Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.3-General-trinomial/130-1.2.3.4-b

Nasser M. Abbasi

May 18, 2024 Compiled on May 18, 2024 at 10:02pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**119**]. This is test number [130].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	90.76 (108)	9.24 (11)
Mathematica	84.03 (100)	15.97 (19)
Maple	61.34 (73)	38.66 (46)
Mupad	43.70 (52)	56.30 (67)
Giac	43.70 (52)	56.30 (67)
Fricas	35.29 (42)	64.71 (77)
Sympy	21.01 (25)	78.99 (94)
Reduce	10.92 (13)	89.08 (106)
Maxima	1.68 (2)	98.32 (117)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

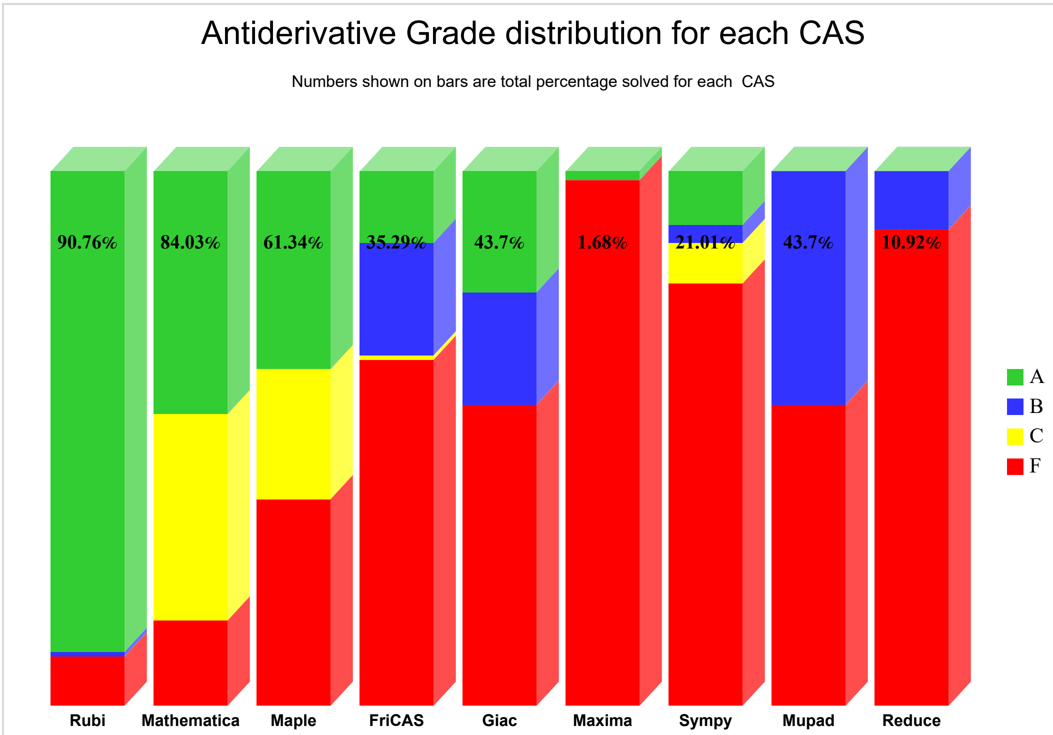
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

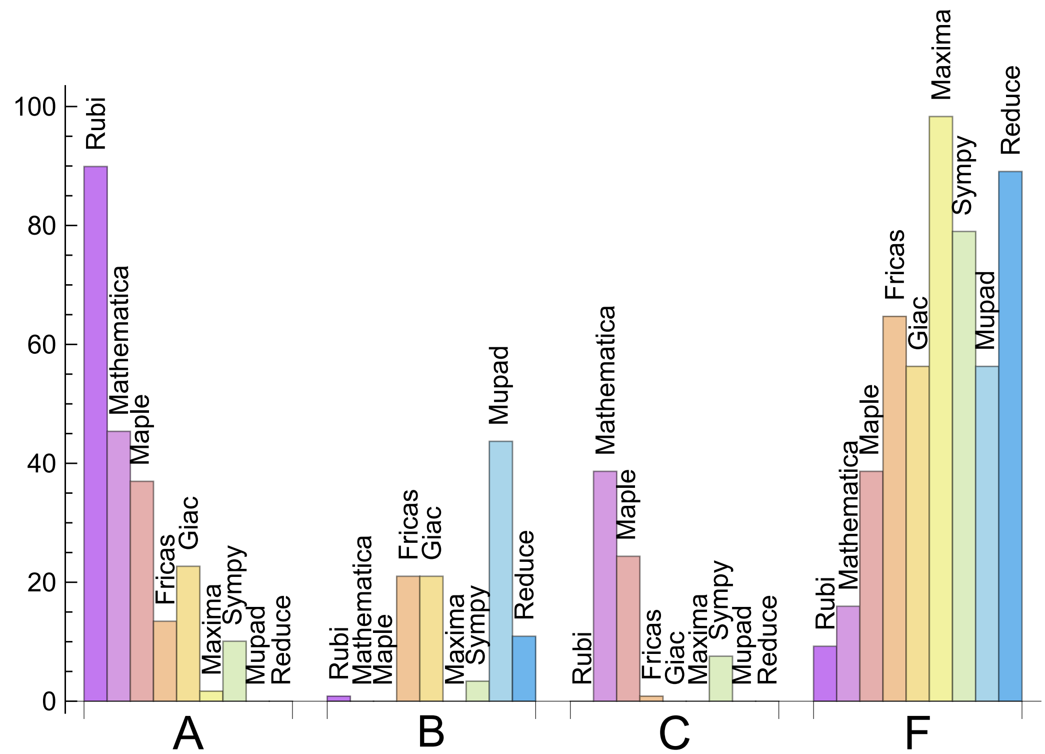
System	% A grade	% B grade	% C grade	% F grade
Rubi	89.916	0.840	0.000	9.244
Mathematica	45.378	0.000	38.655	15.966
Maple	36.975	0.000	24.370	38.655
Giac	22.689	21.008	0.000	56.303
Fricas	13.445	21.008	0.840	64.706
Sympy	10.084	3.361	7.563	78.992
Maxima	1.681	0.000	0.000	98.319
Mupad	0.000	43.697	0.000	56.303
Reduce	0.000	10.924	0.000	89.076

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	11	100.00	0.00	0.00
Mathematica	19	100.00	0.00	0.00
Maple	46	100.00	0.00	0.00
Mupad	67	0.00	100.00	0.00
Giac	67	74.63	25.37	0.00
Fricas	77	59.74	40.26	0.00
Sympy	94	0.00	100.00	0.00
Reduce	106	100.00	0.00	0.00
Maxima	117	73.50	0.00	26.50

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.11
Rubi	0.65
Mathematica	0.88
Reduce	1.27
Maple	2.55
Giac	2.70
Fricas	18.30
Mupad	33.49
Sympy	52.86

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	35.00	0.87	35.00	0.87
Sympy	108.76	1.15	76.00	0.79
Mathematica	217.88	0.79	140.50	0.81
Maple	261.52	0.78	196.00	0.90
Rubi	298.84	1.06	235.00	1.00
Reduce	739.08	7.40	331.00	2.77
Giac	3940.73	8.96	292.50	1.33
Fricas	6076.79	15.94	1262.00	6.98
Mupad	68497.81	150.70	26772.50	98.99

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

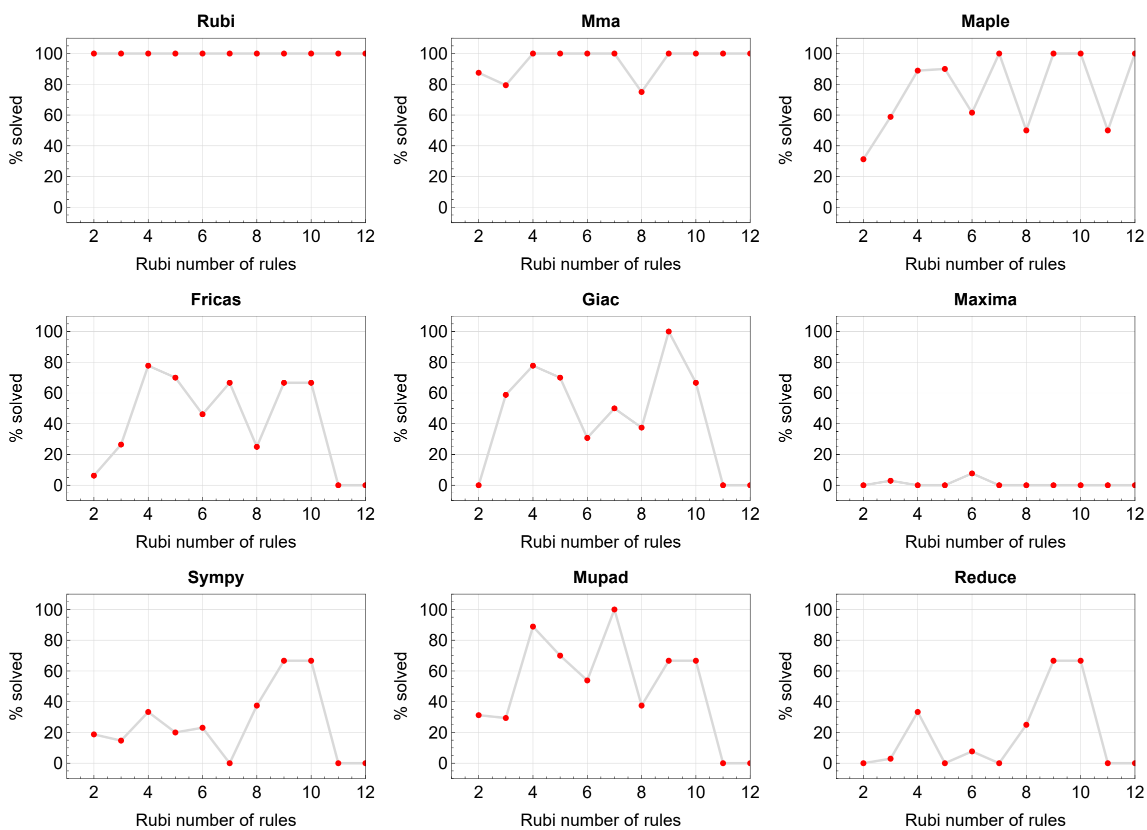


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

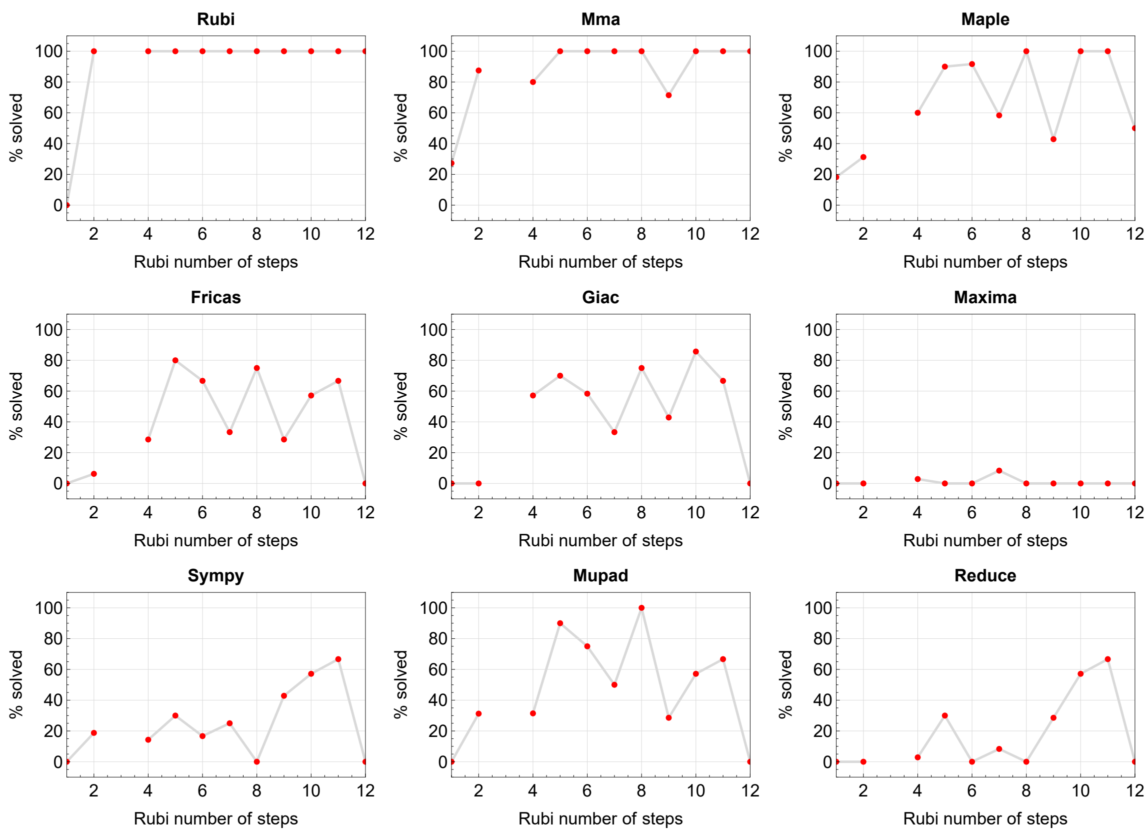


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

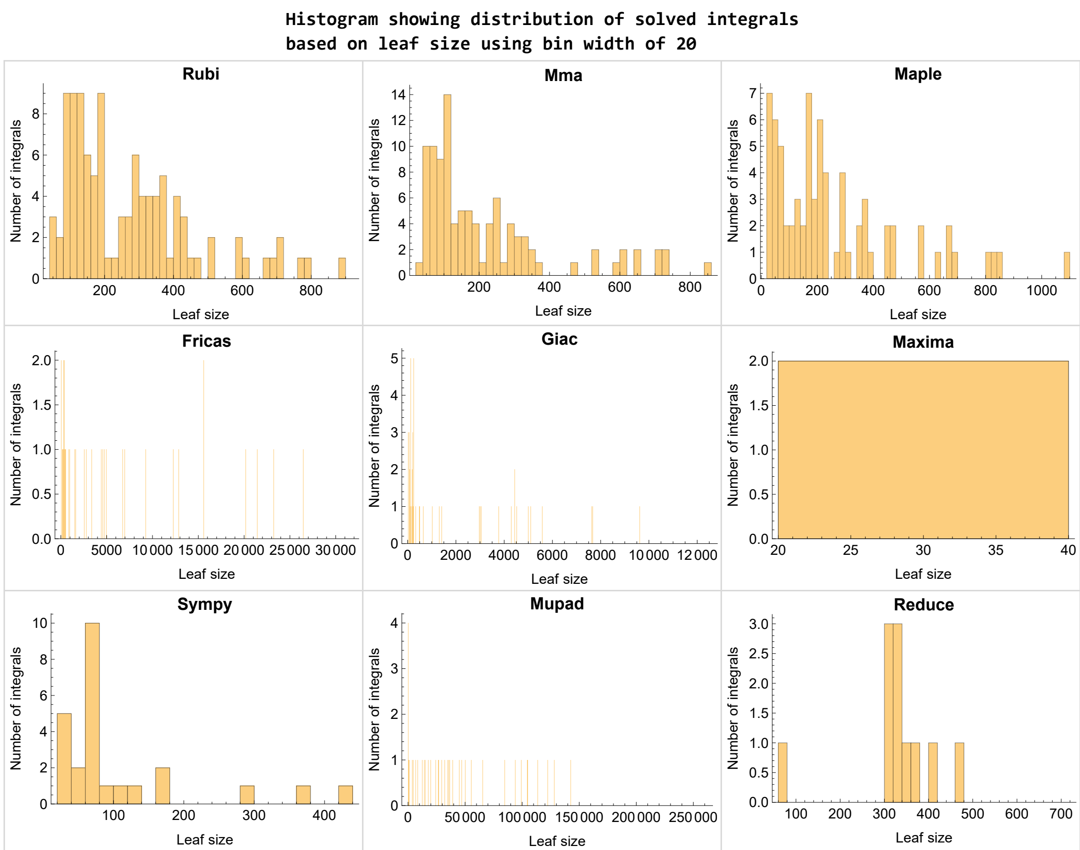


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

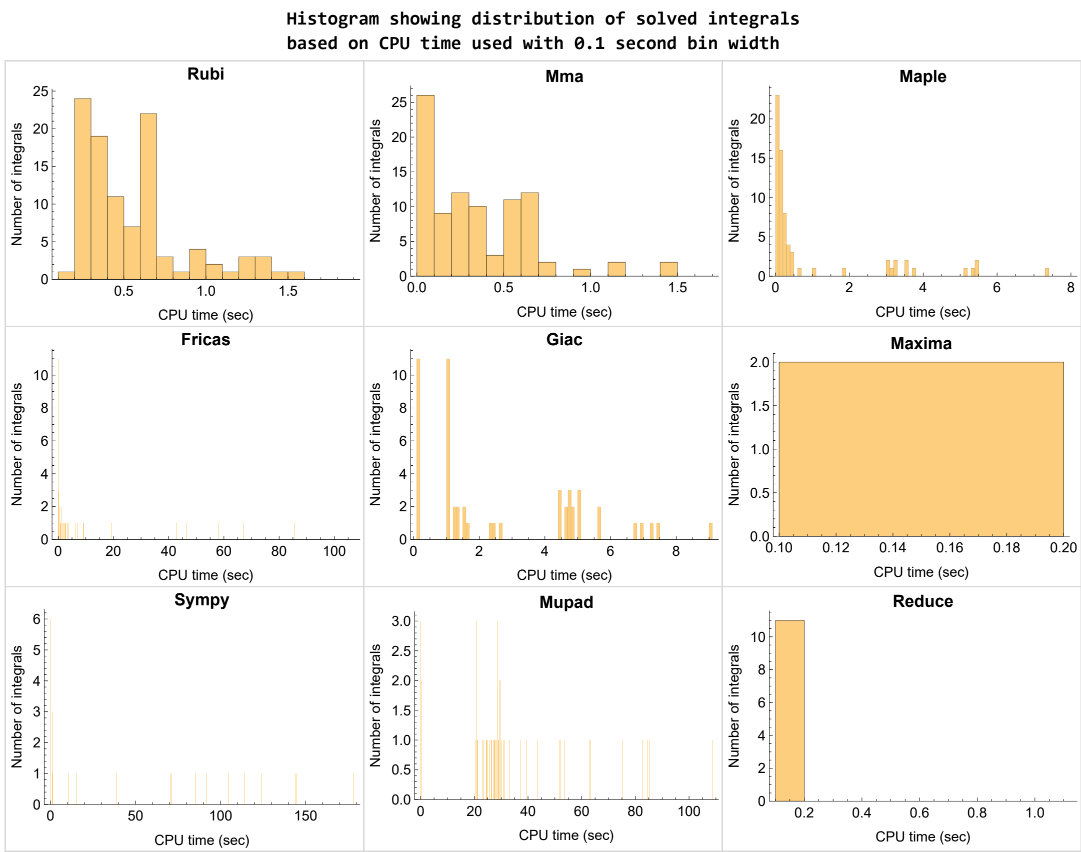


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

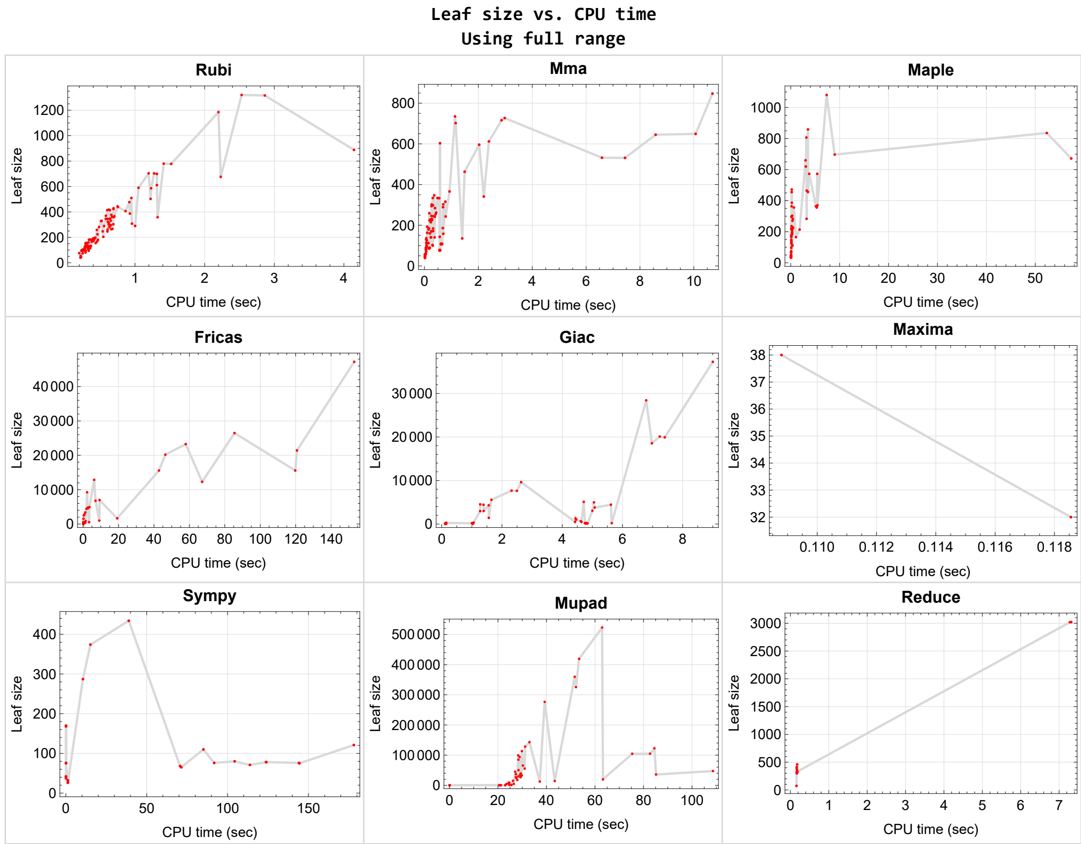


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{35, 41\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

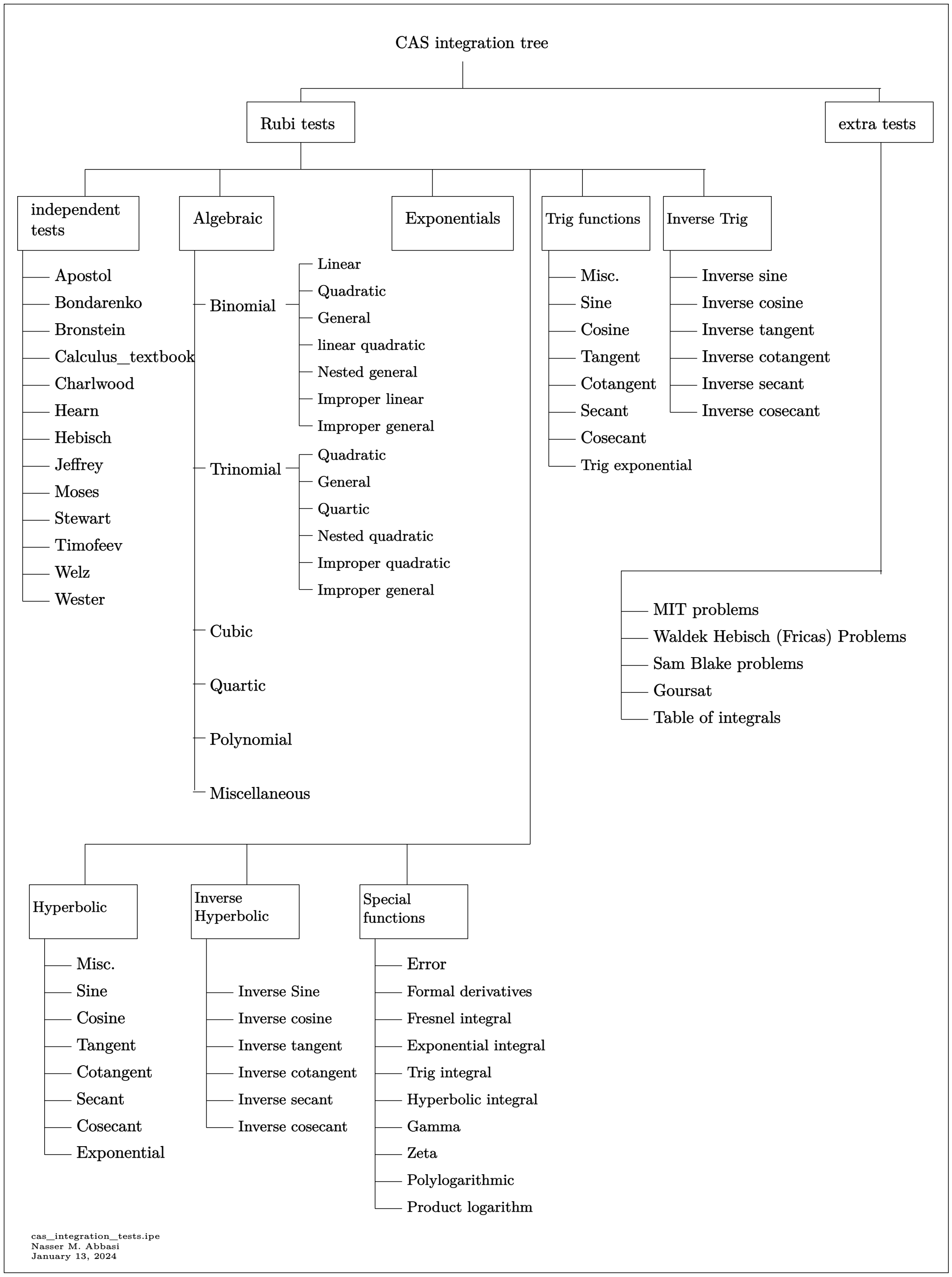
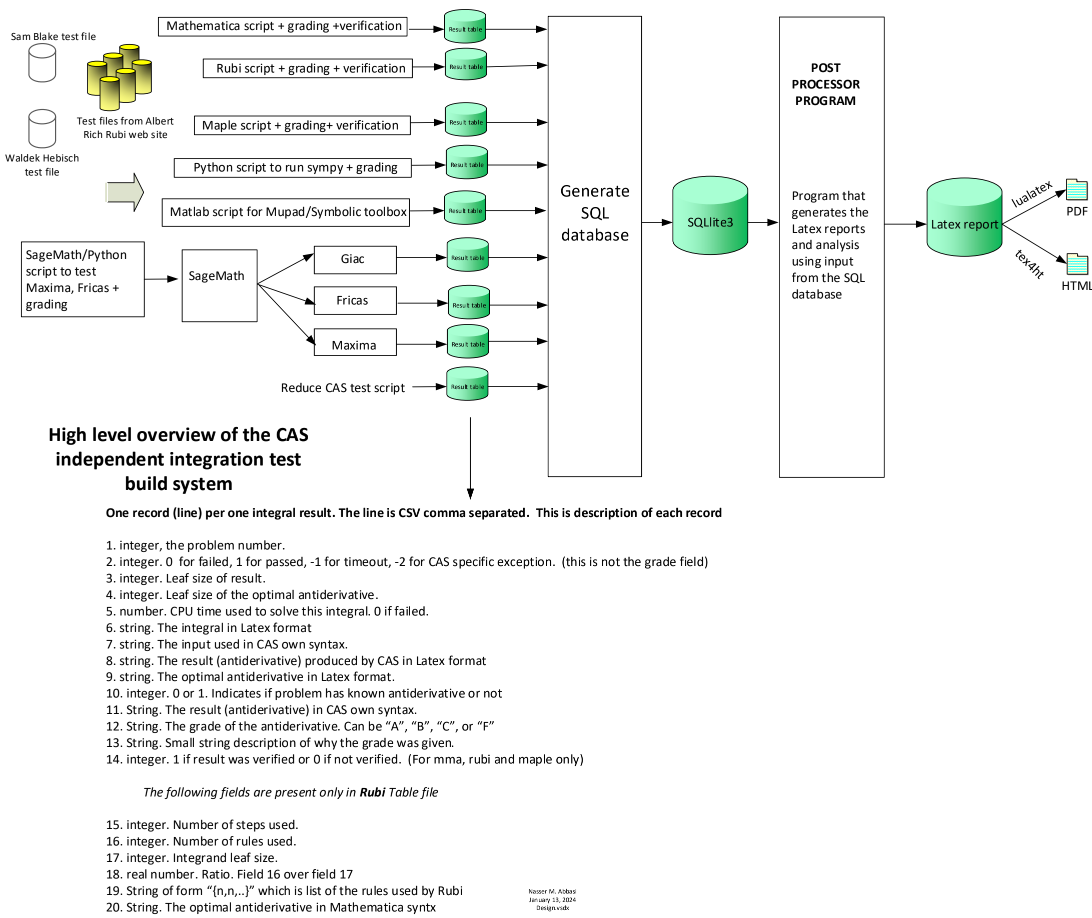


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 35, 36, 37, 38, 39, 40, 41, 44, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117 }

B grade { 65 }

C grade { }

F normal fail { 20, 30, 31, 33, 34, 42, 43, 45, 46, 118, 119 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 35, 37, 41, 47, 48, 49, 53, 54, 55, 64, 65, 74, 75, 79, 80, 81, 89, 90, 91, 94, 95, 96, 97, 105, 106, 107, 110, 111, 112, 113 }

B grade { }

C grade { 50, 51, 52, 56, 57, 58, 59, 60, 61, 62, 63, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 82, 83, 84, 85, 86, 87, 88, 92, 93, 98, 99, 100, 101, 102, 103, 104, 108, 109, 114, 115, 116, 117, 118, 119 }

F normal fail { 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 38, 39, 40, 42, 43, 44, 45, 46 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 64, 65, 66, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114 }

B grade { }

C grade { 58, 59, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 84, 85, 86, 87, 88, 99, 100, 101, 102, 103, 104, 115, 116, 117, 118, 119 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 47, 48, 49, 50, 51, 52, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73 }

B grade { 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 79, 80, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101 }

C grade { 84 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46 }

F(-1) timedout fail { 74, 75, 76, 77, 78, 81, 82, 83, 85, 86, 87, 88, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119 }

F(-2) exception fail { }

Maxima

A grade { 64, 66 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 67, 68, 69, 70, 71, 72, 73, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 115, 116, 117, 118, 119 }

F(-1) timedout fail { }

F(-2) exception fail { 47, 48, 49, 50, 51, 52, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 93, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114 }

Giac

A grade { 47, 48, 49, 50, 51, 52, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 89, 90, 91, 92, 93 }

B grade { 53, 54, 55, 56, 57, 79, 80, 81, 82, 83, 94, 95, 96, 97, 98, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 99, 102, 103, 104 }

F(-1) timedout fail { 58, 59, 60, 61, 62, 84, 85, 86, 87, 88, 100, 101, 115, 116, 117, 118, 119 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 2, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 79, 80, 81, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 74, 75, 76, 77, 78, 82, 83, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119 }

F(-2) exception fail { }

Sympy

A grade { 2, 15, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73 }

B grade { 48, 49, 65, 91 }

C grade { 1, 3, 4, 5, 6, 9, 10, 11, 14 }

F normal fail { }

F(-1) timedout fail { 7, 8, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 90, 91 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	78	0	0	0	76	0	990	0
N.S.	1	1.04	0.77	0.00	0.00	0.00	0.75	0.00	9.80	0.00
time (sec)	N/A	0.258	0.562	0.000	0.000	0.000	144.144	0.000	0.374	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	76	100	0	0	0	65	0	236	68
N.S.	1	1.01	1.33	0.00	0.00	0.00	0.87	0.00	3.15	0.91
time (sec)	N/A	0.200	0.304	0.000	0.000	0.000	71.382	0.000	0.247	22.994

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	102	77	0	0	0	76	0	543	0
N.S.	1	1.01	0.76	0.00	0.00	0.00	0.75	0.00	5.38	0.00
time (sec)	N/A	0.250	0.557	0.000	0.000	0.000	91.711	0.000	0.331	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	94	89	0	0	0	68	0	261	0
N.S.	1	1.01	0.96	0.00	0.00	0.00	0.73	0.00	2.81	0.00
time (sec)	N/A	0.238	0.198	0.000	0.000	0.000	70.632	0.000	0.319	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	100	77	0	0	0	80	0	486	0
N.S.	1	0.99	0.76	0.00	0.00	0.00	0.79	0.00	4.81	0.00
time (sec)	N/A	0.257	0.582	0.000	0.000	0.000	104.365	0.000	0.413	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	95	89	0	0	0	71	0	301	0
N.S.	1	1.02	0.96	0.00	0.00	0.00	0.76	0.00	3.24	0.00
time (sec)	N/A	0.245	0.194	0.000	0.000	0.000	113.819	0.000	0.363	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	78	0	0	0	0	0	980	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	9.70	0.00
time (sec)	N/A	0.243	0.569	0.000	0.000	0.000	0.000	0.000	0.369	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	78	0	0	0	0	0	552	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	5.47	0.00
time (sec)	N/A	0.246	0.570	0.000	0.000	0.000	0.000	0.000	0.318	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	75	0	0	0	75	0	535	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.78	0.00	5.57	0.00
time (sec)	N/A	0.232	0.562	0.000	0.000	0.000	144.457	0.000	0.323	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	77	0	0	0	78	0	581	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.79	0.00	5.87	0.00
time (sec)	N/A	0.238	0.574	0.000	0.000	0.000	123.984	0.000	0.399	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	78	0	0	0	78	0	584	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.81	0.00	6.08	0.00
time (sec)	N/A	0.232	0.576	0.000	0.000	0.000	123.757	0.000	0.441	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	109	0	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	0.650	0.000	0.000	0.000	0.000	0.000	0.562	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	134	135	0	0	0	0	0	846	0
N.S.	1	1.01	1.02	0.00	0.00	0.00	0.00	0.00	6.36	0.00
time (sec)	N/A	0.290	1.401	0.000	0.000	0.000	0.000	0.000	0.301	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	166	108	0	0	0	121	0	1531	0
N.S.	1	1.04	0.68	0.00	0.00	0.00	0.76	0.00	9.63	0.00
time (sec)	N/A	0.378	0.626	0.000	0.000	0.000	178.093	0.000	0.452	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	124	103	0	0	0	110	0	489	0
N.S.	1	1.02	0.84	0.00	0.00	0.00	0.90	0.00	4.01	0.00
time (sec)	N/A	0.278	0.297	0.000	0.000	0.000	85.045	0.000	0.435	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	156	107	0	0	0	0	0	0	0
N.S.	1	1.01	0.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.657	0.000	0.000	0.000	0.000	0.000	1.004	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	133	140	0	0	0	0	0	656	0
N.S.	1	0.98	1.03	0.00	0.00	0.00	0.00	0.00	4.82	0.00
time (sec)	N/A	0.295	0.375	0.000	0.000	0.000	0.000	0.000	0.642	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	156	109	0	0	0	0	0	0	0
N.S.	1	0.99	0.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	0.616	0.000	0.000	0.000	0.000	0.000	0.725	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	156	109	0	0	0	0	0	0	0
N.S.	1	0.99	0.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	0.615	0.000	0.000	0.000	0.000	0.000	0.688	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	0	106	0	0	0	0	0	0	0
N.S.	1	0.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.611	0.000	0.000	0.000	0.000	0.000	0.497	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	154	108	0	0	0	0	0	0	0
N.S.	1	1.01	0.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	0.614	0.000	0.000	0.000	0.000	0.000	0.662	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	108	0	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.624	0.000	0.000	0.000	0.000	0.000	0.780	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	135	186	0	0	0	0	0	583	0
N.S.	1	1.03	1.42	0.00	0.00	0.00	0.00	0.00	4.45	0.00
time (sec)	N/A	0.319	0.678	0.000	0.000	0.000	0.000	0.000	10.393	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	178	0	0	0	0	0	0	535	0
N.S.	1	1.39	0.00	0.00	0.00	0.00	0.00	0.00	4.18	0.00
time (sec)	N/A	0.382	0.000	0.000	0.000	0.000	0.000	0.000	6.406	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	131	143	0	0	0	0	0	24	0
N.S.	1	1.01	1.10	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.303	0.566	0.000	0.000	0.000	0.000	0.000	1.455	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	129	0	0	0	0	0	0	22	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.336	0.000	0.000	0.000	0.000	0.000	0.000	0.775	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	183	0	0	0	0	0	0	23	0
N.S.	1	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.356	0.000	0.000	0.000	0.000	0.000	0.000	2.388	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	183	0	0	0	0	0	0	25	0
N.S.	1	1.43	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.412	0.000	0.000	0.000	0.000	0.000	0.000	4.117	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	182	0	0	0	0	0	0	25	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.380	0.000	0.000	0.000	0.000	0.000	0.000	4.438	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	0	0	0	0	0	0	0	530	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.14	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	6.610	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	0	0	0	0	0	0	0	24	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.777	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	21	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.307	0.000	0.000	0.000	0.000	0.000	0.000	0.772	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	0	0	0	0	0	0	0	25	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	4.020	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	0	0	0	0	0	0	0	25	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	4.775	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	259	243	0	0	0	0	0	0	0
N.S.	1	0.90	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.453	0.786	0.000	0.000	0.000	0.000	0.000	78.809	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	322	0	0	0	0	0	0	35	0
N.S.	1	1.66	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.610	0.000	0.000	0.000	0.000	0.000	0.000	11.634	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	198	197	158	0	0	0	0	0	1055	0
N.S.	1	0.99	0.80	0.00	0.00	0.00	0.00	0.00	5.33	0.00
time (sec)	N/A	0.431	0.699	0.000	0.000	0.000	0.000	0.000	40.331	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	195	0	0	0	0	0	0	33	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.417	0.000	0.000	0.000	0.000	0.000	0.000	11.223	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	312	0	0	0	0	0	0	34	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.625	0.000	0.000	0.000	0.000	0.000	0.000	15.827	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	377	0	0	0	0	0	0	36	0
N.S.	1	1.96	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.661	0.000	0.000	0.000	0.000	0.000	0.000	25.455	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	302	341	0	0	0	0	0	36	0
N.S.	1	0.97	1.10	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.623	2.202	0.000	0.000	0.000	0.000	0.000	66.499	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	0	0	0	0	0	0	0	35	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	11.793	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	0	0	0	0	0	0	0	35	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	12.225	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0	32	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.396	0.000	0.000	0.000	0.000	0.000	0.000	11.534	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	22.669	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	71.399	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	131	126	136	0	430	0	125	27	8521
N.S.	1	0.99	0.95	1.03	0.00	3.26	0.00	0.95	0.20	64.55
time (sec)	N/A	0.368	0.084	0.136	0.000	0.988	0.000	1.020	200.020	24.714

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	95	93	98	0	305	434	91	27	6587
N.S.	1	0.98	0.96	1.01	0.00	3.14	4.47	0.94	0.28	67.91
time (sec)	N/A	0.291	0.092	0.087	0.000	0.377	38.895	1.020	200.036	24.310

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	74	71	66	0	216	287	68	27	3704
N.S.	1	1.03	0.99	0.92	0.00	3.00	3.99	0.94	0.38	51.44
time (sec)	N/A	0.249	0.065	0.088	0.000	0.182	10.548	1.031	200.031	23.410

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	81	80	74	0	240	0	77	27	8454
N.S.	1	1.04	1.03	0.95	0.00	3.08	0.00	0.99	0.35	108.38
time (sec)	N/A	0.290	0.050	0.083	0.000	0.471	0.000	1.020	200.025	24.512

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	114	130	125	0	385	0	125	27	14100
N.S.	1	1.02	1.16	1.12	0.00	3.44	0.00	1.12	0.24	125.89
time (sec)	N/A	0.355	0.061	0.115	0.000	1.248	0.000	1.014	200.017	43.396

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	193	179	0	533	0	181	27	19687
N.S.	1	1.00	1.25	1.15	0.00	3.44	0.00	1.17	0.17	127.01
time (sec)	N/A	0.435	0.098	0.154	0.000	3.373	0.000	1.012	200.038	63.283

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	280	333	298	0	4744	0	4524	27	26868
N.S.	1	1.03	1.22	1.09	0.00	17.38	0.00	16.57	0.10	98.42
time (sec)	N/A	0.631	0.547	0.119	0.000	2.565	0.000	1.276	200.020	27.389

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	224	259	219	0	2520	0	2966	27	17577
N.S.	1	1.01	1.17	0.99	0.00	11.40	0.00	13.42	0.12	79.53
time (sec)	N/A	0.461	0.181	0.095	0.000	0.364	0.000	1.275	200.022	27.963

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	182	179	168	0	1535	0	1404	25	4501
N.S.	1	0.99	0.97	0.91	0.00	8.34	0.00	7.63	0.14	24.46
time (sec)	N/A	0.339	0.150	0.091	0.000	0.207	0.000	1.563	200.018	26.531

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	195	89	177	0	2772	0	3003	27	15013
N.S.	1	0.98	0.45	0.89	0.00	13.93	0.00	15.09	0.14	75.44
time (sec)	N/A	0.418	0.048	0.105	0.000	0.728	0.000	1.392	200.014	26.218

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	247	136	236	0	4924	0	4428	27	24353
N.S.	1	0.96	0.53	0.92	0.00	19.16	0.00	17.23	0.11	94.76
time (sec)	N/A	0.536	0.064	0.138	0.000	3.579	0.000	1.388	200.019	27.554

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	366	88	67	0	12866	0	0	27	50213
N.S.	1	0.85	0.20	0.15	0.00	29.71	0.00	0.00	0.06	115.97
time (sec)	N/A	0.704	0.069	0.063	0.000	6.242	0.000	0.000	200.028	28.468

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	330	59	51	0	15561	0	0	27	29445
N.S.	1	0.88	0.16	0.14	0.00	41.50	0.00	0.00	0.07	78.52
time (sec)	N/A	0.521	0.049	0.052	0.000	42.815	0.000	0.000	200.498	28.428

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	328	61	47	0	9245	0	0	24	36707
N.S.	1	0.87	0.16	0.13	0.00	24.65	0.00	0.00	0.06	97.89
time (sec)	N/A	0.512	0.046	0.048	0.000	2.250	0.000	0.000	200.016	27.401

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	345	85	73	0	21400	0	0	27	39028
N.S.	1	0.88	0.22	0.19	0.00	54.59	0.00	0.00	0.07	99.56
time (sec)	N/A	0.610	0.063	0.083	0.000	120.721	0.000	0.000	200.013	29.399

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	345	86	68	0	20184	0	0	27	65350
N.S.	1	0.88	0.22	0.17	0.00	51.23	0.00	0.00	0.07	165.86
time (sec)	N/A	0.583	0.067	0.088	0.000	46.406	0.000	0.000	200.023	30.279

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	106	85	38	0	67	75	69	341	58
N.S.	1	1.61	1.29	0.58	0.00	1.02	1.14	1.05	5.17	0.88
time (sec)	N/A	0.327	0.173	0.066	0.000	0.062	0.095	0.132	0.178	0.113

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	40	39	33	32	32	37	32	328	34
N.S.	1	1.03	1.00	0.85	0.82	0.82	0.95	0.82	8.41	0.87
time (sec)	N/A	0.217	0.018	0.046	0.119	0.061	0.060	0.110	0.165	0.052

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	54	44	39	0	41	42	31	72	20
N.S.	1	2.08	1.69	1.50	0.00	1.58	1.62	1.19	2.77	0.77
time (sec)	N/A	0.221	0.020	0.052	0.000	0.058	0.054	0.138	0.159	21.131

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	44	44	33	38	34	41	38	330	36
N.S.	1	1.07	1.07	0.80	0.93	0.83	1.00	0.93	8.05	0.88
time (sec)	N/A	0.225	0.018	0.056	0.109	0.059	0.073	0.141	0.172	21.249

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	95	49	40	0	79	76	81	376	56
N.S.	1	1.44	0.74	0.61	0.00	1.20	1.15	1.23	5.70	0.85
time (sec)	N/A	0.336	0.021	0.074	0.000	0.068	0.101	0.117	0.165	0.061

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	414	48	46	0	378	32	258	412	291
N.S.	1	1.48	0.17	0.16	0.00	1.35	0.11	0.92	1.47	1.04
time (sec)	N/A	0.602	0.022	0.056	0.000	0.072	1.253	0.154	0.163	0.155

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	414	46	34	0	132	170	208	318	56
N.S.	1	1.86	0.21	0.15	0.00	0.59	0.77	0.94	1.43	0.25
time (sec)	N/A	0.640	0.021	0.048	0.000	0.070	0.096	0.137	0.181	20.829

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	360	55	46	0	373	27	253	300	248
N.S.	1	1.26	0.19	0.16	0.00	1.31	0.09	0.89	1.05	0.87
time (sec)	N/A	0.671	0.020	0.063	0.000	0.070	1.254	0.127	0.163	20.822

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	347	57	44	0	337	26	253	300	208
N.S.	1	1.22	0.20	0.15	0.00	1.18	0.09	0.89	1.05	0.73
time (sec)	N/A	0.641	0.017	0.047	0.000	0.070	1.183	0.148	0.169	0.083

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	418	47	38	0	140	168	210	331	58
N.S.	1	1.87	0.21	0.17	0.00	0.62	0.75	0.94	1.48	0.26
time (sec)	N/A	0.625	0.022	0.063	0.000	0.075	0.098	0.141	0.182	20.852

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	370	47	38	0	335	32	258	460	479
N.S.	1	1.32	0.17	0.14	0.00	1.20	0.11	0.92	1.64	1.71
time (sec)	N/A	0.582	0.020	0.073	0.000	0.070	1.242	0.142	0.177	20.513

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	131	114	111	0	0	0	131	29	0
N.S.	1	0.99	0.86	0.84	0.00	0.00	0.00	0.99	0.22	0.00
time (sec)	N/A	0.355	0.078	0.342	0.000	0.000	0.000	1.019	200.027	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	116	112	112	0	0	0	133	29	0
N.S.	1	0.87	0.84	0.84	0.00	0.00	0.00	1.00	0.22	0.00
time (sec)	N/A	0.325	0.069	0.329	0.000	0.000	0.000	1.044	200.034	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	168	163	165	0	0	0	170	29	0
N.S.	1	1.01	0.98	0.99	0.00	0.00	0.00	1.02	0.17	0.00
time (sec)	N/A	0.457	0.101	1.053	0.000	0.000	0.000	1.002	200.037	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	204	224	214	0	0	0	232	29	0
N.S.	1	1.00	1.09	1.04	0.00	0.00	0.00	1.13	0.14	0.00
time (sec)	N/A	0.545	0.187	1.808	0.000	0.000	0.000	1.044	200.034	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	266	302	283	0	0	0	327	29	0
N.S.	1	0.99	1.13	1.06	0.00	0.00	0.00	1.22	0.11	0.00
time (sec)	N/A	0.676	0.264	3.237	0.000	0.000	0.000	1.058	200.035	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	290	334	271	0	15577	0	5569	29	122445
N.S.	1	0.99	1.14	0.92	0.00	53.16	0.00	19.01	0.10	417.90
time (sec)	N/A	1.002	0.481	0.349	0.000	119.776	0.000	1.649	200.029	84.468

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	261	289	219	0	12293	0	4297	29	104563
N.S.	1	0.98	1.09	0.83	0.00	46.39	0.00	16.22	0.11	394.58
time (sec)	N/A	0.606	0.687	0.286	0.000	67.197	0.000	1.560	200.032	75.337

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	264	284	220	0	0	0	7671	27	104763
N.S.	1	0.99	1.06	0.82	0.00	0.00	0.00	28.73	0.10	392.37
time (sec)	N/A	0.659	0.392	0.316	0.000	0.000	0.000	2.317	200.024	82.698

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	308	226	282	0	0	0	7626	29	0
N.S.	1	0.99	0.72	0.90	0.00	0.00	0.00	24.44	0.09	0.00
time (sec)	N/A	0.956	0.282	0.426	0.000	0.000	0.000	2.496	200.036	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	358	298	356	0	0	0	9619	29	0
N.S.	1	0.98	0.81	0.97	0.00	0.00	0.00	26.28	0.08	0.00
time (sec)	N/A	1.324	0.313	0.636	0.000	0.000	0.000	2.635	200.031	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	699	215	186	0	47212	0	0	29	359899
N.S.	1	1.11	0.34	0.30	0.00	74.94	0.00	0.00	0.05	571.27
time (sec)	N/A	1.314	0.236	0.290	0.000	153.049	0.000	0.000	200.028	51.637

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	703	229	195	0	0	0	0	29	325549
N.S.	1	1.11	0.36	0.31	0.00	0.00	0.00	0.00	0.05	513.48
time (sec)	N/A	1.274	0.227	0.262	0.000	0.000	0.000	0.000	200.029	52.159

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	703	231	196	0	0	0	0	26	419203
N.S.	1	1.11	0.36	0.31	0.00	0.00	0.00	0.00	0.04	661.20
time (sec)	N/A	1.196	0.272	0.274	0.000	0.000	0.000	0.000	200.030	53.441

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	709	778	334	228	0	0	0	0	29	276728
N.S.	1	1.10	0.47	0.32	0.00	0.00	0.00	0.00	0.04	390.31
time (sec)	N/A	1.520	0.307	0.424	0.000	0.000	0.000	0.000	200.032	39.252

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	711	780	347	224	0	0	0	0	29	522994
N.S.	1	1.10	0.49	0.32	0.00	0.00	0.00	0.00	0.04	735.58
time (sec)	N/A	1.412	0.363	0.457	0.000	0.000	0.000	0.000	200.037	62.918

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	171	160	210	0	844	0	194	27	12336
N.S.	1	1.16	1.08	1.42	0.00	5.70	0.00	1.31	0.18	83.35
time (sec)	N/A	0.402	0.201	0.279	0.000	1.410	0.000	5.644	200.027	37.248

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	109	112	125	0	542	0	120	3021	1081
N.S.	1	0.99	1.02	1.14	0.00	4.93	0.00	1.09	27.46	9.83
time (sec)	N/A	0.279	0.147	0.121	0.000	0.112	0.000	4.841	7.317	24.651

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	97	100	96	0	478	374	104	3019	1036
N.S.	1	0.98	1.01	0.97	0.00	4.83	3.78	1.05	30.49	10.46
time (sec)	N/A	0.263	0.077	0.115	0.000	0.103	15.181	4.785	7.282	25.572

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	180	185	211	0	989	0	201	27	35718
N.S.	1	1.21	1.24	1.42	0.00	6.64	0.00	1.35	0.18	239.72
time (sec)	N/A	0.468	0.164	0.276	0.000	9.126	0.000	4.753	200.016	85.155

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	246	294	303	0	1647	0	254	27	47178
N.S.	1	1.09	1.31	1.35	0.00	7.32	0.00	1.13	0.12	209.68
time (sec)	N/A	0.604	0.262	0.294	0.000	19.233	0.000	4.804	200.022	108.625

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	406	463	455	0	6774	0	4984	27	45062
N.S.	1	1.00	1.15	1.13	0.00	16.77	0.00	12.34	0.07	111.54
time (sec)	N/A	0.865	1.490	0.180	0.000	7.000	0.000	5.054	200.018	27.102

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	331	366	364	0	4420	0	3779	27	34824
N.S.	1	0.99	1.09	1.08	0.00	13.15	0.00	11.25	0.08	103.64
time (sec)	N/A	0.650	0.929	0.162	0.000	1.878	0.000	5.078	200.021	29.172

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	281	303	296	0	3377	0	3053	27	26677
N.S.	1	0.98	1.06	1.03	0.00	11.81	0.00	10.67	0.09	93.28
time (sec)	N/A	0.484	0.696	0.152	0.000	1.214	0.000	5.001	200.025	29.353

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	289	316	472	0	4583	0	4435	25	32336
N.S.	1	0.97	1.06	1.58	0.00	15.33	0.00	14.83	0.08	108.15
time (sec)	N/A	0.556	0.775	0.197	0.000	2.708	0.000	5.616	200.017	29.822

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	360	246	387	0	6960	0	5103	27	55298
N.S.	1	0.96	0.65	1.03	0.00	18.51	0.00	13.57	0.07	147.07
time (sec)	N/A	0.697	0.259	0.209	0.000	9.292	0.000	4.715	200.020	31.018

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	619	477	171	167	0	26466	0	0	27	113499
N.S.	1	0.77	0.28	0.27	0.00	42.76	0.00	0.00	0.04	183.36
time (sec)	N/A	0.919	0.321	0.109	0.000	85.482	0.000	0.000	200.020	29.870

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	429	140	148	0	0	0	0	27	84889
N.S.	1	0.81	0.27	0.28	0.00	0.00	0.00	0.00	0.05	161.08
time (sec)	N/A	0.692	0.236	0.106	0.000	0.000	0.000	0.000	200.022	28.437

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	423	141	141	0	23255	0	0	27	99213
N.S.	1	0.81	0.27	0.27	0.00	44.30	0.00	0.00	0.05	188.98
time (sec)	N/A	0.690	0.242	0.090	0.000	57.925	0.000	0.000	200.024	28.340

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	439	184	172	0	0	0	0	27	93859
N.S.	1	0.81	0.34	0.32	0.00	0.00	0.00	0.00	0.05	173.17
time (sec)	N/A	0.755	0.319	0.102	0.000	0.000	0.000	0.000	200.023	28.933

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	443	170	168	0	0	0	0	24	128217
N.S.	1	0.81	0.31	0.31	0.00	0.00	0.00	0.00	0.04	233.97
time (sec)	N/A	0.750	0.254	0.097	0.000	0.000	0.000	0.000	200.020	31.209

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	510	244	214	0	0	0	0	27	142799
N.S.	1	0.77	0.37	0.32	0.00	0.00	0.00	0.00	0.04	216.69
time (sec)	N/A	0.948	0.340	0.167	0.000	0.000	0.000	0.000	200.019	32.987

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	303	262	370	0	0	0	643	29	0
N.S.	1	1.17	1.01	1.43	0.00	0.00	0.00	2.48	0.11	0.00
time (sec)	N/A	0.689	0.435	5.454	0.000	0.000	0.000	4.617	200.032	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	296	254	356	0	0	0	498	29	0
N.S.	1	1.17	1.00	1.40	0.00	0.00	0.00	1.96	0.11	0.00
time (sec)	N/A	0.652	0.414	5.306	0.000	0.000	0.000	4.427	200.032	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	288	242	364	0	0	0	500	29	0
N.S.	1	1.19	1.00	1.50	0.00	0.00	0.00	2.06	0.12	0.00
time (sec)	N/A	0.639	0.384	5.140	0.000	0.000	0.000	4.640	200.029	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	387	603	672	0	0	0	1019	29	0
N.S.	1	0.99	1.54	1.72	0.00	0.00	0.00	2.61	0.07	0.00
time (sec)	N/A	0.927	0.576	57.368	0.000	0.000	0.000	4.465	200.035	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	502	735	836	0	0	0	1303	29	0
N.S.	1	0.99	1.45	1.65	0.00	0.00	0.00	2.57	0.06	0.00
time (sec)	N/A	1.224	1.134	52.365	0.000	0.000	0.000	4.441	200.030	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	633	611	727	807	0	0	0	20106	29	0
N.S.	1	0.97	1.15	1.27	0.00	0.00	0.00	31.76	0.05	0.00
time (sec)	N/A	1.315	2.978	3.191	0.000	0.000	0.000	7.240	200.048	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	586	596	620	0	0	0	18563	29	0
N.S.	1	1.10	1.12	1.17	0.00	0.00	0.00	34.96	0.05	0.00
time (sec)	N/A	1.232	2.031	3.078	0.000	0.000	0.000	6.978	200.044	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	619	590	612	660	0	0	0	19913	29	0
N.S.	1	0.95	0.99	1.07	0.00	0.00	0.00	32.17	0.05	0.00
time (sec)	N/A	1.050	2.391	3.019	0.000	0.000	0.000	7.405	200.054	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	759	676	716	859	0	0	0	37269	27	0
N.S.	1	0.89	0.94	1.13	0.00	0.00	0.00	49.10	0.04	0.00
time (sec)	N/A	2.231	2.861	3.517	0.000	0.000	0.000	9.001	200.277	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	928	888	702	1081	0	0	0	28419	29	0
N.S.	1	0.96	0.76	1.16	0.00	0.00	0.00	30.62	0.03	0.00
time (sec)	N/A	4.145	1.157	7.333	0.000	0.000	0.000	6.789	202.668	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1267	1316	532	465	0	0	0	0	29	0
N.S.	1	1.04	0.42	0.37	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	2.864	6.589	3.250	0.000	0.000	0.000	0.000	200.038	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1265	1185	531	456	0	0	0	0	29	0
N.S.	1	0.94	0.42	0.36	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	2.199	7.446	3.522	0.000	0.000	0.000	0.000	200.033	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1702	1320	645	573	0	0	0	0	29	0
N.S.	1	0.78	0.38	0.34	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	2.531	8.581	3.770	0.000	0.000	0.000	0.000	200.030	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1534	0	649	573	0	0	0	0	26	0
N.S.	1	0.00	0.42	0.37	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	10.067	5.410	0.000	0.000	0.000	0.000	200.031	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2110	0	847	697	0	0	0	0	29	0
N.S.	1	0.00	0.40	0.33	0.00	0.00	0.00	0.00	0.01	0.00
time (sec)	N/A	0.000	10.693	8.977	0.000	0.000	0.000	0.000	200.028	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [35] had the largest ratio of [.500000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.04	20	0.150
2	A	5	4	1.01	20	0.200
3	A	4	3	1.01	18	0.167
4	A	7	6	1.01	20	0.300
5	A	4	3	0.99	20	0.150
6	A	7	6	1.02	20	0.300
7	A	2	2	1.00	20	0.100
8	A	2	2	1.00	20	0.100
9	A	2	2	1.00	17	0.118
10	A	2	2	1.00	20	0.100
11	A	2	2	1.00	20	0.100
12	A	4	3	1.00	22	0.136
13	A	7	6	1.01	22	0.273
14	A	6	5	1.04	20	0.250
15	A	9	8	1.02	22	0.364
16	A	4	3	1.01	22	0.136
17	A	9	8	0.98	22	0.364
18	A	2	2	0.99	22	0.091
19	A	2	2	0.99	22	0.091
20	F	0	0	N/A	0.000	N/A
21	A	2	2	1.01	22	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	22	0.091
23	A	7	6	1.03	22	0.273
24	A	4	3	1.39	22	0.136
25	A	7	6	1.01	22	0.273
26	A	4	3	1.01	20	0.150
27	A	9	8	1.02	22	0.364
28	A	4	3	1.43	22	0.136
29	A	9	8	1.01	22	0.364
30	F	0	0	N/A	0.000	N/A
31	F	0	0	N/A	0.000	N/A
32	A	2	2	1.00	19	0.105
33	F	0	0	N/A	0.000	N/A
34	F	0	0	N/A	0.000	N/A
35	A	12	11	0.90	22	0.500
36	A	4	3	1.66	22	0.136
37	A	4	3	0.99	22	0.136
38	A	4	3	1.01	20	0.150
39	A	4	3	1.01	22	0.136
40	A	4	3	1.96	22	0.136
41	A	4	3	0.97	22	0.136
42	F	0	0	N/A	0.000	N/A
43	F	0	0	N/A	0.000	N/A
44	A	2	2	1.00	19	0.105
45	F	0	0	N/A	0.000	N/A
46	F	0	0	N/A	0.000	N/A
47	A	4	3	0.99	25	0.120
48	A	4	3	0.98	25	0.120
49	A	6	5	1.03	25	0.200
50	A	4	3	1.04	25	0.120
51	A	4	3	1.02	25	0.120
52	A	4	3	1.00	25	0.120

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	A	8	7	1.03	25	0.280
54	A	5	4	1.01	25	0.160
55	A	4	3	0.99	23	0.130
56	A	5	4	0.98	25	0.160
57	A	7	6	0.96	25	0.240
58	A	5	5	0.85	25	0.200
59	A	5	5	0.88	25	0.200
60	A	4	4	0.87	22	0.182
61	A	6	6	0.88	25	0.240
62	A	6	6	0.88	25	0.240
63	A	10	9	1.61	23	0.391
64	A	7	6	1.03	23	0.261
65	B	5	4	2.08	21	0.190
66	A	4	3	1.07	23	0.130
67	A	10	9	1.44	23	0.391
68	A	10	9	1.48	23	0.391
69	A	10	9	1.86	23	0.391
70	A	11	10	1.26	23	0.435
71	A	9	8	1.22	20	0.400
72	A	9	8	1.87	23	0.348
73	A	11	10	1.32	23	0.435
74	A	4	3	0.99	27	0.111
75	A	7	6	0.87	27	0.222
76	A	4	3	1.01	27	0.111
77	A	4	3	1.00	27	0.111
78	A	4	3	0.99	27	0.111
79	A	4	3	0.99	27	0.111
80	A	4	3	0.98	27	0.111
81	A	4	3	0.99	25	0.120
82	A	4	3	0.99	27	0.111
83	A	4	3	0.98	27	0.111
84	A	2	2	1.11	27	0.074

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
85	A	2	2	1.11	27	0.074
86	A	2	2	1.11	24	0.083
87	A	2	2	1.10	27	0.074
88	A	2	2	1.10	27	0.074
89	A	8	7	1.16	25	0.280
90	A	5	4	0.99	25	0.160
91	A	5	4	0.98	25	0.160
92	A	6	5	1.21	25	0.200
93	A	6	5	1.09	25	0.200
94	A	8	7	1.00	25	0.280
95	A	6	5	0.99	25	0.200
96	A	5	4	0.98	25	0.160
97	A	6	5	0.97	23	0.217
98	A	7	6	0.96	25	0.240
99	A	7	7	0.77	25	0.280
100	A	7	7	0.81	25	0.280
101	A	6	6	0.81	25	0.240
102	A	7	7	0.81	25	0.280
103	A	6	6	0.81	22	0.273
104	A	8	8	0.77	25	0.320
105	A	6	5	1.17	27	0.185
106	A	6	5	1.17	27	0.185
107	A	5	4	1.19	27	0.148
108	A	4	3	0.99	27	0.111
109	A	4	3	0.99	27	0.111
110	A	9	8	0.97	27	0.296
111	A	10	9	1.10	27	0.333
112	A	10	9	0.95	27	0.333
113	A	4	3	0.89	25	0.120
114	A	4	3	0.96	27	0.111
115	A	12	12	1.04	27	0.444
116	A	10	10	0.94	27	0.370

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
117	A	11	11	0.78	27	0.407
118	F	0	0	N/A	0.000	N/A
119	F	0	0	N/A	0.000	N/A

CHAPTER 3

Listing of integrals

3.1 $\int x^5(c+dx^4)(a+bx^8)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int x^5(c+dx^4)(a+bx^8)^p\,dx=\frac{1}{6}cx^6(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^8}{a}\right)+\frac{1}{10}dx^{10}(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^8}{a}\right)$$

output
```
1/6*c*x^6*(b*x^8+a)^p*hypergeom([3/4, -p],[7/4],-b*x^8/a)/((1+b*x^8/a)^p)+
1/10*d*x^10*(b*x^8+a)^p*hypergeom([5/4, -p],[9/4],-b*x^8/a)/((1+b*x^8/a)^p
)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

$$\int x^5(c+dx^4)(a+bx^8)^p\,dx=\frac{1}{30}x^6(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(5c\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^8}{a}\right)+3dx^4\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^8}{a}\right)\right)$$

input
```
Integrate[x^5*(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
(x^6*(a + b*x^8)^p*(5*c*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^8)/a)] + 3*
d*x^4*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^8)/a)]))/(30*(1 + (b*x^8)/a)^
p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1815, 1675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(c+dx^4\right) \left(a+bx^8\right)^p \, dx$$

$$\Big\downarrow \text{1815}$$

$$\frac{1}{2}\int x^4 \left(dx^4+c\right) \left(bx^8+a\right)^p dx^2$$

$$\Big\downarrow \text{1675}$$

$$\frac{1}{2}\int \left(dx^8 \left(bx^8+a\right)^p + cx^4 \left(bx^8+a\right)^p\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{3}cx^6 \left(a+bx^8\right)^p \left(\frac{bx^8}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^8}{a}\right)+\frac{1}{5}dx^{10}\left(a+bx^8\right)^p \left(\frac{bx^8}{a}+1\right)^{-p} \text{Hyp}\right.$$

input
```
Int[x^5*(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
((c*x^6*(a + b*x^8)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^8)/a)])/(3*(1
 + (b*x^8)/a)^p) + (d*x^10*(a + b*x^8)^p*Hypergeometric2F1[5/4, -p, 9/4, -
((b*x^8)/a)])/(5*(1 + (b*x^8)/a)^p))/2
```

Defintions of rubi rules used

rule 1675
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

rule 1815
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/
k - 1)*(d + e*x^(n/k))^q*(a + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1]
/; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^5 \left(x^4 d+c\right) \left(b\,x^8+a\right)^p dx$$

input
```
int(x^5*(d*x^4+c)*(b*x^8+a)^p,x)
```

output
```
int(x^5*(d*x^4+c)*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int x^5 \left(c+dx^4\right) \left(a+bx^8\right)^p \, dx = \int \left(dx^4+c\right) \left(bx^8+a\right)^p x^5 \, dx$$

input
```
integrate(x^5*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x^9 + c*x^5)*(b*x^8 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 144.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int x^5(c+dx^4)\,(a+bx^8)^p\,dx = \frac{a^pcx^6\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{7}{4}\right)} + \frac{a^pdx^{10}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**5*(d*x**4+c)*(b*x**8+a)**p,x)
```

output
```
a**p*c*x**6*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**8*exp_polar(I*pi)/a)/
(8*gamma(7/4)) + a**p*d*x**10*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**8*e
xp_polar(I*pi)/a)/(8*gamma(9/4))
```

Maxima [F]

$$\int x^5(c+dx^4)\,(a+bx^8)^p\,dx = \int (dx^4+c)\,(bx^8+a)^p x^5\,dx$$

input
```
integrate(x^5*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p*x^5, x)
```

Giac [F]

$$\int x^5 \left(c+dx^4\right) \left(a+bx^8\right)^p \, dx = \int \left(dx^4+c\right)\left(bx^8+a\right)^p x^5 \, dx$$

input
```
integrate(x^5*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(c+dx^4\right) \left(a+bx^8\right)^p \, dx = \int x^5 \left(b\,x^8+a\right)^p \left(d\,x^4+c\right) \, dx$$

input
```
int(x^5*(a + b*x^8)^p*(c + d*x^4),x)
```

output
```
int(x^5*(a + b*x^8)^p*(c + d*x^4), x)
```

Reduce [F]

$$\int x^5 \left(c+dx^4\right) \left(a+bx^8\right)^p \, dx = \text{Too large to display}$$

input
```
int(x^5*(d*x^4+c)*(b*x^8+a)^p,x)
```

output

```
(16*(a + b*x**8)**p*a*d*p**2*x**2 + 12*(a + b*x**8)**p*a*d*p*x**2 + 16*(a
+ b*x**8)**p*b*c*p**2*x**6 + 24*(a + b*x**8)**p*b*c*p*x**6 + 5*(a + b*x**8
)**p*b*c*x**6 + 16*(a + b*x**8)**p*b*d*p**2*x**10 + 16*(a + b*x**8)**p*b*d
*p*x**10 + 3*(a + b*x**8)**p*b*d*x**10 + 8192*int(((a + b*x**8)**p*x**5)/(
64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b*p**2*x**8
+ 92*b*p*x**8 + 15*b*x**8),x)*a*b*c*p**6 + 30720*int(((a + b*x**8)**p*x**5
)/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b*p**2*x*
*8 + 92*b*p*x**8 + 15*b*x**8),x)*a*b*c*p**5 + 41984*int(((a + b*x**8)**p*x
**5)/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b*p**2
*x**8 + 92*b*p*x**8 + 15*b*x**8),x)*a*b*c*p**4 + 25344*int(((a + b*x**8)**
p*x**5)/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b*p
**2*x**8 + 92*b*p*x**8 + 15*b*x**8),x)*a*b*c*p**3 + 6560*int(((a + b*x**8)
**p*x**5)/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b
*p**2*x**8 + 92*b*p*x**8 + 15*b*x**8),x)*a*b*c*p**2 + 600*int(((a + b*x**8
)**p*x**5)/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*
b*p**2*x**8 + 92*b*p*x**8 + 15*b*x**8),x)*a*b*c*p - 2048*int(((a + b*x**8)
**p*x)/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b*p*
*2*x**8 + 92*b*p*x**8 + 15*b*x**8),x)*a**2*d*p**5 - 6144*int(((a + b*x**8)
**p*x)/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b*p*
*2*x**8 + 92*b*p*x**8 + 15*b*x**8),x)*a**2*d*p**4 - 6400*int(((a + b*x*...
```

3.2 $\int x^3(c+dx^4)\left(a+bx^8\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int x^3\left(c+dx^4\right)\left(a+bx^8\right)^p\,dx=\frac{d\left(a+bx^8\right)^{1+p}}{8b(1+p)}+\frac{1}{4}cx^4\left(a+bx^8\right)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)$$

output
```
1/8*d*(b*x^8+a)^(p+1)/b/(p+1)+1/4*c*x^4*(b*x^8+a)^p*hypergeom([1/2, -p],[3
/2],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.33

$$\int x^3\left(c+dx^4\right)\left(a+bx^8\right)^p\,dx=\frac{\left(a+bx^8\right)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(bdx^8\left(1+\frac{bx^8}{a}\right)^p+ad\left(-1+\left(1+\frac{bx^8}{a}\right)^p\right)+2bc(1+p)x^4\,\text{Hypergeometric2F1}\left(\right.\right.}{8b(1+p)}$$

input
```
Integrate[x^3*(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
((a + b*x^8)^p*(b*d*x^8*(1 + (b*x^8)/a)^p + a*d*(-1 + (1 + (b*x^8)/a)^p) +
 2*b*c*(1 + p)*x^4*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^8)/a)]))/(8*b*(1
 + p)*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1799, 455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (c + dx^4) (a + bx^8)^p \, dx$$

$$\Big\downarrow \text{1799}$$

$$\frac{1}{4}\int (dx^4 + c) (bx^8 + a)^p \, dx^4$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{4}\left(c\int (bx^8 + a)^p \, dx^4 + \frac{d(a + bx^8)^{p+1}}{2b(p+1)}\right)$$

$$\Big\downarrow \text{238}$$

$$\frac{1}{4}\left(c(a + bx^8)^p \left(\frac{bx^8}{a} + 1\right)^{-p} \int \left(\frac{bx^8}{a} + 1\right)^p dx^4 + \frac{d(a + bx^8)^{p+1}}{2b(p+1)}\right)$$

$$\Big\downarrow \text{237}$$

$$\frac{1}{4}\left(cx^4 (a + bx^8)^p \left(\frac{bx^8}{a} + 1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, -\frac{bx^8}{a}\right) + \frac{d(a + bx^8)^{p+1}}{2b(p+1)}\right)$$

input
```
Int[x^3*(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
((d*(a + b*x^8)^(1 + p))/(2*b*(1 + p)) + (c*x^4*(a + b*x^8)^p*Hypergeometr
ic2F1[1/2, -p, 3/2, -((b*x^8)/a)])/(1 + (b*x^8)/a)^p)/4
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

Maple [F]

$$\int x^{3} \left(x^{4} d+c\right) \left(b \,x^{8}+a\right)^{p}d x$$

input
```
int(x^3*(d*x^4+c)*(b*x^8+a)^p,x)
```

output
```
int(x^3*(d*x^4+c)*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int x^3 (c + dx^4) (a + bx^8)^p \, dx = \int (dx^4 + c)(bx^8 + a)^p x^3 \, dx$$

input
```
integrate(x^3*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x^7 + c*x^3)*(b*x^8 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 71.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int x^3 (c + dx^4) (a + bx^8)^p \, dx = \frac{a^p c x^4 {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^8 e^{i\pi}}{a}\right)}{4} + d \left(\begin{cases} \frac{a^p x^8}{8} & \text{for } b = 0 \\ \frac{\begin{cases} \frac{(a+bx^8)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a + bx^8) & \text{otherwise} \end{cases}}{8b} & \text{otherwise} \end{cases} \right)$$

input
```
integrate(x**3*(d*x**4+c)*(b*x**8+a)**p,x)
```

output
```
a**p*c*x**4*hyper((1/2, -p), (3/2,), b*x**8*exp_polar(I*pi)/a)/4 + d*Piece
wise((a**p*x**8/8, Eq(b, 0)), (Piecewise(((a + b*x**8)**(p + 1)/(p + 1), N
e(p, -1)), (log(a + b*x**8), True))/(8*b), True))
```

Maxima [F]

$$\int x^3(c+dx^4)\left(a+bx^8\right)^p\,dx=\int \left(dx^4+c\right)\left(bx^8+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p*x^3, x)
```

Giac [F]

$$\int x^3(c+dx^4)\left(a+bx^8\right)^p\,dx=\int \left(dx^4+c\right)\left(bx^8+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p*x^3, x)
```

Mupad [B] (verification not implemented)

Time = 22.99 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int x^3(c+dx^4)\left(a+bx^8\right)^p\,dx=\frac{d\,(b\,x^8+a)^{p+1}}{8\,b\,(p+1)}+\frac{c\,x^4\,(b\,x^8+a)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,x^8}{a}\right)}{4\left(\frac{b\,x^8}{a}+1\right)^p}$$

input
```
int(x^3*(a + b*x^8)^p*(c + d*x^4),x)
```

output
```
(d*(a + b*x^8)^(p + 1))/(8*b*(p + 1)) + (c*x^4*(a + b*x^8)^p*hypergeom([1/
2, -p], 3/2, -(b*x^8)/a))/(4*((b*x^8)/a + 1)^p)
```

Reduce [F]

$$\int x^3(c+dx^4)\left(a+bx^8\right)^p\,dx$$

$$= \frac{2(b\,x^8+a)^p\,adp+(b\,x^8+a)^p\,ad+2(b\,x^8+a)^p\,bcp\,x^4+2(b\,x^8+a)^p\,bc\,x^4+2(b\,x^8+a)^p\,bdp\,x^8+(b\,x^8+}{8b\,(2}$$

input
```
int(x^3*(d*x^4+c)*(b*x^8+a)^p,x)
```

output
```
(2*(a + b*x**8)**p*a*d*p + (a + b*x**8)**p*a*d + 2*(a + b*x**8)**p*b*c*p*x
**4 + 2*(a + b*x**8)**p*b*c*x**4 + 2*(a + b*x**8)**p*b*d*p*x**8 + (a + b*x
**8)**p*b*d*x**8 + 32*int(((a + b*x**8)**p*x**3)/(2*a*p + a + 2*b*p*x**8 +
 b*x**8),x)*a*b*c*p**3 + 48*int(((a + b*x**8)**p*x**3)/(2*a*p + a + 2*b*p*
x**8 + b*x**8),x)*a*b*c*p**2 + 16*int(((a + b*x**8)**p*x**3)/(2*a*p + a +
2*b*p*x**8 + b*x**8),x)*a*b*c*p)/(8*b*(2*p**2 + 3*p + 1))
```

3.3 $\int x(c+dx^4)(a+bx^8)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 101

$$\int x(c+dx^4)(a+bx^8)^p\,dx = \frac{1}{2}cx^2(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)+\frac{1}{6}dx^6(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^8}{a}\right)$$

output
```
1/2*c*x^2*(b*x^8+a)^p*hypergeom([1/4, -p],[5/4],-b*x^8/a)/((1+b*x^8/a)^p)+
1/6*d*x^6*(b*x^8+a)^p*hypergeom([3/4, -p],[7/4],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int x(c+dx^4)(a+bx^8)^p\,dx = \frac{1}{6}x^2(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(3c\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)+dx^4\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^8}{a}\right)\right)$$

input
```
Integrate[x*(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
(x^2*(a + b*x^8)^p*(3*c*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^8)/a)] + d*
x^4*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^8)/a)]))/(6*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1815, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c+dx^4\right)\left(a+bx^8\right)^p\,dx$$

$$\Big\downarrow \text{1815}$$

$$\frac{1}{2}\int \left(dx^4+c\right)\left(bx^8+a\right)^p dx^2$$

$$\Big\downarrow \text{1516}$$

$$\frac{1}{2}\int \left(dx^4\left(bx^8+a\right)^p+c\left(bx^8+a\right)^p\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(cx^2\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)+\frac{1}{3}dx^6\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hyperg}\right.$$

input
```
Int[x*(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
((c*x^2*(a + b*x^8)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^8)/a)])/(1 +
(b*x^8)/a)^p + (d*x^6*(a + b*x^8)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x
^8)/a)])/(3*(1 + (b*x^8)/a)^p))/2
```

Defintions of rubi rules used

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 1815
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/
k - 1)*(d + e*x^(n/k))^q*(a + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1]
/; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x\left(x^{4} d+c\right) \left(b\,x^{8}+a\right)^{p}d x$$

input
```
int(x*(d*x^4+c)*(b*x^8+a)^p,x)
```

output
```
int(x*(d*x^4+c)*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int x\left(c+dx^4\right)\left(a+bx^8\right)^p\,dx=\int\left(dx^4+c\right)\left(bx^8+a\right)^px\,dx$$

input
```
integrate(x*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x^5 + c*x)*(b*x^8 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 91.71 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int x(c+dx^4)\left(a+bx^8\right)^p\,dx = \frac{a^p c x^2 \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4}, -p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{5}{4}\right)} + \frac{a^p d x^6 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}\frac{3}{4}, -p\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x*(d*x**4+c)*(b*x**8+a)**p,x)
```

output
```
a**p*c*x**2*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**8*exp_polar(I*pi)/a)/
(8*gamma(5/4)) + a**p*d*x**6*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**8*ex
p_polar(I*pi)/a)/(8*gamma(7/4))
```

Maxima [F]

$$\int x(c+dx^4)\left(a+bx^8\right)^p\,dx = \int \left(dx^4+c\right)\left(bx^8+a\right)^p x\,dx$$

input
```
integrate(x*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p*x, x)
```

Giac [F]

$$\int x(c+dx^4)(a+bx^8)^p\,dx = \int (dx^4+c)(bx^8+a)^p x\,dx$$

input
```
integrate(x*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+dx^4)(a+bx^8)^p\,dx = \int x\,(b\,x^8+a)^p\,(d\,x^4+c)\,dx$$

input
```
int(x*(a + b*x^8)^p*(c + d*x^4),x)
```

output
```
int(x*(a + b*x^8)^p*(c + d*x^4), x)
```

Reduce [F]

$$\int x(c+dx^4)(a+bx^8)^p\,dx$$
$$= \frac{4(b\,x^8+a)^p\,cp\,x^2+3(b\,x^8+a)^p\,c\,x^2+4(b\,x^8+a)^p\,dp\,x^6+(b\,x^8+a)^p\,d\,x^6+512\Big(\int \frac{(b\,x^8+a)^p\,x}{16b\,p^2x^8+16bp\,x^8+3b\,x^8+\cdots}}{\cdots}$$

input
```
int(x*(d*x^4+c)*(b*x^8+a)^p,x)
```

output

```
(4*(a + b*x**8)**p*c*p*x**2 + 3*(a + b*x**8)**p*c*x**2 + 4*(a + b*x**8)**p
*d*p*x**6 + (a + b*x**8)**p*d*x**6 + 512*int(((a + b*x**8)**p*x**5)/(16*a*
p**2 + 16*a*p + 3*a + 16*b*p**2*x**8 + 16*b*p*x**8 + 3*b*x**8),x)*a*d*p**4
 + 640*int(((a + b*x**8)**p*x**5)/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x*
*8 + 16*b*p*x**8 + 3*b*x**8),x)*a*d*p**3 + 224*int(((a + b*x**8)**p*x**5)/
(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**8 + 16*b*p*x**8 + 3*b*x**8),x)*a*
d*p**2 + 24*int(((a + b*x**8)**p*x**5)/(16*a*p**2 + 16*a*p + 3*a + 16*b*p*
*2*x**8 + 16*b*p*x**8 + 3*b*x**8),x)*a*d*p + 512*int(((a + b*x**8)**p*x)/(
16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**8 + 16*b*p*x**8 + 3*b*x**8),x)*a*c
*p**4 + 896*int(((a + b*x**8)**p*x)/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*
x**8 + 16*b*p*x**8 + 3*b*x**8),x)*a*c*p**3 + 480*int(((a + b*x**8)**p*x)/(
16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**8 + 16*b*p*x**8 + 3*b*x**8),x)*a*c
*p**2 + 72*int(((a + b*x**8)**p*x)/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x
**8 + 16*b*p*x**8 + 3*b*x**8),x)*a*c*p)/(2*(16*p**2 + 16*p + 3))
```

3.4 $\int \frac{(c+dx^4)(a+bx^8)^p}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x}\,dx = \frac{1}{4}dx^4(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right) - \frac{c(a+bx^8)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^8}{a}\right)}{8a(1+p)}$$

output

```
1/4*d*x^4*(b*x^8+a)^p*hypergeom([1/2, -p],[3/2],-b*x^8/a)/((1+b*x^8/a)^p)-
1/8*c*(b*x^8+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^8/a)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x} dx$$
$$= \frac{1}{8}(a+bx^8)^p \left(2dx^4\left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right) - \frac{c(a+bx^8)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^8}{a}\right)}{a(1+p)}\right)$$

input
```
Integrate[((c + d*x^4)*(a + b*x^8)^p)/x,x]
```

output
```
((a + b*x^8)^p*((2*d*x^4*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^8)/a)])/(1
 + (b*x^8)/a)^p - (c*(a + b*x^8)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*x^8)/a])/(a*(1 + p))))/8
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1803, 542, 238, 237, 243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x} dx$$
$$\Big\downarrow \text{1803}$$
$$\frac{1}{4}\int \frac{(dx^4+c)(bx^8+a)^p}{x^4}dx^4$$
$$\Big\downarrow \text{542}$$

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{x^4}dx^4+d\int (bx^8+a)^p\,dx^4\right)$$

$$\Big\downarrow \; 238$$

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{x^4}dx^4+d(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \left(\frac{bx^8}{a}+1\right)^p dx^4\right)$$

$$\Big\downarrow \; 237$$

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{x^4}dx^4+dx^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)\right)$$

$$\Big\downarrow \; 243$$

$$\frac{1}{4}\left(\frac{1}{2}c\int \frac{(bx^8+a)^p}{x^4}dx^8+dx^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)\right)$$

$$\Big\downarrow \; 75$$

$$\frac{1}{4}\left(dx^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)-\frac{c(a+bx^8)^{p+1}\text{Hypergeometric2F1}\left(1\right.}{2a(p+1)}\right.$$

input
```
Int[((c + d*x^4)*(a + b*x^8)^p)/x,x]
```

output
```
((d*x^4*(a + b*x^8)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^8)/a)])/(1 +
(b*x^8)/a)^p - (c*(a + b*x^8)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1
 + (b*x^8)/a])/(2*a*(1 + p)))/4
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(x^4d+c)\,(b\,x^8+a)^p}{x}dx$$

input
```
int((d*x^4+c)*(b*x^8+a)^p/x,x)
```

output
```
int((d*x^4+c)*(b*x^8+a)^p/x,x)
```

Fricas [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x}\,dx=\int \frac{(dx^4+c)(bx^8+a)^p}{x}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x,x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)*(b*x^8 + a)^p/x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 70.63 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x}\,dx=\frac{a^p dx^4\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\right|\frac{bx^8e^{i\pi}}{a}\right)}{4}-\frac{b^p cx^{8p}\Gamma(-p)\,{}_2F_1\left(\left.\begin{matrix}-p,-p\\ 1-p\end{matrix}\right|\frac{ae^{i\pi}}{bx^8}\right)}{8\Gamma(1-p)}$$

input
```
integrate((d*x**4+c)*(b*x**8+a)**p/x,x)
```

output
```
a**p*d*x**4*hyper((1/2, -p), (3/2,), b*x**8*exp_polar(I*pi)/a)/4 - b**p*c*
x**(8*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**8))/(
8*gamma(1 - p))
```

Maxima [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x}\,dx=\int \frac{(dx^4+c)(bx^8+a)^p}{x}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p/x, x)
```

Giac [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x}\,dx = \int \frac{(dx^4+c)(bx^8+a)^p}{x}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x}\,dx = \int \frac{(b\,x^8+a)^p\,(d\,x^4+c)}{x}\,dx$$

input
```
int(((a + b*x^8)^p*(c + d*x^4))/x,x)
```

output
```
int(((a + b*x^8)^p*(c + d*x^4))/x, x)
```

Reduce [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x}\,dx = \frac{2(b\,x^8+a)^p\,cp+(b\,x^8+a)^p\,c+2(b\,x^8+a)^p\,dp\,x^4+32\left(\int \frac{(b\,x^8+a)^p}{2bp\,x^9+b\,x^9+2apx+ax}dx\right)ac\,p^3+32\left(\int \frac{(b\,x^8+}{2bp\,x^9+b\,x^9+}\right.}{}$$

input
```
int((d*x^4+c)*(b*x^8+a)^p/x,x)
```

output
```
(2*(a + b*x**8)**p*c*p + (a + b*x**8)**p*c + 2*(a + b*x**8)**p*d*p*x**4 +
32*int((a + b*x**8)**p/(2*a*p*x + a*x + 2*b*p*x**9 + b*x**9),x)*a*c*p**3 +
 32*int((a + b*x**8)**p/(2*a*p*x + a*x + 2*b*p*x**9 + b*x**9),x)*a*c*p**2
+ 8*int((a + b*x**8)**p/(2*a*p*x + a*x + 2*b*p*x**9 + b*x**9),x)*a*c*p + 3
2*int(((a + b*x**8)**p*x**3)/(2*a*p + a + 2*b*p*x**8 + b*x**8),x)*a*d*p**3
 + 16*int(((a + b*x**8)**p*x**3)/(2*a*p + a + 2*b*p*x**8 + b*x**8),x)*a*d*
p**2)/(8*p*(2*p + 1))
```

3.5 $\int \frac{(c+dx^4)(a+bx^8)^p}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^3} dx = -\frac{c(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{4},-p,\frac{3}{4},-\frac{bx^8}{a}\right)}{2x^2} + \frac{1}{2}dx^2(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)$$

output
```
-1/2*c*(b*x^8+a)^p*hypergeom([-1/4, -p],[3/4],-b*x^8/a)/x^2/((1+b*x^8/a)^p
)+1/2*d*x^2*(b*x^8+a)^p*hypergeom([1/4, -p],[5/4],-b*x^8/a)/((1+b*x^8/a)^p
)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^3} dx = \frac{(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(-c\,\text{Hypergeometric2F1}\left(-\frac{1}{4},-p,\frac{3}{4},-\frac{bx^8}{a}\right)+dx^4\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4}\right.\right.}{2x^2}$$

input
```
Integrate[((c + d*x^4)*(a + b*x^8)^p)/x^3,x]
```

output
```
((a + b*x^8)^p*(-(c*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^8)/a)]) + d*x^
4*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^8)/a)]))/(2*x^2*(1 + (b*x^8)/a)^p
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1815, 1675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^3}dx$$

$$\Big\downarrow \text{1815}$$

$$\frac{1}{2}\int \frac{(dx^4+c)(bx^8+a)^p}{x^4}dx^2$$

$$\Big\downarrow \text{1675}$$

$$\frac{1}{2}\int \left(d(bx^8+a)^p+\frac{c(bx^8+a)^p}{x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(dx^2(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)-\frac{c(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeom}}{x^2}\right.$$

input
```
Int[((c + d*x^4)*(a + b*x^8)^p)/x^3,x]
```

output
```
(-((c*(a + b*x^8)^p*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^8)/a)])/(x^2*(
1 + (b*x^8)/a)^p)) + (d*x^2*(a + b*x^8)^p*Hypergeometric2F1[1/4, -p, 5/4,
-((b*x^8)/a)])/(1 + (b*x^8)/a)^p)/2
```

Defintions of rubi rules used

rule 1675
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

rule 1815
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/
k - 1)*(d + e*x^(n/k))^q*(a + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1]
/; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(x^4d+c)\,(b\,x^8+a)^p}{x^3}dx$$

input
```
int((d*x^4+c)*(b*x^8+a)^p/x^3,x)
```

output
```
int((d*x^4+c)*(b*x^8+a)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^3}\,dx = \int \frac{(dx^4+c)(bx^8+a)^p}{x^3}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x^3,x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)*(b*x^8 + a)^p/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 104.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^3}\,dx = \frac{a^p c\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, -p \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^8 e^{i\pi}}{a}\right)}{8x^2\Gamma\left(\frac{3}{4}\right)} + \frac{a^p dx^2\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^8 e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**4+c)*(b*x**8+a)**p/x**3,x)
```

output
```
a**p*c*gamma(-1/4)*hyper((-1/4, -p), (3/4,), b*x**8*exp_polar(I*pi)/a)/(8*
x**2*gamma(3/4)) + a**p*d*x**2*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**8*
exp_polar(I*pi)/a)/(8*gamma(5/4))
```

Maxima [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^3}\,dx = \int \frac{(dx^4+c)(bx^8+a)^p}{x^3}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^3}\,dx = \int \frac{(dx^4+c)(bx^8+a)^p}{x^3}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x^3,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^3}\,dx = \int \frac{(b\,x^8+a)^p\,(d\,x^4+c)}{x^3}\,dx$$

input
```
int(((a + b*x^8)^p*(c + d*x^4))/x^3,x)
```

output
```
int(((a + b*x^8)^p*(c + d*x^4))/x^3, x)
```

Reduce [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^3}\,dx$$
$$= \frac{4(b\,x^8+a)^p\,cp+(b\,x^8+a)^p\,c+4(b\,x^8+a)^p\,dp\,x^4-(b\,x^8+a)^p\,d\,x^4+512\Big(\int \frac{(b\,x^8+a)^p}{16b\,p^2x^{11}-b\,x^{11}+16a\,p^2x^3-a\,x^3}dx$$

input
```
int((d*x^4+c)*(b*x^8+a)^p/x^3,x)
```

output
```
(4*(a + b*x**8)**p*c*p + (a + b*x**8)**p*c + 4*(a + b*x**8)**p*d*p*x**4 -
(a + b*x**8)**p*d*x**4 + 512*int((a + b*x**8)**p/(16*a*p**2*x**3 - a*x**3
+ 16*b*p**2*x**11 - b*x**11),x)*a*c*p**4*x**2 + 128*int((a + b*x**8)**p/(1
6*a*p**2*x**3 - a*x**3 + 16*b*p**2*x**11 - b*x**11),x)*a*c*p**3*x**2 - 32*
int((a + b*x**8)**p/(16*a*p**2*x**3 - a*x**3 + 16*b*p**2*x**11 - b*x**11),
x)*a*c*p**2*x**2 - 8*int((a + b*x**8)**p/(16*a*p**2*x**3 - a*x**3 + 16*b*p
**2*x**11 - b*x**11),x)*a*c*p*x**2 + 512*int(((a + b*x**8)**p*x)/(16*a*p**
2 - a + 16*b*p**2*x**8 - b*x**8),x)*a*d*p**4*x**2 - 128*int(((a + b*x**8)*
*p*x)/(16*a*p**2 - a + 16*b*p**2*x**8 - b*x**8),x)*a*d*p**3*x**2 - 32*int(
((a + b*x**8)**p*x)/(16*a*p**2 - a + 16*b*p**2*x**8 - b*x**8),x)*a*d*p**2*
x**2 + 8*int(((a + b*x**8)**p*x)/(16*a*p**2 - a + 16*b*p**2*x**8 - b*x**8)
,x)*a*d*p*x**2)/(2*x**2*(16*p**2 - 1))
```

3.6 $\int \frac{(c+dx^4)(a+bx^8)^p}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^5}\,dx = -\frac{c(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^8}{a}\right)}{4x^4} - \frac{d(a+bx^8)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^8}{a}\right)}{8a(1+p)}$$

output
```
-1/4*c*(b*x^8+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^8/a)/x^4/((1+b*x^8/a)^p
)-1/8*d*(b*x^8+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^8/a)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^5}\,dx$$
$$=\frac{1}{8}(a+bx^8)^p\left(-\frac{2c\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^8}{a}\right)}{x^4}-\frac{d(a+bx^8)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^8}{a}\right)}{a(1+p)}\right)$$

input
```
Integrate[((c + d*x^4)*(a + b*x^8)^p)/x^5,x]
```

output
```
((a + b*x^8)^p*((-2*c*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^8)/a)])/(x^4
*(1 + (b*x^8)/a)^p) - (d*(a + b*x^8)*Hypergeometric2F1[1, 1 + p, 2 + p, 1
+ (b*x^8)/a])/(a*(1 + p))))/8
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1803, 542, 243, 75, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^5}\,dx$$
$$\Big\downarrow \text{1803}$$
$$\frac{1}{4}\int \frac{(dx^4+c)(bx^8+a)^p}{x^8}dx^4$$
$$\Big\downarrow \text{542}$$

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{x^8}dx^4+d\int \frac{(bx^8+a)^p}{x^4}dx^4\right)$$

$\downarrow$ 243

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{x^8}dx^4+\frac{1}{2}d\int \frac{(bx^8+a)^p}{x^4}dx^8\right)$$

$\downarrow$ 75

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{x^8}dx^4-\frac{d(a+bx^8)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^8}{a}+1\right)}{2a(p+1)}\right)$$

$\downarrow$ 279

$$\frac{1}{4}\left(c(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^8}{a}+1\right)^p}{x^8}dx^4-\frac{d(a+bx^8)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^8}{a}+1\right)}{2a(p+1)}\right)$$

$\downarrow$ 278

$$\frac{1}{4}\left(-\frac{c(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^8}{a}\right)}{x^4}-\frac{d(a+bx^8)^{p+1}\text{Hypergeometric2F1}\left(1,\right.}{2a(p+1)}\right.$$

input
```
Int[((c + d*x^4)*(a + b*x^8)^p)/x^5,x]
```

output
```
(-((c*(a + b*x^8)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^8)/a)])/(x^4*(
1 + (b*x^8)/a)^p)) - (d*(a + b*x^8)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 + (b*x^8)/a])/(2*a*(1 + p)))/4
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(x^4d + c)\,(b\,x^8 + a)^p}{x^5}dx$$

input
```
int((d*x^4+c)*(b*x^8+a)^p/x^5,x)
```

output
```
int((d*x^4+c)*(b*x^8+a)^p/x^5,x)
```

Fricas [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^5}\,dx = \int \frac{(dx^4+c)(bx^8+a)^p}{x^5}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x^5,x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)*(b*x^8 + a)^p/x^5, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 113.82 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.76

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^5}\,dx = -\frac{a^p c\,{}_2F_1\left(\begin{matrix}-\frac{1}{2}, -p\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{4x^4} - \frac{b^p dx^{8p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p, -p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^8}\right)}{8\Gamma(1-p)}$$

input
```
integrate((d*x**4+c)*(b*x**8+a)**p/x**5,x)
```

output
```
-a**p*c*hyper((-1/2, -p), (1/2,), b*x**8*exp_polar(I*pi)/a)/(4*x**4) - b**
p*d*x**(8*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**8
))/(8*gamma(1 - p))
```

Maxima [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^5}\,dx = \int \frac{(dx^4+c)(bx^8+a)^p}{x^5}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x^5,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p/x^5, x)
```

Giac [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^5}\,dx = \int \frac{(dx^4+c)(bx^8+a)^p}{x^5}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x^5,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^5}\,dx = \int \frac{(b\,x^8+a)^p\,(d\,x^4+c)}{x^5}\,dx$$

input
```
int(((a + b*x^8)^p*(c + d*x^4))/x^5,x)
```

output
```
int(((a + b*x^8)^p*(c + d*x^4))/x^5, x)
```

Reduce [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^5}\,dx$$
$$= \frac{2(b\,x^8+a)^p\,cp+2(b\,x^8+a)^p\,dp\,x^4-(b\,x^8+a)^p\,d\,x^4+32\left(\int \frac{(b\,x^8+a)^p}{2bp\,x^{13}-b\,x^{13}+2ap\,x^5-a\,x^5}dx\right)ac\,p^3x^4-16\left(\int \frac{}{2}\right.}{}$$

input
```
int((d*x^4+c)*(b*x^8+a)^p/x^5,x)
```

output
```
(2*(a + b*x**8)**p*c*p + 2*(a + b*x**8)**p*d*p*x**4 - (a + b*x**8)**p*d*x*
*4 + 32*int((a + b*x**8)**p/(2*a*p*x**5 - a*x**5 + 2*b*p*x**13 - b*x**13),
x)*a*c*p**3*x**4 - 16*int((a + b*x**8)**p/(2*a*p*x**5 - a*x**5 + 2*b*p*x**
13 - b*x**13),x)*a*c*p**2*x**4 + 32*int((a + b*x**8)**p/(2*a*p*x - a*x + 2
*b*p*x**9 - b*x**9),x)*a*d*p**3*x**4 - 32*int((a + b*x**8)**p/(2*a*p*x - a
*x + 2*b*p*x**9 - b*x**9),x)*a*d*p**2*x**4 + 8*int((a + b*x**8)**p/(2*a*p*
x - a*x + 2*b*p*x**9 - b*x**9),x)*a*d*p*x**4)/(8*p*x**4*(2*p - 1))
```

3.7 $\int x^4(c+dx^4)(a+bx^8)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int x^4(c+dx^4)(a+bx^8)^p\,dx=\frac{1}{5}cx^5(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)+\frac{1}{9}dx^9(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{9}{8},-p,\frac{17}{8},-\frac{bx^8}{a}\right)$$

output
```
1/5*c*x^5*(b*x^8+a)^p*hypergeom([5/8, -p],[13/8],-b*x^8/a)/((1+b*x^8/a)^p)
+1/9*d*x^9*(b*x^8+a)^p*hypergeom([9/8, -p],[17/8],-b*x^8/a)/((1+b*x^8/a)^p
)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

$$\int x^4(c+dx^4)(a+bx^8)^p\,dx=\frac{1}{45}x^5(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(9c\,\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)+5dx^4\,\text{Hypergeometric2F1}\left(\frac{9}{8},-p,\frac{17}{8},-\frac{bx^8}{a}\right)\right)$$

input
```
Integrate[x^4*(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
(x^5*(a + b*x^8)^p*(9*c*Hypergeometric2F1[5/8, -p, 13/8, -((b*x^8)/a)] + 5
*d*x^4*Hypergeometric2F1[9/8, -p, 17/8, -((b*x^8)/a)]))/(45*(1 + (b*x^8)/a
)^p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(c+dx^4\right) \left(a+bx^8\right)^p \, dx$$

$$\Big\downarrow \text{1865}$$

$$\int \left(cx^4 \left(a+bx^8\right)^p+dx^8 \left(a+bx^8\right)^p\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5} c x^5 \left(a+bx^8\right)^p \left(\frac{bx^8}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)+\frac{1}{9} d x^9 \left(a+bx^8\right)^p \left(\frac{bx^8}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{9}{8},-p,\frac{17}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[x^4*(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
(c*x^5*(a + b*x^8)^p*Hypergeometric2F1[5/8, -p, 13/8, -((b*x^8)/a)])/(5*(1
 + (b*x^8)/a)^p) + (d*x^9*(a + b*x^8)^p*Hypergeometric2F1[9/8, -p, 17/8, -
((b*x^8)/a)])/(9*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1865
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (f*x)^m*(d
 + e*x^n)^q, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && I
GtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^{4}\left(x^{4} d+c\right)\left(b\,x^{8}+a\right)^{p}dx$$

input
```
int(x^4*(d*x^4+c)*(b*x^8+a)^p,x)
```

output
```
int(x^4*(d*x^4+c)*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int x^{4}(c+dx^{4})\left(a+bx^{8}\right)^{p}\,dx=\int\left(dx^{4}+c\right)\left(bx^{8}+a\right)^{p}x^{4}\,dx$$

input
```
integrate(x^4*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x^8 + c*x^4)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4(c+dx^4)\left(a+bx^8\right)^p\,dx = \text{Timed out}$$

input
```
integrate(x**4*(d*x**4+c)*(b*x**8+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^4(c+dx^4)\left(a+bx^8\right)^p\,dx = \int (dx^4+c)\left(bx^8+a\right)^p x^4\,dx$$

input
```
integrate(x^4*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p*x^4, x)
```

Giac [F]

$$\int x^4(c+dx^4)\left(a+bx^8\right)^p\,dx = \int (dx^4+c)\left(bx^8+a\right)^p x^4\,dx$$

input
```
integrate(x^4*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(c+dx^4)\left(a+bx^8\right)^p\,dx=\int x^4\left(b\,x^8+a\right)^p\left(d\,x^4+c\right)\,dx$$

input
```
int(x^4*(a + b*x^8)^p*(c + d*x^4),x)
```

output
```
int(x^4*(a + b*x^8)^p*(c + d*x^4), x)
```

Reduce [F]

$$\int x^4(c+dx^4)\left(a+bx^8\right)^p\,dx=\text{Too large to display}$$

input
```
int(x^4*(d*x^4+c)*(b*x^8+a)^p,x)
```

output
```
(64*(a + b*x**8)**p*a*d*p**2*x + 40*(a + b*x**8)**p*a*d*p*x + 64*(a + b*x*
*8)**p*b*c*p**2*x**5 + 80*(a + b*x**8)**p*b*c*p*x**5 + 9*(a + b*x**8)**p*b
*c*x**5 + 64*(a + b*x**8)**p*b*d*p**2*x**9 + 48*(a + b*x**8)**p*b*d*p*x**9
 + 5*(a + b*x**8)**p*b*d*x**9 - 32768*int((a + b*x**8)**p/(512*a*p**3 + 96
0*a*p**2 + 472*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x*
*8 + 45*b*x**8),x)*a**2*d*p**5 - 81920*int((a + b*x**8)**p/(512*a*p**3 + 9
60*a*p**2 + 472*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x
**8 + 45*b*x**8),x)*a**2*d*p**4 - 68608*int((a + b*x**8)**p/(512*a*p**3 +
960*a*p**2 + 472*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*
x**8 + 45*b*x**8),x)*a**2*d*p**3 - 21760*int((a + b*x**8)**p/(512*a*p**3 +
 960*a*p**2 + 472*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p
*x**8 + 45*b*x**8),x)*a**2*d*p**2 - 1800*int((a + b*x**8)**p/(512*a*p**3 +
 960*a*p**2 + 472*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p
*x**8 + 45*b*x**8),x)*a**2*d*p + 262144*int(((a + b*x**8)**p*x**4)/(512*a*
p**3 + 960*a*p**2 + 472*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 4
72*b*p*x**8 + 45*b*x**8),x)*a*b*c*p**6 + 819200*int(((a + b*x**8)**p*x**4)
/(512*a*p**3 + 960*a*p**2 + 472*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*
x**8 + 472*b*p*x**8 + 45*b*x**8),x)*a*b*c*p**5 + 892928*int(((a + b*x**8)*
*p*x**4)/(512*a*p**3 + 960*a*p**2 + 472*a*p + 45*a + 512*b*p**3*x**8 + 960
*b*p**2*x**8 + 472*b*p*x**8 + 45*b*x**8),x)*a*b*c*p**4 + 394240*int(((a...
```

3.8 $\int x^2(c+dx^4)(a+bx^8)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int x^2(c+dx^4)(a+bx^8)^p\,dx=\frac{1}{3}cx^3(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)+\frac{1}{7}dx^7(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{8},-p,\frac{15}{8},-\frac{bx^8}{a}\right)$$

output
```
1/3*c*x^3*(b*x^8+a)^p*hypergeom([3/8, -p],[11/8],-b*x^8/a)/((1+b*x^8/a)^p)
+1/7*d*x^7*(b*x^8+a)^p*hypergeom([7/8, -p],[15/8],-b*x^8/a)/((1+b*x^8/a)^p
)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

$$\int x^2(c+dx^4)(a+bx^8)^p\,dx=\frac{1}{21}x^3(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(7c\,\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)+3dx^4\,\text{Hypergeometric2F1}\left(\frac{7}{8},-p,\frac{15}{8},-\frac{bx^8}{a}\right)\right)$$

input
```
Integrate[x^2*(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
(x^3*(a + b*x^8)^p*(7*c*Hypergeometric2F1[3/8, -p, 11/8, -((b*x^8)/a)] + 3
*d*x^4*Hypergeometric2F1[7/8, -p, 15/8, -((b*x^8)/a)]))/(21*(1 + (b*x^8)/a
)^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (c+dx^4) (a+bx^8)^p \, dx$$

$$\Big\downarrow \text{ 1865}$$

$$\int \left(cx^2 (a+bx^8)^p + dx^6 (a+bx^8)^p\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} cx^3 (a+bx^8)^p \left(\frac{bx^8}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right) + \frac{1}{7} dx^7 (a+bx^8)^p \left(\frac{bx^8}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{7}{8},-p,\frac{15}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[x^2*(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
(c*x^3*(a + b*x^8)^p*Hypergeometric2F1[3/8, -p, 11/8, -((b*x^8)/a)])/(3*(1
 + (b*x^8)/a)^p) + (d*x^7*(a + b*x^8)^p*Hypergeometric2F1[7/8, -p, 15/8, -
((b*x^8)/a)])/(7*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1865
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (f*x)^m*(d
 + e*x^n)^q, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && I
GtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^{2}\left(x^{4} d+c\right)\left(b\,x^{8}+a\right)^{p}dx$$

input
```
int(x^2*(d*x^4+c)*(b*x^8+a)^p,x)
```

output
```
int(x^2*(d*x^4+c)*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int x^2\left(c+dx^4\right)\left(a+bx^8\right)^p\,dx=\int\left(dx^4+c\right)\left(bx^8+a\right)^px^2\,dx$$

input
```
integrate(x^2*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x^6 + c*x^2)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(c+dx^4)\left(a+bx^8\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**2*(d*x**4+c)*(b*x**8+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(c+dx^4)\left(a+bx^8\right)^p\,dx=\int (dx^4+c)\left(bx^8+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(c+dx^4)\left(a+bx^8\right)^p\,dx=\int (dx^4+c)\left(bx^8+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(d*x^4+c)*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (c+dx^4) (a+bx^8)^p \, dx = \int x^2 (b x^8 + a)^p (d x^4 + c) \, dx$$

input
```
int(x^2*(a + b*x^8)^p*(c + d*x^4),x)
```

output
```
int(x^2*(a + b*x^8)^p*(c + d*x^4), x)
```

Reduce [F]

$$\int x^2 (c+dx^4) (a+bx^8)^p \, dx = \frac{8(b x^8 + a)^p cp x^3 + 7(b x^8 + a)^p c x^3 + 8(b x^8 + a)^p dp x^7 + 3(b x^8 + a)^p d x^7 + 4096\left(\int \frac{(b x^8 +}{64b p^2 x^8 + 80bp x^8 + 21b}\right.}{}$$

input
```
int(x^2*(d*x^4+c)*(b*x^8+a)^p,x)
```

output
```
(8*(a + b*x**8)**p*c*p*x**3 + 7*(a + b*x**8)**p*c*x**3 + 8*(a + b*x**8)**p
*d*p*x**7 + 3*(a + b*x**8)**p*d*x**7 + 4096*int(((a + b*x**8)**p*x**6)/(64
*a*p**2 + 80*a*p + 21*a + 64*b*p**2*x**8 + 80*b*p*x**8 + 21*b*x**8),x)*a*d
*p**4 + 6656*int(((a + b*x**8)**p*x**6)/(64*a*p**2 + 80*a*p + 21*a + 64*b*
p**2*x**8 + 80*b*p*x**8 + 21*b*x**8),x)*a*d*p**3 + 3264*int(((a + b*x**8)*
*p*x**6)/(64*a*p**2 + 80*a*p + 21*a + 64*b*p**2*x**8 + 80*b*p*x**8 + 21*b*
x**8),x)*a*d*p**2 + 504*int(((a + b*x**8)**p*x**6)/(64*a*p**2 + 80*a*p + 2
1*a + 64*b*p**2*x**8 + 80*b*p*x**8 + 21*b*x**8),x)*a*d*p + 4096*int(((a +
b*x**8)**p*x**2)/(64*a*p**2 + 80*a*p + 21*a + 64*b*p**2*x**8 + 80*b*p*x**8
 + 21*b*x**8),x)*a*c*p**4 + 8704*int(((a + b*x**8)**p*x**2)/(64*a*p**2 + 8
0*a*p + 21*a + 64*b*p**2*x**8 + 80*b*p*x**8 + 21*b*x**8),x)*a*c*p**3 + 582
4*int(((a + b*x**8)**p*x**2)/(64*a*p**2 + 80*a*p + 21*a + 64*b*p**2*x**8 +
 80*b*p*x**8 + 21*b*x**8),x)*a*c*p**2 + 1176*int(((a + b*x**8)**p*x**2)/(6
4*a*p**2 + 80*a*p + 21*a + 64*b*p**2*x**8 + 80*b*p*x**8 + 21*b*x**8),x)*a*
c*p)/(64*p**2 + 80*p + 21)
```

3.9 $\int (c+dx^4)(a+bx^8)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 96

$$\int (c+dx^4)(a+bx^8)^p \, dx = cx(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+\frac{1}{5}dx^5(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)$$

output
```
c*x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)+1/5*d*
x^5*(b*x^8+a)^p*hypergeom([5/8, -p],[13/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int (c+dx^4)(a+bx^8)^p \, dx = \frac{1}{5}x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \left(5c\,\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+dx^4\,\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)\right)$$

input
```
Integrate[(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*(5*c*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] + d*x^
4*Hypergeometric2F1[5/8, -p, 13/8, -((b*x^8)/a)]))/(5*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1763, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c + dx^4) (a + bx^8)^p \, dx$$

$$\Big\downarrow \text{1763}$$

$$\int \left(c(a + bx^8)^p + dx^4 (a + bx^8)^p\right) dx$$

$$\Big\downarrow \text{2009}$$

$$cx(a + bx^8)^p \left(\frac{bx^8}{a} + 1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8}, -p, \frac{9}{8}, -\frac{bx^8}{a}\right) + \frac{1}{5} dx^5 (a + bx^8)^p \left(\frac{bx^8}{a} + 1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{8}, -p, \frac{13}{8}, -\frac{bx^8}{a}\right)$$

input
```
Int[(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
(c*x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (b*
x^8)/a)^p + (d*x^5*(a + b*x^8)^p*Hypergeometric2F1[5/8, -p, 13/8, -((b*x^8
)/a)])/(5*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1763
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int (x^4 d + c) (b x^8 + a)^p dx$$

input
```
int((d*x^4+c)*(b*x^8+a)^p,x)
```

output
```
int((d*x^4+c)*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int (c + dx^4) (a + bx^8)^p dx = \int (dx^4 + c) (bx^8 + a)^p dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)*(b*x^8 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 144.46 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int (c+dx^4)(a+bx^8)^p\,dx = \frac{a^p c x \Gamma\left(\frac{1}{8}\right) {}_2F_1\left(\begin{matrix}\frac{1}{8}, -p \\ \frac{9}{8}\end{matrix}\middle| \frac{bx^8 e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{9}{8}\right)} + \frac{a^p d x^5 \Gamma\left(\frac{5}{8}\right) {}_2F_1\left(\begin{matrix}\frac{5}{8}, -p \\ \frac{13}{8}\end{matrix}\middle| \frac{bx^8 e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{13}{8}\right)}$$

input
```
integrate((d*x**4+c)*(b*x**8+a)**p,x)
```

output
```
a**p*c*x*gamma(1/8)*hyper((1/8, -p), (9/8,), b*x**8*exp_polar(I*pi)/a)/(8*
gamma(9/8)) + a**p*d*x**5*gamma(5/8)*hyper((5/8, -p), (13/8,), b*x**8*exp_
polar(I*pi)/a)/(8*gamma(13/8))
```

Maxima [F]

$$\int (c+dx^4)(a+bx^8)^p\,dx = \int (dx^4+c)(bx^8+a)^p\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int (c+dx^4) (a+bx^8)^p \, dx = \int (dx^4+c)(bx^8+a)^p \, dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx^4) (a+bx^8)^p \, dx = \int (b\,x^8+a)^p\,(d\,x^4+c) \, dx$$

input
```
int((a + b*x^8)^p*(c + d*x^4),x)
```

output
```
int((a + b*x^8)^p*(c + d*x^4), x)
```

Reduce [F]

$$\int (c+dx^4) (a+bx^8)^p \, dx$$
$$= \frac{8(b\,x^8+a)^p\,cpx+5(b\,x^8+a)^p\,cx+8(b\,x^8+a)^p\,dp\,x^5+(b\,x^8+a)^p\,d\,x^5+4096\left(\int \frac{(b\,x^8+a)^p}{64b\,p^2x^8+48bp\,x^8+5b\,x^8+64}\right.}{}$$

input
```
int((d*x^4+c)*(b*x^8+a)^p,x)
```

output
```
(8*(a + b*x**8)**p*c*p*x + 5*(a + b*x**8)**p*c*x + 8*(a + b*x**8)**p*d*p*x
**5 + (a + b*x**8)**p*d*x**5 + 4096*int((a + b*x**8)**p/(64*a*p**2 + 48*a*
p + 5*a + 64*b*p**2*x**8 + 48*b*p*x**8 + 5*b*x**8),x)*a*c*p**4 + 5632*int(
(a + b*x**8)**p/(64*a*p**2 + 48*a*p + 5*a + 64*b*p**2*x**8 + 48*b*p*x**8 +
 5*b*x**8),x)*a*c*p**3 + 2240*int((a + b*x**8)**p/(64*a*p**2 + 48*a*p + 5*
a + 64*b*p**2*x**8 + 48*b*p*x**8 + 5*b*x**8),x)*a*c*p**2 + 200*int((a + b*
x**8)**p/(64*a*p**2 + 48*a*p + 5*a + 64*b*p**2*x**8 + 48*b*p*x**8 + 5*b*x*
*8),x)*a*c*p + 4096*int(((a + b*x**8)**p*x**4)/(64*a*p**2 + 48*a*p + 5*a +
 64*b*p**2*x**8 + 48*b*p*x**8 + 5*b*x**8),x)*a*d*p**4 + 3584*int(((a + b*x
**8)**p*x**4)/(64*a*p**2 + 48*a*p + 5*a + 64*b*p**2*x**8 + 48*b*p*x**8 + 5
*b*x**8),x)*a*d*p**3 + 704*int(((a + b*x**8)**p*x**4)/(64*a*p**2 + 48*a*p
+ 5*a + 64*b*p**2*x**8 + 48*b*p*x**8 + 5*b*x**8),x)*a*d*p**2 + 40*int(((a
+ b*x**8)**p*x**4)/(64*a*p**2 + 48*a*p + 5*a + 64*b*p**2*x**8 + 48*b*p*x**
8 + 5*b*x**8),x)*a*d*p)/(64*p**2 + 48*p + 5)
```

3.10 $\int \frac{(c+dx^4)(a+bx^8)^p}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 99

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^2}\,dx = -\frac{c(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{8},-p,\frac{7}{8},-\frac{bx^8}{a}\right)}{x} + \frac{1}{3}dx^3(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)$$

output
```
-c*(b*x^8+a)^p*hypergeom([-1/8, -p],[7/8],-b*x^8/a)/x/((1+b*x^8/a)^p)+1/3*
d*x^3*(b*x^8+a)^p*hypergeom([3/8, -p],[11/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^2}\,dx = \frac{(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(-3c\,\text{Hypergeometric2F1}\left(-\frac{1}{8},-p,\frac{7}{8},-\frac{bx^8}{a}\right)+dx^4\,\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\right.\right.}{3x}$$

input
```
Integrate[((c + d*x^4)*(a + b*x^8)^p)/x^2,x]
```

output
```
((a + b*x^8)^p*(-3*c*Hypergeometric2F1[-1/8, -p, 7/8, -((b*x^8)/a)] + d*x^
4*Hypergeometric2F1[3/8, -p, 11/8, -((b*x^8)/a)]))/(3*x*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^2}\,dx$$

$$\Big\downarrow \text{1865}$$

$$\int \left(\frac{c(a+bx^8)^p}{x^2}+dx^2(a+bx^8)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}dx^3(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)-\frac{c(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{8},-p,\frac{7}{8},-\frac{bx^8}{a}\right)}{x}$$

input
```
Int[((c + d*x^4)*(a + b*x^8)^p)/x^2,x]
```

output
```
-((c*(a + b*x^8)^p*Hypergeometric2F1[-1/8, -p, 7/8, -((b*x^8)/a)])/(x*(1 +
 (b*x^8)/a)^p)) + (d*x^3*(a + b*x^8)^p*Hypergeometric2F1[3/8, -p, 11/8, -(
(b*x^8)/a)])/(3*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1865
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (f*x)^m*(d
 + e*x^n)^q, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && I
GtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(x^4 d + c) (b x^8 + a)^p}{x^2} dx$$

input
```
int((d*x^4+c)*(b*x^8+a)^p/x^2,x)
```

output
```
int((d*x^4+c)*(b*x^8+a)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(c + dx^4) (a + bx^8)^p}{x^2} dx = \int \frac{(dx^4 + c)(bx^8 + a)^p}{x^2} dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)*(b*x^8 + a)^p/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 123.98 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^2}\,dx = \frac{a^p c\Gamma\left(-\frac{1}{8}\right) {}_2F_1\left(\begin{matrix}-\frac{1}{8}, -p \\ \frac{7}{8}\end{matrix}\middle| \frac{bx^8 e^{i\pi}}{a}\right)}{8x\Gamma\left(\frac{7}{8}\right)} + \frac{a^p dx^3\Gamma\left(\frac{3}{8}\right) {}_2F_1\left(\begin{matrix}\frac{3}{8}, -p \\ \frac{11}{8}\end{matrix}\middle| \frac{bx^8 e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{11}{8}\right)}$$

input
```
integrate((d*x**4+c)*(b*x**8+a)**p/x**2,x)
```

output
```
a**p*c*gamma(-1/8)*hyper((-1/8, -p), (7/8,), b*x**8*exp_polar(I*pi)/a)/(8*
x*gamma(7/8)) + a**p*d*x**3*gamma(3/8)*hyper((3/8, -p), (11/8,), b*x**8*ex
p_polar(I*pi)/a)/(8*gamma(11/8))
```

Maxima [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^2}\,dx = \int \frac{(dx^4+c)(bx^8+a)^p}{x^2}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^2}\,dx=\int \frac{(dx^4+c)(bx^8+a)^p}{x^2}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^2}\,dx=\int \frac{(b\,x^8+a)^p\,(d\,x^4+c)}{x^2}\,dx$$

input
```
int(((a + b*x^8)^p*(c + d*x^4))/x^2,x)
```

output
```
int(((a + b*x^8)^p*(c + d*x^4))/x^2, x)
```

Reduce [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^2}\,dx$$
$$=\frac{8(b\,x^8+a)^p\,cp+3(b\,x^8+a)^p\,c+8(b\,x^8+a)^p\,dp\,x^4-(b\,x^8+a)^p\,d\,x^4+4096\left(\int \frac{(b\,x^8+a)^p}{64b\,p^2x^{10}+16bp\,x^{10}-3b\,x^{10}+64a}\right.}{}$$

input
```
int((d*x^4+c)*(b*x^8+a)^p/x^2,x)
```

output

