	0	0	0	0	690	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	2.38	0.00
time (sec)	N/A	0.434	0.739	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	332	300	0	0	0	0	0	1473	0
N.S.	1	0.99	0.89	0.00	0.00	0.00	0.00	0.00	4.38	0.00
time (sec)	N/A	0.486	0.696	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	235	0	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.467	0.622	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	235	0	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.433	0.575	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	234	0	0	0	0	0	1409	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	5.09	0.00
time (sec)	N/A	0.440	0.659	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	234	0	0	0	0	0	1347	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	4.86	0.00
time (sec)	N/A	0.445	0.679	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	235	0	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.445	0.703	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	235	0	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.458	0.663	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	232	0	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	0.640	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	235	0	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.408	0.679	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	235	0	0	0	0	0	1341	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	4.81	0.00
time (sec)	N/A	0.456	0.693	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	378	387	470	0	0	0	0	0	0	0
N.S.	1	1.02	1.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.667	2.127	0.000	0.000	0.000	0.000	0.000	0.741	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	320	0	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	1.249	0.000	0.000	0.000	0.000	0.000	0.644	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	200	0	0	0	0	0	944	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	4.70	0.00
time (sec)	N/A	0.350	0.699	0.000	0.000	0.000	0.000	0.000	0.339	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	362	361	0	0	0	0	0	0	28	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.662	0.000	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	531	528	0	0	0	0	0	0	30	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.822	0.000	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	0	29	1304	29
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.00	1.07	48.30	1.07
time (sec)	N/A	0.171	6.668	0.044	0.073	0.074	0.000	0.186	0.269	22.013

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	27	27
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08	1.08
time (sec)	N/A	0.167	6.452	0.040	0.070	0.068	0.000	0.144	0.183	21.725

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	30	0	29	30	29
N.S.	1	1.00	1.07	1.00	1.07	1.11	0.00	1.07	1.11	1.07
time (sec)	N/A	0.176	6.134	0.051	0.070	0.072	0.000	0.160	0.240	21.881

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	30	0	29	30	29
N.S.	1	1.00	1.07	1.00	1.07	1.11	0.00	1.07	1.11	1.07
time (sec)	N/A	0.175	6.202	0.031	0.070	0.069	0.000	0.149	0.244	22.220

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	0	29	6857	29
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.00	1.07	253.96	1.07
time (sec)	N/A	0.174	6.549	0.052	0.075	0.073	0.000	0.168	0.479	21.500

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	0	29	1299	29
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.00	1.07	48.11	1.07
time (sec)	N/A	0.172	6.211	0.041	0.071	0.070	0.000	0.187	0.267	21.022

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08	1.08
time (sec)	N/A	0.163	5.935	0.039	0.066	0.072	0.000	0.147	0.186	20.962

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	30	0	29	30	29
N.S.	1	1.00	1.07	1.00	1.07	1.11	0.00	1.07	1.11	1.07
time (sec)	N/A	0.171	6.172	0.056	0.073	0.069	0.000	0.176	0.245	21.034

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	30	0	29	30	29
N.S.	1	1.00	1.07	1.00	1.07	1.11	0.00	1.07	1.11	1.07
time (sec)	N/A	0.173	6.148	0.030	0.070	0.067	0.000	0.137	0.234	21.029

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	538	524	520	0	0	0	0	0	0	0
N.S.	1	0.97	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.865	2.443	0.000	0.000	0.000	0.000	0.000	5.503	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	450	429	327	0	0	0	0	0	0	0
N.S.	1	0.95	0.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.618	1.555	0.000	0.000	0.000	0.000	0.000	1.907	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	212	211	0	0	0	0	0	0	0
N.S.	1	0.99	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.357	0.993	0.000	0.000	0.000	0.000	0.000	0.833	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	576	572	0	0	0	0	0	0	39	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.901	0.000	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	738	733	626	0	0	0	0	0	41	0
N.S.	1	0.99	0.85	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.024	3.014	0.000	0.000	0.000	0.000	0.000	0.274	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	40	0	29	19068	29
N.S.	1	1.00	1.07	1.00	1.07	1.48	0.00	1.07	706.22	1.07
time (sec)	N/A	0.169	1.823	0.053	0.073	0.110	0.000	0.150	1.260	21.862

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	38	0	27	38	27
N.S.	1	1.00	1.08	1.00	1.08	1.52	0.00	1.08	1.52	1.08
time (sec)	N/A	0.164	1.158	0.052	0.070	0.096	0.000	0.149	0.260	21.532

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	41	0	29	41	29
N.S.	1	1.00	1.07	1.00	1.07	1.52	0.00	1.07	1.52	1.07
time (sec)	N/A	0.169	0.989	0.030	0.079	0.117	0.000	0.143	0.267	28.247

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	41	0	29	41	29
N.S.	1	1.00	1.07	1.00	1.07	1.52	0.00	1.07	1.52	1.07
time (sec)	N/A	0.175	2.939	0.030	0.080	0.154	0.000	0.152	0.272	29.153

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	40	0	29	79840	29
N.S.	1	1.00	1.07	1.00	1.07	1.48	0.00	1.07	2957.04	1.07
time (sec)	N/A	0.168	1.429	0.055	0.071	0.077	0.000	0.193	3.467	21.746

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	40	0	29	18802	29
N.S.	1	1.00	1.07	1.00	1.07	1.48	0.00	1.07	696.37	1.07
time (sec)	N/A	0.172	1.240	0.054	0.075	0.079	0.000	0.151	1.491	21.184

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	37	0	26	37	26
N.S.	1	1.00	1.08	1.00	1.08	1.54	0.00	1.08	1.54	1.08
time (sec)	N/A	0.163	0.726	0.053	0.071	0.077	0.000	0.167	0.272	20.967

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	41	0	29	41	29
N.S.	1	1.00	1.07	1.00	1.07	1.52	0.00	1.07	1.52	1.07
time (sec)	N/A	0.167	0.999	0.031	0.075	0.151	0.000	0.169	0.264	29.462

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	41	0	29	41	29
N.S.	1	1.00	1.07	1.00	1.07	1.52	0.00	1.07	1.52	1.07
time (sec)	N/A	0.170	1.132	0.030	0.076	0.150	0.000	0.146	0.281	30.207

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [17] had the largest ratio of [.800000000000000044]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	0.99	21	0.143
2	A	4	3	0.98	21	0.143
3	A	5	4	1.02	21	0.190
4	A	4	3	1.05	21	0.143
5	A	4	3	1.02	21	0.143
6	A	13	12	0.85	21	0.571
7	A	12	11	0.91	21	0.524
8	A	11	10	0.86	19	0.526
9	A	11	10	0.86	18	0.556
10	A	13	12	0.85	21	0.571
11	A	13	12	0.91	21	0.571
12	A	4	3	0.99	20	0.150
13	A	4	3	0.98	20	0.150
14	A	5	4	1.02	20	0.200
15	A	4	3	1.06	20	0.150
16	A	4	3	1.03	20	0.150
17	A	17	16	1.17	20	0.800
18	A	13	12	1.12	20	0.600
19	A	12	11	1.15	17	0.647
20	A	14	13	1.11	20	0.650
21	A	16	15	1.16	20	0.750

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	15	14	1.12	20	0.700
23	A	13	12	1.09	20	0.600
24	A	11	10	1.11	18	0.556
25	A	13	12	1.09	20	0.600
26	A	15	14	1.12	20	0.700
27	A	10	9	1.20	18	0.500
28	A	2	2	1.12	22	0.091
29	A	2	2	1.12	22	0.091
30	A	2	2	1.36	22	0.091
31	A	2	2	1.37	22	0.091
32	F	0	0	N/A	0.000	N/A
33	F	0	0	N/A	0.000	N/A
34	F	0	0	N/A	0.000	N/A
35	F	0	0	N/A	0.000	N/A
36	A	4	3	0.99	25	0.120
37	A	4	3	0.98	25	0.120
38	A	6	5	1.03	25	0.200
39	A	4	3	1.04	25	0.120
40	A	4	3	1.02	25	0.120
41	A	13	12	0.80	25	0.480
42	A	12	11	0.77	25	0.440
43	A	11	10	0.84	23	0.435
44	A	11	10	0.81	22	0.455
45	A	12	11	0.84	25	0.440
46	A	13	12	0.81	25	0.480
47	A	4	3	1.04	23	0.130
48	A	4	3	1.00	23	0.130
49	A	7	6	1.03	23	0.261
50	A	4	3	1.07	23	0.130
51	A	4	3	1.00	23	0.130
52	A	11	10	0.85	23	0.435
53	A	11	10	0.95	23	0.435

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	11	10	0.90	23	0.435
55	A	9	8	0.89	21	0.381
56	A	9	8	0.85	20	0.400
57	A	10	9	0.89	23	0.391
58	A	11	10	0.85	23	0.435
59	A	8	7	1.14	21	0.333
60	A	4	3	1.15	21	0.143
61	A	5	4	1.15	18	0.222
62	A	2	2	1.00	27	0.074
63	A	2	2	1.00	27	0.074
64	A	2	2	1.12	27	0.074
65	A	2	2	1.11	27	0.074
66	A	6	6	0.96	27	0.222
67	A	5	5	1.00	27	0.185
68	A	4	4	1.03	24	0.167
69	A	4	4	1.03	27	0.148
70	A	5	5	1.09	27	0.185
71	A	6	6	1.07	27	0.222
72	A	6	5	1.04	25	0.200
73	A	4	3	1.00	25	0.120
74	A	4	3	1.01	25	0.120
75	A	6	5	1.00	25	0.200
76	A	8	7	0.99	25	0.280
77	A	2	2	1.00	25	0.080
78	A	2	2	1.00	23	0.087
79	A	2	2	1.00	25	0.080
80	A	2	2	1.00	25	0.080
81	A	2	2	1.00	25	0.080
82	A	2	2	1.00	25	0.080
83	A	2	2	1.00	22	0.091
84	A	2	2	1.00	25	0.080
85	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	9	8	1.02	27	0.296
87	A	6	5	1.00	27	0.185
88	A	4	3	1.00	27	0.111
89	A	4	3	1.00	27	0.111
90	A	4	3	0.99	27	0.111
91	N/A	1	0	1.00	27	0.000
92	N/A	1	0	1.00	25	0.000
93	N/A	1	0	1.00	27	0.000
94	N/A	1	0	1.00	27	0.000
95	N/A	1	0	1.00	27	0.000
96	N/A	1	0	1.00	27	0.000
97	N/A	1	0	1.00	24	0.000
98	N/A	1	0	1.00	27	0.000
99	N/A	1	0	1.00	27	0.000
100	A	10	9	0.97	27	0.333
101	A	8	7	0.95	27	0.259
102	A	4	3	0.99	27	0.111
103	A	4	3	0.99	27	0.111
104	A	4	3	0.99	27	0.111
105	N/A	1	0	1.00	27	0.000
106	N/A	1	0	1.00	25	0.000
107	N/A	1	0	1.00	27	0.000
108	N/A	1	0	1.00	27	0.000
109	N/A	1	0	1.00	27	0.000
110	N/A	1	0	1.00	27	0.000
111	N/A	1	0	1.00	24	0.000
112	N/A	1	0	1.00	27	0.000
113	N/A	1	0	1.00	27	0.000

CHAPTER 3

Listing of integrals

3.1 $\int \frac{x^8(d+ex^3)}{a-cx^6} dx$

Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{x^8(d+ex^3)}{a-cx^6} dx = -\frac{dx^3}{3c} - \frac{ex^6}{6c} + \frac{\sqrt{a}d\text{arctanh}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3c^{3/2}} - \frac{ae\log(a-cx^6)}{6c^2}$$

output
```
-1/3*d*x^3/c-1/6*e*x^6/c+1/3*a^(1/2)*d*arctanh(c^(1/2)*x^3/a^(1/2))/c^(3/2
)-1/6*a*e*ln(-c*x^6+a)/c^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{x^8(d+ex^3)}{a-cx^6} dx = -\frac{2cdx^3 + cex^6 - 2\sqrt{a}\sqrt{c}d\text{arctanh}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right) + ae\log(a-cx^6)}{6c^2}$$

input
```
Integrate[(x^8*(d + e*x^3))/(a - c*x^6),x]
```

output
```
-1/6*(2*c*d*x^3 + c*e*x^6 - 2*Sqrt[a]*Sqrt[c]*d*ArcTanh[(Sqrt[c]*x^3)/Sqrt
[a]] + a*e*Log[a - c*x^6])/c^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1803, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(d+ex^3)}{a-cx^6}dx$$

$$\Big\downarrow \text{ 1803}$$

$$\frac{1}{3}\int \frac{x^6(ex^3+d)}{a-cx^6}dx^3$$

$$\Big\downarrow \text{ 523}$$

$$\frac{1}{3}\int \left(-\frac{ex^3}{c}-\frac{d}{c}+\frac{aex^3+ad}{c(a-cx^6)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{\sqrt{a}d\text{arctanh}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{c^{3/2}}-\frac{ae\log\left(a-cx^6\right)}{2c^2}-\frac{dx^3}{c}-\frac{ex^6}{2c}\right)$$

input
```
Int[(x^8*(d + e*x^3))/(a - c*x^6),x]
```

output
```
(-((d*x^3)/c) - (e*x^6)/(2*c) + (Sqrt[a]*d*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a]])
/c^(3/2) - (a*e*Log[a - c*x^6])/(2*c^2))/3
```

Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\frac{1}{2}e\,x^{6}+d\,x^{3}}{3c}-\frac{a\left(\frac{e\ln\left(-c\,x^{6}+a\right)}{2c}-\frac{d\,\operatorname{arctanh}\left(\frac{c\,x^{3}}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{3c}$	60
risch	$-\frac{e\,x^{6}}{6c}-\frac{d\,x^{3}}{3c}-\frac{d^{2}}{6ec}+\frac{\ln\left(c\,x^{3}+\sqrt{ac}\right)d\sqrt{ac}}{6c^{2}}-\frac{\ln\left(c\,x^{3}+\sqrt{ac}\right)ae}{6c^{2}}-\frac{\ln\left(c\,x^{3}-\sqrt{ac}\right)d\sqrt{ac}}{6c^{2}}-\frac{\ln\left(c\,x^{3}-\sqrt{ac}\right)ae}{6c^{2}}$	119

input
```
int(x^8*(e*x^3+d)/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/c*(1/2*e*x^6+d*x^3)-1/3*a/c*(1/2*e*ln(-c*x^6+a)/c-d/(a*c)^(1/2)*arcta
nh(c*x^3/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.94

$$\int \frac{x^8(d+ex^3)}{a-cx^6}\,dx = \left[-\frac{cex^6+2\,cdx^3-cd\sqrt{\frac{a}{c}}\log\left(\frac{cx^6+2\,cx^3\sqrt{\frac{a}{c}}+a}{cx^6-a}\right)+ae\log\left(cx^6-a\right)}{6\,c^2},\right.$$
$$\left.-\frac{cex^6+2\,cdx^3+2\,cd\sqrt{-\frac{a}{c}}\arctan\left(\frac{cx^3\sqrt{-\frac{a}{c}}}{a}\right)+ae\log\left(cx^6-a\right)}{6\,c^2}\right]$$

input
```
integrate(x^8*(e*x^3+d)/(-c*x^6+a),x, algorithm="fricas")
```

output
```
[-1/6*(c*e*x^6 + 2*c*d*x^3 - c*d*sqrt(a/c)*log((c*x^6 + 2*c*x^3*sqrt(a/c)
+ a)/(c*x^6 - a)) + a*e*log(c*x^6 - a))/c^2, -1/6*(c*e*x^6 + 2*c*d*x^3 + 2
*c*d*sqrt(-a/c)*arctan(c*x^3*sqrt(-a/c)/a) + a*e*log(c*x^6 - a))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(61) = 122.

Time = 0.52 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.06

$$\int \frac{x^8(d+ex^3)}{a-cx^6}\,dx = -\left(\frac{ae}{6c^2}-\frac{d\sqrt{ac^5}}{6c^4}\right)\log\left(x^3+\frac{ae-6c^2\left(\frac{ae}{6c^2}-\frac{d\sqrt{ac^5}}{6c^4}\right)}{cd}\right) - \left(\frac{ae}{6c^2}+\frac{d\sqrt{ac^5}}{6c^4}\right)\log\left(x^3+\frac{ae-6c^2\left(\frac{ae}{6c^2}+\frac{d\sqrt{ac^5}}{6c^4}\right)}{cd}\right) - \frac{dx^3}{3c} - \frac{ex^6}{6c}$$

input
```
integrate(x**8*(e*x**3+d)/(-c*x**6+a),x)
```

output
```
-(a*e/(6*c**2) - d*sqrt(a*c**5)/(6*c**4))*log(x**3 + (a*e - 6*c**2*(a*e/(6
*c**2) - d*sqrt(a*c**5)/(6*c**4)))/(c*d)) - (a*e/(6*c**2) + d*sqrt(a*c**5)
/(6*c**4))*log(x**3 + (a*e - 6*c**2*(a*e/(6*c**2) + d*sqrt(a*c**5)/(6*c**4
)))/(c*d)) - d*x**3/(3*c) - e*x**6/(6*c)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{x^8(d+ex^3)}{a-cx^6}\,dx = -\frac{ad\log\left(\frac{cx^3-\sqrt{ac}}{cx^3+\sqrt{ac}}\right)}{6\sqrt{ac}c} - \frac{ae\log\left(cx^6-a\right)}{6\,c^2} - \frac{ex^6+2\,dx^3}{6\,c}$$

input
```
integrate(x^8*(e*x^3+d)/(-c*x^6+a),x, algorithm="maxima")
```

output
```
-1/6*a*d*log((c*x^3 - sqrt(a*c))/(c*x^3 + sqrt(a*c)))/(sqrt(a*c)*c) - 1/6*
a*e*log(c*x^6 - a)/c^2 - 1/6*(e*x^6 + 2*d*x^3)/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{x^8(d+ex^3)}{a-cx^6}\,dx = -\frac{ad\arctan\left(\frac{cx^3}{\sqrt{-ac}}\right)}{3\sqrt{-ac}c} - \frac{ae\log\left(cx^6-a\right)}{6\,c^2} - \frac{cex^6+2\,cdx^3}{6\,c^2}$$

input
```
integrate(x^8*(e*x^3+d)/(-c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*a*d*arctan(c*x^3/sqrt(-a*c))/(sqrt(-a*c)*c) - 1/6*a*e*log(c*x^6 - a)/
c^2 - 1/6*(c*e*x^6 + 2*c*d*x^3)/c^2
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.34

$$\int \frac{x^8(d+ex^3)}{a-cx^6}\,dx = \frac{\ln\left(\sqrt{a\,c^5}+c^3\,x^3\right)\,\left(d\,\sqrt{a\,c^5}-a\,c^2\,e\right)}{6\,c^4} - \frac{e\,x^6}{6\,c} - \frac{d\,x^3}{3\,c} - \frac{\ln\left(\sqrt{a\,c^5}-c^3\,x^3\right)\,\left(d\,\sqrt{a\,c^5}+a\,c^2\,e\right)}{6\,c^4}$$

input
```
int((x^8*(d + e*x^3))/(a - c*x^6),x)
```

output
```
(log((a*c^5)^(1/2) + c^3*x^3)*(d*(a*c^5)^(1/2) - a*c^2*e))/(6*c^4) - (e*x^
6)/(6*c) - (d*x^3)/(3*c) - (log((a*c^5)^(1/2) - c^3*x^3)*(d*(a*c^5)^(1/2)
+ a*c^2*e))/(6*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.48

$$\int \frac{x^8(d+ex^3)}{a-cx^6}\,dx$$
$$= \frac{c^{\frac{7}{6}}a^{\frac{7}{6}}\log\left(-c^{\frac{1}{6}}a^{\frac{1}{6}}x+a^{\frac{1}{3}}+c^{\frac{1}{3}}x^2\right)d+c^{\frac{7}{6}}a^{\frac{7}{6}}\log\left(-c^{\frac{1}{6}}a^{\frac{1}{6}}-c^{\frac{1}{3}}x\right)d-c^{\frac{7}{6}}a^{\frac{7}{6}}\log\left(c^{\frac{1}{6}}a^{\frac{1}{6}}x+a^{\frac{1}{3}}+c^{\frac{1}{3}}x^2\right)d-c^{\frac{7}{6}}a}{}$$

input
```
int(x^8*(e*x^3+d)/(-c*x^6+a),x)
```

output
```
(c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*
a*c*d + c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*a*c*d - c
**(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*a*c*d
 - c**(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*a*c*d - c**(2/3)*
a**(2/3)*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*a*e - c**(
2/3)*a**(2/3)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*a*e - c**(2/3)*a**(2/
3)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*a*e - c**(2/3)*a**(
2/3)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*a*e - 2*c**(2/3)*a**(2/3)*c*d*x**
3 - c**(2/3)*a**(2/3)*c*e*x**6)/(6*c**(2/3)*a**(2/3)*c**2)
```

3.2 $\int \frac{x^5(d+ex^3)}{a-cx^6} dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{x^5(d+ex^3)}{a-cx^6} dx = -\frac{ex^3}{3c} + \frac{\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3c^{3/2}} - \frac{d\log(a-cx^6)}{6c}$$

output
```
-1/3*e*x^3/c+1/3*a^(1/2)*e*arctanh(c^(1/2)*x^3/a^(1/2))/c^(3/2)-1/6*d*ln(-
c*x^6+a)/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{x^5(d+ex^3)}{a-cx^6} dx = \frac{\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3c^{3/2}} - \frac{2ex^3+d\log(a-cx^6)}{6c}$$

input
```
Integrate[(x^5*(d + e*x^3))/(a - c*x^6),x]
```

output
```
(Sqrt[a]*e*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a]])/(3*c^(3/2)) - (2*e*x^3 + d*Log[
a - c*x^6])/(6*c)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1803, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(d+e x^3\right)}{a-c x^6} \, dx$$

$$\Big\downarrow \text{1803}$$

$$\frac{1}{3} \int \frac{x^3 \left(e x^3+d\right)}{a-c x^6}dx^3$$

$$\Big\downarrow \text{523}$$

$$\frac{1}{3} \int \left(\frac{c d x^3+a e}{c \left(a-c x^6\right)}-\frac{e}{c}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{\sqrt{a} e \text{arctanh}\left(\frac{\sqrt{c} x^3}{\sqrt{a}}\right)}{c^{3/2}}-\frac{d \log \left(a-c x^6\right)}{2 c}-\frac{e x^3}{c}\right)$$

input
```
Int[(x^5*(d + e*x^3))/(a - c*x^6),x]
```

output
```
(-((e*x^3)/c) + (Sqrt[a]*e*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a]])/c^(3/2) - (d*Lo
g[a - c*x^6])/(2*c))/3
```

Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{e\,x^{3}}{3c}+\frac{-\frac{d\ln\left(-c\,x^{6}+a\right)}{2}+\frac{ae\,\operatorname{arctanh}\left(\frac{c\,x^{3}}{\sqrt{ac}}\right)}{\sqrt{ac}}}{3c}$	48
risch	$-\frac{e\,x^{3}}{3c}-\frac{\ln\left(c\,x^{3}-\sqrt{ac}\right)e\sqrt{ac}}{6c^{2}}-\frac{\ln\left(c\,x^{3}-\sqrt{ac}\right)d}{6c}+\frac{\ln\left(c\,x^{3}+\sqrt{ac}\right)e\sqrt{ac}}{6c^{2}}-\frac{\ln\left(c\,x^{3}+\sqrt{ac}\right)d}{6c}$	97

input
```
int(x^5*(e*x^3+d)/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*e*x^3/c+1/3/c*(-1/2*d*ln(-c*x^6+a)+a*e/(a*c)^(1/2)*arctanh(c*x^3/(a*c
)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.03

$$\int \frac{x^5(d+ex^3)}{a-cx^6}\,dx = \left[-\frac{2\,ex^3-e\sqrt{\frac{a}{c}}\log\left(\frac{cx^6+2\,cx^3\sqrt{\frac{a}{c}}+a}{cx^6-a}\right)+d\log\left(cx^6-a\right)}{6\,c},\ -\frac{2\,ex^3+2\,e\sqrt{-\frac{a}{c}}\arctan\left(\frac{cx^3\sqrt{-\frac{a}{c}}}{a}\right)+d\log\left(cx^6-a\right)}{6\,c}\right]$$

input
```
integrate(x^5*(e*x^3+d)/(-c*x^6+a),x, algorithm="fricas")
```

output
```
[-1/6*(2*e*x^3 - e*sqrt(a/c)*log((c*x^6 + 2*c*x^3*sqrt(a/c) + a)/(c*x^6 -
a)) + d*log(c*x^6 - a))/c, -1/6*(2*e*x^3 + 2*e*sqrt(-a/c)*arctan(c*x^3*sqr
t(-a/c)/a) + d*log(c*x^6 - a))/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(49) = 98$.

Time = 0.49 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.93

$$\int \frac{x^5(d+ex^3)}{a-cx^6}\,dx = -\left(\frac{d}{6c}-\frac{e\sqrt{ac^3}}{6c^3}\right)\log\left(x^3+\frac{-6c\left(\frac{d}{6c}-\frac{e\sqrt{ac^3}}{6c^3}\right)+d}{e}\right) - \left(\frac{d}{6c}+\frac{e\sqrt{ac^3}}{6c^3}\right)\log\left(x^3+\frac{-6c\left(\frac{d}{6c}+\frac{e\sqrt{ac^3}}{6c^3}\right)+d}{e}\right) - \frac{ex^3}{3c}$$

input
```
integrate(x**5*(e*x**3+d)/(-c*x**6+a),x)
```

output
```
-(d/(6*c) - e*sqrt(a*c**3)/(6*c**3))*log(x**3 + (-6*c*(d/(6*c) - e*sqrt(a*
c**3)/(6*c**3)) + d)/e) - (d/(6*c) + e*sqrt(a*c**3)/(6*c**3))*log(x**3 + (
-6*c*(d/(6*c) + e*sqrt(a*c**3)/(6*c**3)) + d)/e) - e*x**3/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{x^5(d+ex^3)}{a-cx^6}\,dx = -\frac{ex^3}{3\,c}-\frac{ae\log\left(\frac{cx^3-\sqrt{ac}}{cx^3+\sqrt{ac}}\right)}{6\,\sqrt{ac}c}-\frac{d\log\left(cx^6-a\right)}{6\,c}$$

input
```
integrate(x^5*(e*x^3+d)/(-c*x^6+a),x, algorithm="maxima")
```

output
```
-1/3*e*x^3/c - 1/6*a*e*log((c*x^3 - sqrt(a*c))/(c*x^3 + sqrt(a*c)))/(sqrt(
a*c)*c) - 1/6*d*log(c*x^6 - a)/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{x^5(d+ex^3)}{a-cx^6}\,dx = -\frac{ex^3}{3\,c} - \frac{ae\arctan\left(\frac{cx^3}{\sqrt{-ac}}\right)}{3\sqrt{-ac}c} - \frac{d\log\left(cx^6-a\right)}{6\,c}$$

input
```
integrate(x^5*(e*x^3+d)/(-c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*e*x^3/c - 1/3*a*e*arctan(c*x^3/sqrt(-a*c))/(sqrt(-a*c)*c) - 1/6*d*log
(c*x^6 - a)/c
```

Mupad [B] (verification not implemented)

Time = 21.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.93

$$\int \frac{x^5(d+ex^3)}{a-cx^6}\,dx = \frac{e\,\ln\left(x^3\,\sqrt{a\,c^3}+a\,c\right)\,\sqrt{a\,c^3}}{6\,c^3} - \frac{d\,\ln\left(x^3\,\sqrt{a\,c^3}+a\,c\right)}{6\,c} - \frac{d\,\ln\left(a\,c-x^3\,\sqrt{a\,c^3}\right)}{6\,c} - \frac{e\,x^3}{3\,c} - \frac{e\,\ln\left(a\,c-x^3\,\sqrt{a\,c^3}\right)\,\sqrt{a\,c^3}}{6\,c^3}$$

input
```
int((x^5*(d + e*x^3))/(a - c*x^6),x)
```

output
```
(e*log(x^3*(a*c^3)^(1/2) + a*c)*(a*c^3)^(1/2))/(6*c^3) - (d*log(x^3*(a*c^3
)^(1/2) + a*c))/(6*c) - (d*log(a*c - x^3*(a*c^3)^(1/2)))/(6*c) - (e*x^3)/(
3*c) - (e*log(a*c - x^3*(a*c^3)^(1/2))*(a*c^3)^(1/2))/(6*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.98

$$\int \frac{x^5(d+ex^3)}{a-cx^6}\,dx$$
$$= \frac{c^{\frac{1}{6}}a^{\frac{7}{6}}\log\left(-c^{\frac{1}{6}}a^{\frac{1}{6}}x+a^{\frac{1}{3}}+c^{\frac{1}{3}}x^2\right)e+c^{\frac{1}{6}}a^{\frac{7}{6}}\log\left(-c^{\frac{1}{6}}a^{\frac{1}{6}}-c^{\frac{1}{3}}x\right)e-c^{\frac{1}{6}}a^{\frac{7}{6}}\log\left(c^{\frac{1}{6}}a^{\frac{1}{6}}x+a^{\frac{1}{3}}+c^{\frac{1}{3}}x^2\right)e-c^{\frac{1}{6}}a}{}$$

input
```
int(x^5*(e*x^3+d)/(-c*x^6+a),x)
```

output
```
(c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*
a*e + c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*a*e - c**(1
/6)*a**(1/6)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*a*e - c**
(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*a*e - c**(2/3)*a**(2/3)
*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*d - c**(2/3)*a**(2
/3)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*d - c**(2/3)*a**(2/3)*log(c**(1
/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*d - c**(2/3)*a**(2/3)*log(c**(1
/6)*a**(1/6) - c**(1/3)*x)*d - 2*c**(2/3)*a**(2/3)*e*x**3)/(6*c**(2/3)*a**
(2/3)*c)
```

3.3 $\int \frac{x^2(d+ex^3)}{a-cx^6} dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \frac{x^2(d+ex^3)}{a-cx^6} dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{c}} - \frac{e\log(a-cx^6)}{6c}$$

output
```
1/3*d*arctanh(c^(1/2)*x^3/a^(1/2))/a^(1/2)/c^(1/2)-1/6*e*ln(-c*x^6+a)/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{x^2(d+ex^3)}{a-cx^6} dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{c}} - \frac{e\log(a-cx^6)}{6c}$$

input
```
Integrate[(x^2*(d + e*x^3))/(a - c*x^6),x]
```

output
```
(d*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a]])/(3*Sqrt[a]*Sqrt[c]) - (e*Log[a - c*x^6]
)/(6*c)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1799, 452, 221, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (d+e x^3)}{a-c x^6} \, dx$$

$$\Big\downarrow \text{1799}$$

$$\frac{1}{3} \int \frac{e x^3+d}{a-c x^6}dx^3$$

$$\Big\downarrow \text{452}$$

$$\frac{1}{3} \left(d \int \frac{1}{a-c x^6}dx^3+e \int \frac{x^3}{a-c x^6}dx^3\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3} \left(e \int \frac{x^3}{a-c x^6}dx^3+\frac{d \text{arctanh}\left(\frac{\sqrt{c} x^3}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{c}}\right)$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{3} \left(\frac{d \text{arctanh}\left(\frac{\sqrt{c} x^3}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{c}}-\frac{e \log \left(a-c x^6\right)}{2 c}\right)$$

input
```
Int[(x^2*(d + e*x^3))/(a - c*x^6),x]
```

output
```
((d*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - (e*Log[a - c*x^6])
/(2*c))/3
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{e\ln\left(-c\,x^{6}+a\right)}{6c}+\frac{d\,\operatorname{arctanh}\left(\frac{c\,x^{3}}{\sqrt{ac}}\right)}{3\sqrt{ac}}$	36
risch	$\frac{\ln\left(c\,x^{3}+\sqrt{ac}\right)d\sqrt{ac}}{6ac}-\frac{\ln\left(c\,x^{3}+\sqrt{ac}\right)e}{6c}-\frac{\ln\left(c\,x^{3}-\sqrt{ac}\right)d\sqrt{ac}}{6ac}-\frac{\ln\left(c\,x^{3}-\sqrt{ac}\right)e}{6c}$	94

input
```
int(x^2*(e*x^3+d)/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*e*ln(-c*x^6+a)/c+1/3*d/(a*c)^(1/2)*arctanh(c*x^3/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.21

$$\int \frac{x^2(d+ex^3)}{a-cx^6}\,dx = \left[-\frac{ae\log\left(cx^6-a\right)-\sqrt{ac}d\log\left(\frac{cx^6+2\sqrt{ac}x^3+a}{cx^6-a}\right)}{6\,ac},\; -\frac{ae\log\left(cx^6-a\right)+2\sqrt{-ac}d\arctan\left(\frac{\sqrt{-ac}x^3}{a}\right)}{6\,ac}\right]$$

input
```
integrate(x^2*(e*x^3+d)/(-c*x^6+a),x, algorithm="fricas")
```

output
```
[-1/6*(a*e*log(c*x^6 - a) - sqrt(a*c)*d*log((c*x^6 + 2*sqrt(a*c)*x^3 + a)/
(c*x^6 - a)))/(a*c), -1/6*(a*e*log(c*x^6 - a) + 2*sqrt(-a*c)*d*arctan(sqrt
(-a*c)*x^3/a))/(a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(41) = 82$.

Time = 0.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int \frac{x^2(d+ex^3)}{a-cx^6}\,dx = -\left(\frac{e}{6c}-\frac{d\sqrt{ac^3}}{6ac^2}\right)\log\left(x^3+\frac{-6ac\left(\frac{e}{6c}-\frac{d\sqrt{ac^3}}{6ac^2}\right)+ae}{cd}\right) - \left(\frac{e}{6c}+\frac{d\sqrt{ac^3}}{6ac^2}\right)\log\left(x^3+\frac{-6ac\left(\frac{e}{6c}+\frac{d\sqrt{ac^3}}{6ac^2}\right)+ae}{cd}\right)$$

input
```
integrate(x**2*(e*x**3+d)/(-c*x**6+a),x)
```

output
```
-(e/(6*c) - d*sqrt(a*c**3)/(6*a*c**2))*log(x**3 + (-6*a*c*(e/(6*c) - d*sqr
t(a*c**3)/(6*a*c**2)) + a*e)/(c*d)) - (e/(6*c) + d*sqrt(a*c**3)/(6*a*c**2)
)*log(x**3 + (-6*a*c*(e/(6*c) + d*sqrt(a*c**3)/(6*a*c**2)) + a*e)/(c*d))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{x^2(d+ex^3)}{a-cx^6}\,dx = -\frac{d\log\left(\frac{cx^3-\sqrt{ac}}{cx^3+\sqrt{ac}}\right)}{6\sqrt{ac}} - \frac{e\log\left(cx^6-a\right)}{6\,c}$$

input
```
integrate(x^2*(e*x^3+d)/(-c*x^6+a),x, algorithm="maxima")
```

output
```
-1/6*d*log((c*x^3 - sqrt(a*c))/(c*x^3 + sqrt(a*c)))/sqrt(a*c) - 1/6*e*log(
c*x^6 - a)/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{x^2(d+ex^3)}{a-cx^6}\,dx = -\frac{d\arctan\left(\frac{cx^3}{\sqrt{-ac}}\right)}{3\sqrt{-ac}} - \frac{e\log\left(cx^6-a\right)}{6\,c}$$

input
```
integrate(x^2*(e*x^3+d)/(-c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*d*arctan(c*x^3/sqrt(-a*c))/sqrt(-a*c) - 1/6*e*log(c*x^6 - a)/c
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.31

$$\int \frac{x^2(d+ex^3)}{a-cx^6}\,dx = \frac{d\,\ln\left(x^3\,\sqrt{a\,c^3}+a\,c\right)\,\sqrt{a\,c^3}}{6\,a\,c^2} - \frac{e\,\ln\left(a\,c-x^3\,\sqrt{a\,c^3}\right)}{6\,c} - \frac{e\,\ln\left(x^3\,\sqrt{a\,c^3}+a\,c\right)}{6\,c} - \frac{d\,\ln\left(a\,c-x^3\,\sqrt{a\,c^3}\right)\,\sqrt{a\,c^3}}{6\,a\,c^2}$$

input
```
int((x^2*(d + e*x^3))/(a - c*x^6),x)
```

output
```
(d*log(x^3*(a*c^3)^(1/2) + a*c)*(a*c^3)^(1/2))/(6*a*c^2) - (e*log(a*c - x^
3*(a*c^3)^(1/2)))/(6*c) - (e*log(x^3*(a*c^3)^(1/2) + a*c))/(6*c) - (d*log(
a*c - x^3*(a*c^3)^(1/2))*(a*c^3)^(1/2))/(6*a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 223, normalized size of antiderivative = 4.65

$$\int \frac{x^2(d+ex^3)}{a-cx^6}\,dx = \frac{c^{\frac{7}{6}}a^{\frac{1}{6}}\log\left(-c^{\frac{1}{6}}a^{\frac{1}{6}}x+a^{\frac{1}{3}}+c^{\frac{1}{3}}x^2\right)d+c^{\frac{7}{6}}a^{\frac{1}{6}}\log\left(-c^{\frac{1}{6}}a^{\frac{1}{6}}-c^{\frac{1}{3}}x\right)d-c^{\frac{7}{6}}a^{\frac{1}{6}}\log\left(c^{\frac{1}{6}}a^{\frac{1}{6}}x+a^{\frac{1}{3}}+c^{\frac{1}{3}}x^2\right)d-c^{\frac{7}{6}}a}{}$$

input
```
int(x^2*(e*x^3+d)/(-c*x^6+a),x)
```

output
```
(c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*
c*d + c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*c*d - c**(1
/6)*a**(1/6)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*c*d - c**
(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*c*d - c**(2/3)*a**(2/3)
*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*e - c**(2/3)*a**(2
/3)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*e - c**(2/3)*a**(2/3)*log(c**(1
/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*e - c**(2/3)*a**(2/3)*log(c**(1
/6)*a**(1/6) - c**(1/3)*x)*e)/(6*c**(2/3)*a**(2/3)*c)
```

3.4 $\int \frac{d+ex^3}{x(a-cx^6)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{d+ex^3}{x\left(a-cx^6\right)}\,dx=\frac{e\operatorname{arctanh}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{c}}+\frac{d\log(x)}{a}-\frac{d\log\left(a-cx^6\right)}{6a}$$

output
```
1/3*e*arctanh(c^(1/2)*x^3/a^(1/2))/a^(1/2)/c^(1/2)+d*ln(x)/a-1/6*d*ln(-c*x
^6+a)/a
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 156 vs. 2(55) = 110.

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.84

$$\int \frac{d+ex^3}{x\left(a-cx^6\right)}\,dx$$
$$=\frac{6\sqrt{c}d\log(x)-\sqrt{a}e\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)+\sqrt{a}e\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)+\sqrt{a}e\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)-\sqrt{a}e\ldots}{6a\sqrt{c}}$$

input
```
Integrate[(d + e*x^3)/(x*(a - c*x^6)),x]
```

output
```
(6*Sqrt[c]*d*Log[x] - Sqrt[a]*e*Log[a^(1/6) - c^(1/6)*x] + Sqrt[a]*e*Log[a
^(1/6) + c^(1/6)*x] + Sqrt[a]*e*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*
x^2] - Sqrt[a]*e*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] - Sqrt[c]*
d*Log[a - c*x^6])/(6*a*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1803, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x(a-cx^6)}dx$$

$$\Big\downarrow \text{1803}$$

$$\frac{1}{3}\int \frac{ex^3+d}{x^3(a-cx^6)}dx^3$$

$$\Big\downarrow \text{523}$$

$$\frac{1}{3}\int \left(\frac{d}{ax^3}+\frac{cdx^3+ae}{a(a-cx^6)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{e\,\text{arctanh}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}-\frac{d\log\left(a-cx^6\right)}{2a}+\frac{d\log\left(x^3\right)}{a}\right)$$

input
```
Int[(d + e*x^3)/(x*(a - c*x^6)),x]
```

output
```
((e*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + (d*Log[x^3])/a - (
d*Log[a - c*x^6])/(2*a))/3
```

Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\frac{d\ln\left(-c\,x^{6}+a\right)}{2}-\frac{ae\ \operatorname{arctanh}\left(\frac{c\,x^{3}}{\sqrt{ac}}\right)}{\sqrt{ac}}}{3a}+\frac{d\ln\left(x\right)}{a}$	47
risch	$\frac{d\ln\left(x\right)}{a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{2}c\_Z^{2}+2acd\_Z-a\,e^{2}+c\,d^{2}\right)}\_R\ln\left(\left(-7ac\_R^{2}-7cd\_R+6e^{2}\right)x^{3}-ae\_R+6de\right)\right)}{6}$	76

input
```
int((e*x^3+d)/x/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a*(1/2*d*ln(-c*x^6+a)-a*e/(a*c)^(1/2)*arctanh(c*x^3/(a*c)^(1/2)))+d*l
n(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.15

$$\int \frac{d+ex^3}{x(a-cx^6)}\,dx = \left[-\frac{cd\log\left(cx^6-a\right)-6\,cd\log\left(x\right)-\sqrt{ac}e\log\left(\frac{cx^6+2\,\sqrt{ac}x^3+a}{cx^6-a}\right)}{6\,ac},\right.$$
$$\left.-\frac{cd\log\left(cx^6-a\right)-6\,cd\log\left(x\right)+2\,\sqrt{-ac}e\arctan\left(\frac{\sqrt{-ac}x^3}{a}\right)}{6\,ac}\right]$$

input
```
integrate((e*x^3+d)/x/(-c*x^6+a),x, algorithm="fricas")
```

output
```
[-1/6*(c*d*log(c*x^6 - a) - 6*c*d*log(x) - sqrt(a*c)*e*log((c*x^6 + 2*sqrt
(a*c)*x^3 + a)/(c*x^6 - a)))/(a*c), -1/6*(c*d*log(c*x^6 - a) - 6*c*d*log(x
) + 2*sqrt(-a*c)*e*arctan(sqrt(-a*c)*x^3/a))/(a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(48) = 96.

Time = 1.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.13

$$\int \frac{d+ex^3}{x(a-cx^6)}\,dx = -\left(\frac{d}{6a}-\frac{e\sqrt{a^3c}}{6a^2c}\right)\log\left(x^3+\frac{-6a\left(\frac{d}{6a}-\frac{e\sqrt{a^3c}}{6a^2c}\right)+d}{e}\right)$$
$$-\left(\frac{d}{6a}+\frac{e\sqrt{a^3c}}{6a^2c}\right)\log\left(x^3+\frac{-6a\left(\frac{d}{6a}+\frac{e\sqrt{a^3c}}{6a^2c}\right)+d}{e}\right)+\frac{d\log\left(x\right)}{a}$$

input
```
integrate((e*x**3+d)/x/(-c*x**6+a),x)
```

output
```
-(d/(6*a) - e*sqrt(a**3*c)/(6*a**2*c))*log(x**3 + (-6*a*(d/(6*a) - e*sqrt(
a**3*c)/(6*a**2*c)) + d)/e) - (d/(6*a) + e*sqrt(a**3*c)/(6*a**2*c))*log(x*
*3 + (-6*a*(d/(6*a) + e*sqrt(a**3*c)/(6*a**2*c)) + d)/e) + d*log(x)/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{d+ex^3}{x\left(a-cx^6\right)}\,dx = -\frac{e\log\left(\frac{cx^3-\sqrt{ac}}{cx^3+\sqrt{ac}}\right)}{6\sqrt{ac}} - \frac{d\log\left(cx^6-a\right)}{6\,a} + \frac{d\log\left(x^3\right)}{3\,a}$$

input
```
integrate((e*x^3+d)/x/(-c*x^6+a),x, algorithm="maxima")
```

output
```
-1/6*e*log((c*x^3 - sqrt(a*c))/(c*x^3 + sqrt(a*c)))/sqrt(a*c) - 1/6*d*log(
c*x^6 - a)/a + 1/3*d*log(x^3)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{d+ex^3}{x\left(a-cx^6\right)}\,dx = -\frac{e\arctan\left(\frac{cx^3}{\sqrt{-ac}}\right)}{3\sqrt{-ac}} - \frac{d\log\left(cx^6-a\right)}{6\,a} + \frac{d\log\left(|x|\right)}{a}$$

input
```
integrate((e*x^3+d)/x/(-c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*e*arctan(c*x^3/sqrt(-a*c))/sqrt(-a*c) - 1/6*d*log(c*x^6 - a)/a + d*lo
g(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 21.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 4.11

$$\int \frac{d+ex^3}{x\left(a-cx^6\right)}\,dx = \frac{d\ln(x)}{a} - \frac{d\ln\left(a\,e\,\sqrt{a^3\,c}+7\,a^2\,c\,d-a^2\,c\,e\,x^3-7\,c\,d\,x^3\,\sqrt{a^3\,c}\right)}{6\,a} - \frac{d\ln\left(7\,a^2\,c\,d-a\,e\,\sqrt{a^3\,c}-a^2\,c\,e\,x^3+7\,c\,d\,x^3\,\sqrt{a^3\,c}\right)}{6\,a} - \frac{e\ln\left(a\,e\,\sqrt{a^3\,c}+7\,a^2\,c\,d-a^2\,c\,e\,x^3-7\,c\,d\,x^3\,\sqrt{a^3\,c}\right)\,\sqrt{a^3\,c}}{6\,a^2\,c} + \frac{e\ln\left(7\,a^2\,c\,d-a\,e\,\sqrt{a^3\,c}-a^2\,c\,e\,x^3+7\,c\,d\,x^3\,\sqrt{a^3\,c}\right)\,\sqrt{a^3\,c}}{6\,a^2\,c}$$

input
```
int((d + e*x^3)/(x*(a - c*x^6)),x)
```

output
```
(d*log(x))/a - (d*log(a*e*(a^3*c)^(1/2) + 7*a^2*c*d - a^2*c*e*x^3 - 7*c*d*
x^3*(a^3*c)^(1/2)))/(6*a) - (d*log(7*a^2*c*d - a*e*(a^3*c)^(1/2) - a^2*c*e
*x^3 + 7*c*d*x^3*(a^3*c)^(1/2)))/(6*a) - (e*log(a*e*(a^3*c)^(1/2) + 7*a^2*
c*d - a^2*c*e*x^3 - 7*c*d*x^3*(a^3*c)^(1/2))*(a^3*c)^(1/2))/(6*a^2*c) + (e
*log(7*a^2*c*d - a*e*(a^3*c)^(1/2) - a^2*c*e*x^3 + 7*c*d*x^3*(a^3*c)^(1/2)
)*(a^3*c)^(1/2))/(6*a^2*c)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 234, normalized size of antiderivative = 4.25

$$\int \frac{d+ex^3}{x\left(a-cx^6\right)}\,dx = \frac{c^{\frac{1}{6}}a^{\frac{7}{6}}\log\left(-c^{\frac{1}{6}}a^{\frac{1}{6}}x+a^{\frac{1}{3}}+c^{\frac{1}{3}}x^2\right)e+c^{\frac{1}{6}}a^{\frac{7}{6}}\log\left(-c^{\frac{1}{6}}a^{\frac{1}{6}}-c^{\frac{1}{3}}x\right)e-c^{\frac{1}{6}}a^{\frac{7}{6}}\log\left(c^{\frac{1}{6}}a^{\frac{1}{6}}x+a^{\frac{1}{3}}+c^{\frac{1}{3}}x^2\right)e-c^{\frac{1}{6}}a^{\frac{7}{6}}}{}$$

input
```
int((e*x^3+d)/x/(-c*x^6+a),x)
```

output
```
(c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*
a*e + c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*a*e - c**(1
/6)*a**(1/6)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*a*e - c**
(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*a*e - c**(2/3)*a**(2/3)
*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*d - c**(2/3)*a**(2
/3)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*d - c**(2/3)*a**(2/3)*log(c**(1
/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*d - c**(2/3)*a**(2/3)*log(c**(1
/6)*a**(1/6) - c**(1/3)*x)*d + 6*c**(2/3)*a**(2/3)*log(x)*d)/(6*c**(2/3)*a
**(2/3)*a)
```

3.5 $\int \frac{d+ex^3}{x^4(a-cx^6)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{d+ex^3}{x^4\left(a-cx^6\right)}\,dx=-\frac{d}{3ax^3}+\frac{\sqrt{c}d\,\text{arctanh}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3a^{3/2}}+\frac{e\log(x)}{a}-\frac{e\log\left(a-cx^6\right)}{6a}$$

output
```
-1/3*d/a/x^3+1/3*c^(1/2)*d*arctanh(c^(1/2)*x^3/a^(1/2))/a^(3/2)+e*ln(x)/a-
1/6*e*ln(-c*x^6+a)/a
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(66) = 132.

Time = 0.03 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.76

$$\begin{aligned}\int \frac{d+ex^3}{x^4\left(a-cx^6\right)}\,dx &= -\frac{d}{3ax^3}+\frac{e\log(x)}{a}-\frac{\sqrt{c}d\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{3/2}}\\ &\quad+\frac{\sqrt{c}d\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{3/2}}+\frac{\sqrt{c}d\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{6a^{3/2}}\\ &\quad-\frac{\sqrt{c}d\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{6a^{3/2}}-\frac{e\log\left(a-cx^6\right)}{6a}\end{aligned}$$

input
```
Integrate[(d + e*x^3)/(x^4*(a - c*x^6)),x]
```

output
```
-1/3*d/(a*x^3) + (e*Log[x])/a - (Sqrt[c]*d*Log[a^(1/6) - c^(1/6)*x])/(6*a^
(3/2)) + (Sqrt[c]*d*Log[a^(1/6) + c^(1/6)*x])/(6*a^(3/2)) + (Sqrt[c]*d*Log
[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(6*a^(3/2)) - (Sqrt[c]*d*Log[
a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(6*a^(3/2)) - (e*Log[a - c*x^6
])/(6*a)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1803, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^4\left(a-cx^6\right)}dx$$

$$\Big\downarrow \; 1803$$

$$\frac{1}{3}\int \frac{ex^3+d}{x^6\left(a-cx^6\right)}dx^3$$

$$\Big\downarrow \; 523$$

$$\frac{1}{3}\int \left(\frac{d}{ax^6}+\frac{c\left(ex^3+d\right)}{a\left(a-cx^6\right)}+\frac{e}{ax^3}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(\frac{\sqrt{c}d\text{arctanh}\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{a^{3/2}}-\frac{e\log\left(a-cx^6\right)}{2a}-\frac{d}{ax^3}+\frac{e\log\left(x^3\right)}{a}\right)$$

input
```
Int[(d + e*x^3)/(x^4*(a - c*x^6)),x]
```

output
```
(-(d/(a*x^3)) + (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a]])/a^(3/2) + (e*Lo
g[x^3])/a - (e*Log[a - c*x^6])/(2*a))/3
```

Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

method	result
default	$-\frac{c\left(\frac{e\ln\left(-c\,x^{6}+a\right)}{2c}-\frac{d\,\operatorname{arctanh}\left(\frac{c\,x^{3}}{\sqrt{a c}}\right)}{\sqrt{a c}}\right)}{3a}-\frac{d}{3a\,x^{3}}+\frac{e\ln\left(x\right)}{a}$
risch	$-\frac{d}{3a\,x^{3}}+\frac{e\ln\left(x\right)}{a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{2}a^{3}+2\_Z e\,a^{2}+a\,e^{2}-c\,d^{2}\right)}\_R\ln\left(\left(-7a^{3}\_R^{2}-7a^{2}e\_R+6c\,d^{2}\right)x^{3}-a^{2}d\_R+6ade\right)\right)}{6}$

input
```
int((e*x^3+d)/x^4/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c/a*(1/2*e*ln(-c*x^6+a)/c-d/(a*c)^(1/2)*arctanh(c*x^3/(a*c)^(1/2)))-1
/3*d/a/x^3+e*ln(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.18

$$\int \frac{d+ex^3}{x^4(a-cx^6)}\,dx = \left[\frac{dx^3\sqrt{\frac{c}{a}}\log\left(\frac{cx^6+2\,ax^3\sqrt{\frac{c}{a}}+a}{cx^6-a}\right)-ex^3\log\left(cx^6-a\right)+6\,ex^3\log\left(x\right)-2\,d}{6\,ax^3},\ -\frac{2\,dx^3\sqrt{-\frac{c}{a}}\arctan\left(x^3\sqrt{-\frac{c}{a}}\right)+ex^3\log\left(cx^6-a\right)-6\,ex^3\log\left(x\right)+2\,d}{6\,ax^3}\right]$$

input
```
integrate((e*x^3+d)/x^4/(-c*x^6+a),x, algorithm="fricas")
```

output
```
[1/6*(d*x^3*sqrt(c/a)*log((c*x^6 + 2*a*x^3*sqrt(c/a) + a)/(c*x^6 - a)) - e
*x^3*log(c*x^6 - a) + 6*e*x^3*log(x) - 2*d)/(a*x^3), -1/6*(2*d*x^3*sqrt(-c
/a)*arctan(x^3*sqrt(-c/a)) + e*x^3*log(c*x^6 - a) - 6*e*x^3*log(x) + 2*d)/
(a*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(56) = 112$.

Time = 1.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.95

$$\int \frac{d+ex^3}{x^4(a-cx^6)}\,dx = -\left(\frac{e}{6a}-\frac{d\sqrt{a^3c}}{6a^3}\right)\log\left(x^3+\frac{-6a^2\left(\frac{e}{6a}-\frac{d\sqrt{a^3c}}{6a^3}\right)+ae}{cd}\right) - \left(\frac{e}{6a}+\frac{d\sqrt{a^3c}}{6a^3}\right)\log\left(x^3+\frac{-6a^2\left(\frac{e}{6a}+\frac{d\sqrt{a^3c}}{6a^3}\right)+ae}{cd}\right) - \frac{d}{3ax^3}+\frac{e\log\left(x\right)}{a}$$

input
```
integrate((e*x**3+d)/x**4/(-c*x**6+a),x)
```

output
```
-(e/(6*a) - d*sqrt(a**3*c)/(6*a**3))*log(x**3 + (-6*a**2*(e/(6*a) - d*sqrt
(a**3*c)/(6*a**3)) + a*e)/(c*d)) - (e/(6*a) + d*sqrt(a**3*c)/(6*a**3))*log
(x**3 + (-6*a**2*(e/(6*a) + d*sqrt(a**3*c)/(6*a**3)) + a*e)/(c*d)) - d/(3*
a*x**3) + e*log(x)/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{d+ex^3}{x^4(a-cx^6)}\,dx = -\frac{cd\log\left(\frac{cx^3-\sqrt{ac}}{cx^3+\sqrt{ac}}\right)}{6\sqrt{ac}a} - \frac{e\log\left(cx^6-a\right)}{6\,a} + \frac{e\log\left(x^3\right)}{3\,a} - \frac{d}{3\,ax^3}$$

input
```
integrate((e*x^3+d)/x^4/(-c*x^6+a),x, algorithm="maxima")
```

output
```
-1/6*c*d*log((c*x^3 - sqrt(a*c))/(c*x^3 + sqrt(a*c)))/(sqrt(a*c)*a) - 1/6*
e*log(c*x^6 - a)/a + 1/3*e*log(x^3)/a - 1/3*d/(a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{d+ex^3}{x^4(a-cx^6)}\,dx = -\frac{cd\arctan\left(\frac{cx^3}{\sqrt{-ac}}\right)}{3\sqrt{-ac}a} - \frac{e\log\left(cx^6-a\right)}{6\,a} + \frac{e\log\left(|x|\right)}{a} - \frac{ex^3+d}{3\,ax^3}$$

input
```
integrate((e*x^3+d)/x^4/(-c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*c*d*arctan(c*x^3/sqrt(-a*c))/(sqrt(-a*c)*a) - 1/6*e*log(c*x^6 - a)/a
+ e*log(abs(x))/a - 1/3*(e*x^3 + d)/(a*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.05

$$\int \frac{d+ex^3}{x^4(a-cx^6)}\,dx = \frac{e\ln(x)}{a} - \frac{d}{3\,a\,x^3} - \frac{\ln\left(7\,a^2\,e + d\,\sqrt{a^3\,c} - 7\,e\,x^3\,\sqrt{a^3\,c} - a\,c\,d\,x^3\right)\,\left(a^2\,e + d\,\sqrt{a^3\,c}\right)}{6\,a^3} - \frac{\ln\left(7\,a^2\,e - d\,\sqrt{a^3\,c} + 7\,e\,x^3\,\sqrt{a^3\,c} - a\,c\,d\,x^3\right)\,\left(a^2\,e - d\,\sqrt{a^3\,c}\right)}{6\,a^3}$$

input
```
int((d + e*x^3)/(x^4*(a - c*x^6)),x)
```

output
```
(e*log(x))/a - d/(3*a*x^3) - (log(7*a^2*e + d*(a^3*c)^(1/2) - 7*e*x^3*(a^3
*c)^(1/2) - a*c*d*x^3)*(a^2*e + d*(a^3*c)^(1/2)))/(6*a^3) - (log(7*a^2*e -
 d*(a^3*c)^(1/2) + 7*e*x^3*(a^3*c)^(1/2) - a*c*d*x^3)*(a^2*e - d*(a^3*c)^(
1/2)))/(6*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 273, normalized size of antiderivative = 4.14

$$\int \frac{d+ex^3}{x^4(a-cx^6)}\,dx = \frac{c^{\frac{7}{6}}a^{\frac{1}{6}}\log\left(-c^{\frac{1}{6}}a^{\frac{1}{6}}x + a^{\frac{1}{3}} + c^{\frac{1}{3}}x^2\right)d\,x^3 + c^{\frac{7}{6}}a^{\frac{1}{6}}\log\left(-c^{\frac{1}{6}}a^{\frac{1}{6}} - c^{\frac{1}{3}}x\right)d\,x^3 - c^{\frac{7}{6}}a^{\frac{1}{6}}\log\left(c^{\frac{1}{6}}a^{\frac{1}{6}}x + a^{\frac{1}{3}} + c^{\frac{1}{3}}x^2\right)d\,x}{}$$

input
```
int((e*x^3+d)/x^4/(-c*x^6+a),x)
```

output
```
(c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*
c*d*x**3 + c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*c*d*x*
*3 - c**(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)
*c*d*x**3 - c**(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*c*d*x**3
 - c**(2/3)*a**(2/3)*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2
)*e*x**3 - c**(2/3)*a**(2/3)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*e*x**3
 - c**(2/3)*a**(2/3)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*e
*x**3 - c**(2/3)*a**(2/3)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*e*x**3 + 6*c
**(2/3)*a**(2/3)*log(x)*e*x**3 - 2*c**(2/3)*a**(2/3)*d)/(6*c**(2/3)*a**(2/
3)*a*x**3)
```

3.6 $\int \frac{x^4(d+ex^3)}{a-cx^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 337

$$\begin{aligned}\int \frac{x^4(d+ex^3)}{a-cx^6}\,dx &= -\frac{ex^2}{2c}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}\sqrt[6]{a}c^{4/3}}\\
&\quad-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}\sqrt[6]{a}c^{4/3}}\\
&\quad-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6\sqrt[6]{a}c^{4/3}}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6\sqrt[6]{a}c^{4/3}}\\
&\quad-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12\sqrt[6]{a}c^{4/3}}\\
&\quad+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12\sqrt[6]{a}c^{4/3}}\end{aligned}$$

output
```
-1/2*e*x^2/c+1/6*(c^(1/2)*d-a^(1/2)*e)*arctan(1/3*(a^(1/6)-2*c^(1/6)*x)*3^
(1/2)/a^(1/6))*3^(1/2)/a^(1/6)/c^(4/3)-1/6*(c^(1/2)*d+a^(1/2)*e)*arctan(1/
3*(a^(1/6)+2*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2)/a^(1/6)/c^(4/3)-1/6*(c^(1
/2)*d+a^(1/2)*e)*ln(a^(1/6)-c^(1/6)*x)/a^(1/6)/c^(4/3)+1/6*(c^(1/2)*d-a^(1
/2)*e)*ln(a^(1/6)+c^(1/6)*x)/a^(1/6)/c^(4/3)-1/12*(c^(1/2)*d-a^(1/2)*e)*ln
(a^(1/3)-a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(1/6)/c^(4/3)+1/12*(c^(1/2)*d+a^
(1/2)*e)*ln(a^(1/3)+a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(1/6)/c^(4/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.05

$$\int \frac{x^4(d+ex^3)}{a-cx^6}\,dx$$

$$= \frac{-6\sqrt[6]{a}\sqrt[3]{c}ex^2 + 2\sqrt{3}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right) - 2\sqrt{3}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{1+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right) - 2\sqrt{c}d\,\mathrm{l}}{}$$

input
```
Integrate[(x^4*(d + e*x^3))/(a - c*x^6),x]
```

output
```
(-6*a^(1/6)*c^(1/3)*e*x^2 + 2*Sqrt[3]*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(1 -
(2*c^(1/6)*x)/a^(1/6))/Sqrt[3]] - 2*Sqrt[3]*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan
[(1 + (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]] - 2*Sqrt[c]*d*Log[a^(1/6) - c^(1/6)*
x] - 2*Sqrt[a]*e*Log[a^(1/6) - c^(1/6)*x] + 2*Sqrt[c]*d*Log[a^(1/6) + c^(1
/6)*x] - 2*Sqrt[a]*e*Log[a^(1/6) + c^(1/6)*x] - Sqrt[c]*d*Log[a^(1/3) - a^
(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[a]*e*Log[a^(1/3) - a^(1/6)*c^(1/6)*x
 + c^(1/3)*x^2] + Sqrt[c]*d*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]
 + Sqrt[a]*e*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(1/6)*c
^(4/3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.85, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1827, 27, 1835, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex^3)}{a-cx^6}\,dx$$

$$\Big\downarrow \text{1827}$$

$$\frac{\int \frac{2x(cdx^3+ae)}{a-cx^6}dx}{2c}-\frac{ex^2}{2c}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x(cdx^3+ae)}{a-cx^6}dx}{c}-\frac{ex^2}{2c}$$

$$\Big\downarrow \; 1835$$

$$\frac{\frac{1}{2}\sqrt{c}\left(\sqrt{a}e+\sqrt{c}d\right)\int \frac{x}{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^3\right)}dx-\frac{1}{2}\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)\int \frac{x}{\sqrt{c}\left(\sqrt{c}x^3+\sqrt{a}\right)}dx}{c}-\frac{ex^2}{2c}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\int \frac{x}{\sqrt{a}-\sqrt{c}x^3}dx-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\int \frac{x}{\sqrt{c}x^3+\sqrt{a}}dx}{c}-\frac{ex^2}{2c}$$

$$\Big\downarrow \; 821$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\int \frac{1}{\sqrt[6]{a}-\sqrt[6]{c}x}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\cdots\right)}{c}-\frac{ex^2}{2c}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{\int \frac{\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\cdots\right)}{\cdots}\right)}{c}-\frac{ex^2}{2c}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\int\frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\cdots\right.}{c}$$

$$\frac{ex^2}{2c}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\int\frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\cdots\right.}{c}$$

$$\frac{ex^2}{2c}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\cdots\right.}{c}$$

$$\frac{ex^2}{2c}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{-\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{3\int\frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}{\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\cdots}{c}$$

$$\frac{ex^2}{2c}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{-\frac{1}{2}\int\frac{\cdots}{\sqrt[3]{c}\cdots}}{\cdots}\cdots\right)}{c}$$

$$\frac{ex^2}{2c}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\frac{\log\left(-\sqrt[6]{a}\sqrt[6]{\cdots}\right)}{\cdots}}{\cdots}\cdots\right)}{c}$$

$$\frac{ex^2}{2c}$$

input
```
Int[(x^4*(d + e*x^3))/(a - c*x^6),x]
```

output
```
-1/2*(e*x^2)/c + (-1/2*((Sqrt[c]*d - Sqrt[a]*e)*(-1/3*Log[a^(1/6) + c^(1/6
)*x]/(a^(1/6)*c^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sq
rt[3]])/c^(1/6)) + Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/
6)))/(3*a^(1/6)*c^(1/6)))) + ((Sqrt[c]*d + Sqrt[a]*e)*(-1/3*Log[a^(1/6) -
c^(1/6)*x]/(a^(1/6)*c^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6)
)/Sqrt[3]])/c^(1/6) - Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*c^
(1/6)))/(3*a^(1/6)*c^(1/6))))/2)/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1827
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a + c*x^(2*n))^(p +
1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(m + n*(2*p + 1) + 1))   In
t[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

rule 1835
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[-(e/2 + c*(d/(2*q)))   Int[(f
*x)^m/(q - c*x^n), x], x] + Simp[(e/2 - c*(d/(2*q)))   Int[(f*x)^m/(q + c*x
^n), x], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.15

method	result
risch	$-\frac{e\,x^{2}}{2c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c-a\right)}\frac{\left(-\_R^{4}cd-ae\_R\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6c^{2}}$
default	$-\frac{e\,x^{2}}{2c}-\frac{\frac{c\ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}}x-x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{5}{6}}d}{12a}-\frac{\ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}}x-x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{1}{3}}e}{12}+\frac{\sqrt{3}\,d\arctan\left(-\frac{\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{3}}e\arctan\left(-\frac{\sqrt{3}}{3}\right.}{6}}{}$

input
```
int(x^4*(e*x^3+d)/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*e*x^2/c+1/6/c^2*sum((-_R^4*c*d-_R*a*e)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c
-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1785 vs. $2(235) = 470$.

Time = 0.32 (sec) , antiderivative size = 1785, normalized size of antiderivative = 5.30

$$\int \frac{x^4(d+ex^3)}{a-cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x^3+d)/(-c*x^6+a),x, algorithm="fricas")
```

output
```
-1/12*(6*e*x^2 - (sqrt(-3)*c - c)*(-(c^4*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9
*a^2*d^2*e^4)/(a*c^7)) + 3*c*d^2*e + a*e^3)/c^4)^(1/3)*log((c^3*d^7 + a*c^
2*d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x - 1/2*(2*a*c^4*d^4*e + 6*a^2*
c^3*d^2*e^3 + 2*sqrt(-3)*(a*c^4*d^4*e + 3*a^2*c^3*d^2*e^3) - (a*c^7*d^2 +
a^2*c^6*e^2 + sqrt(-3)*(a*c^7*d^2 + a^2*c^6*e^2))*sqrt((c^2*d^6 + 6*a*c*d^
4*e^2 + 9*a^2*d^2*e^4)/(a*c^7)))*(-(c^4*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*
a^2*d^2*e^4)/(a*c^7)) + 3*c*d^2*e + a*e^3)/c^4)^(2/3)) + (sqrt(-3)*c + c)*
(-(c^4*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a*c^7)) + 3*c*d^2*e
 + a*e^3)/c^4)^(1/3)*log((c^3*d^7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*a^
3*d*e^6)*x - 1/2*(2*a*c^4*d^4*e + 6*a^2*c^3*d^2*e^3 - 2*sqrt(-3)*(a*c^4*d^
4*e + 3*a^2*c^3*d^2*e^3) - (a*c^7*d^2 + a^2*c^6*e^2 - sqrt(-3)*(a*c^7*d^2
+ a^2*c^6*e^2))*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a*c^7)))*(
-(c^4*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a*c^7)) + 3*c*d^2*e
+ a*e^3)/c^4)^(2/3)) - 2*c*(-(c^4*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^
2*e^4)/(a*c^7)) + 3*c*d^2*e + a*e^3)/c^4)^(1/3)*log((c^3*d^7 + a*c^2*d^5*e
^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x + (2*a*c^4*d^4*e + 6*a^2*c^3*d^2*e^3
 - (a*c^7*d^2 + a^2*c^6*e^2)*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4
)/(a*c^7)))*(-(c^4*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a*c^7))
 + 3*c*d^2*e + a*e^3)/c^4)^(2/3)) - (sqrt(-3)*c - c)*((c^4*sqrt((c^2*d^6 +
 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a*c^7)) - 3*c*d^2*e - a*e^3)/c^4)^(1/3...
```

Sympy [A] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.64

$$\int \frac{x^4(d+ex^3)}{a-cx^6}\,dx =$$
$$-\operatorname{RootSum}\left(46656t^6ac^8+t^3\left(-432a^2c^4e^3-1296ac^5d^2e\right)+a^3e^6-3a^2cd^2e^4+3ac^2d^4e^2-c^3d^6,\left(t\mapsto t\right.\right.$$
$$-\frac{ex^2}{2c}$$

input
```
integrate(x**4*(e*x**3+d)/(-c*x**6+a),x)
```

output
```
-RootSum(46656*_t**6*a*c**8 + _t**3*(-432*a**2*c**4*e**3 - 1296*a*c**5*d**
2*e) + a**3*e**6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d**6, La
mbda(_t, _t*log(x + (-7776*_t**5*a**2*c**6*e**2 - 7776*_t**5*a*c**7*d**2 +
 36*_t**2*a**3*c**2*e**5 + 360*_t**2*a**2*c**3*d**2*e**3 + 180*_t**2*a*c**
4*d**4*e)/(3*a**3*d*e**6 - 5*a**2*c*d**3*e**4 + a*c**2*d**5*e**2 + c**3*d*
*7)))) - e*x**2/(2*c)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.01

$$\int \frac{x^4(d+ex^3)}{a-cx^6}\,dx = -\frac{ex^2}{2\,c}$$
$$-\frac{\frac{2\sqrt{3}\left(\sqrt{a}cd+a\sqrt{c}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\left(\sqrt{a}cd-a\sqrt{c}e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}-\frac{\left(\sqrt{a}cd+a\sqrt{c}e\right)\log\left(x^2+x\left(\frac{\sqrt{a}}{\sqrt{c}}\right.\right.}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}}{}$$

input
```
integrate(x^4*(e*x^3+d)/(-c*x^6+a),x, algorithm="maxima")
```

output
```
-1/2*e*x^2/c - 1/12*(2*sqrt(3)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/3*sqrt
(3)*(2*x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(s
qrt(a)/sqrt(c))^(1/3)) + 2*sqrt(3)*(sqrt(a)*c*d - a*sqrt(c)*e)*arctan(1/3*
sqrt(3)*(2*x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*
c*(sqrt(a)/sqrt(c))^(1/3)) - (sqrt(a)*c*d + a*sqrt(c)*e)*log(x^2 + x*(sqrt
(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))
^(1/3)) + (sqrt(a)*c*d - a*sqrt(c)*e)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3)
+ (sqrt(a)/sqrt(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(1/3)) - 2*(sqrt(a
)*c*d - a*sqrt(c)*e)*log(x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/
sqrt(c))^(1/3)) + 2*(sqrt(a)*c*d + a*sqrt(c)*e)*log(x - (sqrt(a)/sqrt(c))^
(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(1/3)))/c
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.87

$$
\begin{aligned}
\int \frac{x^4(d+ex^3)}{a-cx^6}\,dx = & -\frac{ex^2}{2\,c} - \frac{d\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,\left(-ac^5\right)^{\frac{1}{6}}} \\
& + \frac{\left(\sqrt{3}ac^2e - \sqrt{-ac}c^2d\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,\left(-ac^5\right)^{\frac{2}{3}}} \\
& - \frac{\left(\sqrt{3}ac^2e + \sqrt{-ac}c^2d\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,\left(-ac^5\right)^{\frac{2}{3}}} \\
& - \frac{\left(\sqrt{3}\sqrt{-ac}c^2d - ac^2e\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,\left(-ac^5\right)^{\frac{2}{3}}} \\
& - \frac{\left(\sqrt{3}\sqrt{-ac}c^2d - ac^2e\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,\left(-ac^5\right)^{\frac{2}{3}}} \\
& + \frac{\left(-ac^5\right)^{\frac{1}{3}}e\log\left(x^2+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,c^3}
\end{aligned}
$$

input
```
integrate(x^4*(e*x^3+d)/(-c*x^6+a),x, algorithm="giac")
```

output
```
-1/2*e*x^2/c - 1/3*d*arctan(x/(-a/c)^(1/6))/(-a*c^5)^(1/6) + 1/6*(sqrt(3)*
a*c^2*e - sqrt(-a*c)*c^2*d)*arctan((2*x + sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/
6))/(-a*c^5)^(2/3) - 1/6*(sqrt(3)*a*c^2*e + sqrt(-a*c)*c^2*d)*arctan((2*x
- sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(-a*c^5)^(2/3) - 1/12*(sqrt(3)*sqrt(
-a*c)*c^2*d - a*c^2*e)*log(x^2 + sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(-
a*c^5)^(2/3) - 1/12*(sqrt(3)*sqrt(-a*c)*c^2*d - a*c^2*e)*log(x^2 - sqrt(3)
*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(-a*c^5)^(2/3) + 1/6*(-a*c^5)^(1/3)*e*log(
x^2 + (-a/c)^(1/3))/c^3
```

Mupad [B] (verification not implemented)

Time = 22.45 (sec) , antiderivative size = 1273, normalized size of antiderivative = 3.78

$$\int \frac{x^4(d+ex^3)}{a-cx^6} \, dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x^3))/(a - c*x^6),x)
```

output
```
log(a*c^7*((c*d^3*(a*c^9)^(1/2) - a^2*c^4*e^3 - 3*a*c^5*d^2*e + 3*a*d*e^2*
(a*c^9)^(1/2))/(a*c^8))^(2/3) + a*e^2*x*(a*c^9)^(1/2) + c*d^2*x*(a*c^9)^(1
/2) - 2*a*c^5*d*e*x)*((c*d^3*(a*c^9)^(1/2) - a^2*c^4*e^3 - 3*a*c^5*d^2*e +
 3*a*d*e^2*(a*c^9)^(1/2))/(216*a*c^8))^(1/3) + log(a*e^2*x*(a*c^9)^(1/2) -
 a*c^7*(-(c*d^3*(a*c^9)^(1/2) + a^2*c^4*e^3 + 3*a*c^5*d^2*e + 3*a*d*e^2*(a
*c^9)^(1/2))/(a*c^8))^(2/3) + c*d^2*x*(a*c^9)^(1/2) + 2*a*c^5*d*e*x)*(-(c*
d^3*(a*c^9)^(1/2) + a^2*c^4*e^3 + 3*a*c^5*d^2*e + 3*a*d*e^2*(a*c^9)^(1/2))
/(216*a*c^8))^(1/3) - (e*x^2)/(2*c) - log((3^(1/2)*a*c^7*((c*d^3*(a*c^9)^(
1/2) - a^2*c^4*e^3 - 3*a*c^5*d^2*e + 3*a*d*e^2*(a*c^9)^(1/2))/(a*c^8))^(2/
3)*1i)/2 - (a*c^7*((c*d^3*(a*c^9)^(1/2) - a^2*c^4*e^3 - 3*a*c^5*d^2*e + 3*
a*d*e^2*(a*c^9)^(1/2))/(a*c^8))^(2/3))/2 + a*e^2*x*(a*c^9)^(1/2) + c*d^2*x
*(a*c^9)^(1/2) - 2*a*c^5*d*e*x)*((3^(1/2)*1i)/2 + 1/2)*((c*d^3*(a*c^9)^(1/
2) - a^2*c^4*e^3 - 3*a*c^5*d^2*e + 3*a*d*e^2*(a*c^9)^(1/2))/(216*a*c^8))^(
1/3) - log(a*c^7*(-(c*d^3*(a*c^9)^(1/2) + a^2*c^4*e^3 + 3*a*c^5*d^2*e + 3*
a*d*e^2*(a*c^9)^(1/2))/(a*c^8))^(2/3) - 3^(1/2)*a*c^7*(-(c*d^3*(a*c^9)^(1/
2) + a^2*c^4*e^3 + 3*a*c^5*d^2*e + 3*a*d*e^2*(a*c^9)^(1/2))/(a*c^8))^(2/3)
*1i + 2*a*e^2*x*(a*c^9)^(1/2) + 2*c*d^2*x*(a*c^9)^(1/2) + 4*a*c^5*d*e*x)*(
(3^(1/2)*1i)/2 + 1/2)*(-(c*d^3*(a*c^9)^(1/2) + a^2*c^4*e^3 + 3*a*c^5*d^2*e
 + 3*a*d*e^2*(a*c^9)^(1/2))/(216*a*c^8))^(1/3) + log(a*c^7*(-(c*d^3*(a*c^9
)^(1/2) + a^2*c^4*e^3 + 3*a*c^5*d^2*e + 3*a*d*e^2*(a*c^9)^(1/2))/(a*c^8...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.13

$$\int \frac{x^4(d+ex^3)}{a-cx^6}\,dx$$
$$= \frac{2c^{\frac{7}{6}}a^{\frac{1}{6}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)d-2c^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)e-2c^{\frac{7}{6}}a^{\frac{1}{6}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)d-2c^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}}{}$$

input
```
int(x^4*(e*x^3+d)/(-c*x^6+a),x)
```

output
```
(2*c**(1/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(
1/6)*a**(1/6)*sqrt(3)))*c*d - 2*c**(2/3)*a**(2/3)*sqrt(3)*atan((c**(1/6)*a
**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*e - 2*c**(1/6)*a**(1/
6)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt
(3)))*c*d - 2*c**(2/3)*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/
3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*e - c**(1/6)*a**(1/6)*log( - c**(1/6)*a
**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*c*d + 2*c**(1/6)*a**(1/6)*log( - c**
(1/6)*a**(1/6) - c**(1/3)*x)*c*d + c**(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6)
*x + a**(1/3) + c**(1/3)*x**2)*c*d - 2*c**(1/6)*a**(1/6)*log(c**(1/6)*a**(
1/6) - c**(1/3)*x)*c*d + c**(2/3)*a**(2/3)*log( - c**(1/6)*a**(1/6)*x + a*
*(1/3) + c**(1/3)*x**2)*e - 2*c**(2/3)*a**(2/3)*log( - c**(1/6)*a**(1/6) -
 c**(1/3)*x)*e + c**(2/3)*a**(2/3)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c*
*(1/3)*x**2)*e - 2*c**(2/3)*a**(2/3)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*e
 - 6*a**(1/3)*c*e*x**2)/(12*a**(1/3)*c**2)
```

3.7 $\int \frac{x^3(d+ex^3)}{a-cx^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 333

$$\int \frac{x^3(d+ex^3)}{a-cx^6}\,dx = -\frac{ex}{c}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}\sqrt[3]{a}c^{7/6}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}\sqrt[3]{a}c^{7/6}}-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6\sqrt[3]{a}c^{7/6}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6\sqrt[3]{a}c^{7/6}}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}$$

output
```
-e*x/c+1/6*(c^(1/2)*d-a^(1/2)*e)*arctan(1/3*(a^(1/6)-2*c^(1/6)*x)*3^(1/2)/
a^(1/6))*3^(1/2)/a^(1/3)/c^(7/6)+1/6*(c^(1/2)*d+a^(1/2)*e)*arctan(1/3*(a^(
1/6)+2*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2)/a^(1/3)/c^(7/6)-1/6*(c^(1/2)*d+
a^(1/2)*e)*ln(a^(1/6)-c^(1/6)*x)/a^(1/3)/c^(7/6)-1/6*(c^(1/2)*d-a^(1/2)*e)
*ln(a^(1/6)+c^(1/6)*x)/a^(1/3)/c^(7/6)+1/12*(c^(1/2)*d-a^(1/2)*e)*ln(a^(1/
3)-a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(1/3)/c^(7/6)+1/12*(c^(1/2)*d+a^(1/2)*
e)*ln(a^(1/3)+a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(1/3)/c^(7/6)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.05

$$\int \frac{x^3(d+ex^3)}{a-cx^6}\,dx$$

$$= \frac{-12\sqrt[3]{a}\sqrt[6]{c}ex + 2\sqrt{3}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right) + 2\sqrt{3}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{1+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right) - 2\sqrt{c}d}{}$$

input
```
Integrate[(x^3*(d + e*x^3))/(a - c*x^6),x]
```

output
```
(-12*a^(1/3)*c^(1/6)*e*x + 2*Sqrt[3]*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(1 - (
2*c^(1/6)*x)/a^(1/6))/Sqrt[3]] + 2*Sqrt[3]*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[
(1 + (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]] - 2*Sqrt[c]*d*Log[a^(1/6) - c^(1/6)*x
] - 2*Sqrt[a]*e*Log[a^(1/6) - c^(1/6)*x] - 2*Sqrt[c]*d*Log[a^(1/6) + c^(1/
6)*x] + 2*Sqrt[a]*e*Log[a^(1/6) + c^(1/6)*x] + Sqrt[c]*d*Log[a^(1/3) - a^(
1/6)*c^(1/6)*x + c^(1/3)*x^2] - Sqrt[a]*e*Log[a^(1/3) - a^(1/6)*c^(1/6)*x
+ c^(1/3)*x^2] + Sqrt[c]*d*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]
+ Sqrt[a]*e*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(1/3)*c^
(7/6))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {1827, 1747, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(d+ex^3\right)}{a-cx^6}\,dx$$

$$\Big\downarrow \; 1827$$

$$\frac{\int \frac{cdx^3+ae}{a-cx^6}dx}{c}-\frac{ex}{c}$$

$$\Big\downarrow \; 1747$$

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\int \frac{1}{a-\sqrt{a}\sqrt{c}x^3}dx-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\sqrt{a}\sqrt{c}x^3+a}dx}{c}-\frac{ex}{c}$$

$$\Big\downarrow \; 750$$

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(\sqrt[6]{c}x+2\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x}dx}{3a^{2/3}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}-\sqrt[6]{c}x\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a \cdots}}{3a^{2/3}} \cdots\right.}{c} \cdots \frac{ex}{c}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(\sqrt[6]{c}x+2\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}-\sqrt[6]{c}x\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}} \cdots}{3a^{2/3}} \cdots\right.}{c} \cdots \frac{ex}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\int \frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}} \cdots}{3\sqrt{a}} \cdots\right.}{c} \cdots \frac{ex}{c}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\right.}{\frac{ex}{c}}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\right.}{\frac{ex}{c}}$$

$\downarrow$ 27

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d\right.}{\frac{ex}{c}}$$

$\downarrow$ 1082

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx-\frac{3\int\frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\right.}{\frac{ex}{c}}$$

$\Big\downarrow$ 217

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\cdots\right)}{c}$$

$$\frac{ex}{c}$$

$\Big\downarrow$ 1103

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}+\frac{\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\log\left(\sqrt[6]{\cdots}\right.}{3a\cdots}\cdots\right)}{c}$$

$$\frac{ex}{c}$$

input
```
Int[(x^3*(d + e*x^3))/(a - c*x^6),x]
```

output
```
-((e*x)/c) + (-1/2*(Sqrt[a]*(Sqrt[c]*d - Sqrt[a]*e)*(Log[a^(1/6) + c^(1/6)
*x]/(3*a^(5/6)*c^(1/6)) + (-((Sqrt[3]*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/S
qrt[3]])/(a^(1/3)*c^(1/6))) - Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^
2]/(2*a^(1/3)*c^(1/6)))/(3*Sqrt[a]))) + (Sqrt[a]*(Sqrt[c]*d + Sqrt[a]*e)*(
-1/3*Log[a^(1/6) - c^(1/6)*x]/(a^(5/6)*c^(1/6)) + ((Sqrt[3]*ArcTan[(1 + (2
*c^(1/6)*x)/a^(1/6))/Sqrt[3]])/(a^(1/3)*c^(1/6)) + Log[a^(1/3) + a^(1/6)*c
^(1/6)*x + c^(1/3)*x^2]/(2*a^(1/3)*c^(1/6)))/(3*Sqrt[a])))/2)/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1747
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-a/c, 2]}, Simp[(d + e*q)/2   Int[1/(a + c*q*x^n), x], x] + Simp[(d
- e*q)/2   Int[1/(a - c*q*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ
[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NegQ[a*c] && IntegerQ[n]
```

rule 1827
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a + c*x^(2*n))^(p +
1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(m + n*(2*p + 1) + 1))   In
t[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.15

method	result
risch	$-\frac{ex}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6} c-a\right)}\frac{\left(-\_R^{3} c d-a e\right) \ln\left(x-\_R\right)}{\_R^{5}}}{6 c^{2}}$
default	$-\frac{ex}{c}+\frac{\frac{c\left(\frac{a}{c}\right)^{\frac{2}{3}} \ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}} x-x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right) d}{12 a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}} \ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}} x-x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right) e}{12}-\frac{c\left(\frac{a}{c}\right)^{\frac{2}{3}} \sqrt{3}\, d \arctan\left(-\frac{\sqrt{3}}{3}+\frac{2 x \sqrt{3}}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6 a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}} \sqrt{3}\, e \arctan}{}}{}$

input
```
int(x^3*(e*x^3+d)/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-e*x/c+1/6/c^2*sum((-_R^3*c*d-a*e)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1596 vs. $2(233) = 466$.

Time = 0.11 (sec) , antiderivative size = 1596, normalized size of antiderivative = 4.79

$$\int \frac{x^3(d+ex^3)}{a-cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^3+d)/(-c*x^6+a),x, algorithm="fricas")
```

output
```
1/12*(2*c*(-(a*c^3*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^7))
 + c*d^3 + 3*a*d*e^2)/(a*c^3))^(1/3)*log(-(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a
^2*e^5)*x - (a*c^5*d*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^7
)) - 3*a*c^2*d^2*e^2 - a^2*c*e^4)*(-(a*c^3*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2
*e^4 + a^2*e^6)/(a*c^7)) + c*d^3 + 3*a*d*e^2)/(a*c^3))^(1/3)) - (sqrt(-3)*
c + c)*(-(a*c^3*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^7)) +
c*d^3 + 3*a*d*e^2)/(a*c^3))^(1/3)*log(-(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*
e^5)*x - 1/2*(3*a*c^2*d^2*e^2 + a^2*c*e^4 + sqrt(-3)*(3*a*c^2*d^2*e^2 + a^
2*c*e^4) - (sqrt(-3)*a*c^5*d + a*c^5*d)*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^
4 + a^2*e^6)/(a*c^7)))*(-(a*c^3*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*
e^6)/(a*c^7)) + c*d^3 + 3*a*d*e^2)/(a*c^3))^(1/3)) + (sqrt(-3)*c - c)*(-(a
*c^3*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^7)) + c*d^3 + 3*a
*d*e^2)/(a*c^3))^(1/3)*log(-(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*e^5)*x - 1/
2*(3*a*c^2*d^2*e^2 + a^2*c*e^4 - sqrt(-3)*(3*a*c^2*d^2*e^2 + a^2*c*e^4) +
(sqrt(-3)*a*c^5*d - a*c^5*d)*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6
)/(a*c^7)))*(-(a*c^3*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^7
)) + c*d^3 + 3*a*d*e^2)/(a*c^3))^(1/3)) + 2*c*((a*c^3*sqrt((9*c^2*d^4*e^2
+ 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^7)) - c*d^3 - 3*a*d*e^2)/(a*c^3))^(1/3)*lo
g(-(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*e^5)*x + (a*c^5*d*sqrt((9*c^2*d^4*e^
2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^7)) + 3*a*c^2*d^2*e^2 + a^2*c*e^4)*((...
```

Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.51

$$\int \frac{x^3(d+ex^3)}{a-cx^6}\,dx = -\operatorname{RootSum}\left(46656t^6a^2c^7+t^3\left(-1296a^2c^4de^2-432ac^5d^3\right)-a^3e^6+3a^2cd^2e^4-3ac^2d^4e^2+c^3d^6,\left(t\mapsto\right.\right.$$
$$-\frac{ex}{c}$$

input
```
integrate(x**3*(e*x**3+d)/(-c*x**6+a),x)
```

output
```
-RootSum(46656*_t**6*a**2*c**7 + _t**3*(-1296*a**2*c**4*d*e**2 - 432*a*c**
5*d**3) - a**3*e**6 + 3*a**2*c*d**2*e**4 - 3*a*c**2*d**4*e**2 + c**3*d**6,
 Lambda(_t, _t*log(x + (1296*_t**4*a*c**5*d - 6*_t*a**2*c*e**4 - 36*_t*a*c
**2*d**2*e**2 - 6*_t*c**3*d**4)/(a**2*e**5 + 2*a*c*d**2*e**3 - 3*c**2*d**4
*e)))) - e*x/c
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.01

$$\int \frac{x^3(d+ex^3)}{a-cx^6}\,dx = -\frac{ex}{c}$$
$$+\frac{\frac{2\sqrt{3}(\sqrt{a}cd+a\sqrt{c}e)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}-\frac{2\sqrt{3}(\sqrt{a}cd-a\sqrt{c}e)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{(\sqrt{a}cd+a\sqrt{c}e)\log\left(x^2+x\left(\frac{\sqrt{a}}{\sqrt{c}}\right.\right.}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}}{}$$

input
```
integrate(x^3*(e*x^3+d)/(-c*x^6+a),x, algorithm="maxima")
```

output
```
-e*x/c + 1/12*(2*sqrt(3)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/3*sqrt(3)*(2
*x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)
/sqrt(c))^(2/3)) - 2*sqrt(3)*(sqrt(a)*c*d - a*sqrt(c)*e)*arctan(1/3*sqrt(3
)*(2*x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqr
t(a)/sqrt(c))^(2/3)) + (sqrt(a)*c*d + a*sqrt(c)*e)*log(x^2 + x*(sqrt(a)/sq
rt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)
) + (sqrt(a)*c*d - a*sqrt(c)*e)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sqr
t(a)/sqrt(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - 2*(sqrt(a)*c*d
- a*sqrt(c)*e)*log(x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c
))^(2/3)) - 2*(sqrt(a)*c*d + a*sqrt(c)*e)*log(x - (sqrt(a)/sqrt(c))^(1/3))
/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)))/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{x^3(d+ex^3)}{a-cx^6}\,dx &= \frac{d|c|\log\left(x^2+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\left(-ac^5\right)^{\frac{1}{3}}}-\frac{ex}{c}+\frac{\left(-ac^5\right)^{\frac{1}{6}}e\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,c^2}\\
&\quad+\frac{\left(\left(-ac^5\right)^{\frac{1}{6}}ac^2e-\sqrt{3}\left(-ac^5\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4}\\
&\quad+\frac{\left(\left(-ac^5\right)^{\frac{1}{6}}ac^2e+\sqrt{3}\left(-ac^5\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4}\\
&\quad+\frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}ac^2e+\left(-ac^5\right)^{\frac{2}{3}}d\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}\\
&\quad-\frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}ac^2e-\left(-ac^5\right)^{\frac{2}{3}}d\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}\end{aligned}$$

input
```
integrate(x^3*(e*x^3+d)/(-c*x^6+a),x, algorithm="giac")
```

output
```
1/6*d*abs(c)*log(x^2 + (-a/c)^(1/3))/(-a*c^5)^(1/3) - e*x/c + 1/3*(-a*c^5)
^(1/6)*e*arctan(x/(-a/c)^(1/6))/c^2 + 1/6*((-a*c^5)^(1/6)*a*c^2*e - sqrt(3
)*(-a*c^5)^(2/3)*d)*arctan((2*x + sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c
^4) + 1/6*((-a*c^5)^(1/6)*a*c^2*e + sqrt(3)*(-a*c^5)^(2/3)*d)*arctan((2*x
- sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(-a*c^5)^(1/
6)*a*c^2*e + (-a*c^5)^(2/3)*d)*log(x^2 + sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(
1/3))/(a*c^4) - 1/12*(sqrt(3)*(-a*c^5)^(1/6)*a*c^2*e - (-a*c^5)^(2/3)*d)*l
og(x^2 - sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4)
```

Mupad [B] (verification not implemented)

Time = 22.24 (sec) , antiderivative size = 1267, normalized size of antiderivative = 3.80

$$\int \frac{x^3(d+ex^3)}{a-cx^6}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x^3))/(a - c*x^6),x)
```

output
```
log(d*x*(a^3*c^7)^(1/2) + a^2*c^4*(-(a*c^5*d^3 + a*e^3*(a^3*c^7)^(1/2) + 3
*a^2*c^4*d*e^2 + 3*c*d^2*e*(a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) + a^2*c^3*e*x
)*(-(a*c^5*d^3 + a*e^3*(a^3*c^7)^(1/2) + 3*a^2*c^4*d*e^2 + 3*c*d^2*e*(a^3*
c^7)^(1/2))/(216*a^2*c^7))^(1/3) + log(a^2*c^4*(-(a*c^5*d^3 - a*e^3*(a^3*c
^7)^(1/2) + 3*a^2*c^4*d*e^2 - 3*c*d^2*e*(a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)
- d*x*(a^3*c^7)^(1/2) + a^2*c^3*e*x)*(-(a*c^5*d^3 - a*e^3*(a^3*c^7)^(1/2)
+ 3*a^2*c^4*d*e^2 - 3*c*d^2*e*(a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) - log(
d*x*(a^3*c^7)^(1/2) - (a^2*c^4*(-(a*c^5*d^3 + a*e^3*(a^3*c^7)^(1/2) + 3*a^
2*c^4*d*e^2 + 3*c*d^2*e*(a^3*c^7)^(1/2))/(a^2*c^7))^(1/3))/2 - (3^(1/2)*a^
2*c^4*(-(a*c^5*d^3 + a*e^3*(a^3*c^7)^(1/2) + 3*a^2*c^4*d*e^2 + 3*c*d^2*e*(
a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1i)/2 + a^2*c^3*e*x)*((3^(1/2)*1i)/2 + 1/
2)*(-(a*c^5*d^3 + a*e^3*(a^3*c^7)^(1/2) + 3*a^2*c^4*d*e^2 + 3*c*d^2*e*(a^3
*c^7)^(1/2))/(216*a^2*c^7))^(1/3) + log(d*x*(a^3*c^7)^(1/2) - (a^2*c^4*(-(
a*c^5*d^3 + a*e^3*(a^3*c^7)^(1/2) + 3*a^2*c^4*d*e^2 + 3*c*d^2*e*(a^3*c^7)^
(1/2))/(a^2*c^7))^(1/3))/2 + (3^(1/2)*a^2*c^4*(-(a*c^5*d^3 + a*e^3*(a^3*c^
7)^(1/2) + 3*a^2*c^4*d*e^2 + 3*c*d^2*e*(a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1
i)/2 + a^2*c^3*e*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a*c^5*d^3 + a*e^3*(a^3*c^7)^
(1/2) + 3*a^2*c^4*d*e^2 + 3*c*d^2*e*(a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3)
+ log(2*d*x*(a^3*c^7)^(1/2) + a^2*c^4*(-(a*c^5*d^3 - a*e^3*(a^3*c^7)^(1/2)
 + 3*a^2*c^4*d*e^2 - 3*c*d^2*e*(a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) - 3^(1...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.02

$$\int \frac{x^3(d+ex^3)}{a-cx^6}\,dx$$
$$= \frac{-2\sqrt{c}\,\sqrt{a}\,\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)e+2\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)cd+2\sqrt{c}\,\sqrt{a}\,\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)e+2\sqrt{3}\,a}{}$$

input
```
int(x^3*(e*x^3+d)/(-c*x^6+a),x)
```

output
```
( - 2*sqrt(c)*sqrt(a)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**
(1/6)*a**(1/6)*sqrt(3)))*e + 2*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3
)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*c*d + 2*sqrt(c)*sqrt(a)*sqrt(3)*atan((c*
*(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*e + 2*sqrt(3)
*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*c*d
- sqrt(c)*sqrt(a)*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*e
 + 2*sqrt(c)*sqrt(a)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*e + sqrt(c)*sq
rt(a)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*e - 2*sqrt(c)*sq
rt(a)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*e - 12*c**(2/3)*a**(1/3)*e*x + l
og( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*c*d - 2*log( - c**(1
/6)*a**(1/6) - c**(1/3)*x)*c*d + log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(
1/3)*x**2)*c*d - 2*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*c*d)/(12*c**(2/3)*a
**(1/3)*c)
```

3.8 $\int \frac{x(d+ex^3)}{a-cx^6}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 326

$$\begin{aligned}\int \frac{x(d+ex^3)}{a-cx^6}\,dx = &-\frac{(\sqrt{c}d-\sqrt{a}e)\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{2/3}c^{5/6}} \\ &-\frac{(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{2/3}c^{5/6}} \\ &-\frac{(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{2/3}c^{5/6}}-\frac{(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{2/3}c^{5/6}} \\ &+\frac{(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{2/3}c^{5/6}} \\ &+\frac{(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{2/3}c^{5/6}}\end{aligned}$$

output
```
-1/6*(c^(1/2)*d-a^(1/2)*e)*arctan(1/3*(a^(1/6)-2*c^(1/6)*x)*3^(1/2)/a^(1/6
))*3^(1/2)/a^(2/3)/c^(5/6)-1/6*(c^(1/2)*d+a^(1/2)*e)*arctan(1/3*(a^(1/6)+2
*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2)/a^(2/3)/c^(5/6)-1/6*(c^(1/2)*d+a^(1/2
)*e)*ln(a^(1/6)-c^(1/6)*x)/a^(2/3)/c^(5/6)-1/6*(c^(1/2)*d-a^(1/2)*e)*ln(a^
(1/6)+c^(1/6)*x)/a^(2/3)/c^(5/6)+1/12*(c^(1/2)*d-a^(1/2)*e)*ln(a^(1/3)-a^(
1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(2/3)/c^(5/6)+1/12*(c^(1/2)*d+a^(1/2)*e)*ln(
a^(1/3)+a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(2/3)/c^(5/6)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.03

$$\int \frac{x(d+ex^3)}{a-cx^6}\,dx$$

$$= \frac{-2\sqrt{3}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)-2\sqrt{3}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{1+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)-2\sqrt{c}d\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)\ldots}{}$$

input
```
Integrate[(x*(d + e*x^3))/(a - c*x^6),x]
```

output
```
(-2*Sqrt[3]*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqr
t[3]] - 2*Sqrt[3]*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6
))/Sqrt[3]] - 2*Sqrt[c]*d*Log[a^(1/6) - c^(1/6)*x] - 2*Sqrt[a]*e*Log[a^(1/
6) - c^(1/6)*x] - 2*Sqrt[c]*d*Log[a^(1/6) + c^(1/6)*x] + 2*Sqrt[a]*e*Log[a
^(1/6) + c^(1/6)*x] + Sqrt[c]*d*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*
x^2] - Sqrt[a]*e*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[c]*
d*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[a]*e*Log[a^(1/3) +
 a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(2/3)*c^(5/6))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1835, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(d+ex^3\right)}{a-cx^6}dx$$

$$\Big\downarrow \text{1835}$$

$$\frac{1}{2}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{x}{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^3\right)}dx+\frac{1}{2}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{x}{\sqrt{c}\left(\sqrt{c}x^3+\sqrt{a}\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{x}{\sqrt{a}-\sqrt{c}x^3}dx}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{x}{\sqrt{c}x^3+\sqrt{a}}dx}{2\sqrt{c}}$$

$$\Big\downarrow \text{821}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\int \frac{1}{\sqrt[6]{c}x+\sqrt[6]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int \frac{1}{\sqrt[6]{a}-\sqrt[6]{c}x}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(-\frac{\int \frac{\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\int -\frac{\sqrt[6]{c}\left(\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(-\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\int\frac{\sqrt[6]{c}\left(\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(-\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\int\frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{1}{2}\int\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(-\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{c}}-\frac{1}{2}\int \frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(-\frac{-\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}{\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

$\downarrow$ 217

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

$\downarrow$ 1103

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\frac{\log\left(-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

input
```
Int[(x*(d + e*x^3))/(a - c*x^6),x]
```

output
```
(((Sqrt[c]*d)/Sqrt[a] - e)*(-1/3*Log[a^(1/6) + c^(1/6)*x]/(a^(1/6)*c^(1/3)
) + (-((Sqrt[3]*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]])/c^(1/6)) + Lo
g[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/6)))/(3*a^(1/6)*c^(1/
6))))/(2*Sqrt[c]) + (((Sqrt[c]*d)/Sqrt[a] + e)*(-1/3*Log[a^(1/6) - c^(1/6)
*x]/(a^(1/6)*c^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6))/Sqrt[
3]])/c^(1/6) - Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/6)))
/(3*a^(1/6)*c^(1/6))))/(2*Sqrt[c])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1835
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[-(e/2 + c*(d/(2*q)))   Int[(f
*x)^m/(q - c*x^n), x], x] + Simp[(e/2 - c*(d/(2*q)))   Int[(f*x)^m/(q + c*x
^n), x], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.12

method	result
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c-a\right)}\frac{\left(\_R^{4}e+\_Rd\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6c}$
default	$-\frac{\ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}}x-x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{5}{6}}e}{12a}+\frac{\ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}}x-x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{1}{3}}d}{12a}-\frac{\sqrt{3}\,e\arctan\left(-\frac{\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6c\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{3}}d\arctan\left(\cdots\right.}{6a}$

input
```
int(x*(e*x^3+d)/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6/c*sum((_R^4*e+_R*d)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1874 vs. 2(226) = 452.

Time = 0.13 (sec) , antiderivative size = 1874, normalized size of antiderivative = 5.75

$$\int \frac{x(d+ex^3)}{a-cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^3+d)/(-c*x^6+a),x, algorithm="fricas")
```

output
```
1/12*(sqrt(-3) - 1)*(-(a^2*c^2*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e
^6)/(a^3*c^5)) + c*d^3 + 3*a*d*e^2)/(a^2*c^2))^(1/3)*log((3*c^3*d^6*e - 5*
a*c^2*d^4*e^3 + a^2*c*d^2*e^5 + a^3*e^7)*x - 1/2*(6*a^2*c^3*d^3*e^2 + 2*a^
3*c^2*d*e^4 + 2*sqrt(-3)*(3*a^2*c^3*d^3*e^2 + a^3*c^2*d*e^4) - (a^3*c^5*d^
2 + a^4*c^4*e^2 + sqrt(-3)*(a^3*c^5*d^2 + a^4*c^4*e^2))*sqrt((9*c^2*d^4*e^
2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a^3*c^5)))*(-(a^2*c^2*sqrt((9*c^2*d^4*e^2 +
6*a*c*d^2*e^4 + a^2*e^6)/(a^3*c^5)) + c*d^3 + 3*a*d*e^2)/(a^2*c^2))^(2/3))
 - 1/12*(sqrt(-3) + 1)*(-(a^2*c^2*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^
2*e^6)/(a^3*c^5)) + c*d^3 + 3*a*d*e^2)/(a^2*c^2))^(1/3)*log((3*c^3*d^6*e -
 5*a*c^2*d^4*e^3 + a^2*c*d^2*e^5 + a^3*e^7)*x - 1/2*(6*a^2*c^3*d^3*e^2 + 2
*a^3*c^2*d*e^4 - 2*sqrt(-3)*(3*a^2*c^3*d^3*e^2 + a^3*c^2*d*e^4) - (a^3*c^5
*d^2 + a^4*c^4*e^2 - sqrt(-3)*(a^3*c^5*d^2 + a^4*c^4*e^2))*sqrt((9*c^2*d^4
*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a^3*c^5)))*(-(a^2*c^2*sqrt((9*c^2*d^4*e^2
 + 6*a*c*d^2*e^4 + a^2*e^6)/(a^3*c^5)) + c*d^3 + 3*a*d*e^2)/(a^2*c^2))^(2/
3)) + 1/12*(sqrt(-3) - 1)*((a^2*c^2*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 +
a^2*e^6)/(a^3*c^5)) - c*d^3 - 3*a*d*e^2)/(a^2*c^2))^(1/3)*log((3*c^3*d^6*e
 - 5*a*c^2*d^4*e^3 + a^2*c*d^2*e^5 + a^3*e^7)*x - 1/2*(6*a^2*c^3*d^3*e^2 +
 2*a^3*c^2*d*e^4 + 2*sqrt(-3)*(3*a^2*c^3*d^3*e^2 + a^3*c^2*d*e^4) + (a^3*c
^5*d^2 + a^4*c^4*e^2 + sqrt(-3)*(a^3*c^5*d^2 + a^4*c^4*e^2))*sqrt((9*c^2*d
^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a^3*c^5)))*((a^2*c^2*sqrt((9*c^2*d^4...
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.65

$$\int \frac{x(d+ex^3)}{a-cx^6}\,dx =$$

$$-\operatorname{RootSum}\left(46656t^6a^4c^5+t^3\left(-1296a^3c^3de^2-432a^2c^4d^3\right)-a^3e^6+3a^2cd^2e^4-3ac^2d^4e^2+c^3d^6,\left(t\mapsto\right.\right.$$

input
```
integrate(x*(e*x**3+d)/(-c*x**6+a),x)
```

output
```
-RootSum(46656*_t**6*a**4*c**5 + _t**3*(-1296*a**3*c**3*d*e**2 - 432*a**2*
c**4*d**3) - a**3*e**6 + 3*a**2*c*d**2*e**4 - 3*a*c**2*d**4*e**2 + c**3*d*
*6, Lambda(_t, _t*log(x + (-7776*_t**5*a**4*c**4*e**2 - 7776*_t**5*a**3*c*
*5*d**2 + 180*_t**2*a**3*c**2*d*e**4 + 360*_t**2*a**2*c**3*d**3*e**2 + 36*
_t**2*a*c**4*d**5)/(a**3*e**7 + a**2*c*d**2*e**5 - 5*a*c**2*d**4*e**3 + 3*
c**3*d**6*e))))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.96

$$\int \frac{x(d+ex^3)}{a-cx^6}\,dx = -\frac{\sqrt{3}(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{6\,\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}(\sqrt{c}d-\sqrt{a}e)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{6\,\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}} + \frac{(\sqrt{c}d+\sqrt{a}e)\log\left(x^2+x\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}\right)}{12\,\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}} + \frac{(\sqrt{c}d-\sqrt{a}e)\log\left(x^2-x\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}\right)}{12\,\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}} - \frac{(\sqrt{c}d-\sqrt{a}e)\log\left(x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{6\,\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}} - \frac{(\sqrt{c}d+\sqrt{a}e)\log\left(x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{6\,\sqrt{ac}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}$$

input
```
integrate(x*(e*x^3+d)/(-c*x^6+a),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/3*sqrt(3)*(2*x + (sqrt(a)/sq
rt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(1/3))
 + 1/6*sqrt(3)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/3*sqrt(3)*(2*x - (sqrt(a)/
sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(1/3
)) + 1/12*(sqrt(c)*d + sqrt(a)*e)*log(x^2 + x*(sqrt(a)/sqrt(c))^(1/3) + (s
qrt(a)/sqrt(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(1/3)) + 1/12*(sqrt(c)
*d - sqrt(a)*e)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2
/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(1/3)) - 1/6*(sqrt(c)*d - sqrt(a)*e)*log
(x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(1/3)) - 1/6*(s
qrt(c)*d + sqrt(a)*e)*log(x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)
/sqrt(c))^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.86

$$\int \frac{x(d+ex^3)}{a-cx^6}\,dx = -\frac{e\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\left(-ac^5\right)^{\frac{1}{6}}} + \frac{\left(\sqrt{3}c^3d-\sqrt{-ac}c^2e\right)\arctan\left(\frac{2x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\left(-ac^5\right)^{\frac{2}{3}}} - \frac{\left(\sqrt{3}c^3d+\sqrt{-ac}c^2e\right)\arctan\left(\frac{2x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\left(-ac^5\right)^{\frac{2}{3}}} + \frac{\left(c^3d+\sqrt{3}\sqrt{-ac}c^2e\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\left(-ac^5\right)^{\frac{2}{3}}} + \frac{\left(c^3d+\sqrt{3}\sqrt{-ac}c^2e\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\left(-ac^5\right)^{\frac{2}{3}}} + \frac{\left(-ac^5\right)^{\frac{1}{3}}d\log\left(x^2+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,ac^2}$$

input
```
integrate(x*(e*x^3+d)/(-c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*e*arctan(x/(-a/c)^(1/6))/(-a*c^5)^(1/6) + 1/6*(sqrt(3)*c^3*d - sqrt(-
a*c)*c^2*e)*arctan((2*x + sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(-a*c^5)^(2/
3) - 1/6*(sqrt(3)*c^3*d + sqrt(-a*c)*c^2*e)*arctan((2*x - sqrt(3)*(-a/c)^(
1/6))/(-a/c)^(1/6))/(-a*c^5)^(2/3) + 1/12*(c^3*d + sqrt(3)*sqrt(-a*c)*c^2*
e)*log(x^2 + sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(-a*c^5)^(2/3) + 1/12*
(c^3*d + sqrt(3)*sqrt(-a*c)*c^2*e)*log(x^2 - sqrt(3)*x*(-a/c)^(1/6) + (-a/
c)^(1/3))/(-a*c^5)^(2/3) + 1/6*(-a*c^5)^(1/3)*d*log(x^2 + (-a/c)^(1/3))/(a
*c^2)
```

Mupad [B] (verification not implemented)

Time = 23.13 (sec) , antiderivative size = 1498, normalized size of antiderivative = 4.60

$$\int \frac{x(d+ex^3)}{a-cx^6}\,dx = \text{Too large to display}$$

input
```
int((x*(d + e*x^3))/(a - c*x^6),x)
```

output
```
log(a*e^2*x*(a^5*c^5)^(1/2) - a^4*c^4*(-(a^2*c^4*d^3 + a*e^3*(a^5*c^5)^(1/
2) + 3*a^3*c^3*d*e^2 + 3*c*d^2*e*(a^5*c^5)^(1/2))/(a^4*c^5))^(2/3) + c*d^2
*x*(a^5*c^5)^(1/2) + 2*a^3*c^3*d*e*x)*(-(a^2*c^4*d^3 + a*e^3*(a^5*c^5)^(1/
2) + 3*a^3*c^3*d*e^2 + 3*c*d^2*e*(a^5*c^5)^(1/2))/(216*a^4*c^5))^(1/3) + l
og(a^4*c^4*(-(a^2*c^4*d^3 - a*e^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d*e^2 - 3*c*
d^2*e*(a^5*c^5)^(1/2))/(a^4*c^5))^(2/3) + a*e^2*x*(a^5*c^5)^(1/2) + c*d^2*
x*(a^5*c^5)^(1/2) - 2*a^3*c^3*d*e*x)*(-(a^2*c^4*d^3 - a*e^3*(a^5*c^5)^(1/2
) + 3*a^3*c^3*d*e^2 - 3*c*d^2*e*(a^5*c^5)^(1/2))/(216*a^4*c^5))^(1/3) - lo
g(- (((3^(1/2)*1i)/2 - 1/2)*(36*a^3*c^3*e^3 + 108*a^2*c^4*d^2*e - 36*a^2*c
^4*x*((3^(1/2)*1i)/2 + 1/2)*(a*e^2 + c*d^2)*(-(a^2*c^4*d^3 + a*e^3*(a^5*c^
5)^(1/2) + 3*a^3*c^3*d*e^2 + 3*c*d^2*e*(a^5*c^5)^(1/2))/(a^4*c^5))^(1/3))*
(-(a^2*c^4*d^3 + a*e^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d*e^2 + 3*c*d^2*e*(a^5*
c^5)^(1/2))/(a^4*c^5))^(2/3))/36 - c^2*d*x*(a*e^2 - c*d^2)^2)*((3^(1/2)*1i
)/2 + 1/2)*(-(a^2*c^4*d^3 + a*e^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d*e^2 + 3*c*
d^2*e*(a^5*c^5)^(1/2))/(216*a^4*c^5))^(1/3) + log((((3^(1/2)*1i)/2 + 1/2)*
(36*a^3*c^3*e^3 + 108*a^2*c^4*d^2*e + 36*a^2*c^4*x*((3^(1/2)*1i)/2 - 1/2)*
(a*e^2 + c*d^2)*(-(a^2*c^4*d^3 + a*e^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d*e^2 +
 3*c*d^2*e*(a^5*c^5)^(1/2))/(a^4*c^5))^(1/3))*(-(a^2*c^4*d^3 + a*e^3*(a^5*
c^5)^(1/2) + 3*a^3*c^3*d*e^2 + 3*c*d^2*e*(a^5*c^5)^(1/2))/(a^4*c^5))^(2/3)
)/36 - c^2*d*x*(a*e^2 - c*d^2)^2)*((3^(1/2)*1i)/2 - 1/2)*(-(a^2*c^4*d^3...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.13

$$\int \frac{x(d+ex^3)}{a-cx^6}\,dx$$
$$= \frac{2c^{\frac{1}{6}}a^{\frac{7}{6}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)e-2c^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)d-2c^{\frac{1}{6}}a^{\frac{7}{6}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)e-2c^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}}{}$$

input
```
int(x*(e*x^3+d)/(-c*x^6+a),x)
```

output
```
(2*c**(1/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(
1/6)*a**(1/6)*sqrt(3)))*a*e - 2*c**(2/3)*a**(2/3)*sqrt(3)*atan((c**(1/6)*a
**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*d - 2*c**(1/6)*a**(1/
6)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt
(3)))*a*e - 2*c**(2/3)*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/
3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*d - c**(1/6)*a**(1/6)*log( - c**(1/6)*a
**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*a*e + 2*c**(1/6)*a**(1/6)*log( - c**
(1/6)*a**(1/6) - c**(1/3)*x)*a*e + c**(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6)
*x + a**(1/3) + c**(1/3)*x**2)*a*e - 2*c**(1/6)*a**(1/6)*log(c**(1/6)*a**(
1/6) - c**(1/3)*x)*a*e + c**(2/3)*a**(2/3)*log( - c**(1/6)*a**(1/6)*x + a*
*(1/3) + c**(1/3)*x**2)*d - 2*c**(2/3)*a**(2/3)*log( - c**(1/6)*a**(1/6) -
 c**(1/3)*x)*d + c**(2/3)*a**(2/3)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c*
*(1/3)*x**2)*d - 2*c**(2/3)*a**(2/3)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*d
)/(12*a**(1/3)*a*c)
```

3.9 $\int \frac{d+ex^3}{a-cx^6} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 323

$$\begin{aligned}\int \frac{d+ex^3}{a-cx^6} \, dx = & -\frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right) \arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}\sqrt[6]{c}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right) \arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}c^{2/3}} \\ & - \frac{\left(\sqrt{c}d+\sqrt{a}e\right) \log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{5/6}c^{2/3}} + \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right) \log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{5/6}\sqrt[6]{c}} \\ & - \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right) \log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}\sqrt[6]{c}} \\ & + \frac{\left(\sqrt{c}d+\sqrt{a}e\right) \log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}\end{aligned}$$

output

```
-1/6*(d-a^(1/2)*e/c^(1/2))*arctan(1/3*(a^(1/6)-2*c^(1/6)*x)*3^(1/2)/a^(1/6
))*3^(1/2)/a^(5/6)/c^(1/6)+1/6*(c^(1/2)*d+a^(1/2)*e)*arctan(1/3*(a^(1/6)+2
*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2)/a^(5/6)/c^(2/3)-1/6*(c^(1/2)*d+a^(1/2
)*e)*ln(a^(1/6)-c^(1/6)*x)/a^(5/6)/c^(2/3)+1/6*(d-a^(1/2)*e/c^(1/2))*ln(a^
(1/6)+c^(1/6)*x)/a^(5/6)/c^(1/6)-1/12*(d-a^(1/2)*e/c^(1/2))*ln(a^(1/3)-a^(
1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(5/6)/c^(1/6)+1/12*(c^(1/2)*d+a^(1/2)*e)*ln(
a^(1/3)+a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(5/6)/c^(2/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.04

$$\int \frac{d+ex^3}{a-cx^6}\,dx$$
$$= \frac{-2\sqrt{3}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)+2\sqrt{3}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{1+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)-2\sqrt{c}d\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right.}{}$$

input
```
Integrate[(d + e*x^3)/(a - c*x^6),x]
```

output
```
(-2*Sqrt[3]*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqr
t[3]] + 2*Sqrt[3]*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6
))/Sqrt[3]] - 2*Sqrt[c]*d*Log[a^(1/6) - c^(1/6)*x] - 2*Sqrt[a]*e*Log[a^(1/
6) - c^(1/6)*x] + 2*Sqrt[c]*d*Log[a^(1/6) + c^(1/6)*x] - 2*Sqrt[a]*e*Log[a
^(1/6) + c^(1/6)*x] - Sqrt[c]*d*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*
x^2] + Sqrt[a]*e*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[c]*
d*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + Sqrt[a]*e*Log[a^(1/3) +
 a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(2/3))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1747, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{a-cx^6}\,dx$$
$$\Big\downarrow \text{1747}$$
$$\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{a-\sqrt{a}\sqrt{c}x^3}dx+\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a}\sqrt{c}x^3+a}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}-\sqrt[6]{c}x\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(\sqrt[6]{c}x+2\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x}dx}{3a^{2/3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}-\sqrt[6]{c}x\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(\sqrt[6]{c}x+2\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\int \frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{a}\sqrt[6]{c}\left(\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[6]{c}\left(\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[3]{a}}}{3\sqrt{a}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{\int\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[3]{a}}+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx-\frac{3\int\frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

$\downarrow$ 217

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\frac{\int\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}+\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)+\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

$\downarrow$ 1103

$$\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}+\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}-\frac{\log\left(-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}\right)+$$

$$\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}+\frac{\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)$$

input
```
Int[(d + e*x^3)/(a - c*x^6),x]
```

output
```
((d - (Sqrt[a]*e)/Sqrt[c])*(Log[a^(1/6) + c^(1/6)*x]/(3*a^(5/6)*c^(1/6)) +
 (-((Sqrt[3]*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]])/(a^(1/3)*c^(1/6)
)) - Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*a^(1/3)*c^(1/6)))/(
3*Sqrt[a])))/2 + ((d + (Sqrt[a]*e)/Sqrt[c])*(-1/3*Log[a^(1/6) - c^(1/6)*x]
/(a^(5/6)*c^(1/6)) + ((Sqrt[3]*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]]
)/(a^(1/3)*c^(1/6)) + Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*a^
(1/3)*c^(1/6)))/(3*Sqrt[a])))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1747
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-a/c, 2]}, Simp[(d + e*q)/2   Int[1/(a + c*q*x^n), x], x] + Simp[(d
- e*q)/2   Int[1/(a - c*q*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ
[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NegQ[a*c] && IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.11

method	result
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c-a\right)}\frac{\left(\_R^{3}e+d\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6c}$
default	$\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}}x-x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)e}{12a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}}x-x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)d}{12a}-\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}\sqrt{3}\,e\arctan\left(-\frac{\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\sqrt{3}\,d\,\mathrm{arcta}}{}$

input
```
int((e*x^3+d)/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6/c*sum((_R^3*e+d)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1613 vs. $2(223) = 446$.

Time = 0.16 (sec) , antiderivative size = 1613, normalized size of antiderivative = 4.99

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(-3) + 1)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*
e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 + 2*a*
c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 + 3*a^2*c*d^2*e^2 + sqrt(-3)*(
a*c^2*d^4 + 3*a^2*c*d^2*e^2) - (sqrt(-3)*a^4*c^2*e + a^4*c^2*e)*sqrt((c^2*
d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt((c^2*d^6
+ 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2)
)^(1/3)) + 1/12*(sqrt(-3) - 1)*(-(a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 +
9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*
d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 + 3*a^2*c*d^2*e^2 -
sqrt(-3)*(a*c^2*d^4 + 3*a^2*c*d^2*e^2) + (sqrt(-3)*a^4*c^2*e - a^4*c^2*e)*
sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(-(a^2*c^2*sqrt
((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e + a*e^3)
/(a^2*c^2))^(1/3)) - 1/12*(sqrt(-3) + 1)*((a^2*c^2*sqrt((c^2*d^6 + 6*a*c*d
^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*l
og(-(c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 + 3*a^2*c*d
^2*e^2 + sqrt(-3)*(a*c^2*d^4 + 3*a^2*c*d^2*e^2) + (sqrt(-3)*a^4*c^2*e + a^
4*c^2*e)*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*
c^2*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e
- a*e^3)/(a^2*c^2))^(1/3)) + 1/12*(sqrt(-3) - 1)*((a^2*c^2*sqrt((c^2*d^6 +
 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e - a*e^3)/(a^2*c^...
```

Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.52

$$\int \frac{d+ex^3}{a-cx^6}\,dx =$$
$$-\operatorname{RootSum}\left(46656t^6a^5c^4 + t^3\left(-432a^4c^2e^3 - 1296a^3c^3d^2e\right) + a^3e^6 - 3a^2cd^2e^4 + 3ac^2d^4e^2 - c^3d^6, \left(t \mapsto\right.\right.$$

input
```
integrate((e*x**3+d)/(-c*x**6+a),x)
```

output
```
-RootSum(46656*_t**6*a**5*c**4 + _t**3*(-432*a**4*c**2*e**3 - 1296*a**3*c*
*3*d**2*e) + a**3*e**6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d*
*6, Lambda(_t, _t*log(x + (-1296*_t**4*a**4*c**2*e + 6*_t*a**3*e**4 + 36*_
t*a**2*c*d**2*e**2 + 6*_t*a*c**2*d**4)/(3*a**2*d*e**4 - 2*a*c*d**3*e**2 -
c**2*d**5))))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.97

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \frac{\sqrt{3}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{6\,\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{6\,\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(x^2+x\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}\right)}{12\,\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(x^2-x\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}\right)}{12\,\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} + \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{6\,\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{6\,\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/3*sqrt(3)*(2*x + (sqrt(a)/sqr
t(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3))
+ 1/6*sqrt(3)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/3*sqrt(3)*(2*x - (sqrt(a)/s
qrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)
) + 1/12*(sqrt(c)*d + sqrt(a)*e)*log(x^2 + x*(sqrt(a)/sqrt(c))^(1/3) + (sq
rt(a)/sqrt(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - 1/12*(sqrt(c)*
d - sqrt(a)*e)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2/
3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) + 1/6*(sqrt(c)*d - sqrt(a)*e)*log(
x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - 1/6*(sq
rt(c)*d + sqrt(a)*e)*log(x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/
sqrt(c))^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.94

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \frac{e|c|\log\left(x^2+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\left(-ac^5\right)^{\frac{1}{3}}} + \frac{\left(-ac^5\right)^{\frac{1}{6}} d\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac} + \frac{\left(\left(-ac^5\right)^{\frac{1}{6}} c^3d - \sqrt{3}\left(-ac^5\right)^{\frac{2}{3}} e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\left(-ac^5\right)^{\frac{1}{6}} c^3d + \sqrt{3}\left(-ac^5\right)^{\frac{2}{3}} e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}} c^3d + \left(-ac^5\right)^{\frac{2}{3}} e\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4} - \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}} c^3d - \left(-ac^5\right)^{\frac{2}{3}} e\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$

input
```
integrate((e*x^3+d)/(-c*x^6+a),x, algorithm="giac")
```

output
```
1/6*e*abs(c)*log(x^2 + (-a/c)^(1/3))/(-a*c^5)^(1/3) + 1/3*(-a*c^5)^(1/6)*d
*arctan(x/(-a/c)^(1/6))/(a*c) + 1/6*((-a*c^5)^(1/6)*c^3*d - sqrt(3)*(-a*c^
5)^(2/3)*e)*arctan((2*x + sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/
6*((-a*c^5)^(1/6)*c^3*d + sqrt(3)*(-a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*
(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*d +
 (-a*c^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4
) - 1/12*(sqrt(3)*(-a*c^5)^(1/6)*c^3*d - (-a*c^5)^(2/3)*e)*log(x^2 - sqrt(
3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4)
```

Mupad [B] (verification not implemented)

Time = 21.13 (sec) , antiderivative size = 1293, normalized size of antiderivative = 4.00

$$\int \frac{d+ex^3}{a-cx^6}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^3)/(a - c*x^6),x)
```

output
```
log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a
*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + e*x*(a^5*c^5)^(1/2) + a^2*c^3*d
*x)*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(
a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*
(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^
(1/3) - e*x*(a^5*c^5)^(1/2) + a^2*c^3*d*x)*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5
)^(1/2) + 3*a^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3
) - log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e +
 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) - 2*e*x*(a^5*c^5)^(1/2) + 3^(
1/2)*a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*
a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*x)*((3^(1/2)*1i
)/2 + 1/2)*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*
d*e^2*(a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(e*x*(a^5*c^5)^(1/2) - (a
^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^
2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3))/2 + (3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 +
 c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5
*c^4))^(1/3)*1i)/2 + a^2*c^3*d*x)*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*c^2*e^3 +
c*d^3*(a^5*c^5)^(1/2) + 3*a^3*c^3*d^2*e + 3*a*d*e^2*(a^5*c^5)^(1/2))/(216*
a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d^3*(a^5*c^5)^(1/2) + 3*a
^3*c^3*d^2*e - 3*a*d*e^2*(a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + 2*e*x*(a^5...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.02

$$\int \frac{d+ex^3}{a-cx^6}\,dx$$
$$= \frac{-2\sqrt{c}\,\sqrt{a}\,\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)d+2\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)ae+2\sqrt{c}\,\sqrt{a}\,\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)d+2\sqrt{3}\,a}{}$$

input
```
int((e*x^3+d)/(-c*x^6+a),x)
```

output
```
( - 2*sqrt(c)*sqrt(a)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**
(1/6)*a**(1/6)*sqrt(3)))*d + 2*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3
)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*e + 2*sqrt(c)*sqrt(a)*sqrt(3)*atan((c*
*(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*d + 2*sqrt(3)
*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*e
- sqrt(c)*sqrt(a)*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*d
 + 2*sqrt(c)*sqrt(a)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*d + sqrt(c)*sq
rt(a)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*d - 2*sqrt(c)*sq
rt(a)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*d + log( - c**(1/6)*a**(1/6)*x +
 a**(1/3) + c**(1/3)*x**2)*a*e - 2*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*
a*e + log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*a*e - 2*log(c**(
1/6)*a**(1/6) - c**(1/3)*x)*a*e)/(12*c**(2/3)*a**(1/3)*a)
```

3.10 $\int \frac{d+ex^3}{x^2(a-cx^6)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 335

$$\int \frac{d+ex^3}{x^2(a-cx^6)}\,dx = -\frac{d}{ax} + \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{7/6}\sqrt[3]{c}} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{7/6}\sqrt[3]{c}} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{7/6}\sqrt[3]{c}} + \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{7/6}\sqrt[3]{c}} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{7/6}\sqrt[3]{c}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{7/6}\sqrt[3]{c}}$$

output
```
-d/a/x+1/6*(c^(1/2)*d-a^(1/2)*e)*arctan(1/3*(a^(1/6)-2*c^(1/6)*x)*3^(1/2)/
a^(1/6))*3^(1/2)/a^(7/6)/c^(1/3)-1/6*(c^(1/2)*d+a^(1/2)*e)*arctan(1/3*(a^(
1/6)+2*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2)/a^(7/6)/c^(1/3)-1/6*(c^(1/2)*d+
a^(1/2)*e)*ln(a^(1/6)-c^(1/6)*x)/a^(7/6)/c^(1/3)+1/6*(c^(1/2)*d-a^(1/2)*e)
*ln(a^(1/6)+c^(1/6)*x)/a^(7/6)/c^(1/3)-1/12*(c^(1/2)*d-a^(1/2)*e)*ln(a^(1/
3)-a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(7/6)/c^(1/3)+1/12*(c^(1/2)*d+a^(1/2)*
e)*ln(a^(1/3)+a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(7/6)/c^(1/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.08

$$\int \frac{d+ex^3}{x^2\left(a-cx^6\right)}\,dx = -\frac{d}{ax} - \frac{\left(a^{5/6}cd-a^{4/3}\sqrt{c}e\right)\arctan\left(\frac{-\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^2c^{5/6}} - \frac{\left(a^{5/6}cd+a^{4/3}\sqrt{c}e\right)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^2c^{5/6}} + \frac{\left(-a^{5/6}cd-a^{4/3}\sqrt{c}e\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^2c^{5/6}} + \frac{\left(a^{5/6}cd-a^{4/3}\sqrt{c}e\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^2c^{5/6}} + \frac{\left(-a^{5/6}cd+a^{4/3}\sqrt{c}e\right)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^2c^{5/6}} + \frac{\left(a^{5/6}cd+a^{4/3}\sqrt{c}e\right)\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^2c^{5/6}}$$

input
```
Integrate[(d + e*x^3)/(x^2*(a - c*x^6)),x]
```

output
```
-(d/(a*x)) - ((a^(5/6)*c*d - a^(4/3)*Sqrt[c]*e)*ArcTan[(-a^(1/6) + 2*c^(1/
6)*x)/(Sqrt[3]*a^(1/6))]])/(2*Sqrt[3]*a^2*c^(5/6)) - ((a^(5/6)*c*d + a^(4/3
)*Sqrt[c]*e)*ArcTan[(a^(1/6) + 2*c^(1/6)*x)/(Sqrt[3]*a^(1/6))]])/(2*Sqrt[3]
*a^2*c^(5/6)) + ((-(a^(5/6)*c*d) - a^(4/3)*Sqrt[c]*e)*Log[a^(1/6) - c^(1/6
)*x]])/(6*a^2*c^(5/6)) + ((a^(5/6)*c*d - a^(4/3)*Sqrt[c]*e)*Log[a^(1/6) + c
^(1/6)*x]])/(6*a^2*c^(5/6)) + ((-(a^(5/6)*c*d) + a^(4/3)*Sqrt[c]*e)*Log[a^(
1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]])/(12*a^2*c^(5/6)) + ((a^(5/6)*c*d
+ a^(4/3)*Sqrt[c]*e)*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]])/(12*a
^2*c^(5/6))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.85, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1829, 25, 1835, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^2\left(a-cx^6\right)}dx$$

$$\Big\downarrow \ 1829$$

$$-\frac{\int -\frac{x\left(cdx^3+ae\right)}{a-cx^6}dx}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{x\left(cdx^3+ae\right)}{a-cx^6}dx}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 1835$$

$$\frac{\frac{1}{2}\sqrt{c}\left(\sqrt{a}e+\sqrt{c}d\right)\int \frac{x}{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^3\right)}dx-\frac{1}{2}\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)\int \frac{x}{\sqrt{c}\left(\sqrt{c}x^3+\sqrt{a}\right)}dx}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\int \frac{x}{\sqrt{a}-\sqrt{c}x^3}dx-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\int \frac{x}{\sqrt{c}x^3+\sqrt{a}}dx}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 821$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\int \frac{1}{\sqrt[6]{a}-\sqrt[6]{c}x}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\int \frac{\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\cdots\right.}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{\int \frac{\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log(}{3}\right.}{a}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\right.}{a}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\right.}{a}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\right.}{a}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{-\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}{\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)}{a}-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)$$

$$\frac{d}{ax}$$

$\Big\downarrow$ 217

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{-\frac{1}{2}\int \frac{}{\sqrt[3]{c}x}\right.}{a}$$

$$\frac{d}{ax}$$

$\Big\downarrow$ 1103

$$\frac{\frac{1}{2}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{3\sqrt[6]{a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\log\left(-\sqrt[6]{a}\sqrt[6]{}\right.}{}\right.}{a}$$

$$\frac{d}{ax}$$

input
```
Int[(d + e*x^3)/(x^2*(a - c*x^6)),x]
```

output
```
-(d/(a*x)) + (-1/2*((Sqrt[c]*d - Sqrt[a]*e)*(-1/3*Log[a^(1/6) + c^(1/6)*x]
/(a^(1/6)*c^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt[3
]])/c^(1/6)) + Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/6)))
/(3*a^(1/6)*c^(1/6)))) + ((Sqrt[c]*d + Sqrt[a]*e)*(-1/3*Log[a^(1/6) - c^(1
/6)*x]/(a^(1/6)*c^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6))/Sq
rt[3]])/c^(1/6) - Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/6
)))/(3*a^(1/6)*c^(1/6))))/2)/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1829
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^(2*n))^(p + 1)/(a*f*(m + 1
))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m + n)*(a + c*x^(2*n))^p*(a*e
*(m + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f,
p}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

rule 1835
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[-(e/2 + c*(d/(2*q)))   Int[(f
*x)^m/(q - c*x^n), x], x] + Simp[(e/2 - c*(d/(2*q)))   Int[(f*x)^m/(q + c*x
^n), x], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{d}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{7}c^{2}\_Z^{6}+\left(2a^{5}c\,e^{3}+6a^{4}c^{2}d^{2}e\right)\_Z^{3}+a^{3}e^{6}-3a^{2}c\,d^{2}e^{4}+3a\,c^{2}d^{4}e^{2}-c^{3}d^{6}\right)}\_R\ln\left(\left(7\_R^{6}a^{7}c^{2}+\left(13a^{5}c\,e^{3}+39\right.\right.\right.\right.}{6}$
default	$-\frac{d}{ax}-\frac{\frac{c\ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}}x-x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{5}{6}}d}{12a}-\frac{\ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}}x-x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{1}{3}}e}{12}+\frac{\sqrt{3}\,d\arctan\left(-\frac{\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{3}}e\arctan\left(-\frac{\sqrt{3}}{3}\right.}{6}}{}$

input
```
int((e*x^3+d)/x^2/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-d/a/x+1/6*sum(_R*ln((7*_R^6*a^7*c^2+(13*a^5*c*e^3+39*a^4*c^2*d^2*e)*_R^3+
6*a^3*e^6-18*a^2*c*d^2*e^4+18*a*c^2*d^4*e^2-6*c^3*d^6)*x+a^6*c^2*d*_R^5+(-
2*a^4*c*d*e^3+2*a^3*c^2*d^3*e)*_R^2),_R=RootOf(a^7*c^2*_Z^6+(2*a^5*c*e^3+6
*a^4*c^2*d^2*e)*_Z^3+a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1845 vs. $2(235) = 470$.

Time = 0.32 (sec) , antiderivative size = 1845, normalized size of antiderivative = 5.51

$$\int \frac{d+ex^3}{x^2\,(a-cx^6)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^3+d)/x^2/(-c*x^6+a),x, algorithm="fricas")
```

output
```
1/12*(2*a*x*(-(a^3*c*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^7*c
)) + 3*c*d^2*e + a*e^3)/(a^3*c))^(1/3)*log((c^3*d^7 + a*c^2*d^5*e^2 - 5*a^
2*c*d^3*e^4 + 3*a^3*d*e^6)*x + (2*a^3*c^2*d^4*e + 6*a^4*c*d^2*e^3 - (a^6*c
^2*d^2 + a^7*c*e^2)*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^7*c)
))*(-(a^3*c*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^7*c)) + 3*c*
d^2*e + a*e^3)/(a^3*c))^(2/3)) + 2*a*x*((a^3*c*sqrt((c^2*d^6 + 6*a*c*d^4*e
^2 + 9*a^2*d^2*e^4)/(a^7*c)) - 3*c*d^2*e - a*e^3)/(a^3*c))^(1/3)*log((c^3*
d^7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*e^6)*x + (2*a^3*c^2*d^4*e
+ 6*a^4*c*d^2*e^3 + (a^6*c^2*d^2 + a^7*c*e^2)*sqrt((c^2*d^6 + 6*a*c*d^4*e^
2 + 9*a^2*d^2*e^4)/(a^7*c)))*((a^3*c*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2
*d^2*e^4)/(a^7*c)) - 3*c*d^2*e - a*e^3)/(a^3*c))^(2/3)) + (sqrt(-3)*a*x -
a*x)*(-(a^3*c*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^7*c)) + 3*
c*d^2*e + a*e^3)/(a^3*c))^(1/3)*log((c^3*d^7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3
*e^4 + 3*a^3*d*e^6)*x - 1/2*(2*a^3*c^2*d^4*e + 6*a^4*c*d^2*e^3 + 2*sqrt(-3
)*(a^3*c^2*d^4*e + 3*a^4*c*d^2*e^3) - (a^6*c^2*d^2 + a^7*c*e^2 + sqrt(-3)*
(a^6*c^2*d^2 + a^7*c*e^2))*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/
(a^7*c)))*(-(a^3*c*sqrt((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^7*c))
 + 3*c*d^2*e + a*e^3)/(a^3*c))^(2/3)) - (sqrt(-3)*a*x + a*x)*(-(a^3*c*sqrt
((c^2*d^6 + 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^7*c)) + 3*c*d^2*e + a*e^3)/(
a^3*c))^(1/3)*log((c^3*d^7 + a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 + 3*a^3*d*...
```

Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.62

$$\int \frac{d+ex^3}{x^2\,(a-cx^6)}\,dx =$$
$$-\operatorname{RootSum}\left(46656t^6a^7c^2+t^3\left(-432a^5ce^3-1296a^4c^2d^2e\right)+a^3e^6-3a^2cd^2e^4+3ac^2d^4e^2-c^3d^6,\left(t\mapsto\right.\right.$$
$$-\frac{d}{ax}$$

input
```
integrate((e*x**3+d)/x**2/(-c*x**6+a),x)
```

output
```
-RootSum(46656*_t**6*a**7*c**2 + _t**3*(-432*a**5*c*e**3 - 1296*a**4*c**2*
d**2*e) + a**3*e**6 - 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 - c**3*d**6,
 Lambda(_t, _t*log(x + (-7776*_t**5*a**7*c*e**2 - 7776*_t**5*a**6*c**2*d**
2 + 36*_t**2*a**5*e**5 + 360*_t**2*a**4*c*d**2*e**3 + 180*_t**2*a**3*c**2*
d**4*e)/(3*a**3*d*e**6 - 5*a**2*c*d**3*e**4 + a*c**2*d**5*e**2 + c**3*d**7
)))) - d/(a*x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.01

$$\int \frac{d+ex^3}{x^2\,(a-cx^6)}\,dx =$$
$$-\frac{\frac{2\sqrt{3}\left(\sqrt{a}cd+a\sqrt{c}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\left(\sqrt{a}cd-a\sqrt{c}e\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}-\frac{\left(\sqrt{a}cd+a\sqrt{c}e\right)\log\left(x^2+x\left(\frac{\sqrt{a}}{\sqrt{c}}\right.\right.}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}}{}$$
$$-\frac{d}{ax}$$

input
```
integrate((e*x^3+d)/x^2/(-c*x^6+a),x, algorithm="maxima")
```

output
```
-1/12*(2*sqrt(3)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/3*sqrt(3)*(2*x + (sq
rt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c)
)^(1/3)) + 2*sqrt(3)*(sqrt(a)*c*d - a*sqrt(c)*e)*arctan(1/3*sqrt(3)*(2*x -
 (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqr
t(c))^(1/3)) - (sqrt(a)*c*d + a*sqrt(c)*e)*log(x^2 + x*(sqrt(a)/sqrt(c))^(
1/3) + (sqrt(a)/sqrt(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(1/3)) + (sqr
t(a)*c*d - a*sqrt(c)*e)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqr
t(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(1/3)) - 2*(sqrt(a)*c*d - a*sqrt
(c)*e)*log(x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(1/3)
) + 2*(sqrt(a)*c*d + a*sqrt(c)*e)*log(x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a
)*c*(sqrt(a)/sqrt(c))^(1/3)))/a - d/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.96

$$\int \frac{d+ex^3}{x^2(a-cx^6)}\,dx = \frac{\left(-ac^5\right)^{\frac{1}{3}}e\log\left(x^2+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,ac^2} - \frac{d}{ax} + \frac{\left(-ac^5\right)^{\frac{5}{6}}d\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,a^2c^4} - \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{3}}ac^2e - \left(-ac^5\right)^{\frac{5}{6}}d\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,a^2c^4} + \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{3}}ac^2e + \left(-ac^5\right)^{\frac{5}{6}}d\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,a^2c^4} - \frac{\left(\left(-ac^5\right)^{\frac{1}{3}}ac^2e + \sqrt{3}\left(-ac^5\right)^{\frac{5}{6}}d\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,a^2c^4} - \frac{\left(\left(-ac^5\right)^{\frac{1}{3}}ac^2e - \sqrt{3}\left(-ac^5\right)^{\frac{5}{6}}d\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,a^2c^4}$$

input
```
integrate((e*x^3+d)/x^2/(-c*x^6+a),x, algorithm="giac")
```

output
```
1/6*(-a*c^5)^(1/3)*e*log(x^2 + (-a/c)^(1/3))/(a*c^2) - d/(a*x) + 1/3*(-a*c
^5)^(5/6)*d*arctan(x/(-a/c)^(1/6))/(a^2*c^4) - 1/6*(sqrt(3)*(-a*c^5)^(1/3)
*a*c^2*e - (-a*c^5)^(5/6)*d)*arctan((2*x + sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1
/6))/(a^2*c^4) + 1/6*(sqrt(3)*(-a*c^5)^(1/3)*a*c^2*e + (-a*c^5)^(5/6)*d)*a
rctan((2*x - sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a^2*c^4) - 1/12*((-a*c^5
)^(1/3)*a*c^2*e + sqrt(3)*(-a*c^5)^(5/6)*d)*log(x^2 + sqrt(3)*x*(-a/c)^(1/
6) + (-a/c)^(1/3))/(a^2*c^4) - 1/12*((-a*c^5)^(1/3)*a*c^2*e - sqrt(3)*(-a*
c^5)^(5/6)*d)*log(x^2 - sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 22.62 (sec) , antiderivative size = 1487, normalized size of antiderivative = 4.44

$$\int \frac{d+ex^3}{x^2(a-cx^6)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^3)/(x^2*(a - c*x^6)),x)
```

output
```
log(a*e^2*x*(a^7*c^3)^(1/2) - a^6*c^2*(-(a^5*c*e^3 + c*d^3*(a^7*c^3)^(1/2)
 + 3*a^4*c^2*d^2*e + 3*a*d*e^2*(a^7*c^3)^(1/2))/(a^7*c^2))^(2/3) + c*d^2*x
*(a^7*c^3)^(1/2) + 2*a^4*c^2*d*e*x)*(-(a^5*c*e^3 + c*d^3*(a^7*c^3)^(1/2) +
 3*a^4*c^2*d^2*e + 3*a*d*e^2*(a^7*c^3)^(1/2))/(216*a^7*c^2))^(1/3) + log(a
^6*c^2*(-(a^5*c*e^3 - c*d^3*(a^7*c^3)^(1/2) + 3*a^4*c^2*d^2*e - 3*a*d*e^2*
(a^7*c^3)^(1/2))/(a^7*c^2))^(2/3) + a*e^2*x*(a^7*c^3)^(1/2) + c*d^2*x*(a^7
*c^3)^(1/2) - 2*a^4*c^2*d*e*x)*(-(a^5*c*e^3 - c*d^3*(a^7*c^3)^(1/2) + 3*a^
4*c^2*d^2*e - 3*a*d*e^2*(a^7*c^3)^(1/2))/(216*a^7*c^2))^(1/3) - d/(a*x) -
log(- (((3^(1/2)*1i)/2 - 1/2)*(-(a^5*c*e^3 + c*d^3*(a^7*c^3)^(1/2) + 3*a^4
*c^2*d^2*e + 3*a*d*e^2*(a^7*c^3)^(1/2))/(a^7*c^2))^(2/3)*(36*a^9*c^6*d^3 +
 108*a^10*c^5*d*e^2 - 36*a^10*c^5*x*((3^(1/2)*1i)/2 + 1/2)*(a*e^2 + c*d^2)
*(-(a^5*c*e^3 + c*d^3*(a^7*c^3)^(1/2) + 3*a^4*c^2*d^2*e + 3*a*d*e^2*(a^7*c
^3)^(1/2))/(a^7*c^2))^(1/3)))/36 - a^7*c^4*e*x*(a*e^2 - c*d^2)^2)*((3^(1/2
)*1i)/2 + 1/2)*(-(a^5*c*e^3 + c*d^3*(a^7*c^3)^(1/2) + 3*a^4*c^2*d^2*e + 3*
a*d*e^2*(a^7*c^3)^(1/2))/(216*a^7*c^2))^(1/3) + log((((3^(1/2)*1i)/2 + 1/2
)*(-(a^5*c*e^3 + c*d^3*(a^7*c^3)^(1/2) + 3*a^4*c^2*d^2*e + 3*a*d*e^2*(a^7*
c^3)^(1/2))/(a^7*c^2))^(2/3)*(36*a^9*c^6*d^3 + 108*a^10*c^5*d*e^2 + 36*a^1
0*c^5*x*((3^(1/2)*1i)/2 - 1/2)*(a*e^2 + c*d^2)*(-(a^5*c*e^3 + c*d^3*(a^7*c
^3)^(1/2) + 3*a^4*c^2*d^2*e + 3*a*d*e^2*(a^7*c^3)^(1/2))/(a^7*c^2))^(1/3))
)/36 - a^7*c^4*e*x*(a*e^2 - c*d^2)^2)*((3^(1/2)*1i)/2 - 1/2)*(-(a^5*c*e...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.05

$$\int \frac{d+ex^3}{x^2\left(a-cx^6\right)}\,dx$$
$$= \frac{2\sqrt{c}\,\sqrt{a}\,\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)dx - 2\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)aex - 2\sqrt{c}\,\sqrt{a}\,\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)dx - 2\sqrt{3}}{}$$

input
```
int((e*x^3+d)/x^2/(-c*x^6+a),x)
```

output
```
(2*sqrt(c)*sqrt(a)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/
6)*a**(1/6)*sqrt(3)))*d*x - 2*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)
*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*e*x - 2*sqrt(c)*sqrt(a)*sqrt(3)*atan((c
**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*d*x - 2*sqrt
(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a
*e*x - sqrt(c)*sqrt(a)*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x*
*2)*d*x + 2*sqrt(c)*sqrt(a)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*d*x + s
qrt(c)*sqrt(a)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*d*x - 2
*sqrt(c)*sqrt(a)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*d*x - 12*c**(1/3)*a**
(2/3)*d + log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*a*e*x - 2
*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*a*e*x + log(c**(1/6)*a**(1/6)*x +
a**(1/3) + c**(1/3)*x**2)*a*e*x - 2*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*a*
e*x)/(12*c**(1/3)*a**(2/3)*a*x)
```

3.11 $\int \frac{d+ex^3}{x^3(a-cx^6)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 337

$$\begin{aligned}\int \frac{d+ex^3}{x^3(a-cx^6)}\,dx = & -\frac{d}{2ax^2}+\frac{(\sqrt{c}d-\sqrt{a}e)\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{4/3}\sqrt[6]{c}} \\ & +\frac{(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{4/3}\sqrt[6]{c}} \\ & -\frac{(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{4/3}\sqrt[6]{c}}-\frac{(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{4/3}\sqrt[6]{c}} \\ & +\frac{(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{4/3}\sqrt[6]{c}} \\ & +\frac{(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{4/3}\sqrt[6]{c}}\end{aligned}$$

output

```
-1/2*d/a/x^2+1/6*(c^(1/2)*d-a^(1/2)*e)*arctan(1/3*(a^(1/6)-2*c^(1/6)*x)*3^
(1/2)/a^(1/6))*3^(1/2)/a^(4/3)/c^(1/6)+1/6*(c^(1/2)*d+a^(1/2)*e)*arctan(1/
3*(a^(1/6)+2*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2)/a^(4/3)/c^(1/6)-1/6*(c^(1
/2)*d+a^(1/2)*e)*ln(a^(1/6)-c^(1/6)*x)/a^(4/3)/c^(1/6)-1/6*(c^(1/2)*d-a^(1
/2)*e)*ln(a^(1/6)+c^(1/6)*x)/a^(4/3)/c^(1/6)+1/12*(c^(1/2)*d-a^(1/2)*e)*ln
(a^(1/3)-a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(4/3)/c^(1/6)+1/12*(c^(1/2)*d+a^
(1/2)*e)*ln(a^(1/3)+a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(4/3)/c^(1/6)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.96

$$\int \frac{d+ex^3}{x^3(a-cx^6)}\,dx$$

$$= \frac{-6ac^{2/3}d - 2\sqrt{3}\left(-a^{2/3}cd+a^{7/6}\sqrt{c}e\right)x^2\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)+2\sqrt{3}\left(a^{2/3}cd+a^{7/6}\sqrt{c}e\right)x^2\arctan\left(\frac{1+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{}$$

input
```
Integrate[(d + e*x^3)/(x^3*(a - c*x^6)),x]
```

output
```
(-6*a*c^(2/3)*d - 2*Sqrt[3]*(-(a^(2/3)*c*d) + a^(7/6)*Sqrt[c]*e)*x^2*ArcTa
n[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]] + 2*Sqrt[3]*(a^(2/3)*c*d + a^(7/6)*
Sqrt[c]*e)*x^2*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]] - 2*(a^(2/3)*c*
d + a^(7/6)*Sqrt[c]*e)*x^2*Log[a^(1/6) - c^(1/6)*x] + 2*(-(a^(2/3)*c*d) +
a^(7/6)*Sqrt[c]*e)*x^2*Log[a^(1/6) + c^(1/6)*x] + (a^(2/3)*c*d - a^(7/6)*S
qrt[c]*e)*x^2*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + (a^(2/3)*c*
d + a^(7/6)*Sqrt[c]*e)*x^2*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])
/(12*a^2*c^(2/3)*x^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.91, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1829, 27, 1747, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^3(a-cx^6)}\,dx$$

$$\Big\downarrow \; 1829$$

$$-\frac{\int -\frac{2(cdx^3+ae)}{a-cx^6}dx}{2a}-\frac{d}{2ax^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{cdx^3+ae}{a-cx^6}dx}{a}-\frac{d}{2ax^2}$$

$\Big\downarrow$ 1747

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\int \frac{1}{a-\sqrt{a}\sqrt{c}x^3}dx-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\sqrt{a}\sqrt{c}x^3+a}dx}{a}-\frac{d}{2ax^2}$$

$\Big\downarrow$ 750

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(\sqrt[6]{c}x+2\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x}dx}{3a^{2/3}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}-\sqrt[6]{c}x\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a\cdots}}{3a^{2/3}}\cdots\right.}{a}$$

$$\frac{d}{2ax^2}$$

$\Big\downarrow$ 16

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(\sqrt[6]{c}x+2\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}-\sqrt[6]{c}x\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}\cdots}{3a^{2/3}}\cdots\right.}{a}$$

$$\frac{d}{2ax^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\int \frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}}\cdots}{3\sqrt{a}}\cdots\right.}{a}$$

$$\frac{d}{2ax^2}$$

$\Big\downarrow$ 1142

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\right.}{a}$$

$$\frac{d}{2ax^2}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\right.}{a}$$

$$\frac{d}{2ax^2}$$

$\downarrow$ 27

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d\right.}{a}$$

$$\frac{d}{2ax^2}$$

$\downarrow$ 1082

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx-\frac{3\int\frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\right.}{a}$$

$$\frac{d}{2ax^2}$$

$\downarrow$ 217

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\cdots\right)}{a}$$

$$\frac{d}{2ax^2}$$

$\downarrow$ 1103

$$\frac{\frac{1}{2}\sqrt{a}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}+\frac{\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\frac{\log\left(\sqrt[6]{\cdots}\right.}{3a\cdots}\right.}{a}$$

$$\frac{d}{2ax^2}$$

input

```
Int[(d + e*x^3)/(x^3*(a - c*x^6)),x]
```

output

```
-1/2*d/(a*x^2) + (-1/2*(Sqrt[a]*(Sqrt[c]*d - Sqrt[a]*e)*(Log[a^(1/6) + c^(
1/6)*x]/(3*a^(5/6)*c^(1/6)) + (-((Sqrt[3]*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6
))/Sqrt[3]])/(a^(1/3)*c^(1/6))) - Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3
)*x^2]/(2*a^(1/3)*c^(1/6)))/(3*Sqrt[a]))) + (Sqrt[a]*(Sqrt[c]*d + Sqrt[a]*
e)*(-1/3*Log[a^(1/6) - c^(1/6)*x]/(a^(5/6)*c^(1/6)) + ((Sqrt[3]*ArcTan[(1
+ (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]])/(a^(1/3)*c^(1/6)) + Log[a^(1/3) + a^(1/
6)*c^(1/6)*x + c^(1/3)*x^2]/(2*a^(1/3)*c^(1/6)))/(3*Sqrt[a])))/2)/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1747
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-a/c, 2]}, Simp[(d + e*q)/2   Int[1/(a + c*q*x^n), x], x] + Simp[(d
- e*q)/2   Int[1/(a - c*q*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ
[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NegQ[a*c] && IntegerQ[n]
```

rule 1829
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^(2*n))^(p + 1)/(a*f*(m + 1
))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m + n)*(a + c*x^(2*n))^p*(a*e
*(m + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f,
p}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{d}{2a\,x^{2}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{8}c\,\_Z^{6}+\left(6a^{5}cd\,e^{2}+2a^{4}c^{2}d^{3}\right)\_Z^{3}-a^{3}e^{6}+3a^{2}c\,d^{2}e^{4}-3a\,c^{2}d^{4}e^{2}+c^{3}d^{6}\right)}\_R\ln\left(\left(7\_R^{6}a^{8}c+(39a^{5}cd\,e^{2}+\right.\right.\right.}{6}$
default	$-\frac{d}{2a\,x^{2}}+\frac{\frac{c\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}}x-x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)d}{12a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}}x-x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)e}{12}-\frac{c\left(\frac{a}{c}\right)^{\frac{2}{3}}\sqrt{3}\,d\arctan\left(-\frac{\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\sqrt{3}\,e\,\mathrm{arc}}{}}{}$

input
```
int((e*x^3+d)/x^3/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/a/x^2+1/6*sum(_R*ln((7*_R^6*a^8*c+(39*a^5*c*d*e^2+13*a^4*c^2*d^3)*_
R^3-6*a^3*e^6+18*a^2*c*d^2*e^4-18*a*c^2*d^4*e^2+6*c^3*d^6)*x-2*a^6*c*d*e*_
R^4+(a^4*e^5-2*a^3*c*d^2*e^3+a^2*c^2*d^4*e)*_R),_R=RootOf(a^8*c*_Z^6+(6*a^
5*c*d*e^2+2*a^4*c^2*d^3)*_Z^3-a^3*e^6+3*a^2*c*d^2*e^4-3*a*c^2*d^4*e^2+c^3*
d^6))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1572 vs. $2(235) = 470$.

Time = 0.11 (sec) , antiderivative size = 1572, normalized size of antiderivative = 4.66

$$\int \frac{d + ex^3}{x^3 (a - cx^6)} \, dx = \text{Too large to display}$$

input
```
integrate((e*x^3+d)/x^3/(-c*x^6+a),x, algorithm="fricas")
```

output
```
1/12*(2*a*x^2*(-(a^4*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c
)) + c*d^3 + 3*a*d*e^2)/a^4)^(1/3)*log(-(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2
*e^5)*x - (a^5*c*d*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c))
 - 3*a^2*c*d^2*e^2 - a^3*e^4)*(-(a^4*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 +
 a^2*e^6)/(a^7*c)) + c*d^3 + 3*a*d*e^2)/a^4)^(1/3)) + 2*a*x^2*((a^4*sqrt((
9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)) - c*d^3 - 3*a*d*e^2)/a^4
)^(1/3)*log(-(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*e^5)*x + (a^5*c*d*sqrt((9*
c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)) + 3*a^2*c*d^2*e^2 + a^3*e^
4)*((a^4*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)) - c*d^3 -
 3*a*d*e^2)/a^4)^(1/3)) - (sqrt(-3)*a*x^2 + a*x^2)*(-(a^4*sqrt((9*c^2*d^4*
e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)) + c*d^3 + 3*a*d*e^2)/a^4)^(1/3)*lo
g(-(3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*e^5)*x - 1/2*(3*a^2*c*d^2*e^2 + a^3*
e^4 + sqrt(-3)*(3*a^2*c*d^2*e^2 + a^3*e^4) - (sqrt(-3)*a^5*c*d + a^5*c*d)*
sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)))*(-(a^4*sqrt((9*c^
2*d^4*e^2 + 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)) + c*d^3 + 3*a*d*e^2)/a^4)^(1
/3)) + (sqrt(-3)*a*x^2 - a*x^2)*(-(a^4*sqrt((9*c^2*d^4*e^2 + 6*a*c*d^2*e^4
 + a^2*e^6)/(a^7*c)) + c*d^3 + 3*a*d*e^2)/a^4)^(1/3)*log(-(3*c^2*d^4*e - 2
*a*c*d^2*e^3 - a^2*e^5)*x - 1/2*(3*a^2*c*d^2*e^2 + a^3*e^4 - sqrt(-3)*(3*a
^2*c*d^2*e^2 + a^3*e^4) + (sqrt(-3)*a^5*c*d - a^5*c*d)*sqrt((9*c^2*d^4*e^2
 + 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)))*(-(a^4*sqrt((9*c^2*d^4*e^2 + 6*a*...
```

Sympy [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.51

$$\int \frac{d+ex^3}{x^3(a-cx^6)}\,dx = -\operatorname{RootSum}\left(46656t^6a^8c+t^3\left(-1296a^5cde^2-432a^4c^2d^3\right)-a^3e^6+3a^2cd^2e^4-3ac^2d^4e^2+c^3d^6,\left(t\mapsto t\right.\right.$$
$$-\frac{d}{2ax^2}$$

input
```
integrate((e*x**3+d)/x**3/(-c*x**6+a),x)
```

output
```
-RootSum(46656*_t**6*a**8*c + _t**3*(-1296*a**5*c*d*e**2 - 432*a**4*c**2*d
**3) - a**3*e**6 + 3*a**2*c*d**2*e**4 - 3*a*c**2*d**4*e**2 + c**3*d**6, La
mbda(_t, _t*log(x + (1296*_t**4*a**5*c*d - 6*_t*a**3*e**4 - 36*_t*a**2*c*d
**2*e**2 - 6*_t*a*c**2*d**4)/(a**2*e**5 + 2*a*c*d**2*e**3 - 3*c**2*d**4*e)
))) - d/(2*a*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00

$$\int \frac{d+ex^3}{x^3(a-cx^6)}\,dx$$
$$= \frac{\frac{2\sqrt{3}(\sqrt{a}cd+a\sqrt{c}e)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}-\frac{2\sqrt{3}(\sqrt{a}cd-a\sqrt{c}e)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{(\sqrt{a}cd+a\sqrt{c}e)\log\left(x^2+x\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right.}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}}{12}$$
$$-\frac{d}{2\,ax^2}$$

input
```
integrate((e*x^3+d)/x^3/(-c*x^6+a),x, algorithm="maxima")
```

output
```
1/12*(2*sqrt(3)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/3*sqrt(3)*(2*x + (sqr
t(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))
^(2/3)) - 2*sqrt(3)*(sqrt(a)*c*d - a*sqrt(c)*e)*arctan(1/3*sqrt(3)*(2*x -
(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt
(c))^(2/3)) + (sqrt(a)*c*d + a*sqrt(c)*e)*log(x^2 + x*(sqrt(a)/sqrt(c))^(1
/3) + (sqrt(a)/sqrt(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) + (sqrt
(a)*c*d - a*sqrt(c)*e)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt
(c))^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - 2*(sqrt(a)*c*d - a*sqrt(
c)*e)*log(x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3))
 - 2*(sqrt(a)*c*d + a*sqrt(c)*e)*log(x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)
*c*(sqrt(a)/sqrt(c))^(2/3)))/a - 1/2*d/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.96

$$\int \frac{d+ex^3}{x^3(a-cx^6)}\,dx = \frac{(-ac^5)^{\frac{1}{6}}\,e\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac} - \frac{d}{2\,ax^2} - \frac{(-ac^5)^{\frac{2}{3}}\,d|c|\log\left(x^2+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,a^2c^4} + \frac{\left((-ac^5)^{\frac{1}{6}}\,ac^2e-\sqrt{3}(-ac^5)^{\frac{2}{3}}\,d\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,a^2c^3} + \frac{\left((-ac^5)^{\frac{1}{6}}\,ac^2e+\sqrt{3}(-ac^5)^{\frac{2}{3}}\,d\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,a^2c^3} + \frac{\left(\sqrt{3}(-ac^5)^{\frac{1}{6}}\,ac^2e+(-ac^5)^{\frac{2}{3}}\,d\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,a^2c^3} - \frac{\left(\sqrt{3}(-ac^5)^{\frac{1}{6}}\,ac^2e-(-ac^5)^{\frac{2}{3}}\,d\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,a^2c^3}$$

input
```
integrate((e*x^3+d)/x^3/(-c*x^6+a),x, algorithm="giac")
```

output
```
1/3*(-a*c^5)^(1/6)*e*arctan(x/(-a/c)^(1/6))/(a*c) - 1/2*d/(a*x^2) - 1/6*(-
a*c^5)^(2/3)*d*abs(c)*log(x^2 + (-a/c)^(1/3))/(a^2*c^4) + 1/6*((-a*c^5)^(1
/6)*a*c^2*e - sqrt(3)*(-a*c^5)^(2/3)*d)*arctan((2*x + sqrt(3)*(-a/c)^(1/6)
)/(-a/c)^(1/6))/(a^2*c^3) + 1/6*((-a*c^5)^(1/6)*a*c^2*e + sqrt(3)*(-a*c^5)
^(2/3)*d)*arctan((2*x - sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a^2*c^3) + 1/
12*(sqrt(3)*(-a*c^5)^(1/6)*a*c^2*e + (-a*c^5)^(2/3)*d)*log(x^2 + sqrt(3)*x
*(-a/c)^(1/6) + (-a/c)^(1/3))/(a^2*c^3) - 1/12*(sqrt(3)*(-a*c^5)^(1/6)*a*c
^2*e - (-a*c^5)^(2/3)*d)*log(x^2 - sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/
(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 21.16 (sec) , antiderivative size = 1149, normalized size of antiderivative = 3.41

$$\int \frac{d+ex^3}{x^3\,(a-cx^6)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^3)/(x^3*(a - c*x^6)),x)
```

output
```
log(a^6*(-(a*e^3*(a^9*c)^(1/2) + a^4*c^2*d^3 + 3*a^5*c*d*e^2 + 3*c*d^2*e*(
a^9*c)^(1/2))/(a^8*c))^(1/3) + d*x*(a^9*c)^(1/2) + a^5*e*x)*(-(a*e^3*(a^9*
c)^(1/2) + a^4*c^2*d^3 + 3*a^5*c*d*e^2 + 3*c*d^2*e*(a^9*c)^(1/2))/(216*a^8
*c))^(1/3) + log(a^6*((a*e^3*(a^9*c)^(1/2) - a^4*c^2*d^3 - 3*a^5*c*d*e^2 +
 3*c*d^2*e*(a^9*c)^(1/2))/(a^8*c))^(1/3) - d*x*(a^9*c)^(1/2) + a^5*e*x)*((
a*e^3*(a^9*c)^(1/2) - a^4*c^2*d^3 - 3*a^5*c*d*e^2 + 3*c*d^2*e*(a^9*c)^(1/2
))/(216*a^8*c))^(1/3) - d/(2*a*x^2) - log(a^6*(-(a*e^3*(a^9*c)^(1/2) + a^4
*c^2*d^3 + 3*a^5*c*d*e^2 + 3*c*d^2*e*(a^9*c)^(1/2))/(a^8*c))^(1/3) - 2*d*x
*(a^9*c)^(1/2) + 3^(1/2)*a^6*(-(a*e^3*(a^9*c)^(1/2) + a^4*c^2*d^3 + 3*a^5*
c*d*e^2 + 3*c*d^2*e*(a^9*c)^(1/2))/(a^8*c))^(1/3)*1i - 2*a^5*e*x)*((3^(1/2
)*1i)/2 + 1/2)*(-(a*e^3*(a^9*c)^(1/2) + a^4*c^2*d^3 + 3*a^5*c*d*e^2 + 3*c*
d^2*e*(a^9*c)^(1/2))/(216*a^8*c))^(1/3) + log(a^6*((a*e^3*(a^9*c)^(1/2) -
a^4*c^2*d^3 - 3*a^5*c*d*e^2 + 3*c*d^2*e*(a^9*c)^(1/2))/(a^8*c))^(1/3) + 2*
d*x*(a^9*c)^(1/2) - 3^(1/2)*a^6*((a*e^3*(a^9*c)^(1/2) - a^4*c^2*d^3 - 3*a^
5*c*d*e^2 + 3*c*d^2*e*(a^9*c)^(1/2))/(a^8*c))^(1/3)*1i - 2*a^5*e*x)*((3^(1
/2)*1i)/2 - 1/2)*((a*e^3*(a^9*c)^(1/2) - a^4*c^2*d^3 - 3*a^5*c*d*e^2 + 3*c
*d^2*e*(a^9*c)^(1/2))/(216*a^8*c))^(1/3) - log(a^6*((a*e^3*(a^9*c)^(1/2) -
 a^4*c^2*d^3 - 3*a^5*c*d*e^2 + 3*c*d^2*e*(a^9*c)^(1/2))/(a^8*c))^(1/3) + 2
*d*x*(a^9*c)^(1/2) + 3^(1/2)*a^6*((a*e^3*(a^9*c)^(1/2) - a^4*c^2*d^3 - 3*a
^5*c*d*e^2 + 3*c*d^2*e*(a^9*c)^(1/2))/(a^8*c))^(1/3)*1i - 2*a^5*e*x)*((...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.18

$$\int \frac{d+ex^3}{x^3\left(a-cx^6\right)}\,dx$$

$$= \frac{-2\sqrt{c}\,a^{\frac{7}{6}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)e\,x^2+2a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)cd\,x^2+2\sqrt{c}\,a^{\frac{7}{6}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)e\,x^2 \cdots}{\cdots}$$

input
```
int((e*x^3+d)/x^3/(-c*x^6+a),x)
```

output

```
( - 2*sqrt(c)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c*
*(1/6)*a**(1/6)*sqrt(3)))*a*e*x**2 + 2*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**
(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*c*d*x**2 + 2*sqrt(c)*a*
*(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*
sqrt(3)))*a*e*x**2 + 2*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/
3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*c*d*x**2 - sqrt(c)*a**(1/6)*log( - c**(
1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*a*e*x**2 + 2*sqrt(c)*a**(1/6)*
log( - c**(1/6)*a**(1/6) - c**(1/3)*x)*a*e*x**2 + sqrt(c)*a**(1/6)*log(c**
(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*a*e*x**2 - 2*sqrt(c)*a**(1/6)
*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*a*e*x**2 + a**(2/3)*log( - c**(1/6)*a
**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*c*d*x**2 - 2*a**(2/3)*log( - c**(1/6
)*a**(1/6) - c**(1/3)*x)*c*d*x**2 + a**(2/3)*log(c**(1/6)*a**(1/6)*x + a**
(1/3) + c**(1/3)*x**2)*c*d*x**2 - 2*a**(2/3)*log(c**(1/6)*a**(1/6) - c**(1
/3)*x)*c*d*x**2 - 6*c**(2/3)*a*d)/(12*c**(2/3)*a**2*x**2)
```

3.12 $\int \frac{x^8(d+ex^3)}{a+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{x^8(d+ex^3)}{a+cx^6}\,dx=\frac{dx^3}{3c}+\frac{ex^6}{6c}-\frac{\sqrt{a}d\arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3c^{3/2}}-\frac{ae\log\left(a+cx^6\right)}{6c^2}$$

output
```
1/3*d*x^3/c+1/6*e*x^6/c-1/3*a^(1/2)*d*arctan(c^(1/2)*x^3/a^(1/2))/c^(3/2)-
1/6*a*e*ln(c*x^6+a)/c^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{x^8(d+ex^3)}{a+cx^6}\,dx=\frac{2cdx^3+cex^6-2\sqrt{a}\sqrt{c}d\arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)-ae\log\left(a+cx^6\right)}{6c^2}$$

input
```
Integrate[(x^8*(d + e*x^3))/(a + c*x^6),x]
```

output
```
(2*c*d*x^3 + c*e*x^6 - 2*Sqrt[a]*Sqrt[c]*d*ArcTan[(Sqrt[c]*x^3)/Sqrt[a]] -
 a*e*Log[a + c*x^6])/(6*c^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1803, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (d+ex^3)}{a+cx^6} \, dx$$

$$\Big\downarrow \text{ 1803}$$

$$\frac{1}{3}\int \frac{x^6 (ex^3+d)}{cx^6+a}dx^3$$

$$\Big\downarrow \text{ 523}$$

$$\frac{1}{3}\int \left(\frac{ex^3}{c}+\frac{d}{c}-\frac{aex^3+ad}{c\,(cx^6+a)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{a}d\arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{c^{3/2}}-\frac{ae\log\left(a+cx^6\right)}{2c^2}+\frac{dx^3}{c}+\frac{ex^6}{2c}\right)$$

input
```
Int[(x^8*(d + e*x^3))/(a + c*x^6),x]
```

output
```
((d*x^3)/c + (e*x^6)/(2*c) - (Sqrt[a]*d*ArcTan[(Sqrt[c]*x^3)/Sqrt[a]])/c^(
3/2) - (a*e*Log[a + c*x^6])/(2*c^2))/3
```

Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

method	result
default	$\frac{\frac{1}{2}e\,x^{6}+d\,x^{3}}{3c}-\frac{a\left(\frac{e\ln\left(c\,x^{6}+a\right)}{2c}+\frac{d\arctan\left(\frac{c\,x^{3}}{\sqrt{a c}}\right)}{\sqrt{a c}}\right)}{3c}$
risch	$\frac{e\,x^{6}}{6c}+\frac{d\,x^{3}}{3c}+\frac{d^{2}}{6ec}+\frac{\ln\left(-c\,x^{3}+\sqrt{-ac}\right)d\sqrt{-ac}}{6c^{2}}-\frac{\ln\left(-c\,x^{3}+\sqrt{-ac}\right)ae}{6c^{2}}-\frac{\ln\left(-c\,x^{3}-\sqrt{-ac}\right)d\sqrt{-ac}}{6c^{2}}-\frac{\ln\left(-c\,x^{3}-\sqrt{-ac}\right)ae}{6c^{2}}$

input
```
int(x^8*(e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/c*(1/2*e*x^6+d*x^3)-1/3*a/c*(1/2*e*ln(c*x^6+a)/c+d/(a*c)^(1/2)*arctan(
c*x^3/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.93

$$\int \frac{x^8(d+ex^3)}{a+cx^6}\,dx$$
$$=\left[\frac{cex^6+2\,cdx^3+cd\sqrt{-\frac{a}{c}}\log\left(\frac{cx^6-2\,cx^3\sqrt{-\frac{a}{c}}-a}{cx^6+a}\right)-ae\log\left(cx^6+a\right)}{6\,c^2},\frac{cex^6+2\,cdx^3-2\,cd\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^3}{\cdots}\right.}{6\,c^2}\right.$$

input
```
integrate(x^8*(e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

output
```
[1/6*(c*e*x^6 + 2*c*d*x^3 + c*d*sqrt(-a/c)*log((c*x^6 - 2*c*x^3*sqrt(-a/c)
 - a)/(c*x^6 + a)) - a*e*log(c*x^6 + a))/c^2, 1/6*(c*e*x^6 + 2*c*d*x^3 - 2
*c*d*sqrt(a/c)*arctan(c*x^3*sqrt(a/c)/a) - a*e*log(c*x^6 + a))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(61) = 122$.

Time = 0.51 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.26

$$\int \frac{x^8(d+ex^3)}{a+cx^6}\,dx = \left(-\frac{ae}{6c^2}-\frac{d\sqrt{-ac^5}}{6c^4}\right)\log\left(x^3+\frac{-ae-6c^2\left(-\frac{ae}{6c^2}-\frac{d\sqrt{-ac^5}}{6c^4}\right)}{cd}\right) + \left(-\frac{ae}{6c^2}+\frac{d\sqrt{-ac^5}}{6c^4}\right)\log\left(x^3+\frac{-ae-6c^2\left(-\frac{ae}{6c^2}+\frac{d\sqrt{-ac^5}}{6c^4}\right)}{cd}\right) + \frac{dx^3}{3c}+\frac{ex^6}{6c}$$

input
```
integrate(x**8*(e*x**3+d)/(c*x**6+a),x)
```

output
```
(-a*e/(6*c**2) - d*sqrt(-a*c**5)/(6*c**4))*log(x**3 + (-a*e - 6*c**2*(-a*e
/(6*c**2) - d*sqrt(-a*c**5)/(6*c**4)))/(c*d)) + (-a*e/(6*c**2) + d*sqrt(-a
*c**5)/(6*c**4))*log(x**3 + (-a*e - 6*c**2*(-a*e/(6*c**2) + d*sqrt(-a*c**5
)/(6*c**4)))/(c*d)) + d*x**3/(3*c) + e*x**6/(6*c)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{x^8(d+ex^3)}{a+cx^6}\,dx = -\frac{ad\arctan\left(\frac{cx^3}{\sqrt{ac}}\right)}{3\sqrt{ac}c}-\frac{ae\log\left(cx^6+a\right)}{6\,c^2}+\frac{ex^6+2\,dx^3}{6\,c}$$

input
```
integrate(x^8*(e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

output
```
-1/3*a*d*arctan(c*x^3/sqrt(a*c))/(sqrt(a*c)*c) - 1/6*a*e*log(c*x^6 + a)/c^
2 + 1/6*(e*x^6 + 2*d*x^3)/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{x^8(d+ex^3)}{a+cx^6}\,dx = -\frac{ad\arctan\left(\frac{cx^3}{\sqrt{ac}}\right)}{3\sqrt{ac}c} - \frac{ae\log\left(cx^6+a\right)}{6c^2} + \frac{cex^6+2cdx^3}{6c^2}$$

input
```
integrate(x^8*(e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*a*d*arctan(c*x^3/sqrt(a*c))/(sqrt(a*c)*c) - 1/6*a*e*log(c*x^6 + a)/c^
2 + 1/6*(c*e*x^6 + 2*c*d*x^3)/c^2
```

Mupad [B] (verification not implemented)

Time = 19.58 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{x^8(d+ex^3)}{a+cx^6}\,dx = \frac{d\,x^3}{3\,c} + \frac{e\,x^6}{6\,c} - \frac{\sqrt{a}\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^3}{\sqrt{a}}\right)}{3\,c^{3/2}} - \frac{a\,e\,\ln\left(c\,x^6+a\right)}{6\,c^2}$$

input
```
int((x^8*(d + e*x^3))/(a + c*x^6),x)
```

output
```
(d*x^3)/(3*c) + (e*x^6)/(6*c) - (a^(1/2)*d*atan((c^(1/2)*x^3)/a^(1/2)))/(3
*c^(3/2)) - (a*e*log(a + c*x^6))/(6*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.86

$$\int \frac{x^8(d+ex^3)}{a+cx^6}\,dx$$
$$= \frac{2c^{\frac{7}{6}}a^{\frac{7}{6}}atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d-2c^{\frac{7}{6}}a^{\frac{7}{6}}atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d+2c^{\frac{7}{6}}a^{\frac{7}{6}}atan\left(\frac{c^{\frac{1}{6}}x}{a^{\frac{1}{6}}}\right)d-c^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(a^{\frac{1}{3}}+c^{\frac{1}{3}}x^2\right)}{6c^{\frac{8}{3}}}$$

input
```
int(x^8*(e*x^3+d)/(c*x^6+a),x)
```

output
```
(2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(
1/6)*a**(1/6)))*a*c*d - 2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3
) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c*d + 2*c**(1/6)*a**(1/6)*atan((c
**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c*d - c**(2/3)*a**(2/3)*log(a**(1/3) + c
**(1/3)*x**2)*a*e - c**(2/3)*a**(2/3)*log( - c**(1/6)*a**(1/6)*sqrt(3)*x +
 a**(1/3) + c**(1/3)*x**2)*a*e - c**(2/3)*a**(2/3)*log(c**(1/6)*a**(1/6)*s
qrt(3)*x + a**(1/3) + c**(1/3)*x**2)*a*e + 2*c**(2/3)*a**(2/3)*c*d*x**3 +
c**(2/3)*a**(2/3)*c*e*x**6)/(6*c**(2/3)*a**(2/3)*c**2)
```

3.13 $\int \frac{x^5(d+ex^3)}{a+cx^6} dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{x^5(d+ex^3)}{a+cx^6} dx = \frac{ex^3}{3c} - \frac{\sqrt{a}e \arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3c^{3/2}} + \frac{d\log(a+cx^6)}{6c}$$

output
```
1/3*e*x^3/c-1/3*a^(1/2)*e*arctan(c^(1/2)*x^3/a^(1/2))/c^(3/2)+1/6*d*ln(c*x
^6+a)/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{x^5(d+ex^3)}{a+cx^6} dx = \frac{ex^3}{3c} - \frac{\sqrt{a}e \arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3c^{3/2}} + \frac{d\log(a+cx^6)}{6c}$$

input
```
Integrate[(x^5*(d + e*x^3))/(a + c*x^6),x]
```

output
```
(e*x^3)/(3*c) - (Sqrt[a]*e*ArcTan[(Sqrt[c]*x^3)/Sqrt[a]])/(3*c^(3/2)) + (d
*Log[a + c*x^6])/(6*c)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1803, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(d+ex^3)}{a+cx^6}dx$$

$$\Big\downarrow \text{ 1803}$$

$$\frac{1}{3}\int \frac{x^3(ex^3+d)}{cx^6+a}dx^3$$

$$\Big\downarrow \text{ 523}$$

$$\frac{1}{3}\int \left(\frac{e}{c}-\frac{ae-cdx^3}{c(cx^6+a)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{a}e\arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{c^{3/2}}+\frac{d\log(a+cx^6)}{2c}+\frac{ex^3}{c}\right)$$

input
```
Int[(x^5*(d + e*x^3))/(a + c*x^6),x]
```

output
```
((e*x^3)/c - (Sqrt[a]*e*ArcTan[(Sqrt[c]*x^3)/Sqrt[a]])/c^(3/2) + (d*Log[a
+ c*x^6])/(2*c))/3
```

Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{e\,x^{3}}{3c}+\frac{\frac{d\ln\left(c\,x^{6}+a\right)}{2}-\frac{a e\arctan\left(\frac{c\,x^{3}}{\sqrt{a c}}\right)}{\sqrt{a c}}}{3c}$	48
risch	$\frac{e\,x^{3}}{3c}+\frac{\ln\left(c\,x^{3}-\sqrt{-a c}\right)e\sqrt{-a c}}{6c^{2}}+\frac{\ln\left(c\,x^{3}-\sqrt{-a c}\right)d}{6c}-\frac{\ln\left(c\,x^{3}+\sqrt{-a c}\right)e\sqrt{-a c}}{6c^{2}}+\frac{\ln\left(c\,x^{3}+\sqrt{-a c}\right)d}{6c}$	103

input
```
int(x^5*(e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*e*x^3/c+1/3/c*(1/2*d*ln(c*x^6+a)-a*e/(a*c)^(1/2)*arctan(c*x^3/(a*c)^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.98

$$\int \frac{x^5(d+ex^3)}{a+cx^6}\,dx = \left[\frac{2\,ex^3+e\sqrt{-\frac{a}{c}}\log\left(\frac{cx^6-2\,cx^3\sqrt{-\frac{a}{c}}-a}{cx^6+a}\right)+d\log\left(cx^6+a\right)}{6\,c}, \frac{2\,ex^3-2\,e\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^3\sqrt{\frac{a}{c}}}{a}\right)+d\log\left(cx^6+\right.}{6\,c}\right.$$

input
```
integrate(x^5*(e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

output
```
[1/6*(2*e*x^3 + e*sqrt(-a/c)*log((c*x^6 - 2*c*x^3*sqrt(-a/c) - a)/(c*x^6 +
 a)) + d*log(c*x^6 + a))/c, 1/6*(2*e*x^3 - 2*e*sqrt(a/c)*arctan(c*x^3*sqrt
(a/c)/a) + d*log(c*x^6 + a))/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(49) = 98$.

Time = 0.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.05

$$\int \frac{x^5(d+ex^3)}{a+cx^6}\,dx = \left(\frac{d}{6c} - \frac{e\sqrt{-ac^3}}{6c^3}\right)\log\left(x^3 + \frac{-6c\left(\frac{d}{6c} - \frac{e\sqrt{-ac^3}}{6c^3}\right)+d}{e}\right) + \left(\frac{d}{6c} + \frac{e\sqrt{-ac^3}}{6c^3}\right)\log\left(x^3 + \frac{-6c\left(\frac{d}{6c} + \frac{e\sqrt{-ac^3}}{6c^3}\right)+d}{e}\right) + \frac{ex^3}{3c}$$

input
```
integrate(x**5*(e*x**3+d)/(c*x**6+a),x)
```

output
```
(d/(6*c) - e*sqrt(-a*c**3)/(6*c**3))*log(x**3 + (-6*c*(d/(6*c) - e*sqrt(-a
*c**3)/(6*c**3)) + d)/e) + (d/(6*c) + e*sqrt(-a*c**3)/(6*c**3))*log(x**3 +
 (-6*c*(d/(6*c) + e*sqrt(-a*c**3)/(6*c**3)) + d)/e) + e*x**3/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{x^5(d+ex^3)}{a+cx^6}\,dx = \frac{ex^3}{3\,c} - \frac{ae\arctan\left(\frac{cx^3}{\sqrt{ac}}\right)}{3\,\sqrt{ac}c} + \frac{d\log\left(cx^6+a\right)}{6\,c}$$

input
```
integrate(x^5*(e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

output
```
1/3*e*x^3/c - 1/3*a*e*arctan(c*x^3/sqrt(a*c))/(sqrt(a*c)*c) + 1/6*d*log(c*
x^6 + a)/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{x^5(d+ex^3)}{a+cx^6}\,dx = \frac{ex^3}{3\,c} - \frac{ae\arctan\left(\frac{cx^3}{\sqrt{ac}}\right)}{3\,\sqrt{ac}c} + \frac{d\log\left(cx^6+a\right)}{6\,c}$$

input
```
integrate(x^5*(e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

output
```
1/3*e*x^3/c - 1/3*a*e*arctan(c*x^3/sqrt(a*c))/(sqrt(a*c)*c) + 1/6*d*log(c*
x^6 + a)/c
```

Mupad [B] (verification not implemented)

Time = 20.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{x^5(d+ex^3)}{a+cx^6}\,dx = \frac{d\,\ln\left(c\,x^6+a\right)}{6\,c} + \frac{e\,x^3}{3\,c} - \frac{\sqrt{a}\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^3}{\sqrt{a}}\right)}{3\,c^{3/2}}$$

input
```
int((x^5*(d + e*x^3))/(a + c*x^6),x)
```

output
```
(d*log(a + c*x^6))/(6*c) + (e*x^3)/(3*c) - (a^(1/2)*e*atan((c^(1/2)*x^3)/a
^(1/2)))/(3*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.21

$$\int \frac{x^5(d+ex^3)}{a+cx^6}\,dx = \frac{2c^{\frac{1}{6}}a^{\frac{7}{6}}atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e - 2c^{\frac{1}{6}}a^{\frac{7}{6}}atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e + 2c^{\frac{1}{6}}a^{\frac{7}{6}}atan\left(\frac{c^{\frac{1}{6}}x}{a^{\frac{1}{6}}}\right)e + c^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{1}{3}}+c^{\frac{1}{3}}x^2\right)}{6c^{\frac{5}{3}}a^{\frac{2}{3}}}$$

input
```
int(x^5*(e*x^3+d)/(c*x^6+a),x)
```

output

```
(2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(
1/6)*a**(1/6)))*a*e - 2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3)
+ 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*e + 2*c**(1/6)*a**(1/6)*atan((c**(1
/3)*x)/(c**(1/6)*a**(1/6)))*a*e + c**(2/3)*a**(2/3)*log(a**(1/3) + c**(1/3
)*x**2)*d + c**(2/3)*a**(2/3)*log( - c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3
) + c**(1/3)*x**2)*d + c**(2/3)*a**(2/3)*log(c**(1/6)*a**(1/6)*sqrt(3)*x +
 a**(1/3) + c**(1/3)*x**2)*d + 2*c**(2/3)*a**(2/3)*e*x**3)/(6*c**(2/3)*a**
(2/3)*c)
```

3.14 $\int \frac{x^2(d+ex^3)}{a+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{x^2(d+ex^3)}{a+cx^6}\,dx = \frac{d\arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{c}} + \frac{e\log(a+cx^6)}{6c}$$

output
```
1/3*d*arctan(c^(1/2)*x^3/a^(1/2))/a^(1/2)/c^(1/2)+1/6*e*ln(c*x^6+a)/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{x^2(d+ex^3)}{a+cx^6}\,dx = \frac{d\arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{c}} + \frac{e\log(a+cx^6)}{6c}$$

input
```
Integrate[(x^2*(d + e*x^3))/(a + c*x^6),x]
```

output
```
(d*ArcTan[(Sqrt[c]*x^3)/Sqrt[a]])/(3*Sqrt[a]*Sqrt[c]) + (e*Log[a + c*x^6])
/(6*c)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1799, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex^3)}{a+cx^6}dx$$

$$\Big\downarrow \text{1799}$$

$$\frac{1}{3}\int \frac{ex^3+d}{cx^6+a}dx^3$$

$$\Big\downarrow \text{452}$$

$$\frac{1}{3}\left(d\int \frac{1}{cx^6+a}dx^3+e\int \frac{x^3}{cx^6+a}dx^3\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{3}\left(e\int \frac{x^3}{cx^6+a}dx^3+\frac{d\arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}\right)$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{3}\left(\frac{d\arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}+\frac{e\log\left(a+cx^6\right)}{2c}\right)$$

input
```
Int[(x^2*(d + e*x^3))/(a + c*x^6),x]
```

output
```
((d*ArcTan[(Sqrt[c]*x^3)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + (e*Log[a + c*x^6])/
(2*c))/3
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{e\ln\left(c\,x^{6}+a\right)}{6c}+\frac{d\arctan\left(\frac{c\,x^{3}}{\sqrt{ac}}\right)}{3\sqrt{ac}}$	35
risch	$\frac{\ln\left(c\,x^{3}+\sqrt{-ac}\right)d\sqrt{-ac}}{6ac}+\frac{\ln\left(c\,x^{3}+\sqrt{-ac}\right)e}{6c}-\frac{\ln\left(c\,x^{3}-\sqrt{-ac}\right)d\sqrt{-ac}}{6ac}+\frac{\ln\left(c\,x^{3}-\sqrt{-ac}\right)e}{6c}$	100

input
```
int(x^2*(e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*e*ln(c*x^6+a)/c+1/3*d/(a*c)^(1/2)*arctan(c*x^3/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.17

$$\int \frac{x^2(d+ex^3)}{a+cx^6}\,dx = \left[\frac{ae\log\left(cx^6+a\right)-\sqrt{-ac}\,d\log\left(\frac{cx^6-2\sqrt{-ac}\,x^3-a}{cx^6+a}\right)}{6\,ac}, \frac{ae\log\left(cx^6+a\right)+2\sqrt{ac}\,d\arctan\left(\frac{\sqrt{ac}\,x^3}{a}\right)}{6\,ac}\right]$$

input
```
integrate(x^2*(e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

output
```
[1/6*(a*e*log(c*x^6 + a) - sqrt(-a*c)*d*log((c*x^6 - 2*sqrt(-a*c)*x^3 - a)
/(c*x^6 + a)))/(a*c), 1/6*(a*e*log(c*x^6 + a) + 2*sqrt(a*c)*d*arctan(sqrt(
a*c)*x^3/a))/(a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(41) = 82$.

Time = 0.45 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.72

$$\int \frac{x^2(d+ex^3)}{a+cx^6}\,dx = \left(\frac{e}{6c}-\frac{d\sqrt{-ac^3}}{6ac^2}\right)\log\left(x^3+\frac{6ac\left(\frac{e}{6c}-\frac{d\sqrt{-ac^3}}{6ac^2}\right)-ae}{cd}\right) + \left(\frac{e}{6c}+\frac{d\sqrt{-ac^3}}{6ac^2}\right)\log\left(x^3+\frac{6ac\left(\frac{e}{6c}+\frac{d\sqrt{-ac^3}}{6ac^2}\right)-ae}{cd}\right)$$

input
```
integrate(x**2*(e*x**3+d)/(c*x**6+a),x)
```

output
```
(e/(6*c) - d*sqrt(-a*c**3)/(6*a*c**2))*log(x**3 + (6*a*c*(e/(6*c) - d*sqrt
(-a*c**3)/(6*a*c**2)) - a*e)/(c*d)) + (e/(6*c) + d*sqrt(-a*c**3)/(6*a*c**2
))*log(x**3 + (6*a*c*(e/(6*c) + d*sqrt(-a*c**3)/(6*a*c**2)) - a*e)/(c*d))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{x^2(d+ex^3)}{a+cx^6}\,dx = \frac{d\arctan\left(\frac{cx^3}{\sqrt{ac}}\right)}{3\sqrt{ac}} + \frac{e\log\left(cx^6+a\right)}{6\,c}$$

input
```
integrate(x^2*(e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

output
```
1/3*d*arctan(c*x^3/sqrt(a*c))/sqrt(a*c) + 1/6*e*log(c*x^6 + a)/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{x^2(d+ex^3)}{a+cx^6}\,dx = \frac{d\arctan\left(\frac{cx^3}{\sqrt{ac}}\right)}{3\sqrt{ac}} + \frac{e\log\left(cx^6+a\right)}{6\,c}$$

input
```
integrate(x^2*(e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

output
```
1/3*d*arctan(c*x^3/sqrt(a*c))/sqrt(a*c) + 1/6*e*log(c*x^6 + a)/c
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{x^2(d+ex^3)}{a+cx^6}\,dx = \frac{e\,\ln\left(c\,x^6+a\right)}{6\,c} + \frac{d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^3}{\sqrt{a}}\right)}{3\,\sqrt{a}\,\sqrt{c}}$$

input
```
int((x^2*(d + e*x^3))/(a + c*x^6),x)
```

output
```
(e*log(a + c*x^6))/(6*c) + (d*atan((c^(1/2)*x^3)/a^(1/2)))/(3*a^(1/2)*c^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.70

$$\int \frac{x^2(d+ex^3)}{a+cx^6}\,dx$$
$$= \frac{-2c^{\frac{7}{6}}a^{\frac{1}{6}}atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d+2c^{\frac{7}{6}}a^{\frac{1}{6}}atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d-2c^{\frac{7}{6}}a^{\frac{1}{6}}atan\left(\frac{c^{\frac{1}{6}}x}{a^{\frac{1}{6}}}\right)d+c^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{1}{3}}+c^{\frac{1}{3}}x^2\right.}{6c^{\frac{5}{3}}a^{\frac{2}{3}}}$$

input
```
int(x^2*(e*x^3+d)/(c*x^6+a),x)
```

output
```
( - 2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c
**(1/6)*a**(1/6)))*c*d + 2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(
3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d - 2*c**(1/6)*a**(1/6)*atan((c*
*(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d + c**(2/3)*a**(2/3)*log(a**(1/3) + c**(
1/3)*x**2)*e + c**(2/3)*a**(2/3)*log( - c**(1/6)*a**(1/6)*sqrt(3)*x + a**(
1/3) + c**(1/3)*x**2)*e + c**(2/3)*a**(2/3)*log(c**(1/6)*a**(1/6)*sqrt(3)*
x + a**(1/3) + c**(1/3)*x**2)*e)/(6*c**(2/3)*a**(2/3)*c)
```

3.15 $\int \frac{d+ex^3}{x(a+cx^6)} dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{d+ex^3}{x(a+cx^6)} dx = \frac{e\arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{c}} + \frac{d\log(x)}{a} - \frac{d\log(a+cx^6)}{6a}$$

output
```
1/3*e*arctan(c^(1/2)*x^3/a^(1/2))/a^(1/2)/c^(1/2)+d*ln(x)/a-1/6*d*ln(c*x^6
+a)/a
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(54) = 108.

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.13

$$\int \frac{d+ex^3}{x(a+cx^6)} dx = -\frac{2\sqrt{a}e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right) + 2\sqrt{a}e\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right) - 2\sqrt{a}e\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right) - 6\sqrt{c}d\log(x) +}{6a\sqrt{c}}$$

input
```
Integrate[(d + e*x^3)/(x*(a + c*x^6)),x]
```

output
```
-1/6*(2*Sqrt[a]*e*ArcTan[(c^(1/6)*x)/a^(1/6)] + 2*Sqrt[a]*e*ArcTan[Sqrt[3]
 - (2*c^(1/6)*x)/a^(1/6)] - 2*Sqrt[a]*e*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(
1/6)] - 6*Sqrt[c]*d*Log[x] + Sqrt[c]*d*Log[a + c*x^6])/(a*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1803, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x\left(a+cx^6\right)}dx$$

$$\Big\downarrow \text{1803}$$

$$\frac{1}{3}\int \frac{ex^3+d}{x^3\left(cx^6+a\right)}dx^3$$

$$\Big\downarrow \text{523}$$

$$\frac{1}{3}\int \left(\frac{d}{ax^3}+\frac{ae-cdx^3}{a\left(cx^6+a\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{e\arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}-\frac{d\log\left(a+cx^6\right)}{2a}+\frac{d\log\left(x^3\right)}{a}\right)$$

input
```
Int[(d + e*x^3)/(x*(a + c*x^6)),x]
```

output
```
((e*ArcTan[(Sqrt[c]*x^3)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + (d*Log[x^3])/a - (d
*Log[a + c*x^6])/(2*a))/3
```

Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{d\ln(x)}{a}+\frac{-\frac{d\ln\left(c\,x^{6}+a\right)}{2}+\frac{ae\arctan\left(\frac{c\,x^{3}}{\sqrt{ac}}\right)}{\sqrt{ac}}}{3a}$	45
risch	$\frac{d\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{2}c\_Z^{2}+2acd\_Z+a\,e^{2}+c\,d^{2}\right)}\_R\ln\left(\left(-7ac\_R^{2}-7cd\_R-6e^{2}\right)x^{3}+ae\_R-6de\right)\right)}{6}$	74

input
```
int((e*x^3+d)/x/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
d*ln(x)/a+1/3/a*(-1/2*d*ln(c*x^6+a)+a*e/(a*c)^(1/2)*arctan(c*x^3/(a*c)^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.09

$$\int \frac{d+ex^3}{x\left(a+cx^6\right)}\,dx = \left[-\frac{cd\log\left(cx^6+a\right)-6\,cd\log\left(x\right)+\sqrt{-ac}e\log\left(\frac{cx^6-2\,\sqrt{-ac}x^3-a}{cx^6+a}\right)}{6\,ac},\ -\frac{cd\log\left(cx^6+a\right)-6\,cd\log\left(x\right)-2\,\sqrt{ac}e\arctan\left(\frac{\sqrt{ac}x^3}{a}\right)}{6\,ac}\right]$$

input
```
integrate((e*x^3+d)/x/(c*x^6+a),x, algorithm="fricas")
```

output
```
[-1/6*(c*d*log(c*x^6 + a) - 6*c*d*log(x) + sqrt(-a*c)*e*log((c*x^6 - 2*sqr
t(-a*c)*x^3 - a)/(c*x^6 + a)))/(a*c), -1/6*(c*d*log(c*x^6 + a) - 6*c*d*log
(x) - 2*sqrt(a*c)*e*arctan(sqrt(a*c)*x^3/a))/(a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(48) = 96$.

Time = 1.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.37

$$\int \frac{d+ex^3}{x\left(a+cx^6\right)}\,dx = \left(-\frac{d}{6a}-\frac{e\sqrt{-a^3c}}{6a^2c}\right)\log\left(x^3+\frac{6a\left(-\frac{d}{6a}-\frac{e\sqrt{-a^3c}}{6a^2c}\right)+d}{e}\right)+\left(-\frac{d}{6a}+\frac{e\sqrt{-a^3c}}{6a^2c}\right)\log\left(x^3+\frac{6a\left(-\frac{d}{6a}+\frac{e\sqrt{-a^3c}}{6a^2c}\right)+d}{e}\right)+\frac{d\log\left(x\right)}{a}$$

input
```
integrate((e*x**3+d)/x/(c*x**6+a),x)
```

output
```
(-d/(6*a) - e*sqrt(-a**3*c)/(6*a**2*c))*log(x**3 + (6*a*(-d/(6*a) - e*sqrt
(-a**3*c)/(6*a**2*c)) + d)/e) + (-d/(6*a) + e*sqrt(-a**3*c)/(6*a**2*c))*lo
g(x**3 + (6*a*(-d/(6*a) + e*sqrt(-a**3*c)/(6*a**2*c)) + d)/e) + d*log(x)/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{d+ex^3}{x\left(a+cx^6\right)}\,dx=\frac{e\arctan\left(\frac{cx^3}{\sqrt{ac}}\right)}{3\sqrt{ac}}-\frac{d\log\left(cx^6+a\right)}{6\,a}+\frac{d\log\left(x^3\right)}{3\,a}$$

input
```
integrate((e*x^3+d)/x/(c*x^6+a),x, algorithm="maxima")
```

output
```
1/3*e*arctan(c*x^3/sqrt(a*c))/sqrt(a*c) - 1/6*d*log(c*x^6 + a)/a + 1/3*d*l
og(x^3)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{d+ex^3}{x\left(a+cx^6\right)}\,dx=\frac{e\arctan\left(\frac{cx^3}{\sqrt{ac}}\right)}{3\sqrt{ac}}-\frac{d\log\left(cx^6+a\right)}{6\,a}+\frac{d\log\left(|x|\right)}{a}$$

input
```
integrate((e*x^3+d)/x/(c*x^6+a),x, algorithm="giac")
```

output
```
1/3*e*arctan(c*x^3/sqrt(a*c))/sqrt(a*c) - 1/6*d*log(c*x^6 + a)/a + d*log(a
bs(x))/a
```

Mupad [B] (verification not implemented)

Time = 20.55 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{d+ex^3}{x\left(a+cx^6\right)}\,dx=\frac{d\,\ln\left(x\right)}{a}-\frac{d\,\ln\left(c\,x^6+a\right)}{6\,a}+\frac{e\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^3}{\sqrt{a}}\right)}{3\,\sqrt{a}\,\sqrt{c}}$$

input
```
int((d + e*x^3)/(x*(a + c*x^6)),x)
```

output
```
(d*log(x))/a - (d*log(a + c*x^6))/(6*a) + (e*atan((c^(1/2)*x^3)/a^(1/2)))/
(3*a^(1/2)*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.48

$$\int \frac{d+ex^3}{x(a+cx^6)}\,dx = \frac{-2c^{\frac{1}{6}}a^{\frac{7}{6}}atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e+2c^{\frac{1}{6}}a^{\frac{7}{6}}atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e-2c^{\frac{1}{6}}a^{\frac{7}{6}}atan\left(\frac{c^{\frac{1}{6}}x}{a^{\frac{1}{6}}}\right)e-c^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{1}{3}}+c^{\frac{1}{3}}x^2\right.}{6c^{\frac{2}{3}}a^{\frac{5}{3}}}$$

input
```
int((e*x^3+d)/x/(c*x^6+a),x)
```

output
```
( - 2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c
**(1/6)*a**(1/6)))*a*e + 2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(
3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*e - 2*c**(1/6)*a**(1/6)*atan((c*
*(1/3)*x)/(c**(1/6)*a**(1/6)))*a*e - c**(2/3)*a**(2/3)*log(a**(1/3) + c**(
1/3)*x**2)*d - c**(2/3)*a**(2/3)*log( - c**(1/6)*a**(1/6)*sqrt(3)*x + a**(
1/3) + c**(1/3)*x**2)*d - c**(2/3)*a**(2/3)*log(c**(1/6)*a**(1/6)*sqrt(3)*
x + a**(1/3) + c**(1/3)*x**2)*d + 6*c**(2/3)*a**(2/3)*log(x)*d)/(6*c**(2/3
)*a**(2/3)*a)
```

3.16 $\int \frac{d+ex^3}{x^4(a+cx^6)} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{d+ex^3}{x^4\left(a+cx^6\right)} \, dx = -\frac{d}{3ax^3} - \frac{\sqrt{c}d \arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{3a^{3/2}} + \frac{e\log(x)}{a} - \frac{e\log\left(a+cx^6\right)}{6a}$$

output
```
-1/3*d/a/x^3-1/3*c^(1/2)*d*arctan(c^(1/2)*x^3/a^(1/2))/a^(3/2)+e*ln(x)/a-1
/6*e*ln(c*x^6+a)/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.91

$$\int \frac{d+ex^3}{x^4\left(a+cx^6\right)} \, dx$$
$$= \frac{-\frac{2\sqrt{a}d}{x^3} + 2\sqrt{c}d \arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right) + 2\sqrt{c}d \arctan\left(\sqrt{3} - \frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right) - 2\sqrt{c}d \arctan\left(\sqrt{3} + \frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right) + 6\sqrt{a}e \log}{6a^{3/2}}$$

input
```
Integrate[(d + e*x^3)/(x^4*(a + c*x^6)),x]
```

output
```
((-2*Sqrt[a]*d)/x^3 + 2*Sqrt[c]*d*ArcTan[(c^(1/6)*x)/a^(1/6)] + 2*Sqrt[c]*
d*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)] - 2*Sqrt[c]*d*ArcTan[Sqrt[3] + (
2*c^(1/6)*x)/a^(1/6)] + 6*Sqrt[a]*e*Log[x] - Sqrt[a]*e*Log[a + c*x^6])/(6*
a^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1803, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^4(a+cx^6)}dx$$

$$\Big\downarrow \text{ 1803}$$

$$\frac{1}{3}\int \frac{ex^3+d}{x^6(cx^6+a)}dx^3$$

$$\Big\downarrow \text{ 523}$$

$$\frac{1}{3}\int \left(\frac{d}{ax^6}-\frac{c(ex^3+d)}{a(cx^6+a)}+\frac{e}{ax^3}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{\sqrt{c}d\arctan\left(\frac{\sqrt{c}x^3}{\sqrt{a}}\right)}{a^{3/2}}-\frac{e\log(a+cx^6)}{2a}-\frac{d}{ax^3}+\frac{e\log(x^3)}{a}\right)$$

input
```
Int[(d + e*x^3)/(x^4*(a + c*x^6)),x]
```

output
```
(-(d/(a*x^3)) - (Sqrt[c]*d*ArcTan[(Sqrt[c]*x^3)/Sqrt[a]])/a^(3/2) + (e*Log
[x^3])/a - (e*Log[a + c*x^6])/(2*a))/3
```

Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

method	result
default	$-\frac{d}{3a\,x^{3}}+\frac{e\ln\left(x\right)}{a}-\frac{c\left(\frac{e\ln\left(c\,x^{6}+a\right)}{2c}+\frac{d\arctan\left(\frac{c\,x^{3}}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{3a}$
risch	$-\frac{d}{3a\,x^{3}}+\frac{e\ln\left(x\right)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{2}a^{3}+2\_Ze\,a^{2}+a\,e^{2}+c\,d^{2}\right)}\_R\ln\left(\left(-7a^{3}\_R^{2}-7a^{2}e\_R-6c\,d^{2}\right)x^{3}-a^{2}d\_R+6ade\right)\right)}{6}$

input
```
int((e*x^3+d)/x^4/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*d/a/x^3+e*ln(x)/a-1/3*c/a*(1/2*e*ln(c*x^6+a)/c+d/(a*c)^(1/2)*arctan(c
*x^3/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.15

$$\int \frac{d+ex^3}{x^4\left(a+cx^6\right)}\,dx = \left[\frac{dx^3\sqrt{-\frac{c}{a}}\log\left(\frac{cx^6-2\,ax^3\sqrt{-\frac{c}{a}}-a}{cx^6+a}\right)-ex^3\log\left(cx^6+a\right)+6\,ex^3\log\left(x\right)-2\,d}{6\,ax^3},\ -\frac{2\,dx^3\sqrt{\frac{c}{a}}\arctan\left(x^3\sqrt{\frac{c}{a}}\right)+ex^3\log\left(cx^6+a\right)-6\,ex^3\log\left(x\right)+2\,d}{6\,ax^3}\right]$$

input
```
integrate((e*x^3+d)/x^4/(c*x^6+a),x, algorithm="fricas")
```

output
```
[1/6*(d*x^3*sqrt(-c/a)*log((c*x^6 - 2*a*x^3*sqrt(-c/a) - a)/(c*x^6 + a)) -
 e*x^3*log(c*x^6 + a) + 6*e*x^3*log(x) - 2*d)/(a*x^3), -1/6*(2*d*x^3*sqrt(
c/a)*arctan(x^3*sqrt(c/a)) + e*x^3*log(c*x^6 + a) - 6*e*x^3*log(x) + 2*d)/
(a*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(56) = 112$.

Time = 1.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.20

$$\int \frac{d+ex^3}{x^4\left(a+cx^6\right)}\,dx = \left(-\frac{e}{6a}-\frac{d\sqrt{-a^3c}}{6a^3}\right)\log\left(x^3+\frac{-6a^2\left(-\frac{e}{6a}-\frac{d\sqrt{-a^3c}}{6a^3}\right)-ae}{cd}\right)+\left(-\frac{e}{6a}+\frac{d\sqrt{-a^3c}}{6a^3}\right)\log\left(x^3+\frac{-6a^2\left(-\frac{e}{6a}+\frac{d\sqrt{-a^3c}}{6a^3}\right)-ae}{cd}\right)-\frac{d}{3ax^3}+\frac{e\log\left(x\right)}{a}$$

input
```
integrate((e*x**3+d)/x**4/(c*x**6+a),x)
```

output
```
(-e/(6*a) - d*sqrt(-a**3*c)/(6*a**3))*log(x**3 + (-6*a**2*(-e/(6*a) - d*sq
rt(-a**3*c)/(6*a**3)) - a*e)/(c*d)) + (-e/(6*a) + d*sqrt(-a**3*c)/(6*a**3)
)*log(x**3 + (-6*a**2*(-e/(6*a) + d*sqrt(-a**3*c)/(6*a**3)) - a*e)/(c*d))
- d/(3*a*x**3) + e*log(x)/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{d+ex^3}{x^4(a+cx^6)}\,dx = -\frac{cd\arctan\left(\frac{cx^3}{\sqrt{ac}}\right)}{3\sqrt{ac}a} - \frac{e\log(cx^6+a)}{6a} + \frac{e\log(x^3)}{3a} - \frac{d}{3ax^3}$$

input
```
integrate((e*x^3+d)/x^4/(c*x^6+a),x, algorithm="maxima")
```

output
```
-1/3*c*d*arctan(c*x^3/sqrt(a*c))/(sqrt(a*c)*a) - 1/6*e*log(c*x^6 + a)/a +
1/3*e*log(x^3)/a - 1/3*d/(a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{d+ex^3}{x^4(a+cx^6)}\,dx = -\frac{cd\arctan\left(\frac{cx^3}{\sqrt{ac}}\right)}{3\sqrt{ac}a} - \frac{e\log(cx^6+a)}{6a} + \frac{e\log(|x|)}{a} - \frac{ex^3+d}{3ax^3}$$

input
```
integrate((e*x^3+d)/x^4/(c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*c*d*arctan(c*x^3/sqrt(a*c))/(sqrt(a*c)*a) - 1/6*e*log(c*x^6 + a)/a +
e*log(abs(x))/a - 1/3*(e*x^3 + d)/(a*x^3)
```

Mupad [B] (verification not implemented)

Time = 20.99 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int \frac{d+ex^3}{x^4\,(a+cx^6)}\,dx = \frac{e\,\ln(x)}{a} - \frac{d}{3\,a\,x^3} - \frac{e\,\ln\left(c\,x^6+a\right)}{6\,a} - \frac{\sqrt{c}\,d\,\mathrm{atan}\left(\frac{c^{3/2}\,d^2\,x^3}{\sqrt{a}\,(c\,d^2+49\,a\,e^2)} + \frac{49\,\sqrt{a}\,\sqrt{c}\,e^2\,x^3}{c\,d^2+49\,a\,e^2}\right)}{3\,a^{3/2}}$$

input
```
int((d + e*x^3)/(x^4*(a + c*x^6)),x)
```

output
```
(e*log(x))/a - d/(3*a*x^3) - (e*log(a + c*x^6))/(6*a) - (c^(1/2)*d*atan((c
^(3/2)*d^2*x^3)/(a^(1/2)*(49*a*e^2 + c*d^2)) + (49*a^(1/2)*c^(1/2)*e^2*x^3
)/(49*a*e^2 + c*d^2)))/(3*a^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.40

$$\int \frac{d+ex^3}{x^4\,(a+cx^6)}\,dx$$
$$= \frac{2c^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d\,x^3 - 2c^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d\,x^3 + 2c^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{c^{\frac{1}{6}}x}{a^{\frac{1}{6}}}\right)d\,x^3 - c^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{1}{3}} + \cdots\right.}{\cdots}$$

input
```
int((e*x^3+d)/x^4/(c*x^6+a),x)
```

output
```
(2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(
1/6)*a**(1/6)))*c*d*x**3 - 2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqr
t(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d*x**3 + 2*c**(1/6)*a**(1/6)*a
tan((c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d*x**3 - c**(2/3)*a**(2/3)*log(a**
(1/3) + c**(1/3)*x**2)*e*x**3 - c**(2/3)*a**(2/3)*log( - c**(1/6)*a**(1/6)
*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*e*x**3 - c**(2/3)*a**(2/3)*log(c**(
1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*e*x**3 + 6*c**(2/3)*a*
*(2/3)*log(x)*e*x**3 - 2*c**(2/3)*a**(2/3)*d)/(6*c**(2/3)*a**(2/3)*a*x**3)
```

3.17 $\int \frac{x^6(d+ex^3)}{a+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 322

$$\begin{aligned}\int \frac{x^6(d+ex^3)}{a+cx^6}\,dx &= \frac{dx}{c}+\frac{ex^4}{4c}-\frac{\sqrt[6]{a}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{7/6}}\\
&+\frac{\sqrt[6]{a}\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6c^{5/3}}\\
&-\frac{\sqrt[6]{a}\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6c^{5/3}}\\
&+\frac{a^{2/3}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6c^{5/3}}\\
&+\frac{\sqrt[6]{a}\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12c^{5/3}}\\
&-\frac{\sqrt[6]{a}\left(\sqrt{3}\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12c^{5/3}}\end{aligned}$$

output
```
d*x/c+1/4*e*x^4/c-1/3*a^(1/6)*d*arctan(c^(1/6)*x/a^(1/6))/c^(7/6)-1/6*a^(1
/6)*(c^(1/2)*d+3^(1/2)*a^(1/2)*e)*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/c^(
5/3)-1/6*a^(1/6)*(c^(1/2)*d-3^(1/2)*a^(1/2)*e)*arctan(3^(1/2)+2*c^(1/6)*x/
a^(1/6))/c^(5/3)+1/6*a^(2/3)*e*ln(a^(1/3)+c^(1/3)*x^2)/c^(5/3)+1/12*a^(1/6
)*(3^(1/2)*c^(1/2)*d-a^(1/2)*e)*ln(a^(1/3)-3^(1/2)*a^(1/6)*c^(1/6)*x+c^(1/
3)*x^2)/c^(5/3)-1/12*a^(1/6)*(3^(1/2)*c^(1/2)*d+a^(1/2)*e)*ln(a^(1/3)+3^(1
/2)*a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/c^(5/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.92

$$\int \frac{x^6(d+ex^3)}{a+cx^6}\,dx$$
$$= \frac{12c^{2/3}dx + 3c^{2/3}ex^4 - 4\sqrt[6]{a}\sqrt{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right) + 2\sqrt[6]{a}\left(\sqrt{c}d + \sqrt{3}\sqrt{a}e\right)\arctan\left(\sqrt{3} - \frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right) + 2\sqrt[6]{a}(-}{}$$

input
```
Integrate[(x^6*(d + e*x^3))/(a + c*x^6),x]
```

output
```
(12*c^(2/3)*d*x + 3*c^(2/3)*e*x^4 - 4*a^(1/6)*Sqrt[c]*d*ArcTan[(c^(1/6)*x)
/a^(1/6)] + 2*a^(1/6)*(Sqrt[c]*d + Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3] - (2*
c^(1/6)*x)/a^(1/6)] + 2*a^(1/6)*(-(Sqrt[c]*d) + Sqrt[3]*Sqrt[a]*e)*ArcTan[
Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)] + 2*a^(2/3)*e*Log[a^(1/3) + c^(1/3)*x^2]
 + (Sqrt[3]*a^(1/6)*Sqrt[c]*d - a^(2/3)*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^
(1/6)*x + c^(1/3)*x^2] - (Sqrt[3]*a^(1/6)*Sqrt[c]*d + a^(2/3)*e)*Log[a^(1/
3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*c^(5/3))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.17, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {1827, 27, 1827, 25, 27, 1746, 27, 452, 218, 240, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (d+ex^3)}{a+cx^6}dx$$

$$\Big\downarrow \; 1827$$

$$\frac{ex^4}{4c}-\frac{\int \frac{4x^3(ae-cdx^3)}{cx^6+a}dx}{4c}$$

$$\Big\downarrow \; 27$$

$$\frac{ex^4}{4c}-\frac{\int \frac{x^3(ae-cdx^3)}{cx^6+a}dx}{c}$$

$$\Big\downarrow \; 1827$$

$$\frac{ex^4}{4c}-\frac{-\frac{\int -\frac{ac(ex^3+d)}{cx^6+a}dx}{c}-dx}{c}$$

$$\Big\downarrow \; 25$$

$$\frac{ex^4}{4c}-\frac{\frac{\int \frac{ac(ex^3+d)}{cx^6+a}dx}{c}-dx}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{ex^4}{4c}-\frac{a\int \frac{ex^3+d}{cx^6+a}dx-dx}{c}$$

$$\Big\downarrow \; 1746$$

$$\frac{ex^4}{4c}-\frac{a\left(\frac{\int \frac{\sqrt[3]{c}d-\sqrt[3]{a}ex}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{6a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{6a^{2/3}\sqrt[3]{c}}\right)-dx}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{ex^4}{4c}-\frac{a\left(\frac{\int \frac{\sqrt[3]{c}d-\sqrt[3]{a}ex}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}\right)-dx}{c}$$

$$\Big\downarrow \; 452$$

$$\frac{\frac{ex^4}{4c}-a\left(\frac{\sqrt[3]{c}d\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx-\sqrt[3]{a}e\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}\right)-dx}{c}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{ex^4}{4c}-a\left(\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\sqrt[3]{a}e\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}\right)-dx}{c}$$

$$\Big\downarrow \ 240$$

$$\frac{\frac{ex^4}{4c}-a\left(\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}\right)-dx}{c}$$

$$\Big\downarrow \ 1142$$

$$\frac{\frac{ex^4}{4c}-a\left(\frac{\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}-\frac{a^{2/3}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int -\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+}{}\right.}{c}$$

$$\Big\downarrow \ 25$$

$$\frac{ex^4}{4c}-\frac{a\left(\frac{\frac{a^{2/3}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[3]{c}}+\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int\frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int\frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\cdots}{\cdots}\cdots\right)}{c}$$

$\big\downarrow$ 27

$$\frac{ex^4}{4c}-\frac{a\left(\frac{\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int\frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int\frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\left(\sqrt{a}e+\sqrt{3}\cdots\right)\cdots}{\cdots}}{6a\sqrt[3]{c}}\cdots\right)}{c}$$

$\big\downarrow$ 1082

$$\frac{ex^4}{4c}-\frac{a\left(\frac{\frac{\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\sqrt[6]{a}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int\frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int\frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}\cdots}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}\cdots}\cdots}{2\sqrt[6]{a}\sqrt[6]{c}}\cdots}{\cdots}\cdots\right)}{c}$$

$\big\downarrow$ 217

$$\frac{ex^4}{4c} - \frac{a\left(\frac{\frac{\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}} - \frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)}{\sqrt[3]{c}}}{6a\sqrt[3]{c}} + \frac{\frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int \frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{a}\sqrt[6]{c}} + \cdots}{6a\sqrt[3]{c}} \cdots\right)}{c}$$

$\Big\downarrow$ 1103

$$\frac{ex^4}{4c} - \frac{a\left(\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}} - \frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}} + \frac{-\frac{a^{2/3}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}} - \frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(\cdots\right.}{\sqrt[3]{c}}}{6a\sqrt[3]{c}} \cdots\right)}{c}$$

input
```
Int[(x^6*(d + e*x^3))/(a + c*x^6),x]
```

output
```
(e*x^4)/(4*c) - (-(d*x) + a*(((c^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/a^(1
/6) - (a^(1/3)*e*Log[a^(1/3) + c^(1/3)*x^2])/(2*c^(1/3)))/(3*a^(2/3)*c^(1/
3)) + (-((a^(1/6)*(Sqrt[c]*d + Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3]*(1 - (2*c
^(1/6)*x)/(Sqrt[3]*a^(1/6)))])/c^(1/3)) - (a^(2/3)*((Sqrt[3]*Sqrt[c]*d)/Sq
rt[a] - e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1
/3)))/(6*a*c^(1/3)) + ((a^(1/6)*(Sqrt[c]*d - Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqr
t[3]*(1 + (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))])/c^(1/3) + (a^(1/6)*(Sqrt[3]*S
qrt[c]*d + Sqrt[a]*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^
2])/(2*c^(1/3)))/(6*a*c^(1/3))))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1746
```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Simp[1/(3*a*q^2)   Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Sim
p[1/(6*a*q^2)   Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^
2*x^2), x], x] + Simp[1/(6*a*q^2)   Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(
1 + Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
 + a*e^2, 0] && PosQ[c/a]
```

rule 1827
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a + c*x^(2*n))^(p +
1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(m + n*(2*p + 1) + 1))   In
t[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.17

method	result
risch	$\frac{e\,x^{4}}{4c}+\frac{dx}{c}+\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+a\right)}\frac{\left(-e\_R^{3}-d\right)\ln\left(x-\_R\right)}{\_R^{5}}\right)}{6c^{2}}$
default	$\frac{\frac{1}{4}x^{4}e+dx}{c}-\frac{\left(\frac{\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{2}{3}}e}{12a}-\frac{\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}d}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)\sqrt{3}\,e}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^6*(e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*e*x^4/c+d*x/c+1/6/c^2*a*sum((-_R^3*e-d)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c
+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1582 vs. $2(222) = 444$.

Time = 0.15 (sec) , antiderivative size = 1582, normalized size of antiderivative = 4.91

$$\int \frac{x^6(d+ex^3)}{a+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

output
```
1/12*(3*e*x^4 + 2*c*(-(c^5*sqrt(-(a*c^2*d^6 - 6*a^2*c*d^4*e^2 + 9*a^3*d^2*
e^4)/c^9) + 3*a*c*d^2*e - a^2*e^3)/c^5)^(1/3)*log(-(c^2*d^5 - 2*a*c*d^3*e^
2 - 3*a^2*d*e^4)*x + (c^6*e*sqrt(-(a*c^2*d^6 - 6*a^2*c*d^4*e^2 + 9*a^3*d^2
*e^4)/c^9) + c^3*d^4 - 3*a*c^2*d^2*e^2)*(-(c^5*sqrt(-(a*c^2*d^6 - 6*a^2*c*
d^4*e^2 + 9*a^3*d^2*e^4)/c^9) + 3*a*c*d^2*e - a^2*e^3)/c^5)^(1/3)) - (sqrt
(-3)*c + c)*(-(c^5*sqrt(-(a*c^2*d^6 - 6*a^2*c*d^4*e^2 + 9*a^3*d^2*e^4)/c^9
) + 3*a*c*d^2*e - a^2*e^3)/c^5)^(1/3)*log(-(c^2*d^5 - 2*a*c*d^3*e^2 - 3*a^
2*d*e^4)*x - 1/2*(c^3*d^4 - 3*a*c^2*d^2*e^2 + sqrt(-3)*(c^3*d^4 - 3*a*c^2*
d^2*e^2) + (sqrt(-3)*c^6*e + c^6*e)*sqrt(-(a*c^2*d^6 - 6*a^2*c*d^4*e^2 + 9
*a^3*d^2*e^4)/c^9))*(-(c^5*sqrt(-(a*c^2*d^6 - 6*a^2*c*d^4*e^2 + 9*a^3*d^2*
e^4)/c^9) + 3*a*c*d^2*e - a^2*e^3)/c^5)^(1/3)) + (sqrt(-3)*c - c)*(-(c^5*s
qrt(-(a*c^2*d^6 - 6*a^2*c*d^4*e^2 + 9*a^3*d^2*e^4)/c^9) + 3*a*c*d^2*e - a^
2*e^3)/c^5)^(1/3)*log(-(c^2*d^5 - 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x - 1/2*(c^
3*d^4 - 3*a*c^2*d^2*e^2 - sqrt(-3)*(c^3*d^4 - 3*a*c^2*d^2*e^2) - (sqrt(-3)
*c^6*e - c^6*e)*sqrt(-(a*c^2*d^6 - 6*a^2*c*d^4*e^2 + 9*a^3*d^2*e^4)/c^9))*
(-(c^5*sqrt(-(a*c^2*d^6 - 6*a^2*c*d^4*e^2 + 9*a^3*d^2*e^4)/c^9) + 3*a*c*d^
2*e - a^2*e^3)/c^5)^(1/3)) + 2*c*((c^5*sqrt(-(a*c^2*d^6 - 6*a^2*c*d^4*e^2
+ 9*a^3*d^2*e^4)/c^9) - 3*a*c*d^2*e + a^2*e^3)/c^5)^(1/3)*log(-(c^2*d^5 -
2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x - (c^6*e*sqrt(-(a*c^2*d^6 - 6*a^2*c*d^4*e^2
 + 9*a^3*d^2*e^4)/c^9) - c^3*d^4 + 3*a*c^2*d^2*e^2)*((c^5*sqrt(-(a*c^2*...
```

Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.54

$$\int \frac{x^6(d+ex^3)}{a+cx^6}\,dx$$
$$= \operatorname{RootSum}\left(46656t^6c^{10}+t^3\left(-432a^2c^5e^3+1296ac^6d^2e\right)+a^4e^6+3a^3cd^2e^4+3a^2c^2d^4e^2+ac^3d^6,\left(t\mapsto t\,\mathrm{l}\right.\right.$$
$$+\frac{dx}{c}+\frac{ex^4}{4c}$$

input
```
integrate(x**6*(e*x**3+d)/(c*x**6+a),x)
```

output
```
RootSum(46656*_t**6*c**10 + _t**3*(-432*a**2*c**5*e**3 + 1296*a*c**6*d**2*
e) + a**4*e**6 + 3*a**3*c*d**2*e**4 + 3*a**2*c**2*d**4*e**2 + a*c**3*d**6,
 Lambda(_t, _t*log(x + (-1296*_t**4*c**6*e + 6*_t*a**2*c*e**4 - 36*_t*a*c*
*2*d**2*e**2 + 6*_t*c**3*d**4)/(3*a**2*d*e**4 + 2*a*c*d**3*e**2 - c**2*d**
5)))) + d*x/c + e*x**4/(4*c)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.94

$$\int \frac{x^6(d+ex^3)}{a+cx^6}\,dx$$
$$= \frac{a\left(\frac{2\,e\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}c^{\frac{2}{3}}}-\frac{4\,d\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}-\frac{\left(\sqrt{3}a^{\frac{1}{6}}\sqrt{c}d+a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{ac^{\frac{2}{3}}}+\frac{\left(\sqrt{3}a^{\frac{1}{6}}\sqrt{c}d-a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2-\sqrt{3}a\right.}{ac^{\frac{2}{3}}}\right.}{12\,c}$$
$$+\frac{ex^4+4\,dx}{4\,c}$$

input
```
integrate(x^6*(e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

output
```
1/12*a*(2*e*log(c^(1/3)*x^2 + a^(1/3))/(a^(1/3)*c^(2/3)) - 4*d*arctan(c^(1
/3)*x/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*sqrt(a^(1/3)*c^(1/3))) - (sqrt(3)*a^
(1/6)*sqrt(c)*d + a^(2/3)*e)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x +
 a^(1/3))/(a*c^(2/3)) + (sqrt(3)*a^(1/6)*sqrt(c)*d - a^(2/3)*e)*log(c^(1/3
)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) + 2*(sqrt(3)*a^(5
/6)*c^(1/6)*e - a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6)*c
^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))) - 2*(sqrt
(3)*a^(5/6)*c^(1/6)*e + a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x - sqrt(3)*a
^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))))/
c + 1/4*(e*x^4 + 4*d*x)/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.91

$$\int \frac{x^6(d+ex^3)}{a+cx^6}\,dx = -\frac{(ac^5)^{\frac{1}{6}}\,d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,c^2} + \frac{(ac^5)^{\frac{2}{3}}\,e|c|\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,c^6} + \frac{c^3ex^4+4\,c^3dx}{4\,c^4} - \frac{\left((ac^5)^{\frac{1}{6}}\,c^3d-\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,c^5} - \frac{\left((ac^5)^{\frac{1}{6}}\,c^3d+\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,c^5} - \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,c^3d+(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,c^5} + \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,c^3d-(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,c^5}$$

input
```
integrate(x^6*(e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*(a*c^5)^(1/6)*d*arctan(x/(a/c)^(1/6))/c^2 + 1/6*(a*c^5)^(2/3)*e*abs(c
)*log(x^2 + (a/c)^(1/3))/c^6 + 1/4*(c^3*e*x^4 + 4*c^3*d*x)/c^4 - 1/6*((a*c
^5)^(1/6)*c^3*d - sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/
6))/(a/c)^(1/6))/c^5 - 1/6*((a*c^5)^(1/6)*c^3*d + sqrt(3)*(a*c^5)^(2/3)*e)
*arctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/c^5 - 1/12*(sqrt(3)*(a*c^
5)^(1/6)*c^3*d + (a*c^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^
(1/3))/c^5 + 1/12*(sqrt(3)*(a*c^5)^(1/6)*c^3*d - (a*c^5)^(2/3)*e)*log(x^2
- sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/c^5
```

Mupad [B] (verification not implemented)

Time = 22.54 (sec) , antiderivative size = 1147, normalized size of antiderivative = 3.56

$$\int \frac{x^6(d+ex^3)}{a+cx^6} dx = \text{Too large to display}$$

input
```
int((x^6*(d + e*x^3))/(a + c*x^6),x)
```

output
```
log(e*x*(-a*c^11)^(1/2) - c^7*((c*d^3*(-a*c^11)^(1/2) + a^2*c^5*e^3 - 3*a*
c^6*d^2*e - 3*a*d*e^2*(-a*c^11)^(1/2))/c^10)^(1/3) + c^6*d*x)*((c*d^3*(-a*
c^11)^(1/2) + a^2*c^5*e^3 - 3*a*c^6*d^2*e - 3*a*d*e^2*(-a*c^11)^(1/2))/(21
6*c^10))^(1/3) + log(c^7*(-(c*d^3*(-a*c^11)^(1/2) - a^2*c^5*e^3 + 3*a*c^6*
d^2*e - 3*a*d*e^2*(-a*c^11)^(1/2))/c^10)^(1/3) + e*x*(-a*c^11)^(1/2) - c^6
*d*x)*(-(c*d^3*(-a*c^11)^(1/2) - a^2*c^5*e^3 + 3*a*c^6*d^2*e - 3*a*d*e^2*(
-a*c^11)^(1/2))/(216*c^10))^(1/3) + log(c^7*((c*d^3*(-a*c^11)^(1/2) + a^2*
c^5*e^3 - 3*a*c^6*d^2*e - 3*a*d*e^2*(-a*c^11)^(1/2))/c^10)^(1/3) + 2*e*x*(
-a*c^11)^(1/2) - 3^(1/2)*c^7*((c*d^3*(-a*c^11)^(1/2) + a^2*c^5*e^3 - 3*a*c
^6*d^2*e - 3*a*d*e^2*(-a*c^11)^(1/2))/c^10)^(1/3)*1i + 2*c^6*d*x)*((3^(1/2
)*1i)/2 - 1/2)*((c*d^3*(-a*c^11)^(1/2) + a^2*c^5*e^3 - 3*a*c^6*d^2*e - 3*a
*d*e^2*(-a*c^11)^(1/2))/(216*c^10))^(1/3) - log(c^7*((c*d^3*(-a*c^11)^(1/2
) + a^2*c^5*e^3 - 3*a*c^6*d^2*e - 3*a*d*e^2*(-a*c^11)^(1/2))/c^10)^(1/3) +
 2*e*x*(-a*c^11)^(1/2) + 3^(1/2)*c^7*((c*d^3*(-a*c^11)^(1/2) + a^2*c^5*e^3
 - 3*a*c^6*d^2*e - 3*a*d*e^2*(-a*c^11)^(1/2))/c^10)^(1/3)*1i + 2*c^6*d*x)*
((3^(1/2)*1i)/2 + 1/2)*((c*d^3*(-a*c^11)^(1/2) + a^2*c^5*e^3 - 3*a*c^6*d^2
*e - 3*a*d*e^2*(-a*c^11)^(1/2))/(216*c^10))^(1/3) + log(c^7*(-(c*d^3*(-a*c
^11)^(1/2) - a^2*c^5*e^3 + 3*a*c^6*d^2*e - 3*a*d*e^2*(-a*c^11)^(1/2))/c^10
)^(1/3) - 2*e*x*(-a*c^11)^(1/2) - 3^(1/2)*c^7*(-(c*d^3*(-a*c^11)^(1/2) - a
^2*c^5*e^3 + 3*a*c^6*d^2*e - 3*a*d*e^2*(-a*c^11)^(1/2))/c^10)^(1/3)*1i ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.93

$$\int \frac{x^6(d+ex^3)}{a+cx^6}\,dx = \frac{2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d+2\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)ae-2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d+2\sqrt{3}\,ata}{}$$

input
```
int(x^6*(e*x^3+d)/(c*x^6+a),x)
```

output
```
(2*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/
6)*a**(1/6)))*d + 2*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x
)/(c**(1/6)*a**(1/6)))*a*e - 2*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqr
t(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*d + 2*sqrt(3)*atan((c**(1/6)*a**
(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*e - 4*sqrt(c)*sqrt(a)
*atan((c**(1/3)*x)/(c**(1/6)*a**(1/6)))*d + sqrt(c)*sqrt(a)*sqrt(3)*log( -
 c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*d - sqrt(c)*sqrt(
a)*sqrt(3)*log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*d +
 12*c**(2/3)*a**(1/3)*d*x + 3*c**(2/3)*a**(1/3)*e*x**4 + 2*log(a**(1/3) +
c**(1/3)*x**2)*a*e - log( - c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1
/3)*x**2)*a*e - log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2
)*a*e)/(12*c**(2/3)*a**(1/3)*c)
```

3.18 $\int \frac{x^3(d+ex^3)}{a+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 311

$$\begin{aligned}\int \frac{x^3(d+ex^3)}{a+cx^6}\,dx =& \frac{ex}{c}-\frac{\sqrt[6]{a}e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{7/6}}-\frac{\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}c^{7/6}} \\ &-\frac{\left(\sqrt{3}\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[3]{a}c^{7/6}}-\frac{d\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}} \\ &+\frac{\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}} \\ &+\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}\end{aligned}$$

output
```
e*x/c-1/3*a^(1/6)*e*arctan(c^(1/6)*x/a^(1/6))/c^(7/6)+1/6*(3^(1/2)*c^(1/2)
*d-a^(1/2)*e)*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(1/3)/c^(7/6)-1/6*(3^
(1/2)*c^(1/2)*d+a^(1/2)*e)*arctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(1/3)/c^(
7/6)-1/6*d*ln(a^(1/3)+c^(1/3)*x^2)/a^(1/3)/c^(2/3)+1/12*(c^(1/2)*d+3^(1/2)
*a^(1/2)*e)*ln(a^(1/3)-3^(1/2)*a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(1/3)/c^(7
/6)+1/12*(c^(1/2)*d-3^(1/2)*a^(1/2)*e)*ln(a^(1/3)+3^(1/2)*a^(1/6)*c^(1/6)*
x+c^(1/3)*x^2)/a^(1/3)/c^(7/6)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.11

$$\int \frac{x^3(d+ex^3)}{a+cx^6}\,dx = \frac{ex}{c} - \frac{\sqrt[6]{a}e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{7/6}} + \frac{\left(\sqrt{3}a^{2/3}cd - a^{7/6}\sqrt{c}e\right)\arctan\left(\frac{-\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{5/3}} + \frac{\left(-\sqrt{3}a^{2/3}cd - a^{7/6}\sqrt{c}e\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{5/3}} - \frac{d\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}} - \frac{\left(-a^{2/3}cd - \sqrt{3}a^{7/6}\sqrt{c}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{5/3}} - \frac{\left(-a^{2/3}cd + \sqrt{3}a^{7/6}\sqrt{c}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{5/3}}$$

input
```
Integrate[(x^3*(d + e*x^3))/(a + c*x^6),x]
```

output
```
(e*x)/c - (a^(1/6)*e*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*c^(7/6)) + ((Sqrt[3]*
a^(2/3)*c*d - a^(7/6)*Sqrt[c]*e)*ArcTan[(-(Sqrt[3]*a^(1/6)) + 2*c^(1/6)*x)
/a^(1/6)])/(6*a*c^(5/3)) + ((-(Sqrt[3]*a^(2/3)*c*d) - a^(7/6)*Sqrt[c]*e)*A
rcTan[(Sqrt[3]*a^(1/6) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^(5/3)) - (d*Log[a^(
1/3) + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) - ((-(a^(2/3)*c*d) - Sqrt[3]*a^(7
/6)*Sqrt[c]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12
*a*c^(5/3)) - ((-(a^(2/3)*c*d) + Sqrt[3]*a^(7/6)*Sqrt[c]*e)*Log[a^(1/3) +
Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(5/3))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1827, 1746, 27, 452, 218, 240, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(d+e x^3\right)}{a+c x^6} \, dx$$

$$\Big\downarrow \text{1827}$$

$$\frac{e x}{c}-\frac{\int \frac{a e-c d x^3}{c x^6+a}dx}{c}$$

$$\Big\downarrow \text{1746}$$

$$\frac{e x}{c}-\frac{\frac{\int \frac{\sqrt[3]{a} \sqrt[3]{c} \left(a^{2/3} e+c^{2/3} d x\right)}{\sqrt[3]{c} x^2+\sqrt[3]{a}}dx}{3 a^{2/3} \sqrt[3]{c}}+\frac{\int \frac{\sqrt[3]{c} \left(2 a^{2/3} e-\sqrt[6]{c} \left(\sqrt{c} d+\sqrt{3} \sqrt{a} e\right) x\right)}{\frac{\sqrt[3]{c} x^2}{\sqrt[3]{a}}-\frac{\sqrt{3} \sqrt[6]{c} x}{\sqrt[6]{a}}+1}dx}{6 a^{2/3} \sqrt[3]{c}}+\frac{\int \frac{\sqrt[3]{c} \left(2 a^{2/3} e-\sqrt[6]{c} \left(\sqrt{c} d-\sqrt{3} \sqrt{a} e\right) x\right)}{\frac{\sqrt[3]{c} x^2}{\sqrt[3]{a}}+\frac{\sqrt{3} \sqrt[6]{c} x}{\sqrt[6]{a}}+1}dx}{6 a^{2/3} \sqrt[3]{c}}}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{e x}{c}-\frac{\frac{\int \frac{a^{2/3} e+c^{2/3} d x}{\sqrt[3]{c} x^2+\sqrt[3]{a}}dx}{3 \sqrt[3]{a}}+\frac{\int \frac{2 a^{2/3} e-\sqrt[6]{c} \left(\sqrt{c} d+\sqrt{3} \sqrt{a} e\right) x}{\frac{\sqrt[3]{c} x^2}{\sqrt[3]{a}}-\frac{\sqrt{3} \sqrt[6]{c} x}{\sqrt[6]{a}}+1}dx}{6 a^{2/3}}+\frac{\int \frac{2 a^{2/3} e-\sqrt[6]{c} \left(\sqrt{c} d-\sqrt{3} \sqrt{a} e\right) x}{\frac{\sqrt[3]{c} x^2}{\sqrt[3]{a}}+\frac{\sqrt{3} \sqrt[6]{c} x}{\sqrt[6]{a}}+1}dx}{6 a^{2/3}}}{c}$$

$$\Big\downarrow \text{452}$$

$$\frac{e x}{c}-\frac{\frac{a^{2/3} e \int \frac{1}{\sqrt[3]{c} x^2+\sqrt[3]{a}}dx+c^{2/3} d \int \frac{x}{\sqrt[3]{c} x^2+\sqrt[3]{a}}dx}{3 \sqrt[3]{a}}+\frac{\int \frac{2 a^{2/3} e-\sqrt[6]{c} \left(\sqrt{c} d+\sqrt{3} \sqrt{a} e\right) x}{\frac{\sqrt[3]{c} x^2}{\sqrt[3]{a}}-\frac{\sqrt{3} \sqrt[6]{c} x}{\sqrt[6]{a}}+1}dx}{6 a^{2/3}}+\frac{\int \frac{2 a^{2/3} e-\sqrt[6]{c} \left(\sqrt{c} d-\sqrt{3} \sqrt{a} e\right) x}{\frac{\sqrt[3]{c} x^2}{\sqrt[3]{a}}+\frac{\sqrt{3} \sqrt[6]{c} x}{\sqrt[6]{a}}+1}dx}{6 a^{2/3}}}{c}$$

$$\Big\downarrow \text{218}$$

$$\frac{ex}{c}-\frac{\frac{\int \frac{2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\int \frac{2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{c^{2/3}d\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx+\frac{\sqrt{a}e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{c}}}{3\sqrt[3]{a}}}{c}$$

$$\Big\downarrow \; 240$$

$$\frac{ex}{c}-\frac{\frac{\int \frac{2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\int \frac{2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\frac{\sqrt{a}e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{c}}+\frac{1}{2}\sqrt[3]{c}d\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{3\sqrt[3]{a}}}{c}$$

$$\Big\downarrow \; 1142$$

$$\frac{ex}{c}-\frac{\frac{-\frac{1}{2}\sqrt[6]{a}\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{\sqrt[3]{a}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int -\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[6]{c}}}{6a^{2/3}}+\frac{\frac{1}{2}\sqrt[6]{a}\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int \frac{\cdots}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\cdots}}{\cdots}}{c}$$

$$\Big\downarrow \; 25$$

$$\frac{ex}{c}-\frac{\frac{\frac{\sqrt[3]{a}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[6]{c}}-\frac{1}{2}\sqrt[6]{a}\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\frac{1}{2}\sqrt[6]{a}\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\cdots}{\cdots}}\cdots}{\cdots}}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{ex}{c}-\frac{\frac{\frac{1}{2}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{1}{2}\sqrt[6]{a}\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\frac{1}{2}\sqrt[6]{a}\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}\cdots}{6\cdots}}{c}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{ex}{c}-\frac{\frac{\frac{1}{2}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{\sqrt[3]{a}\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\int\frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{c}}}{6a^{2/3}}+\frac{-\frac{1}{2}\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int\frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{\ldots}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{\ldots}}{\sqrt[6]{\ldots}}}\ldots}{\ldots}}{c}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{ex}{c}-\frac{\frac{\frac{1}{2}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx+\frac{\sqrt[3]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)}{\sqrt[6]{c}}}{6a^{2/3}}+\frac{\frac{\sqrt[3]{a}\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)}{\sqrt[6]{c}}-\frac{1}{2}\ldots}{6a^{2/3}}}{c}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{ex}{c}-\frac{\frac{\sqrt[3]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)}{\sqrt[6]{c}}-\frac{\sqrt[3]{a}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{6a^{2/3}}+\frac{\frac{\sqrt[3]{a}\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)\left(\sqrt{a}e+\sqrt{\ldots}\right)}{\sqrt[6]{c}}\ldots}{\ldots}}{c}$$

input
```
Int[(x^3*(d + e*x^3))/(a + c*x^6),x]
```

output
```
(e*x)/c - (((Sqrt[a]*e*ArcTan[(c^(1/6)*x)/a^(1/6)])/c^(1/6) + (c^(1/3)*d*L
og[a^(1/3) + c^(1/3)*x^2])/2)/(3*a^(1/3)) + ((a^(1/3)*(Sqrt[3]*Sqrt[c]*d -
 Sqrt[a]*e)*ArcTan[Sqrt[3]*(1 - (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))])/c^(1/6)
 - (a^(1/3)*(Sqrt[c]*d + Sqrt[3]*Sqrt[a]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*
c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6)))/(6*a^(2/3)) + ((a^(1/3)*(Sqrt[3]*Sq
rt[c]*d + Sqrt[a]*e)*ArcTan[Sqrt[3]*(1 + (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))]
)/c^(1/6) - (a^(1/3)*(Sqrt[c]*d - Sqrt[3]*Sqrt[a]*e)*Log[a^(1/3) + Sqrt[3]
*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6)))/(6*a^(2/3)))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1746
```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Simp[1/(3*a*q^2)   Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Sim
p[1/(6*a*q^2)   Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^
2*x^2), x], x] + Simp[1/(6*a*q^2)   Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(
1 + Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
 + a*e^2, 0] && PosQ[c/a]
```

rule 1827
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a + c*x^(2*n))^(p +
1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(m + n*(2*p + 1) + 1))   In
t[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.14

method	result
risch	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+a\right)}\frac{\left(\_R^{3}cd-ae\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6c^{2}}$
default	$\frac{ex}{c}+\frac{c\left(\frac{a}{c}\right)^{\frac{7}{6}}\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\,e}{12a}+\frac{c\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)d}{12a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)e}{6}+\frac{c\left(\frac{a}{c}\right)^{\frac{2}{3}}\arctan\left(\ldots\right.}{6\ldots}$

input
```
int(x^3*(e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
e*x/c+1/6/c^2*sum((_R^3*c*d-a*e)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1622 vs. $2(213) = 426$.