```
(8*(a + b*x**8)**p*c*p + 3*(a + b*x**8)**p*c + 8*(a + b*x**8)**p*d*p*x**4
- (a + b*x**8)**p*d*x**4 + 4096*int((a + b*x**8)**p/(64*a*p**2*x**2 + 16*a
*p*x**2 - 3*a*x**2 + 64*b*p**2*x**10 + 16*b*p*x**10 - 3*b*x**10),x)*a*c*p*
*4*x + 2560*int((a + b*x**8)**p/(64*a*p**2*x**2 + 16*a*p*x**2 - 3*a*x**2 +
 64*b*p**2*x**10 + 16*b*p*x**10 - 3*b*x**10),x)*a*c*p**3*x + 192*int((a +
b*x**8)**p/(64*a*p**2*x**2 + 16*a*p*x**2 - 3*a*x**2 + 64*b*p**2*x**10 + 16
*b*p*x**10 - 3*b*x**10),x)*a*c*p**2*x - 72*int((a + b*x**8)**p/(64*a*p**2*
x**2 + 16*a*p*x**2 - 3*a*x**2 + 64*b*p**2*x**10 + 16*b*p*x**10 - 3*b*x**10
),x)*a*c*p*x + 4096*int(((a + b*x**8)**p*x**2)/(64*a*p**2 + 16*a*p - 3*a +
 64*b*p**2*x**8 + 16*b*p*x**8 - 3*b*x**8),x)*a*d*p**4*x + 512*int(((a + b*
x**8)**p*x**2)/(64*a*p**2 + 16*a*p - 3*a + 64*b*p**2*x**8 + 16*b*p*x**8 -
3*b*x**8),x)*a*d*p**3*x - 320*int(((a + b*x**8)**p*x**2)/(64*a*p**2 + 16*a
*p - 3*a + 64*b*p**2*x**8 + 16*b*p*x**8 - 3*b*x**8),x)*a*d*p**2*x + 24*int
(((a + b*x**8)**p*x**2)/(64*a*p**2 + 16*a*p - 3*a + 64*b*p**2*x**8 + 16*b*
p*x**8 - 3*b*x**8),x)*a*d*p*x)/(x*(64*p**2 + 16*p - 3))
```

3.11 $\int \frac{(c+dx^4)(a+bx^8)^p}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^4}\,dx = -\frac{c(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{8},-p,\frac{5}{8},-\frac{bx^8}{a}\right)}{3x^3} + dx(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

output
```
-1/3*c*(b*x^8+a)^p*hypergeom([-3/8, -p],[5/8],-b*x^8/a)/x^3/((1+b*x^8/a)^p
)+d*x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^4}\,dx = \frac{(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(-c\,\text{Hypergeometric2F1}\left(-\frac{3}{8},-p,\frac{5}{8},-\frac{bx^8}{a}\right)+3dx^4\,\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\right.\right.}{3x^3}$$

input
```
Integrate[((c + d*x^4)*(a + b*x^8)^p)/x^4,x]
```

output
```
((a + b*x^8)^p*(-(c*Hypergeometric2F1[-3/8, -p, 5/8, -((b*x^8)/a)]) + 3*d*
x^4*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)]))/(3*x^3*(1 + (b*x^8)/a)
^p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^4}dx$$

$$\Big\downarrow \text{1865}$$

$$\int \left(\frac{c(a+bx^8)^p}{x^4}+d(a+bx^8)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$dx(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)-\frac{c(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{8},-p,\frac{5}{8},-\frac{bx^8}{a}\right)}{3x^3}$$

input
```
Int[((c + d*x^4)*(a + b*x^8)^p)/x^4,x]
```

output
```
-1/3*(c*(a + b*x^8)^p*Hypergeometric2F1[-3/8, -p, 5/8, -((b*x^8)/a)])/(x^3
*(1 + (b*x^8)/a)^p) + (d*x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -
((b*x^8)/a)])/(1 + (b*x^8)/a)^p
```

Defintions of rubi rules used

rule 1865
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (f*x)^m*(d
 + e*x^n)^q, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && I
GtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(x^4d+c)\,(b\,x^8+a)^p}{x^4}dx$$

input
```
int((d*x^4+c)*(b*x^8+a)^p/x^4,x)
```

output
```
int((d*x^4+c)*(b*x^8+a)^p/x^4,x)
```

Fricas [F]

$$\int \frac{(c+dx^4)\,(a+bx^8)^p}{x^4}\,dx = \int \frac{(dx^4+c)(bx^8+a)^p}{x^4}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x^4,x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)*(b*x^8 + a)^p/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 123.76 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^4}\,dx = \frac{a^p c\Gamma\left(-\frac{3}{8}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{8}, -p \\ \frac{5}{8} \end{matrix}\middle| \frac{bx^8e^{i\pi}}{a}\right)}{8x^3\Gamma\left(\frac{5}{8}\right)} + \frac{a^p dx\Gamma\left(\frac{1}{8}\right) {}_2F_1\left(\begin{matrix} \frac{1}{8}, -p \\ \frac{9}{8} \end{matrix}\middle| \frac{bx^8e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{9}{8}\right)}$$

input
```
integrate((d*x**4+c)*(b*x**8+a)**p/x**4,x)
```

output
```
a**p*c*gamma(-3/8)*hyper((-3/8, -p), (5/8,), b*x**8*exp_polar(I*pi)/a)/(8*
x**3*gamma(5/8)) + a**p*d*x*gamma(1/8)*hyper((1/8, -p), (9/8,), b*x**8*exp
_polar(I*pi)/a)/(8*gamma(9/8))
```

Maxima [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^4}\,dx = \int \frac{(dx^4+c)(bx^8+a)^p}{x^4}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x^4,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^4}\,dx=\int \frac{(dx^4+c)(bx^8+a)^p}{x^4}\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p/x^4,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^4}\,dx=\int \frac{(b\,x^8+a)^p\,(d\,x^4+c)}{x^4}\,dx$$

input
```
int(((a + b*x^8)^p*(c + d*x^4))/x^4,x)
```

output
```
int(((a + b*x^8)^p*(c + d*x^4))/x^4, x)
```

Reduce [F]

$$\int \frac{(c+dx^4)(a+bx^8)^p}{x^4}\,dx$$
$$=\frac{8(b\,x^8+a)^p\,cp+(b\,x^8+a)^p\,c+8(b\,x^8+a)^p\,dp\,x^4-3(b\,x^8+a)^p\,d\,x^4+4096\left(\int \frac{(b\,x^8+a)^p}{64b\,p^2x^{12}-16bp\,x^{12}-3b\,x^{12}+64a}\right.}{}$$

input
```
int((d*x^4+c)*(b*x^8+a)^p/x^4,x)
```

output
```
(8*(a + b*x**8)**p*c*p + (a + b*x**8)**p*c + 8*(a + b*x**8)**p*d*p*x**4 -
3*(a + b*x**8)**p*d*x**4 + 4096*int((a + b*x**8)**p/(64*a*p**2*x**4 - 16*a
*p*x**4 - 3*a*x**4 + 64*b*p**2*x**12 - 16*b*p*x**12 - 3*b*x**12),x)*a*c*p*
*4*x**3 - 512*int((a + b*x**8)**p/(64*a*p**2*x**4 - 16*a*p*x**4 - 3*a*x**4
 + 64*b*p**2*x**12 - 16*b*p*x**12 - 3*b*x**12),x)*a*c*p**3*x**3 - 320*int(
(a + b*x**8)**p/(64*a*p**2*x**4 - 16*a*p*x**4 - 3*a*x**4 + 64*b*p**2*x**12
 - 16*b*p*x**12 - 3*b*x**12),x)*a*c*p**2*x**3 - 24*int((a + b*x**8)**p/(64
*a*p**2*x**4 - 16*a*p*x**4 - 3*a*x**4 + 64*b*p**2*x**12 - 16*b*p*x**12 - 3
*b*x**12),x)*a*c*p*x**3 + 4096*int((a + b*x**8)**p/(64*a*p**2 - 16*a*p - 3
*a + 64*b*p**2*x**8 - 16*b*p*x**8 - 3*b*x**8),x)*a*d*p**4*x**3 - 2560*int(
(a + b*x**8)**p/(64*a*p**2 - 16*a*p - 3*a + 64*b*p**2*x**8 - 16*b*p*x**8 -
 3*b*x**8),x)*a*d*p**3*x**3 + 192*int((a + b*x**8)**p/(64*a*p**2 - 16*a*p
- 3*a + 64*b*p**2*x**8 - 16*b*p*x**8 - 3*b*x**8),x)*a*d*p**2*x**3 + 72*int
((a + b*x**8)**p/(64*a*p**2 - 16*a*p - 3*a + 64*b*p**2*x**8 - 16*b*p*x**8
- 3*b*x**8),x)*a*d*p*x**3)/(x**3*(64*p**2 - 16*p - 3))
```

3.12 $\int x^5(c+dx^4)^2(a+bx^8)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int x^5(c+dx^4)^2(a+bx^8)^p\,dx=\frac{d^2x^6(a+bx^8)^{1+p}}{2b(7+4p)}-\frac{(3ad^2-bc^2(7+4p))\,x^6(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^8}{a}\right)}{6b(7+4p)}+\frac{1}{5}cdx^{10}(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^8}{a}\right)$$

output
```
1/2*d^2*x^6*(b*x^8+a)^(p+1)/b/(7+4*p)-1/6*(3*a*d^2-b*c^2*(7+4*p))*x^6*(b*x
^8+a)^p*hypergeom([3/4, -p],[7/4],-b*x^8/a)/b/(7+4*p)/((1+b*x^8/a)^p)+1/5*
c*d*x^10*(b*x^8+a)^p*hypergeom([5/4, -p],[9/4],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.68

$$\int x^5\left(c+dx^4\right)^2\left(a+bx^8\right)^p\,dx=\frac{1}{210}x^6\left(a+bx^8\right)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(35c^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^8}{a}\right)+3dx^4\left(14c\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^8}{a}\right)+5dx^4\,\text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{bx^8}{a}\right)\right)\right)$$

input
```
Integrate[x^5*(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
(x^6*(a + b*x^8)^p*(35*c^2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^8)/a)] +
 3*d*x^4*(14*c*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^8)/a)] + 5*d*x^4*Hyp
ergeometric2F1[7/4, -p, 11/4, -((b*x^8)/a)])))/(210*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1815, 1675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(c+dx^4\right)^2\left(a+bx^8\right)^p\,dx$$

$$\Big\downarrow \text{1815}$$

$$\frac{1}{2}\int x^4\left(dx^4+c\right)^2\left(bx^8+a\right)^p\,dx^2$$

$$\Big\downarrow \text{1675}$$

$$\frac{1}{2}\int \left(d^2x^{12}\left(bx^8+a\right)^p+2cdx^8\left(bx^8+a\right)^p+c^2x^4\left(bx^8+a\right)^p\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{3}c^2x^6\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^8}{a}\right)+\frac{2}{5}cdx^{10}\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hy}\right.$$

input
```
Int[x^5*(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
((c^2*x^6*(a + b*x^8)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^8)/a)])/(3*
(1 + (b*x^8)/a)^p) + (2*c*d*x^10*(a + b*x^8)^p*Hypergeometric2F1[5/4, -p,
9/4, -((b*x^8)/a)])/(5*(1 + (b*x^8)/a)^p) + (d^2*x^14*(a + b*x^8)^p*Hyperg
eometric2F1[7/4, -p, 11/4, -((b*x^8)/a)])/(7*(1 + (b*x^8)/a)^p))/2
```

Defintions of rubi rules used

rule 1675
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

rule 1815
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/
k - 1)*(d + e*x^(n/k))^q*(a + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1]
/; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^{5} \left(x^{4} d+c\right)^{2} \left(b \,x^{8}+a\right)^{p}d x$$

input
```
int(x^5*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

output
```
int(x^5*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int x^5 \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \int \left(dx^{4} + c\right)^{2} {\left(bx^{8} + a\right)}^{p} x^{5} \,d x$$

input
```
integrate(x^5*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^13 + 2*c*d*x^9 + c^2*x^5)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^5 \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x**5*(d*x**4+c)**2*(b*x**8+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^5 (c+dx^4)^2 (a+bx^8)^p \, dx = \int (dx^4+c)^2 (bx^8+a)^p x^5 \, dx$$

input
```
integrate(x^5*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p*x^5, x)
```

Giac [F]

$$\int x^5 (c+dx^4)^2 (a+bx^8)^p \, dx = \int (dx^4+c)^2 (bx^8+a)^p x^5 \, dx$$

input
```
integrate(x^5*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 (c+dx^4)^2 (a+bx^8)^p \, dx = \int x^5 (b x^8+a)^p (d x^4+c)^2 \, dx$$

input
```
int(x^5*(a + b*x^8)^p*(c + d*x^4)^2,x)
```

output
```
int(x^5*(a + b*x^8)^p*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int x^5 (c + dx^4)^2 (a + bx^8)^p \, dx = \text{too large to display}$$

input
```
int(x^5*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

output
```
(128*(a + b*x**8)**p*a*c*d*p**3*x**2 + 320*(a + b*x**8)**p*a*c*d*p**2*x**2
 + 168*(a + b*x**8)**p*a*c*d*p*x**2 + 64*(a + b*x**8)**p*a*d**2*p**3*x**6
+ 96*(a + b*x**8)**p*a*d**2*p**2*x**6 + 20*(a + b*x**8)**p*a*d**2*p*x**6 +
 64*(a + b*x**8)**p*b*c**2*p**3*x**6 + 208*(a + b*x**8)**p*b*c**2*p**2*x**
6 + 188*(a + b*x**8)**p*b*c**2*p*x**6 + 35*(a + b*x**8)**p*b*c**2*x**6 + 1
28*(a + b*x**8)**p*b*c*d*p**3*x**10 + 352*(a + b*x**8)**p*b*c*d*p**2*x**10
 + 248*(a + b*x**8)**p*b*c*d*p*x**10 + 42*(a + b*x**8)**p*b*c*d*x**10 + 64
*(a + b*x**8)**p*b*d**2*p**3*x**14 + 144*(a + b*x**8)**p*b*d**2*p**2*x**14
 + 92*(a + b*x**8)**p*b*d**2*p*x**14 + 15*(a + b*x**8)**p*b*d**2*x**14 - 9
8304*int(((a + b*x**8)**p*x**5)/(256*a*p**4 + 1024*a*p**3 + 1376*a*p**2 +
704*a*p + 105*a + 256*b*p**4*x**8 + 1024*b*p**3*x**8 + 1376*b*p**2*x**8 +
704*b*p*x**8 + 105*b*x**8),x)*a**2*d**2*p**7 - 540672*int(((a + b*x**8)**p
*x**5)/(256*a*p**4 + 1024*a*p**3 + 1376*a*p**2 + 704*a*p + 105*a + 256*b*p
**4*x**8 + 1024*b*p**3*x**8 + 1376*b*p**2*x**8 + 704*b*p*x**8 + 105*b*x**8
),x)*a**2*d**2*p**6 - 1148928*int(((a + b*x**8)**p*x**5)/(256*a*p**4 + 102
4*a*p**3 + 1376*a*p**2 + 704*a*p + 105*a + 256*b*p**4*x**8 + 1024*b*p**3*x
**8 + 1376*b*p**2*x**8 + 704*b*p*x**8 + 105*b*x**8),x)*a**2*d**2*p**5 - 11
85792*int(((a + b*x**8)**p*x**5)/(256*a*p**4 + 1024*a*p**3 + 1376*a*p**2 +
 704*a*p + 105*a + 256*b*p**4*x**8 + 1024*b*p**3*x**8 + 1376*b*p**2*x**8 +
 704*b*p*x**8 + 105*b*x**8),x)*a**2*d**2*p**4 - 610944*int(((a + b*x**8...
```

3.13 $\int x^3(c+dx^4)^2(a+bx^8)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 133

$$\int x^3(c+dx^4)^2(a+bx^8)^p\,dx = \frac{cd(a+bx^8)^{1+p}}{4b(1+p)}+\frac{d^2x^4(a+bx^8)^{1+p}}{4b(3+2p)} - \frac{(ad^2-bc^2(3+2p))\,x^4(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)}{4b(3+2p)}$$

output
```
1/4*c*d*(b*x^8+a)^(p+1)/b/(p+1)+1/4*d^2*x^4*(b*x^8+a)^(p+1)/b/(3+2*p)-1/4*
(a*d^2-b*c^2*(3+2*p))*x^4*(b*x^8+a)^p*hypergeom([1/2, -p],[3/2],-b*x^8/a)/
b/(3+2*p)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

$$\int x^3(c+dx^4)^2(a+bx^8)^p\,dx = \frac{(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(3bc^2(1+p)x^4\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)+d\left(3c\left(bx^8\left(1+\frac{bx^8}{a}\right)^p+a\right.\right.\right.}{12b(1+p)}$$

input
```
Integrate[x^3*(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
((a + b*x^8)^p*(3*b*c^2*(1 + p)*x^4*Hypergeometric2F1[1/2, -p, 3/2, -((b*x
^8)/a)] + d*(3*c*(b*x^8*(1 + (b*x^8)/a)^p + a*(-1 + (1 + (b*x^8)/a)^p)) +
b*d*(1 + p)*x^12*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^8)/a)])))/(12*b*(1
 + p)*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1799, 497, 25, 455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx$$

$$\Big\downarrow \text{1799}$$

$$\frac{1}{4} \int \left(dx^4+c\right)^2 \left(bx^8+a\right)^p dx^4$$

$$\Big\downarrow \text{497}$$

$$\frac{1}{4} \left(\frac{\int -\left(\left(-2bcd(p+2)x^4+ad^2-bc^2(2p+3)\right) \left(bx^8+a\right)^p\right) dx^4}{b(2p+3)}+\frac{d\left(c+dx^4\right) \left(a+bx^8\right)^{p+1}}{b(2p+3)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4} \left(\frac{d\left(c+dx^4\right) \left(a+bx^8\right)^{p+1}}{b(2p+3)}-\frac{\int \left(-2bcd(p+2)x^4+ad^2-bc^2(2p+3)\right) \left(bx^8+a\right)^p dx^4}{b(2p+3)}\right)$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{4} \left(\frac{d\left(c+dx^4\right) \left(a+bx^8\right)^{p+1}}{b(2p+3)}-\frac{\left(ad^2-bc^2(2p+3)\right) \int \left(bx^8+a\right)^p dx^4-\frac{cd(p+2)\left(a+bx^8\right)^{p+1}}{p+1}}{b(2p+3)}\right)$$

$$\Big\downarrow \text{238}$$

$$\frac{1}{4}\left(\frac{d(c+dx^4)\,(a+bx^8)^{p+1}}{b(2p+3)}-\frac{(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}(ad^2-bc^2(2p+3))\int\left(\frac{bx^8}{a}+1\right)^p dx^4-\frac{cd(p+2)(a+bx^8)^{p+1}}{p+1}}{b(2p+3)}\right.$$

$$\Big\downarrow \text{237}$$

$$\frac{1}{4}\left(\frac{d(c+dx^4)\,(a+bx^8)^{p+1}}{b(2p+3)}-\frac{x^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}(ad^2-bc^2(2p+3))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b}{}\right.}{b(2p+3)}\right.$$

input
```
Int[x^3*(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
((d*(c + d*x^4)*(a + b*x^8)^(1 + p))/(b*(3 + 2*p)) - (-(((c*d*(2 + p)*(a + 
b*x^8)^(1 + p))/(1 + p)) + ((a*d^2 - b*c^2*(3 + 2*p))*x^4*(a + b*x^8)^p*Hy
pergeometric2F1[1/2, -p, 3/2, -((b*x^8)/a)])/(1 + (b*x^8)/a)^p)/(b*(3 + 2*
p)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

Maple [F]

$$\int x^{3} \left(x^{4} d+c\right)^{2} \left(b \,x^{8}+a\right)^{p}d x$$

input
```
int(x^3*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

output
```
int(x^3*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int x^3 \left(c+d x^4\right)^2 \left(a+b x^8\right)^p \, dx=\int { {\left(d x^{4} + c\right)}^{2} {\left(b x^{8} + a\right)}^{p} x^{3} \,d x }$$

input
```
integrate(x^3*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^11 + 2*c*d*x^7 + c^2*x^3)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(c + dx^4\right)^2 \left(a + bx^8\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x**3*(d*x**4+c)**2*(b*x**8+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3 \left(c + dx^4\right)^2 \left(a + bx^8\right)^p \, dx = \int \left(dx^4 + c\right)^2 \left(bx^8 + a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p*x^3, x)
```

Giac [F]

$$\int x^3 \left(c + dx^4\right)^2 \left(a + bx^8\right)^p \, dx = \int \left(dx^4 + c\right)^2 \left(bx^8 + a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \int x^3 \left(b\,x^8+a\right)^p \left(d\,x^4+c\right)^2 dx$$

input
```
int(x^3*(a + b*x^8)^p*(c + d*x^4)^2,x)
```

output
```
int(x^3*(a + b*x^8)^p*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int x^3 \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \text{Too large to display}$$

input
```
int(x^3*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

output

```
(4*(a + b*x**8)**p*a*c*d*p**2 + 8*(a + b*x**8)**p*a*c*d*p + 3*(a + b*x**8)
**p*a*c*d + 2*(a + b*x**8)**p*a*d**2*p**2*x**4 + 2*(a + b*x**8)**p*a*d**2*
p*x**4 + 2*(a + b*x**8)**p*b*c**2*p**2*x**4 + 5*(a + b*x**8)**p*b*c**2*p*x
**4 + 3*(a + b*x**8)**p*b*c**2*x**4 + 4*(a + b*x**8)**p*b*c*d*p**2*x**8 +
8*(a + b*x**8)**p*b*c*d*p*x**8 + 3*(a + b*x**8)**p*b*c*d*x**8 + 2*(a + b*x
**8)**p*b*d**2*p**2*x**12 + 3*(a + b*x**8)**p*b*d**2*p*x**12 + (a + b*x**8
)**p*b*d**2*x**12 - 32*int(((a + b*x**8)**p*x**3)/(4*a*p**2 + 8*a*p + 3*a
+ 4*b*p**2*x**8 + 8*b*p*x**8 + 3*b*x**8),x)*a**2*d**2*p**4 - 96*int(((a +
b*x**8)**p*x**3)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**8 + 8*b*p*x**8 + 3*
b*x**8),x)*a**2*d**2*p**3 - 88*int(((a + b*x**8)**p*x**3)/(4*a*p**2 + 8*a*
p + 3*a + 4*b*p**2*x**8 + 8*b*p*x**8 + 3*b*x**8),x)*a**2*d**2*p**2 - 24*in
t(((a + b*x**8)**p*x**3)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**8 + 8*b*p*x
**8 + 3*b*x**8),x)*a**2*d**2*p + 64*int(((a + b*x**8)**p*x**3)/(4*a*p**2 +
 8*a*p + 3*a + 4*b*p**2*x**8 + 8*b*p*x**8 + 3*b*x**8),x)*a*b*c**2*p**5 + 2
88*int(((a + b*x**8)**p*x**3)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**8 + 8*
b*p*x**8 + 3*b*x**8),x)*a*b*c**2*p**4 + 464*int(((a + b*x**8)**p*x**3)/(4*
a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**8 + 8*b*p*x**8 + 3*b*x**8),x)*a*b*c**2*
p**3 + 312*int(((a + b*x**8)**p*x**3)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x
**8 + 8*b*p*x**8 + 3*b*x**8),x)*a*b*c**2*p**2 + 72*int(((a + b*x**8)**p*x*
*3)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**8 + 8*b*p*x**8 + 3*b*x**8),x)...
```

3.14 $\int x(c+dx^4)^2 (a+bx^8)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 159

$$\int x(c+dx^4)^2 (a+bx^8)^p \, dx = \frac{d^2x^2(a+bx^8)^{1+p}}{2b(5+4p)} - \frac{(ad^2-bc^2(5+4p))\, x^2(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)}{2b(5+4p)} + \frac{1}{3}cdx^6(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^8}{a}\right)$$

output
```
1/2*d^2*x^2*(b*x^8+a)^(p+1)/b/(5+4*p)-1/2*(a*d^2-b*c^2*(5+4*p))*x^2*(b*x^8
+a)^p*hypergeom([1/4, -p],[5/4],-b*x^8/a)/b/(5+4*p)/((1+b*x^8/a)^p)+1/3*c*
d*x^6*(b*x^8+a)^p*hypergeom([3/4, -p],[7/4],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.68

$$\int x(c+dx^4)^2(a+bx^8)^p\,dx = \frac{1}{30}x^2(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(15c^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)+dx^4\left(10c\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^8}{a}\right)+3dx^4\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^8}{a}\right)\right)\right)$$

input
```
Integrate[x*(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
(x^2*(a + b*x^8)^p*(15*c^2*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^8)/a)] +
 d*x^4*(10*c*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^8)/a)] + 3*d*x^4*Hyper
geometric2F1[5/4, -p, 9/4, -((b*x^8)/a)])))/(30*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1815, 1519, 25, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(c+dx^4)^2(a+bx^8)^p\,dx$$

$$\Big\downarrow \text{1815}$$

$$\frac{1}{2}\int (dx^4+c)^2(bx^8+a)^p\,dx^2$$

$$\Big\downarrow \text{1519}$$

$$\frac{1}{2}\left(\frac{\int -\left(\left(-2bcd(4p+5)x^4+ad^2-bc^2(4p+5)\right)\left(bx^8+a\right)^p\right)dx^2}{b(4p+5)}+\frac{d^2x^2\left(a+bx^8\right)^{p+1}}{b(4p+5)}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{d^2x^2\left(a+bx^8\right)^{p+1}}{b(4p+5)}-\frac{\int \left(-2bcd(4p+5)x^4+ad^2-bc^2(4p+5)\right)\left(bx^8+a\right)^p dx^2}{b(4p+5)}\right)$$

$\Big\downarrow$ 1516

$$\frac{1}{2}\left(\frac{d^2x^2\left(a+bx^8\right)^{p+1}}{b(4p+5)}-\frac{\int \left(ad^2\left(1-\frac{bc^2(4p+5)}{ad^2}\right)\left(bx^8+a\right)^p-2bcd(4p+5)x^4\left(bx^8+a\right)^p\right)dx^2}{b(4p+5)}\right)$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{d^2x^2\left(a+bx^8\right)^{p+1}}{b(4p+5)}-\frac{x^2\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\left(ad^2-bc^2(4p+5)\right)\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)-}{b(4p+5)}\right.$$

input
```
Int[x*(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
((d^2*x^2*(a + b*x^8)^(1 + p))/(b*(5 + 4*p)) - (((a*d^2 - b*c^2*(5 + 4*p))
*x^2*(a + b*x^8)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^8)/a)])/(1 + (b*
x^8)/a)^p - (2*b*c*d*(5 + 4*p)*x^6*(a + b*x^8)^p*Hypergeometric2F1[3/4, -p
, 7/4, -((b*x^8)/a)])/(3*(1 + (b*x^8)/a)^p))/(b*(5 + 4*p)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 1815
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/
k - 1)*(d + e*x^(n/k))^q*(a + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1]
/; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x \left(x^{4} d+c \right)^{2} \left(b \,x^{8}+a \right)^{p}d x$$

input
```
int(x*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

output
```
int(x*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int x \left(c+d x^4\right)^2 \left(a+b x^8\right)^p \, dx=\int { {\left(d x^{4} + c\right)}^{2} {\left(b x^{8} + a\right)}^{p} x \,d x}$$

input
```
integrate(x*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^9 + 2*c*d*x^5 + c^2*x)*(b*x^8 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 178.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.76

$$\int x(c+dx^4)^2 (a+bx^8)^p \, dx = \frac{a^p c^2 x^2 \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix} \middle| \frac{bx^8 e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{5}{4}\right)} + \frac{a^p c d x^6 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix} \middle| \frac{bx^8 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{a^p d^2 x^{10} \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, -p \\ \frac{9}{4} \end{matrix} \middle| \frac{bx^8 e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x*(d*x**4+c)**2*(b*x**8+a)**p,x)
```

output
```
a**p*c**2*x**2*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**8*exp_polar(I*pi)/
a)/(8*gamma(5/4)) + a**p*c*d*x**6*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x*
*8*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**p*d**2*x**10*gamma(5/4)*hyper((5
/4, -p), (9/4,), b*x**8*exp_polar(I*pi)/a)/(8*gamma(9/4))
```

Maxima [F]

$$\int x(c+dx^4)^2 (a+bx^8)^p \, dx = \int (dx^4+c)^2 (bx^8+a)^p x \, dx$$

input
```
integrate(x*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p*x, x)
```

Giac [F]

$$\int x\left(c+dx^4\right)^2\left(a+bx^8\right)^p\,dx=\int \left(dx^4+c\right)^2\left(bx^8+a\right)^p x\,dx$$

input
```
integrate(x*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(c+dx^4\right)^2\left(a+bx^8\right)^p\,dx=\int x\left(b\,x^8+a\right)^p\left(d\,x^4+c\right)^2\,dx$$

input
```
int(x*(a + b*x^8)^p*(c + d*x^4)^2,x)
```

output
```
int(x*(a + b*x^8)^p*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int x\left(c+dx^4\right)^2\left(a+bx^8\right)^p\,dx=\text{Too large to display}$$

input
```
int(x*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

output

```
(16*(a + b*x**8)**p*a*d**2*p**2*x**2 + 12*(a + b*x**8)**p*a*d**2*p*x**2 +
16*(a + b*x**8)**p*b*c**2*p**2*x**2 + 32*(a + b*x**8)**p*b*c**2*p*x**2 + 1
5*(a + b*x**8)**p*b*c**2*x**2 + 32*(a + b*x**8)**p*b*c*d*p**2*x**6 + 48*(a
 + b*x**8)**p*b*c*d*p*x**6 + 10*(a + b*x**8)**p*b*c*d*x**6 + 16*(a + b*x**
8)**p*b*d**2*p**2*x**10 + 16*(a + b*x**8)**p*b*d**2*p*x**10 + 3*(a + b*x**
8)**p*b*d**2*x**10 + 16384*int(((a + b*x**8)**p*x**5)/(64*a*p**3 + 144*a*p
**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b*p**2*x**8 + 92*b*p*x**8 + 15*
b*x**8),x)*a*b*c*d*p**6 + 61440*int(((a + b*x**8)**p*x**5)/(64*a*p**3 + 14
4*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b*p**2*x**8 + 92*b*p*x**8
+ 15*b*x**8),x)*a*b*c*d*p**5 + 83968*int(((a + b*x**8)**p*x**5)/(64*a*p**3
 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b*p**2*x**8 + 92*b*p*
x**8 + 15*b*x**8),x)*a*b*c*d*p**4 + 50688*int(((a + b*x**8)**p*x**5)/(64*a
*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b*p**2*x**8 + 92
*b*p*x**8 + 15*b*x**8),x)*a*b*c*d*p**3 + 13120*int(((a + b*x**8)**p*x**5)/
(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b*p**2*x**8
 + 92*b*p*x**8 + 15*b*x**8),x)*a*b*c*d*p**2 + 1200*int(((a + b*x**8)**p*x*
*5)/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b*p**2*
x**8 + 92*b*p*x**8 + 15*b*x**8),x)*a*b*c*d*p - 2048*int(((a + b*x**8)**p*x
)/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**8 + 144*b*p**2*x*
*8 + 92*b*p*x**8 + 15*b*x**8),x)*a**2*d**2*p**5 - 6144*int(((a + b*x**8...
```

3.15 $\int \frac{(c+dx^4)^2(a+bx^8)^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\begin{aligned}\int \frac{(c+dx^4)^2(a+bx^8)^p}{x}\,dx &= \frac{d^2(a+bx^8)^{1+p}}{8b(1+p)} \\ &\quad + \frac{1}{2}cdx^4(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right) \\ &\quad - \frac{c^2(a+bx^8)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^8}{a}\right)}{8a(1+p)}\end{aligned}$$

output
```
1/8*d^2*(b*x^8+a)^(p+1)/b/(p+1)+1/2*c*d*x^4*(b*x^8+a)^p*hypergeom([1/2, -p
],[3/2],-b*x^8/a)/((1+b*x^8/a)^p)-1/8*c^2*(b*x^8+a)^(p+1)*hypergeom([1, p+
1],[2+p],1+b*x^8/a)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx^4)^2(a+bx^8)^p}{x}\,dx$$
$$= \frac{1}{8}(a+bx^8)^p\left(4cdx^4\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)\right.$$
$$\left.+\frac{(a+bx^8)\left(ad^2-bc^2\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^8}{a}\right)\right)}{ab(1+p)}\right)$$

input
```
Integrate[((c + d*x^4)^2*(a + b*x^8)^p)/x,x]
```

output
```
((a + b*x^8)^p*((4*c*d*x^4*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^8)/a)])/
(1 + (b*x^8)/a)^p + ((a + b*x^8)*(a*d^2 - b*c^2*Hypergeometric2F1[1, 1 + p
, 2 + p, 1 + (b*x^8)/a]))/(a*b*(1 + p))))/8
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1803, 543, 27, 238, 237, 354, 90, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2(a+bx^8)^p}{x}\,dx$$
$$\Big\downarrow \; 1803$$
$$\frac{1}{4}\int \frac{(dx^4+c)^2(bx^8+a)^p}{x^4}dx^4$$
$$\Big\downarrow \; 543$$

$$\frac{1}{4}\left(\int \frac{(bx^8+a)^p\,(d^2x^8+c^2)}{x^4}dx^4+\int 2cd(bx^8+a)^p\,dx^4\right)$$

$\Big\downarrow$ 27

$$\frac{1}{4}\left(\int \frac{(bx^8+a)^p\,(d^2x^8+c^2)}{x^4}dx^4+2cd\int (bx^8+a)^p\,dx^4\right)$$

$\Big\downarrow$ 238

$$\frac{1}{4}\left(\int \frac{(bx^8+a)^p\,(d^2x^8+c^2)}{x^4}dx^4+2cd(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \left(\frac{bx^8}{a}+1\right)^p dx^4\right)$$

$\Big\downarrow$ 237

$$\frac{1}{4}\left(\int \frac{(bx^8+a)^p\,(d^2x^8+c^2)}{x^4}dx^4+2cdx^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)\right)$$

$\Big\downarrow$ 354

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{(bx^8+a)^p\,(d^2x^8+c^2)}{x^4}dx^8+2cdx^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)\right)$$

$\Big\downarrow$ 90

$$\frac{1}{4}\left(\frac{1}{2}\left(c^2\int \frac{(bx^8+a)^p}{x^4}dx^8+\frac{d^2(a+bx^8)^{p+1}}{b(p+1)}\right)+2cdx^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2}\right.\right.$$

$\Big\downarrow$ 75

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{d^2(a+bx^8)^{p+1}}{b(p+1)}-\frac{c^2(a+bx^8)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^8}{a}+1\right)}{a(p+1)}\right)+2cdx^4(a+bx^8)^p\left(\right.\right.$$

input
```
Int[((c + d*x^4)^2*(a + b*x^8)^p)/x,x]
```

output
```
((2*c*d*x^4*(a + b*x^8)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^8)/a)])/(
1 + (b*x^8)/a)^p + ((d^2*(a + b*x^8)^(1 + p))/(b*(1 + p)) - (c^2*(a + b*x^
8)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^8)/a])/(a*(1 + p)))
/2)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(x^{4} d+c\right)^{2} \left(b\,x^{8}+a\right)^{p}}{x}dx$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p/x,x)
```

output
```
int((d*x^4+c)^2*(b*x^8+a)^p/x,x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^2\,(a+bx^8)^p}{x}\,dx=\int \frac{(dx^4+c)^2(bx^8+a)^p}{x}\,dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x,x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^8 + a)^p/x, x)
```

Sympy [A] (verification not implemented)

Time = 85.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x}\,dx = \frac{a^p c d x^4 {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^8 e^{i\pi}}{a}\right)}{2} - \frac{b^p c^2 x^{8p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^8}\right)}{8\Gamma(1-p)} + d^2\left(\begin{cases} \frac{a^p x^8}{8} & \text{for}\ b=0 \\ \frac{\begin{cases} \frac{(a+bx^8)^{p+1}}{p+1} & \text{for}\ p \neq -1 \\ \log(a+bx^8) & \text{otherwise} \end{cases}}{8b} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x**4+c)**2*(b*x**8+a)**p/x,x)
```

output
```
a**p*c*d*x**4*hyper((1/2, -p), (3/2,), b*x**8*exp_polar(I*pi)/a)/2 - b**p*
c**2*x**(8*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**
8))/(8*gamma(1 - p)) + d**2*Piecewise((a**p*x**8/8, Eq(b, 0)), (Piecewise(
((a + b*x**8)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**8), True))/(8*b)
, True))
```

Maxima [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x}\,dx = \int \frac{(dx^4+c)^2 (bx^8+a)^p}{x}\,dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p/x, x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x}\,dx = \int \frac{(dx^4+c)^2(bx^8+a)^p}{x}\,dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x}\,dx = \int \frac{(b\,x^8+a)^p\,(d\,x^4+c)^2}{x}\,dx$$

input
```
int(((a + b*x^8)^p*(c + d*x^4)^2)/x,x)
```

output
```
int(((a + b*x^8)^p*(c + d*x^4)^2)/x, x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x}\,dx$$
$$= \frac{2(b\,x^8+a)^p\,a\,d^2p^2+(b\,x^8+a)^p\,a\,d^2p+2(b\,x^8+a)^p\,b\,c^2p^2+3(b\,x^8+a)^p\,b\,c^2p+(b\,x^8+a)^p\,b\,c^2+4(b\,x^8}{}$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p/x,x)
```

output
```
(2*(a + b*x**8)**p*a*d**2*p**2 + (a + b*x**8)**p*a*d**2*p + 2*(a + b*x**8)
**p*b*c**2*p**2 + 3*(a + b*x**8)**p*b*c**2*p + (a + b*x**8)**p*b*c**2 + 4*
(a + b*x**8)**p*b*c*d*p**2*x**4 + 4*(a + b*x**8)**p*b*c*d*p*x**4 + 2*(a +
b*x**8)**p*b*d**2*p**2*x**8 + (a + b*x**8)**p*b*d**2*p*x**8 + 32*int((a +
b*x**8)**p/(2*a*p*x + a*x + 2*b*p*x**9 + b*x**9),x)*a*b*c**2*p**4 + 64*int
((a + b*x**8)**p/(2*a*p*x + a*x + 2*b*p*x**9 + b*x**9),x)*a*b*c**2*p**3 +
40*int((a + b*x**8)**p/(2*a*p*x + a*x + 2*b*p*x**9 + b*x**9),x)*a*b*c**2*p
**2 + 8*int((a + b*x**8)**p/(2*a*p*x + a*x + 2*b*p*x**9 + b*x**9),x)*a*b*c
**2*p + 64*int(((a + b*x**8)**p*x**3)/(2*a*p + a + 2*b*p*x**8 + b*x**8),x)
*a*b*c*d*p**4 + 96*int(((a + b*x**8)**p*x**3)/(2*a*p + a + 2*b*p*x**8 + b*
x**8),x)*a*b*c*d*p**3 + 32*int(((a + b*x**8)**p*x**3)/(2*a*p + a + 2*b*p*x
**8 + b*x**8),x)*a*b*c*d*p**2)/(8*b*p*(2*p**2 + 3*p + 1))
```

3.16 $\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 155

$$\begin{aligned}\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^3}\,dx &= \frac{d^2(a+bx^8)^{1+p}}{2b(3+4p)x^2} \\ &\quad - \frac{(ad^2+bc^2(3+4p))\,(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{4},-p,\frac{3}{4},-\frac{bx^8}{a}\right)}{2b(3+4p)x^2} \\ &\quad + cdx^2(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)\end{aligned}$$

output
```
1/2*d^2*(b*x^8+a)^(p+1)/b/(3+4*p)/x^2-1/2*(a*d^2+b*c^2*(3+4*p))*(b*x^8+a)^
p*hypergeom([-1/4, -p],[3/4],-b*x^8/a)/b/(3+4*p)/x^2/((1+b*x^8/a)^p)+c*d*x
^2*(b*x^8+a)^p*hypergeom([1/4, -p],[5/4],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^3}\,dx$$
$$= \frac{(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \left(-3c^2 \operatorname{Hypergeometric2F1}\left(-\frac{1}{4},-p,\frac{3}{4},-\frac{bx^8}{a}\right)+dx^4\left(6c \operatorname{Hypergeometric2F1}\left(\frac{1}{4}\right.\right.\right.}{6x^2}$$

input
```
Integrate[((c + d*x^4)^2*(a + b*x^8)^p)/x^3,x]
```

output
```
((a + b*x^8)^p*(-3*c^2*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^8)/a)] + d*
x^4*(6*c*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^8)/a)] + d*x^4*Hypergeomet
ric2F1[3/4, -p, 7/4, -((b*x^8)/a)])))/(6*x^2*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1815, 1675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^3}\,dx$$

$$\Big\downarrow \; 1815$$

$$\frac{1}{2}\int \frac{(dx^4+c)^2 (bx^8+a)^p}{x^4}dx^2$$

$$\Big\downarrow \; 1675$$

$$\frac{1}{2}\int \left(d^2x^4(bx^8+a)^p+2cd(bx^8+a)^p+\frac{c^2(bx^8+a)^p}{x^4}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{c^2(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{4},-p,\frac{3}{4},-\frac{bx^8}{a}\right)}{x^2}+2cdx^2(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hyperg}\right.$$

input
```
Int[((c + d*x^4)^2*(a + b*x^8)^p)/x^3,x]
```

output
```
(-((c^2*(a + b*x^8)^p*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^8)/a)])/(x^2
*(1 + (b*x^8)/a)^p)) + (2*c*d*x^2*(a + b*x^8)^p*Hypergeometric2F1[1/4, -p,
 5/4, -((b*x^8)/a)])/(1 + (b*x^8)/a)^p + (d^2*x^6*(a + b*x^8)^p*Hypergeome
tric2F1[3/4, -p, 7/4, -((b*x^8)/a)])/(3*(1 + (b*x^8)/a)^p))/2
```

Defintions of rubi rules used

rule 1675
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

rule 1815
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/
k - 1)*(d + e*x^(n/k))^q*(a + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1]
/; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{\left(x^{4} d+c\right)^{2} \left(b \,x^{8}+a\right)^{p}}{x^{3}}d x$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p/x^3,x)
```

output
```
int((d*x^4+c)^2*(b*x^8+a)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^3}\,dx = \int \frac{(dx^4+c)^2(bx^8+a)^p}{x^3}\,dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x^3,x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^8 + a)^p/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)**2*(b*x**8+a)**p/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^3}\,dx = \int \frac{(dx^4+c)^2(bx^8+a)^p}{x^3}\,dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^3}\,dx = \int \frac{(dx^4+c)^2(bx^8+a)^p}{x^3}\,dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x^3,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^3}\,dx = \int \frac{(b\,x^8+a)^p\,(d\,x^4+c)^2}{x^3}\,dx$$

input
```
int(((a + b*x^8)^p*(c + d*x^4)^2)/x^3,x)
```

output
```
int(((a + b*x^8)^p*(c + d*x^4)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^3} \, dx = \text{too large to display}$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p/x^3,x)
```

output
```
(16*(a + b*x**8)**p*a*d**2*p**2 + 4*(a + b*x**8)**p*a*d**2*p + 16*(a + b*x
**8)**p*b*c**2*p**2 + 16*(a + b*x**8)**p*b*c**2*p + 3*(a + b*x**8)**p*b*c*
*2 + 32*(a + b*x**8)**p*b*c*d*p**2*x**4 + 16*(a + b*x**8)**p*b*c*d*p*x**4
- 6*(a + b*x**8)**p*b*c*d*x**4 + 16*(a + b*x**8)**p*b*d**2*p**2*x**8 - (a
+ b*x**8)**p*b*d**2*x**8 + 2048*int((a + b*x**8)**p/(64*a*p**3*x**3 + 48*a
*p**2*x**3 - 4*a*p*x**3 - 3*a*x**3 + 64*b*p**3*x**11 + 48*b*p**2*x**11 - 4
*b*p*x**11 - 3*b*x**11),x)*a**2*d**2*p**5*x**2 + 2048*int((a + b*x**8)**p/
(64*a*p**3*x**3 + 48*a*p**2*x**3 - 4*a*p*x**3 - 3*a*x**3 + 64*b*p**3*x**11
 + 48*b*p**2*x**11 - 4*b*p*x**11 - 3*b*x**11),x)*a**2*d**2*p**4*x**2 + 256
*int((a + b*x**8)**p/(64*a*p**3*x**3 + 48*a*p**2*x**3 - 4*a*p*x**3 - 3*a*x
**3 + 64*b*p**3*x**11 + 48*b*p**2*x**11 - 4*b*p*x**11 - 3*b*x**11),x)*a**2
*d**2*p**3*x**2 - 128*int((a + b*x**8)**p/(64*a*p**3*x**3 + 48*a*p**2*x**3
 - 4*a*p*x**3 - 3*a*x**3 + 64*b*p**3*x**11 + 48*b*p**2*x**11 - 4*b*p*x**11
 - 3*b*x**11),x)*a**2*d**2*p**2*x**2 - 24*int((a + b*x**8)**p/(64*a*p**3*x
**3 + 48*a*p**2*x**3 - 4*a*p*x**3 - 3*a*x**3 + 64*b*p**3*x**11 + 48*b*p**2
*x**11 - 4*b*p*x**11 - 3*b*x**11),x)*a**2*d**2*p*x**2 + 8192*int((a + b*x*
*8)**p/(64*a*p**3*x**3 + 48*a*p**2*x**3 - 4*a*p*x**3 - 3*a*x**3 + 64*b*p**
3*x**11 + 48*b*p**2*x**11 - 4*b*p*x**11 - 3*b*x**11),x)*a*b*c**2*p**6*x**2
 + 14336*int((a + b*x**8)**p/(64*a*p**3*x**3 + 48*a*p**2*x**3 - 4*a*p*x**3
 - 3*a*x**3 + 64*b*p**3*x**11 + 48*b*p**2*x**11 - 4*b*p*x**11 - 3*b*x**...
```

3.17 $\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\begin{aligned}\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^5}\,dx &= -\frac{c^2(a+bx^8)^{1+p}}{4ax^4} \\ &+ \frac{(ad^2+bc^2(1+2p))\,x^4(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)}{4a} \\ &- \frac{cd(a+bx^8)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^8}{a}\right)}{4a(1+p)}\end{aligned}$$

output

```
-1/4*c^2*(b*x^8+a)^(p+1)/a/x^4+1/4*(a*d^2+b*c^2*(1+2*p))*x^4*(b*x^8+a)^p*h
ypergeom([1/2, -p],[3/2],-b*x^8/a)/a/((1+b*x^8/a)^p)-1/4*c*d*(b*x^8+a)^(p+
1)*hypergeom([1, p+1],[2+p],1+b*x^8/a)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^5}\,dx =$$
$$-\frac{(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(ac^2(1+p)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^8}{a}\right)+dx^4\left(-ad(1+p)x^4\,\text{Hyp}\right.\right.}{4a(}$$

input
```
Integrate[((c + d*x^4)^2*(a + b*x^8)^p)/x^5,x]
```

output
```
-1/4*((a + b*x^8)^p*(a*c^2*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x
^8)/a)] + d*x^4*(-(a*d*(1 + p)*x^4*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^
8)/a)]) + c*(a + b*x^8)*(1 + (b*x^8)/a)^p*Hypergeometric2F1[1, 1 + p, 2 +
p, 1 + (b*x^8)/a])))/(a*(1 + p)*x^4*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1803, 543, 27, 243, 75, 359, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^5}\,dx$$

$$\Big\downarrow \; 1803$$

$$\frac{1}{4}\int \frac{(dx^4+c)^2(bx^8+a)^p}{x^8}dx^4$$

$$\Big\downarrow \; 543$$

$$\frac{1}{4}\left(\int \frac{(bx^8+a)^p(d^2x^8+c^2)}{x^8}dx^4+\int \frac{2cd(bx^8+a)^p}{x^4}dx^4\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(\int \frac{(bx^8+a)^p (d^2x^8+c^2)}{x^8}dx^4+2cd\int \frac{(bx^8+a)^p}{x^4}dx^4\right)$$

$\Big\downarrow$ 243

$$\frac{1}{4}\left(\int \frac{(bx^8+a)^p (d^2x^8+c^2)}{x^8}dx^4+cd\int \frac{(bx^8+a)^p}{x^4}dx^8\right)$$

$\Big\downarrow$ 75

$$\frac{1}{4}\left(\int \frac{(bx^8+a)^p (d^2x^8+c^2)}{x^8}dx^4-\frac{cd(a+bx^8)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^8}{a}+1\right)}{a(p+1)}\right)$$

$\Big\downarrow$ 359

$$\frac{1}{4}\left(\frac{(ad^2+bc^2(2p+1))\int (bx^8+a)^p dx^4}{a}-\frac{c^2(a+bx^8)^{p+1}}{ax^4}-\frac{cd(a+bx^8)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+\right.}{a(p+1)}\right.$$

$\Big\downarrow$ 238

$$\frac{1}{4}\left(\frac{(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}(ad^2+bc^2(2p+1))\int \left(\frac{bx^8}{a}+1\right)^p dx^4}{a}-\frac{c^2(a+bx^8)^{p+1}}{ax^4}-\frac{cd(a+bx^8)^{p+1}\text{Hypergeo}}{}\right.$$

$\Big\downarrow$ 237

$$\frac{1}{4}\left(\frac{x^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}(ad^2+bc^2(2p+1))\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)}{a}-\frac{c^2(a+bx^8)^{p+1}}{ax^4}-\frac{cd}{}\right.$$

input
```
Int[((c + d*x^4)^2*(a + b*x^8)^p)/x^5,x]
```

output
```
(-((c^2*(a + b*x^8)^(1 + p))/(a*x^4)) + ((a*d^2 + b*c^2*(1 + 2*p))*x^4*(a
+ b*x^8)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^8)/a)])/(a*(1 + (b*x^8)/
a)^p) - (c*d*(a + b*x^8)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*x^8)/a])/(a*(1 + p)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 543
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Module[{k}, Int[x^m*Sum[Binomial[n, 2*k]*c^(n - 2*k)*d^(2*k)*x^(2*k),
 {k, 0, n/2}]*(a + b*x^2)^p, x] + Int[x^(m + 1)*Sum[Binomial[n, 2*k + 1]*c^
(n - 2*k - 1)*d^(2*k + 1)*x^(2*k), {k, 0, (n - 1)/2}]*(a + b*x^2)^p, x]] /;
 FreeQ[{a, b, c, d, p}, x] && IGtQ[n, 1] && IntegerQ[m] &&  !IntegerQ[2*p]
&&  !(EqQ[m, 1] && EqQ[b*c^2 + a*d^2, 0])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(x^4d+c)^2 (b\,x^8+a)^p}{x^5}dx$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p/x^5,x)
```

output
```
int((d*x^4+c)^2*(b*x^8+a)^p/x^5,x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^5}\,dx = \int \frac{(dx^4+c)^2(bx^8+a)^p}{x^5}\,dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x^5,x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^8 + a)^p/x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)**2*(b*x**8+a)**p/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^5} \, dx = \int \frac{(dx^4+c)^2 (bx^8+a)^p}{x^5} \, dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x^5,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p/x^5, x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^5} \, dx = \int \frac{(dx^4+c)^2 (bx^8+a)^p}{x^5} \, dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x^5,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^5} \, dx = \int \frac{(b\,x^8+a)^p\,(d\,x^4+c)^2}{x^5} \, dx$$

input
```
int(((a + b*x^8)^p*(c + d*x^4)^2)/x^5,x)
```

output
```
int(((a + b*x^8)^p*(c + d*x^4)^2)/x^5, x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^5}\,dx$$
$$= \frac{2(b x^8+a)^p a d^2p^2+2(b x^8+a)^p b c^2p^2+(b x^8+a)^p b c^2p+4(b x^8+a)^p bcd p^2x^4-(b x^8+a)^p bcd x^4+}{}$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p/x^5,x)
```

output
```
(2*(a + b*x**8)**p*a*d**2*p**2 + 2*(a + b*x**8)**p*b*c**2*p**2 + (a + b*x*
*8)**p*b*c**2*p + 4*(a + b*x**8)**p*b*c*d*p**2*x**4 - (a + b*x**8)**p*b*c*
d*x**4 + 2*(a + b*x**8)**p*b*d**2*p**2*x**8 - (a + b*x**8)**p*b*d**2*p*x**
8 + 32*int((a + b*x**8)**p/(4*a*p**2*x**5 - a*x**5 + 4*b*p**2*x**13 - b*x*
*13),x)*a**2*d**2*p**4*x**4 - 8*int((a + b*x**8)**p/(4*a*p**2*x**5 - a*x**
5 + 4*b*p**2*x**13 - b*x**13),x)*a**2*d**2*p**2*x**4 + 64*int((a + b*x**8)
**p/(4*a*p**2*x**5 - a*x**5 + 4*b*p**2*x**13 - b*x**13),x)*a*b*c**2*p**5*x
**4 + 32*int((a + b*x**8)**p/(4*a*p**2*x**5 - a*x**5 + 4*b*p**2*x**13 - b*
x**13),x)*a*b*c**2*p**4*x**4 - 16*int((a + b*x**8)**p/(4*a*p**2*x**5 - a*x
**5 + 4*b*p**2*x**13 - b*x**13),x)*a*b*c**2*p**3*x**4 - 8*int((a + b*x**8)
**p/(4*a*p**2*x**5 - a*x**5 + 4*b*p**2*x**13 - b*x**13),x)*a*b*c**2*p**2*x
**4 + 128*int((a + b*x**8)**p/(4*a*p**2*x - a*x + 4*b*p**2*x**9 - b*x**9),
x)*a*b*c*d*p**5*x**4 - 64*int((a + b*x**8)**p/(4*a*p**2*x - a*x + 4*b*p**2
*x**9 - b*x**9),x)*a*b*c*d*p**3*x**4 + 8*int((a + b*x**8)**p/(4*a*p**2*x -
 a*x + 4*b*p**2*x**9 - b*x**9),x)*a*b*c*d*p*x**4)/(4*b*p*x**4*(4*p**2 - 1)
)
```

3.18 $\int x^4(c+dx^4)^2(a+bx^8)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int x^4(c+dx^4)^2(a+bx^8)^p\,dx = \frac{d^2x^5(a+bx^8)^{1+p}}{b(13+8p)} - \frac{(5ad^2-bc^2(13+8p))\,x^5(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)}{5b(13+8p)} + \frac{2}{9}cdx^9(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{9}{8},-p,\frac{17}{8},-\frac{bx^8}{a}\right)$$

output
```
d^2*x^5*(b*x^8+a)^(p+1)/b/(13+8*p)-1/5*(5*a*d^2-b*c^2*(13+8*p))*x^5*(b*x^8
+a)^p*hypergeom([5/8, -p],[13/8],-b*x^8/a)/b/(13+8*p)/((1+b*x^8/a)^p)+2/9*
c*d*x^9*(b*x^8+a)^p*hypergeom([9/8, -p],[17/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69

$$\int x^4\left(c+dx^4\right)^2\left(a+bx^8\right)^p\,dx=\frac{1}{585}x^5\left(a+bx^8\right)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(117c^2\,\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)+5dx^4\left(26c\,\text{Hypergeometric2F1}\left(\frac{9}{8},-p,\frac{17}{8},-\frac{bx^8}{a}\right)+9dx^4\,\text{Hypergeometric2F1}\left(\frac{13}{8},-p,\frac{21}{8},-\frac{bx^8}{a}\right)\right)\right)$$

input
```
Integrate[x^4*(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
(x^5*(a + b*x^8)^p*(117*c^2*Hypergeometric2F1[5/8, -p, 13/8, -((b*x^8)/a)]
 + 5*d*x^4*(26*c*Hypergeometric2F1[9/8, -p, 17/8, -((b*x^8)/a)] + 9*d*x^4*
Hypergeometric2F1[13/8, -p, 21/8, -((b*x^8)/a)])))/(585*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(c+dx^4\right)^2\left(a+bx^8\right)^p\,dx$$

$$\Big\downarrow \text{1865}$$

$$\int \left(c^2x^4\left(a+bx^8\right)^p+2cdx^8\left(a+bx^8\right)^p+d^2x^{12}\left(a+bx^8\right)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}c^2x^5(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)+\frac{2}{9}cdx^9(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{9}{8},-p,\frac{17}{8},-\frac{bx^8}{a}\right)+\frac{1}{13}d^2x^{13}(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{13}{8},-p,\frac{21}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[x^4*(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
(c^2*x^5*(a + b*x^8)^p*Hypergeometric2F1[5/8, -p, 13/8, -((b*x^8)/a)])/(5*
(1 + (b*x^8)/a)^p) + (2*c*d*x^9*(a + b*x^8)^p*Hypergeometric2F1[9/8, -p, 1
7/8, -((b*x^8)/a)])/(9*(1 + (b*x^8)/a)^p) + (d^2*x^13*(a + b*x^8)^p*Hyperg
eometric2F1[13/8, -p, 21/8, -((b*x^8)/a)])/(13*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1865
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (f*x)^m*(d
 + e*x^n)^q, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && I
GtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^4\left(x^4d+c\right)^2\left(b\,x^8+a\right)^p dx$$

input
```
int(x^4*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

output
```
int(x^4*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int x^4 (c + dx^4)^2 (a + bx^8)^p \, dx = \int (dx^4 + c)^2 (bx^8 + a)^p x^4 \, dx$$

input
```
integrate(x^4*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^12 + 2*c*d*x^8 + c^2*x^4)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4 (c + dx^4)^2 (a + bx^8)^p \, dx = \text{Timed out}$$

input
```
integrate(x**4*(d*x**4+c)**2*(b*x**8+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^4 (c + dx^4)^2 (a + bx^8)^p \, dx = \int (dx^4 + c)^2 (bx^8 + a)^p x^4 \, dx$$

input
```
integrate(x^4*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p*x^4, x)
```

Giac [F]

$$\int x^4(c+dx^4)^2(a+bx^8)^p\,dx = \int (dx^4+c)^2(bx^8+a)^p x^4\,dx$$

input
```
integrate(x^4*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(c+dx^4)^2(a+bx^8)^p\,dx = \int x^4\,(b\,x^8+a)^p\,(d\,x^4+c)^2\,dx$$

input
```
int(x^4*(a + b*x^8)^p*(c + d*x^4)^2,x)
```

output
```
int(x^4*(a + b*x^8)^p*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int x^4(c+dx^4)^2(a+bx^8)^p\,dx = \text{too large to display}$$

input
```
int(x^4*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

output
```
(1024*(a + b*x**8)**p*a*c*d*p**3*x + 2304*(a + b*x**8)**p*a*c*d*p**2*x + 1
040*(a + b*x**8)**p*a*c*d*p*x + 512*(a + b*x**8)**p*a*d**2*p**3*x**5 + 640
*(a + b*x**8)**p*a*d**2*p**2*x**5 + 72*(a + b*x**8)**p*a*d**2*p*x**5 + 512
*(a + b*x**8)**p*b*c**2*p**3*x**5 + 1472*(a + b*x**8)**p*b*c**2*p**2*x**5
+ 1112*(a + b*x**8)**p*b*c**2*p*x**5 + 117*(a + b*x**8)**p*b*c**2*x**5 + 1
024*(a + b*x**8)**p*b*c*d*p**3*x**9 + 2432*(a + b*x**8)**p*b*c*d*p**2*x**9
 + 1328*(a + b*x**8)**p*b*c*d*p*x**9 + 130*(a + b*x**8)**p*b*c*d*x**9 + 51
2*(a + b*x**8)**p*b*d**2*p**3*x**13 + 960*(a + b*x**8)**p*b*d**2*p**2*x**1
3 + 472*(a + b*x**8)**p*b*d**2*p*x**13 + 45*(a + b*x**8)**p*b*d**2*x**13 -
 4194304*int((a + b*x**8)**p/(4096*a*p**4 + 14336*a*p**3 + 16256*a*p**2 +
6496*a*p + 585*a + 4096*b*p**4*x**8 + 14336*b*p**3*x**8 + 16256*b*p**2*x**
8 + 6496*b*p*x**8 + 585*b*x**8),x)*a**2*c*d*p**7 - 24117248*int((a + b*x**
8)**p/(4096*a*p**4 + 14336*a*p**3 + 16256*a*p**2 + 6496*a*p + 585*a + 4096
*b*p**4*x**8 + 14336*b*p**3*x**8 + 16256*b*p**2*x**8 + 6496*b*p*x**8 + 585
*b*x**8),x)*a**2*c*d*p**6 - 53936128*int((a + b*x**8)**p/(4096*a*p**4 + 14
336*a*p**3 + 16256*a*p**2 + 6496*a*p + 585*a + 4096*b*p**4*x**8 + 14336*b*
p**3*x**8 + 16256*b*p**2*x**8 + 6496*b*p*x**8 + 585*b*x**8),x)*a**2*c*d*p*
*5 - 59015168*int((a + b*x**8)**p/(4096*a*p**4 + 14336*a*p**3 + 16256*a*p*
*2 + 6496*a*p + 585*a + 4096*b*p**4*x**8 + 14336*b*p**3*x**8 + 16256*b*p**
2*x**8 + 6496*b*p*x**8 + 585*b*x**8),x)*a**2*c*d*p**4 - 32472064*int((a...
```

3.19 $\int x^2(c+dx^4)^2 (a+bx^8)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int x^2\left(c+dx^4\right)^2\left(a+bx^8\right)^p\,dx=\frac{d^2x^3\left(a+bx^8\right)^{1+p}}{b(11+8p)}-\frac{\left(3ad^2-bc^2(11+8p)\right)x^3\left(a+bx^8\right)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)}{3b(11+8p)}+\frac{2}{7}cdx^7\left(a+bx^8\right)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{8},-p,\frac{15}{8},-\frac{bx^8}{a}\right)$$

output
```
d^2*x^3*(b*x^8+a)^(p+1)/b/(11+8*p)-1/3*(3*a*d^2-b*c^2*(11+8*p))*x^3*(b*x^8
+a)^p*hypergeom([3/8, -p],[11/8],-b*x^8/a)/b/(11+8*p)/((1+b*x^8/a)^p)+2/7*
c*d*x^7*(b*x^8+a)^p*hypergeom([7/8, -p],[15/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69

$$\int x^2(c+dx^4)^2(a+bx^8)^p\,dx = \frac{1}{231}x^3(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(77c^2\,\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)+3dx^4\left(22c\,\text{Hypergeometric2F1}\left(\frac{7}{8},-p,\frac{15}{8},-\frac{bx^8}{a}\right)+7dx^4\,\text{Hypergeometric2F1}\left(\frac{11}{8},-p,\frac{19}{8},-\frac{bx^8}{a}\right)\right)\right)$$

input
```
Integrate[x^2*(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
(x^3*(a + b*x^8)^p*(77*c^2*Hypergeometric2F1[3/8, -p, 11/8, -((b*x^8)/a)]
+ 3*d*x^4*(22*c*Hypergeometric2F1[7/8, -p, 15/8, -((b*x^8)/a)] + 7*d*x^4*H
ypergeometric2F1[11/8, -p, 19/8, -((b*x^8)/a)])))/(231*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(c+dx^4)^2(a+bx^8)^p\,dx$$

$$\Big\downarrow \text{1865}$$

$$\int \left(c^2x^2(a+bx^8)^p+2cdx^6(a+bx^8)^p+d^2x^{10}(a+bx^8)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}c^2x^3(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)+\frac{2}{7}cdx^7(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{8},-p,\frac{15}{8},-\frac{bx^8}{a}\right)+\frac{1}{11}d^2x^{11}(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{11}{8},-p,\frac{19}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[x^2*(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
(c^2*x^3*(a + b*x^8)^p*Hypergeometric2F1[3/8, -p, 11/8, -((b*x^8)/a)])/(3*
(1 + (b*x^8)/a)^p) + (2*c*d*x^7*(a + b*x^8)^p*Hypergeometric2F1[7/8, -p, 1
5/8, -((b*x^8)/a)])/(7*(1 + (b*x^8)/a)^p) + (d^2*x^11*(a + b*x^8)^p*Hyperg
eometric2F1[11/8, -p, 19/8, -((b*x^8)/a)])/(11*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1865
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (f*x)^m*(d
 + e*x^n)^q, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && I
GtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^2(x^4d+c)^2(b\,x^8+a)^p\,dx$$

input
```
int(x^2*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

output
```
int(x^2*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int x^2(c+dx^4)^2\left(a+bx^8\right)^p\,dx=\int \left(dx^4+c\right)^2\left(bx^8+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^10 + 2*c*d*x^6 + c^2*x^2)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(c+dx^4)^2\left(a+bx^8\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**2*(d*x**4+c)**2*(b*x**8+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(c+dx^4)^2\left(a+bx^8\right)^p\,dx=\int \left(dx^4+c\right)^2\left(bx^8+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p*x^2, x)
```

Giac [F]

$$\int x^2 \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \int \left(dx^4+c\right)^2 \left(bx^8+a\right)^p x^2 \, dx$$

input
```
integrate(x^2*(d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \int x^2 \left(b\,x^8+a\right)^p \left(d\,x^4+c\right)^2 \, dx$$

input
```
int(x^2*(a + b*x^8)^p*(c + d*x^4)^2,x)
```

output
```
int(x^2*(a + b*x^8)^p*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int x^2 \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \text{too large to display}$$

input
```
int(x^2*(d*x^4+c)^2*(b*x^8+a)^p,x)
```

output

```
(64*(a + b*x**8)**p*a*d**2*p**2*x**3 + 56*(a + b*x**8)**p*a*d**2*p*x**3 +
64*(a + b*x**8)**p*b*c**2*p**2*x**3 + 144*(a + b*x**8)**p*b*c**2*p*x**3 +
77*(a + b*x**8)**p*b*c**2*x**3 + 128*(a + b*x**8)**p*b*c*d*p**2*x**7 + 224
*(a + b*x**8)**p*b*c*d*p*x**7 + 66*(a + b*x**8)**p*b*c*d*x**7 + 64*(a + b*
x**8)**p*b*d**2*p**2*x**11 + 80*(a + b*x**8)**p*b*d**2*p*x**11 + 21*(a + b
*x**8)**p*b*d**2*x**11 + 524288*int(((a + b*x**8)**p*x**6)/(512*a*p**3 + 1
344*a*p**2 + 1048*a*p + 231*a + 512*b*p**3*x**8 + 1344*b*p**2*x**8 + 1048*
b*p*x**8 + 231*b*x**8),x)*a*b*c*d*p**6 + 2293760*int(((a + b*x**8)**p*x**6
)/(512*a*p**3 + 1344*a*p**2 + 1048*a*p + 231*a + 512*b*p**3*x**8 + 1344*b*
p**2*x**8 + 1048*b*p*x**8 + 231*b*x**8),x)*a*b*c*d*p**5 + 3751936*int(((a
+ b*x**8)**p*x**6)/(512*a*p**3 + 1344*a*p**2 + 1048*a*p + 231*a + 512*b*p*
*3*x**8 + 1344*b*p**2*x**8 + 1048*b*p*x**8 + 231*b*x**8),x)*a*b*c*d*p**4 +
 2824192*int(((a + b*x**8)**p*x**6)/(512*a*p**3 + 1344*a*p**2 + 1048*a*p +
 231*a + 512*b*p**3*x**8 + 1344*b*p**2*x**8 + 1048*b*p*x**8 + 231*b*x**8),
x)*a*b*c*d*p**3 + 967296*int(((a + b*x**8)**p*x**6)/(512*a*p**3 + 1344*a*p
**2 + 1048*a*p + 231*a + 512*b*p**3*x**8 + 1344*b*p**2*x**8 + 1048*b*p*x**
8 + 231*b*x**8),x)*a*b*c*d*p**2 + 121968*int(((a + b*x**8)**p*x**6)/(512*a
*p**3 + 1344*a*p**2 + 1048*a*p + 231*a + 512*b*p**3*x**8 + 1344*b*p**2*x**
8 + 1048*b*p*x**8 + 231*b*x**8),x)*a*b*c*d*p - 98304*int(((a + b*x**8)**p*
x**2)/(512*a*p**3 + 1344*a*p**2 + 1048*a*p + 231*a + 512*b*p**3*x**8 + ...
```

3.20 $\int (c+dx^4)^2 (a+bx^8)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 150

$$\int (c+dx^4)^2 (a+bx^8)^p \, dx = \frac{d^2x(a+bx^8)^{1+p}}{b(9+8p)} - \frac{(ad^2-bc^2(9+8p))\,x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)}{b(9+8p)} + \frac{2}{5}cdx^5(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)$$

output
```
d^2*x*(b*x^8+a)^(p+1)/b/(9+8*p)-(a*d^2-b*c^2*(9+8*p))*x*(b*x^8+a)^p*hyperg
eom([1/8, -p],[9/8],-b*x^8/a)/b/(9+8*p)/((1+b*x^8/a)^p)+2/5*c*d*x^5*(b*x^8
+a)^p*hypergeom([5/8, -p],[13/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \frac{1}{45}x\left(a+bx^8\right)^p \left(1+\frac{bx^8}{a}\right)^{-p} \left(45c^2 \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+dx^4\left(18c\,\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)+5dx^4\,\text{Hypergeometric2F1}\left(\frac{9}{8},-p,\frac{17}{8},-\frac{bx^8}{a}\right)\right)\right)$$

input
```
Integrate[(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*(45*c^2*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] + d
*x^4*(18*c*Hypergeometric2F1[5/8, -p, 13/8, -((b*x^8)/a)] + 5*d*x^4*Hyperg
eometric2F1[9/8, -p, 17/8, -((b*x^8)/a)])))/(45*(1 + (b*x^8)/a)^p)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx$$

$$\Big\downarrow \text{1770}$$

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p dx$$

input
```
Int[(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1770
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Unintegrable[(d + e*x^n)^q*(a + c*x^(2*n))^p, x] /; FreeQ[{a, c, d, e,
n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \left(x^{4} d+c\right)^{2} \left(b \,x^{8}+a\right)^{p}d x$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p,x)
```

output
```
int((d*x^4+c)^2*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \int \left(dx^{4} + c\right)^{2} {\left(bx^{8} + a\right)}^{p} \,d x$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)**2*(b*x**8+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \int \left(dx^4+c\right)^2 \left(bx^8+a\right)^p \, dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \int \left(dx^4+c\right)^2 \left(bx^8+a\right)^p \, dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \int \left(b\,x^8+a\right)^p \left(d\,x^4+c\right)^2 \, dx$$

input
```
int((a + b*x^8)^p*(c + d*x^4)^2,x)
```

output
```
int((a + b*x^8)^p*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int (c+dx^4)^2 (a+bx^8)^p \, dx = \text{too large to display}$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p,x)
```

output
```
(64*(a + b*x**8)**p*a*d**2*p**2*x + 40*(a + b*x**8)**p*a*d**2*p*x + 64*(a
+ b*x**8)**p*b*c**2*p**2*x + 112*(a + b*x**8)**p*b*c**2*p*x + 45*(a + b*x*
*8)**p*b*c**2*x + 128*(a + b*x**8)**p*b*c*d*p**2*x**5 + 160*(a + b*x**8)**
p*b*c*d*p*x**5 + 18*(a + b*x**8)**p*b*c*d*x**5 + 64*(a + b*x**8)**p*b*d**2
*p**2*x**9 + 48*(a + b*x**8)**p*b*d**2*p*x**9 + 5*(a + b*x**8)**p*b*d**2*x
**9 - 32768*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p**2 + 472*a*p + 45*a
+ 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 + 45*b*x**8),x)*a**2*d*
*2*p**5 - 81920*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p**2 + 472*a*p + 4
5*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 + 45*b*x**8),x)*a**
2*d**2*p**4 - 68608*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p**2 + 472*a*p
 + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 + 45*b*x**8),x)
*a**2*d**2*p**3 - 21760*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p**2 + 472
*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 + 45*b*x**8
),x)*a**2*d**2*p**2 - 1800*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p**2 +
472*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 + 45*b*x
**8),x)*a**2*d**2*p + 262144*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p**2
+ 472*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 + 45*b
*x**8),x)*a*b*c**2*p**6 + 950272*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p
**2 + 472*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 +
45*b*x**8),x)*a*b*c**2*p**5 + 1286144*int((a + b*x**8)**p/(512*a*p**3 +...
```

3.21 $\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^2}\,dx = \frac{d^2(a+bx^8)^{1+p}}{b(7+8p)x} - \frac{(ad^2+bc^2(7+8p))(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{8},-p,\frac{7}{8},-\frac{bx^8}{a}\right)}{b(7+8p)x} + \frac{2}{3}cdx^3(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)$$

output
```
d^2*(b*x^8+a)^(p+1)/b/(7+8*p)/x-(a*d^2+b*c^2*(7+8*p))*(b*x^8+a)^p*hypergeo
m([-1/8, -p],[7/8],-b*x^8/a)/b/(7+8*p)/x/((1+b*x^8/a)^p)+2/3*c*d*x^3*(b*x^
8+a)^p*hypergeom([3/8, -p],[11/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.71

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^2} \, dx = \frac{(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \left(-21c^2 \text{Hypergeometric2F1}\left(-\frac{1}{8},-p,\frac{7}{8},-\frac{bx^8}{a}\right)+dx^4\left(14c \text{Hypergeometric2F1}\left(\right.\right.\right.}{21x}$$

input
```
Integrate[((c + d*x^4)^2*(a + b*x^8)^p)/x^2,x]
```

output
```
((a + b*x^8)^p*(-21*c^2*Hypergeometric2F1[-1/8, -p, 7/8, -((b*x^8)/a)] + d
*x^4*(14*c*Hypergeometric2F1[3/8, -p, 11/8, -((b*x^8)/a)] + 3*d*x^4*Hyperg
eometric2F1[7/8, -p, 15/8, -((b*x^8)/a)])))/(21*x*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^2} \, dx$$

$$\Big\downarrow \text{1865}$$

$$\int \left(\frac{c^2 (a+bx^8)^p}{x^2} + 2cdx^2 (a+bx^8)^p + d^2x^6 (a+bx^8)^p\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{8},-p,\frac{7}{8},-\frac{bx^8}{a}\right)}{x}+\frac{2}{3}cdx^3(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)+\frac{1}{7}d^2x^7(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{8},-p,\frac{15}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[((c + d*x^4)^2*(a + b*x^8)^p)/x^2,x]
```

output
```
-((c^2*(a + b*x^8)^p*Hypergeometric2F1[-1/8, -p, 7/8, -((b*x^8)/a)])/(x*(1
 + (b*x^8)/a)^p)) + (2*c*d*x^3*(a + b*x^8)^p*Hypergeometric2F1[3/8, -p, 11
/8, -((b*x^8)/a)])/(3*(1 + (b*x^8)/a)^p) + (d^2*x^7*(a + b*x^8)^p*Hypergeo
metric2F1[7/8, -p, 15/8, -((b*x^8)/a)])/(7*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1865
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (f*x)^m*(d
 + e*x^n)^q, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && I
GtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(x^4d+c)^2\,(b\,x^8+a)^p}{x^2}dx$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p/x^2,x)
```

output
```
int((d*x^4+c)^2*(b*x^8+a)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^2}\,dx = \int \frac{(dx^4+c)^2(bx^8+a)^p}{x^2}\,dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^8 + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)**2*(b*x**8+a)**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^2}\,dx = \int \frac{(dx^4+c)^2(bx^8+a)^p}{x^2}\,dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^2}\,dx=\int \frac{(dx^4+c)^2(bx^8+a)^p}{x^2}\,dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^2}\,dx=\int \frac{(b\,x^8+a)^p\,(d\,x^4+c)^2}{x^2}\,dx$$

input
```
int(((a + b*x^8)^p*(c + d*x^4)^2)/x^2,x)
```

output
```
int(((a + b*x^8)^p*(c + d*x^4)^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^2}\,dx=\text{too large to display}$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p/x^2,x)
```

output

```
(64*(a + b*x**8)**p*a*d**2*p**2 + 24*(a + b*x**8)**p*a*d**2*p + 64*(a + b*
x**8)**p*b*c**2*p**2 + 80*(a + b*x**8)**p*b*c**2*p + 21*(a + b*x**8)**p*b*
c**2 + 128*(a + b*x**8)**p*b*c*d*p**2*x**4 + 96*(a + b*x**8)**p*b*c*d*p*x*
*4 - 14*(a + b*x**8)**p*b*c*d*x**4 + 64*(a + b*x**8)**p*b*d**2*p**2*x**8 +
 16*(a + b*x**8)**p*b*d**2*p*x**8 - 3*(a + b*x**8)**p*b*d**2*x**8 + 32768*
int((a + b*x**8)**p/(512*a*p**3*x**2 + 576*a*p**2*x**2 + 88*a*p*x**2 - 21*
a*x**2 + 512*b*p**3*x**10 + 576*b*p**2*x**10 + 88*b*p*x**10 - 21*b*x**10),
x)*a**2*d**2*p**5*x + 49152*int((a + b*x**8)**p/(512*a*p**3*x**2 + 576*a*p
**2*x**2 + 88*a*p*x**2 - 21*a*x**2 + 512*b*p**3*x**10 + 576*b*p**2*x**10 +
 88*b*p*x**10 - 21*b*x**10),x)*a**2*d**2*p**4*x + 19456*int((a + b*x**8)**
p/(512*a*p**3*x**2 + 576*a*p**2*x**2 + 88*a*p*x**2 - 21*a*x**2 + 512*b*p**
3*x**10 + 576*b*p**2*x**10 + 88*b*p*x**10 - 21*b*x**10),x)*a**2*d**2*p**3*
x + 768*int((a + b*x**8)**p/(512*a*p**3*x**2 + 576*a*p**2*x**2 + 88*a*p*x*
*2 - 21*a*x**2 + 512*b*p**3*x**10 + 576*b*p**2*x**10 + 88*b*p*x**10 - 21*b
*x**10),x)*a**2*d**2*p**2*x - 504*int((a + b*x**8)**p/(512*a*p**3*x**2 + 5
76*a*p**2*x**2 + 88*a*p*x**2 - 21*a*x**2 + 512*b*p**3*x**10 + 576*b*p**2*x
**10 + 88*b*p*x**10 - 21*b*x**10),x)*a**2*d**2*p*x + 262144*int((a + b*x**
8)**p/(512*a*p**3*x**2 + 576*a*p**2*x**2 + 88*a*p*x**2 - 21*a*x**2 + 512*b
*p**3*x**10 + 576*b*p**2*x**10 + 88*b*p*x**10 - 21*b*x**10),x)*a*b*c**2*p*
*6*x + 622592*int((a + b*x**8)**p/(512*a*p**3*x**2 + 576*a*p**2*x**2 + ...
```

3.22 $\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^4}\,dx = \frac{d^2(a+bx^8)^{1+p}}{b(5+8p)x^3} - \frac{(3ad^2+bc^2(5+8p))(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{8},-p,\frac{5}{8},-\frac{bx^8}{a}\right)}{3b(5+8p)x^3} + 2cdx(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

output
```
d^2*(b*x^8+a)^(p+1)/b/(5+8*p)/x^3-1/3*(3*a*d^2+b*c^2*(5+8*p))*(b*x^8+a)^p*
hypergeom([-3/8, -p],[5/8],-b*x^8/a)/b/(5+8*p)/x^3/((1+b*x^8/a)^p)+2*c*d*x
*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.71

$$\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^4}\,dx$$
$$= \frac{(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(-5c^2\,\text{Hypergeometric2F1}\left(-\frac{3}{8},-p,\frac{5}{8},-\frac{bx^8}{a}\right)+3dx^4\left(10c\,\text{Hypergeometric2F1}\left(\right.\right.\right.}{15x^3}$$

input
```
Integrate[((c + d*x^4)^2*(a + b*x^8)^p)/x^4,x]
```

output
```
((a + b*x^8)^p*(-5*c^2*Hypergeometric2F1[-3/8, -p, 5/8, -((b*x^8)/a)] + 3*
d*x^4*(10*c*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] + d*x^4*Hypergeo
metric2F1[5/8, -p, 13/8, -((b*x^8)/a)])))/(15*x^3*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2(a+bx^8)^p}{x^4}\,dx$$
$$\Big\downarrow \text{1865}$$
$$\int \left(\frac{c^2(a+bx^8)^p}{x^4}+2cd(a+bx^8)^p+d^2x^4(a+bx^8)^p\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{c^2(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{8},-p,\frac{5}{8},-\frac{bx^8}{a}\right)}{3x^3}+ 2cdx(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+ \frac{1}{5}d^2x^5(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[((c + d*x^4)^2*(a + b*x^8)^p)/x^4,x]
```

output
```
-1/3*(c^2*(a + b*x^8)^p*Hypergeometric2F1[-3/8, -p, 5/8, -((b*x^8)/a)])/(x
^3*(1 + (b*x^8)/a)^p) + (2*c*d*x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p,
9/8, -((b*x^8)/a)])/(1 + (b*x^8)/a)^p + (d^2*x^5*(a + b*x^8)^p*Hypergeomet
ric2F1[5/8, -p, 13/8, -((b*x^8)/a)])/(5*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1865
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (f*x)^m*(d
 + e*x^n)^q, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && I
GtQ[n, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(x^4d+c)^2\,(b\,x^8+a)^p}{x^4}dx$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p/x^4,x)
```

output
```
int((d*x^4+c)^2*(b*x^8+a)^p/x^4,x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^4} \, dx = \int \frac{(dx^4+c)^2 (bx^8+a)^p}{x^4} \, dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x^4,x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^8 + a)^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^4} \, dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)**2*(b*x**8+a)**p/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^4} \, dx = \int \frac{(dx^4+c)^2 (bx^8+a)^p}{x^4} \, dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x^4,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^4} \, dx = \int \frac{(dx^4+c)^2(bx^8+a)^p}{x^4} \, dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p/x^4,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^4} \, dx = \int \frac{(b\,x^8+a)^p\,(d\,x^4+c)^2}{x^4} \, dx$$

input
```
int(((a + b*x^8)^p*(c + d*x^4)^2)/x^4,x)
```

output
```
int(((a + b*x^8)^p*(c + d*x^4)^2)/x^4, x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2 (a+bx^8)^p}{x^4} \, dx = \text{too large to display}$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p/x^4,x)
```

output

```
(64*(a + b*x**8)**p*a*d**2*p**2 + 8*(a + b*x**8)**p*a*d**2*p + 64*(a + b*x
**8)**p*b*c**2*p**2 + 48*(a + b*x**8)**p*b*c**2*p + 5*(a + b*x**8)**p*b*c*
*2 + 128*(a + b*x**8)**p*b*c*d*p**2*x**4 + 32*(a + b*x**8)**p*b*c*d*p*x**4
 - 30*(a + b*x**8)**p*b*c*d*x**4 + 64*(a + b*x**8)**p*b*d**2*p**2*x**8 - 1
6*(a + b*x**8)**p*b*d**2*p*x**8 - 3*(a + b*x**8)**p*b*d**2*x**8 + 98304*in
t((a + b*x**8)**p/(512*a*p**3*x**4 + 192*a*p**2*x**4 - 104*a*p*x**4 - 15*a
*x**4 + 512*b*p**3*x**12 + 192*b*p**2*x**12 - 104*b*p*x**12 - 15*b*x**12),
x)*a**2*d**2*p**5*x**3 + 49152*int((a + b*x**8)**p/(512*a*p**3*x**4 + 192*
a*p**2*x**4 - 104*a*p*x**4 - 15*a*x**4 + 512*b*p**3*x**12 + 192*b*p**2*x**
12 - 104*b*p*x**12 - 15*b*x**12),x)*a**2*d**2*p**4*x**3 - 15360*int((a + b
*x**8)**p/(512*a*p**3*x**4 + 192*a*p**2*x**4 - 104*a*p*x**4 - 15*a*x**4 +
512*b*p**3*x**12 + 192*b*p**2*x**12 - 104*b*p*x**12 - 15*b*x**12),x)*a**2*
d**2*p**3*x**3 - 5376*int((a + b*x**8)**p/(512*a*p**3*x**4 + 192*a*p**2*x*
*4 - 104*a*p*x**4 - 15*a*x**4 + 512*b*p**3*x**12 + 192*b*p**2*x**12 - 104*
b*p*x**12 - 15*b*x**12),x)*a**2*d**2*p**2*x**3 - 360*int((a + b*x**8)**p/(
512*a*p**3*x**4 + 192*a*p**2*x**4 - 104*a*p*x**4 - 15*a*x**4 + 512*b*p**3*
x**12 + 192*b*p**2*x**12 - 104*b*p*x**12 - 15*b*x**12),x)*a**2*d**2*p*x**3
 + 262144*int((a + b*x**8)**p/(512*a*p**3*x**4 + 192*a*p**2*x**4 - 104*a*p
*x**4 - 15*a*x**4 + 512*b*p**3*x**12 + 192*b*p**2*x**12 - 104*b*p*x**12 -
15*b*x**12),x)*a*b*c**2*p**6*x**3 + 294912*int((a + b*x**8)**p/(512*a*p...
```

3.23 $\int \frac{x^7(a+bx^8)^p}{c+dx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 131

$$\int \frac{x^7(a+bx^8)^p}{c+dx^4}\,dx = -\frac{dx^{12}(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},-p,1,\frac{5}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{12c^2} + \frac{c(a+bx^8)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{d^2(a+bx^8)}{bc^2+ad^2}\right)}{8\left(bc^2+ad^2\right)(1+p)}$$

output
```
-1/12*d*x^12*(b*x^8+a)^p*AppellF1(3/2,1,-p,5/2,d^2*x^8/c^2,-b*x^8/a)/c^2/(
(1+b*x^8/a)^p)+1/8*c*(b*x^8+a)^(p+1)*hypergeom([1, p+1],[2+p],d^2*(b*x^8+a
)/(a*d^2+b*c^2))/(a*d^2+b*c^2)/(p+1)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.42

$$\int \frac{x^7(a+bx^8)^p}{c+dx^4}\,dx = \frac{(a+bx^8)^p\left(-\frac{c\left(\frac{d\left(-\sqrt{-\frac{a}{b}}+x^4\right)}{c+dx^4}\right)^{-p}\left(\frac{d\left(\sqrt{-\frac{a}{b}}+x^4\right)}{c+dx^4}\right)^{-p}\operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c-\sqrt{-\frac{a}{b}}d}{c+dx^4},\frac{c+\sqrt{-\frac{a}{b}}d}{c+dx^4}\right)}{p} + 2dx^4\left(1+\frac{bx^8}{a}\right)^{-p}\right.}{8d^2}$$

input
```
Integrate[(x^7*(a + b*x^8)^p)/(c + d*x^4),x]
```

output
```
((a + b*x^8)^p*(-((c*AppellF1[-2*p, -p, -p, 1 - 2*p, (c - Sqrt[-(a/b)]*d)/
(c + d*x^4), (c + Sqrt[-(a/b)]*d)/(c + d*x^4)])/(p*((d*(-Sqrt[-(a/b)] + x^
4))/(c + d*x^4))^p*((d*(Sqrt[-(a/b)] + x^4))/(c + d*x^4))^p)) + (2*d*x^4*H
ypergeometric2F1[1/2, -p, 3/2, -((b*x^8)/a)])/(1 + (b*x^8)/a)^p))/(8*d^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1803, 621, 353, 78, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(a+b x^8\right)^p}{c+d x^4} \, dx$$

$$\Big\downarrow \text{1803}$$

$$\frac{1}{4} \int \frac{x^4 \left(b x^8+a\right)^p}{d x^4+c} dx^4$$

$$\Big\downarrow \text{621}$$

$$\frac{1}{4} \left(c \int \frac{x^4 \left(b x^8+a\right)^p}{c^2-d^2 x^8} dx^4-d \int \frac{x^8 \left(b x^8+a\right)^p}{c^2-d^2 x^8} dx^4\right)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{4} \left(\frac{1}{2} c \int \frac{\left(b x^8+a\right)^p}{c^2-d^2 x^8} dx^8-d \int \frac{x^8 \left(b x^8+a\right)^p}{c^2-d^2 x^8} dx^4\right)$$

$$\Big\downarrow \text{78}$$

$$\frac{1}{4} \left(\frac{c \left(a+b x^8\right)^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{d^2 \left(b x^8+a\right)}{b c^2+a d^2}\right)}{2 (p+1) \left(a d^2+b c^2\right)}-d \int \frac{x^8 \left(b x^8+a\right)^p}{c^2-d^2 x^8} dx^4\right)$$

$$\Big\downarrow \text{395}$$

$$\frac{1}{4}\left(\frac{c(a+bx^8)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{d^2(bx^8+a)}{bc^2+ad^2}\right)}{2(p+1)\left(ad^2+bc^2\right)}-d(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int\frac{x^8\left(\frac{bx^8}{a}+1\right)^p}{c^2-d^2x^8}\right.$$

$$\Big\downarrow \textbf{394}$$

$$\frac{1}{4}\left(\frac{c(a+bx^8)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{d^2(bx^8+a)}{bc^2+ad^2}\right)}{2(p+1)\left(ad^2+bc^2\right)}-\frac{dx^{12}(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},\right.}{3c^2}\right.$$

input
```
Int[(x^7*(a + b*x^8)^p)/(c + d*x^4),x]
```

output
```
(-1/3*(d*x^12*(a + b*x^8)^p*AppellF1[3/2, -p, 1, 5/2, -((b*x^8)/a), (d^2*x
^8)/c^2])/(c^2*(1 + (b*x^8)/a)^p) + (c*(a + b*x^8)^(1 + p)*Hypergeometric2
F1[1, 1 + p, 2 + p, (d^2*(a + b*x^8))/(b*c^2 + a*d^2)])/(2*(b*c^2 + a*d^2)
*(1 + p)))/4
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 621
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[c   Int[x^m*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] - Simp[d   Int[
x^(m + 1)*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, m,
p}, x]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{7} \left(b\,x^{8}+a\right)^{p}}{x^{4} d+c}dx$$

input
```
int(x^7*(b*x^8+a)^p/(d*x^4+c),x)
```

output
```
int(x^7*(b*x^8+a)^p/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{x^7(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(bx^8+a)^p x^7}{dx^4+c}\,dx$$

input
```
integrate(x^7*(b*x^8+a)^p/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p*x^7/(d*x^4 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(a+bx^8)^p}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(b*x**8+a)**p/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^7(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(bx^8+a)^p x^7}{dx^4+c}\,dx$$

input
```
integrate(x^7*(b*x^8+a)^p/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p*x^7/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{x^7(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(bx^8+a)^p x^7}{dx^4+c}\,dx$$

input
```
integrate(x^7*(b*x^8+a)^p/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p*x^7/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{x^7\,(b\,x^8+a)^p}{d\,x^4+c}\,dx$$

input
```
int((x^7*(a + b*x^8)^p)/(c + d*x^4),x)
```

output
```
int((x^7*(a + b*x^8)^p)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{x^7(a+bx^8)^p}{c+dx^4}\,dx$$
$$= \frac{(b\,x^8+a)^p\,ad+(b\,x^8+a)^p\,bc\,x^4-16\left(\int \frac{(b\,x^8+a)^p\,x^{11}}{2bdp\,x^{12}+bd\,x^{12}+2bcp\,x^8+bc\,x^8+2adp\,x^4+ad\,x^4+2acp+ac}dx\right)ab\,d^2p^2-8\left(\int\right.}{}$$

input
```
int(x^7*(b*x^8+a)^p/(d*x^4+c),x)
```

output
```
((a + b*x**8)**p*a*d + (a + b*x**8)**p*b*c*x**4 - 16*int(((a + b*x**8)**p*
x**11)/(2*a*c*p + a*c + 2*a*d*p*x**4 + a*d*x**4 + 2*b*c*p*x**8 + b*c*x**8
+ 2*b*d*p*x**12 + b*d*x**12),x)*a*b*d**2*p**2 - 8*int(((a + b*x**8)**p*x**
11)/(2*a*c*p + a*c + 2*a*d*p*x**4 + a*d*x**4 + 2*b*c*p*x**8 + b*c*x**8 + 2
*b*d*p*x**12 + b*d*x**12),x)*a*b*d**2*p - 16*int(((a + b*x**8)**p*x**11)/(
2*a*c*p + a*c + 2*a*d*p*x**4 + a*d*x**4 + 2*b*c*p*x**8 + b*c*x**8 + 2*b*d*
p*x**12 + b*d*x**12),x)*b**2*c**2*p**2 - 16*int(((a + b*x**8)**p*x**11)/(2
*a*c*p + a*c + 2*a*d*p*x**4 + a*d*x**4 + 2*b*c*p*x**8 + b*c*x**8 + 2*b*d*p
*x**12 + b*d*x**12),x)*b**2*c**2*p - 4*int(((a + b*x**8)**p*x**11)/(2*a*c*
p + a*c + 2*a*d*p*x**4 + a*d*x**4 + 2*b*c*p*x**8 + b*c*x**8 + 2*b*d*p*x**1
2 + b*d*x**12),x)*b**2*c**2 - 8*int(((a + b*x**8)**p*x**3)/(2*a*c*p + a*c
+ 2*a*d*p*x**4 + a*d*x**4 + 2*b*c*p*x**8 + b*c*x**8 + 2*b*d*p*x**12 + b*d*
x**12),x)*a*b*c**2*p - 4*int(((a + b*x**8)**p*x**3)/(2*a*c*p + a*c + 2*a*d
*p*x**4 + a*d*x**4 + 2*b*c*p*x**8 + b*c*x**8 + 2*b*d*p*x**12 + b*d*x**12),
x)*a*b*c**2)/(4*b*c*d*(2*p + 1))
```

3.24 $\int \frac{x^5(a+bx^8)^p}{c+dx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{x^5(a+bx^8)^p}{c+dx^4}\,dx = \frac{x^6(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{6c} - \frac{dx^{10}(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{5}{4},-p,1,\frac{9}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{10c^2}$$

output
```
1/6*x^6*(b*x^8+a)^p*AppellF1(3/4,1,-p,7/4,d^2*x^8/c^2,-b*x^8/a)/c/((1+b*x^
8/a)^p)-1/10*d*x^10*(b*x^8+a)^p*AppellF1(5/4,1,-p,9/4,d^2*x^8/c^2,-b*x^8/a
)/c^2/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{x^5(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{x^5(a+bx^8)^p}{c+dx^4}\,dx$$

input
```
Integrate[(x^5*(a + b*x^8)^p)/(c + d*x^4),x]
```

output
```
Integrate[(x^5*(a + b*x^8)^p)/(c + d*x^4), x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.39, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1815, 1675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (a+bx^8)^p}{c+dx^4} \, dx$$

$$\Big\downarrow \text{1815}$$

$$\frac{1}{2} \int \frac{x^4 (bx^8+a)^p}{dx^4+c} dx^2$$

$$\Big\downarrow \text{1675}$$

$$\frac{1}{2} \int \left(\frac{(bx^8+a)^p}{d} - \frac{c(bx^8+a)^p}{d(dx^4+c)}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{x^6 (a+bx^8)^p \left(\frac{bx^8}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{3c} - \frac{x^2 (a+bx^8)^p \left(\frac{bx^8}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{4},-p,\right.}{d}\right.$$

input
```
Int[(x^5*(a + b*x^8)^p)/(c + d*x^4),x]
```

output
```
(-((x^2*(a + b*x^8)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^8)/a), (d^2*x^8)/c^
2])/(d*(1 + (b*x^8)/a)^p)) + (x^6*(a + b*x^8)^p*AppellF1[3/4, -p, 1, 7/4,
-((b*x^8)/a), (d^2*x^8)/c^2])/(3*c*(1 + (b*x^8)/a)^p) + (x^2*(a + b*x^8)^p
*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^8)/a)])/(d*(1 + (b*x^8)/a)^p))/2
```

Defintions of rubi rules used

rule 1675
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

rule 1815
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/
k - 1)*(d + e*x^(n/k))^q*(a + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1]
/; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^5(b\,x^8+a)^p}{x^4d+c}dx$$

input
```
int(x^5*(b*x^8+a)^p/(d*x^4+c),x)
```

output
```
int(x^5*(b*x^8+a)^p/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{x^5(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(bx^8+a)^p x^5}{dx^4+c}\,dx$$

input
```
integrate(x^5*(b*x^8+a)^p/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p*x^5/(d*x^4 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^8)^p}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(b*x**8+a)**p/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(bx^8+a)^p x^5}{dx^4+c}\,dx$$

input
```
integrate(x^5*(b*x^8+a)^p/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p*x^5/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{x^5(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(bx^8+a)^p x^5}{dx^4+c}\,dx$$

input
```
integrate(x^5*(b*x^8+a)^p/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p*x^5/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{x^5\,{\left(b\,x^8+a\right)}^p}{d\,x^4+c}\,dx$$

input
```
int((x^5*(a + b*x^8)^p)/(c + d*x^4),x)
```

output
```
int((x^5*(a + b*x^8)^p)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{x^5(a+bx^8)^p}{c+dx^4}\,dx = \frac{(b\,x^8+a)^p\,x^2-32\left(\int \frac{(b\,x^8+a)^p x^9}{4bdp\,x^{12}+bd\,x^{12}+4bcp\,x^8+bc\,x^8+4adp\,x^4+ad\,x^4+4acp+ac}dx\right)bc\,p^2-16\left(\int \frac{(b\,x^8}{4bdp\,x^{12}+bd\,x^{12}+4bcp\,x^8+}\right.}{}$$

input
```
int(x^5*(b*x^8+a)^p/(d*x^4+c),x)
```

output
```
((a + b*x**8)**p*x**2 - 32*int(((a + b*x**8)**p*x**9)/(4*a*c*p + a*c + 4*a
*d*p*x**4 + a*d*x**4 + 4*b*c*p*x**8 + b*c*x**8 + 4*b*d*p*x**12 + b*d*x**12
),x)*b*c*p**2 - 16*int(((a + b*x**8)**p*x**9)/(4*a*c*p + a*c + 4*a*d*p*x**
4 + a*d*x**4 + 4*b*c*p*x**8 + b*c*x**8 + 4*b*d*p*x**12 + b*d*x**12),x)*b*c
*p - 2*int(((a + b*x**8)**p*x**9)/(4*a*c*p + a*c + 4*a*d*p*x**4 + a*d*x**4
 + 4*b*c*p*x**8 + b*c*x**8 + 4*b*d*p*x**12 + b*d*x**12),x)*b*c + 32*int(((
a + b*x**8)**p*x**5)/(4*a*c*p + a*c + 4*a*d*p*x**4 + a*d*x**4 + 4*b*c*p*x*
*8 + b*c*x**8 + 4*b*d*p*x**12 + b*d*x**12),x)*a*d*p**2 + 8*int(((a + b*x**
8)**p*x**5)/(4*a*c*p + a*c + 4*a*d*p*x**4 + a*d*x**4 + 4*b*c*p*x**8 + b*c*
x**8 + 4*b*d*p*x**12 + b*d*x**12),x)*a*d*p - 8*int(((a + b*x**8)**p*x)/(4*
a*c*p + a*c + 4*a*d*p*x**4 + a*d*x**4 + 4*b*c*p*x**8 + b*c*x**8 + 4*b*d*p*
x**12 + b*d*x**12),x)*a*c*p - 2*int(((a + b*x**8)**p*x)/(4*a*c*p + a*c + 4
*a*d*p*x**4 + a*d*x**4 + 4*b*c*p*x**8 + b*c*x**8 + 4*b*d*p*x**12 + b*d*x**
12),x)*a*c)/(2*d*(4*p + 1))
```

3.25 $\int \frac{x^3(a+bx^8)^p}{c+dx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 130

$$\int \frac{x^3(a+bx^8)^p}{c+dx^4}\,dx = \frac{x^4(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{4c} - \frac{d(a+bx^8)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{d^2(a+bx^8)}{bc^2+ad^2}\right)}{8(bc^2+ad^2)(1+p)}$$

output
```
1/4*x^4*(b*x^8+a)^p*AppellF1(1/2,1,-p,3/2,d^2*x^8/c^2,-b*x^8/a)/c/((1+b*x^
8/a)^p)-1/8*d*(b*x^8+a)^(p+1)*hypergeom([1, p+1],[2+p],d^2*(b*x^8+a)/(a*d^
2+b*c^2))/(a*d^2+b*c^2)/(p+1)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int \frac{x^3(a+bx^8)^p}{c+dx^4}\,dx = \frac{\left(\frac{d\left(-\sqrt{-\frac{a}{b}}+x^4\right)}{c+dx^4}\right)^{-p}\left(\frac{d\left(\sqrt{-\frac{a}{b}}+x^4\right)}{c+dx^4}\right)^{-p}(a+bx^8)^p\operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c-\sqrt{-\frac{a}{b}}d}{c+dx^4},\frac{c+\sqrt{-\frac{a}{b}}d}{c+dx^4}\right)}{8dp}$$

input
```
Integrate[(x^3*(a + b*x^8)^p)/(c + d*x^4),x]
```

output
```
((a + b*x^8)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (c - Sqrt[-(a/b)]*d)/(c + d
*x^4), (c + Sqrt[-(a/b)]*d)/(c + d*x^4)])/(8*d*p*((d*(-Sqrt[-(a/b)] + x^4)
)/(c + d*x^4))^p*((d*(Sqrt[-(a/b)] + x^4))/(c + d*x^4))^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1799, 504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+bx^8)^p}{c+dx^4} \, dx$$

$$\Big\downarrow \; 1799$$

$$\frac{1}{4}\int \frac{(bx^8+a)^p}{dx^4+c}dx^4$$

$$\Big\downarrow \; 504$$

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{c^2-d^2x^8}dx^4-d\int \frac{x^4(bx^8+a)^p}{c^2-d^2x^8}dx^4\right)$$

$$\Big\downarrow \; 334$$

$$\frac{1}{4}\left(c(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^8}{a}+1\right)^p}{c^2-d^2x^8}dx^4-d\int \frac{x^4(bx^8+a)^p}{c^2-d^2x^8}dx^4\right)$$

$$\Big\downarrow \; 333$$

$$\frac{1}{4}\left(\frac{x^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c}-d\int \frac{x^4(bx^8+a)^p}{c^2-d^2x^8}dx^4\right)$$

$$\Big\downarrow \; 353$$

$$\frac{1}{4}\left(\frac{x^4\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c}-\frac{1}{2}d\int\frac{\left(bx^8+a\right)^p}{c^2-d^2x^8}dx^8\right)$$

$$\Big\downarrow \text{78}$$

$$\frac{1}{4}\left(\frac{x^4\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c}-\frac{d\left(a+bx^8\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1\right.}{2(p+1)\left(ad^2+bc^2\right)}\right.$$

input
```
Int[(x^3*(a + b*x^8)^p)/(c + d*x^4),x]
```

output
```
((x^4*(a + b*x^8)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^8)/a), (d^2*x^8)/c^2]
)/(c*(1 + (b*x^8)/a)^p) - (d*(a + b*x^8)^(1 + p)*Hypergeometric2F1[1, 1 +
p, 2 + p, (d^2*(a + b*x^8))/(b*c^2 + a*d^2)])/(2*(b*c^2 + a*d^2)*(1 + p)))
/4
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

Maple [F]

$$\int \frac{x^{3} \left(b \,x^{8}+a \right)^{p}}{x^{4} d +c}d x$$

input
```
int(x^3*(b*x^8+a)^p/(d*x^4+c),x)
```

output
```
int(x^3*(b*x^8+a)^p/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{x^3(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(bx^8+a)^px^3}{dx^4+c}\,dx$$

input
```
integrate(x^3*(b*x^8+a)^p/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p*x^3/(d*x^4 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^8)^p}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(b*x**8+a)**p/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(bx^8+a)^p x^3}{dx^4+c}\,dx$$

input
```
integrate(x^3*(b*x^8+a)^p/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p*x^3/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{x^3(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(bx^8+a)^p x^3}{dx^4+c}\,dx$$

input
```
integrate(x^3*(b*x^8+a)^p/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p*x^3/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{x^3\,(b\,x^8+a)^p}{d\,x^4+c}\,dx$$

input
```
int((x^3*(a + b*x^8)^p)/(c + d*x^4),x)
```

output
```
int((x^3*(a + b*x^8)^p)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{x^3(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(b\,x^8+a)^p\,x^3}{d\,x^4+c}dx$$

input
```
int(x^3*(b*x^8+a)^p/(d*x^4+c),x)
```

output
```
int(((a + b*x**8)**p*x**3)/(c + d*x**4),x)
```

3.26 $\int \frac{x(a+bx^8)^p}{c+dx^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 128

$$\int \frac{x(a+bx^8)^p}{c+dx^4} \, dx = \frac{x^2(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{2c} - \frac{dx^6(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{6c^2}$$

output
```
1/2*x^2*(b*x^8+a)^p*AppellF1(1/4,1,-p,5/4,d^2*x^8/c^2,-b*x^8/a)/c/((1+b*x^
8/a)^p)-1/6*d*x^6*(b*x^8+a)^p*AppellF1(3/4,1,-p,7/4,d^2*x^8/c^2,-b*x^8/a)/
c^2/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{x(a+bx^8)^p}{c+dx^4} \, dx = \int \frac{x(a+bx^8)^p}{c+dx^4} \, dx$$

input
```
Integrate[(x*(a + b*x^8)^p)/(c + d*x^4),x]
```

output
```
Integrate[(x*(a + b*x^8)^p)/(c + d*x^4), x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1815, 1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx^8\right)^p}{c+dx^4}dx$$

$$\Big\downarrow \text{1815}$$

$$\frac{1}{2}\int \frac{\left(bx^8+a\right)^p}{dx^4+c}dx^2$$

$$\Big\downarrow \text{1569}$$

$$\frac{1}{2}\int \left(\frac{c\left(bx^8+a\right)^p}{c^2-d^2x^8}+\frac{dx^4\left(bx^8+a\right)^p}{d^2x^8-c^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c}-\frac{dx^6\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{4},-p\right.}{3c^2}\right.$$

input
```
Int[(x*(a + b*x^8)^p)/(c + d*x^4),x]
```

output
```
((x^2*(a + b*x^8)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^8)/a), (d^2*x^8)/c^2]
)/(c*(1 + (b*x^8)/a)^p) - (d*x^6*(a + b*x^8)^p*AppellF1[3/4, -p, 1, 7/4, -
((b*x^8)/a), (d^2*x^8)/c^2])/(3*c^2*(1 + (b*x^8)/a)^p))/2
```

Defintions of rubi rules used

rule 1569
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

rule 1815
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/
k - 1)*(d + e*x^(n/k))^q*(a + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1]
/; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x(b\,x^{8}+a)^{p}}{x^{4}d+c}dx$$

input
```
int(x*(b*x^8+a)^p/(d*x^4+c),x)
```

output
```
int(x*(b*x^8+a)^p/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{x(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(bx^8+a)^px}{dx^4+c}\,dx$$

input
```
integrate(x*(b*x^8+a)^p/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p*x/(d*x^4 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx^8)^p}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x**8+a)**p/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(bx^8+a)^p x}{dx^4+c}\,dx$$

input
```
integrate(x*(b*x^8+a)^p/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p*x/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{x(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(bx^8+a)^p x}{dx^4+c}\,dx$$

input
```
integrate(x*(b*x^8+a)^p/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p*x/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{x\,{\left(b\,x^8+a\right)}^p}{d\,x^4+c}\,dx$$

input
```
int((x*(a + b*x^8)^p)/(c + d*x^4),x)
```

output
```
int((x*(a + b*x^8)^p)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{x(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{\left(b\,x^8+a\right)^p x}{d\,x^4+c}dx$$

input
```
int(x*(b*x^8+a)^p/(d*x^4+c),x)
```

output
```
int(((a + b*x**8)**p*x)/(c + d*x**4),x)
```

3.27 $\int \frac{(a+bx^8)^p}{x(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 180

$$\int \frac{(a+bx^8)^p}{x(c+dx^4)}\,dx = -\frac{dx^4(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{4c^2} + \frac{d^2(a+bx^8)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{d^2(a+bx^8)}{bc^2+ad^2}\right)}{8c(bc^2+ad^2)(1+p)} - \frac{(a+bx^8)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^8}{a}\right)}{8ac(1+p)}$$

output