Time = 0.11 (sec) , antiderivative size = 1622, normalized size of antiderivative = 5.22

$$\int \frac{x^3(d+ex^3)}{a+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

output
```
1/12*(2*c*(-(a*c^3*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^7)
) + c*d^3 - 3*a*d*e^2)/(a*c^3))^(1/3)*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 -
a^2*e^5)*x + (a*c^5*d*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c
^7)) + 3*a*c^2*d^2*e^2 - a^2*c*e^4)*(-(a*c^3*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*
d^2*e^4 + a^2*e^6)/(a*c^7)) + c*d^3 - 3*a*d*e^2)/(a*c^3))^(1/3)) - (sqrt(-
3)*c + c)*(-(a*c^3*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^7)
) + c*d^3 - 3*a*d*e^2)/(a*c^3))^(1/3)*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 -
a^2*e^5)*x - 1/2*(3*a*c^2*d^2*e^2 - a^2*c*e^4 + sqrt(-3)*(3*a*c^2*d^2*e^2
- a^2*c*e^4) + (sqrt(-3)*a*c^5*d + a*c^5*d)*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d
^2*e^4 + a^2*e^6)/(a*c^7)))*(-(a*c^3*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4
+ a^2*e^6)/(a*c^7)) + c*d^3 - 3*a*d*e^2)/(a*c^3))^(1/3)) + (sqrt(-3)*c - c
)*(-(a*c^3*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^7)) + c*d^
3 - 3*a*d*e^2)/(a*c^3))^(1/3)*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 - a^2*e^5)
*x - 1/2*(3*a*c^2*d^2*e^2 - a^2*c*e^4 - sqrt(-3)*(3*a*c^2*d^2*e^2 - a^2*c*
e^4) - (sqrt(-3)*a*c^5*d - a*c^5*d)*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 +
 a^2*e^6)/(a*c^7)))*(-(a*c^3*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^
6)/(a*c^7)) + c*d^3 - 3*a*d*e^2)/(a*c^3))^(1/3)) + 2*c*((a*c^3*sqrt(-(9*c^
2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^7)) - c*d^3 + 3*a*d*e^2)/(a*c^3)
)^(1/3)*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 - a^2*e^5)*x - (a*c^5*d*sqrt(-(9
*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a*c^7)) - 3*a*c^2*d^2*e^2 + a^...
```

Sympy [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.54

$$\int \frac{x^3(d+ex^3)}{a+cx^6}\,dx$$
$$= \mathrm{RootSum}\left(46656t^6a^2c^7 + t^3\left(-1296a^2c^4de^2 + 432ac^5d^3\right) + a^3e^6 + 3a^2cd^2e^4 + 3ac^2d^4e^2 + c^3d^6, \left(t \mapsto t\,\mathrm{lo}\right.\right.$$
$$+ \frac{ex}{c}$$

input
```
integrate(x**3*(e*x**3+d)/(c*x**6+a),x)
```

output
```
RootSum(46656*_t**6*a**2*c**7 + _t**3*(-1296*a**2*c**4*d*e**2 + 432*a*c**5
*d**3) + a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6,
Lambda(_t, _t*log(x + (-1296*_t**4*a*c**5*d - 6*_t*a**2*c*e**4 + 36*_t*a*c
**2*d**2*e**2 - 6*_t*c**3*d**4)/(a**2*e**5 - 2*a*c*d**2*e**3 - 3*c**2*d**4
*e)))) + e*x/c
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.95

$$\int \frac{x^3(d+ex^3)}{a+cx^6}\,dx = \frac{ex}{c}$$
$$- \frac{\frac{2\,c^{\frac{1}{3}}\,d\log\left(c^{\frac{1}{3}}\,x^2+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}} + \frac{4\,a^{\frac{1}{3}}\,e\arctan\left(\frac{c^{\frac{1}{3}}\,x}{\sqrt{a^{\frac{1}{3}}\,c^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}\,c^{\frac{1}{3}}}} + \frac{\left(\sqrt{3}a^{\frac{7}{6}}\sqrt{c}e-a^{\frac{2}{3}}\,cd\right)\log\left(c^{\frac{1}{3}}\,x^2+\sqrt{3}a^{\frac{1}{6}}\,c^{\frac{1}{6}}\,x+a^{\frac{1}{3}}\right)}{ac^{\frac{2}{3}}} - \frac{\left(\sqrt{3}a^{\frac{7}{6}}\sqrt{c}e+a^{\frac{2}{3}}\,cd\right)\log\left(c^{\frac{1}{3}}\,x^{\cdots}\right.}{ac^{\frac{2}{3}}}}{12\,c}$$

input
```
integrate(x^3*(e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

output
```
e*x/c - 1/12*(2*c^(1/3)*d*log(c^(1/3)*x^2 + a^(1/3))/a^(1/3) + 4*a^(1/3)*e
*arctan(c^(1/3)*x/sqrt(a^(1/3)*c^(1/3)))/sqrt(a^(1/3)*c^(1/3)) + (sqrt(3)*
a^(7/6)*sqrt(c)*e - a^(2/3)*c*d)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)
*x + a^(1/3))/(a*c^(2/3)) - (sqrt(3)*a^(7/6)*sqrt(c)*e + a^(2/3)*c*d)*log(
c^(1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) + 2*(sqrt(3
)*a^(5/6)*c^(7/6)*d + a^(4/3)*c^(2/3)*e)*arctan((2*c^(1/3)*x + sqrt(3)*a^(
1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))) - 2
*(sqrt(3)*a^(5/6)*c^(7/6)*d - a^(4/3)*c^(2/3)*e)*arctan((2*c^(1/3)*x - sqr
t(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/
3))))/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.93

$$
\begin{aligned}
\int \frac{x^3(d+ex^3)}{a+cx^6}\,dx = &-\frac{d|c|\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\left(ac^5\right)^{\frac{1}{3}}}+\frac{ex}{c}-\frac{\left(ac^5\right)^{\frac{1}{6}}e\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,c^2}\\
&-\frac{\left(\left(ac^5\right)^{\frac{1}{6}}ac^2e+\sqrt{3}\left(ac^5\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4}\\
&-\frac{\left(\left(ac^5\right)^{\frac{1}{6}}ac^2e-\sqrt{3}\left(ac^5\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4}\\
&-\frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{6}}ac^2e-\left(ac^5\right)^{\frac{2}{3}}d\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}\\
&+\frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{6}}ac^2e+\left(ac^5\right)^{\frac{2}{3}}d\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}
\end{aligned}
$$

input
```
integrate(x^3*(e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

output
```
-1/6*d*abs(c)*log(x^2 + (a/c)^(1/3))/(a*c^5)^(1/3) + e*x/c - 1/3*(a*c^5)^(
1/6)*e*arctan(x/(a/c)^(1/6))/c^2 - 1/6*((a*c^5)^(1/6)*a*c^2*e + sqrt(3)*(a
*c^5)^(2/3)*d)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1
/6*((a*c^5)^(1/6)*a*c^2*e - sqrt(3)*(a*c^5)^(2/3)*d)*arctan((2*x - sqrt(3)
*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1/12*(sqrt(3)*(a*c^5)^(1/6)*a*c^2*e -
 (a*c^5)^(2/3)*d)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4) +
 1/12*(sqrt(3)*(a*c^5)^(1/6)*a*c^2*e + (a*c^5)^(2/3)*d)*log(x^2 - sqrt(3)*
x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4)
```

Mupad [B] (verification not implemented)

Time = 22.70 (sec) , antiderivative size = 1308, normalized size of antiderivative = 4.21

$$\int \frac{x^3(d+ex^3)}{a+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x^3))/(a + c*x^6),x)
```

output
```
log(d*x*(-a^3*c^7)^(1/2) - a^2*c^4*(-(a*c^5*d^3 + a*e^3*(-a^3*c^7)^(1/2) -
 3*a^2*c^4*d*e^2 - 3*c*d^2*e*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) + a^2*c^3*
e*x)*(-(a*c^5*d^3 + a*e^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d*e^2 - 3*c*d^2*e*(
-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) + log(d*x*(-a^3*c^7)^(1/2) + a^2*c^4
*(-(a*c^5*d^3 - a*e^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d*e^2 + 3*c*d^2*e*(-a^3
*c^7)^(1/2))/(a^2*c^7))^(1/3) - a^2*c^3*e*x)*(-(a*c^5*d^3 - a*e^3*(-a^3*c^
7)^(1/2) - 3*a^2*c^4*d*e^2 + 3*c*d^2*e*(-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1
/3) + log(2*d*x*(-a^3*c^7)^(1/2) + a^2*c^4*(-(a*c^5*d^3 + a*e^3*(-a^3*c^7)
^(1/2) - 3*a^2*c^4*d*e^2 - 3*c*d^2*e*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) -
3^(1/2)*a^2*c^4*(-(a*c^5*d^3 + a*e^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d*e^2 -
3*c*d^2*e*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3)*1i + 2*a^2*c^3*e*x)*((3^(1/2)
*1i)/2 - 1/2)*(-(a*c^5*d^3 + a*e^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d*e^2 - 3*
c*d^2*e*(-a^3*c^7)^(1/2))/(216*a^2*c^7))^(1/3) - log(2*d*x*(-a^3*c^7)^(1/2
) + a^2*c^4*(-(a*c^5*d^3 + a*e^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d*e^2 - 3*c*
d^2*e*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) + 3^(1/2)*a^2*c^4*(-(a*c^5*d^3 +
a*e^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d*e^2 - 3*c*d^2*e*(-a^3*c^7)^(1/2))/(a^
2*c^7))^(1/3)*1i + 2*a^2*c^3*e*x)*((3^(1/2)*1i)/2 + 1/2)*(-(a*c^5*d^3 + a*
e^3*(-a^3*c^7)^(1/2) - 3*a^2*c^4*d*e^2 - 3*c*d^2*e*(-a^3*c^7)^(1/2))/(216*
a^2*c^7))^(1/3) - log(a^2*c^4*(-(a*c^5*d^3 - a*e^3*(-a^3*c^7)^(1/2) - 3*a^
2*c^4*d*e^2 + 3*c*d^2*e*(-a^3*c^7)^(1/2))/(a^2*c^7))^(1/3) - 2*d*x*(-a^...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.92

$$\int \frac{x^3(d+ex^3)}{a+cx^6}\,dx$$
$$= \frac{2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e-2\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)cd-2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e-2\sqrt{3}\,atan}{}$$

input
```
int(x^3*(e*x^3+d)/(c*x^6+a),x)
```

output
```
(2*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/
6)*a**(1/6)))*e - 2*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x
)/(c**(1/6)*a**(1/6)))*c*d - 2*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqr
t(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*e - 2*sqrt(3)*atan((c**(1/6)*a**
(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d - 4*sqrt(c)*sqrt(a)
*atan((c**(1/3)*x)/(c**(1/6)*a**(1/6)))*e + sqrt(c)*sqrt(a)*sqrt(3)*log( -
 c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*e - sqrt(c)*sqrt(
a)*sqrt(3)*log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*e +
 12*c**(2/3)*a**(1/3)*e*x - 2*log(a**(1/3) + c**(1/3)*x**2)*c*d + log( - c
**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c*d + log(c**(1/6)*
a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c*d)/(12*c**(2/3)*a**(1/3)*
c)
```

3.19 $\int \frac{d+ex^3}{a+cx^6}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 305

$$\int \frac{d+ex^3}{a+cx^6}\,dx = \frac{d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} - \frac{\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}} + \frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{2/3}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}} - \frac{\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}} + \frac{\left(\sqrt{3}\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{2/3}}$$

output
```
1/3*d*arctan(c^(1/6)*x/a^(1/6))/a^(5/6)/c^(1/6)+1/6*(c^(1/2)*d+3^(1/2)*a^(
1/2)*e)*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(5/6)/c^(2/3)+1/6*(c^(1/2)*
d-3^(1/2)*a^(1/2)*e)*arctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(5/6)/c^(2/3)-1
/6*e*ln(a^(1/3)+c^(1/3)*x^2)/a^(1/3)/c^(2/3)-1/12*(3^(1/2)*c^(1/2)*d-a^(1/
2)*e)*ln(a^(1/3)-3^(1/2)*a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(5/6)/c^(2/3)+1/
12*(3^(1/2)*c^(1/2)*d+a^(1/2)*e)*ln(a^(1/3)+3^(1/2)*a^(1/6)*c^(1/6)*x+c^(1
/3)*x^2)/a^(5/6)/c^(2/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.10

$$\int \frac{d+ex^3}{a+cx^6}\,dx = \frac{d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} + \frac{\left(\sqrt[6]{a}\sqrt{c}d+\sqrt{3}a^{2/3}e\right)\arctan\left(\frac{-\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{2/3}} + \frac{\left(\sqrt[6]{a}\sqrt{c}d-\sqrt{3}a^{2/3}e\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{2/3}} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}} - \frac{\left(\sqrt{3}\sqrt[6]{a}\sqrt{c}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{2/3}} - \frac{\left(-\sqrt{3}\sqrt[6]{a}\sqrt{c}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{2/3}}$$

input
```
Integrate[(d + e*x^3)/(a + c*x^6),x]
```

output
```
(d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*c^(1/6)) + ((a^(1/6)*Sqrt[c]*d
+ Sqrt[3]*a^(2/3)*e)*ArcTan[(-(Sqrt[3]*a^(1/6)) + 2*c^(1/6)*x)/a^(1/6)])/(
6*a*c^(2/3)) + ((a^(1/6)*Sqrt[c]*d - Sqrt[3]*a^(2/3)*e)*ArcTan[(Sqrt[3]*a^
(1/6) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^(2/3)) - (e*Log[a^(1/3) + c^(1/3)*x^
2])/(6*a^(1/3)*c^(2/3)) - ((Sqrt[3]*a^(1/6)*Sqrt[c]*d - a^(2/3)*e)*Log[a^(
1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(2/3)) - ((-(Sqrt
[3]*a^(1/6)*Sqrt[c]*d) - a^(2/3)*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*
x + c^(1/3)*x^2])/(12*a*c^(2/3))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1746, 27, 452, 218, 240, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{a+cx^6}\,dx$$

$$\Big\downarrow \text{1746}$$

$$\frac{\int \frac{\sqrt[3]{c}d-\sqrt[3]{a}ex}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{6a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{6a^{2/3}\sqrt[3]{c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt[3]{c}d-\sqrt[3]{a}ex}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}$$

$$\Big\downarrow \text{452}$$

$$\frac{\sqrt[3]{c}d\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx-\sqrt[3]{a}e\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\sqrt[3]{a}e\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}$$

$$\Big\downarrow \text{240}$$

$$\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}-\frac{a^{2/3}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int -\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\sqrt[6]{a}\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{a^{2/3}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[3]{c}}+\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\sqrt[6]{a}\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int \frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{a}\sqrt[6]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{cd}}{\sqrt{a}}-e\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\sqrt[6]{a}\left(\sqrt{3}\sqrt{a}e+\sqrt{cd}\right)\int \frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{cd}\right)\int \frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{a}\sqrt[6]{c}}-\frac{\sqrt[6]{a}\left(\sqrt{cd}-\sqrt{3}\sqrt{a}e\right)\int \frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)^2-\frac{1}{3}}d\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)}{\sqrt{3}\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{cd}}{\sqrt{a}}-e\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}-\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(\sqrt{3}\sqrt{a}e+\sqrt{cd}\right)}{\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{cd}\right)\int \frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{a}\sqrt[6]{c}}+\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)\left(\sqrt{cd}-\sqrt{3}\sqrt{a}e\right)}{\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}+\frac{-\frac{a^{2/3}\left(\frac{\sqrt{3}\sqrt{cd}}{\sqrt{a}}-e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}-\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(\sqrt{3}\sqrt{a}e+\sqrt{cd}\right)}{\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)\left(\sqrt{cd}-\sqrt{3}\sqrt{a}e\right)}{\sqrt[3]{c}}+\frac{\sqrt[6]{a}\left(\sqrt{a}e+\sqrt{3}\sqrt{cd}\right)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{6a\sqrt[3]{c}}$$

input
```
Int[(d + e*x^3)/(a + c*x^6),x]
```

output
```
((c^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])/a^(1/6) - (a^(1/3)*e*Log[a^(1/3)
+ c^(1/3)*x^2])/(2*c^(1/3)))/(3*a^(2/3)*c^(1/3)) + (-((a^(1/6)*(Sqrt[c]*d
+ Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3]*(1 - (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))]
)/c^(1/3)) - (a^(2/3)*((Sqrt[3]*Sqrt[c]*d)/Sqrt[a] - e)*Log[a^(1/3) - Sqrt
[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/3)))/(6*a*c^(1/3)) + ((a^(1/
6)*(Sqrt[c]*d - Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3]*(1 + (2*c^(1/6)*x)/(Sqrt
[3]*a^(1/6)))])/c^(1/3) + (a^(1/6)*(Sqrt[3]*Sqrt[c]*d + Sqrt[a]*e)*Log[a^(
1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/3)))/(6*a*c^(1/3)
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1746
```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Simp[1/(3*a*q^2)   Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Sim
p[1/(6*a*q^2)   Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^
2*x^2), x], x] + Simp[1/(6*a*q^2)   Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(
1 + Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
 + a*e^2, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.11

method	result
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+a\right)}\frac{\left(\_R^{3}e+d\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6c}$
default	$-\frac{\ln\left(-x^{2}+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{1}{6}}\sqrt{3}\,d}{12a}+\frac{\ln\left(-x^{2}+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{2}{3}}e}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)d}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}}{}$

input
```
int((e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/c*sum((_R^3*e+d)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1631 vs. $2(207) = 414$.

Time = 0.15 (sec) , antiderivative size = 1631, normalized size of antiderivative = 5.35

$$\int \frac{d + ex^3}{a + cx^6} \, dx = \text{Too large to display}$$

input
```
integrate((e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(-3) + 1)*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*
e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2*d^5 - 2*a*
c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 - 3*a^2*c*d^2*e^2 + sqrt(-3)*(
a*c^2*d^4 - 3*a^2*c*d^2*e^2) + (sqrt(-3)*a^4*c^2*e + a^4*c^2*e)*sqrt(-(c^2
*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqrt(-(c^2*d^6
 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2
))^(1/3)) + 1/12*(sqrt(-3) - 1)*((a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 +
 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^3)/(a^2*c^2))^(1/3)*log(-(c^2
*d^5 - 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 - 3*a^2*c*d^2*e^2 -
 sqrt(-3)*(a*c^2*d^4 - 3*a^2*c*d^2*e^2) - (sqrt(-3)*a^4*c^2*e - a^4*c^2*e)
*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*((a^2*c^2*sqr
t(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) + 3*c*d^2*e - a*e^
3)/(a^2*c^2))^(1/3)) - 1/12*(sqrt(-3) + 1)*(-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a
*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3)/(a^2*c^2))^(1/
3)*log(-(c^2*d^5 - 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x + 1/2*(a*c^2*d^4 - 3*a^2
*c*d^2*e^2 + sqrt(-3)*(a*c^2*d^4 - 3*a^2*c*d^2*e^2) - (sqrt(-3)*a^4*c^2*e
+ a^4*c^2*e)*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)))*(
-(a^2*c^2*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c
*d^2*e + a*e^3)/(a^2*c^2))^(1/3)) + 1/12*(sqrt(-3) - 1)*(-(a^2*c^2*sqrt(-(
c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^5*c^3)) - 3*c*d^2*e + a*e^3...
```

Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.54

$$\int \frac{d+ex^3}{a+cx^6}\,dx$$
$$= \text{RootSum}\left(46656t^6a^5c^4 + t^3\cdot\left(432a^4c^2e^3 - 1296a^3c^3d^2e\right) + a^3e^6 + 3a^2cd^2e^4 + 3ac^2d^4e^2 + c^3d^6, \left(t \mapsto t\,\text{l}\right.\right.$$

input
```
integrate((e*x**3+d)/(c*x**6+a),x)
```

output
```
RootSum(46656*_t**6*a**5*c**4 + _t**3*(432*a**4*c**2*e**3 - 1296*a**3*c**3
*d**2*e) + a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6
, Lambda(_t, _t*log(x + (-1296*_t**4*a**4*c**2*e - 6*_t*a**3*e**4 + 36*_t*
a**2*c*d**2*e**2 - 6*_t*a*c**2*d**4)/(3*a**2*d*e**4 + 2*a*c*d**3*e**2 - c*
*2*d**5))))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.92

$$\int \frac{d+ex^3}{a+cx^6}\,dx = -\frac{e\log\left(c^{\frac{1}{3}}x^2 + a^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}c^{\frac{2}{3}}} + \frac{d\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{3\,a^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} + \frac{\left(\sqrt{3}a^{\frac{1}{6}}\sqrt{c}d + a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2 + \sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x + a^{\frac{1}{3}}\right)}{12\,ac^{\frac{2}{3}}} - \frac{\left(\sqrt{3}a^{\frac{1}{6}}\sqrt{c}d - a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2 - \sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x + a^{\frac{1}{3}}\right)}{12\,ac^{\frac{2}{3}}} - \frac{\left(\sqrt{3}a^{\frac{5}{6}}c^{\frac{1}{6}}e - a^{\frac{1}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,c^{\frac{1}{3}}x + \sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{6\,ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} + \frac{\left(\sqrt{3}a^{\frac{5}{6}}c^{\frac{1}{6}}e + a^{\frac{1}{3}}c^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,c^{\frac{1}{3}}x - \sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{6\,ac^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}$$

input
```
integrate((e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

output
```
-1/6*e*log(c^(1/3)*x^2 + a^(1/3))/(a^(1/3)*c^(2/3)) + 1/3*d*arctan(c^(1/3)
*x/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*sqrt(a^(1/3)*c^(1/3))) + 1/12*(sqrt(3)*
a^(1/6)*sqrt(c)*d + a^(2/3)*e)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x
 + a^(1/3))/(a*c^(2/3)) - 1/12*(sqrt(3)*a^(1/6)*sqrt(c)*d - a^(2/3)*e)*log
(c^(1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) - 1/6*(sqr
t(3)*a^(5/6)*c^(1/6)*e - a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x + sqrt(3)*
a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3)))
+ 1/6*(sqrt(3)*a^(5/6)*c^(1/6)*e + a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x
- sqrt(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*
c^(1/3)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.93

$$\int \frac{d+ex^3}{a+cx^6}\,dx = -\frac{e|c|\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,(ac^5)^{\frac{1}{3}}} + \frac{(ac^5)^{\frac{1}{6}}\,d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac} + \frac{\left((ac^5)^{\frac{1}{6}}\,c^3d-\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left((ac^5)^{\frac{1}{6}}\,c^3d+\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4} + \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,c^3d+(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4} - \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,c^3d-(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$

input
```
integrate((e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

output
```
-1/6*e*abs(c)*log(x^2 + (a/c)^(1/3))/(a*c^5)^(1/3) + 1/3*(a*c^5)^(1/6)*d*a
rctan(x/(a/c)^(1/6))/(a*c) + 1/6*((a*c^5)^(1/6)*c^3*d - sqrt(3)*(a*c^5)^(2
/3)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) + 1/6*((a*c
^5)^(1/6)*c^3*d + sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)*(a/c)^(1/
6))/(a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(a*c^5)^(1/6)*c^3*d + (a*c^5)^(2/
3)*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^4) - 1/12*(sqrt(
3)*(a*c^5)^(1/6)*c^3*d - (a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*x*(a/c)^(1/6)
+ (a/c)^(1/3))/(a*c^4)
```

Mupad [B] (verification not implemented)

Time = 22.15 (sec) , antiderivative size = 1331, normalized size of antiderivative = 4.36

$$\int \frac{d+ex^3}{a+cx^6}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^3)/(a + c*x^6),x)
```

output
```
log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*
a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) + e*x*(-a^5*c^5)^(1/2) + a^2*c^
3*d*x)*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e
^2*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c
*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5
*c^4))^(1/3) - e*x*(-a^5*c^5)^(1/2) + a^2*c^3*d*x)*(-(a^4*c^2*e^3 - c*d^3*
(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(216*a^5*
c^4))^(1/3) - log(a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*
c^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3) - 2*e*x*(-a^5*c^5
)^(1/2) + 3^(1/2)*a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*
c^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i - 2*a^2*c^3*d*
x)*((3^(1/2)*1i)/2 + 1/2)*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*
c^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(e*x*(-a
^5*c^5)^(1/2) - (a^3*c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c
^3*d^2*e - 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3))/2 + (3^(1/2)*a^3*
c^3*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e^2*
(-a^5*c^5)^(1/2))/(a^5*c^4))^(1/3)*1i)/2 + a^2*c^3*d*x)*((3^(1/2)*1i)/2 -
1/2)*(-(a^4*c^2*e^3 + c*d^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e - 3*a*d*e^2
*(-a^5*c^5)^(1/2))/(216*a^5*c^4))^(1/3) + log(a^3*c^3*(-(a^4*c^2*e^3 - c*d
^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d^2*e + 3*a*d*e^2*(-a^5*c^5)^(1/2))/(a^...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.90

$$\int \frac{d+ex^3}{a+cx^6}\,dx$$
$$= \frac{-2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d-2\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)ae+2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d-2\sqrt{3}\,a}{}$$

input
```
int((e*x^3+d)/(c*x^6+a),x)
```

output
```
( - 2*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**
(1/6)*a**(1/6)))*d - 2*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3
)*x)/(c**(1/6)*a**(1/6)))*a*e + 2*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*
sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*d - 2*sqrt(3)*atan((c**(1/6)*
a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*e + 4*sqrt(c)*sqrt
(a)*atan((c**(1/3)*x)/(c**(1/6)*a**(1/6)))*d - sqrt(c)*sqrt(a)*sqrt(3)*log
( - c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*d + sqrt(c)*sq
rt(a)*sqrt(3)*log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*
d - 2*log(a**(1/3) + c**(1/3)*x**2)*a*e + log( - c**(1/6)*a**(1/6)*sqrt(3)
*x + a**(1/3) + c**(1/3)*x**2)*a*e + log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**
(1/3) + c**(1/3)*x**2)*a*e)/(12*c**(2/3)*a**(1/3)*a)
```

3.20 $\int \frac{d+ex^3}{x^3(a+cx^6)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 316

$$\begin{aligned}\int \frac{d+ex^3}{x^3(a+cx^6)}\,dx &= -\frac{d}{2ax^2}+\frac{e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}} \\ &\quad+\frac{\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{4/3}\sqrt[6]{c}} \\ &\quad+\frac{\left(\sqrt{3}\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{4/3}\sqrt[6]{c}}+\frac{\sqrt[3]{c}d\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6a^{4/3}} \\ &\quad-\frac{\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{4/3}\sqrt[6]{c}} \\ &\quad-\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{4/3}\sqrt[6]{c}}\end{aligned}$$

output
```
-1/2*d/a/x^2+1/3*e*arctan(c^(1/6)*x/a^(1/6))/a^(5/6)/c^(1/6)-1/6*(3^(1/2)*
c^(1/2)*d-a^(1/2)*e)*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(4/3)/c^(1/6)+
1/6*(3^(1/2)*c^(1/2)*d+a^(1/2)*e)*arctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(4
/3)/c^(1/6)+1/6*c^(1/3)*d*ln(a^(1/3)+c^(1/3)*x^2)/a^(4/3)-1/12*(c^(1/2)*d+
3^(1/2)*a^(1/2)*e)*ln(a^(1/3)-3^(1/2)*a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(4/
3)/c^(1/6)-1/12*(c^(1/2)*d-3^(1/2)*a^(1/2)*e)*ln(a^(1/3)+3^(1/2)*a^(1/6)*c
^(1/6)*x+c^(1/3)*x^2)/a^(4/3)/c^(1/6)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.97

$$\int \frac{d+ex^3}{x^3\left(a+cx^6\right)}\,dx$$

$$= \frac{-\frac{6ad}{x^2} + \frac{4a^{7/6}e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{c}} + \frac{2a^{2/3}\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{c}} + \frac{2a^{2/3}\left(\sqrt{3}\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{c}} + 2a^2 \cdots}{\cdots}$$

input
```
Integrate[(d + e*x^3)/(x^3*(a + c*x^6)),x]
```

output
```
((-6*a*d)/x^2 + (4*a^(7/6)*e*ArcTan[(c^(1/6)*x)/a^(1/6)])/c^(1/6) + (2*a^(
2/3)*(Sqrt[3]*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6
)])/c^(1/6) + (2*a^(2/3)*(Sqrt[3]*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[Sqrt[3] +
(2*c^(1/6)*x)/a^(1/6)])/c^(1/6) + 2*a^(2/3)*c^(1/3)*d*Log[a^(1/3) + c^(1/3
)*x^2] - ((a^(2/3)*Sqrt[c]*d + Sqrt[3]*a^(7/6)*e)*Log[a^(1/3) - Sqrt[3]*a^
(1/6)*c^(1/6)*x + c^(1/3)*x^2])/c^(1/6) + ((-(a^(2/3)*Sqrt[c]*d) + Sqrt[3]
*a^(7/6)*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/c^(1/6
))/(12*a^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {1829, 27, 1746, 27, 452, 218, 240, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^3\left(a+cx^6\right)}dx$$

$$\Big\downarrow \ 1829$$

$$-\frac{\int -\frac{2\left(ae-cdx^3\right)}{cx^6+a}dx}{2a}-\frac{d}{2ax^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{ae-cdx^3}{cx^6+a}dx}{a}-\frac{d}{2ax^2}$$

$$\Big\downarrow \ 1746$$

$$\frac{\frac{\int \frac{\sqrt[3]{a}\sqrt[3]{c}\left(a^{2/3}e+c^{2/3}dx\right)}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{\sqrt[3]{c}\left(2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)x\right)}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{\sqrt[3]{c}\left(2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)x\right)}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}\sqrt[3]{c}}}{a}-\frac{d}{2ax^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{a^{2/3}e+c^{2/3}dx}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\int \frac{2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}}{a}-\frac{d}{2ax^2}$$

$$\Big\downarrow \ 452$$

$$\frac{\frac{a^{2/3} e \int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx+c^{2/3} d \int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\int \frac{2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}}{a}-\frac{d}{2ax^2}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\int \frac{2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\int \frac{2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{c^{2/3} d \int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx+\frac{\sqrt{a}e \arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{c}}}{3\sqrt[3]{a}}}{a}-\frac{d}{2ax^2}$$

$$\Big\downarrow \ 240$$

$$\frac{\frac{\int \frac{2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\int \frac{2a^{2/3}e-\sqrt[6]{c}\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\frac{\sqrt{a}e \arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{c}}+\frac{1}{2}\sqrt[3]{c}d\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{3\sqrt[3]{a}}}{a}-\frac{d}{2ax^2}$$

$$\Big\downarrow \ 1142$$

$$\frac{\frac{-\frac{1}{2}\sqrt[6]{a}\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{\sqrt[3]{a}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int -\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[6]{c}}}{6a^{2/3}}+\frac{\frac{1}{2}\sqrt[6]{a}\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int \frac{\cdots}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\cdots}\cdots}{\cdots}}{a}$$

$$\frac{d}{2ax^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{\sqrt[3]{a}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[6]{c}}-\frac{1}{2}\sqrt[6]{a}\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\frac{1}{2}\sqrt[6]{a}\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\ldots}{\ldots}}}{\ldots}}{a}$$

$$\frac{d}{2ax^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{1}{2}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{1}{2}\sqrt[6]{a}\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a^{2/3}}+\frac{\frac{1}{2}\sqrt[6]{a}\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}\ldots}{6\ldots}}{a}$$

$$\frac{d}{2ax^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\frac{1}{2}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{\sqrt[3]{a}\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\int \frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{c}}}{6a^{2/3}}+\frac{-\frac{1}{2}\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int \frac{2\sqrt[6]{c}x+\sqrt{3}\ldots}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\ldots}{\ldots}}\ldots}{\ldots}}{a}$$

$$\frac{d}{2ax^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\frac{1}{2}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx+\frac{\sqrt[3]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)}{\sqrt[6]{c}}}{6a^{2/3}}+\frac{\frac{\sqrt[3]{a}\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)}{\sqrt[6]{c}}-\frac{1}{2}\ldots}{6a^{2/3}}}{a}$$

$$\frac{d}{2ax^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\frac{\sqrt[3]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)}{\sqrt[6]{c}}-\frac{\sqrt[3]{a}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{6a^{2/3}}+\frac{\frac{\sqrt[3]{a}\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)\left(\sqrt{a}e+\sqrt{3}\right.}{\sqrt[6]{c}}}{}}{a}$$

$$\frac{d}{2ax^2}$$

input
```
Int[(d + e*x^3)/(x^3*(a + c*x^6)),x]
```

output
```
-1/2*d/(a*x^2) + (((Sqrt[a]*e*ArcTan[(c^(1/6)*x)/a^(1/6)])/c^(1/6) + (c^(1
/3)*d*Log[a^(1/3) + c^(1/3)*x^2])/2)/(3*a^(1/3)) + ((a^(1/3)*(Sqrt[3]*Sqrt
[c]*d - Sqrt[a]*e)*ArcTan[Sqrt[3]*(1 - (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))])/
c^(1/6) - (a^(1/3)*(Sqrt[c]*d + Sqrt[3]*Sqrt[a]*e)*Log[a^(1/3) - Sqrt[3]*a
^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6)))/(6*a^(2/3)) + ((a^(1/3)*(Sqr
t[3]*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[Sqrt[3]*(1 + (2*c^(1/6)*x)/(Sqrt[3]*a^(
1/6)))])/c^(1/6) - (a^(1/3)*(Sqrt[c]*d - Sqrt[3]*Sqrt[a]*e)*Log[a^(1/3) +
Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6)))/(6*a^(2/3)))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1746
```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Simp[1/(3*a*q^2)   Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Sim
p[1/(6*a*q^2)   Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^
2*x^2), x], x] + Simp[1/(6*a*q^2)   Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(
1 + Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
 + a*e^2, 0] && PosQ[c/a]
```

rule 1829
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^(2*n))^(p + 1)/(a*f*(m + 1
))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m + n)*(a + c*x^(2*n))^p*(a*e
*(m + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f,
p}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{d}{2a\,x^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^8c\_Z^6+\left(6a^5cd\,e^2-2a^4c^2d^3\right)\_Z^3+a^3e^6+3a^2c\,d^2e^4+3a\,c^2d^4e^2+c^3d^6\right)}\_R\ln\left(\left(7\_R^6a^8c+(39a^5cd\,e^2-\right.\right.}{6}$
default	$-\frac{d}{2a\,x^2}+\frac{-\frac{c\left(\frac{a}{c}\right)^{\frac{7}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\,e}{12a}-\frac{c\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)d}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)e}{6}-\frac{c\left(\frac{a}{c}\right)^{\frac{2}{3}}\text{ar}}{}}{}$

input
```
int((e*x^3+d)/x^3/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/a/x^2+1/6*sum(_R*ln((7*_R^6*a^8*c+(39*a^5*c*d*e^2-13*a^4*c^2*d^3)*_
R^3+6*a^3*e^6+18*a^2*c*d^2*e^4+18*a*c^2*d^4*e^2+6*c^3*d^6)*x-2*a^6*c*d*e*_
R^4+(-a^4*e^5-2*a^3*c*d^2*e^3-a^2*c^2*d^4*e)*_R),_R=RootOf(a^8*c*_Z^6+(6*a
^5*c*d*e^2-2*a^4*c^2*d^3)*_Z^3+a^3*e^6+3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2+c^3
*d^6))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1598 vs. 2(216) = 432.

Time = 0.11 (sec) , antiderivative size = 1598, normalized size of antiderivative = 5.06

$$\int \frac{d+ex^3}{x^3\,(a+cx^6)}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^3+d)/x^3/(c*x^6+a),x, algorithm="fricas")
```

output
```
1/12*(2*a*x^2*((a^4*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c
)) + c*d^3 - 3*a*d*e^2)/a^4)^(1/3)*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 - a^2
*e^5)*x - (a^5*c*d*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)
) + 3*a^2*c*d^2*e^2 - a^3*e^4)*((a^4*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4
+ a^2*e^6)/(a^7*c)) + c*d^3 - 3*a*d*e^2)/a^4)^(1/3)) + 2*a*x^2*(-(a^4*sqrt
(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)) - c*d^3 + 3*a*d*e^2)/
a^4)^(1/3)*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 - a^2*e^5)*x + (a^5*c*d*sqrt(
-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)) - 3*a^2*c*d^2*e^2 + a^
3*e^4)*(-(a^4*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)) - c
*d^3 + 3*a*d*e^2)/a^4)^(1/3)) - (sqrt(-3)*a*x^2 + a*x^2)*((a^4*sqrt(-(9*c^
2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)) + c*d^3 - 3*a*d*e^2)/a^4)^(1
/3)*log(-(3*c^2*d^4*e + 2*a*c*d^2*e^3 - a^2*e^5)*x + 1/2*(3*a^2*c*d^2*e^2
- a^3*e^4 + sqrt(-3)*(3*a^2*c*d^2*e^2 - a^3*e^4) + (sqrt(-3)*a^5*c*d + a^5
*c*d)*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)))*((a^4*sqrt
(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)) + c*d^3 - 3*a*d*e^2)/
a^4)^(1/3)) + (sqrt(-3)*a*x^2 - a*x^2)*((a^4*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*
d^2*e^4 + a^2*e^6)/(a^7*c)) + c*d^3 - 3*a*d*e^2)/a^4)^(1/3)*log(-(3*c^2*d^
4*e + 2*a*c*d^2*e^3 - a^2*e^5)*x + 1/2*(3*a^2*c*d^2*e^2 - a^3*e^4 - sqrt(-
3)*(3*a^2*c*d^2*e^2 - a^3*e^4) - (sqrt(-3)*a^5*c*d - a^5*c*d)*sqrt(-(9*c^2
*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a^7*c)))*((a^4*sqrt(-(9*c^2*d^4*e^...
```

Sympy [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.53

$$\int \frac{d+ex^3}{x^3\,(a+cx^6)}\,dx$$
$$= \mathrm{RootSum}\left(46656t^6a^8c + t^3\cdot\left(1296a^5cde^2 - 432a^4c^2d^3\right) + a^3e^6 + 3a^2cd^2e^4 + 3ac^2d^4e^2 + c^3d^6, \left(t \mapsto t\log\right.\right.$$
$$-\frac{d}{2ax^2}$$

input
```
integrate((e*x**3+d)/x**3/(c*x**6+a),x)
```

output
```
RootSum(46656*_t**6*a**8*c + _t**3*(1296*a**5*c*d*e**2 - 432*a**4*c**2*d**
3) + a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6, Lamb
da(_t, _t*log(x + (-1296*_t**4*a**5*c*d + 6*_t*a**3*e**4 - 36*_t*a**2*c*d*
*2*e**2 + 6*_t*a*c**2*d**4)/(a**2*e**5 - 2*a*c*d**2*e**3 - 3*c**2*d**4*e))
)) - d/(2*a*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.94

$$\int \frac{d+ex^3}{x^3\left(a+cx^6\right)}\,dx$$

$$= \frac{\frac{2c^{\frac{1}{3}}d\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}} + \frac{4a^{\frac{1}{3}}e\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} + \frac{\left(\sqrt{3}a^{\frac{7}{6}}\sqrt{c}e-a^{\frac{2}{3}}cd\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{ac^{\frac{2}{3}}} - \frac{\left(\sqrt{3}a^{\frac{7}{6}}\sqrt{c}e+a^{\frac{2}{3}}cd\right)\log\left(c^{\frac{1}{3}}x^2-\cdots\right.}{ac^{\frac{2}{3}}}}{12\,a}$$

$$- \frac{d}{2\,ax^2}$$

input
```
integrate((e*x^3+d)/x^3/(c*x^6+a),x, algorithm="maxima")
```

output
```
1/12*(2*c^(1/3)*d*log(c^(1/3)*x^2 + a^(1/3))/a^(1/3) + 4*a^(1/3)*e*arctan(
c^(1/3)*x/sqrt(a^(1/3)*c^(1/3)))/sqrt(a^(1/3)*c^(1/3)) + (sqrt(3)*a^(7/6)*
sqrt(c)*e - a^(2/3)*c*d)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x + a^(
1/3))/(a*c^(2/3)) - (sqrt(3)*a^(7/6)*sqrt(c)*e + a^(2/3)*c*d)*log(c^(1/3)*
x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) + 2*(sqrt(3)*a^(5/6
)*c^(7/6)*d + a^(4/3)*c^(2/3)*e)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6)*c^(
1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))) - 2*(sqrt(3
)*a^(5/6)*c^(7/6)*d - a^(4/3)*c^(2/3)*e)*arctan((2*c^(1/3)*x - sqrt(3)*a^(
1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))))/a
- 1/2*d/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.96

$$\int \frac{d+ex^3}{x^3(a+cx^6)}\,dx = \frac{(ac^5)^{\frac{1}{6}}e\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac} - \frac{d}{2\,ax^2} + \frac{(ac^5)^{\frac{2}{3}}d|c|\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,a^2c^4} + \frac{\left((ac^5)^{\frac{1}{6}}ac^2e+\sqrt{3}(ac^5)^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,a^2c^3} + \frac{\left((ac^5)^{\frac{1}{6}}ac^2e-\sqrt{3}(ac^5)^{\frac{2}{3}}d\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,a^2c^3} + \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}ac^2e-(ac^5)^{\frac{2}{3}}d\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,a^2c^3} - \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}ac^2e+(ac^5)^{\frac{2}{3}}d\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,a^2c^3}$$

input
```
integrate((e*x^3+d)/x^3/(c*x^6+a),x, algorithm="giac")
```

output
```
1/3*(a*c^5)^(1/6)*e*arctan(x/(a/c)^(1/6))/(a*c) - 1/2*d/(a*x^2) + 1/6*(a*c
^5)^(2/3)*d*abs(c)*log(x^2 + (a/c)^(1/3))/(a^2*c^4) + 1/6*((a*c^5)^(1/6)*a
*c^2*e + sqrt(3)*(a*c^5)^(2/3)*d)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)
^(1/6))/(a^2*c^3) + 1/6*((a*c^5)^(1/6)*a*c^2*e - sqrt(3)*(a*c^5)^(2/3)*d)*
arctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^2*c^3) + 1/12*(sqrt(3)*
(a*c^5)^(1/6)*a*c^2*e - (a*c^5)^(2/3)*d)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) +
 (a/c)^(1/3))/(a^2*c^3) - 1/12*(sqrt(3)*(a*c^5)^(1/6)*a*c^2*e + (a*c^5)^(2
/3)*d)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 22.39 (sec) , antiderivative size = 1187, normalized size of antiderivative = 3.76

$$\int \frac{d+ex^3}{x^3(a+cx^6)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^3)/(x^3*(a + c*x^6)),x)
```

output
```
log(a^6*((a*e^3*(-a^9*c)^(1/2) + a^4*c^2*d^3 - 3*a^5*c*d*e^2 - 3*c*d^2*e*(
-a^9*c)^(1/2))/(a^8*c))^(1/3) + d*x*(-a^9*c)^(1/2) + a^5*e*x)*((a*e^3*(-a^
9*c)^(1/2) + a^4*c^2*d^3 - 3*a^5*c*d*e^2 - 3*c*d^2*e*(-a^9*c)^(1/2))/(216*
a^8*c))^(1/3) + log(a^6*(-(a*e^3*(-a^9*c)^(1/2) - a^4*c^2*d^3 + 3*a^5*c*d*
e^2 - 3*c*d^2*e*(-a^9*c)^(1/2))/(a^8*c))^(1/3) - d*x*(-a^9*c)^(1/2) + a^5*
e*x)*(-(a*e^3*(-a^9*c)^(1/2) - a^4*c^2*d^3 + 3*a^5*c*d*e^2 - 3*c*d^2*e*(-a
^9*c)^(1/2))/(216*a^8*c))^(1/3) - d/(2*a*x^2) - log(a^6*((a*e^3*(-a^9*c)^(
1/2) + a^4*c^2*d^3 - 3*a^5*c*d*e^2 - 3*c*d^2*e*(-a^9*c)^(1/2))/(a^8*c))^(1
/3) - 2*d*x*(-a^9*c)^(1/2) + 3^(1/2)*a^6*((a*e^3*(-a^9*c)^(1/2) + a^4*c^2*
d^3 - 3*a^5*c*d*e^2 - 3*c*d^2*e*(-a^9*c)^(1/2))/(a^8*c))^(1/3)*1i - 2*a^5*
e*x)*((3^(1/2)*1i)/2 + 1/2)*((a*e^3*(-a^9*c)^(1/2) + a^4*c^2*d^3 - 3*a^5*c
*d*e^2 - 3*c*d^2*e*(-a^9*c)^(1/2))/(216*a^8*c))^(1/3) + log(d*x*(-a^9*c)^(
1/2) - (a^6*((a*e^3*(-a^9*c)^(1/2) + a^4*c^2*d^3 - 3*a^5*c*d*e^2 - 3*c*d^2
*e*(-a^9*c)^(1/2))/(a^8*c))^(1/3))/2 + (3^(1/2)*a^6*((a*e^3*(-a^9*c)^(1/2)
 + a^4*c^2*d^3 - 3*a^5*c*d*e^2 - 3*c*d^2*e*(-a^9*c)^(1/2))/(a^8*c))^(1/3)*
1i)/2 + a^5*e*x)*((3^(1/2)*1i)/2 - 1/2)*((a*e^3*(-a^9*c)^(1/2) + a^4*c^2*d
^3 - 3*a^5*c*d*e^2 - 3*c*d^2*e*(-a^9*c)^(1/2))/(216*a^8*c))^(1/3) + log(a^
6*(-(a*e^3*(-a^9*c)^(1/2) - a^4*c^2*d^3 + 3*a^5*c*d*e^2 - 3*c*d^2*e*(-a^9*
c)^(1/2))/(a^8*c))^(1/3) + 2*d*x*(-a^9*c)^(1/2) - 3^(1/2)*a^6*(-(a*e^3*(-a
^9*c)^(1/2) - a^4*c^2*d^3 + 3*a^5*c*d*e^2 - 3*c*d^2*e*(-a^9*c)^(1/2))/(...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.07

$$\int \frac{d+ex^3}{x^3\left(a+cx^6\right)}\,dx$$

$$= \frac{-2\sqrt{c}\,a^{\frac{7}{6}}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e\,x^2+2a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)cd\,x^2+2\sqrt{c}\,a^{\frac{7}{6}}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e\,x^2 -}{}$$

input
```
int((e*x^3+d)/x^3/(c*x^6+a),x)
```

output
```
( - 2*sqrt(c)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c*
*(1/6)*a**(1/6)))*a*e*x**2 + 2*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**(1/6)*sq
rt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d*x**2 + 2*sqrt(c)*a**(1/6)*a
tan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*e*x*
*2 + 2*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c
**(1/6)*a**(1/6)))*c*d*x**2 + 4*sqrt(c)*a**(1/6)*atan((c**(1/3)*x)/(c**(1/
6)*a**(1/6)))*a*e*x**2 - sqrt(c)*a**(1/6)*sqrt(3)*log( - c**(1/6)*a**(1/6)
*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*a*e*x**2 + sqrt(c)*a**(1/6)*sqrt(3)
*log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*a*e*x**2 + 2*
a**(2/3)*log(a**(1/3) + c**(1/3)*x**2)*c*d*x**2 - a**(2/3)*log( - c**(1/6)
*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c*d*x**2 - a**(2/3)*log(c*
*(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c*d*x**2 - 6*c**(2/3
)*a*d)/(12*c**(2/3)*a**2*x**2)
```

3.21 $\int \frac{d+ex^3}{x^6(a+cx^6)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 327

$$\begin{aligned}\int \frac{d+ex^3}{x^6(a+cx^6)}\,dx = & -\frac{d}{5ax^5}-\frac{e}{2ax^2}-\frac{c^{5/6}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{11/6}} \\ & +\frac{\sqrt[3]{c}\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{11/6}} \\ & -\frac{\sqrt[3]{c}\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{11/6}} \\ & +\frac{\sqrt[3]{c}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6a^{4/3}} \\ & +\frac{\sqrt[3]{c}\left(\sqrt{3}\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{11/6}} \\ & -\frac{\sqrt[3]{c}\left(\sqrt{3}\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{11/6}}\end{aligned}$$

output
```
-1/5*d/a/x^5-1/2*e/a/x^2-1/3*c^(5/6)*d*arctan(c^(1/6)*x/a^(1/6))/a^(11/6)-
1/6*c^(1/3)*(c^(1/2)*d+3^(1/2)*a^(1/2)*e)*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1
/6))/a^(11/6)-1/6*c^(1/3)*(c^(1/2)*d-3^(1/2)*a^(1/2)*e)*arctan(3^(1/2)+2*c
^(1/6)*x/a^(1/6))/a^(11/6)+1/6*c^(1/3)*e*ln(a^(1/3)+c^(1/3)*x^2)/a^(4/3)+1
/12*c^(1/3)*(3^(1/2)*c^(1/2)*d-a^(1/2)*e)*ln(a^(1/3)-3^(1/2)*a^(1/6)*c^(1/
6)*x+c^(1/3)*x^2)/a^(11/6)-1/12*c^(1/3)*(3^(1/2)*c^(1/2)*d+a^(1/2)*e)*ln(a
^(1/3)+3^(1/2)*a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(11/6)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.96

$$\int \frac{d+ex^3}{x^6\left(a+cx^6\right)}\,dx$$
$$= \frac{-\frac{12ad}{x^5}-\frac{30ae}{x^2}-20\sqrt[6]{a}c^{5/6}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)+10\sqrt[6]{a}\sqrt[3]{c}\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)+10\sqrt[3]{c}(-}{}$$

input
```
Integrate[(d + e*x^3)/(x^6*(a + c*x^6)),x]
```

output
```
((-12*a*d)/x^5 - (30*a*e)/x^2 - 20*a^(1/6)*c^(5/6)*d*ArcTan[(c^(1/6)*x)/a^
(1/6)] + 10*a^(1/6)*c^(1/3)*(Sqrt[c]*d + Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3]
 - (2*c^(1/6)*x)/a^(1/6)] + 10*c^(1/3)*(-(a^(1/6)*Sqrt[c]*d) + Sqrt[3]*a^(
2/3)*e)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)] + 10*a^(2/3)*c^(1/3)*e*Log
[a^(1/3) + c^(1/3)*x^2] + 5*c^(1/3)*(Sqrt[3]*a^(1/6)*Sqrt[c]*d - a^(2/3)*e
)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] - 5*c^(1/3)*(Sqrt
[3]*a^(1/6)*Sqrt[c]*d + a^(2/3)*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x
 + c^(1/3)*x^2])/(60*a^2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {1829, 27, 1829, 27, 1746, 27, 452, 218, 240, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^6\left(a+cx^6\right)}dx$$

$$\Big\downarrow \text{1829}$$

$$-\frac{\int -\frac{5\left(ae-cdx^3\right)}{x^3\left(cx^6+a\right)}dx}{5a}-\frac{d}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{ae-cdx^3}{x^3\left(cx^6+a\right)}dx}{a}-\frac{d}{5ax^5}$$

$$\Big\downarrow \text{1829}$$

$$\frac{-\frac{\int \frac{2ac\left(ex^3+d\right)}{cx^6+a}dx}{2a}-\frac{e}{2x^2}}{a}-\frac{d}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{-c\int \frac{ex^3+d}{cx^6+a}dx-\frac{e}{2x^2}}{a}-\frac{d}{5ax^5}$$

$$\Big\downarrow \text{1746}$$

$$\frac{-c\left(\frac{\int \frac{\sqrt[3]{c}d-\sqrt[3]{a}ex}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{6a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{6a^{2/3}\sqrt[3]{c}}\right)-\frac{e}{2x^2}}{a}-\frac{d}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{-c\left(\frac{\int \frac{\sqrt[3]{c}d-\sqrt[3]{a}ex}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}\right)-\frac{e}{2x^2}}{a}-\frac{d}{5ax^5}$$

$\Big\downarrow$ 452

$$\frac{-c\left(\frac{\sqrt[3]{c}d\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx-\sqrt[3]{a}e\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}\right)-\frac{e}{2x^2}}{a}-\frac{d}{5ax^5}$$

$\Big\downarrow$ 218

$$\frac{-c\left(\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\sqrt[3]{a}e\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}\right)-\frac{e}{2x^2}}{a}-\frac{d}{5ax^5}$$

$\Big\downarrow$ 240

$$\frac{-c\left(\frac{\int \frac{2\sqrt[3]{c}d-\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\int \frac{2\sqrt[3]{c}d+\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}+e\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a\sqrt[3]{c}}+\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}\right)-\frac{e}{2x^2}}{a}-\frac{d}{5ax^5}$$

$\Big\downarrow$ 1142

$$-c\left(\frac{\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}-\frac{a^{2/3}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int -\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}\cdots}{\cdots}\right.$$

$$a$$

$$\frac{d}{5ax^5}$$

$\big\downarrow$ 25

$$-c\left(\frac{\frac{a^{2/3}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[3]{c}}+\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\cdots}{\cdots}\right.$$

$$a$$

$$\frac{d}{5ax^5}$$

$\big\downarrow$ 27

$$-c\left(\frac{\frac{\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{c}d-\sqrt{3}\sqrt{a}e\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\left(\sqrt{a}e+\cdots\right.}{\cdots}}{6a\sqrt[3]{c}}\right.$$

$$a$$

$$\frac{d}{5ax^5}$$

$\big\downarrow$ 1082

$$-c\left(\frac{\frac{\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}+\frac{\sqrt[6]{a}\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)\int\frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int\frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{a}\sqrt[6]{c}}\cdots}{\cdots}\cdots\right.$$

$$\frac{\cdots}{a}$$

$$\frac{d}{5ax^5}$$

$\downarrow$ 217

$$-c\left(\frac{\frac{\sqrt[3]{a}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{c}}-\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(\sqrt{3}\sqrt{a}e+\sqrt{c}d\right)}{\sqrt[3]{c}}}{6a\sqrt[3]{c}}+\frac{\frac{\left(\sqrt{a}e+\sqrt{3}\sqrt{c}d\right)\int\frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{2\sqrt[6]{a}\sqrt[6]{c}}+\cdots}{6a\sqrt[3]{\cdots}}\cdots\right.$$

$$\frac{\cdots}{a}$$

$$\frac{d}{5ax^5}$$

$\downarrow$ 1103

$$-c\left(\frac{\frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}}{3a^{2/3}\sqrt[3]{c}}+\frac{-\frac{a^{2/3}\left(\frac{\sqrt{3}\sqrt{c}d}{\sqrt{a}}-e\right)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{c}}-\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\cdots\right.}{\sqrt[3]{c}}\cdots}{6a\sqrt[3]{c}}\cdots\right.$$

$$\frac{\cdots}{a}$$

$$\frac{d}{5ax^5}$$

input
```
Int[(d + e*x^3)/(x^6*(a + c*x^6)),x]
```

output
```
-1/5*d/(a*x^5) + (-1/2*e/x^2 - c*(((c^(1/6)*d*ArcTan[(c^(1/6)*x)/a^(1/6)])
/a^(1/6) - (a^(1/3)*e*Log[a^(1/3) + c^(1/3)*x^2])/(2*c^(1/3)))/(3*a^(2/3)*
c^(1/3)) + (-((a^(1/6)*(Sqrt[c]*d + Sqrt[3]*Sqrt[a]*e)*ArcTan[Sqrt[3]*(1 -
 (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))])/c^(1/3)) - (a^(2/3)*((Sqrt[3]*Sqrt[c]*
d)/Sqrt[a] - e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2
*c^(1/3)))/(6*a*c^(1/3)) + ((a^(1/6)*(Sqrt[c]*d - Sqrt[3]*Sqrt[a]*e)*ArcTa
n[Sqrt[3]*(1 + (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))])/c^(1/3) + (a^(1/6)*(Sqrt
[3]*Sqrt[c]*d + Sqrt[a]*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/
3)*x^2])/(2*c^(1/3)))/(6*a*c^(1/3))))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1746
```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Simp[1/(3*a*q^2)   Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Sim
p[1/(6*a*q^2)   Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^
2*x^2), x], x] + Simp[1/(6*a*q^2)   Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(
1 + Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
 + a*e^2, 0] && PosQ[c/a]
```

rule 1829
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^(2*n))^(p + 1)/(a*f*(m + 1
))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m + n)*(a + c*x^(2*n))^p*(a*e
*(m + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f,
p}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.73

method	result
risch	$\frac{-\frac{e\,x^3}{2a}-\frac{d}{5a}}{x^5}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{11}\_Z^6+\left(-2a^7c\,e^3+6a^6c^2d^2e\right)\_Z^3+a^3c^2e^6+3a^2c^3d^2e^4+3a\,c^4d^4e^2+c^5d^6\right)}\_R\ln\left(\left(7\_R^6a^{11}+(-13a\right.\right.}{}$
default	$-\frac{d}{5a\,x^5}-\frac{e}{2a\,x^2}-\frac{\left(\frac{\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{2}{3}}e}{12a}-\frac{\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}d}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{2}{3}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)\sqrt{3}\,e}{6a}\right.}{}$

input
```
int((e*x^3+d)/x^6/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
(-1/2*e/a*x^3-1/5*d/a)/x^5+1/6*sum(_R*ln((7*_R^6*a^11+(-13*a^7*c*e^3+39*a^
6*c^2*d^2*e)*_R^3+6*a^3*c^2*e^6+18*a^2*c^3*d^2*e^4+18*a*c^4*d^4*e^2+6*c^5*
d^6)*x+2*a^8*c*d*e*_R^4+(a^4*c^2*d*e^4+2*a^3*c^3*d^3*e^2+a^2*c^4*d^5)*_R),
_R=RootOf(a^11*_Z^6+(-2*a^7*c*e^3+6*a^6*c^2*d^2*e)*_Z^3+a^3*c^2*e^6+3*a^2*
c^3*d^2*e^4+3*a*c^4*d^4*e^2+c^5*d^6))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1734 vs. 2(225) = 450.

Time = 0.20 (sec) , antiderivative size = 1734, normalized size of antiderivative = 5.30

$$\int \frac{d+ex^3}{x^6\left(a+cx^6\right)}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^3+d)/x^6/(c*x^6+a),x, algorithm="fricas")
```

output
```
1/60*(10*a*x^5*(-(a^5*sqrt(-(c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4
)/a^11) + 3*c^2*d^2*e - a*c*e^3)/a^5)^(1/3)*log(-(c^4*d^5 - 2*a*c^3*d^3*e^
2 - 3*a^2*c^2*d*e^4)*x + (a^8*e*sqrt(-(c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c
^3*d^2*e^4)/a^11) + a^2*c^3*d^4 - 3*a^3*c^2*d^2*e^2)*(-(a^5*sqrt(-(c^5*d^6
 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)/a^11) + 3*c^2*d^2*e - a*c*e^3)/a^5
)^(1/3)) + 10*a*x^5*((a^5*sqrt(-(c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2
*e^4)/a^11) - 3*c^2*d^2*e + a*c*e^3)/a^5)^(1/3)*log(-(c^4*d^5 - 2*a*c^3*d^
3*e^2 - 3*a^2*c^2*d*e^4)*x - (a^8*e*sqrt(-(c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a
^2*c^3*d^2*e^4)/a^11) - a^2*c^3*d^4 + 3*a^3*c^2*d^2*e^2)*((a^5*sqrt(-(c^5*
d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)/a^11) - 3*c^2*d^2*e + a*c*e^3)/
a^5)^(1/3)) - 30*e*x^3 - 5*(sqrt(-3)*a*x^5 + a*x^5)*(-(a^5*sqrt(-(c^5*d^6
- 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)/a^11) + 3*c^2*d^2*e - a*c*e^3)/a^5)
^(1/3)*log(-(c^4*d^5 - 2*a*c^3*d^3*e^2 - 3*a^2*c^2*d*e^4)*x - 1/2*(a^2*c^3
*d^4 - 3*a^3*c^2*d^2*e^2 + sqrt(-3)*(a^2*c^3*d^4 - 3*a^3*c^2*d^2*e^2) + (s
qrt(-3)*a^8*e + a^8*e)*sqrt(-(c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^
4)/a^11))*(-(a^5*sqrt(-(c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)/a^1
1) + 3*c^2*d^2*e - a*c*e^3)/a^5)^(1/3)) + 5*(sqrt(-3)*a*x^5 - a*x^5)*(-(a^
5*sqrt(-(c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)/a^11) + 3*c^2*d^2*
e - a*c*e^3)/a^5)^(1/3)*log(-(c^4*d^5 - 2*a*c^3*d^3*e^2 - 3*a^2*c^2*d*e^4)
*x - 1/2*(a^2*c^3*d^4 - 3*a^3*c^2*d^2*e^2 - sqrt(-3)*(a^2*c^3*d^4 - 3*a...
```

Sympy [A] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.58

$$\int \frac{d+ex^3}{x^6\left(a+cx^6\right)}\,dx$$
$$= \operatorname{RootSum}\left(46656t^6a^{11} + t^3\left(-432a^7ce^3 + 1296a^6c^2d^2e\right) + a^3c^2e^6 + 3a^2c^3d^2e^4 + 3ac^4d^4e^2 + c^5d^6, \left(t \mapsto t\right.\right.$$
$$+ \frac{-2d-5ex^3}{10ax^5}$$

input
```
integrate((e*x**3+d)/x**6/(c*x**6+a),x)
```

output
```
RootSum(46656*_t**6*a**11 + _t**3*(-432*a**7*c*e**3 + 1296*a**6*c**2*d**2*
e) + a**3*c**2*e**6 + 3*a**2*c**3*d**2*e**4 + 3*a*c**4*d**4*e**2 + c**5*d*
*6, Lambda(_t, _t*log(x + (-1296*_t**4*a**8*e + 6*_t*a**4*c*e**4 - 36*_t*a
**3*c**2*d**2*e**2 + 6*_t*a**2*c**3*d**4)/(3*a**2*c**2*d*e**4 + 2*a*c**3*d
**3*e**2 - c**4*d**5)))) + (-2*d - 5*e*x**3)/(10*a*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.94

$$\int \frac{d+ex^3}{x^6\,(a+cx^6)}\,dx$$
$$= \frac{c\left(\frac{2\,e\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}c^{\frac{2}{3}}} - \frac{4\,d\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} - \frac{\left(\sqrt{3}a^{\frac{1}{6}}\sqrt{c}d+a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{ac^{\frac{2}{3}}} + \frac{\left(\sqrt{3}a^{\frac{1}{6}}\sqrt{c}d-a^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2-\sqrt{3}a\right.}{ac^{\frac{2}{3}}}\right.}{12\,a}$$
$$- \frac{5\,ex^3+2\,d}{10\,ax^5}$$

input
```
integrate((e*x^3+d)/x^6/(c*x^6+a),x, algorithm="maxima")
```

output
```
1/12*c*(2*e*log(c^(1/3)*x^2 + a^(1/3))/(a^(1/3)*c^(2/3)) - 4*d*arctan(c^(1
/3)*x/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*sqrt(a^(1/3)*c^(1/3))) - (sqrt(3)*a^
(1/6)*sqrt(c)*d + a^(2/3)*e)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x +
 a^(1/3))/(a*c^(2/3)) + (sqrt(3)*a^(1/6)*sqrt(c)*d - a^(2/3)*e)*log(c^(1/3
)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) + 2*(sqrt(3)*a^(5
/6)*c^(1/6)*e - a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6)*c
^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))) - 2*(sqrt
(3)*a^(5/6)*c^(1/6)*e + a^(1/3)*c^(2/3)*d)*arctan((2*c^(1/3)*x - sqrt(3)*a
^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))))/
a - 1/10*(5*e*x^3 + 2*d)/(a*x^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.93

$$\int \frac{d+ex^3}{x^6(a+cx^6)}\,dx = -\frac{(ac^5)^{\frac{1}{6}}\,d\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,a^2} + \frac{(ac^5)^{\frac{2}{3}}\,e|c|\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,a^2c^4} - \frac{\left((ac^5)^{\frac{1}{6}}\,c^3d-\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,a^2c^3} - \frac{\left((ac^5)^{\frac{1}{6}}\,c^3d+\sqrt{3}(ac^5)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,a^2c^3} - \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,c^3d+(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,a^2c^3} + \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,c^3d-(ac^5)^{\frac{2}{3}}\,e\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,a^2c^3} - \frac{5\,ex^3+2\,d}{10\,ax^5}$$

input
```
integrate((e*x^3+d)/x^6/(c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*(a*c^5)^(1/6)*d*arctan(x/(a/c)^(1/6))/a^2 + 1/6*(a*c^5)^(2/3)*e*abs(c
)*log(x^2 + (a/c)^(1/3))/(a^2*c^4) - 1/6*((a*c^5)^(1/6)*c^3*d - sqrt(3)*(a
*c^5)^(2/3)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^2*c^3) -
 1/6*((a*c^5)^(1/6)*c^3*d + sqrt(3)*(a*c^5)^(2/3)*e)*arctan((2*x - sqrt(3)
*(a/c)^(1/6))/(a/c)^(1/6))/(a^2*c^3) - 1/12*(sqrt(3)*(a*c^5)^(1/6)*c^3*d +
 (a*c^5)^(2/3)*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^2*c^3)
 + 1/12*(sqrt(3)*(a*c^5)^(1/6)*c^3*d - (a*c^5)^(2/3)*e)*log(x^2 - sqrt(3)*
x*(a/c)^(1/6) + (a/c)^(1/3))/(a^2*c^3) - 1/10*(5*e*x^3 + 2*d)/(a*x^5)
```

Mupad [B] (verification not implemented)

Time = 22.33 (sec) , antiderivative size = 1261, normalized size of antiderivative = 3.86

$$\int \frac{d+ex^3}{x^6\left(a+cx^6\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^3)/(x^6*(a + c*x^6)),x)
```

output
```
log(e*x*(-a^11*c^3)^(1/2) - a^7*c*((a^7*c*e^3 + c*d^3*(-a^11*c^3)^(1/2) -
3*a^6*c^2*d^2*e - 3*a*d*e^2*(-a^11*c^3)^(1/2))/a^11)^(1/3) + a^5*c^2*d*x)*
((a^7*c*e^3 + c*d^3*(-a^11*c^3)^(1/2) - 3*a^6*c^2*d^2*e - 3*a*d*e^2*(-a^11
*c^3)^(1/2))/(216*a^11))^(1/3) + log(e*x*(-a^11*c^3)^(1/2) + a^7*c*((a^7*c
*e^3 - c*d^3*(-a^11*c^3)^(1/2) - 3*a^6*c^2*d^2*e + 3*a*d*e^2*(-a^11*c^3)^(
1/2))/a^11)^(1/3) - a^5*c^2*d*x)*((a^7*c*e^3 - c*d^3*(-a^11*c^3)^(1/2) - 3
*a^6*c^2*d^2*e + 3*a*d*e^2*(-a^11*c^3)^(1/2))/(216*a^11))^(1/3) - (d/(5*a)
 + (e*x^3)/(2*a))/x^5 + log(2*e*x*(-a^11*c^3)^(1/2) + a^7*c*((a^7*c*e^3 +
c*d^3*(-a^11*c^3)^(1/2) - 3*a^6*c^2*d^2*e - 3*a*d*e^2*(-a^11*c^3)^(1/2))/a
^11)^(1/3) - 3^(1/2)*a^7*c*((a^7*c*e^3 + c*d^3*(-a^11*c^3)^(1/2) - 3*a^6*c
^2*d^2*e - 3*a*d*e^2*(-a^11*c^3)^(1/2))/a^11)^(1/3)*1i + 2*a^5*c^2*d*x)*((
3^(1/2)*1i)/2 - 1/2)*((a^7*c*e^3 + c*d^3*(-a^11*c^3)^(1/2) - 3*a^6*c^2*d^2
*e - 3*a*d*e^2*(-a^11*c^3)^(1/2))/(216*a^11))^(1/3) - log(2*e*x*(-a^11*c^3
)^(1/2) + a^7*c*((a^7*c*e^3 + c*d^3*(-a^11*c^3)^(1/2) - 3*a^6*c^2*d^2*e -
3*a*d*e^2*(-a^11*c^3)^(1/2))/a^11)^(1/3) + 3^(1/2)*a^7*c*((a^7*c*e^3 + c*d
^3*(-a^11*c^3)^(1/2) - 3*a^6*c^2*d^2*e - 3*a*d*e^2*(-a^11*c^3)^(1/2))/a^11
)^(1/3)*1i + 2*a^5*c^2*d*x)*((3^(1/2)*1i)/2 + 1/2)*((a^7*c*e^3 + c*d^3*(-a
^11*c^3)^(1/2) - 3*a^6*c^2*d^2*e - 3*a*d*e^2*(-a^11*c^3)^(1/2))/(216*a^11)
)^(1/3) - log(a^7*c*((a^7*c*e^3 - c*d^3*(-a^11*c^3)^(1/2) - 3*a^6*c^2*d^2*
e + 3*a*d*e^2*(-a^11*c^3)^(1/2))/a^11)^(1/3) - 2*e*x*(-a^11*c^3)^(1/2) ...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.05

$$\int \frac{d+ex^3}{x^6\left(a+cx^6\right)}\,dx$$
$$= \frac{10\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)cd\,x^5+10\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)ace\,x^5-10\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)c\cdots}{}$$

input
```
int((e*x^3+d)/x^6/(c*x^6+a),x)
```

output
```
(10*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1
/6)*a**(1/6)))*c*d*x**5 + 10*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c
**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c*e*x**5 - 10*sqrt(c)*sqrt(a)*atan((c**(
1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d*x**5 + 10*s
qrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6))
)*a*c*e*x**5 - 20*sqrt(c)*sqrt(a)*atan((c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c
*d*x**5 + 5*sqrt(c)*sqrt(a)*sqrt(3)*log( - c**(1/6)*a**(1/6)*sqrt(3)*x + a
**(1/3) + c**(1/3)*x**2)*c*d*x**5 - 5*sqrt(c)*sqrt(a)*sqrt(3)*log(c**(1/6)
*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c*d*x**5 - 12*c**(2/3)*a**
(1/3)*a*d - 30*c**(2/3)*a**(1/3)*a*e*x**3 + 10*log(a**(1/3) + c**(1/3)*x**
2)*a*c*e*x**5 - 5*log( - c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)
*x**2)*a*c*e*x**5 - 5*log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3
)*x**2)*a*c*e*x**5)/(60*c**(2/3)*a**(1/3)*a**2*x**5)
```

3.22 $\int \frac{x^7(d+ex^3)}{a+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 301

$$\begin{aligned}\int \frac{x^7(d+ex^3)}{a+cx^6}\,dx = {} & \frac{dx^2}{2c}+\frac{ex^5}{5c}-\frac{a^{5/6}e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3c^{11/6}}+\frac{a^{5/6}e\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6c^{11/6}}\\ & -\frac{a^{5/6}e\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6c^{11/6}}+\frac{\sqrt[3]{a}d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{c}x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}c^{4/3}}\\ & +\frac{a^{5/6}e\,\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{2\sqrt{3}c^{11/6}}-\frac{\sqrt[3]{a}d\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6c^{4/3}}\\ & +\frac{\sqrt[3]{a}d\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{c}x^2+c^{2/3}x^4\right)}{12c^{4/3}}\end{aligned}$$

output
```
1/2*d*x^2/c+1/5*e*x^5/c-1/3*a^(5/6)*e*arctan(c^(1/6)*x/a^(1/6))/c^(11/6)-1
/6*a^(5/6)*e*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/c^(11/6)-1/6*a^(5/6)*e*a
rctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/c^(11/6)+1/6*a^(1/3)*d*arctan(1/3*(a^(1
/3)-2*c^(1/3)*x^2)*3^(1/2)/a^(1/3))*3^(1/2)/c^(4/3)+1/6*a^(5/6)*e*arctanh(
3^(1/2)*a^(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)*x^2))*3^(1/2)/c^(11/6)-1/6*a^(1
/3)*d*ln(a^(1/3)+c^(1/3)*x^2)/c^(4/3)+1/12*a^(1/3)*d*ln(a^(2/3)-a^(1/3)*c^
(1/3)*x^2+c^(2/3)*x^4)/c^(4/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.99

$$\int \frac{x^7(d+ex^3)}{a+cx^6}\,dx$$
$$= \frac{30c^{5/6}dx^2+12c^{5/6}ex^5-20a^{5/6}e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)+10\sqrt[3]{a}(\sqrt{3}\sqrt{c}d+\sqrt{a}e)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)+10\sqrt[3]{}}{}$$

input
```
Integrate[(x^7*(d + e*x^3))/(a + c*x^6),x]
```

output
```
(30*c^(5/6)*d*x^2 + 12*c^(5/6)*e*x^5 - 20*a^(5/6)*e*ArcTan[(c^(1/6)*x)/a^(
1/6)] + 10*a^(1/3)*(Sqrt[3]*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[Sqrt[3] - (2*c^(
1/6)*x)/a^(1/6)] + 10*a^(1/3)*(Sqrt[3]*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[Sqrt[
3] + (2*c^(1/6)*x)/a^(1/6)] - 10*a^(1/3)*Sqrt[c]*d*Log[a^(1/3) + c^(1/3)*x
^2] - 5*(-(a^(1/3)*Sqrt[c]*d) + Sqrt[3]*a^(5/6)*e)*Log[a^(1/3) - Sqrt[3]*a
^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + 5*(a^(1/3)*Sqrt[c]*d + Sqrt[3]*a^(5/6)*e
)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(60*c^(11/6))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {1827, 27, 1827, 27, 1835, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(d+ex^3)}{a+cx^6}\,dx$$
$$\Big\downarrow \text{ 1827}$$
$$\frac{ex^5}{5c}-\frac{\int \frac{5x^4(ae-cdx^3)}{cx^6+a}dx}{5c}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{ex^5}{5c}-\frac{\int \frac{x^4\left(ae-cdx^3\right)}{cx^6+a}dx}{c}$$

$\Big\downarrow$ 1827

$$\frac{ex^5}{5c}-\frac{-\frac{\int -\frac{2acx\left(ex^3+d\right)}{cx^6+a}dx}{2c}-\frac{dx^2}{2}}{c}$$

$\Big\downarrow$ 27

$$\frac{ex^5}{5c}-\frac{a\int \frac{x\left(ex^3+d\right)}{cx^6+a}dx-\frac{dx^2}{2}}{c}$$

$\Big\downarrow$ 1835

$$\frac{ex^5}{5c}-\frac{a\left(\frac{1}{2}\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\int \frac{x}{\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x^3\right)}dx+\frac{1}{2}\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\int \frac{x}{\sqrt{c}\left(\sqrt{c}x^3+\sqrt{-a}\right)}dx\right)-\frac{dx^2}{2}}{c}$$

$\Big\downarrow$ 27

$$\frac{ex^5}{5c}-\frac{a\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\int \frac{x}{\sqrt{-a}-\sqrt{c}x^3}dx}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\int \frac{x}{\sqrt{c}x^3+\sqrt{-a}}dx}{2\sqrt{c}}\right)-\frac{dx^2}{2}}{c}$$

$\Big\downarrow$ 821

$$\frac{ex^5}{5c}-$$

$$a\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\int \frac{1}{\sqrt[6]{c}x+\sqrt[6]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(\frac{\int \frac{1}{\sqrt[6]{-a}-\sqrt[6]{c}x}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\int \frac{\sqrt[6]{-a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+}}{3\sqrt[6]{-a}\sqrt[6]{c}}}{2\sqrt{c}}$$

$$\overline{\qquad\qquad c \qquad\qquad}$$

$\Big\downarrow$ 16

$$\frac{ex^5}{5c} - \frac{a\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}} - \frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}} + \frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\frac{\int \frac{\sqrt[6]{-a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}} - \frac{\log(\cdots}{3\cdots}\cdots\right)}{2\sqrt{c}}\cdots\right)}{c}$$

$$\Big\downarrow \text{1142}$$

$$\frac{ex^5}{5c} - \frac{a\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{3}{2}\sqrt[6]{-a}\int \frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx + \frac{\int -\frac{\sqrt[6]{c}\left(\sqrt[6]{-a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}} - \frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}} + \frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\cdots}{\cdots}\right)}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{ex^5}{5c} - \frac{a\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{3}{2}\sqrt[6]{-a}\int \frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx - \frac{\int \frac{\sqrt[6]{c}\left(\sqrt[6]{-a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}} - \frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}} + \frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\cdots}{\cdots}\right)}{c}$$

$\downarrow$ 27

$$\frac{ex^5}{5c}-\frac{a\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{3}{2}\sqrt[6]{-a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{1}{2}\int\frac{\sqrt[6]{-a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\cdots}{\cdots}\cdots\right)}{c}$$

$\downarrow$ 1082

$$\frac{ex^5}{5c}-\frac{a\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}\right)^2-3}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}\right)}{\sqrt[6]{c}}-\frac{1}{2}\int\frac{\sqrt[6]{-a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\cdots\right)}{\cdots}\cdots\right)}{c}$$

$\downarrow$ 217

$$\frac{ex^5}{5c}-\frac{a\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{-\frac{1}{2}\int\frac{\sqrt[6]{-a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\cdots}{\cdots}}{\cdots}\right)\cdots}{\sqrt[6]{\cdots}}\cdots\right)}{\cdots}\right)}{c}$$

$\downarrow$ 1103

$$\frac{ex^5}{5c}-\frac{a\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{\log\left(-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\cdots}{\cdots}}{\cdots}\right)\cdots}{\sqrt[6]{\cdots}}\cdots\right)}{\cdots}\right)}{c}$$

input
```
Int[(x^7*(d + e*x^3))/(a + c*x^6),x]
```

output
```
(e*x^5)/(5*c) - (-1/2*(d*x^2) + a*((((a*Sqrt[c]*d)/(-a)^(3/2) + e)*(-1/3*L
og[(-a)^(1/6) + c^(1/6)*x]/((-a)^(1/6)*c^(1/3)) + (-((Sqrt[3]*ArcTan[(1 -
(2*c^(1/6)*x)/(-a)^(1/6))/Sqrt[3]])/c^(1/6)) + Log[(-a)^(1/3) - (-a)^(1/6)
*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/6)))/(3*(-a)^(1/6)*c^(1/6))))/(2*Sqrt[c]
) + (((a*Sqrt[c]*d)/(-a)^(3/2) - e)*(-1/3*Log[(-a)^(1/6) - c^(1/6)*x]/((-a
)^(1/6)*c^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*c^(1/6)*x)/(-a)^(1/6))/Sqrt[3]
])/c^(1/6) - Log[(-a)^(1/3) + (-a)^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/
6)))/(3*(-a)^(1/6)*c^(1/6))))/(2*Sqrt[c])))/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1827
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a + c*x^(2*n))^(p +
1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(m + n*(2*p + 1) + 1))   In
t[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

rule 1835
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[-(e/2 + c*(d/(2*q)))   Int[(f
*x)^m/(q - c*x^n), x], x] + Simp[(e/2 - c*(d/(2*q)))   Int[(f*x)^m/(q + c*x
^n), x], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.19

method	result
risch	$\frac{e x^5}{5c}+\frac{d x^2}{2c}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6 c+a\right)}\frac{\left(-\_R^4 e-d\_R\right)\ln\left(x-\_R\right)}{\_R^5}\right)}{6c^2}$
default	$\frac{\frac{1}{5}e x^5+\frac{1}{2}d x^2}{c}-\frac{\left(\frac{\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\left(\frac{a}{c}\right)^{\frac{5}{6}}e}{12a}-\frac{\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{1}{3}}d}{12a}+\frac{\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)e}{6c\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{3}}\arctan}{}\right.}{}$

input
```
int(x^7*(e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*e*x^5/c+1/2*d/c*x^2+1/6/c^2*a*sum((-_R^4*e-_R*d)/_R^5*ln(x-_R),_R=Root
Of(_Z^6*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1931 vs. 2(210) = 420.

Time = 0.14 (sec) , antiderivative size = 1931, normalized size of antiderivative = 6.42

$$\int \frac{x^7(d+ex^3)}{a+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

output
```
1/60*(12*e*x^5 + 30*d*x^2 + 5*(sqrt(-3)*c - c)*(-(c^5*sqrt(-(9*a^3*c^2*d^4
*e^2 - 6*a^4*c*d^2*e^4 + a^5*e^6)/c^11) + a*c*d^3 - 3*a^2*d*e^2)/c^5)^(1/3
)*log(-(3*a^2*c^3*d^6*e + 5*a^3*c^2*d^4*e^3 + a^4*c*d^2*e^5 - a^5*e^7)*x +
 1/2*(6*a^2*c^5*d^3*e^2 - 2*a^3*c^4*d*e^4 + 2*sqrt(-3)*(3*a^2*c^5*d^3*e^2
- a^3*c^4*d*e^4) + (c^10*d^2 - a*c^9*e^2 + sqrt(-3)*(c^10*d^2 - a*c^9*e^2)
)*sqrt(-(9*a^3*c^2*d^4*e^2 - 6*a^4*c*d^2*e^4 + a^5*e^6)/c^11))*(-(c^5*sqrt
(-(9*a^3*c^2*d^4*e^2 - 6*a^4*c*d^2*e^4 + a^5*e^6)/c^11) + a*c*d^3 - 3*a^2*
d*e^2)/c^5)^(2/3)) - 5*(sqrt(-3)*c + c)*(-(c^5*sqrt(-(9*a^3*c^2*d^4*e^2 -
6*a^4*c*d^2*e^4 + a^5*e^6)/c^11) + a*c*d^3 - 3*a^2*d*e^2)/c^5)^(1/3)*log(-
(3*a^2*c^3*d^6*e + 5*a^3*c^2*d^4*e^3 + a^4*c*d^2*e^5 - a^5*e^7)*x + 1/2*(6
*a^2*c^5*d^3*e^2 - 2*a^3*c^4*d*e^4 - 2*sqrt(-3)*(3*a^2*c^5*d^3*e^2 - a^3*c
^4*d*e^4) + (c^10*d^2 - a*c^9*e^2 - sqrt(-3)*(c^10*d^2 - a*c^9*e^2))*sqrt(
-(9*a^3*c^2*d^4*e^2 - 6*a^4*c*d^2*e^4 + a^5*e^6)/c^11))*(-(c^5*sqrt(-(9*a^
3*c^2*d^4*e^2 - 6*a^4*c*d^2*e^4 + a^5*e^6)/c^11) + a*c*d^3 - 3*a^2*d*e^2)/
c^5)^(2/3)) + 10*c*(-(c^5*sqrt(-(9*a^3*c^2*d^4*e^2 - 6*a^4*c*d^2*e^4 + a^5
*e^6)/c^11) + a*c*d^3 - 3*a^2*d*e^2)/c^5)^(1/3)*log(-(3*a^2*c^3*d^6*e + 5*
a^3*c^2*d^4*e^3 + a^4*c*d^2*e^5 - a^5*e^7)*x - (6*a^2*c^5*d^3*e^2 - 2*a^3*
c^4*d*e^4 + (c^10*d^2 - a*c^9*e^2)*sqrt(-(9*a^3*c^2*d^4*e^2 - 6*a^4*c*d^2*
e^4 + a^5*e^6)/c^11))*(-(c^5*sqrt(-(9*a^3*c^2*d^4*e^2 - 6*a^4*c*d^2*e^4 +
a^5*e^6)/c^11) + a*c*d^3 - 3*a^2*d*e^2)/c^5)^(2/3)) + 5*(sqrt(-3)*c - c...
```

Sympy [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.74

$$\int \frac{x^7(d+ex^3)}{a+cx^6}\,dx$$
$$= \operatorname{RootSum}\left(46656t^6c^{11} + t^3\left(-1296a^2c^6de^2 + 432ac^7d^3\right) + a^5e^6 + 3a^4cd^2e^4 + 3a^3c^2d^4e^2 + a^2c^3d^6, \left(t \mapsto t\right.\right.$$
$$+ \frac{dx^2}{2c} + \frac{ex^5}{5c}$$

input
```
integrate(x**7*(e*x**3+d)/(c*x**6+a),x)
```

output
```
RootSum(46656*_t**6*c**11 + _t**3*(-1296*a**2*c**6*d*e**2 + 432*a*c**7*d**
3) + a**5*e**6 + 3*a**4*c*d**2*e**4 + 3*a**3*c**2*d**4*e**2 + a**2*c**3*d*
*6, Lambda(_t, _t*log(x + (-7776*_t**5*a*c**9*e**2 + 7776*_t**5*c**10*d**2
 + 180*_t**2*a**3*c**4*d*e**4 - 360*_t**2*a**2*c**5*d**3*e**2 + 36*_t**2*a
*c**6*d**5)/(a**5*e**7 - a**4*c*d**2*e**5 - 5*a**3*c**2*d**4*e**3 - 3*a**2
*c**3*d**6*e)))) + d*x**2/(2*c) + e*x**5/(5*c)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.02

$$\int \frac{x^7(d+ex^3)}{a+cx^6}\,dx =$$

$$-\frac{a\left(\frac{2\,d\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}c^{\frac{1}{3}}}+\frac{4\,e\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{c^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}-\frac{\left(\sqrt{3}\sqrt{a}c^{\frac{1}{6}}\,e+c^{\frac{2}{3}}\,d\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}\,c^{\frac{1}{6}}\,x+a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}c}+\frac{\left(\sqrt{3}\sqrt{a}c^{\frac{1}{6}}\,e-c^{\frac{2}{3}}\,d\right)\log\left(c^{\frac{1}{3}}x^2-\cdots\right.}{a^{\frac{2}{3}}c}\cdots\right.}{12\,c}$$

$$+\frac{2\,ex^5+5\,dx^2}{10\,c}$$

input
```
integrate(x^7*(e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

output
```
-1/12*a*(2*d*log(c^(1/3)*x^2 + a^(1/3))/(a^(2/3)*c^(1/3)) + 4*e*arctan(c^(
1/3)*x/sqrt(a^(1/3)*c^(1/3)))/(c^(2/3)*sqrt(a^(1/3)*c^(1/3))) - (sqrt(3)*s
qrt(a)*c^(1/6)*e + c^(2/3)*d)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x
+ a^(1/3))/(a^(2/3)*c) + (sqrt(3)*sqrt(a)*c^(1/6)*e - c^(2/3)*d)*log(c^(1/
3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a^(2/3)*c) - 2*(sqrt(3)*a^(
1/6)*c^(5/6)*d - a^(2/3)*c^(1/3)*e)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6)*
c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c*sqrt(a^(1/3)*c^(1/3))) + 2*(sqr
t(3)*a^(1/6)*c^(5/6)*d + a^(2/3)*c^(1/3)*e)*arctan((2*c^(1/3)*x - sqrt(3)*
a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c*sqrt(a^(1/3)*c^(1/3))))
/c + 1/10*(2*e*x^5 + 5*d*x^2)/c
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.97

$$\int \frac{x^7(d+ex^3)}{a+cx^6}\,dx = -\frac{e\left(\frac{a}{c}\right)^{\frac{5}{6}}\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,c} - \frac{\left(ac^5\right)^{\frac{1}{3}}d\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,c^3} + \frac{2\,c^4ex^5+5\,c^4dx^2}{10\,c^5} + \frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{3}}c^3d-\left(ac^5\right)^{\frac{5}{6}}e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,c^6} - \frac{\left(\sqrt{3}\left(ac^5\right)^{\frac{1}{3}}c^3d+\left(ac^5\right)^{\frac{5}{6}}e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,c^6} + \frac{\left(\left(ac^5\right)^{\frac{1}{3}}c^3d+\sqrt{3}\left(ac^5\right)^{\frac{5}{6}}e\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,c^6} + \frac{\left(\left(ac^5\right)^{\frac{1}{3}}c^3d-\sqrt{3}\left(ac^5\right)^{\frac{5}{6}}e\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,c^6}$$

input
```
integrate(x^7*(e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*e*(a/c)^(5/6)*arctan(x/(a/c)^(1/6))/c - 1/6*(a*c^5)^(1/3)*d*log(x^2 +
 (a/c)^(1/3))/c^3 + 1/10*(2*c^4*e*x^5 + 5*c^4*d*x^2)/c^5 + 1/6*(sqrt(3)*(a
*c^5)^(1/3)*c^3*d - (a*c^5)^(5/6)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a
/c)^(1/6))/c^6 - 1/6*(sqrt(3)*(a*c^5)^(1/3)*c^3*d + (a*c^5)^(5/6)*e)*arcta
n((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/c^6 + 1/12*((a*c^5)^(1/3)*c^3*d
 + sqrt(3)*(a*c^5)^(5/6)*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))
/c^6 + 1/12*((a*c^5)^(1/3)*c^3*d - sqrt(3)*(a*c^5)^(5/6)*e)*log(x^2 - sqrt
(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/c^6
```

Mupad [B] (verification not implemented)

Time = 23.45 (sec) , antiderivative size = 1461, normalized size of antiderivative = 4.85

$$\int \frac{x^7(d+ex^3)}{a+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x^7*(d + e*x^3))/(a + c*x^6),x)
```

output
```
log(a*c^9*(-(a*c^7*d^3 + a*e^3*(-a^3*c^11)^(1/2) - 3*a^2*c^6*d*e^2 - 3*c*d
^2*e*(-a^3*c^11)^(1/2))/c^11)^(2/3) - a*e^2*x*(-a^3*c^11)^(1/2) + c*d^2*x*
(-a^3*c^11)^(1/2) + 2*a^2*c^6*d*e*x)*(-(a*c^7*d^3 + a*e^3*(-a^3*c^11)^(1/2
) - 3*a^2*c^6*d*e^2 - 3*c*d^2*e*(-a^3*c^11)^(1/2))/(216*c^11))^(1/3) + log
(a*c^9*(-(a*c^7*d^3 - a*e^3*(-a^3*c^11)^(1/2) - 3*a^2*c^6*d*e^2 + 3*c*d^2*
e*(-a^3*c^11)^(1/2))/c^11)^(2/3) + a*e^2*x*(-a^3*c^11)^(1/2) - c*d^2*x*(-a
^3*c^11)^(1/2) + 2*a^2*c^6*d*e*x)*(-(a*c^7*d^3 - a*e^3*(-a^3*c^11)^(1/2) -
 3*a^2*c^6*d*e^2 + 3*c*d^2*e*(-a^3*c^11)^(1/2))/(216*c^11))^(1/3) + (d*x^2
)/(2*c) + (e*x^5)/(5*c) - log(- (((3^(1/2)*1i)/2 - 1/2)*(-(a*c^7*d^3 + a*e
^3*(-a^3*c^11)^(1/2) - 3*a^2*c^6*d*e^2 - 3*c*d^2*e*(-a^3*c^11)^(1/2))/c^11
)^(2/3)*(108*a^5*c*d^2*e - 36*a^6*e^3 + 36*a^4*c^2*x*((3^(1/2)*1i)/2 + 1/2
)*(a*e^2 - c*d^2)*(-(a*c^7*d^3 + a*e^3*(-a^3*c^11)^(1/2) - 3*a^2*c^6*d*e^2
 - 3*c*d^2*e*(-a^3*c^11)^(1/2))/c^11)^(1/3)))/36 - (a^5*d*x*(a*e^2 + c*d^2
)^2)/c^3)*((3^(1/2)*1i)/2 + 1/2)*(-(a*c^7*d^3 + a*e^3*(-a^3*c^11)^(1/2) -
3*a^2*c^6*d*e^2 - 3*c*d^2*e*(-a^3*c^11)^(1/2))/(216*c^11))^(1/3) + log(- (
((3^(1/2)*1i)/2 + 1/2)*(-(a*c^7*d^3 + a*e^3*(-a^3*c^11)^(1/2) - 3*a^2*c^6*
d*e^2 - 3*c*d^2*e*(-a^3*c^11)^(1/2))/c^11)^(2/3)*(36*a^6*e^3 - 108*a^5*c*d
^2*e + 36*a^4*c^2*x*((3^(1/2)*1i)/2 - 1/2)*(a*e^2 - c*d^2)*(-(a*c^7*d^3 +
a*e^3*(-a^3*c^11)^(1/2) - 3*a^2*c^6*d*e^2 - 3*c*d^2*e*(-a^3*c^11)^(1/2))/c
^11)^(1/3)))/36 - (a^5*d*x*(a*e^2 + c*d^2)^2)/c^3)*((3^(1/2)*1i)/2 - 1/...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.11

$$\int \frac{x^7(d+ex^3)}{a+cx^6}\,dx$$

$$= \frac{10c^{\frac{1}{6}}a^{\frac{7}{6}}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e+10c^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d-10c^{\frac{1}{6}}a^{\frac{7}{6}}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e+10c^{\frac{2}{3}}a}{}$$

input
```
int(x^7*(e*x^3+d)/(c*x^6+a),x)
```

output
```
(10*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**
(1/6)*a**(1/6)))*a*e + 10*c**(2/3)*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**(1/6
)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*d - 10*c**(1/6)*a**(1/6)*at
an((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*e + 1
0*c**(2/3)*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x
)/(c**(1/6)*a**(1/6)))*d - 20*c**(1/6)*a**(1/6)*atan((c**(1/3)*x)/(c**(1/6
)*a**(1/6)))*a*e - 5*c**(1/6)*a**(1/6)*sqrt(3)*log( - c**(1/6)*a**(1/6)*sq
rt(3)*x + a**(1/3) + c**(1/3)*x**2)*a*e + 5*c**(1/6)*a**(1/6)*sqrt(3)*log(
c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*a*e - 10*c**(2/3)*
a**(2/3)*log(a**(1/3) + c**(1/3)*x**2)*d + 5*c**(2/3)*a**(2/3)*log( - c**(
1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*d + 5*c**(2/3)*a**(2/3
)*log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*d + 30*a**(1
/3)*c*d*x**2 + 12*a**(1/3)*c*e*x**5)/(60*a**(1/3)*c**2)
```

3.23 $\int \frac{x^4(d+ex^3)}{a+cx^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 290

$$\int \frac{x^4(d+ex^3)}{a+cx^6}\,dx = \frac{ex^2}{2c}+\frac{d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}c^{5/6}}-\frac{d\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[6]{a}c^{5/6}}+\frac{d\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[6]{a}c^{5/6}}+\frac{\sqrt[3]{a}e\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{c}x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}c^{4/3}}-\frac{d\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{2\sqrt{3}\sqrt[6]{a}c^{5/6}}-\frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6c^{4/3}}+\frac{\sqrt[3]{a}e\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{c}x^2+c^{2/3}x^4\right)}{12c^{4/3}}$$

output
```
1/2*e*x^2/c+1/3*d*arctan(c^(1/6)*x/a^(1/6))/a^(1/6)/c^(5/6)+1/6*d*arctan(-
3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(1/6)/c^(5/6)+1/6*d*arctan(3^(1/2)+2*c^(1/6
)*x/a^(1/6))/a^(1/6)/c^(5/6)+1/6*a^(1/3)*e*arctan(1/3*(a^(1/3)-2*c^(1/3)*x
^2)*3^(1/2)/a^(1/3))*3^(1/2)/c^(4/3)-1/6*d*arctanh(3^(1/2)*a^(1/6)*c^(1/6)
*x/(a^(1/3)+c^(1/3)*x^2))*3^(1/2)/a^(1/6)/c^(5/6)-1/6*a^(1/3)*e*ln(a^(1/3)
+c^(1/3)*x^2)/c^(4/3)+1/12*a^(1/3)*e*ln(a^(2/3)-a^(1/3)*c^(1/3)*x^2+c^(2/3
)*x^4)/c^(4/3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.20

$$\int \frac{x^4(d+ex^3)}{a+cx^6}\,dx = \frac{ex^2}{2c} + \frac{d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}c^{5/6}} + \frac{\left(a^{5/6}cd-\sqrt{3}a^{4/3}\sqrt{c}e\right)\arctan\left(\frac{-\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{11/6}} + \frac{\left(a^{5/6}cd+\sqrt{3}a^{4/3}\sqrt{c}e\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{11/6}} - \frac{\sqrt[3]{a}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6c^{4/3}} - \frac{\left(-\sqrt{3}a^{5/6}cd-a^{4/3}\sqrt{c}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{11/6}} - \frac{\left(\sqrt{3}a^{5/6}cd-a^{4/3}\sqrt{c}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{11/6}}$$

input
```
Integrate[(x^4*(d + e*x^3))/(a + c*x^6),x]
```

output
```
(e*x^2)/(2*c) + (d*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(1/6)*c^(5/6)) + ((a^
(5/6)*c*d - Sqrt[3]*a^(4/3)*Sqrt[c]*e)*ArcTan[(-(Sqrt[3]*a^(1/6)) + 2*c^(1
/6)*x)/a^(1/6)])/(6*a*c^(11/6)) + ((a^(5/6)*c*d + Sqrt[3]*a^(4/3)*Sqrt[c]*
e)*ArcTan[(Sqrt[3]*a^(1/6) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^(11/6)) - (a^(1
/3)*e*Log[a^(1/3) + c^(1/3)*x^2])/(6*c^(4/3)) - ((-(Sqrt[3]*a^(5/6)*c*d) -
 a^(4/3)*Sqrt[c]*e)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]
)/(12*a*c^(11/6)) - ((Sqrt[3]*a^(5/6)*c*d - a^(4/3)*Sqrt[c]*e)*Log[a^(1/3)
 + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(11/6))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1827, 27, 1835, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(d+ex^3\right)}{a+cx^6}dx$$

$$\Big\downarrow \ 1827$$

$$\frac{ex^2}{2c}-\frac{\int \frac{2x\left(ae-cdx^3\right)}{cx^6+a}dx}{2c}$$

$$\Big\downarrow \ 27$$

$$\frac{ex^2}{2c}-\frac{\int \frac{x\left(ae-cdx^3\right)}{cx^6+a}dx}{c}$$

$$\Big\downarrow \ 1835$$

$$\frac{ex^2}{2c}-\frac{\frac{1}{2}\sqrt{c}\left(\sqrt{-a}e+\sqrt{c}d\right)\int \frac{x}{\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x^3\right)}dx-\frac{1}{2}\sqrt{c}\left(\sqrt{c}d-\sqrt{-a}e\right)\int \frac{x}{\sqrt{c}\left(\sqrt{c}x^3+\sqrt{-a}\right)}dx}{c}$$

$$\Big\downarrow \ 27$$

$$\frac{ex^2}{2c}-\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\int \frac{x}{\sqrt{-a}-\sqrt{c}x^3}dx-\frac{1}{2}\left(\sqrt{c}d-\sqrt{-a}e\right)\int \frac{x}{\sqrt{c}x^3+\sqrt{-a}}dx}{c}$$

$$\Big\downarrow \ 821$$

$$\frac{ex^2}{2c}-\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(\frac{\int \frac{1}{\sqrt[6]{-a}-\sqrt[6]{c}x}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\int \frac{\sqrt[6]{-a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-}}}{3\sqrt[6]{-a}\sqrt[6]{c}}\right.}{c}$$

$$\Big\downarrow \ 16$$

$$\frac{ex^2}{2c}-\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{\int \frac{\sqrt[6]{-a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{\cdots}}}{3\sqrt[6]{-a}\sqrt[6]{c}}\cdots\right.}{c}$$

$\downarrow$ 1142

$$\frac{ex^2}{2c}-\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{\frac{3}{2}\sqrt[6]{-a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{-a}\right)}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}\cdots\right.}{c}$$

$\downarrow$ 25

$$\frac{ex^2}{2c}-\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{\frac{3}{2}\sqrt[6]{-a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{-a}\right)}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}\cdots\right.}{c}$$

$\downarrow$ 27

$$\frac{ex^2}{2c}-\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{\frac{3}{2}\sqrt[6]{-a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{\cdots}\right.}{c}$$

$\downarrow$ 1082

$$\frac{ex^2}{2c}-\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{-\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{3\int\frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1\right)^2-3}d\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1\right)}{\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d\right.}{c}$$

$\downarrow$ 217

$$\frac{ex^2}{2c}-\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\right.}{c}$$

$\downarrow$ 1103

$$\frac{ex^2}{2c}-\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\right.}{c}$$

input
```
Int[(x^4*(d + e*x^3))/(a + c*x^6),x]
```

output
```
(e*x^2)/(2*c) - (-1/2*((Sqrt[c]*d - Sqrt[-a]*e)*(-1/3*Log[(-a)^(1/6) + c^(
1/6)*x]/((-a)^(1/6)*c^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*c^(1/6)*x)/(-a)^
(1/6))/Sqrt[3]])/c^(1/6)) + Log[(-a)^(1/3) - (-a)^(1/6)*c^(1/6)*x + c^(1/3
)*x^2]/(2*c^(1/6)))/(3*(-a)^(1/6)*c^(1/6)))) + ((Sqrt[c]*d + Sqrt[-a]*e)*(
-1/3*Log[(-a)^(1/6) - c^(1/6)*x]/((-a)^(1/6)*c^(1/3)) - ((Sqrt[3]*ArcTan[(
1 + (2*c^(1/6)*x)/(-a)^(1/6))/Sqrt[3]])/c^(1/6) - Log[(-a)^(1/3) + (-a)^(1
/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/6)))/(3*(-a)^(1/6)*c^(1/6))))/2)/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1827
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a + c*x^(2*n))^(p +
1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(m + n*(2*p + 1) + 1))   In
t[(f*x)^(m - n)*(a + c*x^(2*n))^p*(a*e*(m - n + 1) - c*d*(m + n*(2*p + 1) +
 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && EqQ[n2, 2*n] && IGtQ[n,
 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

rule 1835
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[-(e/2 + c*(d/(2*q)))   Int[(f
*x)^m/(q - c*x^n), x], x] + Simp[(e/2 - c*(d/(2*q)))   Int[(f*x)^m/(q + c*x
^n), x], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.17

method	result
risch	$\frac{e x^2}{2c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^6 c+a\right)}\frac{\left(\_R^4 cd-ae\_R\right)\ln\left(x-\_R\right)}{\_R^5}}{6c^2}$
default	$\frac{e x^2}{2c}-\frac{-\frac{c\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\left(\frac{a}{c}\right)^{\frac{5}{6}}d}{12a}-\frac{c\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{4}{3}}e}{12a}-\frac{\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)d}{6\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\frac{c\left(\frac{a}{c}\right)^{\frac{4}{3}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)\cdots}\right.}{6a}}{\cdots}$

input
```
int(x^4*(e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*e*x^2/c+1/6/c^2*sum((_R^4*c*d-_R*a*e)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c+a
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1825 vs. $2(201) = 402$.

Time = 0.32 (sec) , antiderivative size = 1825, normalized size of antiderivative = 6.29

$$\int \frac{x^4(d+ex^3)}{a+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

output
```
1/12*(6*e*x^2 + (sqrt(-3)*c - c)*((c^4*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*
a^2*d^2*e^4)/(a*c^7)) + 3*c*d^2*e - a*e^3)/c^4)^(1/3)*log(-(c^3*d^7 - a*c^
2*d^5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6)*x - 1/2*(2*a*c^4*d^4*e - 6*a^2*
c^3*d^2*e^3 + 2*sqrt(-3)*(a*c^4*d^4*e - 3*a^2*c^3*d^2*e^3) - (a*c^7*d^2 -
a^2*c^6*e^2 + sqrt(-3)*(a*c^7*d^2 - a^2*c^6*e^2))*sqrt(-(c^2*d^6 - 6*a*c*d
^4*e^2 + 9*a^2*d^2*e^4)/(a*c^7)))*((c^4*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9
*a^2*d^2*e^4)/(a*c^7)) + 3*c*d^2*e - a*e^3)/c^4)^(2/3)) - (sqrt(-3)*c + c)
*((c^4*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a*c^7)) + 3*c*d^2*
e - a*e^3)/c^4)^(1/3)*log(-(c^3*d^7 - a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 - 3*
a^3*d*e^6)*x - 1/2*(2*a*c^4*d^4*e - 6*a^2*c^3*d^2*e^3 - 2*sqrt(-3)*(a*c^4*
d^4*e - 3*a^2*c^3*d^2*e^3) - (a*c^7*d^2 - a^2*c^6*e^2 - sqrt(-3)*(a*c^7*d^
2 - a^2*c^6*e^2))*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a*c^7))
)*((c^4*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a*c^7)) + 3*c*d^2
*e - a*e^3)/c^4)^(2/3)) + 2*c*((c^4*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2
*d^2*e^4)/(a*c^7)) + 3*c*d^2*e - a*e^3)/c^4)^(1/3)*log(-(c^3*d^7 - a*c^2*d
^5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6)*x + (2*a*c^4*d^4*e - 6*a^2*c^3*d^2
*e^3 - (a*c^7*d^2 - a^2*c^6*e^2)*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^
2*e^4)/(a*c^7)))*((c^4*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a*
c^7)) + 3*c*d^2*e - a*e^3)/c^4)^(2/3)) + (sqrt(-3)*c - c)*(-(c^4*sqrt(-(c^
2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a*c^7)) - 3*c*d^2*e + a*e^3)/c^...
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.73

$$\int \frac{x^4(d+ex^3)}{a+cx^6}\,dx$$
$$= \mathrm{RootSum}\left(46656t^6ac^8 + t^3\cdot\left(432a^2c^4e^3 - 1296ac^5d^2e\right) + a^3e^6 + 3a^2cd^2e^4 + 3ac^2d^4e^2 + c^3d^6, \left(t \mapsto t\log\right.\right.$$
$$+ \frac{ex^2}{2c}$$

input
```
integrate(x**4*(e*x**3+d)/(c*x**6+a),x)
```

output
```
RootSum(46656*_t**6*a*c**8 + _t**3*(432*a**2*c**4*e**3 - 1296*a*c**5*d**2*
e) + a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6, Lamb
da(_t, _t*log(x + (7776*_t**5*a**2*c**6*e**2 - 7776*_t**5*a*c**7*d**2 + 36
*_t**2*a**3*c**2*e**5 - 360*_t**2*a**2*c**3*d**2*e**3 + 180*_t**2*a*c**4*d
**4*e)/(3*a**3*d*e**6 + 5*a**2*c*d**3*e**4 + a*c**2*d**5*e**2 - c**3*d**7)
))) + e*x**2/(2*c)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.03

$$\int \frac{x^4(d+ex^3)}{a+cx^6}\,dx = \frac{ex^2}{2\,c}$$
$$- \frac{\frac{2\,a^{\frac{1}{3}}\,e\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}} - \frac{4\,c^{\frac{1}{3}}\,d\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} + \frac{\left(\sqrt{3}\sqrt{a}c^{\frac{7}{6}}d - ac^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}c} - \frac{\left(\sqrt{3}\sqrt{a}c^{\frac{7}{6}}d + ac^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^{\cdots}\right.}{a^{\frac{2}{3}}c}}{12\,c}$$

input
```
integrate(x^4*(e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

output
```
1/2*e*x^2/c - 1/12*(2*a^(1/3)*e*log(c^(1/3)*x^2 + a^(1/3))/c^(1/3) - 4*c^(
1/3)*d*arctan(c^(1/3)*x/sqrt(a^(1/3)*c^(1/3)))/sqrt(a^(1/3)*c^(1/3)) + (sq
rt(3)*sqrt(a)*c^(7/6)*d - a*c^(2/3)*e)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c
^(1/6)*x + a^(1/3))/(a^(2/3)*c) - (sqrt(3)*sqrt(a)*c^(7/6)*d + a*c^(2/3)*e
)*log(c^(1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a^(2/3)*c) - 2*(
sqrt(3)*a^(7/6)*c^(5/6)*e + a^(2/3)*c^(4/3)*d)*arctan((2*c^(1/3)*x + sqrt(
3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c*sqrt(a^(1/3)*c^(1/3)
)) + 2*(sqrt(3)*a^(7/6)*c^(5/6)*e - a^(2/3)*c^(4/3)*d)*arctan((2*c^(1/3)*x
 - sqrt(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c*sqrt(a^(1/3)
*c^(1/3))))/c
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{x^4(d+ex^3)}{a+cx^6}\,dx &= \frac{ex^2}{2\,c}+\frac{d\left(\frac{a}{c}\right)^{\frac{5}{6}}\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,a} \\ &\quad+\frac{\left(\sqrt{3}ac^2e+\sqrt{ac}c^2d\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,\left(ac^5\right)^{\frac{2}{3}}} \\ &\quad-\frac{\left(\sqrt{3}ac^2e-\sqrt{ac}c^2d\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,\left(ac^5\right)^{\frac{2}{3}}} \\ &\quad+\frac{\left(\sqrt{3}\sqrt{ac}c^2d+ac^2e\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,\left(ac^5\right)^{\frac{2}{3}}} \\ &\quad+\frac{\left(\sqrt{3}\sqrt{ac}c^2d+ac^2e\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,\left(ac^5\right)^{\frac{2}{3}}} \\ &\quad-\frac{\left(ac^5\right)^{\frac{1}{3}}e\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,c^3}\end{aligned}$$

input
```
integrate(x^4*(e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

output
```
1/2*e*x^2/c + 1/3*d*(a/c)^(5/6)*arctan(x/(a/c)^(1/6))/a + 1/6*(sqrt(3)*a*c
^2*e + sqrt(a*c)*c^2*d)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a
*c^5)^(2/3) - 1/6*(sqrt(3)*a*c^2*e - sqrt(a*c)*c^2*d)*arctan((2*x - sqrt(3
)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^5)^(2/3) + 1/12*(sqrt(3)*sqrt(a*c)*c^2*d
+ a*c^2*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^5)^(2/3) +
1/12*(sqrt(3)*sqrt(a*c)*c^2*d + a*c^2*e)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) +
 (a/c)^(1/3))/(a*c^5)^(2/3) - 1/6*(a*c^5)^(1/3)*e*log(x^2 + (a/c)^(1/3))/c
^3
```

Mupad [B] (verification not implemented)

Time = 22.52 (sec) , antiderivative size = 1319, normalized size of antiderivative = 4.55

$$\int \frac{x^4(d+ex^3)}{a+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x^3))/(a + c*x^6),x)
```

output
```
log(a*c^7*(-(c*d^3*(-a*c^9)^(1/2) + a^2*c^4*e^3 - 3*a*c^5*d^2*e - 3*a*d*e^
2*(-a*c^9)^(1/2))/(a*c^8))^(2/3) - a*e^2*x*(-a*c^9)^(1/2) + c*d^2*x*(-a*c^
9)^(1/2) - 2*a*c^5*d*e*x)*(-(c*d^3*(-a*c^9)^(1/2) + a^2*c^4*e^3 - 3*a*c^5*
d^2*e - 3*a*d*e^2*(-a*c^9)^(1/2))/(216*a*c^8))^(1/3) + log(a*c^7*((c*d^3*(
-a*c^9)^(1/2) - a^2*c^4*e^3 + 3*a*c^5*d^2*e - 3*a*d*e^2*(-a*c^9)^(1/2))/(a
*c^8))^(2/3) + a*e^2*x*(-a*c^9)^(1/2) - c*d^2*x*(-a*c^9)^(1/2) - 2*a*c^5*d
*e*x)*((c*d^3*(-a*c^9)^(1/2) - a^2*c^4*e^3 + 3*a*c^5*d^2*e - 3*a*d*e^2*(-a
*c^9)^(1/2))/(216*a*c^8))^(1/3) + (e*x^2)/(2*c) - log(a*c^7*(-(c*d^3*(-a*c
^9)^(1/2) + a^2*c^4*e^3 - 3*a*c^5*d^2*e - 3*a*d*e^2*(-a*c^9)^(1/2))/(a*c^8
))^(2/3) - 3^(1/2)*a*c^7*(-(c*d^3*(-a*c^9)^(1/2) + a^2*c^4*e^3 - 3*a*c^5*d
^2*e - 3*a*d*e^2*(-a*c^9)^(1/2))/(a*c^8))^(2/3)*1i + 2*a*e^2*x*(-a*c^9)^(1
/2) - 2*c*d^2*x*(-a*c^9)^(1/2) + 4*a*c^5*d*e*x)*((3^(1/2)*1i)/2 + 1/2)*(-(
c*d^3*(-a*c^9)^(1/2) + a^2*c^4*e^3 - 3*a*c^5*d^2*e - 3*a*d*e^2*(-a*c^9)^(1
/2))/(216*a*c^8))^(1/3) + log(a*c^7*(-(c*d^3*(-a*c^9)^(1/2) + a^2*c^4*e^3
- 3*a*c^5*d^2*e - 3*a*d*e^2*(-a*c^9)^(1/2))/(a*c^8))^(2/3) + 3^(1/2)*a*c^7
*(-(c*d^3*(-a*c^9)^(1/2) + a^2*c^4*e^3 - 3*a*c^5*d^2*e - 3*a*d*e^2*(-a*c^9
)^(1/2))/(a*c^8))^(2/3)*1i + 2*a*e^2*x*(-a*c^9)^(1/2) - 2*c*d^2*x*(-a*c^9)
^(1/2) + 4*a*c^5*d*e*x)*((3^(1/2)*1i)/2 - 1/2)*(-(c*d^3*(-a*c^9)^(1/2) + a
^2*c^4*e^3 - 3*a*c^5*d^2*e - 3*a*d*e^2*(-a*c^9)^(1/2))/(216*a*c^8))^(1/3)
- log(a*c^7*((c*d^3*(-a*c^9)^(1/2) - a^2*c^4*e^3 + 3*a*c^5*d^2*e - 3*a*...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.11

$$\int \frac{x^4(d+ex^3)}{a+cx^6}\,dx$$
$$= \frac{-2c^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d+2c^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e+2c^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d+2c^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{}}{}$$

input
```
int(x^4*(e*x^3+d)/(c*x^6+a),x)
```

output
```
( - 2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c
**(1/6)*a**(1/6)))*c*d + 2*c**(2/3)*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**(1/
6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*e + 2*c**(1/6)*a**(1/6)*at
an((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d + 2
*c**(2/3)*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)
/(c**(1/6)*a**(1/6)))*e + 4*c**(1/6)*a**(1/6)*atan((c**(1/3)*x)/(c**(1/6)*
a**(1/6)))*c*d + c**(1/6)*a**(1/6)*sqrt(3)*log( - c**(1/6)*a**(1/6)*sqrt(3
)*x + a**(1/3) + c**(1/3)*x**2)*c*d - c**(1/6)*a**(1/6)*sqrt(3)*log(c**(1/
6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c*d - 2*c**(2/3)*a**(2/3
)*log(a**(1/3) + c**(1/3)*x**2)*e + c**(2/3)*a**(2/3)*log( - c**(1/6)*a**(
1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*e + c**(2/3)*a**(2/3)*log(c**(1
/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*e + 6*a**(1/3)*c*e*x**2
)/(12*a**(1/3)*c**2)
```

3.24 $\int \frac{x(d+ex^3)}{a+cx^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 279

$$\int \frac{x(d+ex^3)}{a+cx^6}\,dx = \frac{e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}c^{5/6}} - \frac{e\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[6]{a}c^{5/6}} + \frac{e\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6\sqrt[6]{a}c^{5/6}} - \frac{d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{c}x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{2/3}\sqrt[3]{c}} - \frac{e\,\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{2\sqrt{3}\sqrt[6]{a}c^{5/6}} + \frac{d\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6a^{2/3}\sqrt[3]{c}} - \frac{d\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{c}x^2+c^{2/3}x^4\right)}{12a^{2/3}\sqrt[3]{c}}$$

output
```
1/3*e*arctan(c^(1/6)*x/a^(1/6))/a^(1/6)/c^(5/6)+1/6*e*arctan(-3^(1/2)+2*c^
(1/6)*x/a^(1/6))/a^(1/6)/c^(5/6)+1/6*e*arctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))
/a^(1/6)/c^(5/6)-1/6*d*arctan(1/3*(a^(1/3)-2*c^(1/3)*x^2)*3^(1/2)/a^(1/3))
*3^(1/2)/a^(2/3)/c^(1/3)-1/6*e*arctanh(3^(1/2)*a^(1/6)*c^(1/6)*x/(a^(1/3)+
c^(1/3)*x^2))*3^(1/2)/a^(1/6)/c^(5/6)+1/6*d*ln(a^(1/3)+c^(1/3)*x^2)/a^(2/3
)/c^(1/3)-1/12*d*ln(a^(2/3)-a^(1/3)*c^(1/3)*x^2+c^(2/3)*x^4)/a^(2/3)/c^(1/
3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.19

$$\int \frac{x(d+ex^3)}{a+cx^6}\,dx = \frac{e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}c^{5/6}} + \frac{\left(\sqrt{3}\sqrt[3]{a}\sqrt{c}d+a^{5/6}e\right)\arctan\left(\frac{-\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{5/6}} + \frac{\left(-\sqrt{3}\sqrt[3]{a}\sqrt{c}d+a^{5/6}e\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6ac^{5/6}} + \frac{d\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6a^{2/3}\sqrt[3]{c}} - \frac{\left(\sqrt[3]{a}\sqrt{c}d-\sqrt{3}a^{5/6}e\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{5/6}} - \frac{\left(\sqrt[3]{a}\sqrt{c}d+\sqrt{3}a^{5/6}e\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12ac^{5/6}}$$

input
```
Integrate[(x*(d + e*x^3))/(a + c*x^6),x]
```

output
```
(e*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(1/6)*c^(5/6)) + ((Sqrt[3]*a^(1/3)*Sq
rt[c]*d + a^(5/6)*e)*ArcTan[(-(Sqrt[3]*a^(1/6)) + 2*c^(1/6)*x)/a^(1/6)])/(
6*a*c^(5/6)) + ((-(Sqrt[3]*a^(1/3)*Sqrt[c]*d) + a^(5/6)*e)*ArcTan[(Sqrt[3]
*a^(1/6) + 2*c^(1/6)*x)/a^(1/6)])/(6*a*c^(5/6)) + (d*Log[a^(1/3) + c^(1/3)
*x^2])/(6*a^(2/3)*c^(1/3)) - ((a^(1/3)*Sqrt[c]*d - Sqrt[3]*a^(5/6)*e)*Log[
a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a*c^(5/6)) - ((a^(
1/3)*Sqrt[c]*d + Sqrt[3]*a^(5/6)*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*
x + c^(1/3)*x^2])/(12*a*c^(5/6))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1835, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(d+ex^3\right)}{a+cx^6}dx$$

$$\Big\downarrow \text{1835}$$

$$\frac{1}{2}\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\int \frac{x}{\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x^3\right)}dx+\frac{1}{2}\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\int \frac{x}{\sqrt{c}\left(\sqrt{c}x^3+\sqrt{-a}\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\int \frac{x}{\sqrt{-a}-\sqrt{c}x^3}dx}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\int \frac{x}{\sqrt{c}x^3+\sqrt{-a}}dx}{2\sqrt{c}}$$

$$\Big\downarrow \text{821}$$

$$\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\int \frac{1}{\sqrt[6]{c}x+\sqrt[6]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(\frac{\int \frac{1}{\sqrt[6]{-a}-\sqrt[6]{c}x}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\int \frac{\sqrt[6]{-a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\frac{\int \frac{\sqrt[6]{-a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{3}{2}\sqrt[6]{-a}\int \frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx+\frac{\int -\frac{\sqrt[6]{c}\left(\sqrt[6]{-a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\frac{\frac{3}{2}\sqrt[6]{-a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{-a}\right)}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

$$\downarrow 25$$

$$\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{3}{2}\sqrt[6]{-a}\int \frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{\int \frac{\sqrt[6]{c}\left(\sqrt[6]{-a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\frac{\frac{3}{2}\sqrt[6]{-a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{-a}\right)}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

$$\downarrow 27$$

$$\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{3}{2}\sqrt[6]{-a}\int \frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{1}{2}\int \frac{\sqrt[6]{-a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\frac{\frac{3}{2}\sqrt[6]{-a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

$\Big\downarrow$ 1082

$$\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}\right)^2-3}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}\right)}{\sqrt[6]{c}}-\frac{1}{2}\int \frac{\sqrt[6]{-a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\frac{-\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1\right)^2-3}d\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1\right)}{\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

$\Big\downarrow$ 217

$$\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{-\frac{1}{2}\int\frac{\sqrt[6]{-a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{\log\left(-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}$$

input
```
Int[(x*(d + e*x^3))/(a + c*x^6),x]
```

output
```
(((a*Sqrt[c]*d)/(-a)^(3/2) + e)*(-1/3*Log[(-a)^(1/6) + c^(1/6)*x]/((-a)^(1
/6)*c^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*c^(1/6)*x)/(-a)^(1/6))/Sqrt[3]])
/c^(1/6)) + Log[(-a)^(1/3) - (-a)^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/6
)))/(3*(-a)^(1/6)*c^(1/6))))/(2*Sqrt[c]) + (((a*Sqrt[c]*d)/(-a)^(3/2) - e)
*(-1/3*Log[(-a)^(1/6) - c^(1/6)*x]/((-a)^(1/6)*c^(1/3)) - ((Sqrt[3]*ArcTan
[(1 + (2*c^(1/6)*x)/(-a)^(1/6))/Sqrt[3]])/c^(1/6) - Log[(-a)^(1/3) + (-a)^
(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/6)))/(3*(-a)^(1/6)*c^(1/6))))/(2*Sq
rt[c])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1835
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[-(e/2 + c*(d/(2*q)))   Int[(f
*x)^m/(q - c*x^n), x], x] + Simp[(e/2 - c*(d/(2*q)))   Int[(f*x)^m/(q + c*x
^n), x], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.13

method	result
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6} c+a\right)} \frac{\left(\_R^{4} e+d\_R\right) \ln\left(x-\_R\right)}{\_R^{5}}}{6c}$
default	$\frac{\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}} x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\left(\frac{a}{c}\right)^{\frac{5}{6}} e}{12a}-\frac{\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}} x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{1}{3}} d}{12a}+\frac{\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right) e}{6c\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{3}} \arctan\left(\frac{2x}{\left(\frac{a}{c}\right)\right.}$

input
```
int(x*(e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/c*sum((_R^4*e+_R*d)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1920 vs. $2(192) = 384$.