```
-1/4*d*x^4*(b*x^8+a)^p*AppellF1(1/2,1,-p,3/2,d^2*x^8/c^2,-b*x^8/a)/c^2/((1
+b*x^8/a)^p)+1/8*d^2*(b*x^8+a)^(p+1)*hypergeom([1, p+1],[2+p],d^2*(b*x^8+a
)/(a*d^2+b*c^2))/c/(a*d^2+b*c^2)/(p+1)-1/8*(b*x^8+a)^(p+1)*hypergeom([1, p
+1],[2+p],1+b*x^8/a)/a/c/(p+1)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{x(c+dx^4)}\,dx=\int \frac{(a+bx^8)^p}{x(c+dx^4)}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(x*(c + d*x^4)),x]
```

output
```
Integrate[(a + b*x^8)^p/(x*(c + d*x^4)), x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1803, 621, 334, 333, 354, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{x(c+dx^4)}\,dx$$

$$\Big\downarrow \ 1803$$

$$\frac{1}{4}\int \frac{(bx^8+a)^p}{x^4(dx^4+c)}dx^4$$

$$\Big\downarrow \ 621$$

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{x^4(c^2-d^2x^8)}dx^4-d\int \frac{(bx^8+a)^p}{c^2-d^2x^8}dx^4\right)$$

$$\Big\downarrow \ 334$$

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{x^4(c^2-d^2x^8)}dx^4-d(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^8}{a}+1\right)^p}{c^2-d^2x^8}dx^4\right)$$

$$\Big\downarrow \ 333$$

$$\frac{1}{4}\left(c\int \frac{\left(bx^8+a\right)^p}{x^4\left(c^2-d^2x^8\right)}dx^4-\frac{dx^4\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^2}\right)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{4}\left(\frac{1}{2}c\int \frac{\left(bx^8+a\right)^p}{x^4\left(c^2-d^2x^8\right)}dx^8-\frac{dx^4\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^2}\right)$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{4}\left(\frac{1}{2}c\left(\frac{d^2\int \frac{\left(bx^8+a\right)^p}{c^2-d^2x^8}dx^8}{c^2}+\frac{\int \frac{\left(bx^8+a\right)^p}{x^4}dx^8}{c^2}\right)-\frac{dx^4\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^2}\right)$$

$$\Big\downarrow \text{75}$$

$$\frac{1}{4}\left(\frac{1}{2}c\left(\frac{d^2\int \frac{\left(bx^8+a\right)^p}{c^2-d^2x^8}dx^8}{c^2}-\frac{\left(a+bx^8\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^8}{a}+1\right)}{ac^2(p+1)}\right)-\frac{dx^4\left(a+bx^8\right)^p\left(\frac{bx^8}{a}\right.}{}\right.$$

$$\Big\downarrow \text{78}$$

$$\frac{1}{4}\left(\frac{1}{2}c\left(\frac{d^2\left(a+bx^8\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{d^2\left(bx^8+a\right)}{bc^2+ad^2}\right)}{c^2(p+1)\left(ad^2+bc^2\right)}-\frac{\left(a+bx^8\right)^{p+1}\text{Hypergeometric2F1}\left(1,\right.}{ac^2(p+1)}\right.\right.$$

input
```
Int[(a + b*x^8)^p/(x*(c + d*x^4)),x]
```

output
```
(-((d*x^4*(a + b*x^8)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^8)/a), (d^2*x^8)/
c^2])/(c^2*(1 + (b*x^8)/a)^p)) + (c*((d^2*(a + b*x^8)^(1 + p)*Hypergeometr
ic2F1[1, 1 + p, 2 + p, (d^2*(a + b*x^8))/(b*c^2 + a*d^2)])/(c^2*(b*c^2 + a
*d^2)*(1 + p)) - ((a + b*x^8)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1
 + (b*x^8)/a])/(a*c^2*(1 + p))))/2)/4
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 621
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[c   Int[x^m*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] - Simp[d   Int[
x^(m + 1)*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, m,
p}, x]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(b\,x^{8}+a\right)^{p}}{x\left(x^{4} d+c\right)}d x$$

input
```
int((b*x^8+a)^p/x/(d*x^4+c),x)
```

output
```
int((b*x^8+a)^p/x/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{x\left(c+dx^4\right)}\,dx=\int \frac{(bx^8+a)^p}{(dx^4+c)x}\,dx$$

input
```
integrate((b*x^8+a)^p/x/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d*x^5 + c*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x\,(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/x/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{x\,(c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x}\,dx$$

input
```
integrate((b*x^8+a)^p/x/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)*x), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{x\,(c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x}\,dx$$

input
```
integrate((b*x^8+a)^p/x/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x\,(c+dx^4)}\,dx = \int \frac{(b\,x^8+a)^p}{x\,\left(d\,x^4+c\right)}\,dx$$

input
```
int((a + b*x^8)^p/(x*(c + d*x^4)),x)
```

output
```
int((a + b*x^8)^p/(x*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{x\,(c+dx^4)}\,dx = \int \frac{(b\,x^8+a)^p}{d\,x^5+cx}dx$$

input
```
int((b*x^8+a)^p/x/(d*x^4+c),x)
```

output
```
int((a + b*x**8)**p/(c*x + d*x**5),x)
```

3.28 $\int \frac{(a+bx^8)^p}{x^3(c+dx^4)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{(a+bx^8)^p}{x^3(c+dx^4)} \, dx = -\frac{(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(-\frac{1}{4},-p,1,\frac{3}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{2cx^2} - \frac{dx^2(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{2c^2}$$

output
```
-1/2*(b*x^8+a)^p*AppellF1(-1/4,1,-p,3/4,d^2*x^8/c^2,-b*x^8/a)/c/x^2/((1+b*
x^8/a)^p)-1/2*d*x^2*(b*x^8+a)^p*AppellF1(1/4,1,-p,5/4,d^2*x^8/c^2,-b*x^8/a
)/c^2/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{x^3(c+dx^4)} \, dx = \int \frac{(a+bx^8)^p}{x^3(c+dx^4)} \, dx$$

input
```
Integrate[(a + b*x^8)^p/(x^3*(c + d*x^4)),x]
```

output
```
Integrate[(a + b*x^8)^p/(x^3*(c + d*x^4)), x]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.43, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1815, 1675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{x^3(c+dx^4)}dx$$

$$\Big\downarrow \text{1815}$$

$$\frac{1}{2}\int \frac{(bx^8+a)^p}{x^4(dx^4+c)}dx^2$$

$$\Big\downarrow \text{1675}$$

$$\frac{1}{2}\int \left(\frac{(bx^8+a)^p}{cx^4}-\frac{d(bx^8+a)^p}{c(dx^4+c)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{dx^2(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^2}+\frac{d^2x^6(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{4}\right.}{3c^3}\right.$$

input
```
Int[(a + b*x^8)^p/(x^3*(c + d*x^4)),x]
```

output
```
(-((d*x^2*(a + b*x^8)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^8)/a), (d^2*x^8)/
c^2])/(c^2*(1 + (b*x^8)/a)^p)) + (d^2*x^6*(a + b*x^8)^p*AppellF1[3/4, -p,
1, 7/4, -((b*x^8)/a), (d^2*x^8)/c^2])/(3*c^3*(1 + (b*x^8)/a)^p) - ((a + b*
x^8)^p*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^8)/a)])/(c*x^2*(1 + (b*x^8)
/a)^p))/2
```

Defintions of rubi rules used

rule 1675
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

rule 1815
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/
k - 1)*(d + e*x^(n/k))^q*(a + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1]
/; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^{8}+a)^{p}}{x^{3}\left(x^{4} d+c\right)}dx$$

input
```
int((b*x^8+a)^p/x^3/(d*x^4+c),x)
```

output
```
int((b*x^8+a)^p/x^3/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{x^3\,(c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x^3}\,dx$$

input
```
integrate((b*x^8+a)^p/x^3/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d*x^7 + c*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^3(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/x**3/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{x^3(c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x^3}\,dx$$

input
```
integrate((b*x^8+a)^p/x^3/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)*x^3), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{x^3(c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x^3}\,dx$$

input
```
integrate((b*x^8+a)^p/x^3/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^3\,(c+dx^4)}\,dx=\int \frac{{\left(b\,x^8+a\right)}^p}{x^3\,\left(d\,x^4+c\right)}\,dx$$

input
```
int((a + b*x^8)^p/(x^3*(c + d*x^4)),x)
```

output
```
int((a + b*x^8)^p/(x^3*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{x^3\,(c+dx^4)}\,dx=\int \frac{\left(b\,x^8+a\right)^p}{d\,x^7+c\,x^3}dx$$

input
```
int((b*x^8+a)^p/x^3/(d*x^4+c),x)
```

output
```
int((a + b*x**8)**p/(c*x**3 + d*x**7),x)
```

3.29 $\int \frac{(a+bx^8)^p}{x^5(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 180

$$\int \frac{(a+bx^8)^p}{x^5(c+dx^4)}\,dx = -\frac{(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(-\frac{1}{2},-p,1,\frac{1}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{4cx^4} - \frac{d^3(a+bx^8)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{d^2(a+bx^8)}{bc^2+ad^2}\right)}{8c^2(bc^2+ad^2)(1+p)} + \frac{d(a+bx^8)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^8}{a}\right)}{8ac^2(1+p)}$$

output
```
-1/4*(b*x^8+a)^p*AppellF1(-1/2,1,-p,1/2,d^2*x^8/c^2,-b*x^8/a)/c/x^4/((1+b*
x^8/a)^p)-1/8*d^3*(b*x^8+a)^(p+1)*hypergeom([1, p+1],[2+p],d^2*(b*x^8+a)/(
a*d^2+b*c^2))/c^2/(a*d^2+b*c^2)/(p+1)+1/8*d*(b*x^8+a)^(p+1)*hypergeom([1,
p+1],[2+p],1+b*x^8/a)/a/c^2/(p+1)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{x^5\,(c+dx^4)}\,dx = \int \frac{(a+bx^8)^p}{x^5\,(c+dx^4)}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(x^5*(c + d*x^4)),x]
```

output
```
Integrate[(a + b*x^8)^p/(x^5*(c + d*x^4)), x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1803, 621, 354, 97, 75, 78, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{x^5\,(c+dx^4)}\,dx$$

$$\Big\downarrow \text{ 1803}$$

$$\frac{1}{4}\int \frac{(bx^8+a)^p}{x^8\,(dx^4+c)}dx^4$$

$$\Big\downarrow \text{ 621}$$

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{x^8\,(c^2-d^2x^8)}dx^4 - d\int \frac{(bx^8+a)^p}{x^4\,(c^2-d^2x^8)}dx^4\right)$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{x^8\,(c^2-d^2x^8)}dx^4 - \frac{1}{2}d\int \frac{(bx^8+a)^p}{x^4\,(c^2-d^2x^8)}dx^8\right)$$

$$\Big\downarrow \text{ 97}$$

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{x^8\left(c^2-d^2x^8\right)}dx^4-\frac{1}{2}d\left(\frac{d^2\int \frac{(bx^8+a)^p}{c^2-d^2x^8}dx^8}{c^2}+\frac{\int \frac{(bx^8+a)^p}{x^4}dx^8}{c^2}\right)\right)$$

$\downarrow$ 75

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{x^8\left(c^2-d^2x^8\right)}dx^4-\frac{1}{2}d\left(\frac{d^2\int \frac{(bx^8+a)^p}{c^2-d^2x^8}dx^8}{c^2}-\frac{(a+bx^8)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^8}{a}+1\right)}{ac^2(p+1)}\right.$$

$\downarrow$ 78

$$\frac{1}{4}\left(c\int \frac{(bx^8+a)^p}{x^8\left(c^2-d^2x^8\right)}dx^4-\frac{1}{2}d\left(\frac{d^2(a+bx^8)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{d^2(bx^8+a)}{bc^2+ad^2}\right)}{c^2(p+1)\left(ad^2+bc^2\right)}-\frac{(a+bx^8)^{p+}}{}\right.$$

$\downarrow$ 395

$$\frac{1}{4}\left(c(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^8}{a}+1\right)^p}{x^8\left(c^2-d^2x^8\right)}dx^4-\frac{1}{2}d\left(\frac{d^2(a+bx^8)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\right.}{c^2(p+1)\left(ad^2+bc^2\right)}\right.\right.$$

$\downarrow$ 394

$$\frac{1}{4}\left(-\frac{(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-p,1,\frac{1}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{cx^4}-\frac{1}{2}d\left(\frac{d^2(a+bx^8)^{p+1}\text{Hypergeometric2F1}\left(\right.}{c^2(p+1)\left(ad^2+\right.}\right.\right.$$

input
```
Int[(a + b*x^8)^p/(x^5*(c + d*x^4)),x]
```

output
```
(-(((a + b*x^8)^p*AppellF1[-1/2, -p, 1, 1/2, -((b*x^8)/a), (d^2*x^8)/c^2])
/(c*x^4*(1 + (b*x^8)/a)^p)) - (d*((d^2*(a + b*x^8)^(1 + p)*Hypergeometric2
F1[1, 1 + p, 2 + p, (d^2*(a + b*x^8))/(b*c^2 + a*d^2)])/(c^2*(b*c^2 + a*d^
2)*(1 + p)) - ((a + b*x^8)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 +
(b*x^8)/a])/(a*c^2*(1 + p))))/2)/4
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 621
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[c   Int[x^m*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] - Simp[d   Int[
x^(m + 1)*((a + b*x^2)^p/(c^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, m,
p}, x]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(b\,x^{8}+a)^{p}}{x^{5}\,(x^{4}d+c)}dx$$

input
```
int((b*x^8+a)^p/x^5/(d*x^4+c),x)
```

output
```
int((b*x^8+a)^p/x^5/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{x^5\,(c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x^5}\,dx$$

input
```
integrate((b*x^8+a)^p/x^5/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d*x^9 + c*x^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^5 (c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/x**5/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{x^5 (c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x^5}\,dx$$

input
```
integrate((b*x^8+a)^p/x^5/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)*x^5), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{x^5 (c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x^5}\,dx$$

input
```
integrate((b*x^8+a)^p/x^5/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^5\,(c+dx^4)}\,dx = \int \frac{(b\,x^8+a)^p}{x^5\,(d\,x^4+c)}\,dx$$

input
```
int((a + b*x^8)^p/(x^5*(c + d*x^4)),x)
```

output
```
int((a + b*x^8)^p/(x^5*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{x^5\,(c+dx^4)}\,dx = \int \frac{(b\,x^8+a)^p}{d\,x^9+c\,x^5}dx$$

input
```
int((b*x^8+a)^p/x^5/(d*x^4+c),x)
```

output
```
int((a + b*x**8)**p/(c*x**5 + d*x**9),x)
```

3.30 $\int \frac{x^4(a+bx^8)^p}{c+dx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{x^4(a+bx^8)^p}{c+dx^4}\,dx = \frac{x^5(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{8},-p,1,\frac{13}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c} - \frac{dx^9(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{9}{8},-p,1,\frac{17}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{9c^2}$$

output
```
1/5*x^5*(b*x^8+a)^p*AppellF1(5/8,1,-p,13/8,d^2*x^8/c^2,-b*x^8/a)/c/((1+b*x
^8/a)^p)-1/9*d*x^9*(b*x^8+a)^p*AppellF1(9/8,1,-p,17/8,d^2*x^8/c^2,-b*x^8/a
)/c^2/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{x^4(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{x^4(a+bx^8)^p}{c+dx^4}\,dx$$

input
```
Integrate[(x^4*(a + b*x^8)^p)/(c + d*x^4),x]
```

output
```
Integrate[(x^4*(a + b*x^8)^p)/(c + d*x^4), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a+bx^8)^p}{c+dx^4} \, dx$$
$$\Big\downarrow \; 1888$$
$$\int \frac{x^4 (a+bx^8)^p}{c+dx^4} dx$$

input
```
Int[(x^4*(a + b*x^8)^p)/(c + d*x^4),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1888
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n)
)^p, x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x^4 (b\,x^8+a)^p}{x^4 d+c} dx$$

input
```
int(x^4*(b*x^8+a)^p/(d*x^4+c),x)
```

output
```
int(x^4*(b*x^8+a)^p/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{x^4(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(bx^8+a)^p x^4}{dx^4+c}\,dx$$

input
```
integrate(x^4*(b*x^8+a)^p/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p*x^4/(d*x^4 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^8)^p}{c+dx^4}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(b*x**8+a)**p/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(bx^8+a)^p x^4}{dx^4+c}\,dx$$

input
```
integrate(x^4*(b*x^8+a)^p/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p*x^4/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{x^4(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(bx^8+a)^p x^4}{dx^4+c}\,dx$$

input
```
integrate(x^4*(b*x^8+a)^p/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p*x^4/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{x^4\,(b\,x^8+a)^p}{d\,x^4+c}\,dx$$

input
```
int((x^4*(a + b*x^8)^p)/(c + d*x^4),x)
```

output
```
int((x^4*(a + b*x^8)^p)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{x^4(a+bx^8)^p}{c+dx^4}\,dx$$
$$=\frac{(b\,x^8+a)^p\,x-8\left(\int \frac{(b\,x^8+a)^p}{8bdp\,x^{12}+bd\,x^{12}+8bcp\,x^8+bc\,x^8+8adp\,x^4+ad\,x^4+8acp+ac}dx\right)acp-\left(\int \frac{(b\,x^8+a)^p}{8bdp\,x^{12}+bd\,x^{12}+8bcp\,x^8+bc\,x^8+8}\right.}{}$$

input
```
int(x^4*(b*x^8+a)^p/(d*x^4+c),x)
```

output

```
((a + b*x**8)**p*x - 8*int((a + b*x**8)**p/(8*a*c*p + a*c + 8*a*d*p*x**4 +
 a*d*x**4 + 8*b*c*p*x**8 + b*c*x**8 + 8*b*d*p*x**12 + b*d*x**12),x)*a*c*p
- int((a + b*x**8)**p/(8*a*c*p + a*c + 8*a*d*p*x**4 + a*d*x**4 + 8*b*c*p*x
**8 + b*c*x**8 + 8*b*d*p*x**12 + b*d*x**12),x)*a*c - 64*int(((a + b*x**8)*
*p*x**8)/(8*a*c*p + a*c + 8*a*d*p*x**4 + a*d*x**4 + 8*b*c*p*x**8 + b*c*x**
8 + 8*b*d*p*x**12 + b*d*x**12),x)*b*c*p**2 - 16*int(((a + b*x**8)**p*x**8)
/(8*a*c*p + a*c + 8*a*d*p*x**4 + a*d*x**4 + 8*b*c*p*x**8 + b*c*x**8 + 8*b*
d*p*x**12 + b*d*x**12),x)*b*c*p - int(((a + b*x**8)**p*x**8)/(8*a*c*p + a*
c + 8*a*d*p*x**4 + a*d*x**4 + 8*b*c*p*x**8 + b*c*x**8 + 8*b*d*p*x**12 + b*
d*x**12),x)*b*c + 64*int(((a + b*x**8)**p*x**4)/(8*a*c*p + a*c + 8*a*d*p*x
**4 + a*d*x**4 + 8*b*c*p*x**8 + b*c*x**8 + 8*b*d*p*x**12 + b*d*x**12),x)*a
*d*p**2 + 8*int(((a + b*x**8)**p*x**4)/(8*a*c*p + a*c + 8*a*d*p*x**4 + a*d
*x**4 + 8*b*c*p*x**8 + b*c*x**8 + 8*b*d*p*x**12 + b*d*x**12),x)*a*d*p)/(d*
(8*p + 1))
```

3.31 $\int \frac{x^2(a+bx^8)^p}{c+dx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{x^2(a+bx^8)^p}{c+dx^4}\,dx = \frac{x^3(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{8},-p,1,\frac{11}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{3c} - \frac{dx^7(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{7}{8},-p,1,\frac{15}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{7c^2}$$

output
```
1/3*x^3*(b*x^8+a)^p*AppellF1(3/8,1,-p,11/8,d^2*x^8/c^2,-b*x^8/a)/c/((1+b*x
^8/a)^p)-1/7*d*x^7*(b*x^8+a)^p*AppellF1(7/8,1,-p,15/8,d^2*x^8/c^2,-b*x^8/a
)/c^2/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{x^2(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{x^2(a+bx^8)^p}{c+dx^4}\,dx$$

input
```
Integrate[(x^2*(a + b*x^8)^p)/(c + d*x^4),x]
```

output
```
Integrate[(x^2*(a + b*x^8)^p)/(c + d*x^4), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx^8)^p}{c+dx^4}\,dx$$

$$\Big\downarrow \; 1888$$

$$\int \frac{x^2(a+bx^8)^p}{c+dx^4}dx$$

input
```
Int[(x^2*(a + b*x^8)^p)/(c + d*x^4),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1888
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n)
)^p, x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x^2(b\,x^8+a)^p}{x^4d+c}dx$$

input
```
int(x^2*(b*x^8+a)^p/(d*x^4+c),x)
```

output
```
int(x^2*(b*x^8+a)^p/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{x^2(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(bx^8+a)^p x^2}{dx^4+c}\,dx$$

input
```
integrate(x^2*(b*x^8+a)^p/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p*x^2/(d*x^4 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^8)^p}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(b*x**8+a)**p/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(bx^8+a)^p x^2}{dx^4+c}\,dx$$

input
```
integrate(x^2*(b*x^8+a)^p/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p*x^2/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{x^2(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(bx^8+a)^p x^2}{dx^4+c}\,dx$$

input
```
integrate(x^2*(b*x^8+a)^p/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p*x^2/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{x^2\,(b\,x^8+a)^p}{d\,x^4+c}\,dx$$

input
```
int((x^2*(a + b*x^8)^p)/(c + d*x^4),x)
```

output
```
int((x^2*(a + b*x^8)^p)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{x^2(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(b\,x^8+a)^p\,x^2}{d\,x^4+c}dx$$

input
```
int(x^2*(b*x^8+a)^p/(d*x^4+c),x)
```

output
```
int(((a + b*x**8)**p*x**2)/(c + d*x**4),x)
```

3.32 $\int \frac{(a+bx^8)^p}{c+dx^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{(a+bx^8)^p}{c+dx^4} dx = \frac{x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{8},-p,1,\frac{9}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c} - \frac{dx^5(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{5}{8},-p,1,\frac{13}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c^2}$$

output
```
x*(b*x^8+a)^p*AppellF1(1/8,1,-p,9/8,d^2*x^8/c^2,-b*x^8/a)/c/((1+b*x^8/a)^p
)-1/5*d*x^5*(b*x^8+a)^p*AppellF1(5/8,1,-p,13/8,d^2*x^8/c^2,-b*x^8/a)/c^2/(
(1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{c+dx^4} dx = \int \frac{(a+bx^8)^p}{c+dx^4} dx$$

input
```
Integrate[(a + b*x^8)^p/(c + d*x^4),x]
```

output
```
Integrate[(a + b*x^8)^p/(c + d*x^4), x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1768, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{c+dx^4}dx$$

$$\Big\downarrow \text{1768}$$

$$\int \left(\frac{c(a+bx^8)^p}{c^2-d^2x^8}+\frac{dx^4(a+bx^8)^p}{d^2x^8-c^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{8},-p,1,\frac{9}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c}-\frac{dx^5(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{8},-p,1,\frac{13}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c^2}$$

input
```
Int[(a + b*x^8)^p/(c + d*x^4),x]
```

output
```
(x*(a + b*x^8)^p*AppellF1[1/8, -p, 1, 9/8, -((b*x^8)/a), (d^2*x^8)/c^2])/(
c*(1 + (b*x^8)/a)^p) - (d*x^5*(a + b*x^8)^p*AppellF1[5/8, -p, 1, 13/8, -((
b*x^8)/a), (d^2*x^8)/c^2])/(5*c^2*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1768
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^{8}+a)^{p}}{x^{4}d+c}dx$$

input
```
int((b*x^8+a)^p/(d*x^4+c),x)
```

output
```
int((b*x^8+a)^p/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(bx^8+a)^p}{dx^4+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d*x^4 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx=\text{Timed out}$$

input
```
integrate((b*x**8+a)**p/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(bx^8+a)^p}{dx^4+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(bx^8+a)^p}{dx^4+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(b\,x^8+a)^p}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^8)^p/(c + d*x^4),x)
```

output
```
int((a + b*x^8)^p/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(b\,x^8+a)^p}{d\,x^4+c}dx$$

input
```
int((b*x^8+a)^p/(d*x^4+c),x)
```

output
```
int((a + b*x**8)**p/(c + d*x**4),x)
```

3.33 $\int \frac{(a+bx^8)^p}{x^2(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 126

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)}\,dx = -\frac{(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(-\frac{1}{8},-p,1,\frac{7}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{cx} - \frac{dx^3(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{8},-p,1,\frac{11}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{3c^2}$$

output
```
-(b*x^8+a)^p*AppellF1(-1/8,1,-p,7/8,d^2*x^8/c^2,-b*x^8/a)/c/x/((1+b*x^8/a)
^p)-1/3*d*x^3*(b*x^8+a)^p*AppellF1(3/8,1,-p,11/8,d^2*x^8/c^2,-b*x^8/a)/c^2
/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)}\,dx = \int \frac{(a+bx^8)^p}{x^2(c+dx^4)}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(x^2*(c + d*x^4)),x]
```

output
```
Integrate[(a + b*x^8)^p/(x^2*(c + d*x^4)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)}\,dx$$
$$\Big\downarrow \; 1888$$
$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)}dx$$

input
```
Int[(a + b*x^8)^p/(x^2*(c + d*x^4)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1888
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n)
)^p, x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(b\,x^8+a)^p}{x^2\,(x^4d+c)}dx$$

input
```
int((b*x^8+a)^p/x^2/(d*x^4+c),x)
```

output
```
int((b*x^8+a)^p/x^2/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x^2}\,dx$$

input
```
integrate((b*x^8+a)^p/x^2/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d*x^6 + c*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/x**2/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x^2}\,dx$$

input
```
integrate((b*x^8+a)^p/x^2/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)*x^2), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x^2}\,dx$$

input
```
integrate((b*x^8+a)^p/x^2/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)}\,dx = \int \frac{(b\,x^8+a)^p}{x^2\,(d\,x^4+c)}\,dx$$

input
```
int((a + b*x^8)^p/(x^2*(c + d*x^4)),x)
```

output
```
int((a + b*x^8)^p/(x^2*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)}\,dx = \int \frac{(b\,x^8+a)^p}{d\,x^6+c\,x^2}dx$$

input
```
int((b*x^8+a)^p/x^2/(d*x^4+c),x)
```

output
```
int((a + b*x**8)**p/(c*x**2 + d*x**6),x)
```

3.34 $\int \frac{(a+bx^8)^p}{x^4(c+dx^4)} dx$

Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)} dx = -\frac{(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \operatorname{AppellF1}\left(-\frac{3}{8},-p,1,\frac{5}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{3cx^3} - \frac{dx(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{8},-p,1,\frac{9}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^2}$$

output
```
-1/3*(b*x^8+a)^p*AppellF1(-3/8,1,-p,5/8,d^2*x^8/c^2,-b*x^8/a)/c/x^3/((1+b*
x^8/a)^p)-d*x*(b*x^8+a)^p*AppellF1(1/8,1,-p,9/8,d^2*x^8/c^2,-b*x^8/a)/c^2/
((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)} dx = \int \frac{(a+bx^8)^p}{x^4(c+dx^4)} dx$$

input
```
Integrate[(a + b*x^8)^p/(x^4*(c + d*x^4)),x]
```

output
```
Integrate[(a + b*x^8)^p/(x^4*(c + d*x^4)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)} dx$$

$$\Big\downarrow \text{1888}$$

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)} dx$$

input
```
Int[(a + b*x^8)^p/(x^4*(c + d*x^4)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1888
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n)
)^p, x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(b\,x^8+a)^p}{x^4(x^4d+c)} dx$$

input
```
int((b*x^8+a)^p/x^4/(d*x^4+c),x)
```

output
```
int((b*x^8+a)^p/x^4/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x^4}\,dx$$

input
```
integrate((b*x^8+a)^p/x^4/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d*x^8 + c*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/x**4/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x^4}\,dx$$

input
```
integrate((b*x^8+a)^p/x^4/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)*x^4), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)x^4}\,dx$$

input
```
integrate((b*x^8+a)^p/x^4/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)}\,dx = \int \frac{(b\,x^8+a)^p}{x^4\,(d\,x^4+c)}\,dx$$

input
```
int((a + b*x^8)^p/(x^4*(c + d*x^4)),x)
```

output
```
int((a + b*x^8)^p/(x^4*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)}\,dx = \int \frac{(b\,x^8+a)^p}{d\,x^8+c\,x^4}dx$$

input
```
int((b*x^8+a)^p/x^4/(d*x^4+c),x)
```

output
```
int((a + b*x**8)**p/(c*x**4 + d*x**8),x)
```

3.35 $\int \frac{x^7(a+bx^8)^p}{(c+dx^4)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 287

$$\int \frac{x^7(a+bx^8)^p}{(c+dx^4)^2} dx = \frac{c(a+bx^8)^{1+p}}{4(bc^2+ad^2)(c+dx^4)} + \frac{(ad^2+bc^2(1+2p))\,x^4(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{4cd(bc^2+ad^2)} - \frac{bc(1+2p)x^4(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)}{4d(bc^2+ad^2)} - \frac{(ad^2+bc^2(1+2p))\,(a+bx^8)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{d^2(a+bx^8)}{bc^2+ad^2}\right)}{8(bc^2+ad^2)^2(1+p)}$$

output
```
1/4*c*(b*x^8+a)^(p+1)/(a*d^2+b*c^2)/(d*x^4+c)+1/4*(a*d^2+b*c^2*(1+2*p))*x^
4*(b*x^8+a)^p*AppellF1(1/2,1,-p,3/2,d^2*x^8/c^2,-b*x^8/a)/c/d/(a*d^2+b*c^2
)/((1+b*x^8/a)^p)-1/4*b*c*(1+2*p)*x^4*(b*x^8+a)^p*hypergeom([1/2, -p],[3/2
],-b*x^8/a)/d/(a*d^2+b*c^2)/((1+b*x^8/a)^p)-1/8*(a*d^2+b*c^2*(1+2*p))*(b*x
^8+a)^(p+1)*hypergeom([1, p+1],[2+p],d^2*(b*x^8+a)/(a*d^2+b*c^2))/(a*d^2+b
*c^2)^2/(p+1)
```

Mathematica [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.85

$$\int \frac{x^7(a+bx^8)^p}{(c+dx^4)^2}\,dx$$
$$= \frac{\left(\frac{d\left(-\sqrt{-\frac{a}{b}}+x^4\right)}{c+dx^4}\right)^{-p}\left(\frac{d\left(\sqrt{-\frac{a}{b}}+x^4\right)}{c+dx^4}\right)^{-p}(a+bx^8)^p\left(-2cp\operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{c-\sqrt{-\frac{a}{b}}d}{c+dx^4},\frac{c+\sqrt{-\frac{a}{b}}}{c+dx^4}\right.\right.}{8d^2p(-1+2p)\left(c+dx^4\right)}$$

input
```
Integrate[(x^7*(a + b*x^8)^p)/(c + d*x^4)^2,x]
```

output
```
((a + b*x^8)^p*(-2*c*p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (c - Sqrt[-(a/b)
]*d)/(c + d*x^4), (c + Sqrt[-(a/b)]*d)/(c + d*x^4)] + (-1 + 2*p)*(c + d*x^
4)*AppellF1[-2*p, -p, -p, 1 - 2*p, (c - Sqrt[-(a/b)]*d)/(c + d*x^4), (c +
Sqrt[-(a/b)]*d)/(c + d*x^4)]))/(8*d^2*p*(-1 + 2*p)*((d*(-Sqrt[-(a/b)] + x^
4))/(c + d*x^4))^p*((d*(Sqrt[-(a/b)] + x^4))/(c + d*x^4))^p*(c + d*x^4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1803, 594, 25, 719, 238, 237, 504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(a+bx^8)^p}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{1803}$$

$$\frac{1}{4}\int \frac{x^4(bx^8+a)^p}{(dx^4+c)^2}\,dx^4$$

$$\Big\downarrow \text{594}$$

$$\frac{1}{4}\left(\frac{c\left(a+bx^8\right)^{p+1}}{\left(c+dx^4\right)\left(ad^2+bc^2\right)}-\frac{\int -\frac{\left(ad-bc(2p+1)x^4\right)\left(bx^8+a\right)^p}{dx^4+c}dx^4}{ad^2+bc^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\left(\frac{\int \frac{\left(ad-bc(2p+1)x^4\right)\left(bx^8+a\right)^p}{dx^4+c}dx^4}{ad^2+bc^2}+\frac{c\left(a+bx^8\right)^{p+1}}{\left(c+dx^4\right)\left(ad^2+bc^2\right)}\right)$$

$$\Big\downarrow \text{ 719}$$

$$\frac{1}{4}\left(\frac{\frac{\left(ad^2+bc^2(2p+1)\right)\int \frac{\left(bx^8+a\right)^p}{dx^4+c}dx^4}{d}-\frac{bc(2p+1)\int \left(bx^8+a\right)^p dx^4}{d}}{ad^2+bc^2}+\frac{c\left(a+bx^8\right)^{p+1}}{\left(c+dx^4\right)\left(ad^2+bc^2\right)}\right)$$

$$\Big\downarrow \text{ 238}$$

$$\frac{1}{4}\left(\frac{\frac{\left(ad^2+bc^2(2p+1)\right)\int \frac{\left(bx^8+a\right)^p}{dx^4+c}dx^4}{d}-\frac{bc(2p+1)\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \left(\frac{bx^8}{a}+1\right)^p dx^4}{d}}{ad^2+bc^2}+\frac{c\left(a+bx^8\right)^{p+1}}{\left(c+dx^4\right)\left(ad^2+bc^2\right)}\right)$$

$$\Big\downarrow \text{ 237}$$

$$\frac{1}{4}\left(\frac{\frac{\left(ad^2+bc^2(2p+1)\right)\int \frac{\left(bx^8+a\right)^p}{dx^4+c}dx^4}{d}-\frac{bc(2p+1)x^4\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)}{d}}{ad^2+bc^2}+\frac{c\left(a+bx^8\right)^{p+1}}{\left(c+dx^4\right)\left(ad^2+bc\right.}\right.$$

$$\Big\downarrow \text{ 504}$$

$$\frac{1}{4}\left(\frac{\frac{\left(ad^2+bc^2(2p+1)\right)\left(c\int \frac{\left(bx^8+a\right)^p}{c^2-d^2x^8}dx^4-d\int \frac{x^4\left(bx^8+a\right)^p}{c^2-d^2x^8}dx^4\right)}{d}-\frac{bc(2p+1)x^4\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)}{d}}{ad^2+bc^2}+\right.$$

$$\Big\downarrow \text{ 334}$$

$$\frac{1}{4}\left(\frac{\frac{\left(ad^2+bc^2(2p+1)\right)\left(c\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^8}{a}+1\right)^p}{c^2-d^2x^8}dx^4-d\int \frac{x^4\left(bx^8+a\right)^p}{c^2-d^2x^8}dx^4\right)}{d}-\frac{bc(2p+1)x^4\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric}}{d}}{ad^2+bc^2}\right.$$

$$\Big\downarrow \ 333$$

$$\frac{1}{4}\left(\frac{\frac{(ad^2+bc^2(2p+1))\left(\frac{x^4\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c}-d\int\frac{x^4\left(bx^8+a\right)^p}{c^2-d^2x^8}dx^4\right)}{d}-\frac{bc(2p+1)x^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}}{}}{ad^2+bc^2}\right.$$

$$\Big\downarrow \ 353$$

$$\frac{1}{4}\left(\frac{\frac{(ad^2+bc^2(2p+1))\left(\frac{x^4\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c}-\frac{1}{2}d\int\frac{\left(bx^8+a\right)^p}{c^2-d^2x^8}dx^8\right)}{d}-\frac{bc(2p+1)x^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}}{}}{ad^2+bc^2}\right.$$

$$\Big\downarrow \ 78$$

$$\frac{1}{4}\left(\frac{\frac{(ad^2+bc^2(2p+1))\left(\frac{x^4\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c}-\frac{d\left(a+bx^8\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{d^2\left(bx^8+a\right)}{bc^2+ad^2}\right)}{2(p+1)\left(ad^2+bc^2\right)}\right)}{d}-}{ad^2+bc^2}\right.$$

input
```
Int[(x^7*(a + b*x^8)^p)/(c + d*x^4)^2,x]
```

output
```
((c*(a + b*x^8)^(1 + p))/((b*c^2 + a*d^2)*(c + d*x^4)) + (-((b*c*(1 + 2*p)
*x^4*(a + b*x^8)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^8)/a)])/(d*(1 +
(b*x^8)/a)^p)) + ((a*d^2 + b*c^2*(1 + 2*p))*((x^4*(a + b*x^8)^p*AppellF1[1
/2, -p, 1, 3/2, -((b*x^8)/a), (d^2*x^8)/c^2])/(c*(1 + (b*x^8)/a)^p) - (d*(
a + b*x^8)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (d^2*(a + b*x^8))/(b
*c^2 + a*d^2)])/(2*(b*c^2 + a*d^2)*(1 + p))))/d)/(b*c^2 + a*d^2))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^7 (b\,x^8 + a)^p}{(x^4 d + c)^2} dx$$

input
```
int(x^7*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int(x^7*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{x^7(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^7}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^7*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p*x^7/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(a+bx^8)^p}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(b*x**8+a)**p/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^7(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^7}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^7*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p*x^7/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{x^7(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^7}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^7*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p*x^7/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{x^7\,(b\,x^8+a)^p}{(d\,x^4+c)^2}\,dx$$

input
```
int((x^7*(a + b*x^8)^p)/(c + d*x^4)^2,x)
```

output
```
int((x^7*(a + b*x^8)^p)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{x^7(a+bx^8)^p}{(c+dx^4)^2}\,dx = \text{too large to display}$$

input
```
int(x^7*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

output

```
(2*(a + b*x**8)**p*a*c*p + (a + b*x**8)**p*a*c - 2*(a + b*x**8)**p*a*d*p*x
**4 + (a + b*x**8)**p*a*d*x**4 + 32*int(((a + b*x**8)**p*x**15)/(2*a*c**2*
p + a*c**2 + 4*a*c*d*p*x**4 + 2*a*c*d*x**4 + 2*a*d**2*p*x**8 + a*d**2*x**8
 + 2*b*c**2*p*x**8 + b*c**2*x**8 + 4*b*c*d*p*x**12 + 2*b*c*d*x**12 + 2*b*d
**2*p*x**16 + b*d**2*x**16),x)*a*b*c*d**2*p**3 - 8*int(((a + b*x**8)**p*x*
*15)/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x**4 + 2*a*c*d*x**4 + 2*a*d**2*p*x**
8 + a*d**2*x**8 + 2*b*c**2*p*x**8 + b*c**2*x**8 + 4*b*c*d*p*x**12 + 2*b*c*
d*x**12 + 2*b*d**2*p*x**16 + b*d**2*x**16),x)*a*b*c*d**2*p + 32*int(((a +
b*x**8)**p*x**15)/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x**4 + 2*a*c*d*x**4 + 2
*a*d**2*p*x**8 + a*d**2*x**8 + 2*b*c**2*p*x**8 + b*c**2*x**8 + 4*b*c*d*p*x
**12 + 2*b*c*d*x**12 + 2*b*d**2*p*x**16 + b*d**2*x**16),x)*a*b*d**3*p**3*x
**4 - 8*int(((a + b*x**8)**p*x**15)/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x**4
+ 2*a*c*d*x**4 + 2*a*d**2*p*x**8 + a*d**2*x**8 + 2*b*c**2*p*x**8 + b*c**2*
x**8 + 4*b*c*d*p*x**12 + 2*b*c*d*x**12 + 2*b*d**2*p*x**16 + b*d**2*x**16),
x)*a*b*d**3*p*x**4 + 32*int(((a + b*x**8)**p*x**15)/(2*a*c**2*p + a*c**2 +
 4*a*c*d*p*x**4 + 2*a*c*d*x**4 + 2*a*d**2*p*x**8 + a*d**2*x**8 + 2*b*c**2*
p*x**8 + b*c**2*x**8 + 4*b*c*d*p*x**12 + 2*b*c*d*x**12 + 2*b*d**2*p*x**16
+ b*d**2*x**16),x)*b**2*c**3*p**3 + 32*int(((a + b*x**8)**p*x**15)/(2*a*c*
*2*p + a*c**2 + 4*a*c*d*p*x**4 + 2*a*c*d*x**4 + 2*a*d**2*p*x**8 + a*d**2*x
**8 + 2*b*c**2*p*x**8 + b*c**2*x**8 + 4*b*c*d*p*x**12 + 2*b*c*d*x**12 +...
```

3.36 $\int \frac{x^5(a+bx^8)^p}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \frac{x^5(a+bx^8)^p}{(c+dx^4)^2}\,dx = \frac{x^6(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{4},-p,2,\frac{7}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{6c^2} - \frac{dx^{10}(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{4},-p,2,\frac{9}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c^3} + \frac{d^2x^{14}(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{7}{4},-p,2,\frac{11}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{14c^4}$$

output
```
1/6*x^6*(b*x^8+a)^p*AppellF1(3/4,2,-p,7/4,d^2*x^8/c^2,-b*x^8/a)/c^2/((1+b*
x^8/a)^p)-1/5*d*x^10*(b*x^8+a)^p*AppellF1(5/4,2,-p,9/4,d^2*x^8/c^2,-b*x^8/
a)/c^3/((1+b*x^8/a)^p)+1/14*d^2*x^14*(b*x^8+a)^p*AppellF1(7/4,2,-p,11/4,d^
2*x^8/c^2,-b*x^8/a)/c^4/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{x^5(a+bx^8)^p}{(c+dx^4)^2}\,dx=\int \frac{x^5(a+bx^8)^p}{(c+dx^4)^2}\,dx$$

input
```
Integrate[(x^5*(a + b*x^8)^p)/(c + d*x^4)^2,x]
```

output
```
Integrate[(x^5*(a + b*x^8)^p)/(c + d*x^4)^2, x]
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.66, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1815, 1675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+bx^8)^p}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \ 1815$$

$$\frac{1}{2}\int \frac{x^4(bx^8+a)^p}{(dx^4+c)^2}dx^2$$

$$\Big\downarrow \ 1675$$

$$\frac{1}{2}\int \left(\frac{(bx^8+a)^p}{d(dx^4+c)}-\frac{c(bx^8+a)^p}{d(dx^4+c)^2}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{x^6(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{3c^2}+\frac{2x^6(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{4},-\right.}{3c^2}\right.$$

input
```
Int[(x^5*(a + b*x^8)^p)/(c + d*x^4)^2,x]
```

output
```
((x^2*(a + b*x^8)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^8)/a), (d^2*x^8)/c^2]
)/(c*d*(1 + (b*x^8)/a)^p) - (x^2*(a + b*x^8)^p*AppellF1[1/4, -p, 2, 5/4, -
((b*x^8)/a), (d^2*x^8)/c^2])/(c*d*(1 + (b*x^8)/a)^p) - (x^6*(a + b*x^8)^p*
AppellF1[3/4, -p, 1, 7/4, -((b*x^8)/a), (d^2*x^8)/c^2])/(3*c^2*(1 + (b*x^8
)/a)^p) + (2*x^6*(a + b*x^8)^p*AppellF1[3/4, -p, 2, 7/4, -((b*x^8)/a), (d^
2*x^8)/c^2])/(3*c^2*(1 + (b*x^8)/a)^p) - (d*x^10*(a + b*x^8)^p*AppellF1[5/
4, -p, 2, 9/4, -((b*x^8)/a), (d^2*x^8)/c^2])/(5*c^3*(1 + (b*x^8)/a)^p))/2
```

Defintions of rubi rules used

rule 1675
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

rule 1815
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/
k - 1)*(d + e*x^(n/k))^q*(a + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1]
/; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^5(b\,x^8+a)^p}{(x^4d+c)^2}dx$$

input
```
int(x^5*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int(x^5*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{x^5(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^5}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^5*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p*x^5/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^8)^p}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(b*x**8+a)**p/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^5}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^5*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p*x^5/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{x^5(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^5}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^5*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p*x^5/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{x^5\,(b\,x^8+a)^p}{(d\,x^4+c)^2}\,dx$$

input
```
int((x^5*(a + b*x^8)^p)/(c + d*x^4)^2,x)
```

output
```
int((x^5*(a + b*x^8)^p)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{x^5(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^8+a)^p\,x^5}{d^2x^8+2cd\,x^4+c^2}dx$$

input
```
int(x^5*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int(((a + b*x**8)**p*x**5)/(c**2 + 2*c*d*x**4 + d**2*x**8),x)
```

3.37 $\int \frac{x^3(a+bx^8)^p}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 198

$$\int \frac{x^3(a+bx^8)^p}{(c+dx^4)^2}\,dx = \frac{x^4(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{4c^2} + \frac{d^2x^{12}(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{12c^4} - \frac{bcd(a+bx^8)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{d^2(a+bx^8)}{bc^2+ad^2}\right)}{4(bc^2+ad^2)^2(1+p)}$$

output
```
1/4*x^4*(b*x^8+a)^p*AppellF1(1/2,2,-p,3/2,d^2*x^8/c^2,-b*x^8/a)/c^2/((1+b*
x^8/a)^p)+1/12*d^2*x^12*(b*x^8+a)^p*AppellF1(3/2,2,-p,5/2,d^2*x^8/c^2,-b*x
^8/a)/c^4/((1+b*x^8/a)^p)-1/4*b*c*d*(b*x^8+a)^(p+1)*hypergeom([2, p+1],[2+
p],d^2*(b*x^8+a)/(a*d^2+b*c^2))/(a*d^2+b*c^2)^2/(p+1)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.80

$$\int \frac{x^3(a+bx^8)^p}{(c+dx^4)^2}\,dx = \frac{\left(\frac{d\left(-\sqrt{-\frac{a}{b}}+x^4\right)}{c+dx^4}\right)^{-p}\left(\frac{d\left(\sqrt{-\frac{a}{b}}+x^4\right)}{c+dx^4}\right)^{-p}(a+bx^8)^p\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{c-\sqrt{-\frac{a}{b}}d}{c+dx^4},\frac{c+\sqrt{-\frac{a}{b}}d}{c+dx^4}\right)}{4d(-1+2p)\,(c+dx^4)}$$

input
```
Integrate[(x^3*(a + b*x^8)^p)/(c + d*x^4)^2,x]
```

output
```
((a + b*x^8)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (c - Sqrt[-(a/b)]*d)/(c
+ d*x^4), (c + Sqrt[-(a/b)]*d)/(c + d*x^4)])/(4*d*(-1 + 2*p)*((d*(-Sqrt[-(
a/b)] + x^4))/(c + d*x^4))^p*((d*(Sqrt[-(a/b)] + x^4))/(c + d*x^4))^p*(c +
 d*x^4))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1799, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+bx^8\right)^p}{\left(c+dx^4\right)^2}dx$$

$$\Big\downarrow \text{ 1799}$$

$$\frac{1}{4}\int \frac{\left(bx^8+a\right)^p}{\left(dx^4+c\right)^2}dx^4$$

$$\Big\downarrow \text{ 505}$$

$$\frac{1}{4}\int \left(-\frac{2cdx^4\left(bx^8+a\right)^p}{\left(c^2-d^2x^8\right)^2}+\frac{c^2\left(bx^8+a\right)^p}{\left(c^2-d^2x^8\right)^2}+\frac{d^2x^8\left(bx^8+a\right)^p}{\left(d^2x^8-c^2\right)^2}\right)dx^4$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(\frac{x^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^2}+\frac{d^2x^{12}(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-\right.}{3c^4}\right.$$

input
```
Int[(x^3*(a + b*x^8)^p)/(c + d*x^4)^2,x]
```

output
```
((x^4*(a + b*x^8)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^8)/a), (d^2*x^8)/c^2]
)/(c^2*(1 + (b*x^8)/a)^p) + (d^2*x^12*(a + b*x^8)^p*AppellF1[3/2, -p, 2, 5
/2, -((b*x^8)/a), (d^2*x^8)/c^2])/(3*c^4*(1 + (b*x^8)/a)^p) - (b*c*d*(a +
b*x^8)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (d^2*(a + b*x^8))/(b*c^2
 + a*d^2)])/((b*c^2 + a*d^2)^2*(1 + p)))/4
```

Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^{3} \left(b\,x^{8}+a \right)^{p}}{\left(x^{4} d +c \right)^{2}}d x$$

input
```
int(x^3*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int(x^3*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{x^3(a+bx^8)^p}{(c+dx^4)^2}\,dx=\int \frac{(bx^8+a)^p x^3}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^3*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p*x^3/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^8)^p}{(c+dx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(b*x**8+a)**p/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^3}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^3*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p*x^3/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{x^3(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^3}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^3*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p*x^3/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{x^3\,(b\,x^8+a)^p}{(d\,x^4+c)^2}\,dx$$

input
```
int((x^3*(a + b*x^8)^p)/(c + d*x^4)^2,x)
```

output
```
int((x^3*(a + b*x^8)^p)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{x^3(a+bx^8)^p}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^3*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
((a + b*x**8)**p*x**4 - 16*int(((a + b*x**8)**p*x**15)/(2*a*c**2*p + a*c**
2 + 4*a*c*d*p*x**4 + 2*a*c*d*x**4 + 2*a*d**2*p*x**8 + a*d**2*x**8 + 2*b*c*
*2*p*x**8 + b*c**2*x**8 + 4*b*c*d*p*x**12 + 2*b*c*d*x**12 + 2*b*d**2*p*x**
16 + b*d**2*x**16),x)*b*c*d*p**2 - 8*int(((a + b*x**8)**p*x**15)/(2*a*c**2
*p + a*c**2 + 4*a*c*d*p*x**4 + 2*a*c*d*x**4 + 2*a*d**2*p*x**8 + a*d**2*x**
8 + 2*b*c**2*p*x**8 + b*c**2*x**8 + 4*b*c*d*p*x**12 + 2*b*c*d*x**12 + 2*b*
d**2*p*x**16 + b*d**2*x**16),x)*b*c*d*p - 16*int(((a + b*x**8)**p*x**15)/(
2*a*c**2*p + a*c**2 + 4*a*c*d*p*x**4 + 2*a*c*d*x**4 + 2*a*d**2*p*x**8 + a*
d**2*x**8 + 2*b*c**2*p*x**8 + b*c**2*x**8 + 4*b*c*d*p*x**12 + 2*b*c*d*x**1
2 + 2*b*d**2*p*x**16 + b*d**2*x**16),x)*b*d**2*p**2*x**4 - 8*int(((a + b*x
**8)**p*x**15)/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x**4 + 2*a*c*d*x**4 + 2*a*
d**2*p*x**8 + a*d**2*x**8 + 2*b*c**2*p*x**8 + b*c**2*x**8 + 4*b*c*d*p*x**1
2 + 2*b*c*d*x**12 + 2*b*d**2*p*x**16 + b*d**2*x**16),x)*b*d**2*p*x**4 + 16
*int(((a + b*x**8)**p*x**3)/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x**4 + 2*a*c*
d*x**4 + 2*a*d**2*p*x**8 + a*d**2*x**8 + 2*b*c**2*p*x**8 + b*c**2*x**8 + 4
*b*c*d*p*x**12 + 2*b*c*d*x**12 + 2*b*d**2*p*x**16 + b*d**2*x**16),x)*a*c**
2*p**2 + 8*int(((a + b*x**8)**p*x**3)/(2*a*c**2*p + a*c**2 + 4*a*c*d*p*x**
4 + 2*a*c*d*x**4 + 2*a*d**2*p*x**8 + a*d**2*x**8 + 2*b*c**2*p*x**8 + b*c**
2*x**8 + 4*b*c*d*p*x**12 + 2*b*c*d*x**12 + 2*b*d**2*p*x**16 + b*d**2*x**16
),x)*a*c**2*p + 16*int(((a + b*x**8)**p*x**3)/(2*a*c**2*p + a*c**2 + 4*...
```

3.38 $\int \frac{x(a+bx^8)^p}{(c+dx^4)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 194

$$\int \frac{x(a+bx^8)^p}{(c+dx^4)^2} dx = \frac{x^2(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{2c^2} - \frac{dx^6(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{3}{4},-p,2,\frac{7}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{3c^3} + \frac{d^2x^{10}(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{5}{4},-p,2,\frac{9}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{10c^4}$$

output
```
1/2*x^2*(b*x^8+a)^p*AppellF1(1/4,2,-p,5/4,d^2*x^8/c^2,-b*x^8/a)/c^2/((1+b*
x^8/a)^p)-1/3*d*x^6*(b*x^8+a)^p*AppellF1(3/4,2,-p,7/4,d^2*x^8/c^2,-b*x^8/a
)/c^3/((1+b*x^8/a)^p)+1/10*d^2*x^10*(b*x^8+a)^p*AppellF1(5/4,2,-p,9/4,d^2*
x^8/c^2,-b*x^8/a)/c^4/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{x(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{x(a+bx^8)^p}{(c+dx^4)^2}\,dx$$

input
```
Integrate[(x*(a + b*x^8)^p)/(c + d*x^4)^2,x]
```

output
```
Integrate[(x*(a + b*x^8)^p)/(c + d*x^4)^2, x]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1815, 1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^8)^p}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \; 1815$$

$$\frac{1}{2}\int \frac{(bx^8+a)^p}{(dx^4+c)^2}dx^2$$

$$\Big\downarrow \; 1569$$

$$\frac{1}{2}\int \left(-\frac{2cdx^4(bx^8+a)^p}{(c^2-d^2x^8)^2}+\frac{c^2(bx^8+a)^p}{(c^2-d^2x^8)^2}+\frac{d^2x^8(bx^8+a)^p}{(d^2x^8-c^2)^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{x^2(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^2}+\frac{d^2x^{10}(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{4},-\right.}{5c^4}\right.$$

input
```
Int[(x*(a + b*x^8)^p)/(c + d*x^4)^2,x]
```

output
```
((x^2*(a + b*x^8)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^8)/a), (d^2*x^8)/c^2]
)/(c^2*(1 + (b*x^8)/a)^p) - (2*d*x^6*(a + b*x^8)^p*AppellF1[3/4, -p, 2, 7/
4, -((b*x^8)/a), (d^2*x^8)/c^2])/(3*c^3*(1 + (b*x^8)/a)^p) + (d^2*x^10*(a
+ b*x^8)^p*AppellF1[5/4, -p, 2, 9/4, -((b*x^8)/a), (d^2*x^8)/c^2])/(5*c^4*
(1 + (b*x^8)/a)^p))/2
```

Defintions of rubi rules used

rule 1569
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

rule 1815
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/
k - 1)*(d + e*x^(n/k))^q*(a + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1]
/; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x(b\,x^8+a)^p}{(x^4d+c)^2}dx$$

input
```
int(x*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int(x*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{x(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x}{(dx^4+c)^2}\,dx$$

input
```
integrate(x*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p*x/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx^8)^p}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x**8+a)**p/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x}{(dx^4+c)^2}\,dx$$

input
```
integrate(x*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p*x/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{x(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x}{(dx^4+c)^2}\,dx$$

input
```
integrate(x*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p*x/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{x\,(b\,x^8+a)^p}{(d\,x^4+c)^2}\,dx$$

input
```
int((x*(a + b*x^8)^p)/(c + d*x^4)^2,x)
```

output
```
int((x*(a + b*x^8)^p)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{x(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^8+a)^p\,x}{d^2x^8+2cd\,x^4+c^2}dx$$

input
```
int(x*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int(((a + b*x**8)**p*x)/(c**2 + 2*c*d*x**4 + d**2*x**8),x)
```

3.39 $\int \frac{(a+bx^8)^p}{x(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 310

$$\begin{aligned}\int \frac{(a+bx^8)^p}{x(c+dx^4)^2}\,dx &= \frac{d^2(a+bx^8)^{1+p}}{8(bc^2+ad^2)(c^2-d^2x^8)} \\ &\quad - \frac{dx^4(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{2c^3} \\ &\quad + \frac{d^2(ad^2+bc^2(1-p))(a+bx^8)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{d^2(a+bx^8)}{bc^2+ad^2}\right)}{8c^2(bc^2+ad^2)^2(1+p)} \\ &\quad - \frac{(a+bx^8)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^8}{a}\right)}{8ac^2(1+p)} \\ &\quad + \frac{bd^2(a+bx^8)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{d^2(a+bx^8)}{bc^2+ad^2}\right)}{8(bc^2+ad^2)^2(1+p)}\end{aligned}$$

output
```
1/8*d^2*(b*x^8+a)^(p+1)/(a*d^2+b*c^2)/(-d^2*x^8+c^2)-1/2*d*x^4*(b*x^8+a)^p
*AppellF1(1/2,2,-p,3/2,d^2*x^8/c^2,-b*x^8/a)/c^3/((1+b*x^8/a)^p)+1/8*d^2*(
a*d^2+b*c^2*(1-p))*(b*x^8+a)^(p+1)*hypergeom([1, p+1],[2+p],d^2*(b*x^8+a)/
(a*d^2+b*c^2))/c^2/(a*d^2+b*c^2)^2/(p+1)-1/8*(b*x^8+a)^(p+1)*hypergeom([1,
 p+1],[2+p],1+b*x^8/a)/a/c^2/(p+1)+1/8*b*d^2*(b*x^8+a)^(p+1)*hypergeom([2,
 p+1],[2+p],d^2*(b*x^8+a)/(a*d^2+b*c^2))/(a*d^2+b*c^2)^2/(p+1)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{x(c+dx^4)^2}\,dx=\int \frac{(a+bx^8)^p}{x(c+dx^4)^2}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(x*(c + d*x^4)^2),x]
```

output
```
Integrate[(a + b*x^8)^p/(x*(c + d*x^4)^2), x]
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {1803, 622, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{x(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{1803}$$

$$\frac{1}{4}\int \frac{(bx^8+a)^p}{x^4(dx^4+c)^2}dx^4$$

$$\Big\downarrow \text{622}$$

$$\frac{1}{4}\int \left(-\frac{2cd(bx^8+a)^p}{(c^2-d^2x^8)^2}+\frac{c^2(bx^8+a)^p}{x^4(c^2-d^2x^8)^2}+\frac{d^2x^4(bx^8+a)^p}{(d^2x^8-c^2)^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{2dx^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^3}+\frac{d^2(a+bx^8)^{p+1}\left(ad^2+bc^2(1-p)\right)\text{Hype}}{2c^2(p+1)}\right.$$

input
```
Int[(a + b*x^8)^p/(x*(c + d*x^4)^2),x]
```

output
```
((d^2*(a + b*x^8)^(1 + p))/(2*(b*c^2 + a*d^2)*(c^2 - d^2*x^8)) - (2*d*x^4*
(a + b*x^8)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^8)/a), (d^2*x^8)/c^2])/(c^3
*(1 + (b*x^8)/a)^p) + (d^2*(a*d^2 + b*c^2*(1 - p))*(a + b*x^8)^(1 + p)*Hyp
ergeometric2F1[1, 1 + p, 2 + p, (d^2*(a + b*x^8))/(b*c^2 + a*d^2)])/(2*c^2
*(b*c^2 + a*d^2)^2*(1 + p)) - ((a + b*x^8)^(1 + p)*Hypergeometric2F1[1, 1
+ p, 2 + p, 1 + (b*x^8)/a])/(2*a*c^2*(1 + p)) + (b*d^2*(a + b*x^8)^(1 + p)
*Hypergeometric2F1[2, 1 + p, 2 + p, (d^2*(a + b*x^8))/(b*c^2 + a*d^2)])/(2
*(b*c^2 + a*d^2)^2*(1 + p)))/4
```

Defintions of rubi rules used

rule 622
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[x^m*(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2
 - d^2*x^2)))^(-n), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[n, -1]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^{8}+a)^{p}}{x\,(x^{4}d+c)^{2}}dx$$

input
```
int((b*x^8+a)^p/x/(d*x^4+c)^2,x)
```

output
```
int((b*x^8+a)^p/x/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{x\,(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^2x}\,dx$$

input
```
integrate((b*x^8+a)^p/x/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d^2*x^9 + 2*c*d*x^5 + c^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x\,(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/x/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{x\left(c+dx^4\right)^2}\,dx=\int \frac{(bx^8+a)^p}{(dx^4+c)^2x}\,dx$$

input
```
integrate((b*x^8+a)^p/x/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)^2*x), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{x\left(c+dx^4\right)^2}\,dx=\int \frac{(bx^8+a)^p}{(dx^4+c)^2x}\,dx$$

input
```
integrate((b*x^8+a)^p/x/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x\left(c+dx^4\right)^2}\,dx=\int \frac{(b\,x^8+a)^p}{x\,(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^8)^p/(x*(c + d*x^4)^2),x)
```

output
```
int((a + b*x^8)^p/(x*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{x(c+dx^4)^2}\,dx = \int \frac{(b\,x^8+a)^p}{d^2x^9+2cd\,x^5+c^2x}dx$$

input
```
int((b*x^8+a)^p/x/(d*x^4+c)^2,x)
```

output
```
int((a + b*x**8)**p/(c**2*x + 2*c*d*x**5 + d**2*x**9),x)
```

3.40 $\int \frac{(a+bx^8)^p}{x^3(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 192

$$\int \frac{(a+bx^8)^p}{x^3(c+dx^4)^2}\,dx = -\frac{(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(-\frac{1}{4},-p,2,\frac{3}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{2c^2x^2} - \frac{dx^2(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^3} + \frac{d^2x^6(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{4},-p,2,\frac{7}{4},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{6c^4}$$

output
```
-1/2*(b*x^8+a)^p*AppellF1(-1/4,2,-p,3/4,d^2*x^8/c^2,-b*x^8/a)/c^2/x^2/((1+
b*x^8/a)^p)-d*x^2*(b*x^8+a)^p*AppellF1(1/4,2,-p,5/4,d^2*x^8/c^2,-b*x^8/a)/
c^3/((1+b*x^8/a)^p)+1/6*d^2*x^6*(b*x^8+a)^p*AppellF1(3/4,2,-p,7/4,d^2*x^8/
c^2,-b*x^8/a)/c^4/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{x^3 (c+dx^4)^2} \, dx = \int \frac{(a+bx^8)^p}{x^3 (c+dx^4)^2} \, dx$$

input
```
Integrate[(a + b*x^8)^p/(x^3*(c + d*x^4)^2),x]
```

output
```
Integrate[(a + b*x^8)^p/(x^3*(c + d*x^4)^2), x]
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1815, 1675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{x^3 (c+dx^4)^2} \, dx$$

$$\Big\downarrow \ 1815$$

$$\frac{1}{2} \int \frac{(bx^8+a)^p}{x^4 (dx^4+c)^2} dx^2$$

$$\Big\downarrow \ 1675$$

$$\frac{1}{2} \int \left(-\frac{d(bx^8+a)^p}{c^2 (dx^4+c)} - \frac{d(bx^8+a)^p}{c (dx^4+c)^2} + \frac{(bx^8+a)^p}{c^2 x^4} \right) dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2} \left(-\frac{d^3 x^{10} (a+bx^8)^p \left(\frac{bx^8}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{5}{4}, -p, 2, \frac{9}{4}, -\frac{bx^8}{a}, \frac{d^2x^8}{c^2}\right)}{5c^5} + \frac{d^2 x^6 (a+bx^8)^p \left(\frac{bx^8}{a}+1\right)^{-p} \text{AppellF1} (}{3c^4} \right.$$

input
```
Int[(a + b*x^8)^p/(x^3*(c + d*x^4)^2),x]
```

output
```
(-((d*x^2*(a + b*x^8)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^8)/a), (d^2*x^8)/
c^2])/(c^3*(1 + (b*x^8)/a)^p)) - (d*x^2*(a + b*x^8)^p*AppellF1[1/4, -p, 2,
 5/4, -((b*x^8)/a), (d^2*x^8)/c^2])/(c^3*(1 + (b*x^8)/a)^p) + (d^2*x^6*(a
+ b*x^8)^p*AppellF1[3/4, -p, 1, 7/4, -((b*x^8)/a), (d^2*x^8)/c^2])/(3*c^4*
(1 + (b*x^8)/a)^p) + (2*d^2*x^6*(a + b*x^8)^p*AppellF1[3/4, -p, 2, 7/4, -(
(b*x^8)/a), (d^2*x^8)/c^2])/(3*c^4*(1 + (b*x^8)/a)^p) - (d^3*x^10*(a + b*x
^8)^p*AppellF1[5/4, -p, 2, 9/4, -((b*x^8)/a), (d^2*x^8)/c^2])/(5*c^5*(1 +
(b*x^8)/a)^p) - ((a + b*x^8)^p*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^8)/
a)])/(c^2*x^2*(1 + (b*x^8)/a)^p))/2
```

Defintions of rubi rules used

rule 1675
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

rule 1815
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/
k - 1)*(d + e*x^(n/k))^q*(a + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1]
/; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^{8}+a)^{p}}{x^{3}\,(x^{4} d+c)^{2}}dx$$

input
```
int((b*x^8+a)^p/x^3/(d*x^4+c)^2,x)
```

output
```
int((b*x^8+a)^p/x^3/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{x^3\left(c+dx^4\right)^2}\,dx=\int \frac{(bx^8+a)^p}{(dx^4+c)^2x^3}\,dx$$

input
```
integrate((b*x^8+a)^p/x^3/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d^2*x^11 + 2*c*d*x^7 + c^2*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^3\left(c+dx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x**8+a)**p/x**3/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{x^3\left(c+dx^4\right)^2}\,dx=\int \frac{(bx^8+a)^p}{(dx^4+c)^2x^3}\,dx$$

input
```
integrate((b*x^8+a)^p/x^3/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)^2*x^3), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{x^3(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^2x^3}\,dx$$

input
```
integrate((b*x^8+a)^p/x^3/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^3(c+dx^4)^2}\,dx = \int \frac{(b\,x^8+a)^p}{x^3\,(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^8)^p/(x^3*(c + d*x^4)^2),x)
```

output
```
int((a + b*x^8)^p/(x^3*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{x^3(c+dx^4)^2}\,dx = \int \frac{(b\,x^8+a)^p}{d^2x^{11}+2cd\,x^7+c^2x^3}dx$$

input
```
int((b*x^8+a)^p/x^3/(d*x^4+c)^2,x)
```

output
```
int((a + b*x**8)**p/(c**2*x**3 + 2*c*d*x**7 + d**2*x**11),x)
```

3.41 $\int \frac{(a+bx^8)^p}{x^5(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 310

$$\begin{aligned}\int \frac{(a+bx^8)^p}{x^5(c+dx^4)^2}\,dx &= -\frac{d^3(a+bx^8)^{1+p}}{4c(bc^2+ad^2)(c^2-d^2x^8)} \\ &\quad -\frac{(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-p,2,\frac{1}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{4c^2x^4} \\ &\quad +\frac{d^2x^4(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{4c^4} \\ &\quad -\frac{d^3(ad^2+bc^2(1-p))(a+bx^8)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{d^2(a+bx^8)}{bc^2+ad^2}\right)}{4c^3(bc^2+ad^2)^2(1+p)} \\ &\quad +\frac{d(a+bx^8)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^8}{a}\right)}{4ac^3(1+p)}\end{aligned}$$

output
```
-1/4*d^3*(b*x^8+a)^(p+1)/c/(a*d^2+b*c^2)/(-d^2*x^8+c^2)-1/4*(b*x^8+a)^p*Ap
pellF1(-1/2,2,-p,1/2,d^2*x^8/c^2,-b*x^8/a)/c^2/x^4/((1+b*x^8/a)^p)+1/4*d^2
*x^4*(b*x^8+a)^p*AppellF1(1/2,2,-p,3/2,d^2*x^8/c^2,-b*x^8/a)/c^4/((1+b*x^8
/a)^p)-1/4*d^3*(a*d^2+b*c^2*(1-p))*(b*x^8+a)^(p+1)*hypergeom([1, p+1],[2+p
],d^2*(b*x^8+a)/(a*d^2+b*c^2))/c^3/(a*d^2+b*c^2)^2/(p+1)+1/4*d*(b*x^8+a)^(
p+1)*hypergeom([1, p+1],[2+p],1+b*x^8/a)/a/c^3/(p+1)
```

Mathematica [A] (warning: unable to verify)

Time = 2.20 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^8)^p}{x^5(c+dx^4)^2}\,dx = \frac{d\left(\frac{d\left(-\sqrt{-\frac{a}{b}}+x^4\right)}{c+dx^4}\right)^{-p}\left(\frac{d\left(\sqrt{-\frac{a}{b}}+x^4\right)}{c+dx^4}\right)^{-p}(a+bx^8)^p\left(cp\,\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{c-\sqrt{-\frac{a}{b}}d}{c+dx^4},\frac{c+\sqrt{-\frac{a}{b}}d}{c+dx^4}\right)\right.}{4c^3p(-1+2p)\,(c+dx^4)} + \frac{(a+bx^8)^p\left(-\frac{c\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^8}{a}\right)}{x^4}-\frac{d\left(1+\frac{a}{bx^8}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{a}{bx^8}\right)}{p}\right)}{4c^3}$$

input
```
Integrate[(a + b*x^8)^p/(x^5*(c + d*x^4)^2),x]
```

output
```
(d*(a + b*x^8)^p*(c*p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (c - Sqrt[-(a/b)]
*d)/(c + d*x^4), (c + Sqrt[-(a/b)]*d)/(c + d*x^4)] + (-1 + 2*p)*(c + d*x^4
)*AppellF1[-2*p, -p, -p, 1 - 2*p, (c - Sqrt[-(a/b)]*d)/(c + d*x^4), (c + S
qrt[-(a/b)]*d)/(c + d*x^4)]))/(4*c^3*p*(-1 + 2*p)*((d*(-Sqrt[-(a/b)] + x^4
))/(c + d*x^4))^p*((d*(Sqrt[-(a/b)] + x^4))/(c + d*x^4))^p*(c + d*x^4)) +
((a + b*x^8)^p*(-((c*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^8)/a)])/(x^4*
(1 + (b*x^8)/a)^p)) - (d*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^8))])/(
p*(1 + a/(b*x^8))^p)))/(4*c^3)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1803, 622, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{x^5(c+dx^4)^2}dx$$

$$\Big\downarrow \text{ 1803}$$

$$\frac{1}{4}\int \frac{(bx^8+a)^p}{x^8(dx^4+c)^2}dx^4$$

$$\Big\downarrow \text{ 622}$$

$$\frac{1}{4}\int \left(-\frac{2cd(bx^8+a)^p}{x^4(c^2-d^2x^8)^2}+\frac{c^2(bx^8+a)^p}{x^8(c^2-d^2x^8)^2}+\frac{d^2(bx^8+a)^p}{(d^2x^8-c^2)^2}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(-\frac{(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-p,2,\frac{1}{2},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^2x^4}+\frac{d^2x^4(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},\right.}{c^4}\right.$$

input
```
Int[(a + b*x^8)^p/(x^5*(c + d*x^4)^2),x]
```

output
```
(-((d^3*(a + b*x^8)^(1 + p))/(c*(b*c^2 + a*d^2)*(c^2 - d^2*x^8))) - ((a +
b*x^8)^p*AppellF1[-1/2, -p, 2, 1/2, -((b*x^8)/a), (d^2*x^8)/c^2])/(c^2*x^4
*(1 + (b*x^8)/a)^p) + (d^2*x^4*(a + b*x^8)^p*AppellF1[1/2, -p, 2, 3/2, -((
b*x^8)/a), (d^2*x^8)/c^2])/(c^4*(1 + (b*x^8)/a)^p) - (d^3*(a*d^2 + b*c^2*(
1 - p))*(a + b*x^8)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (d^2*(a + b
*x^8))/(b*c^2 + a*d^2)])/(c^3*(b*c^2 + a*d^2)^2*(1 + p)) + (d*(a + b*x^8)^
(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^8)/a])/(a*c^3*(1 + p))
)/4
```

Defintions of rubi rules used

rule 622
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[x^m*(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2
 - d^2*x^2)))^(-n), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[n, -1]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^{8}+a)^{p}}{x^{5}\left(x^{4} d+c\right)^{2}}dx$$

input
```
int((b*x^8+a)^p/x^5/(d*x^4+c)^2,x)
```

output
```
int((b*x^8+a)^p/x^5/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{x^5\,(c+dx^4)^2}\,dx=\int \frac{(bx^8+a)^p}{(dx^4+c)^2x^5}\,dx$$

input
```
integrate((b*x^8+a)^p/x^5/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d^2*x^13 + 2*c*d*x^9 + c^2*x^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^5(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/x**5/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{x^5(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^2x^5}\,dx$$

input
```
integrate((b*x^8+a)^p/x^5/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)^2*x^5), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{x^5(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^2x^5}\,dx$$

input
```
integrate((b*x^8+a)^p/x^5/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)^2*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^5\,(c+dx^4)^2}\,dx = \int \frac{(b\,x^8+a)^p}{x^5\,(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^8)^p/(x^5*(c + d*x^4)^2),x)
```

output
```
int((a + b*x^8)^p/(x^5*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{x^5\,(c+dx^4)^2}\,dx = \int \frac{(b\,x^8+a)^p}{d^2x^{13}+2cd\,x^9+c^2x^5}dx$$

input
```
int((b*x^8+a)^p/x^5/(d*x^4+c)^2,x)
```

output
```
int((a + b*x**8)**p/(c**2*x**5 + 2*c*d*x**9 + d**2*x**13),x)
```

3.42 $\int \frac{x^4(a+bx^8)^p}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \frac{x^4(a+bx^8)^p}{(c+dx^4)^2}\,dx = \frac{x^5(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{8},-p,2,\frac{13}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c^2} - \frac{2dx^9(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{9}{8},-p,2,\frac{17}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{9c^3} + \frac{d^2x^{13}(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{13}{8},-p,2,\frac{21}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{13c^4}$$

output
```
1/5*x^5*(b*x^8+a)^p*AppellF1(5/8,2,-p,13/8,d^2*x^8/c^2,-b*x^8/a)/c^2/((1+b
*x^8/a)^p)-2/9*d*x^9*(b*x^8+a)^p*AppellF1(9/8,2,-p,17/8,d^2*x^8/c^2,-b*x^8
/a)/c^3/((1+b*x^8/a)^p)+1/13*d^2*x^13*(b*x^8+a)^p*AppellF1(13/8,2,-p,21/8,
d^2*x^8/c^2,-b*x^8/a)/c^4/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{x^4(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{x^4(a+bx^8)^p}{(c+dx^4)^2}\,dx$$

input
```
Integrate[(x^4*(a + b*x^8)^p)/(c + d*x^4)^2,x]
```

output
```
Integrate[(x^4*(a + b*x^8)^p)/(c + d*x^4)^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(a+bx^8\right)^p}{\left(c+dx^4\right)^2}\,dx$$
$$\Big\downarrow \ 1888$$
$$\int \frac{x^4\left(a+bx^8\right)^p}{\left(c+dx^4\right)^2}dx$$

input
```
Int[(x^4*(a + b*x^8)^p)/(c + d*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1888
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n)
)^p, x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x^{4} \left(b\,x^{8}+a\right)^{p}}{\left(x^{4} d+c\right)^{2}}d x$$

input
```
int(x^4*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int(x^4*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{x^4(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^4}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^4*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p*x^4/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^8)^p}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(b*x**8+a)**p/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^4}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^4*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p*x^4/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{x^4(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^4}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^4*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p*x^4/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{x^4\,(b\,x^8+a)^p}{(d\,x^4+c)^2}\,dx$$

input
```
int((x^4*(a + b*x^8)^p)/(c + d*x^4)^2,x)
```

output
```
int((x^4*(a + b*x^8)^p)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{x^4(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^8+a)^p\,x^4}{d^2x^8+2cd\,x^4+c^2}dx$$

input
```
int(x^4*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int(((a + b*x**8)**p*x**4)/(c**2 + 2*c*d*x**4 + d**2*x**8),x)
```

3.43 $\int \frac{x^2 (a+bx^8)^p}{(c+dx^4)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \frac{x^2(a+bx^8)^p}{(c+dx^4)^2} \, dx = \frac{x^3(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{3}{8},-p,2,\frac{11}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{3c^2} - \frac{2dx^7(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{7}{8},-p,2,\frac{15}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{7c^3} + \frac{d^2x^{11}(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{11}{8},-p,2,\frac{19}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{11c^4}$$

output

```
1/3*x^3*(b*x^8+a)^p*AppellF1(3/8,2,-p,11/8,d^2*x^8/c^2,-b*x^8/a)/c^2/((1+b
*x^8/a)^p)-2/7*d*x^7*(b*x^8+a)^p*AppellF1(7/8,2,-p,15/8,d^2*x^8/c^2,-b*x^8
/a)/c^3/((1+b*x^8/a)^p)+1/11*d^2*x^11*(b*x^8+a)^p*AppellF1(11/8,2,-p,19/8,
d^2*x^8/c^2,-b*x^8/a)/c^4/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{x^2(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{x^2(a+bx^8)^p}{(c+dx^4)^2}\,dx$$

input
```
Integrate[(x^2*(a + b*x^8)^p)/(c + d*x^4)^2,x]
```

output
```
Integrate[(x^2*(a + b*x^8)^p)/(c + d*x^4)^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx^8)^p}{(c+dx^4)^2}\,dx$$
$$\Big\downarrow \text{ 1888}$$
$$\int \frac{x^2(a+bx^8)^p}{(c+dx^4)^2}dx$$

input
```
Int[(x^2*(a + b*x^8)^p)/(c + d*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1888
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n)
)^p, x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{x^2(b\,x^8+a)^p}{(x^4d+c)^2}dx$$

input
```
int(x^2*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int(x^2*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{x^2(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^2}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^2*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p*x^2/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^8)^p}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(b*x**8+a)**p/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^2}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^2*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p*x^2/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{x^2(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p x^2}{(dx^4+c)^2}\,dx$$

input
```
integrate(x^2*(b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p*x^2/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{x^2\,(b\,x^8+a)^p}{(d\,x^4+c)^2}\,dx$$

input
```
int((x^2*(a + b*x^8)^p)/(c + d*x^4)^2,x)
```

output
```
int((x^2*(a + b*x^8)^p)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{x^2(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^8+a)^p\,x^2}{d^2x^8+2cd\,x^4+c^2}dx$$

input
```
int(x^2*(b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int(((a + b*x**8)**p*x**2)/(c**2 + 2*c*d*x**4 + d**2*x**8),x)
```

3.44 $\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 189

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx = \frac{x(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{8},-p,2,\frac{9}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^2} - \frac{2dx^5(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{5}{8},-p,2,\frac{13}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c^3} + \frac{d^2x^9(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{9}{8},-p,2,\frac{17}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{9c^4}$$

output

```
x*(b*x^8+a)^p*AppellF1(1/8,2,-p,9/8,d^2*x^8/c^2,-b*x^8/a)/c^2/((1+b*x^8/a)
^p)-2/5*d*x^5*(b*x^8+a)^p*AppellF1(5/8,2,-p,13/8,d^2*x^8/c^2,-b*x^8/a)/c^3
/((1+b*x^8/a)^p)+1/9*d^2*x^9*(b*x^8+a)^p*AppellF1(9/8,2,-p,17/8,d^2*x^8/c^
2,-b*x^8/a)/c^4/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(c + d*x^4)^2,x]
```

output
```
Integrate[(a + b*x^8)^p/(c + d*x^4)^2, x]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1768, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{1768}$$

$$\int \left(\frac{c^2(a+bx^8)^p}{(c^2-d^2x^8)^2}+\frac{d^2x^8(a+bx^8)^p}{(d^2x^8-c^2)^2}-\frac{2cdx^4(a+bx^8)^p}{(c^2-d^2x^8)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{8},-p,2,\frac{9}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^2}+\frac{d^2x^9(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{9}{8},-p,2,\frac{17}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{9c^4}-\frac{2dx^5(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{8},-p,2,\frac{13}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c^3}$$

input
```
Int[(a + b*x^8)^p/(c + d*x^4)^2,x]
```

output
```
(x*(a + b*x^8)^p*AppellF1[1/8, -p, 2, 9/8, -((b*x^8)/a), (d^2*x^8)/c^2])/(
c^2*(1 + (b*x^8)/a)^p) - (2*d*x^5*(a + b*x^8)^p*AppellF1[5/8, -p, 2, 13/8,
 -((b*x^8)/a), (d^2*x^8)/c^2])/(5*c^3*(1 + (b*x^8)/a)^p) + (d^2*x^9*(a + b
*x^8)^p*AppellF1[9/8, -p, 2, 17/8, -((b*x^8)/a), (d^2*x^8)/c^2])/(9*c^4*(1
 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1768
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^{8}+a)^{p}}{(x^{4}d+c)^{2}}dx$$

input
```
int((b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int((b*x^8+a)^p/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx=\int \frac{(bx^8+a)^p}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx=\int \frac{{\left(b\,x^8+a\right)}^p}{{\left(d\,x^4+c\right)}^2}\,dx$$

input
```
int((a + b*x^8)^p/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^8)^p/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx=\int \frac{\left(b\,x^{8}+a\right)^{p}}{d^{2}x^{8}+2cd\,x^{4}+c^{2}}dx$$

input
```
int((b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int((a + b*x**8)**p/(c**2 + 2*c*d*x**4 + d**2*x**8),x)
```

3.45 $\int \frac{(a+bx^8)^p}{x^2(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 192

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)^2}\,dx = -\frac{(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(-\frac{1}{8},-p,2,\frac{7}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^2x} - \frac{2dx^3(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{8},-p,2,\frac{11}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{3c^3} + \frac{d^2x^7(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{7}{8},-p,2,\frac{15}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{7c^4}$$

output
```
-(b*x^8+a)^p*AppellF1(-1/8,2,-p,7/8,d^2*x^8/c^2,-b*x^8/a)/c^2/x/((1+b*x^8/
a)^p)-2/3*d*x^3*(b*x^8+a)^p*AppellF1(3/8,2,-p,11/8,d^2*x^8/c^2,-b*x^8/a)/c
^3/((1+b*x^8/a)^p)+1/7*d^2*x^7*(b*x^8+a)^p*AppellF1(7/8,2,-p,15/8,d^2*x^8/
c^2,-b*x^8/a)/c^4/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)^2}\,dx = \int \frac{(a+bx^8)^p}{x^2(c+dx^4)^2}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(x^2*(c + d*x^4)^2),x]
```

output
```
Integrate[(a + b*x^8)^p/(x^2*(c + d*x^4)^2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)^2}\,dx$$
$$\Big\downarrow \text{ 1888}$$
$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)^2}dx$$

input
```
Int[(a + b*x^8)^p/(x^2*(c + d*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1888
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n)
)^p, x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(b\,x^{8}+a)^{p}}{x^{2}\,(x^{4}d+c)^{2}}dx$$

input
```
int((b*x^8+a)^p/x^2/(d*x^4+c)^2,x)
```

output
```
int((b*x^8+a)^p/x^2/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{x^2\,(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^2x^2}\,dx$$

input
```
integrate((b*x^8+a)^p/x^2/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d^2*x^10 + 2*c*d*x^6 + c^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^2\,(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/x**2/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^2x^2}\,dx$$

input
```
integrate((b*x^8+a)^p/x^2/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)^2*x^2), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^2x^2}\,dx$$

input
```
integrate((b*x^8+a)^p/x^2/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^2(c+dx^4)^2}\,dx = \int \frac{(b\,x^8+a)^p}{x^2\,(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^8)^p/(x^2*(c + d*x^4)^2),x)
```

output
```
int((a + b*x^8)^p/(x^2*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{x^2 (c+dx^4)^2} \, dx = \int \frac{(b\,x^8+a)^p}{d^2x^{10}+2cd\,x^6+c^2x^2} dx$$

input
```
int((b*x^8+a)^p/x^2/(d*x^4+c)^2,x)
```

output
```
int((a + b*x**8)**p/(c**2*x**2 + 2*c*d*x**6 + d**2*x**10),x)
```

3.46 $\int \frac{(a+bx^8)^p}{x^4(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 190

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)^2}\,dx = -\frac{(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(-\frac{3}{8},-p,2,\frac{5}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{3c^2x^3} - \frac{2dx(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{8},-p,2,\frac{9}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^3} + \frac{d^2x^5(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{8},-p,2,\frac{13}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c^4}$$

output
```
-1/3*(b*x^8+a)^p*AppellF1(-3/8,2,-p,5/8,d^2*x^8/c^2,-b*x^8/a)/c^2/x^3/((1+
b*x^8/a)^p)-2*d*x*(b*x^8+a)^p*AppellF1(1/8,2,-p,9/8,d^2*x^8/c^2,-b*x^8/a)/
c^3/((1+b*x^8/a)^p)+1/5*d^2*x^5*(b*x^8+a)^p*AppellF1(5/8,2,-p,13/8,d^2*x^8
/c^2,-b*x^8/a)/c^4/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)^2}\,dx = \int \frac{(a+bx^8)^p}{x^4(c+dx^4)^2}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(x^4*(c + d*x^4)^2),x]
```

output
```
Integrate[(a + b*x^8)^p/(x^4*(c + d*x^4)^2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{1888}$$

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)^2}dx$$

input
```
Int[(a + b*x^8)^p/(x^4*(c + d*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1888
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n)
)^p, x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \frac{(b\,x^8+a)^p}{x^4\,(x^4d+c)^2}dx$$

input
```
int((b*x^8+a)^p/x^4/(d*x^4+c)^2,x)
```

output
```
int((b*x^8+a)^p/x^4/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{x^4\,(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^2x^4}\,dx$$

input
```
integrate((b*x^8+a)^p/x^4/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d^2*x^12 + 2*c*d*x^8 + c^2*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^4\,(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/x**4/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)^2}\,dx=\int \frac{(bx^8+a)^p}{(dx^4+c)^2x^4}\,dx$$

input
```
integrate((b*x^8+a)^p/x^4/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)^2*x^4), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)^2}\,dx=\int \frac{(bx^8+a)^p}{(dx^4+c)^2x^4}\,dx$$

input
```
integrate((b*x^8+a)^p/x^4/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/((d*x^4 + c)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{x^4(c+dx^4)^2}\,dx=\int \frac{(b\,x^8+a)^p}{x^4\,(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^8)^p/(x^4*(c + d*x^4)^2),x)
```

output
```
int((a + b*x^8)^p/(x^4*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{x^4 (c+dx^4)^2} \, dx = \int \frac{(b\,x^8+a)^p}{d^2x^{12}+2cd\,x^8+c^2x^4}dx$$

input
```
int((b*x^8+a)^p/x^4/(d*x^4+c)^2,x)
```

output
```
int((a + b*x**8)**p/(c**2*x**4 + 2*c*d*x**8 + d**2*x**12),x)
```

3.47 $\int \frac{x^{11}(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{x^{11}(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{(cd-be)x^4}{4c^2}+\frac{ex^8}{8c} - \frac{(b^2cd-2ac^2d-b^3e+3abce)\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^3\sqrt{b^2-4ac}} - \frac{(bcd-b^2e+ace)\log(a+bx^4+cx^8)}{8c^3}$$

output
```
1/4*(-b*e+c*d)*x^4/c^2+1/8*e*x^8/c-1/4*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)
*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)-1/8*(a*c*e
-b^2*e+b*c*d)*ln(c*x^8+b*x^4+a)/c^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{x^{11}(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{2c(cd-be)x^4+c^2ex^8+\frac{2(b^2cd-2ac^2d-b^3e+3abce)\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(-bcd+b^2e-ace)\log(a+bx^4+cx^8)}{8c^3}$$

input
```
Integrate[(x^11*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(2*c*(c*d - b*e)*x^4 + c^2*e*x^8 + (2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b
*c*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*
c*d) + b^2*e - a*c*e)*Log[a + b*x^4 + c*x^8])/(8*c^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\left(d+ex^4\right)}{a+bx^4+cx^8}dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{4}\int \frac{x^8\left(ex^4+d\right)}{cx^8+bx^4+a}dx^4$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{4}\int \left(\frac{ex^4}{c}+\frac{cd-be}{c^2}-\frac{\left(-eb^2+cdb+ace\right)x^4+a(cd-be)}{c^2\left(cx^8+bx^4+a\right)}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(-\frac{\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)\left(3abce-2ac^2d+b^3(-e)+b^2cd\right)}{c^3\sqrt{b^2-4ac}}-\frac{\left(ace+b^2(-e)+bcd\right)\log\left(a+bx^4+cx^8\right)}{2c^3}+\frac{x^4(cd-\ldots}{c^2}\right.$$

input
```
Int[(x^11*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(((c*d - b*e)*x^4)/c^2 + (e*x^8)/(2*c) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3
*a*b*c*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]
) - ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^4 + c*x^8])/(2*c^3))/4
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{-\frac{1}{2} c e \,x^{8}+b e \,x^{4}-c d \,x^{4}}{4 c^{2}}+\frac{\frac{\left(-a c e +b^{2} e -c b d \right) \ln \left(c \,x^{8}+b \,x^{4}+a \right)}{2 c}+\frac{2 \left(a b e -a c d -\frac{\left(-a c e +b^{2} e -c b d \right) b}{2 c}\right) \arctan \left(\frac{2 c \,x^{4}+b}{\sqrt{4 a c -b^{2}}}\right)}{\sqrt{4 a c -b^{2}}}}{4 c^{2}}$	136
risch	Expression too large to display	2131

input
```
int(x^11*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/c^2*(-1/2*c*e*x^8+b*e*x^4-c*d*x^4)+1/4/c^2*(1/2*(-a*c*e+b^2*e-b*c*d)/
c*ln(c*x^8+b*x^4+a)+2*(a*b*e-a*c*d-1/2*(-a*c*e+b^2*e-b*c*d)*b/c)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.26

$$\int \frac{x^{11}(d+ex^4)}{a+bx^4+cx^8}\,dx$$
$$= \left[\frac{(b^2c^2-4\,ac^3)ex^8+2\,((b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e)x^4+\sqrt{b^2-4\,ac}((b^2c-2\,ac^2)d-(b^3-3\,abc)e}{8\,(b^2c^3}\right.$$

input
```
integrate(x^11*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*((b^2*c^2 - 4*a*c^3)*e*x^8 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*
b*c^2)*e)*x^4 + sqrt(b^2 - 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e
)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c - (2*c*x^4 + b)*sqrt(b^2 - 4*a*
c))/(c*x^8 + b*x^4 + a)) - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*e)*log(c*x^8 + b*x^4 + a))/(b^2*c^3 - 4*a*c^4), 1/8*((b^2*c^2 - 4*
a*c^3)*e*x^8 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*x^4 - 2*s
qrt(-b^2 + 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*arctan(-(2*c*x
^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3*c - 4*a*b*c^2)*d - (b^4
- 5*a*b^2*c + 4*a^2*c^2)*e)*log(c*x^8 + b*x^4 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x**11*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{x^{11}(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{cex^8+2\,cdx^4-2\,bex^4}{8\,c^2} - \frac{(bcd-b^2e+ace)\log\left(cx^8+bx^4+a\right)}{8\,c^3} + \frac{(b^2cd-2\,ac^2d-b^3e+3\,abce)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate(x^11*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/8*(c*e*x^8 + 2*c*d*x^4 - 2*b*e*x^4)/c^2 - 1/8*(b*c*d - b^2*e + a*c*e)*lo
g(c*x^8 + b*x^4 + a)/c^3 + 1/4*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*a
rctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 24.71 (sec) , antiderivative size = 8521, normalized size of antiderivative = 64.55

$$\int \frac{x^{11}(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int((x^11*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

output
```
x^4*(d/(4*c) - (b*e)/(4*c^2)) + (e*x^8)/(8*c) - (log(a + b*x^4 + c*x^8)*(4
*b^4*e + 16*a^2*c^2*e - 4*b^3*c*d + 16*a*b*c^2*d - 20*a*b^2*c*e))/(2*(64*a
*c^4 - 16*b^2*c^3)) + (atan((8*c^8*(4*a*c - b^2)^2*(x^4*(((a*c - b^2)*((((
((((448*b^4*c^10*d - 448*b^5*c^9*e - 384*a*b^2*c^11*d + 832*a*b^3*c^10*e)/
c^8 - (256*b^3*c^4*(4*b^4*e + 16*a^2*c^2*e - 4*b^3*c*d + 16*a*b*c^2*d - 20
*a*b^2*c*e))/(64*a*c^4 - 16*b^2*c^3))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b
*c*e))/(8*c^3*(4*a*c - b^2)^(1/2)) - (32*b^3*c*(b^3*e + 2*a*c^2*d - b^2*c*
d - 3*a*b*c*e)*(4*b^4*e + 16*a^2*c^2*e - 4*b^3*c*d + 16*a*b*c^2*d - 20*a*b
^2*c*e))/((4*a*c - b^2)^(1/2)*(64*a*c^4 - 16*b^2*c^3)))*(4*b^4*e + 16*a^2*
c^2*e - 4*b^3*c*d + 16*a*b*c^2*d - 20*a*b^2*c*e))/(2*(64*a*c^4 - 16*b^2*c^
3)) - (((144*b^5*c^8*d^2 + 144*b^7*c^6*e^2 - 240*a*b^3*c^9*d^2 + 96*a^2*b*
c^10*d^2 - 528*a*b^5*c^7*e^2 + 480*a^2*b^3*c^8*e^2 - 288*b^6*c^7*d*e + 768
*a*b^4*c^8*d*e - 432*a^2*b^2*c^9*d*e)/c^8 - (((448*b^4*c^10*d - 448*b^5*c^
9*e - 384*a*b^2*c^11*d + 832*a*b^3*c^10*e)/c^8 - (256*b^3*c^4*(4*b^4*e + 1
6*a^2*c^2*e - 4*b^3*c*d + 16*a*b*c^2*d - 20*a*b^2*c*e))/(64*a*c^4 - 16*b^2
*c^3))*(4*b^4*e + 16*a^2*c^2*e - 4*b^3*c*d + 16*a*b*c^2*d - 20*a*b^2*c*e))
/(2*(64*a*c^4 - 16*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(
8*c^3*(4*a*c - b^2)^(1/2)))*(4*b^4*e + 16*a^2*c^2*e - 4*b^3*c*d + 16*a*b*c
^2*d - 20*a*b^2*c*e))/(2*(64*a*c^4 - 16*b^2*c^3)) - ((((((448*b^4*c^10*d
- 448*b^5*c^9*e - 384*a*b^2*c^11*d + 832*a*b^3*c^10*e)/c^8 - (256*b^3*c...
```

Reduce [F]

$$\int \frac{x^{11}(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{x^{11}(e\,x^4+d)}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^11*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x^11*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.48 $\int \frac{x^7(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{x^7(d+ex^4)}{a+bx^4+cx^8}\,dx=\frac{ex^4}{4c}+\frac{(bcd-b^2e+2ace)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log(a+bx^4+cx^8)}{8c^2}$$

output
```
1/4*e*x^4/c+1/4*(2*a*c*e-b^2*e+b*c*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/
2))/c^2/(-4*a*c+b^2)^(1/2)+1/8*(-b*e+c*d)*ln(c*x^8+b*x^4+a)/c^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{x^7(d+ex^4)}{a+bx^4+cx^8}\,dx=\frac{2cex^4+\frac{2(-bcd+b^2e-2ace)\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(cd-be)\log(a+bx^4+cx^8)}{8c^2}$$

input
```
Integrate[(x^7*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(2*c*e*x^4 + (2*(-(b*c*d) + b^2*e - 2*a*c*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^
2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*d - b*e)*Log[a + b*x^4 + c*x^8])/(8*c
^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (d+ex^4)}{a+bx^4+cx^8} \, dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{4} \int \frac{x^4 (ex^4+d)}{cx^8+bx^4+a} dx^4$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{4} \int \left(\frac{e}{c} - \frac{ae-(cd-be)x^4}{c\,(cx^8+bx^4+a)}\right) dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)(2ace+b^2(-e)+bcd)}{c^2\sqrt{b^2-4ac}} + \frac{(cd-be)\log\left(a+bx^4+cx^8\right)}{2c^2} + \frac{ex^4}{c}\right)$$

input
```
Int[(x^7*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
((e*x^4)/c + ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4
*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*d - b*e)*Log[a + b*x^4 + c*x^8])/(2*
c^2))/4
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{e\,x^{4}}{4c}+\frac{\frac{\left(-eb+cd\right)\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{2c}+\frac{2\left(-ae-\frac{\left(-eb+cd\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{4c}$	98
risch	Expression too large to display	1400

input
```
int(x^7*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*e*x^4/c+1/4/c*(1/2*(-b*e+c*d)/c*ln(c*x^8+b*x^4+a)+2*(-a*e-1/2*(-b*e+c*
d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.14

$$\int \frac{x^7(d+ex^4)}{a+bx^4+cx^8}\,dx$$
$$= \left[\frac{2\,(b^2c-4\,ac^2)ex^4+(bcd-(b^2-2\,ac)e)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)+((b^2c-}{8\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate(x^7*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(2*(b^2*c - 4*a*c^2)*e*x^4 + (b*c*d - (b^2 - 2*a*c)*e)*sqrt(b^2 - 4*a
*c)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*
a*c))/(c*x^8 + b*x^4 + a)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log
(c*x^8 + b*x^4 + a))/(b^2*c^2 - 4*a*c^3), 1/8*(2*(b^2*c - 4*a*c^2)*e*x^4 +
 2*(b*c*d - (b^2 - 2*a*c)*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^4 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*
log(c*x^8 + b*x^4 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(92) = 184$.

Time = 38.90 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.47

$$\int \frac{x^7(d+ex^4)}{a+bx^4+cx^8}\,dx = \left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{8c^2\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{be-cd}{8c^2}\right)\log\left(x^4+\frac{-abe-16ac^2\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{8c^2\cdot(4ac-b^2)}-\frac{be-cd}{8c^2}\right)+2acd+4b^2c\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-}{8c^2\cdot(4ac-b^2}\right.}{2ace-b^2e+bcd}\right.$$
$$+\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{8c^2\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{be-cd}{8c^2}\right)\log\left(x^4+\frac{-abe-16ac^2\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{8c^2\cdot(4ac-b^2)}-\frac{be-cd}{8c^2}\right)+2acd+4b^2c\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+}{8c^2\cdot(4ac-b^2)}\right.}{2ace-b^2e+bcd}\right.$$
$$+\frac{ex^4}{4c}$$

input
```
integrate(x**7*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
(-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(8*c**2*(4*a*c - b**2)) -
 (b*e - c*d)/(8*c**2))*log(x**4 + (-a*b*e - 16*a*c**2*(-sqrt(-4*a*c + b**2
)*(2*a*c*e - b**2*e + b*c*d)/(8*c**2*(4*a*c - b**2)) - (b*e - c*d)/(8*c**2
)) + 2*a*c*d + 4*b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(
8*c**2*(4*a*c - b**2)) - (b*e - c*d)/(8*c**2)))/(2*a*c*e - b**2*e + b*c*d)
) + (sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(8*c**2*(4*a*c - b**2)
) - (b*e - c*d)/(8*c**2))*log(x**4 + (-a*b*e - 16*a*c**2*(sqrt(-4*a*c + b*
*2)*(2*a*c*e - b**2*e + b*c*d)/(8*c**2*(4*a*c - b**2)) - (b*e - c*d)/(8*c*
*2)) + 2*a*c*d + 4*b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/
(8*c**2*(4*a*c - b**2)) - (b*e - c*d)/(8*c**2)))/(2*a*c*e - b**2*e + b*c*d
)) + e*x**4/(4*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(d + ex^4)}{a + bx^4 + cx^8}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{x^7(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{ex^4}{4\,c} + \frac{(cd-be)\log\left(cx^8+bx^4+a\right)}{8\,c^2} - \frac{(bcd-b^2e+2\,ace)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x^7*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/4*e*x^4/c + 1/8*(c*d - b*e)*log(c*x^8 + b*x^4 + a)/c^2 - 1/4*(b*c*d - b^
2*e + 2*a*c*e)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c
)*c^2)
```

Mupad [B] (verification not implemented)

Time = 24.31 (sec) , antiderivative size = 6587, normalized size of antiderivative = 67.91

$$\int \frac{x^7(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int((x^7*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

output
```
(e*x^4)/(4*c) + (log(a + b*x^4 + c*x^8)*(4*b^3*e + 16*a*c^2*d - 4*b^2*c*d
- 16*a*b*c*e))/(2*(64*a*c^3 - 16*b^2*c^2)) - (atan((8*c^4*x^4*(4*a*c - b^2
)^2*(((a*c - b^2)*((((((((448*b^3*c^7*d - 448*b^4*c^6*e + 384*a*b^2*c^7*e)
/c^4 - (256*b^3*c^4*(4*b^3*e + 16*a*c^2*d - 4*b^2*c*d - 16*a*b*c*e))/(64*a
*c^3 - 16*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/(8*c^2*(4*a*c - b^2)^(1/2))
 - (32*b^3*c^2*(2*a*c*e - b^2*e + b*c*d)*(4*b^3*e + 16*a*c^2*d - 4*b^2*c*d
 - 16*a*b*c*e))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2)))*(4*b^3*e +
16*a*c^2*d - 4*b^2*c*d - 16*a*b*c*e))/(2*(64*a*c^3 - 16*b^2*c^2)) - (((144
*b^3*c^6*d^2 + 144*b^5*c^4*e^2 - 240*a*b^3*c^5*e^2 + 96*a^2*b*c^6*e^2 - 28
8*b^4*c^5*d*e + 240*a*b^2*c^6*d*e)/c^4 - (((448*b^3*c^7*d - 448*b^4*c^6*e
+ 384*a*b^2*c^7*e)/c^4 - (256*b^3*c^4*(4*b^3*e + 16*a*c^2*d - 4*b^2*c*d -
16*a*b*c*e))/(64*a*c^3 - 16*b^2*c^2))*(4*b^3*e + 16*a*c^2*d - 4*b^2*c*d -
16*a*b*c*e))/(2*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c*e - b^2*e + b*c*d))/(8*c^
2*(4*a*c - b^2)^(1/2)))*(4*b^3*e + 16*a*c^2*d - 4*b^2*c*d - 16*a*b*c*e))/(
2*(64*a*c^3 - 16*b^2*c^2)) - (((((((448*b^3*c^7*d - 448*b^4*c^6*e + 384*a*
b^2*c^7*e)/c^4 - (256*b^3*c^4*(4*b^3*e + 16*a*c^2*d - 4*b^2*c*d - 16*a*b*c
*e))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/(8*c^2*(4*a*c - b
^2)^(1/2)) - (32*b^3*c^2*(2*a*c*e - b^2*e + b*c*d)*(4*b^3*e + 16*a*c^2*d -
 4*b^2*c*d - 16*a*b*c*e))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2)))*(
2*a*c*e - b^2*e + b*c*d))/(8*c^2*(4*a*c - b^2)^(1/2)) - (4*b^3*(2*a*c*e...
```

Reduce [F]

$$\int \frac{x^7(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{x^7(e\,x^4+d)}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^7*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x^7*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.49 $\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = -\frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{e\log\left(a+bx^4+cx^8\right)}{8c}$$

output
```
-1/4*(-b*e+2*c*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(
1/2)+1/8*e*ln(c*x^8+b*x^4+a)/c
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{-\frac{2(-2cd+be)\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + e\log\left(a+bx^4+cx^8\right)}{8c}$$

input
```
Integrate[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
4*a*c] + e*Log[a + b*x^4 + c*x^8])/(8*c)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1798, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (d+ex^4)}{a+bx^4+cx^8} \, dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{4} \int \frac{ex^4+d}{cx^8+bx^4+a} dx^4$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4} \left(\frac{(2cd-be) \int \frac{1}{cx^8+bx^4+a} dx^4}{2c} + \frac{e \int \frac{2cx^4+b}{cx^8+bx^4+a} dx^4}{2c} \right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4} \left(\frac{e \int \frac{2cx^4+b}{cx^8+bx^4+a} dx^4}{2c} - \frac{(2cd-be) \int \frac{1}{-x^8+b^2-4ac} d(2cx^4+b)}{c} \right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4} \left(\frac{e \int \frac{2cx^4+b}{cx^8+bx^4+a} dx^4}{2c} - \frac{(2cd-be) \text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} \right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4} \left(\frac{e \log (a+bx^4+cx^8)}{2c} - \frac{(2cd-be) \text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} \right)$$

input
```
Int[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(-(((2*c*d - b*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 
4*a*c])) + (e*Log[a + b*x^4 + c*x^8])/(2*c))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

method	result
default	$\frac{e\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{8c}+\frac{\left(d-\frac{eb}{2c}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{2\sqrt{4ac-b^{2}}}$
risch	$\frac{\ln\left(\left(-4abce+8a\,c^{2}d+b^{3}e-2b^{2}cd+\sqrt{-\left(eb-2cd\right)^{2}\left(4ac-b^{2}\right)}\,b\right)x^{4}+2\sqrt{-\left(eb-2cd\right)^{2}\left(4ac-b^{2}\right)}\,a\right)ae}{8ac-2b^{2}}-\frac{\ln\left(\left(-4abce+8a\,c^{2}d+b^{3}e-\right.\right.}{}$

input
```
int(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*e*ln(c*x^8+b*x^4+a)/c+1/2*(d-1/2*e*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x^4+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.00

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \left[\frac{(b^2-4\,ac)e\log\left(cx^8+bx^4+a\right)-\sqrt{b^2-4\,ac}(2\,cd-be)\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{8\left(b^2c-4\,ac^2\right)}, \frac{(b^2-$$

input
```
integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*((b^2 - 4*a*c)*e*log(c*x^8 + b*x^4 + a) - sqrt(b^2 - 4*a*c)*(2*c*d -
b*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4
*a*c))/(c*x^8 + b*x^4 + a)))/(b^2*c - 4*a*c^2), 1/8*((b^2 - 4*a*c)*e*log(c
*x^8 + b*x^4 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^4 +
b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(63) = 126$.