Time = 0.13 (sec) , antiderivative size = 1920, normalized size of antiderivative = 6.88

$$\int \frac{x(d+ex^3)}{a+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

output
```
1/12*(sqrt(-3) - 1)*((a^2*c^2*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e
^6)/(a^3*c^5)) + c*d^3 - 3*a*d*e^2)/(a^2*c^2))^(1/3)*log(-(3*c^3*d^6*e + 5
*a*c^2*d^4*e^3 + a^2*c*d^2*e^5 - a^3*e^7)*x + 1/2*(6*a^2*c^3*d^3*e^2 - 2*a
^3*c^2*d*e^4 + 2*sqrt(-3)*(3*a^2*c^3*d^3*e^2 - a^3*c^2*d*e^4) + (a^3*c^5*d
^2 - a^4*c^4*e^2 + sqrt(-3)*(a^3*c^5*d^2 - a^4*c^4*e^2))*sqrt(-(9*c^2*d^4*
e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a^3*c^5)))*((a^2*c^2*sqrt(-(9*c^2*d^4*e^2
- 6*a*c*d^2*e^4 + a^2*e^6)/(a^3*c^5)) + c*d^3 - 3*a*d*e^2)/(a^2*c^2))^(2/3
)) - 1/12*(sqrt(-3) + 1)*((a^2*c^2*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 +
a^2*e^6)/(a^3*c^5)) + c*d^3 - 3*a*d*e^2)/(a^2*c^2))^(1/3)*log(-(3*c^3*d^6*
e + 5*a*c^2*d^4*e^3 + a^2*c*d^2*e^5 - a^3*e^7)*x + 1/2*(6*a^2*c^3*d^3*e^2
- 2*a^3*c^2*d*e^4 - 2*sqrt(-3)*(3*a^2*c^3*d^3*e^2 - a^3*c^2*d*e^4) + (a^3*
c^5*d^2 - a^4*c^4*e^2 - sqrt(-3)*(a^3*c^5*d^2 - a^4*c^4*e^2))*sqrt(-(9*c^2
*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a^3*c^5)))*((a^2*c^2*sqrt(-(9*c^2*d^4
*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a^3*c^5)) + c*d^3 - 3*a*d*e^2)/(a^2*c^2))
^(2/3)) + 1/12*(sqrt(-3) - 1)*(-(a^2*c^2*sqrt(-(9*c^2*d^4*e^2 - 6*a*c*d^2*
e^4 + a^2*e^6)/(a^3*c^5)) - c*d^3 + 3*a*d*e^2)/(a^2*c^2))^(1/3)*log(-(3*c^
3*d^6*e + 5*a*c^2*d^4*e^3 + a^2*c*d^2*e^5 - a^3*e^7)*x + 1/2*(6*a^2*c^3*d^
3*e^2 - 2*a^3*c^2*d*e^4 + 2*sqrt(-3)*(3*a^2*c^3*d^3*e^2 - a^3*c^2*d*e^4) -
 (a^3*c^5*d^2 - a^4*c^4*e^2 + sqrt(-3)*(a^3*c^5*d^2 - a^4*c^4*e^2))*sqrt(-
(9*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + a^2*e^6)/(a^3*c^5)))*(-(a^2*c^2*sqrt(-...
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.74

$$\int \frac{x(d+ex^3)}{a+cx^6}\,dx$$
$$= \mathrm{RootSum}\left(46656t^6a^4c^5 + t^3\cdot\left(1296a^3c^3de^2 - 432a^2c^4d^3\right) + a^3e^6 + 3a^2cd^2e^4 + 3ac^2d^4e^2 + c^3d^6, \left(t \mapsto t\,\mathrm{l}\right.\right.$$

input
```
integrate(x*(e*x**3+d)/(c*x**6+a),x)
```

output
```
RootSum(46656*_t**6*a**4*c**5 + _t**3*(1296*a**3*c**3*d*e**2 - 432*a**2*c*
*4*d**3) + a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6
, Lambda(_t, _t*log(x + (7776*_t**5*a**4*c**4*e**2 - 7776*_t**5*a**3*c**5*
d**2 + 180*_t**2*a**3*c**2*d*e**4 - 360*_t**2*a**2*c**3*d**3*e**2 + 36*_t*
*2*a*c**4*d**5)/(a**3*e**7 - a**2*c*d**2*e**5 - 5*a*c**2*d**4*e**3 - 3*c**
3*d**6*e)))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.01

$$\int \frac{x(d+ex^3)}{a+cx^6}\,dx = \frac{d\log\left(c^{\frac{1}{3}}x^2 + a^{\frac{1}{3}}\right)}{6\,a^{\frac{2}{3}}c^{\frac{1}{3}}} + \frac{e\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{3\,c^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} - \frac{\left(\sqrt{3}\sqrt{a}c^{\frac{1}{6}}e + c^{\frac{2}{3}}d\right)\log\left(c^{\frac{1}{3}}x^2 + \sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x + a^{\frac{1}{3}}\right)}{12\,a^{\frac{2}{3}}c} + \frac{\left(\sqrt{3}\sqrt{a}c^{\frac{1}{6}}e - c^{\frac{2}{3}}d\right)\log\left(c^{\frac{1}{3}}x^2 - \sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x + a^{\frac{1}{3}}\right)}{12\,a^{\frac{2}{3}}c} - \frac{\left(\sqrt{3}a^{\frac{1}{6}}c^{\frac{5}{6}}d - a^{\frac{2}{3}}c^{\frac{1}{3}}e\right)\arctan\left(\frac{2\,c^{\frac{1}{3}}x + \sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{6\,a^{\frac{2}{3}}c\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} + \frac{\left(\sqrt{3}a^{\frac{1}{6}}c^{\frac{5}{6}}d + a^{\frac{2}{3}}c^{\frac{1}{3}}e\right)\arctan\left(\frac{2\,c^{\frac{1}{3}}x - \sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{6\,a^{\frac{2}{3}}c\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}$$

input
```
integrate(x*(e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

output
```
1/6*d*log(c^(1/3)*x^2 + a^(1/3))/(a^(2/3)*c^(1/3)) + 1/3*e*arctan(c^(1/3)*
x/sqrt(a^(1/3)*c^(1/3)))/(c^(2/3)*sqrt(a^(1/3)*c^(1/3))) - 1/12*(sqrt(3)*s
qrt(a)*c^(1/6)*e + c^(2/3)*d)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x
+ a^(1/3))/(a^(2/3)*c) + 1/12*(sqrt(3)*sqrt(a)*c^(1/6)*e - c^(2/3)*d)*log(
c^(1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a^(2/3)*c) - 1/6*(sqrt
(3)*a^(1/6)*c^(5/6)*d - a^(2/3)*c^(1/3)*e)*arctan((2*c^(1/3)*x + sqrt(3)*a
^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c*sqrt(a^(1/3)*c^(1/3))) +
 1/6*(sqrt(3)*a^(1/6)*c^(5/6)*d + a^(2/3)*c^(1/3)*e)*arctan((2*c^(1/3)*x -
 sqrt(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c*sqrt(a^(1/3)*c
^(1/3)))
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.94

$$\int \frac{x(d+ex^3)}{a+cx^6}\,dx = \frac{e\left(\frac{a}{c}\right)^{\frac{5}{6}}\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,a} - \frac{\left(\sqrt{3}c^3d-\sqrt{ac}c^2e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,\left(ac^5\right)^{\frac{2}{3}}} + \frac{\left(\sqrt{3}c^3d+\sqrt{ac}c^2e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,\left(ac^5\right)^{\frac{2}{3}}} - \frac{\left(c^3d+\sqrt{3}\sqrt{ac}c^2e\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,\left(ac^5\right)^{\frac{2}{3}}} - \frac{\left(c^3d+\sqrt{3}\sqrt{ac}c^2e\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,\left(ac^5\right)^{\frac{2}{3}}} + \frac{\left(ac^5\right)^{\frac{1}{3}}d\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,ac^2}$$

input
```
integrate(x*(e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

output
```
1/3*e*(a/c)^(5/6)*arctan(x/(a/c)^(1/6))/a - 1/6*(sqrt(3)*c^3*d - sqrt(a*c)
*c^2*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^5)^(2/3) + 1/
6*(sqrt(3)*c^3*d + sqrt(a*c)*c^2*e)*arctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/
c)^(1/6))/(a*c^5)^(2/3) - 1/12*(c^3*d + sqrt(3)*sqrt(a*c)*c^2*e)*log(x^2 +
 sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^5)^(2/3) - 1/12*(c^3*d + sqrt(3
)*sqrt(a*c)*c^2*e)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^5)^
(2/3) + 1/6*(a*c^5)^(1/3)*d*log(x^2 + (a/c)^(1/3))/(a*c^2)
```

Mupad [B] (verification not implemented)

Time = 23.64 (sec) , antiderivative size = 1537, normalized size of antiderivative = 5.51

$$\int \frac{x(d+ex^3)}{a+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x*(d + e*x^3))/(a + c*x^6),x)
```

output
```
log(a^4*c^4*((a^2*c^4*d^3 + a*e^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d*e^2 - 3*c
*d^2*e*(-a^5*c^5)^(1/2))/(a^4*c^5))^(2/3) - a*e^2*x*(-a^5*c^5)^(1/2) + c*d
^2*x*(-a^5*c^5)^(1/2) + 2*a^3*c^3*d*e*x)*((a^2*c^4*d^3 + a*e^3*(-a^5*c^5)^
(1/2) - 3*a^3*c^3*d*e^2 - 3*c*d^2*e*(-a^5*c^5)^(1/2))/(216*a^4*c^5))^(1/3)
 + log(a^4*c^4*((a^2*c^4*d^3 - a*e^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d*e^2 +
3*c*d^2*e*(-a^5*c^5)^(1/2))/(a^4*c^5))^(2/3) + a*e^2*x*(-a^5*c^5)^(1/2) -
c*d^2*x*(-a^5*c^5)^(1/2) + 2*a^3*c^3*d*e*x)*((a^2*c^4*d^3 - a*e^3*(-a^5*c^
5)^(1/2) - 3*a^3*c^3*d*e^2 + 3*c*d^2*e*(-a^5*c^5)^(1/2))/(216*a^4*c^5))^(1
/3) - log(c^2*d*x*(a*e^2 + c*d^2)^2 - (((3^(1/2)*1i)/2 - 1/2)*(36*a^3*c^3*
e^3 - 108*a^2*c^4*d^2*e + 36*a^2*c^4*x*((3^(1/2)*1i)/2 + 1/2)*(a*e^2 - c*d
^2)*((a^2*c^4*d^3 + a*e^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d*e^2 - 3*c*d^2*e*(
-a^5*c^5)^(1/2))/(a^4*c^5))^(1/3))*((a^2*c^4*d^3 + a*e^3*(-a^5*c^5)^(1/2)
- 3*a^3*c^3*d*e^2 - 3*c*d^2*e*(-a^5*c^5)^(1/2))/(a^4*c^5))^(2/3))/36)*((3^
(1/2)*1i)/2 + 1/2)*((a^2*c^4*d^3 + a*e^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d*e^
2 - 3*c*d^2*e*(-a^5*c^5)^(1/2))/(216*a^4*c^5))^(1/3) + log(c^2*d*x*(a*e^2
+ c*d^2)^2 - (((3^(1/2)*1i)/2 + 1/2)*(108*a^2*c^4*d^2*e - 36*a^3*c^3*e^3 +
 36*a^2*c^4*x*((3^(1/2)*1i)/2 - 1/2)*(a*e^2 - c*d^2)*((a^2*c^4*d^3 + a*e^3
*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d*e^2 - 3*c*d^2*e*(-a^5*c^5)^(1/2))/(a^4*c^5
))^(1/3))*((a^2*c^4*d^3 + a*e^3*(-a^5*c^5)^(1/2) - 3*a^3*c^3*d*e^2 - 3*c*d
^2*e*(-a^5*c^5)^(1/2))/(a^4*c^5))^(2/3))/36)*((3^(1/2)*1i)/2 - 1/2)*((a...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.12

$$\int \frac{x(d+ex^3)}{a+cx^6}\,dx$$
$$= \frac{-2c^{\frac{1}{6}}a^{\frac{7}{6}}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e-2c^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)d+2c^{\frac{1}{6}}a^{\frac{7}{6}}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e-2c^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{}}{}$$

input
```
int(x*(e*x^3+d)/(c*x^6+a),x)
```

output
```
( - 2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c
**(1/6)*a**(1/6)))*a*e - 2*c**(2/3)*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**(1/
6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*d + 2*c**(1/6)*a**(1/6)*at
an((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*e - 2
*c**(2/3)*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)
/(c**(1/6)*a**(1/6)))*d + 4*c**(1/6)*a**(1/6)*atan((c**(1/3)*x)/(c**(1/6)*
a**(1/6)))*a*e + c**(1/6)*a**(1/6)*sqrt(3)*log( - c**(1/6)*a**(1/6)*sqrt(3
)*x + a**(1/3) + c**(1/3)*x**2)*a*e - c**(1/6)*a**(1/6)*sqrt(3)*log(c**(1/
6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*a*e + 2*c**(2/3)*a**(2/3
)*log(a**(1/3) + c**(1/3)*x**2)*d - c**(2/3)*a**(2/3)*log( - c**(1/6)*a**(
1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*d - c**(2/3)*a**(2/3)*log(c**(1
/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*d)/(12*a**(1/3)*a*c)
```

3.25 $\int \frac{d+ex^3}{x^2(a+cx^6)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 288

$$\int \frac{d+ex^3}{x^2(a+cx^6)}\,dx = -\frac{d}{ax} - \frac{\sqrt[6]{c}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{7/6}} + \frac{\sqrt[6]{c}d\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{7/6}} - \frac{\sqrt[6]{c}d\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{7/6}} - \frac{e\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{c}x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{2/3}\sqrt[3]{c}} + \frac{\sqrt[6]{c}d\,\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{2\sqrt{3}a^{7/6}} + \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6a^{2/3}\sqrt[3]{c}} - \frac{e\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{c}x^2+c^{2/3}x^4\right)}{12a^{2/3}\sqrt[3]{c}}$$

output
```
-d/a/x-1/3*c^(1/6)*d*arctan(c^(1/6)*x/a^(1/6))/a^(7/6)-1/6*c^(1/6)*d*arcta
n(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(7/6)-1/6*c^(1/6)*d*arctan(3^(1/2)+2*c^(
1/6)*x/a^(1/6))/a^(7/6)-1/6*e*arctan(1/3*(a^(1/3)-2*c^(1/3)*x^2)*3^(1/2)/a
^(1/3))*3^(1/2)/a^(2/3)/c^(1/3)+1/6*c^(1/6)*d*arctanh(3^(1/2)*a^(1/6)*c^(1
/6)*x/(a^(1/3)+c^(1/3)*x^2))*3^(1/2)/a^(7/6)+1/6*e*ln(a^(1/3)+c^(1/3)*x^2)
/a^(2/3)/c^(1/3)-1/12*e*ln(a^(2/3)-a^(1/3)*c^(1/3)*x^2+c^(2/3)*x^4)/a^(2/3
)/c^(1/3)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.05

$$\int \frac{d+ex^3}{x^2(a+cx^6)}\,dx$$
$$= \frac{-12ac^{5/6}d-4a^{5/6}cdx\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)-2\left(-a^{5/6}cd+\sqrt{3}a^{4/3}\sqrt{c}e\right)x\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)-2\left(a^{5/6}cd+\right.}{}$$

input
```
Integrate[(d + e*x^3)/(x^2*(a + c*x^6)),x]
```

output
```
(-12*a*c^(5/6)*d - 4*a^(5/6)*c*d*x*ArcTan[(c^(1/6)*x)/a^(1/6)] - 2*(-(a^(5
/6)*c*d) + Sqrt[3]*a^(4/3)*Sqrt[c]*e)*x*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(
1/6)] - 2*(a^(5/6)*c*d + Sqrt[3]*a^(4/3)*Sqrt[c]*e)*x*ArcTan[Sqrt[3] + (2*
c^(1/6)*x)/a^(1/6)] + 2*a^(4/3)*Sqrt[c]*e*x*Log[a^(1/3) + c^(1/3)*x^2] - (
Sqrt[3]*a^(5/6)*c*d + a^(4/3)*Sqrt[c]*e)*x*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c
^(1/6)*x + c^(1/3)*x^2] + (Sqrt[3]*a^(5/6)*c*d - a^(4/3)*Sqrt[c]*e)*x*Log[
a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^2*c^(5/6)*x)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1829, 25, 1835, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^2(a+cx^6)}\,dx$$
$$\Big\downarrow \ 1829$$
$$-\frac{\int -\frac{x(ae-cdx^3)}{cx^6+a}dx}{a}-\frac{d}{ax}$$
$$\Big\downarrow \ 25$$

$$\frac{\int \frac{x(ae-cdx^3)}{cx^6+a}dx}{a}-\frac{d}{ax}$$

$\big\downarrow$ 1835

$$\frac{\frac{1}{2}\sqrt{c}\left(\sqrt{-a}e+\sqrt{c}d\right)\int \frac{x}{\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x^3\right)}dx-\frac{1}{2}\sqrt{c}\left(\sqrt{c}d-\sqrt{-a}e\right)\int \frac{x}{\sqrt{c}\left(\sqrt{c}x^3+\sqrt{-a}\right)}dx}{a}-\frac{d}{ax}$$

$\big\downarrow$ 27

$$\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\int \frac{x}{\sqrt{-a}-\sqrt{c}x^3}dx-\frac{1}{2}\left(\sqrt{c}d-\sqrt{-a}e\right)\int \frac{x}{\sqrt{c}x^3+\sqrt{-a}}dx}{a}-\frac{d}{ax}$$

$\big\downarrow$ 821

$$\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(\frac{\int \frac{1}{\sqrt[6]{-a}-\sqrt[6]{c}x}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\int \frac{\sqrt[6]{-a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-}}}{3\sqrt[6]{-a}\sqrt[6]{c}}\right.}{a}$$

$$\frac{d}{ax}$$

$\big\downarrow$ 16

$$\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{\int \frac{\sqrt[6]{-a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{}}}{3\sqrt[6]{-a}\sqrt[6]{c}}\right.}{a}$$

$$\frac{d}{ax}$$

$\big\downarrow$ 1142

$$\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{\frac{3}{2}\sqrt[6]{-a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{-a}\right)}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}\right.}{a}$$

$$\frac{d}{ax}$$

$\big\downarrow$ 25

$$\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{\frac{3}{2}\sqrt[6]{-a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{\int\frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{-a}\right)}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}\cdots\right.}{a}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{\frac{3}{2}\sqrt[6]{-a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{\cdots}\right.}{a}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{-\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{3\int\frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1\right)^2-3}d\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1\right)}{\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}c\cdots\right.}{a}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\right.}{a}$$

$$\frac{d}{ax}$$

$\Big\downarrow$ 1103

$$\frac{\frac{1}{2}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)-\frac{1}{2}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\right.}{a}$$

$$\frac{d}{ax}$$

input
```
Int[(d + e*x^3)/(x^2*(a + c*x^6)),x]
```

output
```
-(d/(a*x)) + (-1/2*((Sqrt[c]*d - Sqrt[-a]*e)*(-1/3*Log[(-a)^(1/6) + c^(1/6
)*x]/((-a)^(1/6)*c^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*c^(1/6)*x)/(-a)^(1/
6))/Sqrt[3]])/c^(1/6)) + Log[(-a)^(1/3) - (-a)^(1/6)*c^(1/6)*x + c^(1/3)*x
^2]/(2*c^(1/6)))/(3*(-a)^(1/6)*c^(1/6)))) + ((Sqrt[c]*d + Sqrt[-a]*e)*(-1/
3*Log[(-a)^(1/6) - c^(1/6)*x]/((-a)^(1/6)*c^(1/3)) - ((Sqrt[3]*ArcTan[(1 +
 (2*c^(1/6)*x)/(-a)^(1/6))/Sqrt[3]])/c^(1/6) - Log[(-a)^(1/3) + (-a)^(1/6)
*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/6)))/(3*(-a)^(1/6)*c^(1/6))))/2)/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1829
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^(2*n))^(p + 1)/(a*f*(m + 1
))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m + n)*(a + c*x^(2*n))^p*(a*e
*(m + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f,
p}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

rule 1835
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[-(e/2 + c*(d/(2*q)))   Int[(f
*x)^m/(q - c*x^n), x], x] + Simp[(e/2 - c*(d/(2*q)))   Int[(f*x)^m/(q + c*x
^n), x], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{d}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{7}c^{2}\_Z^{6}+\left(-2a^{5}ce^{3}+6a^{4}c^{2}d^{2}e\right)\_Z^{3}+a^{3}e^{6}+3a^{2}cd^{2}e^{4}+3ac^{2}d^{4}e^{2}+c^{3}d^{6}\right)}\_R\ln\left(\left(7\_R^{6}a^{7}c^{2}+\left(-13a^{5}ce^{3} \ldots\right.\right.\right.}{6}$
default	$-\frac{d}{ax}+\frac{-\frac{c\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\left(\frac{a}{c}\right)^{\frac{5}{6}}d}{12a}-\frac{c\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{4}{3}}e}{12a}-\frac{\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)d}{6\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\frac{c\left(\frac{a}{c}\right)^{\frac{4}{3}}\arctan\left(\ldots\right)}{6a}\ldots}{\ldots}$

input
```
int((e*x^3+d)/x^2/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-d/a/x+1/6*sum(_R*ln((7*_R^6*a^7*c^2+(-13*a^5*c*e^3+39*a^4*c^2*d^2*e)*_R^3
+6*a^3*e^6+18*a^2*c*d^2*e^4+18*a*c^2*d^4*e^2+6*c^3*d^6)*x+a^6*c^2*d*_R^5+(
2*a^4*c*d*e^3+2*a^3*c^2*d^3*e)*_R^2),_R=RootOf(a^7*c^2*_Z^6+(-2*a^5*c*e^3+
6*a^4*c^2*d^2*e)*_Z^3+a^3*e^6+3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2+c^3*d^6))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1885 vs. $2(201) = 402$.

Time = 0.31 (sec) , antiderivative size = 1885, normalized size of antiderivative = 6.55

$$\int \frac{d + ex^3}{x^2 (a + cx^6)} dx = \text{Too large to display}$$

input
```
integrate((e*x^3+d)/x^2/(c*x^6+a),x, algorithm="fricas")
```

output
```
1/12*(2*a*x*(-(a^3*c*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^7*
c)) + 3*c*d^2*e - a*e^3)/(a^3*c))^(1/3)*log(-(c^3*d^7 - a*c^2*d^5*e^2 - 5*
a^2*c*d^3*e^4 - 3*a^3*d*e^6)*x + (2*a^3*c^2*d^4*e - 6*a^4*c*d^2*e^3 - (a^6
*c^2*d^2 - a^7*c*e^2)*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^7
*c)))*(-(a^3*c*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^7*c)) +
3*c*d^2*e - a*e^3)/(a^3*c))^(2/3)) + 2*a*x*((a^3*c*sqrt(-(c^2*d^6 - 6*a*c*
d^4*e^2 + 9*a^2*d^2*e^4)/(a^7*c)) - 3*c*d^2*e + a*e^3)/(a^3*c))^(1/3)*log(
-(c^3*d^7 - a*c^2*d^5*e^2 - 5*a^2*c*d^3*e^4 - 3*a^3*d*e^6)*x + (2*a^3*c^2*
d^4*e - 6*a^4*c*d^2*e^3 + (a^6*c^2*d^2 - a^7*c*e^2)*sqrt(-(c^2*d^6 - 6*a*c
*d^4*e^2 + 9*a^2*d^2*e^4)/(a^7*c)))*((a^3*c*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2
 + 9*a^2*d^2*e^4)/(a^7*c)) - 3*c*d^2*e + a*e^3)/(a^3*c))^(2/3)) + (sqrt(-3
)*a*x - a*x)*(-(a^3*c*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^7
*c)) + 3*c*d^2*e - a*e^3)/(a^3*c))^(1/3)*log(-(c^3*d^7 - a*c^2*d^5*e^2 - 5
*a^2*c*d^3*e^4 - 3*a^3*d*e^6)*x - 1/2*(2*a^3*c^2*d^4*e - 6*a^4*c*d^2*e^3 +
 2*sqrt(-3)*(a^3*c^2*d^4*e - 3*a^4*c*d^2*e^3) - (a^6*c^2*d^2 - a^7*c*e^2 +
 sqrt(-3)*(a^6*c^2*d^2 - a^7*c*e^2))*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^
2*d^2*e^4)/(a^7*c)))*(-(a^3*c*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e
^4)/(a^7*c)) + 3*c*d^2*e - a*e^3)/(a^3*c))^(2/3)) - (sqrt(-3)*a*x + a*x)*(
-(a^3*c*sqrt(-(c^2*d^6 - 6*a*c*d^4*e^2 + 9*a^2*d^2*e^4)/(a^7*c)) + 3*c*d^2
*e - a*e^3)/(a^3*c))^(1/3)*log(-(c^3*d^7 - a*c^2*d^5*e^2 - 5*a^2*c*d^3*...
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.72

$$\int \frac{d+ex^3}{x^2(a+cx^6)}\,dx$$
$$= \operatorname{RootSum}\left(46656t^6a^7c^2 + t^3\left(-432a^5ce^3 + 1296a^4c^2d^2e\right) + a^3e^6 + 3a^2cd^2e^4 + 3ac^2d^4e^2 + c^3d^6, \left(t \mapsto t\,\mathrm{lo}\right.\right.$$
$$- \frac{d}{ax}$$

input
```
integrate((e*x**3+d)/x**2/(c*x**6+a),x)
```

output
```
RootSum(46656*_t**6*a**7*c**2 + _t**3*(-432*a**5*c*e**3 + 1296*a**4*c**2*d
**2*e) + a**3*e**6 + 3*a**2*c*d**2*e**4 + 3*a*c**2*d**4*e**2 + c**3*d**6,
Lambda(_t, _t*log(x + (-7776*_t**5*a**7*c*e**2 + 7776*_t**5*a**6*c**2*d**2
 + 36*_t**2*a**5*e**5 - 360*_t**2*a**4*c*d**2*e**3 + 180*_t**2*a**3*c**2*d
**4*e)/(3*a**3*d*e**6 + 5*a**2*c*d**3*e**4 + a*c**2*d**5*e**2 - c**3*d**7)
))) - d/(a*x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.03

$$\int \frac{d+ex^3}{x^2(a+cx^6)}\,dx$$
$$= \frac{\frac{2\,a^{\frac{1}{3}}\,e\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}} - \frac{4\,c^{\frac{1}{3}}\,d\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} + \frac{\left(\sqrt{3}\sqrt{a}c^{\frac{7}{6}}d-ac^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}c} - \frac{\left(\sqrt{3}\sqrt{a}c^{\frac{7}{6}}d+ac^{\frac{2}{3}}e\right)\log\left(c^{\frac{1}{3}}x^2-\right.}{a^{\frac{2}{3}}c}}{12\,a}$$
$$- \frac{d}{ax}$$

input
```
integrate((e*x^3+d)/x^2/(c*x^6+a),x, algorithm="maxima")
```

output
```
1/12*(2*a^(1/3)*e*log(c^(1/3)*x^2 + a^(1/3))/c^(1/3) - 4*c^(1/3)*d*arctan(
c^(1/3)*x/sqrt(a^(1/3)*c^(1/3)))/sqrt(a^(1/3)*c^(1/3)) + (sqrt(3)*sqrt(a)*
c^(7/6)*d - a*c^(2/3)*e)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x + a^(
1/3))/(a^(2/3)*c) - (sqrt(3)*sqrt(a)*c^(7/6)*d + a*c^(2/3)*e)*log(c^(1/3)*
x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a^(2/3)*c) - 2*(sqrt(3)*a^(7/6
)*c^(5/6)*e + a^(2/3)*c^(4/3)*d)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6)*c^(
1/6))/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c*sqrt(a^(1/3)*c^(1/3))) + 2*(sqrt(3
)*a^(7/6)*c^(5/6)*e - a^(2/3)*c^(4/3)*d)*arctan((2*c^(1/3)*x - sqrt(3)*a^(
1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c*sqrt(a^(1/3)*c^(1/3))))/a
- d/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.03

$$\int \frac{d+ex^3}{x^2(a+cx^6)}\,dx = -\frac{cd\left(\frac{a}{c}\right)^{\frac{5}{6}}\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,a^2} + \frac{(ac^5)^{\frac{1}{3}}\,e\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,ac^2} - \frac{d}{ax} - \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{3}}\,ac^2e+(ac^5)^{\frac{5}{6}}\,d\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,a^2c^4} + \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{3}}\,ac^2e-(ac^5)^{\frac{5}{6}}\,d\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,a^2c^4} - \frac{\left((ac^5)^{\frac{1}{3}}\,ac^2e-\sqrt{3}(ac^5)^{\frac{5}{6}}\,d\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,a^2c^4} - \frac{\left((ac^5)^{\frac{1}{3}}\,ac^2e+\sqrt{3}(ac^5)^{\frac{5}{6}}\,d\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,a^2c^4}$$

input
```
integrate((e*x^3+d)/x^2/(c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*c*d*(a/c)^(5/6)*arctan(x/(a/c)^(1/6))/a^2 + 1/6*(a*c^5)^(1/3)*e*log(x
^2 + (a/c)^(1/3))/(a*c^2) - d/(a*x) - 1/6*(sqrt(3)*(a*c^5)^(1/3)*a*c^2*e +
 (a*c^5)^(5/6)*d)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^2*c^4
) + 1/6*(sqrt(3)*(a*c^5)^(1/3)*a*c^2*e - (a*c^5)^(5/6)*d)*arctan((2*x - sq
rt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^2*c^4) - 1/12*((a*c^5)^(1/3)*a*c^2*e -
sqrt(3)*(a*c^5)^(5/6)*d)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a
^2*c^4) - 1/12*((a*c^5)^(1/3)*a*c^2*e + sqrt(3)*(a*c^5)^(5/6)*d)*log(x^2 -
 sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 23.64 (sec) , antiderivative size = 1526, normalized size of antiderivative = 5.30

$$\int \frac{d+ex^3}{x^2\left(a+cx^6\right)}\,dx=\text{Too large to display}$$

input
```
int((d + e*x^3)/(x^2*(a + c*x^6)),x)
```

output
```
log(a^6*c^2*((a^5*c*e^3 + c*d^3*(-a^7*c^3)^(1/2) - 3*a^4*c^2*d^2*e - 3*a*d
*e^2*(-a^7*c^3)^(1/2))/(a^7*c^2))^(2/3) - a*e^2*x*(-a^7*c^3)^(1/2) + c*d^2
*x*(-a^7*c^3)^(1/2) - 2*a^4*c^2*d*e*x)*((a^5*c*e^3 + c*d^3*(-a^7*c^3)^(1/2
) - 3*a^4*c^2*d^2*e - 3*a*d*e^2*(-a^7*c^3)^(1/2))/(216*a^7*c^2))^(1/3) + l
og(a^6*c^2*((a^5*c*e^3 - c*d^3*(-a^7*c^3)^(1/2) - 3*a^4*c^2*d^2*e + 3*a*d*
e^2*(-a^7*c^3)^(1/2))/(a^7*c^2))^(2/3) + a*e^2*x*(-a^7*c^3)^(1/2) - c*d^2*
x*(-a^7*c^3)^(1/2) - 2*a^4*c^2*d*e*x)*((a^5*c*e^3 - c*d^3*(-a^7*c^3)^(1/2)
 - 3*a^4*c^2*d^2*e + 3*a*d*e^2*(-a^7*c^3)^(1/2))/(216*a^7*c^2))^(1/3) - d/
(a*x) - log((((3^(1/2)*1i)/2 - 1/2)*((a^5*c*e^3 + c*d^3*(-a^7*c^3)^(1/2) -
 3*a^4*c^2*d^2*e - 3*a*d*e^2*(-a^7*c^3)^(1/2))/(a^7*c^2))^(2/3)*(36*a^9*c^
6*d^3 - 108*a^10*c^5*d*e^2 + 36*a^10*c^5*x*((3^(1/2)*1i)/2 + 1/2)*(a*e^2 -
 c*d^2)*((a^5*c*e^3 + c*d^3*(-a^7*c^3)^(1/2) - 3*a^4*c^2*d^2*e - 3*a*d*e^2
*(-a^7*c^3)^(1/2))/(a^7*c^2))^(1/3)))/36 + a^7*c^4*e*x*(a*e^2 + c*d^2)^2)*
((3^(1/2)*1i)/2 + 1/2)*((a^5*c*e^3 + c*d^3*(-a^7*c^3)^(1/2) - 3*a^4*c^2*d^
2*e - 3*a*d*e^2*(-a^7*c^3)^(1/2))/(216*a^7*c^2))^(1/3) + log((((3^(1/2)*1i
)/2 + 1/2)*((a^5*c*e^3 + c*d^3*(-a^7*c^3)^(1/2) - 3*a^4*c^2*d^2*e - 3*a*d*
e^2*(-a^7*c^3)^(1/2))/(a^7*c^2))^(2/3)*(108*a^10*c^5*d*e^2 - 36*a^9*c^6*d^
3 + 36*a^10*c^5*x*((3^(1/2)*1i)/2 - 1/2)*(a*e^2 - c*d^2)*((a^5*c*e^3 + c*d
^3*(-a^7*c^3)^(1/2) - 3*a^4*c^2*d^2*e - 3*a*d*e^2*(-a^7*c^3)^(1/2))/(a^7*c
^2))^(1/3)))/36 + a^7*c^4*e*x*(a*e^2 + c*d^2)^2)*((3^(1/2)*1i)/2 - 1/2)...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.16

$$\int \frac{d+ex^3}{x^2(a+cx^6)}\,dx$$
$$= \frac{2c^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)dx - 2c^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)ex - 2c^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)dx - 2c^{\frac{2}{3}}a}{}$$

input
```
int((e*x^3+d)/x^2/(c*x^6+a),x)
```

output
```
(2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(
1/6)*a**(1/6)))*c*d*x - 2*c**(2/3)*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**(1/6
)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*e*x - 2*c**(1/6)*a**(1/6)*a
tan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d*x
- 2*c**(2/3)*a**(2/3)*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)
*x)/(c**(1/6)*a**(1/6)))*e*x - 4*c**(1/6)*a**(1/6)*atan((c**(1/3)*x)/(c**(
1/6)*a**(1/6)))*c*d*x - c**(1/6)*a**(1/6)*sqrt(3)*log( - c**(1/6)*a**(1/6)
*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c*d*x + c**(1/6)*a**(1/6)*sqrt(3)*l
og(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c*d*x + 2*c**(2
/3)*a**(2/3)*log(a**(1/3) + c**(1/3)*x**2)*e*x - c**(2/3)*a**(2/3)*log( -
c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*e*x - c**(2/3)*a**
(2/3)*log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*e*x - 12
*a**(1/3)*c*d)/(12*a**(1/3)*a*c*x)
```

3.26 $\int \frac{d+ex^3}{x^5(a+cx^6)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 299

$$\int \frac{d+ex^3}{x^5(a+cx^6)}\,dx = -\frac{d}{4ax^4}-\frac{e}{ax}-\frac{\sqrt[6]{c}e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{7/6}}+\frac{\sqrt[6]{c}e\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{7/6}} -\frac{\sqrt[6]{c}e\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{7/6}}+\frac{c^{2/3}d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{c}x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{5/3}} +\frac{\sqrt[6]{c}e\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{2\sqrt{3}a^{7/6}}-\frac{c^{2/3}d\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6a^{5/3}} +\frac{c^{2/3}d\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{c}x^2+c^{2/3}x^4\right)}{12a^{5/3}}$$

output
```
-1/4*d/a/x^4-e/a/x-1/3*c^(1/6)*e*arctan(c^(1/6)*x/a^(1/6))/a^(7/6)-1/6*c^(
1/6)*e*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(7/6)-1/6*c^(1/6)*e*arctan(3
^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(7/6)+1/6*c^(2/3)*d*arctan(1/3*(a^(1/3)-2*c^
(1/3)*x^2)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)+1/6*c^(1/6)*e*arctanh(3^(1/2)*
a^(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)*x^2))*3^(1/2)/a^(7/6)-1/6*c^(2/3)*d*ln(
a^(1/3)+c^(1/3)*x^2)/a^(5/3)+1/12*c^(2/3)*d*ln(a^(2/3)-a^(1/3)*c^(1/3)*x^2
+c^(2/3)*x^4)/a^(5/3)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.04

$$\int \frac{d+ex^3}{x^5(a+cx^6)}\,dx$$
$$= \frac{-\frac{3ad}{x^4}-\frac{12ae}{x}-4a^{5/6}\sqrt[6]{c}e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)+2\sqrt[3]{a}\sqrt[6]{c}(\sqrt{3}\sqrt{c}d+\sqrt{a}e)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)+2\sqrt[3]{a}\sqrt[6]{c}(\sqrt{}}{}$$

input
```
Integrate[(d + e*x^3)/(x^5*(a + c*x^6)),x]
```

output
```
((-3*a*d)/x^4 - (12*a*e)/x - 4*a^(5/6)*c^(1/6)*e*ArcTan[(c^(1/6)*x)/a^(1/6
)] + 2*a^(1/3)*c^(1/6)*(Sqrt[3]*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[Sqrt[3] - (2
*c^(1/6)*x)/a^(1/6)] + 2*a^(1/3)*c^(1/6)*(Sqrt[3]*Sqrt[c]*d - Sqrt[a]*e)*A
rcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)] - 2*a^(1/3)*c^(2/3)*d*Log[a^(1/3) +
 c^(1/3)*x^2] + c^(1/6)*(a^(1/3)*Sqrt[c]*d - Sqrt[3]*a^(5/6)*e)*Log[a^(1/3
) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + c^(1/6)*(a^(1/3)*Sqrt[c]*d
+ Sqrt[3]*a^(5/6)*e)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2
])/(12*a^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {1829, 27, 1829, 27, 1835, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^5(a+cx^6)}\,dx$$
$$\Big\downarrow \text{1829}$$
$$-\frac{\int -\frac{4(ae-cdx^3)}{x^2(cx^6+a)}dx}{4a}-\frac{d}{4ax^4}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{ae-cdx^3}{x^2(cx^6+a)}dx}{a}-\frac{d}{4ax^4}$$

$$\Big\downarrow \; 1829$$

$$\frac{-\frac{\int \frac{acx\left(ex^3+d\right)}{cx^6+a}dx}{a}-\frac{e}{x}}{a}-\frac{d}{4ax^4}$$

$$\Big\downarrow \; 27$$

$$\frac{-c\int \frac{x(ex^3+d)}{cx^6+a}dx-\frac{e}{x}}{a}-\frac{d}{4ax^4}$$

$$\Big\downarrow \; 1835$$

$$\frac{-c\left(\frac{1}{2}\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\int \frac{x}{\sqrt{c}(\sqrt{-a}-\sqrt{c}x^3)}dx+\frac{1}{2}\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\int \frac{x}{\sqrt{c}(\sqrt{c}x^3+\sqrt{-a})}dx\right)-\frac{e}{x}}{a}-\frac{d}{4ax^4}$$

$$\Big\downarrow \; 27$$

$$\frac{-c\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\int \frac{x}{\sqrt{-a}-\sqrt{c}x^3}dx}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\int \frac{x}{\sqrt{c}x^3+\sqrt{-a}}dx}{2\sqrt{c}}\right)-\frac{e}{x}}{a}-\frac{d}{4ax^4}$$

$$\Big\downarrow \; 821$$

$$\frac{-c\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\int \frac{1}{\sqrt[6]{c}x+\sqrt[6]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(\frac{\int \frac{1}{\sqrt[6]{-a}-\sqrt[6]{c}x}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\int \frac{\sqrt[6]{-a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}\right)}{2\sqrt{c}}\right)}{a}$$

$$\frac{d}{4ax^4}$$

$$\Big\downarrow \; 16$$

$$-c\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\int \frac{\sqrt[6]{c}x+\sqrt[6]{-a}}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\frac{\int \frac{\sqrt[6]{-a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log(}{}\right.}{2\sqrt{c}}\right.$$

$$\overline{\qquad a \qquad}$$

$$\frac{d}{4ax^4}$$

$\downarrow$ 1142

$$-c\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{3}{2}\sqrt[6]{-a}\int \frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx+\frac{\int -\frac{\sqrt[6]{c}\left(\sqrt[6]{-a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-\right.}{}\right.$$

$$\overline{\qquad a \qquad}$$

$$\frac{d}{4ax^4}$$

$\downarrow$ 25

$$\frac{-c\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{3}{2}\sqrt[6]{-a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{\int\frac{\sqrt[6]{c}\left(\sqrt[6]{-a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{2\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\cdots}{\cdots}\right)}{a}$$

$$\frac{d}{4ax^4}$$

$$\Big\downarrow \; 27$$

$$\frac{-c\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{3}{2}\sqrt[6]{-a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{1}{2}\int\frac{\sqrt[6]{-a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\cdots}{\cdots}\right)}{a}$$

$$\frac{d}{4ax^4}$$

$$\Big\downarrow \; 1082$$

$$-c\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}\right)^2-3}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}\right)}{\sqrt[6]{c}}-\frac{1}{2}\int\frac{\sqrt[6]{-a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(\cdots\right.}{\cdots}\right.$$

$$a$$

$$\frac{d}{4ax^4}$$

$\big\downarrow$ 217

$$-c\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{-\frac{1}{2}\int\frac{\sqrt[6]{-a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\frac{\sqrt{3}\arctan\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$$a$$

$$\frac{d}{4ax^4}$$

$\big\downarrow$ 1103

$$\frac{-c\left(\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}+e\right)\left(\frac{\frac{\log\left(-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{3\sqrt[6]{-a}\sqrt[6]{c}}-\frac{\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{3\sqrt[6]{-a}\sqrt[3]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{a\sqrt{c}d}{(-a)^{3/2}}-e\right)\left(-\frac{\sqrt{3}\arctan\left(\cdots\right)}{\sqrt[6]{\cdots}}\cdots\right.}{\cdots}\right.}{a}$$

$$\frac{d}{4ax^4}$$

input
```
Int[(d + e*x^3)/(x^5*(a + c*x^6)),x]
```

output
```
-1/4*d/(a*x^4) + (-(e/x) - c*((((a*Sqrt[c]*d)/(-a)^(3/2) + e)*(-1/3*Log[(-
a)^(1/6) + c^(1/6)*x]/((-a)^(1/6)*c^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*c^
(1/6)*x)/(-a)^(1/6))/Sqrt[3]])/c^(1/6)) + Log[(-a)^(1/3) - (-a)^(1/6)*c^(1
/6)*x + c^(1/3)*x^2]/(2*c^(1/6)))/(3*(-a)^(1/6)*c^(1/6))))/(2*Sqrt[c]) + (
((a*Sqrt[c]*d)/(-a)^(3/2) - e)*(-1/3*Log[(-a)^(1/6) - c^(1/6)*x]/((-a)^(1/
6)*c^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*c^(1/6)*x)/(-a)^(1/6))/Sqrt[3]])/c^
(1/6) - Log[(-a)^(1/3) + (-a)^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/6)))/
(3*(-a)^(1/6)*c^(1/6))))/(2*Sqrt[c])))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1829
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(
p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^(2*n))^(p + 1)/(a*f*(m + 1
))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m + n)*(a + c*x^(2*n))^p*(a*e
*(m + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n), x], x] /; FreeQ[{a, c, d, e, f,
p}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

rule 1835
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[-(e/2 + c*(d/(2*q)))   Int[(f
*x)^m/(q - c*x^n), x], x] + Simp[(e/2 - c*(d/(2*q)))   Int[(f*x)^m/(q + c*x
^n), x], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.74

method	result
risch	$\frac{-\frac{e\,x^3}{a}-\frac{d}{4a}}{x^4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}\_Z^6+\left(-6a^6cd\,e^2+2a^5c^2d^3\right)\_Z^3+a^3c\,e^6+3a^2c^2d^2e^4+3a\,c^3d^4e^2+c^4d^6\right)}\_R\ln\left(\left(7\_R^6a^{10}+(-39a\right.\right.\right.}{}$
default	$-\frac{d}{4a\,x^4}-\frac{e}{ax}-\frac{\left(\frac{\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\left(\frac{a}{c}\right)^{\frac{5}{6}}e}{12a}-\frac{\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\left(\frac{a}{c}\right)^{\frac{1}{3}}d}{12a}+\frac{\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)e}{6c\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{3}}\mathrm{arct}\right.}{}$

input
```
int((e*x^3+d)/x^5/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
(-e/a*x^3-1/4*d/a)/x^4+1/6*sum(_R*ln((7*_R^6*a^10+(-39*a^6*c*d*e^2+13*a^5*
c^2*d^3)*_R^3+6*a^3*c*e^6+18*a^2*c^2*d^2*e^4+18*a*c^3*d^4*e^2+6*c^4*d^6)*x
+a^9*e*_R^5+(-2*a^5*c*d*e^3-2*a^4*c^2*d^3*e)*_R^2),_R=RootOf(a^10*_Z^6+(-6
*a^6*c*d*e^2+2*a^5*c^2*d^3)*_Z^3+a^3*c*e^6+3*a^2*c^2*d^2*e^4+3*a*c^3*d^4*e
^2+c^4*d^6))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1893 vs. $2(210) = 420$.

Time = 0.12 (sec) , antiderivative size = 1893, normalized size of antiderivative = 6.33

$$\int \frac{d+ex^3}{x^5(a+cx^6)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^3+d)/x^5/(c*x^6+a),x, algorithm="fricas")
```

output
```
1/12*(2*a*x^4*(-(a^5*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/a
^9) + c^2*d^3 - 3*a*c*d*e^2)/a^5)^(1/3)*log(-(3*c^4*d^6*e + 5*a*c^3*d^4*e^
3 + a^2*c^2*d^2*e^5 - a^3*c*e^7)*x - (6*a^4*c^2*d^3*e^2 - 2*a^5*c*d*e^4 +
(a^8*c*d^2 - a^9*e^2)*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/
a^9))*(-(a^5*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/a^9) + c^
2*d^3 - 3*a*c*d*e^2)/a^5)^(2/3)) + 2*a*x^4*((a^5*sqrt(-(9*c^3*d^4*e^2 - 6*
a*c^2*d^2*e^4 + a^2*c*e^6)/a^9) - c^2*d^3 + 3*a*c*d*e^2)/a^5)^(1/3)*log(-(
3*c^4*d^6*e + 5*a*c^3*d^4*e^3 + a^2*c^2*d^2*e^5 - a^3*c*e^7)*x - (6*a^4*c^
2*d^3*e^2 - 2*a^5*c*d*e^4 - (a^8*c*d^2 - a^9*e^2)*sqrt(-(9*c^3*d^4*e^2 - 6
*a*c^2*d^2*e^4 + a^2*c*e^6)/a^9))*((a^5*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2
*e^4 + a^2*c*e^6)/a^9) - c^2*d^3 + 3*a*c*d*e^2)/a^5)^(2/3)) - 12*e*x^3 + (
sqrt(-3)*a*x^4 - a*x^4)*(-(a^5*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^
2*c*e^6)/a^9) + c^2*d^3 - 3*a*c*d*e^2)/a^5)^(1/3)*log(-(3*c^4*d^6*e + 5*a*
c^3*d^4*e^3 + a^2*c^2*d^2*e^5 - a^3*c*e^7)*x + 1/2*(6*a^4*c^2*d^3*e^2 - 2*
a^5*c*d*e^4 + 2*sqrt(-3)*(3*a^4*c^2*d^3*e^2 - a^5*c*d*e^4) + (a^8*c*d^2 -
a^9*e^2 + sqrt(-3)*(a^8*c*d^2 - a^9*e^2))*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d
^2*e^4 + a^2*c*e^6)/a^9))*(-(a^5*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 +
a^2*c*e^6)/a^9) + c^2*d^3 - 3*a*c*d*e^2)/a^5)^(2/3)) - (sqrt(-3)*a*x^4 + a
*x^4)*(-(a^5*sqrt(-(9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)/a^9) + c^
2*d^3 - 3*a*c*d*e^2)/a^5)^(1/3)*log(-(3*c^4*d^6*e + 5*a*c^3*d^4*e^3 + a...
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.74

$$\int \frac{d+ex^3}{x^5(a+cx^6)}\,dx$$
$$= \mathrm{RootSum}\left(46656t^6a^{10}+t^3\left(-1296a^6cde^2+432a^5c^2d^3\right)+a^3ce^6+3a^2c^2d^2e^4+3ac^3d^4e^2+c^4d^6,\left(t\mapsto t\,\mathrm{l}\right.\right.$$
$$+\frac{-d-4ex^3}{4ax^4}$$

input
```
integrate((e*x**3+d)/x**5/(c*x**6+a),x)
```

output
```
RootSum(46656*_t**6*a**10 + _t**3*(-1296*a**6*c*d*e**2 + 432*a**5*c**2*d**
3) + a**3*c*e**6 + 3*a**2*c**2*d**2*e**4 + 3*a*c**3*d**4*e**2 + c**4*d**6,
 Lambda(_t, _t*log(x + (-7776*_t**5*a**9*e**2 + 7776*_t**5*a**8*c*d**2 + 1
80*_t**2*a**5*c*d*e**4 - 360*_t**2*a**4*c**2*d**3*e**2 + 36*_t**2*a**3*c**
3*d**5)/(a**3*c*e**7 - a**2*c**2*d**2*e**5 - 5*a*c**3*d**4*e**3 - 3*c**4*d
**6*e)))) + (-d - 4*e*x**3)/(4*a*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.02

$$\int \frac{d+ex^3}{x^5(a+cx^6)}\,dx =$$
$$-\frac{c\left(\frac{2\,d\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}c^{\frac{1}{3}}}+\frac{4\,e\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{c^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}-\frac{\left(\sqrt{3}\sqrt{a}c^{\frac{1}{6}}e+c^{\frac{2}{3}}d\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}c}+\frac{\left(\sqrt{3}\sqrt{a}c^{\frac{1}{6}}e-c^{\frac{2}{3}}d\right)\log\left(c^{\frac{1}{3}}x^2-\sqrt{\right.}{a^{\frac{2}{3}}c}}{12\,a}$$
$$-\frac{4\,ex^3+d}{4\,ax^4}$$

input
```
integrate((e*x^3+d)/x^5/(c*x^6+a),x, algorithm="maxima")
```

output
```
-1/12*c*(2*d*log(c^(1/3)*x^2 + a^(1/3))/(a^(2/3)*c^(1/3)) + 4*e*arctan(c^(
1/3)*x/sqrt(a^(1/3)*c^(1/3)))/(c^(2/3)*sqrt(a^(1/3)*c^(1/3))) - (sqrt(3)*s
qrt(a)*c^(1/6)*e + c^(2/3)*d)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x
+ a^(1/3))/(a^(2/3)*c) + (sqrt(3)*sqrt(a)*c^(1/6)*e - c^(2/3)*d)*log(c^(1/
3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a^(2/3)*c) - 2*(sqrt(3)*a^(
1/6)*c^(5/6)*d - a^(2/3)*c^(1/3)*e)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6)*
c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c*sqrt(a^(1/3)*c^(1/3))) + 2*(sqr
t(3)*a^(1/6)*c^(5/6)*d + a^(2/3)*c^(1/3)*e)*arctan((2*c^(1/3)*x - sqrt(3)*
a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c*sqrt(a^(1/3)*c^(1/3))))
/a - 1/4*(4*e*x^3 + d)/(a*x^4)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.01

$$\int \frac{d+ex^3}{x^5(a+cx^6)}\,dx = -\frac{ce\left(\frac{a}{c}\right)^{\frac{5}{6}}\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,a^2} - \frac{(ac^5)^{\frac{1}{3}}\,d\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,a^2c} - \frac{4\,ex^3+d}{4\,ax^4} + \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{3}}\,c^3d-(ac^5)^{\frac{5}{6}}\,e\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,a^2c^4} - \frac{\left(\sqrt{3}(ac^5)^{\frac{1}{3}}\,c^3d+(ac^5)^{\frac{5}{6}}\,e\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,a^2c^4} + \frac{\left((ac^5)^{\frac{1}{3}}\,c^3d+\sqrt{3}(ac^5)^{\frac{5}{6}}\,e\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,a^2c^4} + \frac{\left((ac^5)^{\frac{1}{3}}\,c^3d-\sqrt{3}(ac^5)^{\frac{5}{6}}\,e\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,a^2c^4}$$

input
```
integrate((e*x^3+d)/x^5/(c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*c*e*(a/c)^(5/6)*arctan(x/(a/c)^(1/6))/a^2 - 1/6*(a*c^5)^(1/3)*d*log(x
^2 + (a/c)^(1/3))/(a^2*c) - 1/4*(4*e*x^3 + d)/(a*x^4) + 1/6*(sqrt(3)*(a*c^
5)^(1/3)*c^3*d - (a*c^5)^(5/6)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)
^(1/6))/(a^2*c^4) - 1/6*(sqrt(3)*(a*c^5)^(1/3)*c^3*d + (a*c^5)^(5/6)*e)*ar
ctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^2*c^4) + 1/12*((a*c^5)^(1
/3)*c^3*d + sqrt(3)*(a*c^5)^(5/6)*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/
c)^(1/3))/(a^2*c^4) + 1/12*((a*c^5)^(1/3)*c^3*d - sqrt(3)*(a*c^5)^(5/6)*e)
*log(x^2 - sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 22.40 (sec) , antiderivative size = 1274, normalized size of antiderivative = 4.26

$$\int \frac{d+ex^3}{x^5(a+cx^6)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^3)/(x^5*(a + c*x^6)),x)
```

output
```
log(a^9*(-(a*e^3*(-a^11*c)^(1/2) + a^5*c^2*d^3 - 3*a^6*c*d*e^2 - 3*c*d^2*e
*(-a^11*c)^(1/2))/a^10)^(2/3) - a*e^2*x*(-a^11*c)^(1/2) + c*d^2*x*(-a^11*c
)^(1/2) + 2*a^6*c*d*e*x)*(-(a*e^3*(-a^11*c)^(1/2) + a^5*c^2*d^3 - 3*a^6*c*
d*e^2 - 3*c*d^2*e*(-a^11*c)^(1/2))/(216*a^10))^(1/3) - (d/(4*a) + (e*x^3)/
a)/x^4 + log(a^9*((a*e^3*(-a^11*c)^(1/2) - a^5*c^2*d^3 + 3*a^6*c*d*e^2 - 3
*c*d^2*e*(-a^11*c)^(1/2))/a^10)^(2/3) + a*e^2*x*(-a^11*c)^(1/2) - c*d^2*x*
(-a^11*c)^(1/2) + 2*a^6*c*d*e*x)*((a*e^3*(-a^11*c)^(1/2) - a^5*c^2*d^3 + 3
*a^6*c*d*e^2 - 3*c*d^2*e*(-a^11*c)^(1/2))/(216*a^10))^(1/3) + log(a^9*(-(a
*e^3*(-a^11*c)^(1/2) + a^5*c^2*d^3 - 3*a^6*c*d*e^2 - 3*c*d^2*e*(-a^11*c)^(
1/2))/a^10)^(2/3) + 3^(1/2)*a^9*(-(a*e^3*(-a^11*c)^(1/2) + a^5*c^2*d^3 - 3
*a^6*c*d*e^2 - 3*c*d^2*e*(-a^11*c)^(1/2))/a^10)^(2/3)*1i + 2*a*e^2*x*(-a^1
1*c)^(1/2) - 2*c*d^2*x*(-a^11*c)^(1/2) - 4*a^6*c*d*e*x)*((3^(1/2)*1i)/2 -
1/2)*(-(a*e^3*(-a^11*c)^(1/2) + a^5*c^2*d^3 - 3*a^6*c*d*e^2 - 3*c*d^2*e*(-
a^11*c)^(1/2))/(216*a^10))^(1/3) + log(a^9*((a*e^3*(-a^11*c)^(1/2) - a^5*c
^2*d^3 + 3*a^6*c*d*e^2 - 3*c*d^2*e*(-a^11*c)^(1/2))/a^10)^(2/3) + 3^(1/2)*
a^9*((a*e^3*(-a^11*c)^(1/2) - a^5*c^2*d^3 + 3*a^6*c*d*e^2 - 3*c*d^2*e*(-a^
11*c)^(1/2))/a^10)^(2/3)*1i - 2*a*e^2*x*(-a^11*c)^(1/2) + 2*c*d^2*x*(-a^11
*c)^(1/2) - 4*a^6*c*d*e*x)*((3^(1/2)*1i)/2 - 1/2)*((a*e^3*(-a^11*c)^(1/2)
- a^5*c^2*d^3 + 3*a^6*c*d*e^2 - 3*c*d^2*e*(-a^11*c)^(1/2))/(216*a^10))^(1/
3) - log(3^(1/2)*a^9*(-(a*e^3*(-a^11*c)^(1/2) + a^5*c^2*d^3 - 3*a^6*c*d...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.10

$$\int \frac{d+ex^3}{x^5(a+cx^6)}\,dx$$
$$= \frac{2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e\,x^4+2\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)cd\,x^4-2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}}\right)e\,x^4+2}{}$$

input
```
int((e*x^3+d)/x^5/(c*x^6+a),x)
```

output
```
(2*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/
6)*a**(1/6)))*e*x**4 + 2*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1
/3)*x)/(c**(1/6)*a**(1/6)))*c*d*x**4 - 2*sqrt(c)*sqrt(a)*atan((c**(1/6)*a*
*(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*e*x**4 + 2*sqrt(3)*ata
n((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d*x**4
 - 4*sqrt(c)*sqrt(a)*atan((c**(1/3)*x)/(c**(1/6)*a**(1/6)))*e*x**4 - sqrt(
c)*sqrt(a)*sqrt(3)*log( - c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3
)*x**2)*e*x**4 + sqrt(c)*sqrt(a)*sqrt(3)*log(c**(1/6)*a**(1/6)*sqrt(3)*x +
 a**(1/3) + c**(1/3)*x**2)*e*x**4 - 3*c**(1/3)*a**(2/3)*d - 12*c**(1/3)*a*
*(2/3)*e*x**3 - 2*log(a**(1/3) + c**(1/3)*x**2)*c*d*x**4 + log( - c**(1/6)
*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c*d*x**4 + log(c**(1/6)*a*
*(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c*d*x**4)/(12*c**(1/3)*a**(2/
3)*a*x**4)
```

3.27 $\int \frac{x(27-2x^3)}{729-64x^6} dx$

Optimal result

Integrand size = 18, antiderivative size = 99

$$\int \frac{x(27-2x^3)}{729-64x^6} dx = -\frac{5\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}} - \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{32\sqrt{3}} - \frac{1}{96}\log(3-2x) - \frac{5}{288}\log(3+2x) + \frac{5}{576}\log\left(9-6x+4x^2\right) + \frac{1}{192}\log\left(9+6x+4x^2\right)$$

output
```
-5/288*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)-1/96*arctan(1/9*(3+4*x)*3^(1/2)
)*3^(1/2)-1/96*ln(3-2*x)-5/288*ln(3+2*x)+5/576*ln(4*x^2-6*x+9)+1/192*ln(4*
x^2+6*x+9)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \frac{x(27-2x^3)}{729-64x^6} dx = \frac{1}{576}\left(10\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) - 6\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right) - 6\log(3-2x) - 10\log(3+2x) + 5\log\left(9-6x+4x^2\right) + 3\log\left(9+6x+4x^2\right)\right)$$

input
```
Integrate[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
```

output
```
(10*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*Sqrt[3]*ArcTan[(3 + 4*x)/(3
*Sqrt[3])] - 6*Log[3 - 2*x] - 10*Log[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3
*Log[9 + 6*x + 4*x^2])/576
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1835, 27, 821, 16, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx$$

$$\Big\downarrow \text{1835}$$

$$3\int \frac{x}{8(27-8x^3)}dx+5\int \frac{x}{8(8x^3+27)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{8}\int \frac{x}{27-8x^3}dx+\frac{5}{8}\int \frac{x}{8x^3+27}dx$$

$$\Big\downarrow \text{821}$$

$$\frac{5}{8}\left(\frac{1}{18}\int \frac{2x+3}{4x^2-6x+9}dx-\frac{1}{18}\int \frac{1}{2x+3}dx\right)+\frac{3}{8}\left(\frac{1}{18}\int \frac{1}{3-2x}dx-\frac{1}{18}\int \frac{3-2x}{4x^2+6x+9}dx\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{5}{8}\left(\frac{1}{18}\int \frac{2x+3}{4x^2-6x+9}dx-\frac{1}{36}\log(2x+3)\right)+\frac{3}{8}\left(-\frac{1}{18}\int \frac{3-2x}{4x^2+6x+9}dx-\frac{1}{36}\log(3-2x)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{5}{8}\left(\frac{1}{18}\left(\frac{9}{2}\int \frac{1}{4x^2-6x+9}dx+\frac{1}{4}\int -\frac{2(3-4x)}{4x^2-6x+9}dx\right)-\frac{1}{36}\log(2x+3)\right)+\frac{3}{8}\left(\frac{1}{18}\left(\frac{1}{4}\int \frac{2(4x+3)}{4x^2+6x+9}dx-\frac{9}{2}\int \frac{1}{4x^2+6x+9}dx\right)-\frac{1}{36}\log(3-2x)\right)$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{5}{8}\left(\frac{1}{18}\left(\frac{9}{2}\int \frac{1}{4x^2-6x+9}dx-\frac{1}{2}\int \frac{3-4x}{4x^2-6x+9}dx\right)-\frac{1}{36}\log(2x+3)\right)+\frac{3}{8}\left(\frac{1}{18}\left(\frac{1}{2}\int \frac{4x+3}{4x^2+6x+9}dx-\frac{9}{2}\int \frac{1}{4x^2+6x+9}dx\right)-\frac{1}{36}\log(3-2x)\right)$$

$$\Big\downarrow \textcolor{blue}{1083}$$

$$\frac{5}{8}\left(\frac{1}{18}\left(-\frac{1}{2}\int \frac{3-4x}{4x^2-6x+9}dx-9\int \frac{1}{-(8x-6)^2-108}d(8x-6)\right)-\frac{1}{36}\log(2x+3)\right)+\frac{3}{8}\left(\frac{1}{18}\left(\frac{1}{2}\int \frac{4x+3}{4x^2+6x+9}dx+9\int \frac{1}{-(8x+6)^2-108}d(8x+6)\right)-\frac{1}{36}\log(3-2x)\right)$$

$$\Big\downarrow \textcolor{blue}{217}$$

$$\frac{5}{8}\left(\frac{1}{18}\left(\frac{1}{2}\sqrt{3}\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)-\frac{1}{2}\int \frac{3-4x}{4x^2-6x+9}dx\right)-\frac{1}{36}\log(2x+3)\right)+\frac{3}{8}\left(\frac{1}{18}\left(\frac{1}{2}\int \frac{4x+3}{4x^2+6x+9}dx-\frac{1}{2}\sqrt{3}\arctan\left(\frac{8x+6}{6\sqrt{3}}\right)\right)-\frac{1}{36}\log(3-2x)\right)$$

$$\Big\downarrow \textcolor{blue}{1103}$$

$$\frac{5}{8}\left(\frac{1}{18}\left(\frac{1}{2}\sqrt{3}\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)+\frac{1}{4}\log\left(4x^2-6x+9\right)\right)-\frac{1}{36}\log(2x+3)\right)+\frac{3}{8}\left(\frac{1}{18}\left(\frac{1}{4}\log\left(4x^2+6x+9\right)-\frac{1}{2}\sqrt{3}\arctan\left(\frac{8x+6}{6\sqrt{3}}\right)\right)-\frac{1}{36}\log(3-2x)\right)$$

input
```
Int[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
```

output
```
(5*(-1/36*Log[3 + 2*x] + ((Sqrt[3]*ArcTan[(-6 + 8*x)/(6*Sqrt[3])])/2 + Log
[9 - 6*x + 4*x^2]/4)/18))/8 + (3*(-1/36*Log[3 - 2*x] + (-1/2*(Sqrt[3]*ArcT
an[(6 + 8*x)/(6*Sqrt[3])]) + Log[9 + 6*x + 4*x^2]/4)/18))/8
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1835
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[-(e/2 + c*(d/(2*q)))   Int[(f
*x)^m/(q - c*x^n), x], x] + Simp[(e/2 - c*(d/(2*q)))   Int[(f*x)^m/(q + c*x
^n), x], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.77

method	result
default	$-\frac{5\ln(2x+3)}{288}+\frac{5\ln(4x^2-6x+9)}{576}+\frac{5\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{288}-\frac{\ln(2x-3)}{96}+\frac{\ln(4x^2+6x+9)}{192}-\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{96}$
risch	$-\frac{5\ln(2x+3)}{288}+\frac{\ln(4x^2+6x+9)}{192}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{96}-\frac{\ln(2x-3)}{96}+\frac{5\ln(4x^2-6x+9)}{576}+\frac{5\sqrt{3}\,\arctan\left(\frac{2\left(2x-\frac{3}{2}\right)\sqrt{3}}{9}\right.}{288}$
meijerg	$\frac{x^5\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}+\sqrt{3}\,\mathrm{a}\right.}{288(x^6)^{\frac{5}{6}}}$

input
```
int(x*(-2*x^3+27)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
-5/288*ln(2*x+3)+5/576*ln(4*x^2-6*x+9)+5/288*3^(1/2)*arctan(1/18*(8*x-6)*3
^(1/2))-1/96*ln(2*x-3)+1/192*ln(4*x^2+6*x+9)-1/96*3^(1/2)*arctan(1/18*(8*x
+6)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{1}{96}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{5}{288}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) + \frac{1}{192}\,\log\left(4\,x^2+6\,x+9\right) + \frac{5}{576}\,\log\left(4\,x^2-6\,x+9\right) - \frac{5}{288}\,\log\left(2\,x+3\right) - \frac{1}{96}\,\log\left(2\,x-3\right)$$

input
```
integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
-1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqr
t(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9)
- 5/288*log(2*x + 3) - 1/96*log(2*x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{96} - \frac{5\log\left(x+\frac{3}{2}\right)}{288} + \frac{5\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{576} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{192} + \frac{5\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{288} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{96}$$

input
```
integrate(x*(-2*x**3+27)/(-64*x**6+729),x)
```

output
```
-log(x - 3/2)/96 - 5*log(x + 3/2)/288 + 5*log(x**2 - 3*x/2 + 9/4)/576 + lo
g(x**2 + 3*x/2 + 9/4)/192 + 5*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/288
- sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/96
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{192}\log\left(4x^2+6x+9\right) + \frac{5}{576}\log\left(4x^2-6x+9\right) - \frac{5}{288}\log\left(2x+3\right) - \frac{1}{96}\log\left(2x-3\right)$$

input
```
integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
-1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqr
t(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9)
- 5/288*log(2*x + 3) - 1/96*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right) + \frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right) + \frac{1}{192}\log\left(x^2+\frac{3}{2}\,x+\frac{9}{4}\right) + \frac{5}{576}\log\left(x^2-\frac{3}{2}\,x+\frac{9}{4}\right) - \frac{5}{288}\log\left(\left|x+\frac{3}{2}\right|\right) - \frac{1}{96}\log\left(\left|x-\frac{3}{2}\right|\right)$$

input
```
integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="giac")
```

output
```
-1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqr
t(3)*(4*x - 3)) + 1/192*log(x^2 + 3/2*x + 9/4) + 5/576*log(x^2 - 3/2*x + 9
/4) - 5/288*log(abs(x + 3/2)) - 1/96*log(abs(x - 3/2))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{\ln\left(x-\frac{3}{2}\right)}{96} - \frac{5\,\ln\left(x+\frac{3}{2}\right)}{288} + \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{192}+\frac{\sqrt{3}\,1\mathrm{i}}{192}\right) - \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{192}+\frac{\sqrt{3}\,1\mathrm{i}}{192}\right) - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{5}{576}+\frac{\sqrt{3}\,5\mathrm{i}}{576}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{5}{576}+\frac{\sqrt{3}\,5\mathrm{i}}{576}\right)$$

input
```
int((x*(2*x^3 - 27))/(64*x^6 - 729),x)
```

output
```
log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/192 + 1/192) - (5*log(x + 3/2)
)/288 - log(x - 3/2)/96 - log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/192
- 1/192) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*5i)/576 - 5/576) + log(
x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*5i)/576 + 5/576)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = \frac{5\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)}{288} - \frac{\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)}{96} + \frac{5\log(4x^2-6x+9)}{576} + \frac{\log(4x^2+6x+9)}{192} - \frac{\log(2x-3)}{96} - \frac{5\log(2x+3)}{288}$$

input
```
int(x*(-2*x^3+27)/(-64*x^6+729),x)
```

output
```
(10*sqrt(3)*atan((4*x - 3)/(3*sqrt(3))) - 6*sqrt(3)*atan((4*x + 3)/(3*sqrt
(3))) + 5*log(4*x**2 - 6*x + 9) + 3*log(4*x**2 + 6*x + 9) - 6*log(2*x - 3)
 - 10*log(2*x + 3))/576
```

3.28 $\int \frac{1}{x^2 \left(d+ex^3\right) \left(a+cx^6\right)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 533

$$\int \frac{1}{x^2(d+ex^3)(a+cx^6)}\,dx = -\frac{1}{adx}-\frac{c^{7/6}d\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{7/6}(cd^2+ae^2)}+\frac{c^{7/6}d\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{7/6}(cd^2+ae^2)}-\frac{c^{7/6}d\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{7/6}(cd^2+ae^2)}+\frac{e^{7/3}\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{4/3}(cd^2+ae^2)}+\frac{c^{2/3}e\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{c}x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{2/3}(cd^2+ae^2)}+\frac{c^{7/6}d\,\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{2\sqrt{3}a^{7/6}(cd^2+ae^2)}+\frac{e^{7/3}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{4/3}(cd^2+ae^2)}-\frac{c^{2/3}e\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6a^{2/3}(cd^2+ae^2)}-\frac{e^{7/3}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{4/3}(cd^2+ae^2)}+\frac{c^{2/3}e\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{c}x^2+c^{2/3}x^4\right)}{12a^{2/3}(cd^2+ae^2)}$$

output
```
-1/a/d/x-1/3*c^(7/6)*d*arctan(c^(1/6)*x/a^(1/6))/a^(7/6)/(a*e^2+c*d^2)-1/6
*c^(7/6)*d*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(7/6)/(a*e^2+c*d^2)-1/6*
c^(7/6)*d*arctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(7/6)/(a*e^2+c*d^2)+1/3*e^
(7/3)*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)*3^(1/2)/d^(1/3))*3^(1/2)/d^(4/3)/(a
*e^2+c*d^2)+1/6*c^(2/3)*e*arctan(1/3*(a^(1/3)-2*c^(1/3)*x^2)*3^(1/2)/a^(1/
3))*3^(1/2)/a^(2/3)/(a*e^2+c*d^2)+1/6*c^(7/6)*d*arctanh(3^(1/2)*a^(1/6)*c^
(1/6)*x/(a^(1/3)+c^(1/3)*x^2))*3^(1/2)/a^(7/6)/(a*e^2+c*d^2)+1/3*e^(7/3)*l
n(d^(1/3)+e^(1/3)*x)/d^(4/3)/(a*e^2+c*d^2)-1/6*c^(2/3)*e*ln(a^(1/3)+c^(1/3
)*x^2)/a^(2/3)/(a*e^2+c*d^2)-1/6*e^(7/3)*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2
/3)*x^2)/d^(4/3)/(a*e^2+c*d^2)+1/12*c^(2/3)*e*ln(a^(2/3)-a^(1/3)*c^(1/3)*x
^2+c^(2/3)*x^4)/a^(2/3)/(a*e^2+c*d^2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^2 (d+ex^3)(a+cx^6)}\,dx$$

$$= \frac{-\frac{12cd}{a} - \frac{12e^2}{d} - \frac{4c^{7/6}dx\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{7/6}} + \frac{2c^{2/3}\left(\sqrt{c}d+\sqrt{3}\sqrt{a}e\right)x\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{7/6}} - \frac{2c^{7/6}dx\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{7/6}} + \frac{2\sqrt{3}c}{}}{}$$

input
```
Integrate[1/(x^2*(d + e*x^3)*(a + c*x^6)),x]
```

output
```
((-12*c*d)/a - (12*e^2)/d - (4*c^(7/6)*d*x*ArcTan[(c^(1/6)*x)/a^(1/6)])/a^
(7/6) + (2*c^(2/3)*(Sqrt[c]*d + Sqrt[3]*Sqrt[a]*e)*x*ArcTan[Sqrt[3] - (2*c
^(1/6)*x)/a^(1/6)])/a^(7/6) - (2*c^(7/6)*d*x*ArcTan[Sqrt[3] + (2*c^(1/6)*x
)/a^(1/6)])/a^(7/6) + (2*Sqrt[3]*c^(2/3)*e*x*ArcTan[Sqrt[3] + (2*c^(1/6)*x
)/a^(1/6)])/a^(2/3) + (4*Sqrt[3]*e^(7/3)*x*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/
3))/Sqrt[3]])/d^(4/3) + (4*e^(7/3)*x*Log[d^(1/3) + e^(1/3)*x])/d^(4/3) - (
2*c^(2/3)*e*x*Log[a^(1/3) + c^(1/3)*x^2])/a^(2/3) - (Sqrt[3]*c^(7/6)*d*x*L
og[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/a^(7/6) + (c^(2/3)*
e*x*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/a^(2/3) + (Sqr
t[3]*c^(7/6)*d*x*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/a
^(7/6) + (c^(2/3)*e*x*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^
2])/a^(2/3) - (2*e^(7/3)*x*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])
/d^(4/3))/(12*(c*d^2 + a*e^2)*x)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1837, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+cx^6)(d+ex^3)}\,dx$$

$\Big\downarrow$ 1837

$$\int \left(-\frac{cx(ae+cdx^3)}{a(a+cx^6)(ae^2+cd^2)}-\frac{e^3x}{d(d+ex^3)(ae^2+cd^2)}+\frac{1}{adx^2}\right)dx$$

$\Big\downarrow$ 2009

$$-\frac{c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}}{\sqrt{3}}\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{2\sqrt{3}(-a)^{7/6}(ae^2+cd^2)}+\frac{c^{2/3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1}{\sqrt{3}}\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}{2\sqrt{3}(-a)^{7/6}(ae^2+cd^2)}+\frac{e^{7/3}\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{4/3}(ae^2+cd^2)}+\frac{c^{2/3}\left(\sqrt{-a}e+\sqrt{c}d\right)\log\left(-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}+\sqrt[3]{c}x^2\right)}{12(-a)^{7/6}(ae^2+cd^2)}-\frac{c^{2/3}\left(\sqrt{c}d-\sqrt{-a}e\right)\log\left(\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}+\sqrt[3]{c}x^2\right)}{12(-a)^{7/6}(ae^2+cd^2)}+\frac{c^{2/3}\left(\sqrt{c}d-\sqrt{-a}e\right)\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{6(-a)^{7/6}(ae^2+cd^2)}-\frac{c^{2/3}\left(\sqrt{-a}e+\sqrt{c}d\right)\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{6(-a)^{7/6}(ae^2+cd^2)}-\frac{e^{7/3}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{4/3}(ae^2+cd^2)}+\frac{e^{7/3}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{4/3}(ae^2+cd^2)}-\frac{1}{adx}$$

input
```
Int[1/(x^2*(d + e*x^3)*(a + c*x^6)),x]
```

output
```
-(1/(a*d*x)) - (c^(2/3)*(Sqrt[c]*d + Sqrt[-a]*e)*ArcTan[(1 - (2*c^(1/6)*x)
/(-a)^(1/6))/Sqrt[3]])/(2*Sqrt[3]*(-a)^(7/6)*(c*d^2 + a*e^2)) + (c^(2/3)*(
Sqrt[c]*d - Sqrt[-a]*e)*ArcTan[(1 + (2*c^(1/6)*x)/(-a)^(1/6))/Sqrt[3]])/(2
*Sqrt[3]*(-a)^(7/6)*(c*d^2 + a*e^2)) + (e^(7/3)*ArcTan[(d^(1/3) - 2*e^(1/3
)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(4/3)*(c*d^2 + a*e^2)) + (c^(2/3)*(Sqr
t[c]*d - Sqrt[-a]*e)*Log[(-a)^(1/6) - c^(1/6)*x])/(6*(-a)^(7/6)*(c*d^2 + a
*e^2)) - (c^(2/3)*(Sqrt[c]*d + Sqrt[-a]*e)*Log[(-a)^(1/6) + c^(1/6)*x])/(6
*(-a)^(7/6)*(c*d^2 + a*e^2)) + (e^(7/3)*Log[d^(1/3) + e^(1/3)*x])/(3*d^(4/
3)*(c*d^2 + a*e^2)) + (c^(2/3)*(Sqrt[c]*d + Sqrt[-a]*e)*Log[(-a)^(1/3) - (
-a)^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*(-a)^(7/6)*(c*d^2 + a*e^2)) - (c^(
2/3)*(Sqrt[c]*d - Sqrt[-a]*e)*Log[(-a)^(1/3) + (-a)^(1/6)*c^(1/6)*x + c^(1
/3)*x^2])/(12*(-a)^(7/6)*(c*d^2 + a*e^2)) - (e^(7/3)*Log[d^(2/3) - d^(1/3)
*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(4/3)*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 1837
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^n)^q/(a + c*x^(
2*n))), x], x] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0
] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.91

method	result
default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)e^{3}}{d\left(a\,e^{2}+c\,d^{2}\right)}-\frac{1}{adx}-\frac{\left(\frac{c\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)\sqrt{3}\left(\frac{a}{c}\right.}{12a}\right.}{}$
risch	$-\frac{1}{adx}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{3}d^{4}e^{6}+3a^{2}c\,d^{6}e^{4}+3a\,c^{2}d^{8}e^{2}+c^{3}d^{10}\right)\_Z^{3}-e^{7}\right)}\_R\ln\left(\left(\left(-256e^{12}d^{4}a^{13}-1376e^{10}d^{6}c\,a^{12}-3168e^{8}d^{8}c^{2}a\right.\right.\right.\right.}{}$

input
```
int(1/x^2/(e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3
)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/
3)*x-1)))*e^3/d/(a*e^2+c*d^2)-1/a/d/x-(1/12*c/a*ln(x^2-3^(1/2)*(a/c)^(1/6)
*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*d-1/12*c/a*ln(x^2-3^(1/2)*(a/c)^(1/6)*
x+(a/c)^(1/3))*(a/c)^(4/3)*e+1/6/(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)-3^(1/2
))*d-1/6*c/a*(a/c)^(4/3)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*3^(1/2)*e+1/3*(a/
c)^(1/3)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*3^(1/2)*e-1/12*c/a*ln(x^2+3^(1/2)
*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*d-1/12*ln(x^2+3^(1/2)*(a/c
)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*e+1/6/(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6
)+3^(1/2))*d-1/6*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*3^(1/2)*e+1/6
*(a/c)^(1/3)*e*ln(x^2+(a/c)^(1/3))+1/3*d/(a/c)^(1/6)*arctan(x/(a/c)^(1/6))
)*c/a/(a*e^2+c*d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4880 vs. $2(412) = 824$.

Time = 10.76 (sec) , antiderivative size = 4880, normalized size of antiderivative = 9.16

$$\int \frac{1}{x^2 (d + ex^3) (a + cx^6)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d + ex^3) (a + cx^6)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*x**3+d)/(c*x**6+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (d + ex^3)(a + cx^6)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2 (d + ex^3)(a + cx^6)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

output
```
1/3*e^3*(-d/e)^(2/3)*log(abs(x - (-d/e)^(1/3)))/(c*d^4 + a*d^2*e^2) - 1/3*
(a*c^5)^(5/6)*d*arctan(x/(a/c)^(1/6))/(a^2*c^4*d^2 + a^3*c^3*e^2) + (-d*e^
2)^(2/3)*e*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/(sqrt(3)*
c*d^4 + sqrt(3)*a*d^2*e^2) - 1/6*(-d*e^2)^(2/3)*e*log(x^2 + x*(-d/e)^(1/3)
 + (-d/e)^(2/3))/(c*d^4 + a*d^2*e^2) - 1/6*(a*c^5)^(1/3)*e*log(x^2 + (a/c)
^(1/3))/(a*c^2*d^2 + a^2*c*e^2) + 1/6*(sqrt(3)*(a*c^5)^(1/3)*a*c^2*e - (a*
c^5)^(5/6)*d)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^2*c^4*d^2
 + a^3*c^3*e^2) - 1/6*(sqrt(3)*(a*c^5)^(1/3)*a*c^2*e + (a*c^5)^(5/6)*d)*ar
ctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^2*c^4*d^2 + a^3*c^3*e^2)
+ 1/12*((a*c^5)^(1/3)*a*c^2*e + sqrt(3)*(a*c^5)^(5/6)*d)*log(x^2 + sqrt(3)
*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^2*c^4*d^2 + a^3*c^3*e^2) + 1/12*((a*c^5)^
(1/3)*a*c^2*e - sqrt(3)*(a*c^5)^(5/6)*d)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) +
 (a/c)^(1/3))/(a^2*c^4*d^2 + a^3*c^3*e^2) - 1/(a*d*x)
```

Mupad [B] (verification not implemented)

Time = 61.33 (sec) , antiderivative size = 5967, normalized size of antiderivative = 11.20

$$\int \frac{1}{x^2\left(d+ex^3\right)\left(a+cx^6\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(x^2*(a + c*x^6)*(d + e*x^3)),x)
```

output
```
log(((((-(a^5*c^2*e^3 + c*d^3*(-a^7*c^5)^(1/2) - 3*a^4*c^3*d^2*e - 3*a*d*e
^2*(-a^7*c^5)^(1/2))/(a^7*(a*e^2 + c*d^2)^3))^(2/3)*(((5832*a^16*c^6*d^13*
e^3*x*(a*e^2 + c*d^2)^2*(8*a^4*e^8 + c^4*d^8 - 6*a*c^3*d^6*e^2 + a^2*c^2*d
^4*e^4) + 11664*a^19*c^6*d^16*e^4*(a*e^2 + c*d^2)^4*(a*e^2 - c*d^2)*(-(a^5
*c^2*e^3 + c*d^3*(-a^7*c^5)^(1/2) - 3*a^4*c^3*d^2*e - 3*a*d*e^2*(-a^7*c^5)
^(1/2))/(a^7*(a*e^2 + c*d^2)^3))^(2/3))*(-(a^5*c^2*e^3 + c*d^3*(-a^7*c^5)^
(1/2) - 3*a^4*c^3*d^2*e - 3*a*d*e^2*(-a^7*c^5)^(1/2))/(a^7*(a*e^2 + c*d^2)
^3))^(1/3))/6 + 972*a^15*c^12*d^24*e^3 + 2916*a^16*c^11*d^22*e^5 - 8748*a^
17*c^10*d^20*e^7 - 6804*a^18*c^9*d^18*e^9 + 19440*a^19*c^8*d^16*e^11 + 155
52*a^20*c^7*d^14*e^13))/36 + 27*a^13*c^8*d^13*e^4*x*(8*a^4*e^8 + c^4*d^8 +
 2*a*c^3*d^6*e^2 - 8*a^3*c*d^2*e^6 + a^2*c^2*d^4*e^4))*(-(a^5*c^2*e^3 + c*
d^3*(-a^7*c^5)^(1/2) - 3*a^4*c^3*d^2*e - 3*a*d*e^2*(-a^7*c^5)^(1/2))/(a^7*
(a*e^2 + c*d^2)^3))^(1/3))/6 + 9*a^13*c^11*d^18*e^6 + 9*a^14*c^10*d^16*e^8
 - 36*a^15*c^9*d^14*e^10)*(-(a^5*c^2*e^3 + c*d^3*(-a^7*c^5)^(1/2) - 3*a^4*
c^3*d^2*e - 3*a*d*e^2*(-a^7*c^5)^(1/2))/(216*(a^10*e^6 + a^7*c^3*d^6 + 3*a
^9*c*d^2*e^4 + 3*a^8*c^2*d^4*e^2)))^(1/3) + log(((((-(a^5*c^2*e^3 - c*d^3*
(-a^7*c^5)^(1/2) - 3*a^4*c^3*d^2*e + 3*a*d*e^2*(-a^7*c^5)^(1/2))/(a^7*(a*e
^2 + c*d^2)^3))^(2/3)*(((5832*a^16*c^6*d^13*e^3*x*(a*e^2 + c*d^2)^2*(8*a^4
*e^8 + c^4*d^8 - 6*a*c^3*d^6*e^2 + a^2*c^2*d^4*e^4) + 11664*a^19*c^6*d^16*
e^4*(a*e^2 + c*d^2)^4*(a*e^2 - c*d^2)*(-(a^5*c^2*e^3 - c*d^3*(-a^7*c^5)...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2\left(d+ex^3\right)\left(a+cx^6\right)}\,dx=\text{Too large to display}$$

input
```
int(1/x^2/(e*x^3+d)/(c*x^6+a),x)
```

output

```
(4*e**(1/3)*c**(1/3)*a**(2/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(
1/3)*sqrt(3)))*a*e**2*x + 2*d**(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/
6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d**2*x + 2*d**(1/3)*sqrt
(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a
*c*d*e*x - 2*d**(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*
c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d**2*x + 2*d**(1/3)*sqrt(3)*atan((c**(1
/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c*d*e*x - 4*d*
*(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d**2*x - d
**(1/3)*sqrt(c)*sqrt(a)*sqrt(3)*log( - c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1
/3) + c**(1/3)*x**2)*c*d**2*x + d**(1/3)*sqrt(c)*sqrt(a)*sqrt(3)*log(c**(1
/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c*d**2*x - 12*d**(1/3)*
c**(1/3)*a**(2/3)*a*e**2 - 12*d**(1/3)*c**(1/3)*a**(2/3)*c*d**2 - 2*e**(1/
3)*c**(1/3)*a**(2/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*x + e**(2/3)*x**2)*a
*e**2*x + 4*e**(1/3)*c**(1/3)*a**(2/3)*log(d**(1/3) + e**(1/3)*x)*a*e**2*x
 - 2*d**(1/3)*log(a**(1/3) + c**(1/3)*x**2)*a*c*d*e*x + d**(1/3)*log( - c*
*(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*a*c*d*e*x + d**(1/3)
*log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*a*c*d*e*x)/(1
2*d**(1/3)*c**(1/3)*a**(2/3)*a*d*x*(a*e**2 + c*d**2))
```

3.29 $\int \frac{1}{x^5 \left(d+e x^3\right) \left(a+c x^6\right)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 547

$$\int \frac{1}{x^5(d+ex^3)(a+cx^6)}\,dx = -\frac{1}{4adx^4}+\frac{e}{ad^2x}+\frac{c^{7/6}e\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{7/6}(cd^2+ae^2)} - \frac{c^{7/6}e\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{7/6}(cd^2+ae^2)} + \frac{c^{7/6}e\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{7/6}(cd^2+ae^2)} - \frac{e^{10/3}\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{7/3}(cd^2+ae^2)} + \frac{c^{5/3}d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{c}x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{5/3}(cd^2+ae^2)} - \frac{c^{7/6}e\,\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{2\sqrt{3}a^{7/6}(cd^2+ae^2)} - \frac{e^{10/3}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{7/3}(cd^2+ae^2)} - \frac{c^{5/3}d\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6a^{5/3}(cd^2+ae^2)} + \frac{e^{10/3}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{7/3}(cd^2+ae^2)} + \frac{c^{5/3}d\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{c}x^2+c^{2/3}x^4\right)}{12a^{5/3}(cd^2+ae^2)}$$

output
```
-1/4/a/d/x^4+e/a/d^2/x+1/3*c^(7/6)*e*arctan(c^(1/6)*x/a^(1/6))/a^(7/6)/(a*
e^2+c*d^2)+1/6*c^(7/6)*e*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(7/6)/(a*e
^2+c*d^2)+1/6*c^(7/6)*e*arctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(7/6)/(a*e^2
+c*d^2)-1/3*e^(10/3)*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)*3^(1/2)/d^(1/3))*3^(
1/2)/d^(7/3)/(a*e^2+c*d^2)+1/6*c^(5/3)*d*arctan(1/3*(a^(1/3)-2*c^(1/3)*x^2
)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/(a*e^2+c*d^2)-1/6*c^(7/6)*e*arctanh(3^(
1/2)*a^(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)*x^2))*3^(1/2)/a^(7/6)/(a*e^2+c*d^2
)-1/3*e^(10/3)*ln(d^(1/3)+e^(1/3)*x)/d^(7/3)/(a*e^2+c*d^2)-1/6*c^(5/3)*d*l
n(a^(1/3)+c^(1/3)*x^2)/a^(5/3)/(a*e^2+c*d^2)+1/6*e^(10/3)*ln(d^(2/3)-d^(1/
3)*e^(1/3)*x+e^(2/3)*x^2)/d^(7/3)/(a*e^2+c*d^2)+1/12*c^(5/3)*d*ln(a^(2/3)-
a^(1/3)*c^(1/3)*x^2+c^(2/3)*x^4)/a^(5/3)/(a*e^2+c*d^2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 517, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5 (d+ex^3) (a+cx^6)} \, dx$$

$$= \frac{-3ad^{4/3}(cd^2+ae^2)+12a\sqrt[3]{d}e(cd^2+ae^2)\,x^3+4a^{5/6}c^{7/6}d^{7/3}ex^4\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)+2\sqrt[3]{a}c^{7/6}d^{7/3}(\sqrt{3}\sqrt{c}d\cdots}{\cdots}$$

input
```
Integrate[1/(x^5*(d + e*x^3)*(a + c*x^6)),x]
```

output
```
(-3*a*d^(4/3)*(c*d^2 + a*e^2) + 12*a*d^(1/3)*e*(c*d^2 + a*e^2)*x^3 + 4*a^(
5/6)*c^(7/6)*d^(7/3)*e*x^4*ArcTan[(c^(1/6)*x)/a^(1/6)] + 2*a^(1/3)*c^(7/6)
*d^(7/3)*(Sqrt[3]*Sqrt[c]*d - Sqrt[a]*e)*x^4*ArcTan[Sqrt[3] - (2*c^(1/6)*x
)/a^(1/6)] + 2*a^(1/3)*c^(7/6)*d^(7/3)*(Sqrt[3]*Sqrt[c]*d + Sqrt[a]*e)*x^4
*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)] - 4*Sqrt[3]*a^2*e^(10/3)*x^4*ArcT
an[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 4*a^2*e^(10/3)*x^4*Log[d^(1/3) +
 e^(1/3)*x] - 2*a^(1/3)*c^(5/3)*d^(10/3)*x^4*Log[a^(1/3) + c^(1/3)*x^2] +
c^(7/6)*d^(7/3)*(a^(1/3)*Sqrt[c]*d + Sqrt[3]*a^(5/6)*e)*x^4*Log[a^(1/3) -
Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + c^(7/6)*d^(7/3)*(a^(1/3)*Sqrt[c
]*d - Sqrt[3]*a^(5/6)*e)*x^4*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(
1/3)*x^2] + 2*a^2*e^(10/3)*x^4*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x
^2])/(12*a^2*d^(7/3)*(c*d^2 + a*e^2)*x^4)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 613, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1837, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+cx^6) (d+ex^3)} \, dx$$

$$\Big\downarrow \text{1837}$$

$$\int \left(-\frac{c^2 x (d-ex^3)}{a (a+cx^6) (ae^2+cd^2)}+\frac{e^4 x}{d^2 (d+ex^3) (ae^2+cd^2)}-\frac{e}{ad^2x^2}+\frac{1}{adx^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^{7/6}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}}{\sqrt{3}}\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{2\sqrt{3}(-a)^{5/3}(ae^2+cd^2)}+\frac{c^{7/6}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1}{\sqrt{3}}\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}{2\sqrt{3}(-a)^{5/3}(ae^2+cd^2)}-\frac{e^{10/3}\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{7/3}(ae^2+cd^2)}-\frac{c^{7/6}\left(\sqrt{-a}e+\sqrt{c}d\right)\log\left(-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}+\sqrt[3]{c}x^2\right)}{12(-a)^{5/3}(ae^2+cd^2)}-\frac{c^{7/6}\left(\sqrt{c}d-\sqrt{-a}e\right)\log\left(\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}+\sqrt[3]{c}x^2\right)}{12(-a)^{5/3}(ae^2+cd^2)}+\frac{c^{7/6}\left(\sqrt{c}d-\sqrt{-a}e\right)\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{6(-a)^{5/3}(ae^2+cd^2)}+\frac{c^{7/6}\left(\sqrt{-a}e+\sqrt{c}d\right)\log\left(\sqrt[6]{-a}+\sqrt[6]{c}x\right)}{6(-a)^{5/3}(ae^2+cd^2)}+\frac{e^{10/3}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{7/3}(ae^2+cd^2)}-\frac{e^{10/3}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{7/3}(ae^2+cd^2)}+\frac{e}{ad^2x}-\frac{1}{4adx^4}$$

input
```
Int[1/(x^5*(d + e*x^3)*(a + c*x^6)),x]
```

output
```
-1/4*1/(a*d*x^4) + e/(a*d^2*x) + (c^(7/6)*(Sqrt[c]*d + Sqrt[-a]*e)*ArcTan[
(1 - (2*c^(1/6)*x)/(-a)^(1/6))/Sqrt[3]])/(2*Sqrt[3]*(-a)^(5/3)*(c*d^2 + a*
e^2)) + (c^(7/6)*(Sqrt[c]*d - Sqrt[-a]*e)*ArcTan[(1 + (2*c^(1/6)*x)/(-a)^(
1/6))/Sqrt[3]])/(2*Sqrt[3]*(-a)^(5/3)*(c*d^2 + a*e^2)) - (e^(10/3)*ArcTan[
(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(7/3)*(c*d^2 + a*e^
2)) + (c^(7/6)*(Sqrt[c]*d - Sqrt[-a]*e)*Log[(-a)^(1/6) - c^(1/6)*x])/(6*(-
a)^(5/3)*(c*d^2 + a*e^2)) + (c^(7/6)*(Sqrt[c]*d + Sqrt[-a]*e)*Log[(-a)^(1/
6) + c^(1/6)*x])/(6*(-a)^(5/3)*(c*d^2 + a*e^2)) - (e^(10/3)*Log[d^(1/3) +
e^(1/3)*x])/(3*d^(7/3)*(c*d^2 + a*e^2)) - (c^(7/6)*(Sqrt[c]*d + Sqrt[-a]*e
)*Log[(-a)^(1/3) - (-a)^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*(-a)^(5/3)*(c*
d^2 + a*e^2)) - (c^(7/6)*(Sqrt[c]*d - Sqrt[-a]*e)*Log[(-a)^(1/3) + (-a)^(1
/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*(-a)^(5/3)*(c*d^2 + a*e^2)) + (e^(10/3)*
Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(7/3)*(c*d^2 + a*e^2)
)
```

Defintions of rubi rules used

rule 1837
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^n)^q/(a + c*x^(
2*n))), x], x] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0
] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.88

method	result
default	$\frac{\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)e^{4}}{d^{2}\left(a\,e^{2}+c\,d^{2}\right)}-\frac{1}{4ad\,x^{4}}+\frac{e}{a\,d^{2}x}-\frac{\left(-\frac{\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right.\right.}{12a}\right.}{}$
risch	Expression too large to display

input
```
int(1/x^5/(e*x^3+d)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)
*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3
)*x-1)))*e^4/d^2/(a*e^2+c*d^2)-1/4/a/d/x^4+e/a/d^2/x-(-1/12/a*ln(x^2-3^(1/
2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*e-1/12/a*ln(x^2-3^(1/2)*
(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*d-1/6/c/(a/c)^(1/6)*arctan(2*x/(a/c
)^(1/6)-3^(1/2))*e+1/6/a*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*3^(1/
2)*d+1/12/a*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*
e-1/12/a*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*d-1/6/c/(a/
c)^(1/6)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*e-1/6/a*(a/c)^(1/3)*arctan(2*x/(a
/c)^(1/6)+3^(1/2))*3^(1/2)*d+1/6/a*(a/c)^(1/3)*d*ln(x^2+(a/c)^(1/3))-1/3/c
*e/(a/c)^(1/6)*arctan(x/(a/c)^(1/6)))*c^2/a/(a*e^2+c*d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4962 vs. 2(423) = 846.

Time = 39.30 (sec) , antiderivative size = 4962, normalized size of antiderivative = 9.07

$$\int \frac{1}{x^5\left(d+ex^3\right)\left(a+cx^6\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(e*x^3+d)/(c*x^6+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d + e x^3) (a + c x^6)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**5/(e*x**3+d)/(c*x**6+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 (d + e x^3) (a + c x^6)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(e*x^3+d)/(c*x^6+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5 (d + ex^3) (a + cx^6)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(e*x^3+d)/(c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*e^4*(-d/e)^(2/3)*log(abs(x - (-d/e)^(1/3)))/(c*d^5 + a*d^3*e^2) - (-d
*e^2)^(2/3)*e^2*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/(sqr
t(3)*c*d^5 + sqrt(3)*a*d^3*e^2) + 1/6*(-d*e^2)^(2/3)*e^2*log(x^2 + x*(-d/e
)^(1/3) + (-d/e)^(2/3))/(c*d^5 + a*d^3*e^2) + 1/3*(a*c^5)^(5/6)*e*arctan(x
/(a/c)^(1/6))/(a^2*c^4*d^2 + a^3*c^3*e^2) - 1/6*(a*c^5)^(1/3)*d*log(x^2 +
(a/c)^(1/3))/(a^2*c*d^2 + a^3*e^2) + 1/6*(sqrt(3)*(a*c^5)^(1/3)*c^3*d + (a
*c^5)^(5/6)*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^2*c^4*d^
2 + a^3*c^3*e^2) - 1/6*(sqrt(3)*(a*c^5)^(1/3)*c^3*d - (a*c^5)^(5/6)*e)*arc
tan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^2*c^4*d^2 + a^3*c^3*e^2) +
 1/12*((a*c^5)^(1/3)*c^3*d - sqrt(3)*(a*c^5)^(5/6)*e)*log(x^2 + sqrt(3)*x*
(a/c)^(1/6) + (a/c)^(1/3))/(a^2*c^4*d^2 + a^3*c^3*e^2) + 1/12*((a*c^5)^(1/
3)*c^3*d + sqrt(3)*(a*c^5)^(5/6)*e)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) + (a/c
)^(1/3))/(a^2*c^4*d^2 + a^3*c^3*e^2) + 1/4*(4*e*x^3 - d)/(a*d^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 63.49 (sec) , antiderivative size = 6015, normalized size of antiderivative = 11.00

$$\int \frac{1}{x^5 (d + ex^3) (a + cx^6)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^5*(a + c*x^6)*(d + e*x^3)),x)
```

output
```
log(9*a^10*c^14*d^33*e^6 - ((((-(a^5*c^5*d^3 + a*e^3*(-a^11*c^7)^(1/2) - 3
*a^6*c^4*d*e^2 - 3*c*d^2*e*(-a^11*c^7)^(1/2))/(a^10*(a*e^2 + c*d^2)^3))^(2
/3)*(4860*a^16*c^11*d^34*e^8 - 972*a^14*c^13*d^38*e^4 - 6804*a^15*c^12*d^3
6*e^6 - ((5832*a^15*c^6*d^26*e^3*x*(a*e^2 + c*d^2)^2*(8*a^5*e^10 - c^5*d^1
0 + 6*a*c^4*d^8*e^2 - a^2*c^3*d^6*e^4) + 11664*a^19*c^6*d^31*e^4*(a*e^2 +
c*d^2)^4*(a*e^2 - c*d^2)*(-(a^5*c^5*d^3 + a*e^3*(-a^11*c^7)^(1/2) - 3*a^6*
c^4*d*e^2 - 3*c*d^2*e*(-a^11*c^7)^(1/2))/(a^10*(a*e^2 + c*d^2)^3))^(2/3))*
(-(a^5*c^5*d^3 + a*e^3*(-a^11*c^7)^(1/2) - 3*a^6*c^4*d*e^2 - 3*c*d^2*e*(-a
^11*c^7)^(1/2))/(a^10*(a*e^2 + c*d^2)^3))^(1/3))/6 + 10692*a^17*c^10*d^32*
e^10 + 15552*a^19*c^8*d^28*e^14 + 15552*a^20*c^7*d^26*e^16))/36 + 27*a^10*
c^9*d^25*e^3*x*(8*a^6*e^12 + c^6*d^12 + 2*a*c^5*d^10*e^2 - 8*a^5*c*d^2*e^1
0 + a^2*c^4*d^8*e^4))*(-(a^5*c^5*d^3 + a*e^3*(-a^11*c^7)^(1/2) - 3*a^6*c^4
*d*e^2 - 3*c*d^2*e*(-a^11*c^7)^(1/2))/(a^10*(a*e^2 + c*d^2)^3))^(1/3))/6 +
 9*a^11*c^13*d^31*e^8 + 36*a^14*c^10*d^25*e^14)*(-(a^5*c^5*d^3 + a*e^3*(-a
^11*c^7)^(1/2) - 3*a^6*c^4*d*e^2 - 3*c*d^2*e*(-a^11*c^7)^(1/2))/(216*(a^13
*e^6 + a^10*c^3*d^6 + 3*a^12*c*d^2*e^4 + 3*a^11*c^2*d^4*e^2)))^(1/3) + log
(9*a^10*c^14*d^33*e^6 - ((((-(a^5*c^5*d^3 - a*e^3*(-a^11*c^7)^(1/2) - 3*a^
6*c^4*d*e^2 + 3*c*d^2*e*(-a^11*c^7)^(1/2))/(a^10*(a*e^2 + c*d^2)^3))^(2/3)
*(4860*a^16*c^11*d^34*e^8 - 972*a^14*c^13*d^38*e^4 - 6804*a^15*c^12*d^36*e
^6 - ((5832*a^15*c^6*d^26*e^3*x*(a*e^2 + c*d^2)^2*(8*a^5*e^10 - c^5*d^1...
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 516, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5 (d+ex^3) (a+cx^6)} \, dx = \text{Too large to display}$$

input
```
int(1/x^5/(e*x^3+d)/(c*x^6+a),x)
```

output
```
( - 4*e**(1/3)*c**(1/3)*a**(2/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d
**(1/3)*sqrt(3)))*a*e**3*x**4 - 2*d**(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/6)*
a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d**2*e*x**4 + 2*d*
*(1/3)*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a
**(1/6)))*c**2*d**3*x**4 + 2*d**(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1
/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*d**2*e*x**4 + 2*d**(1/3
)*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/
6)))*c**2*d**3*x**4 + 4*d**(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/3)*x)/(c**(1/
6)*a**(1/6)))*c*d**2*e*x**4 + d**(1/3)*sqrt(c)*sqrt(a)*sqrt(3)*log( - c**(
1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c*d**2*e*x**4 - d**(1/
3)*sqrt(c)*sqrt(a)*sqrt(3)*log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c*
*(1/3)*x**2)*c*d**2*e*x**4 - 3*d**(1/3)*c**(1/3)*a**(2/3)*a*d*e**2 + 12*d*
*(1/3)*c**(1/3)*a**(2/3)*a*e**3*x**3 - 3*d**(1/3)*c**(1/3)*a**(2/3)*c*d**3
 + 12*d**(1/3)*c**(1/3)*a**(2/3)*c*d**2*e*x**3 + 2*e**(1/3)*c**(1/3)*a**(2
/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*x + e**(2/3)*x**2)*a*e**3*x**4 - 4*e*
*(1/3)*c**(1/3)*a**(2/3)*log(d**(1/3) + e**(1/3)*x)*a*e**3*x**4 - 2*d**(1/
3)*log(a**(1/3) + c**(1/3)*x**2)*c**2*d**3*x**4 + d**(1/3)*log( - c**(1/6)
*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c**2*d**3*x**4 + d**(1/3)*
log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c**2*d**3*x**4
)/(12*d**(1/3)*c**(1/3)*a**(2/3)*a*d**2*x**4*(a*e**2 + c*d**2))
```

3.30 $\int \frac{1}{x^2(d+ex^3)^2(a+cx^6)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 654

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+cx^6)}\,dx = -\frac{1}{ad^2x} - \frac{e^3x^2}{3d^2(cd^2+ae^2)(d+ex^3)} - \frac{c^{7/6}(cd^2-ae^2)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{7/6}(cd^2+ae^2)^2} + \frac{c^{7/6}(cd^2-ae^2)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{7/6}(cd^2+ae^2)^2} - \frac{c^{7/6}(cd^2-ae^2)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{7/6}(cd^2+ae^2)^2} + \frac{2e^{7/3}(5cd^2+2ae^2)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{3\sqrt{3}d^{7/3}(cd^2+ae^2)^2} + \frac{c^{5/3}de\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{c}x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(cd^2+ae^2)^2} + \frac{c^{7/6}(cd^2-ae^2)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{2\sqrt{3}a^{7/6}(cd^2+ae^2)^2} + \frac{2e^{7/3}(5cd^2+2ae^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{7/3}(cd^2+ae^2)^2} - \frac{c^{5/3}de\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{3a^{2/3}(cd^2+ae^2)^2} - \frac{e^{7/3}(5cd^2+2ae^2)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{9d^{7/3}(cd^2+ae^2)^2} + \frac{c^{5/3}de\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{c}x^2+c^{2/3}x^4\right)}{6a^{2/3}(cd^2+ae^2)^2}$$

output
```
-1/a/d^2/x-1/3*e^3*x^2/d^2/(a*e^2+c*d^2)/(e*x^3+d)-1/3*c^(7/6)*(-a*e^2+c*d
^2)*arctan(c^(1/6)*x/a^(1/6))/a^(7/6)/(a*e^2+c*d^2)^2-1/6*c^(7/6)*(-a*e^2+
c*d^2)*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(7/6)/(a*e^2+c*d^2)^2-1/6*c^
(7/6)*(-a*e^2+c*d^2)*arctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(7/6)/(a*e^2+c*
d^2)^2+2/9*e^(7/3)*(2*a*e^2+5*c*d^2)*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)*3^(1
/2)/d^(1/3))*3^(1/2)/d^(7/3)/(a*e^2+c*d^2)^2+1/3*c^(5/3)*d*e*arctan(1/3*(a
^(1/3)-2*c^(1/3)*x^2)*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/(a*e^2+c*d^2)^2+1/6
*c^(7/6)*(-a*e^2+c*d^2)*arctanh(3^(1/2)*a^(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)
*x^2))*3^(1/2)/a^(7/6)/(a*e^2+c*d^2)^2+2/9*e^(7/3)*(2*a*e^2+5*c*d^2)*ln(d^
(1/3)+e^(1/3)*x)/d^(7/3)/(a*e^2+c*d^2)^2-1/3*c^(5/3)*d*e*ln(a^(1/3)+c^(1/3
)*x^2)/a^(2/3)/(a*e^2+c*d^2)^2-1/9*e^(7/3)*(2*a*e^2+5*c*d^2)*ln(d^(2/3)-d^
(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(7/3)/(a*e^2+c*d^2)^2+1/6*c^(5/3)*d*e*ln(a^
(2/3)-a^(1/3)*c^(1/3)*x^2+c^(2/3)*x^4)/a^(2/3)/(a*e^2+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+cx^6)}\,dx$$

$$= \frac{-12a^{7/6}\sqrt[3]{d}e^3(cd^2+ae^2)\,x^3 - 36\sqrt[6]{a}\sqrt[3]{d}(cd^2+ae^2)^2\,(d+ex^3) + 12c^{7/6}d^{7/3}(-cd^2+ae^2)\,x(d+ex^3)\,\mathrm{arcta}}{}$$

input
```
Integrate[1/(x^2*(d + e*x^3)^2*(a + c*x^6)),x]
```

output
```
(-12*a^(7/6)*d^(1/3)*e^3*(c*d^2 + a*e^2)*x^3 - 36*a^(1/6)*d^(1/3)*(c*d^2 +
 a*e^2)^2*(d + e*x^3) + 12*c^(7/6)*d^(7/3)*(-(c*d^2) + a*e^2)*x*(d + e*x^3
)*ArcTan[(c^(1/6)*x)/a^(1/6)] + 6*c^(7/6)*d^(7/3)*(c*d^2 + 2*Sqrt[3]*Sqrt[
a]*Sqrt[c]*d*e - a*e^2)*x*(d + e*x^3)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/
6)] + 6*c^(7/6)*d^(7/3)*(-(c*d^2) + 2*Sqrt[3]*Sqrt[a]*Sqrt[c]*d*e + a*e^2)
*x*(d + e*x^3)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)] + 8*Sqrt[3]*a^(7/6)
*e^(7/3)*(5*c*d^2 + 2*a*e^2)*x*(d + e*x^3)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/
3))/Sqrt[3]] + 8*a^(7/6)*e^(7/3)*(5*c*d^2 + 2*a*e^2)*x*(d + e*x^3)*Log[d^(
1/3) + e^(1/3)*x] - 12*Sqrt[a]*c^(5/3)*d^(10/3)*e*x*(d + e*x^3)*Log[a^(1/3
) + c^(1/3)*x^2] + 3*c^(7/6)*d^(7/3)*(-(Sqrt[3]*c*d^2) + 2*Sqrt[a]*Sqrt[c]
*d*e + Sqrt[3]*a*e^2)*x*(d + e*x^3)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*
x + c^(1/3)*x^2] + 3*c^(7/6)*d^(7/3)*(Sqrt[3]*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*
e - Sqrt[3]*a*e^2)*x*(d + e*x^3)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x +
 c^(1/3)*x^2] - 4*a^(7/6)*e^(7/3)*(5*c*d^2 + 2*a*e^2)*x*(d + e*x^3)*Log[d^
(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(36*a^(7/6)*d^(7/3)*(c*d^2 + a*e
^2)^2*x*(d + e*x^3))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 888, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1837, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+cx^6) (d+ex^3)^2} \, dx$$

$$\Big\downarrow \text{1837}$$

$$\int \left(\frac{c^2 x \left(-\left(x^3 \left(cd^2-ae^2\right)\right)-2ade\right)}{a (a+cx^6) (ae^2+cd^2)^2}-\frac{e^3 x (ae^2+3cd^2)}{d^2 (d+ex^3) (ae^2+cd^2)^2}-\frac{e^3 x}{d (d+ex^3)^2 (ae^2+cd^2)}+\frac{1}{ad^2x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{x^2e^3}{3d^2(cd^2+ae^2)(ex^3+d)}+\frac{(3cd^2+ae^2)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)e^{7/3}}{\sqrt{3}d^{7/3}(cd^2+ae^2)^2}+\frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)e^{7/3}}{3\sqrt{3}d^{7/3}(cd^2+ae^2)}+\frac{(3cd^2+ae^2)\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)e^{7/3}}{3d^{7/3}(cd^2+ae^2)^2}+\frac{\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)e^{7/3}}{9d^{7/3}(cd^2+ae^2)}-\frac{(3cd^2+ae^2)\log\left(e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}\right)e^{7/3}}{6d^{7/3}(cd^2+ae^2)^2}-\frac{\log\left(e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}\right)e^{7/3}}{18d^{7/3}(cd^2+ae^2)}-\frac{c^{7/6}\left(cd^2+2\sqrt{-a}\sqrt{c}ed-ae^2\right)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}(-a)^{7/6}(cd^2+ae^2)^2}+\frac{c^{7/6}\left(cd^2-2\sqrt{-a}\sqrt{c}ed-ae^2\right)\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1}{\sqrt{3}}\right)}{2\sqrt{3}(-a)^{7/6}(cd^2+ae^2)^2}+\frac{c^{7/6}\left(cd^2-2\sqrt{-a}\sqrt{c}ed-ae^2\right)\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{6(-a)^{7/6}(cd^2+ae^2)^2}-\frac{c^{7/6}\left(cd^2+2\sqrt{-a}\sqrt{c}ed-ae^2\right)\log\left(\sqrt[6]{c}x+\sqrt[6]{-a}\right)}{6(-a)^{7/6}(cd^2+ae^2)^2}+\frac{c^{7/6}\left(cd^2+2\sqrt{-a}\sqrt{c}ed-ae^2\right)\log\left(\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}\right)}{12(-a)^{7/6}(cd^2+ae^2)^2}-\frac{c^{7/6}\left(cd^2-2\sqrt{-a}\sqrt{c}ed-ae^2\right)\log\left(\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}\right)}{12(-a)^{7/6}(cd^2+ae^2)^2}-\frac{1}{ad^2x}
$$

input
```
Int[1/(x^2*(d + e*x^3)^2*(a + c*x^6)),x]
```

output
```
-(1/(a*d^2*x)) - (e^3*x^2)/(3*d^2*(c*d^2 + a*e^2)*(d + e*x^3)) - (c^(7/6)*
(c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTan[(1 - (2*c^(1/6)*x)/(-a)^(1
/6))/Sqrt[3]])/(2*Sqrt[3]*(-a)^(7/6)*(c*d^2 + a*e^2)^2) + (c^(7/6)*(c*d^2
- 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTan[(1 + (2*c^(1/6)*x)/(-a)^(1/6))/Sq
rt[3]])/(2*Sqrt[3]*(-a)^(7/6)*(c*d^2 + a*e^2)^2) + (e^(7/3)*ArcTan[(d^(1/3
) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(3*Sqrt[3]*d^(7/3)*(c*d^2 + a*e^2)) +
 (e^(7/3)*(3*c*d^2 + a*e^2)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3
))])/(Sqrt[3]*d^(7/3)*(c*d^2 + a*e^2)^2) + (c^(7/6)*(c*d^2 - 2*Sqrt[-a]*Sq
rt[c]*d*e - a*e^2)*Log[(-a)^(1/6) - c^(1/6)*x])/(6*(-a)^(7/6)*(c*d^2 + a*e
^2)^2) - (c^(7/6)*(c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*Log[(-a)^(1/6)
+ c^(1/6)*x])/(6*(-a)^(7/6)*(c*d^2 + a*e^2)^2) + (e^(7/3)*Log[d^(1/3) + e^
(1/3)*x])/(9*d^(7/3)*(c*d^2 + a*e^2)) + (e^(7/3)*(3*c*d^2 + a*e^2)*Log[d^(
1/3) + e^(1/3)*x])/(3*d^(7/3)*(c*d^2 + a*e^2)^2) + (c^(7/6)*(c*d^2 + 2*Sqr
t[-a]*Sqrt[c]*d*e - a*e^2)*Log[(-a)^(1/3) - (-a)^(1/6)*c^(1/6)*x + c^(1/3)
*x^2])/(12*(-a)^(7/6)*(c*d^2 + a*e^2)^2) - (c^(7/6)*(c*d^2 - 2*Sqrt[-a]*Sq
rt[c]*d*e - a*e^2)*Log[(-a)^(1/3) + (-a)^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(
12*(-a)^(7/6)*(c*d^2 + a*e^2)^2) - (e^(7/3)*Log[d^(2/3) - d^(1/3)*e^(1/3)*
x + e^(2/3)*x^2])/(18*d^(7/3)*(c*d^2 + a*e^2)) - (e^(7/3)*(3*c*d^2 + a*e^2
)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(7/3)*(c*d^2 + a*e^
2)^2)
```

Defintions of rubi rules used

rule 1837
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^n)^q/(a + c*x^(
2*n))), x], x] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0
] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.15

method	result	size
default	Expression too large to display	751
risch	Expression too large to display	2257

input
```
int(1/x^2/(e*x^3+d)^2/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-e^3/d^2/(a*e^2+c*d^2)^2*((1/3*a*e^2+1/3*c*d^2)*x^2/(e*x^3+d)+(4/3*a*e^2+1
0/3*c*d^2)*(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-
(d/e)^(1/3)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2
/(d/e)^(1/3)*x-1))))-1/a/d^2/x-(-1/12*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(
1/3))*3^(1/2)*(a/c)^(5/6)*e^2+1/12*c/a*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^
(1/3))*3^(1/2)*(a/c)^(5/6)*d^2-1/6*c/a*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^
(1/3))*(a/c)^(4/3)*d*e-1/6/c*a/(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)-3^(1/2))
*e^2+1/6/(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*d^2-1/3*c/a*(a/c)^(4/
3)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*3^(1/2)*d*e+2/3*(a/c)^(1/3)*arctan(2*x/
(a/c)^(1/6)-3^(1/2))*3^(1/2)*d*e+1/12*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(
1/3))*3^(1/2)*(a/c)^(5/6)*e^2-1/12*c/a*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^
(1/3))*3^(1/2)*(a/c)^(5/6)*d^2-1/6*c/a*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^
(1/3))*(a/c)^(4/3)*d*e-1/6/c*a/(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)+3^(1/2))
*e^2+1/6/(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*d^2+1/3*c/a*(a/c)^(4/
3)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*3^(1/2)*d*e-2/3*(a/c)^(1/3)*arctan(2*x/
(a/c)^(1/6)+3^(1/2))*3^(1/2)*d*e+1/3*(a/c)^(1/3)*d*e*ln(x^2+(a/c)^(1/3))-1
/3/c/(a/c)^(1/6)*arctan(x/(a/c)^(1/6))*a*e^2+1/3/(a/c)^(1/6)*arctan(x/(a/c
)^(1/6))*d^2)/(a*e^2+c*d^2)^2*c^2/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+cx^6)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(e*x^3+d)^2/(c*x^6+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d + ex^3)^2 (a + cx^6)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*x**3+d)**2/(c*x**6+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (d + ex^3)^2 (a + cx^6)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(e*x^3+d)^2/(c*x^6+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 898, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+cx^6)} dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x^3+d)^2/(c*x^6+a),x, algorithm="giac")
```

output
```
-1/3*(a*c^5)^(1/3)*d*e*log(x^2 + (a/c)^(1/3))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2
 + a^3*e^4) + 2/9*(5*c*d^2*e^3*(-d/e)^(1/3) + 2*a*e^5*(-d/e)^(1/3))*(-d/e)
^(1/3)*log(abs(x - (-d/e)^(1/3)))/(c^2*d^7 + 2*a*c*d^5*e^2 + a^2*d^3*e^4)
+ 2/3*(5*(-d*e^2)^(2/3)*c*d^2*e + 2*(-d*e^2)^(2/3)*a*e^3)*arctan(1/3*sqrt(
3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/(sqrt(3)*c^2*d^7 + 2*sqrt(3)*a*c*d^5
*e^2 + sqrt(3)*a^2*d^3*e^4) + 1/6*(2*sqrt(3)*(a*c^5)^(1/3)*a*c^3*d*e - (a*
c^5)^(5/6)*c*d^2 + (a*c^5)^(5/6)*a*e^2)*arctan((2*x + sqrt(3)*(a/c)^(1/6))
/(a/c)^(1/6))/(a^2*c^5*d^4 + 2*a^3*c^4*d^2*e^2 + a^4*c^3*e^4) - 1/6*(2*sqr
t(3)*(a*c^5)^(1/3)*a*c^3*d*e + (a*c^5)^(5/6)*c*d^2 - (a*c^5)^(5/6)*a*e^2)*
arctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^2*c^5*d^4 + 2*a^3*c^4*d
^2*e^2 + a^4*c^3*e^4) - 1/3*((a*c^5)^(5/6)*c*d^2 - (a*c^5)^(5/6)*a*e^2)*ar
ctan(x/(a/c)^(1/6))/(a^2*c^5*d^4 + 2*a^3*c^4*d^2*e^2 + a^4*c^3*e^4) + 1/12
*(2*(a*c^5)^(1/3)*a*c^3*d*e + sqrt(3)*(a*c^5)^(5/6)*c*d^2 - sqrt(3)*(a*c^5
)^(5/6)*a*e^2)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^2*c^5*d^4
 + 2*a^3*c^4*d^2*e^2 + a^4*c^3*e^4) + 1/12*(2*(a*c^5)^(1/3)*a*c^3*d*e - sq
rt(3)*(a*c^5)^(5/6)*c*d^2 + sqrt(3)*(a*c^5)^(5/6)*a*e^2)*log(x^2 - sqrt(3)
*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^2*c^5*d^4 + 2*a^3*c^4*d^2*e^2 + a^4*c^3*e
^4) - 1/9*(5*(-d*e^2)^(2/3)*c*d^2*e + 2*(-d*e^2)^(2/3)*a*e^3)*log(x^2 + x*
(-d/e)^(1/3) + (-d/e)^(2/3))/(c^2*d^7 + 2*a*c*d^5*e^2 + a^2*d^3*e^4) - 1/3
*(3*c*d^2*e*x^3 + 4*a*e^3*x^3 + 3*c*d^3 + 3*a*d*e^2)/((a*c*d^4 + a^2*d^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+cx^6)} dx = \text{Hanged}$$

input
```
int(1/(x^2*(a + c*x^6)*(d + e*x^3)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{x^2 (d + ex^3)^2 (a + cx^6)} \, dx = \int \frac{1}{x^2 (e\,x^3 + d)^2 (c\,x^6 + a)} dx$$

input
```
int(1/x^2/(e*x^3+d)^2/(c*x^6+a),x)
```

output
```
int(1/x^2/(e*x^3+d)^2/(c*x^6+a),x)
```

3.31 $\int \frac{1}{x^5 (d+ex^3)^2 (a+cx^6)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 659

$$\int \frac{1}{x^5 (d+ex^3)^2 (a+cx^6)} \, dx = -\frac{1}{4ad^2x^4} + \frac{2e}{ad^3x} + \frac{e^4x^2}{3d^3 (cd^2+ae^2) (d+ex^3)} + \frac{2c^{13/6}de \arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{7/6} (cd^2+ae^2)^2} - \frac{c^{13/6}de \arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{7/6} (cd^2+ae^2)^2} + \frac{c^{13/6}de \arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{7/6} (cd^2+ae^2)^2} - \frac{e^{10/3}(13cd^2+7ae^2) \arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{3\sqrt{3}d^{10/3} (cd^2+ae^2)^2} + \frac{c^{5/3}(cd^2-ae^2) \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{c}x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{5/3} (cd^2+ae^2)^2} - \frac{c^{13/6}de\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{\sqrt{3}a^{7/6} (cd^2+ae^2)^2} - \frac{e^{10/3}(13cd^2+7ae^2) \log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{10/3} (cd^2+ae^2)^2} - \frac{c^{5/3}(cd^2-ae^2) \log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6a^{5/3} (cd^2+ae^2)^2} + \frac{e^{10/3}(13cd^2+7ae^2) \log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{18d^{10/3} (cd^2+ae^2)^2} + \frac{c^{5/3}(cd^2-ae^2) \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{c}x^2+c^{2/3}x^4\right)}{12a^{5/3} (cd^2+ae^2)^2}$$

output

```
-1/4/a/d^2/x^4+2*e/a/d^3/x+1/3*e^4*x^2/d^3/(a*e^2+c*d^2)/(e*x^3+d)+2/3*c^(
13/6)*d*e*arctan(c^(1/6)*x/a^(1/6))/a^(7/6)/(a*e^2+c*d^2)^2+1/3*c^(13/6)*d
*e*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(7/6)/(a*e^2+c*d^2)^2+1/3*c^(13/
6)*d*e*arctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(7/6)/(a*e^2+c*d^2)^2-1/9*e^(
10/3)*(7*a*e^2+13*c*d^2)*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)*3^(1/2)/d^(1/3))
*3^(1/2)/d^(10/3)/(a*e^2+c*d^2)^2+1/6*c^(5/3)*(-a*e^2+c*d^2)*arctan(1/3*(a
^(1/3)-2*c^(1/3)*x^2)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/(a*e^2+c*d^2)^2-1/3
*c^(13/6)*d*e*arctanh(3^(1/2)*a^(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)*x^2))*3^(
1/2)/a^(7/6)/(a*e^2+c*d^2)^2-1/9*e^(10/3)*(7*a*e^2+13*c*d^2)*ln(d^(1/3)+e^
(1/3)*x)/d^(10/3)/(a*e^2+c*d^2)^2-1/6*c^(5/3)*(-a*e^2+c*d^2)*ln(a^(1/3)+c^
(1/3)*x^2)/a^(5/3)/(a*e^2+c*d^2)^2+1/18*e^(10/3)*(7*a*e^2+13*c*d^2)*ln(d^(
2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(10/3)/(a*e^2+c*d^2)^2+1/12*c^(5/3)*
(-a*e^2+c*d^2)*ln(a^(2/3)-a^(1/3)*c^(1/3)*x^2+c^(2/3)*x^4)/a^(5/3)/(a*e^2+
c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5 (d+ex^3)^2 (a+cx^6)} dx$$

$$= \frac{1}{36} \left(-\frac{9}{ad^2x^4} + \frac{72e}{ad^3x} + \frac{12e^4x^2}{d^3(cd^2+ae^2)(d+ex^3)} + \frac{24c^{13/6}de\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{7/6}(cd^2+ae^2)^2} - \frac{6c^{5/3}(-\sqrt{3}cd^2+2\sqrt{a}\sqrt{c}de+\sqrt{3}ae^2)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{5/3}(cd^2+ae^2)^2} + \frac{6c^{5/3}(\sqrt{3}cd^2+2\sqrt{a}\sqrt{c}de-\sqrt{3}ae^2)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{5/3}(cd^2+ae^2)^2} - \frac{4\sqrt{3}e^{10/3}(13cd^2+7ae^2)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{10/3}(cd^2+ae^2)^2} - \frac{4e^{10/3}(13cd^2+7ae^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{d^{10/3}(cd^2+ae^2)^2} + \frac{6c^{5/3}(-cd^2+ae^2)\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{a^{5/3}(cd^2+ae^2)^2} + \frac{3c^{5/3}(cd^2+2\sqrt{3}\sqrt{a}\sqrt{c}de-ae^2)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{a^{5/3}(cd^2+ae^2)^2} - \frac{3c^{5/3}(-cd^2+2\sqrt{3}\sqrt{a}\sqrt{c}de+ae^2)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{a^{5/3}(cd^2+ae^2)^2} + \frac{2e^{10/3}(13cd^2+7ae^2)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{d^{10/3}(cd^2+ae^2)^2} \right)$$

input
```
Integrate[1/(x^5*(d + e*x^3)^2*(a + c*x^6)),x]
```

output
```
(-9/(a*d^2*x^4) + (72*e)/(a*d^3*x) + (12*e^4*x^2)/(d^3*(c*d^2 + a*e^2)*(d
+ e*x^3)) + (24*c^(13/6)*d*e*ArcTan[(c^(1/6)*x)/a^(1/6)])/(a^(7/6)*(c*d^2
+ a*e^2)^2) - (6*c^(5/3)*(-(Sqrt[3]*c*d^2) + 2*Sqrt[a]*Sqrt[c]*d*e + Sqrt[
3]*a*e^2)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)])/(a^(5/3)*(c*d^2 + a*e^2
)^2) + (6*c^(5/3)*(Sqrt[3]*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - Sqrt[3]*a*e^2)*
ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)])/(a^(5/3)*(c*d^2 + a*e^2)^2) - (4*
Sqrt[3]*e^(10/3)*(13*c*d^2 + 7*a*e^2)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/S
qrt[3]])/(d^(10/3)*(c*d^2 + a*e^2)^2) - (4*e^(10/3)*(13*c*d^2 + 7*a*e^2)*L
og[d^(1/3) + e^(1/3)*x]])/(d^(10/3)*(c*d^2 + a*e^2)^2) + (6*c^(5/3)*(-(c*d^
2) + a*e^2)*Log[a^(1/3) + c^(1/3)*x^2]])/(a^(5/3)*(c*d^2 + a*e^2)^2) + (3*c
^(5/3)*(c*d^2 + 2*Sqrt[3]*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[a^(1/3) - Sqrt[
3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]])/(a^(5/3)*(c*d^2 + a*e^2)^2) - (3*c^(5
/3)*(-(c*d^2) + 2*Sqrt[3]*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[a^(1/3) + Sqrt[
3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]])/(a^(5/3)*(c*d^2 + a*e^2)^2) + (2*e^(1
0/3)*(13*c*d^2 + 7*a*e^2)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]])/
(d^(10/3)*(c*d^2 + a*e^2)^2))/36
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 903, normalized size of antiderivative = 1.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1837, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+cx^6) (d+ex^3)^2} \, dx$$

$$\Big\downarrow \text{1837}$$

$$\int \left(\frac{c^2 x (ae^2-cd^2+2cdex^3)}{a (a+cx^6) (ae^2+cd^2)^2}+\frac{e^4 x}{d^2 (d+ex^3)^2 (ae^2+cd^2)}+\frac{2e^4 x (ae^2+2cd^2)}{d^3 (d+ex^3) (ae^2+cd^2)^2}-\frac{2e}{ad^3x^2}+\frac{1}{ad^2x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2e^4}{3d^3(cd^2+ae^2)(ex^3+d)}-\frac{2(2cd^2+ae^2)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)e^{10/3}}{\sqrt{3}d^{10/3}(cd^2+ae^2)^2}-\frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)e^{10/3}}{3\sqrt{3}d^{10/3}(cd^2+ae^2)}-\frac{2(2cd^2+ae^2)\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)e^{10/3}}{3d^{10/3}(cd^2+ae^2)^2}-\frac{\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)e^{10/3}}{9d^{10/3}(cd^2+ae^2)}+\frac{(2cd^2+ae^2)\log\left(e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}\right)e^{10/3}}{3d^{10/3}(cd^2+ae^2)^2}+\frac{\log\left(e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}\right)e^{10/3}}{18d^{10/3}(cd^2+ae^2)}+\frac{2e}{ad^3x}+\frac{c^{5/3}(cd^2+2\sqrt{-a}\sqrt{c}ed-ae^2)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}(-a)^{5/3}(cd^2+ae^2)^2}+\frac{c^{5/3}(cd^2-2\sqrt{-a}\sqrt{c}ed-ae^2)\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{-a}}+1}{\sqrt{3}}\right)}{2\sqrt{3}(-a)^{5/3}(cd^2+ae^2)^2}+\frac{c^{5/3}(cd^2-2\sqrt{-a}\sqrt{c}ed-ae^2)\log\left(\sqrt[6]{-a}-\sqrt[6]{c}x\right)}{6(-a)^{5/3}(cd^2+ae^2)^2}+\frac{c^{5/3}(cd^2+2\sqrt{-a}\sqrt{c}ed-ae^2)\log\left(\sqrt[6]{c}x+\sqrt[6]{-a}\right)}{6(-a)^{5/3}(cd^2+ae^2)^2}-\frac{c^{5/3}(cd^2+2\sqrt{-a}\sqrt{c}ed-ae^2)\log\left(\sqrt[3]{c}x^2-\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}\right)}{12(-a)^{5/3}(cd^2+ae^2)^2}-\frac{c^{5/3}(cd^2-2\sqrt{-a}\sqrt{c}ed-ae^2)\log\left(\sqrt[3]{c}x^2+\sqrt[6]{-a}\sqrt[6]{c}x+\sqrt[3]{-a}\right)}{12(-a)^{5/3}(cd^2+ae^2)^2}-\frac{1}{4ad^2x^4}$$

input
```
Int[1/(x^5*(d + e*x^3)^2*(a + c*x^6)),x]
```

output
```
-1/4*1/(a*d^2*x^4) + (2*e)/(a*d^3*x) + (e^4*x^2)/(3*d^3*(c*d^2 + a*e^2)*(d
 + e*x^3)) + (c^(5/3)*(c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTan[(1 -
 (2*c^(1/6)*x)/(-a)^(1/6))/Sqrt[3]])/(2*Sqrt[3]*(-a)^(5/3)*(c*d^2 + a*e^2)
^2) + (c^(5/3)*(c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTan[(1 + (2*c^(
1/6)*x)/(-a)^(1/6))/Sqrt[3]])/(2*Sqrt[3]*(-a)^(5/3)*(c*d^2 + a*e^2)^2) - (
e^(10/3)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(3*Sqrt[3]*d^(
10/3)*(c*d^2 + a*e^2)) - (2*e^(10/3)*(2*c*d^2 + a*e^2)*ArcTan[(d^(1/3) - 2
*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(10/3)*(c*d^2 + a*e^2)^2) + (c^
(5/3)*(c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*Log[(-a)^(1/6) - c^(1/6)*x]
)/(6*(-a)^(5/3)*(c*d^2 + a*e^2)^2) + (c^(5/3)*(c*d^2 + 2*Sqrt[-a]*Sqrt[c]*
d*e - a*e^2)*Log[(-a)^(1/6) + c^(1/6)*x])/(6*(-a)^(5/3)*(c*d^2 + a*e^2)^2)
 - (e^(10/3)*Log[d^(1/3) + e^(1/3)*x])/(9*d^(10/3)*(c*d^2 + a*e^2)) - (2*e
^(10/3)*(2*c*d^2 + a*e^2)*Log[d^(1/3) + e^(1/3)*x])/(3*d^(10/3)*(c*d^2 + a
*e^2)^2) - (c^(5/3)*(c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*Log[(-a)^(1/3
) - (-a)^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*(-a)^(5/3)*(c*d^2 + a*e^2)^2)
 - (c^(5/3)*(c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*Log[(-a)^(1/3) + (-a)
^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*(-a)^(5/3)*(c*d^2 + a*e^2)^2) + (e^(1
0/3)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(18*d^(10/3)*(c*d^2 +
 a*e^2)) + (e^(10/3)*(2*c*d^2 + a*e^2)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e
^(2/3)*x^2])/(3*d^(10/3)*(c*d^2 + a*e^2)^2)
```

Defintions of rubi rules used

rule 1837
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^n)^q/(a + c*x^(
2*n))), x], x] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0
] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.05

method	result	size
default	Expression too large to display	695
risch	Expression too large to display	2449

input
```
int(1/x^5/(e*x^3+d)^2/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
e^4/d^3/(a*e^2+c*d^2)^2*((1/3*a*e^2+1/3*c*d^2)*x^2/(e*x^3+d)+(7/3*a*e^2+13
/3*c*d^2)*(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(
d/e)^(1/3)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/
(d/e)^(1/3)*x-1))))-1/4/a/d^2/x^4+2*e/a/d^3/x+(1/6*c/a*ln(x^2-3^(1/2)*(a/c
)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*d*e-1/12*ln(x^2-3^(1/2)*(a/c)^(
1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*e^2+1/12*c/a*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+
(a/c)^(1/3))*(a/c)^(1/3)*d^2+1/3/(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)-3^(1/2
))*d*e+1/6*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*3^(1/2)*e^2-1/6*c/a
*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*3^(1/2)*d^2-1/6*c/a*ln(x^2+3^
(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*d*e-1/12*ln(x^2+3^(1/
2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*e^2+1/12*c/a*ln(x^2+3^(1/2)*(a/c
)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*d^2+1/3/(a/c)^(1/6)*arctan(2*x/(a/c)^(1
/6)+3^(1/2))*d*e-1/6*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*3^(1/2)*e
^2+1/6*c/a*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*3^(1/2)*d^2+1/6*ln(
x^2+(a/c)^(1/3))*(a/c)^(1/3)*e^2-1/6*c/a*ln(x^2+(a/c)^(1/3))*(a/c)^(1/3)*d
^2+2/3*d*e/(a/c)^(1/6)*arctan(x/(a/c)^(1/6)))*c^2/(a*e^2+c*d^2)^2/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d+ex^3)^2 (a+cx^6)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^5/(e*x^3+d)^2/(c*x^6+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d+ex^3)^2 (a+cx^6)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**5/(e*x**3+d)**2/(c*x**6+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 (d+ex^3)^2 (a+cx^6)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(e*x^3+d)^2/(c*x^6+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 897, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^5 (d+ex^3)^2 (a+cx^6)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(e*x^3+d)^2/(c*x^6+a),x, algorithm="giac")
```

output
```
1/3*e^4*x^2/((c*d^5 + a*d^3*e^2)*(e*x^3 + d)) + 2/3*(a*c^5)^(5/6)*d*e*arct
an(x/(a/c)^(1/6))/(a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4) - 1/9*(1
3*c*d^2*e^4*(-d/e)^(1/3) + 7*a*e^6*(-d/e)^(1/3))*(-d/e)^(1/3)*log(abs(x -
(-d/e)^(1/3)))/(c^2*d^8 + 2*a*c*d^6*e^2 + a^2*d^4*e^4) - 1/3*(13*(-d*e^2)^
(2/3)*c*d^2*e^2 + 7*(-d*e^2)^(2/3)*a*e^4)*arctan(1/3*sqrt(3)*(2*x + (-d/e)
^(1/3))/(-d/e)^(1/3))/(sqrt(3)*c^2*d^8 + 2*sqrt(3)*a*c*d^6*e^2 + sqrt(3)*a
^2*d^4*e^4) + 1/6*(sqrt(3)*(a*c^5)^(1/3)*c^3*d^2 - sqrt(3)*(a*c^5)^(1/3)*a
*c^2*e^2 + 2*(a*c^5)^(5/6)*d*e)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(
1/6))/(a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4) - 1/6*(sqrt(3)*(a*c^
5)^(1/3)*c^3*d^2 - sqrt(3)*(a*c^5)^(1/3)*a*c^2*e^2 - 2*(a*c^5)^(5/6)*d*e)*
arctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^2*c^4*d^4 + 2*a^3*c^3*d
^2*e^2 + a^4*c^2*e^4) + 1/12*((a*c^5)^(1/3)*c^3*d^2 - (a*c^5)^(1/3)*a*c^2*
e^2 - 2*sqrt(3)*(a*c^5)^(5/6)*d*e)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)
^(1/3))/(a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4) + 1/12*((a*c^5)^(1
/3)*c^3*d^2 - (a*c^5)^(1/3)*a*c^2*e^2 + 2*sqrt(3)*(a*c^5)^(5/6)*d*e)*log(x
^2 - sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2
 + a^4*c^2*e^4) + 1/18*(13*(-d*e^2)^(2/3)*c*d^2*e^2 + 7*(-d*e^2)^(2/3)*a*e
^4)*log(x^2 + x*(-d/e)^(1/3) + (-d/e)^(2/3))/(c^2*d^8 + 2*a*c*d^6*e^2 + a^
2*d^4*e^4) - 1/6*((a*c^5)^(1/3)*c*d^2 - (a*c^5)^(1/3)*a*e^2)*log(x^2 + (a/
c)^(1/3))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) + 1/4*(8*e*x^3 - d)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d + ex^3)^2 (a + cx^6)} \, dx = \text{Hanged}$$

input
```
int(1/(x^5*(a + c*x^6)*(d + e*x^3)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{x^5 (d+ex^3)^2 (a+cx^6)}\,dx = \int \frac{1}{x^5 (e\,x^3+d)^2 (c\,x^6+a)}dx$$

input
```
int(1/x^5/(e*x^3+d)^2/(c*x^6+a),x)
```

output
```
int(1/x^5/(e*x^3+d)^2/(c*x^6+a),x)
```

3.32 $\int \frac{1}{x^2(d+ex^3)(a+cx^6)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 668

$$\begin{aligned}\int \frac{1}{x^2 (d+ex^3) (a+cx^6)^2}\,dx = & -\frac{1}{a^2dx} - \frac{x(acex+c^2dx^4)}{6a^2 (cd^2+ae^2) (a+cx^6)} \\ & - \frac{c^{7/6}d(7cd^2+13ae^2)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{18a^{13/6} (cd^2+ae^2)^2} \\ & + \frac{c^{7/6}d(7cd^2+13ae^2)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{36a^{13/6} (cd^2+ae^2)^2} \\ & - \frac{c^{7/6}d(7cd^2+13ae^2)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{36a^{13/6} (cd^2+ae^2)^2} \\ & + \frac{e^{13/3}\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{4/3} (cd^2+ae^2)^2} \\ & + \frac{c^{2/3}e(2cd^2+5ae^2)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{c}x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{5/3} (cd^2+ae^2)^2} \\ & + \frac{c^{7/6}d(7cd^2+13ae^2)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{12\sqrt{3}a^{13/6} (cd^2+ae^2)^2} \\ & + \frac{e^{13/3}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{4/3} (cd^2+ae^2)^2} \\ & - \frac{c^{2/3}e(2cd^2+5ae^2)\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{18a^{5/3} (cd^2+ae^2)^2} \\ & - \frac{e^{13/3}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{4/3} (cd^2+ae^2)^2} \\ & + \frac{c^{2/3}e(2cd^2+5ae^2)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{c}x^2+c^{2/3}x^4\right)}{36a^{5/3} (cd^2+ae^2)^2}\end{aligned}$$

output
```
-1/a^2/d/x-1/6*x*(c^2*d*x^4+a*c*e*x)/a^2/(a*e^2+c*d^2)/(c*x^6+a)-1/18*c^(7
/6)*d*(13*a*e^2+7*c*d^2)*arctan(c^(1/6)*x/a^(1/6))/a^(13/6)/(a*e^2+c*d^2)^
2-1/36*c^(7/6)*d*(13*a*e^2+7*c*d^2)*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/a
^(13/6)/(a*e^2+c*d^2)^2-1/36*c^(7/6)*d*(13*a*e^2+7*c*d^2)*arctan(3^(1/2)+2
*c^(1/6)*x/a^(1/6))/a^(13/6)/(a*e^2+c*d^2)^2+1/3*e^(13/3)*arctan(1/3*(d^(1
/3)-2*e^(1/3)*x)*3^(1/2)/d^(1/3))*3^(1/2)/d^(4/3)/(a*e^2+c*d^2)^2+1/18*c^(
2/3)*e*(5*a*e^2+2*c*d^2)*arctan(1/3*(a^(1/3)-2*c^(1/3)*x^2)*3^(1/2)/a^(1/3
))*3^(1/2)/a^(5/3)/(a*e^2+c*d^2)^2+1/36*c^(7/6)*d*(13*a*e^2+7*c*d^2)*arcta
nh(3^(1/2)*a^(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)*x^2))*3^(1/2)/a^(13/6)/(a*e^
2+c*d^2)^2+1/3*e^(13/3)*ln(d^(1/3)+e^(1/3)*x)/d^(4/3)/(a*e^2+c*d^2)^2-1/18
*c^(2/3)*e*(5*a*e^2+2*c*d^2)*ln(a^(1/3)+c^(1/3)*x^2)/a^(5/3)/(a*e^2+c*d^2)
^2-1/6*e^(13/3)*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(4/3)/(a*e^2+c
*d^2)^2+1/36*c^(2/3)*e*(5*a*e^2+2*c*d^2)*ln(a^(2/3)-a^(1/3)*c^(1/3)*x^2+c^
(2/3)*x^4)/a^(5/3)/(a*e^2+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2 (d+ex^3) (a+cx^6)^2} \, dx$$

$$= \frac{-12\sqrt[6]{a}cd^{4/3}(cd^2+ae^2)\,x^3(ae+cdx^3) - 72\sqrt[6]{a}\sqrt[3]{d}(cd^2+ae^2)^2\,(a+cx^6) - 4c^{7/6}d^{7/3}(7cd^2+13ae^2)\,x(a-}{}$$

input
```
Integrate[1/(x^2*(d + e*x^3)*(a + c*x^6)^2),x]
```

output
```
(-12*a^(1/6)*c*d^(4/3)*(c*d^2 + a*e^2)*x^3*(a*e + c*d*x^3) - 72*a^(1/6)*d^
(1/3)*(c*d^2 + a*e^2)^2*(a + c*x^6) - 4*c^(7/6)*d^(7/3)*(7*c*d^2 + 13*a*e^
2)*x*(a + c*x^6)*ArcTan[(c^(1/6)*x)/a^(1/6)] + 2*c^(2/3)*d^(4/3)*(7*c^(3/2
)*d^3 + 4*Sqrt[3]*Sqrt[a]*c*d^2*e + 13*a*Sqrt[c]*d*e^2 + 10*Sqrt[3]*a^(3/2
)*e^3)*x*(a + c*x^6)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)] + 2*c^(2/3)*d
^(4/3)*(-7*c^(3/2)*d^3 + 4*Sqrt[3]*Sqrt[a]*c*d^2*e - 13*a*Sqrt[c]*d*e^2 +
10*Sqrt[3]*a^(3/2)*e^3)*x*(a + c*x^6)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/
6)] + 24*Sqrt[3]*a^(13/6)*e^(13/3)*x*(a + c*x^6)*ArcTan[(1 - (2*e^(1/3)*x)
/d^(1/3))/Sqrt[3]] + 24*a^(13/6)*e^(13/3)*x*(a + c*x^6)*Log[d^(1/3) + e^(1
/3)*x] - 4*Sqrt[a]*c^(2/3)*d^(4/3)*e*(2*c*d^2 + 5*a*e^2)*x*(a + c*x^6)*Log
[a^(1/3) + c^(1/3)*x^2] + c^(2/3)*d^(4/3)*(-7*Sqrt[3]*c^(3/2)*d^3 + 4*Sqrt
[a]*c*d^2*e - 13*Sqrt[3]*a*Sqrt[c]*d*e^2 + 10*a^(3/2)*e^3)*x*(a + c*x^6)*L
og[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + c^(2/3)*d^(4/3)*(7
*Sqrt[3]*c^(3/2)*d^3 + 4*Sqrt[a]*c*d^2*e + 13*Sqrt[3]*a*Sqrt[c]*d*e^2 + 10
*a^(3/2)*e^3)*x*(a + c*x^6)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1
/3)*x^2] - 12*a^(13/6)*e^(13/3)*x*(a + c*x^6)*Log[d^(2/3) - d^(1/3)*e^(1/3
)*x + e^(2/3)*x^2])/(72*a^(13/6)*d^(4/3)*(c*d^2 + a*e^2)^2*x*(a + c*x^6))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+cx^6\right)^2\left(d+ex^3\right)}dx$$

$$\Big\downarrow \; 1888$$

$$\int \frac{1}{x^2\left(a+cx^6\right)^2\left(d+ex^3\right)}dx$$

input
```
Int[1/(x^2*(d + e*x^3)*(a + c*x^6)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1888
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n)
)^p, x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 946, normalized size of antiderivative = 1.42

method	result	size
default	Expression too large to display	946
risch	Expression too large to display	2393

input
```
int(1/x^2/(e*x^3+d)/(c*x^6+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3
)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/
3)*x-1)))*e^5/d/(a*e^2+c*d^2)^2-1/a^2/d/x-c/(a*e^2+c*d^2)^2/a^2*(((1/6*a*c
*d*e^2+1/6*c^2*d^3)*x^5+(1/6*a^2*e^3+1/6*a*c*d^2*e)*x^2)/(c*x^6+a)+13/72*c
*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*d*e^2+7/72*
c^2/a*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*d^3-5/
36*c*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(4/3)*e^3-1/18*c^2/a*
ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(4/3)*d^2*e+13/36*a/(a/c)^
(1/6)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*d*e^2+7/36*c/(a/c)^(1/6)*arctan(2*x/
(a/c)^(1/6)-3^(1/2))*d^3-5/18*c*(a/c)^(4/3)*arctan(2*x/(a/c)^(1/6)-3^(1/2)
)*3^(1/2)*e^3-1/9*c^2/a*(a/c)^(4/3)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*3^(1/2
)*d^2*e+5/9*a*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*3^(1/2)*e^3+2/9*
c*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*3^(1/2)*d^2*e-13/72*c*ln(x^2
+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*d*e^2-7/72*c^2/a*l
n(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*d^3-5/36*a*ln
(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*e^3-1/18*c*ln(x^2+3^(1
/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*d^2*e+13/36*a/(a/c)^(1/6)*arcta
n(2*x/(a/c)^(1/6)+3^(1/2))*d*e^2+7/36*c/(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)
+3^(1/2))*d^3-5/18*a*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*3^(1/2)*e
^3-1/9*c*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*3^(1/2)*d^2*e+5/18...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d + e x^3) (a + c x^6)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(e*x^3+d)/(c*x^6+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d + e x^3) (a + c x^6)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*x**3+d)/(c*x**6+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (d + e x^3) (a + c x^6)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(e*x^3+d)/(c*x^6+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 962, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^2 (d+ex^3) (a+cx^6)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x^3+d)/(c*x^6+a)^2,x, algorithm="giac")
```

output
```
1/3*e^5*(-d/e)^(2/3)*log(abs(x - (-d/e)^(1/3)))/(c^2*d^6 + 2*a*c*d^4*e^2 +
 a^2*d^2*e^4) + (-d*e^2)^(2/3)*e^3*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/3))
/(-d/e)^(1/3))/(sqrt(3)*c^2*d^6 + 2*sqrt(3)*a*c*d^4*e^2 + sqrt(3)*a^2*d^2*
e^4) - 1/6*(-d*e^2)^(2/3)*e^3*log(x^2 + x*(-d/e)^(1/3) + (-d/e)^(2/3))/(c^
2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4) + 1/36*(4*sqrt(3)*(a*c^5)^(1/3)*a*c^3
*d^2*e + 10*sqrt(3)*(a*c^5)^(1/3)*a^2*c^2*e^3 - 7*(a*c^5)^(5/6)*c*d^3 - 13
*(a*c^5)^(5/6)*a*d*e^2)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a
^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4) - 1/36*(4*sqrt(3)*(a*c^5)^(1
/3)*a*c^3*d^2*e + 10*sqrt(3)*(a*c^5)^(1/3)*a^2*c^2*e^3 + 7*(a*c^5)^(5/6)*c
*d^3 + 13*(a*c^5)^(5/6)*a*d*e^2)*arctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^
(1/6))/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4) - 1/18*(7*c^3*d^3*(
a/c)^(5/6) + 13*a*c^2*d*e^2*(a/c)^(5/6))*arctan(x/(a/c)^(1/6))/(a^3*c^2*d^
4 + 2*a^4*c*d^2*e^2 + a^5*e^4) + 1/72*(4*(a*c^5)^(1/3)*a*c^3*d^2*e + 10*(a
*c^5)^(1/3)*a^2*c^2*e^3 + 7*sqrt(3)*(a*c^5)^(5/6)*c*d^3 + 13*sqrt(3)*(a*c^
5)^(5/6)*a*d*e^2)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^3*c^5*
d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4) + 1/72*(4*(a*c^5)^(1/3)*a*c^3*d^2*e
 + 10*(a*c^5)^(1/3)*a^2*c^2*e^3 - 7*sqrt(3)*(a*c^5)^(5/6)*c*d^3 - 13*sqrt(
3)*(a*c^5)^(5/6)*a*d*e^2)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(
a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4) - 1/18*(2*(a*c^5)^(1/3)*c*d
^2*e + 5*(a*c^5)^(1/3)*a*e^3)*log(x^2 + (a/c)^(1/3))/(a^2*c^3*d^4 + 2*a...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d+ex^3) (a+cx^6)^2} \, dx = \text{Hanged}$$

input
```
int(1/(x^2*(a + c*x^6)^2*(d + e*x^3)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 1847, normalized size of antiderivative = 2.76

$$\int \frac{1}{x^2 (d + ex^3) (a + cx^6)^2} \, dx = \text{Too large to display}$$

input
```
int(1/x^2/(e*x^3+d)/(c*x^6+a)^2,x)
```

output
```
(24*e**(1/3)*c**(1/3)*a**(2/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**
(1/3)*sqrt(3)))*a**3*e**4*x + 24*e**(1/3)*c**(1/3)*a**(2/3)*sqrt(3)*atan((
d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*a**2*c*e**4*x**7 + 26*d**(1/3
)*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6
)*a**(1/6)))*a**2*c*d**2*e**2*x + 14*d**(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/
6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c**2*d**4*x + 2
6*d**(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)
/(c**(1/6)*a**(1/6)))*a*c**2*d**2*e**2*x**7 + 14*d**(1/3)*sqrt(c)*sqrt(a)*
atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c**3*
d**4*x**7 + 20*d**(1/3)*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/
3)*x)/(c**(1/6)*a**(1/6)))*a**3*c*d*e**3*x + 8*d**(1/3)*sqrt(3)*atan((c**(
1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a**2*c**2*d**3*
e*x + 20*d**(1/3)*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/
(c**(1/6)*a**(1/6)))*a**2*c**2*d*e**3*x**7 + 8*d**(1/3)*sqrt(3)*atan((c**(
1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c**3*d**3*e*x
**7 - 26*d**(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(
1/3)*x)/(c**(1/6)*a**(1/6)))*a**2*c*d**2*e**2*x - 14*d**(1/3)*sqrt(c)*sqrt
(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a
*c**2*d**4*x - 26*d**(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3)
 + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c**2*d**2*e**2*x**7 - 14*d**(1/...
```

3.33 $\int \frac{1}{x^5(d+ex^3)(a+cx^6)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 684

$$\int \frac{1}{x^5(d+ex^3)(a+cx^6)^2}\,dx = -\frac{1}{4a^2dx^4}+\frac{e}{a^2d^2x}-\frac{x(c^2dx-c^2ex^4)}{6a^2(cd^2+ae^2)(a+cx^6)}+\frac{c^{7/6}e(7cd^2+13ae^2)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{18a^{13/6}(cd^2+ae^2)^2}-\frac{c^{7/6}e(7cd^2+13ae^2)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{36a^{13/6}(cd^2+ae^2)^2}+\frac{c^{7/6}e(7cd^2+13ae^2)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{36a^{13/6}(cd^2+ae^2)^2}-\frac{e^{16/3}\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{7/3}(cd^2+ae^2)^2}+\frac{c^{5/3}d(5cd^2+8ae^2)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{c}x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{8/3}(cd^2+ae^2)^2}-\frac{c^{7/6}e(7cd^2+13ae^2)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{12\sqrt{3}a^{13/6}(cd^2+ae^2)^2}-\frac{e^{16/3}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{7/3}(cd^2+ae^2)^2}-\frac{c^{5/3}d(5cd^2+8ae^2)\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{18a^{8/3}(cd^2+ae^2)^2}+\frac{e^{16/3}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{7/3}(cd^2+ae^2)^2}+\frac{c^{5/3}d(5cd^2+8ae^2)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{c}x^2+c^{2/3}x^4\right)}{36a^{8/3}(cd^2+ae^2)^2}$$

output
```
-1/4/a^2/d/x^4+e/a^2/d^2/x-1/6*x*(-c^2*e*x^4+c^2*d*x)/a^2/(a*e^2+c*d^2)/(c
*x^6+a)+1/18*c^(7/6)*e*(13*a*e^2+7*c*d^2)*arctan(c^(1/6)*x/a^(1/6))/a^(13/
6)/(a*e^2+c*d^2)^2+1/36*c^(7/6)*e*(13*a*e^2+7*c*d^2)*arctan(-3^(1/2)+2*c^(
1/6)*x/a^(1/6))/a^(13/6)/(a*e^2+c*d^2)^2+1/36*c^(7/6)*e*(13*a*e^2+7*c*d^2)
*arctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(13/6)/(a*e^2+c*d^2)^2-1/3*e^(16/3)
*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)*3^(1/2)/d^(1/3))*3^(1/2)/d^(7/3)/(a*e^2+
c*d^2)^2+1/18*c^(5/3)*d*(8*a*e^2+5*c*d^2)*arctan(1/3*(a^(1/3)-2*c^(1/3)*x^
2)*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)/(a*e^2+c*d^2)^2-1/36*c^(7/6)*e*(13*a*e
^2+7*c*d^2)*arctanh(3^(1/2)*a^(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)*x^2))*3^(1/
2)/a^(13/6)/(a*e^2+c*d^2)^2-1/3*e^(16/3)*ln(d^(1/3)+e^(1/3)*x)/d^(7/3)/(a*
e^2+c*d^2)^2-1/18*c^(5/3)*d*(8*a*e^2+5*c*d^2)*ln(a^(1/3)+c^(1/3)*x^2)/a^(8
/3)/(a*e^2+c*d^2)^2+1/6*e^(16/3)*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)
/d^(7/3)/(a*e^2+c*d^2)^2+1/36*c^(5/3)*d*(8*a*e^2+5*c*d^2)*ln(a^(2/3)-a^(1/
3)*c^(1/3)*x^2+c^(2/3)*x^4)/a^(8/3)/(a*e^2+c*d^2)^2
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^5 (d+ex^3) (a+cx^6)^2} \, dx = \frac{1}{72} \left(-\frac{18}{a^2dx^4} + \frac{72e}{a^2d^2x} - \frac{12c^2x^2(d-ex^3)}{a^2 (cd^2+ae^2) (a+cx^6)} + \frac{4c^{7/6}e(7cd^2+13ae^2) \arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{13/6} (cd^2+ae^2)^2} + \frac{2c^{7/6}\left(10\sqrt{3}c^{3/2}d^3-7\sqrt{a}cd^2e+16\sqrt{3}a\sqrt{c}de^2-13a^{3/2}e^3\right) \arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{8/3} (cd^2+ae^2)^2} + \frac{2c^{7/6}\left(10\sqrt{3}c^{3/2}d^3+7\sqrt{a}cd^2e+16\sqrt{3}a\sqrt{c}de^2+13a^{3/2}e^3\right) \arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{8/3} (cd^2+ae^2)^2} - \frac{24\sqrt{3}e^{16/3} \arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{7/3} (cd^2+ae^2)^2} - \frac{24e^{16/3} \log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{d^{7/3} (cd^2+ae^2)^2} - \frac{4c^{5/3}(5cd^3+8ade^2) \log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{a^{8/3} (cd^2+ae^2)^2} + \frac{c^{7/6}\left(10c^{3/2}d^3+7\sqrt{3}\sqrt{a}cd^2e+16a\sqrt{c}de^2+13\sqrt{3}a^{3/2}e^3\right) \log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{a^{8/3} (cd^2+ae^2)^2} + \frac{c^{7/6}\left(10c^{3/2}d^3-7\sqrt{3}\sqrt{a}cd^2e+16a\sqrt{c}de^2-13\sqrt{3}a^{3/2}e^3\right) \log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{a^{8/3} (cd^2+ae^2)^2} + \frac{12e^{16/3} \log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{d^{7/3} (cd^2+ae^2)^2} \right)$$

input
```
Integrate[1/(x^5*(d + e*x^3)*(a + c*x^6)^2),x]
```

output
```
(-18/(a^2*d*x^4) + (72*e)/(a^2*d^2*x) - (12*c^2*x^2*(d - e*x^3))/(a^2*(c*d
^2 + a*e^2)*(a + c*x^6)) + (4*c^(7/6)*e*(7*c*d^2 + 13*a*e^2)*ArcTan[(c^(1/
6)*x)/a^(1/6)])/(a^(13/6)*(c*d^2 + a*e^2)^2) + (2*c^(7/6)*(10*Sqrt[3]*c^(3
/2)*d^3 - 7*Sqrt[a]*c*d^2*e + 16*Sqrt[3]*a*Sqrt[c]*d*e^2 - 13*a^(3/2)*e^3)
*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)])/(a^(8/3)*(c*d^2 + a*e^2)^2) + (2
*c^(7/6)*(10*Sqrt[3]*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e + 16*Sqrt[3]*a*Sqrt[c
]*d*e^2 + 13*a^(3/2)*e^3)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)])/(a^(8/3
)*(c*d^2 + a*e^2)^2) - (24*Sqrt[3]*e^(16/3)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1
/3))/Sqrt[3]])/(d^(7/3)*(c*d^2 + a*e^2)^2) - (24*e^(16/3)*Log[d^(1/3) + e^
(1/3)*x])/(d^(7/3)*(c*d^2 + a*e^2)^2) - (4*c^(5/3)*(5*c*d^3 + 8*a*d*e^2)*L
og[a^(1/3) + c^(1/3)*x^2])/(a^(8/3)*(c*d^2 + a*e^2)^2) + (c^(7/6)*(10*c^(3
/2)*d^3 + 7*Sqrt[3]*Sqrt[a]*c*d^2*e + 16*a*Sqrt[c]*d*e^2 + 13*Sqrt[3]*a^(3
/2)*e^3)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(a^(8/3)*
(c*d^2 + a*e^2)^2) + (c^(7/6)*(10*c^(3/2)*d^3 - 7*Sqrt[3]*Sqrt[a]*c*d^2*e
+ 16*a*Sqrt[c]*d*e^2 - 13*Sqrt[3]*a^(3/2)*e^3)*Log[a^(1/3) + Sqrt[3]*a^(1/
6)*c^(1/6)*x + c^(1/3)*x^2])/(a^(8/3)*(c*d^2 + a*e^2)^2) + (12*e^(16/3)*Lo
g[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(d^(7/3)*(c*d^2 + a*e^2)^2))
/72
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+cx^6\right)^2 \left(d+ex^3\right)} \, dx$$
$$\Big\downarrow \; 1888$$
$$\int \frac{1}{x^5 \left(a+cx^6\right)^2 \left(d+ex^3\right)}dx$$

input
```
Int[1/(x^5*(d + e*x^3)*(a + c*x^6)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1888
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n)
)^p, x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 962, normalized size of antiderivative = 1.41

method	result	size
default	Expression too large to display	962
risch	Expression too large to display	2588

input
```
int(1/x^5/(e*x^3+d)/(c*x^6+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)
*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3
)*x-1)))*e^6/d^2/(a*e^2+c*d^2)^2-1/4/a^2/d/x^4+e/a^2/d^2/x-1/(a*e^2+c*d^2)
^2*c^2/a^2*(((-1/6*e^3*a-1/6*d^2*e*c)*x^5+(1/6*a*d*e^2+1/6*c*d^3)*x^2)/(c*
x^6+a)-13/72*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)
*e^3-7/72*c/a*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6
)*d^2*e-2/9*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*d*e^2-5/
36*c/a*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*d^3-13/36/c*a
/(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*e^3-7/36/(a/c)^(1/6)*arctan(2
*x/(a/c)^(1/6)-3^(1/2))*d^2*e+4/9*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)-3^(1/
2))*3^(1/2)*d*e^2+5/18*c/a*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*3^(
1/2)*d^3+13/72*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/
6)*e^3+7/72*c/a*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5
/6)*d^2*e-2/9*c/a*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(4/3)*d*
e^2-5/36*c^2/a^2*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(4/3)*d^3
-13/36/c*a/(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*e^3-7/36/(a/c)^(1/6
)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*d^2*e+4/9*c/a*(a/c)^(4/3)*arctan(2*x/(a/
c)^(1/6)+3^(1/2))*3^(1/2)*d*e^2+5/18*c^2/a^2*(a/c)^(4/3)*arctan(2*x/(a/c)^
(1/6)+3^(1/2))*3^(1/2)*d^3-8/9*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)+3^(1/2))
*3^(1/2)*d*e^2-5/9*c/a*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*3^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(d+ex^3\right)\left(a+cx^6\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/x^5/(e*x^3+d)/(c*x^6+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(d+ex^3\right)\left(a+cx^6\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/x**5/(e*x**3+d)/(c*x**6+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5\left(d+ex^3\right)\left(a+cx^6\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(e*x^3+d)/(c*x^6+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 939, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^5\left(d+ex^3\right)\left(a+cx^6\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(e*x^3+d)/(c*x^6+a)^2,x, algorithm="giac")
```

output
```
-1/3*e^6*(-d/e)^(2/3)*log(abs(x - (-d/e)^(1/3)))/(c^2*d^7 + 2*a*c*d^5*e^2
+ a^2*d^3*e^4) - (-d*e^2)^(2/3)*e^4*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/3)
)/(-d/e)^(1/3))/(sqrt(3)*c^2*d^7 + 2*sqrt(3)*a*c*d^5*e^2 + sqrt(3)*a^2*d^3
*e^4) + 1/6*(-d*e^2)^(2/3)*e^4*log(x^2 + x*(-d/e)^(1/3) + (-d/e)^(2/3))/(c
^2*d^7 + 2*a*c*d^5*e^2 + a^2*d^3*e^4) + 1/36*(10*sqrt(3)*(a*c^5)^(1/3)*c^4
*d^3 + 16*sqrt(3)*(a*c^5)^(1/3)*a*c^3*d*e^2 + 7*(a*c^5)^(5/6)*c*d^2*e + 13
*(a*c^5)^(5/6)*a*e^3)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^3
*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4) - 1/36*(10*sqrt(3)*(a*c^5)^(1/
3)*c^4*d^3 + 16*sqrt(3)*(a*c^5)^(1/3)*a*c^3*d*e^2 - 7*(a*c^5)^(5/6)*c*d^2*
e - 13*(a*c^5)^(5/6)*a*e^3)*arctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6)
)/(a^3*c^5*d^4 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4) + 1/18*(7*c^3*d^2*e*(a/c
)^(5/6) + 13*a*c^2*e^3*(a/c)^(5/6))*arctan(x/(a/c)^(1/6))/(a^3*c^2*d^4 + 2
*a^4*c*d^2*e^2 + a^5*e^4) + 1/72*(10*(a*c^5)^(1/3)*c^4*d^3 + 16*(a*c^5)^(1
/3)*a*c^3*d*e^2 - 7*sqrt(3)*(a*c^5)^(5/6)*c*d^2*e - 13*sqrt(3)*(a*c^5)^(5/
6)*a*e^3)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^3*c^5*d^4 + 2*
a^4*c^4*d^2*e^2 + a^5*c^3*e^4) + 1/72*(10*(a*c^5)^(1/3)*c^4*d^3 + 16*(a*c^
5)^(1/3)*a*c^3*d*e^2 + 7*sqrt(3)*(a*c^5)^(5/6)*c*d^2*e + 13*sqrt(3)*(a*c^5
)^(5/6)*a*e^3)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^3*c^5*d^4
 + 2*a^4*c^4*d^2*e^2 + a^5*c^3*e^4) - 1/18*(5*(a*c^5)^(1/3)*c*d^3 + 8*(a*c
^5)^(1/3)*a*d*e^2)*log(x^2 + (a/c)^(1/3))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(d+ex^3\right)\left(a+cx^6\right)^2}\,dx = \text{Hanged}$$

input
```
int(1/(x^5*(a + c*x^6)^2*(d + e*x^3)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 1944, normalized size of antiderivative = 2.84

$$\int \frac{1}{x^5 (d + ex^3) (a + cx^6)^2} \, dx = \text{Too large to display}$$

input
```
int(1/x^5/(e*x^3+d)/(c*x^6+a)^2,x)
```

output
```
( - 24*e**(1/3)*c**(1/3)*a**(2/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(
d**(1/3)*sqrt(3)))*a**3*e**5*x**4 - 24*e**(1/3)*c**(1/3)*a**(2/3)*sqrt(3)*
atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*a**2*c*e**5*x**10 - 26*
d**(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(
c**(1/6)*a**(1/6)))*a**2*c*d**2*e**3*x**4 - 14*d**(1/3)*sqrt(c)*sqrt(a)*at
an((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c**2*
d**4*e*x**4 - 26*d**(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3)
- 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c**2*d**2*e**3*x**10 - 14*d**(1/3)*
sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*
a**(1/6)))*c**3*d**4*e*x**10 + 32*d**(1/3)*sqrt(3)*atan((c**(1/6)*a**(1/6)
*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a**2*c**2*d**3*e**2*x**4 + 2
0*d**(1/3)*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/
6)*a**(1/6)))*a*c**3*d**5*x**4 + 32*d**(1/3)*sqrt(3)*atan((c**(1/6)*a**(1/
6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c**3*d**3*e**2*x**10 + 2
0*d**(1/3)*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/
6)*a**(1/6)))*c**4*d**5*x**10 + 26*d**(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/6)
*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a**2*c*d**2*e**3*x*
*4 + 14*d**(1/3)*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1
/3)*x)/(c**(1/6)*a**(1/6)))*a*c**2*d**4*e*x**4 + 26*d**(1/3)*sqrt(c)*sqrt(
a)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))...
```

3.34 $\int \frac{1}{x^2(d+ex^3)^2(a+cx^6)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 824

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+cx^6)^2} \, dx$$
$$= -\frac{1}{a^2d^2x} - \frac{e^5x^2}{3d^2 (cd^2+ae^2)^2 (d+ex^3)} - \frac{x(2ac^2de(cd^2+ae^2)\, x + c^2(c^2d^4-a^2e^4)\, x^4)}{6a^2 (cd^2+ae^2)^3 (a+cx^6)}$$
$$- \frac{c^{7/6}(7c^2d^4+18acd^2e^2-13a^2e^4) \arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{18a^{13/6} (cd^2+ae^2)^3}$$
$$+ \frac{c^{7/6}(7c^2d^4+18acd^2e^2-13a^2e^4) \arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{36a^{13/6} (cd^2+ae^2)^3}$$
$$- \frac{c^{7/6}(7c^2d^4+18acd^2e^2-13a^2e^4) \arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{36a^{13/6} (cd^2+ae^2)^3}$$
$$+ \frac{4e^{13/3}(4cd^2+ae^2) \arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{3\sqrt{3}d^{7/3} (cd^2+ae^2)^3}$$
$$+ \frac{2c^{5/3}de(cd^2+4ae^2) \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{c}x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3} (cd^2+ae^2)^3}$$
$$+ \frac{c^{7/6}(7c^2d^4+18acd^2e^2-13a^2e^4) \operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{12\sqrt{3}a^{13/6} (cd^2+ae^2)^3}$$
$$+ \frac{4e^{13/3}(4cd^2+ae^2) \log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{7/3} (cd^2+ae^2)^3} - \frac{2c^{5/3}de(cd^2+4ae^2) \log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{9a^{5/3} (cd^2+ae^2)^3}$$
$$- \frac{2e^{13/3}(4cd^2+ae^2) \log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{9d^{7/3} (cd^2+ae^2)^3}$$
$$+ \frac{c^{5/3}de(cd^2+4ae^2) \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{c}x^2+c^{2/3}x^4\right)}{9a^{5/3} (cd^2+ae^2)^3}$$

output
```
-1/a^2/d^2/x-1/3*e^5*x^2/d^2/(a*e^2+c*d^2)^2/(e*x^3+d)-1/6*x*(2*a*c^2*d*e*
(a*e^2+c*d^2)*x+c^2*(-a^2*e^4+c^2*d^4)*x^4)/a^2/(a*e^2+c*d^2)^3/(c*x^6+a)-
1/18*c^(7/6)*(-13*a^2*e^4+18*a*c*d^2*e^2+7*c^2*d^4)*arctan(c^(1/6)*x/a^(1/
6))/a^(13/6)/(a*e^2+c*d^2)^3-1/36*c^(7/6)*(-13*a^2*e^4+18*a*c*d^2*e^2+7*c^
2*d^4)*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(13/6)/(a*e^2+c*d^2)^3-1/36*
c^(7/6)*(-13*a^2*e^4+18*a*c*d^2*e^2+7*c^2*d^4)*arctan(3^(1/2)+2*c^(1/6)*x/
a^(1/6))/a^(13/6)/(a*e^2+c*d^2)^3+4/9*e^(13/3)*(a*e^2+4*c*d^2)*arctan(1/3*
(d^(1/3)-2*e^(1/3)*x)*3^(1/2)/d^(1/3))*3^(1/2)/d^(7/3)/(a*e^2+c*d^2)^3+2/9
*c^(5/3)*d*e*(4*a*e^2+c*d^2)*arctan(1/3*(a^(1/3)-2*c^(1/3)*x^2)*3^(1/2)/a^
(1/3))*3^(1/2)/a^(5/3)/(a*e^2+c*d^2)^3+1/36*c^(7/6)*(-13*a^2*e^4+18*a*c*d^
2*e^2+7*c^2*d^4)*arctanh(3^(1/2)*a^(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)*x^2))*
3^(1/2)/a^(13/6)/(a*e^2+c*d^2)^3+4/9*e^(13/3)*(a*e^2+4*c*d^2)*ln(d^(1/3)+e
^(1/3)*x)/d^(7/3)/(a*e^2+c*d^2)^3-2/9*c^(5/3)*d*e*(4*a*e^2+c*d^2)*ln(a^(1/
3)+c^(1/3)*x^2)/a^(5/3)/(a*e^2+c*d^2)^3-2/9*e^(13/3)*(a*e^2+4*c*d^2)*ln(d^
(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(7/3)/(a*e^2+c*d^2)^3+1/9*c^(5/3)*d
*e*(4*a*e^2+c*d^2)*ln(a^(2/3)-a^(1/3)*c^(1/3)*x^2+c^(2/3)*x^4)/a^(5/3)/(a*
e^2+c*d^2)^3
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 894, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(d+ex^3\right)^2\left(a+cx^6\right)^2}\,dx=\frac{1}{72}\left(-\frac{72}{a^2d^2x}-\frac{24e^5x^2}{\left(cd^3+ade^2\right)^2\left(d+ex^3\right)}\right.$$

$$-\frac{12c^2x^2(cd^2x^3+ae(2d-ex^3))}{a^2\left(cd^2+ae^2\right)^2\left(a+cx^6\right)}+\frac{4c^{7/6}(-7c^2d^4-18acd^2e^2+13a^2e^4)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{13/6}\left(cd^2+ae^2\right)^3}$$

$$+\frac{2c^{7/6}\left(7c^2d^4+8\sqrt{3}\sqrt{a}c^{3/2}d^3e+18acd^2e^2+32\sqrt{3}a^{3/2}\sqrt{c}de^3-13a^2e^4\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{13/6}\left(cd^2+ae^2\right)^3}$$

$$+\frac{2c^{7/6}\left(-7c^2d^4+8\sqrt{3}\sqrt{a}c^{3/2}d^3e-18acd^2e^2+32\sqrt{3}a^{3/2}\sqrt{c}de^3+13a^2e^4\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{13/6}\left(cd^2+ae^2\right)^3}$$

$$+\frac{32\sqrt{3}e^{13/3}(4cd^2+ae^2)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{7/3}\left(cd^2+ae^2\right)^3}+\frac{32e^{13/3}(4cd^2+ae^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{d^{7/3}\left(cd^2+ae^2\right)^3}$$

$$-\frac{16c^{5/3}(cd^3e+4ade^3)\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{a^{5/3}\left(cd^2+ae^2\right)^3}$$

$$+\frac{c^{7/6}\left(-7\sqrt{3}c^2d^4+8\sqrt{a}c^{3/2}d^3e-18\sqrt{3}acd^2e^2+32a^{3/2}\sqrt{c}de^3+13\sqrt{3}a^2e^4\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\cdots\right)}{a^{13/6}\left(cd^2+ae^2\right)^3}$$

$$+\frac{c^{7/6}\left(7\sqrt{3}c^2d^4+8\sqrt{a}c^{3/2}d^3e+18\sqrt{3}acd^2e^2+32a^{3/2}\sqrt{c}de^3-13\sqrt{3}a^2e^4\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\cdots\right)}{a^{13/6}\left(cd^2+ae^2\right)^3}$$

$$\left.-\frac{16e^{13/3}(4cd^2+ae^2)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{d^{7/3}\left(cd^2+ae^2\right)^3}\right)$$

input
```
Integrate[1/(x^2*(d + e*x^3)^2*(a + c*x^6)^2),x]
```

output
```
(-72/(a^2*d^2*x) - (24*e^5*x^2)/((c*d^3 + a*d*e^2)^2*(d + e*x^3)) - (12*c^
2*x^2*(c*d^2*x^3 + a*e*(2*d - e*x^3)))/(a^2*(c*d^2 + a*e^2)^2*(a + c*x^6))
 + (4*c^(7/6)*(-7*c^2*d^4 - 18*a*c*d^2*e^2 + 13*a^2*e^4)*ArcTan[(c^(1/6)*x
)/a^(1/6)])/(a^(13/6)*(c*d^2 + a*e^2)^3) + (2*c^(7/6)*(7*c^2*d^4 + 8*Sqrt[
3]*Sqrt[a]*c^(3/2)*d^3*e + 18*a*c*d^2*e^2 + 32*Sqrt[3]*a^(3/2)*Sqrt[c]*d*e
^3 - 13*a^2*e^4)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)])/(a^(13/6)*(c*d^2
 + a*e^2)^3) + (2*c^(7/6)*(-7*c^2*d^4 + 8*Sqrt[3]*Sqrt[a]*c^(3/2)*d^3*e -
18*a*c*d^2*e^2 + 32*Sqrt[3]*a^(3/2)*Sqrt[c]*d*e^3 + 13*a^2*e^4)*ArcTan[Sqr
t[3] + (2*c^(1/6)*x)/a^(1/6)])/(a^(13/6)*(c*d^2 + a*e^2)^3) + (32*Sqrt[3]*
e^(13/3)*(4*c*d^2 + a*e^2)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/(d
^(7/3)*(c*d^2 + a*e^2)^3) + (32*e^(13/3)*(4*c*d^2 + a*e^2)*Log[d^(1/3) + e
^(1/3)*x])/(d^(7/3)*(c*d^2 + a*e^2)^3) - (16*c^(5/3)*(c*d^3*e + 4*a*d*e^3)
*Log[a^(1/3) + c^(1/3)*x^2])/(a^(5/3)*(c*d^2 + a*e^2)^3) + (c^(7/6)*(-7*Sq
rt[3]*c^2*d^4 + 8*Sqrt[a]*c^(3/2)*d^3*e - 18*Sqrt[3]*a*c*d^2*e^2 + 32*a^(3
/2)*Sqrt[c]*d*e^3 + 13*Sqrt[3]*a^2*e^4)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1
/6)*x + c^(1/3)*x^2])/(a^(13/6)*(c*d^2 + a*e^2)^3) + (c^(7/6)*(7*Sqrt[3]*c
^2*d^4 + 8*Sqrt[a]*c^(3/2)*d^3*e + 18*Sqrt[3]*a*c*d^2*e^2 + 32*a^(3/2)*Sqr
t[c]*d*e^3 - 13*Sqrt[3]*a^2*e^4)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x +
 c^(1/3)*x^2])/(a^(13/6)*(c*d^2 + a*e^2)^3) - (16*e^(13/3)*(4*c*d^2 + a*e^
2)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(d^(7/3)*(c*d^2 + a*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+cx^6)^2 (d+ex^3)^2} \, dx$$

$$\Big\downarrow \text{ 1888}$$

$$\int \frac{1}{x^2 (a+cx^6)^2 (d+ex^3)^2} dx$$

input
```
Int[1/(x^2*(d + e*x^3)^2*(a + c*x^6)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1888
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n)
)^p, x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 1256, normalized size of antiderivative = 1.52

method	result	size
default	Expression too large to display	1256
risch	Expression too large to display	3631

input
```
int(1/x^2/(e*x^3+d)^2/(c*x^6+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-e^5/d^2/(a*e^2+c*d^2)^3*((1/3*a*e^2+1/3*c*d^2)*x^2/(e*x^3+d)+(4/3*a*e^2+1
6/3*c*d^2)*(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-
(d/e)^(1/3)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2
/(d/e)^(1/3)*x-1))))-1/a^2/d^2/x-1/(a*e^2+c*d^2)^3*c^2/a^2*(-13/72*a*ln(x^
2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*e^4+7/72*c^2/a*ln
(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*d^4-1/9*c^2/a*
ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(4/3)*d^3*e+1/2*a/(a/c)^(1
/6)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*d^2*e^2+13/72*a*ln(x^2+3^(1/2)*(a/c)^(
1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*e^4+2/9*c^2/a*(a/c)^(4/3)*arctan(2
*x/(a/c)^(1/6)+3^(1/2))*3^(1/2)*d^3*e-16/9*a*(a/c)^(1/3)*arctan(2*x/(a/c)^
(1/6)+3^(1/2))*3^(1/2)*d*e^3-2/9*c^2/a*(a/c)^(4/3)*arctan(2*x/(a/c)^(1/6)-
3^(1/2))*3^(1/2)*d^3*e+16/9*a*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)-3^(1/2))*
3^(1/2)*d*e^3-4/9*c*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(4/3)*
d*e^3-4/9*c*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(4/3)*d*e^3+2/
9*c*ln(x^2+(a/c)^(1/3))*(a/c)^(1/3)*d^3*e-13/36/c*a^2/(a/c)^(1/6)*arctan(2
*x/(a/c)^(1/6)-3^(1/2))*e^4-13/36/c*a^2/(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)
+3^(1/2))*e^4-13/18/c/(a/c)^(1/6)*arctan(x/(a/c)^(1/6))*a^2*e^4-7/72*c^2/a
*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*d^4-1/9*c^2
/a*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(4/3)*d^3*e+1/2*a/(a/c)
^(1/6)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*d^2*e^2+8/9*ln(x^2+(a/c)^(1/3))*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d + e x^3)^2 (a + c x^6)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(e*x^3+d)^2/(c*x^6+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+cx^6)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*x**3+d)**2/(c*x**6+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+cx^6)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(e*x^3+d)^2/(c*x^6+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1400 vs. $2(696) = 1392$.

Time = 0.20 (sec) , antiderivative size = 1400, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+cx^6)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x^3+d)^2/(c*x^6+a)^2,x, algorithm="giac")
```

output
```
4/9*(4*c*d^2*e^5*(-d/e)^(1/3) + a*e^7*(-d/e)^(1/3))*(-d/e)^(1/3)*log(abs(x
 - (-d/e)^(1/3)))/(c^3*d^9 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4 + a^3*d^3*e
^6) + 4/3*(4*(-d*e^2)^(2/3)*c*d^2*e^3 + (-d*e^2)^(2/3)*a*e^5)*arctan(1/3*s
qrt(3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/(sqrt(3)*c^3*d^9 + 3*sqrt(3)*a*c
^2*d^7*e^2 + 3*sqrt(3)*a^2*c*d^5*e^4 + sqrt(3)*a^3*d^3*e^6) + 1/36*(8*sqrt
(3)*(a*c^5)^(1/3)*a*c^4*d^3*e + 32*sqrt(3)*(a*c^5)^(1/3)*a^2*c^3*d*e^3 - 7
*(a*c^5)^(5/6)*c^2*d^4 - 18*(a*c^5)^(5/6)*a*c*d^2*e^2 + 13*(a*c^5)^(5/6)*a
^2*e^4)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^3*c^6*d^6 + 3*a
^4*c^5*d^4*e^2 + 3*a^5*c^4*d^2*e^4 + a^6*c^3*e^6) - 1/36*(8*sqrt(3)*(a*c^5
)^(1/3)*a*c^4*d^3*e + 32*sqrt(3)*(a*c^5)^(1/3)*a^2*c^3*d*e^3 + 7*(a*c^5)^(
5/6)*c^2*d^4 + 18*(a*c^5)^(5/6)*a*c*d^2*e^2 - 13*(a*c^5)^(5/6)*a^2*e^4)*ar
ctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^3*c^6*d^6 + 3*a^4*c^5*d^4
*e^2 + 3*a^5*c^4*d^2*e^4 + a^6*c^3*e^6) - 1/18*(7*(a*c^5)^(5/6)*c^2*d^4 +
18*(a*c^5)^(5/6)*a*c*d^2*e^2 - 13*(a*c^5)^(5/6)*a^2*e^4)*arctan(x/(a/c)^(1
/6))/(a^3*c^6*d^6 + 3*a^4*c^5*d^4*e^2 + 3*a^5*c^4*d^2*e^4 + a^6*c^3*e^6) +
 1/72*(8*(a*c^5)^(1/3)*a*c^4*d^3*e + 32*(a*c^5)^(1/3)*a^2*c^3*d*e^3 + 7*sq
rt(3)*(a*c^5)^(5/6)*c^2*d^4 + 18*sqrt(3)*(a*c^5)^(5/6)*a*c*d^2*e^2 - 13*sq
rt(3)*(a*c^5)^(5/6)*a^2*e^4)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3)
)/(a^3*c^6*d^6 + 3*a^4*c^5*d^4*e^2 + 3*a^5*c^4*d^2*e^4 + a^6*c^3*e^6) + 1/
72*(8*(a*c^5)^(1/3)*a*c^4*d^3*e + 32*(a*c^5)^(1/3)*a^2*c^3*d*e^3 - 7*sq...
```

Mupad [B] (verification not implemented)

Time = 33.45 (sec) , antiderivative size = 8933, normalized size of antiderivative = 10.84

$$\int \frac{1}{x^2\left(d+ex^3\right)^2\left(a+cx^6\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + c*x^6)^2*(d + e*x^3)^2),x)
```

output
```
symsum(log(x*(33724611035136*a^22*c^37*d^74*e^15 + 874430986125312*a^23*c^
36*d^72*e^17 + 10708105655181312*a^24*c^35*d^70*e^19 + 82771182193360896*a
^25*c^34*d^68*e^21 + 455227037013417984*a^26*c^33*d^66*e^23 + 190263394258
7572224*a^27*c^32*d^64*e^25 + 6297652185145417728*a^28*c^31*d^62*e^27 + 16
968924263031717888*a^29*c^30*d^60*e^29 + 37924527247195889664*a^30*c^29*d^
58*e^31 + 71190789260628197376*a^31*c^28*d^56*e^33 + 113134693228808749056
*a^32*c^27*d^54*e^35 + 152860821960189837312*a^33*c^26*d^52*e^37 + 1758427
28647447560192*a^34*c^25*d^50*e^39 + 172038266723869065216*a^35*c^24*d^48*
e^41 + 142691377526124724224*a^36*c^23*d^46*e^43 + 99795343972090281984*a^
37*c^22*d^44*e^45 + 58399266487648370688*a^38*c^21*d^42*e^47 + 28293949036
888915968*a^39*c^20*d^40*e^49 + 11187742618886529024*a^40*c^19*d^38*e^51 +
 3540205713501536256*a^41*c^18*d^36*e^53 + 872068021338710016*a^42*c^17*d^
34*e^55 + 160544113825185792*a^43*c^16*d^32*e^57 + 20696858328932352*a^44*
c^15*d^30*e^59 + 1659003644952576*a^45*c^14*d^28*e^61 + 61915232231424*a^4
6*c^13*d^26*e^63) + root(28563737812992*a^14*c^17*d^41*e^2*z^9 + 135963391
98984192*a^26*c^5*d^17*e^26*z^9 + 13596339198984192*a^18*c^13*d^33*e^10*z^
9 + 242791771410432*a^29*c^2*d^11*e^32*z^9 + 242791771410432*a^15*c^16*d^3
9*e^4*z^9 + 4855835428208640*a^27*c^4*d^15*e^28*z^9 + 4855835428208640*a^1
7*c^14*d^35*e^8*z^9 + 69438446623383552*a^23*c^8*d^23*e^20*z^9 + 694384466
23383552*a^21*c^10*d^27*e^16*z^9 + 29458734931132416*a^25*c^6*d^19*e^24...
```

Reduce [F]

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+cx^6)^2}\,dx = \int \frac{1}{x^2 (e\,x^3+d)^2 (c\,x^6+a)^2}dx$$

input
```
int(1/x^2/(e*x^3+d)^2/(c*x^6+a)^2,x)
```

output
```
int(1/x^2/(e*x^3+d)^2/(c*x^6+a)^2,x)
```

3.35 $\int \frac{1}{x^5 (d+ex^3)^2 (a+cx^6)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 831

$$\int \frac{1}{x^5 \left(d+ex^3\right)^2 \left(a+cx^6\right)^2}\,dx$$
$$= -\frac{1}{4a^2d^2x^4}+\frac{2e}{a^2d^3x}+\frac{e^6x^2}{3d^3\left(cd^2+ae^2\right)^2\left(d+ex^3\right)}$$
$$-\frac{x(c^2(c^2d^4-a^2e^4)\,x-2c^3de(cd^2+ae^2)\,x^4)}{6a^2\left(cd^2+ae^2\right)^3\left(a+cx^6\right)}$$
$$+\frac{c^{13/6}de(7cd^2+19ae^2)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{9a^{13/6}\left(cd^2+ae^2\right)^3}$$
$$-\frac{c^{13/6}de(7cd^2+19ae^2)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{18a^{13/6}\left(cd^2+ae^2\right)^3}$$
$$+\frac{c^{13/6}de(7cd^2+19ae^2)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{18a^{13/6}\left(cd^2+ae^2\right)^3}$$
$$-\frac{e^{16/3}(19cd^2+7ae^2)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{3\sqrt{3}d^{10/3}\left(cd^2+ae^2\right)^3}$$
$$+\frac{c^{5/3}(5c^2d^4+9acd^2e^2-8a^2e^4)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{c}x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{8/3}\left(cd^2+ae^2\right)^3}$$
$$-\frac{c^{13/6}de(7cd^2+19ae^2)\,\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{6\sqrt{3}a^{13/6}\left(cd^2+ae^2\right)^3}$$
$$-\frac{e^{16/3}(19cd^2+7ae^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{10/3}\left(cd^2+ae^2\right)^3}$$
$$-\frac{c^{5/3}(5c^2d^4+9acd^2e^2-8a^2e^4)\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{18a^{8/3}\left(cd^2+ae^2\right)^3}$$
$$+\frac{e^{16/3}(19cd^2+7ae^2)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{18d^{10/3}\left(cd^2+ae^2\right)^3}$$
$$+\frac{c^{5/3}(5c^2d^4+9acd^2e^2-8a^2e^4)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{c}x^2+c^{2/3}x^4\right)}{36a^{8/3}\left(cd^2+ae^2\right)^3}$$

output

```
-1/4/a^2/d^2/x^4+2*e/a^2/d^3/x+1/3*e^6*x^2/d^3/(a*e^2+c*d^2)^2/(e*x^3+d)-1
/6*x*(c^2*(-a^2*e^4+c^2*d^4)*x-2*c^3*d*e*(a*e^2+c*d^2)*x^4)/a^2/(a*e^2+c*d
^2)^3/(c*x^6+a)+1/9*c^(13/6)*d*e*(19*a*e^2+7*c*d^2)*arctan(c^(1/6)*x/a^(1/
6))/a^(13/6)/(a*e^2+c*d^2)^3+1/18*c^(13/6)*d*e*(19*a*e^2+7*c*d^2)*arctan(-
3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(13/6)/(a*e^2+c*d^2)^3+1/18*c^(13/6)*d*e*(1
9*a*e^2+7*c*d^2)*arctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(13/6)/(a*e^2+c*d^2
)^3-1/9*e^(16/3)*(7*a*e^2+19*c*d^2)*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)*3^(1/
2)/d^(1/3))*3^(1/2)/d^(10/3)/(a*e^2+c*d^2)^3+1/18*c^(5/3)*(-8*a^2*e^4+9*a*
c*d^2*e^2+5*c^2*d^4)*arctan(1/3*(a^(1/3)-2*c^(1/3)*x^2)*3^(1/2)/a^(1/3))*3
^(1/2)/a^(8/3)/(a*e^2+c*d^2)^3-1/18*c^(13/6)*d*e*(19*a*e^2+7*c*d^2)*arctan
h(3^(1/2)*a^(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)*x^2))*3^(1/2)/a^(13/6)/(a*e^2
+c*d^2)^3-1/9*e^(16/3)*(7*a*e^2+19*c*d^2)*ln(d^(1/3)+e^(1/3)*x)/d^(10/3)/(
a*e^2+c*d^2)^3-1/18*c^(5/3)*(-8*a^2*e^4+9*a*c*d^2*e^2+5*c^2*d^4)*ln(a^(1/3
)+c^(1/3)*x^2)/a^(8/3)/(a*e^2+c*d^2)^3+1/18*e^(16/3)*(7*a*e^2+19*c*d^2)*ln
(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(10/3)/(a*e^2+c*d^2)^3+1/36*c^(5
/3)*(-8*a^2*e^4+9*a*c*d^2*e^2+5*c^2*d^4)*ln(a^(2/3)-a^(1/3)*c^(1/3)*x^2+c^
(2/3)*x^4)/a^(8/3)/(a*e^2+c*d^2)^3
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 908, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^5 (d+ex^3)^2 (a+cx^6)^2} \, dx = \frac{1}{36}\left(-\frac{9}{a^2d^2x^4}+\frac{72e}{a^2d^3x}+\frac{12e^6x^2}{d^3(cd^2+ae^2)^2(d+ex^3)} + \frac{6c^2x^2(ae^2-cd(d-2ex^3))}{a^2(cd^2+ae^2)^2(a+cx^6)} + \frac{4c^{13/6}de(7cd^2+19ae^2)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{13/6}(cd^2+ae^2)^3} - \frac{2c^{5/3}\left(-5\sqrt{3}c^2d^4+7\sqrt{a}c^{3/2}d^3e-9\sqrt{3}acd^2e^2+19a^{3/2}\sqrt{c}de^3+8\sqrt{3}a^2e^4\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{8/3}(cd^2+ae^2)^3} + \frac{2c^{5/3}\left(5\sqrt{3}c^2d^4+7\sqrt{a}c^{3/2}d^3e+9\sqrt{3}acd^2e^2+19a^{3/2}\sqrt{c}de^3-8\sqrt{3}a^2e^4\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{8/3}(cd^2+ae^2)^3} - \frac{4\sqrt{3}e^{16/3}(19cd^2+7ae^2)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{10/3}(cd^2+ae^2)^3} - \frac{4e^{16/3}(19cd^2+7ae^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{d^{10/3}(cd^2+ae^2)^3} + \frac{2c^{5/3}(-5c^2d^4-9acd^2e^2+8a^2e^4)\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{a^{8/3}(cd^2+ae^2)^3} + \frac{c^{5/3}\left(5c^2d^4+7\sqrt{3}\sqrt{a}c^{3/2}d^3e+9acd^2e^2+19\sqrt{3}a^{3/2}\sqrt{c}de^3-8a^2e^4\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{a^{8/3}(cd^2+ae^2)^3} + \frac{c^{5/3}\left(5c^2d^4-7\sqrt{3}\sqrt{a}c^{3/2}d^3e+9acd^2e^2-19\sqrt{3}a^{3/2}\sqrt{c}de^3-8a^2e^4\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{a^{8/3}(cd^2+ae^2)^3} + \frac{2e^{16/3}(19cd^2+7ae^2)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{d^{10/3}(cd^2+ae^2)^3}\right)$$

input
```
Integrate[1/(x^5*(d + e*x^3)^2*(a + c*x^6)^2),x]
```

output
```
(-9/(a^2*d^2*x^4) + (72*e)/(a^2*d^3*x) + (12*e^6*x^2)/(d^3*(c*d^2 + a*e^2)
^2*(d + e*x^3)) + (6*c^2*x^2*(a*e^2 - c*d*(d - 2*e*x^3)))/(a^2*(c*d^2 + a*
e^2)^2*(a + c*x^6)) + (4*c^(13/6)*d*e*(7*c*d^2 + 19*a*e^2)*ArcTan[(c^(1/6)
*x)/a^(1/6)])/(a^(13/6)*(c*d^2 + a*e^2)^3) - (2*c^(5/3)*(-5*Sqrt[3]*c^2*d^
4 + 7*Sqrt[a]*c^(3/2)*d^3*e - 9*Sqrt[3]*a*c*d^2*e^2 + 19*a^(3/2)*Sqrt[c]*d
*e^3 + 8*Sqrt[3]*a^2*e^4)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)])/(a^(8/3
)*(c*d^2 + a*e^2)^3) + (2*c^(5/3)*(5*Sqrt[3]*c^2*d^4 + 7*Sqrt[a]*c^(3/2)*d
^3*e + 9*Sqrt[3]*a*c*d^2*e^2 + 19*a^(3/2)*Sqrt[c]*d*e^3 - 8*Sqrt[3]*a^2*e^
4)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)])/(a^(8/3)*(c*d^2 + a*e^2)^3) -
(4*Sqrt[3]*e^(16/3)*(19*c*d^2 + 7*a*e^2)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3)
)/Sqrt[3]])/(d^(10/3)*(c*d^2 + a*e^2)^3) - (4*e^(16/3)*(19*c*d^2 + 7*a*e^2
)*Log[d^(1/3) + e^(1/3)*x])/(d^(10/3)*(c*d^2 + a*e^2)^3) + (2*c^(5/3)*(-5*
c^2*d^4 - 9*a*c*d^2*e^2 + 8*a^2*e^4)*Log[a^(1/3) + c^(1/3)*x^2])/(a^(8/3)*
(c*d^2 + a*e^2)^3) + (c^(5/3)*(5*c^2*d^4 + 7*Sqrt[3]*Sqrt[a]*c^(3/2)*d^3*e
 + 9*a*c*d^2*e^2 + 19*Sqrt[3]*a^(3/2)*Sqrt[c]*d*e^3 - 8*a^2*e^4)*Log[a^(1/
3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(a^(8/3)*(c*d^2 + a*e^2)^3)
 + (c^(5/3)*(5*c^2*d^4 - 7*Sqrt[3]*Sqrt[a]*c^(3/2)*d^3*e + 9*a*c*d^2*e^2 -
 19*Sqrt[3]*a^(3/2)*Sqrt[c]*d*e^3 - 8*a^2*e^4)*Log[a^(1/3) + Sqrt[3]*a^(1/
6)*c^(1/6)*x + c^(1/3)*x^2])/(a^(8/3)*(c*d^2 + a*e^2)^3) + (2*e^(16/3)*(19
*c*d^2 + 7*a*e^2)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(d^(1...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+cx^6)^2 (d+ex^3)^2} dx$$

$$\Big\downarrow \text{ 1888}$$

$$\int \frac{1}{x^5 (a+cx^6)^2 (d+ex^3)^2} dx$$

input
```
Int[1/(x^5*(d + e*x^3)^2*(a + c*x^6)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1888
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n)
)^p, x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 1127, normalized size of antiderivative = 1.36

method	result	size
default	Expression too large to display	1127
risch	Expression too large to display	3837

input
```
int(1/x^5/(e*x^3+d)^2/(c*x^6+a)^2,x,method=_RETURNVERBOSE)
```

output
```
e^6/d^3/(a*e^2+c*d^2)^3*((1/3*a*e^2+1/3*c*d^2)*x^2/(e*x^3+d)+(7/3*a*e^2+19
/3*c*d^2)*(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(
d/e)^(1/3)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/
(d/e)^(1/3)*x-1))))-1/4/a^2/d^2/x^4+2*e/a^2/d^3/x+1/(a*e^2+c*d^2)^3*c^2/a^
2*(-2/9*a*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*e^4+5/18*c
^2/a*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*3^(1/2)*d^4+19/9*a/(a/c)^
(1/6)*arctan(x/(a/c)^(1/6))*d*e^3-19/36*c*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/
c)^(1/3))*3^(1/2)*(a/c)^(5/6)*d*e^3-7/36*c^2/a*ln(x^2+3^(1/2)*(a/c)^(1/6)*
x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*d^3*e+7/36*c^2/a*ln(x^2-3^(1/2)*(a/c)^(
1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5/6)*d^3*e+5/36*c^2/a*ln(x^2+3^(1/2)*(a
/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*d^4+4/9*a*ln(x^2+(a/c)^(1/3))*(a/c)^(
1/3)*e^4-5/18*c^2/a*ln(x^2+(a/c)^(1/3))*(a/c)^(1/3)*d^4-1/2*c*ln(x^2+(a/c)
^(1/3))*(a/c)^(1/3)*d^2*e^2+7/9*c/(a/c)^(1/6)*arctan(x/(a/c)^(1/6))*d^3*e-
2/9*a*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*e^4+5/36*c^2/a
*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*d^4+1/4*c*ln(x^2-3^
(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*(a/c)^(1/3)*d^2*e^2+7/18*c/(a/c)^(1/6)*ar
ctan(2*x/(a/c)^(1/6)-3^(1/2))*d^3*e+1/4*c*ln(x^2+3^(1/2)*(a/c)^(1/6)*x+(a/
c)^(1/3))*(a/c)^(1/3)*d^2*e^2+7/18*c/(a/c)^(1/6)*arctan(2*x/(a/c)^(1/6)+3^
(1/2))*d^3*e+1/2*c*(a/c)^(1/3)*arctan(2*x/(a/c)^(1/6)+3^(1/2))*3^(1/2)*d^2
*e^2+19/36*c*ln(x^2-3^(1/2)*(a/c)^(1/6)*x+(a/c)^(1/3))*3^(1/2)*(a/c)^(5...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(d+ex^3\right)^2 \left(a+cx^6\right)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x^5/(e*x^3+d)^2/(c*x^6+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d + ex^3)^2 (a + cx^6)^2} dx = \text{Timed out}$$

input
```
integrate(1/x**5/(e*x**3+d)**2/(c*x**6+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 (d + ex^3)^2 (a + cx^6)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(e*x^3+d)^2/(c*x^6+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 1352, normalized size of antiderivative = 1.63

$$\int \frac{1}{x^5 (d + ex^3)^2 (a + cx^6)^2} dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(e*x^3+d)^2/(c*x^6+a)^2,x, algorithm="giac")
```

output
```
-1/9*(19*c*d^2*e^6*(-d/e)^(1/3) + 7*a*e^8*(-d/e)^(1/3))*(-d/e)^(1/3)*log(a
bs(x - (-d/e)^(1/3)))/(c^3*d^10 + 3*a*c^2*d^8*e^2 + 3*a^2*c*d^6*e^4 + a^3*
d^4*e^6) - 1/3*(19*(-d*e^2)^(2/3)*c*d^2*e^4 + 7*(-d*e^2)^(2/3)*a*e^6)*arct
an(1/3*sqrt(3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/(sqrt(3)*c^3*d^10 + 3*sq
rt(3)*a*c^2*d^8*e^2 + 3*sqrt(3)*a^2*c*d^6*e^4 + sqrt(3)*a^3*d^4*e^6) + 1/1
8*(5*sqrt(3)*(a*c^5)^(1/3)*c^4*d^4 + 9*sqrt(3)*(a*c^5)^(1/3)*a*c^3*d^2*e^2
 - 8*sqrt(3)*(a*c^5)^(1/3)*a^2*c^2*e^4 + 7*(a*c^5)^(5/6)*c*d^3*e + 19*(a*c
^5)^(5/6)*a*d*e^3)*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^3*c^
5*d^6 + 3*a^4*c^4*d^4*e^2 + 3*a^5*c^3*d^2*e^4 + a^6*c^2*e^6) - 1/18*(5*sqr
t(3)*(a*c^5)^(1/3)*c^4*d^4 + 9*sqrt(3)*(a*c^5)^(1/3)*a*c^3*d^2*e^2 - 8*sqr
t(3)*(a*c^5)^(1/3)*a^2*c^2*e^4 - 7*(a*c^5)^(5/6)*c*d^3*e - 19*(a*c^5)^(5/6
)*a*d*e^3)*arctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^3*c^5*d^6 +
3*a^4*c^4*d^4*e^2 + 3*a^5*c^3*d^2*e^4 + a^6*c^2*e^6) + 1/9*(7*c^4*d^3*e*(a
/c)^(5/6) + 19*a*c^3*d*e^3*(a/c)^(5/6))*arctan(x/(a/c)^(1/6))/(a^3*c^3*d^6
 + 3*a^4*c^2*d^4*e^2 + 3*a^5*c*d^2*e^4 + a^6*e^6) + 1/36*(5*(a*c^5)^(1/3)*
c^4*d^4 + 9*(a*c^5)^(1/3)*a*c^3*d^2*e^2 - 8*(a*c^5)^(1/3)*a^2*c^2*e^4 - 7*
sqrt(3)*(a*c^5)^(5/6)*c*d^3*e - 19*sqrt(3)*(a*c^5)^(5/6)*a*d*e^3)*log(x^2
+ sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^3*c^5*d^6 + 3*a^4*c^4*d^4*e^2 +
3*a^5*c^3*d^2*e^4 + a^6*c^2*e^6) + 1/36*(5*(a*c^5)^(1/3)*c^4*d^4 + 9*(a*c^
5)^(1/3)*a*c^3*d^2*e^2 - 8*(a*c^5)^(1/3)*a^2*c^2*e^4 + 7*sqrt(3)*(a*c^5...
```

Mupad [B] (verification not implemented)

Time = 31.91 (sec) , antiderivative size = 9445, normalized size of antiderivative = 11.37

$$\int \frac{1}{x^5 (d + ex^3)^2 (a + cx^6)^2} dx = \text{Too large to display}$$

input
```
int(1/(x^5*(a + c*x^6)^2*(d + e*x^3)^2),x)
```

output
```
((7*e*x^3)/(4*a*d^2) - 1/(4*a*d) + (x^6*(28*a^3*e^6 - 5*c^3*d^6 + 20*a*c^2
*d^4*e^2 + 45*a^2*c*d^2*e^4))/(12*a^2*d^3*(a*e^2 + c*d^2)^2) + (e*x^9*(23*
c^2*d^2 + 21*a*c*e^2))/(12*a^2*d^2*(a*e^2 + c*d^2)) + (c*e^2*x^12*(7*a^2*e
^4 + 7*c^2*d^4 + 12*a*c*d^2*e^2))/(3*a^2*d^3*(a^2*e^4 + c^2*d^4 + 2*a*c*d^
2*e^2)))/(a*d*x^4 + a*e*x^7 + c*d*x^10 + c*e*x^13) + symsum(log(root(10849
75728490368*a^26*c^8*d^26*e^20*z^9 + 1084975728490368*a^24*c^10*d^30*e^16*
z^9 + 446308403328*a^33*c*d^12*e^34*z^9 + 789073257083904*a^27*c^7*d^24*e^
22*z^9 + 789073257083904*a^23*c^11*d^32*e^14*z^9 + 75872428565760*a^30*c^4
*d^18*e^28*z^9 + 75872428565760*a^20*c^14*d^38*e^8*z^9 + 3793621428288*a^3
2*c^2*d^14*e^32*z^9 + 3793621428288*a^18*c^16*d^42*e^4*z^9 + 4602927332989
44*a^28*c^6*d^22*e^24*z^9 + 460292733298944*a^22*c^12*d^34*e^12*z^9 + 2124
42799984128*a^29*c^5*d^20*e^26*z^9 + 212442799984128*a^21*c^13*d^36*e^10*z
^9 + 1205528587211520*a^25*c^9*d^28*e^18*z^9 + 446308403328*a^17*c^17*d^44
*e^2*z^9 + 20232647617536*a^31*c^3*d^16*e^30*z^9 + 20232647617536*a^19*c^1
5*d^40*e^6*z^9 + 24794911296*a^34*d^10*e^36*z^9 + 24794911296*a^16*c^18*d^
46*z^9 + 14000298710112*a^18*c^10*d^20*e^20*z^6 - 397892002464*a^11*c^17*d
^34*e^6*z^6 - 14292013052304*a^16*c^12*d^24*e^16*z^6 + 65083683088512*a^22
*c^6*d^12*e^28*z^6 - 6275255328*a^10*c^18*d^36*e^4*z^6 + 43492655268864*a^
23*c^5*d^10*e^30*z^6 - 18770360071104*a^15*c^13*d^26*e^14*z^6 - 2152982282
256*a^17*c^11*d^22*e^18*z^6 + 20831704918848*a^24*c^4*d^8*e^32*z^6 - 23...
```

Reduce [F]

$$\int \frac{1}{x^5 (d+ex^3)^2 (a+cx^6)^2} \, dx = \int \frac{1}{x^5 (e\,x^3+d)^2 (c\,x^6+a)^2} dx$$

input
```
int(1/x^5/(e*x^3+d)^2/(c*x^6+a)^2,x)
```

output
```
int(1/x^5/(e*x^3+d)^2/(c*x^6+a)^2,x)
```

3.36 $\int \frac{x^8\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c} - \frac{(b^2cd-2ac^2d-b^3e+3abce)\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}} - \frac{(bcd-b^2e+ace)\log\left(a+bx^3+cx^6\right)}{6c^3}$$

output
```
1/3*(-b*e+c*d)*x^3/c^2+1/6*e*x^6/c-1/3*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)
*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)-1/6*(a*c*e
-b^2*e+b*c*d)*ln(c*x^6+b*x^3+a)/c^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{2c(cd-be)x^3+c^2ex^6+\frac{2(b^2cd-2ac^2d-b^3e+3abce)\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(-bcd+b^2e-ace)\log\left(a+bx^3+cx^6\right)}{6c^3}$$

input
```
Integrate[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(2*c*(c*d - b*e)*x^3 + c^2*e*x^6 + (2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b
*c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*
c*d) + b^2*e - a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (d+ex^3)}{a+bx^3+cx^6} \, dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{3} \int \frac{x^6 (ex^3+d)}{cx^6+bx^3+a} dx^3$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{3} \int \left(\frac{ex^3}{c} + \frac{cd-be}{c^2} - \frac{(-eb^2+cdb+ace) x^3 + a(cd-be)}{c^2 (cx^6+bx^3+a)}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right) (3abce-2ac^2d+b^3(-e)+b^2cd)}{c^3\sqrt{b^2-4ac}} - \frac{(ace+b^2(-e)+bcd) \log(a+bx^3+cx^6)}{2c^3} + \frac{x^3(cd-\ldots}{c^2 \ldots}\right.$$

input
```
Int[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(((c*d - b*e)*x^3)/c^2 + (e*x^6)/(2*c) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3
*a*b*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]
) - ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^3 + c*x^6])/(2*c^3))/3
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{-\frac{1}{2} c e \,x^{6}+b e \,x^{3}-c d \,x^{3}}{3 c^{2}}+\frac{\frac{\left(-a c e +b^{2} e -c b d \right) \ln \left(c \,x^{6}+b \,x^{3}+a \right)}{2 c}+\frac{2 \left(a b e -a c d -\frac{\left(-a c e +b^{2} e -c b d \right) b}{2 c}\right) \arctan \left(\frac{2 c \,x^{3}+b}{\sqrt{4 a c -b^{2}}}\right)}{\sqrt{4 a c -b^{2}}}}{3 c^{2}}$	136
risch	Expression too large to display	2131

input
```
int(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/c^2*(-1/2*c*e*x^6+b*e*x^3-c*d*x^3)+1/3/c^2*(1/2*(-a*c*e+b^2*e-b*c*d)/
c*ln(c*x^6+b*x^3+a)+2*(a*b*e-a*c*d-1/2*(-a*c*e+b^2*e-b*c*d)*b/c)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.26

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx$$
$$= \left[\frac{(b^2c^2-4\,ac^3)ex^6+2\,((b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e)x^3+\sqrt{b^2-4\,ac}((b^2c-2\,ac^2)d-(b^3-3\,abc)e}{6\,(b^2c^3}\right.$$

input
```
integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*((b^2*c^2 - 4*a*c^3)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*
b*c^2)*e)*x^3 + sqrt(b^2 - 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e
)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4*a*
c))/(c*x^6 + b*x^3 + a)) - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4), 1/6*((b^2*c^2 - 4*
a*c^3)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*x^3 - 2*s
qrt(-b^2 + 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*arctan(-(2*c*x
^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3*c - 4*a*b*c^2)*d - (b^4
- 5*a*b^2*c + 4*a^2*c^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(131) = 262$.

Time = 35.29 (sec) , antiderivative size = 620, normalized size of antiderivative = 4.70

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = x^3\left(-\frac{be}{3c^2}+\frac{d}{3c}\right)+\left(-\frac{\sqrt{-4ac+b^2}\cdot(3abce-2ac^2d-b^3e+b^2cd)}{6c^3\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{ace-b^2e+bcd}{6c^3}\right)\log\left(x^3+\frac{2a^2ce-ab^2e+abcd+12ac^3\left(-\frac{\sqrt{-4ac+b^2}\cdot(3abce-2ac^2d-b^3e+b^2cd)}{6c^3\cdot(4ac-b^2)}-\frac{ace-b^2e+b}{6c^3}\right.}{3abce-2ac^2d-b^3e}\right.$$
$$+\left(\frac{\sqrt{-4ac+b^2}\cdot(3abce-2ac^2d-b^3e+b^2cd)}{6c^3\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{ace-b^2e+bcd}{6c^3}\right)\log\left(x^3+\frac{2a^2ce-ab^2e+abcd+12ac^3\left(\frac{\sqrt{-4ac+b^2}\cdot(3abce-2ac^2d-b^3e+b^2cd)}{6c^3\cdot(4ac-b^2)}-\frac{ace-b^2e+bcd}{6c^3}\right.}{3abce-2ac^2d-b^3e+}\right.$$
$$+\frac{ex^6}{6c}$$

input
```
integrate(x**8*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
x**3*(-b*e/(3*c**2) + d/(3*c)) + (-sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c*
*2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*
d)/(6*c**3))*log(x**3 + (2*a**2*c*e - a*b**2*e + a*b*c*d + 12*a*c**3*(-sqr
t(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a
*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3)) - 3*b**2*c**2*(-sqrt(-4*a
*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b
**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3)))/(3*a*b*c*e - 2*a*c**2*d - b**3
*e + b**2*c*d)) + (sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e +
b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3))*log
(x**3 + (2*a**2*c*e - a*b**2*e + a*b*c*d + 12*a*c**3*(sqrt(-4*a*c + b**2)*
(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*
c*e - b**2*e + b*c*d)/(6*c**3)) - 3*b**2*c**2*(sqrt(-4*a*c + b**2)*(3*a*b*
c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b
**2*e + b*c*d)/(6*c**3)))/(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)) +
e*x**6/(6*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{cex^6+2\,cdx^3-2\,bex^3}{6\,c^2} - \frac{(bcd-b^2e+ace)\log\left(cx^6+bx^3+a\right)}{6\,c^3} + \frac{(b^2cd-2\,ac^2d-b^3e+3\,abce)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/6*(c*e*x^6 + 2*c*d*x^3 - 2*b*e*x^3)/c^2 - 1/6*(b*c*d - b^2*e + a*c*e)*lo
g(c*x^6 + b*x^3 + a)/c^3 + 1/3*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*a
rctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 3586, normalized size of antiderivative = 27.17

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6} \, dx = \text{Too large to display}$$

input
```
int((x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

output
```
x^3*(d/(3*c) - (b*e)/(3*c^2)) + (e*x^6)/(6*c) - (log(a + b*x^3 + c*x^6)*(3
*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a
*c^4 - 9*b^2*c^3)) - (atan((4*c^6*(4*a*c - b^2)^(3/2)*(x^3*((b*((b^5*c^3*d
^3 - b^8*e^3 - 2*a*b^3*c^4*d^3 + a^2*b*c^5*d^3 + a^3*c^5*d^2*e - 3*b^6*c^2
*d^2*e - 8*a^2*b^4*c^2*e^3 + 4*a^3*b^2*c^3*e^3 + 5*a*b^6*c*e^3 + 3*b^7*c*d
*e^2 + 9*a*b^4*c^3*d^2*e - 12*a*b^5*c^2*d*e^2 - 4*a^3*b*c^4*d*e^2 - 7*a^2*
b^2*c^4*d^2*e + 14*a^2*b^3*c^3*d*e^2)/c^6 - (((6*a^2*c^7*d^2 + 12*b^4*c^5*
d^2 + 12*b^6*c^3*e^2 - 18*a*b^2*c^6*d^2 - 42*a*b^4*c^4*e^2 + 36*a^2*b^2*c^
5*e^2 - 24*b^5*c^4*d*e + 60*a*b^3*c^5*d*e - 30*a^2*b*c^6*d*e)/c^6 - (((45*
b^3*c^7*d - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^
3*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*
a*c^4 - 9*b^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 1
5*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*
c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((((45*b
^3*c^7*d - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3
*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a
*c^4 - 9*b^2*c^3))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*
c - b^2)^(1/2)) - (9*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*
e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(4*a*c - b
^2)^(1/2)*(36*a*c^4 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b...
```

Reduce [F]

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6} \, dx = \frac{9\left(\int \frac{x^2}{c\,x^6+b\,x^3+a}dx\right) abce - 6\left(\int \frac{x^2}{c\,x^6+b\,x^3+a}dx\right) a\,c^2 d - 3\left(\int \frac{x^2}{c\,x^6+b\,x^3+a}dx\right) b^3 e + 3\left(\int \frac{x^2}{c\,x^6+b\,x^3+a}dx\right) b^2 cd - \text{l}}{6c}$$

input
```
int(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

output
```
(9*int(x**2/(a + b*x**3 + c*x**6),x)*a*b*c*e - 6*int(x**2/(a + b*x**3 + c*
x**6),x)*a*c**2*d - 3*int(x**2/(a + b*x**3 + c*x**6),x)*b**3*e + 3*int(x**
2/(a + b*x**3 + c*x**6),x)*b**2*c*d - log(a + b*x**3 + c*x**6)*a*c*e + log
(a + b*x**3 + c*x**6)*b**2*e - log(a + b*x**3 + c*x**6)*b*c*d - 2*b*c*e*x*
*3 + 2*c**2*d*x**3 + c**2*e*x**6)/(6*c**3)
```

3.37 $\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx=\frac{ex^3}{3c}+\frac{(bcd-b^2e+2ace)\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}$$

output
```
1/3*e*x^3/c+1/3*(2*a*c*e-b^2*e+b*c*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/
2))/c^2/(-4*a*c+b^2)^(1/2)+1/6*(-b*e+c*d)*ln(c*x^6+b*x^3+a)/c^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx=\frac{2cex^3+\frac{2(-bcd+b^2e-2ace)\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}$$

input
```
Integrate[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(2*c*e*x^3 + (2*(-(b*c*d) + b^2*e - 2*a*c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^
2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*d - b*e)*Log[a + b*x^3 + c*x^6])/(6*c
^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{3}\int \frac{x^3(ex^3+d)}{cx^6+bx^3+a}dx^3$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{3}\int \left(\frac{e}{c}-\frac{ae-(cd-be)x^3}{c(cx^6+bx^3+a)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)(2ace+b^2(-e)+bcd)}{c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{2c^2}+\frac{ex^3}{c}\right)$$

input
```
Int[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
((e*x^3)/c + ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4
*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*d - b*e)*Log[a + b*x^3 + c*x^6])/(2*
c^2))/3
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{e\,x^{3}}{3c}+\frac{\frac{(-eb+cd)\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{2c}+\frac{2\left(-ae-\frac{(-eb+cd)b}{2c}\right)\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{3c}$	98
risch	Expression too large to display	1400

input
```
int(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*e*x^3/c+1/3/c*(1/2*(-b*e+c*d)/c*ln(c*x^6+b*x^3+a)+2*(-a*e-1/2*(-b*e+c*
d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.14

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx$$
$$= \left[\frac{2\,(b^2c-4\,ac^2)ex^3+(bcd-(b^2-2\,ac)e)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+((b^2c-}{6\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 + (b*c*d - (b^2 - 2*a*c)*e)*sqrt(b^2 - 4*a
*c)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*
a*c))/(c*x^6 + b*x^3 + a)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log
(c*x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3), 1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 +
 2*(b*c*d - (b^2 - 2*a*c)*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^3 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*
log(c*x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(94) = 188$.

Time = 10.45 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.47

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{be-cd}{6c^2}\right)\log\left(x^3+\frac{-abe-12ac^2\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}-\frac{be-cd}{6c^2}\right)+2acd+3b^2c\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-}{6c^2\cdot(4ac-b^2}\right.}{2ace-b^2e+bcd}\right.$$
$$+\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{be-cd}{6c^2}\right)\log\left(x^3+\frac{-abe-12ac^2\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}-\frac{be-cd}{6c^2}\right)+2acd+3b^2c\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+}{6c^2\cdot(4ac-b^2)}\right.}{2ace-b^2e+bcd}\right.$$
$$+\frac{ex^3}{3c}$$

input
```
integrate(x**5*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
(-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) -
 (b*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(-sqrt(-4*a*c + b**2
)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2
)) + 2*a*c*d + 3*b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(
6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d)
) + (sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)
) - (b*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(sqrt(-4*a*c + b*
*2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c*
*2)) + 2*a*c*d + 3*b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/
(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d
)) + e*x**3/(3*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex^3}{3\,c} + \frac{(cd-be)\log\left(cx^6+bx^3+a\right)}{6\,c^2} - \frac{(bcd-b^2e+2\,ace)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/3*e*x^3/c + 1/6*(c*d - b*e)*log(c*x^6 + b*x^3 + a)/c^2 - 1/3*(b*c*d - b^
2*e + 2*a*c*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c
)*c^2)
```

Mupad [B] (verification not implemented)

Time = 22.39 (sec) , antiderivative size = 2624, normalized size of antiderivative = 27.05

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

output
```
(e*x^3)/(3*c) + (log(a + b*x^3 + c*x^6)*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d
 - 12*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) + (atan((4*c^3*(4*a*c - b^2)^(3/
2)*(x^3*((b*((b^2*c^3*d^3 - b^5*e^3 - a^2*b*c^2*e^3 + a^2*c^3*d*e^2 - 3*b^
3*c^2*d^2*e + 2*a*b^3*c*e^3 + 3*b^4*c*d*e^2 + 2*a*b*c^3*d^2*e - 4*a*b^2*c^
2*d*e^2)/c^3 - (((6*a^2*c^4*e^2 + 12*b^2*c^4*d^2 + 12*b^4*c^2*e^2 - 18*a*b
^2*c^3*e^2 - 24*b^3*c^3*d*e + 18*a*b*c^4*d*e)/c^3 - (((45*b^2*c^5*d - 45*b
^3*c^4*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c
*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*
d - 12*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2
*c*d - 12*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) - (((((45*b^2*c^5*d - 45*b^
3*c^4*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*
d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2
*(4*a*c - b^2)^(1/2)) - (9*b^2*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e + 12*a
*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2
*c^2)))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^(1/2)) + (3*b^2*(2
*a*c*e - b^2*e + b*c*d)^2*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))
/(4*c*(4*a*c - b^2)*(36*a*c^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*(
(((((45*b^2*c^5*d - 45*b^3*c^4*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*
e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e
 - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^(1/2)) - (9*b^2*c*(2*a*c*e - b^...
```

Reduce [F]

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx$$
$$= \frac{-6\left(\int \frac{x^2}{c\,x^6+b\,x^3+a}dx\right)ace+3\left(\int \frac{x^2}{c\,x^6+b\,x^3+a}dx\right)b^2e-3\left(\int \frac{x^2}{c\,x^6+b\,x^3+a}dx\right)bcd-\log(c\,x^6+b\,x^3+a)\,be+\log}{6c^2}$$

input
```
int(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

output
```
( - 6*int(x**2/(a + b*x**3 + c*x**6),x)*a*c*e + 3*int(x**2/(a + b*x**3 + c
*x**6),x)*b**2*e - 3*int(x**2/(a + b*x**3 + c*x**6),x)*b*c*d - log(a + b*x
**3 + c*x**6)*b*e + log(a + b*x**3 + c*x**6)*c*d + 2*c*e*x**3)/(6*c**2)
```

3.38 $\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx=-\frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{e\log(a+bx^3+cx^6)}{6c}$$

output
```
-1/3*(-b*e+2*c*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(
1/2)+1/6*e*ln(c*x^6+b*x^3+a)/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx=\frac{-\frac{2(-2cd+be)\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+e\log(a+bx^3+cx^6)}{6c}$$

input
```
Integrate[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
4*a*c] + e*Log[a + b*x^3 + c*x^6])/(6*c)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1798, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (d+ex^3)}{a+bx^3+cx^6} \, dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{3} \int \frac{ex^3+d}{cx^6+bx^3+a} dx^3$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3} \left(\frac{(2cd-be) \int \frac{1}{cx^6+bx^3+a} dx^3}{2c} + \frac{e \int \frac{2cx^3+b}{cx^6+bx^3+a} dx^3}{2c} \right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3} \left(\frac{e \int \frac{2cx^3+b}{cx^6+bx^3+a} dx^3}{2c} - \frac{(2cd-be) \int \frac{1}{-x^6+b^2-4ac} d(2cx^3+b)}{c} \right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3} \left(\frac{e \int \frac{2cx^3+b}{cx^6+bx^3+a} dx^3}{2c} - \frac{(2cd-be) \text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} \right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3} \left(\frac{e \log(a+bx^3+cx^6)}{2c} - \frac{(2cd-be) \text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} \right)$$

input
```
Int[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(-(((2*c*d - b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 -
4*a*c])) + (e*Log[a + b*x^3 + c*x^6])/(2*c))/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

method	result
default	$\frac{e\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{6c}+\frac{2\left(d-\frac{eb}{2c}\right)\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{3\sqrt{4ac-b^{2}}}$
risch	$\frac{2\ln\left(\left(\left(-4abce+8a\,c^{2}d+b^{3}e-2b^{2}cd+\sqrt{-\left(eb-2cd\right)^{2}\left(4ac-b^{2}\right)}\,b\right)x^{3}+2\sqrt{-\left(eb-2cd\right)^{2}\left(4ac-b^{2}\right)}\,a\right)ae}{3\left(4ac-b^{2}\right)}-\ln\left(\left(-4abce+8a\,c^{2}d+b^{3}e\right.\right.$

input
```
int(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*e*ln(c*x^6+b*x^3+a)/c+2/3*(d-1/2*e*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x^3+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.00

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \left[\frac{(b^2-4\,ac)e\log\left(cx^6+bx^3+a\right)-\sqrt{b^2-4\,ac}(2\,cd-be)\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{6\left(b^2c-4\,ac^2\right)}, \frac{(b^2 -}{}\right.$$

input
```
integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*((b^2 - 4*a*c)*e*log(c*x^6 + b*x^3 + a) - sqrt(b^2 - 4*a*c)*(2*c*d -
b*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4
*a*c))/(c*x^6 + b*x^3 + a)))/(b^2*c - 4*a*c^2), 1/6*((b^2 - 4*a*c)*e*log(c
*x^6 + b*x^3 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^3 +
b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(65) = 130$.

Time = 3.55 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.99

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \left(\frac{e}{6c} - \frac{\sqrt{-4ac+b^2}(be-2cd)}{6c\left(4ac-b^2\right)}\right)\log\left(x^3+\frac{-12ac\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)+2ae+3b^2\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)}{be-2cd}\right.$$
$$+\left(\frac{e}{6c} + \frac{\sqrt{-4ac+b^2}(be-2cd)}{6c\left(4ac-b^2\right)}\right)\log\left(x^3+\frac{-12ac\left(\frac{e}{6c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)+2ae+3b^2\left(\frac{e}{6c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)}{be-2cd}\right.$$

input
```
integrate(x**2*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2)))*log(x**
3 + (-12*a*c*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b*
*2))) + 2*a*e + 3*b**2*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(
4*a*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(6*c) + sqrt(-4*a*c + b**2)*(b*
e - 2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3 + (-12*a*c*(e/(6*c) + sqrt(-4*a*
c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) + 2*a*e + 3*b**2*(e/(6*c) +
sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*
d))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{e\log\left(cx^6+bx^3+a\right)}{6\,c} + \frac{(2\,cd-be)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c}$$

input
```
integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/6*e*log(c*x^6 + b*x^3 + a)/c + 1/3*(2*c*d - b*e)*arctan((2*c*x^3 + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

Mupad [B] (verification not implemented)

Time = 21.82 (sec) , antiderivative size = 1632, normalized size of antiderivative = 22.67

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

output
```
- (log(a + b*x^3 + c*x^6)*(3*b^2*e - 12*a*c*e))/(2*(36*a*c^2 - 9*b^2*c)) -
 (atan((b*(4*a*c - b^2)^(3/2)*(a*c*d*e^2 - a*b*e^3 - ((3*b^2*e - 12*a*c*e)
*(((3*b^2*e - 12*a*c*e)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e
- 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 15*a*b*c*e^
2 - 12*a*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) + ((((b*e - 2*c*d)*(72*a*b*c^2
*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))
/(6*c*(4*a*c - b^2)^(1/2)) + (9*a*b*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)
)/((36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(6*c*(4*a*c -
 b^2)^(1/2)) + (3*a*b*c*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)^2)/(2*(36*a*c^2
 - 9*b^2*c)*(4*a*c - b^2))))/(a^2*c*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^2*e
- 6*b^2*c*d*e^2)) - (4*x^3*((b*(b^2*e^3 + c^2*d^2*e + ((3*b^2*e - 12*a*c*e
)*(6*c^3*d^2 + ((3*b^2*e - 12*a*c*e)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*
c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c))
+ 12*b^2*c*e^2 - 18*b*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) - 2*b*c*d*e^2 - (
(((b*e - 2*c*d)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e - 12*a*c
*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) + (9*b^2*c^2*(3*b^2*
e - 12*a*c*e)*(b*e - 2*c*d))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))
*(b*e - 2*c*d))/(6*c*(4*a*c - b^2)^(1/2)) - (3*b^2*c*(3*b^2*e - 12*a*c*e)*
(b*e - 2*c*d)^2)/(4*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2))))/(4*a^2*c) - ((2*
a*c - b^2)*(((3*b^2*e - 12*a*c*e)*(((b*e - 2*c*d)*(45*b^2*c^2*e - 36*b*...
```

Reduce [F]

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx$$
$$= \frac{-3\left(\int \frac{x^2}{c\,x^6+b\,x^3+a}dx\right)be+6\left(\int \frac{x^2}{c\,x^6+b\,x^3+a}dx\right)cd+\log(c\,x^6+b\,x^3+a)\,e}{6c}$$

input
```
int(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

output
```
( - 3*int(x**2/(a + b*x**3 + c*x**6),x)*b*e + 6*int(x**2/(a + b*x**3 + c*x
**6),x)*c*d + log(a + b*x**3 + c*x**6)*e)/(6*c)
```

3.39 $\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx=\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}$$

output
```
1/3*(-2*a*e+b*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1
/2)+d*ln(x)/a-1/6*d*ln(c*x^6+b*x^3+a)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx$$
$$=\frac{d\log(x)}{a}$$
$$-\frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^3}{b+2c\#1^3}\&\right]}{3a}$$

input
```
Integrate[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

output
```
(d*Log[x])/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[
x - #1] + c*d*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{3}\int \frac{ex^3+d}{x^3\left(cx^6+bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{3}\int \left(\frac{d}{ax^3}+\frac{-cdx^3-bd+ae}{a\left(cx^6+bx^3+a\right)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^3+cx^6\right)}{2a}+\frac{d\log\left(x^3\right)}{a}\right)$$

input
```
Int[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

output
```
(((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x^3])/a - (d*Log[a + b*x^3 + c*x^6])/(2*a))/3
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

method	result
default	$\frac{-\frac{d\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{2}+\frac{2\left(ae-\frac{bd}{2}\right)\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{3a}+\frac{d\ln\left(x\right)}{a}$
risch	$\frac{d\ln\left(x\right)}{a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4a^{2}c-b^{2}a\right)\_Z^{2}+\left(4acd-d\,b^{2}\right)\_Z+a\,e^{2}-bde+c\,d^{2}\right)}\_R\ln\left(\left(\left(-14ac+4b^{2}\right)\_R^{2}+\left(eb-7cd\right)\_R-3e^{2}\right)x^{3}\right.\right.}{3}$

input
```
int((e*x^3+d)/x/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*(-1/2*d*ln(c*x^6+b*x^3+a)+2*(a*e-1/2*b*d)/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^3+b)/(4*a*c-b^2)^(1/2)))+d*ln(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.08

$$\int \frac{d+ex^3}{x\,(a+bx^3+cx^6)}\,dx$$
$$= \left[-\frac{(b^2-4\,ac)d\log\left(cx^6+bx^3+a\right)-6\,(b^2-4\,ac)d\log\left(x\right)+\sqrt{b^2-4\,ac}(bd-2\,ae)\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2}{cx^6+}\right.}{6\,(ab^2-4\,a^2c)}\right.$$
$$-\frac{(b^2-4\,ac)d\log\left(cx^6+bx^3+a\right)-6\,(b^2-4\,ac)d\log\left(x\right)-2\,\sqrt{-b^2+4\,ac}(bd-2\,ae)\arctan\left(-\frac{(2\,cx^3+b)}{b^2-}\right.}{6\,(ab^2-4\,a^2c)}$$

input
```
integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) +
 sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c
- (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)))/(a*b^2 - 4*a^2*c)
, -1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x)
- 2*sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a
*c)/(b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{x\,(a+bx^3+cx^6)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)/x/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}\,dx = -\frac{d\log\left(cx^6+bx^3+a\right)}{6\,a} + \frac{d\log\left(|x|\right)}{a} - \frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a}$$

input
```
integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
-1/6*d*log(c*x^6 + b*x^3 + a)/a + d*log(abs(x))/a - 1/3*(b*d - 2*a*e)*arct
an((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

Mupad [B] (verification not implemented)

Time = 28.54 (sec) , antiderivative size = 4149, normalized size of antiderivative = 53.19

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x)
```

output
```
(d*log(x))/a - (log(a + b*x^3 + c*x^6)*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 -
 36*a^2*c)) - (atan((48*a^4*x^3*(4*a*c - b^2)^2*(((((((3*b^2*d - 12*a*c*d)
*(((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*
(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a
*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4
)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a
*b^2 - 36*a^2*c)) - ((2*a*e - b*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2
*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9
*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*
a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e))/(6*a*(4*a*c - b^2)^(1/2)))*(3*b^2*d -
12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c)) + ((2*a*e - b*d)*(5*b*c^3*e^3 - ((3*b^
2*d - 12*a*c*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3
*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c))
 + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c))
+ 42*b*c^4*d*e))/(2*(9*a*b^2 - 36*a^2*c)) + 7*c^4*d*e^2))/(6*a*(4*a*c - b^
2)^(1/2)) - ((((2*a*e - b*d)*(((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(108*b
^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*
c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*
(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4
*a*c - b^2)^(1/2))))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*...
```

Reduce [F]

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx = \frac{-6\left(\int \frac{1}{c\,x^7+b\,x^4+ax}dx\right)ae+3\left(\int \frac{1}{c\,x^7+b\,x^4+ax}dx\right)bd-\log(c\,x^6+b\,x^3+a)\,e+6\log(x)\,e}{3b}$$

input
```
int((e*x^3+d)/x/(c*x^6+b*x^3+a),x)
```

output
```
( - 6*int(1/(a*x + b*x**4 + c*x**7),x)*a*e + 3*int(1/(a*x + b*x**4 + c*x**
7),x)*b*d - log(a + b*x**3 + c*x**6)*e + 6*log(x)*e)/(3*b)
```

3.40 $\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx = -\frac{d}{3ax^3} - \frac{\left(b^2d-2acd-abe\right)\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} - \frac{(bd-ae)\log(x)}{a^2} + \frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}$$

output
```
-1/3*d/a/x^3-1/3*(-a*b*e-2*a*c*d+b^2*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(
1/2))/a^2/(-4*a*c+b^2)^(1/2)-(-a*e+b*d)*ln(x)/a^2+1/6*(-a*e+b*d)*ln(c*x^6+
b*x^3+a)/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.16

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx = -\frac{d}{3ax^3} + \frac{(-bd+ae)\log(x)}{a^2} + \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{b^2d\log(x-\#1)-acd\log(x-\#1)-abe\log(x-\#1)+bcd\log(x-\#1)\#1^3-ace\log(x-\#1)\#}{b+2c\#1^3}\right.}{3a^2}$$

input
```
Integrate[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/3*d/(a*x^3) + ((-(b*d) + a*e)*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6
 & , (b^2*d*Log[x - #1] - a*c*d*Log[x - #1] - a*b*e*Log[x - #1] + b*c*d*Lo
g[x - #1]*#1^3 - a*c*e*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{3}\int \frac{ex^3+d}{x^6\left(cx^6+bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{3}\int \left(\frac{d}{ax^6}+\frac{c(bd-ae)x^3+b^2d-acd-abe}{a^2\left(cx^6+bx^3+a\right)}+\frac{ae-bd}{a^2x^3}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)\left(-abe-2acd+b^2d\right)}{a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{2a^2}-\frac{\log\left(x^3\right)(bd-ae)}{a^2}-\frac{d}{ax^3}\right)$$

input
```
Int[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

output
```
(-(d/(a*x^3)) - ((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2
- 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x^3])/a^2 + ((b*d -
a*e)*Log[a + b*x^3 + c*x^6])/(2*a^2))/3
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

method	result
default	$-\frac{\frac{(ace-cbd)\ln\left(c\,x^6+b\,x^3+a\right)}{2c}+\frac{2\left(abe+acd-d\,b^2-\frac{(ace-cbd)b}{2c}\right)\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{3a^2}-\frac{d}{3a\,x^3}+\frac{(ae-bd)\ln(x)}{a^2}$
risch	$-\frac{d}{3a\,x^3}+\frac{e\ln(x)}{a}-\frac{\ln(x)bd}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c-a^2b^2\right)\_Z^2+\left(4a^2ce-a\,b^2e-4abcd+b^3d\right)\_Z+ac\,e^2-bcde+c^2d^2\right)}\_R\ln\left(\left(\left(-1\right.\right.\right.}{}$

input
```
int((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2*(1/2*(a*c*e-b*c*d)/c*ln(c*x^6+b*x^3+a)+2*(a*b*e+a*c*d-d*b^2-1/2*(
a*c*e-b*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
-1/3*d/a/x^3+(a*e-b*d)/a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 385, normalized size of antiderivative = 3.44

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx$$
$$= \left[\frac{(abe-(b^2-2\,ac)d)\sqrt{b^2-4\,ac}x^3\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+((b^3-4\,abc)d-(ab^2-4\,a}{6\,(a^2b^2-4\,a^3c}\right.$$

input
```
integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*((a*b*e - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b
*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a
)) + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(c*x^6 + b*x^3 + a)
- 6*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(x) - 2*(a*b^2 - 4*a^
2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^3), 1/6*(2*(a*b*e - (b^2 - 2*a*c)*d)*sqrt(-
b^2 + 4*a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(c*x^6 + b*x^3 + a) - 6*
((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(x) - 2*(a*b^2 - 4*a^2*c)
*d)/((a^2*b^2 - 4*a^3*c)*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)/x**4/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx = \frac{(bd-ae)\log\left(cx^6+bx^3+a\right)}{6\,a^2} - \frac{(bd-ae)\log\left(|x|\right)}{a^2} + \frac{\left(b^2d-2\,acd-abe\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a^2} + \frac{bdx^3-aex^3-ad}{3\,a^2x^3}$$

input
```
integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
1/6*(b*d - a*e)*log(c*x^6 + b*x^3 + a)/a^2 - (b*d - a*e)*log(abs(x))/a^2 +
 1/3*(b^2*d - 2*a*c*d - a*b*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(s
qrt(-b^2 + 4*a*c)*a^2) + 1/3*(b*d*x^3 - a*e*x^3 - a*d)/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 29.85 (sec) , antiderivative size = 7282, normalized size of antiderivative = 65.02

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x)
```

output
```
(log(x)*(a*e - b*d))/a^2 - (log(((((((((a*e - b*d + a^2*(-(a*b*e - b^2*d +
2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((27*b^2*c^3*(a*b*e - b^2*d + a*c*d
))/a + (9*b*c^4*x^3*(2*b^2*d + 7*a*b*e - 28*a*c*d))/a + (9*b^2*c^3*(a*b +
4*b^2*x^3 - 14*a*c*x^3)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^
4*(4*a*c - b^2)))^(1/2)))/(2*a^2)))/(6*a^2) - (3*c^5*d*x^3*(11*b^2*d - 14*
a*b*e + 14*a*c*d))/a^2 + (9*b*c^4*d*(3*a*b*e - 3*b^2*d + a*c*d))/a^2)*(a*e
 - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6
*a^2) + (c^5*d^2*(9*a*b*e - 9*b^2*d + a*c*d))/a^3 + (c^6*d^2*x^3*(7*a*e -
12*b*d))/a^3)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c -
 b^2)))^(1/2)))/(6*a^2) + (c^6*d^3*(a*e - b*d))/a^4 - (c^7*d^4*x^3)/a^4)*(
((((((b*d - a*e + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(
1/2))*((27*b^2*c^3*(a*b*e - b^2*d + a*c*d))/a + (9*b*c^4*x^3*(2*b^2*d + 7*
a*b*e - 28*a*c*d))/a - (9*b^2*c^3*(a*b + 4*b^2*x^3 - 14*a*c*x^3)*(b*d - a*
e + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(2*a^2)
))/(6*a^2) + (3*c^5*d*x^3*(11*b^2*d - 14*a*b*e + 14*a*c*d))/a^2 - (9*b*c^4
*d*(3*a*b*e - 3*b^2*d + a*c*d))/a^2)*(b*d - a*e + a^2*(-(a*b*e - b^2*d + 2
*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6*a^2) + (c^5*d^2*(9*a*b*e - 9*b^2
*d + a*c*d))/a^3 + (c^6*d^2*x^3*(7*a*e - 12*b*d))/a^3)*(b*d - a*e + a^2*(-
(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6*a^2) - (c^6*d^
3*(a*e - b*d))/a^4 + (c^7*d^4*x^3)/a^4))*(3*b^3*d - 3*a*b^2*e + 12*a^2*...
```

Reduce [F]

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx$$

$$= \frac{6\left(\int \frac{1}{2a\,c^2x^{10}-b^2c\,x^{10}+2abc\,x^7-b^3x^7+2a^2c\,x^4-a\,b^2x^4}dx\right)a^2bce\,x^3+12\left(\int \frac{1}{2a\,c^2x^{10}-b^2c\,x^{10}+2abc\,x^7-b^3x^7+2a^2c\,x^4-a\,b^2x^4}dx\right)a\cdots}{}$$

input
```
int((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x)
```

output
```
(6*int(1/(2*a**2*c*x**4 - a*b**2*x**4 + 2*a*b*c*x**7 + 2*a*c**2*x**10 - b*
*3*x**7 - b**2*c*x**10),x)*a**2*b*c*e*x**3 + 12*int(1/(2*a**2*c*x**4 - a*b
**2*x**4 + 2*a*b*c*x**7 + 2*a*c**2*x**10 - b**3*x**7 - b**2*c*x**10),x)*a*
*2*c**2*d*x**3 - 3*int(1/(2*a**2*c*x**4 - a*b**2*x**4 + 2*a*b*c*x**7 + 2*a
*c**2*x**10 - b**3*x**7 - b**2*c*x**10),x)*a*b**3*e*x**3 - 12*int(1/(2*a**
2*c*x**4 - a*b**2*x**4 + 2*a*b*c*x**7 + 2*a*c**2*x**10 - b**3*x**7 - b**2*
c*x**10),x)*a*b**2*c*d*x**3 + 3*int(1/(2*a**2*c*x**4 - a*b**2*x**4 + 2*a*b
*c*x**7 + 2*a*c**2*x**10 - b**3*x**7 - b**2*c*x**10),x)*b**4*d*x**3 - log(
a + b*x**3 + c*x**6)*c*e*x**3 + 6*log(x)*c*e*x**3 + b*e)/(3*x**3*(2*a*c -
b**2))
```

3.41 $\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 723

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$-\frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$+\frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*e*x^2/c-1/6*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*arctan(
1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*
3^(1/2)/c^(5/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/6*(c*d-b*e+(2*a*c*e-b^2*e+b
*c*d)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2
)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/c^(5/3)/(b+(-4*a*c+b^2)^(1/2))^(1
/3)-1/6*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b
^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)^(1/2))
^(1/3)-1/6*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*
c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/c^(5/3)/(b+(-4*a*c+b^2)^(1/
2))^(1/3)+1/12*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*ln((b-(-
4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(
2/3)*c^(2/3)*x^2)*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/12*(c*d-b
*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(2/
3)-2^(1/3)*c^(1/3)*(b+(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(
1/3)/c^(5/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.12

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{3ex^2-2\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{ae\log(x-\#1)-cd\log(x-\#1)\#1^3+be\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{6c}$$

input
```
Integrate[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(3*e*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x
- #1]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.80, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {1826, 27, 1834, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(d+e x^3\right)}{a+b x^3+c x^6} \, dx$$

$$\Big\downarrow \text{ 1826}$$

$$\frac{e x^2}{2 c}-\frac{\int \frac{2 x \left(a e-(c d-b e) x^3\right)}{c x^6+b x^3+a}dx}{2 c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e x^2}{2 c}-\frac{\int \frac{x \left(a e-(c d-b e) x^3\right)}{c x^6+b x^3+a}dx}{c}$$

$$\Big\downarrow \text{ 1834}$$

$$\frac{e x^2}{2 c}-\frac{-\frac{1}{2} \left(-\frac{2 a c e+b^2 (-e)+b c d}{\sqrt{b^2-4 a c}}-b e+c d\right) \int \frac{2 x}{2 c x^3+b-\sqrt{b^2-4 a c}}dx-\frac{1}{2} \left(\frac{2 a c e+b^2 (-e)+b c d}{\sqrt{b^2-4 a c}}-b e+c d\right) \int \frac{2 x}{2 c x^3+b+\sqrt{b^2-4 a c}}dx}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e x^2}{2 c}-\frac{-\left(\left(-\frac{2 a c e+b^2 (-e)+b c d}{\sqrt{b^2-4 a c}}-b e+c d\right) \int \frac{x}{2 c x^3+b-\sqrt{b^2-4 a c}}dx\right)-\left(\frac{2 a c e+b^2 (-e)+b c d}{\sqrt{b^2-4 a c}}-b e+c d\right) \int \frac{x}{2 c x^3+b+\sqrt{b^2-4 a c}}dx}{c}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{e x^2}{2 c}-\frac{-\left(\left(-\frac{2 a c e+b^2 (-e)+b c d}{\sqrt{b^2-4 a c}}-b e+c d\right) \left(\frac{\int \frac{\sqrt[3]{2} \sqrt[3]{c} x+\sqrt[3]{b-\sqrt{b^2-4 a c}}}{2^{2/3} c^{2/3} x^2-\sqrt[3]{2} \sqrt[3]{c} \sqrt[3]{b-\sqrt{b^2-4 a c}} x+\left(b-\sqrt{b^2-4 a c}\right)^{2/3}}dx}{3 \sqrt[3]{2} \sqrt[3]{c} \sqrt[3]{b-\sqrt{b^2-4 a c}}}-\frac{\int \frac{1}{\sqrt[3]{2} \sqrt[3]{c} x+\sqrt[3]{b-\sqrt{\ldots}}}}{3 \sqrt[3]{2} \sqrt[3]{c} \sqrt[3]{b-\sqrt{\ldots}}}\right.\right.}{}$$

$\downarrow$ 16

$$\frac{ex^2}{2c} - \left(\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd \right) \left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4a}}\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{}}} \right.\right.$$

$\downarrow$ 1142

$$\frac{ex^2}{2c} - \left(\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd \right) \left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx + \frac{\int -}{}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}} \right.\right.$$

$\downarrow$ 25

$$\frac{ex^2}{2c} - \left(\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd \right) \left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx - \frac{\int -}{2^{}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}} \right.\right.$$

$\downarrow$ 27

$$\frac{ex^2}{2c}-$$

$$-\left(\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.\right.$$

$\downarrow$ 1082

$$\frac{ex^2}{2c}-$$

$$-\left(\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int\frac{\sqrt[3]{b-}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.\right.$$

$\downarrow$ 217

$$\frac{ex^2}{2c}-$$

$$-\left(\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}}{\sqrt[3]{b-}}}{\sqrt{}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.\right.$$

$$\Big\downarrow \; 1103$$

$$\frac{ex^2}{2c} - \frac{-\left(\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\ldots}{\sqrt[3]{b-\sqrt{\ldots}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\ldots\right)\ldots}{\ldots}$$

input
```
Int[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(e*x^2)/(2*c) - (-((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c
])*(-1/3*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c
^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)
*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) +
Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c]
)^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(
b - Sqrt[b^2 - 4*a*c])^(1/3)))) - (c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/S
qrt[b^2 - 4*a*c])*(-1/3*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3
)*x]/(2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (-((Sqrt[3]*ArcTan[
(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/
3)*c^(1/3))) + Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sq
rt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^
(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))))/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{e\,x^2}{2c}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6 c+\_Z^3 b+a\right)}\frac{\left((eb-cd)\_R^4+ae\_R\right)\ln\left(x-\_R\right)}{2\_R^5 c+\_R^2 b}}{3c}$	70
risch	$\frac{e\,x^2}{2c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6 c+\_Z^3 b+a\right)}\frac{\left((-eb+cd)\_R^4-ae\_R\right)\ln\left(x-\_R\right)}{2\_R^5 c+\_R^2 b}}{3c}$	71

input
```
int(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*e*x^2/c-1/3/c*sum(((b*e-c*d)*_R^4+a*e*_R)/(2*_R^5*c+_R^2*b)*ln(x-_R),_
R=RootOf(_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13535 vs. 2(583) = 1166.

Time = 45.78 (sec) , antiderivative size = 13535, normalized size of antiderivative = 18.72

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \int \frac{(ex^3+d)x^4}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
1/2*e*x^2/c - integrate(-((c*d - b*e)*x^4 - a*e*x)/(c*x^6 + b*x^3 + a), x)
/c
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 69.11 (sec) , antiderivative size = 13112, normalized size of antiderivative = 18.14

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

output
```
log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*
a*c*e^2 - 2*b*c*d*e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8*e^3 + 16
*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^
4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c
^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2
*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b
^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^
2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*
c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d
*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3
*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^8*e
^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a
*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4
8*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^
3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2
- 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c
^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5
*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^
3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*...
```

Reduce [F]

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{-2\left(\int \frac{x^4}{c\,x^6+b\,x^3+a}dx\right)be+2\left(\int \frac{x^4}{c\,x^6+b\,x^3+a}dx\right)cd-2\left(\int \frac{x}{c\,x^6+b\,x^3+a}dx\right)ae+e\,x^2}{2c}$$

input
```
int(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

output
```
( - 2*int(x**4/(a + b*x**3 + c*x**6),x)*b*e + 2*int(x**4/(a + b*x**3 + c*x
**6),x)*c*d - 2*int(x/(a + b*x**3 + c*x**6),x)*a*e + e*x**2)/(2*c)
```

3.42 $\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 718

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex}{c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+ \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+ \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

output
```
e*x/c-1/6*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1
-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(2/3)*3^(1/2
)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/
(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2
))^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/
6*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(1
/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)
+1/6*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)
^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2
/3)-1/12*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+
b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c
^(2/3)*x^2)*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/12*(c*d-b*e+(2*
a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(2/3)-2^(
1/3)*c^(1/3)*(b+(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(2/3)/c
^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.12

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx$$
$$= \frac{ex}{c} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{ae\log(x-\#1)-cd\log(x-\#1)\#1^3+be\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

input
```
Integrate[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(e*x)/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x - #
1]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 550, normalized size of antiderivative = 0.77, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1826, 1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (d+ex^3)}{a+bx^3+cx^6}dx$$

$$\Big\downarrow \text{ 1826}$$

$$\frac{ex}{c}-\frac{\int \frac{ae-(cd-be)x^3}{cx^6+bx^3+a}dx}{c}$$

$$\Big\downarrow \text{ 1752}$$

$$\frac{ex}{c}-\frac{-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{ex}{c}-\frac{-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{\cdots}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\cdots}}{\cdots}}}{3\left(b-\sqrt{b^2-\cdots}\right.}\cdots\right.}{\cdots}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{ex}{c}-\frac{2^{2/3}\int\frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{\cdots}}\right.}{3\sqrt[3]{c}\left(b-\cdots\right.}\right.$$

$\downarrow$ 27

$$-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{ex}{c}-\frac{2\ 2^{2/3}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\cdots}\right.}{3\sqrt[3]{c}\left(\cdots\right.}\right.$$

$\downarrow$ 1142

$$-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{ex}{c}-\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots}{2\sqrt[3]{2}}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\right.$$

$\downarrow$ 25

$$-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\frac{ex}{c}-}{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}}{2\sqrt[3]{2}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$\downarrow$ 27

$$-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\frac{ex}{c}-}{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}}{2\sqrt[3]{2}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$\downarrow$ 1082

$$-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\frac{ex}{c}-\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{}{-\left(1-\frac{2}{\sqrt[3]{b-}}\right.}}{}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}}{}\right.$$

$$\downarrow 217$$

$$-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(\frac{\frac{ex}{c}-\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\frac{\left(1-\frac{}{\sqrt[3]{}}\right.}{}}{}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}}{}\right.$$

$$\Big\downarrow \text{1103}$$

$$\frac{ex}{c} - \frac{-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}} - \frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2 \cdots}\right.\right.}{4\sqrt[3]{c}}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\cdots}$$

input
```
Int[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(e*x)/c - (-1/2*((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])
*((2^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1
/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) -
Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c]
)^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c])^
(2/3)))) - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*((2^
(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(
b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2
^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(
b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/
3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b + Sqrt[b^2 - 4*a*c])^(2/3)
)))/2)/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-eb+cd\right)\_R^{3}-ae\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67
risch	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-eb+cd\right)\_R^{3}-ae\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67

input
```
int(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
e*x/c+1/3/c*sum(((-b*e+c*d)*_R^3-a*e)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf
(_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8705 vs. $2(580) = 1160$.

Time = 3.50 (sec) , antiderivative size = 8705, normalized size of antiderivative = 12.12

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \int \frac{(ex^3+d)x^3}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
e*x/c - integrate(-((c*d - b*e)*x^3 - a*e)/(c*x^6 + b*x^3 + a), x)/c
```

Giac [F]

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \int \frac{(ex^3+d)x^3}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*x^3/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 52.54 (sec) , antiderivative size = 11453, normalized size of antiderivative = 15.95

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

output
```
log((3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^3*
d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3*
e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/
3)*((2^(1/3)*(81*a*c^3*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c -
b^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2
*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b
^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^
2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*
d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4
*a*c - b^2)^3))^(1/3))/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d
^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*
b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*
b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e
+ 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 +
 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - ...
```

Reduce [F]

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{-\left(\int \frac{x^3}{c\,x^6+b\,x^3+a}dx\right)be+\left(\int \frac{x^3}{c\,x^6+b\,x^3+a}dx\right)cd-\left(\int \frac{1}{c\,x^6+b\,x^3+a}dx\right)ae+ex}{c}$$

input
```
int(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

output
```
( - int(x**3/(a + b*x**3 + c*x**6),x)*b*e + int(x**3/(a + b*x**3 + c*x**6)
,x)*c*d - int(1/(a + b*x**3 + c*x**6),x)*a*e + e*x)/c
```

3.43 $\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 634

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$$
$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$- \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

output
```
-1/6*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x
/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/c^(2/3)/(b-(-4*a*c
+b^2)^(1/2))^(1/3)-1/6*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2
*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/
c^(2/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/6*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2
))*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/c^(2/3)/(b-(
-4*a*c+b^2)^(1/2))^(1/3)-1/6*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*ln((b+(-4
*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/c^(2/3)/(b+(-4*a*c+b^2)^
(1/2))^(1/3)+1/12*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(
1/2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)
*x^2)*2^(1/3)/c^(2/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/12*(e-(-b*e+2*c*d)/(-
4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b+(-4*a
*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(1/3)/c^(2/3)/(b+(-4*a*c+b^2
)^(1/2))^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.09

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{RootSum}\left[a+b\#1^3 + c\#1^6\&, \frac{d\log(x-\#1)+e\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]$$

input
```
Integrate[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1
 + 2*c*#1^4) & ]/3
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 532, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1834, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \text{ 1834}$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{2x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^3+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^3+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{ 821}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \text{ 16}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right.}{3\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$$\downarrow 1142$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{2}\ \cdots}{2^{2/3}c^{2/3}x^2-\cdots}}{\cdots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{2}\ \cdots}{2^{2/3}c^{2/3}x^2-\cdots}}{\cdots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$$\downarrow 25$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{\cdots}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{\cdots}}}{\cdots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{\cdots}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{\cdots}}}{\cdots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$$\downarrow 27$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int \frac{\cdots}{2^{2/3}c^{2/3}x^2-\cdots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3}{2}\sqrt[3]{\sqrt{b^2-4ac}+b}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int \frac{\cdots}{2^{2/3}c^{2/3}x^2-\cdots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$$\downarrow 1082$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}-}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-}}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b+\sqrt{b^2-4ac}-}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-}}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

$$\downarrow 217$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

↓ 1103

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\sqrt{b^2-4ac}+b}}\right.$$

input
```
Int[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

output
```
(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b - Sqrt[b^2 - 4*a*c])^(1
/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) +
 (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/
3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(
1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(
1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3))) + (e - (
2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) +
2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (-((S
qrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sq
rt[3]])/(2^(1/3)*c^(1/3))) + Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c
^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(2*2^(1/3)*c
^(1/3)))/(3*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{4}e+d\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	49
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{4}e+d\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	49

input
```
int(x*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((_R^4*e+_R*d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8268 vs. 2(496) = 992.

Time = 12.24 (sec) , antiderivative size = 8268, normalized size of antiderivative = 13.04

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

input
```
integrate(x*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \int \frac{(ex^3+d)x}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \int \frac{(ex^3+d)x}{cx^6+bx^3+a}\,dx$$

input
```
integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 52.26 (sec) , antiderivative size = 7457, normalized size of antiderivative = 11.76

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
int((x*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

output
```
log((2^(1/3)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3
+ a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3
- b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2
) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*
c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - (2^(2/3)*(27*c^3*x*(4*a*c -
b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*b*c^3*(
4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^
3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^
3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1
/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(
a*c^2*(4*a*c - b^2)^3))^(2/3))/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d
^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^
3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(1/3))/6 - 108*a^2*c^4*d^2*e -
45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e
^2 + 108*a^2*b*c^3*d*e^2))/18 + c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)...
```

Reduce [F]

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \left(\int \frac{x^4}{c\,x^6+b\,x^3+a}dx\right)e+\left(\int \frac{x}{c\,x^6+b\,x^3+a}dx\right)d$$

input
```
int(x*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

output
```
int(x**4/(a + b*x**3 + c*x**6),x)*e + int(x/(a + b*x**3 + c*x**6),x)*d
```

3.44 $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 634

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

output
```
-1/6*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x
/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)/c^(1/3)/(b-(-4*a*c
+b^2)^(1/2))^(2/3)-1/6*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2
*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)/
c^(1/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2
))*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(2/3)/c^(1/3)/(b-(
-4*a*c+b^2)^(1/2))^(2/3)+1/6*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*ln((b+(-4
*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(2/3)/c^(1/3)/(b+(-4*a*c+b^2)^
(1/2))^(2/3)-1/12*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(
1/2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)
*x^2)*2^(2/3)/c^(1/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/12*(e-(-b*e+2*c*d)/(-
4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b+(-4*a
*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(2/3)/c^(1/3)/(b+(-4*a*c+b^2
)^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.10

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{RootSum}\left[a+b\#1^3 + c\#1^6\&, \frac{d\log(x-\#1)+e\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]$$

input
```
Integrate[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

output
```
RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1
^2 + 2*c*#1^5) & ]/3
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 512, normalized size of antiderivative = 0.81, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$$

$$\Big\downarrow \ 1752$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx$$

$$\Big\downarrow \ 750$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-}}}{\sqrt[3]{2}}}}{3\left(b-\sqrt{b^2-4ac}\right)^{2}}\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b+\sqrt{b^2-}}}{\sqrt[3]{2}}}}{3\left(\sqrt{b^2-4ac}+b\right)^{2}}\right.$$

$$\Big\downarrow \ 16$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2^{2/3}\int\frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4\right.}}\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int\frac{2\left(2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+}\right.}{3\sqrt[3]{c}\left(\sqrt{b^2-4ac}+\right.}\right.$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2\ 2^{2/3}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-}}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-}\right.}\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\int\frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}}\right.}{3\sqrt[3]{c}\left(\sqrt{b^2-4ac}\right.}\right.$$

$\downarrow$ 1142

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int-\cdots}{2c^{2/\cdots}\cdots}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\cdots\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int-\cdots}{2c^{2/\cdots}\cdots}\cdots\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}\cdots}\cdots\right.$$

$\downarrow$ 25

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int \quad}{2c^{2/3}\ldots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int \quad}{2c^{2/3}\ldots}\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

$$\downarrow 27$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{\sqrt{b^2-4ac}+b}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\right.$$

↓ 1082

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\cdots}{\sqrt[3]{b-\sqrt{b^2-\cdots}}}\right.\cdots}\cdots}{\cdots}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\cdots\right)$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\cdots}{\sqrt[3]{b+\sqrt{b^2-\cdots}}}\right.\cdots}\cdots}{\cdots}\cdots\right)}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}\cdots}\cdots\right)$$

↓ 217

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\cdots}{\sqrt[3]{b-\sqrt{\cdots}}}}{\sqrt{\cdots}}\right)\cdots}{2\sqrt[3]{c}\cdots}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\cdots}{\sqrt[3]{\sqrt{b^2\cdots}}}}{\sqrt{\cdots}}\right)\cdots}{2\sqrt[3]{c}\cdots}\cdots\right)}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}\cdots}$$

↓ 1103

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/\cdots}\right.}{4\sqrt[3]{c}}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/\cdots}\right.}{4\sqrt[3]{c}}\cdots\right.}{3\left(\sqrt{b^2-4ac}+b\right)^{2/3}}\cdots\right.$$

input
```
Int[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

output
```
((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c
])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) +
 (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2
 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) -
2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*
c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/2 + ((e - (2*c*d - b*e)/Sqr
t[b^2 - 4*a*c])*((2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1
/3)*x])/(3*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt
[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[
3]])/c^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sq
rt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b + Sqrt
[b^2 - 4*a*c])^(2/3))))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(\_R^3e+d\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}\right)}{3}$	47
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(\_R^3e+d\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}\right)}{3}$	47

input
```
int((e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((_R^3*e+d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6748 vs. $2(496) = 992$.

Time = 1.44 (sec) , antiderivative size = 6748, normalized size of antiderivative = 10.64

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \int \frac{ex^3+d}{cx^6+bx^3+a}\,dx$$

input
```
integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx=\int \frac{ex^3+d}{cx^6+bx^3+a}\,dx$$

input
```
integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 40.59 (sec) , antiderivative size = 7469, normalized size of antiderivative = 11.78

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx=\text{Too large to display}$$

input
```
int((d + e*x^3)/(a + b*x^3 + c*x^6),x)
```

output
```
log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2
*e^2 - 4*b*c^2*d^3*e) - (2^(2/3)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e
^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(
1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-
(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*((2^(1/3)*(81*c^3*x*(4*a*
c - b^2)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^5*c*d^3
+ a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*
c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*
a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*
a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^
2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/
3))/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*
a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4
8*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(
4*a*c - b^2)^3))^(2/3))/18 - 36*a*c^5*d^3 + 9*b^2*c^4*d^3 + 9*a*b^3*c^2*e^
3 - 36*a^2*b*c^3*e^3 + 108*a^2*c^4*d*e^2 - 27*a*b^2*c^3*d*e^2))/6)*(-(b...
```

Reduce [F]

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx=\left(\int \frac{x^3}{c\,x^6+b\,x^3+a}dx\right)e+\left(\int \frac{1}{c\,x^6+b\,x^3+a}dx\right)d$$

input
```
int((e*x^3+d)/(c*x^6+b*x^3+a),x)
```

output
```
int(x**3/(a + b*x**3 + c*x**6),x)*e + int(1/(a + b*x**3 + c*x**6),x)*d
```

3.45 $\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 653

$$\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx = -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

output
```
-d/a/x+1/6*c^(1/3)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(
1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/a/(b
-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*(d-(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))
*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*
2^(1/3)*3^(1/2)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*(d+(-2*a*e+b*d)
/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^
(1/3)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*(d-(-2*a*e+b*d)/(-4*a*c+b
^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/a/(b
+(-4*a*c+b^2)^(1/2))^(1/3)-1/12*c^(1/3)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2)
)*ln((b-(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-4*a*c+b^2)^(1/2))^(
1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(1/3)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/12*c^
(1/3)*(d-(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(2/3)-
2^(1/3)*c^(1/3)*(b+(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(1/3
)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.13

$$\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx = -\frac{d}{ax} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{3a}$$

input
```
Integrate[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

output
```
-(d/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x
- #1] + c*d*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 547, normalized size of antiderivative = 0.84, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1828, 1834, 27, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}dx$$

$$\Big\downarrow \ 1828$$

$$-\frac{\int \frac{x\left(cdx^3+bd-ae\right)}{cx^6+bx^3+a}dx}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 1834$$

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int \frac{2x}{2cx^3+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 27$$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int \frac{x}{2cx^3+b-\sqrt{b^2-4ac}}dx+c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^3+b+\sqrt{b^2-4ac}}dx}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 821$$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\int \frac{1}{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)+\cdots}{a}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \ 16$$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\ldots}$$

$$\frac{d}{ax}$$

$\Big\downarrow$ 1142

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\int -\frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{\ldots}\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\ldots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\ldots\right)}{\ldots}$$

$$\frac{d}{ax}$$

$\Big\downarrow$ 25

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\int \frac{\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt[3]{\ldots}\right.}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\ldots}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\ldots\right)}{\ldots}$$

$$\frac{d}{ax}$$

$\Big\downarrow$ 27

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\frac{3}{2}\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{b\cdots}}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{\cdots}}\cdots}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\cdots\right.}{\cdots}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \; 1082$$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}-\frac{1}{2}\int \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\cdots}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}\cdots}\cdots}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}\cdots\right.}{\cdots}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \; 217$$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}-2\sqrt[3]{2}\sqrt[3]{c}x}{2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\ldots\right)}{\frac{d}{ax}}$$

$\big\downarrow$ 1103

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{2\sqrt[3]{2}\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{c}}}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\ldots\right)}{\frac{d}{ax}}$$

input
```
Int[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

output
```
-(d/(a*x)) - (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b - Sqrt[
b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b - Sqrt[b^2 -
4*a*c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b
^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b - Sqrt[b^2 - 4*a
*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(
2/3)*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^
(1/3))) + c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*(-1/3*Log[(b + Sqrt[b^2
- 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*
c])^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 -
 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*c^(1/3))) + Log[(b + Sqrt[b^2 - 4*a*c])
^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)
*x^2]/(2*2^(1/3)*c^(1/3)))/(3*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3
))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.11

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(-\_R^{4}cd+\left(ae-bd\right)\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3a}-\frac{d}{ax}$
risch	$-\frac{d}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^{7}c^{3}-48b^{2}c^{2}a^{6}+12b^{4}c\,a^{5}-a^{4}b^{6}\right)\_Z^{6}+\left(-16a^{5}c^{2}e^{3}+8a^{4}b^{2}c\,e^{3}+48a^{4}b\,c^{2}d\,e^{2}+48a^{4}c^{3}d^{2}e-a^{3}b^{4}e^{3}-24a^{3}b^{3}cd\,e\right.\right.}\right.}{}$

input
```
int((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*sum((-_R^4*c*d+(a*e-b*d)*_R)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z
^6*c+_Z^3*b+a))-d/a/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11285 vs. 2(517) = 1034.

Time = 29.04 (sec) , antiderivative size = 11285, normalized size of antiderivative = 17.28

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx=\text{Timed out}$$

input
```
integrate((e*x**3+d)/x**2/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx=\int \frac{ex^3+d}{(cx^6+bx^3+a)x^2}\,dx$$

input
```
integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((c*d*x^4 + (b*d - a*e)*x)/(c*x^6 + b*x^3 + a), x)/a - d/(a*x)
```

Giac [F]

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx=\int \frac{ex^3+d}{(cx^6+bx^3+a)x^2}\,dx$$

input
```
integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 64.24 (sec) , antiderivative size = 11174, normalized size of antiderivative = 17.11

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x)
```

output
```
log((2^(1/3)*(-(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) +
8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3
+ 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e
- 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6
*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/
(a^4*(4*a*c - b^2)^3))^(2/3)*((2^(2/3)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^
2 - 2*a^3*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^
2 + 6*a^2*b*c*d*e) - (27*2^(1/3)*a^10*b*c^3*(4*a*c - b^2)^2*(-(b^7*d^3 - a
^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*
c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5
*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*
e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2
*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3
))/2)*(-(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*...
```

Reduce [F]

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx = \frac{-\left(\int \frac{x^4}{cx^6+bx^3+a}dx\right)cdx+\left(\int \frac{x}{cx^6+bx^3+a}dx\right)aex-\left(\int \frac{x}{cx^6+bx^3+a}dx\right)bdx-d}{ax}$$

input
```
int((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x)
```

output
```
( - int(x**4/(a + b*x**3 + c*x**6),x)*c*d*x + int(x/(a + b*x**3 + c*x**6),
x)*a*e*x - int(x/(a + b*x**3 + c*x**6),x)*b*d*x - d)/(a*x)
```

3.46 $\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 655

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx = -\frac{d}{2ax^2} + \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+ \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+ \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+ \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

output
```
-1/2*d/a/x^2+1/6*c^(2/3)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1
-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(2/3)*3^(1/2
)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*(d-(-2*a*e+b*d)/(-4*a*c+b^2)^
(1/2))*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(
1/2))*2^(2/3)*3^(1/2)/a/(b+(-4*a*c+b^2)^(1/2))^(2/3)-1/6*c^(2/3)*(d+(-2*a*
e+b*d)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)
*x)*2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*c^(2/3)*(d-(-2*a*e+b*d)/(-4
*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(2/3
)/a/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/12*c^(2/3)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)
^(1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-4*a*c+b^2)^(1
/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1
/12*c^(2/3)*(d-(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^
(2/3)-2^(1/3)*c^(1/3)*(b+(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*
2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.14

$$\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx = -\frac{d}{2ax^2} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3a}$$

input
```
Integrate[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/2*d/(a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Lo
g[x - #1] + c*d*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.81, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {1828, 27, 1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}dx$$

$$\Big\downarrow \text{ 1828}$$

$$-\frac{\int \frac{2\left(cdx^3+bd-ae\right)}{cx^6+bx^3+a}dx}{2a}-\frac{d}{2ax^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{cdx^3+bd-ae}{cx^6+bx^3+a}dx}{a}-\frac{d}{2ax^2}$$

$$\Big\downarrow \text{ 1752}$$

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{d}{2ax^2}$$

$$\Big\downarrow \text{ 750}$$

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}d}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}$$

$$\frac{d}{2ax^2}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\cdots\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$$\frac{d}{2ax^2}$$

$\downarrow$ 27

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{\cdots}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$$\frac{d}{2ax^2}$$

$\downarrow$ 1142

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int -\cdots}{2c^{2/3}x^2-\cdots}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.}{\cdots}$$

$$\frac{d}{2ax^2}$$

$\downarrow$ 25

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int \cdots}{2c^{2/3}x^2-2^{2/3}\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots}{\cdots}$$

$$\frac{d}{2ax^2}$$

$\downarrow$ 27

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\,2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{\cdots}{2c^{2/3}x^2-\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots}{\cdots}$$

$$\frac{d}{2ax^2}$$

$\downarrow$ 1082

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^{2}\cdots}}{2\sqrt[3]{\cdots}}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right)}{\cdots}$$

$$\frac{d}{2ax^2}$$

$\downarrow$ 217

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\,2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4a}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{}$$

$$\frac{d}{2ax^2}$$

$\downarrow$ 1103

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/}\right.}{4\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}$$

$$\frac{d}{2ax^2}$$

input
```
Int[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/2*d/(a*x^2) - ((c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(
b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b - Sqrt[b^
2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1
/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b - Sqrt[b^
2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2
/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/2 + (c*
(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c]
)^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) +
(2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2
- 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2
^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c
^(1/3))))/(3*(b + Sqrt[b^2 - 4*a*c])^(2/3))))/2)/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.10

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(-\_R^{3}cd+ae-bd\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3a}-\frac{d}{2a\,x^{2}}$
risch	$-\frac{d}{2a\,x^{2}}+\frac{\left(\_R=\mathrm{RootOf}\left(\left(64c^{3}a^{8}-48b^{2}c^{2}a^{7}+12b^{4}c\,a^{6}-a^{5}b^{6}\right)\_Z^{6}+\left(16a^{5}b\,c^{2}e^{3}+48a^{5}c^{3}d\,e^{2}-8a^{4}b^{3}c\,e^{3}-72a^{4}b^{2}c^{2}d\,e^{2}-96a^{4}b\,c^{3}d^{2}e-16\right.\right.\right.}{}$

input
```
int((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*sum((-_R^3*c*d+a*e-b*d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+
_Z^3*b+a))-1/2*d/a/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11459 vs. 2(517) = 1034.