Time = 10.55 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.99

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \left(\frac{e}{8c} - \frac{\sqrt{-4ac+b^2}(be-2cd)}{8c\left(4ac-b^2\right)}\right)\log\left(x^4+\frac{-16ac\left(\frac{e}{8c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)+2ae+4b^2\left(\frac{e}{8c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)}{be-2cd}\right.$$
$$+\left(\frac{e}{8c} + \frac{\sqrt{-4ac+b^2}(be-2cd)}{8c\left(4ac-b^2\right)}\right)\log\left(x^4+\frac{-16ac\left(\frac{e}{8c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)+2ae+4b^2\left(\frac{e}{8c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)}{be-2cd}\right.$$

input
```
integrate(x**3*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2)))*log(x**
4 + (-16*a*c*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b*
*2))) + 2*a*e + 4*b**2*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(
4*a*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(8*c) + sqrt(-4*a*c + b**2)*(b*
e - 2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4 + (-16*a*c*(e/(8*c) + sqrt(-4*a*
c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) + 2*a*e + 4*b**2*(e/(8*c) +
sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*
d))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{e\log\left(cx^8+bx^4+a\right)}{8\,c} + \frac{(2\,cd-be)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}c}$$

input
```
integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/8*e*log(c*x^8 + b*x^4 + a)/c + 1/4*(2*c*d - b*e)*arctan((2*c*x^4 + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

Mupad [B] (verification not implemented)

Time = 23.41 (sec) , antiderivative size = 3704, normalized size of antiderivative = 51.44

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

output
```
- (log(a + b*x^4 + c*x^8)*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c))
- (atan((8*x^4*(((a*c - b^2)*(((((4*b^2*e - 16*a*c*e)*(((b*e - 2*c*d)*(448
*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2
- 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e)
*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2
 - 16*b^2*c)) + ((b*e - 2*c*d)*(96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(448*
b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 -
 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e
))/(8*c*(4*a*c - b^2)^(1/2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*
c)) - ((((b*e - 2*c*d)*(((b*e - 2*c*d)*(448*b^3*c^3*e - 384*b^2*c^4*d + (2
56*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2
)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16
*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) + (4*b^3*c^2*(4*b
^2*e - 16*a*c*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)))*(
b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1/2)) + ((b*e - 2*c*d)*(((4*b^2*e - 16*a
*c*e)*(96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d
 + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2
 - 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*
c)) - 8*c^4*d^3 + 20*b^3*c*e^3 - 48*b^2*c^2*d*e^2 + 36*b*c^3*d^2*e))/(8*c*
(4*a*c - b^2)^(1/2)) - (b^3*c*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^3)/(2*...
```

Reduce [F]

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{x^3(e\,x^4+d)}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.50 $\int \frac{d+ex^4}{x(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{d+ex^4}{x(a+bx^4+cx^8)}\,dx=\frac{(bd-2ae)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}$$

output
```
1/4*(-2*a*e+b*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1
/2)+d*ln(x)/a-1/8*d*ln(c*x^8+b*x^4+a)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int \frac{d+ex^4}{x(a+bx^4+cx^8)}\,dx=\frac{d\log(x)}{a}-\frac{\operatorname{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b+2c\#1^4}\&\right]}{4a}$$

input
```
Integrate[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

output
```
(d*Log[x])/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[
x - #1] + c*d*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{4}\int \frac{ex^4+d}{x^4\left(cx^8+bx^4+a\right)}dx^4$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{4}\int \left(\frac{d}{ax^4}+\frac{-cdx^4-bd+ae}{a\left(cx^8+bx^4+a\right)}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^4+cx^8\right)}{2a}+\frac{d\log\left(x^4\right)}{a}\right)$$

input
```
Int[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

output
```
(((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x^4])/a - (d*Log[a + b*x^4 + c*x^8])/(2*a))/4
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

method	result
default	$\frac{-\frac{d \ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4}+\frac{\left(a e-\frac{b d}{2}\right) \arctan\left(\frac{2 c\,x^{4}+b}{\sqrt{4 a c-b^{2}}}\right)}{\sqrt{4 a c-b^{2}}}}{2 a}+\frac{d \ln\left(x\right)}{a}$
risch	$\frac{d \ln\left(x\right)}{a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4 a^{2} c-b^{2} a\right)\_Z^{2}+\left(4 a c d-d\,b^{2}\right)\_Z+a\,e^{2}-b d e+c\,d^{2}\right)}\_R \ln\left(\left(\left(18 a c-5 b^{2}\right)\_R^{2}+\left(-e b+9 c d\right)\_R+4 e^{2}\right) x^{4}\right.\right.}{4}$

input
```
int((e*x^4+d)/x/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-1/4*d*ln(c*x^8+b*x^4+a)+(a*e-1/2*b*d)/(4*a*c-b^2)^(1/2)*arctan((2*
c*x^4+b)/(4*a*c-b^2)^(1/2)))+d*ln(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.08

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx$$
$$= \left[-\frac{(b^2-4\,ac)d\log\left(cx^8+bx^4+a\right)-8\left(b^2-4\,ac\right)d\log\left(x\right)+\sqrt{b^2-4\,ac}(bd-2\,ae)\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2}{cx^8+}\right.}{8\left(ab^2-4\,a^2c\right)}\right.$$
$$-\frac{(b^2-4\,ac)d\log\left(cx^8+bx^4+a\right)-8\left(b^2-4\,ac\right)d\log\left(x\right)-2\sqrt{-b^2+4\,ac}(bd-2\,ae)\arctan\left(-\frac{(2\,cx^4+b)}{b^2-}\right.}{8\left(ab^2-4\,a^2c\right)}$$

input
```
integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[-1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) +
 sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c
- (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)))/(a*b^2 - 4*a^2*c)
, -1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x)
- 2*sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a
*c)/(b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/x/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^4}{x\,(a+bx^4+cx^8)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int \frac{d+ex^4}{x\,(a+bx^4+cx^8)}\,dx = -\frac{d\log\left(cx^8+bx^4+a\right)}{8\,a}+\frac{d\log\left(x^4\right)}{4\,a} - \frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}a}$$

input
```
integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/8*d*log(c*x^8 + b*x^4 + a)/a + 1/4*d*log(x^4)/a - 1/4*(b*d - 2*a*e)*arc
tan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

Mupad [B] (verification not implemented)

Time = 24.51 (sec) , antiderivative size = 8454, normalized size of antiderivative = 108.38

$$\int \frac{d+ex^4}{x(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x)
```

output
```
(d*log(x))/a - (log(a + b*x^4 + c*x^8)*(4*b^2*d - 16*a*c*d))/(2*(16*a*b^2
- 64*a^2*c)) + (atan((128*a^5*x^4*(((c^4*e^5 - ((4*b^2*d - 16*a*c*d)*(11*b
*c^4*e^4 + 9*c^5*d*e^3 - ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4
*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))
/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c
^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432
*b^2*c^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^2*c^4*e^3 +
 108*b*c^5*d*e^2))/(2*(16*a*b^2 - 64*a^2*c))))/(2*(16*a*b^2 - 64*a^2*c)) -
 ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*d)*(((4*b^2*d
 - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) +
576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/
2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))
/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*
a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*
(2*a*e - b*d)^2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2*(16*a*
b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)
*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*
d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^
2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a
*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d))/(2*(16*a*b...
```

Reduce [F]

$$\int \frac{d+ex^4}{x(a+bx^4+cx^8)}\,dx = \int \frac{e\,x^4+d}{x\,(c\,x^8+b\,x^4+a)}dx$$

input
```
int((e*x^4+d)/x/(c*x^8+b*x^4+a),x)
```

output
```
int((e*x^4+d)/x/(c*x^8+b*x^4+a),x)
```

3.51 $\int \frac{d+ex^4}{x^5(a+bx^4+cx^8)} dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{d+ex^4}{x^5(a+bx^4+cx^8)} dx = -\frac{d}{4ax^4} - \frac{(b^2d-2acd-abe)\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}} - \frac{(bd-ae)\log(x)}{a^2} + \frac{(bd-ae)\log(a+bx^4+cx^8)}{8a^2}$$

output
```
-1/4*d/a/x^4-1/4*(-a*b*e-2*a*c*d+b^2*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(
1/2))/a^2/(-4*a*c+b^2)^(1/2)-(-a*e+b*d)*ln(x)/a^2+1/8*(-a*e+b*d)*ln(c*x^8+
b*x^4+a)/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.16

$$\int \frac{d+ex^4}{x^5(a+bx^4+cx^8)} dx = -\frac{d}{4ax^4} + \frac{(-bd+ae)\log(x)}{a^2} + \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b^2d\log(x-\#1)-acd\log(x-\#1)-abe\log(x-\#1)+bcd\log(x-\#1)\#1^4-ace\log(x-\#1)\#}{b+2c\#1^4}\right.}{4a^2}$$

input
```
Integrate[(d + e*x^4)/(x^5*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/4*d/(a*x^4) + ((-(b*d) + a*e)*Log[x])/a^2 + RootSum[a + b*#1^4 + c*#1^8
 & , (b^2*d*Log[x - #1] - a*c*d*Log[x - #1] - a*b*e*Log[x - #1] + b*c*d*Lo
g[x - #1]*#1^4 - a*c*e*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x^5\left(a+bx^4+cx^8\right)}dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{4}\int \frac{ex^4+d}{x^8\left(cx^8+bx^4+a\right)}dx^4$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{4}\int \left(\frac{d}{ax^8}+\frac{c(bd-ae)x^4+b^2d-acd-abe}{a^2\left(cx^8+bx^4+a\right)}+\frac{ae-bd}{a^2x^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)\left(-abe-2acd+b^2d\right)}{a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^4+cx^8\right)}{2a^2}-\frac{\log\left(x^4\right)(bd-ae)}{a^2}-\frac{d}{ax^4}\right)$$

input
```
Int[(d + e*x^4)/(x^5*(a + b*x^4 + c*x^8)),x]
```

output
```
(-(d/(a*x^4)) - ((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2
- 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x^4])/a^2 + ((b*d -
a*e)*Log[a + b*x^4 + c*x^8])/(2*a^2))/4
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.12

method	result
default	$-\frac{\frac{(ace-cbd)\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4c}+\frac{\left(abe+acd-d\,b^{2}-\frac{(ace-cbd)b}{2c}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{2}}-\frac{d}{4a\,x^{4}}+\frac{(ae-bd)\ln(x)}{a^{2}}$
risch	$-\frac{d}{4a\,x^{4}}+\frac{e\ln(x)}{a}-\frac{\ln(x)bd}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(4a^{2}ce-a\,b^{2}e-4abcd+b^{3}d\right)\_Z+ac\,e^{2}-bcde+c^{2}d^{2}\right)}\_R\ln\left(\left(\left((18\right.\right.\right.}{}$

input
```
int((e*x^4+d)/x^5/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2*(1/4*(a*c*e-b*c*d)/c*ln(c*x^8+b*x^4+a)+(a*b*e+a*c*d-d*b^2-1/2*(a*
c*e-b*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))-1
/4*d/a/x^4+(a*e-b*d)/a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 385, normalized size of antiderivative = 3.44

$$\int \frac{d+ex^4}{x^5(a+bx^4+cx^8)}\,dx$$
$$= \left[\frac{(abe-(b^2-2\,ac)d)\sqrt{b^2-4\,ac}x^4\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)+((b^3-4\,abc)d-(ab^2-4\,a}{8\,(a^2b^2-4\,a^3c}\right.$$

input
```
integrate((e*x^4+d)/x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*((a*b*e - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^8 + 2*b
*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a
)) + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^4*log(c*x^8 + b*x^4 + a)
- 8*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^4*log(x) - 2*(a*b^2 - 4*a^
2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^4), 1/8*(2*(a*b*e - (b^2 - 2*a*c)*d)*sqrt(-
b^2 + 4*a*c)*x^4*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^4*log(c*x^8 + b*x^4 + a) - 8*
((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^4*log(x) - 2*(a*b^2 - 4*a^2*c)
*d)/((a^2*b^2 - 4*a^3*c)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^5(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/x**5/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^4}{x^5(a+bx^4+cx^8)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^4+d)/x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.12

$$\int \frac{d+ex^4}{x^5(a+bx^4+cx^8)}\,dx = \frac{(bd-ae)\log(cx^8+bx^4+a)}{8a^2} - \frac{(bd-ae)\log(x^4)}{4a^2} + \frac{(b^2d-2acd-abe)\arctan\left(\frac{2cx^4+b}{\sqrt{-b^2+4ac}}\right)}{4\sqrt{-b^2+4ac}a^2} + \frac{bdx^4-aex^4-ad}{4a^2x^4}$$

input
```
integrate((e*x^4+d)/x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/8*(b*d - a*e)*log(c*x^8 + b*x^4 + a)/a^2 - 1/4*(b*d - a*e)*log(x^4)/a^2
+ 1/4*(b^2*d - 2*a*c*d - a*b*e)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(
sqrt(-b^2 + 4*a*c)*a^2) + 1/4*(b*d*x^4 - a*e*x^4 - a*d)/(a^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 43.40 (sec) , antiderivative size = 14100, normalized size of antiderivative = 125.89

$$\int \frac{d+ex^4}{x^5(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x^5*(a + b*x^4 + c*x^8)),x)
```

output
```
(log(x)*(a*e - b*d))/a^2 - d/(4*a*x^4) - (log(((((((((a*e - b*d + a^2*(-(a*
b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((((256*b^3*c^4*(a*b*
e - b^2*d + a*c*d))/a + (64*b^2*c^5*x^4*(7*b^2*d + 9*a*b*e - 54*a*c*d))/a
+ (32*b^3*c^4*(a*b + 5*b^2*x^4 - 18*a*c*x^4)*(a*e - b*d + a^2*(-(a*b*e - b
^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/a^2)*(a*e - b*d + a^2*(-(a*
b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(8*a^2) + (32*b^2*c^
5*d*(8*a*b*e - 8*b^2*d + 3*a*c*d))/a^2 - (16*b*c^6*d*x^4*(13*b^2*d - 27*a*
b*e + 54*a*c*d))/a^2))/(8*a^2) - (4*c^7*d^2*x^4*(31*b^2*d - 27*a*b*e + 18*
a*c*d))/a^3 + (16*b*c^6*d^2*(6*a*b*e - 6*b^2*d + a*c*d))/a^3)*(a*e - b*d +
 a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(8*a^2) +
(c^7*d^3*(16*a*b*e - 16*b^2*d + a*c*d))/a^4 + (c^8*d^3*x^4*(9*a*e - 20*b*d
))/a^4)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2))
)^(1/2)))/(8*a^2) + (c^8*d^4*(a*e - b*d))/a^5 - (c^9*d^5*x^4)/a^5)*((((c^7
*d^3*(16*a*b*e - 16*b^2*d + a*c*d))/a^4 - ((((b*d - a*e + a^2*(-(a*b*e - b
^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((((256*b^3*c^4*(a*b*e - b^2
*d + a*c*d))/a + (64*b^2*c^5*x^4*(7*b^2*d + 9*a*b*e - 54*a*c*d))/a - (32*b
^3*c^4*(a*b + 5*b^2*x^4 - 18*a*c*x^4)*(b*d - a*e + a^2*(-(a*b*e - b^2*d +
2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/a^2)*(b*d - a*e + a^2*(-(a*b*e - b
^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(8*a^2) - (32*b^2*c^5*d*(8*
a*b*e - 8*b^2*d + 3*a*c*d))/a^2 + (16*b*c^6*d*x^4*(13*b^2*d - 27*a*b*e ...
```

Reduce [F]

$$\int \frac{d+ex^4}{x^5(a+bx^4+cx^8)}\,dx = \int \frac{e\,x^4+d}{x^5\,(c\,x^8+b\,x^4+a)}dx$$

input
```
int((e*x^4+d)/x^5/(c*x^8+b*x^4+a),x)
```

output
```
int((e*x^4+d)/x^5/(c*x^8+b*x^4+a),x)
```

3.52 $\int \frac{d+ex^4}{x^9(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{d+ex^4}{x^9(a+bx^4+cx^8)}\,dx = -\frac{d}{8ax^8}+\frac{bd-ae}{4a^2x^4} + \frac{(b^3d-3abcd-ab^2e+2a^2ce)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^3\sqrt{b^2-4ac}} + \frac{(b^2d-acd-abe)\log(x)}{a^3} - \frac{(b^2d-acd-abe)\log(a+bx^4+cx^8)}{8a^3}$$

output
```
-1/8*d/a/x^8+1/4*(-a*e+b*d)/a^2/x^4+1/4*(2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d
)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)+(-a*b*e-a
*c*d+b^2*d)*ln(x)/a^3-1/8*(-a*b*e-a*c*d+b^2*d)*ln(c*x^8+b*x^4+a)/a^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.25

$$\int \frac{d+ex^4}{x^9\left(a+bx^4+cx^8\right)}\,dx=-\frac{d}{8ax^8}+\frac{bd-ae}{4a^2x^4}+\frac{(b^2d-acd-abe)\log(x)}{a^3}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^3d\log(x-\#1)-2abcd\log(x-\#1)-ab^2e\log(x-\#1)+a^2ce\log(x-\#1)+b^2cd\log(x-\#1)\#}{b+2c\#1^4}\right.}{4a^3}$$

input
```
Integrate[(d + e*x^4)/(x^9*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/8*d/(a*x^8) + (b*d - a*e)/(4*a^2*x^4) + ((b^2*d - a*c*d - a*b*e)*Log[x]
)/a^3 - RootSum[a + b*#1^4 + c*#1^8 & , (b^3*d*Log[x - #1] - 2*a*b*c*d*Log
[x - #1] - a*b^2*e*Log[x - #1] + a^2*c*e*Log[x - #1] + b^2*c*d*Log[x - #1]
*#1^4 - a*c^2*d*Log[x - #1]*#1^4 - a*b*c*e*Log[x - #1]*#1^4)/(b + 2*c*#1^4
) & ]/(4*a^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x^9\left(a+bx^4+cx^8\right)}dx$$

$$\Big\downarrow \ 1802$$

$$\frac{1}{4}\int \frac{ex^4+d}{x^{12}\left(cx^8+bx^4+a\right)}dx^4$$

$$\Big\downarrow \ 1200$$

$$\frac{1}{4}\int\left(\frac{d}{ax^{12}}+\frac{-c\left(db^2-aeb-acd\right)x^4-b^3d+2abcd+ab^2e-a^2ce}{a^3\left(cx^8+bx^4+a\right)}+\frac{db^2-aeb-acd}{a^3x^4}+\frac{ae-bd}{a^2x^8}\right)dx^4$$

$\Big\downarrow$ 2009

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)\left(-abe-acd+b^2d\right)}{a^3}-\frac{\left(-abe-acd+b^2d\right)\log\left(a+bx^4+cx^8\right)}{2a^3}+\frac{bd-ae}{a^2x^4}+\frac{\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)\left(2a\right.}{a^3\sqrt{b}}\right.$$

input
```
Int[(d + e*x^4)/(x^9*(a + b*x^4 + c*x^8)),x]
```

output
```
(-1/2*d/(a*x^8) + (b*d - a*e)/(a^2*x^4) + ((b^3*d - 3*a*b*c*d - a*b^2*e +
2*a^2*c*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c
]) + ((b^2*d - a*c*d - a*b*e)*Log[x^4])/a^3 - ((b^2*d - a*c*d - a*b*e)*Log
[a + b*x^4 + c*x^8])/(2*a^3))/4
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.15

method	result
default	$-\frac{\frac{\left(-abce-a\,c^{2}d+b^{2}cd\right)\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4c}+\frac{\left(a^{2}ce-a\,b^{2}e-2abcd+b^{3}d-\frac{\left(-abce-a\,c^{2}d+b^{2}cd\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{3}}-\frac{d}{8a\,x^{8}}-\frac{ae-bd}{4a^{2}x^{4}}$
risch	$\frac{-\frac{(ae-bd)x^{4}}{4a^{2}}-\frac{d}{8a}}{x^{8}}-\frac{\ln(x)be}{a^{2}}-\frac{\ln(x)cd}{a^{2}}+\frac{\ln(x)d\,b^{2}}{a^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{4}c-a^{3}b^{2}\right)\_Z^{2}+\left(-4a^{2}bce-4a^{2}c^{2}d+a\,b^{3}e+5a\,b^{2}cd-b^{4}d\right)\_\right.}\right.}{}$

input
```
int((e*x^4+d)/x^9/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^3*(1/4*(-a*b*c*e-a*c^2*d+b^2*c*d)/c*ln(c*x^8+b*x^4+a)+(a^2*c*e-a*b^
2*e-2*a*b*c*d+b^3*d-1/2*(-a*b*c*e-a*c^2*d+b^2*c*d)*b/c)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))-1/8*d/a/x^8-1/4*(a*e-b*d)/a^2/x^4+(
-a*b*e-a*c*d+b^2*d)*ln(x)/a^3
```

Fricas [A] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 533, normalized size of antiderivative = 3.44

$$\int \frac{d+ex^4}{x^9\left(a+bx^4+cx^8\right)}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}((b^3-3\,abc)d-(ab^2-2\,a^2c)e)x^8\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-((b^4-5\,ab^2c+}{}\right.$$

input
```
integrate((e*x^4+d)/x^9/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(b^2 - 4*a*c)*((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^8*log(
(2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c
*x^8 + b*x^4 + a)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c
)*e)*x^8*log(c*x^8 + b*x^4 + a) + 8*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*
b^3 - 4*a^2*b*c)*e)*x^8*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a
^3*c)*e)*x^4 - (a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^8), 1/8*(2*sq
rt(-b^2 + 4*a*c)*((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^8*arctan(-(2*
c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4 - 5*a*b^2*c + 4*a^2*c
^2)*d - (a*b^3 - 4*a^2*b*c)*e)*x^8*log(c*x^8 + b*x^4 + a) + 8*((b^4 - 5*a*
b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e)*x^8*log(x) + 2*((a*b^3 - 4*a
^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^4 - (a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2
- 4*a^4*c)*x^8)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^9\left(a+bx^4+cx^8\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/x**9/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^4}{x^9\left(a+bx^4+cx^8\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^4+d)/x^9/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.17

$$\int \frac{d+ex^4}{x^9\left(a+bx^4+cx^8\right)}\,dx = -\frac{\left(b^2d-acd-abe\right)\log\left(cx^8+bx^4+a\right)}{8\,a^3} + \frac{\left(b^2d-acd-abe\right)\log\left(x^4\right)}{4\,a^3} - \frac{\left(b^3d-3\,abcd-ab^2e+2\,a^2ce\right)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}a^3} - \frac{3\,b^2dx^8-3\,acdx^8-3\,abex^8-2\,abdx^4+2\,a^2ex^4+a^2d}{8\,a^3x^8}$$

input
```
integrate((e*x^4+d)/x^9/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/8*(b^2*d - a*c*d - a*b*e)*log(c*x^8 + b*x^4 + a)/a^3 + 1/4*(b^2*d - a*c
*d - a*b*e)*log(x^4)/a^3 - 1/4*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*a
rctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^3) - 1/8*(3*
b^2*d*x^8 - 3*a*c*d*x^8 - 3*a*b*e*x^8 - 2*a*b*d*x^4 + 2*a^2*e*x^4 + a^2*d)
/(a^3*x^8)
```

Mupad [B] (verification not implemented)

Time = 63.28 (sec) , antiderivative size = 19687, normalized size of antiderivative = 127.01

$$\int \frac{d+ex^4}{x^9\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x^9*(a + b*x^4 + c*x^8)),x)
```

output
```
(log((((((((((64*b^2*c^5*x^4*(7*b^3*d - 7*a*b^2*e + 54*a^2*c*e - 45*a*b*c
*d))/a^2 - (256*b^3*c^4*(b^3*d - a*b^2*e + a^2*c*e - 2*a*b*c*d))/a^2 + (32
*b^3*c^4*(a*b + 5*b^2*x^4 - 18*a*c*x^4)*(a^3*(-(b^3*d - a*b^2*e + 2*a^2*c*
e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - b^2*d + a*b*e + a*c*d))/a^3)
*(a^3*(-(b^3*d - a*b^2*e + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(
1/2) - b^2*d + a*b*e + a*c*d))/(8*a^3) + (32*b^2*c^5*(a*e - b*d)*(8*b^3*d
- 8*a*b^2*e + 3*a^2*c*e - 11*a*b*c*d))/a^4 + (16*b*c^6*x^4*(a*e - b*d)*(13
*b^3*d - 13*a*b^2*e - 54*a^2*c*e + 27*a*b*c*d))/a^4)*(a^3*(-(b^3*d - a*b^2
*e + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - b^2*d + a*b*e +
 a*c*d))/(8*a^3) + (4*c^7*x^4*(a*e - b*d)^2*(31*a*b^2*e - 31*b^3*d + 18*a^
2*c*e + 9*a*b*c*d))/a^6 - (16*b*c^6*(a*e - b*d)^2*(6*b^3*d - 6*a*b^2*e + a
^2*c*e - 7*a*b*c*d))/a^6)*(a^3*(-(b^3*d - a*b^2*e + 2*a^2*c*e - 3*a*b*c*d)
^2/(a^6*(4*a*c - b^2)))^(1/2) - b^2*d + a*b*e + a*c*d))/(8*a^3) + (c^7*(a*
e - b*d)^3*(16*b^3*d - 16*a*b^2*e + a^2*c*e - 17*a*b*c*d))/a^8 - (c^8*x^4*
(a*e - b*d)^3*(20*a*b*e - 20*b^2*d + 9*a*c*d))/a^8)*(a^3*(-(b^3*d - a*b^2*
e + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - b^2*d + a*b*e +
a*c*d))/(8*a^3) + (c^9*x^4*(a*e - b*d)^5)/a^10 + (c^8*(a*e - b*d)^4*(a*b*e
 - b^2*d + a*c*d))/a^10)*((c^9*x^4*(a*e - b*d)^5)/a^10 - ((((((((256*b^3*
c^4*(b^3*d - a*b^2*e + a^2*c*e - 2*a*b*c*d))/a^2 - (64*b^2*c^5*x^4*(7*b^3*
d - 7*a*b^2*e + 54*a^2*c*e - 45*a*b*c*d))/a^2 + (32*b^3*c^4*(a*b + 5*b^...
```

Reduce [F]

$$\int \frac{d+ex^4}{x^9\left(a+bx^4+cx^8\right)}\,dx=\int \frac{e\,x^4+d}{x^9\left(c\,x^8+b\,x^4+a\right)}dx$$

input
```
int((e*x^4+d)/x^9/(c*x^8+b*x^4+a),x)
```

output
```
int((e*x^4+d)/x^9/(c*x^8+b*x^4+a),x)
```

3.53 $\int \frac{x^9(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 273

$$\int \frac{x^9(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{(cd-be)x^2}{2c^2}+\frac{ex^6}{6c} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*(-b*e+c*d)*x^2/c^2+1/6*e*x^6/c-1/4*(b*c*d-b^2*e+a*c*e-(3*a*b*c*e-2*a*c
^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b-(-4*
a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*(b
*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*a
rctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(b
+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.22

$$\int \frac{x^9(d+ex^4)}{a+bx^4+cx^8}\,dx$$
$$= \frac{6\sqrt{c}(cd-be)x^2+2c^{3/2}ex^6+\frac{3\sqrt{2}\left(-b^3e+bc\left(-\sqrt{b^2-4ac}d+3ae\right)+b^2\left(cd+\sqrt{b^2-4ace}\right)-ac\left(2cd+\sqrt{b^2-4ace}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4a}}}\right.}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{12c^{5/2}}$$

input
```
Integrate[(x^9*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(6*Sqrt[c]*(c*d - b*e)*x^2 + 2*c^(3/2)*e*x^6 + (3*Sqrt[2]*(-(b^3*e) + b*c*
(-(Sqrt[b^2 - 4*a*c]*d) + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*c*(
2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2
]*(b^3*e - b*c*(Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a
*c]*e) + b^2*(-(c*d) + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(12*c^(5/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1814, 1602, 27, 1602, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9(d+ex^4)}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1814}$$

$$\frac{1}{2}\int \frac{x^8(ex^4+d)}{cx^8+bx^4+a}dx^2$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{2}\left(\frac{ex^6}{3c}-\frac{\int \frac{3x^4\left(ae-(cd-be)x^4\right)}{cx^8+bx^4+a}dx^2}{3c}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(\frac{ex^6}{3c}-\frac{\int \frac{x^4\left(ae-(cd-be)x^4\right)}{cx^8+bx^4+a}dx^2}{c}\right)$$

$\Big\downarrow$ 1602

$$\frac{1}{2}\left(\frac{ex^6}{3c}-\frac{-\frac{\int -\frac{\left(-eb^2+cdb+ace\right)x^4+a(cd-be)}{cx^8+bx^4+a}dx^2}{c}-\frac{x^2(cd-be)}{c}}{c}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{ex^6}{3c}-\frac{\frac{\int \frac{\left(-eb^2+cdb+ace\right)x^4+a(cd-be)}{cx^8+bx^4+a}dx^2}{c}-\frac{x^2(cd-be)}{c}}{c}\right)$$

$\Big\downarrow$ 1480

$$\frac{1}{2}\left(\frac{ex^6}{3c}-\frac{\frac{\frac{1}{2}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{c}}{c}\right.$$

$\Big\downarrow$ 218

$$\frac{1}{2}\left(\frac{ex^6}{3c}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^{\cdots}\right.}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{c}\right.$$

input
```
Int[(x^9*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
((e*x^6)/(3*c) - (-(((c*d - b*e)*x^2)/c) + (((b*c*d - b^2*e + a*c*e - (b^2
*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) + ((b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3
*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/c)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.09

method	result
default	$-\frac{-\frac{1}{3}ce\,x^{6}+x^{2}eb-cd\,x^{2}}{2c^{2}}+\frac{-\frac{\left(-ace\sqrt{-4ac+b^{2}}+b^{2}e\sqrt{-4ac+b^{2}}-cbd\sqrt{-4ac+b^{2}}+3abce-2a\,c^{2}d-b^{3}e+b^{2}cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^{2}\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b}\right.}}\right.}{4c\sqrt{-4ac+b^{2}}\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}}{}$
risch	$\frac{e\,x^{6}}{6c}-\frac{x^{2}eb}{2c^{2}}+\frac{d\,x^{2}}{2c}+\frac{\_R=\operatorname{RootOf}\left(\left(16a^{2}c^{3}-8a\,b^{2}c^{2}+c\,b^{4}\right)\_Z^{4}+\left(-20a^{3}b\,c^{3}e^{2}+16a^{3}c^{4}de+25a^{2}b^{3}c^{2}e^{2}-36a^{2}c^{3}b^{2}de+12b\,c^{4}a^{2}d^{2}-9a\right.\right.}{}$

input
```
int(x^9*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2*(-1/3*c*e*x^6+x^2*e*b-c*d*x^2)+2/c*(-1/8*(-a*c*e*(-4*a*c+b^2)^(1/
2)+b^2*e*(-4*a*c+b^2)^(1/2)-c*b*d*(-4*a*c+b^2)^(1/2)+3*a*b*c*e-2*a*c^2*d-b
^3*e+b^2*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-a*c*e*(-
4*a*c+b^2)^(1/2)+b^2*e*(-4*a*c+b^2)^(1/2)-c*b*d*(-4*a*c+b^2)^(1/2)-3*a*b*c
*e+2*a*c^2*d+b^3*e-b^2*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4744 vs. 2(229) = 458.

Time = 2.56 (sec) , antiderivative size = 4744, normalized size of antiderivative = 17.38

$$\int \frac{x^{9}(d+ex^{4})}{a+bx^{4}+cx^{8}}\,dx = \text{Too large to display}$$

input
```
integrate(x^9*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x**9*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^9(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{(ex^4+d)x^9}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^9*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
1/6*(c*e*x^6 + 3*(c*d - b*e)*x^2)/c^2 - integrate(((b*c*d - (b^2 - a*c)*e)
*x^4 + a*c*d - a*b*e)*x/(c*x^8 + b*x^4 + a), x)/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4524 vs. $2(229) = 458$.

Time = 1.28 (sec) , antiderivative size = 4524, normalized size of antiderivative = 16.57

$$\int \frac{x^9(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^9*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b
^2 - 4*a*c)*a*b*c^3)*d*x^4*abs(c) - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^6 - 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2
- 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 - 48*a^2*b^2*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*c^4 + 32*a^3*c^4 + 2*(b^2 - 4*a*c)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2
+ 8*(b^2 - 4*a*c)*a^2*c^3)*e*x^4*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*...
```

Mupad [B] (verification not implemented)

Time = 27.39 (sec) , antiderivative size = 26868, normalized size of antiderivative = 98.42

$$\int \frac{x^9(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int((x^9*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

output
```
x^2*(d/(2*c) - (b*e)/(2*c^2)) + atan(-((((16*(a*b^12*e^4 + 4*a^5*c^8*d^4 +
 4*a^7*c^6*e^4 + a*b^8*c^4*d^4 - 12*a^2*b^10*c*e^4 - 8*a^2*b^6*c^5*d^4 + 2
0*a^3*b^4*c^6*d^4 - 16*a^4*b^2*c^7*d^4 + 54*a^3*b^8*c^2*e^4 - 112*a^4*b^6*
c^3*e^4 + 105*a^5*b^4*c^4*e^4 - 36*a^6*b^2*c^5*e^4 - 8*a^6*c^7*d^2*e^2 - 4
*a*b^11*c*d*e^3 - 60*a^2*b^8*c^3*d^2*e^2 + 210*a^3*b^6*c^4*d^2*e^2 - 300*a
^4*b^4*c^5*d^2*e^2 + 152*a^5*b^2*c^6*d^2*e^2 - 4*a*b^9*c^3*d^3*e - 40*a^5*
b*c^7*d^3*e + 40*a^6*b*c^6*d*e^3 + 6*a*b^10*c^2*d^2*e^2 + 36*a^2*b^7*c^4*d
^3*e + 44*a^2*b^9*c^2*d*e^3 - 108*a^3*b^5*c^5*d^3*e - 176*a^3*b^7*c^3*d*e^
3 + 120*a^4*b^3*c^6*d^3*e + 308*a^4*b^5*c^4*d*e^3 - 220*a^5*b^3*c^5*d*e^3)
)/c^6 + ((((4*x^2*(256*a^2*b^5*c^9*e - 2048*a^3*b^3*c^10*e + 4096*a^4*b*c^
11*e))/c^6 + (16*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1
/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*
c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2)*(256*a*b^6*c^10 - 2048*a^2*b^4*c^11 + 4096*a^3*b^2*c^12))/c^6)*(-
(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^
2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^...
```

Reduce [F]

$$\int \frac{x^9(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{x^9(e\,x^4+d)}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^9*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x^9*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.54 $\int \frac{x^5(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 221

$$\int \frac{x^5(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{ex^2}{2c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
1/2*e*x^2/c+1/4*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*arctan(
2^(1/2)*c^(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2)+1/4*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2)
)*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.17

$$\int \frac{x^5(d+ex^4)}{a+bx^4+cx^8}\,dx$$
$$= \frac{2\sqrt{c}ex^2 - \frac{\sqrt{2}\left(bcd-c\sqrt{b^2-4ac}d-b^2e+2ace+b\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(-bcd-c\sqrt{b^2-4ac}d+b^2e-2ace+b\sqrt{b^2-4ac}e\right)\text{ar}}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

input
```
Integrate[(x^5*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(2*Sqrt[c]*e*x^2 - (Sqrt[2]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c
*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(-
(b*c*d) - c*Sqrt[b^2 - 4*a*c]*d + b^2*e - 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)
*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1814, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(d+ex^4)}{a+bx^4+cx^8}\,dx$$
$$\Big\downarrow \text{1814}$$
$$\frac{1}{2}\int \frac{x^4(ex^4+d)}{cx^8+bx^4+a}dx^2$$
$$\Big\downarrow \text{1602}$$

$$\frac{1}{2}\left(\frac{ex^2}{c}-\frac{\int \frac{ae-(cd-be)x^4}{cx^8+bx^4+a}dx^2}{c}\right)$$

$\downarrow$ 1480

$$\frac{1}{2}\left(\frac{ex^2}{c}-\frac{-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2-\frac{1}{2}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{}\right.}}{c}\right.$$

$\downarrow$ 218

$$\frac{1}{2}\left(\frac{ex^2}{c}-\frac{-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}\right)$$

input
```
Int[(x^5*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
((e*x^2)/c - (-((((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[
c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.99

method	result
default	$\frac{e\,x^2}{2c}-\frac{\left(-eb\sqrt{-4ac+b^2}+cd\sqrt{-4ac+b^2}-2ace+b^2e-cbd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-eb\sqrt{-4ac+b^2}+cd\sqrt{-4ac+b\right.}{4c\sqrt{-4}}$
risch	$\frac{e\,x^2}{2c}+\frac{\sum\_R=\mathrm{RootOf}\left(\left(16a^2c^3-8a\,b^2c^2+c\,b^4\right)\_Z^4+\left(12a^2c^2e^2b-16c^3de\,a^2-7ac\,e^2b^3+12b^2c^2dea-4b\,c^3d^2a+e^2b^5-2dc\,b^4e+b^3c^2d^2\right)\_Z^2+c^2}{}$

input
```
int(x^5*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*e*x^2/c-1/4*(-e*b*(-4*a*c+b^2)^(1/2)+c*d*(-4*a*c+b^2)^(1/2)-2*a*c*e+b^
2*e-c*b*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/4*(-e*b*(-4*a*c
+b^2)^(1/2)+c*d*(-4*a*c+b^2)^(1/2)+2*a*c*e-b^2*e+c*b*d)/c/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2520 vs. $2(179) = 358$.

Time = 0.36 (sec) , antiderivative size = 2520, normalized size of antiderivative = 11.40

$$\int \frac{x^5(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(2*e*x^2 - sqrt(1/2)*c*sqrt(-(b*c^2*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b
^3 - 3*a*b*c)*e^2 + (b^2*c^3 - 4*a*c^4)*sqrt((c^4*d^4 - 4*b*c^3*d^3*e + 2*
(3*b^2*c^2 - a*c^3)*d^2*e^2 - 4*(b^3*c - a*b*c^2)*d*e^3 + (b^4 - 2*a*b^2*c
 + a^2*c^2)*e^4)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(c^3*d^4
- 3*b*c^2*d^3*e + 3*b^2*c*d^2*e^2 - (b^3 + a*b*c)*d*e^3 + (a*b^2 - a^2*c)*
e^4)*x^2 + 1/2*sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*d^2*e - 2*(b^3*c - 4*a*b*c^2
)*d*e^2 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^3 + (2*(b^2*c^4 - 4*a*c^5)*d - (
b^3*c^3 - 4*a*b*c^4)*e)*sqrt((c^4*d^4 - 4*b*c^3*d^3*e + 2*(3*b^2*c^2 - a*c
^3)*d^2*e^2 - 4*(b^3*c - a*b*c^2)*d*e^3 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^4)
/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b*c^2*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^3 -
 3*a*b*c)*e^2 + (b^2*c^3 - 4*a*c^4)*sqrt((c^4*d^4 - 4*b*c^3*d^3*e + 2*(3*b
^2*c^2 - a*c^3)*d^2*e^2 - 4*(b^3*c - a*b*c^2)*d*e^3 + (b^4 - 2*a*b^2*c + a
^2*c^2)*e^4)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqr
t(-(b*c^2*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^3 - 3*a*b*c)*e^2 + (b^2*c^3 -
 4*a*c^4)*sqrt((c^4*d^4 - 4*b*c^3*d^3*e + 2*(3*b^2*c^2 - a*c^3)*d^2*e^2 -
4*(b^3*c - a*b*c^2)*d*e^3 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^4)/(b^2*c^6 - 4*
a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(c^3*d^4 - 3*b*c^2*d^3*e + 3*b^2*c*d^2*
e^2 - (b^3 + a*b*c)*d*e^3 + (a*b^2 - a^2*c)*e^4)*x^2 - 1/2*sqrt(1/2)*((b^2
*c^2 - 4*a*c^3)*d^2*e - 2*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*b^2*c + 4
*a^2*c^2)*e^3 + (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqr...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{(ex^4+d)x^5}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^5*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*e*x^2/c - integrate(-((c*d - b*e)*x^4 - a*e)*x/(c*x^8 + b*x^4 + a), x)
/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2966 vs. $2(179) = 358$.

Time $= 1.27$ (sec) , antiderivative size $= 2966$, normalized size of antiderivative $= 13.42$

$$\int \frac{x^5(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/2*e*x^2/c + 1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^2 - 2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 16*a*b^2*c^3 - 4*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 32*a^2*c^4 + 2*(b^2 - 4*a*c)*b^2*c^2 -
 8*(b^2 - 4*a*c)*a*c^3)*d*x^4*abs(c) - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2
- 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^
2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*e*x^4*abs(c) + (2*b^3*c^4 - 8*
a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a
*c)*b*c^4)*d*x^4 - (2*b^4*c^3 - 8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*...
```

Mupad [B] (verification not implemented)

Time = 27.96 (sec) , antiderivative size = 17577, normalized size of antiderivative = 79.53

$$\int \frac{x^5(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int((x^5*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

output
```
(e*x^2)/(2*c) - atan((((((16*(a*b^8*e^4 + 4*a^3*c^6*d^4 + 4*a^5*c^4*e^4 + a
*b^4*c^4*d^4 - 8*a^2*b^6*c*e^4 - 4*a^2*b^2*c^5*d^4 + 20*a^3*b^4*c^2*e^4 -
16*a^4*b^2*c^3*e^4 - 8*a^4*c^5*d^2*e^2 - 4*a*b^7*c*d*e^3 - 36*a^2*b^4*c^3*
d^2*e^2 + 56*a^3*b^2*c^4*d^2*e^2 - 4*a*b^5*c^3*d^3*e - 24*a^3*b*c^5*d^3*e
+ 24*a^4*b*c^4*d*e^3 + 6*a*b^6*c^2*d^2*e^2 + 20*a^2*b^3*c^4*d^3*e + 28*a^2
*b^5*c^2*d*e^3 - 56*a^3*b^3*c^3*d*e^3))/c^2 + ((((16*(-(b^5*e^2 + b^3*c^2*
d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
*(256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8))/c^2 - (4*x^2*(256*
a*b^5*c^6*d - 2048*a^2*b^3*c^7*d + 4096*a^3*b*c^8*d))/c^2)*(-(b^5*e^2 + b^
3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*
c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2) + (16*(32*a*b^5*c^5*d^2 + 256*a^3*b*c^7*d^2 + 32*a*b^7*c^3*e^2 - 25
6*a^4*b*c^6*e^2 - 192*a^2*b^3*c^6*d^2 - 256*a^2*b^5*c^4*e^2 + 576*a^3*b^3*
c^5*e^2 - 64*a*b^6*c^4*d*e + 448*a^2*b^4*c^5*d*e - 768*a^3*b^2*c^6*d*e))/c
^2)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d...
```

Reduce [F]

$$\int \frac{x^5(d+ex^4)}{a+bx^4+cx^8}\,dx=\int \frac{x^5(e\,x^4+d)}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^5*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x^5*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.55 $\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 184

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/4*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b-(-4*
a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*(e
-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^
2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.97

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

input
```
Integrate[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt
[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1814, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}dx$$

$$\Big\downarrow \text{1814}$$

$$\frac{1}{2}\int \frac{ex^4+d}{cx^8+bx^4+a}dx^2$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^2\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

input
```
Int[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.91

method	result
default	$2c\left(-\frac{\left(\sqrt{-4ac+b^2}\,e-eb+2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(\sqrt{-4ac+b^2}\,e+eb-2cd\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right.}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right.$
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^3c^3-8a^2b^2c^2+a\,b^4c\right)\_Z^4+\left(-4a^2bc\,e^2+16a^2c^2de+b^3e^2a-4a\,b^2cde-4ab\,c^2d^2+b^3c\,d^2\right)\_Z^2+a^2e^4-2abd\,e^3+2ac\,d^2e^2+b^2\right.}\right.}{}$

input
```
int(x*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
2*c*(-1/8*((-4*a*c+b^2)^(1/2)*e-e*b+2*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))+1/8*((-4*a*c+b^2)^(1/2)*e+e*b-2*c*d)/c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1535 vs. $2(144) = 288$.

Time = 0.21 (sec) , antiderivative size = 1535, normalized size of antiderivative = 8.34

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)
*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b
^2*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 +
1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c
 - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a
*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/4*sqrt(1/2)*sqrt(-(b
*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c
*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log
(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 - 1/2*sqrt(1/2)*((b^2*c
- 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*
(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b
^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*
a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3
)))/(a*b^2*c - 4*a^2*c^2))) + 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a
*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e
 + a*b*d*e^3 - a^2*e^4)*x^2 + 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^
2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(d + ex^4)}{a + bx^4 + cx^8}\, dx = \int \frac{(ex^4 + d)x}{cx^8 + bx^4 + a}\, dx$$

input
```
integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate((e*x^4 + d)*x/(c*x^8 + b*x^4 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1404 vs. 2(144) = 288.

Time = 1.56 (sec) , antiderivative size = 1404, normalized size of antiderivative = 7.63

$$\int \frac{x(d + ex^4)}{a + bx^4 + cx^8}\, dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output

```
1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^
2 - 4*a*c)*b*c^2)*d + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arctan(2*sqrt(1/2)*x^2/sq
rt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*
c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/8*((sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sq...
```

Mupad [B] (verification not implemented)

Time = 26.53 (sec) , antiderivative size = 4501, normalized size of antiderivative = 24.46

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input

```
int((x*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

output
```
atan((b^4*c*d^3*x^2*1i + a^2*b^3*e^3*x^2*1i + a^2*c^3*d^3*x^2*8i - a^2*e^3
*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a^3*b*c*e
^3*x^2*4i - a*b^4*d*e^2*x^2*1i - b*c*d^3*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^
2*c^2 - 12*a*b^4*c)^(1/2)*1i - a*b^2*c^2*d^3*x^2*6i - a^3*c^2*d*e^2*x^2*8i
 + a*b*d*e^2*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i
 + a*c*d^2*e*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i
 + a^2*b*c^2*d^2*e*x^2*4i + a^2*b^2*c*d*e^2*x^2*6i - a*b^3*c*d^2*e*x^2*1i)
/(8*a^2*b^4*e^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*
b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 1024*a^3*
b^3*c^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*
e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) - 64*a^3*c^3*d^2*(-
(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)
 - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^
3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) + 64*a^4*c^2*e^2*(-(a*b^3*e^2
 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(...
```

Reduce [F]

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx=\int \frac{x(e\,x^4+d)}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.56 $\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 199

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = -\frac{d}{2ax^2} - \frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/2*d/a/x^2-1/4*c^(1/2)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2
)*c^(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(b-(-4*a*c+b^2)^(1/2
))^(1/2)-1/4*c^(1/2)*(d-(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^
(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(b+(-4*a*c+b^2)^(1/2))^(
1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.45

$$\int \frac{d+ex^4}{x^3 (a+bx^4+cx^8)} \, dx = -\frac{d}{2ax^2} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b\#1^2+2c\#1^6}\&\right]}{4a}$$

input
```
Integrate[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/2*d/(a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Lo
g[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1814, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x^3 (a+bx^4+cx^8)} \, dx$$

$$\Big\downarrow \text{ 1814}$$

$$\frac{1}{2}\int \frac{ex^4+d}{x^4 (cx^8+bx^4+a)}dx^2$$

$$\Big\downarrow \text{ 1604}$$

$$\frac{1}{2}\left(-\frac{\int \frac{cdx^4+bd-ae}{cx^8+bx^4+a}dx^2}{a}-\frac{d}{ax^2}\right)$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{1}{2}\left(-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int\frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int\frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^2}{a}-\frac{d}{ax^2}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(-\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{d}{ax^2}\right)$$

input
```
Int[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

output
```
(-(d/(a*x^2)) - ((Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2
- 4*a*c]]))/a)/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.89

method	result
default	$-\frac{d}{2a\,x^2}+\frac{2c\left(-\frac{\left(-bd-\sqrt{-4ac+b^2}\,d+2ae\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-\sqrt{-4ac+b^2}\,d-2ae+bd\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right.}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right.}{a}$
risch	$-\frac{d}{2a\,x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16c^2a^5-8a^4b^2c+a^3b^4\right)\_Z^4+\left(-4a^3b\,e^2c-16a^3de\,c^2+a^2b^3e^2+12a^2b^2dec+12a^2b\,c^2d^2-2a\,b^4de-7a\,b^3c\,d^2+b^5d^2\right.\right.}\right.}{}$

input
```
int((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/a/x^2+2/a*c*(-1/8*(-b*d-(-4*a*c+b^2)^(1/2)*d+2*a*e)/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-(-4*a*c+b^2)^(1/2)*d-2*a*e+b*d)/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2772 vs. $2(157) = 314$.

Time = 0.73 (sec) , antiderivative size = 2772, normalized size of antiderivative = 13.93

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2
*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 -
2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*
c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*
e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c
^2)*d^3*e)*x^2 + 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a
*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (
a^3*b^2 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3
 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c
^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*
b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2
*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*
b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d
^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt
(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 -
4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4
+ 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*
a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a
^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*sqrt
(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/x**3/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \int \frac{ex^4+d}{(cx^8+bx^4+a)x^3}\,dx$$

input
```
integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((c*d*x^4 + b*d - a*e)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2*d/(a*x^
2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3003 vs. $2(157) = 314$.

Time = 1.39 (sec) , antiderivative size = 3003, normalized size of antiderivative = 15.09

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3*c^2 - 2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^
3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*c^4 - 32*a^2*c^4 + 2*(b^2 - 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a
*c)*a*c^3)*d*x^4*abs(a) - (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*d*x^4 + (s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2
*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^
2)*d*abs(a) - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^3*c - 2*a*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c...
```

Mupad [B] (verification not implemented)

Time = 26.22 (sec) , antiderivative size = 15013, normalized size of antiderivative = 75.44

$$\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x)
```

output

```
- atan((((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2
*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c
*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e +
 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*
a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a
*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 -
 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*
c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(
1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*
b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*
d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(
1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(
9216*a^11*b^5*c^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^
11*b^6*c^4*e - 8192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c
^7*d))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 1
2*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16...
```

Reduce [F]

$$\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)}\,dx=\int \frac{e\,x^4+d}{x^3\left(c\,x^8+b\,x^4+a\right)}dx$$

input

```
int((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x)
```

output

```
int((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x)
```

3.57 $\int \frac{d+ex^4}{x^7\left(a+bx^4+cx^8\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 257

$$\int \frac{d+ex^4}{x^7\left(a+bx^4+cx^8\right)}\,dx = -\frac{d}{6ax^6}+\frac{bd-ae}{2a^2x^2}+\frac{\sqrt{c}\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2d-2acd-abe-\sqrt{b^2-4ac}(bd-ae)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/6*d/a/x^6+1/2*(-a*e+b*d)/a^2/x^2+1/4*c^(1/2)*(b*d-a*e+(-a*b*e-2*a*c*d+b
^2*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2)
)^(1/2))*2^(1/2)/a^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*c^(1/2)*(b^2*d-2*a*c
*d-a*b*e-(-4*a*c+b^2)^(1/2)*(-a*e+b*d))*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*
a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.53

$$\int \frac{d+ex^4}{x^7\left(a+bx^4+cx^8\right)}\,dx$$
$$= \frac{6bdx^4-2a(d+3ex^4)+3x^6\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^2d\log(x-\#1)-acd\log(x-\#1)-abe\log(x-\#1)+bcd\log(}{b\#1^2+2c\#1^6}\right.}{12a^2x^6}$$

input
```
Integrate[(d + e*x^4)/(x^7*(a + b*x^4 + c*x^8)),x]
```

output
```
(6*b*d*x^4 - 2*a*(d + 3*e*x^4) + 3*x^6*RootSum[a + b*#1^4 + c*#1^8 & , (b^
2*d*Log[x - #1] - a*c*d*Log[x - #1] - a*b*e*Log[x - #1] + b*c*d*Log[x - #1
]*#1^4 - a*c*e*Log[x - #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ])/(12*a^2*x^6)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1814, 1604, 27, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x^7\left(a+bx^4+cx^8\right)}\,dx$$
$$\Big\downarrow \ 1814$$
$$\frac{1}{2}\int \frac{ex^4+d}{x^8\left(cx^8+bx^4+a\right)}dx^2$$
$$\Big\downarrow \ 1604$$
$$\frac{1}{2}\left(-\frac{\int \frac{3\left(cdx^4+bd-ae\right)}{x^4\left(cx^8+bx^4+a\right)}dx^2}{3a}-\frac{d}{3ax^6}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(-\frac{\int \frac{cdx^4+bd-ae}{x^4(cx^8+bx^4+a)}dx^2}{a}-\frac{d}{3ax^6}\right)$$

$\Big\downarrow$ 1604

$$\frac{1}{2}\left(-\frac{-\frac{\int \frac{c(bd-ae)x^4+b^2d-acd-abe}{cx^8+bx^4+a}dx^2}{a}-\frac{bd-ae}{ax^2}}{a}-\frac{d}{3ax^6}\right)$$

$\Big\downarrow$ 1480

$$\frac{1}{2}\left(-\frac{-\frac{\frac{1}{2}c\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}c\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^2}{a}-\frac{bd-ae}{ax^2}}{a}-\frac{}{3a}\right.$$

$\Big\downarrow$ 218

$$\frac{1}{2}\left(-\frac{-\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{bd-ae}{ax^2}}{a}-\frac{d}{3ax^6}\right)$$

input

```
Int[(d + e*x^4)/(x^7*(a + b*x^4 + c*x^8)),x]
```

output

```
(-1/3*d/(a*x^6) - (-((b*d - a*e)/(a*x^2)) - ((Sqrt[c]*(b*d - a*e + (b^2*d
- 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b
*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*
c]]))/a)/a)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.92

method	result
default	$-\frac{2c\left(-\frac{\left(a\sqrt{-4ac+b^2}\,e-b\sqrt{-4ac+b^2}\,d+abe+2acd-d\,b^2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(a\sqrt{-4ac+b^2}\,e-b\sqrt{-4ac+b^2}\,d-abe-2acd+a}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\right.}}\right.}{a^2}$
risch	$\frac{-\frac{(ae-bd)x^4}{2a^2}-\frac{d}{6a}}{x^6}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^7c^2-8a^6b^2c+a^5b^4\right)\_Z^4+\left(12a^4b\,c^2e^2+16a^4c^3de-7a^3b^3c\,e^2-36a^3b^2c^2de-20a^3b\,c^3d^2+a^2b^5e^2+1\right.\right.}\right.}{}$

input
```
int((e*x^4+d)/x^7/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-2/a^2*c*(-1/8*(a*(-4*a*c+b^2)^(1/2)*e-b*(-4*a*c+b^2)^(1/2)*d+a*b*e+2*a*c*
d-d*b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(a*(-4*a*c+b^2)^(
1/2)*e-b*(-4*a*c+b^2)^(1/2)*d-a*b*e-2*a*c*d+d*b^2)/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)))-1/6*d/a/x^6-1/2*(a*e-b*d)/a^2/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4924 vs. $2(211) = 422$.

Time = 3.58 (sec) , antiderivative size = 4924, normalized size of antiderivative = 19.16

$$\int \frac{d+ex^4}{x^7\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/x^7/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^7 (a+bx^4+cx^8)} \, dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/x**7/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^4}{x^7 (a+bx^4+cx^8)} \, dx = \int \frac{ex^4+d}{(cx^8+bx^4+a)x^7} \, dx$$

input
```
integrate((e*x^4+d)/x^7/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-integrate(-((b*c*d - a*c*e)*x^4 - a*b*e + (b^2 - a*c)*d)*x/(c*x^8 + b*x^4
 + a), x)/a^2 + 1/6*(3*(b*d - a*e)*x^4 - a*d)/(a^2*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4428 vs. $2(211) = 422$.

Time = 1.39 (sec) , antiderivative size = 4428, normalized size of antiderivative = 17.23

$$\int \frac{d+ex^4}{x^7 (a+bx^4+cx^8)} \, dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/x^7/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c
^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 2*b^4*c^3 - 4*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 8*a*b^2*c^4 + sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 2*(b^2 - 4*a*c)*b^3*c^2 -
 8*(b^2 - 4*a*c)*a*b*c^3 + 2*(b^2 - 4*a*c)*b^2*c^3)*d*x^4 - (sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2
*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 2*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 8*a^2*b*c^4 + sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b^2 - 4*...
```

Mupad [B] (verification not implemented)

Time = 27.55 (sec) , antiderivative size = 24353, normalized size of antiderivative = 94.76

$$\int \frac{d+ex^4}{x^7\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x^7*(a + b*x^4 + c*x^8)),x)
```

output
```
- atan(((x^2*(8*a^17*c^11*d^5 - 8*a^19*c^9*d*e^4 + 4*a^15*b^4*c^9*d^5 - 16
*a^16*b^2*c^10*d^5 + 12*a^16*b^4*c^8*d^3*e^2 - 36*a^17*b^2*c^9*d^3*e^2 - 1
2*a^17*b^3*c^8*d^2*e^3 + 16*a^17*b*c^10*d^4*e - 4*a^15*b^5*c^8*d^4*e + 8*a
^16*b^3*c^9*d^4*e + 32*a^18*b*c^9*d^2*e^3 + 4*a^18*b^2*c^8*d*e^4) - (-(b^7
*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 -
 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2
*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(32*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*((x^2*(512*a^2
2*c^8*e^3 + 1280*a^20*b*c^9*d^3 - 512*a^21*c^9*d^2*e + 64*a^16*b^9*c^5*d^3
 - 640*a^17*b^7*c^6*d^3 + 2176*a^18*b^5*c^7*d^3 - 2880*a^19*b^3*c^8*d^3 -
64*a^19*b^6*c^5*e^3 + 448*a^20*b^4*c^6*e^3 - 896*a^21*b^2*c^7*e^3 - 2304*a
^21*b*c^8*d*e^2 - 192*a^17*b^8*c^5*d^2*e + 1728*a^18*b^6*c^6*d^2*e + 192*a
^18*b^7*c^5*d*e^2 - 4928*a^19*b^4*c^7*d^2*e - 1536*a^19*b^5*c^6*d*e^2 + 44
80*a^20*b^2*c^8*d^2*e + 3648*a^20*b^3*c^7*d*e^2) + (-(b^7*d^2 + a^2*b^5*e^
2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2
+ 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25
*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(...
```

Reduce [F]

$$\int \frac{d+ex^4}{x^7(a+bx^4+cx^8)}\,dx = \int \frac{e\,x^4+d}{x^7(c\,x^8+b\,x^4+a)}dx$$

input
```
int((e*x^4+d)/x^7/(c*x^8+b*x^4+a),x)
```

output
```
int((e*x^4+d)/x^7/(c*x^8+b*x^4+a),x)
```

3.58 $\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 433

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{ex}{c} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
e*x/c-1/4*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4
)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2
)^(1/2))^(3/4)-1/4*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*arct
an(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-b+(-
4*a*c+b^2)^(1/2))^(3/4)-1/4*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1
/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(5
/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/4*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a
*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^
(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.20

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{ex}{c} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{ae\log(x-\#1)-cd\log(x-\#1)\#1^4+be\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

input
```
Integrate[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(e*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*e*Log[x - #1] - c*d*Log[x - #
1]*#1^4 + b*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1826, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \; 1826$$

$$\frac{ex}{c}-\frac{\int \frac{ae-(cd-be)x^4}{cx^8+bx^4+a}dx}{c}$$

$$\Big\downarrow \; 1752$$

$$\frac{ex}{c}-\frac{-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \; 756$$

$$\frac{ex}{c}-\frac{-\frac{1}{2}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-b\right.}{c}$$

$$\Big\downarrow \; 218$$

$$\frac{ex}{c}-\frac{-\frac{1}{2}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)-\frac{1}{2}\left(-\frac{2ace+b^2(-e)+b}{\sqrt{b^2-4ac}}\right.}{c}$$

$$\Big\downarrow \; 221$$

$$\frac{ex}{c}-\frac{-\frac{1}{2}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)-\frac{1}{2}\left(-\frac{2ace+}{}\right.}{c}$$

input
```
Int[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(e*x)/c - (-1/2*((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])
*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^
(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b -
 Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)
))) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*(-(ArcTan
[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b
+ Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2
- 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2)/c
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
 + b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\left((-eb+cd)\_R^{4}-ae\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	67
risch	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\left((-eb+cd)\_R^{4}-ae\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	67

input
```
int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
e*x/c+1/4/c*sum(((-b*e+c*d)*_R^4-a*e)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf
(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12866 vs. 2(353) = 706.

Time = 6.24 (sec) , antiderivative size = 12866, normalized size of antiderivative = 29.71

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{(ex^4+d)x^4}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
e*x/c - integrate(-((c*d - b*e)*x^4 - a*e)/(c*x^8 + b*x^4 + a), x)/c
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 28.47 (sec) , antiderivative size = 50213, normalized size of antiderivative = 115.97

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

output
```
atan((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d
^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 1
6384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(
-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^
4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c
^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^
7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 +
 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2
)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^
3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*
d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
 + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 -
 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d
 - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^
9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 +
128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e
^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7...
```

Reduce [F]

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx=\int \frac{x^4(e\,x^4+d)}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.59 $\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 375

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
1/4*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a
*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/4*(e
+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2
)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/4*(e-(-b*e
+2*c*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/
2))^(1/4))*2^(1/4)/c^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*(e+(-b*e+2*c*
d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(
1/4))*2^(1/4)/c^(3/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.16

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4 + c\#1^8\&, \frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]$$

input
```
Integrate[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1
 + 2*c*#1^5) & ]/4
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1834}$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{827}$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)$$

$$\Big\downarrow \text{218}$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)$$

$$\Big\downarrow \text{221}$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)$$

input
```
Int[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

output
```
(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
 - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*
c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (e + (2*c*d - b*e)/Sqrt[b^2 - 4
*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3
/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c]
)^(1/4)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8 c+\_Z^4 b+a\right)} \frac{\left(\_R^6 e+\_R^2 d\right) \ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}}{4}$	51
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8 c+\_Z^4 b+a\right)} \frac{\left(\_R^6 e+\_R^2 d\right) \ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}}{4}$	51

input
```
int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R^6*e+_R^2*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b
+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15561 vs. $2(295) = 590$.

Time = 42.82 (sec) , antiderivative size = 15561, normalized size of antiderivative = 41.50

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{(ex^4+d)x^2}{cx^8+bx^4+a}\,dx$$

input
```
integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate((e*x^4 + d)*x^2/(c*x^8 + b*x^4 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 28.43 (sec) , antiderivative size = 29445, normalized size of antiderivative = 78.52

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

output
```
2*atan(((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4
*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 +
 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b
^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^
4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*
c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^
5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c
^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*
c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2
- 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*
b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2
*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^
2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^
(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)
^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*
b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2
*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2
- 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*...
```

Reduce [F]

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{x^2(e\,x^4+d)}{c\,x^8+b\,x^4+a}dx$$

input
```
int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.60 $\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 375

$$
\begin{aligned}
\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = &-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\
&-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} \\
&-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\
&-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}
\end{aligned}
$$

output
```
-1/4*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*
a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/4*(
e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^
2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/4*(e-(-b*
e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1
/2))^(1/4))*2^(3/4)/c^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/4*(e+(-b*e+2*c
*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^
(1/4))*2^(3/4)/c^(1/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.16

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4 + c\#1^8\&, \frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]$$

input
```
Integrate[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1
^3 + 2*c*#1^7) & ]/4
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx$$

$$\Big\downarrow \text{1752}$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}dx}{\sqrt{\sqrt{b^2-4ac}-b}}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)$$

input
```
Int[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

output
```
((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b -
 Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)
)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*
c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(
2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a
*c])^(3/4))))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	47
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\left(\_R^{4}e+d\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{4}$	47

input
```
int((e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R^4*e+d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9245 vs. 2(295) = 590.

Time = 2.25 (sec) , antiderivative size = 9245, normalized size of antiderivative = 24.65

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \int \frac{ex^4+d}{cx^8+bx^4+a}\,dx$$

input
```
integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
integrate((e*x^4 + d)/(c*x^8 + b*x^4 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

input
```
integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 27.40 (sec) , antiderivative size = 36707, normalized size of antiderivative = 97.89

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(a + b*x^4 + c*x^8),x)
```

output
```
- atan((((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 1
1*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) -
8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3
*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6
*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4
*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 +
 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 +
a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a
*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b
^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
+ 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^
4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d
^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*
a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^
7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(
1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096
*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c
^4*d - 262144*a^4*b*c^7*d) + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 ...
```

Reduce [F]

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \int \frac{e\,x^4+d}{c\,x^8+b\,x^4+a}dx$$

input
```
int((e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int((e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.61 $\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 392

$$\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx = -\frac{d}{ax} - \frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
-d/a/x-1/4*c^(1/4)*(d-(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1
/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/
4)-1/4*c^(1/4)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*
x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1
/4*c^(1/4)*(d-(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(
-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/4*
c^(1/4)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+
(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.22

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx = -\frac{d}{ax} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{4a}$$

input
```
Integrate[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

output
```
-(d/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x
- #1] + c*d*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1828, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx$$

$$\Big\downarrow \ 1828$$

$$-\frac{\int \frac{x^2\left(cdx^4+bd-ae\right)}{cx^8+bx^4+a}dx}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 1834$$

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 27$$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{a}-\frac{d}{ax}$$

$$\Big\downarrow \ 827$$

$$-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\left(\frac{\int\frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\int\frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{\sqrt{b^2-4ac}-b}}d}{2\sqrt{2}\sqrt{c}}\right.}{a}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \ 218$$

$$-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{\sqrt{b^2}}}\right.}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b}}}\right.}{a}$$

$$\frac{d}{ax}$$

$$\Big\downarrow \ 221$$

$$-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(\frac{\arctan\left(\right.}{2\,2^{3/4}c}\right.}{a}$$

$$\frac{d}{ax}$$

input
```
Int[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

output
```
-(d/(a*x)) - (c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(
1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1
/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + c*(d + (b*d - 2
*a*e)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*
c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2
^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b +
 Sqrt[b^2 - 4*a*c])^(1/4))))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.19

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^8 c+\_Z^4 b+a\right)} \frac{\left(-cd\_R^6+(ae-bd)\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}}{4a}-\frac{d}{ax}$	73
risch	Expression too large to display	1333

input
```
int((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*sum((-c*d*_R^6+(a*e-b*d)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(
_Z^8*c+_Z^4*b+a))-d/a/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21400 vs. $2(312) = 624$.

Time = 120.72 (sec) , antiderivative size = 21400, normalized size of antiderivative = 54.59

$$\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/x**2/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx = \int \frac{ex^4+d}{(cx^8+bx^4+a)x^2}\,dx$$

input
```
integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((c*d*x^6 + (b*d - a*e)*x^2)/(c*x^8 + b*x^4 + a), x)/a - d/(a*x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 29.40 (sec) , antiderivative size = 39028, normalized size of antiderivative = 99.56

$$\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x)
```

output
```
atan((((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d
^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*
b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a
^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2
) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200
*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^
5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a
^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^
9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 -
4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d
^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^
7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c -
 b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*...
```

Reduce [F]

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx=\int \frac{e\,x^4+d}{x^2\left(c\,x^8+b\,x^4+a\right)}dx$$

input
```
int((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x)
```

output
```
int((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x)
```

3.62 $\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 394

$$\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx = -\frac{d}{3ax^3}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
-1/3*d/a/x^3+1/4*c^(3/4)*(d-(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4
)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2
))^(3/4)+1/4*c^(3/4)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^
(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(
3/4)+1/4*c^(3/4)*(d-(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/
4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4
)+1/4*c^(3/4)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*
x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.22

$$\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx = -\frac{\frac{4d}{x^3}+3\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{12a}$$

input
```
Integrate[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/12*((4*d)/x^3 + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*
e*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/a
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1828, 27, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}dx$$

$$\Big\downarrow \; 1828$$

$$-\frac{\int \frac{3\left(cdx^4+bd-ae\right)}{cx^8+bx^4+a}dx}{3a}-\frac{d}{3ax^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{cdx^4+bd-ae}{cx^8+bx^4+a}dx}{a}-\frac{d}{3ax^3}$$

$$\Big\downarrow \; 1752$$

$$-\frac{\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{d}{3ax^3}$$

$$\Big\downarrow \; 756$$

$$-\frac{\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\sqrt{\cdots}}}}{\sqrt{\sqrt{b^2-4ac}\cdots}}\right.}{a}$$

$$\frac{d}{3ax^3}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(-\frac{\int \frac{\cdots}{\sqrt{\sqrt{b^2-\cdots}}}}{\sqrt{\cdots}}\right.}{a}$$

$$\frac{d}{3ax^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{1}{2}c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\left(-\right.}{a}$$

$$\frac{d}{3ax^3}$$

input
```
Int[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/3*d/(a*x^3) - ((c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1
/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt
[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*
c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + (c*(d +
(b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[
b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - A
rcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4
)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2)/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.17

method	result	size
default	$-\frac{d}{3a\,x^{3}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}cd+ae-bd\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4a}$	68
risch	Expression too large to display	1633

input
```
int((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*d/a/x^3+1/4/a*sum((-_R^4*c*d+a*e-b*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=R
ootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20184 vs. $2(312) = 624$.

Time = 46.41 (sec) , antiderivative size = 20184, normalized size of antiderivative = 51.23

$$\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/x**4/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx = \int \frac{ex^4+d}{(cx^8+bx^4+a)x^4}\,dx$$

input
```
integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-integrate((c*d*x^4 + b*d - a*e)/(c*x^8 + b*x^4 + a), x)/a - 1/3*d/(a*x^3)
```

Giac [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 30.28 (sec) , antiderivative size = 65350, normalized size of antiderivative = 165.86

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x)
```

output
```
atan((((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*
a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c -
 b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 +
 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*
d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a
^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a
^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2)
 - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^
6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3
*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e
- 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 +
16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*
c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^
6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e
^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e
^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e
^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a...
```

Reduce [F]

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx=\int \frac{e\,x^4+d}{x^4\left(c\,x^8+b\,x^4+a\right)}dx$$

input
```
int((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x)
```

output
```
int((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x)
```

3.63 $\int \frac{x^5(1-x^4)}{1-x^4+x^8} dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{x^5(1-x^4)}{1-x^4+x^8} dx = -\frac{x^2}{2} - \frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right) + \frac{1}{4}\arctan\left(\sqrt{3}+2x^2\right) + \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x^2}{1+x^4}\right)}{4\sqrt{3}}$$

output
```
-1/2*x^2+1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(3^(1/2)+2*x^2)+1/12*arctanh(
3^(1/2)*x^2/(x^4+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.29

$$\int \frac{x^5(1-x^4)}{1-x^4+x^8} dx = \frac{1}{12}i\left(6ix^2 + \sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right) - \sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)\right)$$

input
```
Integrate[(x^5*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
(I/12)*((6*I)*x^2 + Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x^2)/
2] - Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x^2)/2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.61, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1814, 1602, 25, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(1-x^4)}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{ 1814}$$

$$\frac{1}{2}\int \frac{x^4(1-x^4)}{x^8-x^4+1}dx^2$$

$$\Big\downarrow \text{ 1602}$$

$$\frac{1}{2}\left(-\int -\frac{1}{x^8-x^4+1}dx^2-x^2\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\int \frac{1}{x^8-x^4+1}dx^2-x^2\right)$$

$$\Big\downarrow \text{ 1407}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\int \frac{x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}-x^2\right)$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4-\sqrt{3}x^2+1}dx^2-\frac{1}{2}\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4+\sqrt{3}x^2+1}dx^2+\frac{1}{2}\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}-x^2\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4-\sqrt{3}x^2+1}dx^2+\frac{1}{2}\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4+\sqrt{3}x^2+1}dx^2+\frac{1}{2}\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}-x^2\right)$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2-\sqrt{3}\int \frac{1}{-x^4-1}d(2x^2-\sqrt{3})}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2-\sqrt{3}\int \frac{1}{-x^4-1}d(2x^2+\sqrt{3})}{2\sqrt{3}}-x^2\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2-\sqrt{3}\arctan\left(\sqrt{3}-2x^2\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2+\sqrt{3}\arctan\left(2x^2+\sqrt{3}\right)}{2\sqrt{3}}-x^2\right)$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{2}\left(\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x^2\right)-\frac{1}{2}\log\left(x^4-\sqrt{3}x^2+1\right)}{2\sqrt{3}}+\frac{\sqrt{3}\arctan\left(2x^2+\sqrt{3}\right)+\frac{1}{2}\log\left(x^4+\sqrt{3}x^2+1\right)}{2\sqrt{3}}-x^2\right)$$

input

```
Int[(x^5*(1 - x^4))/(1 - x^4 + x^8),x]
```

output

```
(-x^2 + (-(Sqrt[3]*ArcTan[Sqrt[3] - 2*x^2]) - Log[1 - Sqrt[3]*x^2 + x^4]/2
)/(2*Sqrt[3]) + (Sqrt[3]*ArcTan[Sqrt[3] + 2*x^2] + Log[1 + Sqrt[3]*x^2 + x
^4]/2)/(2*Sqrt[3]))/2
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

method	result	size
risch	$-\frac{x^2}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-3\_R^3+x^2+\_R\right)\right)}{4}$	38
default	$-\frac{x^2}{2}+\frac{\sqrt{3}\,\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{24}+\frac{\arctan\left(2x^2+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\,\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{24}+\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4}$	70

input
```
int(x^5*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2+1/4*sum(_R*ln(-3*_R^3+x^2+_R),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{x^5(1-x^4)}{1-x^4+x^8}\,dx=-\frac{1}{2}\,x^2+\frac{1}{24}\,\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)-\frac{1}{24}\,\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)+\frac{1}{4}\,\arctan\left(2\,x^2+\sqrt{3}\right)-\frac{1}{4}\,\arctan\left(-2\,x^2+\sqrt{3}\right)$$

input
```
integrate(x^5*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/2*x^2 + 1/24*sqrt(3)*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*log(x^4
- sqrt(3)*x^2 + 1) + 1/4*arctan(2*x^2 + sqrt(3)) - 1/4*arctan(-2*x^2 + sqr
t(3))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{x^5(1-x^4)}{1-x^4+x^8}\,dx = -\frac{x^2}{2} - \frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24} + \frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24} + \frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4} + \frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

input
```
integrate(x**5*(-x**4+1)/(x**8-x**4+1),x)
```

output
```
-x**2/2 - sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqr
t(3)*x**2 + 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

Maxima [F]

$$\int \frac{x^5(1-x^4)}{1-x^4+x^8}\,dx = \int -\frac{(x^4-1)x^5}{x^8-x^4+1}\,dx$$

input
```
integrate(x^5*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/2*x^2 + integrate(x/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{x^5(1-x^4)}{1-x^4+x^8}\,dx = -\frac{1}{2}\,x^2 + \frac{1}{24}\,\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right) - \frac{1}{24}\,\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right) + \frac{1}{4}\,\arctan\left(2\,x^2+\sqrt{3}\right) + \frac{1}{4}\,\arctan\left(2\,x^2-\sqrt{3}\right)$$

input
```
integrate(x^5*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/2*x^2 + 1/24*sqrt(3)*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*log(x^4
- sqrt(3)*x^2 + 1) + 1/4*arctan(2*x^2 + sqrt(3)) + 1/4*arctan(2*x^2 - sqrt
(3))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{x^5(1-x^4)}{1-x^4+x^8}\,dx = -\mathrm{atan}\left(-\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \mathrm{atan}\left(\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{x^2}{2}$$

input
```
int(-(x^5*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

output
```
- atan((3^(1/2)*x^2*1i)/2 - x^2/2)*((3^(1/2)*1i)/12 + 1/4) - atan((3^(1/2)
*x^2*1i)/2 + x^2/2)*((3^(1/2)*1i)/12 - 1/4) - x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.17

$$\int \frac{x^5(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} + \frac{\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} - \frac{\sqrt{3}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24} - \frac{\sqrt{3}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24} - \frac{x^2}{2}$$

input
```
int(x^5*(-x^4+1)/(x^8-x^4+1),x)
```

output
```
( - 3*sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt(
 - sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(2)
 - 4*x)/(2*sqrt( - sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(6)*atan((s
qrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)
*sqrt(2)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 3*sqrt
( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sq
rt(2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*
x)/(sqrt(6) + sqrt(2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - s
qrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*a
tan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) + sqrt(3)*log( - s
qrt( - sqrt(3) + 2)*x + x**2 + 1) + sqrt(3)*log(sqrt( - sqrt(3) + 2)*x + x
**2 + 1) - sqrt(3)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) - sqrt(3
)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2) - 12*x**2)/24
```

3.64 $\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x^4+x^8\right)$$

output
```
-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)-1/8*ln(x^8-x^4+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = \frac{\arctan\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x^4+x^8\right)$$

input
```
Integrate[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1798, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(1-x^4)}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{4}\int \frac{1-x^4}{x^8-x^4+1}dx^4$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^8-x^4+1}dx^4-\frac{1}{2}\int -\frac{1-2x^4}{x^8-x^4+1}dx^4\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^8-x^4+1}dx^4+\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4-\int \frac{1}{-x^8-3}d(2x^4-1)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4+\frac{\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^8-x^4+1\right)\right)$$

input

```
Int[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
(ArcTan[(-1 + 2*x^4)/Sqrt[3]]/Sqrt[3] - Log[1 - x^4 + x^8]/2)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\ln(x^8-x^4+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	33
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(4x^8-4x^4+4)}{8}$	35

input
```
int(x^3*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx=\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)-\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx=-\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**3*(-x**4+1)/(x**8-x**4+1),x)
```

output
```
-log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right) - \frac{1}{8}\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right) - \frac{1}{8}\log\left(x^8-x^4+1\right)$$

input
```
integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\ln\left(x^8-x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12}$$

input
```
int(-(x^3*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

output
```
- log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 328, normalized size of antiderivative = 8.41

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} - \frac{\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} - \frac{\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{8} - \frac{\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{8}$$

input
```
int(x^3*(-x^4+1)/(x^8-x^4+1),x)
```

output
```
( - sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( -
 sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(2) -
 4*x)/(2*sqrt( - sqrt(3) + 2))) - sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(
6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 3*sqrt( - sqrt(3) + 2)*sqr
t(2)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + sqrt( - sq
rt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))
) + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sq
rt(6) + sqrt(2))) + sqrt( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) +
 2) + 4*x)/(sqrt(6) + sqrt(2))) + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*s
qrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) - 3*log( - sqrt( - sqrt(3)
 + 2)*x + x**2 + 1) - 3*log(sqrt( - sqrt(3) + 2)*x + x**2 + 1) - 3*log(( -
 sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) - 3*log((sqrt(6)*x + sqrt(2)*x + 2
*x**2 + 2)/2))/24
```

3.65 $\int \frac{x(1-x^4)}{1-x^4+x^8} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \frac{x(1-x^4)}{1-x^4+x^8} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x^2}{1+x^4}\right)}{2\sqrt{3}}$$

output
```
1/6*arctanh(3^(1/2)*x^2/(x^4+1))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{x(1-x^4)}{1-x^4+x^8} \, dx = \frac{-\log\left(-1+\sqrt{3}x^2-x^4\right)+\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}$$

input
```
Integrate[(x*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
(-Log[-1 + Sqrt[3]*x^2 - x^4] + Log[1 + Sqrt[3]*x^2 + x^4])/(4*Sqrt[3])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 54 vs. 2(26) = 52.

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1814, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1-x^4)}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{1814}$$

$$\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}-\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{2\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[(x*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
(-1/2*Log[1 - Sqrt[3]*x^2 + x^4]/Sqrt[3] + Log[1 + Sqrt[3]*x^2 + x^4]/(2*S
qrt[3]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\sqrt{3}\,\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{12}-\frac{\sqrt{3}\,\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{12}$	39
risch	$\frac{\sqrt{3}\,\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{12}-\frac{\sqrt{3}\,\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{12}$	39

input
```
int(x*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)-1/12*3^(1/2)*ln(x^4-3^(1/2)*x^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{12}\,\sqrt{3}\log\left(\frac{x^8+5\,x^4+2\,\sqrt{3}(x^6+x^2)+1}{x^8-x^4+1}\right)$$

input
```
integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4 + 1)
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

input
```
integrate(x*(-x**4+1)/(x**8-x**4+1),x)
```

output
```
-sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2
 + 1)/12
```

Maxima [F]

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = \int -\frac{(x^4-1)x}{x^8-x^4+1}\,dx$$

input
```
integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = -\frac{1}{12}\sqrt{3}\log\left(\frac{x^2-\sqrt{3}+\frac{1}{x^2}}{x^2+\sqrt{3}+\frac{1}{x^2}}\right)$$

input
```
integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*log((x^2 - sqrt(3) + 1/x^2)/(x^2 + sqrt(3) + 1/x^2))
```

Mupad [B] (verification not implemented)

Time = 21.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x^2}{x^4+1}\right)}{6}$$

input
```
int(-(x*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

output
```
(3^(1/2)*atanh((3^(1/2)*x^2)/(x^4 + 1)))/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.77

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx$$
$$= \frac{\sqrt{3}\left(\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)+\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)-\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)-\log\right.}{12}$$

input
```
int(x*(-x^4+1)/(x^8-x^4+1),x)
```

output
```
(sqrt(3)*(log( - sqrt( - sqrt(3) + 2)*x + x**2 + 1) + log(sqrt( - sqrt(3)
+ 2)*x + x**2 + 1) - log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) - log(
(sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2)))/12
```

3.66 $\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx=\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)$$

output
```
1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)+ln(x)-1/8*ln(x^8-x^4+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx=\log(x)-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^4}{-1+2\#1^4}\&\right]$$

input
```
Integrate[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
```

output
```
Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 + 2*#1^4) & ]/
4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x\left(x^8-x^4+1\right)}dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{4}\int \frac{1-x^4}{x^4\left(x^8-x^4+1\right)}dx^4$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{4}\int \left(\frac{1}{x^4}-\frac{x^4}{x^8-x^4+1}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^4\right)-\frac{1}{2}\log\left(x^8-x^4+1\right)\right)$$

input
```
Int[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
```

output
```
(ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[x^4] - Log[1 - x^4 + x^8]/2)/4
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln\left(x\right)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{4}-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x^{8}-x^{4}+1\right)}{8}$	33
default	$-\frac{\ln\left(x^{8}-x^{4}+1\right)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)}{12}+\ln\left(x\right)$	35

input
```
int((-x^4+1)/x/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/12*3^(1/2)*arctan(2/3*(x^4-1/2)*3^(1/2))-1/8*ln(x^8-x^4+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx=-\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)-\frac{1}{8}\,\log\left(x^8-x^4+1\right)+\log\left(x\right)$$

input
```
integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + l
og(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx = \log(x) - \frac{\log(x^8-x^4+1)}{8} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate((-x**4+1)/x/(x**8-x**4+1),x)
```

output
```
log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/
3)/12
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right) - \frac{1}{8}\log(x^8-x^4+1) + \frac{1}{4}\log(x^4)$$

input
```
integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1
/4*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right) - \frac{1}{8}\log(x^8-x^4+1) + \frac{1}{4}\log(x^4)$$

input
```
integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1
/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 21.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1-x^4}{x\,(1-x^4+x^8)}\,dx = \ln(x) - \frac{\ln(x^8-x^4+1)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}$$

input
```
int(-(x^4 - 1)/(x*(x^8 - x^4 + 1)),x)
```

output
```
log(x) - log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/
3))/12
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 330, normalized size of antiderivative = 8.05

$$\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx = \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} - \frac{\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} - \frac{\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{8} - \frac{\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{8} + \log(x)$$

input
```
int((-x^4+1)/x/(x^8-x^4+1),x)
```

output
```
(sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( - sq
rt(3) + 2))) + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(2) - 4*
x)/(2*sqrt( - sqrt(3) + 2))) + sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6)
+ sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + 3*sqrt( - sqrt(3) + 2)*sqrt(2
)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - sqrt( - sqrt(
3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) -
 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(
6) + sqrt(2))) - sqrt( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2)
 + 4*x)/(sqrt(6) + sqrt(2))) - 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt
( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) - 3*log( - sqrt( - sqrt(3) +
2)*x + x**2 + 1) - 3*log(sqrt( - sqrt(3) + 2)*x + x**2 + 1) - 3*log(( - sq
rt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) - 3*log((sqrt(6)*x + sqrt(2)*x + 2*x*
*2 + 2)/2) + 24*log(x))/24
```

3.67 $\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = -\frac{1}{2x^2}+\frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\arctan\left(\sqrt{3}+2x^2\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x^2}{1+x^4}\right)}{4\sqrt{3}}$$

output
```
-1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(3^(1/2)+2*x^2)+1/12*arctanh(
3^(1/2)*x^2/(x^4+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = -\frac{1}{2x^2}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^4}\&\right]$$

input
```
Integrate[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

output
```
-1/2*1/x^2 - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^4)
& ]/4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.44, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1814, 1604, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^3 (x^8-x^4+1)} \, dx$$

$$\Big\downarrow \text{1814}$$

$$\frac{1}{2}\int \frac{1-x^4}{x^4 (x^8-x^4+1)}dx^2$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{2}\left(-\int \frac{x^4}{x^8-x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2-\frac{1}{2}\int \frac{x^4+1}{x^8-x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^4-\sqrt{3}x^2+1}dx^2-\frac{1}{2}\int \frac{1}{x^4+\sqrt{3}x^2+1}dx^2\right)+\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{1}{-x^4-1}d\left(2x^2-\sqrt{3}\right)+\int \frac{1}{-x^4-1}d\left(2x^2+\sqrt{3}\right)\right)+\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^4}{x^8-x^4+1}dx^2+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}-\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}+\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx^2}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x^2\right)-\arctan\left(2x^2+\sqrt{3}\right)\right)-\frac{1}{x^2}+\frac{1}{2}\left(\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{2\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{2\sqrt{3}}\right)\right)$$

input
```
Int[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

output
```
(-x^(-2) + (ArcTan[Sqrt[3] - 2*x^2] - ArcTan[Sqrt[3] + 2*x^2])/2 + (-1/2*L
og[1 - Sqrt[3]*x^2 + x^4]/Sqrt[3] + Log[1 + Sqrt[3]*x^2 + x^4]/(2*Sqrt[3])
)/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

method	result	size
risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-6\_R^3+x^2-\_R\right)\right)}{4}$	40
default	$-\frac{1}{2x^2}+\frac{\sqrt{3}\left(\frac{\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{2}-\sqrt{3}\,\arctan\left(2x^2+\sqrt{3}\right)\right)}{12}+\frac{\sqrt{3}\left(-\frac{\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{2}-\sqrt{3}\,\arctan\left(2x^2-\sqrt{3}\right)\right)}{12}$	82

input
```
int((-x^4+1)/x^3/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+1/4*sum(_R*ln(-6*_R^3+x^2-_R),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx$$
$$=\frac{\sqrt{3}x^2\log\left(x^4+\sqrt{3}x^2+1\right)-\sqrt{3}x^2\log\left(x^4-\sqrt{3}x^2+1\right)-6\,x^2\arctan\left(2\,x^2+\sqrt{3}\right)+6\,x^2\arctan\left(-2\,x^2\right.}{24\,x^2}$$

input
```
integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/24*(sqrt(3)*x^2*log(x^4 + sqrt(3)*x^2 + 1) - sqrt(3)*x^2*log(x^4 - sqrt(
3)*x^2 + 1) - 6*x^2*arctan(2*x^2 + sqrt(3)) + 6*x^2*arctan(-2*x^2 + sqrt(3
)) - 12)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = -\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24} - \frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}-\frac{1}{2x^2}$$

input
```
integrate((-x**4+1)/x**3/(x**8-x**4+1),x)
```

output
```
-sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
 + 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 - 1/(2*x**2
)
```

Maxima [F]

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = \int -\frac{x^4-1}{(x^8-x^4+1)x^3}\,dx$$

input
```
integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/2/x^2 - integrate(x^5/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = -\frac{1}{24}\sqrt{3}x^4\log\left(x^4+\sqrt{3}x^2+1\right)+\frac{1}{24}\sqrt{3}x^4\log\left(x^4-\sqrt{3}x^2+1\right) - \frac{1}{4}x^4\arctan\left(2x^2+\sqrt{3}\right)-\frac{1}{4}x^4\arctan\left(2x^2-\sqrt{3}\right)-\frac{1}{2x^2}$$

input
```
integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/24*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*sqrt(3)*x^4*log(x^4 -
sqrt(3)*x^2 + 1) - 1/4*x^4*arctan(2*x^2 + sqrt(3)) - 1/4*x^4*arctan(2*x^2
- sqrt(3)) - 1/2/x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = \operatorname{atan}\left(\frac{2\,x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \operatorname{atan}\left(\frac{2\,x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{1}{2\,x^2}$$

input
```
int(-(x^4 - 1)/(x^3*(x^8 - x^4 + 1)),x)
```

output
```
atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x^2)/(3^(
1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - 1/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 376, normalized size of antiderivative = 5.70

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = \frac{3\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x^2+3\sqrt{-\sqrt{3}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x^2+3\sqrt{-\sqrt{3}+2}\,\sqrt{6}\,atan\left(\frac{\sqrt{6}}{2}\right.}{}$$

input
```
int((-x^4+1)/x^3/(x^8-x^4+1),x)
```

output

```
(3*sqrt( - sqrt(3) + 2)*sqrt(6)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( -
sqrt(3) + 2)))*x**2 + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(
2) - 4*x)/(2*sqrt( - sqrt(3) + 2)))*x**2 + 3*sqrt( - sqrt(3) + 2)*sqrt(6)*
atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*x**2 + 3*sqrt( -
sqrt(3) + 2)*sqrt(2)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2
)))*x**2 + 3*sqrt( - sqrt(3) + 2)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4
*x)/(sqrt(6) + sqrt(2)))*x**2 + 3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqr
t( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2)))*x**2 + 3*sqrt( - sqrt(3) + 2
)*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2)))*x**2 +
3*sqrt( - sqrt(3) + 2)*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6
) + sqrt(2)))*x**2 + sqrt(3)*log( - sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x**
2 + sqrt(3)*log(sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x**2 - sqrt(3)*log(( -
sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2)*x**2 - sqrt(3)*log((sqrt(6)*x + sqr
t(2)*x + 2*x**2 + 2)/2)*x**2 - 12)/(24*x**2)
```

3.68 $\int \frac{x^6(1-x^4)}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 280

$$\int \frac{x^6(1-x^4)}{1-x^4+x^8}\,dx = -\frac{x^3}{3}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

output
```
-1/3*x^3+1/4*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1/2*6^(1/2)+1/2*2^(1/2)
))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*arctan((1/2*6^(1/2)+1/2*2^(1/2)-2*x)/(1/2
*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/4*arctan((1/2*6^(1/2)-1
/2*2^(1/2)+2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*a
rctan((1/2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2
)-1/2*6^(1/2))-1/4*arctanh((1/2*6^(1/2)-1/2*2^(1/2))*x/(x^2+1))/(3/2*2^(1/
2)-1/2*6^(1/2))+1/4*arctanh((1/2*6^(1/2)+1/2*2^(1/2))*x/(x^2+1))/(3/2*2^(1
/2)+1/2*6^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.17

$$\int \frac{x^6(1-x^4)}{1-x^4+x^8}\,dx = -\frac{x^3}{3}+\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1+2\#1^5}\&\right]$$

input
```
Integrate[(x^6*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
-1/3*x^3 + RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1 + 2*#1^5) & ]/4
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.48, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1826, 27, 1709, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(1-x^4)}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{ 1826}$$

$$-\frac{1}{3}\int -\frac{3x^2}{x^8-x^4+1}dx-\frac{x^3}{3}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^2}{x^8-x^4+1}dx-\frac{x^3}{3}$$

$$\Big\downarrow \text{ 1709}$$

$$\frac{\int \frac{1}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int \frac{1}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{x^3}{3}$$

$$\Big\downarrow \text{ 1407}$$

$$-\frac{\frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{x^3}{3}$$

$\Big\downarrow$ 1142

$$-\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{x^3}{3}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{x^3}{3}$$

$\Big\downarrow$ 1083

$$-\frac{\frac{\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x+\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x+\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{x^3}{3}$$

$\Big\downarrow$ 217

$$-\frac{\frac{\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{x^3}{3}$$

$\Big\downarrow$ 1103

$$-\frac{\frac{\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{x^3}{3}$$

input
```
Int[(x^6*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
-1/3*x^3 - ((Sqrt[(2 - Sqrt[3])/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]]
+ 2*x)/Sqrt[2 + Sqrt[3]]] - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/2)/(2*Sqrt[
2 - Sqrt[3]]) + (Sqrt[(2 - Sqrt[3])/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3
]] + 2*x)/Sqrt[2 + Sqrt[3]]] + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/2)/(2*Sq
rt[2 - Sqrt[3]]))/(2*Sqrt[3]) + ((Sqrt[(2 + Sqrt[3])/(2 - Sqrt[3])]*ArcTan
[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - Log[1 - Sqrt[2 + Sqrt[3]]
*x + x^2]/2)/(2*Sqrt[2 + Sqrt[3]]) + (Sqrt[(2 + Sqrt[3])/(2 - Sqrt[3])]*Ar
cTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + Log[1 + Sqrt[2 + Sqrt[
3]]*x + x^2]/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.16

method	result	size
default	$-\frac{x^3}{3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	46
risch	$-\frac{x^3}{3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	46

input
```
int(x^6*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3+1/4*sum(_R^2/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.35

$$\int \frac{x^6(1-x^4)}{1-x^4+x^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/3*x^3 + 1/4*sqrt(1/3)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/3)
*sqrt(3/2*sqrt(-1/3) + 1/2)*(sqrt(-1/3) + 1)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1
/2)) + 2*x) - 1/4*sqrt(1/3)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(
1/3)*sqrt(3/2*sqrt(-1/3) + 1/2)*(sqrt(-1/3) + 1)*sqrt(-sqrt(3/2*sqrt(-1/3)
 + 1/2)) + 2*x) - 1/4*sqrt(1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(3*s
qrt(1/3)*(sqrt(-1/3) - 1)*sqrt(-3/2*sqrt(-1/3) + 1/2)*sqrt(-sqrt(-3/2*sqrt
(-1/3) + 1/2)) + 2*x) + 1/4*sqrt(1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*l
og(-3*sqrt(1/3)*(sqrt(-1/3) - 1)*sqrt(-3/2*sqrt(-1/3) + 1/2)*sqrt(-sqrt(-3
/2*sqrt(-1/3) + 1/2)) + 2*x) - 1/4*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4)*
log(3*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(3/4)*(sqrt(-1/3) + 1) + 2*x) + 1/4
*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*(3/2*sqrt(-1/3) +
 1/2)^(3/4)*(sqrt(-1/3) + 1) + 2*x) + 1/4*sqrt(1/3)*(-3/2*sqrt(-1/3) + 1/2
)^(1/4)*log(3*sqrt(1/3)*(sqrt(-1/3) - 1)*(-3/2*sqrt(-1/3) + 1/2)^(3/4) + 2
*x) - 1/4*sqrt(1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*(sqrt(-
1/3) - 1)*(-3/2*sqrt(-1/3) + 1/2)^(3/4) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.11

$$\int \frac{x^6(1-x^4)}{1-x^4+x^8}\,dx = -\frac{x^3}{3} - \operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(442368t^7 + 192t^3 + x\right)\right)\right)$$

input
```
integrate(x**6*(-x**4+1)/(x**8-x**4+1),x)
```

output
```
-x**3/3 - RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(442368
*_t**7 + 192*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^6(1-x^4)}{1-x^4+x^8}\,dx = \int -\frac{(x^4-1)x^6}{x^8-x^4+1}\,dx$$

input
```
integrate(x^6*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/3*x^3 + integrate(x^2/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{x^6(1-x^4)}{1-x^4+x^8}\,dx = & -\frac{1}{3}x^3 + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\ & + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) \\ & + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\ & + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) \\ & - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\ & + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) \\ & - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) \\ & + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

input
```
integrate(x^6*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/3*x^3 + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sq
rt(6) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqr
t(2))/(sqrt(6) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt
(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4
*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*
log(x^2 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(
x^2 - 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2
+ 1/2*x*(sqrt(6) - sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/
2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.04

$$\int \frac{x^6(1-x^4)}{1-x^4+x^8}\,dx$$
$$= -\frac{x^3}{3} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)} + \frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12}$$
$$+ \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)} - \frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12}$$
$$- \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)} - \frac{2^{3/4}\sqrt{3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12}$$
$$+ \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)} + \frac{2^{3/4}\sqrt{3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(-(x^6*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

output

```
(3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)) - (3^(1/2
)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1/4))/
12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)) + (3^(1
/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1
/4)*1i)/12 - x^3/3 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/
4))/(2*(3^(1/2)*1i - 1)) - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(
2*(3^(1/2)*1i - 1)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 + (2^(3/4)*3^(1/2)*atan
((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) + (2^(3/4)*3^(
1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*(3^(1/2)*1i - 1)))*(3^(1/2)*1i + 1)^(1/4
))/12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.47

$$\int \frac{x^6(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{6} - \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{6} + \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} - \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{24} - \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{12} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{12} + \frac{\sqrt{-\sqrt{3}+2}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} - \frac{\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} + \frac{\sqrt{6}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{48} - \frac{\sqrt{6}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{48} - \frac{\sqrt{2}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{16} + \frac{\sqrt{2}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{16} - \frac{x^3}{3}$$

input
```
int(x^6*(-x^4+1)/(x^8-x^4+1),x)
```

output

```
( - 8*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt(
 - sqrt(3) + 2))) - 12*sqrt( - sqrt(3) + 2)*atan((sqrt(6) + sqrt(2) - 4*x)
/(2*sqrt( - sqrt(3) + 2))) + 8*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6)
+ sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + 12*sqrt( - sqrt(3) + 2)*atan(
(sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 2*sqrt(6)*atan((2*sq
rt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) + 6*sqrt(2)*atan((2*sqrt( -
 sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) + 2*sqrt(6)*atan((2*sqrt( - sqrt
(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) - 6*sqrt(2)*atan((2*sqrt( - sqrt(3) +
 2) + 4*x)/(sqrt(6) + sqrt(2))) + 4*sqrt( - sqrt(3) + 2)*sqrt(3)*log( - sq
rt( - sqrt(3) + 2)*x + x**2 + 1) - 4*sqrt( - sqrt(3) + 2)*sqrt(3)*log(sqrt
( - sqrt(3) + 2)*x + x**2 + 1) + 6*sqrt( - sqrt(3) + 2)*log( - sqrt( - sqr
t(3) + 2)*x + x**2 + 1) - 6*sqrt( - sqrt(3) + 2)*log(sqrt( - sqrt(3) + 2)*
x + x**2 + 1) + sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) - s
qrt(6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2) - 3*sqrt(2)*log(( - sqr
t(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) + 3*sqrt(2)*log((sqrt(6)*x + sqrt(2)*x
 + 2*x**2 + 2)/2) - 16*x**3)/48
```

3.69 $\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 222

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -x - \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right)}{2\sqrt{6}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)}{2\sqrt{6}}$$

output
```
-x-1/12*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))*6^
(1/2)-1/12*arctan((1/2*6^(1/2)+1/2*2^(1/2)-2*x)/(1/2*6^(1/2)-1/2*2^(1/2)))
*6^(1/2)+1/12*arctan((1/2*6^(1/2)-1/2*2^(1/2)+2*x)/(1/2*6^(1/2)+1/2*2^(1/2
)))*6^(1/2)+1/12*arctan((1/2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)-1/2*2^(
1/2)))*6^(1/2)+1/12*arctanh((1/2*6^(1/2)-1/2*2^(1/2))*x/(x^2+1))*6^(1/2)+1
/12*arctanh((1/2*6^(1/2)+1/2*2^(1/2))*x/(x^2+1))*6^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.21

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -x+\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
-x + RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1826, 25, 1684, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(1-x^4)}{x^8-x^4+1}dx$$

$$\Big\downarrow \text{ 1826}$$

$$-\int -\frac{1}{x^8-x^4+1}dx-x$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{1}{x^8-x^4+1}dx-x$$

$$\Big\downarrow \text{ 1684}$$

$$\frac{\int \frac{\sqrt{3}-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-x$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{\frac{\int \frac{(1-\sqrt{3})x+\sqrt{3(2-\sqrt{3})}}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{3(2-\sqrt{3})}-(1-\sqrt{3})x}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{3(2+\sqrt{3})}-(1+\sqrt{3})x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{(1+\sqrt{3})x+\sqrt{3(2+\sqrt{3})}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-x$$

$\downarrow$ 1142

$$\frac{\frac{\frac{\int\frac{1}{x^2-\sqrt{2-\sqrt3}x+1}dx}{\sqrt2}+\frac12(1-\sqrt3)\int -\frac{\sqrt{2-\sqrt3}-2x}{x^2-\sqrt{2-\sqrt3}x+1}dx}{2\sqrt{2-\sqrt3}}+\frac{\frac{\int\frac{1}{x^2+\sqrt{2-\sqrt3}x+1}dx}{\sqrt2}-\frac12(1-\sqrt3)\int\frac{2x+\sqrt{2-\sqrt3}}{x^2+\sqrt{2-\sqrt3}x+1}dx}{2\sqrt{2-\sqrt3}}}{2\sqrt3}+\frac{\frac{\frac{\int\frac{1}{x^2-\sqrt{2+\sqrt3}x+1}dx}{\sqrt2}-\frac12(1+\sqrt3)\int-\frac{\sqrt{2+\sqrt3}-2x}{x^2-\sqrt{2+\sqrt3}x+1}dx}{2\sqrt{2+\sqrt3}}+\frac{\frac{\int\frac{1}{x^2+\sqrt{2+\sqrt3}x+1}dx}{\sqrt2}+\frac12(1+\sqrt3)\int\frac{2x+\sqrt{2+\sqrt3}}{x^2+\sqrt{2+\sqrt3}x+1}dx}{2\sqrt{2+\sqrt3}}}{2\sqrt3}-x$$

$\downarrow$ 25

$$\frac{\frac{\frac{\int\frac{1}{x^2-\sqrt{2-\sqrt3}x+1}dx}{\sqrt2}-\frac12(1-\sqrt3)\int \frac{\sqrt{2-\sqrt3}-2x}{x^2-\sqrt{2-\sqrt3}x+1}dx}{2\sqrt{2-\sqrt3}}+\frac{\frac{\int\frac{1}{x^2+\sqrt{2-\sqrt3}x+1}dx}{\sqrt2}-\frac12(1-\sqrt3)\int\frac{2x+\sqrt{2-\sqrt3}}{x^2+\sqrt{2-\sqrt3}x+1}dx}{2\sqrt{2-\sqrt3}}}{2\sqrt3}+\frac{\frac{\frac{\int\frac{1}{x^2-\sqrt{2+\sqrt3}x+1}dx}{\sqrt2}+\frac12(1+\sqrt3)\int\frac{\sqrt{2+\sqrt3}-2x}{x^2-\sqrt{2+\sqrt3}x+1}dx}{2\sqrt{2+\sqrt3}}+\frac{\frac{\int\frac{1}{x^2+\sqrt{2+\sqrt3}x+1}dx}{\sqrt2}+\frac12(1+\sqrt3)\int\frac{2x+\sqrt{2+\sqrt3}}{x^2+\sqrt{2+\sqrt3}x+1}dx}{2\sqrt{2+\sqrt3}}}{2\sqrt3}-x$$

$\downarrow$ 1083

$$\frac{\frac{-\frac12(1-\sqrt3)\int\frac{\sqrt{2-\sqrt3}-2x}{x^2-\sqrt{2-\sqrt3}x+1}dx-\sqrt2\int\frac{1}{-\left(2x-\sqrt{2-\sqrt3}\right)^2-\sqrt3-2}d\left(2x-\sqrt{2-\sqrt3}\right)}{2\sqrt{2-\sqrt3}}+\frac{-\frac12(1-\sqrt3)\int\frac{2x+\sqrt{2-\sqrt3}}{x^2+\sqrt{2-\sqrt3}x+1}dx-\sqrt2\int\frac{1}{-\left(2x+\sqrt{2-\sqrt3}\right)^2-\cdots}}{2\sqrt{2-\sqrt3}}}{2\sqrt3}$$

$$\frac{\frac{\frac12(1+\sqrt3)\int\frac{\sqrt{2+\sqrt3}-2x}{x^2-\sqrt{2+\sqrt3}x+1}dx-\sqrt2\int\frac{1}{-\left(2x-\sqrt{2+\sqrt3}\right)^2+\sqrt3-2}d\left(2x-\sqrt{2+\sqrt3}\right)}{2\sqrt{2+\sqrt3}}+\frac{\frac12(1+\sqrt3)\int\frac{2x+\sqrt{2+\sqrt3}}{x^2+\sqrt{2+\sqrt3}x+1}dx-\sqrt2\int\frac{1}{-\left(2x+\sqrt{2+\sqrt3}\right)^2+\sqrt3-2\cdots}}{2\sqrt{2+\sqrt3}}}{2\sqrt3}$$

x

$\downarrow$ 217

$$\frac{\frac{\sqrt{\frac{2}{2+\sqrt3}}\arctan\left(\frac{2x-\sqrt{2-\sqrt3}}{\sqrt{2+\sqrt3}}\right)-\frac12(1-\sqrt3)\int\frac{\sqrt{2-\sqrt3}-2x}{x^2-\sqrt{2-\sqrt3}x+1}dx}{2\sqrt{2-\sqrt3}}+\frac{\sqrt{\frac{2}{2+\sqrt3}}\arctan\left(\frac{2x+\sqrt{2-\sqrt3}}{\sqrt{2+\sqrt3}}\right)-\frac12(1-\sqrt3)\int\frac{2x+\sqrt{2-\sqrt3}}{x^2+\sqrt{2-\sqrt3}x+1}dx}{2\sqrt{2-\sqrt3}}}{2\sqrt3}+\frac{\frac{\frac12(1+\sqrt3)\int\frac{\sqrt{2+\sqrt3}-2x}{x^2-\sqrt{2+\sqrt3}x+1}dx+\sqrt{\frac{2}{2-\sqrt3}}\arctan\left(\frac{2x-\sqrt{2+\sqrt3}}{\sqrt{2-\sqrt3}}\right)}{2\sqrt{2+\sqrt3}}+\frac{\frac12(1+\sqrt3)\int\frac{2x+\sqrt{2+\sqrt3}}{x^2+\sqrt{2+\sqrt3}x+1}dx+\sqrt{\frac{2}{2-\sqrt3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt3}}{\sqrt{2-\sqrt3}}\right)}{2\sqrt{2+\sqrt3}}}{2\sqrt3}-x$$

$\downarrow$ 1103

$$\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+$$

$$\frac{\frac{\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-$$

$$x$$

input
```
Int[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
-x + ((Sqrt[2/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sq
rt[3]]] + ((1 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2
- Sqrt[3]]) + (Sqrt[2/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt
[2 + Sqrt[3]]] - ((1 - Sqrt[3])*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*
Sqrt[2 - Sqrt[3]]))/(2*Sqrt[3]) + ((Sqrt[2/(2 - Sqrt[3])]*ArcTan[(-Sqrt[2
+ Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((1 + Sqrt[3])*Log[1 - Sqrt[2 + Sqr
t[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (Sqrt[2/(2 - Sqrt[3])]*ArcTan[(
Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + ((1 + Sqrt[3])*Log[1 + Sqrt[
2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1684
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x^(
n/2))/(q - r*x^(n/2) + x^n), x], x] + Simp[1/(2*c*q*r)   Int[(r + x^(n/2))/
(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.15

method	result	size
default	$-x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^{4}+1\right)}\_R\ln\left(3\_R^{2}+3\_Rx+x^{2}\right)\right)}{4}$	34
risch	$-x+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^{4}+1\right)}\_R\ln\left(3\_R^{2}+3\_Rx+x^{2}\right)\right)}{4}$	34

input
```
int(x^4*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-x+1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.59

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx=\frac{1}{4}\sqrt{\frac{2}{3}}\arctan\left(4\,x^2+3\sqrt{\frac{2}{3}}(x^3+2\,x)+3\right)+\frac{1}{4}\sqrt{\frac{2}{3}}\arctan\left(-4\,x^2+3\sqrt{\frac{2}{3}}(x^3+2\,x)-3\right)+\frac{1}{4}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{2}{3}}x+\frac{1}{3}\right)+\frac{1}{4}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{2}{3}}x-\frac{1}{3}\right)+\frac{1}{8}\sqrt{\frac{2}{3}}\log\left(x^4+3\,x^2+3\sqrt{\frac{2}{3}}(x^3+x)+1\right)-\frac{1}{8}\sqrt{\frac{2}{3}}\log\left(x^4+3\,x^2-3\sqrt{\frac{2}{3}}(x^3+x)+1\right)-x$$

input
```
integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(2/3)*arctan(4*x^2 + 3*sqrt(2/3)*(x^3 + 2*x) + 3) + 1/4*sqrt(2/3)*
arctan(-4*x^2 + 3*sqrt(2/3)*(x^3 + 2*x) - 3) + 1/4*sqrt(2/3)*arctan(sqrt(2
/3)*x + 1/3) + 1/4*sqrt(2/3)*arctan(sqrt(2/3)*x - 1/3) + 1/8*sqrt(2/3)*log
(x^4 + 3*x^2 + 3*sqrt(2/3)*(x^3 + x) + 1) - 1/8*sqrt(2/3)*log(x^4 + 3*x^2
- 3*sqrt(2/3)*(x^3 + x) + 1) - x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.77

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -x - \frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} - \frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} - \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} + \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

input
```
integrate(x**4*(-x**4+1)/(x**8-x**4+1),x)
```

output
```
-x - sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 +
2*sqrt(6)*x - 3))/24 - sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6
)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 +
 3*x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + s
qrt(6)*x + 1)/24
```

Maxima [F]

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = \int -\frac{(x^4-1)x^4}{x^8-x^4+1}\,dx$$

input
```
integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-x + integrate(1/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.94

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - x$$

input
```
integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(
6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*ar
ctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2
 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6)
+ sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) -
1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - x
```

Mupad [B] (verification not implemented)

Time = 20.83 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.25

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -x+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

input
```
int(-(x^4*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

output
```
- x - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i
/12) - 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1
i/12)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.43

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} - \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} + \frac{\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{4} - \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{12} + \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{12} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{8} - \frac{\sqrt{6}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24} + \frac{\sqrt{6}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{24} - x$$

input
```
int(x^4*(-x^4+1)/(x^8-x^4+1),x)
```

output
```
( - 2*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt(
 - sqrt(3) + 2))) - 6*sqrt( - sqrt(3) + 2)*atan((sqrt(6) + sqrt(2) - 4*x)/
(2*sqrt( - sqrt(3) + 2))) + 2*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) +
 sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) + 6*sqrt( - sqrt(3) + 2)*atan((s
qrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2))) - 2*sqrt(6)*atan((2*sqrt
( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2))) + 2*sqrt(6)*atan((2*sqrt( - s
qrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2))) - sqrt( - sqrt(3) + 2)*sqrt(3)*log
( - sqrt( - sqrt(3) + 2)*x + x**2 + 1) + sqrt( - sqrt(3) + 2)*sqrt(3)*log(
sqrt( - sqrt(3) + 2)*x + x**2 + 1) - 3*sqrt( - sqrt(3) + 2)*log( - sqrt( -
 sqrt(3) + 2)*x + x**2 + 1) + 3*sqrt( - sqrt(3) + 2)*log(sqrt( - sqrt(3) +
 2)*x + x**2 + 1) - sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2)
 + sqrt(6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2) - 24*x)/24
```

3.70 $\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 285

$$\begin{aligned}\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx &= \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)\end{aligned}$$

output
```
1/4*(1/2*2^(1/2)+1/6*6^(1/2))*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1/2*6^
(1/2)+1/2*2^(1/2)))-1/4*(1/2*2^(1/2)-1/6*6^(1/2))*arctan((1/2*6^(1/2)+1/2*
2^(1/2)-2*x)/(1/2*6^(1/2)-1/2*2^(1/2)))-1/4*(1/2*2^(1/2)+1/6*6^(1/2))*arct
an((1/2*6^(1/2)-1/2*2^(1/2)+2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))+1/4*(1/2*2^(1/
2)-1/6*6^(1/2))*arctan((1/2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)-1/2*2^(1
/2)))-1/4*(1/2*2^(1/2)-1/6*6^(1/2))*arctanh((1/2*6^(1/2)-1/2*2^(1/2))*x/(x
^2+1))+1/4*(1/2*2^(1/2)+1/6*6^(1/2))*arctanh((1/2*6^(1/2)+1/2*2^(1/2))*x/(
x^2+1))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.19

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = -\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1+2\#1^5}\&\right]$$

input
```
Integrate[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
-1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1 +
2*#1^5) & ]
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1830, 1602, 25, 1483, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-x^4)}{x^8-x^4+1}\,dx$$

$$\Big\downarrow \text{ 1830}$$

$$\frac{\int \frac{x^2\left(\sqrt{3}-2x^2\right)}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{x^2\left(2x^2+\sqrt{3}\right)}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \; 1602$$

$$\frac{-\int -\frac{2-\sqrt{3}x^2}{x^4-\sqrt{3}x^2+1}dx-2x}{2\sqrt{3}}+\frac{2x-\int \frac{\sqrt{3}x^2+2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2-\sqrt{3}x^2}{x^4-\sqrt{3}x^2+1}dx-2x}{2\sqrt{3}}+\frac{2x-\int \frac{\sqrt{3}x^2+2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \; 1483$$

$$\frac{-\frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}-\frac{\int \frac{\sqrt{2-\sqrt{3}}\left(\sqrt{2-\sqrt{3}}x+2\right)}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+2x}{2\sqrt{3}}+\frac{\frac{\int \frac{2\sqrt{2+\sqrt{3}}-\left(2+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}\left(\sqrt{2+\sqrt{3}}x+2\right)}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}-2x}{2\sqrt{3}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}-\frac{1}{2}\int \frac{\sqrt{2-\sqrt{3}}x+2}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+2x}{2\sqrt{3}}+\frac{\frac{\int \frac{2\sqrt{2+\sqrt{3}}-\left(2+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\int \frac{\sqrt{2+\sqrt{3}}x+2}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-2x}{2\sqrt{3}}$$

$$\Big\downarrow \; 1142$$

$$\frac{-\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{2x+\cdots}{x^2+\sqrt{\cdots}}\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\left(2+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\cdots}}{x^2+\sqrt{2+\sqrt{\cdots}}}\right.}{2\sqrt{3}}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{2x+\sqrt{\cdots}}{x^2+\sqrt{2-\cdots}}\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\cdots}}{x^2+\sqrt{2+\sqrt{3}}x\cdots}\right.}{2\sqrt{3}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(\left(2+\sqrt{3}\right)\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x\cdots\right.\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-(2-\sqrt{\cdots}\right.}{2\sqrt{3}}$$

$$\Big\downarrow \text{217}$$

$$\frac{-\frac{\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2-\cdots}}{\sqrt{2+\sqrt{3}}}\right.\right.}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)\cdots\right.}{2\sqrt{3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{1}{2}\left(-\sqrt{2+\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\sqrt{2-\sqrt{3}}\log\left(x^2+\sqrt{2\cdots}\right.\right.}{2\sqrt{3}}$$

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{1}{2}\left(\sqrt{2-\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\sqrt{2+\sqrt{3}}\log\left(x^2+\sqrt{2+\cdots}\right.\right.}{2\sqrt{3}}$$

input

```
Int[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

output
```
(2*x - (ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] - ((2 - Sqrt[
3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]) + (-(Sqrt
[2 + Sqrt[3]]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]) - (Sqrt
[2 - Sqrt[3]]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/2)/(2*Sqrt[3]) + (-2*
x + (ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((2 + Sqrt[3])
*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (Sqrt[2 -
Sqrt[3]]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] + (Sqrt[2 + S
qrt[3]]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/2)/2)/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1830
```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Simp[c/(2*q*r)   Int[(f*x)^m*(Simp[d*r - (c*d - e*q)*x^(n/2), x]
/(q - r*x^(n/2) + c*x^n)), x], x] + Simp[c/(2*q*r)   Int[(f*x)^m*(Simp[d*r
+ (c*d - e*q)*x^(n/2), x]/(q + r*x^(n/2) + c*x^n)), x], x]] /;  !LtQ[2*c*q
- b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a
*c, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.16

method	result	size
default	$-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{6}-\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	46
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(-\_R^{6}+\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	46

input
```
int(x^2*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*sum((_R^6-_R^2)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.31

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/3)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/3)*sqrt(3/2*s
qrt(-1/3) + 1/2)*(sqrt(-1/3) - 1)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2)) + 2*x)
 - 1/4*sqrt(1/3)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3)*sqrt(3
/2*sqrt(-1/3) + 1/2)*(sqrt(-1/3) - 1)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2)) +
2*x) - 1/4*sqrt(1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/3)*(s
qrt(-1/3) + 1)*sqrt(-3/2*sqrt(-1/3) + 1/2)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/
2)) + 2*x) + 1/4*sqrt(1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(
1/3)*(sqrt(-1/3) + 1)*sqrt(-3/2*sqrt(-1/3) + 1/2)*sqrt(-sqrt(-3/2*sqrt(-1/
3) + 1/2)) + 2*x) - 1/4*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4)*log(3*sqrt(
1/3)*(3/2*sqrt(-1/3) + 1/2)^(3/4)*(sqrt(-1/3) - 1) + 2*x) + 1/4*sqrt(1/3)*
(3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(3/4)
*(sqrt(-1/3) - 1) + 2*x) + 1/4*sqrt(1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1/4)*log
(3*sqrt(1/3)*(sqrt(-1/3) + 1)*(-3/2*sqrt(-1/3) + 1/2)^(3/4) + 2*x) - 1/4*s
qrt(1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*(sqrt(-1/3) + 1)*(
-3/2*sqrt(-1/3) + 1/2)^(3/4) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.09

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$$
$$= -\operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(442368t^7-384t^3+x\right)\right)\right)$$

input
```
integrate(x**2*(-x**4+1)/(x**8-x**4+1),x)
```

output
```
-RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(442368*_t**7 -
384*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = \int -\frac{(x^4-1)x^2}{x^8-x^4+1}\,dx$$

input
```
integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)*x^2/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.89

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = -\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

input
```
integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sq
rt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt
(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(
6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*
x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(s
qrt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(
6) - sqrt(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 20.82 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.87

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} + \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}-\frac{2^{3/4}\sqrt{3}\,x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{12} - \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}+\frac{2^{3/4}\sqrt{3}\,x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(-(x^2*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

output
```
(3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)) + (3^(1/2)*x
*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4)*1
i)/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) -
 (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)
^(1/4))/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x)/(2*(3^(1/2)*1i + 1)^(3/4))
- (2^(3/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4
)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4))
 + (2^(3/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4))
/12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.05

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} + \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} - \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{24} - \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} - \frac{\sqrt{6}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{48} + \frac{\sqrt{6}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{48} - \frac{\sqrt{2}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{16} + \frac{\sqrt{2}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{16}$$

input
```
int(x^2*(-x^4+1)/(x^8-x^4+1),x)
```

output
```
( - 4*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt(
 - sqrt(3) + 2))) + 4*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2)
 + 4*x)/(2*sqrt( - sqrt(3) + 2))) + 2*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2)
 - 4*x)/(sqrt(6) + sqrt(2))) + 6*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*
x)/(sqrt(6) + sqrt(2))) - 2*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(s
qrt(6) + sqrt(2))) - 6*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6
) + sqrt(2))) + 2*sqrt( - sqrt(3) + 2)*sqrt(3)*log( - sqrt( - sqrt(3) + 2)
*x + x**2 + 1) - 2*sqrt( - sqrt(3) + 2)*sqrt(3)*log(sqrt( - sqrt(3) + 2)*x
 + x**2 + 1) - sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) + sq
rt(6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2) - 3*sqrt(2)*log(( - sqrt
(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) + 3*sqrt(2)*log((sqrt(6)*x + sqrt(2)*x
+ 2*x**2 + 2)/2))/48
```

3.71 $\int \frac{1-x^4}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 285

$$
\begin{aligned}
\int \frac{1-x^4}{1-x^4+x^8}\,dx = & -\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) \\
& +\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) \\
& +\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right) \\
& -\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right) \\
& -\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right) \\
& +\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)
\end{aligned}
$$

output
```
-1/4*(1/2*2^(1/2)+1/6*6^(1/2))*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1/2*6
^(1/2)+1/2*2^(1/2)))+1/4*(1/2*2^(1/2)-1/6*6^(1/2))*arctan((1/2*6^(1/2)+1/2
*2^(1/2)-2*x)/(1/2*6^(1/2)-1/2*2^(1/2)))+1/4*(1/2*2^(1/2)+1/6*6^(1/2))*arc
tan((1/2*6^(1/2)-1/2*2^(1/2)+2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))-1/4*(1/2*2^(1
/2)-1/6*6^(1/2))*arctan((1/2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)-1/2*2^(
1/2)))-1/4*(1/2*2^(1/2)-1/6*6^(1/2))*arctanh((1/2*6^(1/2)-1/2*2^(1/2))*x/(
x^2+1))+1/4*(1/2*2^(1/2)+1/6*6^(1/2))*arctanh((1/2*6^(1/2)+1/2*2^(1/2))*x/
(x^2+1))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.20

$$\int \frac{1-x^4}{1-x^4+x^8} \, dx = -\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

input
```
Integrate[(1 - x^4)/(1 - x^4 + x^8),x]
```

output
```
-1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3
+ 2*#1^7) & ]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1751, 25, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^8-x^4+1} \, dx$$

$$\Big\downarrow \text{ 1751}$$

$$-\frac{\int -\frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\sqrt{3}-2x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}$$

$\Big\downarrow$ 1483

$$\frac{\frac{\int \frac{\left(2-\sqrt{3}\right)x+\sqrt{3\left(2-\sqrt{3}\right)}}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(2-\sqrt{3}\right)x}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(2+\sqrt{3}\right)x+\sqrt{3\left(2+\sqrt{3}\right)}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\Big\downarrow$ 1142

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\left(2-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\left(2+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{3}}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{1}{2}\sqrt{2-\sqrt{3}}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$\Big\downarrow$ 1083

$$\frac{\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{-\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2+\sqrt{3}}\int \frac{\cdots}{-\left(2x+\cdots\right.}\cdots}{2\sqrt{2-\sqrt{3}}\cdots}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2-\sqrt{3}}\int \frac{1\cdots}{-\left(2x+\sqrt{2+\cdots}\right.}\cdots}{2\sqrt{2+\sqrt{3}}\cdots}}{2\sqrt{3}}$$

$\Big\downarrow$ 217

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\int\frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\int\frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(2-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(2+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

input
```
Int[(1 - x^4)/(1 - x^4 + x^8),x]
```

output
```
((ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] + ((2 - Sqrt[3])*Lo
g[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]) + (ArcTan[(Sqrt
[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] - ((2 - Sqrt[3])*Log[1 + Sqrt[2 -
Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]))/(2*Sqrt[3]) + ((-ArcTan[(-Sqr
t[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]] - ((2 + Sqrt[3])*Log[1 - Sqrt[2 +
 Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (-ArcTan[(Sqrt[2 + Sqrt[3]]
 + 2*x)/Sqrt[2 - Sqrt[3]]] + ((2 + Sqrt[3])*Log[1 + Sqrt[2 + Sqrt[3]]*x +
x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1751
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x
^(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Simp[e/(2*c*q)   Int[(q +
2*x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGt
Q[n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(-\_R^{4}+1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44

input
```
int((-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.18

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = \text{Too large to display}$$

input
```
integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/4*sqrt(1/3)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/3)*(sqrt(-1/
3) - 1)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2)) + 2*x) + 1/4*sqrt(1/3)*sqrt(-sqr
t(3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3)*(sqrt(-1/3) - 1)*sqrt(-sqrt(3/2*
sqrt(-1/3) + 1/2)) + 2*x) + 1/4*sqrt(1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2
))*log(3*sqrt(1/3)*(sqrt(-1/3) + 1)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2)) + 2
*x) - 1/4*sqrt(1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3)*(s
qrt(-1/3) + 1)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2)) + 2*x) - 1/4*sqrt(1/3)*(
3/2*sqrt(-1/3) + 1/2)^(1/4)*log(3*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4)*(
sqrt(-1/3) - 1) + 2*x) + 1/4*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3
*sqrt(1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4)*(sqrt(-1/3) - 1) + 2*x) + 1/4*sqrt
(1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1/4)*log(3*sqrt(1/3)*(sqrt(-1/3) + 1)*(-3/2
*sqrt(-1/3) + 1/2)^(1/4) + 2*x) - 1/4*sqrt(1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1
/4)*log(-3*sqrt(1/3)*(sqrt(-1/3) + 1)*(-3/2*sqrt(-1/3) + 1/2)^(1/4) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.09

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(9216t^5 - 8t + x\right)\right)\right)$$

input
```
integrate((-x**4+1)/(x**8-x**4+1),x)
```

output
```
-RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 8*
_t + x)))
```

Maxima [F]

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-x^4+1}\,dx$$

input
```
integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.89

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

input
```
integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

output
```
1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(
2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6
) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sq
rt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6
) - sqrt(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.73