Time = 16.02 (sec) , antiderivative size = 11459, normalized size of antiderivative = 17.49

$$\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)/x**3/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx = \int \frac{ex^3+d}{(cx^6+bx^3+a)x^3}\,dx$$

input
```
integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((c*d*x^3 + b*d - a*e)/(c*x^6 + b*x^3 + a), x)/a - 1/2*d/(a*x^2)
```

Giac [F]

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx=\int \frac{ex^3+d}{(cx^6+bx^3+a)x^3}\,dx$$

input
```
integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 62.82 (sec) , antiderivative size = 13466, normalized size of antiderivative = 20.56

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx=\text{Too large to display}$$

input
```
int((d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x)
```

output
```
log(- (2^(2/3)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c
 - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d
^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*
c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 +
96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*
d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e
^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c
- b^2)^3))^(1/3)*((2^(1/3)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e - b^2*d +
a*c*d) + (81*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8*d^3 - a^3*b^5*e^3 +
16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a
^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48
*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^
5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4
*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b
^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3...
```

Reduce [F]

$$\int \frac{d + ex^3}{x^3 (a + bx^3 + cx^6)} \, dx = \frac{-2\left(\int \frac{x^3}{c\,x^6+b\,x^3+a}dx\right) cd\,x^2 + 2\left(\int \frac{1}{c\,x^6+b\,x^3+a}dx\right) ae\,x^2 - 2\left(\int \frac{1}{c\,x^6+b\,x^3+a}dx\right) bd\,x^2 - d}{2a\,x^2}$$

input
```
int((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x)
```

output
```
( - 2*int(x**3/(a + b*x**3 + c*x**6),x)*c*d*x**2 + 2*int(1/(a + b*x**3 + c
*x**6),x)*a*e*x**2 - 2*int(1/(a + b*x**3 + c*x**6),x)*b*d*x**2 - d)/(2*a*x
**2)
```

3.47 $\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^6}{6} - \frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
-1/6*x^6-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)+1/6*ln(x^6-x^3+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^6}{6} + \frac{\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{6}\log\left(1-x^3+x^6\right)$$

input
```
Integrate[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/6*x^6 + ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(1-x^3)}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{3}\int \frac{x^6(1-x^3)}{x^6-x^3+1}dx^3$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{3}\int \left(\frac{x^3}{x^6-x^3+1}-x^3\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{x^6}{2}+\frac{1}{2}\log\left(x^6-x^3+1\right)\right)$$

input
```
Int[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
(-1/2*x^6 - ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/2)/3
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{x^6}{6}+\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	38
risch	$-\frac{x^6}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}+\frac{\ln(4x^6-4x^3+4)}{6}$	40

input
```
int(x^8*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x^6+1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{6}\,x^6+\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^6}{6} + \frac{\log(x^6-x^3+1)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**8*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-x**6/6 + log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)
/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{6}\,x^6 + \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right) + \frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
 + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{6}\,x^6 + \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right) + \frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = \frac{\ln\left(x^6-x^3+1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9} - \frac{x^6}{6}$$

input
```
int(-(x^8*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - x
^6/6
```

Reduce [F]

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = \frac{\left(\int \frac{x^2}{x^6-x^3+1}dx\right)}{2} + \frac{\log(x^6-x^3+1)}{6} - \frac{x^6}{6}$$

input
```
int(x^8*(-x^3+1)/(x^6-x^3+1),x)
```

output
```
(3*int(x**2/(x**6 - x**3 + 1),x) + log(x**6 - x**3 + 1) - x**6)/6
```

3.48 $\int \frac{x^5(1-x^3)}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6} \, dx = -\frac{x^3}{3} - \frac{2\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
-1/3*x^3-2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6} \, dx = -\frac{x^3}{3} + \frac{2\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Integrate[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/3*x^3 + (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (1-x^3)}{x^6-x^3+1} \, dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{3} \int \frac{x^3 (1-x^3)}{x^6-x^3+1} dx^3$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{3} \int \left(\frac{1}{x^6-x^3+1}-1\right) dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(-\frac{2 \arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-x^3\right)$$

input
```
Int[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
(-x^3 - (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/Sqrt[3])/3
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}$	25
risch	$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}$	25

input
```
int(x^5*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3+2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx=-\frac{1}{3}\,x^3+\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

input
```
integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^3}{3} + \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**5*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-x**3/3 + 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\,x^3 + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

input
```
integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\,x^3 + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

input
```
integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}-\frac{x^3}{3}$$

input
```
int(-(x^5*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
- (2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - x^3/3
```

Reduce [F]

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = \int \frac{x^2}{x^6-x^3+1}dx-\frac{x^3}{3}$$

input
```
int(x^5*(-x^3+1)/(x^6-x^3+1),x)
```

output
```
(3*int(x**2/(x**6 - x**3 + 1),x) - x**3)/3
```

3.49 $\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx=-\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)-1/6*ln(x^6-x^3+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx=\frac{\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

input
```
Integrate[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1798, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-x^3)}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{3}\int \frac{1-x^3}{x^6-x^3+1}dx^3$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^6-x^3+1}dx^3-\frac{1}{2}\int -\frac{1-2x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^6-x^3+1}dx^3+\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3-\int \frac{1}{-x^6-3}d(2x^3-1)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3+\frac{\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\frac{\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^6-x^3+1\right)\right)$$

input
```
Int[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3] - Log[1 - x^3 + x^6]/2)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	33
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}-\frac{\ln(4x^6-4x^3+4)}{6}$	35

input
```
int(x^2*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^3-1)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = -\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**2*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = -\frac{\ln\left(x^6-x^3+1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

input
```
int(-(x^2*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
- log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

Reduce [F]

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = \frac{\left(\int \frac{x^2}{x^6-x^3+1}dx\right)}{2} - \frac{\log(x^6-x^3+1)}{6}$$

input
```
int(x^2*(-x^3+1)/(x^6-x^3+1),x)
```

output
```
(3*int(x**2/(x**6 - x**3 + 1),x) - log(x**6 - x**3 + 1))/6
```

3.50 $\int \frac{1-x^3}{x(1-x^3+x^6)} dx$

Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{1-x^3}{x(1-x^3+x^6)} dx = \frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)+ln(x)-1/6*ln(x^6-x^3+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1-x^3}{x(1-x^3+x^6)} dx = \log(x) - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

input
```
Integrate[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
```

output
```
Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^3) & ]/
3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^3}{x\left(x^6-x^3+1\right)}dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{3}\int \frac{1-x^3}{x^3\left(x^6-x^3+1\right)}dx^3$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{3}\int \left(\frac{1}{x^3}-\frac{x^3}{x^6-x^3+1}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^3\right)-\frac{1}{2}\log\left(x^6-x^3+1\right)\right)$$

input
```
Int[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
```

output
```
(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[x^3] - Log[1 - x^3 + x^6]/2)/3
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln\left(x\right)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{3}-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}$	33
default	$-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}+\ln\left(x\right)$	35

input
```
int((-x^3+1)/x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/9*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))-1/6*ln(x^6-x^3+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx=-\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)-\frac{1}{6}\,\log\left(x^6-x^3+1\right)+\log\left(x\right)$$

input
```
integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + lo
g(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx = \log\left(x\right) - \frac{\log\left(x^6-x^3+1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((-x**3+1)/x/(x**6-x**3+1),x)
```

output
```
log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \frac{1}{3}\log\left(x^3\right)$$

input
```
integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/
3*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log\left(|x|\right)$$

input
```
integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + lo
g(abs(x))
```

Mupad [B] (verification not implemented)

Time = 20.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1-x^3}{x\,(1-x^3+x^6)}\,dx = \ln(x) - \frac{\ln(x^6-x^3+1)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

input
```
int(-(x^3 - 1)/(x*(x^6 - x^3 + 1)),x)
```

output
```
log(x) - log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/
3))/9
```

Reduce [F]

$$\int \frac{1-x^3}{x\,(1-x^3+x^6)}\,dx = -\left(\int \frac{1}{x^7-x^4+x}dx\right) - \frac{\log(x^6-x^3+1)}{3} + 2\log(x)$$

input
```
int((-x^3+1)/x/(x^6-x^3+1),x)
```

output
```
( - 3*int(1/(x**7 - x**4 + x),x) - log(x**6 - x**3 + 1) + 6*log(x))/3
```

3.51 $\int \frac{1-x^3}{x^4(1-x^3+x^6)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=-\frac{1}{3x^3}+\frac{2\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
-1/3/x^3+2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=-\frac{1}{3x^3}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-1+2\#1^3}\&\right]$$

input
```
Integrate[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
```

output
```
-1/3*1/x^3 - RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-1 + 2*#1^3) & ]/3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^3}{x^4 (x^6-x^3+1)} \, dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{3} \int \frac{1-x^3}{x^6 (x^6-x^3+1)}dx^3$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{3} \int \left(\frac{1}{x^6}+\frac{1}{-x^6+x^3-1}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{2 \arctan \left(\frac{1-2 x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{x^3}\right)$$

input
```
Int[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
```

output
```
(-x^(-3) + (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/Sqrt[3])/3
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^{3}}$	25
risch	$-\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^{3}}$	25

input
```
int((-x^3+1)/x^4/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))-1/3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=-\frac{2\sqrt{3}x^3\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^3-1)\right)+3}{9\,x^3}$$

input
```
integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/9*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx = -\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

input
```
integrate((-x**3+1)/x**4/(x**6-x**3+1),x)
```

output
```
-2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9 - 1/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx = -\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)-\frac{1}{3x^3}$$

input
```
integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx = -\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)-\frac{1}{3x^3}$$

input
```
integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

Mupad [B] (verification not implemented)

Time = 20.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}-\frac{1}{3\,x^3}$$

input
```
int(-(x^3 - 1)/(x^4*(x^6 - x^3 + 1)),x)
```

output
```
(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - 1/(3*x^3)
```

Reduce [F]

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=\frac{6\left(\int \frac{1}{x^{10}-x^7+x^4}dx\right)x^3+\log\left(x^6-x^3+1\right)x^3-6\log(x)\,x^3+1}{3x^3}$$

input
```
int((-x^3+1)/x^4/(x^6-x^3+1),x)
```

output
```
(6*int(1/(x**10 - x**7 + x**4),x)*x**3 + log(x**6 - x**3 + 1)*x**3 - 6*log
(x)*x**3 + 1)/(3*x**3)
```

3.52 $\int \frac{x^6(1-x^3)}{1-x^3+x^6} dx$

Optimal result

Integrand size = 23, antiderivative size = 418

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^4}{4} - \frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{(3+i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(3-i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

output

```
-1/4*x^4-1/6*(3^(1/2)+I)*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*3^(1
/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/6*(I-3^(1/2))*arctan(1/3*(1+2*x/(1/2+1/
2*I*3^(1/2))^(1/3))*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/18*(3+I*3^(1/2)
)*ln((1-I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/18*(3-I*
3^(1/2))*ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/3
6*(3+I*3^(1/2))*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2
)*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/36*(3-I*3^(1/2))*ln((1+I*3^(1/2))^(2/3)+(2
+2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.11

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx=-\frac{x^4}{4}+\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{-1+2\#1^3}\&\right]$$

input
```
Integrate[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/4*x^4 + RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/
3
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1826, 27, 1710, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(1-x^3\right)}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{1826}$$

$$-\frac{1}{4}\int -\frac{4x^3}{x^6-x^3+1}dx-\frac{x^4}{4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^3}{x^6-x^3+1}dx-\frac{x^4}{4}$$

$$\Big\downarrow \text{1710}$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx-\frac{x^4}{4}$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{x^4}{4}$$

$$\Big\downarrow \; 16$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{x^4}{4}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)+\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{x^4}{4}$$

$\big\downarrow$ 1142

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int \cdots}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int \cdots}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{x^4}{4}$$

$\big\downarrow$ 1082

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{x^4}{4}$$

$\downarrow$ 217

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{\cdots}{\sqrt[3]{\frac{1}{2}\left(1\cdots\right.}}}{\sqrt{\cdots}}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{\cdots}{\sqrt[3]{\frac{1}{2}\left(1\cdots\right.}}}{\sqrt{\cdots}}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{x^4}{4}$$

$\downarrow$ 1103

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{x^4}{4}$$

input
```
Int[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/4*x^4 + ((3 + I*Sqrt[3])*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1
 - I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(
1/3))/Sqrt[3]] + Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x +
 2^(2/3)*x^2]/2)/(3*((1 - I*Sqrt[3])/2)^(2/3))))/6 + ((3 - I*Sqrt[3])*(Log
[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqrt[
3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sqrt
[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt
[3])/2)^(2/3))))/6
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.11

method	result	size
default	$-\frac{x^4}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	46
risch	$-\frac{x^4}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	46

input
```
int(x^6*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^4+1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.53

$$
\begin{aligned}
\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx
&= -\frac{1}{4}\,x^4 \\
&\quad -\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(3\sqrt{-\frac{1}{3}}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)+2\,x\right) \\
&\quad -\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(-3\sqrt{-\frac{1}{3}}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right. \\
&\qquad \left.+2\,x\right) \\
&\quad +\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(-3\sqrt{-\frac{1}{3}}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\right. \\
&\qquad \left.+2\,x\right) \\
&\quad +\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(3\sqrt{-\frac{1}{3}}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\right. \\
&\qquad \left.+2\,x\right)+\frac{1}{3}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\log\left(x-3\sqrt{-\frac{1}{3}}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\right) \\
&\quad +\frac{1}{3}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\log\left(x+3\sqrt{-\frac{1}{3}}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\right)
\end{aligned}
$$

input
```
integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/4*x^4 - 1/6*(1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) + 1)*log(3*sqrt(-1/3
)*(1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) + 1) + 2*x) - 1/6*(-1/6*sqrt(-1/3
) - 1/6)^(1/3)*(sqrt(-3) + 1)*log(-3*sqrt(-1/3)*(-1/6*sqrt(-1/3) - 1/6)^(1
/3)*(sqrt(-3) + 1) + 2*x) + 1/6*(1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) - 1
)*log(-3*sqrt(-1/3)*(1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) - 1) + 2*x) + 1
/6*(-1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) - 1)*log(3*sqrt(-1/3)*(-1/6*sqr
t(-1/3) - 1/6)^(1/3)*(sqrt(-3) - 1) + 2*x) + 1/3*(1/6*sqrt(-1/3) - 1/6)^(1
/3)*log(x - 3*sqrt(-1/3)*(1/6*sqrt(-1/3) - 1/6)^(1/3)) + 1/3*(-1/6*sqrt(-1
/3) - 1/6)^(1/3)*log(x + 3*sqrt(-1/3)*(-1/6*sqrt(-1/3) - 1/6)^(1/3))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.07

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^4}{4} - \operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(-1458t^4 + 9t + x\right)\right)\right)$$

input
```
integrate(x**6*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-x**4/4 - RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t*
*4 + 9*_t + x)))
```

Maxima [F]

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = \int -\frac{(x^3-1)x^6}{x^6-x^3+1}\,dx$$

input
```
integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/4*x^4 + integrate(x^3/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(260) = 520.

Time = 0.13 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.54

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/4*x^4 - 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9
*pi)^2 + 2*sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9
*pi)*sin(4/9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(1/2*((-I*sq
rt(3) - 1)*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(
2*sqrt(3)*cos(2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(
3)*sin(2/9*pi)^4 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)
^3 + sqrt(3)*cos(2/9*pi) + sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2
/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/
9*pi)^4 - 12*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4
 - 8*cos(1/9*pi)^3*sin(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos
(1/9*pi) + sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/(
(1/2*I*sqrt(3) + 1/2)*sin(1/9*pi))) - 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4/
9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 + 12*cos(4/9
*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9*pi) - cos(4/9*pi)
)*log((-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3
)*cos(2/9*pi)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(
2/9*pi)^4 + 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin
(2/9*pi) - cos(2/9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2
 + 1) + 1/18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*
sin(1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*si...
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.79

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+\frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{x^4}{4}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(-(x^6*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*12i -
 36)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/6)*(
3^(1/2)*12i - 36)^(1/3))/18 - x^4/4 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(-
 3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(4/3))/12)
*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2
^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i -
3)^(4/3))/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)
*log(x - (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/4 - (2^(2/3)*3^(5/6)*(
- 3^(1/2)*1i - 3)^(1/3)*1i)/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6
)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/4 +
(2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^
(1/3) + 3^(5/6)*1i))/36
```

Reduce [F]

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = \int \frac{x^3}{x^6-x^3+1}dx - \frac{x^4}{4}$$

input
```
int(x^6*(-x^3+1)/(x^6-x^3+1),x)
```

output
```
(4*int(x**3/(x**6 - x**3 + 1),x) - x**4)/4
```

3.53 $\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 382

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^2}{2} + \frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} - \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}} + \frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}$$

output

```
-1/2*x^2+1/3*I*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*3^(1/2))/(1/2-
1/2*I*3^(1/2))^(1/3)-1/3*I*arctan(1/3*(1+2*x/(1/2+1/2*I*3^(1/2))^(1/3))*3^
(1/2))/(1/2+1/2*I*3^(1/2))^(1/3)+1/9*I*ln((1-I*3^(1/2))^(1/3)-2^(1/3)*x)*3
^(1/2)/(1/2-1/2*I*3^(1/2))^(1/3)-1/9*I*ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*3
^(1/2)/(1/2+1/2*I*3^(1/2))^(1/3)-1/18*I*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1
/2))^(1/3)*x+2^(2/3)*x^2)*2^(1/3)*3^(1/2)/(1-I*3^(1/2))^(1/3)+1/18*I*ln((1
+I*3^(1/2))^(2/3)+(2+2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(1/3)*3^(1/2)/(1+
I*3^(1/2))^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.13

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx=-\frac{x^2}{2}+\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-\#1+2\#1^4}\&\right]$$

input
```
Integrate[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/2*x^2 + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.435$, Rules used = {1826, 27, 1711, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(1-x^3)}{x^6-x^3+1}\,dx$$

$$\Big\downarrow \; 1826$$

$$-\frac{1}{2}\int -\frac{2x}{x^6-x^3+1}dx-\frac{x^2}{2}$$

$$\Big\downarrow \; 27$$

$$\int \frac{x}{x^6-x^3+1}dx-\frac{x^2}{2}$$

$$\Big\downarrow \; 1711$$

$$\frac{i\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{i\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{x^2}{2}$$

$$\Big\downarrow \; 27$$

$$-\frac{2i\int \frac{x}{-2x^3-i\sqrt{3}+1}dx}{\sqrt{3}}+\frac{2i\int \frac{x}{-2x^3+i\sqrt{3}+1}dx}{\sqrt{3}}-\frac{x^2}{2}$$

$$\Big\downarrow \text{ 821}$$

$$-\frac{2i\left(\frac{\int \frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{3}}+\frac{2i\left(\frac{\int \frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{3}}-\frac{x^2}{2}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{2i\left(-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)}{\sqrt{3}}+\frac{2i\left(-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)}{\sqrt{3}}-\frac{x^2}{2}$$

$$\Big\downarrow \text{ 1142}$$

$$-\frac{2i\left(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int \frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int \frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{\cdots}\right.}{3\,2^{2/3}\sqrt[3]{1-\cdots}}\cdots\right)}{\sqrt{3}}\cdots$$

$$\frac{2i\left(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int \frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int \frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+\cdots}\right.}{3\,2^{2/3}\sqrt[3]{1+\cdots}}\cdots\right)}{\sqrt{3}}\cdots$$

$$\frac{x^2}{2}$$

$$\downarrow 1082$$

$$-\frac{2i\left(-\frac{-\frac{\int \frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\cdots\right.}{3\;2^{2/3}\sqrt[3]{\cdots}}\cdots\right)}{\sqrt{3}}$$

$$\frac{2i\left(-\frac{-\frac{\int \frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{\cdots}\right.}{3\;2^{2/3}\sqrt[3]{1\cdots}}\cdots\right)}{\sqrt{3}}$$

$$\frac{x^2}{2}$$

$\downarrow$ 217

$$
-\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)}{\sqrt{3}}+\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)}{\sqrt{3}}-\frac{x^2}{2}
$$

$\downarrow$ 1103

$$-\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)}{\sqrt{3}}+\frac{2i\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)}{\sqrt{3}}-\frac{x^2}{2}$$

input
```
Int[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/2*x^2 - ((2*I)*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1
 - I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/
3))/Sqrt[3]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1
/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3]))^(1/3))))/Sqrt[3]
+ ((2*I)*(-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1 + I*Sqrt
[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/2^(1/3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2
^(2/3)*x^2]/(2*2^(1/3))))/(3*(2*(1 + I*Sqrt[3]))^(1/3))))/Sqrt[3]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.12

method	result	size
default	$-\frac{x^2}{2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44
risch	$-\frac{x^2}{2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44

input
```
int(x^4*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2+1/3*sum(_R/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.70

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx$$

$$=\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}(\sqrt{-3}-1)\log\left(-3\left(3\sqrt{-\frac{1}{3}}(\sqrt{-3}+1)+\sqrt{-3}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+4x\right)$$
$$-\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}(\sqrt{-3}+1)\log\left(3\left(3\sqrt{-\frac{1}{3}}(\sqrt{-3}-1)+\sqrt{-3}-1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+4x\right)$$
$$+\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}(\sqrt{-3}-1)\log\left(3\left(3\sqrt{-\frac{1}{3}}(\sqrt{-3}+1)-\sqrt{-3}-1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+4x\right)$$
$$-\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}(\sqrt{-3}+1)\log\left(-3\left(3\sqrt{-\frac{1}{3}}(\sqrt{-3}-1)-\sqrt{-3}+1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+4x\right)-\frac{1}{2}x^2+\frac{1}{3}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(3\left(3\sqrt{-\frac{1}{3}}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+2x\right)$$
$$+\frac{1}{3}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(-3\left(3\sqrt{-\frac{1}{3}}-1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+2x\right)$$

input
```
integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/6*(1/6*sqrt(-1/3) + 1/6)^(1/3)*(sqrt(-3) - 1)*log(-3*(3*sqrt(-1/3)*(sqrt
(-3) + 1) + sqrt(-3) + 1)*(1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) - 1/6*(1/6*s
qrt(-1/3) + 1/6)^(1/3)*(sqrt(-3) + 1)*log(3*(3*sqrt(-1/3)*(sqrt(-3) - 1) +
 sqrt(-3) - 1)*(1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) + 1/6*(-1/6*sqrt(-1/3)
+ 1/6)^(1/3)*(sqrt(-3) - 1)*log(3*(3*sqrt(-1/3)*(sqrt(-3) + 1) - sqrt(-3)
- 1)*(-1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) - 1/6*(-1/6*sqrt(-1/3) + 1/6)^(1
/3)*(sqrt(-3) + 1)*log(-3*(3*sqrt(-1/3)*(sqrt(-3) - 1) - sqrt(-3) + 1)*(-1
/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) - 1/2*x^2 + 1/3*(1/6*sqrt(-1/3) + 1/6)^(
1/3)*log(3*(3*sqrt(-1/3) + 1)*(1/6*sqrt(-1/3) + 1/6)^(2/3) + 2*x) + 1/3*(-
1/6*sqrt(-1/3) + 1/6)^(1/3)*log(-3*(3*sqrt(-1/3) - 1)*(-1/6*sqrt(-1/3) + 1
/6)^(2/3) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.08

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx \\ = -\frac{x^2}{2} - \operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-6561t^5 - 27t^2 + x\right)\right)\right)$$

input
```
integrate(x**4*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-x**2/2 - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t*
*5 - 27*_t**2 + x)))
```

Maxima [F]

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = \int -\frac{(x^3-1)x^4}{x^6-x^3+1}\,dx$$

input
```
integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/2*x^2 + integrate(x/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(222) = 444$.

Time = 0.14 (sec) , antiderivative size = 820, normalized size of antiderivative = 2.15

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/2*x^2 - 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*p
i)^2 + 5*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) +
 10*cos(4/9*pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 +
sqrt(3)*sin(4/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3)
 - 1)*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(
3)*cos(2/9*pi)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(
2/9*pi)*sin(2/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin
(2/9*pi)^3 - sin(2/9*pi)^5 - sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2
 + 2*cos(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2
*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10
*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4
 + 5*cos(1/9*pi)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*
pi)^5 + sqrt(3)*cos(1/9*pi)^2 - sqrt(3)*sin(1/9*pi)^2 + 2*cos(1/9*pi)*sin(
1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3)
+ 1/2)*sin(1/9*pi))) - 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqrt
(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 -
10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 - 2*sqrt(3)*c
os(4/9*pi)*sin(4/9*pi) - cos(4/9*pi)^2 + sin(4/9*pi)^2)*log((-I*sqrt(3)*co
s(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(2/9*pi)^4*sin(
2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9*pi)^...
```

Mupad [B] (verification not implemented)

Time = 20.76 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.81

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\left(81\,x-\frac{27\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18} + \frac{\ln\left(x-\left(81\,x-\frac{27\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{x^2}{2} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

input
```
int(-(x^4*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/16
2))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i + 36)
^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(3^(1/2)*12i + 36)^(1/3))/18 - x^2/
2 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 + (2^(1/3
)*3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) +
3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3)
)/12 - (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/
3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(
1/2)*1i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^
(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)
^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

Reduce [F]

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = \int \frac{x}{x^6-x^3+1}dx - \frac{x^2}{2}$$

input
```
int(x^4*(-x^3+1)/(x^6-x^3+1),x)
```

output
```
(2*int(x/(x**6 - x**3 + 1),x) - x**2)/2
```

3.54 $\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 378

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = -x - \frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} + \frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}$$

output
```
-x-1/3*I*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*3^(1/2))/(1/2-1/2*I*
3^(1/2))^(2/3)+1/3*I*arctan(1/3*(1+2*x/(1/2+1/2*I*3^(1/2))^(1/3))*3^(1/2))
/(1/2+1/2*I*3^(1/2))^(2/3)+1/9*I*ln((1-I*3^(1/2))^(1/3)-2^(1/3)*x)*3^(1/2)
/(1/2-1/2*I*3^(1/2))^(2/3)-1/9*I*ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*3^(1/2)
/(1/2+1/2*I*3^(1/2))^(2/3)-1/18*I*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1/2))^(
1/3)*x+2^(2/3)*x^2)*2^(2/3)*3^(1/2)/(1-I*3^(1/2))^(2/3)+1/18*I*ln((1+I*3^(
1/2))^(2/3)+(2+2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(2/3)*3^(1/2)/(1+I*3^(1
/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.12

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = -x + \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{\log(x-\#1)}{-\#1^2+2\#1^5}\&\right]$$

input
```
Integrate[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-x + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1826, 25, 1685, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(1-x^3)}{x^6-x^3+1}\,dx$$

$$\Big\downarrow \; 1826$$

$$-\int -\frac{1}{x^6-x^3+1}dx-x$$

$$\Big\downarrow \; 25$$

$$\int \frac{1}{x^6-x^3+1}dx-x$$

$$\Big\downarrow \; 1685$$

$$-\frac{i\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx}{\sqrt{3}}+\frac{i\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx}{\sqrt{3}}-x$$

$$\Big\downarrow \; 750$$

$$\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-x$$

$$\Big\downarrow \; 16$$

$$\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-x$$

$$\Big\downarrow \; 25$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}}\left(1-i\sqrt{3}\right)x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}}\left(1+i\sqrt{3}\right)x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-x$$

$\downarrow$ 1142

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}}\left(1-i\sqrt{3}\right)x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}}\left(1-i\sqrt{\cdots}\right.}{x^2+\sqrt[3]{\frac{1}{2}}\left(1-i\sqrt{3}\right)x+\left(\frac{1}{2}\cdots\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.}{\sqrt{3}}$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}}\left(1+i\sqrt{3}\right)x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}}\left(1+i\sqrt{\cdots}\right.}{x^2+\sqrt[3]{\frac{1}{2}}\left(1+i\sqrt{3}\right)x+\left(\frac{1}{2}\cdots\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.}{\sqrt{3}}$$

x

$\downarrow$ 1082

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\right.}}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.}{\sqrt{3}}$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\right.}}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.}{\sqrt{3}}$$

x

$\downarrow$ 217

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-$$

$$x$$

$$\Big\downarrow \; 1103$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-$$

$$\frac{i\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)}{\sqrt{3}}-$$

$$x$$

input
```
Int[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-x + (I*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 - I*Sqrt[3])/2)^(2/
3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log
[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*
((1 - I*Sqrt[3])/2)^(2/3))))/Sqrt[3] - (I*(Log[(1 + I*Sqrt[3])^(1/3) - 2^(
1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I
*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[
3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[3])/2)^(2/3))))/Sqrt[3]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.11

method	result	size
default	$-x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	41
risch	$-x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	41

input
```
int(x^3*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-x+1/3*sum(1/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.68

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(-3\left(\sqrt{-\frac{1}{3}}\left(\sqrt{-3}+1\right)+\sqrt{-3}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+4\,x\right)$$
$$+\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(3\left(\sqrt{-\frac{1}{3}}\left(\sqrt{-3}-1\right)+\sqrt{-3}-1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+4\,x\right)$$
$$-\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(3\left(\sqrt{-\frac{1}{3}}\left(\sqrt{-3}+1\right)-\sqrt{-3}-1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+4\,x\right)$$
$$+\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(-3\left(\sqrt{-\frac{1}{3}}\left(\sqrt{-3}-1\right)-\sqrt{-3}+1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+4\,x\right)+\frac{1}{3}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(3\left(\sqrt{-\frac{1}{3}}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+2\,x\right)$$
$$+\frac{1}{3}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(-3\left(\sqrt{-\frac{1}{3}}-1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}+2\,x\right)-x$$

input
```
integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/6*(1/6*sqrt(-1/3) + 1/6)^(1/3)*(sqrt(-3) + 1)*log(-3*(sqrt(-1/3)*(sqrt(
-3) + 1) + sqrt(-3) + 1)*(1/6*sqrt(-1/3) + 1/6)^(1/3) + 4*x) + 1/6*(1/6*sq
rt(-1/3) + 1/6)^(1/3)*(sqrt(-3) - 1)*log(3*(sqrt(-1/3)*(sqrt(-3) - 1) + sq
rt(-3) - 1)*(1/6*sqrt(-1/3) + 1/6)^(1/3) + 4*x) - 1/6*(-1/6*sqrt(-1/3) + 1
/6)^(1/3)*(sqrt(-3) + 1)*log(3*(sqrt(-1/3)*(sqrt(-3) + 1) - sqrt(-3) - 1)*
(-1/6*sqrt(-1/3) + 1/6)^(1/3) + 4*x) + 1/6*(-1/6*sqrt(-1/3) + 1/6)^(1/3)*(
sqrt(-3) - 1)*log(-3*(sqrt(-1/3)*(sqrt(-3) - 1) - sqrt(-3) + 1)*(-1/6*sqrt
(-1/3) + 1/6)^(1/3) + 4*x) + 1/3*(1/6*sqrt(-1/3) + 1/6)^(1/3)*log(3*(sqrt(
-1/3) + 1)*(1/6*sqrt(-1/3) + 1/6)^(1/3) + 2*x) + 1/3*(-1/6*sqrt(-1/3) + 1/
6)^(1/3)*log(-3*(sqrt(-1/3) - 1)*(-1/6*sqrt(-1/3) + 1/6)^(1/3) + 2*x) - x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = -x - \text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 + x\right)\right)\right)$$

input
```
integrate(x**3*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-x - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x)
))
```

Maxima [F]

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = \int -\frac{(x^3-1)x^3}{x^6-x^3+1}\,dx$$

input
```
integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-x + integrate(1/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 635 vs. 2(220) = 440.

Time = 0.14 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.68

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt
(3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi
)^3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(
4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9
*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4
*cos(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9
*pi) - sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*
I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*co
s(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/
9*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*a
rctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin
(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9
*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4
/9*pi)^4 - sqrt(3)*sin(4/9*pi) + cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi)
- cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) -
4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(
2/9*pi)^2 - sin(2/9*pi)^4 - sqrt(3)*sin(2/9*pi) + cos(2/9*pi))*log((-I*sqr
t(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*pi)
^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*c
os(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 + sqrt(3)*sin(1/9*pi) + cos(...
```

Mupad [B] (verification not implemented)

Time = 20.86 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.87

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = -x+\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

input
```
int(-(x^3*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
(log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3
- 3^(1/2)*1i)^(1/3)*1i)/12)*(36 - 3^(1/2)*12i)^(1/3))/18 - x + (log(x + (2
^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i +
3)^(1/3)*1i)/12)*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*
3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3
))/12)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x
- (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1
i + 3)^(4/3))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(
2/3)*log(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1
i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3
^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))
/36
```

Reduce [F]

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx=\int \frac{1}{x^6-x^3+1}dx-x$$

input
```
int(x^3*(-x^3+1)/(x^6-x^3+1),x)
```

output
```
int(1/(x**6 - x**3 + 1),x) - x
```

3.55 $\int \frac{x(1-x^3)}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 411

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

output
```
1/6*(I-3^(1/2))*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*3^(1/2))*2^(1
/3)/(1-I*3^(1/2))^(1/3)-1/6*(3^(1/2)+I)*arctan(1/3*(1+2*x/(1/2+1/2*I*3^(1/
2))^(1/3))*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/18*(3-I*3^(1/2))*ln((1-I
*3^(1/2))^(1/3)-2^(1/3)*x)*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/18*(3+I*3^(1/2))*
ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/36*(3-I*3^
(1/2))*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(1/3)
/(1-I*3^(1/2))^(1/3)+1/36*(3+I*3^(1/2))*ln((1+I*3^(1/2))^(2/3)+(2+2*I*3^(1
/2))^(1/3)*x+2^(2/3)*x^2)*2^(1/3)/(1+I*3^(1/2))^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.13

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1+2\#1^4}\&\right]$$

input
```
Integrate[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
-1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1 +
2*#1^4) & ]
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1834, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1-x^3)}{x^6-x^3+1}dx$$

$$\Big\downarrow \text{ 1834}$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\int \frac{x}{-2x^3-i\sqrt{3}+1}dx+\frac{1}{3}\left(3+i\sqrt{3}\right)\int \frac{x}{-2x^3+i\sqrt{3}+1}dx$$

$$\Big\downarrow \ 821$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)+\frac{1}{3}\left(3+i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)+\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\cdots\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\cdots\right.$$

$\downarrow$ 1082

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\right.$$

$$\Big\downarrow \ 217$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\right.}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\right.}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right.$$

↓ 1103

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1-i\cdots}}\right.$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1+i\cdots}}\right.$$

input
```
Int[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

output
```
((3 - I*Sqrt[3])*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1
 - I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3
))/Sqrt[3]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/
3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3]))^(1/3))))/3 + ((3 +
 I*Sqrt[3])*(-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3)*(1 + I*S
qrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/2^(1/3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x
 + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 + I*Sqrt[3]))^(1/3))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.11

method	result	size
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(\_R^{4}-\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(-\_R^{4}+\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44

input
```
int(x*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.48

$$
\begin{aligned}
&\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx \\
&= \frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(3\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{2}{3}}\left(\sqrt{-3}+1\right)+2\,x\right) \\
&\quad+\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(3\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{2}{3}}\left(\sqrt{-3}+1\right)+2\,x\right) \\
&\quad-\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(-3\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{2}{3}}\left(\sqrt{-3}-1\right)+2\,x\right) \\
&\quad-\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(-3\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{2}{3}}\left(\sqrt{-3}-1\right)+2\,x\right) \\
&\quad+\frac{1}{3}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\log\left(x-3\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{2}{3}}\right) \\
&\quad+\frac{1}{3}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\log\left(x-3\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{2}{3}}\right)
\end{aligned}
$$

input
```
integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/6*(1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) - 1)*log(3*(1/6*sqrt(-1/3) - 1/
6)^(2/3)*(sqrt(-3) + 1) + 2*x) + 1/6*(-1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-
3) - 1)*log(3*(-1/6*sqrt(-1/3) - 1/6)^(2/3)*(sqrt(-3) + 1) + 2*x) - 1/6*(1
/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) + 1)*log(-3*(1/6*sqrt(-1/3) - 1/6)^(2
/3)*(sqrt(-3) - 1) + 2*x) - 1/6*(-1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) +
1)*log(-3*(-1/6*sqrt(-1/3) - 1/6)^(2/3)*(sqrt(-3) - 1) + 2*x) + 1/3*(1/6*s
qrt(-1/3) - 1/6)^(1/3)*log(x - 3*(1/6*sqrt(-1/3) - 1/6)^(2/3)) + 1/3*(-1/6
*sqrt(-1/3) - 1/6)^(1/3)*log(x - 3*(-1/6*sqrt(-1/3) - 1/6)^(2/3))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.05

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = -\operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(-27t^2+x\right)\right)\right)$$

input
```
integrate(x*(-x**3+1)/(x**6-x**3+1),x)
```

output
```
-RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x)))
```

Maxima [F]

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \int -\frac{(x^3-1)x}{x^6-x^3+1}\,dx$$

input
```
integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-integrate((x^3 - 1)*x/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 824 vs. 2(255) = 510.

Time = 0.15 (sec) , antiderivative size = 824, normalized size of antiderivative = 2.00

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output

```
1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sq
rt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9
*pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)
*sin(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*c
os(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(
2/9*pi)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)
*sin(2/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi
)^3 - sin(2/9*pi)^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 -
4*cos(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)
/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sq
rt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 +
5*cos(1/9*pi)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)
^5 - 2*sqrt(3)*cos(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin
(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3)
 + 1/2)*sin(1/9*pi))) + 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqr
t(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 -
 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 + 4*sqrt(3)*
cos(4/9*pi)*sin(4/9*pi) + 2*cos(4/9*pi)^2 - 2*sin(4/9*pi)^2)*log((-I*sqrt(
3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(5*sqrt(3)*cos(2/9*pi)^4
*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9...
```

Mupad [B] (verification not implemented)

Time = 20.87 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.68

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$$
$$=\frac{\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x-\frac{\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{12}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(-(x*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

output
```
(log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1
/3))/18 + (log(x - (- 3^(1/2)*12i - 36)^(2/3)/12)*(- 3^(1/2)*12i - 36)^(1/
3))/18 - (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(
5/6)*1i)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(
2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/2
4)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x -
(2^(1/3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i
- 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)
*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/
3) + 3^(5/6)*1i))/36
```

Reduce [F]

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = -\left(\int \frac{x^4}{x^6-x^3+1}dx\right) + \int \frac{x}{x^6-x^3+1}dx$$

input

```
int(x*(-x^3+1)/(x^6-x^3+1),x)
```

output

```
- int(x**4/(x**6 - x**3 + 1),x) + int(x/(x**6 - x**3 + 1),x)
```

3.56 $\int \frac{1-x^3}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 411

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx=-\frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

output
```
-1/6*(I-3^(1/2))*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*3^(1/2))*2^(
2/3)/(1-I*3^(1/2))^(2/3)+1/6*(3^(1/2)+I)*arctan(1/3*(1+2*x/(1/2+1/2*I*3^(1
/2))^(1/3))*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/18*(3-I*3^(1/2))*ln((1-
I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/18*(3+I*3^(1/2))
*ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/36*(3-I*3
^(1/2))*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(2/3
)/(1-I*3^(1/2))^(2/3)+1/36*(3+I*3^(1/2))*ln((1+I*3^(1/2))^(2/3)+(2+2*I*3^(
1/2))^(1/3)*x+2^(2/3)*x^2)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.14

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = -\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1^2+2\#1^5}\&\right]$$

input
```
Integrate[(1 - x^3)/(1 - x^3 + x^6),x]
```

output
```
-1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2
+ 2*#1^5) & ]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1752, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^3}{x^6-x^3+1}\,dx$$

$$\Big\downarrow \text{1752}$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx-\frac{1}{6}\left(3-i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx$$

$$\Big\downarrow \text{750}$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)$$

$$\Big\downarrow \; 16$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)$$

$\downarrow$ 1142

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int -}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int -}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$\downarrow$ 1082

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right.}}\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

↓ 217

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{\cdots}{\sqrt[3]{\frac{1}{2}\cdots}}}{\cdots}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{\cdots}{\sqrt[3]{\frac{1}{2}\left(1\cdots\right.}}}{\sqrt{\cdots}}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\Big\downarrow \ 1103$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{\cdots}\right.}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)\cdots}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

input
```
Int[(1 - x^3)/(1 - x^3 + x^6),x]
```

output
```
-1/6*((3 - I*Sqrt[3])*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 - I*S
qrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/
Sqrt[3]] + Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/
3)*x^2]/2)/(3*((1 - I*Sqrt[3])/2)^(2/3)))) - ((3 + I*Sqrt[3])*(Log[(1 + I*
Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTa
n[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sqrt[3])^(2/
3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sqrt[3])/2)^
(2/3))))/6
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.11

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(-\_R^{3}+1\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(-\_R^{3}+1\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44

input
```
int((-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((-_R^3+1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.62

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx =$$

$$-\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(-3\left(\sqrt{-\frac{1}{3}}\left(\sqrt{-3}+1\right)+\sqrt{-3}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}+4\,x\right)$$

$$+\frac{1}{6}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(3\left(\sqrt{-\frac{1}{3}}\left(\sqrt{-3}-1\right)+\sqrt{-3}-1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}+4\,x\right)$$

$$-\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(3\left(\sqrt{-\frac{1}{3}}\left(\sqrt{-3}+1\right)-\sqrt{-3}-1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}+4\,x\right)$$

$$+\frac{1}{6}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(-3\left(\sqrt{-\frac{1}{3}}\left(\sqrt{-3}-1\right)-\sqrt{-3}+1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}+4\,x\right)+\frac{1}{3}\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\log\left(3\left(\sqrt{-\frac{1}{3}}+1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}+2\,x\right)$$

$$+\frac{1}{3}\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}\log\left(-3\left(\sqrt{-\frac{1}{3}}-1\right)\left(-\frac{1}{6}\sqrt{-\frac{1}{3}}-\frac{1}{6}\right)^{\frac{1}{3}}+2\,x\right)$$

input
```
integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/6*(1/6*sqrt(-1/3) - 1/6)^(1/3)*(sqrt(-3) + 1)*log(-3*(sqrt(-1/3)*(sqrt(
-3) + 1) + sqrt(-3) + 1)*(1/6*sqrt(-1/3) - 1/6)^(1/3) + 4*x) + 1/6*(1/6*sq
rt(-1/3) - 1/6)^(1/3)*(sqrt(-3) - 1)*log(3*(sqrt(-1/3)*(sqrt(-3) - 1) + sq
rt(-3) - 1)*(1/6*sqrt(-1/3) - 1/6)^(1/3) + 4*x) - 1/6*(-1/6*sqrt(-1/3) - 1
/6)^(1/3)*(sqrt(-3) + 1)*log(3*(sqrt(-1/3)*(sqrt(-3) + 1) - sqrt(-3) - 1)*
(-1/6*sqrt(-1/3) - 1/6)^(1/3) + 4*x) + 1/6*(-1/6*sqrt(-1/3) - 1/6)^(1/3)*(
sqrt(-3) - 1)*log(-3*(sqrt(-1/3)*(sqrt(-3) - 1) - sqrt(-3) + 1)*(-1/6*sqrt
(-1/3) - 1/6)^(1/3) + 4*x) + 1/3*(1/6*sqrt(-1/3) - 1/6)^(1/3)*log(3*(sqrt(
-1/3) + 1)*(1/6*sqrt(-1/3) - 1/6)^(1/3) + 2*x) + 1/3*(-1/6*sqrt(-1/3) - 1/
6)^(1/3)*log(-3*(sqrt(-1/3) - 1)*(-1/6*sqrt(-1/3) - 1/6)^(1/3) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = -\operatorname{RootSum}\left(19683t^6-243t^3+1, \left(t\mapsto t\log\left(729t^4-9t+x\right)\right)\right)$$

input
```
integrate((-x**3+1)/(x**6-x**3+1),x)
```

output
```
-RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t +
 x)))
```

Maxima [F]

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \int -\frac{x^3-1}{x^6-x^3+1}\,dx$$

input
```
integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(255) = 510.

Time = 0.13 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.56

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \text{Too large to display}$$

input
```
integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)
^3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*c
os(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(
2/9*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4
+ 4*cos(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*co
s(2/9*pi) + 2*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)
/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqr
t(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3
*sin(1/9*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin
(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3)
 + 1/2)*sin(1/9*pi))) + 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt
(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi
)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) - 2*cos(4/9*pi))*log((-I*sqrt(
3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(2/9*pi)^3
*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos
(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(2
/9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(4*
sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 +
cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 - 2*sqrt(...
```

Mupad [B] (verification not implemented)

Time = 20.96 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.78

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \frac{\ln\left(x-\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+\frac{\ln\left(x+\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^3}{16}+27\right)}{108}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^3}{16}-27\right)}{108}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

input
```
int(-(x^3 - 1)/(x^6 - x^3 + 1),x)
```

output
```
(log(x - (((3^(1/2)*9i)/2 - 27/2)*(- 3^(1/2)*12i - 36)^(1/3))/54)*(- 3^(1/
2)*12i - 36)^(1/3))/18 + (log(x + (((3^(1/2)*9i)/2 + 27/2)*(3^(1/2)*12i -
36)^(1/3))/54)*(3^(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*(-
 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)*((3*(3^(1/2)*1i + 3)*(3^(1/3
) + 3^(5/6)*1i)^3)/16 + 27))/108)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5
/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) -
3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^(5/6)*1i)^3)/16 - 27))/108)*
(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2
/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1
/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)
^(1/3)*1i)/6)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

Reduce [F]

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = -\left(\int \frac{x^3}{x^6-x^3+1}dx\right) + \int \frac{1}{x^6-x^3+1}dx$$

input
```
int((-x^3+1)/(x^6-x^3+1),x)
```

output
```
 - int(x**3/(x**6 - x**3 + 1),x) + int(1/(x**6 - x**3 + 1),x)
```

3.57 $\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 416

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx=-\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

output
```
-1/x-1/6*(3^(1/2)+I)*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*3^(1/2))
*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/6*(I-3^(1/2))*arctan(1/3*(1+2*x/(1/2+1/2*I*
3^(1/2))^(1/3))*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/18*(3+I*3^(1/2))*ln
((1-I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/18*(3-I*3^(1
/2))*ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/36*(3
+I*3^(1/2))*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^
(1/3)/(1-I*3^(1/2))^(1/3)+1/36*(3-I*3^(1/2))*ln((1+I*3^(1/2))^(2/3)+(2+2*I
*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(1/3)/(1+I*3^(1/2))^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.11

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx=-\frac{1}{x}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^3}\&\right]$$

input
```
Integrate[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

output
```
-x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]
/3
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1828, 1710, 27, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^3}{x^2\left(x^6-x^3+1\right)}\,dx$$

$$\Big\downarrow \text{1828}$$

$$-\int \frac{x^4}{x^6-x^3+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{1710}$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\int -\frac{2x}{-2x^3-i\sqrt{3}+1}dx-\frac{1}{6}\left(3-i\sqrt{3}\right)\int -\frac{2x}{-2x^3+i\sqrt{3}+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\int \frac{x}{-2x^3-i\sqrt{3}+1}dx+\frac{1}{3}\left(3-i\sqrt{3}\right)\int \frac{x}{-2x^3+i\sqrt{3}+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}\right)+\frac{1}{3}\left(3-i\sqrt{3}\right)\left(\frac{\int \frac{1}{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}\right)-\frac{1}{x}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)+\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\int \frac{\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right)-\frac{1}{x}$$

$$\downarrow 1142$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\Bigg(-\frac{\frac{3}{2}\sqrt[3]{1-i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\Bigg(-\frac{\frac{3}{2}\sqrt[3]{1+i\sqrt{3}}\int\frac{1}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx-\frac{\int\frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-$$

$$\frac{1}{x}$$

$$\downarrow 1082$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{-\frac{\int \frac{2\ 2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+1\right)}{\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\right.$$

$$\frac{1}{x}$$

$$\Big\downarrow \ 217$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1-i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\right.}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\int\frac{2\,2^{2/3}x+\sqrt[3]{2\left(1+i\sqrt{3}\right)}}{2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}dx}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\right.}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}\right.$$

$$\frac{1}{x}$$

$$\downarrow 1103$$

$$\frac{1}{3}\left(3+i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1-i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1-i\cdots}}\right.$$

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{2\sqrt[3]{2}}}{3\sqrt[3]{2\left(1+i\sqrt{3}\right)}}-\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1\cdots}\right.}{3\ 2^{2/3}\sqrt[3]{1+i\cdots}}\right.$$

$$\frac{1}{x}$$

input
```
Int[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

output
```
-x^(-1) + ((3 + I*Sqrt[3])*(-1/3*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2
^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3
])/2)^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqr
t[3]))^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 - I*Sqrt[3]))^(1/3))))
/3 + ((3 - I*Sqrt[3])*(-1/3*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(2^(2/3
)*(1 + I*Sqrt[3])^(1/3)) - ((Sqrt[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)
^(1/3))/Sqrt[3]])/2^(1/3) - Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2]/(2*2^(1/3)))/(3*(2*(1 + I*Sqrt[3]))^(1/3))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, 
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.10

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^{6}-9\_Z^{3}+1\right)}\_R\ln\left(-27\_R^{5}+6\_R^{2}+x\right)\right)}{3}$	40
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}-\frac{1}{x}$	46

input
```
int((-x^3+1)/x^2/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/3*sum(_R*ln(-27*_R^5+6*_R^2+x),_R=RootOf(27*_Z^6-9*_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.67

$$\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx$$
$$= \frac{(\sqrt{-3}x-x)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(3\left(3\sqrt{-\frac{1}{3}}(\sqrt{-3}+1)-\sqrt{-3}-1\right)\left(\frac{1}{6}\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{2}{3}}+4x\right)-(\sqrt{-3}x}{}$$

input
```
integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/6*((sqrt(-3)*x - x)*(1/6*sqrt(-1/3) + 1/6)^(1/3)*log(3*(3*sqrt(-1/3)*(sq
rt(-3) + 1) - sqrt(-3) - 1)*(1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) - (sqrt(-3
)*x + x)*(1/6*sqrt(-1/3) + 1/6)^(1/3)*log(-3*(3*sqrt(-1/3)*(sqrt(-3) - 1)
- sqrt(-3) + 1)*(1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) + 2*x*(1/6*sqrt(-1/3)
+ 1/6)^(1/3)*log(-3*(3*sqrt(-1/3) - 1)*(1/6*sqrt(-1/3) + 1/6)^(2/3) + 2*x)
 + (sqrt(-3)*x - x)*(-1/6*sqrt(-1/3) + 1/6)^(1/3)*log(-3*(3*sqrt(-1/3)*(sq
rt(-3) + 1) + sqrt(-3) + 1)*(-1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) - (sqrt(-
3)*x + x)*(-1/6*sqrt(-1/3) + 1/6)^(1/3)*log(3*(3*sqrt(-1/3)*(sqrt(-3) - 1)
 + sqrt(-3) - 1)*(-1/6*sqrt(-1/3) + 1/6)^(2/3) + 4*x) + 2*x*(-1/6*sqrt(-1/
3) + 1/6)^(1/3)*log(3*(3*sqrt(-1/3) + 1)*(-1/6*sqrt(-1/3) + 1/6)^(2/3) + 2
*x) - 6)/x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.07

$$\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx$$
$$= -\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(6561t^5+54t^2+x\right)\right)\right)-\frac{1}{x}$$

input
```
integrate((-x**3+1)/x**2/(x**6-x**3+1),x)
```

output
```
-RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t
**2 + x))) - 1/x
```

Maxima [F]

$$\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx = \int -\frac{x^3-1}{(x^6-x^3+1)x^2}\,dx$$

input
```
integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/x - integrate(x^4/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(260) = 520$.

Time = 0.14 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.00

$$\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx = \text{Too large to display}$$

input
```
integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="giac")
```

output

```
1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10
*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos
(4/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(
3)*sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)
*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*
cos(2/9*pi)^5 - 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/
9*pi)*sin(2/9*pi)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(
2/9*pi)^3 - 2*sin(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^
2 - 2*cos(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) +
2*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 -
 20*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*p
i)^4 + 10*cos(1/9*pi)^4*sin(1/9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*s
in(1/9*pi)^5 - sqrt(3)*cos(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*p
i)*sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*s
qrt(3) + 1/2)*sin(1/9*pi))) + 1/18*(10*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) -
 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqrt(3)*sin(4/9*pi)^5 + 2*cos(
4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*cos(4/9*pi)*sin(4/9*pi)^4
+ 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi)^2 - sin(4/9*pi)^2)*log((
-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(
2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + 2*sqrt...
```

Mupad [B] (verification not implemented)

Time = 20.76 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.75

$$\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx$$
$$= \frac{\ln\left(-x+\left(162\,x+\frac{27\,\left(36+\sqrt{3}\,12{}\mathrm{i}\right)^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}$$
$$+\frac{\ln\left(-x-\left(162\,x+\frac{27\,\left(36-\sqrt{3}\,12{}\mathrm{i}\right)^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{1}{x}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,\left(3-\sqrt{3}\,1{}\mathrm{i}\right)^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,\left(3-\sqrt{3}\,1{}\mathrm{i}\right)^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,\left(3+\sqrt{3}\,1{}\mathrm{i}\right)^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,\left(3+\sqrt{3}\,1{}\mathrm{i}\right)^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(3-\sqrt{3}\,1{}\mathrm{i}\right)^{2/3}}{6}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$
$$-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(3+\sqrt{3}\,1{}\mathrm{i}\right)^{2/3}}{6}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

input
```
int(-(x^3 - 1)/(x^2*(x^6 - x^3 + 1)),x)
```

output
```
(log((162*x + (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162)
- x)*(3^(1/2)*12i + 36)^(1/3))/18 + (log(- x - (162*x + (27*(36 - 3^(1/2)*
12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 -
1/x - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 - (2^(1
/3)*3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3)
- 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/
3))/12 + (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(
1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3
^(1/2)*1i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (
2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i +
3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

Reduce [F]

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx=\frac{-\left(\int \frac{x^4}{x^6-x^3+1}dx\right)x-1}{x}$$

input
```
int((-x^3+1)/x^2/(x^6-x^3+1),x)
```

output
```
( - (int(x**4/(x**6 - x**3 + 1),x)*x + 1))/x
```

3.58 $\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 418

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx = -\frac{1}{2x^2} + \frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

output

```
-1/2/x^2+1/6*(3^(1/2)+I)*arctan(1/3*(1+2*x/(1/2-1/2*I*3^(1/2))^(1/3))*3^(1
/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/6*(I-3^(1/2))*arctan(1/3*(1+2*x/(1/2+1/
2*I*3^(1/2))^(1/3))*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/18*(3+I*3^(1/2)
)*ln((1-I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/18*(3-I*
3^(1/2))*ln((1+I*3^(1/2))^(1/3)-2^(1/3)*x)*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/3
6*(3+I*3^(1/2))*ln((1-I*3^(1/2))^(2/3)+(2-2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2
)*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/36*(3-I*3^(1/2))*ln((1+I*3^(1/2))^(2/3)+(2
+2*I*3^(1/2))^(1/3)*x+2^(2/3)*x^2)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.11

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx = -\frac{1}{2x^2}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{-1+2\#1^3}\&\right]$$

input
```
Integrate[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

output
```
-1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) &
]/3
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1828, 27, 1710, 750, 16, 25, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^3}{x^3\left(x^6-x^3+1\right)}dx$$

$$\Big\downarrow \text{ 1828}$$

$$-\frac{1}{2}\int \frac{2x^3}{x^6-x^3+1}dx-\frac{1}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{x^3}{x^6-x^3+1}dx-\frac{1}{2x^2}$$

$$\Big\downarrow \text{ 1710}$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1-i\sqrt{3}\right)}dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{x^3+\frac{1}{2}\left(-1+i\sqrt{3}\right)}dx-\frac{1}{2x^2}$$

$$\Big\downarrow \text{ 750}$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{2x^2}$$

$$\Big\downarrow \; 16$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\int -\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-$$

$$\frac{1}{2x^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right)-$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\int\frac{x+2^{2/3}\sqrt[3]{1+i\sqrt{3}}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right)-\frac{1}{2x^2}$$

$\downarrow$ 1142

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int -}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\int\frac{1}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\frac{1}{2}\int -}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

$\downarrow$ 1082

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right.}}\right.}}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx-3\int\frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right.}}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{\cdots}{\sqrt[3]{\frac{1}{2}\cdots}}}{\cdots}\right)\cdots}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\frac{1}{2}\int\frac{2x+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}{x^2+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}dx+\sqrt{3}\arctan\left(\frac{1+\frac{\cdots}{\sqrt[3]{\frac{1}{2}\left(1\cdots\right.}}}{\sqrt{\cdots}}\right)\cdots}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{1}{6}\left(3+i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{\right.}}\right.}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{6}\left(3-i\sqrt{3}\right)\left(\frac{\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\right.}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\right.$$

$$\frac{1}{2x^2}$$

input

```
Int[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

output

```
-1/2*1/x^2 - ((3 + I*Sqrt[3])*(Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*(
(1 - I*Sqrt[3])/2)^(2/3)) - (Sqrt[3]*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)
^(1/3))/Sqrt[3]] + Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x
 + 2^(2/3)*x^2]/2)/(3*((1 - I*Sqrt[3])/2)^(2/3))))/6 - ((3 - I*Sqrt[3])*(L
og[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x]/(3*((1 + I*Sqrt[3])/2)^(2/3)) - (Sqr
t[3]*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]] + Log[(1 + I*Sq
rt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2]/2)/(3*((1 + I*Sq
rt[3])/2)^(2/3))))/6
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.09

method	result	size
risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(27\_Z^6-9\_Z^3+1\right)}\_R\ln\left(-18\_R^4+3\_R+x\right)\right)}{3}$	38
default	$-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}-\frac{1}{2x^2}$	46

input
```
int((-x^3+1)/x^3/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+1/3*sum(_R*ln(-18*_R^4+3*_R+x),_R=RootOf(27*_Z^6-9*_Z^3+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.61

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx = \frac{2\,x^2\left(\frac{1}{6}\,\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(x-3\,\sqrt{-\frac{1}{3}}\left(\frac{1}{6}\,\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\right)+2\,x^2\left(-\frac{1}{6}\,\sqrt{-\frac{1}{3}}+\frac{1}{6}\right)^{\frac{1}{3}}\log\left(x+3\,\sqrt{-\frac{1}{3}}\left(-\frac{1}{6}\,\sqrt{\right.\right.}{}$$

input
```
integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/6*(2*x^2*(1/6*sqrt(-1/3) + 1/6)^(1/3)*log(x - 3*sqrt(-1/3)*(1/6*sqrt(-1/
3) + 1/6)^(1/3)) + 2*x^2*(-1/6*sqrt(-1/3) + 1/6)^(1/3)*log(x + 3*sqrt(-1/3
)*(-1/6*sqrt(-1/3) + 1/6)^(1/3)) - (sqrt(-3)*x^2 + x^2)*(1/6*sqrt(-1/3) +
1/6)^(1/3)*log(3*sqrt(-1/3)*(1/6*sqrt(-1/3) + 1/6)^(1/3)*(sqrt(-3) + 1) +
2*x) - (sqrt(-3)*x^2 + x^2)*(-1/6*sqrt(-1/3) + 1/6)^(1/3)*log(-3*sqrt(-1/3
)*(-1/6*sqrt(-1/3) + 1/6)^(1/3)*(sqrt(-3) + 1) + 2*x) + (sqrt(-3)*x^2 - x^
2)*(1/6*sqrt(-1/3) + 1/6)^(1/3)*log(-3*sqrt(-1/3)*(1/6*sqrt(-1/3) + 1/6)^(
1/3)*(sqrt(-3) - 1) + 2*x) + (sqrt(-3)*x^2 - x^2)*(-1/6*sqrt(-1/3) + 1/6)^
(1/3)*log(3*sqrt(-1/3)*(-1/6*sqrt(-1/3) + 1/6)^(1/3)*(sqrt(-3) - 1) + 2*x)
 - 3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.08

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx = -\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(-1458t^4-9t+x\right)\right)\right)-\frac{1}{2x^2}$$

input
```
integrate((-x**3+1)/x**3/(x**6-x**3+1),x)
```

output
```
-RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t
 + x))) - 1/(2*x**2)
```

Maxima [F]

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx = \int -\frac{x^3-1}{(x^6-x^3+1)x^3}\,dx$$

input
```
integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/2/x^2 - integrate(x^3/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(260) = 520$.

Time = 0.14 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.54

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx = \text{Too large to display}$$

input
```
integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2*
sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4/
9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*
cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*c
os(2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*
pi)^4 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3
)*cos(2/9*pi) + sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*
x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 1
2*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/
9*pi)^3*sin(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) +
sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt
(3) + 1/2)*sin(1/9*pi))) + 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 8*s
qrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 + 12*cos(4/9*pi)^2*sin(
4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9*pi) - cos(4/9*pi))*log((-I*s
qrt(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(8*sqrt(3)*cos(2/9*p
i)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 +
 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) -
cos(2/9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/1
8*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)
^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sin(1/9*pi)^4...
```

Mupad [B] (verification not implemented)

Time = 20.88 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.79

$$\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx = \frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{1}{2\,x^2} - \frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{1/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{1/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{1/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{12}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{1/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{12}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

input

```
int(-(x^3 - 1)/(x^3*(x^6 - x^3 + 1)),x)
```

output

```
(log(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(36 - 3^(1/2)*12i)
^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(1
/2)*12i + 36)^(1/3))/18 - 1/(2*x^2) - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3
 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12)*(3
- 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)
*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(4/
3))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x
 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 - 3^(1
/2)*1i)^(1/3)*1i)/12)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 -
(2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(
5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5
/6)*1i))/36
```

Reduce [F]

$$\int \frac{1-x^3}{x^3 \left(1-x^3+x^6\right)} \, dx = \frac{-2\left(\int \frac{x^3}{x^6-x^3+1}dx\right)x^2-1}{2x^2}$$

input
```
int((-x^3+1)/x^3/(x^6-x^3+1),x)
```

output
```
( - 2*int(x**3/(x**6 - x**3 + 1),x)*x**2 - 1)/(2*x**2)
```

3.59 $\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx=\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)+1/6*ln(x^6-x^3+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx=-\frac{\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)$$

input
```
Integrate[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
```

output
```
-(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1798, 25, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(x^3-2)}{x^6-x^3+1}dx$$

$$\Big\downarrow \; 1798$$

$$\frac{1}{3}\int -\frac{2-x^3}{x^6-x^3+1}dx^3$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{3}\int \frac{2-x^3}{x^6-x^3+1}dx^3$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{3}\left(\frac{1}{2}\int -\frac{1-2x^3}{x^6-x^3+1}dx^3-\frac{3}{2}\int \frac{1}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^6-x^3+1}dx^3-\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{3}\left(3\int \frac{1}{-x^6-3}d(2x^3-1)-\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(-\frac{1}{2}\int \frac{1-2x^3}{x^6-x^3+1}dx^3-\sqrt{3}\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)\right)$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{3}\left(\frac{1}{2}\log\left(x^6-x^3+1\right)-\sqrt{3}\arctan\left(\frac{2x^3-1}{\sqrt{3}}\right)\right)$$

input
```
Int[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
```

output
```
(-(Sqrt[3]*ArcTan[(-1 + 2*x^3)/Sqrt[3]]) + Log[1 - x^3 + x^6]/2)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\ln(x^6-x^3+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{3}$	33
risch	$\frac{\ln(4x^6-4x^3+4)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{3}$	35

input
```
int(x^2*(x^3-2)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(x^6-x^3+1)-1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = \frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x**2*(x**3-2)/(x**6-x**3+1),x)
```

output
```
log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

input
```
integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = \frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{3}$$

input
```
int((x^2*(x^3 - 2))/(x^6 - x^3 + 1),x)
```

output
```
log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/3
```

Reduce [F]

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = -\frac{3\left(\int \frac{x^2}{x^6-x^3+1}dx\right)}{2} + \frac{\log(x^6-x^3+1)}{6}$$

input
```
int(x^2*(x^3-2)/(x^6-x^3+1),x)
```

output
```
( - 9*int(x**2/(x**6 - x**3 + 1),x) + log(x**6 - x**3 + 1))/6
```

3.60 $\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 39

$$\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
-1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)+ln(x)-1/6*ln(x^6-x^3+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41

$$\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx = \log(x) - \frac{1}{3}\text{RootSum}\left[1-\#1^3 + \#1^6\&, \frac{-2\log(x-\#1)+\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

input
```
Integrate[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

output
```
Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(
-1 + 2*#1^3) & ]/3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+1}{x\left(x^6-x^3+1\right)}dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{3}\int \frac{x^3+1}{x^3\left(x^6-x^3+1\right)}dx^3$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{3}\int \left(\frac{2-x^3}{x^6-x^3+1}+\frac{1}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)+\log\left(x^3\right)-\frac{1}{2}\log\left(x^6-x^3+1\right)\right)$$

input
```
Int[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

output
```
(-(Sqrt[3]*ArcTan[(1 - 2*x^3)/Sqrt[3]]) + Log[x^3] - Log[1 - x^3 + x^6]/2)
/3
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
risch	$\ln\left(x\right)-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{3}-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{3}+\ln\left(x\right)$	35

input
```
int((x^3+1)/x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log(x)$$

input
```
integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx = \log(x) - \frac{\log(x^6-x^3+1)}{6} + \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x**3+1)/x/(x**6-x**3+1),x)
```

output
```
log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \frac{1}{3}\log\left(x^3\right)$$

input
```
integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3
*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx=\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

input
```
integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx=\ln\left(x\right)-\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{3}$$

input
```
int((x^3 + 1)/(x*(x^6 - x^3 + 1)),x)
```

output
```
log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/
3))/3
```

Reduce [F]

$$\int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx=3\left(\int \frac{1}{x^7-x^4+x}dx\right)+\frac{\log(x^6-x^3+1)}{3}-2\log(x)$$

input
```
int((x^3+1)/x/(x^6-x^3+1),x)
```

output
```
(9*int(1/(x**7 - x**4 + x),x) + log(x**6 - x**3 + 1) - 6*log(x))/3
```

3.61 $\int \frac{1+x^3}{x-x^4+x^7} dx$

Optimal result

Integrand size = 18, antiderivative size = 39

$$\int \frac{1+x^3}{x-x^4+x^7} dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

output
```
-1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)+ln(x)-1/6*ln(x^6-x^3+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41

$$\int \frac{1+x^3}{x-x^4+x^7} dx = \log(x) - \frac{1}{3}\text{RootSum}\Big[1-\#1^3 + \#1^6\&, \frac{-2\log(x-\#1)+\log(x-\#1)\#1^3}{-1+2\#1^3}\&\Big]$$

input
```
Integrate[(1 + x^3)/(x - x^4 + x^7),x]
```

output
```
Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(
-1 + 2*#1^3) & ]/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1979, 1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+1}{x^7-x^4+x}dx$$

$$\Big\downarrow \text{1979}$$

$$\int \frac{x^3+1}{x\left(x^6-x^3+1\right)}dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{3}\int \frac{x^3+1}{x^3\left(x^6-x^3+1\right)}dx^3$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{3}\int \left(\frac{2-x^3}{x^6-x^3+1}+\frac{1}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)+\log\left(x^3\right)-\frac{1}{2}\log\left(x^6-x^3+1\right)\right)$$

input
```
Int[(1 + x^3)/(x - x^4 + x^7),x]
```

output
```
(-(Sqrt[3]*ArcTan[(1 - 2*x^3)/Sqrt[3]]) + Log[x^3] - Log[1 - x^3 + x^6]/2)
/3
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1979
```
Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) +
(B_.)*(x_)^(r_.)), x_Symbol] :> Int[x^(p*q)*(A + B*x^(n - q))*(a + b*x^(n -
 q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[r, n
 - q] && EqQ[j, 2*n - q] && IntegerQ[p] && PosQ[n - q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
risch	$\ln(x)-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{3}+\ln(x)$	35

input
```
int((x^3+1)/(x^7-x^4+x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)-\frac{1}{6}\,\log\left(x^6-x^3+1\right)+\log\left(x\right)$$

input
```
integrate((x^3+1)/(x^7-x^4+x),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \log\left(x\right)-\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x**3+1)/(x**7-x**4+x),x)
```

output
```
log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/3
```

Maxima [F]

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \int \frac{x^3+1}{x^7-x^4+x}\,dx$$

input
```
integrate((x^3+1)/(x^7-x^4+x),x, algorithm="maxima")
```

output
```
-integrate((x^5 - 2*x^2)/(x^6 - x^3 + 1), x) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log\left(|x|\right)$$

input
```
integrate((x^3+1)/(x^7-x^4+x),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \ln(x) - \frac{\ln\left(x^6-x^3+1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{3}$$

input
```
int((x^3 + 1)/(x - x^4 + x^7),x)
```

output
```
log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/
3))/3
```

Reduce [F]

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = 3\left(\int \frac{1}{x^7-x^4+x}dx\right) + \frac{\log(x^6-x^3+1)}{3} - 2\log(x)$$

input
```
int((x^3+1)/(x^7-x^4+x),x)
```

output
```
(9*int(1/(x**7 - x**4 + x),x) + log(x**6 - x**3 + 1) - 6*log(x))/3
```

3.62 $\int \frac{1}{x^2(d+ex^3)(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 1014

$$\int \frac{1}{x^2(d+ex^3)(a+bx^3+cx^6)}\,dx$$

$$= -\frac{1}{adx}+\frac{\sqrt[3]{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$+\frac{\sqrt[3]{c}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$+\frac{e^{7/3}\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{4/3}(cd^2-bde+ae^2)}$$
$$+\frac{\sqrt[3]{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$+\frac{\sqrt[3]{c}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$+\frac{e^{7/3}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{4/3}(cd^2-bde+ae^2)}$$
$$-\frac{\sqrt[3]{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$-\frac{\sqrt[3]{c}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$-\frac{e^{7/3}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{4/3}(cd^2-bde+ae^2)}$$

output
```
-1/a/d/x+1/6*c^(1/3)*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*ar
ctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(
1/3)*3^(1/2)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)+1/6*c^(1/3
)*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/
3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/a/(b+(
-4*a*c+b^2)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)+1/3*e^(7/3)*arctan(1/3*(d^(1/
3)-2*e^(1/3)*x)*3^(1/2)/d^(1/3))*3^(1/2)/d^(4/3)/(a*e^2-b*d*e+c*d^2)+1/6*c
^(1/3)*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^
2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)/
(a*e^2-b*d*e+c*d^2)+1/6*c^(1/3)*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2
)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/a/(b+(
-4*a*c+b^2)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)+1/3*e^(7/3)*ln(d^(1/3)+e^(1/3
)*x)/d^(4/3)/(a*e^2-b*d*e+c*d^2)-1/12*c^(1/3)*(c*d-b*e+(2*a*c*e-b^2*e+b*c*
d)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b-
(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(1/3)/a/(b-(-4*a*c+b^2)
^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)-1/12*c^(1/3)*(c*d-b*e-(2*a*c*e-b^2*e+b*c
*d)/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b
+(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(1/3)/a/(b+(-4*a*c+b^2
)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)-1/6*e^(7/3)*ln(d^(2/3)-d^(1/3)*e^(1/3)*
x+e^(2/3)*x^2)/d^(4/3)/(a*e^2-b*d*e+c*d^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^2\left(d+ex^3\right)\left(a+bx^3+cx^6\right)}\,dx$$

$$= \frac{-6cd+6be-\frac{6ae^2}{d}+\frac{2\sqrt{3}ae^{7/3}x\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{4/3}}+\frac{2ae^{7/3}x\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{d^{4/3}}-\frac{ae^{7/3}x\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{d^{4/3}}+}{6acd^2x-}$$

input
```
Integrate[1/(x^2*(d + e*x^3)*(a + b*x^3 + c*x^6)),x]
```

output
```
(-6*c*d + 6*b*e - (6*a*e^2)/d + (2*Sqrt[3]*a*e^(7/3)*x*ArcTan[(1 - (2*e^(1
/3)*x)/d^(1/3))/Sqrt[3]])/d^(4/3) + (2*a*e^(7/3)*x*Log[d^(1/3) + e^(1/3)*x
])/d^(4/3) - (a*e^(7/3)*x*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/
d^(4/3) + 2*x*RootSum[a + b*#1^3 + c*#1^6 & , (-(b*c*d*Log[x - #1]) + b^2*
e*Log[x - #1] - a*c*e*Log[x - #1] - c^2*d*Log[x - #1]*#1^3 + b*c*e*Log[x -
 #1]*#1^3)/(b*#1 + 2*c*#1^4) & ])/(6*a*c*d^2*x - 6*a*b*d*e*x + 6*a^2*e^2*x
)
```

Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 1014, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1836, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (d+ex^3) (a+bx^3+cx^6)} \, dx$$

$$\Big\downarrow \text{1836}$$

$$\int \left(\frac{x(-ace+b^2e-cx^3(cd-be)-bcd)}{a (a+bx^3+cx^6) (ae^2-bde+cd^2)}-\frac{e^3x}{d (d+ex^3) (ae^2-bde+cd^2)}+\frac{1}{adx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)e^{7/3}}{\sqrt{3}d^{4/3}(cd^2-bed+ae^2)}+\frac{\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)e^{7/3}}{3d^{4/3}(cd^2-bed+ae^2)}-\frac{\log\left(e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}\right)e^{7/3}}{6d^{4/3}(cd^2-bed+ae^2)}+$$

$$\frac{\sqrt[3]{c}\left(cd-be+\frac{-eb^2+cdb+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}(cd^2-bed+ae^2)}+$$

$$\frac{\sqrt[3]{c}\left(cd-be-\frac{-eb^2+cdb+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}(cd^2-bed+ae^2)}+$$

$$\frac{\sqrt[3]{c}\left(cd-be+\frac{-eb^2+cdb+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}(cd^2-bed+ae^2)}+$$

$$\frac{\sqrt[3]{c}\left(cd-be-\frac{-eb^2+cdb+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}(cd^2-bed+ae^2)}-$$

$$\frac{\sqrt[3]{c}\left(cd-be+\frac{-eb^2+cdb+2ace}{\sqrt{b^2-4ac}}\right)\log\left(2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}x}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}(cd^2-bed+ae^2)}-$$

$$\frac{\sqrt[3]{c}\left(cd-be-\frac{-eb^2+cdb+2ace}{\sqrt{b^2-4ac}}\right)\log\left(2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}x}+\left(b+\sqrt{b^2-4ac}\right)^{2/3}\right)}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}(cd^2-bed+ae^2)}-$$

$$\frac{1}{adx}$$

input
```
Int[1/(x^2*(d + e*x^3)*(a + b*x^3 + c*x^6)),x]
```

output
```
-(1/(a*d*x)) + (c^(1/3)*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/S
qrt[3]])/(2^(2/3)*Sqrt[3]*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*e +
 a*e^2)) + (c^(1/3)*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[
3]])/(2^(2/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*e + a*e
^2)) + (e^(7/3)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3
]*d^(4/3)*(c*d^2 - b*d*e + a*e^2)) + (c^(1/3)*(c*d - b*e + (b*c*d - b^2*e
+ 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*
c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*e + a*
e^2)) + (c^(1/3)*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])
*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b +
 Sqrt[b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*e + a*e^2)) + (e^(7/3)*Log[d^(1/3)
+ e^(1/3)*x])/(3*d^(4/3)*(c*d^2 - b*d*e + a*e^2)) - (c^(1/3)*(c*d - b*e +
(b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(
2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x
^2])/(6*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*e + a*e^2)) -
 (c^(1/3)*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b
 + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3
)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)*...
```

Defintions of rubi rules used

rule 1836
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e
*x^n)^q/(a + b*x^n + c*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[q] && Int
egerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.22

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6 c+\_Z^3 b+a\right)} \frac{\left(c(-eb+cd)\_R^4+\left(ace-b^2e+cbd\right)\_R\right)\ln\left(x-\_R\right)}{2\_R^5 c+\_R^2 b}}{3(a\,e^2-bde+c\,d^2)a}-\frac{\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right.}{d(a\,e^2-bd}$
risch	Expression too large to display

input
```
int(1/x^2/(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(a*e^2-b*d*e+c*d^2)/a*sum((c*(-b*e+c*d)*_R^4+(a*c*e-b^2*e+b*c*d)*_R)/
(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))-(-1/3/e/(d/e)^(1/3)
*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))+1/3
*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1)))*e^3/d/(a*e
^2-b*d*e+c*d^2)-1/a/d/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(d+ex^3\right)\left(a+bx^3+cx^6\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d + ex^3) (a + bx^3 + cx^6)} dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (d + ex^3) (a + bx^3 + cx^6)} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{1}{x^2 (d + ex^3) (a + bx^3 + cx^6)} dx = \int \frac{1}{(cx^6 + bx^3 + a)(ex^3 + d)x^2} dx$$

input
```
integrate(1/x^2/(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)*(e*x^3 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d+ex^3) (a+bx^3+cx^6)} dx = \text{Hanged}$$

input
```
int(1/(x^2*(d + e*x^3)*(a + b*x^3 + c*x^6)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{x^2 (d+ex^3) (a+bx^3+cx^6)} dx = \int \frac{1}{ce\,x^{11}+be\,x^8+cd\,x^8+ae\,x^5+bd\,x^5+ad\,x^2} dx$$

input
```
int(1/x^2/(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

output
```
int(1/(a*d*x**2 + a*e*x**5 + b*d*x**5 + b*e*x**8 + c*d*x**8 + c*e*x**11),x
)
```

3.63 $\int \frac{1}{x^5(d+ex^3)(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 1138

$$\int \frac{1}{x^5\left(d+ex^3\right)\left(a+bx^3+cx^6\right)}\,dx = \text{Too large to display}$$

output
```
-1/4/a/d/x^4+(a*e+b*d)/a^2/d^2/x-1/6*c^(1/3)*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e
-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/3)*c^(1
/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/a^2/(b-(-4*a*
c+b^2)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)-1/6*c^(1/3)*(b*c*d-b^2*e+a*c*e-(3*
a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/
3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/a^2/(b
+(-4*a*c+b^2)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)-1/3*e^(10/3)*arctan(1/3*(d^
(1/3)-2*e^(1/3)*x)*3^(1/2)/d^(1/3))*3^(1/2)/d^(7/3)/(a*e^2-b*d*e+c*d^2)-1/
6*c^(1/3)*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b
^2)^(1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/a^2/
(b-(-4*a*c+b^2)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)-1/6*c^(1/3)*(b*c*d-b^2*e+
a*c*e-(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*
c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/a^2/(b+(-4*a*c+b^2)^(1/2))^
(1/3)/(a*e^2-b*d*e+c*d^2)-1/3*e^(10/3)*ln(d^(1/3)+e^(1/3)*x)/d^(7/3)/(a*e^
2-b*d*e+c*d^2)+1/12*c^(1/3)*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^2*d-b^3*e+
b^2*c*d)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/
3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(1/3)/a^2/(b-(-4*
a*c+b^2)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)+1/12*c^(1/3)*(b*c*d-b^2*e+a*c*e-
(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)
^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b+(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^5\left(d+ex^3\right)\left(a+bx^3+cx^6\right)}\,dx$$

$$= \frac{-3ad^{4/3}(cd^2+e(-bd+ae))+12\sqrt[3]{d}(bd+ae)\left(cd^2+e(-bd+ae)\right)x^3-4\sqrt{3}a^2e^{10/3}x^4\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right.}{}$$

input
```
Integrate[1/(x^5*(d + e*x^3)*(a + b*x^3 + c*x^6)),x]
```

output
```
(-3*a*d^(4/3)*(c*d^2 + e*(-(b*d) + a*e)) + 12*d^(1/3)*(b*d + a*e)*(c*d^2 +
 e*(-(b*d) + a*e))*x^3 - 4*Sqrt[3]*a^2*e^(10/3)*x^4*ArcTan[(1 - (2*e^(1/3)
*x)/d^(1/3))/Sqrt[3]] - 4*a^2*e^(10/3)*x^4*Log[d^(1/3) + e^(1/3)*x] + 2*a^
2*e^(10/3)*x^4*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2] - 4*d^(7/3)*
x^4*RootSum[a + b*#1^3 + c*#1^6 & , (-(b^2*c*d*Log[x - #1]) + a*c^2*d*Log[
x - #1] + b^3*e*Log[x - #1] - 2*a*b*c*e*Log[x - #1] - b*c^2*d*Log[x - #1]*
#1^3 + b^2*c*e*Log[x - #1]*#1^3 - a*c^2*e*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1
^4) & ])/(12*a^2*d^(7/3)*(c*d^2 + e*(-(b*d) + a*e))*x^4)
```

Rubi [A] (verified)

Time = 2.76 (sec) , antiderivative size = 1138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1836, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(d+e x^3\right) \left(a+b x^3+c x^6\right)} \, dx$$

$$\Big\downarrow \text{1836}$$

$$\int \left(\frac{x \left(c x^3 \left(a c e+b^2 (-e)+b c d\right)+2 a b c e-a c^2 d+b^3 (-e)+b^2 c d\right)}{a^2 \left(a+b x^3+c x^6\right) \left(a e^2-b d e+c d^2\right)}+\frac{-a e-b d}{a^2 d^2 x^2}+\frac{e^4 x}{d^2 \left(d+e x^3\right) \left(a e^2-b d e+c d^2\right)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)e^{10/3}}{\sqrt{3}d^{7/3}(cd^2-bed+ae^2)}-\frac{\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)e^{10/3}}{3d^{7/3}(cd^2-bed+ae^2)}+\frac{\log\left(e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}\right)e^{10/3}}{6d^{7/3}(cd^2-bed+ae^2)}-$$

$$\frac{\sqrt[3]{c}\left(-eb^2+cdb+ace+\frac{-eb^3+cdb^2+3aceb-2ac^2d}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^2\sqrt[3]{b-\sqrt{b^2-4ac}}(cd^2-bed+ae^2)}-$$

$$\frac{\sqrt[3]{c}\left(-eb^2+cdb+ace-\frac{-eb^3+cdb^2+3aceb-2ac^2d}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^2\sqrt[3]{b+\sqrt{b^2-4ac}}(cd^2-bed+ae^2)}-$$

$$\frac{\sqrt[3]{c}\left(-eb^2+cdb+ace+\frac{-eb^3+cdb^2+3aceb-2ac^2d}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}\right)}{3\,2^{2/3}a^2\sqrt[3]{b-\sqrt{b^2-4ac}}(cd^2-bed+ae^2)}-$$

$$\frac{\sqrt[3]{c}\left(-eb^2+cdb+ace-\frac{-eb^3+cdb^2+3aceb-2ac^2d}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}\right)}{3\,2^{2/3}a^2\sqrt[3]{b+\sqrt{b^2-4ac}}(cd^2-bed+ae^2)}+$$

$$\frac{\sqrt[3]{c}\left(-eb^2+cdb+ace+\frac{-eb^3+cdb^2+3aceb-2ac^2d}{\sqrt{b^2-4ac}}\right)\log\left(2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\left(b-\sqrt{b^2-4ac}\right)^{2/}\right.}{6\,2^{2/3}a^2\sqrt[3]{b-\sqrt{b^2-4ac}}(cd^2-bed+ae^2)}$$

$$\frac{\sqrt[3]{c}\left(-eb^2+cdb+ace-\frac{-eb^3+cdb^2+3aceb-2ac^2d}{\sqrt{b^2-4ac}}\right)\log\left(2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\left(b+\sqrt{b^2-4ac}\right)^{2/}\right.}{6\,2^{2/3}a^2\sqrt[3]{b+\sqrt{b^2-4ac}}(cd^2-bed+ae^2)}$$

$$\frac{bd+ae}{a^2d^2x}-\frac{1}{4adx^4}$$

input
```
Int[1/(x^5*(d + e*x^3)*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/4*1/(a*d*x^4) + (b*d + a*e)/(a^2*d^2*x) - (c^(1/3)*(b*c*d - b^2*e + a*c
*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(
1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3
)*Sqrt[3]*a^2*(b - Sqrt[b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*e + a*e^2)) - (c^
(1/3)*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/S
qrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c]
)^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a^2*(b + Sqrt[b^2 - 4*a*c])^(1/3)*(c*d
^2 - b*d*e + a*e^2)) - (e^(10/3)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d
^(1/3))])/(Sqrt[3]*d^(7/3)*(c*d^2 - b*d*e + a*e^2)) - (c^(1/3)*(b*c*d - b^
2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])
*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a^2*(b
 - Sqrt[b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*e + a*e^2)) - (c^(1/3)*(b*c*d - b
^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c]
)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a^2*(
b + Sqrt[b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*e + a*e^2)) - (e^(10/3)*Log[d^(1
/3) + e^(1/3)*x])/(3*d^(7/3)*(c*d^2 - b*d*e + a*e^2)) + (c^(1/3)*(b*c*d -
b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c
])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a
*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a^2*(b - Sqrt[b^2 - 4*a*c]
)^(1/3)*(c*d^2 - b*d*e + a*e^2)) + (c^(1/3)*(b*c*d - b^2*e + a*c*e - (b...
```

Defintions of rubi rules used

rule 1836
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e
*x^n)^q/(a + b*x^n + c*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[q] && Int
egerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.32 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.23

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(c\left(ace-b^{2}e+cbd\right)\_R^{4}+\left(2abce-a\,c^{2}d-b^{3}e+b^{2}cd\right)\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3\left(a\,e^{2}-bde+c\,d^{2}\right)a^{2}}+\frac{\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\right.\right.}{6}\right.}{}$
risch	Expression too large to display

input
```
int(1/x^5/(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a*e^2-b*d*e+c*d^2)/a^2*sum((c*(a*c*e-b^2*e+b*c*d)*_R^4+(2*a*b*c*e-a*c
^2*d-b^3*e+b^2*c*d)*_R)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b
+a))+(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(d/e)^
(1/3)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)
^(1/3)*x-1)))*e^4/d^2/(a*e^2-b*d*e+c*d^2)-1/4/a/d/x^4-1/d^2/a^2*(-a*e-b*d)
/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(d+ex^3\right)\left(a+bx^3+cx^6\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^5/(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d + ex^3) (a + bx^3 + cx^6)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**5/(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 (d + ex^3) (a + bx^3 + cx^6)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{1}{x^5 (d + ex^3) (a + bx^3 + cx^6)} \, dx = \int \frac{1}{(cx^6 + bx^3 + a)(ex^3 + d)x^5} \, dx$$

input
```
integrate(1/x^5/(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)*(e*x^3 + d)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d + ex^3) (a + bx^3 + cx^6)} \, dx = \text{Hanged}$$

input
```
int(1/(x^5*(d + e*x^3)*(a + b*x^3 + c*x^6)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{x^5 (d + ex^3) (a + bx^3 + cx^6)} \, dx = \int \frac{1}{ce\, x^{14} + be\, x^{11} + cd\, x^{11} + ae\, x^8 + bd\, x^8 + ad\, x^5} dx$$

input
```
int(1/x^5/(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

output
```
int(1/(a*d*x**5 + a*e*x**8 + b*d*x**8 + b*e*x**11 + c*d*x**11 + c*e*x**14)
,x)
```

3.64 $\int \frac{1}{x^2(d+ex^3)^2(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 1378

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+bx^3+cx^6)}\,dx = \text{Too large to display}$$

output
```
-1/a/d^2/x-1/3*e^3*x^2/d^2/(a*e^2-b*d*e+c*d^2)/(e*x^3+d)+1/6*c^(1/3)*(c^2*
d^2+b^2*e^2-c*e*(a*e+2*b*d)-(2*b^2*c*d*e-4*a*c^2*d*e-b^3*e^2-b*c*(-3*a*e^2
+c*d^2))/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+
b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)
/(a*e^2-b*d*e+c*d^2)^2+1/6*c^(1/3)*(c^2*d^2+b^2*e^2-c*e*(a*e+2*b*d)+(2*b^2
*c*d*e-4*a*c^2*d*e-b^3*e^2-b*c*(-3*a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))*arcta
n(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(1/3
)*3^(1/2)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)^2+1/9*e^(7/3)
*(10*c*d^2-e*(-4*a*e+7*b*d))*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)*3^(1/2)/d^(1
/3))*3^(1/2)/d^(7/3)/(a*e^2-b*d*e+c*d^2)^2+1/6*c^(1/3)*(c^2*d^2+b^2*e^2-c*
e*(a*e+2*b*d)-(2*b^2*c*d*e-4*a*c^2*d*e-b^3*e^2-b*c*(-3*a*e^2+c*d^2))/(-4*a
*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/
a/(b-(-4*a*c+b^2)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)^2+1/6*c^(1/3)*(c^2*d^2+
b^2*e^2-c*e*(a*e+2*b*d)+(2*b^2*c*d*e-4*a*c^2*d*e-b^3*e^2-b*c*(-3*a*e^2+c*d
^2))/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x
)*2^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)^2+1/9*e^(7/3)
*(10*c*d^2-e*(-4*a*e+7*b*d))*ln(d^(1/3)+e^(1/3)*x)/d^(7/3)/(a*e^2-b*d*e+c*
d^2)^2-1/12*c^(1/3)*(c^2*d^2+b^2*e^2-c*e*(a*e+2*b*d)-(2*b^2*c*d*e-4*a*c^2*
d*e-b^3*e^2-b*c*(-3*a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2)^(
1/2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 475, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^2 (d + ex^3)^2 (a + bx^3 + cx^6)} \, dx$$

$$= \frac{-6a\sqrt[3]{d}e^3(cd^2 + e(-bd + ae))\,x^3 - 18\sqrt[3]{d}(cd^2 + e(-bd + ae))^2\,(d + ex^3) + 2\sqrt{3}ae^{7/3}(10cd^2 + e(-7bd + \cdots}{\cdots}$$

input
```
Integrate[1/(x^2*(d + e*x^3)^2*(a + b*x^3 + c*x^6)),x]
```

output
```
(-6*a*d^(1/3)*e^3*(c*d^2 + e*(-(b*d) + a*e))*x^3 - 18*d^(1/3)*(c*d^2 + e*(
-(b*d) + a*e))^2*(d + e*x^3) + 2*Sqrt[3]*a*e^(7/3)*(10*c*d^2 + e*(-7*b*d +
 4*a*e))*x*(d + e*x^3)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] + 2*a*e
^(7/3)*(10*c*d^2 + e*(-7*b*d + 4*a*e))*x*(d + e*x^3)*Log[d^(1/3) + e^(1/3)
*x] - a*e^(7/3)*(10*c*d^2 + e*(-7*b*d + 4*a*e))*x*(d + e*x^3)*Log[d^(2/3)
- d^(1/3)*e^(1/3)*x + e^(2/3)*x^2] - 6*d^(7/3)*x*(d + e*x^3)*RootSum[a + b
*#1^3 + c*#1^6 & , (b*c^2*d^2*Log[x - #1] - 2*b^2*c*d*e*Log[x - #1] + 2*a*
c^2*d*e*Log[x - #1] + b^3*e^2*Log[x - #1] - 2*a*b*c*e^2*Log[x - #1] + c^3*
d^2*Log[x - #1]*#1^3 - 2*b*c^2*d*e*Log[x - #1]*#1^3 + b^2*c*e^2*Log[x - #1
]*#1^3 - a*c^2*e^2*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ])/(18*a*d^(7/3)*
(c*d^2 + e*(-(b*d) + a*e))^2*x*(d + e*x^3))
```

Rubi [A] (verified)

Time = 3.98 (sec) , antiderivative size = 1545, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1836, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+bx^3+cx^6)} \, dx$$

$$\Big\downarrow \text{1836}$$

$$\int \left(\frac{x\left(-cx^3\left(-ce(ae+2bd)+b^2e^2+c^2d^2\right)-(cd-be)\left(2ace+b^2(-e)+bcd\right)\right)}{a\left(a+bx^3+cx^6\right)\left(ae^2-bde+cd^2\right)^2}+\frac{e^3x\left(e(2bd-ae)-3cd^2\right)}{d^2\left(d+ex^3\right)\left(ae^2-bde+cd^2\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x^2e^3}{3d^2\left(cd^2-bed+ae^2\right)\left(ex^3+d\right)}+\frac{\left(3cd^2-e(2bd-ae)\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)e^{7/3}}{\sqrt{3}d^{7/3}\left(cd^2-bed+ae^2\right)^2}+\frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)e^{7/3}}{3\sqrt{3}d^{7/3}\left(cd^2-bed+ae^2\right)}+\frac{\left(3cd^2-e(2bd-ae)\right)\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)e^{7/3}}{3d^{7/3}\left(cd^2-bed+ae^2\right)^2}+\frac{\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)e^{7/3}}{9d^{7/3}\left(cd^2-bed+ae^2\right)}-\frac{\left(3cd^2-e(2bd-ae)\right)\log\left(e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}\right)e^{7/3}}{6d^{7/3}\left(cd^2-bed+ae^2\right)^2}-\frac{\log\left(e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}\right)e^{7/3}}{18d^{7/3}\left(cd^2-bed+ae^2\right)}+$$

$$\frac{\sqrt[3]{c}\left(c^2d^2+b^2e^2-ce(2bd+ae)-\frac{-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^2}+$$

$$\frac{\sqrt[3]{c}\left(c^2d^2+b^2e^2-ce(2bd+ae)+\frac{-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^2}+$$

$$\frac{\sqrt[3]{c}\left(c^2d^2+b^2e^2-ce(2bd+ae)-\frac{-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2-4ac}}\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^2}+$$

$$\frac{\sqrt[3]{c}\left(c^2d^2+b^2e^2-ce(2bd+ae)+\frac{-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2-4ac}}\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^2}-$$

$$\frac{\sqrt[3]{c}\left(c^2d^2+b^2e^2-ce(2bd+ae)-\frac{-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de}{\sqrt{b^2-4ac}}\right)\log\left(2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\right.}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^2}$$

$$\frac{\sqrt[3]{c}\left(c^2d^2+b^2e^2-ce(2bd+ae)+\frac{-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de}{\sqrt{b^2-4ac}}\right)\log\left(2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+\right.}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^2}$$

$$\frac{1}{ad^2x}$$

input
```
Int[1/(x^2*(d + e*x^3)^2*(a + b*x^3 + c*x^6)),x]
```

output
```
-(1/(a*d^2*x)) - (e^3*x^2)/(3*d^2*(c*d^2 - b*d*e + a*e^2)*(d + e*x^3)) + (
c^(1/3)*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e) - (2*b^2*c*d*e - 4*a*c^2*d*
e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(
1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*
a*(b - Sqrt[b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*e + a*e^2)^2) + (c^(1/3)*(c^2
*d^2 + b^2*e^2 - c*e*(2*b*d + a*e) + (2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2
- b*c*(c*d^2 - 3*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a*(b + Sqrt[
b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*e + a*e^2)^2) + (e^(7/3)*ArcTan[(d^(1/3)
- 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(3*Sqrt[3]*d^(7/3)*(c*d^2 - b*d*e + a*e
^2)) + (e^(7/3)*(3*c*d^2 - e*(2*b*d - a*e))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)
/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(7/3)*(c*d^2 - b*d*e + a*e^2)^2) + (c^(1/3
)*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e) - (2*b^2*c*d*e - 4*a*c^2*d*e - b^
3*e^2 - b*c*(c*d^2 - 3*a*e^2))/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*
c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)
*(c*d^2 - b*d*e + a*e^2)^2) + (c^(1/3)*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a
*e) + (2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))/Sqrt[b
^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(2/3)*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*e + a*e^2)^2) + (e^(7/3
)*Log[d^(1/3) + e^(1/3)*x])/(9*d^(7/3)*(c*d^2 - b*d*e + a*e^2)) + (e^(7...
```

Defintions of rubi rules used

rule 1836
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e
*x^n)^q/(a + b*x^n + c*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[q] && Int
egerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.73 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.23

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6 c+\_Z^3 b+a\right)} \frac{\left(c\left(a c e^2-b^2 e^2+2 b c d e-c^2 d^2\right)\_R^4+\left(2 a b c e^2-2 a c^2 d e-b^3 e^2+2 b^2 c d e-b c^2 d^2\right)\_R\right) \ln\left(x-\_R\right)}{2\_R^5 c+\_R^2 b}}{3 a\left(a e^2-b d e+c d^2\right)^2} - \frac{e^3 \left(\right.}{}$
risch	Expression too large to display

input
```
int(1/x^2/(e*x^3+d)^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/a/(a*e^2-b*d*e+c*d^2)^2*sum((c*(a*c*e^2-b^2*e^2+2*b*c*d*e-c^2*d^2)*_R^
4+(2*a*b*c*e^2-2*a*c^2*d*e-b^3*e^2+2*b^2*c*d*e-b*c^2*d^2)*_R)/(2*_R^5*c+_R
^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))-e^3/d^2/(a*e^2-b*d*e+c*d^2)^2*(
(1/3*a*e^2-1/3*b*d*e+1/3*c*d^2)*x^2/(e*x^3+d)+(4/3*a*e^2-7/3*b*d*e+10/3*c*
d^2)*(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(d/e)^
(1/3)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)
^(1/3)*x-1))))-1/a/d^2/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(d+e x^3\right)^2 \left(a+b x^3+c x^6\right)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(e*x^3+d)^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d + ex^3)^2 (a + bx^3 + cx^6)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*x**3+d)**2/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (d + ex^3)^2 (a + bx^3 + cx^6)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(e*x^3+d)^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{1}{x^2 (d + ex^3)^2 (a + bx^3 + cx^6)}\,dx = \int \frac{1}{(cx^6 + bx^3 + a)(ex^3 + d)^2 x^2}\,dx$$

input
```
integrate(1/x^2/(e*x^3+d)^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)*(e*x^3 + d)^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 95.39 (sec) , antiderivative size = 79010, normalized size of antiderivative = 57.34

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+bx^3+cx^6)} dx = \text{Too large to display}$$

input
```
int(1/(x^2*(d + e*x^3)^2*(a + b*x^3 + c*x^6)),x)
```

output
```
symsum(log(root(8453100546*a^14*b^6*c^2*d^11*e^20*z^9 + 8453100546*a^6*b^6
*c^10*d^27*e^4*z^9 + 218868661440*a^14*b^3*c^5*d^14*e^17*z^9 + 21886866144
0*a^9*b^3*c^10*d^24*e^7*z^9 + 4115479104*a^16*b^3*c^3*d^10*e^21*z^9 + 4115
479104*a^7*b^3*c^12*d^28*e^3*z^9 + 1332854028*a^14*b^7*c*d^10*e^21*z^9 + 3
998562084*a^5*b^9*c^8*d^26*e^5*z^9 - 25394376744*a^9*b^10*c^3*d^17*e^14*z^
9 - 25394376744*a^7*b^10*c^5*d^21*e^10*z^9 - 3788111448*a^5*b^10*c^7*d^25*
e^6*z^9 + 149770702620*a^10*b^6*c^6*d^19*e^12*z^9 + 22448067840*a^16*b*c^5
*d^12*e^19*z^9 + 22448067840*a^9*b*c^12*d^26*e^5*z^9 + 158258878272*a^9*b^
8*c^5*d^19*e^12*z^9 - 7926973956*a^13*b^7*c^2*d^12*e^19*z^9 - 7926973956*a
^6*b^7*c^9*d^26*e^5*z^9 - 20764462752*a^16*b^2*c^4*d^11*e^20*z^9 - 2076446
2752*a^8*b^2*c^12*d^27*e^4*z^9 - 3577660812*a^8*b^12*c^2*d^17*e^14*z^9 - 3
577660812*a^6*b^12*c^4*d^21*e^10*z^9 + 119535961248*a^12*b^5*c^5*d^16*e^15
*z^9 + 119535961248*a^9*b^5*c^8*d^22*e^9*z^9 + 527529594240*a^13*b^3*c^6*d
^16*e^15*z^9 + 527529594240*a^10*b^3*c^9*d^22*e^9*z^9 + 50765370084*a^13*b
^6*c^3*d^13*e^18*z^9 + 50765370084*a^7*b^6*c^9*d^25*e^6*z^9 - 701502120*a^
8*b^13*c*d^16*e^15*z^9 - 256749775920*a^14*b^2*c^6*d^15*e^16*z^9 - 2567497
75920*a^10*b^2*c^10*d^23*e^8*z^9 + 631351908*a^7*b^14*c*d^17*e^14*z^9 + 50
508152640*a^15*b^3*c^4*d^12*e^19*z^9 + 50508152640*a^8*b^3*c^11*d^26*e^5*z
^9 - 414523980*a^15*b^6*c*d^9*e^22*z^9 + 408146688*a^18*b*c^3*d^8*e^23*z^9
 + 114748209279*a^12*b^6*c^4*d^15*e^16*z^9 + 114748209279*a^8*b^6*c^8*d...
```

Reduce [F]

$$\int \frac{1}{x^2 (d+ex^3)^2 (a+bx^3+cx^6)} dx$$
$$= \int \frac{1}{c e^2 x^{14} + b e^2 x^{11} + 2cde\, x^{11} + a e^2 x^8 + 2bde\, x^8 + c d^2 x^8 + 2ade\, x^5 + b d^2 x^5 + a d^2 x^2} dx$$

input
```
int(1/x^2/(e*x^3+d)^2/(c*x^6+b*x^3+a),x)
```

output
```
int(1/(a*d**2*x**2 + 2*a*d*e*x**5 + a*e**2*x**8 + b*d**2*x**5 + 2*b*d*e*x*
*8 + b*e**2*x**11 + c*d**2*x**8 + 2*c*d*e*x**11 + c*e**2*x**14),x)
```

3.65 $\int \frac{1}{x^5(d+ex^3)^2(a+bx^3+cx^6)} dx$

Optimal result

Integrand size = 27, antiderivative size = 1520

$$\int \frac{1}{x^5 (d+ex^3)^2 (a+bx^3+cx^6)} \, dx = \text{Too large to display}$$

output
```
-1/4/a/d^2/x^4+(2*a*e+b*d)/a^2/d^3/x+1/3*e^4*x^2/d^3/(a*e^2-b*d*e+c*d^2)/(
e*x^3+d)-1/6*c^(1/3)*((-b*e+c*d)*(2*a*c*e-b^2*e+b*c*d)-(2*b^3*c*d*e-6*a*b*
c^2*d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(-a*e^2+c*d^2))/(-4*a*c+b^2
)^(1/2))*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3
^(1/2))*2^(1/3)*3^(1/2)/a^2/(b-(-4*a*c+b^2)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^
2)^2-1/6*c^(1/3)*((-b*e+c*d)*(2*a*c*e-b^2*e+b*c*d)+(2*b^3*c*d*e-6*a*b*c^2*
d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1
/2))*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/
2))*2^(1/3)*3^(1/2)/a^2/(b+(-4*a*c+b^2)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)^2
-1/9*e^(10/3)*(13*c*d^2-e*(-7*a*e+10*b*d))*arctan(1/3*(d^(1/3)-2*e^(1/3)*x
)*3^(1/2)/d^(1/3))*3^(1/2)/d^(10/3)/(a*e^2-b*d*e+c*d^2)^2-1/6*c^(1/3)*((-b
*e+c*d)*(2*a*c*e-b^2*e+b*c*d)-(2*b^3*c*d*e-6*a*b*c^2*d*e-b^4*e^2-b^2*c*(-4
*a*e^2+c*d^2)+2*a*c^2*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^
2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(1/3)/a^2/(b-(-4*a*c+b^2)^(1/2))^(1/3
)/(a*e^2-b*d*e+c*d^2)^2-1/6*c^(1/3)*((-b*e+c*d)*(2*a*c*e-b^2*e+b*c*d)+(2*b
^3*c*d*e-6*a*b*c^2*d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(-a*e^2+c*d^
2))/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)
*2^(1/3)/a^2/(b+(-4*a*c+b^2)^(1/2))^(1/3)/(a*e^2-b*d*e+c*d^2)^2-1/9*e^(10/
3)*(13*c*d^2-e*(-7*a*e+10*b*d))*ln(d^(1/3)+e^(1/3)*x)/d^(10/3)/(a*e^2-b*d*
e+c*d^2)^2+1/12*c^(1/3)*((-b*e+c*d)*(2*a*c*e-b^2*e+b*c*d)-(2*b^3*c*d*e-...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.02 (sec) , antiderivative size = 554, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^5 (d+ex^3)^2 (a+bx^3+cx^6)}\,dx$$

$$= \frac{1}{36}\left(-\frac{9}{ad^2x^4} + \frac{36(bd+2ae)}{a^2d^3x} + \frac{12e^4x^2}{d^3(cd^2+e(-bd+ae))(d+ex^3)} - \frac{4\sqrt{3}e^{10/3}(13cd^2+e(-10bd+7ae))\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{10/3}(cd^2+e(-bd+ae))^2} - \frac{4e^{10/3}(13cd^2+e(-10bd+7ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{d^{10/3}(cd^2+e(-bd+ae))^2} + \frac{2e^{10/3}(13cd^2+e(-10bd+7ae))\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{d^{10/3}(cd^2+e(-bd+ae))^2} + \frac{12\text{RootSum}\left[a+b\#1^3+c\#1^6\&, \frac{b^2c^2d^2\log(x-\#1)-ac^3d^2\log(x-\#1)-2b^3cde\log(x-\#1)+4abc^2de\log(x-\#1)+b^4e^2\log}{}\right.}{}\right.$$

input
```
Integrate[1/(x^5*(d + e*x^3)^2*(a + b*x^3 + c*x^6)),x]
```

output
```
(-9/(a*d^2*x^4) + (36*(b*d + 2*a*e))/(a^2*d^3*x) + (12*e^4*x^2)/(d^3*(c*d^
2 + e*(-(b*d) + a*e))*(d + e*x^3)) - (4*Sqrt[3]*e^(10/3)*(13*c*d^2 + e*(-1
0*b*d + 7*a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/(d^(10/3)*(c*
d^2 + e*(-(b*d) + a*e))^2) - (4*e^(10/3)*(13*c*d^2 + e*(-10*b*d + 7*a*e))*
Log[d^(1/3) + e^(1/3)*x])/(d^(10/3)*(c*d^2 + e*(-(b*d) + a*e))^2) + (2*e^(
10/3)*(13*c*d^2 + e*(-10*b*d + 7*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e
^(2/3)*x^2])/(d^(10/3)*(c*d^2 + e*(-(b*d) + a*e))^2) + (12*RootSum[a + b*#
1^3 + c*#1^6 & , (b^2*c^2*d^2*Log[x - #1] - a*c^3*d^2*Log[x - #1] - 2*b^3*
c*d*e*Log[x - #1] + 4*a*b*c^2*d*e*Log[x - #1] + b^4*e^2*Log[x - #1] - 3*a*
b^2*c*e^2*Log[x - #1] + a^2*c^2*e^2*Log[x - #1] + b*c^3*d^2*Log[x - #1]*#1
^3 - 2*b^2*c^2*d*e*Log[x - #1]*#1^3 + 2*a*c^3*d*e*Log[x - #1]*#1^3 + b^3*c
*e^2*Log[x - #1]*#1^3 - 2*a*b*c^2*e^2*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4)
& ])/(a^2*(c*d^2 + e*(-(b*d) + a*e))^2))/36
```

Rubi [A] (verified)

Time = 3.36 (sec) , antiderivative size = 1688, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1836, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (d+ex^3)^2 (a+bx^3+cx^6)} \, dx$$

$$\Big\downarrow \text{ 1836}$$

$$\int \left(\frac{x \left(b^2 c (cd^2-3ae^2)+cx^3(cd-be)\left(2ace+b^2(-e)+bcd\right)+4abc^2de-ac^2(cd^2-ae^2)+b^4e^2-2b^3cde\right)}{a^2 (a+bx^3+cx^6) (ae^2-bde+cd^2)^2}+-\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2e^4}{3d^3\left(cd^2-bed+ae^2\right)\left(ex^3+d\right)}-\frac{\left(4cd^2-e(3bd-2ae)\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)e^{10/3}}{\sqrt{3}d^{10/3}\left(cd^2-bed+ae^2\right)^2}-$$

$$\frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)e^{10/3}}{3\sqrt{3}d^{10/3}\left(cd^2-bed+ae^2\right)}-\frac{\left(4cd^2-e(3bd-2ae)\right)\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)e^{10/3}}{3d^{10/3}\left(cd^2-bed+ae^2\right)^2}-$$

$$\frac{\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)e^{10/3}}{9d^{10/3}\left(cd^2-bed+ae^2\right)}+\frac{\left(4cd^2-e(3bd-2ae)\right)\log\left(e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}\right)e^{10/3}}{6d^{10/3}\left(cd^2-bed+ae^2\right)^2}+$$

$$\frac{\log\left(e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}\right)e^{10/3}}{18d^{10/3}\left(cd^2-bed+ae^2\right)}-$$

$$\frac{\sqrt[3]{c}\left((cd-be)\left(-eb^2+cdb+2ace\right)-\frac{-e^2b^4+2cdeb^3-c\left(cd^2-4ae^2\right)b^2-6ac^2deb+2ac^2\left(cd^2-ae^2\right)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4}}}}{\sqrt{3}}\right.}{2^{2/3}\sqrt{3}a^2\sqrt[3]{b-\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^2}$$

$$\frac{\sqrt[3]{c}\left((cd-be)\left(-eb^2+cdb+2ace\right)+\frac{-e^2b^4+2cdeb^3-c\left(cd^2-4ae^2\right)b^2-6ac^2deb+2ac^2\left(cd^2-ae^2\right)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4}}}}{\sqrt{3}}\right.}{2^{2/3}\sqrt{3}a^2\sqrt[3]{b+\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^2}$$

$$\frac{\sqrt[3]{c}\left((cd-be)\left(-eb^2+cdb+2ace\right)-\frac{-e^2b^4+2cdeb^3-c\left(cd^2-4ae^2\right)b^2-6ac^2deb+2ac^2\left(cd^2-ae^2\right)}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{b^2}}\right.}{3\ 2^{2/3}a^2\sqrt[3]{b-\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^2}$$

$$\frac{\sqrt[3]{c}\left((cd-be)\left(-eb^2+cdb+2ace\right)+\frac{-e^2b^4+2cdeb^3-c\left(cd^2-4ae^2\right)b^2-6ac^2deb+2ac^2\left(cd^2-ae^2\right)}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{b^2}}\right.}{3\ 2^{2/3}a^2\sqrt[3]{b+\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^2}$$

$$\frac{\sqrt[3]{c}\left((cd-be)\left(-eb^2+cdb+2ace\right)-\frac{-e^2b^4+2cdeb^3-c\left(cd^2-4ae^2\right)b^2-6ac^2deb+2ac^2\left(cd^2-ae^2\right)}{\sqrt{b^2-4ac}}\right)\log\left(2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\ }\right.}{6\ 2^{2/3}a^2\sqrt[3]{b-\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^2}$$

$$\frac{\sqrt[3]{c}\left((cd-be)\left(-eb^2+cdb+2ace\right)+\frac{-e^2b^4+2cdeb^3-c\left(cd^2-4ae^2\right)b^2-6ac^2deb+2ac^2\left(cd^2-ae^2\right)}{\sqrt{b^2-4ac}}\right)\log\left(2^{2/3}c^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{\ }\right.}{6\ 2^{2/3}a^2\sqrt[3]{b+\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^2}$$

$$\frac{bd+2ae}{a^2d^3x}-\frac{1}{4ad^2x^4}$$

input
```
Int[1/(x^5*(d + e*x^3)^2*(a + b*x^3 + c*x^6)),x]
```

output
```
-1/4*1/(a*d^2*x^4) + (b*d + 2*a*e)/(a^2*d^3*x) + (e^4*x^2)/(3*d^3*(c*d^2 -
 b*d*e + a*e^2)*(d + e*x^3)) - (c^(1/3)*((c*d - b*e)*(b*c*d - b^2*e + 2*a*
c*e) - (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) +
2*a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a^2*(b - Sqr
t[b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*e + a*e^2)^2) - (c^(1/3)*((c*d - b*e)*(
b*c*d - b^2*e + 2*a*c*e) + (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*
(c*d^2 - 4*a*e^2) + 2*a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(1
- (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*
Sqrt[3]*a^2*(b + Sqrt[b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*e + a*e^2)^2) - (e^
(10/3)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(3*Sqrt[3]*d^(10
/3)*(c*d^2 - b*d*e + a*e^2)) - (e^(10/3)*(4*c*d^2 - e*(3*b*d - 2*a*e))*Arc
Tan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(10/3)*(c*d^2 -
 b*d*e + a*e^2)^2) - (c^(1/3)*((c*d - b*e)*(b*c*d - b^2*e + 2*a*c*e) - (2*
b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a*c^2*(c
*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x]])/(3*2^(2/3)*a^2*(b - Sqrt[b^2 - 4*a*c])^(1/3)*(c*d^2 - b*d*
e + a*e^2)^2) - (c^(1/3)*((c*d - b*e)*(b*c*d - b^2*e + 2*a*c*e) + (2*b^3*c
*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a*c^2*(c*d^2
- a*e^2))/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3...
```

Defintions of rubi rules used

rule 1836
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e
*x^n)^q/(a + b*x^n + c*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[q] && Int
egerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.02 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.24

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6} c+\_Z^{3} b+a\right)} \frac{\left(c\left(-2abc\,e^{2}+2a\,c^{2} de+b^{3} e^{2}-2b^{2} cde+b\,c^{2} d^{2}\right)\_R^{4}+\left(a^{2} c^{2} e^{2}-3a\,b^{2} c\,e^{2}+4ab\,c^{2} de-a\,c^{3} d^{2}+b^{4} e^{2}-2b^{3} cde+b^{2} c \right.}{2\_R^{5} c+\_R^{2} b}}{3a^{2}\left(a\,e^{2}-bde+c\,d^{2}\right)^{2}}$
risch	Expression too large to display

input
```
int(1/x^5/(e*x^3+d)^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2/(a*e^2-b*d*e+c*d^2)^2*sum((c*(-2*a*b*c*e^2+2*a*c^2*d*e+b^3*e^2-2*b
^2*c*d*e+b*c^2*d^2)*_R^4+(a^2*c^2*e^2-3*a*b^2*c*e^2+4*a*b*c^2*d*e-a*c^3*d^
2+b^4*e^2-2*b^3*c*d*e+b^2*c^2*d^2)*_R)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootO
f(_Z^6*c+_Z^3*b+a))+e^4/d^3/(a*e^2-b*d*e+c*d^2)^2*((1/3*a*e^2-1/3*b*d*e+1/
3*c*d^2)*x^2/(e*x^3+d)+(7/3*a*e^2-10/3*b*d*e+13/3*c*d^2)*(-1/3/e/(d/e)^(1/
3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))+1
/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))))-1/4/a/d
^2/x^4-(-2*a*e-b*d)/d^3/a^2/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(d+ex^3\right)^2 \left(a+bx^3+cx^6\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^5/(e*x^3+d)^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5\,(d+ex^3)^2\,(a+bx^3+cx^6)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**5/(e*x**3+d)**2/(c*x**6+b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5\,(d+ex^3)^2\,(a+bx^3+cx^6)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(e*x^3+d)^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{1}{x^5\left(d+ex^3\right)^2\left(a+bx^3+cx^6\right)}\,dx=\int \frac{1}{(cx^6+bx^3+a)(ex^3+d)^2x^5}\,dx$$

input
```
integrate(1/x^5/(e*x^3+d)^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^6 + b*x^3 + a)*(e*x^3 + d)^2*x^5), x)
```

Mupad [B] (verification not implemented)

Time = 114.71 (sec) , antiderivative size = 93066, normalized size of antiderivative = 61.23

$$\int \frac{1}{x^5\left(d+ex^3\right)^2\left(a+bx^3+cx^6\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(x^5*(d + e*x^3)^2*(a + b*x^3 + c*x^6)),x)
```

output
```
((x^3*(7*a*e + 4*b*d))/(4*a^2*d^2) - 1/(4*a*d) + (e*x^6*(7*a^2*e^3 - 3*b^2
*d^2*e + 3*b*c*d^3 - 3*a*b*d*e^2 + 6*a*c*d^2*e))/(3*a^2*d^3*(a*e^2 + c*d^2
 - b*d*e)))/(d*x^4 + e*x^7) + symsum(log(- root(8453100546*a^17*b^6*c^2*d^
14*e^20*z^9 + 8453100546*a^9*b^6*c^10*d^30*e^4*z^9 + 218868661440*a^17*b^3
*c^5*d^17*e^17*z^9 + 218868661440*a^12*b^3*c^10*d^27*e^7*z^9 + 4115479104*
a^19*b^3*c^3*d^13*e^21*z^9 + 4115479104*a^10*b^3*c^12*d^31*e^3*z^9 + 13328
54028*a^17*b^7*c*d^13*e^21*z^9 + 3998562084*a^8*b^9*c^8*d^29*e^5*z^9 - 253
94376744*a^12*b^10*c^3*d^20*e^14*z^9 - 25394376744*a^10*b^10*c^5*d^24*e^10
*z^9 - 3788111448*a^8*b^10*c^7*d^28*e^6*z^9 + 149770702620*a^13*b^6*c^6*d^
22*e^12*z^9 + 22448067840*a^19*b*c^5*d^15*e^19*z^9 + 22448067840*a^12*b*c^
12*d^29*e^5*z^9 + 158258878272*a^12*b^8*c^5*d^22*e^12*z^9 - 7926973956*a^1
6*b^7*c^2*d^15*e^19*z^9 - 7926973956*a^9*b^7*c^9*d^29*e^5*z^9 - 2076446275
2*a^19*b^2*c^4*d^14*e^20*z^9 - 20764462752*a^11*b^2*c^12*d^30*e^4*z^9 - 35
77660812*a^11*b^12*c^2*d^20*e^14*z^9 - 3577660812*a^9*b^12*c^4*d^24*e^10*z
^9 + 119535961248*a^15*b^5*c^5*d^19*e^15*z^9 + 119535961248*a^12*b^5*c^8*d
^25*e^9*z^9 + 527529594240*a^16*b^3*c^6*d^19*e^15*z^9 + 527529594240*a^13*
b^3*c^9*d^25*e^9*z^9 + 50765370084*a^16*b^6*c^3*d^16*e^18*z^9 + 5076537008
4*a^10*b^6*c^9*d^28*e^6*z^9 - 701502120*a^11*b^13*c*d^19*e^15*z^9 - 256749
775920*a^17*b^2*c^6*d^18*e^16*z^9 - 256749775920*a^13*b^2*c^10*d^26*e^8*z^
9 + 631351908*a^10*b^14*c*d^20*e^14*z^9 + 50508152640*a^18*b^3*c^4*d^15...
```

Reduce [F]

$$\int \frac{1}{x^5 (d+ex^3)^2 (a+bx^3+cx^6)}\,dx = \int \frac{1}{c e^2x^{17}+b e^2x^{14}+2cde\,x^{14}+a e^2x^{11}+2bde\,x^{11}+c d^2x^{11}+2ade\,x^8+b d^2x^8+a d^2x^5}dx$$

input
```
int(1/x^5/(e*x^3+d)^2/(c*x^6+b*x^3+a),x)
```

output
```
int(1/(a*d**2*x**5 + 2*a*d*e*x**8 + a*e**2*x**11 + b*d**2*x**8 + 2*b*d*e*x
**11 + b*e**2*x**14 + c*d**2*x**11 + 2*c*d*e*x**14 + c*e**2*x**17),x)
```

3.66 $\int \frac{x^6(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 360

$$\int \frac{x^6(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx = -\frac{16d(280cd^2-322bde+391ae^2)\,x\sqrt{d+ex^3}}{21505e^4} + \frac{2(280cd^2-322bde+391ae^2)\,x^4\sqrt{d+ex^3}}{4301e^3} - \frac{2(20cd-23be)x^7\sqrt{d+ex^3}}{391e^2} + \frac{2cx^{10}\sqrt{d+ex^3}}{23e} + \frac{32\sqrt{2+\sqrt{3}}\,d^2(280cd^2-322bde+391ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{(1\cdots}{(1\cdots}\right.\right.}{21505\sqrt[4]{3}e^{13/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

output
```
-16/21505*d*(391*a*e^2-322*b*d*e+280*c*d^2)*x*(e*x^3+d)^(1/2)/e^4+2/4301*(
391*a*e^2-322*b*d*e+280*c*d^2)*x^4*(e*x^3+d)^(1/2)/e^3-2/391*(-23*b*e+20*c
*d)*x^7*(e*x^3+d)^(1/2)/e^2+2/23*c*x^10*(e*x^3+d)^(1/2)/e+32/64515*(1/2*6^
(1/2)+1/2*2^(1/2))*d^2*(391*a*e^2-322*b*d*e+280*c*d^2)*(d^(1/3)+e^(1/3)*x)
*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/((1+3^(1/2))*d^(1/3)+e^(1/3)*x)^
2)^(1/2)*EllipticF(((1-3^(1/2))*d^(1/3)+e^(1/3)*x)/((1+3^(1/2))*d^(1/3)+e^
(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/e^(13/3)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/((1+
3^(1/2))*d^(1/3)+e^(1/3)*x)^2)^(1/2)/(e*x^3+d)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.45

$$\int \frac{x^6(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx$$
$$= \frac{2x\Big(-((d+ex^3)\,(5c(448d^3-280d^2ex^3+220de^2x^6-187e^3x^9)-23e(17ae(-8d+5ex^3)+b(112d^2-70}{21505e^4\surd}$$

input
```
Integrate[(x^6*(a + b*x^3 + c*x^6))/Sqrt[d + e*x^3],x]
```

output
```
(2*x*(-((d + e*x^3)*(5*c*(448*d^3 - 280*d^2*e*x^3 + 220*d*e^2*x^6 - 187*e^
3*x^9) - 23*e*(17*a*e*(-8*d + 5*e*x^3) + b*(112*d^2 - 70*d*e*x^3 + 55*e^2*
x^6)))) + 8*d^2*(280*c*d^2 + 23*e*(-14*b*d + 17*a*e))*Sqrt[1 + (e*x^3)/d]*
Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)]))/(21505*e^4*Sqrt[d + e*x^3
])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1810, 27, 959, 843, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx$$
$$\Big\downarrow \text{ 1810}$$
$$\frac{2\int \frac{x^6(23ae-(20cd-23be)x^3)}{2\sqrt{ex^3+d}}dx}{23e}+\frac{2cx^{10}\sqrt{d+ex^3}}{23e}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{x^6\left(23ae-(20cd-23be)x^3\right)}{\sqrt{ex^3+d}}dx}{23e}+\frac{2cx^{10}\sqrt{d+ex^3}}{23e}$$

$$\Big\downarrow \; 959$$

$$\frac{\frac{\left(280cd^2-23e(14bd-17ae)\right)\int \frac{x^6}{\sqrt{ex^3+d}}dx}{17e}-\frac{2x^7\sqrt{d+ex^3}(20cd-23be)}{17e}}{23e}+\frac{2cx^{10}\sqrt{d+ex^3}}{23e}$$

$$\Big\downarrow \; 843$$

$$\frac{\frac{\left(280cd^2-23e(14bd-17ae)\right)\left(\frac{2x^4\sqrt{d+ex^3}}{11e}-\frac{8d\int \frac{x^3}{\sqrt{ex^3+d}}dx}{11e}\right)}{17e}-\frac{2x^7\sqrt{d+ex^3}(20cd-23be)}{17e}}{23e}+\frac{2cx^{10}\sqrt{d+ex^3}}{23e}$$

$$\Big\downarrow \; 843$$

$$\frac{\frac{\left(280cd^2-23e(14bd-17ae)\right)\left(\frac{2x^4\sqrt{d+ex^3}}{11e}-\frac{8d\left(\frac{2x\sqrt{d+ex^3}}{5e}-\frac{2d\int \frac{1}{\sqrt{ex^3+d}}dx}{5e}\right)}{11e}\right)}{17e}-\frac{2x^7\sqrt{d+ex^3}(20cd-23be)}{17e}}{23e}+\frac{2cx^{10}\sqrt{d+ex^3}}{23e}$$

$$\Big\downarrow \; 759$$

$$\frac{\frac{\left(280cd^2-23e(14bd-17ae)\right)\left(\frac{2x^4\sqrt{d+ex^3}}{11e}-\frac{8d\left(\frac{2x\sqrt{d+ex^3}}{5e}-\frac{4\sqrt{2+\sqrt{3}}d\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\cdots\right)\cdots\right)}{5\sqrt[4]{3}e^{4/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}\cdots\right)}{11e}\cdots\right)}{17e}\cdots}{23e}\cdots$$

$$\frac{2cx^{10}\sqrt{d+ex^3}}{23e}$$

input
```
Int[(x^6*(a + b*x^3 + c*x^6))/Sqrt[d + e*x^3],x]
```

output
```
(2*c*x^10*Sqrt[d + e*x^3])/(23*e) + ((-2*(20*c*d - 23*b*e)*x^7*Sqrt[d + e*
x^3])/(17*e) + ((280*c*d^2 - 23*e*(14*b*d - 17*a*e))*((2*x^4*Sqrt[d + e*x^
3])/(11*e) - (8*d*((2*x*Sqrt[d + e*x^3])/(5*e) - (4*Sqrt[2 + Sqrt[3]]*d*(d
^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 +
 Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3)
+ e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(
1/4)*e^(4/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) +
 e^(1/3)*x)^2]*Sqrt[d + e*x^3])))/(11*e)))/(17*e))/(23*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1810
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 2*n*p - n +
 1)*((d + e*x^n)^(q + 1)/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1))), x] +
 Simp[1/(e*(m + 2*n*p + n*q + 1))   Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e
*(m + 2*n*p + n*q + 1)*((a + b*x^n + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*
(m + 2*n*p - n + 1)*x^(2*n*p - n), x], x], x] /; FreeQ[{a, b, c, d, e, f, m
, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[p, 0]
 && GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n*p + n*q + 1, 0]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{2x\left(-935c\,e^3x^9-1265b\,e^3x^6+1100cd\,e^2x^6-1955e^3a\,x^3+1610bd\,e^2x^3-1400d^2ec\,x^3+3128ad\,e^2-2576b\,d^2e+2240c\,d^3\right)\sqrt{e\,x^3+d}}{21505e^4}$
elliptic	$\frac{2c\,x^{10}\sqrt{e\,x^3+d}}{23e}+\frac{2\left(b-\frac{20dc}{23e}\right)x^7\sqrt{e\,x^3+d}}{17e}+\frac{2\left(a-\frac{14d\left(b-\frac{20dc}{23e}\right)}{17e}\right)x^4\sqrt{e\,x^3+d}}{11e}-\frac{16d\left(a-\frac{14d\left(b-\frac{20dc}{23e}\right)}{17e}\right)x\sqrt{e\,x^3+d}}{55e^2}-$ $32id^2($
default	Expression too large to display

input
```
int(x^6*(c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21505*x*(-935*c*e^3*x^9-1265*b*e^3*x^6+1100*c*d*e^2*x^6-1955*a*e^3*x^3+
1610*b*d*e^2*x^3-1400*c*d^2*e*x^3+3128*a*d*e^2-2576*b*d^2*e+2240*c*d^3)/e^
4*(e*x^3+d)^(1/2)-32/64515*I*(391*a*e^2-322*b*d*e+280*c*d^2)*d^2/e^5*3^(1/
2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3
))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2
)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)
+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-
d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/
(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.38

$$\int \frac{x^6(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx = \frac{2\left(16\left(280\,cd^4-322\,bd^3e+391\,ad^2e^2\right)\sqrt{e}\,\text{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)+\left(935\,ce^4x^{10}-55\left(20\,cde^3-2\cdots\right.\right.}{21505\,e\cdots}$$

input
```
integrate(x^6*(c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="fricas")
```

output
```
2/21505*(16*(280*c*d^4 - 322*b*d^3*e + 391*a*d^2*e^2)*sqrt(e)*weierstrassP
Inverse(0, -4*d/e, x) + (935*c*e^4*x^10 - 55*(20*c*d*e^3 - 23*b*e^4)*x^7 +
 5*(280*c*d^2*e^2 - 322*b*d*e^3 + 391*a*e^4)*x^4 - 8*(280*c*d^3*e - 322*b*
d^2*e^2 + 391*a*d*e^3)*x)*sqrt(e*x^3 + d))/e^5
```

Sympy [A] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.34

$$\int \frac{x^6(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx = \frac{ax^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{10}{3}\right)} + \frac{bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{13}{3}\right)} + \frac{cx^{13}\Gamma\left(\frac{13}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{13}{3}\\ \frac{16}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{16}{3}\right)}$$

input
```
integrate(x**6*(c*x**6+b*x**3+a)/(e*x**3+d)**(1/2),x)
```

output
```
a*x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*
sqrt(d)*gamma(10/3)) + b*x**10*gamma(10/3)*hyper((1/2, 10/3), (13/3,), e*x
**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(13/3)) + c*x**13*gamma(13/3)*hyper
((1/2, 13/3), (16/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(16/3))
```

Maxima [F]

$$\int \frac{x^6(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx = \int \frac{(cx^6+bx^3+a)x^6}{\sqrt{ex^3+d}}\,dx$$

input
```
integrate(x^6*(c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)*x^6/sqrt(e*x^3 + d), x)
```

Giac [F]

$$\int \frac{x^6(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx=\int \frac{(cx^6+bx^3+a)x^6}{\sqrt{ex^3+d}}\,dx$$

input
```
integrate(x^6*(c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)*x^6/sqrt(e*x^3 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx=\int \frac{x^6\,(c\,x^6+b\,x^3+a)}{\sqrt{e\,x^3+d}}\,dx$$

input
```
int((x^6*(a + b*x^3 + c*x^6))/(d + e*x^3)^(1/2),x)
```

output
```
int((x^6*(a + b*x^3 + c*x^6))/(d + e*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx$$

$$=\frac{-\frac{16\sqrt{e\,x^3+d}\,ad\,e^2x}{55}+\frac{2\sqrt{e\,x^3+d}\,a\,e^3x^4}{11}+\frac{224\sqrt{e\,x^3+d}\,b\,d^2ex}{935}-\frac{28\sqrt{e\,x^3+d}\,bd\,e^2x^4}{187}+\frac{2\sqrt{e\,x^3+d}\,b\,e^3x^7}{17}-\frac{896\sqrt{e\,x^3+d}\,c\,d^3x}{4301}+560}{}$$

input
```
int(x^6*(c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x)
```

output
```
(2*( - 3128*sqrt(d + e*x**3)*a*d*e**2*x + 1955*sqrt(d + e*x**3)*a*e**3*x**
4 + 2576*sqrt(d + e*x**3)*b*d**2*e*x - 1610*sqrt(d + e*x**3)*b*d*e**2*x**4
 + 1265*sqrt(d + e*x**3)*b*e**3*x**7 - 2240*sqrt(d + e*x**3)*c*d**3*x + 14
00*sqrt(d + e*x**3)*c*d**2*e*x**4 - 1100*sqrt(d + e*x**3)*c*d*e**2*x**7 +
935*sqrt(d + e*x**3)*c*e**3*x**10 + 3128*int(sqrt(d + e*x**3)/(d + e*x**3)
,x)*a*d**2*e**2 - 2576*int(sqrt(d + e*x**3)/(d + e*x**3),x)*b*d**3*e + 224
0*int(sqrt(d + e*x**3)/(d + e*x**3),x)*c*d**4))/(21505*e**4)
```

3.67 $\int \frac{x^3(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 318

$$\int \frac{x^3(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx = \frac{2(112cd^2-136bde+187ae^2)\,x\sqrt{d+ex^3}}{935e^3} - \frac{2(14cd-17be)x^4\sqrt{d+ex^3}}{187e^2} + \frac{2cx^7\sqrt{d+ex^3}}{17e} - \frac{4\sqrt{2+\sqrt{3}}d(112cd^2-136bde+187ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right),-7-4\sqrt{3}\right)}{935\sqrt[4]{3}e^{10/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

output
```
2/935*(187*a*e^2-136*b*d*e+112*c*d^2)*x*(e*x^3+d)^(1/2)/e^3-2/187*(-17*b*e
+14*c*d)*x^4*(e*x^3+d)^(1/2)/e^2+2/17*c*x^7*(e*x^3+d)^(1/2)/e-4/2805*(1/2*
6^(1/2)+1/2*2^(1/2))*d*(187*a*e^2-136*b*d*e+112*c*d^2)*(d^(1/3)+e^(1/3)*x)
*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/((1+3^(1/2))*d^(1/3)+e^(1/3)*x)^
2)^(1/2)*EllipticF(((1-3^(1/2))*d^(1/3)+e^(1/3)*x)/((1+3^(1/2))*d^(1/3)+e^
(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/e^(10/3)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/((1+
3^(1/2))*d^(1/3)+e^(1/3)*x)^2)^(1/2)/(e*x^3+d)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.39

$$\int \frac{x^3(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx$$
$$= \frac{2x\Big((d+ex^3)\left(17e(-8bd+11ae+5bex^3)+c(112d^2-70dex^3+55e^2x^6)\right)+d(-112cd^2+17e(8bd-11}{935e^3\sqrt{d+ex^3}}$$

input
```
Integrate[(x^3*(a + b*x^3 + c*x^6))/Sqrt[d + e*x^3],x]
```

output
```
(2*x*((d + e*x^3)*(17*e*(-8*b*d + 11*a*e + 5*b*e*x^3) + c*(112*d^2 - 70*d*
e*x^3 + 55*e^2*x^6)) + d*(-112*c*d^2 + 17*e*(8*b*d - 11*a*e))*Sqrt[1 + (e*
x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)]))/(935*e^3*Sqrt[d +
 e*x^3])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1810, 27, 959, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx$$
$$\Big\downarrow \; 1810$$
$$\frac{2\int \frac{x^3(17ae-(14cd-17be)x^3)}{2\sqrt{ex^3+d}}dx}{17e}+\frac{2cx^7\sqrt{d+ex^3}}{17e}$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{x^3(17ae-(14cd-17be)x^3)}{\sqrt{ex^3+d}}dx}{17e}+\frac{2cx^7\sqrt{d+ex^3}}{17e}$$

$$\Big\downarrow \text{959}$$

$$\frac{\frac{(112cd^2-17e(8bd-11ae))\int \frac{x^3}{\sqrt{ex^3+d}}dx}{11e}-\frac{2x^4\sqrt{d+ex^3}(14cd-17be)}{11e}}{17e}+\frac{2cx^7\sqrt{d+ex^3}}{17e}$$

$$\Big\downarrow \text{843}$$

$$\frac{\frac{(112cd^2-17e(8bd-11ae))\left(\frac{2x\sqrt{d+ex^3}}{5e}-\frac{2d\int \frac{1}{\sqrt{ex^3+d}}dx}{5e}\right)}{11e}-\frac{2x^4\sqrt{d+ex^3}(14cd-17be)}{11e}}{17e}+\frac{2cx^7\sqrt{d+ex^3}}{17e}$$

$$\Big\downarrow \text{759}$$

$$\frac{\frac{(112cd^2-17e(8bd-11ae))\left(\frac{2x\sqrt{d+ex^3}}{5e}-\frac{4\sqrt{2+\sqrt{3}}d\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}e^{4/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}\right)}{11e}-\dots}{17e}$$

$$\frac{2cx^7\sqrt{d+ex^3}}{17e}$$

input
```
Int[(x^3*(a + b*x^3 + c*x^6))/Sqrt[d + e*x^3],x]
```

output
```
(2*c*x^7*Sqrt[d + e*x^3])/(17*e) + ((-2*(14*c*d - 17*b*e)*x^4*Sqrt[d + e*x
^3])/(11*e) + ((112*c*d^2 - 17*e*(8*b*d - 11*a*e))*((2*x*Sqrt[d + e*x^3])/
(5*e) - (4*Sqrt[2 + Sqrt[3]]*d*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/
3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(
1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*e^(4/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(
1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])))/(11*e))
/(17*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1810
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 2*n*p - n +
 1)*((d + e*x^n)^(q + 1)/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1))), x] +
 Simp[1/(e*(m + 2*n*p + n*q + 1))   Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e
*(m + 2*n*p + n*q + 1)*((a + b*x^n + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*
(m + 2*n*p - n + 1)*x^(2*n*p - n), x], x], x] /; FreeQ[{a, b, c, d, e, f, m
, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[p, 0]
 && GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n*p + n*q + 1, 0]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.14

method	result
risch	$\frac{2x\left(55c\,e^{2}x^{6}+85b\,e^{2}x^{3}-70cde\,x^{3}+187a\,e^{2}-136bde+112c\,d^{2}\right)\sqrt{e\,x^{3}+d}}{935e^{3}}+\frac{4i\left(187a\,e^{2}-136bde+112c\,d^{2}\right)d\sqrt{3}\,\left(-d\,e^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-d\right.}{}\right.}{}}}{}$
elliptic	$\frac{2c\,x^{7}\sqrt{e\,x^{3}+d}}{17e}+\frac{2\left(b-\frac{14dc}{17e}\right)x^{4}\sqrt{e\,x^{3}+d}}{11e}+\frac{2\left(a-\frac{8d\left(b-\frac{14dc}{17e}\right)}{11e}\right)x\sqrt{e\,x^{3}+d}}{5e}+\frac{4id\left(a-\frac{8d\left(b-\frac{14dc}{17e}\right)}{11e}\right)\sqrt{3}\,\left(-d\,e^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-d\,e\right.}{2}\right.}{}}}{}$
default	Expression too large to display

input
```
int(x^3*(c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/935*x*(55*c*e^2*x^6+85*b*e^2*x^3-70*c*d*e*x^3+187*a*e^2-136*b*d*e+112*c*
d^2)/e^3*(e*x^3+d)^(1/2)+4/2805*I*(187*a*e^2-136*b*d*e+112*c*d^2)*d/e^4*3^
(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(
1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*
e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1
/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3
+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e
*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/
3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.33

$$\int \frac{x^3(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx = -\frac{2\left(2\left(112\,cd^3-136\,bd^2e+187\,ade^2\right)\sqrt{e}\text{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)-\left(55\,ce^3x^7-5\left(14\,cde^2-17\,b\right.\right.\right.}{935\,e^4}$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="fricas")
```

output
```
-2/935*(2*(112*c*d^3 - 136*b*d^2*e + 187*a*d*e^2)*sqrt(e)*weierstrassPInve
rse(0, -4*d/e, x) - (55*c*e^3*x^7 - 5*(14*c*d*e^2 - 17*b*e^3)*x^4 + (112*c
*d^2*e - 136*b*d*e^2 + 187*a*e^3)*x)*sqrt(e*x^3 + d))/e^4
```

Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.38

$$\int \frac{x^3(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx = \frac{ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{7}{3}\right)} + \frac{bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{10}{3}\right)} + \frac{cx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate(x**3*(c*x**6+b*x**3+a)/(e*x**3+d)**(1/2),x)
```

output
```
a*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*s
qrt(d)*gamma(7/3)) + b*x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), e*x**3*e
xp_polar(I*pi)/d)/(3*sqrt(d)*gamma(10/3)) + c*x**10*gamma(10/3)*hyper((1/2
, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(13/3))
```

Maxima [F]

$$\int \frac{x^3(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx = \int \frac{(cx^6+bx^3+a)x^3}{\sqrt{ex^3+d}}\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)*x^3/sqrt(e*x^3 + d), x)
```

Giac [F]

$$\int \frac{x^3(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx = \int \frac{(cx^6+bx^3+a)x^3}{\sqrt{ex^3+d}}\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)*x^3/sqrt(e*x^3 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx = \int \frac{x^3\,(c\,x^6+b\,x^3+a)}{\sqrt{e\,x^3+d}}\,dx$$

input
```
int((x^3*(a + b*x^3 + c*x^6))/(d + e*x^3)^(1/2),x)
```

output
```
int((x^3*(a + b*x^3 + c*x^6))/(d + e*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+bx^3+cx^6)}{\sqrt{d+ex^3}}\,dx$$

$$= \frac{\frac{2\sqrt{e\,x^3+d}\,a\,e^2x}{5} - \frac{16\sqrt{e\,x^3+d}\,bdex}{55} + \frac{2\sqrt{e\,x^3+d}\,b\,e^2x^4}{11} + \frac{224\sqrt{e\,x^3+d}\,c\,d^2x}{935} - \frac{28\sqrt{e\,x^3+d}\,cde\,x^4}{187} + \frac{2\sqrt{e\,x^3+d}\,c\,e^2x^7}{17} - \frac{2\left(\int \frac{\sqrt{e\,x^3+d}}{e\,x^3+d} \cdots\right)}{5} \cdots}{e^3}$$

input
```
int(x^3*(c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x)
```

output
```
(2*(187*sqrt(d + e*x**3)*a*e**2*x - 136*sqrt(d + e*x**3)*b*d*e*x + 85*sqrt
(d + e*x**3)*b*e**2*x**4 + 112*sqrt(d + e*x**3)*c*d**2*x - 70*sqrt(d + e*x
**3)*c*d*e*x**4 + 55*sqrt(d + e*x**3)*c*e**2*x**7 - 187*int(sqrt(d + e*x**
3)/(d + e*x**3),x)*a*d*e**2 + 136*int(sqrt(d + e*x**3)/(d + e*x**3),x)*b*d
**2*e - 112*int(sqrt(d + e*x**3)/(d + e*x**3),x)*c*d**3))/(935*e**3)
```

3.68 $\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 278

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = -\frac{2(8cd-11be)x\sqrt{d+ex^3}}{55e^2}+\frac{2cx^4\sqrt{d+ex^3}}{11e} + \frac{2\sqrt{2+\sqrt{3}}(16cd^2-22bde+55ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right),\ldots\right)}{55\sqrt[4]{3}e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

output
```
-2/55*(-11*b*e+8*c*d)*x*(e*x^3+d)^(1/2)/e^2+2/11*c*x^4*(e*x^3+d)^(1/2)/e+2
/165*(1/2*6^(1/2)+1/2*2^(1/2))*(55*a*e^2-22*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/
3)*x)*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/((1+3^(1/2))*d^(1/3)+e^(1/3
)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*d^(1/3)+e^(1/3)*x)/((1+3^(1/2))*d^(1/
3)+e^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/e^(7/3)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/
((1+3^(1/2))*d^(1/3)+e^(1/3)*x)^2)^(1/2)/(e*x^3+d)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.35

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx$$
$$= \frac{x\left(-2(d+ex^3)\left(8cd-11be-5cex^3\right)+\left(16cd^2+11e(-2bd+5ae)\right)\sqrt{1+\frac{ex^3}{d}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2}\right.\right.}{55e^2\sqrt{d+ex^3}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
```

output
```
(x*(-2*(d + e*x^3)*(8*c*d - 11*b*e - 5*c*e*x^3) + (16*c*d^2 + 11*e*(-2*b*d
 + 5*a*e))*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/
d)]))/(55*e^2*Sqrt[d + e*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1741, 27, 913, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx$$

$$\Big\downarrow \; 1741$$

$$\frac{2\int \frac{11ae-(8cd-11be)x^3}{2\sqrt{ex^3+d}}dx}{11e}+\frac{2cx^4\sqrt{d+ex^3}}{11e}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{11ae-(8cd-11be)x^3}{\sqrt{ex^3+d}}dx}{11e}+\frac{2cx^4\sqrt{d+ex^3}}{11e}$$

$$\Big\downarrow \ 913$$

$$\frac{\frac{\left(16cd^2-11e(2bd-5ae)\right)\int \frac{1}{\sqrt{ex^3+d}}dx}{5e}-\frac{2x\sqrt{d+ex^3}(8cd-11be)}{5e}}{11e}+\frac{2cx^4\sqrt{d+ex^3}}{11e}$$

$$\Big\downarrow \ 759$$

$$\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(16cd^2-11e(2bd-5ae)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}e^{4/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{2x\sqrt{d+ex}}{}}{11e}$$

$$\frac{2cx^4\sqrt{d+ex^3}}{11e}$$

input
```
Int[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
```

output
```
(2*c*x^4*Sqrt[d + e*x^3])/(11*e) + ((-2*(8*c*d - 11*b*e)*x*Sqrt[d + e*x^3]
)/(5*e) + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 11*e*(2*b*d - 5*a*e))*(d^(1/3)
+ e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3
])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/
3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*e^
(4/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3
)*x)^2]*Sqrt[d + e*x^3]))/(11*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.20

method	result
risch	$\frac{2x(5ce\,x^3+11eb-8cd)\sqrt{e\,x^3+d}}{55e^2}-\frac{2i(55a\,e^2-22bde+16c\,d^2)\sqrt{3}\,(-d\,e^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-d\,e^2)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,(-d\,e^2)^{\frac{1}{3}}}{2e}\right)\sqrt{3}\,e}{(-d\,e^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-}{}}{-\frac{3(-d\,e^2)^{\frac{1}{3}}}{2e}}}}{}$
elliptic	$\frac{2c\,x^4\sqrt{e\,x^3+d}}{11e}+\frac{2\left(b-\frac{8dc}{11e}\right)x\sqrt{e\,x^3+d}}{5e}-\frac{2i\left(a-\frac{2d\left(b-\frac{8dc}{11e}\right)}{5e}\right)\sqrt{3}\,(-d\,e^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-d\,e^2)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,(-d\,e^2)^{\frac{1}{3}}}{2e}\right)\sqrt{3}\,e}{(-d\,e^2)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3(-d\,e^2)}{2e}}}}{}$
default	Expression too large to display

input
```
int((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/55*x*(5*c*e*x^3+11*b*e-8*c*d)/e^2*(e*x^3+d)^(1/2)-2/165*I*(55*a*e^2-22*b
*d*e+16*c*d^2)/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I
*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2
)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I
*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2
)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^
(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3
^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1
/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.26

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx$$
$$=\frac{2\left((16\,cd^2-22\,bde+55\,ae^2)\sqrt{e}\text{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)+(5\,ce^2x^4-(8\,cde-11\,be^2)x)\sqrt{ex^3+d}\right.}{55\,e^3}$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="fricas")
```

output
```
2/55*((16*c*d^2 - 22*b*d*e + 55*a*e^2)*sqrt(e)*weierstrassPInverse(0, -4*d
/e, x) + (5*c*e^2*x^4 - (8*c*d*e - 11*b*e^2)*x)*sqrt(e*x^3 + d))/e^3
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.43

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(1/2),x)
```

output
```
a*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt
(d)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), e*x**3*exp_p
olar(I*pi)/d)/(3*sqrt(d)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((1/2, 7/3),
 (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(10/3))
```

Maxima [F]

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \int \frac{cx^6+bx^3+a}{\sqrt{ex^3+d}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

Giac [F]

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx=\int \frac{cx^6+bx^3+a}{\sqrt{ex^3+d}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx=\int \frac{c\,x^6+b\,x^3+a}{\sqrt{e\,x^3+d}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^(1/2),x)
```

output
```
int((a + b*x^3 + c*x^6)/(d + e*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx
=\frac{22\sqrt{e\,x^3+d}\,bex-16\sqrt{e\,x^3+d}\,cdx+10\sqrt{e\,x^3+d}\,ce\,x^4+55\left(\int \frac{\sqrt{e\,x^3+d}}{e\,x^3+d}dx\right)a\,e^2-22\left(\int \frac{\sqrt{e\,x^3+d}}{e\,x^3+d}dx\right)bde}{55e^2}$$

input
```
int((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x)
```

output
```
(22*sqrt(d + e*x**3)*b*e*x - 16*sqrt(d + e*x**3)*c*d*x + 10*sqrt(d + e*x**
3)*c*e*x**4 + 55*int(sqrt(d + e*x**3)/(d + e*x**3),x)*a*e**2 - 22*int(sqrt
(d + e*x**3)/(d + e*x**3),x)*b*d*e + 16*int(sqrt(d + e*x**3)/(d + e*x**3),
x)*c*d**2)/(55*e**2)
```

3.69 $\int \frac{a+bx^3+cx^6}{x^3\sqrt{d+ex^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 274

$$\int \frac{a+bx^3+cx^6}{x^3\sqrt{d+ex^3}}\,dx = -\frac{a\sqrt{d+ex^3}}{2dx^2}+\frac{2cx\sqrt{d+ex^3}}{5e}$$
$$-\frac{\sqrt{2+\sqrt{3}}(8cd^2-5e(4bd-ae))\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right),\ldots\right)}{10\sqrt[4]{3}de^{4/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

output
```
-1/2*a*(e*x^3+d)^(1/2)/d/x^2+2/5*c*x*(e*x^3+d)^(1/2)/e-1/30*(1/2*6^(1/2)+1
/2*2^(1/2))*(8*c*d^2-5*e*(-a*e+4*b*d))*(d^(1/3)+e^(1/3)*x)*((d^(2/3)-d^(1/
3)*e^(1/3)*x+e^(2/3)*x^2)/((1+3^(1/2))*d^(1/3)+e^(1/3)*x)^2)^(1/2)*Ellipti
cF(((1-3^(1/2))*d^(1/3)+e^(1/3)*x)/((1+3^(1/2))*d^(1/3)+e^(1/3)*x),I*3^(1/
2)+2*I)*3^(3/4)/d/e^(4/3)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/((1+3^(1/2))*d^(1/3
)+e^(1/3)*x)^2)^(1/2)/(e*x^3+d)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.43

$$\int \frac{a+bx^3+cx^6}{x^3\sqrt{d+ex^3}}\,dx = \frac{-5ae\sqrt{1+\frac{ex^3}{d}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},-\frac{ex^3}{d}\right)+2x^3\left(2c(d+ex^3)+(-2cd+5be)\sqrt{1+\frac{ex^3}{d}}\,\text{Hype}\right.}{10ex^2\sqrt{d+ex^3}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)/(x^3*Sqrt[d + e*x^3]),x]
```

output
```
(-5*a*e*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((e*x^3)/d)
] + 2*x^3*(2*c*(d + e*x^3) + (-2*c*d + 5*b*e)*Sqrt[1 + (e*x^3)/d]*Hypergeo
metric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)]))/(10*e*x^2*Sqrt[d + e*x^3])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1810, 27, 955, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3+cx^6}{x^3\sqrt{d+ex^3}}\,dx$$

$$\Big\downarrow \text{ 1810}$$

$$\frac{2\int \frac{5ae-(2cd-5be)x^3}{2x^3\sqrt{ex^3+d}}dx}{5e}+\frac{2cx\sqrt{d+ex^3}}{5e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{5ae-(2cd-5be)x^3}{x^3\sqrt{ex^3+d}}dx}{5e}+\frac{2cx\sqrt{d+ex^3}}{5e}$$

$$\Big\downarrow \; 955$$

$$\frac{-\frac{(8cd^2-5e(4bd-ae))\int \frac{1}{\sqrt{ex^3+d}}dx}{4d}-\frac{5ae\sqrt{d+ex^3}}{2dx^2}}{5e}+\frac{2cx\sqrt{d+ex^3}}{5e}$$

$$\Big\downarrow \; 759$$

$$\frac{-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}(8cd^2-5e(4bd-ae))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}d\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{5ae\sqrt{d+ex^3}}{2dx^2}}{5e}$$

$$\frac{2cx\sqrt{d+ex^3}}{5e}$$

input
```
Int[(a + b*x^3 + c*x^6)/(x^3*Sqrt[d + e*x^3]),x]
```

output
```
(2*c*x*Sqrt[d + e*x^3])/(5*e) + ((-5*a*e*Sqrt[d + e*x^3])/(2*d*x^2) - (Sqr
t[2 + Sqrt[3]]*(8*c*d^2 - 5*e*(4*b*d - a*e))*(d^(1/3) + e^(1/3)*x)*Sqrt[(d
^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])
*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*d*e^(1/3)*Sqrt[(d^(1/3
)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e
*x^3]))/(5*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1810
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 2*n*p - n +
 1)*((d + e*x^n)^(q + 1)/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1))), x] +
 Simp[1/(e*(m + 2*n*p + n*q + 1))   Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e
*(m + 2*n*p + n*q + 1)*((a + b*x^n + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*
(m + 2*n*p - n + 1)*x^(2*n*p - n), x], x], x] /; FreeQ[{a, b, c, d, e, f, m
, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[p, 0]
 && GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n*p + n*q + 1, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.22

method	result
elliptic	$-\frac{a\sqrt{e\,x^3+d}}{2d\,x^2}+\frac{2cx\sqrt{e\,x^3+d}}{5e}-\frac{2i\left(b-\frac{ea}{4d}-\frac{2dc}{5e}\right)\sqrt{3}\,\left(-d\,e^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-d\,e^2\right)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,\left(-d\,e^2\right)^{\frac{1}{3}}}{2e}\right)\sqrt{3}\,e}{\left(-d\,e^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-d\,e^2\right)^{\frac{1}{3}}}{e}}{-\frac{3\left(-d\,e^2\right)^{\frac{1}{3}}}{2e}+\frac{i\sqrt{3}\,\left(-d\,e\right.}{2e}}}}{}$
	$i\left(5a\,e^2-20bde+8c\,d^2\right)\sqrt{3}\,\left(-d\,e^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-d\,e^2\right)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,\left(-d\,e^2\right)^{\frac{1}{3}}}{2e}\right)\sqrt{3}\,e}{\left(-d\,e^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-d\,e^2\right)^{\frac{1}{3}}}{e}}{-\frac{3\left(-d\,e^2\right)^{\frac{1}{3}}}{2e}+\frac{i\sqrt{3}\,\left(\right.}{}}}$
risch	$-\frac{\sqrt{e\,x^3+d}\,\left(-4cd\,x^3+5ae\right)}{10ed\,x^2}+\frac{}{}$
default	Expression too large to display

input
```
int((c*x^6+b*x^3+a)/x^3/(e*x^3+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(e*x^3+d)^(1/2)/d/x^2+2/5*c*x*(e*x^3+d)^(1/2)/e-2/3*I*(b-1/4*e/d*a-
2/5*d/e*c)*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/
2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3
))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/
2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3
))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-
1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)
/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.26

$$\int \frac{a+bx^3+cx^6}{x^3\sqrt{d+ex^3}}\,dx = -\frac{\left(8\,cd^2-20\,bde+5\,ae^2\right)\sqrt{e}x^2\mathrm{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)-\left(4\,cdex^3-5\,ae^2\right)\sqrt{ex^3+d}}{10\,de^2x^2}$$

input
```
integrate((c*x^6+b*x^3+a)/x^3/(e*x^3+d)^(1/2),x, algorithm="fricas")
```

output
```
-1/10*((8*c*d^2 - 20*b*d*e + 5*a*e^2)*sqrt(e)*x^2*weierstrassPInverse(0, -
4*d/e, x) - (4*c*d*e*x^3 - 5*a*e^2)*sqrt(e*x^3 + d))/(d*e^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.45

$$\int \frac{a+bx^3+cx^6}{x^3\sqrt{d+ex^3}}\,dx = \frac{a\Gamma\left(-\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}x^2\Gamma\left(\frac{1}{3}\right)} + \frac{bx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{4}{3}\right)} + \frac{cx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((c*x**6+b*x**3+a)/x**3/(e*x**3+d)**(1/2),x)
```

output
```
a*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt
(d)*x**2*gamma(1/3)) + b*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), e*x**3*exp
_polar(I*pi)/d)/(3*sqrt(d)*gamma(4/3)) + c*x**4*gamma(4/3)*hyper((1/2, 4/3
), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(7/3))
```

Maxima [F]

$$\int \frac{a+bx^3+cx^6}{x^3\sqrt{d+ex^3}}\,dx = \int \frac{cx^6+bx^3+a}{\sqrt{ex^3+d}x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/x^3/(e*x^3+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(sqrt(e*x^3 + d)*x^3), x)
```

Giac [F]

$$\int \frac{a+bx^3+cx^6}{x^3\sqrt{d+ex^3}}\,dx=\int \frac{cx^6+bx^3+a}{\sqrt{ex^3+d}x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/x^3/(e*x^3+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(sqrt(e*x^3 + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{x^3\sqrt{d+ex^3}}\,dx=\int \frac{c\,x^6+b\,x^3+a}{x^3\,\sqrt{e\,x^3+d}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)/(x^3*(d + e*x^3)^(1/2)),x)
```

output
```
int((a + b*x^3 + c*x^6)/(x^3*(d + e*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+bx^3+cx^6}{x^3\sqrt{d+ex^3}}\,dx = \frac{-10\sqrt{e\,x^3+d}\,be+4\sqrt{e\,x^3+d}\,cd+2\sqrt{e\,x^3+d}\,ce\,x^3+5\left(\int \frac{\sqrt{e\,x^3+d}}{e\,x^6+d\,x^3}dx\right)a\,e^2x^2-20\left(\int \frac{\sqrt{e\,x^3+d}}{e\,x^6+d\,x^3}dx\right)bde\,x}{5e^2x^2}$$

input
```
int((c*x^6+b*x^3+a)/x^3/(e*x^3+d)^(1/2),x)
```

output
```
( - 10*sqrt(d + e*x**3)*b*e + 4*sqrt(d + e*x**3)*c*d + 2*sqrt(d + e*x**3)*
c*e*x**3 + 5*int(sqrt(d + e*x**3)/(d*x**3 + e*x**6),x)*a*e**2*x**2 - 20*in
t(sqrt(d + e*x**3)/(d*x**3 + e*x**6),x)*b*d*e*x**2 + 8*int(sqrt(d + e*x**3
)/(d*x**3 + e*x**6),x)*c*d**2*x**2)/(5*e**2*x**2)
```

3.70 $\int \frac{a+bx^3+cx^6}{x^6\sqrt{d+ex^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 284

$$\int \frac{a+bx^3+cx^6}{x^6\sqrt{d+ex^3}}\,dx = -\frac{a\sqrt{d+ex^3}}{5dx^5} - \frac{(10bd-7ae)\sqrt{d+ex^3}}{20d^2x^2}$$

$$+\frac{\sqrt{2+\sqrt{3}}(40cd^2-e(10bd-7ae))\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}-}{\left(1+\sqrt{3}\right)\sqrt[3]{d}-}\right.\right.}{20\sqrt[4]{3}d^2\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

output
```
-1/5*a*(e*x^3+d)^(1/2)/d/x^5-1/20*(-7*a*e+10*b*d)*(e*x^3+d)^(1/2)/d^2/x^2+
1/60*(1/2*6^(1/2)+1/2*2^(1/2))*(40*c*d^2-e*(-7*a*e+10*b*d))*(d^(1/3)+e^(1/
3)*x)*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/((1+3^(1/2))*d^(1/3)+e^(1/3
)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*d^(1/3)+e^(1/3)*x)/((1+3^(1/2))*d^(1/
3)+e^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/d^2/e^(1/3)/(d^(1/3)*(d^(1/3)+e^(1/3)
*x)/((1+3^(1/2))*d^(1/3)+e^(1/3)*x)^2)^(1/2)/(e*x^3+d)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.37

$$\int \frac{a+bx^3+cx^6}{x^6\sqrt{d+ex^3}}\,dx = \frac{\sqrt{1+\frac{ex^3}{d}}\left(-2a\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},-\frac{ex^3}{d}\right)-5bx^3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},-\frac{ex^3}{d}\right)+\right.}{10x^5\sqrt{d+ex^3}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)/(x^6*Sqrt[d + e*x^3]),x]
```

output
```
(Sqrt[1 + (e*x^3)/d]*(-2*a*Hypergeometric2F1[-5/3, 1/2, -2/3, -((e*x^3)/d)
] - 5*b*x^3*Hypergeometric2F1[-2/3, 1/2, 1/3, -((e*x^3)/d)] + 10*c*x^6*Hyp
ergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)]))/(10*x^5*Sqrt[d + e*x^3])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1810, 27, 955, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3+cx^6}{x^6\sqrt{d+ex^3}}\,dx$$

$$\Big\downarrow \text{1810}$$

$$-\frac{2\int -\frac{ae-(4cd-be)x^3}{2x^6\sqrt{ex^3+d}}dx}{e}-\frac{2c\sqrt{d+ex^3}}{ex^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{ae-(4cd-be)x^3}{x^6\sqrt{ex^3+d}}dx}{e}-\frac{2c\sqrt{d+ex^3}}{ex^2}$$

$$\Big\downarrow \text{955}$$

$$\frac{-\frac{\left(40cd^2-e(10bd-7ae)\right)\int \frac{1}{x^3\sqrt{ex^3+d}}dx}{10d}-\frac{ae\sqrt{d+ex^3}}{5dx^5}}{e}-\frac{2c\sqrt{d+ex^3}}{ex^2}$$

$\Big\downarrow$ 847

$$\frac{-\frac{\left(40cd^2-e(10bd-7ae)\right)\left(-\frac{e\int \frac{1}{\sqrt{ex^3+d}}dx}{4d}-\frac{\sqrt{d+ex^3}}{2dx^2}\right)}{10d}-\frac{ae\sqrt{d+ex^3}}{5dx^5}}{e}-\frac{2c\sqrt{d+ex^3}}{ex^2}$$

$\Big\downarrow$ 759

$$-\frac{\frac{\left(40cd^2-e(10bd-7ae)\right)\left(-\frac{\sqrt{2+\sqrt{3}}e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}d\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{\sqrt{d+ex^3}}{2dx^2}\right)}{10d}}{e}$$

$$\frac{2c\sqrt{d+ex^3}}{ex^2}$$

input

```
Int[(a + b*x^3 + c*x^6)/(x^6*Sqrt[d + e*x^3]),x]
```

output

```
(-2*c*Sqrt[d + e*x^3])/(e*x^2) + (-1/5*(a*e*Sqrt[d + e*x^3])/(d*x^5) - ((4
0*c*d^2 - e*(10*b*d - 7*a*e))*(-1/2*Sqrt[d + e*x^3]/(d*x^2) - (Sqrt[2 + Sq
rt[3]]*e^(2/3)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e
^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4
*Sqrt[3]])/(2*3^(1/4)*d*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3]
)*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])))/(10*d))/e
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1810
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 2*n*p - n +
 1)*((d + e*x^n)^(q + 1)/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1))), x] +
 Simp[1/(e*(m + 2*n*p + n*q + 1))   Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e
*(m + 2*n*p + n*q + 1)*((a + b*x^n + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*
(m + 2*n*p - n + 1)*x^(2*n*p - n), x], x], x] /; FreeQ[{a, b, c, d, e, f, m
, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[p, 0]
 && GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n*p + n*q + 1, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{\sqrt{e\,x^{3}+d}\,\left(-7ae\,x^{3}+10bd\,x^{3}+4ad\right)}{20d^{2}x^{5}}-\frac{i\left(7a\,e^{2}-10bde+40c\,d^{2}\right)\sqrt{3}\,\left(-d\,e^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}\right)\sqrt{3}\,e}{\left(-d\,e^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x\cdots}{-\frac{3\left(-d\,e^{2}\cdots\right.}{2e}\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{a\sqrt{e\,x^{3}+d}}{5d\,x^{5}}+\frac{(7ae-10bd)\sqrt{e\,x^{3}+d}}{20d^{2}x^{2}}-\frac{2i\left(c+\frac{e(7ae-10bd)}{40d^{2}}\right)\sqrt{3}\,\left(-d\,e^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}\right)\sqrt{3}\,e}{\left(-d\,e^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\cdots}{-\frac{3\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}\cdots}}\cdots}{\cdots}$
default	Expression too large to display

input
```
int((c*x^6+b*x^3+a)/x^6/(e*x^3+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(e*x^3+d)^(1/2)*(-7*a*e*x^3+10*b*d*x^3+4*a*d)/d^2/x^5-1/60*I*(7*a*e^
2-10*b*d*e+40*c*d^2)/d^2*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/
3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e
*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(
1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e
/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(
-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1
/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d
*e^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.28

$$\int \frac{a+bx^3+cx^6}{x^6\sqrt{d+ex^3}}\,dx = \frac{(40\,cd^2-10\,bde+7\,ae^2)\sqrt{e}x^5\text{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)-((10\,bde-7\,ae^2)x^3+4\,ade)\sqrt{ex^3+d}}{20\,d^2ex^5}$$

input
```
integrate((c*x^6+b*x^3+a)/x^6/(e*x^3+d)^(1/2),x, algorithm="fricas")
```

output
```
1/20*((40*c*d^2 - 10*b*d*e + 7*a*e^2)*sqrt(e)*x^5*weierstrassPInverse(0, -
4*d/e, x) - ((10*b*d*e - 7*a*e^2)*x^3 + 4*a*d*e)*sqrt(e*x^3 + d))/(d^2*e*x
^5)
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.45

$$\int \frac{a+bx^3+cx^6}{x^6\sqrt{d+ex^3}}\,dx = \frac{a\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{3},\frac{1}{2}\\-\frac{2}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{b\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\\frac{1}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}x^2\Gamma\left(\frac{1}{3}\right)} + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((c*x**6+b*x**3+a)/x**6/(e*x**3+d)**(1/2),x)
```

output
```
a*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqr
t(d)*x**5*gamma(-2/3)) + b*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), e*x**3*e
xp_polar(I*pi)/d)/(3*sqrt(d)*x**2*gamma(1/3)) + c*x*gamma(1/3)*hyper((1/3,
 1/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(4/3))
```

Maxima [F]

$$\int \frac{a+bx^3+cx^6}{x^6\sqrt{d+ex^3}}\,dx=\int \frac{cx^6+bx^3+a}{\sqrt{ex^3+dx^6}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/x^6/(e*x^3+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(sqrt(e*x^3 + d)*x^6), x)
```

Giac [F]

$$\int \frac{a+bx^3+cx^6}{x^6\sqrt{d+ex^3}}\,dx=\int \frac{cx^6+bx^3+a}{\sqrt{ex^3+dx^6}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/x^6/(e*x^3+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(sqrt(e*x^3 + d)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{x^6\sqrt{d+ex^3}}\,dx=\int \frac{c\,x^6+b\,x^3+a}{x^6\,\sqrt{e\,x^3+d}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)/(x^6*(d + e*x^3)^(1/2)),x)
```

output
```
int((a + b*x^3 + c*x^6)/(x^6*(d + e*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+bx^3+cx^6}{x^6\sqrt{d+ex^3}}\,dx$$
$$= \frac{-2\sqrt{e\,x^3+d}\,be+8\sqrt{e\,x^3+d}\,cd-14\sqrt{e\,x^3+d}\,ce\,x^3+7\left(\int \frac{\sqrt{e\,x^3+d}}{e\,x^9+d\,x^6}dx\right)a\,e^2x^5-10\left(\int \frac{\sqrt{e\,x^3+d}}{e\,x^9+d\,x^6}dx\right)bde\,x}{7e^2x^5}$$

input
```
int((c*x^6+b*x^3+a)/x^6/(e*x^3+d)^(1/2),x)
```

output
```
( - 2*sqrt(d + e*x**3)*b*e + 8*sqrt(d + e*x**3)*c*d - 14*sqrt(d + e*x**3)*
c*e*x**3 + 7*int(sqrt(d + e*x**3)/(d*x**6 + e*x**9),x)*a*e**2*x**5 - 10*in
t(sqrt(d + e*x**3)/(d*x**6 + e*x**9),x)*b*d*e*x**5 + 40*int(sqrt(d + e*x**
3)/(d*x**6 + e*x**9),x)*c*d**2*x**5)/(7*e**2*x**5)
```

3.71 $\int \frac{a+bx^3+cx^6}{x^9\sqrt{d+ex^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 322

$$
\begin{aligned}
\int \frac{a+bx^3+cx^6}{x^9\sqrt{d+ex^3}}\,dx
&= -\frac{a\sqrt{d+ex^3}}{8dx^8}-\frac{(16bd-13ae)\sqrt{d+ex^3}}{80d^2x^5}-\frac{(160cd^2-112bde+91ae^2)\sqrt{d+ex^3}}{320d^3x^2} \\
&\quad - \frac{\sqrt{2+\sqrt{3}}e^{2/3}(160cd^2-112bde+91ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{\ldots}\right.}{\left(1+\sqrt{\ldots}\right.}\right.\right.}{320\sqrt[4]{3}d^3\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}
\end{aligned}
$$

output
```
-1/8*a*(e*x^3+d)^(1/2)/d/x^8-1/80*(-13*a*e+16*b*d)*(e*x^3+d)^(1/2)/d^2/x^5
-1/320*(91*a*e^2-112*b*d*e+160*c*d^2)*(e*x^3+d)^(1/2)/d^3/x^2-1/960*(1/2*6
^(1/2)+1/2*2^(1/2))*e^(2/3)*(91*a*e^2-112*b*d*e+160*c*d^2)*(d^(1/3)+e^(1/3
)*x)*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/((1+3^(1/2))*d^(1/3)+e^(1/3)
*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*d^(1/3)+e^(1/3)*x)/((1+3^(1/2))*d^(1/3
)+e^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/d^3/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/((1+3
^(1/2))*d^(1/3)+e^(1/3)*x)^2)^(1/2)/(e*x^3+d)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.33

$$\int \frac{a+bx^3+cx^6}{x^9\sqrt{d+ex^3}}\,dx =$$
$$-\frac{\sqrt{1+\frac{ex^3}{d}}\left(5a\,\text{Hypergeometric2F1}\left(-\frac{8}{3},\frac{1}{2},-\frac{5}{3},-\frac{ex^3}{d}\right)+8bx^3\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},-\frac{ex^3}{d}\right)\right.}{40x^8\sqrt{d+ex^3}}$$

input
```
Integrate[(a + b*x^3 + c*x^6)/(x^9*Sqrt[d + e*x^3]),x]
```

output
```
-1/40*(Sqrt[1 + (e*x^3)/d]*(5*a*Hypergeometric2F1[-8/3, 1/2, -5/3, -((e*x^
3)/d)] + 8*b*x^3*Hypergeometric2F1[-5/3, 1/2, -2/3, -((e*x^3)/d)] + 20*c*x
^6*Hypergeometric2F1[-2/3, 1/2, 1/3, -((e*x^3)/d)]))/(x^8*Sqrt[d + e*x^3])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1810, 27, 955, 847, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3+cx^6}{x^9\sqrt{d+ex^3}}\,dx$$

$$\Big\downarrow \text{1810}$$

$$-\frac{2\int -\frac{7ae-(10cd-7be)x^3}{2x^9\sqrt{ex^3+d}}dx}{7e}-\frac{2c\sqrt{d+ex^3}}{7ex^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7ae-(10cd-7be)x^3}{x^9\sqrt{ex^3+d}}dx}{7e}-\frac{2c\sqrt{d+ex^3}}{7ex^5}$$

$$\Big\downarrow \text{955}$$

$$\frac{-\frac{\left(91ae^2-112bde+160cd^2\right)\int \frac{1}{x^6\sqrt{ex^3+d}}dx}{16d}-\frac{7ae\sqrt{d+ex^3}}{8dx^8}}{7e}-\frac{2c\sqrt{d+ex^3}}{7ex^5}$$

$\Big\downarrow$ 847

$$\frac{-\frac{\left(91ae^2-112bde+160cd^2\right)\left(-\frac{7e\int \frac{1}{x^3\sqrt{ex^3+d}}dx}{10d}-\frac{\sqrt{d+ex^3}}{5dx^5}\right)}{16d}-\frac{7ae\sqrt{d+ex^3}}{8dx^8}}{7e}-\frac{2c\sqrt{d+ex^3}}{7ex^5}$$

$\Big\downarrow$ 847

$$\frac{-\frac{\left(91ae^2-112bde+160cd^2\right)\left(-\frac{7e\left(-\frac{e\int \frac{1}{\sqrt{ex^3+d}}dx}{4d}-\frac{\sqrt{d+ex^3}}{2dx^2}\right)}{10d}-\frac{\sqrt{d+ex^3}}{5dx^5}\right)}{16d}-\frac{7ae\sqrt{d+ex^3}}{8dx^8}}{7e}-\frac{2c\sqrt{d+ex^3}}{7ex^5}$$

$\Big\downarrow$ 759

$$\frac{-\frac{\left(91ae^2-112bde+160cd^2\right)\left(-\frac{7e\left(-\frac{\sqrt{2+\sqrt{3}}\,e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,d\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{\sqrt{d\cdots}}{2\cdots}\cdots\right)}{10d}\cdots\right)}{16d}\cdots}{7e}\cdots$$

$$\frac{2c\sqrt{d+ex^3}}{7ex^5}$$

input
```
Int[(a + b*x^3 + c*x^6)/(x^9*Sqrt[d + e*x^3]),x]
```

output
```
(-2*c*Sqrt[d + e*x^3])/(7*e*x^5) + ((-7*a*e*Sqrt[d + e*x^3])/(8*d*x^8) - (
(160*c*d^2 - 112*b*d*e + 91*a*e^2)*(-1/5*Sqrt[d + e*x^3]/(d*x^5) - (7*e*(-
1/2*Sqrt[d + e*x^3]/(d*x^2) - (Sqrt[2 + Sqrt[3]]*e^(2/3)*(d^(1/3) + e^(1/3
)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/
3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((
1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*d*Sqrt[(d^
(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d
 + e*x^3])))/(10*d)))/(16*d))/(7*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1810
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 2*n*p - n +
 1)*((d + e*x^n)^(q + 1)/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1))), x] +
 Simp[1/(e*(m + 2*n*p + n*q + 1))   Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e
*(m + 2*n*p + n*q + 1)*((a + b*x^n + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*
(m + 2*n*p - n + 1)*x^(2*n*p - n), x], x], x] /; FreeQ[{a, b, c, d, e, f, m
, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[p, 0]
 && GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n*p + n*q + 1, 0]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{\sqrt{e\,x^{3}+d}\,\left(91 a\,e^{2} x^{6}-112 b d e\,x^{6}+160 c\,d^{2} x^{6}-52 a d e\,x^{3}+64 b\,d^{2} x^{3}+40 a\,d^{2}\right)}{320 d^{3} x^{8}}+\frac{i \left(91 a\,e^{2}-112 b d e+160 c\,d^{2}\right) \sqrt{3}\,\left(-d\,e^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{(\cdots}{\cdots}\right)}{\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{a\sqrt{e\,x^{3}+d}}{8 d\,x^{8}}+\frac{\left(13 a e-16 b d\right)\sqrt{e\,x^{3}+d}}{80 d^{2} x^{5}}-\frac{\left(91 a\,e^{2}-112 b d e+160 c\,d^{2}\right)\sqrt{e\,x^{3}+d}}{320 d^{3} x^{2}}+\frac{i \left(91 a\,e^{2}-112 b d e+160 c\,d^{2}\right) \sqrt{3}\,\left(-d\,e^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(\cdots\right)}{\cdots}}\cdots}{\cdots}$
default	Expression too large to display

input
```
int((c*x^6+b*x^3+a)/x^9/(e*x^3+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/320*(e*x^3+d)^(1/2)*(91*a*e^2*x^6-112*b*d*e*x^6+160*c*d^2*x^6-52*a*d*e*
x^3+64*b*d^2*x^3+40*a*d^2)/d^3/x^8+1/960*I*(91*a*e^2-112*b*d*e+160*c*d^2)/
d^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*
e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/
e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e
^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/
(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(
1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^
2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.31

$$\int \frac{a+bx^3+cx^6}{x^9\sqrt{d+ex^3}}\,dx = -\frac{(160\,cd^2-112\,bde+91\,ae^2)\sqrt{e}x^8\text{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)+\left((160\,cd^2-112\,bde+91\,ae^2)x^6+\right.}{320\,d^3x^8}$$

input
```
integrate((c*x^6+b*x^3+a)/x^9/(e*x^3+d)^(1/2),x, algorithm="fricas")
```

output
```
-1/320*((160*c*d^2 - 112*b*d*e + 91*a*e^2)*sqrt(e)*x^8*weierstrassPInverse
(0, -4*d/e, x) + ((160*c*d^2 - 112*b*d*e + 91*a*e^2)*x^6 + 4*(16*b*d^2 - 1
3*a*d*e)*x^3 + 40*a*d^2)*sqrt(e*x^3 + d))/(d^3*x^8)
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.43

$$\int \frac{a+bx^3+cx^6}{x^9\sqrt{d+ex^3}}\,dx = \frac{a\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{8}{3},\frac{1}{2}\\-\frac{5}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}x^8\Gamma\left(-\frac{5}{3}\right)}+\frac{b\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{3},\frac{1}{2}\\-\frac{2}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{c\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\\frac{1}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((c*x**6+b*x**3+a)/x**9/(e*x**3+d)**(1/2),x)
```

output
```
a*gamma(-8/3)*hyper((-8/3, 1/2), (-5/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqr
t(d)*x**8*gamma(-5/3)) + b*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), e*x**3*
exp_polar(I*pi)/d)/(3*sqrt(d)*x**5*gamma(-2/3)) + c*gamma(-2/3)*hyper((-2/
3, 1/2), (1/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{a+bx^3+cx^6}{x^9\sqrt{d+ex^3}}\,dx = \int \frac{cx^6+bx^3+a}{\sqrt{ex^3+dx^9}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/x^9/(e*x^3+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(sqrt(e*x^3 + d)*x^9), x)
```

Giac [F]

$$\int \frac{a+bx^3+cx^6}{x^9\sqrt{d+ex^3}}\,dx = \int \frac{cx^6+bx^3+a}{\sqrt{ex^3+dx^9}}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)/x^9/(e*x^3+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)/(sqrt(e*x^3 + d)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{x^9\sqrt{d+ex^3}}\,dx = \int \frac{c\,x^6+b\,x^3+a}{x^9\,\sqrt{e\,x^3+d}}\,dx$$

input
```
int((a + b*x^3 + c*x^6)/(x^9*(d + e*x^3)^(1/2)),x)
```

output
```
int((a + b*x^3 + c*x^6)/(x^9*(d + e*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+bx^3+cx^6}{x^9\sqrt{d+ex^3}}\,dx$$
$$= \frac{-14\sqrt{e\,x^3+d}\,be+20\sqrt{e\,x^3+d}\,cd-26\sqrt{e\,x^3+d}\,ce\,x^3+91\left(\int \frac{\sqrt{e\,x^3+d}}{e\,x^{12}+d\,x^9}dx\right)a\,e^2x^8-112\left(\int \frac{\sqrt{e\,x^3+d}}{e\,x^{12}+d\,x^9}dx\right.}{91e^2x^8}$$

input
```
int((c*x^6+b*x^3+a)/x^9/(e*x^3+d)^(1/2),x)
```

output
```
( - 14*sqrt(d + e*x**3)*b*e + 20*sqrt(d + e*x**3)*c*d - 26*sqrt(d + e*x**3
)*c*e*x**3 + 91*int(sqrt(d + e*x**3)/(d*x**9 + e*x**12),x)*a*e**2*x**8 - 1
12*int(sqrt(d + e*x**3)/(d*x**9 + e*x**12),x)*b*d*e*x**8 + 160*int(sqrt(d
+ e*x**3)/(d*x**9 + e*x**12),x)*c*d**2*x**8)/(91*e**2*x**8)
```

3.72 $\int x^8(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 300

$$\int x^8\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\frac{ex^6\left(a+bx^3+cx^6\right)^{1+p}}{6c(2+p)}$$
$$-\frac{\left(2ace(3+2p)+b(2+p)(2cd(2+p)-be(3+p))-2c(1+p)(2cd(2+p)-be(3+p))x^3\right)\left(a+bx^3+c\right.}{12c^3(1+p)(2+p)(3+2p)}$$
$$+\frac{2^{-1+p}\left(4ac^2d-6abce-2b^2cd(2+p)+b^3e(3+p)\right)\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}\text{Hyperge}}{3c^3\sqrt{b^2-4ac}(1+p)(3+2p)}$$

output
```
1/6*e*x^6*(c*x^6+b*x^3+a)^(p+1)/c/(2+p)-1/12*(2*a*c*e*(3+2*p)+b*(2+p)*(2*c
*d*(2+p)-b*e*(3+p))-2*c*(p+1)*(2*c*d*(2+p)-b*e*(3+p))*x^3)*(c*x^6+b*x^3+a)
^(p+1)/c^3/(p+1)/(2+p)/(3+2*p)+1/3*2^(-1+p)*(4*a*c^2*d-6*a*b*c*e-2*b^2*c*d
*(2+p)+b^3*e*(3+p))*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))^(
-1-p)*(c*x^6+b*x^3+a)^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+(-4*a*c+b^2)^
(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)/(p+1)/(3+2*p)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.14 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.76

$$\int x^8\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx$$
$$=\frac{1}{36}x^9\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(4d\operatorname{AppellF1}\left(3,-p,-p,4,-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+3ex^3\operatorname{AppellF1}\left(4,-p,-p,5,-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)$$

input
```
Integrate[x^8*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^9*(a + b*x^3 + c*x^6)^p*(4*d*AppellF1[3, -p, -p, 4, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*e*x^3*AppellF1[4,
 -p, -p, 5, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
 4*a*c])]))/(36*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])
)^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {1802, 1236, 25, 1225, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx$$
$$\Big\downarrow \text{1802}$$
$$\frac{1}{3}\int x^6\left(ex^3+d\right)\left(cx^6+bx^3+a\right)^p\,dx^3$$

$$\Big\downarrow 1236$$

$$\frac{1}{3}\left(\frac{\int -x^3\left(2ae-(2cd(p+2)-be(p+3))x^3\right)\left(cx^6+bx^3+a\right)^p dx^3}{2c(p+2)}+\frac{ex^6\left(a+bx^3+cx^6\right)^{p+1}}{2c(p+2)}\right)$$

$$\Big\downarrow 25$$

$$\frac{1}{3}\left(\frac{ex^6\left(a+bx^3+cx^6\right)^{p+1}}{2c(p+2)}-\frac{\int x^3\left(2ae-(2cd(p+2)-be(p+3))x^3\right)\left(cx^6+bx^3+a\right)^p dx^3}{2c(p+2)}\right)$$

$$\Big\downarrow 1225$$

$$\frac{1}{3}\left(\frac{ex^6\left(a+bx^3+cx^6\right)^{p+1}}{2c(p+2)}-\frac{\frac{(p+2)\left(-6abce+4ac^2d+b^3e(p+3)-2b^2cd(p+2)\right)\int\left(cx^6+bx^3+a\right)^p dx^3}{2c^2(2p+3)}+\frac{\left(a+bx^3+cx^6\right)^{p+1}(2ace(2p+3)-}{}}{2c(p+2)}\right.$$

$$\Big\downarrow 1096$$

$$\frac{1}{3}\left(\frac{ex^6\left(a+bx^3+cx^6\right)^{p+1}}{2c(p+2)}-\frac{\frac{\left(a+bx^3+cx^6\right)^{p+1}\left(2ace(2p+3)-2c(p+1)x^3(2cd(p+2)-be(p+3))+b(p+2)(2cd(p+2)-be(p+3))\right)}{2c^2(p+1)(2p+3)}-\frac{2^p(p+}{}}{}\right.$$

input
```
Int[x^8*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
((e*x^6*(a + b*x^3 + c*x^6)^(1 + p))/(2*c*(2 + p)) - (((2*a*c*e*(3 + 2*p)
+ b*(2 + p)*(2*c*d*(2 + p) - b*e*(3 + p)) - 2*c*(1 + p)*(2*c*d*(2 + p) - b
*e*(3 + p))*x^3)*(a + b*x^3 + c*x^6)^(1 + p))/(2*c^2*(1 + p)*(3 + 2*p)) -
(2^p*(2 + p)*(4*a*c^2*d - 6*a*b*c*e - 2*b^2*c*d*(2 + p) + b^3*e*(3 + p))*(
-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^
3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 +
 2*p)))/(2*c*(2 + p)))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^8 \left(e\,x^3+d\right) \left(c\,x^6+b\,x^3+a\right)^p dx$$

input
```
int(x^8*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^8*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^8 \left(d+ex^3\right) \left(a+bx^3+cx^6\right)^p \, dx = \int \left(ex^3+d\right)\left(cx^6+bx^3+a\right)^p x^8 \, dx$$

input
```
integrate(x^8*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^11 + d*x^8)*(c*x^6 + b*x^3 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^8 \left(d+ex^3\right) \left(a+bx^3+cx^6\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x**8*(e*x**3+d)*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^8 (d + ex^3) (a + bx^3 + cx^6)^p \, dx = \int (ex^3 + d)(cx^6 + bx^3 + a)^p x^8 \, dx$$

input
```
integrate(x^8*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x^8, x)
```

Giac [F]

$$\int x^8 (d + ex^3) (a + bx^3 + cx^6)^p \, dx = \int (ex^3 + d)(cx^6 + bx^3 + a)^p x^8 \, dx$$

input
```
integrate(x^8*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 (d + ex^3) (a + bx^3 + cx^6)^p \, dx = \int x^8 (e x^3 + d) (c x^6 + b x^3 + a)^p \, dx$$

input
```
int(x^8*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^8*(d + e*x^3)*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int x^8(d+ex^3)\,(a+bx^3+cx^6)^p\,dx = \text{too large to display}$$

input
```
int(x^8*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
(4*(a + b*x**3 + c*x**6)**p*a**2*b*c*e*p**2 + 20*(a + b*x**3 + c*x**6)**p*
a**2*b*c*e*p + 18*(a + b*x**3 + c*x**6)**p*a**2*b*c*e - 8*(a + b*x**3 + c*
x**6)**p*a**2*c**2*d*p**2 - 24*(a + b*x**3 + c*x**6)**p*a**2*c**2*d*p - 16
*(a + b*x**3 + c*x**6)**p*a**2*c**2*d - (a + b*x**3 + c*x**6)**p*a*b**3*e*
p**2 - 5*(a + b*x**3 + c*x**6)**p*a*b**3*e*p - 6*(a + b*x**3 + c*x**6)**p*
a*b**3*e + 2*(a + b*x**3 + c*x**6)**p*a*b**2*c*d*p**2 + 8*(a + b*x**3 + c*
x**6)**p*a*b**2*c*d*p + 8*(a + b*x**3 + c*x**6)**p*a*b**2*c*d - 4*(a + b*x
**3 + c*x**6)**p*a*b**2*c*e*p**3*x**3 - 20*(a + b*x**3 + c*x**6)**p*a*b**2
*c*e*p**2*x**3 - 18*(a + b*x**3 + c*x**6)**p*a*b**2*c*e*p*x**3 + 8*(a + b*
x**3 + c*x**6)**p*a*b*c**2*d*p**3*x**3 + 24*(a + b*x**3 + c*x**6)**p*a*b*c
**2*d*p**2*x**3 + 16*(a + b*x**3 + c*x**6)**p*a*b*c**2*d*p*x**3 + 8*(a + b
*x**3 + c*x**6)**p*a*b*c**2*e*p**3*x**6 + 16*(a + b*x**3 + c*x**6)**p*a*b*
c**2*e*p**2*x**6 + 6*(a + b*x**3 + c*x**6)**p*a*b*c**2*e*p*x**6 + (a + b*x
**3 + c*x**6)**p*b**4*e*p**3*x**3 + 5*(a + b*x**3 + c*x**6)**p*b**4*e*p**2
*x**3 + 6*(a + b*x**3 + c*x**6)**p*b**4*e*p*x**3 - 2*(a + b*x**3 + c*x**6)
**p*b**3*c*d*p**3*x**3 - 8*(a + b*x**3 + c*x**6)**p*b**3*c*d*p**2*x**3 - 8
*(a + b*x**3 + c*x**6)**p*b**3*c*d*p*x**3 - 2*(a + b*x**3 + c*x**6)**p*b**
3*c*e*p**3*x**6 - 7*(a + b*x**3 + c*x**6)**p*b**3*c*e*p**2*x**6 - 3*(a + b
*x**3 + c*x**6)**p*b**3*c*e*p*x**6 + 4*(a + b*x**3 + c*x**6)**p*b**2*c**2*
d*p**3*x**6 + 10*(a + b*x**3 + c*x**6)**p*b**2*c**2*d*p**2*x**6 + 4*(a ...
```

3.73 $\int x^5(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 224

$$\int x^5(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$$
$$= -\frac{\left(be(2+p)-cd(3+2p)-2ce(1+p)x^3\right)\left(a+bx^3+cx^6\right)^{1+p}}{6c^2(1+p)(3+2p)}$$
$$+\frac{2^p\left(2ace-b^2e(2+p)+bcd(3+2p)\right)\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}\text{Hypergeometric2F1}\left(\right.}{3c^2\sqrt{b^2-4ac}(1+p)(3+2p)}$$

output
```
-1/6*(b*e*(2+p)-c*d*(3+2*p)-2*c*e*(p+1)*x^3)*(c*x^6+b*x^3+a)^(p+1)/c^2/(p+
1)/(3+2*p)+1/3*2^p*(2*a*c*e-b^2*e*(2+p)+b*c*d*(3+2*p))*(-(b-(-4*a*c+b^2)^(
1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))^(-1-p)*(c*x^6+b*x^3+a)^(p+1)*hypergeom([
-p, p+1],[2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))/c^2/
(-4*a*c+b^2)^(1/2)/(p+1)/(3+2*p)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.98 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.01

$$\int x^5(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$$
$$=\frac{1}{18}x^6\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\left(3d\,\text{AppellF1}\left(2,-p,-p,3,-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+2ex^3\,\text{AppellF1}\left(3,-p,-p,4,-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)$$

input
```
Integrate[x^5*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^6*(a + b*x^3 + c*x^6)^p*(3*d*AppellF1[2, -p, -p, 3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 2*e*x^3*AppellF1[3,
 -p, -p, 4, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
 4*a*c])]))/(18*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])
)^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1802, 1225, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$$
$$\Big\downarrow \text{1802}$$
$$\frac{1}{3}\int x^3(ex^3+d)\left(cx^6+bx^3+a\right)^p\,dx^3$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{3}\left(-\frac{(2ace+b^2(-e)(p+2)+bcd(2p+3))\int \left(cx^6+bx^3+a\right)^p dx^3}{2c^2(2p+3)}-\frac{\left(a+bx^3+cx^6\right)^{p+1}(be(p+2)-cd(2p+3$$

$$\Big\downarrow \text{1096}$$

$$\frac{1}{3}\left(\frac{2^p\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^3+cx^6\right)^{p+1}(2ace+b^2(-e)(p+2)+bcd(2p+3))\,\text{Hypergeometric2F1}\Big(}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}}\right.$$

input
```
Int[x^5*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(-1/2*((b*e*(2 + p) - c*d*(3 + 2*p) - 2*c*e*(1 + p)*x^3)*(a + b*x^3 + c*x^
6)^(1 + p))/(c^2*(1 + p)*(3 + 2*p)) + (2^p*(2*a*c*e - b^2*e*(2 + p) + b*c*
d*(3 + 2*p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1
- p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b +
Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c
]*(1 + p)*(3 + 2*p)))/3
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{5}\left(e\,x^{3}+d\right)\left(c\,x^{6}+b\,x^{3}+a\right)^{p}dx$$

input
```
int(x^5*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^5*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^{5}\left(d+ex^{3}\right)\left(a+bx^{3}+cx^{6}\right)^{p}\,dx=\int\left(ex^{3}+d\right)\left(cx^{6}+bx^{3}+a\right)^{p}x^{5}\,dx$$

input
```
integrate(x^5*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^8 + d*x^5)*(c*x^6 + b*x^3 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{5}\left(d+ex^{3}\right)\left(a+bx^{3}+cx^{6}\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate(x**5*(e*x**3+d)*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^5(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int (ex^3+d)\left(cx^6+bx^3+a\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x^5, x)
```

Giac [F]

$$\int x^5(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int (ex^3+d)\left(cx^6+bx^3+a\right)^p x^5\,dx$$

input
```
integrate(x^5*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int x^5\,\left(e\,x^3+d\right)\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

input
```
int(x^5*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^5*(d + e*x^3)*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int x^5(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx = \text{too large to display}$$

input
```
int(x^5*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
( - 4*(a + b*x**3 + c*x**6)**p*a**2*c*e*p - 4*(a + b*x**3 + c*x**6)**p*a**
2*c*e + (a + b*x**3 + c*x**6)**p*a*b**2*e*p + 2*(a + b*x**3 + c*x**6)**p*a
*b**2*e - 2*(a + b*x**3 + c*x**6)**p*a*b*c*d*p - 3*(a + b*x**3 + c*x**6)**
p*a*b*c*d + 4*(a + b*x**3 + c*x**6)**p*a*b*c*e*p**2*x**3 + 4*(a + b*x**3 +
 c*x**6)**p*a*b*c*e*p*x**3 - (a + b*x**3 + c*x**6)**p*b**3*e*p**2*x**3 - 2
*(a + b*x**3 + c*x**6)**p*b**3*e*p*x**3 + 2*(a + b*x**3 + c*x**6)**p*b**2*
c*d*p**2*x**3 + 3*(a + b*x**3 + c*x**6)**p*b**2*c*d*p*x**3 + 2*(a + b*x**3
 + c*x**6)**p*b**2*c*e*p**2*x**6 + (a + b*x**3 + c*x**6)**p*b**2*c*e*p*x**
6 + 4*(a + b*x**3 + c*x**6)**p*b*c**2*d*p**2*x**6 + 8*(a + b*x**3 + c*x**6
)**p*b*c**2*d*p*x**6 + 3*(a + b*x**3 + c*x**6)**p*b*c**2*d*x**6 + 4*(a + b
*x**3 + c*x**6)**p*b*c**2*e*p**2*x**9 + 6*(a + b*x**3 + c*x**6)**p*b*c**2*
e*p*x**9 + 2*(a + b*x**3 + c*x**6)**p*b*c**2*e*x**9 + 96*int(((a + b*x**3
+ c*x**6)**p*x**5)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**3 + 8*b*p*x**3 +
3*b*x**3 + 4*c*p**2*x**6 + 8*c*p*x**6 + 3*c*x**6),x)*a**2*c**2*e*p**4 + 28
8*int(((a + b*x**3 + c*x**6)**p*x**5)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x
**3 + 8*b*p*x**3 + 3*b*x**3 + 4*c*p**2*x**6 + 8*c*p*x**6 + 3*c*x**6),x)*a*
*2*c**2*e*p**3 + 264*int(((a + b*x**3 + c*x**6)**p*x**5)/(4*a*p**2 + 8*a*p
 + 3*a + 4*b*p**2*x**3 + 8*b*p*x**3 + 3*b*x**3 + 4*c*p**2*x**6 + 8*c*p*x**
6 + 3*c*x**6),x)*a**2*c**2*e*p**2 + 72*int(((a + b*x**3 + c*x**6)**p*x**5)
/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**3 + 8*b*p*x**3 + 3*b*x**3 + 4*c*...
```

3.74 $\int x^2(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 170

$$\int x^2\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\frac{e\left(a+bx^3+cx^6\right)^{1+p}}{6c(1+p)}$$
$$-\frac{2^p(2cd-be)\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-}}{2\sqrt{b^2}}\right.}{3c\sqrt{b^2-4ac}(1+p)}$$

output
```
1/6*e*(c*x^6+b*x^3+a)^(p+1)/c/(p+1)-1/3*2^p*(-b*e+2*c*d)*(-(b-(-4*a*c+b^2)
^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))^(-1-p)*(c*x^6+b*x^3+a)^(p+1)*hypergeom
([-p, p+1],[2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))/c/
(-4*a*c+b^2)^(1/2)/(p+1)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.82 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.68

$$\int x^2\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{6}\left(a+bx^3+cx^6\right)^p\left(ex^6\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\text{AppellF1}\left(2,-p,-p,3,-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+\frac{2^pd\left(b-\sqrt{b^2-4ac}+2cx^3\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p}\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{-b+\sqrt{b^2-4ac}-2cx^3}{2\sqrt{b^2-4ac}}\right)}{c(1+p)}\right)$$

input
```
Integrate[x^2*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
((a + b*x^3 + c*x^6)^p*((e*x^6*AppellF1[2, -p, -p, 3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c]))^p) + (2^p*d*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*
Hypergeometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)/(2*
Sqrt[b^2 - 4*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^
2 - 4*a*c])^p)))/6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1798, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{3}\int \left(ex^3+d\right)\left(cx^6+bx^3+a\right)^p\,dx^3$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{(2cd-be)\int \left(cx^6+bx^3+a\right)^p\,dx^3}{2c}+\frac{e\left(a+bx^3+cx^6\right)^{p+1}}{2c(p+1)}\right)$$

$$\Big\downarrow \text{1096}$$

$$\frac{1}{3}\left(\frac{e\left(a+bx^3+cx^6\right)^{p+1}}{2c(p+1)}-\frac{2^p(2cd-be)\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^3+cx^6\right)^{p+1}\text{Hypergeometric2F1}\left(-p,\right.}{c(p+1)\sqrt{b^2-4ac}}\right.$$

input
```
Int[x^2*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
((e*(a + b*x^3 + c*x^6)^(1 + p))/(2*c*(1 + p)) - (2^p*(2*c*d - b*e)*(-((b
- Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 + c
*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p)))/3
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [F]

$$\int x^{2} \left(e\,x^{3}+d \right) \left(c\,x^{6}+b\,x^{3}+a \right)^{p}d x$$

input
```
int(x^2*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^2*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^2 \left(d+e x^3\right) \left(a+b x^3+c x^6\right)^p \, dx=\int { {\left(e x^{3} + d\right)} {\left(c x^{6} + b x^{3} + a\right)}^{p} x^{2} \,d x }$$

input
```
integrate(x^2*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^5 + d*x^2)*(c*x^6 + b*x^3 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(d+e x^3\right) \left(a+b x^3+c x^6\right)^p \, dx=\text{Timed out}$$

input
```
integrate(x**2*(e*x**3+d)*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int\left(ex^3+d\right)\left(cx^6+bx^3+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int\left(ex^3+d\right)\left(cx^6+bx^3+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int x^2\,\left(e\,x^3+d\right)\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

input
```
int(x^2*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^2*(d + e*x^3)*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int x^2\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\text{Too large to display}$$

input
```
int(x^2*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
( - (a + b*x**3 + c*x**6)**p*a*b*e + 4*(a + b*x**3 + c*x**6)**p*a*c*d*p +
4*(a + b*x**3 + c*x**6)**p*a*c*d + (a + b*x**3 + c*x**6)**p*b**2*e*p*x**3
+ 2*(a + b*x**3 + c*x**6)**p*b*c*d*p*x**3 + 2*(a + b*x**3 + c*x**6)**p*b*c
*d*x**3 + 2*(a + b*x**3 + c*x**6)**p*b*c*e*p*x**6 + (a + b*x**3 + c*x**6)*
*p*b*c*e*x**6 + 24*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*
x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*a*b*c*e*p**3 + 36*int(((a + b*x**3
 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**6
),x)*a*b*c*e*p**2 + 12*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*
b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*a*b*c*e*p - 48*int(((a + b*x**
3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**
6),x)*a*c**2*d*p**3 - 72*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a +
2*b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*a*c**2*d*p**2 - 24*int(((a +
 b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 +
 c*x**6),x)*a*c**2*d*p - 6*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a
+ 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*b**3*e*p**3 - 9*int(((a +
b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 +
c*x**6),x)*b**3*e*p**2 - 3*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a
+ 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*b**3*e*p + 12*int(((a + b*
x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*
x**6),x)*b**2*c*d*p**3 + 18*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p ...
```

3.75 $\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 290

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x}\,dx = \frac{2^{-1+2p}d\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2}}{2cx}\right.}{3p} - \frac{2^{1+p}e\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^3+cx^6)^{1+p}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-4ac}+2cx^3}{2\sqrt{b^2-4ac}}\right.}{3\sqrt{b^2-4ac}(1+p)}$$

output
```
1/3*2^(-1+2*p)*d*(c*x^6+b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*a
*c+b^2)^(1/2))/c/x^3,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^3)/p/(((b-(-4*a*c+b^2
)^(1/2)+2*c*x^3)/c/x^3)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)-1/3*
2^(p+1)*e*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))^(-1-p)*(c*x
^6+b*x^3+a)^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*
x^3)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/(p+1)
```

Mathematica [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x}\,dx = \frac{1}{6}(a+bx^3+cx^6)^p\left(\frac{4^p d\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\frac{-b+\sqrt{b}}{2c}\right.}{p} + \frac{2^p e\left(b-\sqrt{b^2-4ac}+2cx^3\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p}\operatorname{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{-b+\sqrt{b^2-4ac}-2cx^3}{2\sqrt{b^2-4ac}}\right.}{c(1+p)}$$

input
```
Integrate[((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x,x]
```

output
```
((a + b*x^3 + c*x^6)^p*((4^p*d*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b + S
qrt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)]])/(p*((b - S
qrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(c*x^3))^p) + (2^p*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*Hypergeometric2F1[-
p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])]
)/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c])^p)))/6
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1802, 1269, 1096, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x}dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{3}\int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^3}dx^3$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{3}\left(d\int \frac{(cx^6+bx^3+a)^p}{x^3}dx^3+e\int (cx^6+bx^3+a)^p\,dx^3\right)$$

$$\Big\downarrow \text{1096}$$

$$\frac{1}{3}\left(d\int \frac{(cx^6+bx^3+a)^p}{x^3}dx^3-\frac{e2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}(a+bx^3+cx^6)^{p+1}\,\text{Hypergeometric2F1}\left(-p,p+\right.}{(p+1)\sqrt{b^2-4ac}}\right.$$

$$\Big\downarrow \text{1178}$$

$$\frac{1}{3}\left(-d4^p\left(\frac{1}{x^3}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\int\left(\frac{b-\sqrt{b^2-4a}}{2cx^3}\right.\right.$$

$$\Big\downarrow \text{150}$$

$$\frac{1}{3}\left(\frac{d2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-}}{2cx^3}\right.}{p}\right.$$

input
```
Int[((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x,x]
```

output
```
((2^(-1 + 2*p)*d*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/
2*(b - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/
(p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(c*x^3))^p) - (2^(1 + p)*e*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1
[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])
])/(Sqrt[b^2 - 4*a*c]*(1 + p)))/3
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(e\,x^3+d)\,(c\,x^6+b\,x^3+a)^p}{x}dx$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x,x)
```

output
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x,x)
```

Fricas [F]

$$\int \frac{(d+ex^3)\,(a+bx^3+cx^6)^p}{x}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x,x, algorithm="fricas")
```

output
```
integral((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^3)\,(a+bx^3+cx^6)^p}{x}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)*(c*x**6+b*x**3+a)**p/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^3)\,(a+bx^3+cx^6)^p}{x}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x, x)
```

Giac [F]

$$\int \frac{(d+ex^3)\,(a+bx^3+cx^6)^p}{x}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^3)\,(a+bx^3+cx^6)^p}{x}\,dx = \int \frac{(e\,x^3+d)\,\left(c\,x^6+b\,x^3+a\right)^p}{x}\,dx$$

input
```
int(((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x,x)
```

output
```
int(((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x, x)
```

Reduce [F]

$$\int \frac{(d+ex^3)\,(a+bx^3+cx^6)^p}{x}\,dx$$
$$= \frac{2(c\,x^6+b\,x^3+a)^p\,aep+2(c\,x^6+b\,x^3+a)^p\,bdp+(c\,x^6+b\,x^3+a)^p\,bd+(c\,x^6+b\,x^3+a)^p\,bep\,x^3+12\Big(}{}$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x,x)
```

output
```
(2*(a + b*x**3 + c*x**6)**p*a*e*p + 2*(a + b*x**3 + c*x**6)**p*b*d*p + (a
+ b*x**3 + c*x**6)**p*b*d + (a + b*x**3 + c*x**6)**p*b*e*p*x**3 + 12*int((
a + b*x**3 + c*x**6)**p/(2*a*p*x + a*x + 2*b*p*x**4 + b*x**4 + 2*c*p*x**7
+ c*x**7),x)*a*b*d*p**3 + 12*int((a + b*x**3 + c*x**6)**p/(2*a*p*x + a*x +
 2*b*p*x**4 + b*x**4 + 2*c*p*x**7 + c*x**7),x)*a*b*d*p**2 + 3*int((a + b*x
**3 + c*x**6)**p/(2*a*p*x + a*x + 2*b*p*x**4 + b*x**4 + 2*c*p*x**7 + c*x**
7),x)*a*b*d*p - 24*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*
x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*a*c*e*p**3 - 12*int(((a + b*x**3 +
 c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),
x)*a*c*e*p**2 + 6*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x
**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*b**2*e*p**3 + 3*int(((a + b*x**3 +
c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x
)*b**2*e*p**2 - 12*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*
x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*b*c*d*p**3 - 12*int(((a + b*x**3 +
 c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x**3 + b*x**3 + 2*c*p*x**6 + c*x**6),
x)*b*c*d*p**2 - 3*int(((a + b*x**3 + c*x**6)**p*x**5)/(2*a*p + a + 2*b*p*x
**3 + b*x**3 + 2*c*p*x**6 + c*x**6),x)*b*c*d*p)/(3*b*p*(2*p + 1))
```

3.76 $\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 336

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^4}\,dx = -\frac{d(a+bx^3+cx^6)^{1+p}}{3ax^3} + \frac{2^{-1+2p}(ae+bdp)\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-\right.}{3ap} - \frac{2^{1+p}cd(1+2p)\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^3+cx^6)^{1+p}\operatorname{Hypergeometric2F1}\left(-p,1+p,2+p,\right.}{3a\sqrt{b^2-4ac}(1+p)}$$

output
```
-1/3*d*(c*x^6+b*x^3+a)^(p+1)/a/x^3+1/3*2^(-1+2*p)*(b*d*p+a*e)*(c*x^6+b*x^3
+a)^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*a*c+b^2)^(1/2))/c/x^3,-1/2*(b+
(-4*a*c+b^2)^(1/2))/c/x^3)/a/p/(((b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)/
(((b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)-1/3*2^(p+1)*c*d*(1+2*p)*(-(b-(-
4*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))^(-1-p)*(c*x^6+b*x^3+a)^(p+1)
*hypergeom([-p, p+1],[2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)
^(1/2))/a/(-4*a*c+b^2)^(1/2)/(p+1)
```

Mathematica [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^4}\,dx$$
$$= \frac{\left(1+\frac{b-\sqrt{b^2-4ac}}{2cx^3}\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^3\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{c}\right)^{p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\left(2dp\,\mathrm{App}\right.}{}$$

input
```
Integrate[((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^4,x]
```

output
```
(((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/c)^p*(a + b*x^3 + c*x^6)^p*(2*d*p*Appe
llF1[1 - 2*p, -p, -p, 2 - 2*p, -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b +
 Sqrt[b^2 - 4*a*c])/(2*c*x^3)] + e*(-1 + 2*p)*x^3*AppellF1[-2*p, -p, -p, 1
 - 2*p, -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*
c*x^3)]))/(6*p*(-1 + 2*p)*(1 + (b - Sqrt[b^2 - 4*a*c])/(2*c*x^3))^p*x^3*((
b - Sqrt[b^2 - 4*a*c])/(2*c) + x^3)^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
c*x^3))^p)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1802, 1237, 25, 1269, 1096, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^4}\,dx$$
$$\Big\downarrow \text{ 1802}$$
$$\frac{1}{3}\int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^6}dx^3$$
$$\Big\downarrow \text{ 1237}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{(cd(2p+1)x^3+ae+bdp)(cx^6+bx^3+a)^p}{x^3}dx^3}{a}-\frac{d(a+bx^3+cx^6)^{p+1}}{ax^3}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{3}\left(\frac{\int \frac{(cd(2p+1)x^3+ae+bdp)(cx^6+bx^3+a)^p}{x^3}dx^3}{a}-\frac{d(a+bx^3+cx^6)^{p+1}}{ax^3}\right)$$

$\Big\downarrow$ 1269

$$\frac{1}{3}\left(\frac{(ae+bdp)\int \frac{(cx^6+bx^3+a)^p}{x^3}dx^3+cd(2p+1)\int (cx^6+bx^3+a)^p\,dx^3}{a}-\frac{d(a+bx^3+cx^6)^{p+1}}{ax^3}\right)$$

$\Big\downarrow$ 1096

$$\frac{1}{3}\left(\frac{(ae+bdp)\int \frac{(cx^6+bx^3+a)^p}{x^3}dx^3-\frac{cd2^{p+1}(2p+1)\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}(a+bx^3+cx^6)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\right.}{(p+1)\sqrt{b^2-4ac}}}{a}\right.$$

$\Big\downarrow$ 1178

$$\frac{1}{3}\left(\frac{-4^p\left(\frac{1}{x^3}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p(ae+bdp)\int\left(\frac{b-\sqrt{b^2-4ac}}{2cx^3}+1\right)^p\left(\frac{b+\sqrt{b}}{2c}\right.}{}\right.$$

$\Big\downarrow$ 150

$$\frac{1}{3}\left(\frac{\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p(ae+bdp)\,\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{p}}{a}\right.$$

input
```
Int[((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^4,x]
```

output
```
(-((d*(a + b*x^3 + c*x^6)^(1 + p))/(a*x^3)) + ((2^(-1 + 2*p)*(a*e + b*d*p)
*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b - Sqrt[b^2
- 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/(p*((b - Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3
))^p) - (2^(1 + p)*c*d*(1 + 2*p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt
[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p,
 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(
Sqrt[b^2 - 4*a*c]*(1 + p)))/a)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(e\,x^3+d)\,(c\,x^6+b\,x^3+a)^p}{x^4}dx$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^4,x)
```

output
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^4,x)
```

Fricas [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^4}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^4}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^4,x, algorithm="fricas")
```

output
```
integral((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)*(c*x**6+b*x**3+a)**p/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^4}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^4}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^4,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^4}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^4}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^4}\,dx = \int \frac{(e\,x^3+d)\,(c\,x^6+b\,x^3+a)^p}{x^4}\,dx$$

input
```
int(((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^4,x)
```

output
```
int(((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^4, x)
```

Reduce [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^4}\,dx = \text{Too large to display}$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^4,x)
```

output

```
( - 2*(a + b*x**3 + c*x**6)**p*a*c*e*p*x**3 + (a + b*x**3 + c*x**6)**p*a*c
*e*x**3 - (a + b*x**3 + c*x**6)**p*b**2*d*p**2 + (a + b*x**3 + c*x**6)**p*
b**2*d*p + (a + b*x**3 + c*x**6)**p*b**2*e*p*x**3 - (a + b*x**3 + c*x**6)*
*p*b**2*e*x**3 - (a + b*x**3 + c*x**6)**p*b*c*d*p*x**3 + 3*int((a + b*x**3
 + c*x**6)**p/(a*p*x - a*x + b*p*x**4 - b*x**4 + c*p*x**7 - c*x**7),x)*a*b
**2*e*p**3*x**3 - 6*int((a + b*x**3 + c*x**6)**p/(a*p*x - a*x + b*p*x**4 -
 b*x**4 + c*p*x**7 - c*x**7),x)*a*b**2*e*p**2*x**3 + 3*int((a + b*x**3 + c
*x**6)**p/(a*p*x - a*x + b*p*x**4 - b*x**4 + c*p*x**7 - c*x**7),x)*a*b**2*
e*p*x**3 + 3*int((a + b*x**3 + c*x**6)**p/(a*p*x - a*x + b*p*x**4 - b*x**4
 + c*p*x**7 - c*x**7),x)*b**3*d*p**4*x**3 - 6*int((a + b*x**3 + c*x**6)**p
/(a*p*x - a*x + b*p*x**4 - b*x**4 + c*p*x**7 - c*x**7),x)*b**3*d*p**3*x**3
 + 3*int((a + b*x**3 + c*x**6)**p/(a*p*x - a*x + b*p*x**4 - b*x**4 + c*p*x
**7 - c*x**7),x)*b**3*d*p**2*x**3 + 12*int(((a + b*x**3 + c*x**6)**p*x**5)
/(a*p - a + b*p*x**3 - b*x**3 + c*p*x**6 - c*x**6),x)*a*c**2*e*p**3*x**3 -
 18*int(((a + b*x**3 + c*x**6)**p*x**5)/(a*p - a + b*p*x**3 - b*x**3 + c*p
*x**6 - c*x**6),x)*a*c**2*e*p**2*x**3 + 6*int(((a + b*x**3 + c*x**6)**p*x*
*5)/(a*p - a + b*p*x**3 - b*x**3 + c*p*x**6 - c*x**6),x)*a*c**2*e*p*x**3 -
 3*int(((a + b*x**3 + c*x**6)**p*x**5)/(a*p - a + b*p*x**3 - b*x**3 + c*p*
x**6 - c*x**6),x)*b**2*c*e*p**3*x**3 + 6*int(((a + b*x**3 + c*x**6)**p*x**
5)/(a*p - a + b*p*x**3 - b*x**3 + c*p*x**6 - c*x**6),x)*b**2*c*e*p**2*x...
```

3.77 $\int x^4(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 279

$$\int x^4\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{5}dx^5\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)+\frac{1}{8}ex^8\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{8}{3},-p,-p,\frac{11}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/5*d*x^5*(c*x^6+b*x^3+a)^p*AppellF1(5/3,-p,-p,8/3,-2*c*x^3/(b-(-4*a*c+b^2
)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2
)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)+1/8*e*x^8*(c*x^6+b*x^3+a)^p*
AppellF1(8/3,-p,-p,11/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*
c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-4*a
*c+b^2)^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.84

$$\int x^4(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$$
$$=\frac{1}{40}x^5\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(8d\,\text{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+5ex^3\,\text{AppellF1}\left(\frac{8}{3},-p,-p,\frac{11}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)$$

input
```
Integrate[x^4*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^5*(a + b*x^3 + c*x^6)^p*(8*d*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 5*e*x^3*AppellF
1[8/3, -p, -p, 11/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + S
qrt[b^2 - 4*a*c])]))/(40*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2
- 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p
)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$$

$$\Big\downarrow \text{ 1864}$$

$$\int \left(dx^4\left(a+bx^3+cx^6\right)^p+ex^7\left(a+bx^3+cx^6\right)^p\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}dx^5\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b-\sqrt{b^2-4\ldots}}\right.$$
$$\frac{1}{8}ex^8\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\text{AppellF1}\left(\frac{8}{3},-p,-p,\frac{11}{3},-\frac{2cx^3}{b-\sqrt{b^2-\ldots}}\right.$$

input
```
Int[x^4*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(d*x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e*
x^8*(a + b*x^3 + c*x^6)^p*AppellF1[8/3, -p, -p, 11/3, (-2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(8*(1 + (2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1864
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n
+ c*x^(2*n))^p, (f*x)^m*(d + e*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m
, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^{4}\left(e\,x^{3}+d\right)\left(c\,x^{6}+b\,x^{3}+a\right)^{p}dx$$

input
```
int(x^4*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^4*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^4\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\int\left(ex^3+d\right)\left(cx^6+bx^3+a\right)^px^4\,dx$$

input
```
integrate(x^4*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^7 + d*x^4)*(c*x^6 + b*x^3 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**4*(e*x**3+d)*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^4(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int (ex^3+d)\left(cx^6+bx^3+a\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x^4, x)
```

Giac [F]

$$\int x^4(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int (ex^3+d)\left(cx^6+bx^3+a\right)^p x^4\,dx$$

input
```
integrate(x^4*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d+ex^3) \left(a+bx^3+cx^6\right)^p \, dx = \int x^4 \left(e\,x^3+d\right) \left(c\,x^6+b\,x^3+a\right)^p dx$$

input
```
int(x^4*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^4*(d + e*x^3)*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int x^4 (d+ex^3) \left(a+bx^3+cx^6\right)^p \, dx = \text{too large to display}$$

input
```
int(x^4*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output

```
(36*(a + b*x**3 + c*x**6)**p*a*c*e*p**2*x**2 + 30*(a + b*x**3 + c*x**6)**p
*a*c*e*p*x**2 - 9*(a + b*x**3 + c*x**6)**p*b**2*e*p**2*x**2 - 15*(a + b*x*
*3 + c*x**6)**p*b**2*e*p*x**2 + 18*(a + b*x**3 + c*x**6)**p*b*c*d*p**2*x**
2 + 24*(a + b*x**3 + c*x**6)**p*b*c*d*p*x**2 + 18*(a + b*x**3 + c*x**6)**p
*b*c*e*p**2*x**5 + 6*(a + b*x**3 + c*x**6)**p*b*c*e*p*x**5 + 36*(a + b*x**
3 + c*x**6)**p*c**2*d*p**2*x**5 + 60*(a + b*x**3 + c*x**6)**p*c**2*d*p*x**
5 + 16*(a + b*x**3 + c*x**6)**p*c**2*d*x**5 + 36*(a + b*x**3 + c*x**6)**p*
c**2*e*p**2*x**8 + 42*(a + b*x**3 + c*x**6)**p*c**2*e*p*x**8 + 10*(a + b*x
**3 + c*x**6)**p*c**2*e*x**8 - 5832*int(((a + b*x**3 + c*x**6)**p*x**4)/(5
4*a*p**3 + 135*a*p**2 + 99*a*p + 20*a + 54*b*p**3*x**3 + 135*b*p**2*x**3 +
 99*b*p*x**3 + 20*b*x**3 + 54*c*p**3*x**6 + 135*c*p**2*x**6 + 99*c*p*x**6
+ 20*c*x**6),x)*a*b*c*e*p**6 - 28188*int(((a + b*x**3 + c*x**6)**p*x**4)/(
54*a*p**3 + 135*a*p**2 + 99*a*p + 20*a + 54*b*p**3*x**3 + 135*b*p**2*x**3
+ 99*b*p*x**3 + 20*b*x**3 + 54*c*p**3*x**6 + 135*c*p**2*x**6 + 99*c*p*x**6
 + 20*c*x**6),x)*a*b*c*e*p**5 - 49572*int(((a + b*x**3 + c*x**6)**p*x**4)/
(54*a*p**3 + 135*a*p**2 + 99*a*p + 20*a + 54*b*p**3*x**3 + 135*b*p**2*x**3
 + 99*b*p*x**3 + 20*b*x**3 + 54*c*p**3*x**6 + 135*c*p**2*x**6 + 99*c*p*x**
6 + 20*c*x**6),x)*a*b*c*e*p**4 - 39258*int(((a + b*x**3 + c*x**6)**p*x**4)
/(54*a*p**3 + 135*a*p**2 + 99*a*p + 20*a + 54*b*p**3*x**3 + 135*b*p**2*x**
3 + 99*b*p*x**3 + 20*b*x**3 + 54*c*p**3*x**6 + 135*c*p**2*x**6 + 99*c*p...
```

3.78 $\int x(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 279

$$\int x(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{2}dx^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)+\frac{1}{5}ex^5\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/2*d*x^2*(c*x^6+b*x^3+a)^p*AppellF1(2/3,-p,-p,5/3,-2*c*x^3/(b-(-4*a*c+b^2
)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2
)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)+1/5*e*x^5*(c*x^6+b*x^3+a)^p*
AppellF1(5/3,-p,-p,8/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c
+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-4*a*
c+b^2)^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.84

$$\int x(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx = \frac{1}{10}x^2\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(5d\,\text{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+2ex^3\,\text{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)$$

input
```
Integrate[x*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^2*(a + b*x^3 + c*x^6)^p*(5*d*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 2*e*x^3*AppellF
1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sq
rt[b^2 - 4*a*c])]))/(10*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx$$

$$\Big\downarrow \text{1864}$$

$$\int \left(dx\left(a+bx^3+cx^6\right)^p+ex^4\left(a+bx^3+cx^6\right)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}dx^2\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\text{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4}}\right.$$
$$\frac{1}{5}ex^5\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{3},-p,-p,\frac{8}{3},-\frac{2cx^3}{b-\sqrt{b^2-4}}\right.$$

input
```
Int[x*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(d*x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*(1 + (2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e*
x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1864
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n
+ c*x^(2*n))^p, (f*x)^m*(d + e*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m
, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x\left(e\,x^{3}+d\right)\left(c\,x^{6}+b\,x^{3}+a\right)^{p}dx$$

input
```
int(x*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int\left(ex^3+d\right)\left(cx^6+bx^3+a\right)^p x\,dx$$

input
```
integrate(x*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^4 + d*x)*(c*x^6 + b*x^3 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx = \text{Timed out}$$

input
```
integrate(x*(e*x**3+d)*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx = \int (ex^3+d)\left(cx^6+bx^3+a\right)^p x\,dx$$

input
```
integrate(x*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x, x)
```

Giac [F]

$$\int x(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx = \int (ex^3+d)\left(cx^6+bx^3+a\right)^p x\,dx$$

input
```
integrate(x*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int x\left(e\,x^3+d\right)\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

input
```
int(x*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x*(d + e*x^3)*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int x(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\text{too large to display}$$

input
```
int(x*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
(3*(a + b*x**3 + c*x**6)**p*b*e*p*x**2 + 6*(a + b*x**3 + c*x**6)**p*c*d*p*
x**2 + 5*(a + b*x**3 + c*x**6)**p*c*d*x**2 + 6*(a + b*x**3 + c*x**6)**p*c*
e*p*x**5 + 2*(a + b*x**3 + c*x**6)**p*c*e*x**5 + 648*int(((a + b*x**3 + c*
x**6)**p*x**4)/(18*a*p**2 + 21*a*p + 5*a + 18*b*p**2*x**3 + 21*b*p*x**3 +
5*b*x**3 + 18*c*p**2*x**6 + 21*c*p*x**6 + 5*c*x**6),x)*a*c*e*p**4 + 972*in
t(((a + b*x**3 + c*x**6)**p*x**4)/(18*a*p**2 + 21*a*p + 5*a + 18*b*p**2*x*
*3 + 21*b*p*x**3 + 5*b*x**3 + 18*c*p**2*x**6 + 21*c*p*x**6 + 5*c*x**6),x)*
a*c*e*p**3 + 432*int(((a + b*x**3 + c*x**6)**p*x**4)/(18*a*p**2 + 21*a*p +
 5*a + 18*b*p**2*x**3 + 21*b*p*x**3 + 5*b*x**3 + 18*c*p**2*x**6 + 21*c*p*x
**6 + 5*c*x**6),x)*a*c*e*p**2 + 60*int(((a + b*x**3 + c*x**6)**p*x**4)/(18
*a*p**2 + 21*a*p + 5*a + 18*b*p**2*x**3 + 21*b*p*x**3 + 5*b*x**3 + 18*c*p*
*2*x**6 + 21*c*p*x**6 + 5*c*x**6),x)*a*c*e*p - 162*int(((a + b*x**3 + c*x*
*6)**p*x**4)/(18*a*p**2 + 21*a*p + 5*a + 18*b*p**2*x**3 + 21*b*p*x**3 + 5*
b*x**3 + 18*c*p**2*x**6 + 21*c*p*x**6 + 5*c*x**6),x)*b**2*e*p**4 - 297*int
(((a + b*x**3 + c*x**6)**p*x**4)/(18*a*p**2 + 21*a*p + 5*a + 18*b*p**2*x**
3 + 21*b*p*x**3 + 5*b*x**3 + 18*c*p**2*x**6 + 21*c*p*x**6 + 5*c*x**6),x)*b
**2*e*p**3 - 171*int(((a + b*x**3 + c*x**6)**p*x**4)/(18*a*p**2 + 21*a*p +
 5*a + 18*b*p**2*x**3 + 21*b*p*x**3 + 5*b*x**3 + 18*c*p**2*x**6 + 21*c*p*x
**6 + 5*c*x**6),x)*b**2*e*p**2 - 30*int(((a + b*x**3 + c*x**6)**p*x**4)/(1
8*a*p**2 + 21*a*p + 5*a + 18*b*p**2*x**3 + 21*b*p*x**3 + 5*b*x**3 + 18*...
```

3.79 $\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 277

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^2}\,dx = -\frac{d\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\mathrm{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x} + \frac{1}{2}ex^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,\mathrm{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
-d*(c*x^6+b*x^3+a)^p*AppellF1(-1/3,-p,-p,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)+1/2*e*x^2*(c*x^6+b*x^3+a)^p*Appe
llF1(2/3,-p,-p,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2
)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^
2)^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^2}\,dx$$
$$= \frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\left(-2d\,\text{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{}{-b\cdots}\right.\right.}{2x}$$

input
```
Integrate[((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^2,x]
```

output
```
((a + b*x^3 + c*x^6)^p*(-2*d*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + e*x^3*AppellF1[2/
3, -p, -p, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b
^2 - 4*a*c])]))/(2*x*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*
a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^2}\,dx$$
$$\Big\downarrow \text{1864}$$
$$\int \left(\frac{d(a+bx^3+cx^6)^p}{x^2}+ex(a+bx^3+cx^6)^p\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}ex^2\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\text{AppellF1}\left(\frac{2}{3},-p,-p,\frac{5}{3},-\frac{2cx^3}{b-\sqrt{b^2-4a}}\right.$$

$$\frac{d\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\text{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

input
```
Int[((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^2,x]
```

output
```
-((d*(a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)) + (e*
x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*(1 + (2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1864
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n
+ c*x^(2*n))^p, (f*x)^m*(d + e*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m
, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{\left(e\,x^{3}+d\right)\left(c\,x^{6}+b\,x^{3}+a\right)^{p}}{x^{2}}dx$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^2,x)
```

output
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^2}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)*(c*x**6+b*x**3+a)**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^2}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^2}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^2}\,dx = \int \frac{(e\,x^3+d)\,(c\,x^6+b\,x^3+a)^p}{x^2}\,dx$$

input
```
int(((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^2,x)
```

output
```
int(((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^2}\,dx = \text{Too large to display}$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^2,x)
```

output
```
(6*(a + b*x**3 + c*x**6)**p*a*e*p + 6*(a + b*x**3 + c*x**6)**p*b*d*p + 2*(
a + b*x**3 + c*x**6)**p*b*d + 3*(a + b*x**3 + c*x**6)**p*b*e*p*x**3 - (a +
 b*x**3 + c*x**6)**p*b*e*x**3 + 54*int((a + b*x**3 + c*x**6)**p/(9*a*p**2*
x**2 - a*x**2 + 9*b*p**2*x**5 - b*x**5 + 9*c*p**2*x**8 - c*x**8),x)*a**2*e
*p**3*x - 6*int((a + b*x**3 + c*x**6)**p/(9*a*p**2*x**2 - a*x**2 + 9*b*p**
2*x**5 - b*x**5 + 9*c*p**2*x**8 - c*x**8),x)*a**2*e*p*x + 162*int((a + b*x
**3 + c*x**6)**p/(9*a*p**2*x**2 - a*x**2 + 9*b*p**2*x**5 - b*x**5 + 9*c*p*
*2*x**8 - c*x**8),x)*a*b*d*p**4*x + 54*int((a + b*x**3 + c*x**6)**p/(9*a*p
**2*x**2 - a*x**2 + 9*b*p**2*x**5 - b*x**5 + 9*c*p**2*x**8 - c*x**8),x)*a*
b*d*p**3*x - 18*int((a + b*x**3 + c*x**6)**p/(9*a*p**2*x**2 - a*x**2 + 9*b
*p**2*x**5 - b*x**5 + 9*c*p**2*x**8 - c*x**8),x)*a*b*d*p**2*x - 6*int((a +
 b*x**3 + c*x**6)**p/(9*a*p**2*x**2 - a*x**2 + 9*b*p**2*x**5 - b*x**5 + 9*
c*p**2*x**8 - c*x**8),x)*a*b*d*p*x - 324*int(((a + b*x**3 + c*x**6)**p*x**
4)/(9*a*p**2 - a + 9*b*p**2*x**3 - b*x**3 + 9*c*p**2*x**6 - c*x**6),x)*a*c
*e*p**4*x + 54*int(((a + b*x**3 + c*x**6)**p*x**4)/(9*a*p**2 - a + 9*b*p**
2*x**3 - b*x**3 + 9*c*p**2*x**6 - c*x**6),x)*a*c*e*p**3*x + 36*int(((a + b
*x**3 + c*x**6)**p*x**4)/(9*a*p**2 - a + 9*b*p**2*x**3 - b*x**3 + 9*c*p**2
*x**6 - c*x**6),x)*a*c*e*p**2*x - 6*int(((a + b*x**3 + c*x**6)**p*x**4)/(9
*a*p**2 - a + 9*b*p**2*x**3 - b*x**3 + 9*c*p**2*x**6 - c*x**6),x)*a*c*e*p*
x + 81*int(((a + b*x**3 + c*x**6)**p*x**4)/(9*a*p**2 - a + 9*b*p**2*x**...
```

3.80 $\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^5} dx$

Optimal result

Integrand size = 25, antiderivative size = 277

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^5} dx = -\frac{d\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{4}{3},-p,-p,-\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4x^4} - \frac{e\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

output
```
-1/4*d*(c*x^6+b*x^3+a)^p*AppellF1(-4/3,-p,-p,-1/3,-2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^4/((1+2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)-e*(c*x^6+b*x^3+a)^p*Appel
lF1(-1/3,-p,-p,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2
)^(1/2)))/x/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-4*a*c+
b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^5}\,dx =$$

$$-\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\left(d\,\text{AppellF1}\left(-\frac{4}{3},-p,-p,-\frac{1}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},-\right.\right.}{4x^4}$$

input
```
Integrate[((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^5,x]
```

output
```
-1/4*((a + b*x^3 + c*x^6)^p*(d*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 4*e*x^3*Appell
F1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
Sqrt[b^2 - 4*a*c])]))/(x^4*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^
2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))
^p)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^5}\,dx$$

$$\Big\downarrow \text{1864}$$

$$\int \left(\frac{d(a+bx^3+cx^6)^p}{x^5}+\frac{e(a+bx^3+cx^6)^p}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\text{AppellF1}\left(-\frac{4}{3},-p,-p,-\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4x^4}$$
$$\frac{e\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{3},-p,-p,\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

input
```
Int[((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^5,x]
```

output
```
-1/4*(d*(a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^4*(1 + (2*c*x
^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
- (e*(a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1864
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n
+ c*x^(2*n))^p, (f*x)^m*(d + e*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m
, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{\left(e\,x^{3}+d\right)\left(c\,x^{6}+b\,x^{3}+a\right)^{p}}{x^{5}}dx$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^5,x)
```

output
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^5,x)
```

Fricas [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^5}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^5,x, algorithm="fricas")
```

output
```
integral((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)*(c*x**6+b*x**3+a)**p/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^5}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^5,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^5, x)
```

Giac [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^5}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^5,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^5}\,dx = \int \frac{(e\,x^3+d)\,(c\,x^6+b\,x^3+a)^p}{x^5}\,dx$$

input
```
int(((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^5,x)
```

output
```
int(((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^5, x)
```

Reduce [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^5}\,dx = \text{Too large to display}$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^5,x)
```

output
```
( - 6*(a + b*x**3 + c*x**6)**p*d*p + (a + b*x**3 + c*x**6)**p*d + 4*(a + b
*x**3 + c*x**6)**p*e*x**3 + 432*int((a + b*x**3 + c*x**6)**p/(18*a*p**2*x*
*2 - 27*a*p*x**2 + 4*a*x**2 + 18*b*p**2*x**5 - 27*b*p*x**5 + 4*b*x**5 + 18
*c*p**2*x**8 - 27*c*p*x**8 + 4*c*x**8),x)*a*e*p**3*x**4 - 648*int((a + b*x
**3 + c*x**6)**p/(18*a*p**2*x**2 - 27*a*p*x**2 + 4*a*x**2 + 18*b*p**2*x**5
 - 27*b*p*x**5 + 4*b*x**5 + 18*c*p**2*x**8 - 27*c*p*x**8 + 4*c*x**8),x)*a*
e*p**2*x**4 + 96*int((a + b*x**3 + c*x**6)**p/(18*a*p**2*x**2 - 27*a*p*x**
2 + 4*a*x**2 + 18*b*p**2*x**5 - 27*b*p*x**5 + 4*b*x**5 + 18*c*p**2*x**8 -
27*c*p*x**8 + 4*c*x**8),x)*a*e*p*x**4 + 324*int((a + b*x**3 + c*x**6)**p/(
18*a*p**2*x**2 - 27*a*p*x**2 + 4*a*x**2 + 18*b*p**2*x**5 - 27*b*p*x**5 + 4
*b*x**5 + 18*c*p**2*x**8 - 27*c*p*x**8 + 4*c*x**8),x)*b*d*p**4*x**4 - 540*
int((a + b*x**3 + c*x**6)**p/(18*a*p**2*x**2 - 27*a*p*x**2 + 4*a*x**2 + 18
*b*p**2*x**5 - 27*b*p*x**5 + 4*b*x**5 + 18*c*p**2*x**8 - 27*c*p*x**8 + 4*c
*x**8),x)*b*d*p**3*x**4 + 153*int((a + b*x**3 + c*x**6)**p/(18*a*p**2*x**2
 - 27*a*p*x**2 + 4*a*x**2 + 18*b*p**2*x**5 - 27*b*p*x**5 + 4*b*x**5 + 18*c
*p**2*x**8 - 27*c*p*x**8 + 4*c*x**8),x)*b*d*p**2*x**4 - 12*int((a + b*x**3
 + c*x**6)**p/(18*a*p**2*x**2 - 27*a*p*x**2 + 4*a*x**2 + 18*b*p**2*x**5 -
27*b*p*x**5 + 4*b*x**5 + 18*c*p**2*x**8 - 27*c*p*x**8 + 4*c*x**8),x)*b*d*p
*x**4 + 216*int(((a + b*x**3 + c*x**6)**p*x)/(18*a*p**2 - 27*a*p + 4*a + 1
8*b*p**2*x**3 - 27*b*p*x**3 + 4*b*x**3 + 18*c*p**2*x**6 - 27*c*p*x**6 +...
```

3.81 $\int x^6(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 279

$$\int x^6\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{7}dx^7\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{7}{3},-p,-p,\frac{10}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)+\frac{1}{10}ex^{10}\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{10}{3},-p,-p,\frac{13}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/7*d*x^7*(c*x^6+b*x^3+a)^p*AppellF1(7/3,-p,-p,10/3,-2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)+1/10*e*x^10*(c*x^6+b*x^3+a)
^p*AppellF1(10/3,-p,-p,13/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-
4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(
-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.84

$$\int x^6(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx = \frac{1}{70}x^7\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(10d\,\text{AppellF1}\left(\frac{7}{3},-p,-p,\frac{10}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+7ex^3\,\text{AppellF1}\left(\frac{10}{3},-p,-p,\frac{13}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)$$

input
```
Integrate[x^6*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^7*(a + b*x^3 + c*x^6)^p*(10*d*AppellF1[7/3, -p, -p, 10/3, (-2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 7*e*x^3*Appel
lF1[10/3, -p, -p, 13/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
+ Sqrt[b^2 - 4*a*c])]))/(70*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
)^p)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 (d+ex^3) (a+bx^3+cx^6)^p \, dx$$

$$\Big\downarrow \text{ 1864}$$

$$\int \left(dx^6 (a+bx^3+cx^6)^p + ex^9 (a+bx^3+cx^6)^p\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{7} dx^7 \left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p} (a+bx^3+cx^6)^p \left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \text{AppellF1}\left(\frac{7}{3},-p,-p,\frac{10}{3},-\frac{2cx^3}{b-\sqrt{b^2-}}\right.$$
$$\frac{1}{10} ex^{10} \left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p} (a+bx^3+cx^6)^p \left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \text{AppellF1}\left(\frac{10}{3},-p,-p,\frac{13}{3},-\frac{2cx}{b-\sqrt{b^2}}\right.$$

input
```
Int[x^6*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(d*x^7*(a + b*x^3 + c*x^6)^p*AppellF1[7/3, -p, -p, 10/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(7*(1 + (2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e
*x^10*(a + b*x^3 + c*x^6)^p*AppellF1[10/3, -p, -p, 13/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(10*(1 + (2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1864
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n
+ c*x^(2*n))^p, (f*x)^m*(d + e*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m
, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^6 \left(e\,x^3+d\right) \left(c\,x^6+b\,x^3+a\right)^p dx$$

input
```
int(x^6*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^6*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^6 \left(d+ex^3\right) \left(a+bx^3+cx^6\right)^p \, dx = \int \left(ex^3+d\right) \left(cx^6+bx^3+a\right)^p x^6 \, dx$$

input
```
integrate(x^6*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^9 + d*x^6)*(c*x^6 + b*x^3 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^6\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**6*(e*x**3+d)*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^6\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\int\left(ex^3+d\right)\left(cx^6+bx^3+a\right)^px^6\,dx$$

input
```
integrate(x^6*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x^6, x)
```

Giac [F]

$$\int x^6\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\int\left(ex^3+d\right)\left(cx^6+bx^3+a\right)^px^6\,dx$$

input
```
integrate(x^6*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 (d+ex^3) \left(a+bx^3+cx^6\right)^p \, dx = \int x^6 \left(e\,x^3+d\right) \left(c\,x^6+b\,x^3+a\right)^p \,dx$$

input
```
int(x^6*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^6*(d + e*x^3)*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int x^6 (d+ex^3) \left(a+bx^3+cx^6\right)^p \, dx = \text{too large to display}$$

input
```
int(x^6*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output

```
( - 108*(a + b*x**3 + c*x**6)**p*a*b*c*e*p**3*x - 396*(a + b*x**3 + c*x**6
)**p*a*b*c*e*p**2*x - 252*(a + b*x**3 + c*x**6)**p*a*b*c*e*p*x + 216*(a +
b*x**3 + c*x**6)**p*a*c**2*d*p**3*x + 504*(a + b*x**3 + c*x**6)**p*a*c**2*
d*p**2*x + 240*(a + b*x**3 + c*x**6)**p*a*c**2*d*p*x + 216*(a + b*x**3 + c
*x**6)**p*a*c**2*e*p**3*x**4 + 288*(a + b*x**3 + c*x**6)**p*a*c**2*e*p**2*
x**4 + 42*(a + b*x**3 + c*x**6)**p*a*c**2*e*p*x**4 + 27*(a + b*x**3 + c*x*
*6)**p*b**3*e*p**3*x + 99*(a + b*x**3 + c*x**6)**p*b**3*e*p**2*x + 84*(a +
 b*x**3 + c*x**6)**p*b**3*e*p*x - 54*(a + b*x**3 + c*x**6)**p*b**2*c*d*p**
3*x - 162*(a + b*x**3 + c*x**6)**p*b**2*c*d*p**2*x - 120*(a + b*x**3 + c*x
**6)**p*b**2*c*d*p*x - 54*(a + b*x**3 + c*x**6)**p*b**2*c*e*p**3*x**4 - 13
5*(a + b*x**3 + c*x**6)**p*b**2*c*e*p**2*x**4 - 21*(a + b*x**3 + c*x**6)**
p*b**2*c*e*p*x**4 + 108*(a + b*x**3 + c*x**6)**p*b*c**2*d*p**3*x**4 + 198*
(a + b*x**3 + c*x**6)**p*b*c**2*d*p**2*x**4 + 30*(a + b*x**3 + c*x**6)**p*
b*c**2*d*p*x**4 + 108*(a + b*x**3 + c*x**6)**p*b*c**2*e*p**3*x**7 + 90*(a
+ b*x**3 + c*x**6)**p*b*c**2*e*p**2*x**7 + 12*(a + b*x**3 + c*x**6)**p*b*c
**2*e*p*x**7 + 216*(a + b*x**3 + c*x**6)**p*c**3*d*p**3*x**7 + 540*(a + b*
x**3 + c*x**6)**p*c**3*d*p**2*x**7 + 324*(a + b*x**3 + c*x**6)**p*c**3*d*p
*x**7 + 40*(a + b*x**3 + c*x**6)**p*c**3*d*x**7 + 216*(a + b*x**3 + c*x**6
)**p*c**3*e*p**3*x**10 + 432*(a + b*x**3 + c*x**6)**p*c**3*e*p**2*x**10 +
234*(a + b*x**3 + c*x**6)**p*c**3*e*p*x**10 + 28*(a + b*x**3 + c*x**6)*...
```

3.82 $\int x^3(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 279

$$\int x^3\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{4}dx^4\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)+\frac{1}{7}ex^7\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\operatorname{AppellF1}\left(\frac{7}{3},-p,-p,\frac{10}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/4*d*x^4*(c*x^6+b*x^3+a)^p*AppellF1(4/3,-p,-p,7/3,-2*c*x^3/(b-(-4*a*c+b^2
)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2
)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)+1/7*e*x^7*(c*x^6+b*x^3+a)^p*
AppellF1(7/3,-p,-p,10/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*
c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-4*a
*c+b^2)^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.84

$$\int x^3(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx = \frac{1}{28}x^4\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(7d\,\text{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+4ex^3\,\text{AppellF1}\left(\frac{7}{3},-p,-p,\frac{10}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)$$

input
```
Integrate[x^3*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x^4*(a + b*x^3 + c*x^6)^p*(7*d*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 4*e*x^3*AppellF
1[7/3, -p, -p, 10/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + S
qrt[b^2 - 4*a*c])]))/(28*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2
- 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p
)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx$$

$$\Big\downarrow \text{1864}$$

$$\int \left(dx^3\left(a+bx^3+cx^6\right)^p+ex^6\left(a+bx^3+cx^6\right)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}dx^4\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\text{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4}}\right.$$
$$\frac{1}{7}ex^7\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\text{AppellF1}\left(\frac{7}{3},-p,-p,\frac{10}{3},-\frac{2cx^3}{b-\sqrt{b^2-}}\right.$$

input
```
Int[x^3*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(d*x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*(1 + (2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e*
x^7*(a + b*x^3 + c*x^6)^p*AppellF1[7/3, -p, -p, 10/3, (-2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(7*(1 + (2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1864
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n
+ c*x^(2*n))^p, (f*x)^m*(d + e*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m
, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^{3}\left(e\,x^{3}+d\right)\left(c\,x^{6}+b\,x^{3}+a\right)^{p}dx$$

input
```
int(x^3*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
int(x^3*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int x^3\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\int\left(ex^3+d\right)\left(cx^6+bx^3+a\right)^px^3\,dx$$

input
```
integrate(x^3*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^6 + d*x^3)*(c*x^6 + b*x^3 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**3*(e*x**3+d)*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int (ex^3+d)(cx^6+bx^3+a)^p x^3\,dx$$

input
```
integrate(x^3*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x^3, x)
```

Giac [F]

$$\int x^3(d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int (ex^3+d)(cx^6+bx^3+a)^p x^3\,dx$$

input
```
integrate(x^3*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\int x^3\,\left(e\,x^3+d\right)\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

input
```
int(x^3*(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int(x^3*(d + e*x^3)*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int x^3\left(d+ex^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\text{too large to display}$$

input
```
int(x^3*(e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
(36*(a + b*x**3 + c*x**6)**p*a*c*e*p**2*x + 24*(a + b*x**3 + c*x**6)**p*a*
c*e*p*x - 9*(a + b*x**3 + c*x**6)**p*b**2*e*p**2*x - 12*(a + b*x**3 + c*x*
*6)**p*b**2*e*p*x + 18*(a + b*x**3 + c*x**6)**p*b*c*d*p**2*x + 21*(a + b*x
**3 + c*x**6)**p*b*c*d*p*x + 18*(a + b*x**3 + c*x**6)**p*b*c*e*p**2*x**4 +
 3*(a + b*x**3 + c*x**6)**p*b*c*e*p*x**4 + 36*(a + b*x**3 + c*x**6)**p*c**
2*d*p**2*x**4 + 48*(a + b*x**3 + c*x**6)**p*c**2*d*p*x**4 + 7*(a + b*x**3
+ c*x**6)**p*c**2*d*x**4 + 36*(a + b*x**3 + c*x**6)**p*c**2*e*p**2*x**7 +
30*(a + b*x**3 + c*x**6)**p*c**2*e*p*x**7 + 4*(a + b*x**3 + c*x**6)**p*c**
2*e*x**7 - 3888*int((a + b*x**3 + c*x**6)**p/(108*a*p**3 + 216*a*p**2 + 11
7*a*p + 14*a + 108*b*p**3*x**3 + 216*b*p**2*x**3 + 117*b*p*x**3 + 14*b*x**
3 + 108*c*p**3*x**6 + 216*c*p**2*x**6 + 117*c*p*x**6 + 14*c*x**6),x)*a**2*
c*e*p**5 - 10368*int((a + b*x**3 + c*x**6)**p/(108*a*p**3 + 216*a*p**2 + 1
17*a*p + 14*a + 108*b*p**3*x**3 + 216*b*p**2*x**3 + 117*b*p*x**3 + 14*b*x*
*3 + 108*c*p**3*x**6 + 216*c*p**2*x**6 + 117*c*p*x**6 + 14*c*x**6),x)*a**2
*c*e*p**4 - 9396*int((a + b*x**3 + c*x**6)**p/(108*a*p**3 + 216*a*p**2 + 1
17*a*p + 14*a + 108*b*p**3*x**3 + 216*b*p**2*x**3 + 117*b*p*x**3 + 14*b*x*
*3 + 108*c*p**3*x**6 + 216*c*p**2*x**6 + 117*c*p*x**6 + 14*c*x**6),x)*a**2
*c*e*p**3 - 3312*int((a + b*x**3 + c*x**6)**p/(108*a*p**3 + 216*a*p**2 + 1
17*a*p + 14*a + 108*b*p**3*x**3 + 216*b*p**2*x**3 + 117*b*p*x**3 + 14*b*x*
*3 + 108*c*p**3*x**6 + 216*c*p**2*x**6 + 117*c*p*x**6 + 14*c*x**6),x)*a...
```

3.83 $\int (d+ex^3)(a+bx^3+cx^6)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$\int (d+ex^3)(a+bx^3+cx^6)^p\,dx = dx\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(\frac{1}{3},-p,-p,\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)+\frac{1}{4}ex^4\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
d*x*(c*x^6+b*x^3+a)^p*AppellF1(1/3,-p,-p,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)
/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)+1/4*e*x^4*(c*x^6+b*x^3+a)^p*Appell
F1(4/3,-p,-p,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^
(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)
^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.85

$$\int (d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$$
$$=\frac{1}{4}x\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(4d\,\text{AppellF1}\left(\frac{1}{3},-p,-p,\frac{4}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+ex^3\,\text{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)$$

input
```
Integrate[(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(x*(a + b*x^3 + c*x^6)^p*(4*d*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + e*x^3*AppellF1[4/
3, -p, -p, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b
^2 - 4*a*c])]))/(4*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*
c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1762, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx$$
$$\Big\downarrow \; 1762$$
$$\int \left(d\left(a+bx^3+cx^6\right)^p+ex^3\left(a+bx^3+cx^6\right)^p\right)dx$$
$$\Big\downarrow \; 2009$$

$$dx\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{3},-p,-p,\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},\right.$$
$$\frac{1}{4}ex^4\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\text{AppellF1}\left(\frac{4}{3},-p,-p,\frac{7}{3},-\frac{2cx^3}{b-\sqrt{b^2-4a}}\right.$$

input
```
Int[(d + e*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

output
```
(d*x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e*x^4*
(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b - Sqrt[b^2
- 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*(1 + (2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1762
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(e\,x^{3}+d\right)\left(c\,x^{6}+b\,x^{3}+a\right)^{p}dx$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