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}-\frac{\sqrt{3}\,x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\,\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1\mathrm{i}}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

input
```
int(-(x^4 - 1)/(x^8 - x^4 + 1),x)
```

output
```
(2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^
(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3
^(1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(
1/4))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4)
+ (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (
2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*
3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.05

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+4x}{2\sqrt{-\sqrt{3}+2}}\right)}{12} - \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{24} - \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}-4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{6}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{24} + \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{-\sqrt{3}+2}+4x}{\sqrt{6}+\sqrt{2}}\right)}{8} + \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(-\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} - \frac{\sqrt{-\sqrt{3}+2}\,\sqrt{3}\log\left(\sqrt{-\sqrt{3}+2}\,x+x^2+1\right)}{24} - \frac{\sqrt{6}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{48} + \frac{\sqrt{6}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{48} - \frac{\sqrt{2}\log\left(-\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{16} + \frac{\sqrt{2}\log\left(\frac{\sqrt{6}\,x}{2}+\frac{\sqrt{2}\,x}{2}+x^2+1\right)}{16}$$

input
```
int((-x^4+1)/(x^8-x^4+1),x)
```

output
```
(4*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( -
sqrt(3) + 2))) - 4*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) +
4*x)/(2*sqrt( - sqrt(3) + 2))) - 2*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) -
4*x)/(sqrt(6) + sqrt(2))) - 6*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/
(sqrt(6) + sqrt(2))) + 2*sqrt(6)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt
(6) + sqrt(2))) + 6*sqrt(2)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) +
 sqrt(2))) + 2*sqrt( - sqrt(3) + 2)*sqrt(3)*log( - sqrt( - sqrt(3) + 2)*x
+ x**2 + 1) - 2*sqrt( - sqrt(3) + 2)*sqrt(3)*log(sqrt( - sqrt(3) + 2)*x +
x**2 + 1) - sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2) + sqrt(
6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2) - 3*sqrt(2)*log(( - sqrt(6)
*x - sqrt(2)*x + 2*x**2 + 2)/2) + 3*sqrt(2)*log((sqrt(6)*x + sqrt(2)*x + 2
*x**2 + 2)/2))/48
```

3.72 $\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 224

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx = -\frac{1}{x}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right)}{2\sqrt{6}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)}{2\sqrt{6}}$$

output
```
-1/x+1/12*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))*
6^(1/2)+1/12*arctan((1/2*6^(1/2)+1/2*2^(1/2)-2*x)/(1/2*6^(1/2)-1/2*2^(1/2)
))*6^(1/2)-1/12*arctan((1/2*6^(1/2)-1/2*2^(1/2)+2*x)/(1/2*6^(1/2)+1/2*2^(1
/2)))*6^(1/2)-1/12*arctan((1/2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)-1/2*2
^(1/2)))*6^(1/2)+1/12*arctanh((1/2*6^(1/2)-1/2*2^(1/2))*x/(x^2+1))*6^(1/2)
+1/12*arctanh((1/2*6^(1/2)+1/2*2^(1/2))*x/(x^2+1))*6^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.21

$$\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx = -\frac{1}{x}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^3}{-1+2\#1^4}\&\right]$$

input
```
Integrate[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

output
```
-x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]
/4
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1828, 1708, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^2(x^8-x^4+1)}dx$$

$$\Big\downarrow \text{1828}$$

$$-\int \frac{x^6}{x^8-x^4+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{1708}$$

$$\frac{\int \frac{1-\sqrt{3}x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{\int \frac{\sqrt{3}x^2+1}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{1483}$$

$$-\frac{\frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int \frac{\left(1-\sqrt{3}\right)x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int \frac{\left(1+\sqrt{3}\right)x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{x}$$

$\Big\downarrow$ 1142

$$-\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{3}\right)\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}-\frac{1}{2}\left(1+\sqrt{3}\right)\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{x}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\frac{\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{\sqrt{2}}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{x}$$

$\Big\downarrow$ 1083

$$-\frac{\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx-\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{\cdots}}}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2}\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx+\sqrt{2}\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-\cdots}}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}$$

$$\frac{1}{x}$$

$\Big\downarrow$ 217

$$-\frac{\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx+\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+$$

$$\frac{\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-$$

$$\frac{1}{x}$$

$\Big\downarrow$ 1103

$$-\frac{\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}+\frac{\sqrt{\frac{2}{2+\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(1-\sqrt{3}\right)\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}}{2\sqrt{3}}+\frac{\frac{-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}+\frac{\frac{1}{2}\left(1+\sqrt{3}\right)\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\sqrt{\frac{2}{2-\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}}{2\sqrt{3}}-\frac{1}{x}$$

input
```
Int[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

output
```
-x^(-1) - ((Sqrt[2/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2
 + Sqrt[3]]] - ((1 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sq
rt[2 - Sqrt[3]]) + (Sqrt[2/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)
/Sqrt[2 + Sqrt[3]]] + ((1 - Sqrt[3])*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2
)/(2*Sqrt[2 - Sqrt[3]]))/(2*Sqrt[3]) + ((-(Sqrt[2/(2 - Sqrt[3])]*ArcTan[(-
Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]) - ((1 + Sqrt[3])*Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (-(Sqrt[2/(2 - Sqrt[3])
]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]) + ((1 + Sqrt[3])*Lo
g[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]))/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1708
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Simp[1/(2*c*r)   Int[x^
(m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Simp[1/(2*c
*r)   Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]]
 /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2,
 0] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.17

method	result	size
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}-\frac{1}{x}$	38
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(-9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	38

input
```
int((-x^4+1)/x^2/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))-1/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.62

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx =$$
$$-\frac{2\sqrt{\frac{2}{3}}x\arctan\left(4x^2+3\sqrt{\frac{2}{3}}(x^3+2x)+3\right)+2\sqrt{\frac{2}{3}}x\arctan\left(-4x^2+3\sqrt{\frac{2}{3}}(x^3+2x)-3\right)+2\sqrt{\frac{2}{3}}x}{}$$

input
```
integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(2/3)*x*arctan(4*x^2 + 3*sqrt(2/3)*(x^3 + 2*x) + 3) + 2*sqrt(2
/3)*x*arctan(-4*x^2 + 3*sqrt(2/3)*(x^3 + 2*x) - 3) + 2*sqrt(2/3)*x*arctan(
sqrt(2/3)*x + 1/3) + 2*sqrt(2/3)*x*arctan(sqrt(2/3)*x - 1/3) - sqrt(2/3)*x
*log(x^4 + 3*x^2 + 3*sqrt(2/3)*(x^3 + x) + 1) + sqrt(2/3)*x*log(x^4 + 3*x^
2 - 3*sqrt(2/3)*(x^3 + x) + 1) + 8)/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.75

$$\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx = -\frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} - \frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} - \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} + \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24} - \frac{1}{x}$$

input
```
integrate((-x**4+1)/x**2/(x**8-x**4+1),x)
```

output
```
-sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqr
t(6)*x - 3))/24 - sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3
 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**
2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)
*x + 1)/24 - 1/x
```

Maxima [F]

$$\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx = \int -\frac{x^4-1}{(x^8-x^4+1)x^2}\,dx$$

input
```
integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/x - integrate(x^6/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.94

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx = -\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{x}$$

input
```
integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12
*sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt
(6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*a
rctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^
2 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6)
 + sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) -
 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/x
```

Mupad [B] (verification not implemented)

Time = 20.85 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.26

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx = -\frac{1}{x}+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right) + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

input
```
int(-(x^4 - 1)/(x^2*(x^8 - x^4 + 1)),x)
```

output
```
6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) +
 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12)
- 1/x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.48

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx = \frac{2\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x+6\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x-2\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}+}{2\sqrt{-\sqrt{3}+}}\right.}{}$$

input
```
int((-x^4+1)/x^2/(x^8-x^4+1),x)
```

output
```
(2*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( -
sqrt(3) + 2)))*x + 6*sqrt( - sqrt(3) + 2)*atan((sqrt(6) + sqrt(2) - 4*x)/(
2*sqrt( - sqrt(3) + 2)))*x - 2*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6)
+ sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*x - 6*sqrt( - sqrt(3) + 2)*atan
((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*x + 2*sqrt(6)*atan((2
*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2)))*x - 2*sqrt(6)*atan((2*sq
rt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2)))*x - sqrt( - sqrt(3) + 2)*sq
rt(3)*log( - sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x + sqrt( - sqrt(3) + 2)*s
qrt(3)*log(sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x - 3*sqrt( - sqrt(3) + 2)*l
og( - sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x + 3*sqrt( - sqrt(3) + 2)*log(sq
rt( - sqrt(3) + 2)*x + x**2 + 1)*x - sqrt(6)*log(( - sqrt(6)*x - sqrt(2)*x
 + 2*x**2 + 2)/2)*x + sqrt(6)*log((sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2)*
x - 24)/(24*x)
```

3.73 $\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 280

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx=-\frac{1}{3x^3}-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}x}{1+x^2}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{3}}x}{1+x^2}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

output
```
-1/3/x^3-1/4*arctan((1/2*6^(1/2)-1/2*2^(1/2)-2*x)/(1/2*6^(1/2)+1/2*2^(1/2)
))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((1/2*6^(1/2)+1/2*2^(1/2)-2*x)/(1/2
*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))+1/4*arctan((1/2*6^(1/2)-1
/2*2^(1/2)+2*x)/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*a
rctan((1/2*6^(1/2)+1/2*2^(1/2)+2*x)/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2
)-1/2*6^(1/2))-1/4*arctanh((1/2*6^(1/2)-1/2*2^(1/2))*x/(x^2+1))/(3/2*2^(1/
2)-1/2*6^(1/2))+1/4*arctanh((1/2*6^(1/2)+1/2*2^(1/2))*x/(x^2+1))/(3/2*2^(1
/2)+1/2*6^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.17

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx=-\frac{1}{3x^3}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1}{-1+2\#1^4}\&\right]$$

input
```
Integrate[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) &
]/4
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1828, 27, 1709, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^4\left(x^8-x^4+1\right)}dx$$

$$\Big\downarrow \text{ 1828}$$

$$-\frac{1}{3}\int \frac{3x^4}{x^8-x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{x^4}{x^8-x^4+1}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{ 1709}$$

$$-\frac{\int \frac{x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\int \frac{x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{ 1447}$$

$$-\frac{\frac{1}{2}\int \frac{x^2+1}{x^4-\sqrt{3}x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{x^2+1}{x^4+\sqrt{3}x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1475}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2+\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2+\sqrt{3}}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2-\sqrt{3}}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2-\sqrt{3}}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x-\sqrt{2-\sqrt{3}}\right)-\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{3}}\right)^2-\sqrt{3}-2}d\left(2x+\sqrt{2-\sqrt{3}}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}}{2\sqrt{3}}$$

$$\frac{\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x-\sqrt{2+\sqrt{3}}\right)-\int \frac{1}{-\left(2x+\sqrt{2+\sqrt{3}}\right)^2+\sqrt{3}-2}d\left(2x+\sqrt{2+\sqrt{3}}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}}{2\sqrt{3}}$$

$$\frac{1}{3x^3}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-\sqrt{3}x^2+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+\sqrt{3}x^2+1}dx}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1478}$$

$$\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}+\frac{\int -\frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-$$

$$\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}+\frac{\int -\frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-$$

$$\frac{1}{3x^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2-\sqrt{3}}-2x}{x^2-\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}-\frac{\int \frac{2x+\sqrt{2-\sqrt{3}}}{x^2+\sqrt{2-\sqrt{3}}x+1}dx}{2\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2+\sqrt{3}}-2x}{x^2-\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}-\frac{\int \frac{2x+\sqrt{2+\sqrt{3}}}{x^2+\sqrt{2+\sqrt{3}}x+1}dx}{2\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{2\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{2\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}-\frac{1}{3x^3}$$

input
```
Int[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

output
```
-1/3*1/x^3 + ((ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[2
 + Sqrt[3]] + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[2 +
 Sqrt[3]])/2 + (Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2 - Sqrt[3]]) -
 Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(2*Sqrt[2 - Sqrt[3]]))/2)/(2*Sqrt[3])
- ((ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2 - Sqrt[3]]
 + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/Sqrt[2 - Sqrt[3]])/
2 + (Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(2*Sqrt[2 + Sqrt[3]]) - Log[1 + Sq
rt[2 + Sqrt[3]]*x + x^2]/(2*Sqrt[2 + Sqrt[3]]))/2)/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1709
```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(
m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Simp[1/(2*c*r)   Int[x^(m - n/2)/(
q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*
(n/2)] && NegQ[b^2 - 4*a*c]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.14

method	result	size
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(18\_R^5-\_R+x\right)\right)}{4}$	38
default	$-\frac{1}{3x^3}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	46

input
```
int((-x^4+1)/x^4/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+1/4*sum(_R*ln(18*_R^5-_R+x),_R=RootOf(81*_Z^8-9*_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.20

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx$$
$$= \frac{3\sqrt{\frac{1}{3}}x^3\sqrt{-\sqrt{\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}\log\left(3\sqrt{\frac{1}{3}}\sqrt{-\frac{1}{3}}\sqrt{-\sqrt{\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}+x\right)-3\sqrt{\frac{1}{3}}x^3\sqrt{-\sqrt{\frac{3}{2}\sqrt{-\frac{1}{3}}+\frac{1}{2}}}\log\left(-\right.}{}$$

input
```
integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="fricas")
```

output
```
1/12*(3*sqrt(1/3)*x^3*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2))*log(3*sqrt(1/3)*sq
rt(-1/3)*sqrt(-sqrt(3/2*sqrt(-1/3) + 1/2)) + x) - 3*sqrt(1/3)*x^3*sqrt(-sq
rt(3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3)*sqrt(-1/3)*sqrt(-sqrt(3/2*sqrt(
-1/3) + 1/2)) + x) - 3*sqrt(1/3)*x^3*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*lo
g(3*sqrt(1/3)*sqrt(-1/3)*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2)) + x) + 3*sqrt(
1/3)*x^3*sqrt(-sqrt(-3/2*sqrt(-1/3) + 1/2))*log(-3*sqrt(1/3)*sqrt(-1/3)*sq
rt(-sqrt(-3/2*sqrt(-1/3) + 1/2)) + x) + 3*sqrt(1/3)*x^3*(3/2*sqrt(-1/3) +
1/2)^(1/4)*log(3*sqrt(1/3)*sqrt(-1/3)*(3/2*sqrt(-1/3) + 1/2)^(1/4) + x) -
3*sqrt(1/3)*x^3*(3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*sqrt(-1/3)*(
3/2*sqrt(-1/3) + 1/2)^(1/4) + x) - 3*sqrt(1/3)*x^3*(-3/2*sqrt(-1/3) + 1/2)
^(1/4)*log(3*sqrt(1/3)*sqrt(-1/3)*(-3/2*sqrt(-1/3) + 1/2)^(1/4) + x) + 3*s
qrt(1/3)*x^3*(-3/2*sqrt(-1/3) + 1/2)^(1/4)*log(-3*sqrt(1/3)*sqrt(-1/3)*(-3
/2*sqrt(-1/3) + 1/2)^(1/4) + x) - 4)/x^3
```

Sympy [A] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.11

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx$$
$$= -\operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-18432t^5+4t+x\right)\right)\right)-\frac{1}{3x^3}$$

input
```
integrate((-x**4+1)/x**4/(x**8-x**4+1),x)
```

output
```
-RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 +
4*_t + x))) - 1/(3*x**3)
```

Maxima [F]

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx = \int -\frac{x^4-1}{(x^8-x^4+1)x^4}\,dx$$

input
```
integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="maxima")
```

output
```
-1/3/x^3 - integrate(x^4/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.92

$$\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx = -\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{3x^3}$$

input
```
integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="giac")
```

output
```
-1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sq
rt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt
(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(
6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*
x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(s
qrt(6) - sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(
6) - sqrt(2)) + 1) - 1/3/x^3
```

Mupad [B] (verification not implemented)

Time = 20.51 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.71

$$\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx = \text{Too large to display}$$

input
```
int(-(x^4 - 1)/(x^4*(x^8 - x^4 + 1)),x)
```

output
```
(3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/
2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*
1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4))
)*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 - 1/(3*x^3) + (3^(1/2)*atan((x*(8 - 3^(1/2
)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8
i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2
)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12
 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(
3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2))
 - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i
+ 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*
1i + 1)^(1/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(
1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2
)*1i + 1)^(1/2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((
2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2
)*1i)/2)))*(3^(1/2)*1i + 1)^(1/4))/12
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.64

$$\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx = \frac{8\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x^3+12\sqrt{-\sqrt{3}+2}\,atan\left(\frac{\sqrt{6}+\sqrt{2}-4x}{2\sqrt{-\sqrt{3}+2}}\right)x^3-8\sqrt{-\sqrt{3}+2}\,\sqrt{3}\,atan\left(\frac{\sqrt{6}+}{2\sqrt{-}}\right.}{}$$

input
```
int((-x^4+1)/x^4/(x^8-x^4+1),x)
```

output
```
(8*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((sqrt(6) + sqrt(2) - 4*x)/(2*sqrt( -
sqrt(3) + 2)))*x**3 + 12*sqrt( - sqrt(3) + 2)*atan((sqrt(6) + sqrt(2) - 4*
x)/(2*sqrt( - sqrt(3) + 2)))*x**3 - 8*sqrt( - sqrt(3) + 2)*sqrt(3)*atan((s
qrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*x**3 - 12*sqrt( - sqrt(3
) + 2)*atan((sqrt(6) + sqrt(2) + 4*x)/(2*sqrt( - sqrt(3) + 2)))*x**3 + 2*s
qrt(6)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2)))*x**3 - 6*s
qrt(2)*atan((2*sqrt( - sqrt(3) + 2) - 4*x)/(sqrt(6) + sqrt(2)))*x**3 - 2*s
qrt(6)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2)))*x**3 + 6*s
qrt(2)*atan((2*sqrt( - sqrt(3) + 2) + 4*x)/(sqrt(6) + sqrt(2)))*x**3 + 4*s
qrt( - sqrt(3) + 2)*sqrt(3)*log( - sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x**3
 - 4*sqrt( - sqrt(3) + 2)*sqrt(3)*log(sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x
**3 + 6*sqrt( - sqrt(3) + 2)*log( - sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x**
3 - 6*sqrt( - sqrt(3) + 2)*log(sqrt( - sqrt(3) + 2)*x + x**2 + 1)*x**3 + s
qrt(6)*log(( - sqrt(6)*x - sqrt(2)*x + 2*x**2 + 2)/2)*x**3 - sqrt(6)*log((
sqrt(6)*x + sqrt(2)*x + 2*x**2 + 2)/2)*x**3 - 3*sqrt(2)*log(( - sqrt(6)*x
- sqrt(2)*x + 2*x**2 + 2)/2)*x**3 + 3*sqrt(2)*log((sqrt(6)*x + sqrt(2)*x +
 2*x**2 + 2)/2)*x**3 - 16)/(48*x**3)
```

3.74 $\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 132

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \frac{(bd-2ae)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} - \frac{d\log(d+ex^4)}{4(cd^2-bde+ae^2)} + \frac{d\log(a+bx^4+cx^8)}{8(cd^2-bde+ae^2)}$$

output
```
1/4*(-2*a*e+b*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
)/(a*e^2-b*d*e+c*d^2)-d*ln(e*x^4+d)/(4*a*e^2-4*b*d*e+4*c*d^2)+d*ln(c*x^8+b
*x^4+a)/(8*a*e^2-8*b*d*e+8*c*d^2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \frac{2(bd-2ae)\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right) + \sqrt{-b^2+4ac}\,d(2\log(d+ex^4)-\log(a+bx^4+cx^8))}{8\sqrt{-b^2+4ac}\,(-cd^2+e(bd-ae))}$$

input
```
Integrate[x^7/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(2*(b*d - 2*a*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*
a*c]*d*(2*Log[d + e*x^4] - Log[a + b*x^4 + c*x^8]))/(8*Sqrt[-b^2 + 4*a*c]*
(-(c*d^2) + e*(b*d - a*e)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)}dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{4}\int \frac{x^4}{(ex^4+d)(cx^8+bx^4+a)}dx^4$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{4}\int \left(\frac{cdx^4+ae}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}-\frac{de}{(cd^2-bed+ae^2)(ex^4+d)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}(ae^2-bde+cd^2)}-\frac{d\log(d+ex^4)}{ae^2-bde+cd^2}+\frac{d\log(a+bx^4+cx^8)}{2(ae^2-bde+cd^2)}\right)$$

input
```
Int[x^7/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)) - (d*Log[d + e*x^4])/(c*d^2 - b*d*e + a*e^2) +
 (d*Log[a + b*x^4 + c*x^8])/(2*(c*d^2 - b*d*e + a*e^2)))/4
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{d\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4}+\frac{\left(ae-\frac{bd}{2}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a\,e^{2}-2bde+2c\,d^{2}}-\frac{d\ln\left(x^{4}e+d\right)}{4\left(a\,e^{2}-bde+c\,d^{2}\right)}$
risch	$-\frac{d\ln\left(x^{4}e+d\right)}{4\left(a\,e^{2}-bde+c\,d^{2}\right)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4a^{2}c\,e^{2}-a\,b^{2}e^{2}-4abcde+4a\,c^{2}d^{2}+b^{3}de-b^{2}c\,d^{2}\right)\_Z^{2}+\left(-4acd+d\,b^{2}\right)\_Z+a\right)}\_R\ln\left(\left(\left(4a^{2}c\,e^{3}\right.\right.\right.$

input
```
int(x^7/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*e^2-b*d*e+c*d^2)*(1/4*d*ln(c*x^8+b*x^4+a)+(a*e-1/2*b*d)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))-1/4*d/(a*e^2-b*d*e+c*d^2)*ln
(e*x^4+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(x^7/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(x**7/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)}\,dx = -\frac{de\log(|ex^4+d|)}{4(cd^2e-bde^2+ae^3)} + \frac{d\log(cx^8+bx^4+a)}{8(cd^2-bde+ae^2)} - \frac{(bd-2ae)\arctan\left(\frac{2cx^4+b}{\sqrt{-b^2+4ac}}\right)}{4(cd^2-bde+ae^2)\sqrt{-b^2+4ac}}$$

input
```
integrate(x^7/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*d*e*log(abs(e*x^4 + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/8*d*log(c*x^8
 + b*x^4 + a)/(c*d^2 - b*d*e + a*e^2) - 1/4*(b*d - 2*a*e)*arctan((2*c*x^4
+ b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Hanged}$$

input
```
int(x^7/((d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \int \frac{x^7}{(e\,x^4+d)\,(c\,x^8+b\,x^4+a)}dx$$

input
```
int(x^7/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x^7/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.75 $\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 133

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)}\,dx = -\frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{e\log(d+ex^4)}{4(cd^2-bde+ae^2)} - \frac{e\log(a+bx^4+cx^8)}{8(cd^2-bde+ae^2)}$$

output

```
-1/4*(-b*e+2*c*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2)/(a*e^2-b*d*e+c*d^2)+e*ln(e*x^4+d)/(4*a*e^2-4*b*d*e+4*c*d^2)-e*ln(c*x^8+
b*x^4+a)/(8*a*e^2-8*b*d*e+8*c*d^2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \frac{(-4cd+2be)\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,e(-2\log(d+ex^4)+\log(a+bx^4+cx^8))}{8\sqrt{-b^2+4ac}\,(-cd^2+e(bd-ae))}$$

input
```
Integrate[x^3/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
((-4*c*d + 2*b*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4
*a*c]*e*(-2*Log[d + e*x^4] + Log[a + b*x^4 + c*x^8]))/(8*Sqrt[-b^2 + 4*a*c
]*(-(c*d^2) + e*(b*d - a*e)))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1798, 1144, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)}dx$$

$$\Big\downarrow \textbf{1798}$$

$$\frac{1}{4}\int \frac{1}{(ex^4+d)(cx^8+bx^4+a)}dx^4$$

$$\Big\downarrow \textbf{1144}$$

$$\frac{1}{4}\left(\frac{\int \frac{-cex^4+cd-be}{cx^8+bx^4+a}dx^4}{ae^2-bde+cd^2}+\frac{e\log\left(d+ex^4\right)}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \textbf{1142}$$

$$\frac{1}{4}\left(\frac{\frac{1}{2}(2cd-be)\int \frac{1}{cx^8+bx^4+a}dx^4-\frac{1}{2}e\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4}{ae^2-bde+cd^2}+\frac{e\log\left(d+ex^4\right)}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \textbf{1083}$$

$$\frac{1}{4}\left(\frac{-\left((2cd-be)\int \frac{1}{-x^8+b^2-4ac}d\left(2cx^4+b\right)\right)-\frac{1}{2}e\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4}{ae^2-bde+cd^2}+\frac{e\log\left(d+ex^4\right)}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \textbf{219}$$

$$\frac{1}{4}\left(\frac{-\frac{1}{2}e\int\frac{2cx^4+b}{cx^8+bx^4+a}dx^4-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{ae^2-bde+cd^2}+\frac{e\log\left(d+ex^4\right)}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{4}\left(\frac{-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{1}{2}e\log\left(a+bx^4+cx^8\right)}{ae^2-bde+cd^2}+\frac{e\log\left(d+ex^4\right)}{ae^2-bde+cd^2}\right)$$

input
```
Int[x^3/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
((e*Log[d + e*x^4])/(c*d^2 - b*d*e + a*e^2) + (-(((2*c*d - b*e)*ArcTanh[(b
 + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]) - (e*Log[a + b*x^4 + c*
x^8])/2)/(c*d^2 - b*d*e + a*e^2))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\frac{e\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4}+\frac{\left(\frac{eb}{2}-cd\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2\left(a\,e^{2}-bde+c\,d^{2}\right)}+\frac{e\ln\left(x^{4}e+d\right)}{4a\,e^{2}-4bde+4c\,d^{2}}$
risch	$\frac{e\ln\left(x^{4}e+d\right)}{4a\,e^{2}-4bde+4c\,d^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{2}c\,e^{2}-a\,b^{2}e^{2}-4abcde+4a\,c^{2}d^{2}+b^{3}de-b^{2}c\,d^{2}\right)\_Z^{2}+\left(4ace-b^{2}e\right)\_Z+c\right)}\_R\ln\left(\left(\left(4a^{2}c\,e^{3}-a\,b\right.\right.\right.}{}$

input
```
int(x^3/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*e^2-b*d*e+c*d^2)*(1/4*e*ln(c*x^8+b*x^4+a)+(1/2*e*b-c*d)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))+1/4*e/(a*e^2-b*d*e+c*d^2)*l
n(e*x^4+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \frac{e^2\log(|ex^4+d|)}{4(cd^2e-bde^2+ae^3)} - \frac{e\log(cx^8+bx^4+a)}{8(cd^2-bde+ae^2)} + \frac{(2\,cd-be)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4(cd^2-bde+ae^2)\sqrt{-b^2+4\,ac}}$$

input
```
integrate(x^3/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/4*e^2*log(abs(e*x^4 + d))/(c*d^2*e - b*d*e^2 + a*e^3) - 1/8*e*log(c*x^8
+ b*x^4 + a)/(c*d^2 - b*d*e + a*e^2) + 1/4*(2*c*d - b*e)*arctan((2*c*x^4 +
 b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Hanged}$$

input
```
int(x^3/((d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \int \frac{x^3}{(e\,x^4+d)(c\,x^8+b\,x^4+a)}dx$$

input
```
int(x^3/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x^3/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.76 $\int \frac{1}{x(d+ex^4)(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 167

$$\int \frac{1}{x(d+ex^4)(a+bx^4+cx^8)}\,dx = \frac{(bcd-b^2e+2ace)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{\log(x)}{ad} - \frac{e^2\log(d+ex^4)}{4d(cd^2-bde+ae^2)} - \frac{(cd-be)\log(a+bx^4+cx^8)}{8a(cd^2-bde+ae^2)}$$

output
```
1/4*(2*a*c*e-b^2*e+b*c*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*
c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)+ln(x)/a/d-1/4*e^2*ln(e*x^4+d)/d/(a*e^2-b*
d*e+c*d^2)-1/8*(-b*e+c*d)*ln(c*x^8+b*x^4+a)/a/(a*e^2-b*d*e+c*d^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx = \frac{4(cd^2-bde+ae^2)\log(x)-ae^2\log\left(d+ex^4\right)+d\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{-bcd\log(x-\#1)+b^2e\log(x-\#}{}\right.}{4ad\left(cd^2+e(-bd+ae)\right)}$$

input
```
Integrate[1/(x*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(4*(c*d^2 - b*d*e + a*e^2)*Log[x] - a*e^2*Log[d + e*x^4] + d*RootSum[a + b
*#1^4 + c*#1^8 & , (-(b*c*d*Log[x - #1]) + b^2*e*Log[x - #1] - a*c*e*Log[x
 - #1] - c^2*d*Log[x - #1]*#1^4 + b*c*e*Log[x - #1]*#1^4)/(b + 2*c*#1^4) &
 ])/(4*a*d*(c*d^2 + e*(-(b*d) + a*e)))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(ex^4+d\right)\left(cx^8+bx^4+a\right)}dx^4$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{4}\int \left(-\frac{e^3}{d\left(cd^2-bed+ae^2\right)\left(ex^4+d\right)}+\frac{-c(cd-be)x^4-bcd+b^2e-ace}{a\left(cd^2-bed+ae^2\right)\left(cx^8+bx^4+a\right)}+\frac{1}{adx^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)\left(2ace+b^2(-e)+bcd\right)}{a\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{e^2\log\left(d+ex^4\right)}{d\left(ae^2-bde+cd^2\right)}-\frac{(cd-be)\log\left(a+bx^4+cx^8\right)}{2a\left(ae^2-bde+cd^2\right)}+\frac{\log\left(x^4\right)}{ad}\right)$$

input
```
Int[1/(x*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(a*S
qrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + Log[x^4]/(a*d) - (e^2*Log[d +
e*x^4])/(d*(c*d^2 - b*d*e + a*e^2)) - ((c*d - b*e)*Log[a + b*x^4 + c*x^8])
/(2*a*(c*d^2 - b*d*e + a*e^2)))/4
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

method	result
default	$-\frac{\frac{\left(-bce+c^{2}d\right)\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4c}+\frac{\left(ace-b^{2}e+cbd-\frac{\left(-bce+c^{2}d\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2(a\,e^{2}-bde+c\,d^{2})a}-\frac{e^{2}\ln\left(x^{4}e+d\right)}{4d(a\,e^{2}-bde+c\,d^{2})}+\frac{\ln(x)}{ad}$
risch	$\frac{\ln(x)}{ad}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^{3}c\,e^{2}-a^{2}b^{2}e^{2}-4a^{2}bcde+4a^{2}c^{2}d^{2}+a\,b^{3}de-a\,b^{2}c\,d^{2}\right)\_Z^{2}+\left(-4abce+4a\,c^{2}d+b^{3}e-b^{2}cd\right)\_Z+c^{2}\right)}\_R\ln\left(\left(\left(-2\right.\right.\right.}{}$

input
```
int(1/x/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*e^2-b*d*e+c*d^2)/a*(1/4*(-b*c*e+c^2*d)/c*ln(c*x^8+b*x^4+a)+(a*c*e-
b^2*e+c*b*d-1/2*(-b*c*e+c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(
4*a*c-b^2)^(1/2)))-1/4*e^2*ln(e*x^4+d)/d/(a*e^2-b*d*e+c*d^2)+ln(x)/a/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^{4}\right)\left(a+bx^{4}+cx^{8}\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^{4}\right)\left(a+bx^{4}+cx^{8}\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{1}{x\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx = & -\frac{e^3\log\left(\left|ex^4+d\right|\right)}{4\left(cd^3e-bd^2e^2+ade^3\right)} \\ & -\frac{(cd-be)\log\left(cx^8+bx^4+a\right)}{8\left(acd^2-abde+a^2e^2\right)} \\ & -\frac{\left(bcd-b^2e+2\,ace\right)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\left(acd^2-abde+a^2e^2\right)\sqrt{-b^2+4\,ac}}+\frac{\log\left(x^4\right)}{4\,ad}\end{aligned}$$

input
```
integrate(1/x/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*e^3*log(abs(e*x^4 + d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3) - 1/8*(c*d -
b*e)*log(c*x^8 + b*x^4 + a)/(a*c*d^2 - a*b*d*e + a^2*e^2) - 1/4*(b*c*d - b
^2*e + 2*a*c*e)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/((a*c*d^2 - a*b*d
*e + a^2*e^2)*sqrt(-b^2 + 4*a*c)) + 1/4*log(x^4)/(a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx = \text{Hanged}$$

input
```
int(1/(x*(d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{x\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx = \int \frac{1}{x\left(e\,x^4+d\right)\left(c\,x^8+b\,x^4+a\right)}dx$$

input
```
int(1/x/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(1/x/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.77 $\int \frac{1}{x^5(d+ex^4)(a+bx^4+cx^8)} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 205

$$\begin{aligned}\int \frac{1}{x^5 (d+ex^4) (a+bx^4+cx^8)} \, dx &= -\frac{1}{4adx^4} \\ &\quad - \frac{(b^2cd-2ac^2d-b^3e+3abce) \operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} \\ &\quad - \frac{(bd+ae)\log(x)}{a^2d^2} + \frac{e^3\log(d+ex^4)}{4d^2(cd^2-bde+ae^2)} \\ &\quad + \frac{(bcd-b^2e+ace)\log(a+bx^4+cx^8)}{8a^2(cd^2-bde+ae^2)}\end{aligned}$$

output

```
-1/4/a/d/x^4-1/4*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*arctanh((2*c*x^4+b)/(
-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)-(a*e+b*d)*ln
(x)/a^2/d^2+1/4*e^3*ln(e*x^4+d)/d^2/(a*e^2-b*d*e+c*d^2)+1/8*(a*c*e-b^2*e+b
*c*d)*ln(c*x^8+b*x^4+a)/a^2/(a*e^2-b*d*e+c*d^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^5 (d+ex^4)(a+bx^4+cx^8)}\,dx$$
$$= \frac{1}{4}\left(-\frac{1}{adx^4}-\frac{4(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log(d+ex^4)}{cd^4+d^2e(-bd+ae)}\right.$$
$$-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{-b^2cd\log(x-\#1)+ac^2d\log(x-\#1)+b^3e\log(x-\#1)-2abce\log(x-\#1)-bc^2d\log(x-\#1)\ldots}{b+2c\#1^4}\right.}{a^2(cd^2+e(-bd+ae))}$$

input
```
Integrate[1/(x^5*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(-(1/(a*d*x^4)) - (4*(b*d + a*e)*Log[x])/(a^2*d^2) + (e^3*Log[d + e*x^4])/
(c*d^4 + d^2*e*(-(b*d) + a*e)) - RootSum[a + b*#1^4 + c*#1^8 & , (-(b^2*c*
d*Log[x - #1]) + a*c^2*d*Log[x - #1] + b^3*e*Log[x - #1] - 2*a*b*c*e*Log[x
 - #1] - b*c^2*d*Log[x - #1]*#1^4 + b^2*c*e*Log[x - #1]*#1^4 - a*c^2*e*Log
[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(a^2*(c*d^2 + e*(-(b*d) + a*e))))/4
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (d+ex^4)(a+bx^4+cx^8)}\,dx$$
$$\Big\downarrow \; 1802$$

$$\frac{1}{4}\int \frac{1}{x^8\left(ex^4+d\right)\left(cx^8+bx^4+a\right)}dx^4$$

$$\Big\downarrow \ 1200$$

$$\frac{1}{4}\int \left(\frac{e^4}{d^2\left(cd^2-bed+ae^2\right)\left(ex^4+d\right)}+\frac{c\left(-eb^2+cdb+ace\right)x^4-ac^2d+b^2cd-b^3e+2abce}{a^2\left(cd^2-bed+ae^2\right)\left(cx^8+bx^4+a\right)}+\frac{-bd-ae}{a^2d^2x^4}+\frac{1}{adx\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)\left(3abce-2ac^2d+b^3(-e)+b^2cd\right)}{a^2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{\left(ace+b^2(-e)+bcd\right)\log\left(a+bx^4+cx^8\right)}{2a^2\left(ae^2-bde+cd^2\right)}-\frac{\log\left(x^4\right.$$

input
```
Int[1/(x^5*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(-(1/(a*d*x^4)) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*ArcTanh[(b +
2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2
)) - ((b*d + a*e)*Log[x^4])/(a^2*d^2) + (e^3*Log[d + e*x^4])/(d^2*(c*d^2 -
 b*d*e + a*e^2)) + ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^4 + c*x^8])/(2*a^2
*(c*d^2 - b*d*e + a*e^2)))/4
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.04

method	result
default	$\frac{\frac{\left(a\,c^{2}e-b^{2}ce+b\,c^{2}d\right)\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4c}+\frac{\left(2abce-a\,c^{2}d-b^{3}e+b^{2}cd-\frac{\left(a\,c^{2}e-b^{2}ce+b\,c^{2}d\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{2}\left(a\,e^{2}-bde+c\,d^{2}\right)}+\frac{e^{3}\ln\left(x^{4}e+d\right)}{4d^{2}\left(a\,e^{2}-bde+c\,d^{2}\right)}-$
risch	$-\frac{1}{4ad\,x^{4}}-\frac{\ln\left(x\right)e}{a\,d^{2}}-\frac{\ln\left(x\right)b}{a^{2}d}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{4}c\,e^{2}-a^{3}b^{2}e^{2}-4a^{3}bcde+4a^{3}c^{2}d^{2}+a^{2}b^{3}de-a^{2}b^{2}c\,d^{2}\right)\_Z^{2}+\left(-4a^{2}c^{2}e+5a\,b^{2}ce-4d\right.\right.}\right.}{}$

input
```
int(1/x^5/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2/(a*e^2-b*d*e+c*d^2)*(1/4*(a*c^2*e-b^2*c*e+b*c^2*d)/c*ln(c*x^8+b*x^
4+a)+(2*a*b*c*e-a*c^2*d-b^3*e+b^2*c*d-1/2*(a*c^2*e-b^2*c*e+b*c^2*d)*b/c)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))+1/4*e^3*ln(e*x^4+d
)/d^2/(a*e^2-b*d*e+c*d^2)-1/4/a/d/x^4+1/a^2/d^2*(-a*e-b*d)*ln(x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^5/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**5/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{1}{x^5\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx=&\frac{e^4\log\left(\left|ex^4+d\right|\right)}{4\left(cd^4e-bd^3e^2+ad^2e^3\right)}\\&+\frac{\left(bcd-b^2e+ace\right)\log\left(cx^8+bx^4+a\right)}{8\left(a^2cd^2-a^2bde+a^3e^2\right)}\\&+\frac{\left(b^2cd-2\,ac^2d-b^3e+3\,abce\right)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\left(a^2cd^2-a^2bde+a^3e^2\right)\sqrt{-b^2+4\,ac}}\\&-\frac{\left(bd+ae\right)\log\left(x^4\right)}{4\,a^2d^2}+\frac{bdx^4+aex^4-ad}{4\,a^2d^2x^4}\end{aligned}$$

input
```
integrate(1/x^5/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/4*e^4*log(abs(e*x^4 + d))/(c*d^4*e - b*d^3*e^2 + a*d^2*e^3) + 1/8*(b*c*d
 - b^2*e + a*c*e)*log(c*x^8 + b*x^4 + a)/(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)
 + 1/4*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*arctan((2*c*x^4 + b)/sqrt
(-b^2 + 4*a*c))/((a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*sqrt(-b^2 + 4*a*c)) - 1
/4*(b*d + a*e)*log(x^4)/(a^2*d^2) + 1/4*(b*d*x^4 + a*e*x^4 - a*d)/(a^2*d^2
*x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d+ex^4) (a+bx^4+cx^8)} \, dx = \text{Hanged}$$

input
```
int(1/(x^5*(d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{x^5 (d+ex^4) (a+bx^4+cx^8)} \, dx = \int \frac{1}{x^5 (e\,x^4+d) (c\,x^8+b\,x^4+a)}dx$$

input
```
int(1/x^5/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(1/x^5/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.78 $\int \frac{1}{x^9(d+ex^4)(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 268

$$\begin{aligned}\int \frac{1}{x^9(d+ex^4)(a+bx^4+cx^8)}\,dx &= -\frac{1}{8adx^8}+\frac{bd+ae}{4a^2d^2x^4}+\frac{(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^3\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}\\ &\quad+\frac{(b^2d^2+abde-a(cd^2-ae^2))\log(x)}{a^3d^3}-\frac{e^4\log(d+ex^4)}{4d^3(cd^2-bde+ae^2)}\\ &\quad-\frac{(b^2cd-ac^2d-b^3e+2abce)\log(a+bx^4+cx^8)}{8a^3(cd^2-bde+ae^2)}\end{aligned}$$

output
```
-1/8/a/d/x^8+1/4*(a*e+b*d)/a^2/d^2/x^4+1/4*(-2*a^2*c^2*e+4*a*b^2*c*e-3*a*b
*c^2*d-b^4*e+b^3*c*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+
b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)+(b^2*d^2+a*b*d*e-a*(-a*e^2+c*d^2))*ln(x)/a^
3/d^3-1/4*e^4*ln(e*x^4+d)/d^3/(a*e^2-b*d*e+c*d^2)-1/8*(2*a*b*c*e-a*c^2*d-b
^3*e+b^2*c*d)*ln(c*x^8+b*x^4+a)/a^3/(a*e^2-b*d*e+c*d^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^9 (d+ex^4) (a+bx^4+cx^8)} dx = \frac{1}{8}\left(-\frac{1}{adx^8}+\frac{2(bd+ae)}{a^2d^2x^4} + \frac{8(b^2d^2+abde+a(-cd^2+ae^2))\log(x)}{a^3d^3} - \frac{2e^4\log(d+ex^4)}{cd^5+d^3e(-bd+ae)} + \frac{2\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{-b^3cd\log(x-\#1)+2abc^2d\log(x-\#1)+b^4e\log(x-\#1)-3ab^2ce\log(x-\#1)+a^2c^2e\log(x-\right.}{a^3(cd^2+e(-bd+ae)}\right.$$

input
```
Integrate[1/(x^9*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(-(1/(a*d*x^8)) + (2*(b*d + a*e))/(a^2*d^2*x^4) + (8*(b^2*d^2 + a*b*d*e +
a*(-(c*d^2) + a*e^2))*Log[x])/(a^3*d^3) - (2*e^4*Log[d + e*x^4])/(c*d^5 +
d^3*e*(-(b*d) + a*e)) + (2*RootSum[a + b*#1^4 + c*#1^8 & , (-(b^3*c*d*Log[
x - #1]) + 2*a*b*c^2*d*Log[x - #1] + b^4*e*Log[x - #1] - 3*a*b^2*c*e*Log[x
 - #1] + a^2*c^2*e*Log[x - #1] - b^2*c^2*d*Log[x - #1]*#1^4 + a*c^3*d*Log[
x - #1]*#1^4 + b^3*c*e*Log[x - #1]*#1^4 - 2*a*b*c^2*e*Log[x - #1]*#1^4)/(b
 + 2*c*#1^4) & ])/(a^3*(c*d^2 + e*(-(b*d) + a*e))))/8
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 (d+ex^4) (a+bx^4+cx^8)} dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{4}\int \frac{1}{x^{12}\left(ex^4+d\right)\left(cx^8+bx^4+a\right)}dx^4$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{4}\int \left(-\frac{e^5}{d^3\left(cd^2-bed+ae^2\right)\left(ex^4+d\right)}+\frac{eb^4-cdb^3-3aceb^2+2ac^2db-c\left(-eb^3+cdb^2+2aceb-ac^2d\right)x^4+a}{a^3\left(cd^2-bed+ae^2\right)\left(cx^8+bx^4+a\right)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)\left(abde-a\left(cd^2-ae^2\right)+b^2d^2\right)}{a^3d^3}-\frac{\left(2abce-ac^2d+b^3(-e)+b^2cd\right)\log\left(a+bx^4+cx^8\right)}{2a^3\left(ae^2-bde+cd^2\right)}+\frac{ae+bd}{a^2d^2x^4}+\right.$$

input
```
Int[1/(x^9*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(-1/2*1/(a*d*x^8) + (b*d + a*e)/(a^2*d^2*x^4) + ((b^3*c*d - 3*a*b*c^2*d -
b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]
])/(a^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d^2 + a*b*d*e -
 a*(c*d^2 - a*e^2))*Log[x^4])/(a^3*d^3) - (e^4*Log[d + e*x^4])/(d^3*(c*d^2
 - b*d*e + a*e^2)) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Log[a + b*x^
4 + c*x^8])/(2*a^3*(c*d^2 - b*d*e + a*e^2)))/4
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.06

method	result
default	$\frac{\frac{\left(-2ab\,c^2e+a\,c^3d+b^3ce-b^2c^2d\right)\ln\left(c\,x^8+b\,x^4+a\right)}{4c}+\frac{\left(a^2c^2e-3a\,b^2ce+2d\,c^2ba+b^4e-dc\,b^3-\frac{\left(-2ab\,c^2e+a\,c^3d+b^3ce-b^2c^2d\right)b}{2c}\right)\arctan\left(\frac{2c\,x^4+}{\sqrt{4ac-b}}\right.}{\sqrt{4ac-b^2}}}{2(a\,e^2-bde+c\,d^2)a^3}$
risch	Expression too large to display

input
```
int(1/x^9/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*e^2-b*d*e+c*d^2)/a^3*(1/4*(-2*a*b*c^2*e+a*c^3*d+b^3*c*e-b^2*c^2*d)/
c*ln(c*x^8+b*x^4+a)+(a^2*c^2*e-3*a*b^2*c*e+2*d*c^2*b*a+b^4*e-d*c*b^3-1/2*(
-2*a*b*c^2*e+a*c^3*d+b^3*c*e-b^2*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x^4+b)/(4*a*c-b^2)^(1/2)))-1/4*e^4*ln(e*x^4+d)/d^3/(a*e^2-b*d*e+c*d^2)-1/
8/a/d/x^8-1/4*(-a*e-b*d)/a^2/d^2/x^4+(a^2*e^2+a*b*d*e-a*c*d^2+b^2*d^2)/d^3
/a^3*ln(x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^9\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^9/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^9 (d + ex^4)(a + bx^4 + cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**9/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^9 (d + ex^4)(a + bx^4 + cx^8)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^9/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^9 (d+ex^4) (a+bx^4+cx^8)}\,dx = -\frac{e^5\log\left(|ex^4+d|\right)}{4\left(cd^5e-bd^4e^2+ad^3e^3\right)} - \frac{(b^2cd-ac^2d-b^3e+2\,abce)\log\left(cx^8+bx^4+a\right)}{8\left(a^3cd^2-a^3bde+a^4e^2\right)} - \frac{(b^3cd-3\,abc^2d-b^4e+4\,ab^2ce-2\,a^2c^2e)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\left(a^3cd^2-a^3bde+a^4e^2\right)\sqrt{-b^2+4\,ac}} + \frac{(b^2d^2-acd^2+abde+a^2e^2)\log\left(x^4\right)}{4\,a^3d^3} - \frac{3\,b^2d^2x^8-3\,acd^2x^8+3\,abdex^8+3\,a^2e^2x^8-2\,abd^2x^4-2\,a^2dex^4+a^2d^2}{8\,a^3d^3x^8}$$

input
```
integrate(1/x^9/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*e^5*log(abs(e*x^4 + d))/(c*d^5*e - b*d^4*e^2 + a*d^3*e^3) - 1/8*(b^2*
c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*log(c*x^8 + b*x^4 + a)/(a^3*c*d^2 - a^3
*b*d*e + a^4*e^2) - 1/4*(b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a
^2*c^2*e)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/((a^3*c*d^2 - a^3*b*d*e
 + a^4*e^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(b^2*d^2 - a*c*d^2 + a*b*d*e + a^2*e
^2)*log(x^4)/(a^3*d^3) - 1/8*(3*b^2*d^2*x^8 - 3*a*c*d^2*x^8 + 3*a*b*d*e*x^
8 + 3*a^2*e^2*x^8 - 2*a*b*d^2*x^4 - 2*a^2*d*e*x^4 + a^2*d^2)/(a^3*d^3*x^8)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9 (d+ex^4) (a+bx^4+cx^8)}\,dx = \text{Hanged}$$

input
```
int(1/(x^9*(d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{x^9 (d + ex^4) (a + bx^4 + cx^8)} \, dx = \int \frac{1}{x^9 (e\,x^4 + d) (c\,x^8 + b\,x^4 + a)} dx$$

input
```
int(1/x^9/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(1/x^9/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.79 $\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 293

$$\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)}\,dx = -\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{2\sqrt{e}\,(cd^2-bde+ae^2)}$$

output
```
-1/4*(b*d-a*e-(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^
(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2)/(a*e^2-b*d*e+c*d^2)-1/4*(b*d-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c
+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1
/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^2-b*d*e+c*d^2)+1/2*d^(3/2)*a
rctan(e^(1/2)*x^2/d^(1/2))/e^(1/2)/(a*e^2-b*d*e+c*d^2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.14

$$\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)}\,dx$$
$$=\frac{1}{4}\left(\frac{\sqrt{2}\left(-b^2d+2acd+b\sqrt{b^2-4ac}d+abe-a\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+e(bd-ae))}+\frac{\sqrt{2}\left(b^2d-2acd+b\sqrt{b^2-4ac}d-abe-a\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+e(bd-ae))}+\frac{2d^{3/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2-bde+ae^2)}\right)$$

input
```
Integrate[x^9/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
((Sqrt[2]*(-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2
 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(S
qrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d -
a*e))) + (Sqrt[2]*(b^2*d - 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - a*b*e - a*Sqr
t[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b
*d - a*e))) + (2*d^(3/2)*ArcTan[(Sqrt[e]*x^2)/Sqrt[d]])/(Sqrt[e]*(c*d^2 -
b*d*e + a*e^2)))/4
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1814, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)}\,dx$$
$$\Big\downarrow \; 1814$$
$$\frac{1}{2}\int \frac{x^8}{(ex^4+d)(cx^8+bx^4+a)}dx^2$$
$$\Big\downarrow \; 1610$$
$$\frac{1}{2}\int \left(\frac{d^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-((bd-ae)x^4)-ad}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx^2$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\right.$$

input
```
Int[x^9/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(-(((b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - S
qrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - ((b*d - a*e + (b^2*d - 2*a*c
*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*
e + a*e^2)) + (d^(3/2)*ArcTan[(Sqrt[e]*x^2)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*
d*e + a*e^2)))/2
```

Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.92

method	result
default	$-\frac{2c\left(-\frac{\left(abe+2acd-a\sqrt{-4ac+b^2}\,e-d\,b^2+b\sqrt{-4ac+b^2}\,d\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-a\sqrt{-4ac+b^2}\,e+b\sqrt{-4ac+b^2}\,d-abe-2acd+\right.}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\right.}}\right.}{a\,e^2-bde+c\,d^2}$
risch	Expression too large to display

input
```
int(x^9/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-2/(a*e^2-b*d*e+c*d^2)*c*(-1/8*(a*b*e+2*a*c*d-a*(-4*a*c+b^2)^(1/2)*e-d*b^2
+b*(-4*a*c+b^2)^(1/2)*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8
*(-a*(-4*a*c+b^2)^(1/2)*e+b*(-4*a*c+b^2)^(1/2)*d-a*b*e-2*a*c*d+d*b^2)/c/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/2*d^2/(a*e^2-b*d*e+c*d^2)/(d*e)^
(1/2)*arctan(e*x^2/(d*e)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7779 vs. $2(243) = 486$.

Time = 119.78 (sec) , antiderivative size = 15577, normalized size of antiderivative = 53.16

$$\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
integrate(x^9/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(x**9/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^9/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5569 vs. $2(243) = 486$.

Time = 1.65 (sec) , antiderivative size = 5569, normalized size of antiderivative = 19.01

$$\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
integrate(x^9/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/2*d^2*arctan(e*x^2/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) - 1/8*
((2*b^4*c^3 - 8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
2*c^3 - 2*(b^2 - 4*a*c)*b^2*c^3)*d^3*x^4 - (2*b^5*c^2 - 6*a*b^3*c^3 - 8*a^
2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 +
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 -
2*(b^2 - 4*a*c)*a*b*c^3)*d^2*e*x^4 + 2*(2*a*b^4*c^2 - 8*a^2*b^2*c^3 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*
c^2)*d*e^2*x^4 - (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s...
```

Mupad [B] (verification not implemented)

Time = 84.47 (sec) , antiderivative size = 122445, normalized size of antiderivative = 417.90

$$\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
int(x^9/((d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
atan(((x^2*(8*a^6*c^4*d^6*e^4 - 8*a^5*c^5*d^8*e^2 - 4*a^7*c^3*d^4*e^6 + 4*
a^8*c^2*d^2*e^8 + 4*a^2*b^6*c^2*d^8*e^2 - 20*a^3*b^4*c^3*d^8*e^2 + 4*a^3*b
^5*c^2*d^7*e^3 + 24*a^4*b^2*c^4*d^8*e^2 - 16*a^4*b^3*c^3*d^7*e^3 + 4*a^4*b
^4*c^2*d^6*e^4 - 16*a^5*b^2*c^3*d^6*e^4 + 4*a^6*b^2*c^2*d^4*e^6 + 8*a^5*b*
c^4*d^7*e^3 + 4*a^7*b*c^2*d^3*e^7) - (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d
^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^
3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^
3)^(1/2))/(32*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4
*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4
 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^
4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)
))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b
^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3
*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e
 + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5*
d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b
^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*
a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e
^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(((-(b^5*d^2 +...
```

Reduce [F]

$$\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)}\,dx=\int \frac{x^9}{(e\,x^4+d)(c\,x^8+b\,x^4+a)}dx$$

input
```
int(x^9/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x^9/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.80 $\int \frac{x^5}{(d+ex^4)(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 265

$$\int \frac{x^5}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{2\,(cd^2-bde+ae^2)}$$

output
```
1/4*c^(1/2)*(d-(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2
/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^2
-b*d*e+c*d^2)+1/4*c^(1/2)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/
2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)^(1/2)/(a*e^2-b*d*e+c*d^2)-d^(1/2)*e^(1/2)*arctan(e^(1/2)*x^2/d^(1/2))/(2
*a*e^2-2*b*d*e+2*c*d^2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(d+ex^4)(a+bx^4+cx^8)}\,dx = -\frac{\sqrt{c}\left(-bd+\sqrt{b^2-4ac}d+2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} - \frac{\sqrt{c}\left(bd+\sqrt{b^2-4ac}d-2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} - \frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{2\,(cd^2-bde+ae^2)}$$

input
```
Integrate[x^5/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/2*(Sqrt[c]*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[
c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b -
Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) - (Sqrt[c]*(b*d + Sqrt[b^2
- 4*a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) +
b*d*e - a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x^2)/Sqrt[d]])/(2*(c*d^
2 - b*d*e + a*e^2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1814, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(d+ex^4)(a+bx^4+cx^8)}\,dx$$

$$\Big\downarrow \; 1814$$

$$\frac{1}{2}\int \frac{x^4}{(ex^4+d)(cx^8+bx^4+a)}dx^2$$

$$\Big\downarrow \text{1610}$$

$$\frac{1}{2}\int \left(\frac{cdx^4+ae}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}-\frac{de}{(cd^2-bed+ae^2)(ex^4+d)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{ae^2-bde+cd^2}\right.$$

input
```
Int[x^5/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
((Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^
2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d
^2 - b*d*e + a*e^2)) + (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + S
qrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt
[e]*x^2)/Sqrt[d]])/(c*d^2 - b*d*e + a*e^2))/2
```

Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.83

method	result
default	$\frac{2c\left(-\frac{\left(\sqrt{-4ac+b^{2}}\,d+2ae-bd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^{2}\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\left(bd+\sqrt{-4ac+b^{2}}\,d-2ae\right)\sqrt{2}\,\arctan\left(\frac{c\,x^{2}\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{a\,e^{2}-bde+c\,d^{2}}-\frac{de}{2(a\,e^{2}$
risch	Expression too large to display

input
```
int(x^5/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
2/(a*e^2-b*d*e+c*d^2)*c*(-1/8*((-4*a*c+b^2)^(1/2)*d+2*a*e-b*d)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/2*d*e/(a*e^2-b*d*e+c*d^2)/(d*e)^(1
/2)*arctan(e*x^2/(d*e)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6137 vs. $2(215) = 430$.

Time = 67.20 (sec) , antiderivative size = 12293, normalized size of antiderivative = 46.39

$$\int \frac{x^5}{(d+ex^4)\,(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4297 vs. $2(215) = 430$.

Time $= 1.56$ (sec) , antiderivative size $= 4297$, normalized size of antiderivative $= 16.22$

$$\int \frac{x^5}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/2*d*e*arctan(e*x^2/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) + 1/8
*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c
^4 - 2*(b^2 - 4*a*c)*b*c^4)*d^3*x^4 - (2*b^4*c^3 - 8*a*b^2*c^4 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 4*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 2*(b^2 - 4*a*c)*b^2*c^3)*d^2*
e*x^4 + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*d*e^2*x^4 + (sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 2*b^4*c^2
 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^2*c^3 + 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^...
```

Mupad [B] (verification not implemented)

Time = 75.34 (sec) , antiderivative size = 104563, normalized size of antiderivative = 394.58

$$\int \frac{x^5}{(d+ex^4)\,(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
int(x^5/((d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
atan(((x^2*(4*a^4*c^6*d*e^9 - 8*a^3*c^7*d^3*e^7 + 4*a^2*b^2*c^6*d^3*e^7 +
4*a*b^3*c^6*d^4*e^6 - 8*a^2*b*c^7*d^4*e^6 + 4*a^3*b*c^6*d^2*e^8) + (-(a*b^
3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)
^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e
)/(32*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d
^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*
d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*
c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x^2*(256*a^
3*c^9*d^6*e^6 - 3648*a^4*c^8*d^4*e^8 + 1984*a^5*c^7*d^2*e^10 + 1344*a^2*b^
3*c^7*d^5*e^7 - 1280*a^2*b^4*c^6*d^4*e^8 + 128*a^2*b^5*c^5*d^3*e^9 + 3712*
a^3*b^2*c^7*d^4*e^8 - 1152*a^3*b^3*c^6*d^3*e^9 + 128*a^3*b^4*c^5*d^2*e^10
- 1024*a^4*b^2*c^6*d^2*e^10 - 512*a^5*b*c^6*d*e^11 + 128*a*b^3*c^8*d^7*e^5
 - 128*a*b^4*c^7*d^6*e^6 - 128*a*b^5*c^6*d^5*e^7 + 128*a*b^6*c^5*d^4*e^8 -
 256*a^2*b*c^9*d^7*e^5 - 2240*a^3*b*c^8*d^5*e^7 + 2688*a^4*b*c^7*d^3*e^9 +
 128*a^4*b^3*c^5*d*e^11) + (-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^
2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(32*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16
*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e
^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3
*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a...
```

Reduce [F]

$$\int \frac{x^5}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \int \frac{x^5}{(e\,x^4+d)\,(c\,x^8+b\,x^4+a)}dx$$

input
```
int(x^5/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x^5/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.81 $\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 267

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)}\,dx = -\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{2\sqrt{d}\,(cd^2-bde+ae^2)}$$

output
```
-1/4*c^(1/2)*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^
2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^
2-b*d*e+c*d^2)-1/4*c^(1/2)*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1
/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2)/(a*e^2-b*d*e+c*d^2)+1/2*e^(3/2)*arctan(e^(1/2)*x^2/d^(1/2))/d^(1/
2)/(a*e^2-b*d*e+c*d^2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.06

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)}\,dx$$
$$=\frac{1}{4}\left(\frac{\sqrt{2}\sqrt{c}\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+e(bd-ae))}+\frac{\sqrt{2}\sqrt{c}\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+e(bd-ae))}+\frac{2e^{3/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)}\right)$$

input
```
Integrate[x/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
((Sqrt[2]*Sqrt[c]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqr
t[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + (Sqrt[2]*Sqrt[c]*(2*c*d + (-b +
 Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d
- a*e))) + (2*e^(3/2)*ArcTan[(Sqrt[e]*x^2)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d
*e + a*e^2)))/4
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1814, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)}\,dx$$

$$\Big\downarrow \text{1814}$$

$$\frac{1}{2}\int \frac{1}{(ex^4+d)(cx^8+bx^4+a)}dx^2$$

$$\Big\downarrow \text{1484}$$

$$\frac{1}{2}\int \left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}\right)$$

input
```
Int[x/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(-((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c
*d^2 - b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x
^2)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)))/2
```

Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.82

method	result
default	$-\frac{2c\left(-\frac{\left(\sqrt{-4ac+b^{2}}\,e+eb-2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^{2}\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\left(\sqrt{-4ac+b^{2}}\,e-eb+2cd\right)\sqrt{2}\,\arctan\left(\frac{c\,x^{2}\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{a\,e^{2}-bde+c\,d^{2}}+\frac{e}{2(a}$
risch	Expression too large to display

input
```
int(x/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-2/(a*e^2-b*d*e+c*d^2)*c*(-1/8*((-4*a*c+b^2)^(1/2)*e+e*b-2*c*d)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*((-4*a*c+b^2)^(1/2)*e-e*b+2*c*d)/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/2*e^2/(a*e^2-b*d*e+c*d^2)/(d*e)^(
1/2)*arctan(e*x^2/(d*e)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^4)\,(a+bx^4+cx^8)}\,dx=\text{Timed out}$$

input
```
integrate(x/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(x/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7671 vs. $2(217) = 434$.

Time = 2.32 (sec) , antiderivative size = 7671, normalized size of antiderivative = 28.73

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/2*e^2*arctan(e*x^2/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) + 1/16
*(2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^
4*e + 4*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b*c^4)*d^3*
e^2 - (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr...
```

Mupad [B] (verification not implemented)

Time = 82.70 (sec) , antiderivative size = 104763, normalized size of antiderivative = 392.37

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
int(x/((d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
(atan((((((-d*e^3)^(1/2)*(((-d*e^3)^(1/2)*(x^2*(320*b^5*c^7*e^12 + 320*c^1
2*d^5*e^7 - 2240*a*b^3*c^8*e^12 + 3840*a^2*b*c^9*e^12 + 448*a*c^11*d^3*e^9
 - 4224*a^2*c^10*d*e^11 - 640*b*c^11*d^4*e^8 - 640*b^4*c^8*d*e^11 + 320*b^
2*c^10*d^3*e^9 + 320*b^3*c^9*d^2*e^10 - 1728*a*b*c^10*d^2*e^10 + 3648*a*b^
2*c^9*d*e^11) - ((-d*e^3)^(1/2)*(3072*a*c^12*d^6*e^7 - ((-d*e^3)^(1/2)*(x^
2*(65536*a^5*c^9*e^14 + 2048*b^10*c^4*e^14 + 2048*c^14*d^10*e^4 - 28672*a*
b^8*c^5*e^14 + 6144*a*c^13*d^8*e^6 - 8192*b*c^13*d^9*e^5 - 8192*b^9*c^5*d*
e^13 + 149504*a^2*b^6*c^6*e^14 - 339968*a^3*b^4*c^7*e^14 + 262144*a^4*b^2*
c^8*e^14 - 104448*a^2*c^12*d^6*e^8 - 120832*a^3*c^11*d^4*e^10 + 446464*a^4
*c^10*d^2*e^12 + 12288*b^2*c^12*d^8*e^6 - 30720*b^4*c^10*d^6*e^8 + 49152*b
^5*c^9*d^5*e^9 - 30720*b^6*c^8*d^4*e^10 + 12288*b^8*c^6*d^2*e^12 - 106496*
a^2*b^2*c^10*d^4*e^10 - 331776*a^2*b^3*c^9*d^3*e^11 + 618496*a^2*b^4*c^8*d
^2*e^12 - 979968*a^3*b^2*c^9*d^2*e^12 - 51200*a*b*c^12*d^7*e^7 + 104448*a*
b^7*c^6*d*e^13 - 733184*a^4*b*c^9*d*e^13 + 177152*a*b^2*c^11*d^6*e^8 - 271
360*a*b^3*c^10*d^5*e^9 + 158720*a*b^4*c^9*d^4*e^10 + 54272*a*b^5*c^8*d^3*e
^11 - 149504*a*b^6*c^7*d^2*e^12 + 262144*a^2*b*c^11*d^5*e^9 - 487424*a^2*b
^5*c^7*d*e^13 + 456704*a^3*b*c^10*d^3*e^11 + 986112*a^3*b^3*c^8*d*e^13) +
((-d*e^3)^(1/2)*(((-d*e^3)^(1/2)*(x^2*(9437184*a^7*b*c^8*e^16 - 11010048*a
^7*c^9*d*e^15 + 49152*a^3*b^9*c^4*e^16 - 737280*a^4*b^7*c^5*e^16 + 4128768
*a^5*b^5*c^6*e^16 - 10223616*a^6*b^3*c^7*e^16 - 655360*a^2*c^14*d^11*e^...
```

Reduce [F]

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \int \frac{x}{(e\,x^4+d)\,(c\,x^8+b\,x^4+a)}dx$$

input
```
int(x/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.82 $\int \frac{1}{x^3(d+ex^4)(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 312

$$\int \frac{1}{x^3(d+ex^4)(a+bx^4+cx^8)}\,dx = -\frac{1}{2adx^2} - \frac{\sqrt{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{e^{5/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{2d^{3/2}(cd^2-bde+ae^2)}$$

output
```
-1/2/a/d/x^2-1/4*c^(1/2)*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2)
)*arctan(2^(1/2)*c^(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(b-(-
4*a*c+b^2)^(1/2))^(1/2)/(a*e^2-b*d*e+c*d^2)-1/4*c^(1/2)*(c*d-b*e-(2*a*c*e-
b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2
)^(1/2))^(1/2))*2^(1/2)/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^2-b*d*e+c*d^2)
-1/2*e^(5/2)*arctan(e^(1/2)*x^2/d^(1/2))/d^(3/2)/(a*e^2-b*d*e+c*d^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^3\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx$$
$$= \frac{2\left(-cd+be-\frac{ae^2}{d}+\frac{ae^{5/2}x^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{d^{3/2}}+\frac{ae^{5/2}x^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{d^{3/2}}\right)+x^2\mathrm{RootSum}\left[a+b\#1^4+c\#\right.}{4a\left(cd^2+e(-bd+ae)\right)x}$$

input
```
Integrate[1/(x^3*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(2*(-(c*d) + b*e - (a*e^2)/d + (a*e^(5/2)*x^2*ArcTan[1 - (Sqrt[2]*e^(1/4)*
x)/d^(1/4)])/d^(3/2) + (a*e^(5/2)*x^2*ArcTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/
4)])/d^(3/2)) + x^2*RootSum[a + b*#1^4 + c*#1^8 & , (-(b*c*d*Log[x - #1])
+ b^2*e*Log[x - #1] - a*c*e*Log[x - #1] - c^2*d*Log[x - #1]*#1^4 + b*c*e*L
og[x - #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ])/(4*a*(c*d^2 + e*(-(b*d) + a*e))*
x^2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1814, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx$$
$$\Big\downarrow \text{1814}$$
$$\frac{1}{2}\int \frac{1}{x^4\left(ex^4+d\right)\left(cx^8+bx^4+a\right)}dx^2$$

$$\Big\downarrow \text{1610}$$

$$\frac{1}{2}\int \left(-\frac{e^3}{d\left(cd^2-bed+ae^2\right)\left(ex^4+d\right)}+\frac{-c(cd-be)x^4-bcd+b^2e-ace}{a\left(cd^2-bed+ae^2\right)\left(cx^8+bx^4+a\right)}+\frac{1}{adx^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+\right.}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}\right.$$

input
```
Int[1/(x^3*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(-(1/(a*d*x^2)) - (Sqrt[c]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*(c*
d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a
*c]]*(c*d^2 - b*d*e + a*e^2)) - (e^(5/2)*ArcTan[(Sqrt[e]*x^2)/Sqrt[d]])/(d
^(3/2)*(c*d^2 - b*d*e + a*e^2)))/2
```

Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.90

method	result
default	$-\frac{1}{2da\,x^2}-\frac{2c\left(-\frac{\left(-eb\sqrt{-4ac+b^2}+cd\sqrt{-4ac+b^2}+2ace-b^2e+cbd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-eb\sqrt{-4ac+b^2}+cd\sqrt{-4ac+b^2}\right.}{8\sqrt{-4a}}\right.}{(a\,e^2-bde+c\,d^2)a}$
risch	Expression too large to display

input
```
int(1/x^3/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/a/x^2-2/(a*e^2-b*d*e+c*d^2)/a*c*(-1/8*(-e*b*(-4*a*c+b^2)^(1/2)+c*d*
(-4*a*c+b^2)^(1/2)+2*a*c*e-b^2*e+c*b*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))+1/8*(-e*b*(-4*a*c+b^2)^(1/2)+c*d*(-4*a*c+b^2)^(1/2)-2*a*c*e+b^2*
e-c*b*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/2*e^3/d/(a*e^2-b*d*e+
c*d^2)/(d*e)^(1/2)*arctan(e*x^2/(d*e)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (d+ex^4) (a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (d+ex^4) (a+bx^4+cx^8)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7626 vs. $2(260) = 520$.

Time = 2.50 (sec) , antiderivative size = 7626, normalized size of antiderivative = 24.44

$$\int \frac{1}{x^3 (d+ex^4) (a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
-1/2*e^3*arctan(e*x^2/sqrt(d*e))/((c*d^3 - b*d^2*e + a*d*e^2)*sqrt(d*e)) -
 1/8*((2*a*b^3*c^5 - 8*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*a*b*c^5)*d^3*x^4 - 2*(2*a*b^4*c^4 - 8
*a^2*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^4 - 2*(b^2 - 4*a*c)*a*b^2*c^4)*d^2*e*x^4 + (2*a*b^5*c^3 - 6*a^2*b^3*c^4 -
 8*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^5*c + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4
*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c
^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c
^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(
b^2 - 4*a*c)*a*b^3*c^3 - 2*(b^2 - 4*a*c)*a^2*b*c^4)*d*e^2*x^4 - (2*a^2*b^4
*c^3 - 8*a^3*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (d + ex^4) (a + bx^4 + cx^8)} \, dx = \text{Hanged}$$

input
```
int(1/(x^3*(d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{x^3 (d + ex^4) (a + bx^4 + cx^8)} \, dx = \int \frac{1}{x^3 (e\, x^4 + d) (c\, x^8 + b\, x^4 + a)} dx$$

input
```
int(1/x^3/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(1/x^3/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.83 $\int \frac{1}{x^7(d+ex^4)(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 366

$$\begin{aligned}
\int \frac{1}{x^7(d+ex^4)(a+bx^4+cx^8)}\,dx &= -\frac{1}{6adx^6}+\frac{bd+ae}{2a^2d^2x^2}\\
&+\frac{\sqrt{c}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}\\
&+\frac{\sqrt{c}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}\\
&+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{2d^{5/2}(cd^2-bde+ae^2)}
\end{aligned}$$

output
```
-1/6/a/d/x^6+1/2*(a*e+b*d)/a^2/d^2/x^2+1/4*c^(1/2)*(b*c*d-b^2*e+a*c*e+(3*a
*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)
*x^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(b-(-4*a*c+b^2)^(1/2))^(1/2
)/(a*e^2-b*d*e+c*d^2)+1/4*c^(1/2)*(b*c*d-b^2*e+a*c*e-(3*a*b*c*e-2*a*c^2*d-
b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*2^(1/2)/a^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^2-b*d*e+c*
d^2)+1/2*e^(7/2)*arctan(e^(1/2)*x^2/d^(1/2))/d^(5/2)/(a*e^2-b*d*e+c*d^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^7 (d+ex^4)(a+bx^4+cx^8)}\,dx$$
$$= \frac{1}{12}\left(-\frac{2}{adx^6}+\frac{6(bd+ae)}{a^2d^2x^2}-\frac{6e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{d^{5/2}(cd^2-bde+ae^2)}-\frac{6e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{d^{5/2}(cd^2-bde+ae^2)}\right.$$
$$-\frac{3\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{-b^2cd\log(x-\#1)+ac^2d\log(x-\#1)+b^3e\log(x-\#1)-2abce\log(x-\#1)-bc^2d\log(x-\#1}{b\#1^2+2c\#1^6}\right.}{a^2(cd^2+e(-bd+ae))}$$

input
```
Integrate[1/(x^7*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(-2/(a*d*x^6) + (6*(b*d + a*e))/(a^2*d^2*x^2) - (6*e^(7/2)*ArcTan[1 - (Sqr
t[2]*e^(1/4)*x)/d^(1/4)])/(d^(5/2)*(c*d^2 - b*d*e + a*e^2)) - (6*e^(7/2)*A
rcTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(d^(5/2)*(c*d^2 - b*d*e + a*e^2))
- (3*RootSum[a + b*#1^4 + c*#1^8 & , (-(b^2*c*d*Log[x - #1]) + a*c^2*d*Log
[x - #1] + b^3*e*Log[x - #1] - 2*a*b*c*e*Log[x - #1] - b*c^2*d*Log[x - #1]
*#1^4 + b^2*c*e*Log[x - #1]*#1^4 - a*c^2*e*Log[x - #1]*#1^4)/(b*#1^2 + 2*c
*#1^6) & ])/(a^2*(c*d^2 + e*(-(b*d) + a*e))))/12
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1814, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (d+ex^4)(a+bx^4+cx^8)}\,dx$$

$$\Big\downarrow \text{1814}$$

$$\frac{1}{2}\int \frac{1}{x^8\left(ex^4+d\right)\left(cx^8+bx^4+a\right)}dx^2$$

$$\Big\downarrow \text{1610}$$

$$\frac{1}{2}\int \left(\frac{e^4}{d^2\left(cd^2-bed+ae^2\right)\left(ex^4+d\right)}+\frac{c\left(-eb^2+cdb+ace\right)x^4-ac^2d+b^2cd-b^3e+2abce}{a^2\left(cd^2-bed+ae^2\right)\left(cx^8+bx^4+a\right)}+\frac{-bd-ae}{a^2d^2x^4}+\frac{1}{adx\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{3abc\cdots}{\cdots}\right.}{\sqrt{2}a^2\sqrt{\sqrt{b^2-\cdots}}}\right.$$

input
```
Int[1/(x^7*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(-1/3*1/(a*d*x^6) + (b*d + a*e)/(a^2*d^2*x^2) + (Sqrt[c]*(b*c*d - b^2*e +
a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(b*c*d - b^2*e +
a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x
^2)/Sqrt[d]])/(d^(5/2)*(c*d^2 - b*d*e + a*e^2)))/2
```

Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.97

method	result
default	$\frac{2c\left(-\frac{\left(ace\sqrt{-4ac+b^2}-b^2e\sqrt{-4ac+b^2}+cbd\sqrt{-4ac+b^2}+3abce-2a c^2 d-b^3 e+b^2 cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(ace\sqrt{-4ac+b^2}-b^2 e \cdots\right.}{\cdots}\right.}{(a e^2-bde+c d^2)a^2}$
risch	Expression too large to display

input
```
int(1/x^7/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
2/(a*e^2-b*d*e+c*d^2)/a^2*c*(-1/8*(a*c*e*(-4*a*c+b^2)^(1/2)-b^2*e*(-4*a*c+
b^2)^(1/2)+c*b*d*(-4*a*c+b^2)^(1/2)+3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(a*c*e*(-4*a*c+b^2)^(1/2)-b^2
*e*(-4*a*c+b^2)^(1/2)+c*b*d*(-4*a*c+b^2)^(1/2)-3*a*b*c*e+2*a*c^2*d+b^3*e-b
^2*c*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/2*e^4/d^2/(a*e^2-b*d*e
+c*d^2)/(d*e)^(1/2)*arctan(e*x^2/(d*e)^(1/2))-1/6/a/d/x^6-1/2*(-a*e-b*d)/x
^2/d^2/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^7\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^7/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^7\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**7/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^7\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x^7/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9619 vs. $2(312) = 624$.

Time = 2.63 (sec) , antiderivative size = 9619, normalized size of antiderivative = 26.28

$$\int \frac{1}{x^7 (d + ex^4) (a + bx^4 + cx^8)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^7/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
1/2*e^4*arctan(e*x^2/sqrt(d*e))/((c*d^4 - b*d^3*e + a*d^2*e^2)*sqrt(d*e))
+ 1/8*((2*a^2*b^4*c^5 - 8*a^3*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 2*(b^2 - 4*a*c)*a^2*b^2*c^5)*d^3*x^4 -
 (4*a^2*b^5*c^4 - 18*a^3*b^3*c^5 + 8*a^4*b*c^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 9*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 4*(b^2 - 4*a*c)*a^2*b^3*c^4 + 2
*(b^2 - 4*a*c)*a^3*b*c^5)*d^2*e*x^4 + (2*a^2*b^6*c^3 - 8*a^3*b^4*c^4 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*(b^2 -
4*a*c)*a^2*b^4*c^3)*d*e^2*x^4 - (2*a^3*b^5*c^3 - 10*a^4*b^3*c^4 + 8*a^5...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (d+ex^4) (a+bx^4+cx^8)} \, dx = \text{Hanged}$$

input
```
int(1/(x^7*(d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{x^7 (d+ex^4) (a+bx^4+cx^8)} \, dx = \int \frac{1}{x^7 (e\,x^4+d) (c\,x^8+b\,x^4+a)}dx$$

input
```
int(1/x^7/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(1/x^7/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.84 $\int \frac{x^4}{(d+ex^4)(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 630

$$\int \frac{x^4}{(d+ex^4)(a+bx^4+cx^8)}\,dx = -\frac{c^{3/4}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)} - \frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)} + \frac{\sqrt[4]{d}e^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}(cd^2-bde+ae^2)} - \frac{\sqrt[4]{d}e^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}(cd^2-bde+ae^2)} - \frac{c^{3/4}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)} - \frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)} - \frac{\sqrt[4]{d}e^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x}{\sqrt{d}+\sqrt{e}x^2}\right)}{2\sqrt{2}(cd^2-bde+ae^2)}$$

output
```
-1/4*c^(3/4)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/
(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(a*e^
2-b*d*e+c*d^2)-1/4*c^(3/4)*(d-(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1
/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-b+(-4*a*c+b^2)^(1/2
))^(3/4)/(a*e^2-b*d*e+c*d^2)-1/4*d^(1/4)*e^(3/4)*arctan(-1+2^(1/2)*e^(1/4)
*x/d^(1/4))*2^(1/2)/(a*e^2-b*d*e+c*d^2)-1/4*d^(1/4)*e^(3/4)*arctan(1+2^(1/
2)*e^(1/4)*x/d^(1/4))*2^(1/2)/(a*e^2-b*d*e+c*d^2)-1/4*c^(3/4)*(d+(-2*a*e+b
*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^
(1/4))*2^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(a*e^2-b*d*e+c*d^2)-1/4*c^(3/
4)*(d-(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a
*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)/(a*e^2-b*d*e+c
*d^2)-1/4*d^(1/4)*e^(3/4)*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*x/(d^(1/2)+e^(1/
2)*x^2))*2^(1/2)/(a*e^2-b*d*e+c*d^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.34

$$\int \frac{x^4}{(d+ex^4)\,(a+bx^4+cx^8)}\,dx = \frac{\sqrt{2}\sqrt[4]{d}e^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)+\log\left(\sqrt{d}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{e}x^2\right)-\log\left(\right.}{8\,(cd^2+e(-bd}$$

input
```
Integrate[x^4/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(Sqrt[2]*d^(1/4)*e^(3/4)*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)] - 2*Ar
cTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1
/4)*x + Sqrt[e]*x^2] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x
^2]) + 2*RootSum[a + b*#1^4 + c*#1^8 & , (a*e*Log[x - #1] + c*d*Log[x - #1
]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(8*(c*d^2 + e*(-(b*d) + a*e)))
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1836, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex^4)(a+bx^4+cx^8)}dx$$
$$\big\downarrow \text{1836}$$
$$\int \left(\frac{ae+cdx^4}{(a+bx^4+cx^8)(ae^2-bde+cd^2)}-\frac{de}{(d+ex^4)(ae^2-bde+cd^2)}\right)dx$$
$$\big\downarrow \text{2009}$$
$$-\frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2\sqrt[4]{2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}(ae^2-bde+cd^2)}-\frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}(ae^2-bde+cd^2)}+\frac{\sqrt[4]{d}e^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}(ae^2-bde+cd^2)}-\frac{\sqrt[4]{d}e^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}+1\right)}{2\sqrt{2}(ae^2-bde+cd^2)}-\frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2\sqrt[4]{2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}(ae^2-bde+cd^2)}-\frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}(ae^2-bde+cd^2)}+\frac{\sqrt[4]{d}e^{3/4}\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}+\sqrt{e}x^2\right)}{4\sqrt{2}(ae^2-bde+cd^2)}-\frac{\sqrt[4]{d}e^{3/4}\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}+\sqrt{e}x^2\right)}{4\sqrt{2}(ae^2-bde+cd^2)}$$

input
```
Int[x^4/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/2*(c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4
)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3
/4)*(c*d^2 - b*d*e + a*e^2)) - (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)
*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)) + (d^(1/4)*e^(3/4
)*ArcTan[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e
^2)) - (d^(1/4)*e^(3/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2
]*(c*d^2 - b*d*e + a*e^2)) - (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)) - (c^(3/4)*(d - (b
*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e
 + a*e^2)) + (d^(1/4)*e^(3/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*x + Sq
rt[e]*x^2])/(4*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)) - (d^(1/4)*e^(3/4)*Log[Sqr
t[d] + Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2])/(4*Sqrt[2]*(c*d^2 - b*d*e
 + a*e^2))
```

Defintions of rubi rules used

rule 1836
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e
*x^n)^q/(a + b*x^n + c*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[q] && Int
egerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.30

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8 c+\_Z^4 b+a\right)} \frac{\left(\_R^4 cd+ae\right)\ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}}{4a e^2-4bde+4c d^2}-\frac{e\left(\frac{d}{e}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{d}{e}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d}{e}}}{x^2-\left(\frac{d}{e}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d}{e}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d}{e}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arc}}{8\left(a e^2-bde+c d^2\right)}$
risch	Expression too large to display

input
```
int(x^4/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*e^2-b*d*e+c*d^2)*sum((_R^4*c*d+a*e)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=R
ootOf(_Z^8*c+_Z^4*b+a))-1/8*e/(a*e^2-b*d*e+c*d^2)*(d/e)^(1/4)*2^(1/2)*(ln(
(x^2+(d/e)^(1/4)*x*2^(1/2)+(d/e)^(1/2))/(x^2-(d/e)^(1/4)*x*2^(1/2)+(d/e)^(
1/2)))+2*arctan(2^(1/2)/(d/e)^(1/4)*x+1)+2*arctan(2^(1/2)/(d/e)^(1/4)*x-1)
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 153.05 (sec) , antiderivative size = 47212, normalized size of antiderivative = 74.94

$$\int \frac{x^4}{(d+ex^4)\,(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^4)\,(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4}{(d+ex^4)\,(a+bx^4+cx^8)}\,dx = \int \frac{x^4}{(cx^8+bx^4+a)(ex^4+d)}\,dx$$

input
```
integrate(x^4/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-1/8*(sqrt(2)*e^(3/4)*log(sqrt(e)*x^2 + sqrt(2)*d^(1/4)*e^(1/4)*x + sqrt(d
))/d^(3/4) - sqrt(2)*e^(3/4)*log(sqrt(e)*x^2 - sqrt(2)*d^(1/4)*e^(1/4)*x +
 sqrt(d))/d^(3/4) + sqrt(2)*e*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)*sqr
t(e)) + sqrt(2)*d^(1/4)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*sqrt
(e)) + sqrt(2)*d^(1/4)*e^(1/4)))/(sqrt(d)*sqrt(-sqrt(d)*sqrt(e))) + sqrt(2
)*e*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4)*e^
(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4)*e^(
1/4)))/(sqrt(d)*sqrt(-sqrt(d)*sqrt(e))))*d/(c*d^2 - b*d*e + a*e^2) + integ
rate((c*d*x^4 + a*e)/(c*x^8 + b*x^4 + a), x)/(c*d^2 - b*d*e + a*e^2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 51.64 (sec) , antiderivative size = 359899, normalized size of antiderivative = 571.27

$$\int \frac{x^4}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
int(x^4/((d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
(2^(1/2)*atan(((2^(1/2)*(x*(4*a^4*c^7*d^2*e^9 - 8*a^3*c^8*d^4*e^7 + 4*a^2*
b^2*c^7*d^4*e^7) - (2^(1/2)*(-(-(d*e^3)/64)^(1/2))^(1/2)*((2^(1/2)*(x*(204
8*a^3*c^12*d^11*e^4 + 8192*a^4*c^11*d^9*e^6 - 118784*a^5*c^10*d^7*e^8 + 27
0336*a^6*c^9*d^5*e^10 - 129024*a^7*c^8*d^3*e^12 - 1024*a^2*b^2*c^11*d^11*e
^4 + 4096*a^2*b^3*c^10*d^10*e^5 - 6144*a^2*b^4*c^9*d^9*e^6 + 3072*a^2*b^5*
c^8*d^8*e^7 + 3072*a^2*b^6*c^7*d^7*e^8 - 6144*a^2*b^7*c^6*d^6*e^9 + 4096*a
^2*b^8*c^5*d^5*e^10 - 1024*a^2*b^9*c^4*d^4*e^11 + 8192*a^3*b^2*c^10*d^9*e^
6 + 12288*a^3*b^3*c^9*d^8*e^7 - 51200*a^3*b^4*c^8*d^7*e^8 + 70656*a^3*b^5*
c^7*d^6*e^9 - 49152*a^3*b^6*c^6*d^5*e^10 + 13312*a^3*b^7*c^5*d^4*e^11 + 14
9504*a^4*b^2*c^9*d^7*e^8 - 233472*a^4*b^3*c^8*d^6*e^9 + 206848*a^4*b^4*c^7
*d^5*e^10 - 68608*a^4*b^5*c^6*d^4*e^11 + 4096*a^4*b^6*c^5*d^3*e^12 - 1024*
a^4*b^7*c^4*d^2*e^13 - 385024*a^5*b^2*c^8*d^5*e^10 + 176128*a^5*b^3*c^7*d^
4*e^11 - 40960*a^5*b^4*c^6*d^3*e^12 + 11264*a^5*b^5*c^5*d^2*e^13 + 130048*
a^6*b^2*c^7*d^3*e^12 - 40960*a^6*b^3*c^6*d^2*e^13 - 8192*a^3*b*c^11*d^10*e
^5 - 40960*a^4*b*c^10*d^8*e^7 + 253952*a^5*b*c^9*d^6*e^9 - 188416*a^6*b*c^
8*d^4*e^11 + 49152*a^7*b*c^7*d^2*e^13) - (2^(1/2)*(-(-(d*e^3)/64)^(1/2))^(
1/2)*((2^(1/2)*(x*(33554432*a^5*c^14*d^14*e^5 + 100663296*a^6*c^13*d^12*e^
7 + 134217728*a^7*c^12*d^10*e^9 + 201326592*a^8*c^11*d^8*e^11 + 301989888*
a^9*c^10*d^6*e^13 + 234881024*a^10*c^9*d^4*e^15 + 67108864*a^11*c^8*d^2*e^
17 + 262144*a^2*b^5*c^12*d^15*e^4 - 2097152*a^2*b^6*c^11*d^14*e^5 + 734...
```

Reduce [F]

$$\int \frac{x^4}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \int \frac{x^4}{(e\,x^4+d)(c\,x^8+b\,x^4+a)}dx$$

input
```
int(x^4/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x^4/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.85 $\int \frac{x^2}{(d+ex^4)(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 634

$$\int \frac{x^2}{(d+ex^4)(a+bx^4+cx^8)}\,dx = -\frac{\sqrt[4]{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt[4]{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{e^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}\sqrt[4]{d}\,(cd^2-bde+ae^2)} + \frac{e^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}\sqrt[4]{d}\,(cd^2-bde+ae^2)} + \frac{\sqrt[4]{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{\sqrt[4]{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{e^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x}{\sqrt{d}+\sqrt{e}x^2}\right)}{2\sqrt{2}\sqrt[4]{d}\,(cd^2-bde+ae^2)}$$

output
```
-1/4*c^(1/4)*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/
(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(a*e^
2-b*d*e+c*d^2)-1/4*c^(1/4)*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1
/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-b+(-4*a*c+b^2)^(1/2
))^(1/4)/(a*e^2-b*d*e+c*d^2)+1/4*e^(5/4)*arctan(-1+2^(1/2)*e^(1/4)*x/d^(1/
4))*2^(1/2)/d^(1/4)/(a*e^2-b*d*e+c*d^2)+1/4*e^(5/4)*arctan(1+2^(1/2)*e^(1/
4)*x/d^(1/4))*2^(1/2)/d^(1/4)/(a*e^2-b*d*e+c*d^2)+1/4*c^(1/4)*(e+(-b*e+2*c
*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^
(1/4))*2^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(a*e^2-b*d*e+c*d^2)+1/4*c^(1/
4)*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a
*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)/(a*e^2-b*d*e+c
*d^2)-1/4*e^(5/4)*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*x/(d^(1/2)+e^(1/2)*x^2))
*2^(1/2)/d^(1/4)/(a*e^2-b*d*e+c*d^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.36

$$\int \frac{x^2}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \frac{\sqrt{2}e^{5/4}\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)+\log\left(\sqrt{d}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{e}x^2\right)-\log\left(\cdots\right.\right.}{8\sqrt[4]{d}\,(cd^2\cdots}$$

input
```
Integrate[x^2/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(Sqrt[2]*e^(5/4)*(-2*ArcTan[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)] + 2*ArcTan[1
+ (Sqrt[2]*e^(1/4)*x)/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*x +
 Sqrt[e]*x^2] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2]) -
2*d^(1/4)*RootSum[a + b*#1^4 + c*#1^8 & , (-(c*d*Log[x - #1]) + b*e*Log[x
- #1] + c*e*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(8*d^(1/4)*(c*d^2 + e
*(-(b*d) + a*e)))
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1836, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex^4)(a+bx^4+cx^8)}dx$$

$$\Big\downarrow \text{1836}$$

$$\int \left(\frac{e^2x^2}{(d+ex^4)(ae^2-bde+cd^2)}+\frac{x^2(-be+cd-cex^4)}{(a+bx^4+cx^8)(ae^2-bde+cd^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2\,2^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}\,(ae^2-bde+cd^2)}-\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2\,2^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}\,(ae^2-bde+cd^2)}-\frac{e^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}\sqrt[4]{d}(ae^2-bde+cd^2)}+\frac{e^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}+1\right)}{2\sqrt{2}\sqrt[4]{d}(ae^2-bde+cd^2)}+\frac{\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2\,2^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}\,(ae^2-bde+cd^2)}+\frac{\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2\,2^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}\,(ae^2-bde+cd^2)}+\frac{e^{5/4}\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}+\sqrt{e}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(ae^2-bde+cd^2)}-\frac{e^{5/4}\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}+\sqrt{e}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(ae^2-bde+cd^2)}$$

input
```
Int[x^2/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/2*(c^(1/4)*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4
)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1
/4)*(c*d^2 - b*d*e + a*e^2)) - (c^(1/4)*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)
*(-b + Sqrt[b^2 - 4*a*c])^(1/4)*(c*d^2 - b*d*e + a*e^2)) - (e^(5/4)*ArcTan
[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*d^(1/4)*(c*d^2 - b*d*e + a*e
^2)) + (e^(5/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*d^(1/4
)*(c*d^2 - b*d*e + a*e^2)) + (c^(1/4)*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*
(-b - Sqrt[b^2 - 4*a*c])^(1/4)*(c*d^2 - b*d*e + a*e^2)) + (c^(1/4)*(e - (2
*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)*(c*d^2 - b*d*e
 + a*e^2)) + (e^(5/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^
2])/(4*Sqrt[2]*d^(1/4)*(c*d^2 - b*d*e + a*e^2)) - (e^(5/4)*Log[Sqrt[d] + S
qrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2])/(4*Sqrt[2]*d^(1/4)*(c*d^2 - b*d*e
 + a*e^2))
```

Defintions of rubi rules used

rule 1836
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e
*x^n)^q/(a + b*x^n + c*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[q] && Int
egerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.31

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\left(ce\_R^{6}+(eb-cd)\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4(a\,e^{2}-bde+c\,d^{2})}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d}{e}}}{x^{2}+\left(\frac{d}{e}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d}{e}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d}{e}\right)^{\frac{1}{4}}}+1\right.\right.}{8(a\,e^{2}-bde+c\,d^{2})\left(\frac{d}{e}\right)^{\frac{1}{4}}}$
risch	Expression too large to display

input
```
int(x^2/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(a*e^2-b*d*e+c*d^2)*sum((c*e*_R^6+(b*e-c*d)*_R^2)/(2*_R^7*c+_R^3*b)*l
n(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))+1/8*e/(a*e^2-b*d*e+c*d^2)/(d/e)^(1/4)*
2^(1/2)*(ln((x^2-(d/e)^(1/4)*x*2^(1/2)+(d/e)^(1/2))/(x^2+(d/e)^(1/4)*x*2^(
1/2)+(d/e)^(1/2)))+2*arctan(2^(1/2)/(d/e)^(1/4)*x+1)+2*arctan(2^(1/2)/(d/e
)^(1/4)*x-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \int \frac{x^2}{(cx^8+bx^4+a)(ex^4+d)}\,dx$$

input
```
integrate(x^2/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-1/8*e^2*(sqrt(2)*log(sqrt(e)*x^2 + sqrt(2)*d^(1/4)*e^(1/4)*x + sqrt(d))/(
d^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*x^2 - sqrt(2)*d^(1/4)*e^(1/4)*x + s
qrt(d))/(d^(1/4)*e^(3/4)) - sqrt(2)*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(
d)*sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d
)*sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4)))/(sqrt(-sqrt(d)*sqrt(e))*sqrt(e)) -
sqrt(2)*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4
)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4)
*e^(1/4)))/(sqrt(-sqrt(d)*sqrt(e))*sqrt(e)))/(c*d^2 - b*d*e + a*e^2) - int
egrate((c*e*x^6 - (c*d - b*e)*x^2)/(c*x^8 + b*x^4 + a), x)/(c*d^2 - b*d*e
+ a*e^2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 52.16 (sec) , antiderivative size = 325549, normalized size of antiderivative = 513.48

$$\int \frac{x^2}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
int(x^2/((d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
(atan((((x*(4*a^2*c^9*d*e^10 + 4*a*b*c^9*d^2*e^9) - ((-2*(-((b^9*e^4 + b^5
*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(
1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6
*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^
3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 -
 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40
*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(
1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128
*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a
^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*
e^3*(-(4*a*c - b^2)^5)^(1/2))*(a^5*b^8*e^8 + 256*a^5*c^8*d^8 + 256*a^9*c^4
*e^8 + a*b^8*c^4*d^8 - 16*a^6*b^6*c*e^8 + a*b^12*d^4*e^4 - 4*a^4*b^9*d*e^7
 - 16*a^2*b^6*c^5*d^8 + 96*a^3*b^4*c^6*d^8 - 256*a^4*b^2*c^7*d^8 + 96*a^7*
b^4*c^2*e^8 - 256*a^8*b^2*c^3*e^8 - 4*a^2*b^11*d^3*e^5 + 6*a^3*b^10*d^2*e^
6 + 1024*a^6*c^7*d^6*e^2 + 1536*a^7*c^6*d^4*e^4 + 1024*a^8*c^5*d^2*e^6 - 9
2*a^2*b^8*c^3*d^6*e^2 + 52*a^2*b^9*c^2*d^5*e^3 + 512*a^3*b^6*c^4*d^6*e^2 -
 192*a^3*b^7*c^3*d^5*e^3 - 90*a^3*b^8*c^2*d^4*e^4 - 1152*a^4*b^4*c^5*d^6*e
^2 - 128*a^4*b^5*c^4*d^5*e^3 + 800*a^4*b^6*c^3*d^4*e^4 - 192*a^4*b^7*c^2*d
^3*e^5 + 512*a^5*b^2*c^6*d^6*e^2 + 2048*a^5*b^3*c^5*d^5*e^3 - 2240*a^5*...
```

Reduce [F]

$$\int \frac{x^2}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \int \frac{x^2}{(e\,x^4+d)(c\,x^8+b\,x^4+a)}dx$$

input
```
int(x^2/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(x^2/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.86 $\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 634

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \frac{c^{3/4}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)} + \frac{c^{3/4}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)} - \frac{e^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}d^{3/4}(cd^2-bde+ae^2)} + \frac{e^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}d^{3/4}(cd^2-bde+ae^2)} + \frac{c^{3/4}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)} + \frac{c^{3/4}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)} + \frac{e^{7/4}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x}{\sqrt{d}+\sqrt{e}x^2}\right)}{2\sqrt{2}d^{3/4}(cd^2-bde+ae^2)}$$

output
```
1/4*c^(3/4)*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(
-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(a*e^2
-b*d*e+c*d^2)+1/4*c^(3/4)*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/
4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-b+(-4*a*c+b^2)^(1/2)
)^(3/4)/(a*e^2-b*d*e+c*d^2)+1/4*e^(7/4)*arctan(-1+2^(1/2)*e^(1/4)*x/d^(1/4
))*2^(1/2)/d^(3/4)/(a*e^2-b*d*e+c*d^2)+1/4*e^(7/4)*arctan(1+2^(1/2)*e^(1/4
)*x/d^(1/4))*2^(1/2)/d^(3/4)/(a*e^2-b*d*e+c*d^2)+1/4*c^(3/4)*(e+(-b*e+2*c*
d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(
1/4))*2^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(a*e^2-b*d*e+c*d^2)+1/4*c^(3/4
)*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*
c+b^2)^(1/2))^(1/4))*2^(3/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)/(a*e^2-b*d*e+c*
d^2)+1/4*e^(7/4)*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*x/(d^(1/2)+e^(1/2)*x^2))*
2^(1/2)/d^(3/4)/(a*e^2-b*d*e+c*d^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.36

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \frac{\sqrt{2}e^{7/4}\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)-\log\left(\sqrt{d}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{e}x^2\right)+\log\left(\sqrt{\ldots}\right.\right.}{8d^{3/4}\left(cd^{\ldots}\right.}$$

input
```
Integrate[1/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(Sqrt[2]*e^(7/4)*(-2*ArcTan[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)] + 2*ArcTan[1
+ (Sqrt[2]*e^(1/4)*x)/d^(1/4)] - Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*x +
 Sqrt[e]*x^2] + Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2]) -
2*d^(3/4)*RootSum[a + b*#1^4 + c*#1^8 & , (-(c*d*Log[x - #1]) + b*e*Log[x
- #1] + c*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(8*d^(3/4)*(c*d^2 +
 e*(-(b*d) + a*e)))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1754, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)}dx$$

$$\Big\downarrow \text{1754}$$

$$\int \left(\frac{e^2}{(d+ex^4)(ae^2-bde+cd^2)}+\frac{-be+cd-cex^4}{(a+bx^4+cx^8)(ae^2-bde+cd^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2\sqrt[4]{2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}(ae^2-bde+cd^2)}+\frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}(ae^2-bde+cd^2)}-\frac{e^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}d^{3/4}(ae^2-bde+cd^2)}+\frac{e^{7/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}+1\right)}{2\sqrt{2}d^{3/4}(ae^2-bde+cd^2)}+\frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2\sqrt[4]{2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}(ae^2-bde+cd^2)}+\frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}(ae^2-bde+cd^2)}-\frac{e^{7/4}\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}+\sqrt{e}x^2\right)}{4\sqrt{2}d^{3/4}(ae^2-bde+cd^2)}+\frac{e^{7/4}\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}+\sqrt{e}x^2\right)}{4\sqrt{2}d^{3/4}(ae^2-bde+cd^2)}$$

input
```
Int[1/((d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(c^(3/4)*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)
*(c*d^2 - b*d*e + a*e^2)) + (c^(3/4)*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])
*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(-
b + Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)) - (e^(7/4)*ArcTan[1
- (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)
) + (e^(7/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*d^(3/4)*(
c*d^2 - b*d*e + a*e^2)) + (c^(3/4)*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(-b
 - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)) + (c^(3/4)*(e - (2*c*
d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4
*a*c])^(1/4)])/(2*2^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e +
a*e^2)) - (e^(7/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2])
/(4*Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)) + (e^(7/4)*Log[Sqrt[d] + Sqrt
[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2])/(4*Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e +
a*e^2))
```

Defintions of rubi rules used

rule 1754
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.31

method	result
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8} c+\_Z^{4} b+a\right)} \frac{\left(-\_R^{4} ce-eb+cd\right) \ln\left(x-\_R\right)}{2\_R^{7} c+\_R^{3} b}}{4a\,e^{2}-4bde+4c\,d^{2}}+\frac{e^{2} \left(\frac{d}{e}\right)^{\frac{1}{4}} \sqrt{2}\,\left(\ln\left(\frac{x^{2}+\left(\frac{d}{e}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{d}{e}}}{x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{d}{e}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d}{e}\right)^{\frac{1}{4}}}+1\right)\right.}{8(a\,e^{2}-bde+c\,d^{2})d}$
risch	Expression too large to display

input
```
int(1/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*e^2-b*d*e+c*d^2)*sum((-_R^4*c*e-b*e+c*d)/(2*_R^7*c+_R^3*b)*ln(x-_R)
,_R=RootOf(_Z^8*c+_Z^4*b+a))+1/8*e^2/(a*e^2-b*d*e+c*d^2)*(d/e)^(1/4)/d*2^(
1/2)*(ln((x^2+(d/e)^(1/4)*x*2^(1/2)+(d/e)^(1/2))/(x^2-(d/e)^(1/4)*x*2^(1/2
)+(d/e)^(1/2)))+2*arctan(2^(1/2)/(d/e)^(1/4)*x+1)+2*arctan(2^(1/2)/(d/e)^(
1/4)*x-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \int \frac{1}{(cx^8+bx^4+a)(ex^4+d)}\,dx$$

input
```
integrate(1/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*(sqrt(2)*e^(7/4)*log(sqrt(e)*x^2 + sqrt(2)*d^(1/4)*e^(1/4)*x + sqrt(d)
)/d^(3/4) - sqrt(2)*e^(7/4)*log(sqrt(e)*x^2 - sqrt(2)*d^(1/4)*e^(1/4)*x +
sqrt(d))/d^(3/4) + sqrt(2)*e^2*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)*sq
rt(e)) + sqrt(2)*d^(1/4)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*sqr
t(e)) + sqrt(2)*d^(1/4)*e^(1/4)))/(sqrt(d)*sqrt(-sqrt(d)*sqrt(e))) + sqrt(
2)*e^2*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4)
*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4)*
e^(1/4)))/(sqrt(d)*sqrt(-sqrt(d)*sqrt(e))))/(c*d^2 - b*d*e + a*e^2) - inte
grate((c*e*x^4 - c*d + b*e)/(c*x^8 + b*x^4 + a), x)/(c*d^2 - b*d*e + a*e^2
)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 53.44 (sec) , antiderivative size = 419203, normalized size of antiderivative = 661.20

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
(atan((((12*c^11*e^11*x + ((-2*(-((b^11*e^4 + b^7*c^4*d^4 + b^6*e^4*(-(4*a
*c - b^2)^5)^(1/2) - 11*a*b^5*c^5*d^4 - 48*a^3*b*c^7*d^4 - a*c^5*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*e^4 - 128*a^4*c^7*d^3*e + 128*a^5*c^6*
d*e^3 - 4*b^8*c^3*d^3*e + 40*a^2*b^3*c^6*d^4 + 86*a^2*b^7*c^2*e^4 - 231*a^
3*b^5*c^3*e^4 + 280*a^4*b^3*c^4*e^4 - a^3*c^3*e^4*(-(4*a*c - b^2)^5)^(1/2)
 + b^2*c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^9*c^2*d^2*e^2 - 15*a*b^9*c*e
^4 - 4*b^10*c*d*e^3 + 6*a^2*b^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 366*a^2
*b^5*c^4*d^2*e^2 - 720*a^3*b^3*c^5*d^2*e^2 + 6*a^2*c^4*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) + 6*b^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*e^4
*(-(4*a*c - b^2)^5)^(1/2) + 48*a*b^6*c^4*d^3*e + 56*a*b^8*c^2*d*e^3 - 4*b^
5*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 78*a*b^7*c^3*d^2*e^2 - 200*a^2*b^4*c^
5*d^3*e - 292*a^2*b^6*c^3*d*e^3 + 320*a^3*b^2*c^6*d^3*e + 680*a^3*b^4*c^4*
d*e^3 + 480*a^4*b*c^6*d^2*e^2 - 640*a^4*b^2*c^5*d*e^3 - 4*b^3*c^3*d^3*e*(-
(4*a*c - b^2)^5)^(1/2) + 16*a*b^3*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 12*
a^2*b*c^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 18*a*b^2*c^3*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) + 8*a*b*c^4*d^3*e*(-(4*a*c - b^2)^5)^(1/2))*(a^7*b^8*e^8 +
256*a^7*c^8*d^8 + 256*a^11*c^4*e^8 - 16*a^8*b^6*c*e^8 - 4*a^6*b^9*d*e^7 +
a^3*b^8*c^4*d^8 - 16*a^4*b^6*c^5*d^8 + 96*a^5*b^4*c^6*d^8 - 256*a^6*b^2*c^
7*d^8 + 96*a^9*b^4*c^2*e^8 - 256*a^10*b^2*c^3*e^8 + a^3*b^12*d^4*e^4 - 4*a
^4*b^11*d^3*e^5 + 6*a^5*b^10*d^2*e^6 + 1024*a^8*c^7*d^6*e^2 + 1536*a^9*...
```

Reduce [F]

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)}\,dx = \int \frac{1}{(e\,x^4+d)\,(c\,x^8+b\,x^4+a)}dx$$

input
```
int(1/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(1/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.87 $\int \frac{1}{x^2(d+ex^4)(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 709

$$\int \frac{1}{x^2(d+ex^4)(a+bx^4+cx^8)}\,dx$$
$$= -\frac{1}{adx} - \frac{\sqrt[4]{c}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$- \frac{\sqrt[4]{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$+ \frac{e^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}d^{5/4}(cd^2-bde+ae^2)} - \frac{e^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}d^{5/4}(cd^2-bde+ae^2)}$$
$$+ \frac{\sqrt[4]{c}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$+ \frac{\sqrt[4]{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$
$$+ \frac{e^{9/4}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x}{\sqrt{d}+\sqrt{e}x^2}\right)}{2\sqrt{2}d^{5/4}(cd^2-bde+ae^2)}$$

output
```
-1/a/d/x-1/4*c^(1/4)*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*ar
ctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-b-(-4*a*
c+b^2)^(1/2))^(1/4)/(a*e^2-b*d*e+c*d^2)-1/4*c^(1/4)*(c*d-b*e+(2*a*c*e-b^2*
e+b*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/
2))^(1/4))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)/(a*e^2-b*d*e+c*d^2)-1/4
*e^(9/4)*arctan(-1+2^(1/2)*e^(1/4)*x/d^(1/4))*2^(1/2)/d^(5/4)/(a*e^2-b*d*e
+c*d^2)-1/4*e^(9/4)*arctan(1+2^(1/2)*e^(1/4)*x/d^(1/4))*2^(1/2)/d^(5/4)/(a
*e^2-b*d*e+c*d^2)+1/4*c^(1/4)*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^
(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/
(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(a*e^2-b*d*e+c*d^2)+1/4*c^(1/4)*(c*d-b*e+(2*
a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a
*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)/(a*e^2-b*d*e
+c*d^2)+1/4*e^(9/4)*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*x/(d^(1/2)+e^(1/2)*x^2
))*2^(1/2)/d^(5/4)/(a*e^2-b*d*e+c*d^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^2 (d+ex^4) (a+bx^4+cx^8)}\,dx$$
$$= \frac{-8cd^{9/4}+8bd^{5/4}e-8a\sqrt[4]{d}e^2+2\sqrt{2}ae^{9/4}x\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)-2\sqrt{2}ae^{9/4}x\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)-\cdots}{}$$

input
```
Integrate[1/(x^2*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(-8*c*d^(9/4) + 8*b*d^(5/4)*e - 8*a*d^(1/4)*e^2 + 2*Sqrt[2]*a*e^(9/4)*x*Ar
cTan[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)] - 2*Sqrt[2]*a*e^(9/4)*x*ArcTan[1 + (
Sqrt[2]*e^(1/4)*x)/d^(1/4)] - Sqrt[2]*a*e^(9/4)*x*Log[Sqrt[d] - Sqrt[2]*d^
(1/4)*e^(1/4)*x + Sqrt[e]*x^2] + Sqrt[2]*a*e^(9/4)*x*Log[Sqrt[d] + Sqrt[2]
*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2] + 2*d^(5/4)*x*RootSum[a + b*#1^4 + c*#1^
8 & , (-(b*c*d*Log[x - #1]) + b^2*e*Log[x - #1] - a*c*e*Log[x - #1] - c^2*
d*Log[x - #1]*#1^4 + b*c*e*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(8*a*d
^(5/4)*(c*d^2 + e*(-(b*d) + a*e))*x)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 778, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1836, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (d+ex^4) (a+bx^4+cx^8)} \, dx$$

$$\Big\downarrow \text{ 1836}$$

$$\int \left(\frac{x^2 (-ace+b^2e-cx^4(cd-be)-bcd)}{a (a+bx^4+cx^8) (ae^2-bde+cd^2)}-\frac{e^3x^2}{d (d+ex^4) (ae^2-bde+cd^2)}+\frac{1}{adx^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\ 2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}\,(ae^2-bde+cd^2)}-\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\ 2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}\,(ae^2-bde+cd^2)}+\frac{e^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}d^{5/4}(ae^2-bde+cd^2)}-\frac{e^{9/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}+1\right)}{2\sqrt{2}d^{5/4}(ae^2-bde+cd^2)}+\frac{\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\ 2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}\,(ae^2-bde+cd^2)}+\frac{\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\ 2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}\,(ae^2-bde+cd^2)}-\frac{e^{9/4}\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}+\sqrt{e}x^2\right)}{4\sqrt{2}d^{5/4}(ae^2-bde+cd^2)}+\frac{e^{9/4}\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}+\sqrt{e}x^2\right)}{4\sqrt{2}d^{5/4}(ae^2-bde+cd^2)}-\frac{1}{adx}$$

input
```
Int[1/(x^2*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
-(1/(a*d*x)) - (c^(1/4)*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(
3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)*(c*d^2 - b*d*e + a*e^2)) - (c^(1/4)*
(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*
c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 -
4*a*c])^(1/4)*(c*d^2 - b*d*e + a*e^2)) + (e^(9/4)*ArcTan[1 - (Sqrt[2]*e^(1
/4)*x)/d^(1/4)])/(2*Sqrt[2]*d^(5/4)*(c*d^2 - b*d*e + a*e^2)) - (e^(9/4)*Ar
cTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*d^(5/4)*(c*d^2 - b*d*e +
 a*e^2)) + (c^(1/4)*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4
)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)*(c*d^2 - b*d*e + a*e^2)) + (c^(1/4)*(c*
d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^
(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*
a*c])^(1/4)*(c*d^2 - b*d*e + a*e^2)) - (e^(9/4)*Log[Sqrt[d] - Sqrt[2]*d^(1
/4)*e^(1/4)*x + Sqrt[e]*x^2])/(4*Sqrt[2]*d^(5/4)*(c*d^2 - b*d*e + a*e^2))
+ (e^(9/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2])/(4*Sqrt
[2]*d^(5/4)*(c*d^2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 1836
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e
*x^n)^q/(a + b*x^n + c*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[q] && Int
egerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.42 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.32

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\left(c(-eb+cd)\_R^{6}+\left(ace-b^{2}e+cbd\right)\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4\left(a\,e^{2}-bde+c\,d^{2}\right)a}-\frac{1}{adx}-\frac{e^{2}\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d}{e}}}{x^{2}+\left(\frac{d}{e}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d}{e}}}\right.\right.}{8d}$
risch	Expression too large to display

input
```
int(1/x^2/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(a*e^2-b*d*e+c*d^2)/a*sum((c*(-b*e+c*d)*_R^6+(a*c*e-b^2*e+b*c*d)*_R^2
)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))-1/a/d/x-1/8*e^2/d
/(a*e^2-b*d*e+c*d^2)/(d/e)^(1/4)*2^(1/2)*(ln((x^2-(d/e)^(1/4)*x*2^(1/2)+(d
/e)^(1/2))/(x^2+(d/e)^(1/4)*x*2^(1/2)+(d/e)^(1/2)))+2*arctan(2^(1/2)/(d/e)
^(1/4)*x+1)+2*arctan(2^(1/2)/(d/e)^(1/4)*x-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex^4)(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(d+e x^4\right) \left(a+b x^4+c x^8\right)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 \left(d+e x^4\right) \left(a+b x^4+c x^8\right)} \, dx = \int \frac{1}{(c x^8+b x^4+a)(e x^4+d) x^2} \, dx$$

input
```
integrate(1/x^2/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*e^3*(sqrt(2)*log(sqrt(e)*x^2 + sqrt(2)*d^(1/4)*e^(1/4)*x + sqrt(d))/(d
^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*x^2 - sqrt(2)*d^(1/4)*e^(1/4)*x + sq
rt(d))/(d^(1/4)*e^(3/4)) - sqrt(2)*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d
)*sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)
*sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4)))/(sqrt(-sqrt(d)*sqrt(e))*sqrt(e)) - s
qrt(2)*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4)
*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4)*
e^(1/4)))/(sqrt(-sqrt(d)*sqrt(e))*sqrt(e)))/(c*d^3 - b*d^2*e + a*d*e^2) +
integrate(-((c^2*d - b*c*e)*x^6 + (b*c*d - (b^2 - a*c)*e)*x^2)/(c*x^8 + b*
x^4 + a), x)/(a*c*d^2 - a*b*d*e + a^2*e^2) - 1/(a*d*x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 39.25 (sec) , antiderivative size = 276728, normalized size of antiderivative = 390.31

$$\int \frac{1}{x^2\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(x^2*(d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
atan(((-e^9/(256*c^4*d^13 + 256*a^4*d^5*e^8 + 256*b^4*d^9*e^4 - 1024*a*b^3
*d^8*e^5 - 1024*a^3*b*d^6*e^7 + 1024*a*c^3*d^11*e^2 + 1024*a^3*c*d^7*e^6 -
 1024*b^3*c*d^10*e^3 + 1536*a^2*b^2*d^7*e^6 + 1536*a^2*c^2*d^9*e^4 + 1536*
b^2*c^2*d^11*e^2 - 1024*b*c^3*d^12*e - 3072*a*b*c^2*d^10*e^3 + 3072*a*b^2*
c*d^9*e^4 - 3072*a^2*b*c*d^8*e^5))^(1/4)*(x*(4*a^19*c^13*d^20*e^12 + 4*a^1
9*b*c^12*d^19*e^13) + (-e^9/(256*c^4*d^13 + 256*a^4*d^5*e^8 + 256*b^4*d^9*
e^4 - 1024*a*b^3*d^8*e^5 - 1024*a^3*b*d^6*e^7 + 1024*a*c^3*d^11*e^2 + 1024
*a^3*c*d^7*e^6 - 1024*b^3*c*d^10*e^3 + 1536*a^2*b^2*d^7*e^6 + 1536*a^2*c^2
*d^9*e^4 + 1536*b^2*c^2*d^11*e^2 - 1024*b*c^3*d^12*e - 3072*a*b*c^2*d^10*e
^3 + 3072*a*b^2*c*d^9*e^4 - 3072*a^2*b*c*d^8*e^5))^(3/4)*((x*(163840*a^24*
c^12*d^23*e^13 - 327680*a^25*c^11*d^21*e^15 + 32768*a^26*c^10*d^19*e^17 -
4096*a^19*b^2*c^15*d^31*e^5 + 6144*a^19*b^3*c^14*d^30*e^6 - 4096*a^19*b^4*
c^13*d^29*e^7 + 1024*a^19*b^5*c^12*d^28*e^8 + 1024*a^19*b^9*c^8*d^24*e^12
- 4096*a^19*b^10*c^7*d^23*e^13 + 6144*a^19*b^11*c^6*d^22*e^14 - 4096*a^19*
b^12*c^5*d^21*e^15 + 1024*a^19*b^13*c^4*d^20*e^16 - 12288*a^20*b^2*c^14*d^
29*e^7 + 12288*a^20*b^3*c^13*d^28*e^8 - 4096*a^20*b^4*c^12*d^27*e^9 - 1331
2*a^20*b^7*c^9*d^24*e^12 + 58368*a^20*b^8*c^8*d^23*e^13 - 96256*a^20*b^9*c
^7*d^22*e^14 + 71680*a^20*b^10*c^6*d^21*e^15 - 21504*a^20*b^11*c^5*d^20*e^
16 + 1024*a^20*b^12*c^4*d^19*e^17 - 12288*a^21*b^2*c^13*d^27*e^9 + 6144*a^
21*b^3*c^12*d^26*e^10 + 62464*a^21*b^5*c^10*d^24*e^12 - 311296*a^21*b^6...
```

Reduce [F]

$$\int \frac{1}{x^2 (d+ex^4) (a+bx^4+cx^8)} \, dx = \int \frac{1}{x^2 \left(e\,x^4+d\right) \left(c\,x^8+b\,x^4+a\right)} dx$$

input
```
int(1/x^2/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(1/x^2/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.88 $\int \frac{1}{x^4(d+ex^4)(a+bx^4+cx^8)} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 711

$$\int \frac{1}{x^4 (d+ex^4)(a+bx^4+cx^8)}\,dx$$
$$= -\frac{1}{3adx^3} + \frac{c^{3/4}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)}$$
$$+ \frac{c^{3/4}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)}$$
$$+ \frac{e^{11/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}d^{7/4}(cd^2-bde+ae^2)} - \frac{e^{11/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}d^{7/4}(cd^2-bde+ae^2)}$$
$$+ \frac{c^{3/4}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)}$$
$$+ \frac{c^{3/4}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)}$$
$$- \frac{e^{11/4}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x}{\sqrt{d}+\sqrt{e}x^2}\right)}{2\sqrt{2}d^{7/4}(cd^2-bde+ae^2)}$$

output
```
-1/3/a/d/x^3+1/4*c^(3/4)*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2)
)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-b-(-
4*a*c+b^2)^(1/2))^(3/4)/(a*e^2-b*d*e+c*d^2)+1/4*c^(3/4)*(c*d-b*e+(2*a*c*e-
b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)
^(1/2))^(1/4))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)/(a*e^2-b*d*e+c*d^2)
-1/4*e^(11/4)*arctan(-1+2^(1/2)*e^(1/4)*x/d^(1/4))*2^(1/2)/d^(7/4)/(a*e^2-
b*d*e+c*d^2)-1/4*e^(11/4)*arctan(1+2^(1/2)*e^(1/4)*x/d^(1/4))*2^(1/2)/d^(7
/4)/(a*e^2-b*d*e+c*d^2)+1/4*c^(3/4)*(c*d-b*e-(2*a*c*e-b^2*e+b*c*d)/(-4*a*c
+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3
/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(a*e^2-b*d*e+c*d^2)+1/4*c^(3/4)*(c*d-b
*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b
+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)/(a*e^2
-b*d*e+c*d^2)-1/4*e^(11/4)*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*x/(d^(1/2)+e^(1
/2)*x^2))*2^(1/2)/d^(7/4)/(a*e^2-b*d*e+c*d^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^4 (d+ex^4) (a+bx^4+cx^8)} \, dx$$
$$= \frac{-8cd^{11/4}+8bd^{7/4}e-8ad^{3/4}e^2+6\sqrt{2}ae^{11/4}x^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)-6\sqrt{2}ae^{11/4}x^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right.}{}$$

input
```
Integrate[1/(x^4*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
(-8*c*d^(11/4) + 8*b*d^(7/4)*e - 8*a*d^(3/4)*e^2 + 6*Sqrt[2]*a*e^(11/4)*x^
3*ArcTan[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)] - 6*Sqrt[2]*a*e^(11/4)*x^3*ArcTa
n[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)] + 3*Sqrt[2]*a*e^(11/4)*x^3*Log[Sqrt[d]
- Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2] - 3*Sqrt[2]*a*e^(11/4)*x^3*Log[
Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2] + 6*d^(7/4)*x^3*RootSum
[a + b*#1^4 + c*#1^8 & , (-(b*c*d*Log[x - #1]) + b^2*e*Log[x - #1] - a*c*e
*Log[x - #1] - c^2*d*Log[x - #1]*#1^4 + b*c*e*Log[x - #1]*#1^4)/(b*#1^3 +
2*c*#1^7) & ])/(24*a*d^(7/4)*(c*d^2 + e*(-(b*d) + a*e))*x^3)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1836, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (d+ex^4) (a+bx^4+cx^8)} \, dx$$

$$\Big\downarrow \; 1836$$

$$\int \left(\frac{-ace+b^2e-cx^4(cd-be)-bcd}{a (a+bx^4+cx^8) (ae^2-bde+cd^2)}-\frac{e^3}{d (d+ex^4) (ae^2-bde+cd^2)}+\frac{1}{adx^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{c^{3/4} \arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}(ae^2-bde+cd^2)}+\frac{c^{3/4} \arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}(ae^2-bde+cd^2)}+\frac{e^{11/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}d^{7/4}(ae^2-bde+cd^2)}-\frac{e^{11/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}+1\right)}{2\sqrt{2}d^{7/4}(ae^2-bde+cd^2)}+\frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}(ae^2-bde+cd^2)}+\frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}(ae^2-bde+cd^2)}+\frac{e^{11/4}\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}+\sqrt{e}x^2\right)}{4\sqrt{2}d^{7/4}(ae^2-bde+cd^2)}-\frac{e^{11/4}\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}+\sqrt{e}x^2\right)}{4\sqrt{2}d^{7/4}(ae^2-bde+cd^2)}-\frac{1}{3adx^3}$$

input
```
Int[1/(x^4*(d + e*x^4)*(a + b*x^4 + c*x^8)),x]
```

output
```
-1/3*1/(a*d*x^3) + (c^(3/4)*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^
2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2
*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)) + (c^(3
/4)*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1
/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^
2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)) + (e^(11/4)*ArcTan[1 - (Sqrt[2]
*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*d^(7/4)*(c*d^2 - b*d*e + a*e^2)) - (e^(11
/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*d^(7/4)*(c*d^2 - b
*d*e + a*e^2)) + (c^(3/4)*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2
- 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*
2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)) + (c^(3/
4)*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1
/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^
2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)) + (e^(11/4)*Log[Sqrt[d] - Sqrt[
2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2])/(4*Sqrt[2]*d^(7/4)*(c*d^2 - b*d*e + a
*e^2)) - (e^(11/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2])
/(4*Sqrt[2]*d^(7/4)*(c*d^2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 1836
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e
*x^n)^q/(a + b*x^n + c*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[q] && Int
egerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.32

method	result
default	$-\frac{1}{3ad\,x^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(c(eb-cd)\_R^4-ace+b^2e-cbd\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4\left(a\,e^2-bde+c\,d^2\right)a}-\frac{e^3\left(\frac{d}{e}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{d}{e}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d}{e}}}{x^2-\left(\frac{d}{e}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d}{e}}}\right.\right.}{8d^2(}$
risch	Expression too large to display

input
```
int(1/x^4/(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/d/x^3+1/4/(a*e^2-b*d*e+c*d^2)/a*sum((c*(b*e-c*d)*_R^4-a*c*e+b^2*e-c
*b*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))-1/8/d^2*e^3/(
a*e^2-b*d*e+c*d^2)*(d/e)^(1/4)*2^(1/2)*(ln((x^2+(d/e)^(1/4)*x*2^(1/2)+(d/e
)^(1/2))/(x^2-(d/e)^(1/4)*x*2^(1/2)+(d/e)^(1/2)))+2*arctan(2^(1/2)/(d/e)^(
1/4)*x+1)+2*arctan(2^(1/2)/(d/e)^(1/4)*x-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^4/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**4/(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^4\left(d+ex^4\right)\left(a+bx^4+cx^8\right)}\,dx=\int \frac{1}{(cx^8+bx^4+a)(ex^4+d)x^4}\,dx$$

input
```
integrate(1/x^4/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

output
```
-1/8*(sqrt(2)*e^(11/4)*log(sqrt(e)*x^2 + sqrt(2)*d^(1/4)*e^(1/4)*x + sqrt(
d))/d^(3/4) - sqrt(2)*e^(11/4)*log(sqrt(e)*x^2 - sqrt(2)*d^(1/4)*e^(1/4)*x
 + sqrt(d))/d^(3/4) + sqrt(2)*e^3*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)
*sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*
sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4)))/(sqrt(d)*sqrt(-sqrt(d)*sqrt(e))) + sq
rt(2)*e^3*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1
/4)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/
4)*e^(1/4)))/(sqrt(d)*sqrt(-sqrt(d)*sqrt(e))))/(c*d^3 - b*d^2*e + a*d*e^2)
 + integrate(-((c^2*d - b*c*e)*x^4 + b*c*d - (b^2 - a*c)*e)/(c*x^8 + b*x^4
 + a), x)/(a*c*d^2 - a*b*d*e + a^2*e^2) - 1/3/(a*d*x^3)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (d + ex^4) (a + bx^4 + cx^8)} dx = \text{Timed out}$$

input
```
integrate(1/x^4/(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 62.92 (sec) , antiderivative size = 522994, normalized size of antiderivative = 735.58

$$\int \frac{1}{x^4 (d + ex^4) (a + bx^4 + cx^8)} dx = \text{Too large to display}$$

input
```
int(1/(x^4*(d + e*x^4)*(a + b*x^4 + c*x^8)),x)
```

output
```
(2^(1/2)*atan(((2^(1/2)*(-(-e^11/(64*d^7))^(1/2))^(1/2)*(x*(4*a^17*c^15*d^
19*e^13 - 8*a^18*c^14*d^17*e^15 + 4*a^17*b^2*c^13*d^17*e^15) - (2^(1/2)*(-
(-e^11/(64*d^7))^(1/2))^(1/2)*(64*a^20*c^13*d^17*e^16 - 16*a^18*c^15*d^21*
e^12 - 16*a^17*c^16*d^23*e^10 + (2^(1/2)*(-(-e^11/(64*d^7))^(1/2))^(3/2)*(
x*(2048*a^18*c^18*d^32*e^4 + 8192*a^19*c^17*d^30*e^6 + 12288*a^20*c^16*d^2
8*e^8 + 8192*a^21*c^15*d^26*e^10 + 2048*a^22*c^14*d^24*e^12 + 131072*a^23*
c^13*d^22*e^14 - 262144*a^24*c^12*d^20*e^16 + 131072*a^25*c^11*d^18*e^18 -
 1024*a^17*b^2*c^17*d^32*e^4 + 4096*a^17*b^3*c^16*d^31*e^5 - 6144*a^17*b^4
*c^15*d^30*e^6 + 4096*a^17*b^5*c^14*d^29*e^7 - 1024*a^17*b^6*c^13*d^28*e^8
 - 1024*a^17*b^11*c^8*d^23*e^13 + 4096*a^17*b^12*c^7*d^22*e^14 - 6144*a^17
*b^13*c^6*d^21*e^15 + 4096*a^17*b^14*c^5*d^20*e^16 - 1024*a^17*b^15*c^4*d^
19*e^17 + 8192*a^18*b^2*c^16*d^30*e^6 + 4096*a^18*b^3*c^15*d^29*e^7 - 1024
0*a^18*b^4*c^14*d^28*e^8 + 4096*a^18*b^5*c^13*d^27*e^9 + 15360*a^18*b^9*c^
9*d^23*e^13 - 65536*a^18*b^10*c^8*d^22*e^14 + 104448*a^18*b^11*c^7*d^21*e^
15 - 73728*a^18*b^12*c^6*d^20*e^16 + 19456*a^18*b^13*c^5*d^19*e^17 + 18432
*a^19*b^2*c^15*d^28*e^8 + 4096*a^19*b^3*c^14*d^27*e^9 - 6144*a^19*b^4*c^13
*d^26*e^10 - 88064*a^19*b^7*c^10*d^23*e^13 + 409600*a^19*b^8*c^9*d^22*e^14
 - 707584*a^19*b^9*c^8*d^21*e^15 + 540672*a^19*b^10*c^7*d^20*e^16 - 157696
*a^19*b^11*c^6*d^19*e^17 + 4096*a^19*b^12*c^5*d^18*e^18 - 1024*a^19*b^13*c
^4*d^17*e^19 + 8192*a^20*b^2*c^14*d^26*e^10 + 4096*a^20*b^3*c^13*d^25*e...
```

Reduce [F]

$$\int \frac{1}{x^4 (d+ex^4) (a+bx^4+cx^8)} \, dx = \int \frac{1}{x^4 (e\,x^4+d) (c\,x^8+b\,x^4+a)} dx$$

input
```
int(1/x^4/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

output
```
int(1/x^4/(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

3.89 $\int \frac{x^{11}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 148

$$\int \frac{x^{11}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \frac{x^4(a(2cd-be)+(bcd-b^2e+2ace)\,x^4)}{4c\,(b^2-4ac)\,(a+bx^4+cx^8)} + \frac{(b^3e+2ac(2cd-3be))\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\,(b^2-4ac)^{3/2}} + \frac{e\log\left(a+bx^4+cx^8\right)}{8c^2}$$

output
```
1/4*x^4*(a*(-b*e+2*c*d)+(2*a*c*e-b^2*e+b*c*d)*x^4)/c/(-4*a*c+b^2)/(c*x^8+b
*x^4+a)+1/4*(b^3*e+2*a*c*(-3*b*e+2*c*d))*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^
(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/8*e*ln(c*x^8+b*x^4+a)/c^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.08

$$\int \frac{x^{11}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$
$$=\frac{-\frac{2(2a^2ce+b^2(cd-be)x^4+a(-b^2e-2c^2dx^4+bc(d+3ex^4)))}{(b^2-4ac)(a+bx^4+cx^8)}+\frac{2(b^3e+2ac(2cd-3be))\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+e\log\left(a+bx^4+cx^8\right)}{8c^2}$$

input
```
Integrate[(x^11*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
((-2*(2*a^2*c*e + b^2*(c*d - b*e)*x^4 + a*(-(b^2*e) - 2*c^2*d*x^4 + b*c*(d
 + 3*e*x^4))))/((b^2 - 4*a*c)*(a + b*x^4 + c*x^8)) + (2*(b^3*e + 2*a*c*(2*
c*d - 3*b*e))*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/
2) + e*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1802, 1233, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\left(d+ex^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx$$
$$\Big\downarrow \text{1802}$$
$$\frac{1}{4}\int \frac{x^8\left(ex^4+d\right)}{\left(cx^8+bx^4+a\right)^2}dx^4$$
$$\Big\downarrow \text{1233}$$
$$\frac{1}{4}\left(\frac{\int -\frac{a(2cd-be)-(b^2-4ac)ex^4}{cx^8+bx^4+a}dx^4}{c\left(b^2-4ac\right)}+\frac{x^4\left(x^4\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)\right)}{c\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right)$$

$$\downarrow 25$$

$$\frac{1}{4}\left(\frac{x^4\left(x^4\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)\right)}{c\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}-\frac{\int \frac{a(2cd-be)-\left(b^2-4ac\right)ex^4}{cx^8+bx^4+a}dx^4}{c\left(b^2-4ac\right)}\right)$$

$$\downarrow 1142$$

$$\frac{1}{4}\left(\frac{x^4\left(x^4\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)\right)}{c\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}-\frac{\frac{\left(2ac(2cd-3be)+b^3e\right)\int \frac{1}{cx^8+bx^4+a}dx^4}{2c}-\frac{e\left(b^2-4ac\right)\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4}{2c}}{c\left(b^2-4ac\right)}\right)$$

$$\downarrow 1083$$

$$\frac{1}{4}\left(\frac{x^4\left(x^4\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)\right)}{c\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}-\frac{-\frac{e\left(b^2-4ac\right)\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4}{2c}-\frac{\left(2ac(2cd-3be)+b^3e\right)\int \frac{1}{-x^8+b^2-4ac}d(2cx^4}{c}}{c\left(b^2-4ac\right)}\right.$$

$$\downarrow 219$$

$$\frac{1}{4}\left(\frac{x^4\left(x^4\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)\right)}{c\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}-\frac{-\frac{e\left(b^2-4ac\right)\int \frac{2cx^4+b}{cx^8+bx^4+a}dx^4}{2c}-\frac{\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)\left(2ac(2cd-3be)+b^3e\right)}{c\sqrt{b^2-4ac}}}{c\left(b^2-4ac\right)}\right.$$

$$\downarrow 1103$$

$$\frac{1}{4}\left(\frac{x^4\left(x^4\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)\right)}{c\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)\left(2ac(2cd-3be)+b^3e\right)}{c\sqrt{b^2-4ac}}-\frac{e\left(b^2-4ac\right)\log\left(a+bx^4+cx^8\right)}{2c}}{c\left(b^2-4ac\right)}\right.$$

input
```
Int[(x^11*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
((x^4*(a*(2*c*d - b*e) + (b*c*d - b^2*e + 2*a*c*e)*x^4))/(c*(b^2 - 4*a*c)*
(a + b*x^4 + c*x^8)) - (-(((b^3*e + 2*a*c*(2*c*d - 3*b*e))*ArcTanh[(b + 2*
c*x^4)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c])) - ((b^2 - 4*a*c)*e*Log[a
 + b*x^4 + c*x^8])/(2*c))/(c*(b^2 - 4*a*c)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.42

method	result
default	$\frac{\frac{\left(3abce-2a\,c^2d-b^3e+b^2cd\right)x^4}{c^2\left(4ac-b^2\right)}+\frac{a\left(2ace-b^2e+cbd\right)}{c^2\left(4ac-b^2\right)}}{4c\,x^8+4b\,x^4+4a}+\frac{\frac{\left(4ace-b^2e\right)\ln\left(c\,x^8+b\,x^4+a\right)}{2c}+\frac{2\left(-abe+2acd-\frac{\left(4ace-b^2e\right)b}{2c}\right)\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4(4ac-b^2)c}$
risch	Expression too large to display

input
```
int(x^11*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(1/c^2*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(4*a*c-b^2)*x^4+a*(2*a*c*e-
b^2*e+b*c*d)/c^2/(4*a*c-b^2))/(c*x^8+b*x^4+a)+1/4/(4*a*c-b^2)/c*(1/2*(4*a*
c*e-b^2*e)/c*ln(c*x^8+b*x^4+a)+2*(-a*b*e+2*a*c*d-1/2*(4*a*c*e-b^2*e)*b/c)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(138) = 276$.

Time = 1.41 (sec) , antiderivative size = 844, normalized size of antiderivative = 5.70

$$\int \frac{x^{11}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^11*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(2*((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (b^5 - 7*a*b^3*c + 12*a^2*
b*c^2)*e)*x^4 - ((4*a*c^3*d + (b^3*c - 6*a*b*c^2)*e)*x^8 + 4*a^2*c^2*d + (
4*a*b*c^2*d + (b^4 - 6*a*b^2*c)*e)*x^4 + (a*b^3 - 6*a^2*b*c)*e)*sqrt(b^2 -
 4*a*c)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2
- 4*a*c))/(c*x^8 + b*x^4 + a)) + 2*(a*b^3*c - 4*a^2*b*c^2)*d - 2*(a*b^4 -
6*a^2*b^2*c + 8*a^3*c^2)*e - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e*x^8 + (
b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*x^4 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
*e)*log(c*x^8 + b*x^4 + a))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c
^4)*x^4), -1/8*(2*((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (b^5 - 7*a*b^3*c
+ 12*a^2*b*c^2)*e)*x^4 - 2*((4*a*c^3*d + (b^3*c - 6*a*b*c^2)*e)*x^8 + 4*a^
2*c^2*d + (4*a*b*c^2*d + (b^4 - 6*a*b^2*c)*e)*x^4 + (a*b^3 - 6*a^2*b*c)*e)
*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)
) + 2*(a*b^3*c - 4*a^2*b*c^2)*d - 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*e -
((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e*x^8 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*e*x^4 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*e)*log(c*x^8 + b*x^4 + a))/(
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*
a^3*c^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**11*(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{x^{11}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx &= -\frac{(4\,ac^2d+b^3e-6\,abce)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} + \frac{e\log\left(cx^8+bx^4+a\right)}{8\,c^2} \\ &\quad - \frac{b^2cex^8-4\,ac^2ex^8+2\,b^2cdx^4-4\,ac^2dx^4-b^3ex^4+2\,abcex^4+2\,abcd-ab^2e}{8\,(cx^8+bx^4+a)(b^2c^2-4\,ac^3)}\end{aligned}$$

input
```
integrate(x^11*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*(4*a*c^2*d + b^3*e - 6*a*b*c*e)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*
c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/8*e*log(c*x^8 + b*x^4 + a
)/c^2 - 1/8*(b^2*c*e*x^8 - 4*a*c^2*e*x^8 + 2*b^2*c*d*x^4 - 4*a*c^2*d*x^4 -
 b^3*e*x^4 + 2*a*b*c*e*x^4 + 2*a*b*c*d - a*b^2*e)/((c*x^8 + b*x^4 + a)*(b^
2*c^2 - 4*a*c^3))
```

Mupad [B] (verification not implemented)

Time = 37.25 (sec) , antiderivative size = 12336, normalized size of antiderivative = 83.35

$$\int \frac{x^{11}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
int((x^11*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x)
```

output
```
((a*(2*a*c*e - b^2*e + b*c*d))/(4*c^2*(4*a*c - b^2)) - (x^4*(b^3*e + 2*a*c
^2*d - b^2*c*d - 3*a*b*c*e))/(4*c^2*(4*a*c - b^2)))/(a + b*x^4 + c*x^8) -
(log(a + b*x^4 + c*x^8)*(4*b^6*e - 256*a^3*c^3*e + 192*a^2*b^2*c^2*e - 48*
a*b^4*c*e))/(2*(1024*a^3*c^5 - 16*b^6*c^2 + 192*a*b^4*c^3 - 768*a^2*b^2*c^
4)) - (atan(((a*c - b^2)*(512*a^3*c^7*(4*a*c - b^2)^6 - 8*b^6*c^4*(4*a*c -
 b^2)^6 + 96*a*b^4*c^5*(4*a*c - b^2)^6 - 384*a^2*b^2*c^6*(4*a*c - b^2)^6)*
(((((((1024*a^2*b^3*c^8*d - 6656*a^2*b^4*c^7*e + 14336*a^3*b^2*c^8*e - 40
96*a^3*b*c^9*d + 768*a*b^6*c^6*e)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) + (
(1024*a*b^6*c^8 - 8192*a^2*b^4*c^9 + 16384*a^3*b^2*c^10)*(4*b^6*e - 256*a^
3*c^3*e + 192*a^2*b^2*c^2*e - 48*a*b^4*c*e))/(2*(16*a^2*c^6 + b^4*c^4 - 8*
a*b^2*c^5)*(1024*a^3*c^5 - 16*b^6*c^2 + 192*a*b^4*c^3 - 768*a^2*b^2*c^4)))
*(b^3*e + 4*a*c^2*d - 6*a*b*c*e))/(8*c^2*(4*a*c - b^2)^(3/2)) + ((b^3*e +
4*a*c^2*d - 6*a*b*c*e)*(1024*a*b^6*c^8 - 8192*a^2*b^4*c^9 + 16384*a^3*b^2*
c^10)*(4*b^6*e - 256*a^3*c^3*e + 192*a^2*b^2*c^2*e - 48*a*b^4*c*e))/(16*c^
2*(4*a*c - b^2)^(3/2)*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(1024*a^3*c^5 -
 16*b^6*c^2 + 192*a*b^4*c^3 - 768*a^2*b^2*c^4)))*(4*b^6*e - 256*a^3*c^3*e
+ 192*a^2*b^2*c^2*e - 48*a*b^4*c*e))/(2*(1024*a^3*c^5 - 16*b^6*c^2 + 192*a
*b^4*c^3 - 768*a^2*b^2*c^4)) + (((256*a^3*c^8*d^2 + 208*a*b^6*c^4*e^2 - 19
20*a^2*b^4*c^5*e^2 + 4416*a^3*b^2*c^6*e^2 - 2304*a^3*b*c^7*d*e + 512*a^2*b
^3*c^6*d*e)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) + (((1024*a^2*b^3*c^8*...
```

Reduce [F]

$$\int \frac{x^{11}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{x^{11}(e\,x^4+d)}{(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^11*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^11*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.90 $\int \frac{x^7(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 110

$$\int \frac{x^7(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \frac{a(2cd-be)+(bcd-b^2e+2ace)\,x^4}{4c\,(b^2-4ac)\,(a+bx^4+cx^8)} - \frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\,(b^2-4ac)^{3/2}}$$

output
```
1/4*(a*(-b*e+2*c*d)+(2*a*c*e-b^2*e+b*c*d)*x^4)/c/(-4*a*c+b^2)/(c*x^8+b*x^4
+a)-1/2*(-2*a*e+b*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int \frac{x^7(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \frac{1}{4}\left(\frac{abe+b(-cd+be)x^4-2ac(d+ex^4)}{c\,(-b^2+4ac)\,(a+bx^4+cx^8)} - \frac{2(bd-2ae)\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}\right)$$

input
```
Integrate[(x^7*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
((a*b*e + b*(-(c*d) + b*e)*x^4 - 2*a*c*(d + e*x^4))/(c*(-b^2 + 4*a*c)*(a +
 b*x^4 + c*x^8)) - (2*(b*d - 2*a*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c
]])/(-b^2 + 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1802, 1224, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(d+e x^4\right)}{\left(a+b x^4+c x^8\right)^2} \, dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{4} \int \frac{x^4 \left(e x^4+d\right)}{\left(c x^8+b x^4+a\right)^2}dx^4$$

$$\Big\downarrow \text{1224}$$

$$\frac{1}{4} \left(\frac{(b d-2 a e) \int \frac{1}{c x^8+b x^4+a}dx^4}{b^2-4 a c}+\frac{x^4 \left(2 a c e+b^2 (-e)+b c d\right)+a (2 c d-b e)}{c \left(b^2-4 a c\right) \left(a+b x^4+c x^8\right)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4} \left(\frac{x^4 \left(2 a c e+b^2 (-e)+b c d\right)+a (2 c d-b e)}{c \left(b^2-4 a c\right) \left(a+b x^4+c x^8\right)}-\frac{2 (b d-2 a e) \int \frac{1}{-x^8+b^2-4 a c}d\left(2 c x^4+b\right)}{b^2-4 a c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4} \left(\frac{x^4 \left(2 a c e+b^2 (-e)+b c d\right)+a (2 c d-b e)}{c \left(b^2-4 a c\right) \left(a+b x^4+c x^8\right)}-\frac{2 (b d-2 a e) \text{arctanh}\left(\frac{b+2 c x^4}{\sqrt{b^2-4 a c}}\right)}{\left(b^2-4 a c\right)^{3/2}}\right)$$

input
```
Int[(x^7*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
((a*(2*c*d - b*e) + (b*c*d - b^2*e + 2*a*c*e)*x^4)/(c*(b^2 - 4*a*c)*(a + b
*x^4 + c*x^8)) - (2*(b*d - 2*a*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]]
)/(b^2 - 4*a*c)^(3/2))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

method	result
default	$\frac{-\frac{(2ace-b^2e+cbd)x^4}{c(4ac-b^2)}+\frac{a(eb-2cd)}{(4ac-b^2)c}}{4c\,x^8+4b\,x^4+4a}+\frac{(2ae-bd)\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{2(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{(2ace-b^2e+cbd)x^4}{4c(4ac-b^2)}+\frac{a(eb-2cd)}{4(4ac-b^2)c}}{c\,x^8+b\,x^4+a}+\frac{\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)x^4+8a^2c-2b^2a\right)ae}{2(-4ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)x^4+8a\right.}{4(-4ac+b^2)^{\frac{3}{2}}}$

input
```
int(x^7*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-(2*a*c*e-b^2*e+b*c*d)/c/(4*a*c-b^2)*x^4+a*(b*e-2*c*d)/(4*a*c-b^2)/c)
/(c*x^8+b*x^4+a)+1/2*(2*a*e-b*d)/(4*a*c-b^2)^(3/2)*arctan((2*c*x^4+b)/(4*a
*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(102) = 204.

Time = 0.11 (sec) , antiderivative size = 542, normalized size of antiderivative = 4.93

$$\int \frac{x^7(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$
$$=\left[\frac{((b^3c-4\,abc^2)d-(b^4-6\,ab^2c+8\,a^2c^2)e)x^4+((bc^2d-2\,ac^2e)x^8+(b^2cd-2\,abce)x^4+abcd-2\,a^2ce)}{4\,((b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^8+ab^4c-8\,a^2b^2c^2+}\right.$$

input
```
integrate(x^7*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(((b^3*c - 4*a*b*c^2)*d - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*e)*x^4 + ((b*
c^2*d - 2*a*c^2*e)*x^8 + (b^2*c*d - 2*a*b*c*e)*x^4 + a*b*c*d - 2*a^2*c*e)*
sqrt(b^2 - 4*a*c)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c - (2*c*x^4 + b)
*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) + 2*(a*b^2*c - 4*a^2*c^2)*d - (a*
b^3 - 4*a^2*b*c)*e)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + a*b^4*c -
8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4), 1/
4*(((b^3*c - 4*a*b*c^2)*d - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*e)*x^4 - 2*((b*c
^2*d - 2*a*c^2*e)*x^8 + (b^2*c*d - 2*a*b*c*e)*x^4 + a*b*c*d - 2*a^2*c*e)*s
qrt(-b^2 + 4*a*c)*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))
+ 2*(a*b^2*c - 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e)/((b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*x^8 + a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09

$$\int \frac{x^7(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2-4\,ac)\sqrt{-b^2+4\,ac}} + \frac{bcdx^4-b^2ex^4+2\,acex^4+2\,acd-abe}{4\,(cx^8+bx^4+a)(b^2c-4\,ac^2)}$$

input
```
integrate(x^7*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*d - 2*a*e)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*
sqrt(-b^2 + 4*a*c)) + 1/4*(b*c*d*x^4 - b^2*e*x^4 + 2*a*c*e*x^4 + 2*a*c*d -
 a*b*e)/((c*x^8 + b*x^4 + a)*(b^2*c - 4*a*c^2))
```

Mupad [B] (verification not implemented)

Time = 24.65 (sec) , antiderivative size = 1081, normalized size of antiderivative = 9.83

$$\int \frac{x^7(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
int((x^7*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x)
```

output

```
-(4*a^2*c*e*atan((b^7 - 64*a^3*b*c^3 + 2*b^6*c*x^4 + 48*a^2*b^3*c^2 - 128*
a^3*c^4*x^4 - 12*a*b^5*c - 24*a*b^4*c^2*x^4 + 96*a^2*b^2*c^3*x^4)/(4*a*c -
 b^2)^(7/2)) + 4*a^2*c*e*atan((b^3 - 3*a*b*c)/(b^2*(4*a*c - b^2)^(1/2) - a
*c*(4*a*c - b^2)^(1/2))) - b^2*e*x^4*(4*a*c - b^2)^(1/2) - a*b*e*(4*a*c -
b^2)^(1/2) + 2*a*c*d*(4*a*c - b^2)^(1/2) - 2*b^2*c*d*x^4*atan((b^7 - 64*a^
3*b*c^3 + 2*b^6*c*x^4 + 48*a^2*b^3*c^2 - 128*a^3*c^4*x^4 - 12*a*b^5*c - 24
*a*b^4*c^2*x^4 + 96*a^2*b^2*c^3*x^4)/(4*a*c - b^2)^(7/2)) + 4*a*c^2*e*x^8*
atan((b^7 - 64*a^3*b*c^3 + 2*b^6*c*x^4 + 48*a^2*b^3*c^2 - 128*a^3*c^4*x^4
- 12*a*b^5*c - 24*a*b^4*c^2*x^4 + 96*a^2*b^2*c^3*x^4)/(4*a*c - b^2)^(7/2))
 - 2*b*c^2*d*x^8*atan((b^7 - 64*a^3*b*c^3 + 2*b^6*c*x^4 + 48*a^2*b^3*c^2 -
 128*a^3*c^4*x^4 - 12*a*b^5*c - 24*a*b^4*c^2*x^4 + 96*a^2*b^2*c^3*x^4)/(4*
a*c - b^2)^(7/2)) - 2*b^2*c*d*x^4*atan((b^3 - 3*a*b*c)/(b^2*(4*a*c - b^2)^
(1/2) - a*c*(4*a*c - b^2)^(1/2))) + 4*a*c^2*e*x^8*atan((b^3 - 3*a*b*c)/(b^
2*(4*a*c - b^2)^(1/2) - a*c*(4*a*c - b^2)^(1/2))) - 2*b*c^2*d*x^8*atan((b^
3 - 3*a*b*c)/(b^2*(4*a*c - b^2)^(1/2) - a*c*(4*a*c - b^2)^(1/2))) - 2*a*b*
c*d*atan((b^7 - 64*a^3*b*c^3 + 2*b^6*c*x^4 + 48*a^2*b^3*c^2 - 128*a^3*c^4*
x^4 - 12*a*b^5*c - 24*a*b^4*c^2*x^4 + 96*a^2*b^2*c^3*x^4)/(4*a*c - b^2)^(7
/2)) - 2*a*b*c*d*atan((b^3 - 3*a*b*c)/(b^2*(4*a*c - b^2)^(1/2) - a*c*(4*a*
c - b^2)^(1/2))) + 2*a*c*e*x^4*(4*a*c - b^2)^(1/2) + b*c*d*x^4*(4*a*c - b^
2)^(1/2) + 4*a*b*c*e*x^4*atan((b^7 - 64*a^3*b*c^3 + 2*b^6*c*x^4 + 48*a^...
```

Reduce [B] (verification not implemented)

Time = 7.32 (sec) , antiderivative size = 3021, normalized size of antiderivative = 27.46

$$\int \frac{x^7(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input

```
int(x^7*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
(4*sqrt(2*sqrt(c)*sqrt(a) - b)*sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/
4)*a**(1/4))*sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4))*at
an((sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)) - 2*c**(1/4
)*x)/sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)))*a**2*b*e - 2
*sqrt(2*sqrt(c)*sqrt(a) - b)*sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)
*a**(1/4))*sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4))*atan
((sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)) - 2*c**(1/4)*
x)/sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)))*a*b**2*d + 4*s
qrt(2*sqrt(c)*sqrt(a) - b)*sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a
**(1/4))*sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4))*atan((
sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)) - 2*c**(1/4)*x)
/sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)))*a*b**2*e*x**4 +
4*sqrt(2*sqrt(c)*sqrt(a) - b)*sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4
)*a**(1/4))*sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4))*ata
n((sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)) - 2*c**(1/4)
*x)/sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)))*a*b*c*e*x**8
- 2*sqrt(2*sqrt(c)*sqrt(a) - b)*sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1
/4)*a**(1/4))*sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4))*a
tan((sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)) - 2*c**(1/
4)*x)/sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)))*b**3*d*x...
```

3.91 $\int \frac{x^3(d+ex^4)}{(a+bx^4+cx^8)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 99

$$\int \frac{x^3(d+ex^4)}{(a+bx^4+cx^8)^2} dx = \frac{-bd+2ae-(2cd-be)x^4}{4(b^2-4ac)(a+bx^4+cx^8)} + \frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2(b^2-4ac)^{3/2}}$$

output
```
1/4*(-b*d+2*a*e-(-b*e+2*c*d)*x^4)/(-4*a*c+b^2)/(c*x^8+b*x^4+a)+1/2*(-b*e+2
*c*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^3(d+ex^4)}{(a+bx^4+cx^8)^2} dx = \frac{\frac{-bd+2ae-2cdx^4+bex^4}{a+bx^4+cx^8} + \frac{2(-2cd+be)\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4(b^2-4ac)}$$

input
```
Integrate[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
((-(b*d) + 2*a*e - 2*c*d*x^4 + b*e*x^4)/(a + b*x^4 + c*x^8) + (2*(-2*c*d +
 b*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(4*(b^
2 - 4*a*c))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1798, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex^4)}{(a+bx^4+cx^8)^2}dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{4}\int \frac{ex^4+d}{(cx^8+bx^4+a)^2}dx^4$$

$$\Big\downarrow \text{1159}$$

$$\frac{1}{4}\left(-\frac{(2cd-be)\int \frac{1}{cx^8+bx^4+a}dx^4}{b^2-4ac}-\frac{-2ae+x^4(2cd-be)+bd}{(b^2-4ac)(a+bx^4+cx^8)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(\frac{2(2cd-be)\int \frac{1}{-x^8+b^2-4ac}d(2cx^4+b)}{b^2-4ac}-\frac{-2ae+x^4(2cd-be)+bd}{(b^2-4ac)(a+bx^4+cx^8)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{2(2cd-be)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2ae+x^4(2cd-be)+bd}{(b^2-4ac)(a+bx^4+cx^8)}\right)$$

input
```
Int[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
(-((b*d - 2*a*e + (2*c*d - b*e)*x^4)/((b^2 - 4*a*c)*(a + b*x^4 + c*x^8)))
+ (2*(2*c*d - b*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)
^(3/2))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97

method	result
default	$\frac{bd-2ae+(-eb+2cd)x^4}{4(4ac-b^2)(cx^8+bx^4+a)}+\frac{(-eb+2cd)\arctan\left(\frac{2cx^4+b}{\sqrt{4ac-b^2}}\right)}{2(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{(eb-2cd)x^4}{4\left(4ac-b^2\right)}-\frac{2ae-bd}{4\left(4ac-b^2\right)}}{cx^8+bx^4+a}+\frac{\ln\left(\left(\left(-(-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x^4+8a^2c-2b^2a\right)eb}{4(-4ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left(\left(\left(-(-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x^4+8a^2c-\right.\right.}{2(-4ac+b^2)^{\frac{3}{2}}}$

input
```
int(x^3*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*d-2*a*e+(-b*e+2*c*d)*x^4)/(4*a*c-b^2)/(c*x^8+b*x^4+a)+1/2*(-b*e+2*c
*d)/(4*a*c-b^2)^(3/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(89) = 178$.

Time = 0.10 (sec) , antiderivative size = 478, normalized size of antiderivative = 4.83

$$\int \frac{x^3(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$
$$= \Bigg[-\frac{(2\,(b^2c-4\,ac^2)d-(b^3-4\,abc)e)x^4-((2\,c^2d-bce)x^8+(2\,bcd-b^2e)x^4+2\,acd-abe)\sqrt{b^2-4\,ac}\,\mathrm{lo}}{4\,((b^4c-8\,ab^2c^2+16\,a^2c^3)x^8+ab^4-8\,a^2b^2c+16\,a^3c^2}$$
$$-\frac{(2\,(b^2c-4\,ac^2)d-(b^3-4\,abc)e)x^4-2\,((2\,c^2d-bce)x^8+(2\,bcd-b^2e)x^4+2\,acd-abe)\sqrt{-b^2+4\,ac}}{4\,((b^4c-8\,ab^2c^2+16\,a^2c^3)x^8+ab^4-8\,a^2b^2c+16\,a^3c^2+(b^5}$$

input
```
integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*((2*(b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x^4 - ((2*c^2*d - b*c*e
)*x^8 + (2*b*c*d - b^2*e)*x^4 + 2*a*c*d - a*b*e)*sqrt(b^2 - 4*a*c)*log((2*
c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^
8 + b*x^4 + a)) + (b^3 - 4*a*b*c)*d - 2*(a*b^2 - 4*a^2*c)*e)/((b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)*x^8 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*x^4), -1/4*((2*(b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c
)*e)*x^4 - 2*((2*c^2*d - b*c*e)*x^8 + (2*b*c*d - b^2*e)*x^4 + 2*a*c*d - a*
b*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)) + (b^3 - 4*a*b*c)*d - 2*(a*b^2 - 4*a^2*c)*e)/((b^4*c - 8*a*b^2*c^2 +
 16*a^2*c^3)*x^8 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2)*x^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(85) = 170$.

Time = 15.18 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.78

$$\int \frac{x^3(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$

$$= \frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2cd)\log\left(x^4+\frac{-16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2cd)+8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2cd)-b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2cd)+b}{2bce-4c^2d}\right.}{4}$$

$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2cd)\log\left(x^4+\frac{16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2cd)-8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2cd)+b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2cd)+}{2bce-4c^2d}\right.}{4}$$

$$+\frac{-2ae+bd+x^4(-be+2cd)}{16a^2c-4ab^2+x^8\cdot(16ac^2-4b^2c)+x^4\cdot(16abc-4b^3)}$$

input
```
integrate(x**3*(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
sqrt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d)*log(x**4 + (-16*a**2*c**2*sqrt(-1
/(4*a*c - b**2)**3)*(b*e - 2*c*d) + 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*
(b*e - 2*c*d) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d) + b**2*e - 2
*b*c*d)/(2*b*c*e - 4*c**2*d))/4 - sqrt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d)
*log(x**4 + (16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d) - 8*a*b
**2*c*sqrt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d) + b**4*sqrt(-1/(4*a*c - b**
2)**3)*(b*e - 2*c*d) + b**2*e - 2*b*c*d)/(2*b*c*e - 4*c**2*d))/4 + (-2*a*e
 + b*d + x**4*(-b*e + 2*c*d))/(16*a**2*c - 4*a*b**2 + x**8*(16*a*c**2 - 4*
b**2*c) + x**4*(16*a*b*c - 4*b**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

$$\int \frac{x^3(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = -\frac{(2\,cd-be)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,cdx^4-bex^4+bd-2\,ae}{4\,(cx^8+bx^4+a)(b^2-4\,ac)}$$

input
```
integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*c*d - b*e)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)
*sqrt(-b^2 + 4*a*c)) - 1/4*(2*c*d*x^4 - b*e*x^4 + b*d - 2*a*e)/((c*x^8 + b
*x^4 + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 25.57 (sec) , antiderivative size = 1036, normalized size of antiderivative = 10.46

$$\int \frac{x^3(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x)
```

output
```
-(2*a*e*(4*a*c - b^2)^(1/2) - b*d*(4*a*c - b^2)^(1/2) + b*e*x^4*(4*a*c - b
^2)^(1/2) - 2*c*d*x^4*(4*a*c - b^2)^(1/2) - 2*b^2*e*x^4*atan((b^7 - 64*a^3
*b*c^3 + 2*b^6*c*x^4 + 48*a^2*b^3*c^2 - 128*a^3*c^4*x^4 - 12*a*b^5*c - 24*
a*b^4*c^2*x^4 + 96*a^2*b^2*c^3*x^4)/(4*a*c - b^2)^(7/2)) + 4*c^2*d*x^8*ata
n((b^7 - 64*a^3*b*c^3 + 2*b^6*c*x^4 + 48*a^2*b^3*c^2 - 128*a^3*c^4*x^4 - 1
2*a*b^5*c - 24*a*b^4*c^2*x^4 + 96*a^2*b^2*c^3*x^4)/(4*a*c - b^2)^(7/2)) -
2*b^2*e*x^4*atan((b^3 - 3*a*b*c)/(b^2*(4*a*c - b^2)^(1/2) - a*c*(4*a*c - b
^2)^(1/2))) + 4*c^2*d*x^8*atan((b^3 - 3*a*b*c)/(b^2*(4*a*c - b^2)^(1/2) -
a*c*(4*a*c - b^2)^(1/2))) - 2*a*b*e*atan((b^7 - 64*a^3*b*c^3 + 2*b^6*c*x^4
 + 48*a^2*b^3*c^2 - 128*a^3*c^4*x^4 - 12*a*b^5*c - 24*a*b^4*c^2*x^4 + 96*a
^2*b^2*c^3*x^4)/(4*a*c - b^2)^(7/2)) + 4*a*c*d*atan((b^7 - 64*a^3*b*c^3 +
2*b^6*c*x^4 + 48*a^2*b^3*c^2 - 128*a^3*c^4*x^4 - 12*a*b^5*c - 24*a*b^4*c^2
*x^4 + 96*a^2*b^2*c^3*x^4)/(4*a*c - b^2)^(7/2)) - 2*a*b*e*atan((b^3 - 3*a*
b*c)/(b^2*(4*a*c - b^2)^(1/2) - a*c*(4*a*c - b^2)^(1/2))) + 4*a*c*d*atan((
b^3 - 3*a*b*c)/(b^2*(4*a*c - b^2)^(1/2) - a*c*(4*a*c - b^2)^(1/2))) + 4*b*
c*d*x^4*atan((b^7 - 64*a^3*b*c^3 + 2*b^6*c*x^4 + 48*a^2*b^3*c^2 - 128*a^3*
c^4*x^4 - 12*a*b^5*c - 24*a*b^4*c^2*x^4 + 96*a^2*b^2*c^3*x^4)/(4*a*c - b^2
)^(7/2)) - 2*b*c*e*x^8*atan((b^7 - 64*a^3*b*c^3 + 2*b^6*c*x^4 + 48*a^2*b^3
*c^2 - 128*a^3*c^4*x^4 - 12*a*b^5*c - 24*a*b^4*c^2*x^4 + 96*a^2*b^2*c^3*x^
4)/(4*a*c - b^2)^(7/2)) + 4*b*c*d*x^4*atan((b^3 - 3*a*b*c)/(b^2*(4*a*c ...
```

Reduce [B] (verification not implemented)

Time = 7.28 (sec) , antiderivative size = 3019, normalized size of antiderivative = 30.49

$$\int \frac{x^3(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
int(x^3*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
( - 2*sqrt(2*sqrt(c)*sqrt(a) - b)*sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**
(1/4)*a**(1/4))*sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4))
*atan((sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)) - 2*c**(
1/4)*x)/sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)))*a*b**2*e
+ 4*sqrt(2*sqrt(c)*sqrt(a) - b)*sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1
/4)*a**(1/4))*sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4))*a
tan((sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)) - 2*c**(1/
4)*x)/sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)))*a*b*c*d - 2
*sqrt(2*sqrt(c)*sqrt(a) - b)*sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)
*a**(1/4))*sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4))*atan
((sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)) - 2*c**(1/4)*
x)/sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)))*b**3*e*x**4 +
4*sqrt(2*sqrt(c)*sqrt(a) - b)*sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4
)*a**(1/4))*sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4))*ata
n((sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)) - 2*c**(1/4)
*x)/sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)))*b**2*c*d*x**4
 - 2*sqrt(2*sqrt(c)*sqrt(a) - b)*sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(
1/4)*a**(1/4))*sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4))*
atan((sqrt( - sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)) - 2*c**(1
/4)*x)/sqrt(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*c**(1/4)*a**(1/4)))*b**2*c*...
```

3.92 $\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 149

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)^2} dx = \frac{b^2d-2acd-abe+c(bd-2ae)x^4}{4a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)} + \frac{\left(b^3d-6abcd+4a^2ce\right)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\left(b^2-4ac\right)^{3/2}} + \frac{d\log(x)}{a^2} - \frac{d\log\left(a+bx^4+cx^8\right)}{8a^2}$$

output
```
1/4*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^4)/a/(-4*a*c+b^2)/(c*x^8+b*x^4+a
)+1/4*(4*a^2*c*e-6*a*b*c*d+b^3*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/
a^2/(-4*a*c+b^2)^(3/2)+d*ln(x)/a^2-1/8*d*ln(c*x^8+b*x^4+a)/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.24

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)^2}\,dx$$
$$= \frac{\frac{a(b^2d+b(-ae+cdx^4)-2ac(d+ex^4))}{(b^2-4ac)(a+bx^4+cx^8)}+4d\log(x)-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^3d\log(x-\#1)-5abcd\log(x-\#1)+2a^2ce\log(x-\#}{b+2c}\right.}{b^2-4ac}}{4a^2}$$

input
```
Integrate[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((a*(b^2*d + b*(-(a*e) + c*d*x^4) - 2*a*c*(d + e*x^4)))/((b^2 - 4*a*c)*(a
+ b*x^4 + c*x^8)) + 4*d*Log[x] - RootSum[a + b*#1^4 + c*#1^8 & , (b^3*d*Lo
g[x - #1] - 5*a*b*c*d*Log[x - #1] + 2*a^2*c*e*Log[x - #1] + b^2*c*d*Log[x
- #1]*#1^4 - 4*a*c^2*d*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(b^2 - 4*a*c))
/(4*a^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1802, 1235, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)^2}\,dx$$
$$\Big\downarrow \text{1802}$$
$$\frac{1}{4}\int \frac{ex^4+d}{x^4\left(cx^8+bx^4+a\right)^2}dx^4$$
$$\Big\downarrow \text{1235}$$

$$\frac{1}{4}\left(\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}-\frac{\int -\frac{c(bd-2ae)x^4+\left(b^2-4ac\right)d}{x^4\left(cx^8+bx^4+a\right)}dx^4}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{\int \frac{c(bd-2ae)x^4+\left(b^2-4ac\right)d}{x^4\left(cx^8+bx^4+a\right)}dx^4}{a\left(b^2-4ac\right)}+\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{4}\left(\frac{\int \left(\frac{-c\left(b^2-4ac\right)dx^4-b^3d+5abcd-2a^2ce}{a\left(cx^8+bx^4+a\right)}-\frac{\left(4ac-b^2\right)d}{ax^4}\right)dx^4}{a\left(b^2-4ac\right)}+\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{\frac{\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)\left(4a^2ce-6abcd+b^3d\right)}{a\sqrt{b^2-4ac}}+\frac{d\log\left(x^4\right)\left(b^2-4ac\right)}{a}-\frac{d\left(b^2-4ac\right)\log\left(a+bx^4+cx^8\right)}{2a}}{a\left(b^2-4ac\right)}+\frac{cx^4(bd-2ae)-abe-2acd}{a\left(b^2-4ac\right)\left(a+bx^4+c\right.}\right.$$

input
```
Int[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^4)/(a*(b^2 - 4*a*c)*(a + b*x
^4 + c*x^8)) + (((b^3*d - 6*a*b*c*d + 4*a^2*c*e)*ArcTanh[(b + 2*c*x^4)/Sqr
t[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*d*Log[x^4])/a - ((
b^2 - 4*a*c)*d*Log[a + b*x^4 + c*x^8])/(2*a))/(a*(b^2 - 4*a*c)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.42

method	result
default	$\frac{\frac{\frac{ac(2ae-bd)x^4}{4ac-b^2}+\frac{a\left(abe+2acd-d\,b^2\right)}{4ac-b^2}}{2c\,x^8+2b\,x^4+2a}+\frac{\frac{\left(-4a\,c^2d+b^2cd\right)\ln\left(c\,x^8+b\,x^4+a\right)}{2c}+\frac{2\left(2a^2ce-5abcd+b^3d-\frac{\left(-4a\,c^2d+b^2cd\right)b}{2c}\right)\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{8ac-2b^2}}{2a^2}+$
risch	$\frac{\frac{c(2ae-bd)x^4}{4a\left(4ac-b^2\right)}+\frac{abe+2acd-d\,b^2}{4\left(4ac-b^2\right)a}}{c\,x^8+b\,x^4+a}+\frac{d\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(64a^5c^3-48a^4b^2c^2+12a^3b^4c-a^2b^6\right)\_Z^2+\left(64a^3c^3d-48a^2b^2c^2d+12a\,b^4cd-\right.\right.}\right.}{}$

input
```
int((e*x^4+d)/x/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*(1/2*(a*c*(2*a*e-b*d)/(4*a*c-b^2)*x^4+a*(a*b*e+2*a*c*d-b^2*d)/(4*a
*c-b^2))/(c*x^8+b*x^4+a)+1/2/(4*a*c-b^2)*(1/2*(-4*a*c^2*d+b^2*c*d)/c*ln(c*
x^8+b*x^4+a)+2*(2*a^2*c*e-5*a*b*c*d+b^3*d-1/2*(-4*a*c^2*d+b^2*c*d)*b/c)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))))+d*ln(x)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(139) = 278$.

Time = 9.13 (sec) , antiderivative size = 989, normalized size of antiderivative = 6.64

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/x/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/8*(2*((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*x^4 - ((
4*a^2*c^2*e + (b^3*c - 6*a*b*c^2)*d)*x^8 + 4*a^3*c*e + (4*a^2*b*c*e + (b^4
 - 6*a*b^2*c)*d)*x^4 + (a*b^3 - 6*a^2*b*c)*d)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^8 + 2*b*c*x^4 + b^2 - 2*a*c - (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 +
 b*x^4 + a)) + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*a^3*
b*c)*e - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^8 + (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*d*x^4 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(c*x^8 + b*x^
4 + a) + 8*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^8 + (b^5 - 8*a*b^3*c +
16*a^2*b*c^2)*d*x^4 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(x))/((a^2*
b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^4), 1/8*(2*((a*b^3*c - 4*a^2
*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*x^4 + 2*((4*a^2*c^2*e + (b^3*c -
6*a*b*c^2)*d)*x^8 + 4*a^3*c*e + (4*a^2*b*c*e + (b^4 - 6*a*b^2*c)*d)*x^4 +
(a*b^3 - 6*a^2*b*c)*d)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^4 + b)*sqrt(-b^2
+ 4*a*c)/(b^2 - 4*a*c)) + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b
^3 - 4*a^3*b*c)*e - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^8 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*d*x^4 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(c
*x^8 + b*x^4 + a) + 8*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^8 + (b^5 - 8
*a*b^3*c + 16*a^2*b*c^2)*d*x^4 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log
(x))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^8 + a^3*b^4 - 8*a^4*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/x/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^4}{x(a+bx^4+cx^8)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^4+d)/x/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 4.75 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.35

$$\begin{aligned}&\int \frac{d+ex^4}{x(a+bx^4+cx^8)^2}\,dx\\&= -\frac{(b^3d-6\,abcd+4\,a^2ce)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}} - \frac{d\log\left(cx^8+bx^4+a\right)}{8\,a^2} + \frac{d\log\left(x^4\right)}{4\,a^2}\\&\quad+\frac{b^2cdx^8-4\,ac^2dx^8+b^3dx^4-2\,abcdx^4-4\,a^2cex^4+3\,ab^2d-8\,a^2cd-2\,a^2be}{8\,(cx^8+bx^4+a)(a^2b^2-4\,a^3c)}\end{aligned}$$

input
```
integrate((e*x^4+d)/x/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*(b^3*d - 6*a*b*c*d + 4*a^2*c*e)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*
c))/((a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/8*d*log(c*x^8 + b*x^4 + a
)/a^2 + 1/4*d*log(x^4)/a^2 + 1/8*(b^2*c*d*x^8 - 4*a*c^2*d*x^8 + b^3*d*x^4
- 2*a*b*c*d*x^4 - 4*a^2*c*e*x^4 + 3*a*b^2*d - 8*a^2*c*d - 2*a^2*b*e)/((c*x
^8 + b*x^4 + a)*(a^2*b^2 - 4*a^3*c))
```

Mupad [B] (verification not implemented)

Time = 85.15 (sec) , antiderivative size = 35718, normalized size of antiderivative = 239.72

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x*(a + b*x^4 + c*x^8)^2),x)
```

output
```
((a*b*e - b^2*d + 2*a*c*d)/(4*a*(4*a*c - b^2)) + (c*x^4*(2*a*e - b*d))/(4*
a*(4*a*c - b^2)))/(a + b*x^4 + c*x^8) - (log((((d + a^2*(-(b^3*d + 4*a^2*c
*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((d + a^2*(-(b^3*d + 4*a^
2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((d + a^2*(-(b^3*d + 4
*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((d + a^2*(-(b^3*d
+ 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((32*b^3*c^4*(d +
 a^2*(-(b^3*d + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(a*
b + 5*b^2*x^4 - 18*a*c*x^4))/a^2 - (256*b^3*c^4*(b^3*d + 2*a^2*c*e - 5*a*b
*c*d))/(a*(4*a*c - b^2)) + (64*b^2*c^5*x^4*(7*b^3*d - 32*a*b^2*e + 108*a^2
*c*e - 18*a*b*c*d))/(a*(4*a*c - b^2))))/(8*a^2) + (32*b^2*c^5*(2*a*e - b*d
)*(8*b^3*d + 6*a^2*c*e - 35*a*b*c*d))/(a^2*(4*a*c - b^2)^2) + (16*b*c^6*x^
4*(2*a*e - b*d)*(13*b^3*d + 28*a*b^2*e - 108*a^2*c*e - 54*a*b*c*d))/(a^2*(
4*a*c - b^2)^2)))/(8*a^2) - (16*b*c^6*(2*a*e - b*d)^2*(6*b^3*d + 2*a^2*c*e
 - 25*a*b*c*d))/(a^3*(4*a*c - b^2)^3) + (4*c^7*x^4*(2*a*e - b*d)^2*(8*a*b^
2*e - 31*b^3*d + 36*a^2*c*e + 90*a*b*c*d))/(a^3*(4*a*c - b^2)^3)))/(8*a^2)
 + (c^7*(2*a*e - b*d)^3*(16*b^3*d + 2*a^2*c*e - 65*a*b*c*d))/(a^4*(4*a*c -
 b^2)^4) - (2*c^8*x^4*(2*a*e - b*d)^3*(11*a*b*e - 10*b^2*d + 18*a*c*d))/(a
^4*(4*a*c - b^2)^4)))/(8*a^2) + (c^8*d*(2*a*e - b*d)^4)/(a^5*(4*a*c - b^2)
^4) + (c^9*x^4*(2*a*e - b*d)^5)/(a^5*(4*a*c - b^2)^5))*(((d - a^2*(-(b^3*d
 + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((d - a^2*(-...
```

Reduce [F]

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)^2}\,dx=\int \frac{e\,x^4+d}{x\left(c\,x^8+b\,x^4+a\right)^2}dx$$

input
```
int((e*x^4+d)/x/(c*x^8+b*x^4+a)^2,x)
```

output
```
int((e*x^4+d)/x/(c*x^8+b*x^4+a)^2,x)
```

3.93 $\int \frac{d+ex^4}{x^5(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 225

$$
\begin{aligned}
\int \frac{d+ex^4}{x^5(a+bx^4+cx^8)^2}\,dx
&= \frac{-2b^2d+6acd+abe}{4a^2(b^2-4ac)x^4}+\frac{b^2d-2acd-abe+c(bd-2ae)x^4}{4a(b^2-4ac)x^4(a+bx^4+cx^8)} \\
&\quad -\frac{(2b^4d-12ab^2cd+12a^2c^2d-ab^3e+6a^2bce)\,\mathrm{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^3(b^2-4ac)^{3/2}} \\
&\quad -\frac{(2bd-ae)\log(x)}{a^3}+\frac{(2bd-ae)\log(a+bx^4+cx^8)}{8a^3}
\end{aligned}
$$

output
```
1/4*(a*b*e+6*a*c*d-2*b^2*d)/a^2/(-4*a*c+b^2)/x^4+1/4*(b^2*d-2*a*c*d-a*b*e+
c*(-2*a*e+b*d)*x^4)/a/(-4*a*c+b^2)/x^4/(c*x^8+b*x^4+a)-1/4*(6*a^2*b*c*e+12
*a^2*c^2*d-a*b^3*e-12*a*b^2*c*d+2*b^4*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^
(1/2))/a^3/(-4*a*c+b^2)^(3/2)-(-a*e+2*b*d)*ln(x)/a^3+1/8*(-a*e+2*b*d)*ln(c
*x^8+b*x^4+a)/a^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.31

$$\int \frac{d+ex^4}{x^5 (a+bx^4+cx^8)^2} \, dx$$
$$= \frac{-\frac{ad}{x^4} - \frac{a(b^3d+2ac(ae-cdx^4)+b^2(-ae+cdx^4)-abc(3d+ex^4))}{(b^2-4ac)(a+bx^4+cx^8)} + 4(-2bd+ae)\log(x) + \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{2b^4 d \log \cdots}{\cdots}\right]}{\cdots}}{\cdots}$$

input
```
Integrate[(d + e*x^4)/(x^5*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(-((a*d)/x^4) - (a*(b^3*d + 2*a*c*(a*e - c*d*x^4) + b^2*(-(a*e) + c*d*x^4)
 - a*b*c*(3*d + e*x^4)))/((b^2 - 4*a*c)*(a + b*x^4 + c*x^8)) + 4*(-2*b*d +
 a*e)*Log[x] + RootSum[a + b*#1^4 + c*#1^8 & , (2*b^4*d*Log[x - #1] - 10*a
*b^2*c*d*Log[x - #1] + 6*a^2*c^2*d*Log[x - #1] - a*b^3*e*Log[x - #1] + 5*a
^2*b*c*e*Log[x - #1] + 2*b^3*c*d*Log[x - #1]*#1^4 - 8*a*b*c^2*d*Log[x - #1
]*#1^4 - a*b^2*c*e*Log[x - #1]*#1^4 + 4*a^2*c^2*e*Log[x - #1]*#1^4)/(b + 2
*c*#1^4) & ]/(b^2 - 4*a*c))/(4*a^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1802, 1235, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x^5 (a+bx^4+cx^8)^2} \, dx$$
$$\Big\downarrow \; 1802$$
$$\frac{1}{4} \int \frac{ex^4+d}{x^8 (cx^8+bx^4+a)^2} dx^4$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{4}\left(\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{ax^4\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}-\frac{\int -\frac{2c(bd-2ae)x^4+2b^2d-6acd-abe}{x^8\left(cx^8+bx^4+a\right)}dx^4}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{\int \frac{2c(bd-2ae)x^4+2b^2d-6acd-abe}{x^8\left(cx^8+bx^4+a\right)}dx^4}{a\left(b^2-4ac\right)}+\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{ax^4\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{4}\left(\frac{\int \left(-\frac{\left(4ac-b^2\right)(ae-2bd)}{a^2x^4}+\frac{2db^4-aeb^3-10acdb^2+5a^2ceb+c\left(b^2-4ac\right)(2bd-ae)x^4+6a^2c^2d}{a^2\left(cx^8+bx^4+a\right)}+\frac{2db^2-aeb-6acd}{ax^8}\right)dx^4}{a\left(b^2-4ac\right)}+\frac{cx^4(bd-2a}{ax^4\left(b^2-\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)\left(6a^2bce+12a^2c^2d-ab^3e-12ab^2cd+2b^4d\right)}{a^2\sqrt{b^2-4ac}}-\frac{\log\left(x^4\right)\left(b^2-4ac\right)(2bd-ae)}{a^2}+\frac{\left(b^2-4ac\right)(2bd-ae)\log\left(a+bx^4+cx^8\right)}{2a^2}}{a\left(b^2-4ac\right)}\right.$$

input
```
Int[(d + e*x^4)/(x^5*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^4)/(a*(b^2 - 4*a*c)*x^4*(a +
 b*x^4 + c*x^8)) + (-((2*b^2*d - 6*a*c*d - a*b*e)/(a*x^4)) - ((2*b^4*d - 1
2*a*b^2*c*d + 12*a^2*c^2*d - a*b^3*e + 6*a^2*b*c*e)*ArcTanh[(b + 2*c*x^4)/
Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - 4*a*c)*(2*b*d - a*e)
*Log[x^4])/a^2 + ((b^2 - 4*a*c)*(2*b*d - a*e)*Log[a + b*x^4 + c*x^8])/(2*a
^2))/(a*(b^2 - 4*a*c)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\frac{\frac{ac(abe+2acd-d\,b^2)x^4}{4ac-b^2}-\frac{a(2a^2ce-a\,b^2e-3abcd+b^3d)}{4ac-b^2}}{2c\,x^8+2b\,x^4+2a}+\frac{\frac{(4a^2c^2e-a\,b^2ce-8d\,c^2ba+2dc\,b^3)\ln(c\,x^8+b\,x^4+a)}{2c}+\frac{2(5a^2bce+6a^2c^2d-a\,b^3e-1\cdots}{8ac-2b^2}}{\cdots}}{2a^3}$
risch	$\frac{-\frac{c(abe+6acd-2d\,b^2)x^8}{4a^2(4ac-b^2)}+\frac{(2a^2ce-a\,b^2e-7abcd+2b^3d)x^4}{4(4ac-b^2)a^2}-\frac{d}{4a}}{x^4(c\,x^8+b\,x^4+a)}+\frac{\ln(x)e}{a^2}-\frac{2\ln(x)bd}{a^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}((64a^6c^3-48a^5b^2c^2+12a^4b^4c-\cdots}\right.}{\cdots}$

input
```
int((e*x^4+d)/x^5/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^3*(1/2*(a*c*(a*b*e+2*a*c*d-b^2*d)/(4*a*c-b^2)*x^4-a*(2*a^2*c*e-a*b^
2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^2))/(c*x^8+b*x^4+a)+1/2/(4*a*c-b^2)*(1/2*(4*
a^2*c^2*e-a*b^2*c*e-8*a*b*c^2*d+2*b^3*c*d)/c*ln(c*x^8+b*x^4+a)+2*(5*a^2*b*
c*e+6*a^2*c^2*d-a*b^3*e-10*a*b^2*c*d+2*b^4*d-1/2*(4*a^2*c^2*e-a*b^2*c*e-8*
a*b*c^2*d+2*b^3*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)
^(1/2))))-1/4*d/a^2/x^4+(a*e-2*b*d)/a^3*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 812 vs. 2(214) = 428.

Time = 19.23 (sec) , antiderivative size = 1647, normalized size of antiderivative = 7.32

$$\int \frac{d+ex^4}{x^5\left(a+bx^4+cx^8\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^4+d)/x^5/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b^3*c - 4*a^3*
b*c^2)*e)*x^8 + 2*((2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d - (a^2*b^4 -
6*a^3*b^2*c + 8*a^4*c^2)*e)*x^4 + ((2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d
- (a*b^3*c - 6*a^2*b*c^2)*e)*x^12 + (2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d -
 (a*b^4 - 6*a^2*b^2*c)*e)*x^8 + (2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d - (
a^2*b^3 - 6*a^3*b*c)*e)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^8 + 2*b*c*x^4
+ b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) + 2*
(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d - ((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2
*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^12 + (2*(b^6 - 8*a
*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^8 +
 (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a
^4*c^2)*e)*x^4)*log(c*x^8 + b*x^4 + a) + 8*((2*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^12 + (2*(b^6 - 8
*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^8
 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16
*a^4*c^2)*e)*x^4)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^12 +
 (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^8 + (a^4*b^4 - 8*a^5*b^2*c + 16*
a^6*c^2)*x^4), -1/8*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*
b^3*c - 4*a^3*b*c^2)*e)*x^8 + 2*((2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d
 - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*e)*x^4 + 2*((2*(b^4*c - 6*a*b^2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^5\left(a+bx^4+cx^8\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x**4+d)/x**5/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^4}{x^5\left(a+bx^4+cx^8\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^4+d)/x^5/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 4.80 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.13

$$\begin{aligned}
&\int \frac{d+ex^4}{x^5\left(a+bx^4+cx^8\right)^2}\,dx \\
&= \frac{(2\,b^4d-12\,ab^2cd+12\,a^2c^2d-ab^3e+6\,a^2bce)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}} \\
&\quad-\frac{2\,b^2cdx^8-6\,ac^2dx^8-abcex^8+2\,b^3dx^4-7\,abcdx^4-ab^2ex^4+2\,a^2cex^4+ab^2d-4\,a^2cd}{4\,(cx^{12}+bx^8+ax^4)(a^2b^2-4\,a^3c)} \\
&\quad+\frac{(2\,bd-ae)\log\left(cx^8+bx^4+a\right)}{8\,a^3}-\frac{(2\,bd-ae)\log\left(x^4\right)}{4\,a^3}
\end{aligned}$$

input
```
integrate((e*x^4+d)/x^5/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(2*b^4*d - 12*a*b^2*c*d + 12*a^2*c^2*d - a*b^3*e + 6*a^2*b*c*e)*arctan
((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)
) - 1/4*(2*b^2*c*d*x^8 - 6*a*c^2*d*x^8 - a*b*c*e*x^8 + 2*b^3*d*x^4 - 7*a*b
*c*d*x^4 - a*b^2*e*x^4 + 2*a^2*c*e*x^4 + a*b^2*d - 4*a^2*c*d)/((c*x^12 + b
*x^8 + a*x^4)*(a^2*b^2 - 4*a^3*c)) + 1/8*(2*b*d - a*e)*log(c*x^8 + b*x^4 +
 a)/a^3 - 1/4*(2*b*d - a*e)*log(x^4)/a^3
```

Mupad [B] (verification not implemented)

Time = 108.62 (sec) , antiderivative size = 47178, normalized size of antiderivative = 209.68

$$\int \frac{d + ex^4}{x^5 (a + bx^4 + cx^8)^2} \, dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x^5*(a + b*x^4 + c*x^8)^2),x)
```

output
```
(log(x)*(a*e - 2*b*d))/a^3 - (d/(4*a) - (x^4*(2*b^3*d - a*b^2*e + 2*a^2*c*
e - 7*a*b*c*d))/(4*a^2*(4*a*c - b^2)) + (c*x^8*(a*b*e - 2*b^2*d + 6*a*c*d)
)/(4*a^2*(4*a*c - b^2)))/(a*x^4 + b*x^8 + c*x^12) + (log((((a*e - 2*b*d +
a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a
^6*(4*a*c - b^2)^3))^(1/2))*(((a*e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*c^2*d
 - a*b^3*e - 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*(
((a*e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 12*a*b^2*c*d + 6
*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((((256*b^3*c^4*(2*b^4*d + 6*a
^2*c^2*d - a*b^3*e - 10*a*b^2*c*d + 5*a^2*b*c*e))/(a^2*(4*a*c - b^2)) + (3
2*b^3*c^4*(a*e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 12*a*b^
2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 5*b^2*x^4 - 18
*a*c*x^4))/a^3 - (64*b^2*c^5*x^4*(14*b^4*d + 324*a^2*c^2*d - 7*a*b^3*e - 1
32*a*b^2*c*d + 18*a^2*b*c*e))/(a^2*(4*a*c - b^2)))*(a*e - 2*b*d + a^3*(-(2
*b^4*d + 12*a^2*c^2*d - a*b^3*e - 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*
c - b^2)^3))^(1/2)))/(8*a^3) + (32*b^2*c^5*(108*a^3*c^3*d^2 - 8*a^2*b^4*e^
2 - 32*b^6*d^2 + 35*a^3*b^2*c*e^2 + 32*a*b^5*d*e - 456*a^2*b^2*c^2*d^2 + 2
36*a*b^4*c*d^2 - 188*a^2*b^3*c*d*e + 228*a^3*b*c^2*d*e))/(a^4*(4*a*c - b^2
)^2) - (16*b*c^6*x^4*(52*b^6*d^2 + 13*a^2*b^4*e^2 + 1944*a^3*c^3*d^2 - 54*
a^3*b^2*c*e^2 - 52*a*b^5*d*e - 504*a^2*b^2*c^2*d^2 - 204*a*b^4*c*d^2 + 210
*a^2*b^3*c*d*e))/(a^4*(4*a*c - b^2)^2)))/(8*a^3) + (16*b*c^6*(a*b*e - 2...
```

Reduce [F]

$$\int \frac{d + ex^4}{x^5 (a + bx^4 + cx^8)^2} \, dx = \int \frac{e\,x^4 + d}{x^5 \left(c\,x^8 + b\,x^4 + a\right)^2} dx$$

input
```
int((e*x^4+d)/x^5/(c*x^8+b*x^4+a)^2,x)
```

output
```
int((e*x^4+d)/x^5/(c*x^8+b*x^4+a)^2,x)
```

3.94 $\int \frac{x^{13}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 404

$$\begin{aligned}\int \frac{x^{13}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx &= \frac{ex^2}{2c^2}-\frac{x^2(a(bcd-b^2e+2ace)+(b^2cd-2ac^2d-b^3e+3abce)\,x^4)}{4c^2\,(b^2-4ac)\,(a+bx^4+cx^8)}\\ &\quad+\frac{\left(b^2cd-6ac^2d-3b^3e+13abce-\frac{b^3cd-8abc^2d-3b^4e+19ab^2ce-20a^2c^2e}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}}\\ &\quad+\frac{\left(b^2cd-6ac^2d-3b^3e+13abce+\frac{b^3cd-8abc^2d-3b^4e+19ab^2ce-20a^2c^2e}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output

```
1/2*e*x^2/c^2-1/4*x^2*(a*(2*a*c*e-b^2*e+b*c*d)+(3*a*b*c*e-2*a*c^2*d-b^3*e+
b^2*c*d)*x^4)/c^2/(-4*a*c+b^2)/(c*x^8+b*x^4+a)+1/8*(b^2*c*d-6*a*c^2*d-3*b^
3*e+13*a*b*c*e-(-20*a^2*c^2*e+19*a*b^2*c*e-8*a*b*c^2*d-3*b^4*e+b^3*c*d)/(-
4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))
*2^(1/2)/c^(5/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/8*(b^2*c*d-6*
a*c^2*d-3*b^3*e+13*a*b*c*e+(-20*a^2*c^2*e+19*a*b^2*c*e-8*a*b*c^2*d-3*b^4*e
+b^3*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(
1/2))^(1/2))*2^(1/2)/c^(5/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.15

$$\int \frac{x^{13}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$

$$= \frac{4\sqrt{c}ex^2 + \frac{2\sqrt{c}x^2\left(-2a^2ce+b^2(-cd+be)x^4+a\left(b^2e+2c^2dx^4-bc\left(d+3ex^4\right)\right)\right)}{(b^2-4ac)(a+bx^4+cx^8)} - \frac{\sqrt{2}\left(-3b^4e+2ac^2\left(3\sqrt{b^2-4ac}d-10ae\right)+b^2c\left(-\sqrt{b^2-4ac}d+\cdots\right.\right.}{(b^2 \cdots}}{}$$

input
```
Integrate[(x^13*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
(4*Sqrt[c]*e*x^2 + (2*Sqrt[c]*x^2*(-2*a^2*c*e + b^2*(-(c*d) + b*e)*x^4 + a
*(b^2*e + 2*c^2*d*x^4 - b*c*(d + 3*e*x^4))))/((b^2 - 4*a*c)*(a + b*x^4 + c
*x^8)) - (Sqrt[2]*(-3*b^4*e + 2*a*c^2*(3*Sqrt[b^2 - 4*a*c]*d - 10*a*e) + b
^2*c*(-(Sqrt[b^2 - 4*a*c]*d) + 19*a*e) + b^3*(c*d + 3*Sqrt[b^2 - 4*a*c]*e)
 - a*b*c*(8*c*d + 13*Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) - (Sqrt[2]*(3*b^4*e + 2*a*c^2*(3*Sqrt[b^2 - 4*a*c]*d + 10*a*e) - b^2*c
*(Sqrt[b^2 - 4*a*c]*d + 19*a*e) + a*b*c*(8*c*d - 13*Sqrt[b^2 - 4*a*c]*e) +
 b^3*(-(c*d) + 3*Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))
/(8*c^(5/2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1814, 1598, 1602, 27, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}\left(d+ex^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx$$

$$\Big\downarrow \; 1814$$

$$\frac{1}{2}\int \frac{x^{12}\left(ex^4+d\right)}{\left(cx^8+bx^4+a\right)^2}dx^2$$

$$\Big\downarrow \text{ 1598}$$

$$\frac{1}{2}\left(\frac{\int \frac{x^8\left(3(2cd-be)x^4+5(bd-2ae)\right)}{cx^8+bx^4+a}dx^2}{2\left(b^2-4ac\right)}-\frac{x^{10}\left(-2ae+x^4(2cd-be)+bd\right)}{2\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right)$$

$$\Big\downarrow \text{ 1602}$$

$$\frac{1}{2}\left(\frac{\frac{x^6(2cd-be)}{c}-\frac{\int \frac{3x^4\left(\left(-3eb^2+cdb+10ace\right)x^4+3a(2cd-be)\right)}{cx^8+bx^4+a}dx^2}{3c}}{2\left(b^2-4ac\right)}-\frac{x^{10}\left(-2ae+x^4(2cd-be)+bd\right)}{2\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{\frac{x^6(2cd-be)}{c}-\frac{\int \frac{x^4\left(\left(-3eb^2+cdb+10ace\right)x^4+3a(2cd-be)\right)}{cx^8+bx^4+a}dx^2}{c}}{2\left(b^2-4ac\right)}-\frac{x^{10}\left(-2ae+x^4(2cd-be)+bd\right)}{2\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right)$$

$$\Big\downarrow \text{ 1602}$$

$$\frac{1}{2}\left(\frac{\frac{x^6(2cd-be)}{c}-\frac{\frac{x^2\left(10ace-3b^2e+bcd\right)}{c}-\frac{\int \frac{\left(-3eb^3+cdb^2+13aceb-6ac^2d\right)x^4+a\left(-3eb^2+cdb+10ace\right)}{cx^8+bx^4+a}dx^2}{c}}{c}}{2\left(b^2-4ac\right)}-\frac{x^{10}\left(-2ae+x^4(2cd-be)+b\right.}{2\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right.$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{1}{2}\left(\frac{\frac{x^6(2cd-be)}{c}-\frac{\frac{x^2\left(10ace-3b^2e+bcd\right)}{c}-\frac{\frac{1}{2}\left(-\frac{-20a^2c^2e+19ab^2ce-8abc^2d-3b^4e+b^3cd}{\sqrt{b^2-4ac}}+13abce-6ac^2d-3b^3e+b^2cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}\left(\right.}{c}}{c}}{2\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2}\left(\frac{\frac{x^6(2cd-be)}{c}-\frac{\frac{x^2\left(10ace-3b^2e+bcd\right)}{c}-\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-20a^2c^2e+19ab^2ce-8abc^2d-3b^4e+b^3cd}{\sqrt{b^2-4ac}}+13abce-6ac^2d-3b^3e+b^2cd\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\cdots\right.}{\cdots}}{c}}{c}}{2\left(b^2-4ac\right)}\right.$$

input
```
Int[(x^13*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
(-1/2*(x^10*(b*d - 2*a*e + (2*c*d - b*e)*x^4))/((b^2 - 4*a*c)*(a + b*x^4 +
 c*x^8)) + (((2*c*d - b*e)*x^6)/c - (((b*c*d - 3*b^2*e + 10*a*c*e)*x^2)/c
- (((b^2*c*d - 6*a*c^2*d - 3*b^3*e + 13*a*b*c*e - (b^3*c*d - 8*a*b*c^2*d -
 3*b^4*e + 19*a*b^2*c*e - 20*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt
[b^2 - 4*a*c]]) + ((b^2*c*d - 6*a*c^2*d - 3*b^3*e + 13*a*b*c*e + (b^3*c*d
- 8*a*b*c^2*d - 3*b^4*e + 19*a*b^2*c*e - 20*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/c)/(2*(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.13

method	result
default	$\frac{e\,x^2}{2c^2} - \frac{-\frac{\left(3abce-2a\,c^2d-b^3e+b^2cd\right)x^6}{2\left(4ac-b^2\right)}-\frac{a\left(2ace-b^2e+cbd\right)x^2}{2\left(4ac-b^2\right)}}{c\,x^8+b\,x^4+a} + \frac{2c\left(-\frac{\left(13abce\sqrt{-4ac+b^2}-6a\,c^2d\sqrt{-4ac+b^2}-3b^3e\sqrt{-4ac+b^2}+b^2cd\sqrt{-4ac\cdots\right.}{8c\sqrt{-4ac\cdots}}\right.}{\cdots}$
risch	Expression too large to display

input
```
int(x^13*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*e*x^2/c^2-1/2/c^2*((-1/2*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(4*a*c-b^
2)*x^6-1/2*a*(2*a*c*e-b^2*e+b*c*d)/(4*a*c-b^2)*x^2)/(c*x^8+b*x^4+a)+2/(4*a
*c-b^2)*c*(-1/8*(13*a*b*c*e*(-4*a*c+b^2)^(1/2)-6*a*c^2*d*(-4*a*c+b^2)^(1/2
)-3*b^3*e*(-4*a*c+b^2)^(1/2)+b^2*c*d*(-4*a*c+b^2)^(1/2)+20*a^2*c^2*e-19*a*
b^2*c*e+8*d*c^2*b*a+3*b^4*e-d*c*b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))+1/8*(13*a*b*c*e*(-4*a*c+b^2)^(1/2)-6*a*c^2*d*(-4*a*c+b^2)^(1/2)-3
*b^3*e*(-4*a*c+b^2)^(1/2)+b^2*c*d*(-4*a*c+b^2)^(1/2)-20*a^2*c^2*e+19*a*b^2
*c*e-8*d*c^2*b*a-3*b^4*e+d*c*b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6774 vs. $2(360) = 720$.

Time = 7.00 (sec) , antiderivative size = 6774, normalized size of antiderivative = 16.77

$$\int \frac{x^{13}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^13*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**13*(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{13}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{(ex^4+d)x^{13}}{(cx^8+bx^4+a)^2}\,dx$$

input
```
integrate(x^13*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/2*e*x^2/c^2 - 1/4*(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*x^6 + (a*b*
c*d - (a*b^2 - 2*a^2*c)*e)*x^2)/((b^2*c^3 - 4*a*c^4)*x^8 + a*b^2*c^2 - 4*a
^2*c^3 + (b^3*c^2 - 4*a*b*c^3)*x^4) + 1/2*integrate((((b^2*c - 6*a*c^2)*d
- (3*b^3 - 13*a*b*c)*e)*x^4 + a*b*c*d - (3*a*b^2 - 10*a^2*c)*e)*x/(c*x^8 +
 b*x^4 + a), x)/(b^2*c^2 - 4*a*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4984 vs. $2(360) = 720$.

Time = 5.05 (sec) , antiderivative size = 4984, normalized size of antiderivative = 12.34

$$\int \frac{x^{13}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^13*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/2*e*x^2/c^2 - 1/4*(b^2*c*d*x^6 - 2*a*c^2*d*x^6 - b^3*e*x^6 + 3*a*b*c*e*x
^6 + a*b*c*d*x^2 - a*b^2*e*x^2 + 2*a^2*c*e*x^2)/((c*x^8 + b*x^4 + a)*(b^2*
c^2 - 4*a*c^3)) + 1/16*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 1
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 2*b^4*c^2 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*c^3 + 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3
+ sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 20*a*b^2*c^3 - 6*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 48*a^2*c^4 + 2*(b^2 - 4*a*c)*b^
2*c^2 - 12*(b^2 - 4*a*c)*a*c^3)*d*x^4*abs(b^2*c^2 - 4*a*c^3) - (3*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 6*b^5*c
 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 26*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^3*c^2 + 50*a*b^3*c^2 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^3 - 104*a^2*b*c^3 + 6*(b^2 - 4*a*c)*b^3*c - 26*(b^2 - 4*a*c)*a*b*c^
2)*e*x^4*abs(b^2*c^2 - 4*a*c^3) + (2*b^5*c^5 - 20*a*b^3*c^6 + 48*a^2*b*c^7
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 10*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 24*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 12*sqr...
```

Mupad [B] (verification not implemented)

Time = 27.10 (sec) , antiderivative size = 45062, normalized size of antiderivative = 111.54

$$\int \frac{x^{13}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
int((x^13*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x)
```

output
```
atan(((((81*a*b^16*e^4 + 5184*a^7*c^10*d^4 + 40000*a^9*c^8*e^4 + a*b^12*c^
4*d^4 - 2052*a^2*b^14*c*e^4 - 32*a^2*b^10*c^5*d^4 + 404*a^3*b^8*c^6*d^4 -
2512*a^4*b^6*c^7*d^4 + 7780*a^5*b^4*c^8*d^4 - 10656*a^6*b^2*c^9*d^4 + 2165
4*a^3*b^12*c^2*e^4 - 122592*a^4*b^10*c^3*e^4 + 398665*a^5*b^8*c^4*e^4 - 73
6316*a^6*b^6*c^5*e^4 + 707924*a^7*b^4*c^6*e^4 - 287200*a^8*b^2*c^7*e^4 - 2
8800*a^8*c^9*d^2*e^2 - 108*a*b^15*c*d*e^3 - 1548*a^2*b^12*c^3*d^2*e^2 + 17
826*a^3*b^10*c^4*d^2*e^2 - 104988*a^4*b^8*c^5*d^2*e^2 + 332040*a^5*b^6*c^6
*d^2*e^2 - 534984*a^6*b^4*c^7*d^2*e^2 + 355776*a^7*b^2*c^8*d^2*e^2 - 12*a*
b^13*c^3*d^3*e - 67968*a^7*b*c^9*d^3*e + 188800*a^8*b*c^8*d*e^3 + 54*a*b^1
4*c^2*d^2*e^2 + 364*a^2*b^11*c^4*d^3*e + 2916*a^2*b^13*c^2*d*e^3 - 4388*a^
3*b^9*c^5*d^3*e - 32112*a^3*b^11*c^3*d*e^3 + 26504*a^4*b^7*c^6*d^3*e + 185
116*a^4*b^9*c^4*d*e^3 - 82840*a^5*b^5*c^7*d^3*e - 593300*a^5*b^7*c^5*d*e^3
 + 123712*a^6*b^3*c^8*d^3*e + 1025528*a^6*b^5*c^6*d*e^3 - 827392*a^7*b^3*c
^7*d*e^3)/(256*a^4*c^10 + b^8*c^6 - 16*a*b^6*c^7 + 96*a^2*b^4*c^8 - 256*a^
3*b^2*c^9) + (((128*a*b^13*c^7*d^2 + 589824*a^7*b*c^13*d^2 + 1152*a*b^15*c
^5*e^2 - 1638400*a^8*b*c^12*e^2 - 3584*a^2*b^11*c^8*d^2 + 40192*a^3*b^9*c^
9*d^2 - 229376*a^4*b^7*c^10*d^2 + 696320*a^5*b^5*c^11*d^2 - 1048576*a^6*b^
3*c^12*d^2 - 28416*a^2*b^13*c^6*e^2 + 291968*a^3*b^11*c^7*e^2 - 1604864*a^
4*b^9*c^8*e^2 + 5017600*a^5*b^7*c^9*e^2 - 8658944*a^6*b^5*c^10*e^2 + 71106
56*a^7*b^3*c^11*e^2 - 768*a*b^14*c^6*d*e + 20224*a^2*b^12*c^7*d*e - 216...
```

Reduce [F]

$$\int \frac{x^{13}(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{x^{13}(e\,x^4+d)}{(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^13*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^13*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.95 $\int \frac{x^9(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 336

$$\int \frac{x^9(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \frac{x^2\left(a(2cd-be)+\left(bcd-b^2e+2ace\right)x^4\right)}{4c\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)} + \frac{\left(bd-6ae+\frac{b^2e}{c}-\frac{b^2cd+4ac^2d+b^3e-8abce}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(bd-6ae+\frac{b^2e}{c}+\frac{b^2cd+4ac^2d+b^3e-8abce}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
1/4*x^2*(a*(-b*e+2*c*d)+(2*a*c*e-b^2*e+b*c*d)*x^4)/c/(-4*a*c+b^2)/(c*x^8+b
*x^4+a)+1/8*(b*d-6*a*e+b^2*e/c-(-8*a*b*c*e+4*a*c^2*d+b^3*e+b^2*c*d)/c/(-4*
a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2
^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/8*(b*d-6*a*e+b^
2*e/c+(-8*a*b*c*e+4*a*c^2*d+b^3*e+b^2*c*d)/c/(-4*a*c+b^2)^(1/2))*arctan(2^
(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.09

$$\int \frac{x^9(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$
$$= \frac{\frac{2\sqrt{c}x^2(-abe+b(cd-be)x^4+2ac(d+ex^4))}{(b^2-4ac)(a+bx^4+cx^8)} + \frac{\sqrt{2}\left(-b^3e+bc\left(\sqrt{b^2-4ac}d+8ae\right)+b^2\left(-cd+\sqrt{b^2-4ac}e\right)-2ac\left(2cd+3\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\cdots}{\sqrt{b-\cdots}}\cdots\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\cdots}{8c^{3/2}}$$

input
```
Integrate[(x^9*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
((2*Sqrt[c]*x^2*(-(a*b*e) + b*(c*d - b*e)*x^4 + 2*a*c*(d + e*x^4)))/((b^2
- 4*a*c)*(a + b*x^4 + c*x^8)) + (Sqrt[2]*(-(b^3*e) + b*c*(Sqrt[b^2 - 4*a*c
]*d + 8*a*e) + b^2*(-(c*d) + Sqrt[b^2 - 4*a*c]*e) - 2*a*c*(2*c*d + 3*Sqrt[
b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*e + b*
c*(Sqrt[b^2 - 4*a*c]*d - 8*a*e) + 2*a*c*(2*c*d - 3*Sqrt[b^2 - 4*a*c]*e) +
b^2*(c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(8*c^
(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1814, 1598, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9\left(d+ex^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx$$
$$\Big\downarrow \text{ 1814}$$
$$\frac{1}{2}\int \frac{x^8\left(ex^4+d\right)}{\left(cx^8+bx^4+a\right)^2}dx^2$$

$\downarrow$ 1598

$$\frac{1}{2}\left(\frac{\int \frac{x^4\left((2cd-be)x^4+3(bd-2ae)\right)}{cx^8+bx^4+a}dx^2}{2\left(b^2-4ac\right)}-\frac{x^6\left(-2ae+x^4(2cd-be)+bd\right)}{2\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right)$$

$\downarrow$ 1602

$$\frac{1}{2}\left(\frac{\frac{x^2(2cd-be)}{c}-\frac{\int \frac{a(2cd-be)-\left(eb^2+cdb-6ace\right)x^4}{cx^8+bx^4+a}dx^2}{c}}{2\left(b^2-4ac\right)}-\frac{x^6\left(-2ae+x^4(2cd-be)+bd\right)}{2\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right)$$

$\downarrow$ 1480

$$\frac{1}{2}\left(\frac{\frac{x^2(2cd-be)}{c}-\frac{-\frac{1}{2}\left(-\frac{-8abce+4ac^2d+b^3e+b^2cd}{\sqrt{b^2-4ac}}-6ace+b^2e+bcd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2-\frac{1}{2}\left(\frac{-8abce+4ac^2d+b^3e+b^2cd}{\sqrt{b^2-4ac}}-6ace+b^2e+bcd\right)\cdots}{c}}{2\left(b^2-4ac\right)}\cdots\right.$$

$\downarrow$ 218

$$\frac{1}{2}\left(\frac{\frac{x^2(2cd-be)}{c}-\frac{-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-8abce+4ac^2d+b^3e+b^2cd}{\sqrt{b^2-4ac}}-6ace+b^2e+bcd\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-8abce+4ac^2d+b^3e+b^2cd}{\sqrt{b^2-4ac}}-6ac\cdots\right.}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{2\left(b^2-4ac\right)}\cdots\right.$$

input

```
Int[(x^9*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output

```
(-1/2*(x^6*(b*d - 2*a*e + (2*c*d - b*e)*x^4))/((b^2 - 4*a*c)*(a + b*x^4 +
c*x^8)) + (((2*c*d - b*e)*x^2)/c - (-(((b*c*d + b^2*e - 6*a*c*e - (b^2*c*d
 + 4*a*c^2*d + b^3*e - 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 -
 4*a*c]])) - ((b*c*d + b^2*e - 6*a*c*e + (b^2*c*d + 4*a*c^2*d + b^3*e - 8*
a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/(2*(b^2 - 4
*a*c)))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.08

method	result
default	$\frac{-\frac{(2ace-b^2e+cbd)x^6}{2c(4ac-b^2)}+\frac{a(eb-2cd)x^2}{2(4ac-b^2)c}}{2c\,x^8+2b\,x^4+2a}+\frac{-\frac{\left(6ace\sqrt{-4ac+b^2}-b^2e\sqrt{-4ac+b^2}-cbd\sqrt{-4ac+b^2}-8abce+4a\,c^2d+b^3e+b^2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\quad}{\sqrt{(-b+}}\right.}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{}$
risch	$\frac{-\frac{(2ace-b^2e+cbd)x^6}{4c(4ac-b^2)}+\frac{a(eb-2cd)x^2}{4(4ac-b^2)c}}{c\,x^8+b\,x^4+a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4096c^9a^6-6144b^2c^8a^5+3840b^4c^7a^4-1280b^6c^6a^3+240b^8c^5a^2-24b^{10}c^4a+b^{12}c^3\right)\_\right.}\right.}{}$

input
```
int(x^9*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-1/2*(2*a*c*e-b^2*e+b*c*d)/c/(4*a*c-b^2)*x^6+1/2*a*(b*e-2*c*d)/(4*a*c
-b^2)/c*x^2)/(c*x^8+b*x^4+a)+1/(4*a*c-b^2)*(-1/8*(6*a*c*e*(-4*a*c+b^2)^(1/
2)-b^2*e*(-4*a*c+b^2)^(1/2)-c*b*d*(-4*a*c+b^2)^(1/2)-8*a*b*c*e+4*a*c^2*d+b
^3*e+b^2*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(6*a*c*e*(
-4*a*c+b^2)^(1/2)-b^2*e*(-4*a*c+b^2)^(1/2)-c*b*d*(-4*a*c+b^2)^(1/2)+8*a*b*
c*e-4*a*c^2*d-b^3*e-b^2*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4420 vs. 2(294) = 588.

Time = 1.88 (sec) , antiderivative size = 4420, normalized size of antiderivative = 13.15

$$\int \frac{x^9(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^9*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**9*(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^9(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{(ex^4+d)x^9}{(cx^8+bx^4+a)^2}\,dx$$

input
```
integrate(x^9*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*((b*c*d - (b^2 - 2*a*c)*e)*x^6 + (2*a*c*d - a*b*e)*x^2)/((b^2*c^2 - 4*
a*c^3)*x^8 + (b^3*c - 4*a*b*c^2)*x^4 + a*b^2*c - 4*a^2*c^2) - 1/2*integrat
e(-((b*c*d + (b^2 - 6*a*c)*e)*x^4 - 2*a*c*d + a*b*e)*x/(c*x^8 + b*x^4 + a)
, x)/(b^2*c - 4*a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3779 vs. $2(294) = 588$.

Time = 5.08 (sec) , antiderivative size = 3779, normalized size of antiderivative = 11.25

$$\int \frac{x^9(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^9*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(b*c*d*x^6 - b^2*e*x^6 + 2*a*c*e*x^6 + 2*a*c*d*x^2 - a*b*e*x^2)/((c*x^
8 + b*x^4 + a)*(b^2*c - 4*a*c^2)) + 1/16*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 2*b^3*c^2 + sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^3 + 8*a*b*c^3 + 2*(b^2 - 4*a*c)*b*c^2)*d*x^4*
abs(b^2*c - 4*a*c^2) + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 10*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*c^2 + 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 20*a*b^2*c^2 - 6*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 48*a^2*c^3 + 2*(b^2 - 4*a*c)*b^2*c - 12*
(b^2 - 4*a*c)*a*c^2)*e*x^4*abs(b^2*c - 4*a*c^2) + (2*b^4*c^4 - 8*a*b^2*c^5
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)
*b^2*c^4)*d*x^4 + (2*b^5*c^3 - 20*a*b^3*c^4 + 48*a^2*b*c^5 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 10*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 24*sqrt(2)*sqrt(b^2 - 4...
```

Mupad [B] (verification not implemented)

Time = 29.17 (sec) , antiderivative size = 34824, normalized size of antiderivative = 103.64

$$\int \frac{x^9(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
int((x^9*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x)
```

output
```
((x^2*(a*b*e - 2*a*c*d))/(4*c*(4*a*c - b^2)) - (x^6*(2*a*c*e - b^2*e + b*c
*d))/(4*c*(4*a*c - b^2)))/(a + b*x^4 + c*x^8) + atan(((((a*b^12*e^4 + 64*a
^5*c^8*d^4 + 5184*a^7*c^6*e^4 + a*b^8*c^4*d^4 - 32*a^2*b^10*c*e^4 + 4*a^2*
b^6*c^5*d^4 + 20*a^3*b^4*c^6*d^4 + 32*a^4*b^2*c^7*d^4 + 404*a^3*b^8*c^2*e^
4 - 2512*a^4*b^6*c^3*e^4 + 7780*a^5*b^4*c^4*e^4 - 10656*a^6*b^2*c^5*e^4 -
1152*a^6*c^7*d^2*e^2 + 4*a*b^11*c*d*e^3 - 84*a^2*b^8*c^3*d^2*e^2 + 264*a^3
*b^6*c^4*d^2*e^2 + 120*a^4*b^4*c^5*d^2*e^2 + 960*a^5*b^2*c^6*d^2*e^2 + 4*a
*b^9*c^3*d^3*e - 128*a^5*b*c^7*d^3*e + 1152*a^6*b*c^6*d*e^3 + 6*a*b^10*c^2
*d^2*e^2 - 20*a^2*b^7*c^4*d^3*e - 92*a^2*b^9*c^2*d*e^3 - 40*a^3*b^5*c^5*d^
3*e + 728*a^3*b^7*c^3*d*e^3 - 256*a^4*b^3*c^6*d^3*e - 2104*a^4*b^5*c^4*d*e
^3 + 832*a^5*b^3*c^5*d*e^3)/(256*a^4*c^6 + b^8*c^2 - 16*a*b^6*c^3 + 96*a^2
*b^4*c^4 - 256*a^3*b^2*c^5) + ((((x^2*(294912*a^2*b^12*c^7*d - 2949120*a^3
*b^10*c^8*d + 15728640*a^4*b^8*c^9*d - 47185920*a^5*b^6*c^10*d + 75497472*
a^6*b^4*c^11*d - 50331648*a^7*b^2*c^12*d + 24576*a^2*b^13*c^6*e - 589824*a
^3*b^11*c^7*e + 5898240*a^4*b^9*c^8*e - 31457280*a^5*b^7*c^9*e + 94371840*
a^6*b^5*c^10*e - 150994944*a^7*b^3*c^11*e - 12288*a*b^14*c^6*d + 100663296
*a^8*b*c^12*e))/(8*(1024*a^5*c^7 - b^10*c^2 + 20*a*b^8*c^3 - 160*a^2*b^6*c
^4 + 640*a^3*b^4*c^5 - 1280*a^4*b^2*c^6)) + ((-(b^11*e^2 + b^9*c^2*d^2 + b
^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a
^4*b*c^6*d^2 - 3840*a^5*b*c^5*e^2 + 2*b^10*c*d*e - 96*a^2*b^5*c^4*d^2 +...
```

Reduce [F]

$$\int \frac{x^9(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{x^9(e\,x^4+d)}{(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^9*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^9*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.96 $\int \frac{x^5(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 286

$$\int \frac{x^5(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = -\frac{x^2(bd-2ae+(2cd-be)x^4)}{4(b^2-4ac)(a+bx^4+cx^8)} - \frac{\left(2cd-be-\frac{4bcd-b^2e-4ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2cd-be+\frac{4bcd-b^2e-4ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/4*x^2*(b*d-2*a*e+(-b*e+2*c*d)*x^4)/(-4*a*c+b^2)/(c*x^8+b*x^4+a)-1/8*(2*
c*d-b*e-(-4*a*c*e-b^2*e+4*b*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2
)*x^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2)-1/8*(2*c*d-b*e+(-4*a*c*e-b^2*e+4*b*c*d)/(-4*a*c+b^2)^(
1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(
1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.06

$$\int \frac{x^5(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$
$$=\frac{1}{8}\left(\frac{2x^2(-bd+2ae-2cdx^4+bex^4)}{(b^2-4ac)\,(a+bx^4+cx^8)}+\frac{\sqrt{2}\big(4bcd-2c\sqrt{b^2-4ac}d-b^2e-4ace+b\sqrt{b^2-4ac}e\big)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\big(-4bcd-2c\sqrt{b^2-4ac}d+b^2e+4ace+b\sqrt{b^2-4ac}e\big)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

input
```
Integrate[(x^5*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
((2*x^2*(-(b*d) + 2*a*e - 2*c*d*x^4 + b*e*x^4))/((b^2 - 4*a*c)*(a + b*x^4
+ c*x^8)) + (Sqrt[2]*(4*b*c*d - 2*c*Sqrt[b^2 - 4*a*c]*d - b^2*e - 4*a*c*e
+ b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[2]*(-4*b*c*d - 2*c*Sqrt[b^2 - 4*a*c]*d + b^2*e + 4*a*c*e + b*Sqrt[b^2 -
4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/8
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1814, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\,(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$

$$\Big\downarrow \; 1814$$

$$\frac{1}{2}\int \frac{x^4\left(ex^4+d\right)}{\left(cx^8+bx^4+a\right)^2}dx^2$$

$$\Big\downarrow \; 1598$$

$$\frac{1}{2}\left(\frac{\int \frac{-((2cd-be)x^4)+bd-2ae}{cx^8+bx^4+a}dx^2}{2\left(b^2-4ac\right)}-\frac{x^2\left(-2ae+x^4(2cd-be)+bd\right)}{2\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right)$$

$$\Big\downarrow \; 1480$$

$$\frac{1}{2}\left(\frac{-\frac{1}{2}\left(-\frac{-4ace+b^2(-e)+4bcd}{\sqrt{b^2-4ac}}-be+2cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2-\frac{1}{2}\left(\frac{-4ace+b^2(-e)+4bcd}{\sqrt{b^2-4ac}}-be+2cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{\right.}\right.}}{2\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(\frac{-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-4ace+b^2(-e)+4bcd}{\sqrt{b^2-4ac}}-be+2cd\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-4ace+b^2(-e)+4bcd}{\sqrt{b^2-4ac}}-be+2cd\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-\frac{x^2\left(-2ae+\right.}{2\left(b^2-4a\right.}\right.$$

input
```
Int[(x^5*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
(-1/2*(x^2*(b*d - 2*a*e + (2*c*d - b*e)*x^4))/((b^2 - 4*a*c)*(a + b*x^4 +
c*x^8)) + (-(((2*c*d - b*e - (4*b*c*d - b^2*e - 4*a*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt
[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*c*d - b*e + (4*b*c*d - b^2*e - 4*a
*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c))
)/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.03

method	result
default	$\frac{-\frac{(eb-2cd)x^6}{2(4ac-b^2)}-\frac{(2ae-bd)x^2}{2(4ac-b^2)}}{2c\,x^8+2b\,x^4+2a}+\frac{c\left(-\frac{\left(-eb\sqrt{-4ac+b^2}+2cd\sqrt{-4ac+b^2}+4ace+b^2e-4cbd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-eb\sqrt{-4ac\cdots}\right.}{\cdots}\right.}{4ac-b^2}$ …
risch	$\frac{-\frac{(eb-2cd)x^6}{4(4ac-b^2)}-\frac{(2ae-bd)x^2}{4(4ac-b^2)}}{c\,x^8+b\,x^4+a}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4096c^7a^7-6144c^6a^6b^2+3840c^5a^5b^4-1280c^4a^4b^6+240c^3a^3b^8-24c^2a^2b^{10}+ca\,b^{12}\right)\_Z^4+\left(-\cdots\right.\right.}\right.}{\cdots}$ …

input
```
int(x^5*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-1/2*(b*e-2*c*d)/(4*a*c-b^2)*x^6-1/2*(2*a*e-b*d)/(4*a*c-b^2)*x^2)/(c*
x^8+b*x^4+a)+1/(4*a*c-b^2)*c*(-1/8*(-e*b*(-4*a*c+b^2)^(1/2)+2*c*d*(-4*a*c+
b^2)^(1/2)+4*a*c*e+b^2*e-4*c*b*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))+1/8*(-e*b*(-4*a*c+b^2)^(1/2)+2*c*d*(-4*a*c+b^2)^(1/2)-4*a*c*e-b^2*e+
4*c*b*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3377 vs. $2(244) = 488$.

Time = 1.21 (sec) , antiderivative size = 3377, normalized size of antiderivative = 11.81

$$\int \frac{x^5(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{(ex^4+d)x^5}{(cx^8+bx^4+a)^2}\,dx$$

input
```
integrate(x^5*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*((2*c*d - b*e)*x^6 + (b*d - 2*a*e)*x^2)/((b^2*c - 4*a*c^2)*x^8 + (b^3
 - 4*a*b*c)*x^4 + a*b^2 - 4*a^2*c) + 1/2*integrate(-((2*c*d - b*e)*x^4 - b
*d + 2*a*e)*x/(c*x^8 + b*x^4 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3053 vs. $2(244) = 488$.

Time = 5.00 (sec) , antiderivative size = 3053, normalized size of antiderivative = 10.67

$$\int \frac{x^5(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*(2*c*d*x^6 - b*e*x^6 + b*d*x^2 - 2*a*e*x^2)/((c*x^8 + b*x^4 + a)*(b^2
 - 4*a*c)) - 1/16*(2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - 4*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b*c^2 - 2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
c^3 + 8*a*c^3 + 2*(b^2 - 4*a*c)*c^2)*d*x^4*abs(b^2 - 4*a*c) - (sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - 2*b^3*c + sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 8*a*b*c^2 + 2*(b^2 - 4*a*c)*b
*c)*e*x^4*abs(b^2 - 4*a*c) + 2*(2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*d*x^4 - (2*b^4*c^2
- 8*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 2*(b^2 - 4
*a*c)*b^2*c^2)*e*x^4 - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^2*c - 2*b^3*c + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)...
```

Mupad [B] (verification not implemented)

Time = 29.35 (sec) , antiderivative size = 26677, normalized size of antiderivative = 93.28

$$\int \frac{x^5(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
int((x^5*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x)
```

output

```
atan((((((((-(a*b^9*e^2 + a*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^9*c*d^2 - c*
d^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^5*d^2 - 768*a^5*b*c^4*e^2 - 96*
a^2*b^5*c^3*d^2 + 512*a^3*b^3*c^4*d^2 - 96*a^3*b^5*c^2*e^2 + 512*a^4*b^3*c
^3*e^2 + 1024*a^5*c^5*d*e + 128*a^2*b^6*c^2*d*e - 384*a^3*b^4*c^3*d*e - 12
*a*b^8*c*d*e)/(128*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 128
0*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(4
096*a*b^14*c^4 - 98304*a^2*b^12*c^5 + 983040*a^3*b^10*c^6 - 5242880*a^4*b^
8*c^7 + 15728640*a^5*b^6*c^8 - 25165824*a^6*b^4*c^9 + 16777216*a^7*b^2*c^1
0))/(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c) -
(x^2*(4096*b^15*c^4*d + 786432*a^2*b^11*c^6*d - 3276800*a^3*b^9*c^7*d + 52
42880*a^4*b^7*c^8*d + 6291456*a^5*b^5*c^9*d - 33554432*a^6*b^3*c^10*d + 29
4912*a^2*b^12*c^5*e - 2949120*a^3*b^10*c^6*e + 15728640*a^4*b^8*c^7*e - 47
185920*a^5*b^6*c^8*e + 75497472*a^6*b^4*c^9*e - 50331648*a^7*b^2*c^10*e -
90112*a*b^13*c^5*d + 33554432*a^7*b*c^11*d - 12288*a*b^14*c^4*e))/(8*(b^10
 - 1024*a^5*c^5 + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 2
0*a*b^8*c)))*(-(a*b^9*e^2 + a*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^9*c*d^2 - c
*d^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^5*d^2 - 768*a^5*b*c^4*e^2 - 96
*a^2*b^5*c^3*d^2 + 512*a^3*b^3*c^4*d^2 - 96*a^3*b^5*c^2*e^2 + 512*a^4*b^3*
c^3*e^2 + 1024*a^5*c^5*d*e + 128*a^2*b^6*c^2*d*e - 384*a^3*b^4*c^3*d*e - 1
2*a*b^8*c*d*e)/(128*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 -...
```

Reduce [F]

$$\int \frac{x^5(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{x^5(e\,x^4+d)}{(c\,x^8+b\,x^4+a)^2}dx$$

input

```
int(x^5*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output

```
int(x^5*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.97 $\int \frac{x(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 299

$$\begin{aligned}\int \frac{x(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx &= \frac{x^2(b^2d-2acd-abe+c(bd-2ae)x^4)}{4a\,(b^2-4ac)\,(a+bx^4+cx^8)} \\ &\quad + \frac{\sqrt{c}\left(bd-2ae+\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}a\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} \\ &\quad + \frac{\sqrt{c}\left(bd-2ae-\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}a\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
1/4*x^2*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^4)/a/(-4*a*c+b^2)/(c*x^8+b*x
^4+a)+1/8*c^(1/2)*(b*d-2*a*e+(4*a*b*e-12*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))*
arctan(2^(1/2)*c^(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c
+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/8*c^(1/2)*(b*d-2*a*e-(4*a*b*e-12*a*c*
d+b^2*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.06

$$\int \frac{x(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$

$$= \frac{\frac{2x^2(b^2d+b(-ae+cdx^4)-2ac(d+ex^4))}{(b^2-4ac)(a+bx^4+cx^8)} + \frac{\sqrt{2}\sqrt{c}\left(b^2d+b\left(\sqrt{b^2-4ac}d+4ae\right)-2a\left(6cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2d\cdots\right.}{}}{8a}$$

input
```
Integrate[(x*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
((2*x^2*(b^2*d + b*(-(a*e) + c*d*x^4) - 2*a*c*(d + e*x^4)))/((b^2 - 4*a*c)
*(a + b*x^4 + c*x^8)) + (Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2 - 4*a*c]*d +
 4*a*e) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - 4*
a*b*e - 2*a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(8*a)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1814, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$

$$\Big\downarrow \text{ 1814}$$

$$\frac{1}{2}\int \frac{ex^4+d}{(cx^8+bx^4+a)^2}dx^2$$

$$\Big\downarrow \text{ 1492}$$

$$\frac{1}{2}\left(\frac{x^2\left(cx^4(bd-2ae)-abe-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}-\frac{\int -\frac{c(bd-2ae)x^4+b^2d-6acd+abe}{cx^8+bx^4+a}dx^2}{2a\left(b^2-4ac\right)}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \frac{c(bd-2ae)x^4+b^2d-6acd+abe}{cx^8+bx^4+a}dx^2}{2a\left(b^2-4ac\right)}+\frac{x^2\left(cx^4(bd-2ae)-abe-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right)$$

$\Big\downarrow$ 1480

$$\frac{1}{2}\left(\frac{\frac{1}{2}c\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}c\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}\right.}{2a\left(b^2-4ac\right)}$$

$\Big\downarrow$ 218

$$\frac{1}{2}\left(\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x^2\left(cx^4(bd-\right.}{2a\left(b^2-\right.}\right.$$

input
```
Int[(x*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
((x^2*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^4))/(2*a*(b^2 - 4*a*c)*
(a + b*x^4 + c*x^8)) + ((Sqrt[c]*(b*d - 2*a*e + (b^2*d - 12*a*c*d + 4*a*b*
e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*e - (b^
2*d - 12*a*c*d + 4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*
(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.58

method	result
default	$8c^{2}\left(\frac{\frac{\left(-bd-\sqrt{-4ac+b^{2}}\,d+2ae\right)\sqrt{-4ac+b^{2}}\,x^{2}}{16ac\left(x^{4}+\frac{b}{2c}-\frac{\sqrt{-4ac+b^{2}}}{2c}\right)}+\frac{\left(12\sqrt{-4ac+b^{2}}\,acd-3\sqrt{-4ac+b^{2}}\,b^{2}d-8a^{2}ce-6a\,b^{2}e+28abcd-3b^{3}d\right)\left(-2b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,a}{16a\left(4ac+3b^{2}\right)\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}}{4\sqrt{-4ac+b^{2}}\,c(4ac-b^{2})}\right.$
risch	$\frac{\frac{c(2ae-bd)x^{6}}{4a\left(4ac-b^{2}\right)}+\frac{\left(abe+2acd-d\,b^{2}\right)x^{2}}{4\left(4ac-b^{2}\right)a}}{c\,x^{8}+b\,x^{4}+a}+\frac{\left(\_R=\mathrm{RootOf}\left(\left(4096a^{9}c^{6}-6144a^{8}b^{2}c^{5}+3840a^{7}b^{4}c^{4}-1280a^{6}b^{6}c^{3}+240a^{5}b^{8}c^{2}-24a^{4}b^{10}c+a^{3}b^{12}\right)\_\right.\right.}{}$

input
```
int(x*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
8*c^2*(1/4/(-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(1/16*(-b*d-(-4*a*c+b^2)^(1/2)
*d+2*a*e)*(-4*a*c+b^2)^(1/2)/a/c*x^2/(x^4+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c
)+1/16*(12*(-4*a*c+b^2)^(1/2)*a*c*d-3*(-4*a*c+b^2)^(1/2)*b^2*d-8*a^2*c*e-6
*a*b^2*e+28*a*b*c*d-3*b^3*d)*(-2*b+(-4*a*c+b^2)^(1/2))/a/(4*a*c+3*b^2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)))+1/4/(-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(1/16*((-4*a*c
+b^2)^(1/2)*d+2*a*e-b*d)*(-4*a*c+b^2)^(1/2)/a/c*x^2/(x^4+1/2*(-4*a*c+b^2)^
(1/2)/c+1/2*b/c)+1/16*(8*a^2*c*e+6*a*b^2*e-28*a*b*c*d+3*b^3*d+12*(-4*a*c+b
^2)^(1/2)*a*c*d-3*(-4*a*c+b^2)^(1/2)*b^2*d)*(2*b+(-4*a*c+b^2)^(1/2))/a/(4*
a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4583 vs. $2(257) = 514$.

Time = 2.71 (sec) , antiderivative size = 4583, normalized size of antiderivative = 15.33

$$\int \frac{x(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{(ex^4+d)x}{(cx^8+bx^4+a)^2}\,dx$$

input
```
integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*((b*c*d - 2*a*c*e)*x^6 - (a*b*e - (b^2 - 2*a*c)*d)*x^2)/((a*b^2*c - 4*
a^2*c^2)*x^8 + (a*b^3 - 4*a^2*b*c)*x^4 + a^2*b^2 - 4*a^3*c) - 1/2*integrat
e(-((b*c*d - 2*a*c*e)*x^4 + a*b*e + (b^2 - 6*a*c)*d)*x/(c*x^8 + b*x^4 + a)
, x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4435 vs. $2(257) = 514$.