Fricas [F]

$$\int (d+ex^3) (a+bx^3+cx^6)^p \, dx = \int (ex^3+d)(cx^6+bx^3+a)^p \, dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex^3) (a+bx^3+cx^6)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)*(c*x**6+b*x**3+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex^3) (a+bx^3+cx^6)^p \, dx = \int (ex^3+d)(cx^6+bx^3+a)^p \, dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p, x)
```

Giac [F]

$$\int (d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int (ex^3+d)(cx^6+bx^3+a)^p\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\int (e\,x^3+d)\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

input
```
int((d + e*x^3)*(a + b*x^3 + c*x^6)^p,x)
```

output
```
int((d + e*x^3)*(a + b*x^3 + c*x^6)^p, x)
```

Reduce [F]

$$\int (d+ex^3)\left(a+bx^3+cx^6\right)^p\,dx=\text{too large to display}$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p,x)
```

output
```
(3*(a + b*x**3 + c*x**6)**p*b*e*p*x + 6*(a + b*x**3 + c*x**6)**p*c*d*p*x +
 4*(a + b*x**3 + c*x**6)**p*c*d*x + 6*(a + b*x**3 + c*x**6)**p*c*e*p*x**4
+ (a + b*x**3 + c*x**6)**p*c*e*x**4 - 54*int((a + b*x**3 + c*x**6)**p/(18*
a*p**2 + 15*a*p + 2*a + 18*b*p**2*x**3 + 15*b*p*x**3 + 2*b*x**3 + 18*c*p**
2*x**6 + 15*c*p*x**6 + 2*c*x**6),x)*a*b*e*p**3 - 45*int((a + b*x**3 + c*x*
*6)**p/(18*a*p**2 + 15*a*p + 2*a + 18*b*p**2*x**3 + 15*b*p*x**3 + 2*b*x**3
 + 18*c*p**2*x**6 + 15*c*p*x**6 + 2*c*x**6),x)*a*b*e*p**2 - 6*int((a + b*x
**3 + c*x**6)**p/(18*a*p**2 + 15*a*p + 2*a + 18*b*p**2*x**3 + 15*b*p*x**3
+ 2*b*x**3 + 18*c*p**2*x**6 + 15*c*p*x**6 + 2*c*x**6),x)*a*b*e*p + 648*int
((a + b*x**3 + c*x**6)**p/(18*a*p**2 + 15*a*p + 2*a + 18*b*p**2*x**3 + 15*
b*p*x**3 + 2*b*x**3 + 18*c*p**2*x**6 + 15*c*p*x**6 + 2*c*x**6),x)*a*c*d*p*
*4 + 972*int((a + b*x**3 + c*x**6)**p/(18*a*p**2 + 15*a*p + 2*a + 18*b*p**
2*x**3 + 15*b*p*x**3 + 2*b*x**3 + 18*c*p**2*x**6 + 15*c*p*x**6 + 2*c*x**6)
,x)*a*c*d*p**3 + 432*int((a + b*x**3 + c*x**6)**p/(18*a*p**2 + 15*a*p + 2*
a + 18*b*p**2*x**3 + 15*b*p*x**3 + 2*b*x**3 + 18*c*p**2*x**6 + 15*c*p*x**6
 + 2*c*x**6),x)*a*c*d*p**2 + 48*int((a + b*x**3 + c*x**6)**p/(18*a*p**2 +
15*a*p + 2*a + 18*b*p**2*x**3 + 15*b*p*x**3 + 2*b*x**3 + 18*c*p**2*x**6 +
15*c*p*x**6 + 2*c*x**6),x)*a*c*d*p + 648*int(((a + b*x**3 + c*x**6)**p*x**
3)/(18*a*p**2 + 15*a*p + 2*a + 18*b*p**2*x**3 + 15*b*p*x**3 + 2*b*x**3 + 1
8*c*p**2*x**6 + 15*c*p*x**6 + 2*c*x**6),x)*a*c*e*p**4 + 648*int(((a + b...
```

3.84 $\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 274

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^3}\,dx = -\frac{d\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2} + ex\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(\frac{1}{3},-p,-p,\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

output
```
-1/2*d*(c*x^6+b*x^3+a)^p*AppellF1(-2/3,-p,-p,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^2/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1
/2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)+e*x*(c*x^6+b*x^3+a)^p*Appe
llF1(1/3,-p,-p,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2
)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^
2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^3}\,dx$$
$$= \frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\left(-d\,\text{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{}{-b+}\right.\right.}{2x^2}$$

input
```
Integrate[((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^3,x]
```

output
```
((a + b*x^3 + c*x^6)^p*(-(d*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]) + 2*e*x^3*AppellF1[
1/3, -p, -p, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt
[b^2 - 4*a*c])]))/(2*x^2*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2
- 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p
)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^3}\,dx$$
$$\Big\downarrow \text{1864}$$
$$\int \left(\frac{d(a+bx^3+cx^6)^p}{x^3}+e(a+bx^3+cx^6)^p\right)dx$$
$$\Big\downarrow \text{2009}$$

$$ex\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(\frac{1}{3},-p,-p,\frac{4}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},\right.$$

$$\frac{d\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

input
```
Int[((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^3,x]
```

output
```
-1/2*(d*(a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*(1 + (2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p) +
 (e*x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1864
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n
+ c*x^(2*n))^p, (f*x)^m*(d + e*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m
, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(e\,x^3+d)\,(c\,x^6+b\,x^3+a)^p}{x^3}dx$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^3,x)
```

output
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^3}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^3}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^3,x, algorithm="fricas")
```

output
```
integral((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)*(c*x**6+b*x**3+a)**p/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^3}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^3}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^3}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^3}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^3}\,dx = \int \frac{(e\,x^3+d)\,(c\,x^6+b\,x^3+a)^p}{x^3}\,dx$$

input
```
int(((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^3,x)
```

output
```
int(((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^3}\,dx = \text{too large to display}$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^3,x)
```

output
```
(6*(a + b*x**3 + c*x**6)**p*a*e*p + 6*(a + b*x**3 + c*x**6)**p*b*d*p + (a
+ b*x**3 + c*x**6)**p*b*d + 3*(a + b*x**3 + c*x**6)**p*b*e*p*x**3 - 2*(a +
 b*x**3 + c*x**6)**p*b*e*x**3 + 216*int((a + b*x**3 + c*x**6)**p/(18*a*p**
2*x**3 - 9*a*p*x**3 - 2*a*x**3 + 18*b*p**2*x**6 - 9*b*p*x**6 - 2*b*x**6 +
18*c*p**2*x**9 - 9*c*p*x**9 - 2*c*x**9),x)*a**2*e*p**3*x**2 - 108*int((a +
 b*x**3 + c*x**6)**p/(18*a*p**2*x**3 - 9*a*p*x**3 - 2*a*x**3 + 18*b*p**2*x
**6 - 9*b*p*x**6 - 2*b*x**6 + 18*c*p**2*x**9 - 9*c*p*x**9 - 2*c*x**9),x)*a
**2*e*p**2*x**2 - 24*int((a + b*x**3 + c*x**6)**p/(18*a*p**2*x**3 - 9*a*p*
x**3 - 2*a*x**3 + 18*b*p**2*x**6 - 9*b*p*x**6 - 2*b*x**6 + 18*c*p**2*x**9
- 9*c*p*x**9 - 2*c*x**9),x)*a**2*e*p*x**2 + 324*int((a + b*x**3 + c*x**6)*
*p/(18*a*p**2*x**3 - 9*a*p*x**3 - 2*a*x**3 + 18*b*p**2*x**6 - 9*b*p*x**6 -
 2*b*x**6 + 18*c*p**2*x**9 - 9*c*p*x**9 - 2*c*x**9),x)*a*b*d*p**4*x**2 - 1
08*int((a + b*x**3 + c*x**6)**p/(18*a*p**2*x**3 - 9*a*p*x**3 - 2*a*x**3 +
18*b*p**2*x**6 - 9*b*p*x**6 - 2*b*x**6 + 18*c*p**2*x**9 - 9*c*p*x**9 - 2*c
*x**9),x)*a*b*d*p**3*x**2 - 63*int((a + b*x**3 + c*x**6)**p/(18*a*p**2*x**
3 - 9*a*p*x**3 - 2*a*x**3 + 18*b*p**2*x**6 - 9*b*p*x**6 - 2*b*x**6 + 18*c*
p**2*x**9 - 9*c*p*x**9 - 2*c*x**9),x)*a*b*d*p**2*x**2 - 6*int((a + b*x**3
+ c*x**6)**p/(18*a*p**2*x**3 - 9*a*p*x**3 - 2*a*x**3 + 18*b*p**2*x**6 - 9*
b*p*x**6 - 2*b*x**6 + 18*c*p**2*x**9 - 9*c*p*x**9 - 2*c*x**9),x)*a*b*d*p*x
**2 - 648*int(((a + b*x**3 + c*x**6)**p*x**3)/(18*a*p**2 - 9*a*p - 2*a ...
```

3.85 $\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 279

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^6}\,dx = -\frac{d\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{5}{3},-p,-p,-\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{}{b+}\right.}{5x^5} - \frac{e\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2c}{b+\sqrt{b^2}}\right.}{2x^2}$$

output
```
-1/5*d*(c*x^6+b*x^3+a)^p*AppellF1(-5/3,-p,-p,-2/3,-2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^5/((1+2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)-1/2*e*(c*x^6+b*x^3+a)^p*A
ppellF1(-2/3,-p,-p,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c
+b^2)^(1/2)))/x^2/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-
4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^6}\,dx =$$

$$-\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\left(2d\,\text{AppellF1}\left(-\frac{5}{3},-p,-p,-\frac{2}{3},-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\right.\right.}{10x^5}$$

input
```
Integrate[((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^6,x]
```

output
```
-1/10*((a + b*x^3 + c*x^6)^p*(2*d*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 5*e*x^3*App
ellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
 + Sqrt[b^2 - 4*a*c])]))/(x^5*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]))^p)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^6}\,dx$$

$$\Big\downarrow \; 1864$$

$$\int \left(\frac{d(a+bx^3+cx^6)^p}{x^6}+\frac{e(a+bx^3+cx^6)^p}{x^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{d\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}(a+bx^3+cx^6)^p\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\text{AppellF1}\left(-\frac{5}{3},-p,-p,-\frac{2}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$
$$-\frac{e\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}(a+bx^3+cx^6)^p\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\text{AppellF1}\left(-\frac{2}{3},-p,-p,\frac{1}{3},-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

input
```
Int[((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^6,x]
```

output
```
-1/5*(d*(a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^5*(1 + (2*c*x
^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
- (e*(a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*x^2*(1 + (2*c*x^3
)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1864
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n
 + c*x^(2*n))^p, (f*x)^m*(d + e*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m
, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(e\,x^3+d)\,(c\,x^6+b\,x^3+a)^p}{x^6}dx$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^6,x)
```

output
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^6,x)
```

Fricas [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^6}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^6}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^6,x, algorithm="fricas")
```

output
```
integral((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((e*x**3+d)*(c*x**6+b*x**3+a)**p/x**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^6}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^6}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^6,x, algorithm="maxima")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^6, x)
```

Giac [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^6}\,dx = \int \frac{(ex^3+d)(cx^6+bx^3+a)^p}{x^6}\,dx$$

input
```
integrate((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^6,x, algorithm="giac")
```

output
```
integrate((e*x^3 + d)*(c*x^6 + b*x^3 + a)^p/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^6}\,dx = \int \frac{(e\,x^3+d)\,(c\,x^6+b\,x^3+a)^p}{x^6}\,dx$$

input
```
int(((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^6,x)
```

output
```
int(((d + e*x^3)*(a + b*x^3 + c*x^6)^p)/x^6, x)
```

Reduce [F]

$$\int \frac{(d+ex^3)(a+bx^3+cx^6)^p}{x^6}\,dx = \text{Too large to display}$$

input
```
int((e*x^3+d)*(c*x^6+b*x^3+a)^p/x^6,x)
```

output

```
( - 6*(a + b*x**3 + c*x**6)**p*d*p + 2*(a + b*x**3 + c*x**6)**p*d + 5*(a +
 b*x**3 + c*x**6)**p*e*x**3 + 270*int((a + b*x**3 + c*x**6)**p/(9*a*p**2*x
**3 - 18*a*p*x**3 + 5*a*x**3 + 9*b*p**2*x**6 - 18*b*p*x**6 + 5*b*x**6 + 9*
c*p**2*x**9 - 18*c*p*x**9 + 5*c*x**9),x)*a*e*p**3*x**5 - 540*int((a + b*x*
*3 + c*x**6)**p/(9*a*p**2*x**3 - 18*a*p*x**3 + 5*a*x**3 + 9*b*p**2*x**6 -
18*b*p*x**6 + 5*b*x**6 + 9*c*p**2*x**9 - 18*c*p*x**9 + 5*c*x**9),x)*a*e*p*
*2*x**5 + 150*int((a + b*x**3 + c*x**6)**p/(9*a*p**2*x**3 - 18*a*p*x**3 +
5*a*x**3 + 9*b*p**2*x**6 - 18*b*p*x**6 + 5*b*x**6 + 9*c*p**2*x**9 - 18*c*p
*x**9 + 5*c*x**9),x)*a*e*p*x**5 + 162*int((a + b*x**3 + c*x**6)**p/(9*a*p*
*2*x**3 - 18*a*p*x**3 + 5*a*x**3 + 9*b*p**2*x**6 - 18*b*p*x**6 + 5*b*x**6
+ 9*c*p**2*x**9 - 18*c*p*x**9 + 5*c*x**9),x)*b*d*p**4*x**5 - 378*int((a +
b*x**3 + c*x**6)**p/(9*a*p**2*x**3 - 18*a*p*x**3 + 5*a*x**3 + 9*b*p**2*x**
6 - 18*b*p*x**6 + 5*b*x**6 + 9*c*p**2*x**9 - 18*c*p*x**9 + 5*c*x**9),x)*b*
d*p**3*x**5 + 198*int((a + b*x**3 + c*x**6)**p/(9*a*p**2*x**3 - 18*a*p*x**
3 + 5*a*x**3 + 9*b*p**2*x**6 - 18*b*p*x**6 + 5*b*x**6 + 9*c*p**2*x**9 - 18
*c*p*x**9 + 5*c*x**9),x)*b*d*p**2*x**5 - 30*int((a + b*x**3 + c*x**6)**p/(
9*a*p**2*x**3 - 18*a*p*x**3 + 5*a*x**3 + 9*b*p**2*x**6 - 18*b*p*x**6 + 5*b
*x**6 + 9*c*p**2*x**9 - 18*c*p*x**9 + 5*c*x**9),x)*b*d*p*x**5 + 135*int((a
 + b*x**3 + c*x**6)**p/(9*a*p**2 - 18*a*p + 5*a + 9*b*p**2*x**3 - 18*b*p*x
**3 + 5*b*x**3 + 9*c*p**2*x**6 - 18*c*p*x**6 + 5*c*x**6),x)*b*e*p**3*x*...
```

3.86 $\int \frac{x^8(a+bx^3+cx^6)^p}{d+ex^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 378

$$\int \frac{x^8(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \frac{(a+bx^3+cx^6)^{1+p}}{6ce(1+p)} + \frac{2^{-1+2p}d^2\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-2p,-p,-p,1-\dots\right)}{3e^3p} + \frac{2^p(2cd+be)\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^3+cx^6)^{1+p}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-\dots}}{2\sqrt{b^2\dots}}\right)}{3c\sqrt{b^2-4ac}\,e^2(1+p)}$$

output
```
1/6*(c*x^6+b*x^3+a)^(p+1)/c/e/(p+1)+1/3*2^(-1+2*p)*d^2*(c*x^6+b*x^3+a)^p*A
ppellF1(-2*p,-p,-p,1-2*p,(2*d-(b+(-4*a*c+b^2)^(1/2))*e/c)/(2*e*x^3+2*d),1/
2*(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/c/(e*x^3+d))/e^3/p/((e*(b-(-4*a*c+b^2)^
(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/((e*(b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3
+d))^p)+1/3*2^p*(b*e+2*c*d)*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^
(1/2))^(-1-p)*(c*x^6+b*x^3+a)^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+(-4*a
*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)/e^2/(p+1)
```

Mathematica [A] (warning: unable to verify)

Time = 2.13 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.24

$$\int \frac{x^8(a+bx^3+cx^6)^p}{d+ex^3}\,dx$$

$$= \frac{(a+bx^3+cx^6)^p\left(e^2x^6\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\text{AppellF1}\left(2,-p,-p,3,-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2}{-b+\sqrt{}}\right.\right.}{}$$

input
```
Integrate[(x^8*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
((a + b*x^3 + c*x^6)^p*((e^2*x^6*AppellF1[2, -p, -p, 3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c]))^p) + 2^p*d*((2^p*d*AppellF1[-2*p, -p, -p, 1
- 2*p, (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*c*d - b*e
 + Sqrt[b^2 - 4*a*c]*e)/(2*c*d + 2*c*e*x^3)])/(p*((e*(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3))/(c*(d + e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c
*(d + e*x^3)))^p) + (e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)*Hypergeometric2F
1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c
])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c])^p)))
)/(6*e^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1802, 1267, 25, 27, 1269, 1096, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(a+bx^3+cx^6)^p}{d+ex^3}\,dx$$

$$\Big\downarrow \; 1802$$

$$\frac{1}{3}\int \frac{x^6\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3$$

$$\Big\downarrow \textbf{1267}$$

$$\frac{1}{3}\left(\frac{\int -\frac{e(p+1)\left((2cd+be)x^3+bd\right)\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3}{2ce^2(p+1)}+\frac{\left(a+bx^3+cx^6\right)^{p+1}}{2ce(p+1)}\right)$$

$$\Big\downarrow \textbf{25}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{p+1}}{2ce(p+1)}-\frac{\int \frac{e(p+1)\left((2cd+be)x^3+bd\right)\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3}{2ce^2(p+1)}\right)$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{p+1}}{2ce(p+1)}-\frac{\int \frac{\left((2cd+be)x^3+bd\right)\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3}{2ce}\right)$$

$$\Big\downarrow \textbf{1269}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{p+1}}{2ce(p+1)}-\frac{\frac{(be+2cd)\int \left(cx^6+bx^3+a\right)^p dx^3}{e}-\frac{2cd^2\int \frac{\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3}{e}}{2ce}\right)$$

$$\Big\downarrow \textbf{1096}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{p+1}}{2ce(p+1)}-\frac{-\frac{2cd^2\int \frac{\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3}{e}-\frac{2^{p+1}(be+2cd)\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\text{Hypergeometri}}{e(p+1)\sqrt{b^2-4ac}}}{2ce}\right.$$

$$\Big\downarrow \textbf{1178}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3+cx^6\right)^{p+1}}{2ce(p+1)}-\frac{\frac{cd^2 2^{2p+1}\left(\frac{1}{d+ex^3}\right)^{2p}\left(a+bx^3+cx^6\right)^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\int\left(\frac{1}{ex^3+d}\right)^{-2p-1}}{e^2}}{}\right.$$

$$\Big\downarrow \textbf{150}$$

$$\frac{1}{3}\left(\frac{(a+bx^3+cx^6)^{p+1}}{2ce(p+1)}-\frac{-\frac{cd^2 2^{2p}(a+bx^3+cx^6)^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p\right.}{e^2p}}{}\right.$$

input
```
Int[(x^8*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
((a + b*x^3 + c*x^6)^(1 + p)/(2*c*e*(1 + p)) - (-((2^(2*p)*c*d^2*(a + b*x^
3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e)/(2*c*(d + e*x^3)), (2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c)/(2*(d + e*
x^3))])/(e^2*p*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p*(
(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p)) - (2^(1 + p)*(2
*c*d + b*e)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 -
 p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + S
qrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*e*(
1 + p)))/(2*c*e))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^8(c\,x^6+b\,x^3+a)^p}{e\,x^3+d}dx$$

input
```
int(x^8*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
int(x^8*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

Fricas [F]

$$\int \frac{x^8(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^8}{ex^3+d}\,dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^8/(e*x^3 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(c*x**6+b*x**3+a)**p/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^8(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^8}{ex^3+d}\,dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^8/(e*x^3 + d), x)
```

Giac [F]

$$\int \frac{x^8(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^8}{ex^3+d}\,dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^8/(e*x^3 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{x^8\,(c\,x^6+b\,x^3+a)^p}{e\,x^3+d}\,dx$$

input
```
int((x^8*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x)
```

output
```
int((x^8*(a + b*x^3 + c*x^6)^p)/(d + e*x^3), x)
```

Reduce [F]

$$\int \frac{x^8(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{too large to display}$$

input
```
int(x^8*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
( - (a + b*x**3 + c*x**6)**p*a*b*e**2*p - 2*(a + b*x**3 + c*x**6)**p*a*c*d
*e*p - (a + b*x**3 + c*x**6)**p*b**2*d*e*p**2 - (a + b*x**3 + c*x**6)**p*b
**2*d*e*p + (a + b*x**3 + c*x**6)**p*b**2*e**2*p**2*x**3 + 2*(a + b*x**3 +
 c*x**6)**p*b*c*d**2*p**2 + 4*(a + b*x**3 + c*x**6)**p*b*c*d**2*p + 2*(a +
 b*x**3 + c*x**6)**p*b*c*d**2 - 2*(a + b*x**3 + c*x**6)**p*b*c*d*e*p*x**3
+ 2*(a + b*x**3 + c*x**6)**p*b*c*e**2*p**2*x**6 + (a + b*x**3 + c*x**6)**p
*b*c*e**2*p*x**6 - 4*(a + b*x**3 + c*x**6)**p*c**2*d**2*p**2*x**3 - 4*(a +
 b*x**3 + c*x**6)**p*c**2*d**2*p*x**3 + 4*(a + b*x**3 + c*x**6)**p*c**2*d*
e*p**2*x**6 + 2*(a + b*x**3 + c*x**6)**p*c**2*d*e*p*x**6 + 24*int(((a + b*
x**3 + c*x**6)**p*x**8)/(2*a*b*d*e*p + a*b*d*e + 2*a*b*e**2*p*x**3 + a*b*e
**2*x**3 + 4*a*c*d**2*p + 2*a*c*d**2 + 4*a*c*d*e*p*x**3 + 2*a*c*d*e*x**3 +
 2*b**2*d*e*p*x**3 + b**2*d*e*x**3 + 2*b**2*e**2*p*x**6 + b**2*e**2*x**6 +
 4*b*c*d**2*p*x**3 + 2*b*c*d**2*x**3 + 6*b*c*d*e*p*x**6 + 3*b*c*d*e*x**6 +
 2*b*c*e**2*p*x**9 + b*c*e**2*x**9 + 4*c**2*d**2*p*x**6 + 2*c**2*d**2*x**6
 + 4*c**2*d*e*p*x**9 + 2*c**2*d*e*x**9),x)*a*b**2*c*e**4*p**4 + 36*int(((a
 + b*x**3 + c*x**6)**p*x**8)/(2*a*b*d*e*p + a*b*d*e + 2*a*b*e**2*p*x**3 +
a*b*e**2*x**3 + 4*a*c*d**2*p + 2*a*c*d**2 + 4*a*c*d*e*p*x**3 + 2*a*c*d*e*x
**3 + 2*b**2*d*e*p*x**3 + b**2*d*e*x**3 + 2*b**2*e**2*p*x**6 + b**2*e**2*x
**6 + 4*b*c*d**2*p*x**3 + 2*b*c*d**2*x**3 + 6*b*c*d*e*p*x**6 + 3*b*c*d*e*x
**6 + 2*b*c*e**2*p*x**9 + b*c*e**2*x**9 + 4*c**2*d**2*p*x**6 + 2*c**2*d...
```

3.87 $\int \frac{x^5(a+bx^3+cx^6)^p}{d+ex^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 336

$$\int \frac{x^5(a+bx^3+cx^6)^p}{d+ex^3}\,dx = -\frac{2^{-1+2p}d\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(-2p,-p,-p,1-2\ldots\right.}{3e^2p} - \frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^3+cx^6)^{1+p}\operatorname{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-4ac}+2cx^3}{2\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}\,e(1+p)}$$

output
```
-1/3*2^(-1+2*p)*d*(c*x^6+b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(2*d-(b+(-4*
a*c+b^2)^(1/2))*e/c)/(2*e*x^3+2*d),1/2*(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/c/
(e*x^3+d))/e^2/p/((e*(b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/((e*(b
+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)-1/3*2^(p+1)*(-(b-(-4*a*c+b^2)
^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))^(-1-p)*(c*x^6+b*x^3+a)^(p+1)*hypergeom
([-p, p+1],[2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(1/2)/e/(p+1)
```

Mathematica [A] (warning: unable to verify)

Time = 1.25 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.95

$$\int \frac{x^5(a+bx^3+cx^6)^p}{d+ex^3}\,dx$$
$$= \frac{(a+bx^3+cx^6)^p\left(-\frac{4^p d\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)e}{2c\left(d+ex^3\right)},\frac{2cd-}{2}\right.}{p}\right.}{6e^2}$$

input
```
Integrate[(x^5*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
((a + b*x^3 + c*x^6)^p*(-((4^p*d*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)/(2*c*d + 2*c*e*x^3)])/(p*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c
*(d + e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p
)) + (2^p*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*Hypergeometric2F1[-p, 1 + p,
 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(c*(1 +
 p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c])^p)))/(6*e^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1802, 1269, 1096, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+bx^3+cx^6\right)^p}{d+ex^3}\,dx$$

$$\Big\downarrow \ 1802$$

$$\frac{1}{3}\int \frac{x^3\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3$$

$$\Big\downarrow \ 1269$$

$$\frac{1}{3}\left(\frac{\int \left(cx^6+bx^3+a\right)^p dx^3}{e}-\frac{d\int \frac{\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3}{e}\right)$$

$\downarrow$ 1096

$$\frac{1}{3}\left(-\frac{d\int \frac{\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3}{e}-\frac{2^{p+1}\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\text{Hypergeometric2F1}\left(-p,p+1,p\right.}{e(p+1)\sqrt{b^2-4ac}}\right.$$

$\downarrow$ 1178

$$\frac{1}{3}\left(\frac{d4^p\left(\frac{1}{d+ex^3}\right)^{2p}\left(a+bx^3+cx^6\right)^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\int \left(\frac{1}{ex^3+d}\right)^{-2p-1}\left(1-\frac{2d-}{}\right.}{e^2}\right.$$

$\downarrow$ 150

$$\frac{1}{3}\left(-\frac{d2^{2p-1}\left(a+bx^3+cx^6\right)^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p\right.}{e^2p}\right.$$

input
```
Int[(x^5*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
(-((2^(-1 + 2*p)*d*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (
2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*d - ((b + Sqrt[b^
2 - 4*a*c])*e)/c)/(2*(d + e*x^3))])/(e^2*p*((e*(b - Sqrt[b^2 - 4*a*c] + 2*
c*x^3))/(c*(d + e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d +
e*x^3)))^p)) - (2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 -
4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p,
 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^
2 - 4*a*c]*e*(1 + p)))/3
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{5}(c\,x^{6}+b\,x^{3}+a)^{p}}{e\,x^{3}+d}dx$$

input
```
int(x^5*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
int(x^5*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

Fricas [F]

$$\int \frac{x^5(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^5}{ex^3+d}\,dx$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^5/(e*x^3 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(c*x**6+b*x**3+a)**p/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^5}{ex^3+d}\,dx$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^5/(e*x^3 + d), x)
```

Giac [F]

$$\int \frac{x^5(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^5}{ex^3+d}\,dx$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^5/(e*x^3 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{x^5\,(c\,x^6+b\,x^3+a)^p}{e\,x^3+d}\,dx$$

input
```
int((x^5*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x)
```

output
```
int((x^5*(a + b*x^3 + c*x^6)^p)/(d + e*x^3), x)
```

Reduce [F]

$$\int \frac{x^5(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{too large to display}$$

input
```
int(x^5*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
(2*(a + b*x**3 + c*x**6)**p*a*e*p - (a + b*x**3 + c*x**6)**p*b*d*p - (a +
b*x**3 + c*x**6)**p*b*d + (a + b*x**3 + c*x**6)**p*b*e*p*x**3 + 2*(a + b*x
**3 + c*x**6)**p*c*d*p*x**3 - 24*int(((a + b*x**3 + c*x**6)**p*x**8)/(2*a*
b*d*e*p + a*b*d*e + 2*a*b*e**2*p*x**3 + a*b*e**2*x**3 + 4*a*c*d**2*p + 2*a
*c*d**2 + 4*a*c*d*e*p*x**3 + 2*a*c*d*e*x**3 + 2*b**2*d*e*p*x**3 + b**2*d*e
*x**3 + 2*b**2*e**2*p*x**6 + b**2*e**2*x**6 + 4*b*c*d**2*p*x**3 + 2*b*c*d*
*2*x**3 + 6*b*c*d*e*p*x**6 + 3*b*c*d*e*x**6 + 2*b*c*e**2*p*x**9 + b*c*e**2
*x**9 + 4*c**2*d**2*p*x**6 + 2*c**2*d**2*x**6 + 4*c**2*d*e*p*x**9 + 2*c**2
*d*e*x**9),x)*a*b*c*e**3*p**3 - 12*int(((a + b*x**3 + c*x**6)**p*x**8)/(2*
a*b*d*e*p + a*b*d*e + 2*a*b*e**2*p*x**3 + a*b*e**2*x**3 + 4*a*c*d**2*p + 2
*a*c*d**2 + 4*a*c*d*e*p*x**3 + 2*a*c*d*e*x**3 + 2*b**2*d*e*p*x**3 + b**2*d
*e*x**3 + 2*b**2*e**2*p*x**6 + b**2*e**2*x**6 + 4*b*c*d**2*p*x**3 + 2*b*c*
d**2*x**3 + 6*b*c*d*e*p*x**6 + 3*b*c*d*e*x**6 + 2*b*c*e**2*p*x**9 + b*c*e*
*2*x**9 + 4*c**2*d**2*p*x**6 + 2*c**2*d**2*x**6 + 4*c**2*d*e*p*x**9 + 2*c*
*2*d*e*x**9),x)*a*b*c*e**3*p**2 - 48*int(((a + b*x**3 + c*x**6)**p*x**8)/(
2*a*b*d*e*p + a*b*d*e + 2*a*b*e**2*p*x**3 + a*b*e**2*x**3 + 4*a*c*d**2*p +
 2*a*c*d**2 + 4*a*c*d*e*p*x**3 + 2*a*c*d*e*x**3 + 2*b**2*d*e*p*x**3 + b**2
*d*e*x**3 + 2*b**2*e**2*p*x**6 + b**2*e**2*x**6 + 4*b*c*d**2*p*x**3 + 2*b*
c*d**2*x**3 + 6*b*c*d*e*p*x**6 + 3*b*c*d*e*x**6 + 2*b*c*e**2*p*x**9 + b*c*
e**2*x**9 + 4*c**2*d**2*p*x**6 + 2*c**2*d**2*x**6 + 4*c**2*d*e*p*x**9 +...
```

3.88 $\int \frac{x^2(a+bx^3+cx^6)^p}{d+ex^3} dx$

Optimal result

Integrand size = 27, antiderivative size = 201

$$\int \frac{x^2(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \frac{2^{-1+2p}\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\ldots\right.}{3ep}$$

output
```
1/3*2^(-1+2*p)*(c*x^6+b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(2*d-(b+(-4*a*c
+b^2)^(1/2))*e/c)/(2*e*x^3+2*d),1/2*(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/c/(e*
x^3+d))/e/p/((e*(b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/((e*(b+(-4*
a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \frac{2^{-1+2p}\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\ldots\right.}{3ep}$$

input
```
Integrate[(x^2*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*c*d - b*e + Sqrt[b^2 -
 4*a*c]*e)/(2*c*d + 2*c*e*x^3)])/(3*e*p*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x
^3))/(c*(d + e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x
^3)))^p)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1798, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a + bx^3 + cx^6)^p}{d + ex^3} \, dx$$

$$\Big\downarrow \text{ 1798}$$

$$\frac{1}{3} \int \frac{(cx^6 + bx^3 + a)^p}{ex^3 + d} dx^3$$

$$\Big\downarrow \text{ 1178}$$

$$-\frac{4^p \left(\frac{1}{d+ex^3}\right)^{2p} (a + bx^3 + cx^6)^p \left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx^3\right)}{c(d+ex^3)}\right)^{-p} \left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx^3\right)}{c(d+ex^3)}\right)^{-p} \int \left(\frac{1}{ex^3+d}\right)^{-2p-1} \left(1 - \frac{2d - \frac{(b-}{2(e}}{}\right.}{3e}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{2^{2p-1} (a + bx^3 + cx^6)^p \left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx^3\right)}{c(d+ex^3)}\right)^{-p} \left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx^3\right)}{c(d+ex^3)}\right)^{-p} \text{AppellF1}\left(-2p, -p, -p, 1-2p, \frac{2cd-(}{2c}\right.}{3ep}$$

input
```
Int[(x^2*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*d - ((b + Sqrt[b^2 - 4
*a*c])*e)/c)/(2*(d + e*x^3))])/(3*e*p*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3
))/(c*(d + e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3
)))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [F]

$$\int \frac{x^2(c\,x^6+b\,x^3+a)^p}{e\,x^3+d}dx$$

input
```
int(x^2*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
int(x^2*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

Fricas [F]

$$\int \frac{x^2(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^2}{ex^3+d}\,dx$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^2/(e*x^3 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(c*x**6+b*x**3+a)**p/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^2}{ex^3+d}\,dx$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^2/(e*x^3 + d), x)
```

Giac [F]

$$\int \frac{x^2(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^2}{ex^3+d}\,dx$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^2/(e*x^3 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{x^2\,(c\,x^6+b\,x^3+a)^p}{e\,x^3+d}\,dx$$

input
```
int((x^2*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x)
```

output
```
int((x^2*(a + b*x^3 + c*x^6)^p)/(d + e*x^3), x)
```

Reduce [F]

$$\int \frac{x^2(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
int(x^2*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
((a + b*x**3 + c*x**6)**p*b - 3*int(((a + b*x**3 + c*x**6)**p*x**8)/(a*b*d
*e + a*b*e**2*x**3 + 2*a*c*d**2 + 2*a*c*d*e*x**3 + b**2*d*e*x**3 + b**2*e*
*2*x**6 + 2*b*c*d**2*x**3 + 3*b*c*d*e*x**6 + b*c*e**2*x**9 + 2*c**2*d**2*x
**6 + 2*c**2*d*e*x**9),x)*b**2*c*e**2*p + 12*int(((a + b*x**3 + c*x**6)**p
*x**8)/(a*b*d*e + a*b*e**2*x**3 + 2*a*c*d**2 + 2*a*c*d*e*x**3 + b**2*d*e*x
**3 + b**2*e**2*x**6 + 2*b*c*d**2*x**3 + 3*b*c*d*e*x**6 + b*c*e**2*x**9 +
2*c**2*d**2*x**6 + 2*c**2*d*e*x**9),x)*c**3*d**2*p + 3*int(((a + b*x**3 +
c*x**6)**p*x**2)/(a*b*d*e + a*b*e**2*x**3 + 2*a*c*d**2 + 2*a*c*d*e*x**3 +
b**2*d*e*x**3 + b**2*e**2*x**6 + 2*b*c*d**2*x**3 + 3*b*c*d*e*x**6 + b*c*e*
*2*x**9 + 2*c**2*d**2*x**6 + 2*c**2*d*e*x**9),x)*a*b**2*e**2*p + 12*int(((
a + b*x**3 + c*x**6)**p*x**2)/(a*b*d*e + a*b*e**2*x**3 + 2*a*c*d**2 + 2*a*
c*d*e*x**3 + b**2*d*e*x**3 + b**2*e**2*x**6 + 2*b*c*d**2*x**3 + 3*b*c*d*e*
x**6 + b*c*e**2*x**9 + 2*c**2*d**2*x**6 + 2*c**2*d*e*x**9),x)*a*b*c*d*e*p
+ 12*int(((a + b*x**3 + c*x**6)**p*x**2)/(a*b*d*e + a*b*e**2*x**3 + 2*a*c*
d**2 + 2*a*c*d*e*x**3 + b**2*d*e*x**3 + b**2*e**2*x**6 + 2*b*c*d**2*x**3 +
 3*b*c*d*e*x**6 + b*c*e**2*x**9 + 2*c**2*d**2*x**6 + 2*c**2*d*e*x**9),x)*a
*c**2*d**2*p - 3*int(((a + b*x**3 + c*x**6)**p*x**2)/(a*b*d*e + a*b*e**2*x
**3 + 2*a*c*d**2 + 2*a*c*d*e*x**3 + b**2*d*e*x**3 + b**2*e**2*x**6 + 2*b*c
*d**2*x**3 + 3*b*c*d*e*x**6 + b*c*e**2*x**9 + 2*c**2*d**2*x**6 + 2*c**2*d*
e*x**9),x)*b**3*d*e*p - 6*int(((a + b*x**3 + c*x**6)**p*x**2)/(a*b*d*e ...
```

3.89 $\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 362

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)}\,dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-}}{2cx^3}\right.}{3dp} - \frac{2^{-1+2p}\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p\right.}{3dp}$$

output
```
1/3*2^(-1+2*p)*(c*x^6+b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*a*c
+b^2)^(1/2))/c/x^3,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^3)/d/p/(((b-(-4*a*c+b^2
)^(1/2)+2*c*x^3)/c/x^3)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)-1/3*
2^(-1+2*p)*(c*x^6+b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(2*d-(b+(-4*a*c+b^2
)^(1/2))*e/c)/(2*e*x^3+2*d),1/2*(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/c/(e*x^3+
d))/d/p/((e*(b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/((e*(b+(-4*a*c+
b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)
```

Mathematica [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)}\,dx=\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)}\,dx$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(x*(d + e*x^3)),x]
```

output
```
Integrate[(a + b*x^3 + c*x^6)^p/(x*(d + e*x^3)), x]
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1802, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)}\,dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^p}{x^3(ex^3+d)}dx^3$$

$$\Big\downarrow \text{ 1289}$$

$$\frac{1}{3}\int \left(\frac{(cx^6+bx^3+a)^p}{dx^3}-\frac{e(cx^6+bx^3+a)^p}{d(ex^3+d)}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4a}}{2cx^3}\right.}{dp}\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(x*(d + e*x^3)),x]
```

output
```
((2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*
(b - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/(d
*p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(c*x^3))^p) - (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p,
 -p, -p, 1 - 2*p, (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (
2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c)/(2*(d + e*x^3))])/(d*p*((e*(b - Sqrt[
b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^3))/(c*(d + e*x^3)))^p))/3
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x\,(e\,x^3+d)}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x/(e*x^3+d),x)
```

output
```
int((c*x^6+b*x^3+a)^p/x/(e*x^3+d),x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e*x^4 + d*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)*x), x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)}\,dx=\int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{x\,(e\,x^3+d)}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(x*(d + e*x^3)),x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(x*(d + e*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{e\,x^4+dx}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x/(e*x^3+d),x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d*x + e*x**4),x)
```

3.90 $\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 531

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)}\,dx = -\frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\mathrm{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{b-\sqrt{b^2-4ac}}{2c\ldots}\right.}{3d(1-2p)x^3}$$
$$-\frac{2^{-1+2p}e\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\mathrm{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\ldots}{\ldots}\right.}{3d^2p}$$
$$+\frac{2^{-1+2p}e\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p\,\mathrm{AppellF1}\left(-2p,-p,-p,1-2\ldots\right.}{3d^2p}$$

output
```
-1/3*4^p*(c*x^6+b*x^3+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,-1/2*(b-(-4*a*c+b^2)
^(1/2))/c/x^3,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^3)/d/(1-2*p)/x^3/(((b-(-4*a*
c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)
-1/3*2^(-1+2*p)*e*(c*x^6+b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*
a*c+b^2)^(1/2))/c/x^3,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^3)/d^2/p/(((b-(-4*a*
c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)
+1/3*2^(-1+2*p)*e*(c*x^6+b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(2*d-(b+(-4*
a*c+b^2)^(1/2))*e/c)/(2*e*x^3+2*d),1/2*(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/c/
(e*x^3+d))/d^2/p/((e*(b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/((e*(b
+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)
```

Mathematica [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)}\,dx=\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)}\,dx$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^4*(d + e*x^3)),x]
```

output
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^4*(d + e*x^3)), x]
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 528, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1802, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)}\,dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^p}{x^6(ex^3+d)}dx^3$$

$$\Big\downarrow \text{1289}$$

$$\frac{1}{3}\int \left(\frac{e^2\left(cx^6+bx^3+a\right)^p}{d^2\left(ex^3+d\right)}-\frac{e\left(cx^6+bx^3+a\right)^p}{d^2x^3}+\frac{\left(cx^6+bx^3+a\right)^p}{dx^6}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{e2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(a+bx^3+cx^6\right)^p\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2}}{2ca}\right.}{d^2p}\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(x^4*(d + e*x^3)),x]
```

output
```
(-((4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p), -1/2*(b
 - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/(d*(
1 - 2*p)*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2
- 4*a*c] + 2*c*x^3)/(c*x^3))^p)) - (2^(-1 + 2*p)*e*(a + b*x^3 + c*x^6)^p*A
ppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*
(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/(d^2*p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p) + (2^(-1 + 2*p
)*e*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/
c)/(2*(d + e*x^3))])/(d^2*p*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d +
 e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p))/3
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x^{4}\,(e\,x^{3}+d)}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^4/(e*x^3+d),x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^4/(e*x^3+d),x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4\,(d+ex^3)}\,dx=\int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^4/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e*x^7 + d*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**4/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^4/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)*x^4), x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^4/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{x^4\,(e\,x^3+d)}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(x^4*(d + e*x^3)),x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(x^4*(d + e*x^3)), x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{e\,x^7+d\,x^4}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^4/(e*x^3+d),x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d*x**4 + e*x**7),x)
```

3.91 $\int \frac{x^4\left(a+bx^3+cx^6\right)^p}{d+ex^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^4(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{Int}\left(\frac{x^4(a+bx^3+cx^6)^p}{d+ex^3},x\right)$$

output
```
Defer(Int)(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

Mathematica [N/A]

Not integrable

Time = 6.67 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^4(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{x^4(a+bx^3+cx^6)^p}{d+ex^3}\,dx$$

input
```
Integrate[(x^4*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
Integrate[(x^4*(a + b*x^3 + c*x^6)^p)/(d + e*x^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+bx^3+cx^6)^p}{d+ex^3}\,dx$$

$$\Big\downarrow \text{ 1887}$$

$$\int \frac{x^4(a+bx^3+cx^6)^p}{d+ex^3}dx$$

input
```
Int[(x^4*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^4(c\,x^6+b\,x^3+a)^p}{e\,x^3+d}dx$$

input
```
int(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
int(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^4(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^4}{ex^3+d}\,dx$$

input
```
integrate(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^4/(e*x^3 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(c*x**6+b*x**3+a)**p/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^4(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^4}{ex^3+d}\,dx$$

input
```
integrate(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^4/(e*x^3 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^4(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^4}{ex^3+d}\,dx$$

input
```
integrate(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^4/(e*x^3 + d), x)
```

Mupad [N/A]

Not integrable

Time = 22.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^4(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{x^4\,\left(c\,x^6+b\,x^3+a\right)^p}{e\,x^3+d}\,dx$$

input
```
int((x^4*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x)
```

output
```
int((x^4*(a + b*x^3 + c*x^6)^p)/(d + e*x^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 1304, normalized size of antiderivative = 48.30

$$\int \frac{x^4(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
int(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
((a + b*x**3 + c*x**6)**p*x**2 + 9*int(((a + b*x**3 + c*x**6)**p*x**7)/(3*
a*d*p + a*d + 3*a*e*p*x**3 + a*e*x**3 + 3*b*d*p*x**3 + b*d*x**3 + 3*b*e*p*
x**6 + b*e*x**6 + 3*c*d*p*x**6 + c*d*x**6 + 3*c*e*p*x**9 + c*e*x**9),x)*b*
e*p**2 + 3*int(((a + b*x**3 + c*x**6)**p*x**7)/(3*a*d*p + a*d + 3*a*e*p*x*
*3 + a*e*x**3 + 3*b*d*p*x**3 + b*d*x**3 + 3*b*e*p*x**6 + b*e*x**6 + 3*c*d*
p*x**6 + c*d*x**6 + 3*c*e*p*x**9 + c*e*x**9),x)*b*e*p - 18*int(((a + b*x**
3 + c*x**6)**p*x**7)/(3*a*d*p + a*d + 3*a*e*p*x**3 + a*e*x**3 + 3*b*d*p*x*
*3 + b*d*x**3 + 3*b*e*p*x**6 + b*e*x**6 + 3*c*d*p*x**6 + c*d*x**6 + 3*c*e*
p*x**9 + c*e*x**9),x)*c*d*p**2 - 12*int(((a + b*x**3 + c*x**6)**p*x**7)/(3
*a*d*p + a*d + 3*a*e*p*x**3 + a*e*x**3 + 3*b*d*p*x**3 + b*d*x**3 + 3*b*e*p
*x**6 + b*e*x**6 + 3*c*d*p*x**6 + c*d*x**6 + 3*c*e*p*x**9 + c*e*x**9),x)*c
*d*p - 2*int(((a + b*x**3 + c*x**6)**p*x**7)/(3*a*d*p + a*d + 3*a*e*p*x**3
 + a*e*x**3 + 3*b*d*p*x**3 + b*d*x**3 + 3*b*e*p*x**6 + b*e*x**6 + 3*c*d*p*
x**6 + c*d*x**6 + 3*c*e*p*x**9 + c*e*x**9),x)*c*d + 18*int(((a + b*x**3 +
c*x**6)**p*x**4)/(3*a*d*p + a*d + 3*a*e*p*x**3 + a*e*x**3 + 3*b*d*p*x**3 +
 b*d*x**3 + 3*b*e*p*x**6 + b*e*x**6 + 3*c*d*p*x**6 + c*d*x**6 + 3*c*e*p*x*
*9 + c*e*x**9),x)*a*e*p**2 + 6*int(((a + b*x**3 + c*x**6)**p*x**4)/(3*a*d*
p + a*d + 3*a*e*p*x**3 + a*e*x**3 + 3*b*d*p*x**3 + b*d*x**3 + 3*b*e*p*x**6
 + b*e*x**6 + 3*c*d*p*x**6 + c*d*x**6 + 3*c*e*p*x**9 + c*e*x**9),x)*a*e*p
- 9*int(((a + b*x**3 + c*x**6)**p*x**4)/(3*a*d*p + a*d + 3*a*e*p*x**3 +...
```

3.92 $\int \frac{x(a+bx^3+cx^6)^p}{d+ex^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{Int}\left(\frac{x(a+bx^3+cx^6)^p}{d+ex^3},x\right)$$

output
```
Defer(Int)(x*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

Mathematica [N/A]

Not integrable

Time = 6.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{x(a+bx^3+cx^6)^p}{d+ex^3}\,dx$$

input
```
Integrate[(x*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
Integrate[(x*(a + b*x^3 + c*x^6)^p)/(d + e*x^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^3+cx^6)^p}{d+ex^3}\,dx$$

$$\Big\downarrow \text{ 1887}$$

$$\int \frac{x(a+bx^3+cx^6)^p}{d+ex^3}dx$$

input
```
Int[(x*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x(c\,x^{6}+b\,x^{3}+a)^{p}}{e\,x^{3}+d}dx$$

input
```
int(x*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
int(x*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx^3+cx^6)^p}{d+ex^3}\,dx=\int \frac{(cx^6+bx^3+a)^p x}{ex^3+d}\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x/(e*x^3 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3+cx^6)^p}{d+ex^3}\,dx=\text{Timed out}$$

input
```
integrate(x*(c*x**6+b*x**3+a)**p/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x}{ex^3+d}\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x/(e*x^3 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x}{ex^3+d}\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x/(e*x^3 + d), x)
```

Mupad [N/A]

Not integrable

Time = 21.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{x\,\left(c\,x^6+b\,x^3+a\right)^p}{e\,x^3+d}\,dx$$

input
```
int((x*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x)
```

output
```
int((x*(a + b*x^3 + c*x^6)^p)/(d + e*x^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p\,x}{e\,x^3+d}dx$$

input
```
int(x*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
int(((a + b*x**3 + c*x**6)**p*x)/(d + e*x**3),x)
```

3.93 $\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)}\,dx = \text{Int}\left(\frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)},x\right)$$

output
```
Defer(Int)((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d),x)
```

Mathematica [N/A]

Not integrable

Time = 6.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)}\,dx = \int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)}\,dx$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^2*(d + e*x^3)),x]
```

output
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^2*(d + e*x^3)), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)}\,dx$$

$$\Big\downarrow \text{1887}$$

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)}dx$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(x^2*(d + e*x^3)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x^{2}\,(e\,x^{3}+d)}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d),x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3+cx^6)^p}{x^2\,(d+ex^3)}\,dx=\int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e*x^5 + d*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^2\,(d+ex^3)}\,dx=\text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**2/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 21.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^2\,(e\,x^3+d)}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(x^2*(d + e*x^3)),x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(x^2*(d + e*x^3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3+cx^6)^p}{x^2\,(d+ex^3)}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{e\,x^5+d\,x^2}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d),x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d*x**2 + e*x**5),x)
```

3.94 $\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)}\,dx = \text{Int}\left(\frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)},x\right)$$

output
```
Defer(Int)((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d),x)
```

Mathematica [N/A]

Not integrable

Time = 6.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)}\,dx = \int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)}\,dx$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^5*(d + e*x^3)),x]
```

output
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^5*(d + e*x^3)), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)}\,dx$$

$$\Big\downarrow \; 1887$$

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)}dx$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(x^5*(d + e*x^3)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^5\,(e\,x^3+d)}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d),x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3+cx^6)^p}{x^5\,(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^5}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e*x^8 + d*x^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^5\,(d+ex^3)}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**5/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^5}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)*x^5), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^5}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)*x^5), x)
```

Mupad [N/A]

Not integrable

Time = 22.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^5\,(e\,x^3+d)}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(x^5*(d + e*x^3)),x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(x^5*(d + e*x^3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{e\,x^8+d\,x^5}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d),x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d*x**5 + e*x**8),x)
```

3.95 $\int \frac{x^6(a+bx^3+cx^6)^p}{d+ex^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^6(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{Int}\left(\frac{x^6(a+bx^3+cx^6)^p}{d+ex^3},x\right)$$

output
```
Defer(Int)(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

Mathematica [N/A]

Not integrable

Time = 6.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^6(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{x^6(a+bx^3+cx^6)^p}{d+ex^3}\,dx$$

input
```
Integrate[(x^6*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
Integrate[(x^6*(a + b*x^3 + c*x^6)^p)/(d + e*x^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (a+bx^3+cx^6)^p}{d+ex^3} \, dx$$
$$\Big\downarrow \text{ 1887}$$
$$\int \frac{x^6 (a+bx^3+cx^6)^p}{d+ex^3} dx$$

input
```
Int[(x^6*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^{6}\left(c\,x^{6}+b\,x^{3}+a\right)^{p}}{e\,x^{3}+d}dx$$

input
```
int(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
int(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^6(a+bx^3+cx^6)^p}{d+ex^3}\,dx=\int \frac{(cx^6+bx^3+a)^p x^6}{ex^3+d}\,dx$$

input
```
integrate(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^6/(e*x^3 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3+cx^6)^p}{d+ex^3}\,dx=\text{Timed out}$$

input
```
integrate(x**6*(c*x**6+b*x**3+a)**p/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^6(a+bx^3+cx^6)^p}{d+ex^3}\,dx=\int \frac{(cx^6+bx^3+a)^p x^6}{ex^3+d}\,dx$$

input
```
integrate(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^6/(e*x^3 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^6(a+bx^3+cx^6)^p}{d+ex^3}\,dx=\int \frac{(cx^6+bx^3+a)^p x^6}{ex^3+d}\,dx$$

input
```
integrate(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^6/(e*x^3 + d), x)
```

Mupad [N/A]

Not integrable

Time = 21.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^6(a+bx^3+cx^6)^p}{d+ex^3}\,dx=\int \frac{x^6\,(c\,x^6+b\,x^3+a)^p}{e\,x^3+d}\,dx$$

input
```
int((x^6*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x)
```

output
```
int((x^6*(a + b*x^3 + c*x^6)^p)/(d + e*x^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 6857, normalized size of antiderivative = 253.96

$$\int \frac{x^6(a + bx^3 + cx^6)^p}{d + ex^3} \, dx = \text{Too large to display}$$

input
```
int(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
(3*(a + b*x**3 + c*x**6)**p*b*e*p*x - 6*(a + b*x**3 + c*x**6)**p*c*d*p*x -
 4*(a + b*x**3 + c*x**6)**p*c*d*x + 6*(a + b*x**3 + c*x**6)**p*c*e*p*x**4
+ (a + b*x**3 + c*x**6)**p*c*e*x**4 - 54*int((a + b*x**3 + c*x**6)**p/(18*
a*d*p**2 + 15*a*d*p + 2*a*d + 18*a*e*p**2*x**3 + 15*a*e*p*x**3 + 2*a*e*x**
3 + 18*b*d*p**2*x**3 + 15*b*d*p*x**3 + 2*b*d*x**3 + 18*b*e*p**2*x**6 + 15*
b*e*p*x**6 + 2*b*e*x**6 + 18*c*d*p**2*x**6 + 15*c*d*p*x**6 + 2*c*d*x**6 +
18*c*e*p**2*x**9 + 15*c*e*p*x**9 + 2*c*e*x**9),x)*a*b*d*e*p**3 - 45*int((a
 + b*x**3 + c*x**6)**p/(18*a*d*p**2 + 15*a*d*p + 2*a*d + 18*a*e*p**2*x**3
+ 15*a*e*p*x**3 + 2*a*e*x**3 + 18*b*d*p**2*x**3 + 15*b*d*p*x**3 + 2*b*d*x*
*3 + 18*b*e*p**2*x**6 + 15*b*e*p*x**6 + 2*b*e*x**6 + 18*c*d*p**2*x**6 + 15
*c*d*p*x**6 + 2*c*d*x**6 + 18*c*e*p**2*x**9 + 15*c*e*p*x**9 + 2*c*e*x**9),
x)*a*b*d*e*p**2 - 6*int((a + b*x**3 + c*x**6)**p/(18*a*d*p**2 + 15*a*d*p +
 2*a*d + 18*a*e*p**2*x**3 + 15*a*e*p*x**3 + 2*a*e*x**3 + 18*b*d*p**2*x**3
+ 15*b*d*p*x**3 + 2*b*d*x**3 + 18*b*e*p**2*x**6 + 15*b*e*p*x**6 + 2*b*e*x*
*6 + 18*c*d*p**2*x**6 + 15*c*d*p*x**6 + 2*c*d*x**6 + 18*c*e*p**2*x**9 + 15
*c*e*p*x**9 + 2*c*e*x**9),x)*a*b*d*e*p + 108*int((a + b*x**3 + c*x**6)**p/
(18*a*d*p**2 + 15*a*d*p + 2*a*d + 18*a*e*p**2*x**3 + 15*a*e*p*x**3 + 2*a*e
*x**3 + 18*b*d*p**2*x**3 + 15*b*d*p*x**3 + 2*b*d*x**3 + 18*b*e*p**2*x**6 +
 15*b*e*p*x**6 + 2*b*e*x**6 + 18*c*d*p**2*x**6 + 15*c*d*p*x**6 + 2*c*d*x**
6 + 18*c*e*p**2*x**9 + 15*c*e*p*x**9 + 2*c*e*x**9),x)*a*c*d**2*p**3 + 1...
```

3.96 $\int \frac{x^3\left(a+bx^3+cx^6\right)^p}{d+ex^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^3(a+bx^3+cx^6)^p}{d+ex^3}\,dx=\operatorname{Int}\left(\frac{x^3(a+bx^3+cx^6)^p}{d+ex^3},x\right)$$

output
```
Defer(Int)(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

Mathematica [N/A]

Not integrable

Time = 6.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a+bx^3+cx^6)^p}{d+ex^3}\,dx=\int \frac{x^3(a+bx^3+cx^6)^p}{d+ex^3}\,dx$$

input
```
Integrate[(x^3*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
Integrate[(x^3*(a + b*x^3 + c*x^6)^p)/(d + e*x^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+bx^3+cx^6)^p}{d+ex^3} \, dx$$

$$\Big\downarrow \ 1887$$

$$\int \frac{x^3 (a+bx^3+cx^6)^p}{d+ex^3}dx$$

input
```
Int[(x^3*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^3(c\,x^6+b\,x^3+a)^p}{e\,x^3+d}dx$$

input
```
int(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
int(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a+bx^3+cx^6)^p}{d+ex^3}\,dx=\int \frac{(cx^6+bx^3+a)^px^3}{ex^3+d}\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^3/(e*x^3 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3+cx^6)^p}{d+ex^3}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(c*x**6+b*x**3+a)**p/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^3}{ex^3+d}\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^3/(e*x^3 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p x^3}{ex^3+d}\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^3/(e*x^3 + d), x)
```

Mupad [N/A]

Not integrable

Time = 21.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{x^3\,(c\,x^6+b\,x^3+a)^p}{e\,x^3+d}\,dx$$

input
```
int((x^3*(a + b*x^3 + c*x^6)^p)/(d + e*x^3),x)
```

output
```
int((x^3*(a + b*x^3 + c*x^6)^p)/(d + e*x^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 1299, normalized size of antiderivative = 48.11

$$\int \frac{x^3(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
int(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
((a + b*x**3 + c*x**6)**p*x - 6*int((a + b*x**3 + c*x**6)**p/(6*a*d*p + a*
d + 6*a*e*p*x**3 + a*e*x**3 + 6*b*d*p*x**3 + b*d*x**3 + 6*b*e*p*x**6 + b*e
*x**6 + 6*c*d*p*x**6 + c*d*x**6 + 6*c*e*p*x**9 + c*e*x**9),x)*a*d*p - int(
(a + b*x**3 + c*x**6)**p/(6*a*d*p + a*d + 6*a*e*p*x**3 + a*e*x**3 + 6*b*d*
p*x**3 + b*d*x**3 + 6*b*e*p*x**6 + b*e*x**6 + 6*c*d*p*x**6 + c*d*x**6 + 6*
c*e*p*x**9 + c*e*x**9),x)*a*d + 18*int(((a + b*x**3 + c*x**6)**p*x**6)/(6*
a*d*p + a*d + 6*a*e*p*x**3 + a*e*x**3 + 6*b*d*p*x**3 + b*d*x**3 + 6*b*e*p*
x**6 + b*e*x**6 + 6*c*d*p*x**6 + c*d*x**6 + 6*c*e*p*x**9 + c*e*x**9),x)*b*
e*p**2 + 3*int(((a + b*x**3 + c*x**6)**p*x**6)/(6*a*d*p + a*d + 6*a*e*p*x*
*3 + a*e*x**3 + 6*b*d*p*x**3 + b*d*x**3 + 6*b*e*p*x**6 + b*e*x**6 + 6*c*d*
p*x**6 + c*d*x**6 + 6*c*e*p*x**9 + c*e*x**9),x)*b*e*p - 36*int(((a + b*x**
3 + c*x**6)**p*x**6)/(6*a*d*p + a*d + 6*a*e*p*x**3 + a*e*x**3 + 6*b*d*p*x*
*3 + b*d*x**3 + 6*b*e*p*x**6 + b*e*x**6 + 6*c*d*p*x**6 + c*d*x**6 + 6*c*e*
p*x**9 + c*e*x**9),x)*c*d*p**2 - 12*int(((a + b*x**3 + c*x**6)**p*x**6)/(6
*a*d*p + a*d + 6*a*e*p*x**3 + a*e*x**3 + 6*b*d*p*x**3 + b*d*x**3 + 6*b*e*p
*x**6 + b*e*x**6 + 6*c*d*p*x**6 + c*d*x**6 + 6*c*e*p*x**9 + c*e*x**9),x)*c
*d*p - int(((a + b*x**3 + c*x**6)**p*x**6)/(6*a*d*p + a*d + 6*a*e*p*x**3 +
 a*e*x**3 + 6*b*d*p*x**3 + b*d*x**3 + 6*b*e*p*x**6 + b*e*x**6 + 6*c*d*p*x*
*6 + c*d*x**6 + 6*c*e*p*x**9 + c*e*x**9),x)*c*d + 36*int(((a + b*x**3 + c*
x**6)**p*x**3)/(6*a*d*p + a*d + 6*a*e*p*x**3 + a*e*x**3 + 6*b*d*p*x**3 ...
```

3.97 $\int \frac{(a+bx^3+cx^6)^p}{d+ex^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{Int}\left(\frac{(a+bx^3+cx^6)^p}{d+ex^3},x\right)$$

output
```
Defer(Int)((c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

Mathematica [N/A]

Not integrable

Time = 5.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(a+bx^3+cx^6)^p}{d+ex^3}\,dx$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(d + e*x^3),x]
```

output
```
Integrate[(a + b*x^3 + c*x^6)^p/(d + e*x^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{d+ex^3}\,dx$$
$$\Big\downarrow \; 1769$$
$$\int \frac{(a+bx^3+cx^6)^p}{d+ex^3}dx$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(d + e*x^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1769
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Unintegrable[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,
 x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{e\,x^3+d}dx$$

input
```
int((c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
int((c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p}{ex^3+d}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e*x^3 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p}{ex^3+d}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/(e*x^3 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(cx^6+bx^3+a)^p}{ex^3+d}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/(e*x^3 + d), x)
```

Mupad [N/A]

Not integrable

Time = 20.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3+cx^6)^p}{d+ex^3}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{e\,x^3+d}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(d + e*x^3),x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(d + e*x^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3+cx^6)^p}{d+ex^3}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{e\,x^3+d}dx$$

input
```
int((c*x^6+b*x^3+a)^p/(e*x^3+d),x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d + e*x**3),x)
```

3.98 $\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)} dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)} dx = \text{Int}\left(\frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)}, x\right)$$

output
```
Defer(Int)((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d),x)
```

Mathematica [N/A]

Not integrable

Time = 6.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)} dx = \int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)} dx$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^3*(d + e*x^3)),x]
```

output
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^3*(d + e*x^3)), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)}\,dx$$
$$\Big\downarrow \ 1887$$
$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)}dx$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(x^3*(d + e*x^3)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^3\,(e\,x^3+d)}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d),x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3+cx^6)^p}{x^3\,(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e*x^6 + d*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^3\,(d+ex^3)}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**3/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 21.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^3\,(e\,x^3+d)}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(x^3*(d + e*x^3)),x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(x^3*(d + e*x^3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{e\,x^6+d\,x^3}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d),x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d*x**3 + e*x**6),x)
```

3.99 $\int \frac{\left(a+bx^3+cx^6\right)^p}{x^6\left(d+ex^3\right)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)}\,dx=\text{Int}\left(\frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)},x\right)$$

output
```
Defer(Int)((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d),x)
```

Mathematica [N/A]

Not integrable

Time = 6.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)}\,dx=\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)}\,dx$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^6*(d + e*x^3)),x]
```

output
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^6*(d + e*x^3)), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{1887\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)} \, dx$$

$$\Big\downarrow \textbf{1887}$$

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)}dx$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(x^6*(d + e*x^3)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^6\,(e\,x^3+d)}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d),x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3+cx^6)^p}{x^6\,(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^6}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d),x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e*x^9 + d*x^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^6\,(d+ex^3)}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**6/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^6}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d),x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)*x^6), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)x^6}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d),x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)*x^6), x)
```

Mupad [N/A]

Not integrable

Time = 21.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^6\,(e\,x^3+d)}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(x^6*(d + e*x^3)),x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(x^6*(d + e*x^3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{e\,x^9+d\,x^6}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d),x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d*x**6 + e*x**9),x)
```

3.100 $\int \frac{x^8(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 538

$$\int \frac{x^8(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx$$
$$= \frac{(a+bx^3+cx^6)^{1+p}}{3ce(1+2p)(d+ex^3)} + \frac{(e(bd-ae)-2cd^2(1+p))(a+bx^3+cx^6)^{1+p}}{3ce(cd^2-bde+ae^2)(1+2p)(d+ex^3)}$$
$$-\frac{2^{-1+2p}d(2cd^2(1+p)+e(2ae-bd(2+p)))\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+c}{3e^3(cd^2-bde+ae^2)p}$$
$$+\frac{2^{1+p}(e(bd-ae)-2cd^2(1+p))\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^3+cx^6)^{1+p}\,\text{Hypergeometric2F1}\left(-p,1\right.}{3\sqrt{b^2-4ac}\,e^2(cd^2-bde+ae^2)(1+p)}$$

output
```
1/3*(c*x^6+b*x^3+a)^(p+1)/c/e/(1+2*p)/(e*x^3+d)+1/3*(e*(-a*e+b*d)-2*c*d^2*
(p+1))*(c*x^6+b*x^3+a)^(p+1)/c/e/(a*e^2-b*d*e+c*d^2)/(1+2*p)/(e*x^3+d)-1/3
*2^(-1+2*p)*d*(2*c*d^2*(p+1)+e*(2*a*e-b*d*(2+p)))*(c*x^6+b*x^3+a)^p*Appell
F1(-2*p,-p,-p,1-2*p,(2*d-(b+(-4*a*c+b^2)^(1/2))*e/c)/(2*e*x^3+2*d),1/2*(2*
c*d-(b-(-4*a*c+b^2)^(1/2))*e)/c/(e*x^3+d))/e^3/(a*e^2-b*d*e+c*d^2)/p/((e*(
b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/((e*(b+(-4*a*c+b^2)^(1/2)+2*
c*x^3)/c/(e*x^3+d))^p)+1/3*2^(p+1)*(e*(-a*e+b*d)-2*c*d^2*(p+1))*(-(b-(-4*a
*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))^(-1-p)*(c*x^6+b*x^3+a)^(p+1)*hy
pergeom([-p, p+1],[2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)/(p+1)
```

Mathematica [A] (warning: unable to verify)

Time = 2.44 (sec) , antiderivative size = 520, normalized size of antiderivative = 0.97

$$\int \frac{x^8(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx$$

$$= \frac{2^{-1+p}(a+bx^3+cx^6)^p\left(\frac{2^{1+p}d^2\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{2cd-\left(b+\sqrt{b^2-4\right.}\right.}{2c\left(d+ex^3\right)}\right.}{(-1+2p)(d+ex^3)}\right.}{}$$

input
```
Integrate[(x^8*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
(2^(-1 + p)*(a + b*x^3 + c*x^6)^p*((2^(1 + p)*d^2*AppellF1[1 - 2*p, -p, -p
, 2 - 2*p, (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*c*d -
 b*e + Sqrt[b^2 - 4*a*c]*e)/(2*c*d + 2*c*e*x^3)])/((-1 + 2*p)*((e*(b - Sqr
t[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^3))/(c*(d + e*x^3)))^p*(d + e*x^3)) - (2^(1 + p)*d*AppellF1[-2*p, -
p, -p, 1 - 2*p, (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*
c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*c*d + 2*c*e*x^3)])/(p*((e*(b - Sqrt[b^
2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^3))/(c*(d + e*x^3)))^p) + (e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*Hypergeo
metric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)/(2*Sqrt[b^2
 - 4*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*
c])^p)))/(3*e^3)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 524, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1802, 1267, 27, 1237, 27, 1269, 1096, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\left(a+bx^3+cx^6\right)^p}{\left(d+ex^3\right)^2}dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{3}\int \frac{x^6\left(cx^6+bx^3+a\right)^p}{\left(ex^3+d\right)^2}dx^3$$

$$\Big\downarrow \text{1267}$$

$$\frac{1}{3}\left(\frac{\int \frac{e\left(-\left((bep+2cd(p+1))x^3\right)+ae-bd(p+1)\right)\left(cx^6+bx^3+a\right)^p}{\left(ex^3+d\right)^2}dx^3}{ce^2(2p+1)}+\frac{\left(a+bx^3+cx^6\right)^{p+1}}{ce(2p+1)\left(d+ex^3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{\left(-\left((bep+2cd(p+1))x^3\right)+ae-bd(p+1)\right)\left(cx^6+bx^3+a\right)^p}{\left(ex^3+d\right)^2}dx^3}{ce(2p+1)}+\frac{\left(a+bx^3+cx^6\right)^{p+1}}{ce(2p+1)\left(d+ex^3\right)}\right)$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{3}\left(\frac{\frac{\left(a+bx^3+cx^6\right)^{p+1}\left(e(bd-ae)-2cd^2(p+1)\right)}{\left(d+ex^3\right)\left(ae^2-bde+cd^2\right)}-\frac{\int \frac{c(2p+1)\left(\left(e(bd-ae)-2cd^2(p+1)\right)x^3+d(ae-bd(p+1))\right)\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3}{ae^2-bde+cd^2}}{ce(2p+1)}+\frac{\left(a+bx^3+cx^6 \ldots\right.}{ce(2p+1)\left(d+\ldots\right.}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\frac{\left(a+bx^3+cx^6\right)^{p+1}\left(e(bd-ae)-2cd^2(p+1)\right)}{\left(d+ex^3\right)\left(ae^2-bde+cd^2\right)}-\frac{c(2p+1)\int \frac{\left(\left(e(bd-ae)-2cd^2(p+1)\right)x^3+d(ae-bd(p+1))\right)\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3}{ae^2-bde+cd^2}}{ce(2p+1)}+\frac{\left(a+bx^3+c \ldots\right.}{ce(2p+1)\left(d \ldots\right.}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{3}\left(\frac{\frac{(a+bx^3+cx^6)^{p+1}(e(bd-ae)-2cd^2(p+1))}{(d+ex^3)(ae^2-bde+cd^2)}-\frac{c(2p+1)\left(\frac{(e(bd-ae)-2cd^2(p+1))\int(cx^6+bx^3+a)^p dx^3}{e}+\frac{d(e(2ae-bd(p+2))+2cd^2(p+1))\int\frac{(cx^6+bx\cdots}{ex^3\cdots}}{e}\cdots\right.}{ae^2-bde+cd^2}}{ce(2p+1)}\cdots\right.$$

$\downarrow$ 1096

$$\frac{1}{3}\left(\frac{\frac{(a+bx^3+cx^6)^{p+1}(e(bd-ae)-2cd^2(p+1))}{(d+ex^3)(ae^2-bde+cd^2)}-\frac{c(2p+1)\left(\frac{d(e(2ae-bd(p+2))+2cd^2(p+1))\int\frac{(cx^6+bx^3+a)^p}{ex^3+d}dx^3}{e}-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p}\cdots}{\cdots}\right.}{ae^2-b\cdots}}{ce(2p+1)}\cdots\right.$$

$\downarrow$ 1178

$$\frac{1}{3}\left(\frac{\frac{(a+bx^3+cx^6)^{p+1}(e(bd-ae)-2cd^2(p+1))}{(d+ex^3)(ae^2-bde+cd^2)}-\frac{c(2p+1)\left(-\frac{d4^p\left(\frac{1}{d+ex^3}\right)^{2p}(a+bx^3+cx^6)^p(e(2ae-bd(p+2))+2cd^2(p+1))\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx^3\right)}{c(d+ex^3)}\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 150

$$\frac{1}{3}\left(\frac{\frac{(a+bx^3+cx^6)^{p+1}(e(bd-ae)-2cd^2(p+1))}{(d+ex^3)(ae^2-bde+cd^2)}-\frac{c(2p+1)\left(\frac{d2^{2p-1}(a+bx^3+cx^6)^p(e(2ae-bd(p+2))+2cd^2(p+1))\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e(\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

input
```
Int[(x^8*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
((a + b*x^3 + c*x^6)^(1 + p)/(c*e*(1 + 2*p)*(d + e*x^3)) + (((e*(b*d - a*e
) - 2*c*d^2*(1 + p))*(a + b*x^3 + c*x^6)^(1 + p))/((c*d^2 - b*d*e + a*e^2)
*(d + e*x^3)) - (c*(1 + 2*p)*((2^(-1 + 2*p)*d*(2*c*d^2*(1 + p) + e*(2*a*e
- b*d*(2 + p)))*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*d - ((b + Sqrt[b^2 -
 4*a*c])*e)/c)/(2*(d + e*x^3))])/(e^2*p*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x
^3))/(c*(d + e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x
^3)))^p) - (2^(1 + p)*(e*(b*d - a*e) - 2*c*d^2*(1 + p))*(-((b - Sqrt[b^2 -
 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p
)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2
*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*e*(1 + p))))/(c*d^2 - b*d*e + a*e
^2))/(c*e*(1 + 2*p)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^8 (c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}dx$$

input
```
int(x^8*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
int(x^8*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

Fricas [F]

$$\int \frac{x^8(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x^8}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^8/(e^2*x^6 + 2*d*e*x^3 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(c*x**6+b*x**3+a)**p/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^8(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x^8}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^8/(e*x^3 + d)^2, x)
```

Giac [F]

$$\int \frac{x^8(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x^8}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^8*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^8/(e*x^3 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{x^8\,(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}\,dx$$

input
```
int((x^8*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x)
```

output
```
int((x^8*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^8(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \text{too large to display}$$

input
```
int(x^8*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
( - 4*(a + b*x**3 + c*x**6)**p*a*c*d*e*p - (a + b*x**3 + c*x**6)**p*b**2*d
*e*p**2 - (a + b*x**3 + c*x**6)**p*b**2*d*e*p + (a + b*x**3 + c*x**6)**p*b
**2*e**2*p**2*x**3 - (a + b*x**3 + c*x**6)**p*b**2*e**2*p*x**3 + 2*(a + b*
x**3 + c*x**6)**p*b*c*d**2*p**2 + 4*(a + b*x**3 + c*x**6)**p*b*c*d**2*p +
2*(a + b*x**3 + c*x**6)**p*b*c*d**2 + 2*(a + b*x**3 + c*x**6)**p*b*c*d*e*x
**3 + 2*(a + b*x**3 + c*x**6)**p*b*c*e**2*p**2*x**6 - 2*(a + b*x**3 + c*x*
*6)**p*b*c*e**2*p*x**6 - 4*(a + b*x**3 + c*x**6)**p*c**2*d**2*p**2*x**3 -
4*(a + b*x**3 + c*x**6)**p*c**2*d**2*p*x**3 + 4*(a + b*x**3 + c*x**6)**p*c
**2*d*e*p**2*x**6 + 24*int(((a + b*x**3 + c*x**6)**p*x**8)/(2*a*b*d**2*e*p
**2 - a*b*d**2*e*p - a*b*d**2*e + 4*a*b*d*e**2*p**2*x**3 - 2*a*b*d*e**2*p*
x**3 - 2*a*b*d*e**2*x**3 + 2*a*b*e**3*p**2*x**6 - a*b*e**3*p*x**6 - a*b*e*
*3*x**6 + 4*a*c*d**3*p**2 + 2*a*c*d**3*p + 8*a*c*d**2*e*p**2*x**3 + 4*a*c*
d**2*e*p*x**3 + 4*a*c*d*e**2*p**2*x**6 + 2*a*c*d*e**2*p*x**6 + 2*b**2*d**2
*e*p**2*x**3 - b**2*d**2*e*p*x**3 - b**2*d**2*e*x**3 + 4*b**2*d*e**2*p**2*
x**6 - 2*b**2*d*e**2*p*x**6 - 2*b**2*d*e**2*x**6 + 2*b**2*e**3*p**2*x**9 -
 b**2*e**3*p*x**9 - b**2*e**3*x**9 + 4*b*c*d**3*p**2*x**3 + 2*b*c*d**3*p*x
**3 + 10*b*c*d**2*e*p**2*x**6 + 3*b*c*d**2*e*p*x**6 - b*c*d**2*e*x**6 + 8*
b*c*d*e**2*p**2*x**9 - 2*b*c*d*e**2*x**9 + 2*b*c*e**3*p**2*x**12 - b*c*e**
3*p*x**12 - b*c*e**3*x**12 + 4*c**2*d**3*p**2*x**6 + 2*c**2*d**3*p*x**6 +
8*c**2*d**2*e*p**2*x**9 + 4*c**2*d**2*e*p*x**9 + 4*c**2*d*e**2*p**2*x**...
```

3.101 $\int \frac{x^5(a+bx^3+cx^6)^p}{(d+ex^3)^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 450

$$\int \frac{x^5(a+bx^3+cx^6)^p}{(d+ex^3)^2} \, dx = \frac{d(a+bx^3+cx^6)^{1+p}}{3(cd^2-bde+ae^2)(d+ex^3)}$$
$$+ \frac{2^{-1+2p}(cd^2(1+2p)+e(ae-bd(1+p)))\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6}{3e^2(cd^2-bde+ae^2)p}$$
$$+ \frac{2^{1+p}cd(1+2p)\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^3+cx^6)^{1+p}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b}}{2}\right.}{3\sqrt{b^2-4ac}e(cd^2-bde+ae^2)(1+p)}$$

output
```
1/3*d*(c*x^6+b*x^3+a)^(p+1)/(a*e^2-b*d*e+c*d^2)/(e*x^3+d)+1/3*2^(-1+2*p)*(
c*d^2*(1+2*p)+e*(a*e-b*d*(p+1)))*(c*x^6+b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2
*p,(2*d-(b+(-4*a*c+b^2)^(1/2))*e/c)/(2*e*x^3+2*d),1/2*(2*c*d-(b-(-4*a*c+b^
2)^(1/2))*e)/c/(e*x^3+d))/e^2/(a*e^2-b*d*e+c*d^2)/p/((e*(b-(-4*a*c+b^2)^(1
/2)+2*c*x^3)/c/(e*x^3+d))^p)/((e*(b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d
))^p)+1/3*2^(p+1)*c*d*(1+2*p)*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2
)^(1/2))^(-1-p)*(c*x^6+b*x^3+a)^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+(-4
*a*c+b^2)^(1/2)+2*c*x^3)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/e/(a*e^2-b
*d*e+c*d^2)/(p+1)
```

Mathematica [A] (warning: unable to verify)

Time = 1.56 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.73

$$\int \frac{x^5(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx$$
$$= \frac{2^{-1+2p}\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p\left(-2dp\,\text{AppellF1}\left(1-2p,-p,-p\right.\right.}{}$$

input
```
Integrate[(x^5*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*(-2*d*p*AppellF1[1 - 2*p, -p, -p, 2 -
2*p, (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*c*d - b*e +
 Sqrt[b^2 - 4*a*c]*e)/(2*c*d + 2*c*e*x^3)] + (-1 + 2*p)*(d + e*x^3)*Appell
F1[-2*p, -p, -p, 1 - 2*p, (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*
x^3)), (2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*c*d + 2*c*e*x^3)]))/(3*e^2*p
*(-1 + 2*p)*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p*((e*
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p*(d + e*x^3))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1802, 1237, 25, 1269, 1096, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+bx^3+cx^6\right)^p}{\left(d+ex^3\right)^2}dx$$
$$\Big\downarrow \text{1802}$$
$$\frac{1}{3}\int \frac{x^3\left(cx^6+bx^3+a\right)^p}{\left(ex^3+d\right)^2}dx^3$$
$$\Big\downarrow \text{1237}$$

$$\frac{1}{3}\left(\frac{d\left(a+bx^3+cx^6\right)^{p+1}}{\left(d+ex^3\right)\left(ae^2-bde+cd^2\right)}-\frac{\int -\frac{\left(-cd(2p+1)x^3+ae-bd(p+1)\right)\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \textbf{25}$$

$$\frac{1}{3}\left(\frac{\int \frac{\left(-cd(2p+1)x^3+ae-bd(p+1)\right)\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3}{ae^2-bde+cd^2}+\frac{d\left(a+bx^3+cx^6\right)^{p+1}}{\left(d+ex^3\right)\left(ae^2-bde+cd^2\right)}\right)$$

$$\Big\downarrow \textbf{1269}$$

$$\frac{1}{3}\left(\frac{\frac{\left(e(ae-bd(p+1))+cd^2(2p+1)\right)\int \frac{\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3}{e}-\frac{cd(2p+1)\int \left(cx^6+bx^3+a\right)^p dx^3}{e}}{ae^2-bde+cd^2}+\frac{d\left(a+bx^3+cx^6\right)^{p+1}}{\left(d+ex^3\right)\left(ae^2-bde+cd^2\right)}\right)$$

$$\Big\downarrow \textbf{1096}$$

$$\frac{1}{3}\left(\frac{\frac{\left(e(ae-bd(p+1))+cd^2(2p+1)\right)\int \frac{\left(cx^6+bx^3+a\right)^p}{ex^3+d}dx^3}{e}+\frac{cd2^{p+1}(2p+1)\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\text{Hypergeometric2F1}\left(\right.}{e(p+1)\sqrt{b^2-4ac}}}{ae^2-bde+cd^2}\right.$$

$$\Big\downarrow \textbf{1178}$$

$$\frac{1}{3}\left(\frac{\frac{cd2^{p+1}(2p+1)\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^3+cx^6\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{e(p+1)\sqrt{b^2-4ac}}-\frac{4^p\left(\frac{1}{d+ex^3}\right)^{2p}(a+}{}}{}\right.$$

$$\Big\downarrow \textbf{150}$$

$$\frac{1}{3}\left(\frac{\frac{2^{2p-1}\left(a+bx^3+cx^6\right)^p\left(e(ae-bd(p+1))+cd^2(2p+1)\right)\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx^3\right)}{c\left(d+ex^3\right)}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{2}{}\right.}{e^2p}}{}\right.$$

input
```
Int[(x^5*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
((d*(a + b*x^3 + c*x^6)^(1 + p))/((c*d^2 - b*d*e + a*e^2)*(d + e*x^3)) + (
(2^(-1 + 2*p)*(c*d^2*(1 + 2*p) + e*(a*e - b*d*(1 + p)))*(a + b*x^3 + c*x^6
)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2
*c*(d + e*x^3)), (2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c)/(2*(d + e*x^3))])/(
e^2*p*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p*((e*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p) + (2^(1 + p)*c*d*(1 + 2*p
)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b
*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 -
4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*e*(1 + p)))/(
c*d^2 - b*d*e + a*e^2))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^5(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}dx$$

input
```
int(x^5*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
int(x^5*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

Fricas [F]

$$\int \frac{x^5(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x^5}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^5/(e^2*x^6 + 2*d*e*x^3 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(c*x**6+b*x**3+a)**p/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x^5}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^5/(e*x^3 + d)^2, x)
```

Giac [F]

$$\int \frac{x^5(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\int \frac{(cx^6+bx^3+a)^p x^5}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^5*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^5/(e*x^3 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\int \frac{x^5\,(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}\,dx$$

input
```
int((x^5*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x)
```

output
```
int((x^5*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^5(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\text{too large to display}$$

input
```
int(x^5*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
(2*(a + b*x**3 + c*x**6)**p*a*e*p - (a + b*x**3 + c*x**6)**p*b*d*p - (a +
b*x**3 + c*x**6)**p*b*d + (a + b*x**3 + c*x**6)**p*b*e*p*x**3 - (a + b*x**
3 + c*x**6)**p*b*e*x**3 + 2*(a + b*x**3 + c*x**6)**p*c*d*p*x**3 - 12*int((
(a + b*x**3 + c*x**6)**p*x**8)/(a*b*d**2*e*p - a*b*d**2*e + 2*a*b*d*e**2*p
*x**3 - 2*a*b*d*e**2*x**3 + a*b*e**3*p*x**6 - a*b*e**3*x**6 + 2*a*c*d**3*p
 + 4*a*c*d**2*e*p*x**3 + 2*a*c*d*e**2*p*x**6 + b**2*d**2*e*p*x**3 - b**2*d
**2*e*x**3 + 2*b**2*d*e**2*p*x**6 - 2*b**2*d*e**2*x**6 + b**2*e**3*p*x**9
- b**2*e**3*x**9 + 2*b*c*d**3*p*x**3 + 5*b*c*d**2*e*p*x**6 - b*c*d**2*e*x*
*6 + 4*b*c*d*e**2*p*x**9 - 2*b*c*d*e**2*x**9 + b*c*e**3*p*x**12 - b*c*e**3
*x**12 + 2*c**2*d**3*p*x**6 + 4*c**2*d**2*e*p*x**9 + 2*c**2*d*e**2*p*x**12
),x)*a*b*c*d*e**3*p**3 + 18*int(((a + b*x**3 + c*x**6)**p*x**8)/(a*b*d**2*
e*p - a*b*d**2*e + 2*a*b*d*e**2*p*x**3 - 2*a*b*d*e**2*x**3 + a*b*e**3*p*x*
*6 - a*b*e**3*x**6 + 2*a*c*d**3*p + 4*a*c*d**2*e*p*x**3 + 2*a*c*d*e**2*p*x
**6 + b**2*d**2*e*p*x**3 - b**2*d**2*e*x**3 + 2*b**2*d*e**2*p*x**6 - 2*b**
2*d*e**2*x**6 + b**2*e**3*p*x**9 - b**2*e**3*x**9 + 2*b*c*d**3*p*x**3 + 5*
b*c*d**2*e*p*x**6 - b*c*d**2*e*x**6 + 4*b*c*d*e**2*p*x**9 - 2*b*c*d*e**2*x
**9 + b*c*e**3*p*x**12 - b*c*e**3*x**12 + 2*c**2*d**3*p*x**6 + 4*c**2*d**2
*e*p*x**9 + 2*c**2*d*e**2*p*x**12),x)*a*b*c*d*e**3*p**2 - 6*int(((a + b*x*
*3 + c*x**6)**p*x**8)/(a*b*d**2*e*p - a*b*d**2*e + 2*a*b*d*e**2*p*x**3 - 2
*a*b*d*e**2*x**3 + a*b*e**3*p*x**6 - a*b*e**3*x**6 + 2*a*c*d**3*p + 4*a...
```

3.102 $\int \frac{x^2(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 214

$$\int \frac{x^2(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = -\frac{4^p\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(1-2p,-p,-p,2(1-p)\right)}{3e(1-2p)\left(d+ex^3\right)}$$

output
```
-1/3*4^p*(c*x^6+b*x^3+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,(2*d-(b+(-4*a*c+b^2)
^(1/2))*e/c)/(2*e*x^3+2*d),1/2*(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/c/(e*x^3+d
))/e/(1-2*p)/((e*(b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/((e*(b+(-4
*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/(e*x^3+d)
```

Mathematica [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99

$$\int \frac{x^2(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx$$
$$= \frac{4^p\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p\operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{2cd-}{}\right.}{3e(-1+2p)\,(d+ex^3)}$$

input
```
Integrate[(x^2*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
(4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*c*d - b*e + Sqrt[b^2 - 4*a*c
]*e)/(2*c*d + 2*c*e*x^3)])/(3*e*(-1 + 2*p)*((e*(b - Sqrt[b^2 - 4*a*c] + 2*
c*x^3))/(c*(d + e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d +
e*x^3)))^p*(d + e*x^3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1798, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+bx^3+cx^6\right)^p}{(d+ex^3)^2}\,dx$$

$$\Big\downarrow \text{ 1798}$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2}dx^3$$

$$\Big\downarrow \text{ 1178}$$

$$-\frac{4^p\left(\frac{1}{d+ex^3}\right)^{2p}\left(a+bx^3+cx^6\right)^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\int\left(\frac{1}{ex^3+d}\right)^{-2p}\left(1-\frac{2d-\frac{(b-\sqrt{b}}{}}{2(ex^3}\right.}{3e}$$

$\downarrow$ 150

$$-\frac{4^p\left(a+bx^3+cx^6\right)^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{2cd-(}{2}\right.}{3e(1-2p)\left(d+ex^3\right)}$$

input
```
Int[(x^2*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
-1/3*(4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (2*c*d
- (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*d - ((b + Sqrt[b^2 - 4*
a*c])*e)/c)/(2*(d + e*x^3))])/(e*(1 - 2*p)*((e*(b - Sqrt[b^2 - 4*a*c] + 2*
c*x^3))/(c*(d + e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d +
e*x^3)))^p*(d + e*x^3))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [F]

$$\int \frac{x^2(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}dx$$

input
```
int(x^2*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
int(x^2*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

Fricas [F]

$$\int \frac{x^2(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\int \frac{(cx^6+bx^3+a)^px^2}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^2/(e^2*x^6 + 2*d*e*x^3 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(c*x**6+b*x**3+a)**p/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x^2}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^2/(e*x^3 + d)^2, x)
```

Giac [F]

$$\int \frac{x^2(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x^2}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^2*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^2/(e*x^3 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{x^2\,(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}\,dx$$

input
```
int((x^2*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x)
```

output
```
int((x^2*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^2(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \text{too large to display}$$

input
```
int(x^2*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
((a + b*x**3 + c*x**6)**p*b - 3*int(((a + b*x**3 + c*x**6)**p*x**8)/(a*b*d
**2*e*p - a*b*d**2*e + 2*a*b*d*e**2*p*x**3 - 2*a*b*d*e**2*x**3 + a*b*e**3*
p*x**6 - a*b*e**3*x**6 + 2*a*c*d**3*p + 4*a*c*d**2*e*p*x**3 + 2*a*c*d*e**2
*p*x**6 + b**2*d**2*e*p*x**3 - b**2*d**2*e*x**3 + 2*b**2*d*e**2*p*x**6 - 2
*b**2*d*e**2*x**6 + b**2*e**3*p*x**9 - b**2*e**3*x**9 + 2*b*c*d**3*p*x**3
+ 5*b*c*d**2*e*p*x**6 - b*c*d**2*e*x**6 + 4*b*c*d*e**2*p*x**9 - 2*b*c*d*e*
*2*x**9 + b*c*e**3*p*x**12 - b*c*e**3*x**12 + 2*c**2*d**3*p*x**6 + 4*c**2*
d**2*e*p*x**9 + 2*c**2*d*e**2*p*x**12),x)*b**2*c*d*e**2*p**2 + 3*int(((a +
 b*x**3 + c*x**6)**p*x**8)/(a*b*d**2*e*p - a*b*d**2*e + 2*a*b*d*e**2*p*x**
3 - 2*a*b*d*e**2*x**3 + a*b*e**3*p*x**6 - a*b*e**3*x**6 + 2*a*c*d**3*p + 4
*a*c*d**2*e*p*x**3 + 2*a*c*d*e**2*p*x**6 + b**2*d**2*e*p*x**3 - b**2*d**2*
e*x**3 + 2*b**2*d*e**2*p*x**6 - 2*b**2*d*e**2*x**6 + b**2*e**3*p*x**9 - b*
*2*e**3*x**9 + 2*b*c*d**3*p*x**3 + 5*b*c*d**2*e*p*x**6 - b*c*d**2*e*x**6 +
 4*b*c*d*e**2*p*x**9 - 2*b*c*d*e**2*x**9 + b*c*e**3*p*x**12 - b*c*e**3*x**
12 + 2*c**2*d**3*p*x**6 + 4*c**2*d**2*e*p*x**9 + 2*c**2*d*e**2*p*x**12),x)
*b**2*c*d*e**2*p - 3*int(((a + b*x**3 + c*x**6)**p*x**8)/(a*b*d**2*e*p - a
*b*d**2*e + 2*a*b*d*e**2*p*x**3 - 2*a*b*d*e**2*x**3 + a*b*e**3*p*x**6 - a*
b*e**3*x**6 + 2*a*c*d**3*p + 4*a*c*d**2*e*p*x**3 + 2*a*c*d*e**2*p*x**6 + b
**2*d**2*e*p*x**3 - b**2*d**2*e*x**3 + 2*b**2*d*e**2*p*x**6 - 2*b**2*d*e**
2*x**6 + b**2*e**3*p*x**9 - b**2*e**3*x**9 + 2*b*c*d**3*p*x**3 + 5*b*c*...
```

3.103 $\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 576

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)^2}\,dx$$

$$= \frac{4^p\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(1-2p,-p,-p,2(1-p),\frac{2c}{}\right.}{3d(1-2p)(d+ex^3)}$$

$$+\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b}}{2c}\right.}{3d^2p}$$

$$-\frac{2^{-1+2p}\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p\right.}{3d^2p}$$

output
```
1/3*4^p*(c*x^6+b*x^3+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,(2*d-(b+(-4*a*c+b^2)^
(1/2))*e/c)/(2*e*x^3+2*d),1/2*(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/c/(e*x^3+d)
)/d/(1-2*p)/((e*(b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/((e*(b+(-4*
a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/(e*x^3+d)+1/3*2^(-1+2*p)*(c*x^6+b*
x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*a*c+b^2)^(1/2))/c/x^3,-1/2*
(b+(-4*a*c+b^2)^(1/2))/c/x^3)/d^2/p/(((b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3
)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)-1/3*2^(-1+2*p)*(c*x^6+b*x^
3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(2*d-(b+(-4*a*c+b^2)^(1/2))*e/c)/(2*e*x^3
+2*d),1/2*(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/c/(e*x^3+d))/d^2/p/((e*(b-(-4*a
*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/((e*(b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/
c/(e*x^3+d))^p)
```

Mathematica [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)^2}\,dx=\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)^2}\,dx$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(x*(d + e*x^3)^2),x]
```

output
```
Integrate[(a + b*x^3 + c*x^6)^p/(x*(d + e*x^3)^2), x]
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 572, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1802, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)^2}\,dx$$

$$\Big\downarrow \; 1802$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^p}{x^3(ex^3+d)^2}dx^3$$

$$\Big\downarrow \text{ 1289}$$

$$\frac{1}{3}\int \left(-\frac{e(cx^6+bx^3+a)^p}{d^2(ex^3+d)}-\frac{e(cx^6+bx^3+a)^p}{d(ex^3+d)^2}+\frac{(cx^6+bx^3+a)^p}{d^2x^3}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{2^{2p-1}(a+bx^3+cx^6)^p\left(\frac{e\left(-\sqrt{b^2-4ac}+b+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(\sqrt{b^2-4ac}+b+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\right.}{d^2p}\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(x*(d + e*x^3)^2),x]
```

output
```
((4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p), (2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*d - ((b + Sqrt[b^2 - 4*a*
c])*e)/c)/(2*(d + e*x^3))])/(d*(1 - 2*p)*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^3))/(c*(d + e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*
x^3)))^p*(d + e*x^3)) + (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p,
 -p, -p, 1 - 2*p, -1/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2
 - 4*a*c])/(c*x^3)])/(d^2*p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*
((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p) - (2^(-1 + 2*p)*(a + b*x^3
+ c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)/(2*c*(d + e*x^3)), (2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c)/(2*(d + e*x^
3))])/(d^2*p*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p*((e
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p))/3
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x\,(e\,x^{3}+d)^{2}}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x/(e*x^3+d)^2,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x/(e*x^3+d)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x\,(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e^2*x^7 + 2*d*e*x^4 + d^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)^2*x), x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)^2}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{x\,(e\,x^3+d)^2}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(x*(d + e*x^3)^2),x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(x*(d + e*x^3)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x(d+ex^3)^2}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{e^2x^7+2de\,x^4+d^2x}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x/(e*x^3+d)^2,x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d**2*x + 2*d*e*x**4 + e**2*x**7),x)
```

3.104 $\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 738

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)^2} dx =$$

$$-\frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{b-\sqrt{b}}{2c}\right.}{3d^2(1-2p)x^3}$$

$$-\frac{4^pe\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(1-2p,-p,-p,2(1-p\right.}{3d^2(1-2p)(d+ex^3)}$$

$$-\frac{4^pe\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4}}{2cx^3}\right.}{3d^3p}$$

$$+\frac{4^pe\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{2c}{}\right.}{3d^3p}$$

output
```
-1/3*4^p*(c*x^6+b*x^3+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,-1/2*(b-(-4*a*c+b^2)
^(1/2))/c/x^3,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^3)/d^2/(1-2*p)/x^3/(((b-(-4*
a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)^
p)-1/3*4^p*e*(c*x^6+b*x^3+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,(2*d-(b+(-4*a*c+
b^2)^(1/2))*e/c)/(2*e*x^3+2*d),1/2*(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/c/(e*x
^3+d))/d^2/(1-2*p)/((e*(b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/((e*
(b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/(e*x^3+d)-1/3*4^p*e*(c*x^6+
b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*a*c+b^2)^(1/2))/c/x^3,-1/
2*(b+(-4*a*c+b^2)^(1/2))/c/x^3)/d^3/p/(((b-(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x
^3)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/x^3)^p)+1/3*4^p*e*(c*x^6+b*x^3+a
)^p*AppellF1(-2*p,-p,-p,1-2*p,(2*d-(b+(-4*a*c+b^2)^(1/2))*e/c)/(2*e*x^3+2*
d),1/2*(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/c/(e*x^3+d))/d^3/p/((e*(b-(-4*a*c+
b^2)^(1/2)+2*c*x^3)/c/(e*x^3+d))^p)/((e*(b+(-4*a*c+b^2)^(1/2)+2*c*x^3)/c/(
e*x^3+d))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 3.01 (sec) , antiderivative size = 626, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)^2}\,dx$$

$$= \frac{\left(1+\frac{b-\sqrt{b^2-4ac}}{2cx^3}\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^3\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{c}\right)^{p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\left(dp\,\mathrm{Appe}\right.}{}$$

$$+\frac{4^pe\left(\frac{e\left(b-\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx^3\right)}{c(d+ex^3)}\right)^{-p}(a+bx^3+cx^6)^p\left(dp\,\mathrm{AppellF1}\left(1-2p,-p,-p,2-\right.\right.}{}$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^4*(d + e*x^3)^2),x]
```

output
```
(((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/c)^p*(a + b*x^3 + c*x^6)^p*(d*p*Appell
F1[1 - 2*p, -p, -p, 2 - 2*p, -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b + S
qrt[b^2 - 4*a*c])/(2*c*x^3)] + e*(1 - 2*p)*x^3*AppellF1[-2*p, -p, -p, 1 -
2*p, -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x
^3)]))/(3*d^3*p*(-1 + 2*p)*(1 + (b - Sqrt[b^2 - 4*a*c])/(2*c*x^3))^p*x^3*(
(b - Sqrt[b^2 - 4*a*c])/(2*c) + x^3)^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(c*x^3))^p) + (4^p*e*(a + b*x^3 + c*x^6)^p*(d*p*AppellF1[1 - 2*p, -p, -p,
2 - 2*p, (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*c*d - b
*e + Sqrt[b^2 - 4*a*c]*e)/(2*c*d + 2*c*e*x^3)] + (-1 + 2*p)*(d + e*x^3)*Ap
pellF1[-2*p, -p, -p, 1 - 2*p, (2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)/(2*c*(d
+ e*x^3)), (2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*c*d + 2*c*e*x^3)]))/(3*d
^3*p*(-1 + 2*p)*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p*
((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p*(d + e*x^3))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 733, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1802, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)^2}\,dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{3}\int \frac{(cx^6+bx^3+a)^p}{x^6(ex^3+d)^2}dx^3$$

$$\Big\downarrow \text{1289}$$

$$\frac{1}{3}\int \left(\frac{2e^2(cx^6+bx^3+a)^p}{d^3(ex^3+d)}+\frac{e^2(cx^6+bx^3+a)^p}{d^2(ex^3+d)^2}-\frac{2e(cx^6+bx^3+a)^p}{d^3x^3}+\frac{(cx^6+bx^3+a)^p}{d^2x^6}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{e4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4a}}{2cx^3}\right.}{d^3p}\right.$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(x^4*(d + e*x^3)^2),x]
```

output
```
(-((4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p), -1/2*(b
 - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/(d^2
*(1 - 2*p)*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p)) - (4^p*e*(a + b*x^3 + c*x^6)^p*AppellF1
[1 - 2*p, -p, -p, 2*(1 - p), (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d +
 e*x^3)), (2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c)/(2*(d + e*x^3))])/(d^2*(1
- 2*p)*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p*((e*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^3)))^p*(d + e*x^3)) - (4^p*e*(a
+ b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b - Sqrt[b^2 - 4*
a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/(d^3*p*((b - Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3
))^p) + (4^p*e*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*(d + e*x^3)), (2*d - ((b + Sqrt[b^2 -
4*a*c])*e)/c)/(2*(d + e*x^3))])/(d^3*p*((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
3))/(c*(d + e*x^3)))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3))/(c*(d + e*x^
3)))^p))/3
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x^{4}\,(e\,x^{3}+d)^{2}}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^4/(e*x^3+d)^2,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^4/(e*x^3+d)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4\,(d+ex^3)^2}\,dx=\int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^4/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e^2*x^10 + 2*d*e*x^7 + d^2*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**4/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^4/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)^2*x^4), x)
```

Giac [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^4}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^4/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)^2}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{x^4\,(e\,x^3+d)^2}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(x^4*(d + e*x^3)^2),x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(x^4*(d + e*x^3)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^3+cx^6)^p}{x^4(d+ex^3)^2}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{e^2x^{10}+2de\,x^7+d^2x^4}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^4/(e*x^3+d)^2,x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d**2*x**4 + 2*d*e*x**7 + e**2*x**10),x)
```

3.105 $\int \frac{x^4(a+bx^3+cx^6)^p}{(d+ex^3)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^4(a+bx^3+cx^6)^p}{(d+ex^3)^2} dx = \text{Int}\left(\frac{x^4(a+bx^3+cx^6)^p}{(d+ex^3)^2}, x\right)$$

output
```
Defer(Int)(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^4(a+bx^3+cx^6)^p}{(d+ex^3)^2} dx = \int \frac{x^4(a+bx^3+cx^6)^p}{(d+ex^3)^2} dx$$

input
```
Integrate[(x^4*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
Integrate[(x^4*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+bx^3+cx^6)^p}{(d+ex^3)^2}dx$$
$$\Big\downarrow \text{1887}$$
$$\int \frac{x^4(a+bx^3+cx^6)^p}{(d+ex^3)^2}dx$$

input
```
Int[(x^4*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^4(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}dx$$

input
```
int(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
int(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{x^4(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x^4}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^4/(e^2*x^6 + 2*d*e*x^3 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(c*x**6+b*x**3+a)**p/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^4(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\int \frac{(cx^6+bx^3+a)^p x^4}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^4/(e*x^3 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^4(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\int \frac{(cx^6+bx^3+a)^p x^4}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^4/(e*x^3 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 21.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^4(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\int \frac{x^4\,(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}\,dx$$

input
```
int((x^4*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x)
```

output
```
int((x^4*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2, x)
```

Reduce [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 19068, normalized size of antiderivative = 706.22

$$\int \frac{x^4(a + bx^3 + cx^6)^p}{(d + ex^3)^2} \, dx = \text{Too large to display}$$

input
```
int(x^4*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
((a + b*x**3 + c*x**6)**p*b*x**2 - 9*int(((a + b*x**3 + c*x**6)**p*x**10)/
(3*a*b*d**2*e*p - a*b*d**2*e + 6*a*b*d*e**2*p*x**3 - 2*a*b*d*e**2*x**3 + 3
*a*b*e**3*p*x**6 - a*b*e**3*x**6 + 6*a*c*d**3*p + 2*a*c*d**3 + 12*a*c*d**2
*e*p*x**3 + 4*a*c*d**2*e*x**3 + 6*a*c*d*e**2*p*x**6 + 2*a*c*d*e**2*x**6 +
3*b**2*d**2*e*p*x**3 - b**2*d**2*e*x**3 + 6*b**2*d*e**2*p*x**6 - 2*b**2*d*
e**2*x**6 + 3*b**2*e**3*p*x**9 - b**2*e**3*x**9 + 6*b*c*d**3*p*x**3 + 2*b*
c*d**3*x**3 + 15*b*c*d**2*e*p*x**6 + 3*b*c*d**2*e*x**6 + 12*b*c*d*e**2*p*x
**9 + 3*b*c*e**3*p*x**12 - b*c*e**3*x**12 + 6*c**2*d**3*p*x**6 + 2*c**2*d*
*3*x**6 + 12*c**2*d**2*e*p*x**9 + 4*c**2*d**2*e*x**9 + 6*c**2*d*e**2*p*x**
12 + 2*c**2*d*e**2*x**12),x)*b**2*c*d*e**2*p**2 + 3*int(((a + b*x**3 + c*x
**6)**p*x**10)/(3*a*b*d**2*e*p - a*b*d**2*e + 6*a*b*d*e**2*p*x**3 - 2*a*b*
d*e**2*x**3 + 3*a*b*e**3*p*x**6 - a*b*e**3*x**6 + 6*a*c*d**3*p + 2*a*c*d**
3 + 12*a*c*d**2*e*p*x**3 + 4*a*c*d**2*e*x**3 + 6*a*c*d*e**2*p*x**6 + 2*a*c
*d*e**2*x**6 + 3*b**2*d**2*e*p*x**3 - b**2*d**2*e*x**3 + 6*b**2*d*e**2*p*x
**6 - 2*b**2*d*e**2*x**6 + 3*b**2*e**3*p*x**9 - b**2*e**3*x**9 + 6*b*c*d**
3*p*x**3 + 2*b*c*d**3*x**3 + 15*b*c*d**2*e*p*x**6 + 3*b*c*d**2*e*x**6 + 12
*b*c*d*e**2*p*x**9 + 3*b*c*e**3*p*x**12 - b*c*e**3*x**12 + 6*c**2*d**3*p*x
**6 + 2*c**2*d**3*x**6 + 12*c**2*d**2*e*p*x**9 + 4*c**2*d**2*e*x**9 + 6*c*
*2*d*e**2*p*x**12 + 2*c**2*d*e**2*x**12),x)*b**2*c*d*e**2*p - 9*int(((a +
b*x**3 + c*x**6)**p*x**10)/(3*a*b*d**2*e*p - a*b*d**2*e + 6*a*b*d*e**2*...
```

3.106 $\int \frac{x\left(a+bx^3+cx^6\right)^p}{\left(d+ex^3\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \text{Int}\left(\frac{x(a+bx^3+cx^6)^p}{(d+ex^3)^2},x\right)$$

output
```
Defer(Int)(x*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{x(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx$$

input
```
Integrate[(x*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
Integrate[(x*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx$$

$$\Big\downarrow \text{ 1887}$$

$$\int \frac{x(a+bx^3+cx^6)^p}{(d+ex^3)^2}dx$$

input
```
Int[(x*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}dx$$

input
```
int(x*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
int(x*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{x(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x}{(ex^3+d)^2}\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x/(e^2*x^6 + 2*d*e*x^3 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x**6+b*x**3+a)**p/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x}{(ex^3+d)^2}\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x/(e*x^3 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x}{(ex^3+d)^2}\,dx$$

input
```
integrate(x*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x/(e*x^3 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 21.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{x\,(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}\,dx$$

input
```
int((x*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x)
```

output
```
int((x*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{x(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p\,x}{e^2x^6+2de\,x^3+d^2}dx$$

input
```
int(x*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
int(((a + b*x**3 + c*x**6)**p*x)/(d**2 + 2*d*e*x**3 + e**2*x**6),x)
```

3.107 $\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)^2} dx = \text{Int}\left(\frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)^2}, x\right)$$

output
```
Defer(Int)((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)^2} dx = \int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)^2} dx$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^2*(d + e*x^3)^2),x]
```

output
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^2*(d + e*x^3)^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)^2}\,dx$$
$$\Big\downarrow \text{ 1887}$$
$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)^2}dx$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(x^2*(d + e*x^3)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{p}}{x^{2}\left(e\,x^{3}+d\right)^{2}}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d)^2,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^3+cx^6)^p}{x^2\left(d+ex^3\right)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e^2*x^8 + 2*d*e*x^5 + d^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^2\left(d+ex^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**2/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)^2} \, dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^2} \, dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)^2*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)^2} \, dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^2} \, dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 28.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)^2} \, dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^2\,(e\,x^3+d)^2} \, dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(x^2*(d + e*x^3)^2),x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(x^2*(d + e*x^3)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^3+cx^6)^p}{x^2(d+ex^3)^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{e^2x^8+2de\,x^5+d^2x^2}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^2/(e*x^3+d)^2,x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d**2*x**2 + 2*d*e*x**5 + e**2*x**8),x)
```

3.108 $\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)^2}\,dx = \text{Int}\left(\frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)^2},x\right)$$

output
```
Defer(Int)((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)^2}\,dx = \int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)^2}\,dx$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^5*(d + e*x^3)^2),x]
```

output
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^5*(d + e*x^3)^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)^2} \, dx$$
$$\Big\downarrow \ 1887$$
$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)^2}dx$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(x^5*(d + e*x^3)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^5\,(e\,x^3+d)^2}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d)^2,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^3+cx^6)^p}{x^5\,(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^5}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e^2*x^11 + 2*d*e*x^8 + d^2*x^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^5\,(d+ex^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**5/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^5}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)^2*x^5), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^5}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)^2*x^5), x)
```

Mupad [N/A]

Not integrable

Time = 29.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^5\,(e\,x^3+d)^2}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(x^5*(d + e*x^3)^2),x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(x^5*(d + e*x^3)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^3+cx^6)^p}{x^5(d+ex^3)^2}\,dx=\int \frac{(c\,x^6+b\,x^3+a)^p}{e^2x^{11}+2de\,x^8+d^2x^5}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^5/(e*x^3+d)^2,x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d**2*x**5 + 2*d*e*x**8 + e**2*x**11),x)
```

3.109 $\int \frac{x^6\left(a+bx^3+cx^6\right)^p}{\left(d+ex^3\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^6(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\text{Int}\left(\frac{x^6(a+bx^3+cx^6)^p}{(d+ex^3)^2},x\right)$$

output
```
Defer(Int)(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^6(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\int \frac{x^6(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx$$

input
```
Integrate[(x^6*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
Integrate[(x^6*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \left(a+bx^3+cx^6\right)^p}{\left(d+ex^3\right)^2} \, dx$$
$$\big\downarrow \text{ 1887}$$
$$\int \frac{x^6 \left(a+bx^3+cx^6\right)^p}{\left(d+ex^3\right)^2}dx$$

input
```
Int[(x^6*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^6(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}dx$$

input
```
int(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
int(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{x^6(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x^6}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^6/(e^2*x^6 + 2*d*e*x^3 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(c*x**6+b*x**3+a)**p/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^6(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\int \frac{(cx^6+bx^3+a)^p x^6}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^6/(e*x^3 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^6(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\int \frac{(cx^6+bx^3+a)^p x^6}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^6/(e*x^3 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 21.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^6(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\int \frac{x^6\,(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}\,dx$$

input
```
int((x^6*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x)
```

output
```
int((x^6*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2, x)
```

Reduce [N/A]

Not integrable

Time = 3.47 (sec) , antiderivative size = 79840, normalized size of antiderivative = 2957.04

$$\int \frac{x^6(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \text{Too large to display}$$

input
```
int(x^6*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
(6*(a + b*x**3 + c*x**6)**p*a*e*p*x - 3*(a + b*x**3 + c*x**6)**p*b*d*p*x -
 4*(a + b*x**3 + c*x**6)**p*b*d*x + 3*(a + b*x**3 + c*x**6)**p*b*e*p*x**4
- 2*(a + b*x**3 + c*x**6)**p*b*e*x**4 + 6*(a + b*x**3 + c*x**6)**p*c*d*p*x
**4 + (a + b*x**3 + c*x**6)**p*c*d*x**4 - 108*int((a + b*x**3 + c*x**6)**p
/(18*a*b*d**2*e*p**2 - 9*a*b*d**2*e*p - 2*a*b*d**2*e + 36*a*b*d*e**2*p**2*
x**3 - 18*a*b*d*e**2*p*x**3 - 4*a*b*d*e**2*x**3 + 18*a*b*e**3*p**2*x**6 -
9*a*b*e**3*p*x**6 - 2*a*b*e**3*x**6 + 36*a*c*d**3*p**2 + 12*a*c*d**3*p + a
*c*d**3 + 72*a*c*d**2*e*p**2*x**3 + 24*a*c*d**2*e*p*x**3 + 2*a*c*d**2*e*x*
*3 + 36*a*c*d*e**2*p**2*x**6 + 12*a*c*d*e**2*p*x**6 + a*c*d*e**2*x**6 + 18
*b**2*d**2*e*p**2*x**3 - 9*b**2*d**2*e*p*x**3 - 2*b**2*d**2*e*x**3 + 36*b*
*2*d*e**2*p**2*x**6 - 18*b**2*d*e**2*p*x**6 - 4*b**2*d*e**2*x**6 + 18*b**2
*e**3*p**2*x**9 - 9*b**2*e**3*p*x**9 - 2*b**2*e**3*x**9 + 36*b*c*d**3*p**2
*x**3 + 12*b*c*d**3*p*x**3 + b*c*d**3*x**3 + 90*b*c*d**2*e*p**2*x**6 + 15*
b*c*d**2*e*p*x**6 + 72*b*c*d*e**2*p**2*x**9 - 6*b*c*d*e**2*p*x**9 - 3*b*c*
d*e**2*x**9 + 18*b*c*e**3*p**2*x**12 - 9*b*c*e**3*p*x**12 - 2*b*c*e**3*x**
12 + 36*c**2*d**3*p**2*x**6 + 12*c**2*d**3*p*x**6 + c**2*d**3*x**6 + 72*c*
*2*d**2*e*p**2*x**9 + 24*c**2*d**2*e*p*x**9 + 2*c**2*d**2*e*x**9 + 36*c**2
*d*e**2*p**2*x**12 + 12*c**2*d*e**2*p*x**12 + c**2*d*e**2*x**12),x)*a**2*b
*d**2*e**2*p**3 + 54*int((a + b*x**3 + c*x**6)**p/(18*a*b*d**2*e*p**2 - 9*
a*b*d**2*e*p - 2*a*b*d**2*e + 36*a*b*d*e**2*p**2*x**3 - 18*a*b*d*e**2*p...
```

3.110 $\int \frac{x^3(a+bx^3+cx^6)^p}{(d+ex^3)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^3(a+bx^3+cx^6)^p}{(d+ex^3)^2} dx = \text{Int}\left(\frac{x^3(a+bx^3+cx^6)^p}{(d+ex^3)^2}, x\right)$$

output
```
Defer(Int)(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a+bx^3+cx^6)^p}{(d+ex^3)^2} dx = \int \frac{x^3(a+bx^3+cx^6)^p}{(d+ex^3)^2} dx$$

input
```
Integrate[(x^3*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
Integrate[(x^3*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+bx^3+cx^6)^p}{(d+ex^3)^2} \, dx$$

$$\Big\downarrow \text{ 1887}$$

$$\int \frac{x^3 (a+bx^3+cx^6)^p}{(d+ex^3)^2} dx$$

input
```
Int[(x^3*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^3(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}dx$$

input
```
int(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
int(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{x^3(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x^3}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p*x^3/(e^2*x^6 + 2*d*e*x^3 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(c*x**6+b*x**3+a)**p/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x^3}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^3/(e*x^3 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p x^3}{(ex^3+d)^2}\,dx$$

input
```
integrate(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p*x^3/(e*x^3 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 21.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{x^3\,(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}\,dx$$

input
```
int((x^3*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2,x)
```

output
```
int((x^3*(a + b*x^3 + c*x^6)^p)/(d + e*x^3)^2, x)
```

Reduce [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 18802, normalized size of antiderivative = 696.37

$$\int \frac{x^3(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \text{Too large to display}$$

input
```
int(x^3*(c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
((a + b*x**3 + c*x**6)**p*b*x - 3*int((a + b*x**3 + c*x**6)**p/(3*a*b*d**2
*e*p - 2*a*b*d**2*e + 6*a*b*d*e**2*p*x**3 - 4*a*b*d*e**2*x**3 + 3*a*b*e**3
*p*x**6 - 2*a*b*e**3*x**6 + 6*a*c*d**3*p + a*c*d**3 + 12*a*c*d**2*e*p*x**3
 + 2*a*c*d**2*e*x**3 + 6*a*c*d*e**2*p*x**6 + a*c*d*e**2*x**6 + 3*b**2*d**2
*e*p*x**3 - 2*b**2*d**2*e*x**3 + 6*b**2*d*e**2*p*x**6 - 4*b**2*d*e**2*x**6
 + 3*b**2*e**3*p*x**9 - 2*b**2*e**3*x**9 + 6*b*c*d**3*p*x**3 + b*c*d**3*x*
*3 + 15*b*c*d**2*e*p*x**6 + 12*b*c*d*e**2*p*x**9 - 3*b*c*d*e**2*x**9 + 3*b
*c*e**3*p*x**12 - 2*b*c*e**3*x**12 + 6*c**2*d**3*p*x**6 + c**2*d**3*x**6 +
 12*c**2*d**2*e*p*x**9 + 2*c**2*d**2*e*x**9 + 6*c**2*d*e**2*p*x**12 + c**2
*d*e**2*x**12),x)*a*b**2*d**2*e*p + 2*int((a + b*x**3 + c*x**6)**p/(3*a*b*
d**2*e*p - 2*a*b*d**2*e + 6*a*b*d*e**2*p*x**3 - 4*a*b*d*e**2*x**3 + 3*a*b*
e**3*p*x**6 - 2*a*b*e**3*x**6 + 6*a*c*d**3*p + a*c*d**3 + 12*a*c*d**2*e*p*
x**3 + 2*a*c*d**2*e*x**3 + 6*a*c*d*e**2*p*x**6 + a*c*d*e**2*x**6 + 3*b**2*
d**2*e*p*x**3 - 2*b**2*d**2*e*x**3 + 6*b**2*d*e**2*p*x**6 - 4*b**2*d*e**2*
x**6 + 3*b**2*e**3*p*x**9 - 2*b**2*e**3*x**9 + 6*b*c*d**3*p*x**3 + b*c*d**
3*x**3 + 15*b*c*d**2*e*p*x**6 + 12*b*c*d*e**2*p*x**9 - 3*b*c*d*e**2*x**9 +
 3*b*c*e**3*p*x**12 - 2*b*c*e**3*x**12 + 6*c**2*d**3*p*x**6 + c**2*d**3*x*
*6 + 12*c**2*d**2*e*p*x**9 + 2*c**2*d**2*e*x**9 + 6*c**2*d*e**2*p*x**12 +
c**2*d*e**2*x**12),x)*a*b**2*d**2*e - 3*int((a + b*x**3 + c*x**6)**p/(3*a*
b*d**2*e*p - 2*a*b*d**2*e + 6*a*b*d*e**2*p*x**3 - 4*a*b*d*e**2*x**3 + 3...
```

3.111 $\int \frac{(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\text{Int}\left(\frac{(a+bx^3+cx^6)^p}{(d+ex^3)^2},x\right)$$

output
```
Defer(Int)((c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx=\int \frac{(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(d + e*x^3)^2,x]
```

output
```
Integrate[(a + b*x^3 + c*x^6)^p/(d + e*x^3)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{(d+ex^3)^2} \, dx$$

$$\Big\downarrow \; 1769$$

$$\int \frac{(a+bx^3+cx^6)^p}{(d+ex^3)^2}dx$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(d + e*x^3)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1769
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Unintegrable[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,
 x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}dx$$

input
```
int((c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
int((c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e^2*x^6 + 2*d*e*x^3 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/(e*x^3 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/(e*x^3 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 20.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{(e\,x^3+d)^2}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(d + e*x^3)^2,x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(d + e*x^3)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx^3+cx^6)^p}{(d+ex^3)^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{e^2x^6+2de\,x^3+d^2}dx$$

input
```
int((c*x^6+b*x^3+a)^p/(e*x^3+d)^2,x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d**2 + 2*d*e*x**3 + e**2*x**6),x)
```

3.112 $\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)^2} dx = \text{Int}\left(\frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)^2}, x\right)$$

output
```
Defer(Int)((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)^2} dx = \int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)^2} dx$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^3*(d + e*x^3)^2),x]
```

output
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^3*(d + e*x^3)^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)^2} \, dx$$
$$\Big\downarrow \; 1887$$
$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)^2} dx$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(x^3*(d + e*x^3)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^3\,(e\,x^3+d)^2}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d)^2,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^3+cx^6)^p}{x^3\,(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e^2*x^9 + 2*d*e*x^6 + d^2*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^3\,(d+ex^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**3/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)^2*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^3}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)^2*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 29.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^3\,(e\,x^3+d)^2}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(x^3*(d + e*x^3)^2),x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(x^3*(d + e*x^3)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^3+cx^6)^p}{x^3(d+ex^3)^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{e^2x^9+2de\,x^6+d^2x^3}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^3/(e*x^3+d)^2,x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d**2*x**3 + 2*d*e*x**6 + e**2*x**9),x)
```

3.113 $\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)^2}\,dx = \text{Int}\left(\frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)^2},x\right)$$

output
```
Defer(Int)((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)^2}\,dx = \int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)^2}\,dx$$

input
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^6*(d + e*x^3)^2),x]
```

output
```
Integrate[(a + b*x^3 + c*x^6)^p/(x^6*(d + e*x^3)^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1887}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)^2}\,dx$$

$$\Big\downarrow \; 1887$$

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)^2}dx$$

input
```
Int[(a + b*x^3 + c*x^6)^p/(x^6*(d + e*x^3)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^6\,(e\,x^3+d)^2}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d)^2,x)
```

output
```
int((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^3+cx^6)^p}{x^6\,(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^6}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^6 + b*x^3 + a)^p/(e^2*x^12 + 2*d*e*x^9 + d^2*x^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3+cx^6)^p}{x^6\,(d+ex^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**6+b*x**3+a)**p/x**6/(e*x**3+d)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^6}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)^2*x^6), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)^2}\,dx = \int \frac{(cx^6+bx^3+a)^p}{(ex^3+d)^2x^6}\,dx$$

input
```
integrate((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^6 + b*x^3 + a)^p/((e*x^3 + d)^2*x^6), x)
```

Mupad [N/A]

Not integrable

Time = 30.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{x^6\,(e\,x^3+d)^2}\,dx$$

input
```
int((a + b*x^3 + c*x^6)^p/(x^6*(d + e*x^3)^2),x)
```

output
```
int((a + b*x^3 + c*x^6)^p/(x^6*(d + e*x^3)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^3+cx^6)^p}{x^6(d+ex^3)^2}\,dx = \int \frac{(c\,x^6+b\,x^3+a)^p}{e^2x^{12}+2de\,x^9+d^2x^6}dx$$

input
```
int((c*x^6+b*x^3+a)^p/x^6/(e*x^3+d)^2,x)
```

output
```
int((a + b*x**3 + c*x**6)**p/(d**2*x**6 + 2*d*e*x**9 + e**2*x**12),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#    "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file