Time = 5.62 (sec) , antiderivative size = 4435, normalized size of antiderivative = 14.83

$$\int \frac{x(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(b*c*d*x^6 - 2*a*c*e*x^6 + b^2*d*x^2 - 2*a*c*d*x^2 - a*b*e*x^2)/((c*x^
8 + b*x^4 + a)*(a*b^2 - 4*a^2*c)) + 1/32*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2
*c)^2*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*e + 2*(sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^
6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 -
 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3
+ 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^
2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^...
```

Mupad [B] (verification not implemented)

Time = 29.82 (sec) , antiderivative size = 32336, normalized size of antiderivative = 108.15

$$\int \frac{x(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input

```
int((x*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x)
```

output

```
((x^2*(a*b*e - b^2*d + 2*a*c*d))/(4*a*(4*a*c - b^2)) + (c*x^6*(2*a*e - b*d
))/(4*a*(4*a*c - b^2)))/(a + b*x^4 + c*x^8) - atan((((((65536*a^8*b*c^9*e
^2 - 589824*a^7*b*c^10*d^2 + 128*a^2*b^11*c^5*d^2 - 3328*a^3*b^9*c^6*d^2 +
 36864*a^4*b^7*c^7*d^2 - 204800*a^5*b^5*c^8*d^2 + 557056*a^6*b^3*c^9*d^2 +
 1280*a^4*b^9*c^5*e^2 - 16384*a^5*b^7*c^6*e^2 + 73728*a^6*b^5*c^7*e^2 - 13
1072*a^7*b^3*c^8*e^2 + 256*a^3*b^10*c^5*d*e - 8192*a^4*b^8*c^6*d*e + 73728
*a^5*b^6*c^7*d*e - 262144*a^6*b^4*c^8*d*e + 327680*a^7*b^2*c^9*d*e)/(a^3*b
^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3) + ((x^
2*(4096*a^2*b^16*c^4*d - 126976*a^3*b^14*c^5*d + 1671168*a^4*b^12*c^6*d -
12124160*a^5*b^10*c^7*d + 52428800*a^6*b^8*c^8*d - 135266304*a^7*b^6*c^9*d
 + 192937984*a^8*b^4*c^10*d - 117440512*a^9*b^2*c^11*d + 4096*a^3*b^15*c^4
*e - 90112*a^4*b^13*c^5*e + 786432*a^5*b^11*c^6*e - 3276800*a^6*b^9*c^7*e
+ 5242880*a^7*b^7*c^8*e + 6291456*a^8*b^5*c^9*e - 33554432*a^9*b^3*c^10*e
+ 33554432*a^10*b*c^11*e))/(8*(a^3*b^10 - 1024*a^8*c^5 - 20*a^4*b^8*c + 16
0*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4)) + ((-(b^11*d^2 + a^2*
b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1
/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*
c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2
 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/
2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^...
```

Reduce [F]

$$\int \frac{x(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{x(e\,x^4+d)}{(c\,x^8+b\,x^4+a)^2}dx$$

input

```
int(x*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.98 $\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 376

$$\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)^2}\,dx = -\frac{3b^2d-10acd-abe}{4a^2(b^2-4ac)x^2}+\frac{b^2d-2acd-abe+c(bd-2ae)x^4}{4a(b^2-4ac)x^2(a+bx^4+cx^8)} - \frac{\sqrt{c}\left(3b^2d-10acd-abe+\frac{3b^3d-16abcd-ab^2e+12a^2ce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}a^2(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(3b^2d-10acd-abe-\frac{3b^3d-16abcd-ab^2e+12a^2ce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}a^2(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
-1/4*(-a*b*e-10*a*c*d+3*b^2*d)/a^2/(-4*a*c+b^2)/x^2+1/4*(b^2*d-2*a*c*d-a*b
*e+c*(-2*a*e+b*d)*x^4)/a/(-4*a*c+b^2)/x^2/(c*x^8+b*x^4+a)-1/8*c^(1/2)*(3*b
^2*d-10*a*c*d-a*b*e+(12*a^2*c*e-a*b^2*e-16*a*b*c*d+3*b^3*d)/(-4*a*c+b^2)^(
1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2
/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/8*c^(1/2)*(3*b^2*d-10*a*c*d-a
*b*e-(12*a^2*c*e-a*b^2*e-16*a*b*c*d+3*b^3*d)/(-4*a*c+b^2)^(1/2))*arctan(2^
(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)/(
b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.65

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)^2}\,dx = -\frac{\frac{4d}{x^2}+\frac{2x^2\left(b^3d+2ac\left(ae-cdx^4\right)+b^2\left(-ae+cdx^4\right)-abc\left(3d+ex^4\right)\right)}{\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}+\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{3b^3d\log(x-\#1)-13abcd\log(x-\#1}{}\right.}{}}{8a^2}$$

input
```
Integrate[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)^2),x]
```

output
```
-1/8*((4*d)/x^2 + (2*x^2*(b^3*d + 2*a*c*(a*e - c*d*x^4) + b^2*(-(a*e) + c*
d*x^4) - a*b*c*(3*d + e*x^4)))/((b^2 - 4*a*c)*(a + b*x^4 + c*x^8)) + RootS
um[a + b*#1^4 + c*#1^8 & , (3*b^3*d*Log[x - #1] - 13*a*b*c*d*Log[x - #1] -
 a*b^2*e*Log[x - #1] + 6*a^2*c*e*Log[x - #1] + 3*b^2*c*d*Log[x - #1]*#1^4
- 10*a*c^2*d*Log[x - #1]*#1^4 - a*b*c*e*Log[x - #1]*#1^4)/(b*#1^2 + 2*c*#1
^6) & ]/(b^2 - 4*a*c))/a^2
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1814, 1600, 25, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)^2}dx$$

$$\Big\downarrow \; 1814$$

$$\frac{1}{2}\int \frac{ex^4+d}{x^4\left(cx^8+bx^4+a\right)^2}dx^2$$

$$\Big\downarrow \; 1600$$

$$\frac{1}{2}\left(\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{2ax^2\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}-\frac{\int -\frac{3c(bd-2ae)x^4+3b^2d-10acd-abe}{x^4(cx^8+bx^4+a)}dx^2}{2a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int \frac{3c(bd-2ae)x^4+3b^2d-10acd-abe}{x^4(cx^8+bx^4+a)}dx^2}{2a\left(b^2-4ac\right)}+\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{2ax^2\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right)$$

$$\Big\downarrow \; 1604$$

$$\frac{1}{2}\left(\frac{-\frac{\int \frac{c\left(3db^2-aeb-10acd\right)x^4+3b^3d-13abcd-ab^2e+6a^2ce}{cx^8+bx^4+a}dx^2}{a}-\frac{-abe-10acd+3b^2d}{ax^2}}{2a\left(b^2-4ac\right)}+\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{2ax^2\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}\right)$$

$$\Big\downarrow \; 1480$$

$$\frac{1}{2}\left(\frac{-\frac{\frac{1}{2}c\left(\frac{12a^2ce-ab^2e-16abcd+3b^3d}{\sqrt{b^2-4ac}}-abe-10acd+3b^2d\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2+\frac{1}{2}c\left(-\frac{12a^2ce-ab^2e-16abcd+3b^3d}{\sqrt{b^2-4ac}}-abe-10acd+3b^2d\right)\int \frac{}{c}}{a}}{2a\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(\frac{-\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12a^2ce-ab^2e-16abcd+3b^3d}{\sqrt{b^2-4ac}}-abe-10acd+3b^2d\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{12a^2ce-ab^2e-16abcd+3b^3d}{\sqrt{b^2-4ac}}-abe-10\right.}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}}{2a\left(b^2-4ac\right)}\right.$$

input

```
Int[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^4)/(2*a*(b^2 - 4*a*c)*x^2*(a
 + b*x^4 + c*x^8)) + (-((3*b^2*d - 10*a*c*d - a*b*e)/(a*x^2)) - ((Sqrt[c]*
(3*b^2*d - 10*a*c*d - a*b*e + (3*b^3*d - 16*a*b*c*d - a*b^2*e + 12*a^2*c*e
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^2*d - 10*a*c*d
 - a*b*e - (3*b^3*d - 16*a*b*c*d - a*b^2*e + 12*a^2*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]))/a)/(2*a*(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.03

method	result
default	$\frac{-\frac{c\left(abe+2acd-d\,b^{2}\right)x^{6}}{2\left(4ac-b^{2}\right)}+\frac{\left(2a^{2}ce-a\,b^{2}e-3abcd+b^{3}d\right)x^{2}}{8ac-2b^{2}}}{c\,x^{8}+b\,x^{4}+a}+\frac{2c\left(-\frac{\left(-abe\sqrt{-4ac+b^{2}}-10\sqrt{-4ac+b^{2}}\,acd+3\sqrt{-4ac+b^{2}}\,b^{2}d+12a^{2}ce-a\,b^{2}e-16abcd\cdots\right.}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)\cdots}}\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int((e*x^4+d)/x^3/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*((-1/2*c*(a*b*e+2*a*c*d-b^2*d)/(4*a*c-b^2)*x^6+1/2*(2*a^2*c*e-a*b^
2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^2)*x^2)/(c*x^8+b*x^4+a)+2/(4*a*c-b^2)*c*(-1/
8*(-a*b*e*(-4*a*c+b^2)^(1/2)-10*(-4*a*c+b^2)^(1/2)*a*c*d+3*(-4*a*c+b^2)^(1
/2)*b^2*d+12*a^2*c*e-a*b^2*e-16*a*b*c*d+3*b^3*d)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))+1/8*(-a*b*e*(-4*a*c+b^2)^(1/2)-10*(-4*a*c+b^2)^(1/2)*a*
c*d+3*(-4*a*c+b^2)^(1/2)*b^2*d-12*a^2*c*e+a*b^2*e+16*a*b*c*d-3*b^3*d)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))-1/2*d/a^2/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6960 vs. $2(332) = 664$.

Time = 9.29 (sec) , antiderivative size = 6960, normalized size of antiderivative = 18.51

$$\int \frac{d+ex^4}{x^3 (a+bx^4+cx^8)^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^3 (a+bx^4+cx^8)^2} \, dx = \text{Timed out}$$

input
```
integrate((e*x**4+d)/x**3/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^4}{x^3 (a+bx^4+cx^8)^2} \, dx = \int \frac{ex^4+d}{(cx^8+bx^4+a)^2 x^3} \, dx$$

input
```
integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*((a*b*c*e - (3*b^2*c - 10*a*c^2)*d)*x^8 - ((3*b^3 - 11*a*b*c)*d - (a*b
^2 - 2*a^2*c)*e)*x^4 - 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2*c - 4*a^3*c^2)*x^1
0 + (a^2*b^3 - 4*a^3*b*c)*x^6 + (a^3*b^2 - 4*a^4*c)*x^2) + 1/2*integrate((
(a*b*c*e - (3*b^2*c - 10*a*c^2)*d)*x^4 - (3*b^3 - 13*a*b*c)*d + (a*b^2 - 6
*a^2*c)*e)*x/(c*x^8 + b*x^4 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5103 vs. $2(332) = 664$.

Time = 4.72 (sec) , antiderivative size = 5103, normalized size of antiderivative = 13.57

$$\int \frac{d+ex^4}{x^3 (a+bx^4+cx^8)^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*(3*b^2*c*d*x^8 - 10*a*c^2*d*x^8 - a*b*c*e*x^8 + 3*b^3*d*x^4 - 11*a*b*
c*d*x^4 - a*b^2*e*x^4 + 2*a^2*c*e*x^4 + 2*a*b^2*d - 8*a^2*c*d)/((c*x^10 +
b*x^6 + a*x^2)*(a^2*b^2 - 4*a^3*c)) - 1/16*((3*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4*c - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2
- 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 6*b^4*c^2 + 40*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 20*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c^3 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3
+ 44*a*b^2*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 80*a^2
*c^4 + 6*(b^2 - 4*a*c)*b^2*c^2 - 20*(b^2 - 4*a*c)*a*c^3)*d*x^4*abs(a^2*b^2
 - 4*a^3*c) - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 4*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^2 - 2*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 + 8*a^2*b*c^3 + 2*(b^2 - 4*a*c)*a*b*c^2)*e*x^4*abs(a^2*b^2 -
4*a^3*c) + (6*a^2*b^5*c^3 - 44*a^3*b^3*c^4 + 80*a^4*b*c^5 - 3*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c + 22*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 6*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 40*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 20*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 10*sqrt(2)*s...
```

Mupad [B] (verification not implemented)

Time = 31.02 (sec) , antiderivative size = 55298, normalized size of antiderivative = 147.07

$$\int \frac{d+ex^4}{x^3 (a+bx^4+cx^8)^2} \, dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)^2),x)
```

output
```
atan(((x^2*(16307453952*a^28*c^17*e^5 + 62914560000*a^25*b*c^19*d^5 - 1258
29120000*a^26*c^19*d^4*e + 145152*a^15*b^21*c^9*d^5 - 5723136*a^16*b^19*c^
10*d^5 + 100763136*a^17*b^17*c^11*d^5 - 1041678336*a^18*b^15*c^12*d^5 + 69
88922880*a^19*b^13*c^13*d^5 - 31716016128*a^20*b^11*c^14*d^5 + 98230861824
*a^21*b^9*c^15*d^5 - 203931254784*a^22*b^7*c^16*d^5 + 269299482624*a^23*b^
5*c^17*d^5 - 201326592000*a^24*b^3*c^18*d^5 - 7680*a^19*b^18*c^8*e^5 + 347
136*a^20*b^16*c^9*e^5 - 7077888*a^21*b^14*c^10*e^5 + 84738048*a^22*b^12*c^
11*e^5 - 651952128*a^23*b^10*c^12*e^5 + 3326607360*a^24*b^8*c^13*e^5 - 112
23957504*a^25*b^6*c^14*e^5 + 24108859392*a^26*b^4*c^15*e^5 - 29896998912*a
^27*b^2*c^16*e^5 + 311040*a^16*b^21*c^8*d^3*e^2 - 12337920*a^17*b^19*c^9*d
^3*e^2 - 241920*a^17*b^20*c^8*d^2*e^3 + 216852480*a^18*b^17*c^10*d^3*e^2 +
 10083840*a^18*b^18*c^9*d^2*e^3 - 2214543360*a^19*b^15*c^11*d^3*e^2 - 1862
86080*a^19*b^16*c^10*d^2*e^3 + 14460518400*a^20*b^13*c^12*d^3*e^2 + 200097
7920*a^20*b^14*c^11*d^2*e^3 - 62456463360*a^21*b^11*c^13*d^3*e^2 - 1376845
8240*a^21*b^12*c^12*d^2*e^3 + 177529159680*a^22*b^9*c^14*d^3*e^2 + 6292635
6480*a^22*b^10*c^13*d^2*e^3 - 316271493120*a^23*b^7*c^15*d^3*e^2 - 1910086
04160*a^23*b^8*c^14*d^2*e^3 + 307274711040*a^24*b^5*c^16*d^3*e^2 + 3713217
33120*a^24*b^6*c^15*d^2*e^3 - 91855257600*a^25*b^3*c^17*d^3*e^2 - 41951428
6080*a^25*b^4*c^16*d^2*e^3 + 209882972160*a^26*b^2*c^17*d^2*e^3 - 95126814
720*a^27*b*c^17*d*e^4 - 103680*a^15*b^22*c^8*d^4*e + 3732480*a^16*b^20*...
```

Reduce [F]

$$\int \frac{d+ex^4}{x^3 (a+bx^4+cx^8)^2} \, dx = \int \frac{e\,x^4+d}{x^3 (c\,x^8+b\,x^4+a)^2} dx$$

input
```
int((e*x^4+d)/x^3/(c*x^8+b*x^4+a)^2,x)
```

output

```
int((e*x^4+d)/x^3/(c*x^8+b*x^4+a)^2,x)
```

3.99 $\int \frac{x^8(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 619

$$\int \frac{x^8(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$
$$= \frac{x(a(2cd-be)+(bcd-b^2e+2ace)\,x^4)}{4c\,(b^2-4ac)\,(a+bx^4+cx^8)}$$
$$- \frac{\left(3bd-10ae+\frac{b^2e}{c}+\frac{3b^2cd+4ac^2d+b^3e-12abce}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$
$$- \frac{\left(3bd-10ae+\frac{b^2e}{c}-\frac{3b^2cd+4ac^2d+b^3e-12abce}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$
$$- \frac{\left(3bd-10ae+\frac{b^2e}{c}+\frac{3b^2cd+4ac^2d+b^3e-12abce}{c\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$
$$- \frac{\left(3bd-10ae+\frac{b^2e}{c}-\frac{3b^2cd+4ac^2d+b^3e-12abce}{c\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output

```
1/4*x*(a*(-b*e+2*c*d)+(2*a*c*e-b^2*e+b*c*d)*x^4)/c/(-4*a*c+b^2)/(c*x^8+b*x
^4+a)-1/16*(3*b*d-10*a*e+b^2*e/c+(-12*a*b*c*e+4*a*c^2*d+b^3*e+3*b^2*c*d)/c
/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4
))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/16*(3*b*d-
10*a*e+b^2*e/c-(-12*a*b*c*e+4*a*c^2*d+b^3*e+3*b^2*c*d)/c/(-4*a*c+b^2)^(1/2
))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)
/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/16*(3*b*d-10*a*e+b^2*e/c+(-1
2*a*b*c*e+4*a*c^2*d+b^3*e+3*b^2*c*d)/c/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)
*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)/(-b
-(-4*a*c+b^2)^(1/2))^(3/4)-1/16*(3*b*d-10*a*e+b^2*e/c-(-12*a*b*c*e+4*a*c^2
*d+b^3*e+3*b^2*c*d)/c/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-
4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/
2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.28

$$\int \frac{x^8(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$
$$= \frac{\frac{4x(-abe+b(cd-be)x^4+2ac(d+ex^4))}{a+bx^4+cx^8} + \text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{-2acd\log(x-\#1)+abe\log(x-\#1)+3bcd\log(x-\#1)\#}{b\#1^3+2c}\right.}{16c\,(b^2-4ac)}$$

input
```
Integrate[(x^8*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
((4*x*(-(a*b*e) + b*(c*d - b*e)*x^4 + 2*a*c*(d + e*x^4)))/(a + b*x^4 + c*x
^8) + RootSum[a + b*#1^4 + c*#1^8 & , (-2*a*c*d*Log[x - #1] + a*b*e*Log[x
 - #1] + 3*b*c*d*Log[x - #1]*#1^4 + b^2*e*Log[x - #1]*#1^4 - 10*a*c*e*Log[x
 - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(16*c*(b^2 - 4*a*c))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1822, 25, 1826, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\,(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$

$$\Big\downarrow \text{1822}$$

$$-\frac{\int -\frac{x^4((2cd-be)x^4+5(bd-2ae))}{cx^8+bx^4+a}dx}{4\,(b^2-4ac)} - \frac{x^5\,(-2ae+x^4(2cd-be)+bd)}{4\,(b^2-4ac)\,(a+bx^4+cx^8)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^4\left((2cd-be)x^4+5(bd-2ae)\right)}{cx^8+bx^4+a}dx}{4\left(b^2-4ac\right)}-\frac{x^5\left(-2ae+x^4(2cd-be)+bd\right)}{4\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\big\downarrow$ 1826

$$\frac{\frac{x(2cd-be)}{c}-\frac{\int \frac{a(2cd-be)-\left(eb^2+3cdb-10ace\right)x^4}{cx^8+bx^4+a}dx}{c}}{4\left(b^2-4ac\right)}-\frac{x^5\left(-2ae+x^4(2cd-be)+bd\right)}{4\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\big\downarrow$ 1752

$$\frac{\frac{x(2cd-be)}{c}-\frac{-\frac{1}{2}\left(-\frac{-12abce+4ac^2d+b^3e+3b^2cd}{\sqrt{b^2-4ac}}-10ace+b^2e+3bcd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{-12abce+4ac^2d+b^3e+3b^2cd}{\sqrt{b^2-4ac}}-10ace+b^2e+3bc\cdots\right.}{c}}{4\left(b^2-4ac\right)}$$

$$\frac{x^5\left(-2ae+x^4(2cd-be)+bd\right)}{4\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\big\downarrow$ 756

$$\frac{\frac{x(2cd-be)}{c}-\frac{-\frac{1}{2}\left(\frac{-12abce+4ac^2d+b^3e+3b^2cd}{\sqrt{b^2-4ac}}-10ace+b^2e+3bcd\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)-\frac{1}{2}\left(-\frac{-12\cdots}{}\right.}{c}}{4\left(b^2-4ac\right)}$$

$$\frac{x^5\left(-2ae+x^4(2cd-be)+bd\right)}{4\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\big\downarrow$ 218

$$\frac{\frac{x(2cd-be)}{c}-\frac{-\frac{1}{2}\left(\frac{-12abce+4ac^2d+b^3e+3b^2cd}{\sqrt{b^2-4ac}}-10ace+b^2e+3bcd\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)-\frac{1}{2}\left(\cdots\right.}{c}}{4\left(b^2-4ac\right)}$$

$$\frac{x^5\left(-2ae+x^4(2cd-be)+bd\right)}{4\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\big\downarrow$ 221

$$\frac{\frac{x(2cd-be)}{c}-\frac{-\frac{1}{2}\left(\frac{-12abce+4ac^2d+b^3e+3b^2cd}{\sqrt{b^2-4ac}}-10ace+b^2e+3bcd\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{}}{4\left(b^2-4\right.}$$

$$\frac{x^5\left(-2ae+x^4(2cd-be)+bd\right)}{4\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

input
```
Int[(x^8*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
-1/4*(x^5*(b*d - 2*a*e + (2*c*d - b*e)*x^4))/((b^2 - 4*a*c)*(a + b*x^4 + c
*x^8)) + (((2*c*d - b*e)*x)/c - (-1/2*((3*b*c*d + b^2*e - 10*a*c*e + (3*b^
2*c*d + 4*a*c^2*d + b^3*e - 12*a*b*c*e)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1
/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt
[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*
c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)))) - ((3*b*c*d
+ b^2*e - 10*a*c*e - (3*b^2*c*d + 4*a*c^2*d + b^3*e - 12*a*b*c*e)/Sqrt[b^2
 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(
2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a
*c])^(3/4))))/2)/c)/(4*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1822
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[f^(n - 1)*(f*x)^(m - n + 1)*(a +
b*x^n + c*x^(2*n))^(p + 1)*((b*d - 2*a*e - (b*e - 2*c*d)*x^n)/(n*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[f^n/(n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - n)*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2
*n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
 n - 1] && IntegerQ[p]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.27

method	result	size
default	$\frac{-\frac{\left(2ace-b^{2}e+cbd\right)x^{5}}{4c\left(4ac-b^{2}\right)}+\frac{a(eb-2cd)x}{4\left(4ac-b^{2}\right)c}}{c\,x^{8}+b\,x^{4}+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\left(\left(10ace-b^{2}e-3cbd\right)\_R^{4}-abe+2acd\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{16c(4ac-b^{2})}$	167
risch	$\frac{-\frac{\left(2ace-b^{2}e+cbd\right)x^{5}}{4c\left(4ac-b^{2}\right)}+\frac{a(eb-2cd)x}{4\left(4ac-b^{2}\right)c}}{c\,x^{8}+b\,x^{4}+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\left(\frac{\left(10ace-b^{2}e-3cbd\right)\_R^{4}}{4ac-b^{2}}-\frac{a(eb-2cd)}{4ac-b^{2}}\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{16c}$	180

input
```
int(x^8*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*(2*a*c*e-b^2*e+b*c*d)/c/(4*a*c-b^2)*x^5+1/4*a*(b*e-2*c*d)/(4*a*c-b^2
)/c*x)/(c*x^8+b*x^4+a)+1/16/c/(4*a*c-b^2)*sum(((10*a*c*e-b^2*e-3*b*c*d)*_R
^4-a*b*e+2*a*c*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26466 vs. 2(537) = 1074.

Time = 85.48 (sec) , antiderivative size = 26466, normalized size of antiderivative = 42.76

$$\int \frac{x^8(d+ex^4)}{\left(a+bx^4+cx^8\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^8*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^8(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{(ex^4+d)x^8}{(cx^8+bx^4+a)^2}\,dx$$

input
```
integrate(x^8*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*((b*c*d - (b^2 - 2*a*c)*e)*x^5 + (2*a*c*d - a*b*e)*x)/((b^2*c^2 - 4*a*
c^3)*x^8 + (b^3*c - 4*a*b*c^2)*x^4 + a*b^2*c - 4*a^2*c^2) - 1/4*integrate(
-((3*b*c*d + (b^2 - 10*a*c)*e)*x^4 - 2*a*c*d + a*b*e)/(c*x^8 + b*x^4 + a),
 x)/(b^2*c - 4*a*c^2)
```

Giac [F]

$$\int \frac{x^8(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{(ex^4+d)x^8}{(cx^8+bx^4+a)^2}\,dx$$

input
```
integrate(x^8*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x^4 + d)*x^8/(c*x^8 + b*x^4 + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 29.87 (sec) , antiderivative size = 113499, normalized size of antiderivative = 183.36

$$\int \frac{x^8(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
int((x^8*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x)
```

output
```
((x*(a*b*e - 2*a*c*d))/(4*c*(4*a*c - b^2)) - (x^5*(2*a*c*e - b^2*e + b*c*d
))/(4*c*(4*a*c - b^2)))/(a + b*x^4 + c*x^8) + atan(((((397*a^4*b^7*c*e^5 -
 32*a^5*c^7*d^5 - 9*a^3*b^9*e^5 - 130000*a^7*b*c^4*e^5 + 5*a^2*b^10*d*e^4
+ 60000*a^7*c^5*d*e^4 + 405*a^2*b^6*c^4*d^5 + 918*a^3*b^4*c^5*d^5 + 96*a^4
*b^2*c^6*d^5 - 6549*a^5*b^5*c^2*e^5 + 47800*a^6*b^3*c^3*e^5 + 1600*a^6*c^6
*d^3*e^2 + 270*a^2*b^8*c^2*d^3*e^2 - 6660*a^3*b^6*c^3*d^3*e^2 - 2450*a^3*b
^7*c^2*d^2*e^3 + 35940*a^4*b^4*c^4*d^3*e^2 + 31230*a^4*b^5*c^3*d^2*e^3 + 4
7520*a^5*b^2*c^5*d^3*e^2 - 118160*a^5*b^3*c^4*d^2*e^3 - 330*a^3*b^8*c*d*e^
4 - 720*a^5*b*c^6*d^4*e + 540*a^2*b^7*c^3*d^4*e + 60*a^2*b^9*c*d^2*e^3 - 5
805*a^3*b^5*c^4*d^4*e - 10840*a^4*b^3*c^5*d^4*e + 7205*a^4*b^6*c^2*d*e^4 -
 64610*a^5*b^4*c^3*d*e^4 - 90400*a^6*b*c^5*d^2*e^3 + 199200*a^6*b^2*c^4*d*
e^4)/(64*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^
2*c^4)) - (((-(b^21*e^4 + 81*b^17*c^4*d^4 + b^6*e^4*(-(4*a*c - b^2)^15)^(1
/2) - 1184*a*b^15*c^5*d^4 - 983040*a^8*b*c^12*d^4 - 4*a*c^5*d^4*(-(4*a*c -
 b^2)^15)^(1/2) + 73728000*a^10*b*c^10*e^4 + 2621440*a^9*c^12*d^3*e - 6553
6000*a^10*c^11*d*e^3 + 108*b^18*c^3*d^3*e + 960*a^2*b^13*c^6*d^4 + 84480*a
^3*b^11*c^7*d^4 - 719360*a^4*b^9*c^8*d^4 + 2727936*a^5*b^7*c^9*d^4 - 52592
64*a^6*b^5*c^10*d^4 + 4587520*a^7*b^3*c^11*d^4 + 2085*a^2*b^17*c^2*e^4 - 3
6320*a^3*b^15*c^3*e^4 + 404160*a^4*b^13*c^4*e^4 - 3001344*a^5*b^11*c^5*e^4
 + 15064576*a^6*b^9*c^6*e^4 - 50503680*a^7*b^7*c^7*e^4 + 108380160*a^8*...
```

Reduce [F]

$$\int \frac{x^8(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{x^8(e\,x^4+d)}{(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^8*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^8*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.100 $\int \frac{x^6(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 527

$$\int \frac{x^6(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = -\frac{x^3(bd-2ae+(2cd-be)x^4)}{4(b^2-4ac)(a+bx^4+cx^8)} - \frac{\left(2cd-be+\frac{8bcd-b^2e-12ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\,2^{3/4}c^{3/4}(b^2-4ac)\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(2cd-be-\frac{8bcd-b^2e-12ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\,2^{3/4}c^{3/4}(b^2-4ac)\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\left(2cd-be+\frac{8bcd-b^2e-12ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\,2^{3/4}c^{3/4}(b^2-4ac)\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\left(2cd-be-\frac{8bcd-b^2e-12ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\,2^{3/4}c^{3/4}(b^2-4ac)\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
-1/4*x^3*(b*d-2*a*e+(-b*e+2*c*d)*x^4)/(-4*a*c+b^2)/(c*x^8+b*x^4+a)-1/16*(2
*c*d-b*e+(-12*a*c*e-b^2*e+8*b*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1
/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)/(-b-(-4*
a*c+b^2)^(1/2))^(1/4)-1/16*(2*c*d-b*e-(-12*a*c*e-b^2*e+8*b*c*d)/(-4*a*c+b^
2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/
c^(3/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/16*(2*c*d-b*e+(-12*a*
c*e-b^2*e+8*b*c*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a
*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))
^(1/4)+1/16*(2*c*d-b*e-(-12*a*c*e-b^2*e+8*b*c*d)/(-4*a*c+b^2)^(1/2))*arcta
nh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-4*a*
c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.27

$$\int \frac{x^6(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$
$$= \frac{\frac{4x^3(-bd+2ae-2cdx^4+bex^4)}{a+bx^4+cx^8} + \text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{3bd\log(x-\#1)-6ae\log(x-\#1)-2cd\log(x-\#1)\#1^4+be\log(}{b\#1+2c\#1^5}\right.}{16\,(b^2-4ac)}$$

input
```
Integrate[(x^6*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
((4*x^3*(-(b*d) + 2*a*e - 2*c*d*x^4 + b*e*x^4))/(a + b*x^4 + c*x^8) + Root
Sum[a + b*#1^4 + c*#1^8 & , (3*b*d*Log[x - #1] - 6*a*e*Log[x - #1] - 2*c*d
*Log[x - #1]*#1^4 + b*e*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(16*(b^2
- 4*a*c))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1822, 25, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(d+ex^4)}{(a+bx^4+cx^8)^2}dx$$

$$\Big\downarrow \text{1822}$$

$$-\frac{\int -\frac{x^2(3(bd-2ae)-(2cd-be)x^4)}{cx^8+bx^4+a}dx}{4(b^2-4ac)}-\frac{x^3(-2ae+x^4(2cd-be)+bd)}{4(b^2-4ac)(a+bx^4+cx^8)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^2(3(bd-2ae)-(2cd-be)x^4)}{cx^8+bx^4+a}dx}{4(b^2-4ac)}-\frac{x^3(-2ae+x^4(2cd-be)+bd)}{4(b^2-4ac)(a+bx^4+cx^8)}$$

$$\Big\downarrow \text{1834}$$

$$\frac{-\frac{1}{2}\left(-\frac{-12ace+b^2(-e)+8bcd}{\sqrt{b^2-4ac}}-be+2cd\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx-\frac{1}{2}\left(\frac{-12ace+b^2(-e)+8bcd}{\sqrt{b^2-4ac}}-be+2cd\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}d}{4(b^2-4ac)}$$
$$\frac{x^3(-2ae+x^4(2cd-be)+bd)}{4(b^2-4ac)(a+bx^4+cx^8)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\left(\left(-\frac{-12ace+b^2(-e)+8bcd}{\sqrt{b^2-4ac}}-be+2cd\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx\right)-\left(\frac{-12ace+b^2(-e)+8bcd}{\sqrt{b^2-4ac}}-be+2cd\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}d}{4(b^2-4ac)}$$
$$\frac{x^3(-2ae+x^4(2cd-be)+bd)}{4(b^2-4ac)(a+bx^4+cx^8)}$$

$$\Big\downarrow \text{827}$$

$$\frac{-\left(\left(\frac{-12ace+b^2(-e)+8bcd}{\sqrt{b^2-4ac}}-be+2cd\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)\right)-\left(-\frac{-12ace+b^2(-e)+8}{\sqrt{b^2-4ac}}\right.}{4\left(b^2-4ac\right)}$$

$$\frac{x^3\left(-2ae+x^4(2cd-be)+bd\right)}{4\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\big\downarrow$ 218

$$\frac{-\left(\left(\frac{-12ace+b^2(-e)+8bcd}{\sqrt{b^2-4ac}}-be+2cd\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)\right)-\left(-\frac{-12ace+b^2}{\sqrt{b^2-}}\right.}{4\left(b^2-4ac\right)}$$

$$\frac{x^3\left(-2ae+x^4(2cd-be)+bd\right)}{4\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\big\downarrow$ 221

$$\frac{-\left(\left(\frac{-12ace+b^2(-e)+8bcd}{\sqrt{b^2-4ac}}-be+2cd\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\right)-\left(-\right.}{4\left(b^2-4ac\right)}$$

$$\frac{x^3\left(-2ae+x^4(2cd-be)+bd\right)}{4\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

input
```
Int[(x^6*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
-1/4*(x^3*(b*d - 2*a*e + (2*c*d - b*e)*x^4))/((b^2 - 4*a*c)*(a + b*x^4 + c
*x^8)) + (-((2*c*d - b*e + (8*b*c*d - b^2*e - 12*a*c*e)/Sqrt[b^2 - 4*a*c])
*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^
(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b -
Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4
)))) - (2*c*d - b*e - (8*b*c*d - b^2*e - 12*a*c*e)/Sqrt[b^2 - 4*a*c])*(Arc
Tan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)
*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[
b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/
(4*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1822
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[f^(n - 1)*(f*x)^(m - n + 1)*(a +
b*x^n + c*x^(2*n))^(p + 1)*((b*d - 2*a*e - (b*e - 2*c*d)*x^n)/(n*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[f^n/(n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - n)*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2
*n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
 n - 1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.28

method	result	size
default	$\frac{-\frac{(eb-2cd)x^7}{4(4ac-b^2)}-\frac{(2ae-bd)x^3}{4(4ac-b^2)}}{cx^8+bx^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^8c+\_Z^4b+a)}\frac{\left((-eb+2cd)\_R^6+3(2ae-bd)\_R^2\right)\ln(x-\_R)}{2\_R^7c+\_R^3b}}{64ac-16b^2}$	148
risch	$\frac{-\frac{(eb-2cd)x^7}{4(4ac-b^2)}-\frac{(2ae-bd)x^3}{4(4ac-b^2)}}{cx^8+bx^4+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^8c+\_Z^4b+a)}\frac{\left(-\frac{(eb-2cd)\_R^6}{4ac-b^2}+\frac{3(2ae-bd)\_R^2}{4ac-b^2}\right)\ln(x-\_R)}{2\_R^7c+\_R^3b}\right)}{16}$	160

input
```
int(x^6*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*(b*e-2*c*d)/(4*a*c-b^2)*x^7-1/4*(2*a*e-b*d)/(4*a*c-b^2)*x^3)/(c*x^8+
b*x^4+a)+1/16/(4*a*c-b^2)*sum(((-b*e+2*c*d)*_R^6+3*(2*a*e-b*d)*_R^2)/(2*_R
^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^6*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**6*(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^6(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx=\int \frac{(ex^4+d)x^6}{(cx^8+bx^4+a)^2}\,dx$$

input
```
integrate(x^6*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*((2*c*d - b*e)*x^7 + (b*d - 2*a*e)*x^3)/((b^2*c - 4*a*c^2)*x^8 + (b^3
 - 4*a*b*c)*x^4 + a*b^2 - 4*a^2*c) + 1/4*integrate(-((2*c*d - b*e)*x^6 - 3
*(b*d - 2*a*e)*x^2)/(c*x^8 + b*x^4 + a), x)/(b^2 - 4*a*c)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^6(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^6*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 28.44 (sec) , antiderivative size = 84889, normalized size of antiderivative = 161.08

$$\int \frac{x^6(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
int((x^6*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x)
```

output
```
- atan(((((1769472*a*b^16*c^4*d^3 - 2147483648*a^9*c^12*d^3 - 86973087744*
a^10*b*c^10*e^3 + 57982058496*a^10*c^11*d*e^2 - 38928384*a^2*b^14*c^5*d^3
+ 339214336*a^3*b^12*c^6*d^3 - 1402994688*a^4*b^10*c^7*d^3 + 2139095040*a^
5*b^8*c^8*d^3 + 3388997632*a^6*b^6*c^9*d^3 - 16508780544*a^7*b^4*c^10*d^3
+ 17716740096*a^8*b^2*c^11*d^3 + 65536*a^3*b^15*c^3*e^3 - 22806528*a^4*b^1
3*c^4*e^3 + 525336576*a^5*b^11*c^5*e^3 - 5179965440*a^6*b^9*c^6*e^3 + 2743
0748160*a^7*b^7*c^7*e^3 - 81939922944*a^8*b^5*c^8*e^3 + 130728067072*a^9*b
^3*c^9*e^3 - 54760833024*a^9*b*c^11*d^2*e - 12386304*a^2*b^15*c^4*d^2*e +
283901952*a^3*b^13*c^5*d^2*e + 27918336*a^3*b^14*c^4*d*e^2 - 2651848704*a^
4*b^11*c^6*d^2*e - 655884288*a^4*b^12*c^5*d*e^2 + 12645826560*a^5*b^9*c^7*
d^2*e + 6360662016*a^5*b^10*c^6*d*e^2 - 30450647040*a^6*b^7*c^8*d^2*e - 32
338083840*a^6*b^8*c^7*d*e^2 + 24763170816*a^7*b^5*c^9*d^2*e + 89087016960*
a^7*b^6*c^8*d*e^2 + 31406948352*a^8*b^3*c^10*d^2*e - 117172076544*a^8*b^4*
c^9*d*e^2 + 27380416512*a^9*b^2*c^10*d*e^2)/(16384*(b^14 - 16384*a^7*c^7 +
 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^
5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x*(-(a*b^19*e^4 + 81*b^17*c^3*d^4
 - 1184*a*b^15*c^4*d^4 - 983040*a^8*b*c^11*d^4 + a*b^4*e^4*(-(4*a*c - b^2)
^15)^(1/2) + 4*a*c^4*d^4*(-(4*a*c - b^2)^15)^(1/2) - 3*a^2*b^17*c*e^4 + 12
386304*a^10*b*c^9*e^4 + 1572864*a^9*c^11*d^3*e - 14155776*a^10*c^10*d*e^3
+ 960*a^2*b^13*c^5*d^4 + 84480*a^3*b^11*c^6*d^4 - 719360*a^4*b^9*c^7*d^...
```

Reduce [F]

$$\int \frac{x^6(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx=\int \frac{x^6(e\,x^4+d)}{(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^6*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^6*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.101 $\int \frac{x^4(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 525

$$\int \frac{x^4(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = -\frac{x(bd-2ae+(2cd-be)x^4)}{4(b^2-4ac)(a+bx^4+cx^8)} + \frac{\left(6cd-3be+\frac{8bcd-3b^2e-4ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\sqrt[4]{2}\sqrt[4]{c}(b^2-4ac)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(6cd-3be-\frac{8bcd-3b^2e-4ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\sqrt[4]{2}\sqrt[4]{c}(b^2-4ac)\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(6cd-3be+\frac{8bcd-3b^2e-4ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\sqrt[4]{2}\sqrt[4]{c}(b^2-4ac)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(6cd-3be-\frac{8bcd-3b^2e-4ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\sqrt[4]{2}\sqrt[4]{c}(b^2-4ac)\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
-1/4*x*(b*d-2*a*e+(-b*e+2*c*d)*x^4)/(-4*a*c+b^2)/(c*x^8+b*x^4+a)+1/16*(6*c
*d-3*b*e+(-4*a*c*e-3*b^2*e+8*b*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(
1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)/(-b-(-4
*a*c+b^2)^(1/2))^(3/4)+1/16*(6*c*d-3*b*e-(-4*a*c*e-3*b^2*e+8*b*c*d)/(-4*a*
c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3
/4)/c^(1/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/16*(6*c*d-3*b*e+(
-4*a*c*e-3*b^2*e+8*b*c*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-
b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)
^(1/2))^(3/4)+1/16*(6*c*d-3*b*e-(-4*a*c*e-3*b^2*e+8*b*c*d)/(-4*a*c+b^2)^(1
/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1
/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.27

$$\int \frac{x^4(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \frac{\frac{4(-bdx+2aex-2cdx^5+bex^5)}{a+bx^4+cx^8} + \text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{bd\log(x-\#1)-2ae\log(x-\#1)-6cd\log(x-\#1)\#1^4+3be\log}{b\#1^3+2c\#1^7}\right.}{16\left(b^2-4ac\right)}$$

input
```
Integrate[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
((4*(-(b*d*x) + 2*a*e*x - 2*c*d*x^5 + b*e*x^5))/(a + b*x^4 + c*x^8) + Root
Sum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - 2*a*e*Log[x - #1] - 6*c*d*L
og[x - #1]*#1^4 + 3*b*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(16*(b^
2 - 4*a*c))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1822, 25, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex^4)}{(a+bx^4+cx^8)^2}dx$$

$$\Big\downarrow \; 1822$$

$$-\frac{\int -\frac{-3(2cd-be)x^4+bd-2ae}{cx^8+bx^4+a}dx}{4(b^2-4ac)}-\frac{x(-2ae+x^4(2cd-be)+bd)}{4(b^2-4ac)(a+bx^4+cx^8)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-3(2cd-be)x^4+bd-2ae}{cx^8+bx^4+a}dx}{4(b^2-4ac)}-\frac{x(-2ae+x^4(2cd-be)+bd)}{4(b^2-4ac)(a+bx^4+cx^8)}$$

$$\Big\downarrow \; 1752$$

$$\frac{-\frac{1}{2}\left(-\frac{-4ace-3b^2e+8bcd}{\sqrt{b^2-4ac}}-3be+6cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{-4ace-3b^2e+8bcd}{\sqrt{b^2-4ac}}-3be+6cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{4(b^2-4ac)}$$

$$\frac{x(-2ae+x^4(2cd-be)+bd)}{4(b^2-4ac)(a+bx^4+cx^8)}$$

$$\Big\downarrow \; 756$$

$$\frac{-\frac{1}{2}\left(\frac{-4ace-3b^2e+8bcd}{\sqrt{b^2-4ac}}-3be+6cd\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)-\frac{1}{2}\left(-\frac{-4ace-3b^2e+8bcd}{\sqrt{b^2-4ac}}\right.}{4(b^2-4ac)}$$

$$\frac{x(-2ae+x^4(2cd-be)+bd)}{4(b^2-4ac)(a+bx^4+cx^8)}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{1}{2}\left(\frac{-4ace-3b^2e+8bcd}{\sqrt{b^2-4ac}}-3be+6cd\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x^2}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)-\frac{1}{2}\left(-\frac{-4ace-3b^2e}{\sqrt{b^2-4a}}\right.}{4\left(b^2-4ac\right)}$$

$$\frac{x\left(-2ae+x^4(2cd-be)+bd\right)}{4\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\Big\downarrow$ 221

$$-\frac{\frac{1}{2}\left(\frac{-4ace-3b^2e+8bcd}{\sqrt{b^2-4ac}}-3be+6cd\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)-\frac{1}{2}\left(-\right.}{4\left(b^2-4ac\right)}$$

$$\frac{x\left(-2ae+x^4(2cd-be)+bd\right)}{4\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

input
```
Int[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
-1/4*(x*(b*d - 2*a*e + (2*c*d - b*e)*x^4))/((b^2 - 4*a*c)*(a + b*x^4 + c*x
^8)) + (-1/2*((6*c*d - 3*b*e + (8*b*c*d - 3*b^2*e - 4*a*c*e)/Sqrt[b^2 - 4*
a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/
4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^
(3/4)))) - ((6*c*d - 3*b*e - (8*b*c*d - 3*b^2*e - 4*a*c*e)/Sqrt[b^2 - 4*a*
c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)
*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-
b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3
/4))))/2)/(4*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1822
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[f^(n - 1)*(f*x)^(m - n + 1)*(a +
b*x^n + c*x^(2*n))^(p + 1)*((b*d - 2*a*e - (b*e - 2*c*d)*x^n)/(n*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[f^n/(n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - n)*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2
*n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
 n - 1] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.27

method	result	size
default	$\frac{-\frac{(eb-2cd)x^5}{4(4ac-b^2)}-\frac{(2ae-bd)x}{4(4ac-b^2)}}{c\,x^8+b\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^8c+\_Z^4b+a)}\frac{(3(-eb+2cd)\_R^4+2ae-bd)\ln(x-\_R)}{2\_R^7c+\_R^3b}}{64ac-16b^2}$	141
risch	$\frac{-\frac{(eb-2cd)x^5}{4(4ac-b^2)}-\frac{(2ae-bd)x}{4(4ac-b^2)}}{c\,x^8+b\,x^4+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^8c+\_Z^4b+a)}\frac{\left(-\frac{3(eb-2cd)\_R^4}{4ac-b^2}+\frac{2ae-bd}{4ac-b^2}\right)\ln(x-\_R)}{2\_R^7c+\_R^3b}\right)}{16}$	154

input
```
int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*(b*e-2*c*d)/(4*a*c-b^2)*x^5-1/4*(2*a*e-b*d)/(4*a*c-b^2)*x)/(c*x^8+b*
x^4+a)+1/16/(4*a*c-b^2)*sum((3*(-b*e+2*c*d)*_R^4+2*a*e-b*d)/(2*_R^7*c+_R^3
*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23255 vs. 2(443) = 886.

Time = 57.92 (sec) , antiderivative size = 23255, normalized size of antiderivative = 44.30

$$\int \frac{x^4(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{(ex^4+d)x^4}{(cx^8+bx^4+a)^2}\,dx$$

input
```
integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*((2*c*d - b*e)*x^5 + (b*d - 2*a*e)*x)/((b^2*c - 4*a*c^2)*x^8 + (b^3 -
 4*a*b*c)*x^4 + a*b^2 - 4*a^2*c) + 1/4*integrate(-(3*(2*c*d - b*e)*x^4 - b
*d + 2*a*e)/(c*x^8 + b*x^4 + a), x)/(b^2 - 4*a*c)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 28.34 (sec) , antiderivative size = 99213, normalized size of antiderivative = 188.98

$$\int \frac{x^4(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x)
```

output
```
atan(((((((-(b^19*c*d^4 + 81*a^3*b^17*e^4 - 3*a*b^17*c^2*d^4 + 12386304*a^
9*b*c^10*d^4 - 1184*a^4*b^15*c*e^4 - 983040*a^11*b*c^8*e^4 + 4*a^4*c*e^4*(
-(4*a*c - b^2)^15)^(1/2) + b^4*c*d^4*(-(4*a*c - b^2)^15)^(1/2) - 14155776*
a^10*c^10*d^3*e + 1572864*a^11*c^9*d*e^3 - 96*a^2*b^15*c^3*d^4 + 2752*a^3*
b^13*c^4*d^4 - 55296*a^4*b^11*c^5*d^4 + 585216*a^5*b^9*c^6*d^4 - 3350528*a
^6*b^7*c^7*d^4 + 10665984*a^7*b^5*c^8*d^4 - 17891328*a^8*b^3*c^9*d^4 + 324
*a^2*c^3*d^4*(-(4*a*c - b^2)^15)^(1/2) - 81*a^3*b^2*e^4*(-(4*a*c - b^2)^15
)^(1/2) + 960*a^5*b^13*c^2*e^4 + 84480*a^6*b^11*c^3*e^4 - 719360*a^7*b^9*c
^4*e^4 + 2727936*a^8*b^7*c^5*e^4 - 5259264*a^9*b^5*c^6*e^4 + 4587520*a^10*
b^3*c^7*e^4 - 20*a*b^18*c*d^3*e + 576*a^3*b^15*c^2*d^2*e^2 - 58752*a^4*b^1
3*c^3*d^2*e^2 + 678912*a^5*b^11*c^4*d^2*e^2 - 3456000*a^6*b^9*c^5*d^2*e^2
+ 8110080*a^7*b^7*c^6*d^2*e^2 - 4030464*a^8*b^5*c^7*d^2*e^2 - 16515072*a^9
*b^3*c^8*d^2*e^2 - 216*a^3*c^2*d^2*e^2*(-(4*a*c - b^2)^15)^(1/2) - 1256*a^
3*b^16*c*d*e^3 + 27*a*b^2*c^2*d^4*(-(4*a*c - b^2)^15)^(1/2) + 168*a^2*b^16
*c^2*d^3*e + 150*a^2*b^17*c*d^2*e^2 - 3072*a^3*b^14*c^3*d^3*e + 85504*a^4*
b^12*c^4*d^3*e + 26880*a^4*b^14*c^2*d*e^3 - 976896*a^5*b^10*c^5*d^3*e - 22
2720*a^5*b^12*c^3*d*e^3 + 5468160*a^6*b^8*c^6*d^3*e + 815104*a^6*b^10*c^4*
d*e^3 - 15859712*a^7*b^6*c^7*d^3*e - 552960*a^7*b^8*c^5*d*e^3 + 20840448*a
^8*b^4*c^8*d^3*e - 5308416*a^8*b^6*c^6*d*e^3 - 2359296*a^9*b^2*c^9*d^3*e +
 16384000*a^9*b^4*c^7*d*e^3 + 22413312*a^10*b*c^9*d^2*e^2 - 15728640*a^...
```

Reduce [F]

$$\int \frac{x^4(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{x^4(e\,x^4+d)}{(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output

```
int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.102 $\int \frac{x^2(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 542

$$\int \frac{x^2(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \frac{x^3(b^2d-2acd-abe+c(bd-2ae)x^4)}{4a\,(b^2-4ac)\,(a+bx^4+cx^8)} + \frac{\sqrt[4]{c}\left(bd-2ae-\frac{b^2d-20acd+8abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\,2^{3/4}a\,(b^2-4ac)\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(bd-2ae+\frac{b^2d-20acd+8abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\,2^{3/4}a\,(b^2-4ac)\sqrt[4]{-b+\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(bd-2ae-\frac{b^2d-20acd+8abe}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\,2^{3/4}a\,(b^2-4ac)\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(bd-2ae+\frac{b^2d-20acd+8abe}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\,2^{3/4}a\,(b^2-4ac)\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
1/4*x^3*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^4)/a/(-4*a*c+b^2)/(c*x^8+b*x
^4+a)+1/16*c^(1/4)*(b*d-2*a*e-(8*a*b*e-20*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))
*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-4*a*c
+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/16*c^(1/4)*(b*d-2*a*e+(8*a*b*e-20*a*
c*d+b^2*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(
1/2))^(1/4))*2^(1/4)/a/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/16*c^(
1/4)*(b*d-2*a*e-(8*a*b*e-20*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/
4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-4*a*c+b^2)/(-b-(-4
*a*c+b^2)^(1/2))^(1/4)-1/16*c^(1/4)*(b*d-2*a*e+(8*a*b*e-20*a*c*d+b^2*d)/(-
4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))
*2^(1/4)/a/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.34

$$\int \frac{x^2(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$
$$= \frac{4x^3(-b^2d+b(ae-cdx^4)+2ac(d+ex^4))-(a+bx^4+cx^8)\,\mathrm{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^2d\log(x-\#1)}{}\right.}{16a\,(-b^2+4ac)\,(a+bx^4+cx^8)}$$

input
```
Integrate[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
(4*x^3*(-(b^2*d) + b*(a*e - c*d*x^4) + 2*a*c*(d + e*x^4)) - (a + b*x^4 + c
*x^8)*RootSum[a + b*#1^4 + c*#1^8 & , (b^2*d*Log[x - #1] - 10*a*c*d*Log[x
- #1] + 3*a*b*e*Log[x - #1] + b*c*d*Log[x - #1]*#1^4 - 2*a*c*e*Log[x - #1]
*#1^4)/(b*#1 + 2*c*#1^5) & ])/(16*a*(-b^2 + 4*a*c)*(a + b*x^4 + c*x^8))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1824, 25, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\,(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx$$

$$\Big\downarrow \; 1824$$

$$\frac{x^3\,(cx^4(bd-2ae)-abe-2acd+b^2d)}{4a\,(b^2-4ac)\,(a+bx^4+cx^8)}-\frac{\int -\frac{x^2\,(c(bd-2ae)x^4+b^2d-10acd+3abe)}{cx^8+bx^4+a}dx}{4a\,(b^2-4ac)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{x^2\left(c(bd-2ae)x^4+b^2d-10acd+3abe\right)}{cx^8+bx^4+a}dx}{4a\left(b^2-4ac\right)}+\frac{x^3\left(cx^4(bd-2ae)-abe-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\big\downarrow$ 1834

$$\frac{\frac{1}{2}c\left(\frac{8abe-20acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}c\left(-\frac{8abe-20acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{4a\left(b^2-4ac\right)}+\frac{x^3\left(cx^4(bd-2ae)-abe-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\big\downarrow$ 27

$$\frac{c\left(\frac{8abe-20acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+c\left(-\frac{8abe-20acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx}{4a\left(b^2-4ac\right)}+\frac{x^3\left(cx^4(bd-2ae)-abe-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\big\downarrow$ 827

$$\frac{c\left(-\frac{8abe-20acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{8abe-20acd+b^2d}{\sqrt{b^2-4ac}}-2ae+\right.}{4a\left(b^2-4ac\right)}$$

$$\frac{x^3\left(cx^4(bd-2ae)-abe-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\big\downarrow$ 218

$$\frac{c\left(-\frac{8abe-20acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{8abe-20acd+b^2d}{\sqrt{b^2-4ac}}-\right.}{4a\left(b^2-4ac\right)}$$

$$\frac{x^3\left(cx^4(bd-2ae)-abe-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\big\downarrow$ 221

$$\frac{c\left(-\frac{8abe-20acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(\frac{8abe-20ac}{\sqrt{b^2-4}}\right.}{4a\left(b^2-4ac\right)}$$

$$\frac{x^3\left(cx^4(bd-2ae)-abe-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

input
```
Int[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x]
```

output
```
(x^3*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^4))/(4*a*(b^2 - 4*a*c)*(
a + b*x^4 + c*x^8)) + (c*(b*d - 2*a*e - (b^2*d - 20*a*c*d + 8*a*b*e)/Sqrt[
b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/
(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1
/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 -
 4*a*c])^(1/4))) + c*(b*d - 2*a*e + (b^2*d - 20*a*c*d + 8*a*b*e)/Sqrt[b^2
- 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2
^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*
x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a
*c])^(1/4))))/(4*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1824
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m
+ 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.32

method	result	s
default	$\frac{\frac{c(2ae-bd)x^7}{4a\left(4ac-b^2\right)}+\frac{\left(abe+2acd-d\,b^2\right)x^3}{4\left(4ac-b^2\right)a}}{c\,x^8+b\,x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(c(-2ae+bd)\_R^6+\left(3abe-10acd+d\,b^2\right)\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{16a(4ac-b^2)}$	
risch	$\frac{\frac{c(2ae-bd)x^7}{4a\left(4ac-b^2\right)}+\frac{\left(abe+2acd-d\,b^2\right)x^3}{4\left(4ac-b^2\right)a}}{c\,x^8+b\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(\frac{c(2ae-bd)\_R^6}{4ac-b^2}-\frac{\left(3abe-10acd+d\,b^2\right)\_R^2}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{16a}$	

input
```
int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*c*(2*a*e-b*d)/a/(4*a*c-b^2)*x^7+1/4*(a*b*e+2*a*c*d-b^2*d)/(4*a*c-b^2)
/a*x^3)/(c*x^8+b*x^4+a)-1/16/a/(4*a*c-b^2)*sum((c*(-2*a*e+b*d)*_R^6+(3*a*b
*e-10*a*c*d+b^2*d)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*
b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{(ex^4+d)x^2}{(cx^8+bx^4+a)^2}\,dx$$

input
```
integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*((b*c*d - 2*a*c*e)*x^7 - (a*b*e - (b^2 - 2*a*c)*d)*x^3)/((a*b^2*c - 4*
a^2*c^2)*x^8 + (a*b^3 - 4*a^2*b*c)*x^4 + a^2*b^2 - 4*a^3*c) - 1/4*integrat
e(-((b*c*d - 2*a*c*e)*x^6 + (3*a*b*e + (b^2 - 10*a*c)*d)*x^2)/(c*x^8 + b*x
^4 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [F]

$$\int \frac{x^2(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{(ex^4+d)x^2}{(cx^8+bx^4+a)^2}\,dx$$

input
```
integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x^4 + d)*x^2/(c*x^8 + b*x^4 + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 28.93 (sec) , antiderivative size = 93859, normalized size of antiderivative = 173.17

$$\int \frac{x^2(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x^4))/(a + b*x^4 + c*x^8)^2,x)
```

output
```
((x^3*(a*b*e - b^2*d + 2*a*c*d))/(4*a*(4*a*c - b^2)) + (c*x^7*(2*a*e - b*d
))/(4*a*(4*a*c - b^2)))/(a + b*x^4 + c*x^8) - atan(((((65536*b^19*c^4*d^3
- 2147483648*a^11*c^12*e^3 - 3735552*a*b^17*c^5*d^3 - 348966092800*a^9*b*c
^13*d^3 + 161061273600*a^10*c^13*d^2*e + 91291648*a^2*b^15*c^6*d^3 - 12559
31904*a^3*b^13*c^7*d^3 + 10742661120*a^4*b^11*c^8*d^3 - 59437481984*a^5*b^
9*c^9*d^3 + 213456519168*a^6*b^7*c^10*d^3 - 481371881472*a^7*b^5*c^11*d^3
+ 620354338816*a^8*b^3*c^12*d^3 + 1769472*a^3*b^16*c^4*e^3 - 38928384*a^4*
b^14*c^5*e^3 + 339214336*a^5*b^12*c^6*e^3 - 1402994688*a^6*b^10*c^7*e^3 +
2139095040*a^7*b^8*c^8*e^3 + 3388997632*a^8*b^6*c^9*e^3 - 16508780544*a^9*
b^4*c^10*e^3 + 17716740096*a^10*b^2*c^11*e^3 + 589824*a*b^18*c^4*d^2*e - 9
3415538688*a^10*b*c^12*d*e^2 - 26738688*a^2*b^16*c^5*d^2*e + 1769472*a^2*b
^17*c^4*d*e^2 + 506068992*a^3*b^14*c^6*d^2*e - 59572224*a^3*b^15*c^5*d*e^2
 - 5236064256*a^4*b^12*c^7*d^2*e + 812384256*a^4*b^13*c^6*d*e^2 + 32432455
680*a^5*b^10*c^8*d^2*e - 5822742528*a^5*b^11*c^7*d*e^2 - 122532397056*a^6*
b^8*c^9*d^2*e + 23215472640*a^6*b^9*c^8*d*e^2 + 269475643392*a^7*b^6*c^10*
d^2*e - 47362080768*a^7*b^7*c^9*d*e^2 - 284675801088*a^8*b^4*c^11*d^2*e +
24763170816*a^8*b^5*c^10*d*e^2 + 14495514624*a^9*b^2*c^12*d^2*e + 70061654
016*a^9*b^3*c^11*d*e^2)/(16384*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c +
 336*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^
5 + 28672*a^8*b^2*c^6)) - (x*(-(b^21*d^4 + 81*a^4*b^17*e^4 + b^6*d^4*(-...
```

Reduce [F]

$$\int \frac{x^2(d+ex^4)}{(a+bx^4+cx^8)^2}\,dx = \int \frac{x^2(e\,x^4+d)}{(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.103 $\int \frac{d+ex^4}{(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 548

$$\int \frac{d+ex^4}{(a+bx^4+cx^8)^2}\,dx = \frac{x(b^2d-2acd-abe+c(bd-2ae)x^4)}{4a(b^2-4ac)(a+bx^4+cx^8)} - \frac{c^{3/4}\left(3bd-6ae-\frac{3b^2d-28acd+8abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\sqrt[4]{2}a(b^2-4ac)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\left(3bd-6ae+\frac{3b^2d-28acd+8abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\sqrt[4]{2}a(b^2-4ac)\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\left(3bd-6ae-\frac{3b^2d-28acd+8abe}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\sqrt[4]{2}a(b^2-4ac)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\left(3bd-6ae+\frac{3b^2d-28acd+8abe}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\sqrt[4]{2}a(b^2-4ac)\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
1/4*x*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^4)/a/(-4*a*c+b^2)/(c*x^8+b*x^4
+a)-1/16*c^(3/4)*(3*b*d-6*a*e-(8*a*b*e-28*a*c*d+3*b^2*d)/(-4*a*c+b^2)^(1/2
))*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-4*a
*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/16*c^(3/4)*(3*b*d-6*a*e+(8*a*b*e-2
8*a*c*d+3*b^2*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+
b^2)^(1/2))^(1/4))*2^(3/4)/a/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/
16*c^(3/4)*(3*b*d-6*a*e-(8*a*b*e-28*a*c*d+3*b^2*d)/(-4*a*c+b^2)^(1/2))*arc
tanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-4*a*c+b^
2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/16*c^(3/4)*(3*b*d-6*a*e+(8*a*b*e-28*a*c
*d+3*b^2*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)
^(1/2))^(1/4))*2^(3/4)/a/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.31

$$\int \frac{d+ex^4}{(a+bx^4+cx^8)^2}\,dx =$$

$$-\frac{\frac{4x(b^2d+b(-ae+cdx^4)-2ac(d+ex^4))}{a+bx^4+cx^8}+\operatorname{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{3b^2d\log(x-\#1)-14acd\log(x-\#1)+abe\log(x-\#\cdots}{b\#1^3+2\cdots}\right.}{16a\left(-b^2+4ac\right)}$$

input
```
Integrate[(d + e*x^4)/(a + b*x^4 + c*x^8)^2,x]
```

output
```
-1/16*((4*x*(b^2*d + b*(-(a*e) + c*d*x^4) - 2*a*c*(d + e*x^4)))/(a + b*x^4
 + c*x^8) + RootSum[a + b*#1^4 + c*#1^8 & , (3*b^2*d*Log[x - #1] - 14*a*c*
d*Log[x - #1] + a*b*e*Log[x - #1] + 3*b*c*d*Log[x - #1]*#1^4 - 6*a*c*e*Log
[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(a*(-b^2 + 4*a*c))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1760, 25, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{(a+bx^4+cx^8)^2}dx$$

$$\Big\downarrow \text{1760}$$

$$\frac{x(cx^4(bd-2ae)-abe-2acd+b^2d)}{4a(b^2-4ac)(a+bx^4+cx^8)}-\frac{\int -\frac{3c(bd-2ae)x^4+3b^2d-14acd+abe}{cx^8+bx^4+a}dx}{4a(b^2-4ac)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3c(bd-2ae)x^4+3b^2d-14acd+abe}{cx^8+bx^4+a}dx}{4a(b^2-4ac)}+\frac{x(cx^4(bd-2ae)-abe-2acd+b^2d)}{4a(b^2-4ac)(a+bx^4+cx^8)}$$

$$\Big\downarrow \text{1752}$$

$$\frac{\frac{1}{2}c\left(\frac{8abe-28acd+3b^2d}{\sqrt{b^2-4ac}}+3(bd-2ae)\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(3(bd-2ae)-\frac{8abe-28acd+3b^2d}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{4a(b^2-4ac)}+$$
$$\frac{x(cx^4(bd-2ae)-abe-2acd+b^2d)}{4a(b^2-4ac)(a+bx^4+cx^8)}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{1}{2}c\left(3(bd-2ae)-\frac{8abe-28acd+3b^2d}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}c\left(\frac{8abe-28acd+3b^2d}{\sqrt{b^2-4ac}}\right.}{4a(b^2-4ac)}$$
$$\frac{x(cx^4(bd-2ae)-abe-2acd+b^2d)}{4a(b^2-4ac)(a+bx^4+cx^8)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{1}{2}c\left(3(bd-2ae)-\frac{8abe-28acd+3b^2d}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x^2}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(\frac{8abe-28acd}{\sqrt{b^2-4a}}\right.}{4a\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^4(bd-2ae)-abe-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\Big\downarrow$ 221

$$\frac{\frac{1}{2}c\left(3(bd-2ae)-\frac{8abe-28acd+3b^2d}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(8a\right.}{4a\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^4(bd-2ae)-abe-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

input
```
Int[(d + e*x^4)/(a + b*x^4 + c*x^8)^2,x]
```

output
```
(x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^4))/(4*a*(b^2 - 4*a*c)*(a
+ b*x^4 + c*x^8)) + ((c*(3*(b*d - 2*a*e) - (3*b^2*d - 28*a*c*d + 8*a*b*e)/
Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4
)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b
^2 - 4*a*c])^(3/4))))/2 + (c*(3*(b*d - 2*a*e) + (3*b^2*d - 28*a*c*d + 8*a*
b*e)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a
*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2
^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + S
qrt[b^2 - 4*a*c])^(3/4))))/2)/(4*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1760
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/
(a*n*(p + 1)*(b^2 - 4*a*c))   Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*
d*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n
+ c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n
] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.31

method	result	size
default	$\frac{\frac{c(2ae-bd)x^5}{4a\left(4ac-b^2\right)}+\frac{\left(abe+2acd-d\,b^2\right)x}{4\left(4ac-b^2\right)a}}{c\,x^8+b\,x^4+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(3c(2ae-bd)\_R^4-abe+14acd-3d\,b^2\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{16a(4ac-b^2)}$	168
risch	$\frac{\frac{c(2ae-bd)x^5}{4a\left(4ac-b^2\right)}+\frac{\left(abe+2acd-d\,b^2\right)x}{4\left(4ac-b^2\right)a}}{c\,x^8+b\,x^4+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(\frac{3c(2ae-bd)\_R^4}{4ac-b^2}-\frac{abe-14acd+3d\,b^2}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{16a}$	182

input
```
int((e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*c*(2*a*e-b*d)/a/(4*a*c-b^2)*x^5+1/4*(a*b*e+2*a*c*d-b^2*d)/(4*a*c-b^2)
/a*x)/(c*x^8+b*x^4+a)+1/16/a/(4*a*c-b^2)*sum((3*c*(2*a*e-b*d)*_R^4-a*b*e+1
4*a*c*d-3*d*b^2)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{\left(a+bx^4+cx^8\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{(a+bx^4+cx^8)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^4}{(a+bx^4+cx^8)^2}\,dx=\int \frac{ex^4+d}{(cx^8+bx^4+a)^2}\,dx$$

input
```
integrate((e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*((b*c*d - 2*a*c*e)*x^5 - (a*b*e - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^
2*c^2)*x^8 + (a*b^3 - 4*a^2*b*c)*x^4 + a^2*b^2 - 4*a^3*c) - 1/4*integrate(
-(3*(b*c*d - 2*a*c*e)*x^4 + a*b*e + (3*b^2 - 14*a*c)*d)/(c*x^8 + b*x^4 + a
), x)/(a*b^2 - 4*a^2*c)
```

Giac [F]

$$\int \frac{d+ex^4}{(a+bx^4+cx^8)^2}\,dx=\int \frac{ex^4+d}{(cx^8+bx^4+a)^2}\,dx$$

input
```
integrate((e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x^4 + d)/(c*x^8 + b*x^4 + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 31.21 (sec) , antiderivative size = 128217, normalized size of antiderivative = 233.97

$$\int \frac{d+ex^4}{(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^4)/(a + b*x^4 + c*x^8)^2,x)
```

output
```
atan(((((19548*a*b^6*c^8*d^5 - 891*b^8*c^7*d^5 - 537824*a^4*c^11*d^5 - 116
64*a^6*b*c^8*e^5 + 54432*a^6*c^9*d*e^4 + 567*b^9*c^6*d^4*e - 155358*a^2*b^
4*c^9*d^5 + 510384*a^3*b^2*c^10*d^5 + 7*a^4*b^5*c^6*e^5 - 2232*a^5*b^3*c^7
*e^5 + 197568*a^5*c^10*d^3*e^2 - 8802*a^2*b^6*c^7*d^3*e^2 + 378*a^2*b^7*c^
6*d^2*e^3 - 3060*a^3*b^4*c^8*d^3*e^2 + 174*a^3*b^5*c^7*d^2*e^3 + 157696*a^
4*b^2*c^9*d^3*e^2 + 22192*a^4*b^3*c^8*d^2*e^3 - 13959*a*b^7*c^7*d^4*e + 22
5008*a^4*b*c^10*d^4*e + 756*a*b^8*c^6*d^3*e^2 + 118071*a^2*b^5*c^8*d^4*e -
 367080*a^3*b^3*c^9*d^4*e + 84*a^3*b^6*c^6*d*e^4 - 6166*a^4*b^4*c^7*d*e^4
- 320544*a^5*b*c^9*d^2*e^3 + 60912*a^5*b^2*c^8*d*e^4)/(64*(a^4*b^8 + 256*a
^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) + ((x*(32212254
72*a^12*b*c^12*e^2 - 202937204736*a^11*b*c^13*d^2 + 589824*a^2*b^19*c^4*d^
2 - 22609920*a^3*b^17*c^5*d^2 + 382271488*a^4*b^15*c^6*d^2 - 3741057024*a^
5*b^13*c^7*d^2 + 23350738944*a^6*b^11*c^8*d^2 - 96380911616*a^7*b^9*c^9*d^
2 + 262982860800*a^8*b^7*c^10*d^2 - 457212690432*a^9*b^5*c^11*d^2 + 459293
065216*a^10*b^3*c^12*d^2 + 65536*a^4*b^17*c^4*e^2 - 983040*a^5*b^15*c^5*e^
2 + 2359296*a^6*b^13*c^6*e^2 + 38797312*a^7*b^11*c^7*e^2 - 314572800*a^8*b
^9*c^8*e^2 + 855638016*a^9*b^7*c^9*e^2 - 335544320*a^10*b^5*c^10*e^2 - 241
5919104*a^11*b^3*c^11*e^2 + 90194313216*a^12*c^13*d*e + 393216*a^3*b^18*c^
4*d*e - 10485760*a^4*b^16*c^5*d*e + 107741184*a^5*b^14*c^6*d*e - 449839104
*a^6*b^12*c^7*d*e - 507510784*a^7*b^10*c^8*d*e + 13941866496*a^8*b^8*c^...
```

Reduce [F]

$$\int \frac{d+ex^4}{(a+bx^4+cx^8)^2}\,dx = \int \frac{e\,x^4+d}{(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int((e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int((e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.104 $\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 659

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)^2}\,dx$$
$$= -\frac{5b^2d-18acd-abe}{4a^2\left(b^2-4ac\right)x}+\frac{b^2d-2acd-abe+c(bd-2ae)x^4}{4a\left(b^2-4ac\right)x\left(a+bx^4+cx^8\right)}$$
$$-\frac{\sqrt[4]{c}\left(5b^2d-18acd-abe-\frac{5b^3d-28abcd-ab^2e+20a^2ce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\,2^{3/4}a^2\left(b^2-4ac\right)\sqrt[4]{-b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt[4]{c}\left(5b^2d-18acd-abe+\frac{5b^3d-28abcd-ab^2e+20a^2ce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\,2^{3/4}a^2\left(b^2-4ac\right)\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt[4]{c}\left(5b^2d-18acd-abe-\frac{5b^3d-28abcd-ab^2e+20a^2ce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{8\,2^{3/4}a^2\left(b^2-4ac\right)\sqrt[4]{-b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt[4]{c}\left(5b^2d-18acd-abe+\frac{5b^3d-28abcd-ab^2e+20a^2ce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{8\,2^{3/4}a^2\left(b^2-4ac\right)\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
-1/4*(-a*b*e-18*a*c*d+5*b^2*d)/a^2/(-4*a*c+b^2)/x+1/4*(b^2*d-2*a*c*d-a*b*e
+c*(-2*a*e+b*d)*x^4)/a/(-4*a*c+b^2)/x/(c*x^8+b*x^4+a)-1/16*c^(1/4)*(5*b^2*
d-18*a*c*d-a*b*e-(20*a^2*c*e-a*b^2*e-28*a*b*c*d+5*b^3*d)/(-4*a*c+b^2)^(1/2
))*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a^2/(-4
*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/16*c^(1/4)*(5*b^2*d-18*a*c*d-a*b
*e+(20*a^2*c*e-a*b^2*e-28*a*b*c*d+5*b^3*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1
/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a^2/(-4*a*c+b^2)/(-b+
(-4*a*c+b^2)^(1/2))^(1/4)+1/16*c^(1/4)*(5*b^2*d-18*a*c*d-a*b*e-(20*a^2*c*e
-a*b^2*e-28*a*b*c*d+5*b^3*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x
/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a^2/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^
(1/2))^(1/4)+1/16*c^(1/4)*(5*b^2*d-18*a*c*d-a*b*e+(20*a^2*c*e-a*b^2*e-28*a
*b*c*d+5*b^3*d)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+
b^2)^(1/2))^(1/4))*2^(1/4)/a^2/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.37

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)^2}\,dx =$$

$$-\frac{\frac{16d}{x}+\frac{4x^3\left(b^3d+2ac\left(ae-cdx^4\right)+b^2\left(-ae+cdx^4\right)-abc\left(3d+ex^4\right)\right)}{\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}+\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{5b^3d\log(x-\#1)-23abcd\log(x-\#\ldots}{\ldots}\right]}{\ldots}}{16a^2}$$

input
```
Integrate[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)^2),x]
```

output
```
-1/16*((16*d)/x + (4*x^3*(b^3*d + 2*a*c*(a*e - c*d*x^4) + b^2*(-(a*e) + c*
d*x^4) - a*b*c*(3*d + e*x^4)))/((b^2 - 4*a*c)*(a + b*x^4 + c*x^8)) + RootS
um[a + b*#1^4 + c*#1^8 & , (5*b^3*d*Log[x - #1] - 23*a*b*c*d*Log[x - #1] -
 a*b^2*e*Log[x - #1] + 10*a^2*c*e*Log[x - #1] + 5*b^2*c*d*Log[x - #1]*#1^4
 - 18*a*c^2*d*Log[x - #1]*#1^4 - a*b*c*e*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^
5) & ]/(b^2 - 4*a*c))/a^2
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 510, normalized size of antiderivative = 0.77, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1824, 25, 1828, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)^2}dx$$

$$\Big\downarrow \ 1824$$

$$\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}-\frac{\int -\frac{5c(bd-2ae)x^4+5b^2d-18acd-abe}{x^2\left(cx^8+bx^4+a\right)}dx}{4a\left(b^2-4ac\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{5c(bd-2ae)x^4+5b^2d-18acd-abe}{x^2\left(cx^8+bx^4+a\right)}dx}{4a\left(b^2-4ac\right)}+\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$$\Big\downarrow \ 1828$$

$$\frac{-\frac{\int \frac{x^2\left(c\left(5db^2-aeb-18acd\right)x^4+5b^3d-23abcd-ab^2e+10a^2ce\right)}{cx^8+bx^4+a}dx}{a}-\frac{-abe-18acd+5b^2d}{ax}}{4a\left(b^2-4ac\right)}+\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$$\Big\downarrow \ 1834$$

$$\frac{-\frac{\frac{1}{2}c\left(\frac{20a^2ce-ab^2e-28abcd+5b^3d}{\sqrt{b^2-4ac}}-abe-18acd+5b^2d\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}c\left(-\frac{20a^2ce-ab^2e-28abcd+5b^3d}{\sqrt{b^2-4ac}}-abe-18acd+5b^2d\right)\int \frac{2x^2}{2cx^4+b+\sqrt{\cdots}}\cdots}{a}\cdots}{4a\left(b^2-4ac\right)}+\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{c\left(\frac{20a^2ce-ab^2e-28abcd+5b^3d}{\sqrt{b^2-4ac}}-abe-18acd+5b^2d\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+c\left(-\frac{20a^2ce-ab^2e-28abcd+5b^3d}{\sqrt{b^2-4ac}}-abe-18acd+5b^2d\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-\cdots}}\cdots}{a}\cdots}{4a\left(b^2-4ac\right)}+\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\Big\downarrow$ 827

$$-\frac{c\left(-\frac{20a^2ce-ab^2e-28abcd+5b^3d}{\sqrt{b^2-4ac}}-abe-18acd+5b^2d\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{20a^2ce-ab^2e-28abcd+}{\sqrt{b^2-4ac}}\right.}{a}$$
$$4a\left(b^2-4ac\right)$$
$$\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\Big\downarrow$ 218

$$-\frac{c\left(-\frac{20a^2ce-ab^2e-28abcd+5b^3d}{\sqrt{b^2-4ac}}-abe-18acd+5b^2d\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{20a^2ce-ab^2e-2}{\sqrt{b^2-4}}\right.}{a}$$
$$4a\left(b^2-4ac\right)$$
$$\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

$\Big\downarrow$ 221

$$-\frac{c\left(-\frac{20a^2ce-ab^2e-28abcd+5b^3d}{\sqrt{b^2-4ac}}-abe-18acd+5b^2d\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(\frac{20a^2}{}\right.}{a}$$
$$4a\left(b^2-\right.$$
$$\frac{cx^4(bd-2ae)-abe-2acd+b^2d}{4ax\left(b^2-4ac\right)\left(a+bx^4+cx^8\right)}$$

input
```
Int[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^4)/(4*a*(b^2 - 4*a*c)*x*(a +
b*x^4 + c*x^8)) + (-((5*b^2*d - 18*a*c*d - a*b*e)/(a*x)) - (c*(5*b^2*d - 1
8*a*c*d - a*b*e - (5*b^3*d - 28*a*b*c*d - a*b^2*e + 20*a^2*c*e)/Sqrt[b^2 -
 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^
(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*
c])^(1/4))) + c*(5*b^2*d - 18*a*c*d - a*b*e + (5*b^3*d - 28*a*b*c*d - a*b^
2*e + 20*a^2*c*e)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqr
t[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
- ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c
^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/a)/(4*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1824
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m
+ 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.32

method	result
default	$\frac{\frac{-\frac{c\left(abe+2acd-d\,b^{2}\right)x^{7}}{4\left(4ac-b^{2}\right)}+\frac{\left(2a^{2}ce-a\,b^{2}e-3abcd+b^{3}d\right)x^{3}}{16ac-4b^{2}}}{c\,x^{8}+b\,x^{4}+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)}\frac{\left(c\left(-abe-18acd+5d\,b^{2}\right)\_R^{6}+\left(10a^{2}ce-a\,b^{2}e-23\right.\right.}{2\_R^{7}c+\_R^{3}b}}{64ac-16b^{2}}}{a^{2}}$
risch	Expression too large to display

input
```
int((e*x^4+d)/x^2/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*((-1/4*c*(a*b*e+2*a*c*d-b^2*d)/(4*a*c-b^2)*x^7+1/4*(2*a^2*c*e-a*b^2*
e-3*a*b*c*d+b^3*d)/(4*a*c-b^2)*x^3)/(c*x^8+b*x^4+a)+1/16/(4*a*c-b^2)*sum((
c*(-a*b*e-18*a*c*d+5*b^2*d)*_R^6+(10*a^2*c*e-a*b^2*e-23*a*b*c*d+5*b^3*d)*_
R^2)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a)))-d/a^2/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x**4+d)/x**2/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)^2}\,dx=\int \frac{ex^4+d}{\left(cx^8+bx^4+a\right)^2x^2}\,dx$$

input
```
integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*((a*b*c*e - (5*b^2*c - 18*a*c^2)*d)*x^8 - ((5*b^3 - 19*a*b*c)*d - (a*b
^2 - 2*a^2*c)*e)*x^4 - 4*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2*c - 4*a^3*c^2)*x^9
 + (a^2*b^3 - 4*a^3*b*c)*x^5 + (a^3*b^2 - 4*a^4*c)*x) + 1/4*integrate(((a*
b*c*e - (5*b^2*c - 18*a*c^2)*d)*x^6 - ((5*b^3 - 23*a*b*c)*d - (a*b^2 - 10*
a^2*c)*e)*x^2)/(c*x^8 + b*x^4 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Giac [F]

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)^2}\,dx=\int \frac{ex^4+d}{\left(cx^8+bx^4+a\right)^2x^2}\,dx$$

input
```
integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)^2*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 32.99 (sec) , antiderivative size = 142799, normalized size of antiderivative = 216.69

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)^2}\,dx=\text{Too large to display}$$

input
```
int((d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)^2),x)
```

output
```
atan((((-(625*b^25*d^4 + a^4*b^21*e^4 + 625*b^10*d^4*(-(4*a*c - b^2)^15)^(
1/2) + 3105423360*a^12*b*c^12*d^4 - 69*a^5*b^19*c*e^4 + 73728000*a^14*b*c^
10*e^4 - 20*a^3*b^22*d*e^3 - 1911029760*a^13*c^12*d^3*e + 589824000*a^14*c
^11*d*e^3 + 638475*a^2*b^21*c^2*d^4 - 8264990*a^3*b^19*c^3*d^4 + 71483001*
a^4*b^17*c^4*d^4 - 434478624*a^5*b^15*c^5*d^4 + 1898983360*a^6*b^13*c^6*d^
4 - 5996689920*a^7*b^11*c^7*d^4 + 13524825600*a^8*b^9*c^8*d^4 - 2112231014
4*a^9*b^7*c^9*d^4 + 21483012096*a^10*b^5*c^10*d^4 - 12575047680*a^11*b^3*c
^11*d^4 + a^4*b^6*e^4*(-(4*a*c - b^2)^15)^(1/2) - 26244*a^5*c^5*d^4*(-(4*a
*c - b^2)^15)^(1/2) + 2085*a^6*b^17*c^2*e^4 - 36320*a^7*b^15*c^3*e^4 + 404
160*a^8*b^13*c^4*e^4 - 3001344*a^9*b^11*c^5*e^4 + 15064576*a^10*b^9*c^6*e^
4 - 50503680*a^11*b^7*c^7*e^4 + 108380160*a^12*b^5*c^8*e^4 - 134676480*a^1
3*b^3*c^9*e^4 - 2500*a^7*c^3*e^4*(-(4*a*c - b^2)^15)^(1/2) + 150*a^2*b^23*
d^2*e^2 - 29625*a*b^23*c*d^4 - 500*a*b^24*d^3*e + 68475*a^2*b^6*c^2*d^4*(-
(4*a*c - b^2)^15)^(1/2) - 181990*a^3*b^4*c^3*d^4*(-(4*a*c - b^2)^15)^(1/2)
 + 171801*a^4*b^2*c^4*d^4*(-(4*a*c - b^2)^15)^(1/2) + 525*a^6*b^2*c^2*e^4*
(-(4*a*c - b^2)^15)^(1/2) + 150*a^2*b^8*d^2*e^2*(-(4*a*c - b^2)^15)^(1/2)
+ 224244*a^4*b^19*c^2*d^2*e^2 - 3363546*a^5*b^17*c^3*d^2*e^2 + 32811840*a^
6*b^15*c^4*d^2*e^2 - 219012480*a^7*b^13*c^5*d^2*e^2 + 1022161920*a^8*b^11*
c^6*d^2*e^2 - 3338689536*a^9*b^9*c^7*d^2*e^2 + 7481769984*a^10*b^7*c^8*d^2
*e^2 - 10951557120*a^11*b^5*c^9*d^2*e^2 + 9413591040*a^12*b^3*c^10*d^2*...
```

Reduce [F]

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)^2}\,dx=\int \frac{e\,x^4+d}{x^2\left(c\,x^8+b\,x^4+a\right)^2}dx$$

input
```
int((e*x^4+d)/x^2/(c*x^8+b*x^4+a)^2,x)
```

output
```
int((e*x^4+d)/x^2/(c*x^8+b*x^4+a)^2,x)
```

3.105 $\int \frac{x^{11}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 259

$$\begin{aligned}\int \frac{x^{11}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx &= -\frac{a(bd-2ae)+(b^2d-2acd-abe)\,x^4}{4(b^2-4ac)(cd^2-bde+ae^2)(a+bx^4+cx^8)} \\ &\quad - \frac{(b^3d^2e+2a^2e^2(2cd+be)-2ad(2c^2d^2-bcde+2b^2e^2))\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4(b^2-4ac)^{3/2}(cd^2-bde+ae^2)^2} \\ &\quad + \frac{d^2e\log(d+ex^4)}{4(cd^2-bde+ae^2)^2} - \frac{d^2e\log(a+bx^4+cx^8)}{8(cd^2-bde+ae^2)^2}\end{aligned}$$

output
```
-1/4*(a*(-2*a*e+b*d)+(-a*b*e-2*a*c*d+b^2*d)*x^4)/(-4*a*c+b^2)/(a*e^2-b*d*e
+c*d^2)/(c*x^8+b*x^4+a)-1/4*(b^3*d^2*e+2*a^2*e^2*(b*e+2*c*d)-2*a*d*(2*b^2*
e^2-b*c*d*e+2*c^2*d^2))*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(3/2)/(a*e^2-b*d*e+c*d^2)^2+1/4*d^2*e*ln(e*x^4+d)/(a*e^2-b*d*e+c*d^2)^2
-1/8*d^2*e*ln(c*x^8+b*x^4+a)/(a*e^2-b*d*e+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.01

$$\int \frac{x^{11}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \frac{1}{8}\left(\frac{2(-2a^2e+b^2dx^4-2acdx^4+ab(d-ex^4))}{(b^2-4ac)(-cd^2+e(bd-ae))(a+bx^4+cx^8)} - \frac{2(b^3d^2e+2a^2e^2(2cd+be)+2ad(-2c^2d^2+bcde-2b^2e^2))\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}(cd^2+e(-bd+ae))^2} + \frac{2d^2e\log(d+ex^4)}{(cd^2+e(-bd+ae))^2} - \frac{d^2e\log(a+bx^4+cx^8)}{(cd^2+e(-bd+ae))^2}\right)$$

input
```
Integrate[x^11/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((2*(-2*a^2*e + b^2*d*x^4 - 2*a*c*d*x^4 + a*b*(d - e*x^4)))/((b^2 - 4*a*c)
*(-(c*d^2) + e*(b*d - a*e))*(a + b*x^4 + c*x^8)) - (2*(b^3*d^2*e + 2*a^2*e
^2*(2*c*d + b*e) + 2*a*d*(-2*c^2*d^2 + b*c*d*e - 2*b^2*e^2))*ArcTan[(b + 2
*c*x^4)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*
e))^2) + (2*d^2*e*Log[d + e*x^4])/(c*d^2 + e*(-(b*d) + a*e))^2 - (d^2*e*Lo
g[a + b*x^4 + c*x^8])/(c*d^2 + e*(-(b*d) + a*e))^2)/8
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1802, 1264, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$$

$$\Big\downarrow \; 1802$$

$$\frac{1}{4}\int \frac{x^8}{(ex^4+d)\left(cx^8+bx^4+a\right)^2}dx^4$$

$\Big\downarrow$ 1264

$$\frac{1}{4}\left(-\frac{\int \frac{e(db^2-aeb-2acd)x^4+ad(2cd-be)}{(cd^2-bed+ae^2)(ex^4+d)(cx^8+bx^4+a)}dx^4}{b^2-4ac}-\frac{x^4\left(-abe-2acd+b^2d\right)+a(bd-2ae)}{(b^2-4ac)\left(a+bx^4+cx^8\right)\left(ae^2-bde+cd^2\right)}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{4}\left(-\frac{\int \frac{e(db^2-aeb-2acd)x^4+ad(2cd-be)}{(ex^4+d)(cx^8+bx^4+a)}dx^4}{(b^2-4ac)\left(ae^2-bde+cd^2\right)}-\frac{x^4\left(-abe-2acd+b^2d\right)+a(bd-2ae)}{(b^2-4ac)\left(a+bx^4+cx^8\right)\left(ae^2-bde+cd^2\right)}\right)$$

$\Big\downarrow$ 1200

$$\frac{1}{4}\left(-\frac{\int \left(\frac{c(b^2-4ac)d^2ex^4+a(2c^2d^3-ce(3bd+2ae)d+be^2(2bd-ae))}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}-\frac{(b^2-4ac)d^2e^2}{(cd^2-bed+ae^2)(ex^4+d)}\right)dx^4}{(b^2-4ac)\left(ae^2-bde+cd^2\right)}-\frac{x^4\left(-abe-2acd+b^2d\right)}{(b^2-4ac)\left(a+bx^4+cx^8\right)}\right.$$

$\Big\downarrow$ 2009

$$\frac{1}{4}\left(-\frac{\frac{\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)\left(2a^2e^2(be+2cd)-2ad(2b^2e^2-bcde+2c^2d^2)+b^3d^2e\right)}{\sqrt{b^2-4ac}(ae^2-bde+cd^2)}-\frac{d^2e(b^2-4ac)\log(d+ex^4)}{ae^2-bde+cd^2}+\frac{d^2e(b^2-4ac)\log(a+bx^4+cx}{2(ae^2-bde+cd^2)}}{(b^2-4ac)\left(ae^2-bde+cd^2\right)}\right.$$

input
```
Int[x^11/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(-((a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x^4)/((b^2 - 4*a*c)*(c*d^2
 - b*d*e + a*e^2)*(a + b*x^4 + c*x^8))) - (((b^3*d^2*e + 2*a^2*e^2*(2*c*d
+ b*e) - 2*a*d*(2*c^2*d^2 - b*c*d*e + 2*b^2*e^2))*ArcTanh[(b + 2*c*x^4)/Sq
rt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) - ((b^2 - 4*
a*c)*d^2*e*Log[d + e*x^4])/(c*d^2 - b*d*e + a*e^2) + ((b^2 - 4*a*c)*d^2*e*
Log[a + b*x^4 + c*x^8])/(2*(c*d^2 - b*d*e + a*e^2)))/((b^2 - 4*a*c)*(c*d^2
 - b*d*e + a*e^2)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\frac{\left(a^{2} b\,e^{3}+2 a^{2} c d\,e^{2}-2 a\,b^{2} d\,e^{2}-a b c\,d^{2} e+2 a\,c^{2} d^{3}+b^{3} d^{2} e-b^{2} c\,d^{3}\right) x^{4}}{4 a c-b^{2}}+\frac{a \left(2 a^{2} e^{3}-3 a b d\,e^{2}+2 a c\,d^{2} e+b^{2} d^{2} e-b c\,d^{3}\right)}{4 a c-b^{2}}}{2 c\,x^{8}+2 b\,x^{4}+2 a}+\frac{\frac{\left(4 a\,c^{2} d^{2} e-b^{2} c\,d^{2} e\right) \ln \left(c\,x \cdots\right.}{2 c} \cdots}{2 \left(a\,e^{2}-b d e+c\,d^{2}\right)^{2}}$
risch	Expression too large to display

input
```
int(x^11/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*e^2-b*d*e+c*d^2)^2*(1/2*((a^2*b*e^3+2*a^2*c*d*e^2-2*a*b^2*d*e^2-a*
b*c*d^2*e+2*a*c^2*d^3+b^3*d^2*e-b^2*c*d^3)/(4*a*c-b^2)*x^4+a*(2*a^2*e^3-3*
a*b*d*e^2+2*a*c*d^2*e+b^2*d^2*e-b*c*d^3)/(4*a*c-b^2))/(c*x^8+b*x^4+a)+1/2/
(4*a*c-b^2)*(1/2*(4*a*c^2*d^2*e-b^2*c*d^2*e)/c*ln(c*x^8+b*x^4+a)+2*(a^2*b*
e^3+2*a^2*c*d*e^2-2*a*b^2*d*e^2+3*a*b*c*d^2*e-2*a*c^2*d^3-1/2*(4*a*c^2*d^2
*e-b^2*c*d^2*e)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2
))))+1/4*d^2*e*ln(e*x^4+d)/(a*e^2-b*d*e+c*d^2)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{11}}{\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^11/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**11/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(247) = 494$.

Time = 4.62 (sec) , antiderivative size = 643, normalized size of antiderivative = 2.48

$$\int \frac{x^{11}}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx$$
$$= \frac{d^2e^2\log\left(|ex^4+d|\right)}{4\left(c^2d^4e-2\,bcd^3e^2+b^2d^2e^3+2\,acd^2e^3-2\,abde^4+a^2e^5\right)}$$
$$-\frac{d^2e\log\left(cx^8+bx^4+a\right)}{8\left(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4\right)}$$
$$-\frac{\left(4\,ac^2d^3-b^3d^2e-2\,abcd^2e+4\,ab^2de^2-4\,a^2cde^2-2\,a^2be^3\right)\arctan\left(\frac{2\,cx^4+}{\sqrt{-b^2+4}}\right.}{4\left(b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+\right.}$$
$$+\frac{b^2cd^2ex^8-4\,ac^2d^2ex^8-2\,b^2cd^3x^4+4\,ac^2d^3x^4+3\,b^3d^2ex^4-6\,abcd^2ex^4-4\,ab^2de^2x^4+4\,a^2cde^2x^4+2}{8\,(cx^8+bx^4+a)(b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-}$$

input
```
integrate(x^11/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*d^2*e^2*log(abs(e*x^4 + d))/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 +
 2*a*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5) - 1/8*d^2*e*log(c*x^8 + b*x^4 + a)
/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*
e^4) - 1/4*(4*a*c^2*d^3 - b^3*d^2*e - 2*a*b*c*d^2*e + 4*a*b^2*d*e^2 - 4*a^
2*c*d*e^2 - 2*a^2*b*e^3)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^
2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*
b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*
b^2*e^4 - 4*a^3*c*e^4)*sqrt(-b^2 + 4*a*c)) + 1/8*(b^2*c*d^2*e*x^8 - 4*a*c^
2*d^2*e*x^8 - 2*b^2*c*d^3*x^4 + 4*a*c^2*d^3*x^4 + 3*b^3*d^2*e*x^4 - 6*a*b*
c*d^2*e*x^4 - 4*a*b^2*d*e^2*x^4 + 4*a^2*c*d*e^2*x^4 + 2*a^2*b*e^3*x^4 - 2*
a*b*c*d^3 + 3*a*b^2*d^2*e - 6*a^2*b*d*e^2 + 4*a^3*e^3)/((c*x^8 + b*x^4 + a
)*(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e
^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e
^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Hanged}$$

input
```
int(x^11/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x^{11}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \int \frac{x^{11}}{(e\,x^4+d)\,(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^11/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^11/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.106 $\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 254

$$\begin{aligned}\int \frac{x^7}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx &= \frac{a(2cd-be)+c(bd-2ae)x^4}{4\,(b^2-4ac)\,(cd^2-bde+ae^2)\,(a+bx^4+cx^8)} \\ &\quad + \frac{(4b^2cd^2e-b^3de^2-4ace(cd^2-ae^2)-2bcd(cd^2+ae^2))\,\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4\,(b^2-4ac)^{3/2}\,(cd^2-bde+ae^2)^2} \\ &\quad - \frac{de^2\log(d+ex^4)}{4\,(cd^2-bde+ae^2)^2}+\frac{de^2\log(a+bx^4+cx^8)}{8\,(cd^2-bde+ae^2)^2}\end{aligned}$$

output
```
1/4*(a*(-b*e+2*c*d)+c*(-2*a*e+b*d)*x^4)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(
c*x^8+b*x^4+a)+1/4*(4*b^2*c*d^2*e-b^3*d*e^2-4*a*c*e*(-a*e^2+c*d^2)-2*b*c*d
*(a*e^2+c*d^2))*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
/(a*e^2-b*d*e+c*d^2)^2-1/4*d*e^2*ln(e*x^4+d)/(a*e^2-b*d*e+c*d^2)^2+1/8*d*e
^2*ln(c*x^8+b*x^4+a)/(a*e^2-b*d*e+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \frac{1}{8}\left(\frac{2(-bcdx^4+a(-2cd+be+2cex^4))}{(b^2-4ac)(-cd^2+e(bd-ae))(a+bx^4+cx^8)} - \frac{2(-4b^2cd^2e+b^3de^2+4ace(cd^2-ae^2)+2bcd(cd^2+ae^2))\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}(cd^2+e(-bd+ae))^2} - \frac{2de^2\log(d+ex^4)}{(cd^2+e(-bd+ae))^2} + \frac{de^2\log(a+bx^4+cx^8)}{(cd^2+e(-bd+ae))^2}\right)$$

input
```
Integrate[x^7/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((2*(-(b*c*d*x^4) + a*(-2*c*d + b*e + 2*c*e*x^4)))/((b^2 - 4*a*c)*(-(c*d^2
) + e*(b*d - a*e))*(a + b*x^4 + c*x^8)) - (2*(-4*b^2*c*d^2*e + b^3*d*e^2 +
 4*a*c*e*(c*d^2 - a*e^2) + 2*b*c*d*(c*d^2 + a*e^2))*ArcTan[(b + 2*c*x^4)/S
qrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2) -
(2*d*e^2*Log[d + e*x^4])/(c*d^2 + e*(-(b*d) + a*e))^2 + (d*e^2*Log[a + b*x
^4 + c*x^8])/(c*d^2 + e*(-(b*d) + a*e))^2)/8
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1802, 1235, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$$

$$\Big\downarrow \; 1802$$

$$\frac{1}{4}\int \frac{x^4}{(ex^4+d)(cx^8+bx^4+a)^2}dx^4$$

$$\Big\downarrow \; 1235$$

$$\frac{1}{4}\left(\frac{cx^4(bd-2ae)+a(2cd-be)}{(b^2-4ac)(a+bx^4+cx^8)(ae^2-bde+cd^2)}-\frac{\int -\frac{ce(bd-2ae)x^4+d(-eb^2+cdb+2ace)}{(ex^4+d)(cx^8+bx^4+a)}dx^4}{(b^2-4ac)(ae^2-bde+cd^2)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}\left(\frac{\int \frac{ce(bd-2ae)x^4+d(-eb^2+cdb+2ace)}{(ex^4+d)(cx^8+bx^4+a)}dx^4}{(b^2-4ac)(ae^2-bde+cd^2)}+\frac{cx^4(bd-2ae)+a(2cd-be)}{(b^2-4ac)(a+bx^4+cx^8)(ae^2-bde+cd^2)}\right)$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{4}\left(\frac{\int \left(\frac{c(b^2-4ac)de^2x^4+b^3de^2-2b^2cd^2e+bcd(cd^2-ae^2)+2ace(cd^2-ae^2)}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}-\frac{(b^2-4ac)de^3}{(cd^2-bed+ae^2)(ex^4+d)}\right)dx^4}{(b^2-4ac)(ae^2-bde+cd^2)}+\frac{cx^4(bd-2ae)}{(b^2-4ac)(a+bx^4+}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(\frac{\frac{\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)(-2bcd(ae^2+cd^2)-4ace(cd^2-ae^2)+b^3(-d)e^2+4b^2cd^2e)}{\sqrt{b^2-4ac}(ae^2-bde+cd^2)}-\frac{de^2(b^2-4ac)\log(d+ex^4)}{ae^2-bde+cd^2}+\frac{de^2(b^2-4ac)\log(a+bx^4}{2(ae^2-bde+cd^2)}}{(b^2-4ac)(ae^2-bde+cd^2)}\right.$$

input

```
Int[x^7/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output

```
((a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x^4)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a
*e^2)*(a + b*x^4 + c*x^8)) + (((4*b^2*c*d^2*e - b^3*d*e^2 - 4*a*c*e*(c*d^2
 - a*e^2) - 2*b*c*d*(c*d^2 + a*e^2))*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*
c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) - ((b^2 - 4*a*c)*d*e^2*Lo
g[d + e*x^4])/(c*d^2 - b*d*e + a*e^2) + ((b^2 - 4*a*c)*d*e^2*Log[a + b*x^4
 + c*x^8])/(2*(c*d^2 - b*d*e + a*e^2)))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*
e^2)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 5.31 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.40

method	result
default	$\frac{\frac{c\left(2a^2e^3-3abd\,e^2+2ac\,d^2e+b^2d^2e-bc\,d^3\right)x^4}{4ac-b^2}+\frac{a\left(ab\,e^3-2acd\,e^2-b^2d\,e^2+3bc\,d^2e-2c^2d^3\right)}{4ac-b^2}}{2c\,x^8+2b\,x^4+2a}+\frac{\frac{\left(4a\,c^2d\,e^2-b^2cd\,e^2\right)\ln\left(c\,x^8+b\,x^4+a\right)}{2c}+\frac{2\left(2a^2c\,e^3\cdots\right.}{}}{2(a\,e^2-bde+c\,d^2)^2}$
risch	Expression too large to display

input
```
int(x^7/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*e^2-b*d*e+c*d^2)^2*(1/2*(c*(2*a^2*e^3-3*a*b*d*e^2+2*a*c*d^2*e+b^2*d
^2*e-b*c*d^3)/(4*a*c-b^2)*x^4+a*(a*b*e^3-2*a*c*d*e^2-b^2*d*e^2+3*b*c*d^2*e
-2*c^2*d^3)/(4*a*c-b^2))/(c*x^8+b*x^4+a)+1/2/(4*a*c-b^2)*(1/2*(4*a*c^2*d*e
^2-b^2*c*d*e^2)/c*ln(c*x^8+b*x^4+a)+2*(2*a^2*c*e^3+a*b*c*d*e^2-2*a*c^2*d^2
*e-b^3*d*e^2+2*b^2*c*d^2*e-b*c^2*d^3-1/2*(4*a*c^2*d*e^2-b^2*c*d*e^2)*b/c)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))))-1/4*d*e^2*ln(e*x
^4+d)/(a*e^2-b*d*e+c*d^2)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^7/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**7/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(242) = 484$.

Time = 4.43 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.96

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$$
$$= -\frac{de^3\log(|ex^4+d|)}{4(c^2d^4e-2bcd^3e^2+b^2d^2e^3+2acd^2e^3-2abde^4+a^2e^5)} + \frac{de^2\log(cx^8+bx^4+a)}{8(c^2d^4-2bcd^3e+b^2d^2e^2+2acd^2e^2-2abde^3+a^2e^4)} + \frac{(2bc^2d^3-4b^2cd^2e+4ac^2d^2e+b^3de^2+2abcde^2-4a^2ce^3)\arctan\left(\frac{2cx^4+}{\sqrt{-b^2+4}}\right.}{4(b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+} + \frac{2ac^2d^3-3abcd^2e+ab^2de^2+2a^2cde^2-a^2be^3+(bc^2d^3-b^2cd^2e-2ac^2d^2e+3abcde^2-2a^2ce^3)x^4}{4(cx^8+bx^4+a)(cd^2-bde+ae^2)^2(b^2-4ac)}$$

input
```
integrate(x^7/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*d*e^3*log(abs(e*x^4 + d))/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 +
2*a*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5) + 1/8*d*e^2*log(c*x^8 + b*x^4 + a)/
(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e
^4) + 1/4*(2*b*c^2*d^3 - 4*b^2*c*d^2*e + 4*a*c^2*d^2*e + b^3*d*e^2 + 2*a*b
*c*d*e^2 - 4*a^2*c*e^3)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2
*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b
^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b
^2*e^4 - 4*a^3*c*e^4)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*a*c^2*d^3 - 3*a*b*c*d^2
*e + a*b^2*d*e^2 + 2*a^2*c*d*e^2 - a^2*b*e^3 + (b*c^2*d^3 - b^2*c*d^2*e -
2*a*c^2*d^2*e + 3*a*b*c*d*e^2 - 2*a^2*c*e^3)*x^4)/((c*x^8 + b*x^4 + a)*(c*
d^2 - b*d*e + a*e^2)^2*(b^2 - 4*a*c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Hanged}$$

input
```
int(x^7/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x^7}{(d+ex^4)\left(a+bx^4+cx^8\right)^2}\,dx=\int \frac{x^7}{(e\,x^4+d)\,(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^7/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^7/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.107 $\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 243

$$\begin{aligned}\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx &= \frac{-bcd+b^2e-2ace-c(2cd-be)x^4}{4(b^2-4ac)(cd^2-bde+ae^2)(a+bx^4+cx^8)} \\ &+ \frac{(2cd-be)(2c^2d^2-b^2e^2-2ce(bd-3ae))\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4(b^2-4ac)^{3/2}(cd^2-bde+ae^2)^2} \\ &+ \frac{e^3\log(d+ex^4)}{4(cd^2-bde+ae^2)^2} - \frac{e^3\log(a+bx^4+cx^8)}{8(cd^2-bde+ae^2)^2}\end{aligned}$$

output
```
1/4*(-b*c*d+b^2*e-2*a*c*e-c*(-b*e+2*c*d)*x^4)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*
d^2)/(c*x^8+b*x^4+a)+1/4*(-b*e+2*c*d)*(2*c^2*d^2-b^2*e^2-2*c*e*(-3*a*e+b*d
))*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e
+c*d^2)^2+1/4*e^3*ln(e*x^4+d)/(a*e^2-b*d*e+c*d^2)^2-1/8*e^3*ln(c*x^8+b*x^4
+a)/(a*e^2-b*d*e+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \frac{1}{8}\left(\frac{-2b^2e+4c(ae+cdx^4)+2bc(d-ex^4)}{(b^2-4ac)(-cd^2+e(bd-ae))(a+bx^4+cx^8)} + \frac{2(-2cd+be)(-2c^2d^2+b^2e^2+2ce(bd-3ae))\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}(cd^2+e(-bd+ae))^2} + \frac{2e^3\log(d+ex^4)}{(cd^2+e(-bd+ae))^2} - \frac{e^3\log(a+bx^4+cx^8)}{(cd^2+e(-bd+ae))^2}\right)$$

input
```
Integrate[x^3/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((-2*b^2*e + 4*c*(a*e + c*d*x^4) + 2*b*c*(d - e*x^4))/((b^2 - 4*a*c)*(-(c*
d^2) + e*(b*d - a*e))*(a + b*x^4 + c*x^8)) + (2*(-2*c*d + b*e)*(-2*c^2*d^2
 + b^2*e^2 + 2*c*e*(b*d - 3*a*e))*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]]
)/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2) + (2*e^3*Log[d + e*x
^4])/(c*d^2 + e*(-(b*d) + a*e))^2 - (e^3*Log[a + b*x^4 + c*x^8])/(c*d^2 +
e*(-(b*d) + a*e))^2)/8
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1798, 1165, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$$

$$\Big\downarrow \text{1798}$$

$$\frac{1}{4}\int \frac{1}{(ex^4+d)\left(cx^8+bx^4+a\right)^2}dx^4$$

$\big\downarrow$ 1165

$$\frac{1}{4}\left(-\frac{\int \frac{ce(2cd-be)x^4+2c^2d^2-b^2e^2-ce(bd-4ae)}{(ex^4+d)(cx^8+bx^4+a)}dx^4}{(b^2-4ac)\,(ae^2-bde+cd^2)}-\frac{2ace-b^2e+cx^4(2cd-be)+bcd}{(b^2-4ac)\,(a+bx^4+cx^8)\,(ae^2-bde+cd^2)}\right)$$

$\big\downarrow$ 1200

$$\frac{1}{4}\left(-\frac{\int \left(\frac{c(b^2-4ac)e^3x^4+2c^3d^3+b^3e^3-5abce^3-3c^2de(bd-2ae)}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}-\frac{(b^2-4ac)e^4}{(cd^2-bed+ae^2)(ex^4+d)}\right)dx^4}{(b^2-4ac)\,(ae^2-bde+cd^2)}-\frac{2ace-b^2e+cx^4(2cd-}{(b^2-4ac)\,(a+bx^4+cx^8)\,(ae}\right.$$

$\big\downarrow$ 2009

$$\frac{1}{4}\left(-\frac{-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)(6ace^2-b^2e^2-2bcde+2c^2d^2)}{\sqrt{b^2-4ac}(ae^2-bde+cd^2)}-\frac{e^3(b^2-4ac)\log(d+ex^4)}{ae^2-bde+cd^2}+\frac{e^3(b^2-4ac)\log(a+bx^4+cx^8)}{2(ae^2-bde+cd^2)}}{(b^2-4ac)\,(ae^2-bde+cd^2)}-\frac{}{(b^2-}\right.$$

input
```
Int[x^3/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(-((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^4)/((b^2 - 4*a*c)*(c*d^2 -
 b*d*e + a*e^2)*(a + b*x^4 + c*x^8))) - (-(((2*c*d - b*e)*(2*c^2*d^2 - 2*b
*c*d*e - b^2*e^2 + 6*a*c*e^2)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(S
qrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2))) - ((b^2 - 4*a*c)*e^3*Log[d + e*
x^4])/(c*d^2 - b*d*e + a*e^2) + ((b^2 - 4*a*c)*e^3*Log[a + b*x^4 + c*x^8])
/(2*(c*d^2 - b*d*e + a*e^2)))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)))/4
```

Defintions of rubi rules used

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1798
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + b
*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 5.14 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.50

method	result
default	$-\frac{\frac{c\left(ab\,e^{3}-2acd\,e^{2}-b^{2}d\,e^{2}+3bc\,d^{2}e-2c^{2}d^{3}\right)x^{4}}{4ac-b^{2}}-\frac{2a^{2}c\,e^{3}-a\,b^{2}e^{3}-abcd\,e^{2}+2a\,c^{2}d^{2}e+b^{3}d\,e^{2}-2b^{2}c\,d^{2}e+b\,c^{2}d^{3}}{4ac-b^{2}}}{2c\,x^{8}+2b\,x^{4}+2a}+\frac{\frac{\left(4a\,c^{2}e^{3}-b^{2}c\,e^{3}\right)\ln\left(c\,x^{8}+b\right.}{2c}}{2(a\,e^{2}-bde+c\,d^{2})^{2}}$
risch	Expression too large to display

input
```
int(x^3/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*e^2-b*d*e+c*d^2)^2*(1/2*(c*(a*b*e^3-2*a*c*d*e^2-b^2*d*e^2+3*b*c*d^
2*e-2*c^2*d^3)/(4*a*c-b^2)*x^4-(2*a^2*c*e^3-a*b^2*e^3-a*b*c*d*e^2+2*a*c^2*
d^2*e+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)/(4*a*c-b^2))/(c*x^8+b*x^4+a)+1/2/
(4*a*c-b^2)*(1/2*(4*a*c^2*e^3-b^2*c*e^3)/c*ln(c*x^8+b*x^4+a)+2*(5*a*b*c*e^
3-6*a*c^2*d*e^2-b^3*e^3+3*b*c^2*d^2*e-2*c^3*d^3-1/2*(4*a*c^2*e^3-b^2*c*e^3
)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))))+1/4*e^3*l
n(e*x^4+d)/(a*e^2-b*d*e+c*d^2)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(230) = 460$.

Time = 4.64 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.06

$$\int \frac{x^3}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \frac{e^4\log(|ex^4+d|)}{4(c^2d^4e-2bcd^3e^2+b^2d^2e^3+2acd^2e^3-2abde^4+a^2e^5)} - \frac{e^3\log(cx^8+bx^4+a)}{8(c^2d^4-2bcd^3e+b^2d^2e^2+2acd^2e^2-2abde^3+a^2e^4)} - \frac{(4c^3d^3-6bc^2d^2e+12ac^2de^2+b^3e^3-6abce^3)\arctan\left(\frac{2cx^4+b}{\sqrt{-b^2+4ac}}\right)}{4(b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+} - \frac{bc^2d^3-2b^2cd^2e+2ac^2d^2e+b^3de^2-abcde^2-ab^2e^3+2a^2ce^3+(2c^3d^3-3bc^2d^2e+b^2cde^2+2ac^2d}{4(cx^8+bx^4+a)(cd^2-bde+ae^2)^2(b^2-4ac)}$$

input
```
integrate(x^3/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*e^4*log(abs(e*x^4 + d))/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + 2*a
*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5) - 1/8*e^3*log(c*x^8 + b*x^4 + a)/(c^2*
d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4) -
 1/4*(4*c^3*d^3 - 6*b*c^2*d^2*e + 12*a*c^2*d*e^2 + b^3*e^3 - 6*a*b*c*e^3)*
arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b
^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2
*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4)*sq
rt(-b^2 + 4*a*c)) - 1/4*(b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e + b^3*d
*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e
+ b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e^3)*x^4)/((c*x^8 + b*x^4 + a)*(c*d^
2 - b*d*e + a*e^2)^2*(b^2 - 4*a*c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^4)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Hanged}$$

input
```
int(x^3/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x^3}{(d+ex^4)\left(a+bx^4+cx^8\right)^2}\,dx = \int \frac{x^3}{(e\,x^4+d)\left(c\,x^8+b\,x^4+a\right)^2}dx$$

input
```
int(x^3/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^3/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.108 $\int \frac{1}{x(d+ex^4)(a+bx^4+cx^8)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 391

$$\begin{aligned}\int \frac{1}{x(d+ex^4)(a+bx^4+cx^8)^2} dx &= \frac{b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)x^4}{4a(b^2-4ac)(cd^2-bde+ae^2)(a+bx^4+cx^8)} \\ &- \frac{c(bcd-b^2e+2ace)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2a(b^2-4ac)^{3/2}(cd^2-bde+ae^2)} \\ &+ \frac{(b^3de^2+2a^2ce^3+bcd(cd^2+2ae^2)-2b^2(cd^2e+ae^3))\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}(cd^2-bde+ae^2)^2} + \frac{\log(x)}{a^2d} \\ &- \frac{e^4\log(d+ex^4)}{4d(cd^2-bde+ae^2)^2} - \frac{(cd-be)(cd^2-e(bd-2ae))\log(a+bx^4+cx^8)}{8a^2(cd^2-bde+ae^2)^2}\end{aligned}$$

output
```
1/4*(b^2*c*d-2*a*c^2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^4)/a/(-4*
a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^8+b*x^4+a)-1/2*c*(2*a*c*e-b^2*e+b*c*d)*a
rctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c
*d^2)+1/4*(b^3*d*e^2+2*a^2*c*e^3+b*c*d*(2*a*e^2+c*d^2)-2*b^2*(a*e^3+c*d^2*
e))*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/(a*e^2-
b*d*e+c*d^2)^2+ln(x)/a^2/d-1/4*e^4*ln(e*x^4+d)/d/(a*e^2-b*d*e+c*d^2)^2-1/8
*(-b*e+c*d)*(c*d^2-e*(-2*a*e+b*d))*ln(c*x^8+b*x^4+a)/a^2/(a*e^2-b*d*e+c*d^
2)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.58 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.54

$$\int \frac{1}{x(d+ex^4)(a+bx^4+cx^8)^2}\,dx$$
$$= \frac{1}{4}\left(\frac{b^3e-bc(3ae+cdx^4)+2ac^2(d-ex^4)+b^2c(-d+ex^4)}{a(-b^2+4ac)(cd^2+e(-bd+ae))(a+bx^4+cx^8)}+\frac{4\log(x)}{a^2d}\right.$$
$$-\frac{e^4\log(d+ex^4)}{d(cd^2+e(-bd+ae))^2}$$
$$+\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^3c^2d^3\log(x-\#1)-5abc^3d^3\log(x-\#1)-2b^4cd^2e\log(x-\#1)+10ab^2c^2d^2e\log(x-\#1)-2a^2\cdots}{}\right.}{}$$

input
```
Integrate[1/(x*(d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((b^3*e - b*c*(3*a*e + c*d*x^4) + 2*a*c^2*(d - e*x^4) + b^2*c*(-d + e*x^4)
)/(a*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*(a + b*x^4 + c*x^8)) + (4*L
og[x])/(a^2*d) - (e^4*Log[d + e*x^4])/(d*(c*d^2 + e*(-(b*d) + a*e))^2) + R
ootSum[a + b*#1^4 + c*#1^8 & , (b^3*c^2*d^3*Log[x - #1] - 5*a*b*c^3*d^3*Lo
g[x - #1] - 2*b^4*c*d^2*e*Log[x - #1] + 10*a*b^2*c^2*d^2*e*Log[x - #1] - 2
*a^2*c^3*d^2*e*Log[x - #1] + b^5*d*e^2*Log[x - #1] - 3*a*b^3*c*d*e^2*Log[x
 - #1] - 7*a^2*b*c^2*d*e^2*Log[x - #1] - 2*a*b^4*e^3*Log[x - #1] + 10*a^2*
b^2*c*e^3*Log[x - #1] - 6*a^3*c^2*e^3*Log[x - #1] + b^2*c^3*d^3*Log[x - #1
]*#1^4 - 4*a*c^4*d^3*Log[x - #1]*#1^4 - 2*b^3*c^2*d^2*e*Log[x - #1]*#1^4 +
 8*a*b*c^3*d^2*e*Log[x - #1]*#1^4 + b^4*c*d*e^2*Log[x - #1]*#1^4 - 2*a*b^2
*c^2*d*e^2*Log[x - #1]*#1^4 - 8*a^2*c^3*d*e^2*Log[x - #1]*#1^4 - 2*a*b^3*c
*e^3*Log[x - #1]*#1^4 + 8*a^2*b*c^2*e^3*Log[x - #1]*#1^4)/(b + 2*c*#1^4) &
 ]/(a^2*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2))/4
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1802, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(ex^4+d\right)\left(cx^8+bx^4+a\right)^2}dx^4$$

$$\Big\downarrow \text{ 1289}$$

$$\frac{1}{4}\int \left(-\frac{e^5}{d\left(cd^2-bed+ae^2\right)^2\left(ex^4+d\right)}+\frac{-c(cd-be)\left(cd^2-e(bd-2ae)\right)x^4-a^2ce^3-b^3de^2-bcd\left(cd^2+2ae^2\right)}{a^2\left(cd^2-bed+ae^2\right)^2\left(cx^8+bx^4+a\right)}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)\left(2a^2ce^3-2b^2\left(ae^3+cd^2e\right)+bcd\left(2ae^2+cd^2\right)+b^3de^2\right)}{a^2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2}-\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)\text{lo}}{2a^2\left(ae^2-bde+c\right.}\right.$$

input
```
Int[1/(x*(d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x^
4)/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x^4 + c*x^8)) - (2*c*(b
*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(a*(b^2
- 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)) + ((b^3*d*e^2 + 2*a^2*c*e^3 + b*c*
d*(c*d^2 + 2*a*e^2) - 2*b^2*(c*d^2*e + a*e^3))*ArcTanh[(b + 2*c*x^4)/Sqrt[
b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2) + Log[x^4
]/(a^2*d) - (e^4*Log[d + e*x^4])/(d*(c*d^2 - b*d*e + a*e^2)^2) - ((c*d - b
*e)*(c*d^2 - e*(b*d - 2*a*e))*Log[a + b*x^4 + c*x^8])/(2*a^2*(c*d^2 - b*d*
e + a*e^2)^2))/4
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 57.37 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.72

method	result
default	$-\frac{\frac{ac\left(2a^2c\,e^3-a\,b^2e^3-abcd\,e^2+2a\,c^2d^2e+b^3d\,e^2-2b^2c\,d^2e+b\,c^2d^3\right)x^4}{4ac-b^2}+\frac{a\left(3a^2bc\,e^3-2a^2c^2e^2d-a\,b^3e^3-2a\,b^2cd\,e^2+5ab\,c^2d^2e-2a\,c^3d^3+b^4d\,e\right.}{4ac-b^2}}{2c\,x^8+2b\,x^4+2a}$
risch	Expression too large to display

input
```
int(1/x/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/(a*e^2-b*d*e+c*d^2)^2*(1/2*(a*c*(2*a^2*c*e^3-a*b^2*e^3-a*b*c*d*e^
2+2*a*c^2*d^2*e+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)/(4*a*c-b^2)*x^4+a*(3*a^
2*b*c*e^3-2*a^2*c^2*d*e^2-a*b^3*e^3-2*a*b^2*c*d*e^2+5*a*b*c^2*d^2*e-2*a*c^
3*d^3+b^4*d*e^2-2*b^3*c*d^2*e+b^2*c^2*d^3)/(4*a*c-b^2))/(c*x^8+b*x^4+a)+1/
2/(4*a*c-b^2)*(1/2*(-8*a^2*b*c^2*e^3+8*a^2*c^3*d*e^2+2*a*b^3*c*e^3+2*a*b^2
*c^2*d*e^2-8*a*b*c^3*d^2*e+4*a*c^4*d^3-b^4*c*d*e^2+2*b^3*c^2*d^2*e-b^2*c^3
*d^3)/c*ln(c*x^8+b*x^4+a)+2*(6*a^3*c^2*e^3-10*a^2*b^2*c*e^3+7*a^2*b*c^2*d*
e^2+2*a^2*c^3*d^2*e+2*a*b^4*e^3+3*a*b^3*c*d*e^2-10*a*b^2*c^2*d^2*e+5*a*b*c
^3*d^3-b^5*d*e^2+2*b^4*c*d^2*e-b^3*c^2*d^3-1/2*(-8*a^2*b*c^2*e^3+8*a^2*c^3
*d*e^2+2*a*b^3*c*e^3+2*a*b^2*c^2*d*e^2-8*a*b*c^3*d^2*e+4*a*c^4*d^3-b^4*c*d
*e^2+2*b^3*c^2*d^2*e-b^2*c^3*d^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b
)/(4*a*c-b^2)^(1/2))))-1/4*e^4*ln(e*x^4+d)/d/(a*e^2-b*d*e+c*d^2)^2+ln(x)/a
^2/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. 2(373) = 746.

Time = 4.46 (sec) , antiderivative size = 1019, normalized size of antiderivative = 2.61

$$\int \frac{1}{x\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*e^5*log(abs(e*x^4 + d))/(c^2*d^5*e - 2*b*c*d^4*e^2 + b^2*d^3*e^3 + 2*
a*c*d^3*e^3 - 2*a*b*d^2*e^4 + a^2*d*e^5) - 1/8*(c^2*d^3 - 2*b*c*d^2*e + b^
2*d*e^2 + 2*a*c*d*e^2 - 2*a*b*e^3)*log(c*x^8 + b*x^4 + a)/(a^2*c^2*d^4 - 2
*a^2*b*c*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3*c*d^2*e^2 - 2*a^3*b*d*e^3 + a^4*e
^4) - 1/4*(b^3*c^2*d^3 - 6*a*b*c^3*d^3 - 2*b^4*c*d^2*e + 12*a*b^2*c^2*d^2*
e - 4*a^2*c^3*d^2*e + b^5*d*e^2 - 4*a*b^3*c*d*e^2 - 6*a^2*b*c^2*d*e^2 - 2*
a*b^4*e^3 + 12*a^2*b^2*c*e^3 - 12*a^3*c^2*e^3)*arctan((2*c*x^4 + b)/sqrt(-
b^2 + 4*a*c))/((a^2*b^2*c^2*d^4 - 4*a^3*c^3*d^4 - 2*a^2*b^3*c*d^3*e + 8*a^
3*b*c^2*d^3*e + a^2*b^4*d^2*e^2 - 2*a^3*b^2*c*d^2*e^2 - 8*a^4*c^2*d^2*e^2
- 2*a^3*b^3*d*e^3 + 8*a^4*b*c*d*e^3 + a^4*b^2*e^4 - 4*a^5*c*e^4)*sqrt(-b^2
 + 4*a*c)) + 1/8*(b^2*c^3*d^3*x^8 - 4*a*c^4*d^3*x^8 - 2*b^3*c^2*d^2*e*x^8
+ 8*a*b*c^3*d^2*e*x^8 + b^4*c*d*e^2*x^8 - 2*a*b^2*c^2*d*e^2*x^8 - 8*a^2*c^
3*d*e^2*x^8 - 2*a*b^3*c*e^3*x^8 + 8*a^2*b*c^2*e^3*x^8 + b^3*c^2*d^3*x^4 -
2*a*b*c^3*d^3*x^4 - 2*b^4*c*d^2*e*x^4 + 4*a*b^2*c^2*d^2*e*x^4 + 4*a^2*c^3*
d^2*e*x^4 + b^5*d*e^2*x^4 - 10*a^2*b*c^2*d*e^2*x^4 - 2*a*b^4*e^3*x^4 + 6*a
^2*b^2*c*e^3*x^4 + 4*a^3*c^2*e^3*x^4 + 3*a*b^2*c^2*d^3 - 8*a^2*c^3*d^3 - 6
*a*b^3*c*d^2*e + 18*a^2*b*c^2*d^2*e + 3*a*b^4*d*e^2 - 6*a^2*b^2*c*d*e^2 -
12*a^3*c^2*d*e^2 - 4*a^2*b^3*e^3 + 14*a^3*b*c*e^3)/((a^2*b^2*c^2*d^4 - 4*a
^3*c^3*d^4 - 2*a^2*b^3*c*d^3*e + 8*a^3*b*c^2*d^3*e + a^2*b^4*d^2*e^2 - 2*a
^3*b^2*c*d^2*e^2 - 8*a^4*c^2*d^2*e^2 - 2*a^3*b^3*d*e^3 + 8*a^4*b*c*d*e^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx=\text{Hanged}$$

input
```
int(1/(x*(d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{x\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx=\int \frac{1}{x\left(e\,x^4+d\right)\left(c\,x^8+b\,x^4+a\right)^2}dx$$

input
```
int(1/x/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(1/x/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.109 $\int \frac{1}{x^5\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 507

$$\begin{aligned}
&\int \frac{1}{x^5\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx \\
&= -\frac{1}{4a^2dx^4}-\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e+c(b^2cd-2ac^2d-b^3e+3abce)\,x^4}{4a^2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx^4+cx^8\right)} \\
&\quad+\frac{c(b^2cd-2ac^2d-b^3e+3abce)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}\left(cd^2-bde+ae^2\right)} \\
&\quad-\frac{\left(2b^2c^2d^3+2b^4de^2-2ac^2d(cd^2+2ae^2)+abce(5cd^2+6ae^2)-b^3(4cd^2e+3ae^3)\right)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^3\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)^2} \\
&\quad-\frac{(2bd+ae)\log(x)}{a^3d^2}+\frac{e^5\log\left(d+ex^4\right)}{4d^2\left(cd^2-bde+ae^2\right)^2} \\
&\quad+\frac{\left(2b^3de^2+2bcd(cd^2+ae^2)+ace(cd^2+2ae^2)-b^2(4cd^2e+3ae^3)\right)\log\left(a+bx^4+cx^8\right)}{8a^3\left(cd^2-bde+ae^2\right)^2}
\end{aligned}$$

output
```
-1/4/a^2/d/x^4-1/4*(b^3*c*d-3*a*b*c^2*d-b^4*e+4*a*b^2*c*e-2*a^2*c^2*e+c*(3
*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*x^4)/a^2/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2
)/(c*x^8+b*x^4+a)+1/2*c*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*arctanh((2*c*x
^4+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c*d^2)-1/4*(
2*b^2*c^2*d^3+2*b^4*d*e^2-2*a*c^2*d*(2*a*e^2+c*d^2)+a*b*c*e*(6*a*e^2+5*c*d
^2)-b^3*(3*a*e^3+4*c*d^2*e))*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a^3/(
-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)^2-(a*e+2*b*d)*ln(x)/a^3/d^2+1/4*e^5*
ln(e*x^4+d)/d^2/(a*e^2-b*d*e+c*d^2)^2+1/8*(2*b^3*d*e^2+2*b*c*d*(a*e^2+c*d^
2)+a*c*e*(2*a*e^2+c*d^2)-b^2*(3*a*e^3+4*c*d^2*e))*ln(c*x^8+b*x^4+a)/a^3/(a
*e^2-b*d*e+c*d^2)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.13 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^5 (d+ex^4) (a+bx^4+cx^8)^2} \, dx = \frac{1}{4}\left(-\frac{1}{a^2dx^4} + \frac{-b^4e-2ac^2(ae+cdx^4)+b^2c(4ae+cdx^4)+b^3c(d-ex^4)-3abc^2(d-ex^4)}{a^2(b^2-4ac)(-cd^2+e(bd-ae))(a+bx^4+cx^8)} - \frac{4(2bd+ae)\log(x)}{a^3d^2} + \frac{e^5\log(d+ex^4)}{(cd^3+de(-bd+ae))^2} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{2b^4c^2d^3\log(x-\#1)-10ab^2c^3d^3\log(x-\#1)+6a^2c^4d^3\log(x-\#1)-4b^5cd^2e\log(x-\#1)+21a\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
Integrate[1/(x^5*(d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(-(1/(a^2*d*x^4)) + (-(b^4*e) - 2*a*c^2*(a*e + c*d*x^4) + b^2*c*(4*a*e + c
*d*x^4) + b^3*c*(d - e*x^4) - 3*a*b*c^2*(d - e*x^4))/(a^2*(b^2 - 4*a*c)*(-
(c*d^2) + e*(b*d - a*e))*(a + b*x^4 + c*x^8)) - (4*(2*b*d + a*e)*Log[x])/(
a^3*d^2) + (e^5*Log[d + e*x^4])/(c*d^3 + d*e*(-(b*d) + a*e))^2 - RootSum[a
 + b*#1^4 + c*#1^8 & , (2*b^4*c^2*d^3*Log[x - #1] - 10*a*b^2*c^3*d^3*Log[x
 - #1] + 6*a^2*c^4*d^3*Log[x - #1] - 4*b^5*c*d^2*e*Log[x - #1] + 21*a*b^3*
c^2*d^2*e*Log[x - #1] - 17*a^2*b*c^3*d^2*e*Log[x - #1] + 2*b^6*d*e^2*Log[x
 - #1] - 8*a*b^4*c*d*e^2*Log[x - #1] - 4*a^2*b^2*c^2*d*e^2*Log[x - #1] + 1
0*a^3*c^3*d*e^2*Log[x - #1] - 3*a*b^5*e^3*Log[x - #1] + 17*a^2*b^3*c*e^3*L
og[x - #1] - 19*a^3*b*c^2*e^3*Log[x - #1] + 2*b^3*c^3*d^3*Log[x - #1]*#1^4
 - 8*a*b*c^4*d^3*Log[x - #1]*#1^4 - 4*b^4*c^2*d^2*e*Log[x - #1]*#1^4 + 17*
a*b^2*c^3*d^2*e*Log[x - #1]*#1^4 - 4*a^2*c^4*d^2*e*Log[x - #1]*#1^4 + 2*b^
5*c*d*e^2*Log[x - #1]*#1^4 - 6*a*b^3*c^2*d*e^2*Log[x - #1]*#1^4 - 8*a^2*b*
c^3*d*e^2*Log[x - #1]*#1^4 - 3*a*b^4*c*e^3*Log[x - #1]*#1^4 + 14*a^2*b^2*c
^2*e^3*Log[x - #1]*#1^4 - 8*a^3*c^3*e^3*Log[x - #1]*#1^4)/(b + 2*c*#1^4) &
 ]/(a^3*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2))/4
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1802, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (d+ex^4) (a+bx^4+cx^8)^2} \, dx$$

$$\Big\downarrow \text{1802}$$

$$\frac{1}{4} \int \frac{1}{x^8 (ex^4+d) (cx^8+bx^4+a)^2} dx^4$$

$$\Big\downarrow \text{1289}$$

$$\frac{1}{4} \int \left(\frac{e^6}{d^2 (cd^2-bed+ae^2)^2 (ex^4+d)} + \frac{2de^2b^4-(3ae^3+4cd^2e)\,b^3+cd\,(2cd^2+ae^2)\,b^2+ace\,(3cd^2+4ae^2)\,b+}{a^3\,(cd^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)\left(-b^3\left(3ae^3+4cd^2e\right)+abce\left(6ae^2+5cd^2\right)-2ac^2d\left(2ae^2+cd^2\right)+2b^4de^2+2b^2c^2d^3\right)}{a^3\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2}+\right.$$

input
```
Int[1/(x^5*(d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(-(1/(a^2*d*x^4)) - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c
^2*e + c*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*x^4)/(a^2*(b^2 - 4*a*c)
*(c*d^2 - b*d*e + a*e^2)*(a + b*x^4 + c*x^8)) + (2*c*(b^2*c*d - 2*a*c^2*d
- b^3*e + 3*a*b*c*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 -
 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)) - ((2*b^2*c^2*d^3 + 2*b^4*d*e^2 - 2
*a*c^2*d*(c*d^2 + 2*a*e^2) + a*b*c*e*(5*c*d^2 + 6*a*e^2) - b^3*(4*c*d^2*e
+ 3*a*e^3))*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)^2) - ((2*b*d + a*e)*Log[x^4])/(a^3*d^2) + (e^5*
Log[d + e*x^4])/(d^2*(c*d^2 - b*d*e + a*e^2)^2) + ((2*b^3*d*e^2 + 2*b*c*d*
(c*d^2 + a*e^2) + a*c*e*(c*d^2 + 2*a*e^2) - b^2*(4*c*d^2*e + 3*a*e^3))*Log
[a + b*x^4 + c*x^8])/(2*a^3*(c*d^2 - b*d*e + a*e^2)^2))/4
```

Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 52.36 (sec) , antiderivative size = 836, normalized size of antiderivative = 1.65

method	result
default	$\frac{\frac{ac\left(3a^2bc\,e^3-2a^2c^2e^2d-a\,b^3e^3-2a\,b^2cd\,e^2+5ab\,c^2d^2e-2a\,c^3d^3+b^4d\,e^2-2b^3c\,d^2e+b^2c^2d^3\right)x^4}{4ac-b^2}-\frac{a\left(2a^3c^2e^3-4a^2b^2c\,e^3+a^2b\,c^2d\,e^2+2a^2c^3d^2e-\cdots\right.}{\cdots}}{2c\,x^8+2b\,x^4+2a}$
risch	Expression too large to display

input
```
int(1/x^5/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a^3/(a*e^2-b*d*e+c*d^2)^2*(1/2*(a*c*(3*a^2*b*c*e^3-2*a^2*c^2*d*e^2-a*b
^3*e^3-2*a*b^2*c*d*e^2+5*a*b*c^2*d^2*e-2*a*c^3*d^3+b^4*d*e^2-2*b^3*c*d^2*e
+b^2*c^2*d^3)/(4*a*c-b^2)*x^4-a*(2*a^3*c^2*e^3-4*a^2*b^2*c*e^3+a^2*b*c^2*d
*e^2+2*a^2*c^3*d^2*e+a*b^4*e^3+3*a*b^3*c*d*e^2-7*a*b^2*c^2*d^2*e+3*a*b*c^3
*d^3-b^5*d*e^2+2*b^4*c*d^2*e-b^3*c^2*d^3)/(4*a*c-b^2))/(c*x^8+b*x^4+a)+1/2
/(4*a*c-b^2)*(1/2*(8*a^3*c^3*e^3-14*a^2*b^2*c^2*e^3+8*a^2*b*c^3*d*e^2+4*a^
2*c^4*d^2*e+3*a*b^4*c*e^3+6*a*b^3*c^2*d*e^2-17*a*b^2*c^3*d^2*e+8*a*b*c^4*d
^3-2*b^5*c*d*e^2+4*b^4*c^2*d^2*e-2*b^3*c^3*d^3)/c*ln(c*x^8+b*x^4+a)+2*(19*
a^3*b*c^2*e^3-10*a^3*c^3*d*e^2-17*a^2*b^3*c*e^3+4*a^2*b^2*c^2*d*e^2+17*a^2
*b*c^3*d^2*e-6*a^2*c^4*d^3+3*a*b^5*e^3+8*a*b^4*c*d*e^2-21*a*b^3*c^2*d^2*e+
10*a*b^2*c^3*d^3-2*b^6*d*e^2+4*b^5*c*d^2*e-2*b^4*c^2*d^3-1/2*(8*a^3*c^3*e^
3-14*a^2*b^2*c^2*e^3+8*a^2*b*c^3*d*e^2+4*a^2*c^4*d^2*e+3*a*b^4*c*e^3+6*a*b
^3*c^2*d*e^2-17*a*b^2*c^3*d^2*e+8*a*b*c^4*d^3-2*b^5*c*d*e^2+4*b^4*c^2*d^2*
e-2*b^3*c^3*d^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/
2))))+1/4*e^5*ln(e*x^4+d)/d^2/(a*e^2-b*d*e+c*d^2)^2-1/4/a^2/d/x^4+(-a*e-2*
b*d)/a^3/d^2*ln(x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x^5/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d+ex^4) (a+bx^4+cx^8)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**5/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 (d+ex^4) (a+bx^4+cx^8)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1303 vs. $2(487) = 974$.

Time = 4.44 (sec) , antiderivative size = 1303, normalized size of antiderivative = 2.57

$$\int \frac{1}{x^5 (d + ex^4) (a + bx^4 + cx^8)^2} dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*e^6*log(abs(e*x^4 + d))/(c^2*d^6*e - 2*b*c*d^5*e^2 + b^2*d^4*e^3 + 2*a
*c*d^4*e^3 - 2*a*b*d^3*e^4 + a^2*d^2*e^5) + 1/8*(2*b*c^2*d^3 - 4*b^2*c*d^2
*e + a*c^2*d^2*e + 2*b^3*d*e^2 + 2*a*b*c*d*e^2 - 3*a*b^2*e^3 + 2*a^2*c*e^3
)*log(c*x^8 + b*x^4 + a)/(a^3*c^2*d^4 - 2*a^3*b*c*d^3*e + a^3*b^2*d^2*e^2
+ 2*a^4*c*d^2*e^2 - 2*a^4*b*d*e^3 + a^5*e^4) + 1/4*(2*b^4*c^2*d^3 - 12*a*b
^2*c^3*d^3 + 12*a^2*c^4*d^3 - 4*b^5*c*d^2*e + 25*a*b^3*c^2*d^2*e - 30*a^2*
b*c^3*d^2*e + 2*b^6*d*e^2 - 10*a*b^4*c*d*e^2 + 20*a^3*c^3*d*e^2 - 3*a*b^5*
e^3 + 20*a^2*b^3*c*e^3 - 30*a^3*b*c^2*e^3)*arctan((2*c*x^4 + b)/sqrt(-b^2
+ 4*a*c))/((a^3*b^2*c^2*d^4 - 4*a^4*c^3*d^4 - 2*a^3*b^3*c*d^3*e + 8*a^4*b*
c^2*d^3*e + a^3*b^4*d^2*e^2 - 2*a^4*b^2*c*d^2*e^2 - 8*a^5*c^2*d^2*e^2 - 2*
a^4*b^3*d*e^3 + 8*a^5*b*c*d*e^3 + a^5*b^2*e^4 - 4*a^6*c*e^4)*sqrt(-b^2 + 4
*a*c)) + 1/12*(a^2*b^2*c*e^5*x^12 - 4*a^3*c^2*e^5*x^12 - 6*b^2*c^3*d^5*x^8
 + 18*a*c^4*d^5*x^8 + 12*b^3*c^2*d^4*e*x^8 - 39*a*b*c^3*d^4*e*x^8 - 6*b^4*
c*d^3*e^2*x^8 + 12*a*b^2*c^2*d^3*e^2*x^8 + 30*a^2*c^3*d^3*e^2*x^8 + 9*a*b^
3*c*d^2*e^3*x^8 - 33*a^2*b*c^2*d^2*e^3*x^8 - 3*a^2*b^2*c*d*e^4*x^8 + 12*a^
3*c^2*d*e^4*x^8 + a^2*b^3*e^5*x^8 - 4*a^3*b*c*e^5*x^8 - 6*b^3*c^2*d^5*x^4
+ 21*a*b*c^3*d^5*x^4 + 12*b^4*c*d^4*e*x^4 - 45*a*b^2*c^2*d^4*e*x^4 + 6*a^2
*c^3*d^4*e*x^4 - 6*b^5*d^3*e^2*x^4 + 15*a*b^3*c*d^3*e^2*x^4 + 27*a^2*b*c^2
*d^3*e^2*x^4 + 9*a*b^4*d^2*e^3*x^4 - 36*a^2*b^2*c*d^2*e^3*x^4 + 6*a^3*c^2*
d^2*e^3*x^4 - 3*a^2*b^3*d*e^4*x^4 + 12*a^3*b*c*d*e^4*x^4 + a^3*b^2*e^5*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d+ex^4) (a+bx^4+cx^8)^2} \, dx = \text{Hanged}$$

input
```
int(1/(x^5*(d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{x^5 (d+ex^4) (a+bx^4+cx^8)^2} \, dx = \int \frac{1}{x^5 (e\,x^4+d) (c\,x^8+b\,x^4+a)^2} dx$$

input
```
int(1/x^5/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(1/x^5/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.110 $\int \frac{x^{13}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 633

$$\begin{aligned}
\int \frac{x^{13}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = &-\frac{x^2\left(a(bd-2ae)+(b^2d-2acd-abe)\,x^4\right)}{4(b^2-4ac)(cd^2-bde+ae^2)(a+bx^4+cx^8)} \\
&+\frac{\left(b^3d^2e-abe(cd^2-ae^2)-2acd(3cd^2-ae^2)+b^2(cd^3-2ade^2)-\frac{b^4d^2e-ab^2e(3cd^2-ae^2)-4abcd(2cd^2+ae^2)+4a^2}{\sqrt{b^2-4ac}}\right.}{4\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} \\
&+\frac{\left(b^3d^2e-abe(cd^2-ae^2)-2acd(3cd^2-ae^2)+b^2(cd^3-2ade^2)+\frac{b^4d^2e-ab^2e(3cd^2-ae^2)-4abcd(2cd^2+ae^2)+4a^2}{\sqrt{b^2-4ac}}\right.}{4\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} \\
&-\frac{d^{5/2}\sqrt{e}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{2(cd^2-bde+ae^2)^2}
\end{aligned}$$

output
```
-1/4*x^2*(a*(-2*a*e+b*d)+(-a*b*e-2*a*c*d+b^2*d)*x^4)/(-4*a*c+b^2)/(a*e^2-b
*d*e+c*d^2)/(c*x^8+b*x^4+a)+1/8*(b^3*d^2*e-a*b*e*(-a*e^2+c*d^2)-2*a*c*d*(-
a*e^2+3*c*d^2)+b^2*(-2*a*d*e^2+c*d^3)-(b^4*d^2*e-a*b^2*e*(-a*e^2+3*c*d^2)-
4*a*b*c*d*(a*e^2+2*c*d^2)+4*a^2*c*e*(a*e^2+5*c*d^2)+b^3*(-2*a*d*e^2+c*d^3)
)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2))^(1
/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^2-b*d*
e+c*d^2)^2+1/8*(b^3*d^2*e-a*b*e*(-a*e^2+c*d^2)-2*a*c*d*(-a*e^2+3*c*d^2)+b^
2*(-2*a*d*e^2+c*d^3)+(b^4*d^2*e-a*b^2*e*(-a*e^2+3*c*d^2)-4*a*b*c*d*(a*e^2+
2*c*d^2)+4*a^2*c*e*(a*e^2+5*c*d^2)+b^3*(-2*a*d*e^2+c*d^3))/(-4*a*c+b^2)^(1
/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1
/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^2-b*d*e+c*d^2)^2-1/2*d^
(5/2)*e^(1/2)*arctan(e^(1/2)*x^2/d^(1/2))/(a*e^2-b*d*e+c*d^2)^2
```

Mathematica [A] (verified)

Time = 2.98 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.15

$$\int \frac{x^{13}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \frac{1}{8}\left(\frac{2x^2(-2a^2e+b^2dx^4-2acdx^4+ab(d-ex^4))}{(b^2-4ac)(-cd^2+e(bd-ae))(a+bx^4+cx^8)} + \frac{\sqrt{2}\left(-b^4d^2e+b^3\left(-cd^3+de\left(\sqrt{b^2-4ac}d+2ae\right)\right)+b^2\left(-ae^2\left(2\sqrt{b^2-4ac}d+ae\right)+cd^2\left(\sqrt{b^2-4ac}d+\right.\right.\right.}{\sqrt{c}} + \frac{\sqrt{2}\left(b^4d^2e+b^3\left(cd^3+de\left(\sqrt{b^2-4ac}d-2ae\right)\right)+b^2\left(cd^2\left(\sqrt{b^2-4ac}d-3ae\right)+ae^2\left(-2\sqrt{b^2-4ac}d+a\right.\right.\right.}{\sqrt{c}\,(b^2} - \frac{4d^{5/2}\sqrt{e}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{(cd^2+e(-bd+ae))^2}\right)$$

input
```
Integrate[x^13/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((2*x^2*(-2*a^2*e + b^2*d*x^4 - 2*a*c*d*x^4 + a*b*(d - e*x^4)))/((b^2 - 4*
a*c)*(-(c*d^2) + e*(b*d - a*e))*(a + b*x^4 + c*x^8)) + (Sqrt[2]*(-(b^4*d^2
*e) + b^3*(-(c*d^3) + d*e*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + b^2*(-(a*e^2*(2
*Sqrt[b^2 - 4*a*c]*d + a*e)) + c*d^2*(Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + a*b*
(8*c^2*d^3 + a*Sqrt[b^2 - 4*a*c]*e^3 + c*d*e*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a
*e)) - 2*a*c*(a*e^2*(-(Sqrt[b^2 - 4*a*c]*d) + 2*a*e) + c*d^2*(3*Sqrt[b^2 -
 4*a*c]*d + 10*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e
*(-(b*d) + a*e))^2) + (Sqrt[2]*(b^4*d^2*e + b^3*(c*d^3 + d*e*(Sqrt[b^2 - 4
*a*c]*d - 2*a*e)) + b^2*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 3*a*e) + a*e^2*(-2*S
qrt[b^2 - 4*a*c]*d + a*e)) - a*b*(8*c^2*d^3 - a*Sqrt[b^2 - 4*a*c]*e^3 + c*
d*e*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)) + 2*a*c*(a*e^2*(Sqrt[b^2 - 4*a*c]*d + 2
*a*e) + c*d^2*(-3*Sqrt[b^2 - 4*a*c]*d + 10*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*
x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))^2) - (4*d^(5/2)*Sqrt[e]*ArcTa
n[(Sqrt[e]*x^2)/Sqrt[d]])/(c*d^2 + e*(-(b*d) + a*e))^2)/8
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 611, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1814, 1650, 1598, 25, 1480, 218, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx$$

$$\Big\downarrow \text{1814}$$

$$\frac{1}{2}\int \frac{x^{12}}{(ex^4+d)\,(cx^8+bx^4+a)^2}dx^2$$

$$\Big\downarrow \text{1650}$$

$$\frac{1}{2}\left(\frac{d^2\int \frac{x^4}{(ex^4+d)(cx^8+bx^4+a)}dx^2}{ae^2-bde+cd^2}-\frac{\int \frac{x^4((bd-ae)x^4+ad)}{(cx^8+bx^4+a)^2}dx^2}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \text{1598}$$

$$\frac{1}{2}\left(\frac{d^2\int \frac{x^4}{(ex^4+d)(cx^8+bx^4+a)}dx^2}{ae^2-bde+cd^2}-\frac{\frac{\int -\frac{a(bd-2ae)-\left(db^2-aeb-2acd\right)x^4}{cx^8+bx^4+a}dx^2}{2(b^2-4ac)}+\frac{x^2\left(x^4\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{2(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{d^2\int \frac{x^4}{(ex^4+d)(cx^8+bx^4+a)}dx^2}{ae^2-bde+cd^2}-\frac{\frac{x^2\left(x^4\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{2(b^2-4ac)(a+bx^4+cx^8)}-\frac{\int \frac{a(bd-2ae)-\left(db^2-aeb-2acd\right)x^4}{cx^8+bx^4+a}dx^2}{2(b^2-4ac)}}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \; 1480$$

$$\frac{1}{2}\left(\frac{d^2\int \frac{x^4}{(ex^4+d)(cx^8+bx^4+a)}dx^2}{ae^2-bde+cd^2}-\frac{\frac{x^2\left(x^4\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{2(b^2-4ac)(a+bx^4+cx^8)}-\frac{-\frac{1}{2}\left(-\frac{-4a^2ce-ab^2e+b^3d}{\sqrt{b^2-4ac}}-abe-2acd+b^2d\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{\right.}}}{}}{ae^2-bde+cd^2}\right.$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(\frac{d^2\int \frac{x^4}{(ex^4+d)(cx^8+bx^4+a)}dx^2}{ae^2-bde+cd^2}-\frac{\frac{x^2\left(x^4\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{2(b^2-4ac)(a+bx^4+cx^8)}-\frac{-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-4a^2ce-ab^2e+b^3d}{\sqrt{b^2-4ac}}-abe-2acd+b\right.}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}}{2(}}{ae^2-bde+cd^2}\right.$$

$$\Big\downarrow \; 1610$$

$$\frac{1}{2}\left(\frac{d^2\int \left(\frac{cdx^4+ae}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}-\frac{de}{(cd^2-bed+ae^2)(ex^4+d)}\right)dx^2}{ae^2-bde+cd^2}-\frac{\frac{x^2\left(x^4\left(-abe-2acd+b^2d\right)+a(bd-2ae)\right)}{2(b^2-4ac)(a+bx^4+cx^8)}-\frac{-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2}}}\right.}{}}{}}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{d^2\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}(ae^2-bde+cd^2)}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}(ae^2-bde+cd^2)}-\frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{ae^2-bde+cd^2}\right)}{ae^2-bde+cd^2}-\frac{x^2(x^4(-\dots}{2\dots}\right.$$

input
```
Int[x^13/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(-(((x^2*(a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x^4))/(2*(b^2 - 4*a*
c)*(a + b*x^4 + c*x^8)) - (-(((b^2*d - 2*a*c*d - a*b*e - (b^3*d - a*b^2*e
- 4*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - ((b^2*d
- 2*a*c*d - a*b*e + (b^3*d - a*b^2*e - 4*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)))/(c*d^2 - b*d*e + a*e^2)) + (
d^2*((Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*
(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(
Sqrt[e]*x^2)/Sqrt[d]])/(c*d^2 - b*d*e + a*e^2)))/(c*d^2 - b*d*e + a*e^2))/
2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 1650
```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) +
 (e_.)*(x_)^2), x_Symbol] :> Simp[-f^4/(c*d^2 - b*d*e + a*e^2)   Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Simp[d^2*(f
^4/(c*d^2 - b*d*e + a*e^2))   Int[(f*x)^(m - 4)*((a + b*x^2 + c*x^4)^(p + 1
)/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] && LtQ[p, -1] && GtQ[m, 2]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 807, normalized size of antiderivative = 1.27

method	result
default	$\frac{-\frac{\left(a^{2} b\,e^{3}+2 a^{2} c d\,e^{2}-2 a\,b^{2} d\,e^{2}-a b c\,d^{2} e+2 a\,c^{2} d^{3}+b^{3} d^{2} e-b^{2} c\,d^{3}\right) x^{6}}{2\left(4 a c-b^{2}\right)}-\frac{a\left(2 a^{2} e^{3}-3 a b d\,e^{2}+2 a c\,d^{2} e+b^{2} d^{2} e-b c\,d^{3}\right) x^{2}}{2\left(4 a c-b^{2}\right)}}{c\,x^{8}+b\,x^{4}+a}+\frac{2 c\left(-\frac{\left(-a^{2} b\,e^{3} \sqrt{-4 a c+}\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(x^13/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*e^2-b*d*e+c*d^2)^2*((-1/2*(a^2*b*e^3+2*a^2*c*d*e^2-2*a*b^2*d*e^2-a*
b*c*d^2*e+2*a*c^2*d^3+b^3*d^2*e-b^2*c*d^3)/(4*a*c-b^2)*x^6-1/2*a*(2*a^2*e^
3-3*a*b*d*e^2+2*a*c*d^2*e+b^2*d^2*e-b*c*d^3)/(4*a*c-b^2)*x^2)/(c*x^8+b*x^4
+a)+2/(4*a*c-b^2)*c*(-1/8*(-a^2*b*e^3*(-4*a*c+b^2)^(1/2)-2*a^2*c*d*e^2*(-4
*a*c+b^2)^(1/2)+2*a*b^2*d*e^2*(-4*a*c+b^2)^(1/2)+a*b*c*d^2*e*(-4*a*c+b^2)^
(1/2)+6*a*c^2*d^3*(-4*a*c+b^2)^(1/2)-b^3*d^2*e*(-4*a*c+b^2)^(1/2)-b^2*c*d^
3*(-4*a*c+b^2)^(1/2)+4*a^3*e^3*c+a^2*b^2*e^3-4*a^2*b*d*e^2*c+20*a^2*d^2*e*
c^2-2*a*b^3*d*e^2-3*a*b^2*d^2*e*c-8*a*b*c^2*d^3+b^4*d^2*e+b^3*c*d^3)/c/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-a^2*b*e^3*(-4*a*c+b^2)^(1/2
)-2*a^2*c*d*e^2*(-4*a*c+b^2)^(1/2)+2*a*b^2*d*e^2*(-4*a*c+b^2)^(1/2)+a*b*c*
d^2*e*(-4*a*c+b^2)^(1/2)+6*a*c^2*d^3*(-4*a*c+b^2)^(1/2)-b^3*d^2*e*(-4*a*c+
b^2)^(1/2)-b^2*c*d^3*(-4*a*c+b^2)^(1/2)-4*a^3*e^3*c-a^2*b^2*e^3+4*a^2*b*d*
e^2*c-20*a^2*d^2*e*c^2+2*a*b^3*d*e^2+3*a*b^2*d^2*e*c+8*a*b*c^2*d^3-b^4*d^2
*e-b^3*c*d^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))-1/2*d^3*e/(a*e^
2-b*d*e+c*d^2)^2/(d*e)^(1/2)*arctan(e*x^2/(d*e)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^13/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**13/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{13}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^13/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20106 vs. 2(581) = 1162.

Time = 7.24 (sec) , antiderivative size = 20106, normalized size of antiderivative = 31.76

$$\int \frac{x^{13}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^13/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/2*d^3*e*arctan(e*x^2/sqrt(d*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 +
 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(d*e)) - 1/16*((2*b^5*c^5 - 20
*a*b^3*c^6 + 48*a^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^5*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^5 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^3*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 12*(b^2 - 4*a*c)*a*b*c^6)*d^7*x^4 - (2*b^6*
c^4 - 30*a*b^4*c^5 + 88*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^5*c^3 - 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^4 - 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^4*c^4 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 22*(b^2 - 4*a*c)*a*b^2*c^5)*
d^6*e*x^4 - (2*b^7*c^3 - 12*a^2*b^3*c^5 - 80*a^3*b*c^6 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c + 2*sqrt(2)*sqrt(b^2 - 4...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Hanged}$$

input
```
int(x^13/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x^{13}}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \int \frac{x^{13}}{(e\,x^4+d)\,(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^13/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^13/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.111 $\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 531

$$\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \frac{x^2(a(2cd-be)+c(bd-2ae)x^4)}{4(b^2-4ac)(cd^2-bde+ae^2)(a+bx^4+cx^8)}$$
$$-\frac{\sqrt{c}\left(3b^2d^2e-2ae(3cd^2-ae^2)-b(cd^3+3ade^2)-\frac{3b^3d^2e+4a^2be^3-4acd(cd^2-3ae^2)-b^2(cd^3+9ade^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{\cdots}}{\sqrt{b-\cdots}}\right.}{4\sqrt{2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2}$$
$$+\frac{\sqrt{c}\left(bcd^3-3b^2d^2e+6acd^2e+3abde^2-2a^2e^3-\frac{3b^3d^2e+4a^2be^3-4acd(cd^2-3ae^2)-b^2(cd^3+9ade^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\cdots}{\sqrt{b+\cdots}}\right.}{4\sqrt{2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2}$$
$$+\frac{d^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{2(cd^2-bde+ae^2)^2}$$

output
```
1/4*x^2*(a*(-b*e+2*c*d)+c*(-2*a*e+b*d)*x^4)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^
2)/(c*x^8+b*x^4+a)-1/8*c^(1/2)*(3*b^2*d^2*e-2*a*e*(-a*e^2+3*c*d^2)-b*(3*a*
d*e^2+c*d^3)-(3*b^3*d^2*e+4*a^2*b*e^3-4*a*c*d*(-3*a*e^2+c*d^2)-b^2*(9*a*d*
e^2+c*d^3))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b-(-4*a*c+b^2)
^(1/2))^(1/2))*2^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^2-b*
d*e+c*d^2)^2+1/8*c^(1/2)*(b*c*d^3-3*b^2*d^2*e+6*a*c*d^2*e+3*a*b*d*e^2-2*a^
2*e^3-(3*b^3*d^2*e+4*a^2*b*e^3-4*a*c*d*(-3*a*e^2+c*d^2)-b^2*(9*a*d*e^2+c*d
^3))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))
^(1/2))*2^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^2-b*d*e+c*d
^2)^2+1/2*d^(3/2)*e^(3/2)*arctan(e^(1/2)*x^2/d^(1/2))/(a*e^2-b*d*e+c*d^2)^
2
```

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.12

$$\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \frac{1}{8}\left(\frac{-2bcdx^6+2ax^2(-2cd+be+2cex^4)}{(b^2-4ac)(-cd^2+e(bd-ae))(a+bx^4+cx^8)}\right.$$
$$-\frac{\sqrt{2}\sqrt{c}\left(-3b^3d^2e+2a\left(2c^2d^3+a\sqrt{b^2-4ac}e^3-3cde\left(\sqrt{b^2-4ac}d+2ae\right)\right)+b^2\left(cd^3+3de\left(\sqrt{b^2-4ac}d\right.\right.\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2+}$$
$$+\frac{\sqrt{2}\sqrt{c}\left(4ac^2d^3+bc\sqrt{b^2-4ac}d^3-3b^3d^2e-2a^2\sqrt{b^2-4ac}e^3+abe^2\left(3\sqrt{b^2-4ac}d-4ae\right)+6acde\left(\sqrt{b}\right.\right.}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2+e(}$$
$$\left.+\frac{4d^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{(cd^2+e(-bd+ae))^2}\right)$$

input
```
Integrate[x^9/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((-2*b*c*d*x^6 + 2*a*x^2*(-2*c*d + b*e + 2*c*e*x^4))/((b^2 - 4*a*c)*(-(c*d
^2) + e*(b*d - a*e))*(a + b*x^4 + c*x^8)) - (Sqrt[2]*Sqrt[c]*(-3*b^3*d^2*e
 + 2*a*(2*c^2*d^3 + a*Sqrt[b^2 - 4*a*c]*e^3 - 3*c*d*e*(Sqrt[b^2 - 4*a*c]*d
 + 2*a*e)) + b^2*(c*d^3 + 3*d*e*(Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - b*(c*Sqrt
[b^2 - 4*a*c]*d^3 + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTan[(Sqrt[2
]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))^2) + (Sqrt[2]*Sqrt[c]*(4*a*
c^2*d^3 + b*c*Sqrt[b^2 - 4*a*c]*d^3 - 3*b^3*d^2*e - 2*a^2*Sqrt[b^2 - 4*a*c
]*e^3 + a*b*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 4*a*e) + 6*a*c*d*e*(Sqrt[b^2 - 4*
a*c]*d - 2*a*e) + b^2*d*(c*d^2 - 3*Sqrt[b^2 - 4*a*c]*d*e + 9*a*e^2))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))^2) + (4*d^(3/2)*e^
(3/2)*ArcTan[(Sqrt[e]*x^2)/Sqrt[d]])/(c*d^2 + e*(-(b*d) + a*e))^2)/8
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1814, 1650, 1484, 1492, 25, 27, 1480, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx$$

$$\Big\downarrow \ 1814$$

$$\frac{1}{2}\int \frac{x^8}{(ex^4+d)\,(cx^8+bx^4+a)^2}dx^2$$

$$\Big\downarrow \ 1650$$

$$\frac{1}{2}\left(\frac{d^2\int \frac{1}{(ex^4+d)(cx^8+bx^4+a)}dx^2}{ae^2-bde+cd^2}-\frac{\int \frac{(bd-ae)x^4+ad}{(cx^8+bx^4+a)^2}dx^2}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \ 1484$$

$$\frac{1}{2}\left(\frac{d^2\int \left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx^2}{ae^2-bde+cd^2}-\frac{\int \frac{(bd-ae)x^4+ad}{(cx^8+bx^4+a)^2}dx^2}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \text{1492}$$

$$\frac{1}{2}\left(\frac{d^2\int\left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx^2}{ae^2-bde+cd^2}-\frac{-\frac{\int-\frac{a\left(-c(bd-2ae)x^4+2b^2d-6acd-abe\right)}{cx^8+bx^4+a}dx^2}{2a(b^2-4ac)}-\frac{x^2\left(cx^4(bd-2a\right.}{2(b^2-4ac)(}}{ae^2-bde+cd^2}\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{d^2\int\left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx^2}{ae^2-bde+cd^2}-\frac{\frac{\int\frac{a\left(-c(bd-2ae)x^4+2b^2d-6acd-abe\right)}{cx^8+bx^4+a}dx^2}{2a(b^2-4ac)}-\frac{x^2\left(cx^4(bd-2ae)+\right.}{2(b^2-4ac)(a+b}}{ae^2-bde+cd^2}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{d^2\int\left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx^2}{ae^2-bde+cd^2}-\frac{\frac{\int\frac{-c(bd-2ae)x^4+2b^2d-6acd-abe}{cx^8+bx^4+a}dx^2}{2(b^2-4ac)}-\frac{x^2\left(cx^4(bd-2ae)+a(2\right.}{2(b^2-4ac)(a+bx^4-}}{ae^2-bde+cd^2}\right.$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(\frac{d^2\int\left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx^2}{ae^2-bde+cd^2}-\frac{-\frac{1}{2}c\left(-\frac{-4abe-12acd+5b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\int\frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d}{2(b}\right.$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{d^2\int\left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx^2}{ae^2-bde+cd^2}-\frac{-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-4abe-12acd+5b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-}{2(b^2-4a}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{d^2\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}(ae^2-bde+cd^2)}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}(ae^2-bde+cd^2)}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{\sqrt{d}(ae^2-bde+cd^2)}\right)}{ae^2-bde+cd^2}-\cdots\right.$$

input
```
Int[x^9/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(-((-1/2*(x^2*(a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x^4))/((b^2 - 4*a*c)*(a +
 b*x^4 + c*x^8)) + (-((Sqrt[c]*(b*d - 2*a*e - (5*b^2*d - 12*a*c*d - 4*a*b*
e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - (Sqrt[c]*(b*d - 2*a*e + (5
*b^2*d - 12*a*c*d - 4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^
2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2
*(b^2 - 4*a*c)))/(c*d^2 - b*d*e + a*e^2)) + (d^2*(-((Sqrt[c]*(e - (2*c*d -
 b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) -
 (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^
2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d
^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x^2)/Sqrt[d]])/(Sqrt[d]*(c
*d^2 - b*d*e + a*e^2))))/(c*d^2 - b*d*e + a*e^2))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1650
```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) +
 (e_.)*(x_)^2), x_Symbol] :> Simp[-f^4/(c*d^2 - b*d*e + a*e^2)   Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Simp[d^2*(f
^4/(c*d^2 - b*d*e + a*e^2))   Int[(f*x)^(m - 4)*((a + b*x^2 + c*x^4)^(p + 1
)/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] && LtQ[p, -1] && GtQ[m, 2]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.17

method	result
default	$-\frac{-\frac{c\left(2a^2e^3-3abd\,e^2+2ac\,d^2e+b^2d^2e-bc\,d^3\right)x^6}{2\left(4ac-b^2\right)}-\frac{a\left(ab\,e^3-2acd\,e^2-b^2d\,e^2+3bc\,d^2e-2c^2d^3\right)x^2}{2\left(4ac-b^2\right)}}{c\,x^8+b\,x^4+a}+2c\left(-\frac{\left(-2a^2e^3\sqrt{-4ac+b^2}+3abd\,e^2\sqrt{-4ac-}\right.}{}\right.$
risch	Expression too large to display

input
```
int(x^9/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*e^2-b*d*e+c*d^2)^2*((-1/2*c*(2*a^2*e^3-3*a*b*d*e^2+2*a*c*d^2*e+b^2
*d^2*e-b*c*d^3)/(4*a*c-b^2)*x^6-1/2*a*(a*b*e^3-2*a*c*d*e^2-b^2*d*e^2+3*b*c
*d^2*e-2*c^2*d^3)/(4*a*c-b^2)*x^2)/(c*x^8+b*x^4+a)+2/(4*a*c-b^2)*c*(-1/8*(
-2*a^2*e^3*(-4*a*c+b^2)^(1/2)+3*a*b*d*e^2*(-4*a*c+b^2)^(1/2)+6*a*c*d^2*e*(
-4*a*c+b^2)^(1/2)-3*b^2*d^2*e*(-4*a*c+b^2)^(1/2)+b*c*d^3*(-4*a*c+b^2)^(1/2
)+4*a^2*b*e^3+12*a^2*c*d*e^2-9*a*b^2*d*e^2-4*a*c^2*d^3+3*b^3*d^2*e-b^2*c*d
^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c
*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-2*a^2*e^3*(-4*a*c+b^
2)^(1/2)+3*a*b*d*e^2*(-4*a*c+b^2)^(1/2)+6*a*c*d^2*e*(-4*a*c+b^2)^(1/2)-3*b
^2*d^2*e*(-4*a*c+b^2)^(1/2)+b*c*d^3*(-4*a*c+b^2)^(1/2)-4*a^2*b*e^3-12*a^2*
c*d*e^2+9*a*b^2*d*e^2+4*a*c^2*d^3-3*b^3*d^2*e+b^2*c*d^3)/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))))+1/2*d^2*e^2/(a*e^2-b*d*e+c*d^2)^2/(d*e)^(1/2)*ar
ctan(e*x^2/(d*e)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^9}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^9/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**9/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^9}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^9/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18563 vs. $2(479) = 958$.

Time = 6.98 (sec) , antiderivative size = 18563, normalized size of antiderivative = 34.96

$$\int \frac{x^9}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^9/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/2*d^2*e^2*arctan(e*x^2/sqrt(d*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2
+ 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(d*e)) + 1/16*((2*b^4*c^6 - 8
*a*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4
*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^6 - 2*(b^
2 - 4*a*c)*b^2*c^6)*d^7*x^4 - (10*b^5*c^5 - 52*a*b^3*c^6 + 48*a^2*b*c^7 -
5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 26*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 10*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 24*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 12*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 5*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 6*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 10*(b^2 - 4*a*c)*b
^3*c^5 + 12*(b^2 - 4*a*c)*a*b*c^6)*d^6*e*x^4 + 7*(2*b^6*c^4 - 10*a*b^4*c^5
 + 8*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^6*c^2 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \text{Hanged}$$

input
```
int(x^9/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x^9}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \int \frac{x^9}{(e\,x^4+d)\,(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^9/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^9/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.112 $\int \frac{x^5}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 619

$$\int \frac{x^5}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = -\frac{x^2(bcd-b^2e+2ace+c(2cd-be)x^4)}{4(b^2-4ac)(cd^2-bde+ae^2)(a+bx^4+cx^8)}$$

$$+\frac{\sqrt{c}\left(b^3de^2-b^2e\left(7cd^2-e\left(\sqrt{b^2-4ac}d+ae\right)\right)-2c\left(cd^2\left(\sqrt{b^2-4ac}d-2ae\right)+ae^2\left(5\sqrt{b^2-4ac}d+6ae\right.\right.\right.}{4\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(c}$$

$$-\frac{\sqrt{c}\left(b^3de^2+b\left(4c^2d^3-a\sqrt{b^2-4ac}e^3-cde\left(3\sqrt{b^2-4ac}d-8ae\right)\right)-b^2e\left(7cd^2+e\left(\sqrt{b^2-4ac}d-ae\right)\right)\right.}{4\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,(c}$$

$$-\frac{\sqrt{d}e^{5/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{2(cd^2-bde+ae^2)^2}$$

output
```
-1/4*x^2*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x^4)/(-4*a*c+b^2)/(a*e^2-b*d*
e+c*d^2)/(c*x^8+b*x^4+a)+1/8*c^(1/2)*(b^3*d*e^2-b^2*e*(7*c*d^2-e*((-4*a*c+
b^2)^(1/2)*d+a*e))-2*c*(c*d^2*((-4*a*c+b^2)^(1/2)*d-2*a*e)+a*e^2*(5*(-4*a*
c+b^2)^(1/2)*d+6*a*e))+b*(4*c^2*d^3+a*(-4*a*c+b^2)^(1/2)*e^3+c*d*e*(3*(-4*
a*c+b^2)^(1/2)*d+8*a*e)))*arctan(2^(1/2)*c^(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2)
)^(1/2))*2^(1/2)/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^2-b*
d*e+c*d^2)^2-1/8*c^(1/2)*(b^3*d*e^2+b*(4*c^2*d^3-a*(-4*a*c+b^2)^(1/2)*e^3-
c*d*e*(3*(-4*a*c+b^2)^(1/2)*d-8*a*e))-b^2*e*(7*c*d^2+e*((-4*a*c+b^2)^(1/2)
*d-a*e))+2*c*(a*e^2*(5*(-4*a*c+b^2)^(1/2)*d-6*a*e)+c*d^2*((-4*a*c+b^2)^(1/
2)*d+2*a*e)))*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(
1/2)/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^2-b*d*e+c*d^2)^2
-1/2*d^(1/2)*e^(5/2)*arctan(e^(1/2)*x^2/d^(1/2))/(a*e^2-b*d*e+c*d^2)^2
```

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 612, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \frac{1}{8}\left(\frac{2x^2(-b^2e+2c(ae+cdx^4)+bc(d-ex^4))}{(b^2-4ac)(-cd^2+e(bd-ae))(a+bx^4+cx^8)}\right.$$
$$+\frac{\sqrt{2}\sqrt{c}\left(b^3de^2+b^2e\left(-7cd^2+e\left(\sqrt{b^2-4ac}d+ae\right)\right)-2c\left(cd^2\left(\sqrt{b^2-4ac}d-2ae\right)+ae^2\left(5\sqrt{b^2-4ac}d\right.\right.\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd}$$
$$-\frac{\sqrt{2}\sqrt{c}\left(b^3de^2+b^2e\left(-7cd^2+e\left(-\sqrt{b^2-4ac}d+ae\right)\right)+2c\left(ae^2\left(5\sqrt{b^2-4ac}d-6ae\right)+cd^2\left(\sqrt{b^2-4ac}\right.\right.\right.}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,(}$$
$$\left.-\frac{4\sqrt{d}e^{5/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{(cd^2+e(-bd+ae))^2}\right)$$

input
```
Integrate[x^5/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((2*x^2*(-(b^2*e) + 2*c*(a*e + c*d*x^4) + b*c*(d - e*x^4)))/((b^2 - 4*a*c)
*(-(c*d^2) + e*(b*d - a*e))*(a + b*x^4 + c*x^8)) + (Sqrt[2]*Sqrt[c]*(b^3*d
*e^2 + b^2*e*(-7*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) - 2*c*(c*d^2*(Sqrt
[b^2 - 4*a*c]*d - 2*a*e) + a*e^2*(5*Sqrt[b^2 - 4*a*c]*d + 6*a*e)) + b*(4*c
^2*d^3 + a*Sqrt[b^2 - 4*a*c]*e^3 + c*d*e*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e)))
*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)
^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))^2) - (Sqrt[2
]*Sqrt[c]*(b^3*d*e^2 + b^2*e*(-7*c*d^2 + e*(-(Sqrt[b^2 - 4*a*c]*d) + a*e))
 + 2*c*(a*e^2*(5*Sqrt[b^2 - 4*a*c]*d - 6*a*e) + c*d^2*(Sqrt[b^2 - 4*a*c]*d
 + 2*a*e)) + b*(4*c^2*d^3 - a*Sqrt[b^2 - 4*a*c]*e^3 + c*d*e*(-3*Sqrt[b^2 -
 4*a*c]*d + 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d)
+ a*e))^2) - (4*Sqrt[d]*e^(5/2)*ArcTan[(Sqrt[e]*x^2)/Sqrt[d]])/(c*d^2 + e*
(-(b*d) + a*e))^2)/8
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 590, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1814, 1652, 1484, 1492, 25, 27, 1480, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$$

$$\Big\downarrow \text{1814}$$

$$\frac{1}{2}\int \frac{x^4}{(ex^4+d)(cx^8+bx^4+a)^2}dx^2$$

$$\Big\downarrow \text{1652}$$

$$\frac{1}{2}\left(\frac{\int \frac{cdx^4+ae}{(cx^8+bx^4+a)^2}dx^2}{ae^2-bde+cd^2}-\frac{de\int \frac{1}{(ex^4+d)(cx^8+bx^4+a)}dx^2}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \text{1484}$$

$$\frac{1}{2}\left(\frac{\int \frac{cdx^4+ae}{(cx^8+bx^4+a)^2}dx^2}{ae^2-bde+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx^2}{ae^2-bde+cd^2}\right)$$

$\downarrow$ 1492

$$\frac{1}{2}\left(\frac{-\frac{\int -\frac{a\left(-c(2cd-be)x^4+bcd+b^2e-6ace\right)}{cx^8+bx^4+a}dx^2}{2a(b^2-4ac)}-\frac{x^2\left(2ace-b^2e+cx^4(2cd-be)+bcd\right)}{2(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd}{(cd^2-bed+ae^2)(c}\right.}{ae^2-bde+cd^2}\right.$$

$\downarrow$ 25

$$\frac{1}{2}\left(\frac{\frac{\int \frac{a\left(-c(2cd-be)x^4+bcd+b^2e-6ace\right)}{cx^8+bx^4+a}dx^2}{2a(b^2-4ac)}-\frac{x^2\left(2ace-b^2e+cx^4(2cd-be)+bcd\right)}{2(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+}\right.}{ae^2-bde+cd^2}\right.$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{\frac{\int \frac{-c(2cd-be)x^4+bcd+b^2e-6ace}{cx^8+bx^4+a}dx^2}{2(b^2-4ac)}-\frac{x^2\left(2ace-b^2e+cx^4(2cd-be)+bcd\right)}{2(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4}\right.}{ae^2-bde+cd^2}\right.$$

$\downarrow$ 1480

$$\frac{1}{2}\left(\frac{\frac{-\frac{1}{2}c\left(-\frac{-12ace+b^2e+4bcd}{\sqrt{b^2-4ac}}-be+2cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx^2-\frac{1}{2}c\left(\frac{-12ace+b^2e+4bcd}{\sqrt{b^2-4ac}}-be+2cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx^2}{2(b^2-4ac)}-\frac{x^2\left(2ace-b\right.}{2(b^2-}}{ae^2-bde+cd^2}\right.$$

$\downarrow$ 218

$$\frac{1}{2}\left(\frac{\frac{-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-12ace+b^2e+4bcd}{\sqrt{b^2-4ac}}-be+2cd\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-12ace+b^2e+4bcd}{\sqrt{b^2-4ac}}-be+2cd\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-\frac{x^2\left(2ace-b^2e+cx^4(2\right.}{2(b^2-4ac)(a+b}}{ae^2-bde+cd^2}\right.$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{\frac{-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-12ace+b^2e+4bcd}{\sqrt{b^2-4ac}}-be+2cd\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-12ace+b^2e+4bcd}{\sqrt{b^2-4ac}}-be+2cd\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-\frac{x^2(2ace-b^2e+cx^4(2}{2(b^2-4ac)(a+b}}{ae^2-bde+cd^2}\right.$$

input
```
Int[x^5/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((-1/2*(x^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^4))/((b^2 - 4*a*c
)*(a + b*x^4 + c*x^8)) + (-((Sqrt[c]*(2*c*d - b*e - (4*b*c*d + b^2*e - 12*
a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - (Sqrt[c]*(2*c*d - b*e
+ (4*b*c*d + b^2*e - 12*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/
(2*(b^2 - 4*a*c)))/(c*d^2 - b*d*e + a*e^2) - (d*e*(-((Sqrt[c]*(e - (2*c*d
- b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)))
- (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*
d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x^2)/Sqrt[d]])/(Sqrt[d]*(
c*d^2 - b*d*e + a*e^2))))/(c*d^2 - b*d*e + a*e^2))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1652
```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) +
 (e_.)*(x_)^2), x_Symbol] :> Simp[f^2/(c*d^2 - b*d*e + a*e^2)   Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Simp[d*e*(f^2/(c*d^2
 - b*d*e + a*e^2))   Int[(f*x)^(m - 2)*((a + b*x^2 + c*x^4)^(p + 1)/(d + e*
x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.07

method	result
default	$\frac{-\frac{c\left(ab\,e^{3}-2acd\,e^{2}-b^{2}d\,e^{2}+3bc\,d^{2}e-2c^{2}d^{3}\right)x^{6}}{2\left(4ac-b^{2}\right)}+\frac{\left(2a^{2}c\,e^{3}-a\,b^{2}e^{3}-abcd\,e^{2}+2a\,c^{2}d^{2}e+b^{3}d\,e^{2}-2b^{2}c\,d^{2}e+b\,c^{2}d^{3}\right)x^{2}}{8ac-2b^{2}}}{c\,x^{8}+b\,x^{4}+a}+\frac{2c\left(-\frac{\left(-ab\,e^{3}\sqrt{-4ac+b^{2}}\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(x^5/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*e^2-b*d*e+c*d^2)^2*((-1/2*c*(a*b*e^3-2*a*c*d*e^2-b^2*d*e^2+3*b*c*d^
2*e-2*c^2*d^3)/(4*a*c-b^2)*x^6+1/2*(2*a^2*c*e^3-a*b^2*e^3-a*b*c*d*e^2+2*a*
c^2*d^2*e+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)/(4*a*c-b^2)*x^2)/(c*x^8+b*x^4
+a)+2/(4*a*c-b^2)*c*(-1/8*(-a*b*e^3*(-4*a*c+b^2)^(1/2)+10*a*c*d*e^2*(-4*a*
c+b^2)^(1/2)-b^2*d*e^2*(-4*a*c+b^2)^(1/2)-3*b*c*d^2*e*(-4*a*c+b^2)^(1/2)+2
*c^2*d^3*(-4*a*c+b^2)^(1/2)+12*a^2*c*e^3-a*b^2*e^3-8*a*b*c*d*e^2-4*a*c^2*d
^2*e-b^3*d*e^2+7*b^2*c*d^2*e-4*b*c^2*d^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))+1/8*(-a*b*e^3*(-4*a*c+b^2)^(1/2)+10*a*c*d*e^2*(-4*a*c+b^2)^(1/
2)-b^2*d*e^2*(-4*a*c+b^2)^(1/2)-3*b*c*d^2*e*(-4*a*c+b^2)^(1/2)+2*c^2*d^3*(
-4*a*c+b^2)^(1/2)-12*a^2*c*e^3+a*b^2*e^3+8*a*b*c*d*e^2+4*a*c^2*d^2*e+b^3*d
*e^2-7*b^2*c*d^2*e+4*b*c^2*d^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
-1/2*d*e^3/(a*e^2-b*d*e+c*d^2)^2/(d*e)^(1/2)*arctan(e*x^2/(d*e)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^5/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex^4)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(d+ex^4)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19913 vs. 2(550) = 1100.

Time = 7.40 (sec) , antiderivative size = 19913, normalized size of antiderivative = 32.17

$$\int \frac{x^5}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/2*d*e^3*arctan(e*x^2/sqrt(d*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 +
 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(d*e)) - 1/16*(2*(2*b^3*c^7 -
8*a*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^6
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^6 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^7 - 2*(b^2 - 4
*a*c)*b*c^7)*d^7*x^4 - 7*(2*b^4*c^6 - 8*a*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^6)*d^6*e*x^4 + 7*(
2*b^5*c^5 - 4*a*b^3*c^6 - 16*a^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 - 4*(b^2 - 4*a*c)*a*b*c^6)*d^5*e^2*
x^4 - (2*b^6*c^4 + 54*a*b^4*c^5 - 248*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \text{Hanged}$$

input
```
int(x^5/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x^5}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \int \frac{x^5}{(e\,x^4+d)\,(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^5/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^5/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.113 $\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 759

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$$

$$= \frac{x^2(c(b^2-2ac)\,d-b(b^2-3ac)\,e+c(bcd-b^2e+2ace)\,x^4)}{4a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,(a+bx^4+cx^8)}$$

$$+ \frac{\sqrt{c}\left(b^4de^2-2ac\left(6c^2d^3-5a\sqrt{b^2-4ac}e^3-cde\left(\sqrt{b^2-4ac}d-14ae\right)\right)-b^3e\left(2cd^2-e\left(\sqrt{b^2-4ac}d-3\right.\right.\right.}{4\sqrt{2}a\,(}$$

$$- \frac{\sqrt{c}\left(b^4de^2+b^2\left(c^2d^3+3a\sqrt{b^2-4ac}e^3+cde\left(2\sqrt{b^2-4ac}d-3ae\right)\right)-b^3e\left(2cd^2+e\left(\sqrt{b^2-4ac}d+3ae\right)\right.\right.}{4\sqrt{2}a\,(}$$

$$+ \frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{2\sqrt{d}\,(cd^2-bde+ae^2)^2}$$

output
```
1/4*x^2*(c*(-2*a*c+b^2)*d-b*(-3*a*c+b^2)*e+c*(2*a*c*e-b^2*e+b*c*d)*x^4)/a/
(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^8+b*x^4+a)+1/8*c^(1/2)*(b^4*d*e^2-2*
a*c*(6*c^2*d^3-5*a*(-4*a*c+b^2)^(1/2)*e^3-c*d*e*((-4*a*c+b^2)^(1/2)*d-14*a
*e))-b^3*e*(2*c*d^2-e*((-4*a*c+b^2)^(1/2)*d-3*a*e))+b^2*(c^2*d^3-3*a*(-4*a
*c+b^2)^(1/2)*e^3-c*d*e*(2*(-4*a*c+b^2)^(1/2)*d+3*a*e))-b*c*(a*e^2*((-4*a*
c+b^2)^(1/2)*d-16*a*e)-c*d^2*((-4*a*c+b^2)^(1/2)*d+20*a*e)))*arctan(2^(1/2
)*c^(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(3/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^2-b*d*e+c*d^2)^2-1/8*c^(1/2)*(b^4*d*e^2+b
^2*(c^2*d^3+3*a*(-4*a*c+b^2)^(1/2)*e^3+c*d*e*(2*(-4*a*c+b^2)^(1/2)*d-3*a*e
))-b^3*e*(2*c*d^2+e*((-4*a*c+b^2)^(1/2)*d+3*a*e))-2*a*c*(6*c^2*d^3+5*a*(-4
*a*c+b^2)^(1/2)*e^3+c*d*e*((-4*a*c+b^2)^(1/2)*d+14*a*e))-b*c*(c*d^2*((-4*a
*c+b^2)^(1/2)*d-20*a*e)-a*e^2*((-4*a*c+b^2)^(1/2)*d+16*a*e)))*arctan(2^(1/
2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(3/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2)/(c*d^2-e*(-a*e+b*d))^2+1/2*e^(7/2)*arctan(e^(
1/2)*x^2/d^(1/2))/d^(1/2)/(a*e^2-b*d*e+c*d^2)^2
```

Mathematica [A] (verified)

Time = 2.86 (sec) , antiderivative size = 716, normalized size of antiderivative = 0.94

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$$

$$= \frac{\frac{2(cd^2+e(-bd+ae))x^2(b^3e-bc(3ae+cdx^4)+2ac^2(d-ex^4)+b^2c(-d+ex^4))}{a(-b^2+4ac)(a+bx^4+cx^8)} + \frac{\sqrt{2}\sqrt{c}\left(b^4de^2+2ac\left(-6c^2d^3+5a\sqrt{b^2-4ac}e^3+cde\left(\sqrt{b^2-4ac}d-1\right.\right.\right.}{}}{}$$

input
```
Integrate[x/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((2*(c*d^2 + e*(-(b*d) + a*e))*x^2*(b^3*e - b*c*(3*a*e + c*d*x^4) + 2*a*c^
2*(d - e*x^4) + b^2*c*(-d + e*x^4)))/(a*(-b^2 + 4*a*c)*(a + b*x^4 + c*x^8)
) + (Sqrt[2]*Sqrt[c]*(b^4*d*e^2 + 2*a*c*(-6*c^2*d^3 + 5*a*Sqrt[b^2 - 4*a*c
]*e^3 + c*d*e*(Sqrt[b^2 - 4*a*c]*d - 14*a*e)) + b^3*e*(-2*c*d^2 + e*(Sqrt[
b^2 - 4*a*c]*d - 3*a*e)) + b^2*(c^2*d^3 - 3*a*Sqrt[b^2 - 4*a*c]*e^3 - c*d*
e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + b*c*(a*e^2*(-(Sqrt[b^2 - 4*a*c]*d) +
16*a*e) + c*d^2*(Sqrt[b^2 - 4*a*c]*d + 20*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x
^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^4*d*e^2) - b^2*(c^2*d^3 + 3*a*Sqrt[b^2
 - 4*a*c]*e^3 + c*d*e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e)) + b^3*e*(2*c*d^2 +
e*(Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + 2*a*c*(6*c^2*d^3 + 5*a*Sqrt[b^2 - 4*a*c
]*e^3 + c*d*e*(Sqrt[b^2 - 4*a*c]*d + 14*a*e)) + b*c*(c*d^2*(Sqrt[b^2 - 4*a
*c]*d - 20*a*e) - a*e^2*(Sqrt[b^2 - 4*a*c]*d + 16*a*e)))*ArcTan[(Sqrt[2]*S
qrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b +
Sqrt[b^2 - 4*a*c]]) + (4*e^(7/2)*ArcTan[(Sqrt[e]*x^2)/Sqrt[d]])/Sqrt[d])/(
8*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 676, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1814, 1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)^2}dx$$

$$\Big\downarrow \text{1814}$$

$$\frac{1}{2}\int \frac{1}{(ex^4+d)(cx^8+bx^4+a)^2}dx^2$$

$$\Big\downarrow \text{1567}$$

$$\frac{1}{2}\int \left(\frac{e^4}{(cd^2-bed+ae^2)^2(ex^4+d)}-\frac{(cex^4-cd+be)e^2}{(cd^2-bed+ae^2)^2(cx^8+bx^4+a)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\sqrt{c}e^2\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)^2}-\frac{\sqrt{c}e^2\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)^2}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{\ }}{\sqrt{b-}}\right.}{2\sqrt{2}}\right.$$

input
```
Int[x/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((x^2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*
e)*x^4))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x^4 + c*x^8)) -
 (Sqrt[c]*e^2*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*
(c*d^2 - b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e + (b^2*c*d
- 12*a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b -
Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*e^2*(e + (2*c*d - b
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) +
(Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e - (b^2*c*d - 12*a*c^2*d - b^3*e + 8*a*b*
c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b
*d*e + a*e^2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x^2)/Sqrt[d]])/(Sqrt[d]*(c*d^2 -
 b*d*e + a*e^2)^2))/2
```

Defintions of rubi rules used

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 859, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\frac{\frac{c\left(2a^2 c\, e^3-a\, b^2 e^3-abcd\, e^2+2a\, c^2 d^2 e+b^3 d\, e^2-2b^2 c\, d^2 e+b\, c^2 d^3\right)x^6}{2a\left(4ac-b^2\right)}+\frac{\left(3a^2 bc\, e^3-2a^2 c^2 e^2 d-a\, b^3 e^3-2a\, b^2 cd\, e^2+5ab\, c^2 d^2 e-2a\, c^3 d^3+b^4 d\, e^2-\right.}{2a\left(4ac-b^2\right)}}{c\, x^8+b\, x^4+a}}{}$
risch	Expression too large to display

input
```
int(x/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*e^2-b*d*e+c*d^2)^2*((1/2*c*(2*a^2*c*e^3-a*b^2*e^3-a*b*c*d*e^2+2*a*
c^2*d^2*e+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)/a/(4*a*c-b^2)*x^6+1/2*(3*a^2*
b*c*e^3-2*a^2*c^2*d*e^2-a*b^3*e^3-2*a*b^2*c*d*e^2+5*a*b*c^2*d^2*e-2*a*c^3*
d^3+b^4*d*e^2-2*b^3*c*d^2*e+b^2*c^2*d^3)/a/(4*a*c-b^2)*x^2)/(c*x^8+b*x^4+a
)+2/a/(4*a*c-b^2)*c*(-1/8*(10*a^2*c*e^3*(-4*a*c+b^2)^(1/2)-3*a*b^2*e^3*(-4
*a*c+b^2)^(1/2)-a*b*c*d*e^2*(-4*a*c+b^2)^(1/2)+2*a*c^2*d^2*e*(-4*a*c+b^2)^
(1/2)+b^3*d*e^2*(-4*a*c+b^2)^(1/2)-2*b^2*c*d^2*e*(-4*a*c+b^2)^(1/2)+b*c^2*
d^3*(-4*a*c+b^2)^(1/2)+16*a^2*b*c*e^3-28*a^2*c^2*e^2*d-3*a*b^3*e^3-3*a*b^2
*c*d*e^2+20*a*b*c^2*d^2*e-12*a*c^3*d^3+b^4*d*e^2-2*b^3*c*d^2*e+b^2*c^2*d^3
)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(10*a^2*c*e^3*(-4*a*c+b^
2)^(1/2)-3*a*b^2*e^3*(-4*a*c+b^2)^(1/2)-a*b*c*d*e^2*(-4*a*c+b^2)^(1/2)+2*a
*c^2*d^2*e*(-4*a*c+b^2)^(1/2)+b^3*d*e^2*(-4*a*c+b^2)^(1/2)-2*b^2*c*d^2*e*(
-4*a*c+b^2)^(1/2)+b*c^2*d^3*(-4*a*c+b^2)^(1/2)-16*a^2*b*c*e^3+28*a^2*c^2*e
^2*d+3*a*b^3*e^3+3*a*b^2*c*d*e^2-20*a*b*c^2*d^2*e+12*a*c^3*d^3-b^4*d*e^2+2
*b^3*c*d^2*e-b^2*c^2*d^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))+1/2*e
^4/(a*e^2-b*d*e+c*d^2)^2/(d*e)^(1/2)*arctan(e*x^2/(d*e)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37269 vs. 2(678) = 1356.

Time = 9.00 (sec) , antiderivative size = 37269, normalized size of antiderivative = 49.10

$$\int \frac{x}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/2*e^4*arctan(e*x^2/sqrt(d*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*
a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(d*e)) + 1/32*((2*a^2*b^7*c^8 - 4
0*a^3*b^5*c^9 + 224*a^4*b^3*c^10 - 384*a^5*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 20*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 - 112*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 32*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^8 + 192*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 + 96*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 + 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^9 - 48*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^10 - 2*(b^2 - 4*a*c)*a^2*
b^5*c^8 + 32*(b^2 - 4*a*c)*a^3*b^3*c^9 - 96*(b^2 - 4*a*c)*a^4*b*c^10)*d^11
 - 2*(6*a^2*b^8*c^7 - 116*a^3*b^6*c^8 + 640*a^4*b^4*c^9 - 1088*a^5*b^2*c^1
0 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^
5 + 58*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c
^6 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c
^6 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4
*c^7 - 92*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Hanged}$$

input
```
int(x/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x}{\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx = \int \frac{x}{\left(e\,x^4+d\right)\left(c\,x^8+b\,x^4+a\right)^2}dx$$

input
```
int(x/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.114 $\int \frac{1}{x^3(d+ex^4)(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 928

$$\int \frac{1}{x^3\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx = -\frac{1}{2a^2dx^2}$$
$$-\frac{x^2(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e+c(b^2cd-2ac^2d-b^3e+3abce)\,x^4)}{4a^2\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx^4+cx^8\right)}$$
$$-\frac{\sqrt{c}\left(3b^5de^2-b^4e\left(6cd^2-e\left(3\sqrt{b^2-4ac}d-5ae\right)\right)-abc\left(16c^2d^3-19a\sqrt{b^2-4ac}e^3-3cde\left(7\sqrt{b^2-4ac}\right.\right.\right.}{}$$
$$+\frac{\sqrt{c}\left(3b^5de^2+2ac^2\left(ae^2\left(9\sqrt{b^2-4ac}d-14ae\right)+cd^2\left(5\sqrt{b^2-4ac}d-6ae\right)\right)+b^3\left(3c^2d^3+5a\sqrt{b^2-4ac}e\right.\right.}{}$$
$$-\frac{e^{9/2}\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2-bde+ae^2\right)^2}$$

output
```
-1/2/a^2/d/x^2-1/4*x^2*(b^3*c*d-3*a*b*c^2*d-b^4*e+4*a*b^2*c*e-2*a^2*c^2*e+
c*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*x^4)/a^2/(-4*a*c+b^2)/(a*e^2-b*d*e+c
*d^2)/(c*x^8+b*x^4+a)-1/8*c^(1/2)*(3*b^5*d*e^2-b^4*e*(6*c*d^2-e*(3*(-4*a*c
+b^2)^(1/2)*d-5*a*e))-a*b*c*(16*c^2*d^3-19*a*(-4*a*c+b^2)^(1/2)*e^3-3*c*d*
e*(7*(-4*a*c+b^2)^(1/2)*d-4*a*e))+b^3*(3*c^2*d^3-5*a*(-4*a*c+b^2)^(1/2)*e^
3-6*c*d*e*((-4*a*c+b^2)^(1/2)*d+2*a*e))-b^2*c*(a*e^2*(6*(-4*a*c+b^2)^(1/2)
*d-29*a*e)-3*c*d^2*((-4*a*c+b^2)^(1/2)*d+11*a*e))-2*a*c^2*(c*d^2*(5*(-4*a*
c+b^2)^(1/2)*d+6*a*e)+a*e^2*(9*(-4*a*c+b^2)^(1/2)*d+14*a*e)))*arctan(2^(1/
2)*c^(1/2)*x^2/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(3/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(c*d^2-e*(-a*e+b*d))^2+1/8*c^(1/2)*(3*b^5*d
*e^2+2*a*c^2*(a*e^2*(9*(-4*a*c+b^2)^(1/2)*d-14*a*e)+c*d^2*(5*(-4*a*c+b^2)^
(1/2)*d-6*a*e))+b^3*(3*c^2*d^3+5*a*(-4*a*c+b^2)^(1/2)*e^3+6*c*d*e*((-4*a*c
+b^2)^(1/2)*d-2*a*e))-a*b*c*(16*c^2*d^3+19*a*(-4*a*c+b^2)^(1/2)*e^3+3*c*d*
e*(7*(-4*a*c+b^2)^(1/2)*d+4*a*e))-b^4*e*(6*c*d^2+e*(3*(-4*a*c+b^2)^(1/2)*d
+5*a*e))-b^2*c*(3*c*d^2*((-4*a*c+b^2)^(1/2)*d-11*a*e)-a*e^2*(6*(-4*a*c+b^2
)^(1/2)*d+29*a*e)))*arctan(2^(1/2)*c^(1/2)*x^2/(b+(-4*a*c+b^2)^(1/2))^(1/2
))*2^(1/2)/a^2/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(c*d^2-e*(-
a*e+b*d))^2-1/2*e^(9/2)*arctan(e^(1/2)*x^2/d^(1/2))/d^(3/2)/(a*e^2-b*d*e+c
*d^2)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.16 (sec) , antiderivative size = 702, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3 (d+ex^4) (a+bx^4+cx^8)^2} dx = \frac{1}{8} \Bigg(-\frac{4}{a^2 d x^2} - \frac{2x^2(b^4 e + 2ac^2(ae+cdx^4) - b^2c(4ae+cdx^4) + 3abc^2(d-ex^4) + b^3c(-d+ex^4))}{a^2(b^2-4ac)(-cd^2+e(bd-ae))(a+bx^4+cx^8)} + \frac{4e^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{d^{3/2}(cd^2+e(-bd+ae))^2} + \frac{4e^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{d^{3/2}(cd^2+e(-bd+ae))^2} + \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{3b^3c^2d^3\log(x-\#1)-13abc^3d^3\log(x-\#1)-6b^4cd^2e\log(x-\#1)+27ab^2c^2d^2e\log(x-\#1)-6\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/(x^3*(d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(-4/(a^2*d*x^2) - (2*x^2*(b^4*e + 2*a*c^2*(a*e + c*d*x^4) - b^2*c*(4*a*e +
 c*d*x^4) + 3*a*b*c^2*(d - e*x^4) + b^3*c*(-d + e*x^4)))/(a^2*(b^2 - 4*a*c
)*(-(c*d^2) + e*(b*d - a*e))*(a + b*x^4 + c*x^8)) + (4*e^(9/2)*ArcTan[1 -
(Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(d^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2) + (4*
e^(9/2)*ArcTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(d^(3/2)*(c*d^2 + e*(-(b*
d) + a*e))^2) + RootSum[a + b*#1^4 + c*#1^8 & , (3*b^3*c^2*d^3*Log[x - #1]
 - 13*a*b*c^3*d^3*Log[x - #1] - 6*b^4*c*d^2*e*Log[x - #1] + 27*a*b^2*c^2*d
^2*e*Log[x - #1] - 6*a^2*c^3*d^2*e*Log[x - #1] + 3*b^5*d*e^2*Log[x - #1] -
 9*a*b^3*c*d*e^2*Log[x - #1] - 15*a^2*b*c^2*d*e^2*Log[x - #1] - 5*a*b^4*e^
3*Log[x - #1] + 24*a^2*b^2*c*e^3*Log[x - #1] - 14*a^3*c^2*e^3*Log[x - #1]
+ 3*b^2*c^3*d^3*Log[x - #1]*#1^4 - 10*a*c^4*d^3*Log[x - #1]*#1^4 - 6*b^3*c
^2*d^2*e*Log[x - #1]*#1^4 + 21*a*b*c^3*d^2*e*Log[x - #1]*#1^4 + 3*b^4*c*d*
e^2*Log[x - #1]*#1^4 - 6*a*b^2*c^2*d*e^2*Log[x - #1]*#1^4 - 18*a^2*c^3*d*e
^2*Log[x - #1]*#1^4 - 5*a*b^3*c*e^3*Log[x - #1]*#1^4 + 19*a^2*b*c^2*e^3*Lo
g[x - #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(a^2*(-b^2 + 4*a*c)*(c*d^2 + e*(-(
b*d) + a*e))^2))/8
```

Rubi [A] (verified)

Time = 4.15 (sec) , antiderivative size = 888, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1814, 1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (d+ex^4) (a+bx^4+cx^8)^2} \, dx$$

$$\Big\downarrow \text{ 1814}$$

$$\frac{1}{2} \int \frac{1}{x^4 (ex^4+d) (cx^8+bx^4+a)^2} dx^2$$

$$\Big\downarrow \text{ 1674}$$

$$\frac{1}{2}\int\left(-\frac{e^5}{d\left(cd^2-bed+ae^2\right)^2\left(ex^4+d\right)}+\frac{-c(cd-be)\left(cd^2-e(bd-2ae)\right)x^4-a^2ce^3-b^3de^2-bcd\left(cd^2+2ae^2\right)}{a^2\left(cd^2-bed+ae^2\right)^2\left(cx^8+bx^4+a\right)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{\sqrt{e}x^2}{\sqrt{d}}\right)e^{9/2}}{d^{3/2}\left(cd^2-bed+ae^2\right)^2}-\frac{\sqrt{c}\left(-eb^3+cdb^2+3aceb-2ac^2d+\frac{-eb^4+cdb^3+9aceb^2-8ac^2db-12a^2c^2e}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{}}{\sqrt{b}}\right.}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)}\right.$$

input
```
Int[1/(x^3*(d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(-(1/(a^2*d*x^2)) - (x^2*(b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*
a^2*c^2*e + c*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*x^4))/(2*a^2*(b^2
- 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x^4 + c*x^8)) - (Sqrt[c]*(b^2*c*d
- 2*a*c^2*d - b^3*e + 3*a*b*c*e + (b^3*c*d - 8*a*b*c^2*d - b^4*e + 9*a*b^2
*c*e - 12*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 -
4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*((c*d - b*e)*(c*d^2 - e*(b*d -
 2*a*e)) + (b^3*d*e^2 + 2*a^2*c*e^3 + b*c*d*(c*d^2 + 2*a*e^2) - 2*b^2*(c*d
^2*e + a*e^3))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e
 + a*e^2)^2) - (Sqrt[c]*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e - (b^3*c*
d - 8*a*b*c^2*d - b^4*e + 9*a*b^2*c*e - 12*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(
b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[
c]*((c*d - b*e)*(c*d^2 - e*(b*d - 2*a*e)) - (b^3*d*e^2 + 2*a^2*c*e^3 + b*c
*d*(c*d^2 + 2*a*e^2) - 2*b^2*(c*d^2*e + a*e^3))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b +
Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (e^(9/2)*ArcTan[(Sqrt[e]*x
^2)/Sqrt[d]])/(d^(3/2)*(c*d^2 - b*d*e + a*e^2)^2))/2
```

Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 7.33 (sec) , antiderivative size = 1081, normalized size of antiderivative = 1.16

method	result	size
default	Expression too large to display	1081
risch	Expression too large to display	24170

input
```
int(1/x^3/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*e^2-b*d*e+c*d^2)^2/a^2*((-1/2*c*(3*a^2*b*c*e^3-2*a^2*c^2*d*e^2-a*b
^3*e^3-2*a*b^2*c*d*e^2+5*a*b*c^2*d^2*e-2*a*c^3*d^3+b^4*d*e^2-2*b^3*c*d^2*e
+b^2*c^2*d^3)/(4*a*c-b^2)*x^6+1/2*(2*a^3*c^2*e^3-4*a^2*b^2*c*e^3+a^2*b*c^2
*d*e^2+2*a^2*c^3*d^2*e+a*b^4*e^3+3*a*b^3*c*d*e^2-7*a*b^2*c^2*d^2*e+3*a*b*c
^3*d^3-b^5*d*e^2+2*b^4*c*d^2*e-b^3*c^2*d^3)/(4*a*c-b^2)*x^2)/(c*x^8+b*x^4+
a)+2/(4*a*c-b^2)*c*(-1/8*(-19*a^2*b*c*e^3*(-4*a*c+b^2)^(1/2)+18*a^2*c^2*e^
2*d*(-4*a*c+b^2)^(1/2)+5*a*b^3*e^3*(-4*a*c+b^2)^(1/2)+6*a*b^2*c*d*e^2*(-4*
a*c+b^2)^(1/2)-21*a*b*c^2*d^2*e*(-4*a*c+b^2)^(1/2)+10*a*c^3*d^3*(-4*a*c+b^
2)^(1/2)-3*b^4*d*e^2*(-4*a*c+b^2)^(1/2)+6*b^3*c*d^2*e*(-4*a*c+b^2)^(1/2)-3
*b^2*c^2*d^3*(-4*a*c+b^2)^(1/2)+28*a^3*c^2*e^3-29*a^2*b^2*c*e^3+12*a^2*b*c
^2*d*e^2+12*a^2*c^3*d^2*e+5*a*b^4*e^3+12*a*b^3*c*d*e^2-33*a*b^2*c^2*d^2*e+
16*a*b*c^3*d^3-3*b^5*d*e^2+6*b^4*c*d^2*e-3*b^3*c^2*d^3)/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-19*a^2*b*c*e^3*(-4*a*c+b^2)^(1/2)+18*a^2*c
^2*e^2*d*(-4*a*c+b^2)^(1/2)+5*a*b^3*e^3*(-4*a*c+b^2)^(1/2)+6*a*b^2*c*d*e^2
*(-4*a*c+b^2)^(1/2)-21*a*b*c^2*d^2*e*(-4*a*c+b^2)^(1/2)+10*a*c^3*d^3*(-4*a
*c+b^2)^(1/2)-3*b^4*d*e^2*(-4*a*c+b^2)^(1/2)+6*b^3*c*d^2*e*(-4*a*c+b^2)^(1
/2)-3*b^2*c^2*d^3*(-4*a*c+b^2)^(1/2)-28*a^3*c^2*e^3+29*a^2*b^2*c*e^3-12*a^
2*b*c^2*d*e^2-12*a^2*c^3*d^2*e-5*a*b^4*e^3-12*a*b^3*c*d*e^2+33*a*b^2*c^2*d
^2*e-16*a*b*c^3*d^3+3*b^5*d*e^2-6*b^4*c*d^2*e+3*b^3*c^2*d^3)/(-4*a*c+b^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (d+ex^4) (a+bx^4+cx^8)^2} dx = \text{Timed out}$$

input
```
integrate(1/x^3/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (d+ex^4) (a+bx^4+cx^8)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (d+ex^4) (a+bx^4+cx^8)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28419 vs. $2(836) = 1672$.

Time = 6.79 (sec) , antiderivative size = 28419, normalized size of antiderivative = 30.62

$$\int \frac{1}{x^3 (d+ex^4) (a+bx^4+cx^8)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/2*e^5*arctan(e*x^2/sqrt(d*e))/((c^2*d^5 - 2*b*c*d^4*e + b^2*d^3*e^2 + 2
*a*c*d^3*e^2 - 2*a*b*d^2*e^3 + a^2*d*e^4)*sqrt(d*e)) - 1/16*((6*a^2*b^5*c^
7 - 44*a^3*b^3*c^8 + 80*a^4*b*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^5 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^6 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^7 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 - 6*(b^2 - 4*a*c)*a^2*b^3*c^7 + 20*(b^2
- 4*a*c)*a^3*b*c^8)*d^7*x^4 - (24*a^2*b^6*c^6 - 178*a^3*b^4*c^7 + 328*a^4*
b^2*c^8 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^6*c^4 + 89*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^4*c^5 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^5*c^5 - 164*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^4*b^2*c^6 - 82*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^3*c^6 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^4*c^6 + 41*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b^2*c^7 - 24*(b^2 - 4*a*c)*a^2*b^4*c^6 + 82*(b^2 - 4*a*c)*a^3*b^2*
c^7)*d^6*e*x^4 + 2*(18*a^2*b^7*c^5 - 124*a^3*b^5*c^6 + 170*a^4*b^3*c^7 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (d + ex^4) (a + bx^4 + cx^8)^2} \, dx = \text{Hanged}$$

input
```
int(1/(x^3*(d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{x^3 (d + ex^4) (a + bx^4 + cx^8)^2} \, dx = \int \frac{1}{x^3 (e\,x^4 + d) (c\,x^8 + b\,x^4 + a)^2} dx$$

input
```
int(1/x^3/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(1/x^3/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.115 $\int \frac{x^6}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 1267

$$\int \frac{x^6}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

output
```
1/5*d*e*(-5*b*e+c*d)*x^3/(a*e^2-b*d*e+c*d^2)^2/(c*x^8+b*x^4+a)-c*d*e^2*x^7
/(a*e^2-b*d*e+c*d^2)^2/(c*x^8+b*x^4+a)-1/20*x^3*(b*c*d*(5*c*d^2-9*e*(-5*a*
e+b*d))+(-2*a*c+b^2)*e*(3*c*d^2-5*e*(a*e+3*b*d))+5*c*(2*c^2*d^3-3*c*d*e*(-
6*a*e+b*d)-b*e^2*(a*e+3*b*d))*x^4)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x
^8+b*x^4+a)-1/16*c^(1/4)*(2*c^2*d^3-3*c*d*e*(-6*a*e+b*d)-b*e^2*(a*e+3*b*d)
+(3*b^3*d*e^2+4*a*c*e*(-5*a*e^2+3*c*d^2)+4*b*c*d*(3*a*e^2+2*c*d^2)-b^2*(-a
*e^3+15*c*d^2*e))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c
+b^2)^(1/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(a*
e^2-b*d*e+c*d^2)^2-1/16*c^(1/4)*(2*c^2*d^3-3*c*d*e*(-6*a*e+b*d)-b*e^2*(a*e
+3*b*d)-(3*b^3*d*e^2+4*a*c*e*(-5*a*e^2+3*c*d^2)+4*b*c*d*(3*a*e^2+2*c*d^2)-
b^2*(-a*e^3+15*c*d^2*e))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+
(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1
/4)/(a*e^2-b*d*e+c*d^2)^2-1/4*d^(3/4)*e^(9/4)*arctan(-1+2^(1/2)*e^(1/4)*x/
d^(1/4))*2^(1/2)/(a*e^2-b*d*e+c*d^2)^2-1/4*d^(3/4)*e^(9/4)*arctan(1+2^(1/2
)*e^(1/4)*x/d^(1/4))*2^(1/2)/(a*e^2-b*d*e+c*d^2)^2+1/16*c^(1/4)*(2*c^2*d^3
-3*c*d*e*(-6*a*e+b*d)-b*e^2*(a*e+3*b*d)+(3*b^3*d*e^2+4*a*c*e*(-5*a*e^2+3*c
*d^2)+4*b*c*d*(3*a*e^2+2*c*d^2)-b^2*(-a*e^3+15*c*d^2*e))/(-4*a*c+b^2)^(1/2
))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-4*a*
c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(a*e^2-b*d*e+c*d^2)^2+1/16*c^(1/4)*(2
*c^2*d^3-3*c*d*e*(-6*a*e+b*d)-b*e^2*(a*e+3*b*d)-(3*b^3*d*e^2+4*a*c*e*(-...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.59 (sec) , antiderivative size = 532, normalized size of antiderivative = 0.42

$$\int \frac{x^6}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx$$

$$= \frac{-\frac{4(cd^2+e(-bd+ae))x^3(-b^2e+2c(ae+cdx^4)+bc(d-ex^4))}{(b^2-4ac)(a+bx^4+cx^8)}+4\sqrt{2}d^{3/4}e^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)-4\sqrt{2}d^{3/4}e^{9/4}\arctan}{}$$

input
```
Integrate[x^6/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((-4*(c*d^2 + e*(-(b*d) + a*e))*x^3*(-(b^2*e) + 2*c*(a*e + c*d*x^4) + b*c*
(d - e*x^4)))/((b^2 - 4*a*c)*(a + b*x^4 + c*x^8)) + 4*Sqrt[2]*d^(3/4)*e^(9
/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)] - 4*Sqrt[2]*d^(3/4)*e^(9/4)*Ar
cTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)] - 2*Sqrt[2]*d^(3/4)*e^(9/4)*Log[Sqrt
[d] - Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2] + 2*Sqrt[2]*d^(3/4)*e^(9/4)
*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2] + RootSum[a + b*#1
^4 + c*#1^8 & , (3*b*c^2*d^3*Log[x - #1] - 6*b^2*c*d^2*e*Log[x - #1] + 6*a
*c^2*d^2*e*Log[x - #1] + 3*b^3*d*e^2*Log[x - #1] - 3*a*b*c*d*e^2*Log[x - #
1] + a*b^2*e^3*Log[x - #1] - 10*a^2*c*e^3*Log[x - #1] - 2*c^3*d^3*Log[x -
#1]*#1^4 + 3*b*c^2*d^2*e*Log[x - #1]*#1^4 + 3*b^2*c*d*e^2*Log[x - #1]*#1^4
 - 18*a*c^2*d*e^2*Log[x - #1]*#1^4 + a*b*c*e^3*Log[x - #1]*#1^4)/(b*#1 + 2
*c*#1^5) & ]/(b^2 - 4*a*c))/(16*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Rubi [A] (verified)

Time = 2.86 (sec) , antiderivative size = 1316, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1860, 1824, 25, 27, 1828, 1834, 27, 827, 218, 221, 1836, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(d+ex^4)(a+bx^4+cx^8)^2}dx$$

$$\Big\downarrow \text{1860}$$

$$\frac{d^2\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}-\frac{\int \frac{(bd-ae)x^4+ad}{x^2(cx^8+bx^4+a)^2}dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \text{1824}$$

$$\frac{d^2\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}-\frac{-\frac{\int -\frac{a\left(-5c(bd-2ae)x^4+4b^2d-18acd+abe\right)}{x^2\left(cx^8+bx^4+a\right)}dx}{4a(b^2-4ac)}-\frac{cx^4(bd-2ae)+a(2cd-be)}{4x(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}-\frac{\frac{\int \frac{a\left(-5c(bd-2ae)x^4+4b^2d-18acd+abe\right)}{x^2\left(cx^8+bx^4+a\right)}dx}{4a(b^2-4ac)}-\frac{cx^4(bd-2ae)+a(2cd-be)}{4x(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{d^2\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}-\frac{\frac{\int \frac{-5c(bd-2ae)x^4+4b^2d-18acd+abe}{x^2\left(cx^8+bx^4+a\right)}dx}{4(b^2-4ac)}-\frac{cx^4(bd-2ae)+a(2cd-be)}{4x(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}$$

$$\Big\downarrow \ 1828$$

$$\frac{d^2\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}-\frac{\frac{-\frac{\int \frac{x^2\left(c\left(4db^2+aeb-18acd\right)x^4+4b^3d-13abcd+ab^2e-10a^2ce\right)}{cx^8+bx^4+a}dx}{a}-\frac{abe-18acd+4b^2d}{ax}}{4(b^2-4ac)}-\frac{cx^4(bd-2ae)+a(2cd-be)}{4x(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}$$

$$\Big\downarrow \ 1834$$

$$\frac{d^2\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}-\frac{\frac{-\frac{\frac{1}{2}c\left(\frac{-20a^2ce+ab^2e-8abcd+4b^3d}{\sqrt{b^2-4ac}}+abe-18acd+4b^2d\right)\int\frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}c\left(-\frac{-20a^2ce+ab^2e-8abcd+4b^3d}{\sqrt{b^2-4ac}}+abe-18acd+4b^2d\right)\int\frac{2x^2}{2cx^4+b+\sqrt{b^2-4ac}}\cdots}{a}}{4(b^2-4ac)}\cdots}{ae^2-bde+cd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{d^2\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}-\frac{\frac{-\frac{c\left(\frac{-20a^2ce+ab^2e-8abcd+4b^3d}{\sqrt{b^2-4ac}}+abe-18acd+4b^2d\right)\int\frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+c\left(-\frac{-20a^2ce+ab^2e-8abcd+4b^3d}{\sqrt{b^2-4ac}}+abe-18acd+4b^2d\right)\int\frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx\cdots}{a}}{4(b^2-4ac)}\cdots}{ae^2-bde+cd^2}$$

$$\Big\downarrow \ 827$$

$$\frac{d^2\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}-\frac{\frac{-\frac{c\left(-\frac{-20a^2ce+ab^2e-8abcd+4b^3d}{\sqrt{b^2-4ac}}+abe-18acd+4b^2d\right)\left(\frac{\int\frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{-20a^2ce+ab^2e-8abcd+4b^3d}{\sqrt{b^2-4ac}}\cdots\right.}{a}}{4(b^2-4ac)}\cdots}{ae^2-bde+cd^2\cdots}$$

$$\Big\downarrow \ 218$$

$$\frac{d^2\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2} - \frac{-\frac{c\left(-\frac{-20a^2ce+ab^2e-8abcd+4b^3d}{\sqrt{b^2-4ac}}+abe-18acd+4b^2d\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{-20a^2ce+ab^2e-8abc\cdots}{\sqrt{b^2-4ac}}\right.}{a}}{4(b^2-4ac)}}{ae^2-bde\cdots}$$

$\downarrow$ 221

$$\frac{d^2\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2} - \frac{-\frac{c\left(-\frac{-20a^2ce+ab^2e-8abcd+4b^3d}{\sqrt{b^2-4ac}}+abe-18acd+4b^2d\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(\frac{-20a^2ce\cdots}{}\right.}{a}}{4(b^2-4ac)}}{ae\cdots}$$

$\downarrow$ 1836

$$\frac{d^2\int \left(-\frac{x^2e^3}{d(cd^2-bed+ae^2)(ex^4+d)}+\frac{x^2\left(-c(cd-be)x^4-bcd+b^2e-ace\right)}{a(cd^2-bed+ae^2)(cx^8+bx^4+a)}+\frac{1}{adx^2}\right)dx}{ae^2-bde+cd^2} - \frac{-\frac{c\left(-\frac{-20a^2ce+ab^2e-8abcd+4b^3d}{\sqrt{b^2-4ac}}+abe-18acd+4b^2d\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(\frac{-20a^2ce\cdots}{}\right.}{a}}{4(b^2-4ac)}}{ae\cdots}$$

$\downarrow$ 2009

$$\frac{d^2\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)e^{9/4}}{2\sqrt{2}d^{5/4}(cd^2-bed+ae^2)}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}+1\right)e^{9/4}}{2\sqrt{2}d^{5/4}(cd^2-bed+ae^2)}-\frac{\log\left(\sqrt{e}x^2-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}\right)e^{9/4}}{4\sqrt{2}d^{5/4}(cd^2-bed+ae^2)}+\frac{\log\left(\sqrt{e}x^2+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}\right)e^{9/4}}{4\sqrt{2}d^{5/4}(cd^2-bed+ae^2)}\right.}{\frac{-\frac{4db^2+aeb-18acd}{ax}-\frac{c\left(4db^2+aeb-18acd-\frac{4db^3+aeb^2-8acdb-20a^2ce}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right.}{}}{4(b^2-4ac)}}{cd^2}$$

input
```
Int[x^6/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
-((-1/4*(a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x^4)/((b^2 - 4*a*c)*x*(a + b*x^
4 + c*x^8)) + (-((4*b^2*d - 18*a*c*d + a*b*e)/(a*x)) - (c*(4*b^2*d - 18*a*
c*d + a*b*e - (4*b^3*d - 8*a*b*c*d + a*b^2*e - 20*a^2*c*e)/Sqrt[b^2 - 4*a*
c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)
*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(
1/4))) + c*(4*b^2*d - 18*a*c*d + a*b*e + (4*b^3*d - 8*a*b*c*d + a*b^2*e -
20*a^2*c*e)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2
- 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcT
anh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)
*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/a)/(4*(b^2 - 4*a*c)))/(c*d^2 - b*d*e +
a*e^2)) + (d^2*(-(1/(a*d*x)) - (c^(1/4)*(c*d - b*e - (b*c*d - b^2*e + 2*a*
c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)*(c*d^2 - b*d*e + a*e
^2)) - (c^(1/4)*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*
ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(
-b + Sqrt[b^2 - 4*a*c])^(1/4)*(c*d^2 - b*d*e + a*e^2)) + (e^(9/4)*ArcTan[1
 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*d^(5/4)*(c*d^2 - b*d*e + a*e^2
)) - (e^(9/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*d^(5/4)*
(c*d^2 - b*d*e + a*e^2)) + (c^(1/4)*(c*d - b*e - (b*c*d - b^2*e + 2*a*c...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1824
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m
+ 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

rule 1836
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e
*x^n)^q/(a + b*x^n + c*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[q] && Int
egerQ[m]
```

rule 1860
```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_))
/((d_.) + (e_.)*(x_)^(n_)), x_Symbol] :> Simp[-f^(2*n)/(c*d^2 - b*d*e + a*e
^2)   Int[(f*x)^(m - 2*n)*(a*d + (b*d - a*e)*x^n)*(a + b*x^n + c*x^(2*n))^p
, x], x] + Simp[d^2*(f^(2*n)/(c*d^2 - b*d*e + a*e^2))   Int[(f*x)^(m - 2*n)
*((a + b*x^n + c*x^(2*n))^(p + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p,
-1] && GtQ[m, n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.25 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.37

method	result
default	$\frac{-\frac{c\left(ab\,e^{3}-2acd\,e^{2}-b^{2}d\,e^{2}+3bc\,d^{2}e-2c^{2}d^{3}\right)x^{7}}{4\left(4ac-b^{2}\right)}+\frac{\left(2a^{2}c\,e^{3}-a\,b^{2}e^{3}-abcd\,e^{2}+2a\,c^{2}d^{2}e+b^{3}d\,e^{2}-2b^{2}c\,d^{2}e+b\,c^{2}d^{3}\right)x^{3}}{16ac-4b^{2}}}{c\,x^{8}+b\,x^{4}+a}+\frac{\sum_{\_R=\text{RootOf}\left(\_Z^{8}c+\_Z\right.}}{\left(a\,e^{\cdots}\right.}$
risch	Expression too large to display

input
```
int(x^6/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2-b*d*e+c*d^2)^2*((-1/4*c*(a*b*e^3-2*a*c*d*e^2-b^2*d*e^2+3*b*c*d^2*
e-2*c^2*d^3)/(4*a*c-b^2)*x^7+1/4*(2*a^2*c*e^3-a*b^2*e^3-a*b*c*d*e^2+2*a*c^
2*d^2*e+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)/(4*a*c-b^2)*x^3)/(c*x^8+b*x^4+a
)+1/16/(4*a*c-b^2)*sum((c*(-a*b*e^3+18*a*c*d*e^2-3*b^2*d*e^2-3*b*c*d^2*e+2
*c^2*d^3)*_R^6+(10*a^2*c*e^3-a*b^2*e^3+3*a*b*c*d*e^2-6*a*c^2*d^2*e-3*b^3*d
*e^2+6*b^2*c*d^2*e-3*b*c^2*d^3)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf
(_Z^8*c+_Z^4*b+a))-1/8*d*e^2/(a*e^2-b*d*e+c*d^2)^2/(d/e)^(1/4)*2^(1/2)*(l
n((x^2-(d/e)^(1/4)*x*2^(1/2)+(d/e)^(1/2))/(x^2+(d/e)^(1/4)*x*2^(1/2)+(d/e)
^(1/2)))+2*arctan(2^(1/2)/(d/e)^(1/4)*x+1)+2*arctan(2^(1/2)/(d/e)^(1/4)*x-
1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^6/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**6/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^6}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \int \frac{x^6}{(cx^8+bx^4+a)^2(ex^4+d)}\,dx$$

input
```
integrate(x^6/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/8*d*e^3*(sqrt(2)*log(sqrt(e)*x^2 + sqrt(2)*d^(1/4)*e^(1/4)*x + sqrt(d))/
(d^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*x^2 - sqrt(2)*d^(1/4)*e^(1/4)*x +
sqrt(d))/(d^(1/4)*e^(3/4)) - sqrt(2)*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt
(d)*sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(
d)*sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4)))/(sqrt(-sqrt(d)*sqrt(e))*sqrt(e)) -
 sqrt(2)*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/
4)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4
)*e^(1/4)))/(sqrt(-sqrt(d)*sqrt(e))*sqrt(e)))/(c^2*d^4 - 2*b*c*d^3*e - 2*a
*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2) - 1/4*((2*c^2*d - b*c*e)*x^7 +
 (b*c*d - (b^2 - 2*a*c)*e)*x^3)/(((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*c - 4*a*b
*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^8 + ((b^3*c - 4*a*b*c^2)*d^2 - (b
^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^2*b*c)*e^2)*x^4 + (a*b^2*c - 4*a^2*c^2)
*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2) + 1/4*integrate(
-((2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 - 3*(b^2*c - 6*a*c^2)*d*e^2)*x^6
- (3*b*c^2*d^3 - 6*(b^2*c - a*c^2)*d^2*e + 3*(b^3 - a*b*c)*d*e^2 + (a*b^2
- 10*a^2*c)*e^3)*x^2)/(c*x^8 + b*x^4 + a), x)/((b^2*c^2 - 4*a*c^3)*d^4 - 2
*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*
b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Hanged}$$

input
```
int(x^6/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x^6}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \int \frac{x^6}{(e\,x^4+d)\,(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^6/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^6/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.116 $\int \frac{x^4}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 1265

$$\int \frac{x^4}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

output
```
-1/4*x*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x^4)/(-4*a*c+b^2)/(a*e^2-b*d*e+
c*d^2)/(c*x^8+b*x^4+a)+1/16*c^(3/4)*(b^3*d*e^2+b*(8*c^2*d^3-3*a*(-4*a*c+b^
2)^(1/2)*e^3-c*d*e*(9*(-4*a*c+b^2)^(1/2)*d-20*a*e))-b^2*e*(13*c*d^2+e*((-4
*a*c+b^2)^(1/2)*d-3*a*e))+2*c*(a*e^2*(11*(-4*a*c+b^2)^(1/2)*d-14*a*e)+c*d^
2*(3*(-4*a*c+b^2)^(1/2)*d+2*a*e)))*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^
2)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/
(a*e^2-b*d*e+c*d^2)^2-1/16*c^(3/4)*(b^3*d*e^2-b^2*e*(13*c*d^2-e*((-4*a*c+b
^2)^(1/2)*d+3*a*e))-2*c*(c*d^2*(3*(-4*a*c+b^2)^(1/2)*d-2*a*e)+a*e^2*(11*(-
4*a*c+b^2)^(1/2)*d+14*a*e))+b*(8*c^2*d^3+3*a*(-4*a*c+b^2)^(1/2)*e^3+c*d*e*
(9*(-4*a*c+b^2)^(1/2)*d+20*a*e)))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2
)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)/(
a*e^2-b*d*e+c*d^2)^2-1/4*d^(1/4)*e^(11/4)*arctan(-1+2^(1/2)*e^(1/4)*x/d^(1
/4))*2^(1/2)/(a*e^2-b*d*e+c*d^2)^2-1/4*d^(1/4)*e^(11/4)*arctan(1+2^(1/2)*e
^(1/4)*x/d^(1/4))*2^(1/2)/(a*e^2-b*d*e+c*d^2)^2+1/16*c^(3/4)*(b^3*d*e^2+b*
(8*c^2*d^3-3*a*(-4*a*c+b^2)^(1/2)*e^3-c*d*e*(9*(-4*a*c+b^2)^(1/2)*d-20*a*e
))-b^2*e*(13*c*d^2+e*((-4*a*c+b^2)^(1/2)*d-3*a*e))+2*c*(a*e^2*(11*(-4*a*c+
b^2)^(1/2)*d-14*a*e)+c*d^2*(3*(-4*a*c+b^2)^(1/2)*d+2*a*e)))*arctanh(2^(1/4
)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)^(3/2)/(-b-
(-4*a*c+b^2)^(1/2))^(3/4)/(a*e^2-b*d*e+c*d^2)^2-1/16*c^(3/4)*(b^3*d*e^2-b^
2*e*(13*c*d^2-e*((-4*a*c+b^2)^(1/2)*d+3*a*e))-2*c*(c*d^2*(3*(-4*a*c+b^2...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 7.45 (sec) , antiderivative size = 531, normalized size of antiderivative = 0.42

$$\int \frac{x^4}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$$

$$= \frac{-\frac{4(cd^2+e(-bd+ae))x(-b^2e+2c(ae+cdx^4)+bc(d-ex^4))}{(b^2-4ac)(a+bx^4+cx^8)}+4\sqrt{2}\sqrt[4]{d}e^{11/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)-4\sqrt{2}\sqrt[4]{d}e^{11/4}\arctan\left(\right.}{}$$

input
```
Integrate[x^4/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((-4*(c*d^2 + e*(-(b*d) + a*e))*x*(-(b^2*e) + 2*c*(a*e + c*d*x^4) + b*c*(d
 - e*x^4)))/((b^2 - 4*a*c)*(a + b*x^4 + c*x^8)) + 4*Sqrt[2]*d^(1/4)*e^(11/
4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)] - 4*Sqrt[2]*d^(1/4)*e^(11/4)*Ar
cTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)] + 2*Sqrt[2]*d^(1/4)*e^(11/4)*Log[Sqr
t[d] - Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2] - 2*Sqrt[2]*d^(1/4)*e^(11/
4)*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2] + RootSum[a + b*
#1^4 + c*#1^8 & , (b*c^2*d^3*Log[x - #1] - 2*b^2*c*d^2*e*Log[x - #1] + 2*a
*c^2*d^2*e*Log[x - #1] + b^3*d*e^2*Log[x - #1] - a*b*c*d*e^2*Log[x - #1] +
 3*a*b^2*e^3*Log[x - #1] - 14*a^2*c*e^3*Log[x - #1] - 6*c^3*d^3*Log[x - #1
]*#1^4 + 9*b*c^2*d^2*e*Log[x - #1]*#1^4 + b^2*c*d*e^2*Log[x - #1]*#1^4 - 2
2*a*c^2*d*e^2*Log[x - #1]*#1^4 + 3*a*b*c*e^3*Log[x - #1]*#1^4)/(b*#1^3 + 2
*c*#1^7) & ]/(b^2 - 4*a*c))/(16*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 1185, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {1862, 1754, 1760, 25, 27, 1752, 756, 218, 221, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex^4)(a+bx^4+cx^8)^2}dx$$

$$\Big\downarrow \text{1862}$$

$$\frac{\int \frac{cdx^4+ae}{(cx^8+bx^4+a)^2}dx}{ae^2-bde+cd^2}-\frac{de\int \frac{1}{(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \text{1754}$$

$$\frac{\int \frac{cdx^4+ae}{(cx^8+bx^4+a)^2}dx}{ae^2-bde+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \text{1760}$$

$$\frac{-\frac{\int -\frac{a\left(-3c(2cd-be)x^4+bcd+3b^2e-14ace\right)}{cx^8+bx^4+a}dx}{4a(b^2-4ac)}-\frac{x(2ace-b^2e+cx^4(2cd-be)+bcd)}{4(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx}{ae^2-bde+cd^2}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{a\left(-3c(2cd-be)x^4+bcd+3b^2e-14ace\right)}{cx^8+bx^4+a}dx}{4a(b^2-4ac)}-\frac{x\left(2ace-b^2e+cx^4(2cd-be)+bcd\right)}{4(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx}{ae^2-bde+cd^2}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{-3c(2cd-be)x^4+bcd+3b^2e-14ace}{cx^8+bx^4+a}dx}{4(b^2-4ac)}-\frac{x\left(2ace-b^2e+cx^4(2cd-be)+bcd\right)}{4(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx}{ae^2-bde+cd^2}$$

$\downarrow$ 1752

$$\frac{\frac{-\frac{1}{2}c\left(3(2cd-be)-\frac{-28ace+3b^2e+8bcd}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}c\left(\frac{-28ace+3b^2e+8bcd}{\sqrt{b^2-4ac}}+3(2cd-be)\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{4(b^2-4ac)}-\frac{x\left(2ace-b^2e\cdots\right.}{4(b^2-4\cdots}}{ae^2-bde+cd^2}$$

$$-\frac{de\int \left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx}{ae^2-bde+cd^2}$$

$\downarrow$ 756

$$\frac{\frac{-\frac{1}{2}c\left(\frac{-28ace+3b^2e+8bcd}{\sqrt{b^2-4ac}}+3(2cd-be)\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)-\frac{1}{2}c\left(3(2cd-be)-\frac{-28ace+3b^2e+8bcd}{\sqrt{b^2-4ac}}\right)\Big(\cdots}{4(b^2-4ac)}}{ae^2-bde+cd^2}$$

$$-\frac{de\int \left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx}{ae^2-bde+cd^2}$$

$\downarrow$ 218

$$\frac{\frac{-\frac{1}{2}c\left(\frac{-28ace+3b^2e+8bcd}{\sqrt{b^2-4ac}}+3(2cd-be)\right)\left(-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x^2}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)-\frac{1}{2}c\left(3(2cd-be)-\frac{-28ace+3b^2e+8}{\sqrt{b^2-4ac}}\right.}{4(b^2-4ac)}}{ae^2-bde+cd^2}$$

$$\frac{de\int\left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx}{ae^2-bde+cd^2}$$

$\downarrow$ 221

$$\frac{\frac{-\frac{1}{2}c\left(\frac{-28ace+3b^2e+8bcd}{\sqrt{b^2-4ac}}+3(2cd-be)\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)-\frac{1}{2}c\left(3(2cd-be)-\frac{-28a}{}\right.}{4(b^2-4ac)}}{ae^2-bde+c}$$

$$\frac{de\int\left(\frac{e^2}{(cd^2-bed+ae^2)(ex^4+d)}+\frac{-cex^4+cd-be}{(cd^2-bed+ae^2)(cx^8+bx^4+a)}\right)dx}{ae^2-bde+cd^2}$$

$\downarrow$ 2009

$$\frac{\frac{-\frac{1}{2}c\left(3(2cd-be)+\frac{3eb^2+8cdb-28ace}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}\right)-\frac{1}{2}c\left(3(2cd-be)-\frac{3eb^2+}{\sqrt{}}\right.}{4(b^2-4ac)}}{cd^2-bed+ae}$$

$$de\left(-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)e^{7/4}}{2\sqrt{2}d^{3/4}(cd^2-bed+ae^2)}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}+1\right)e^{7/4}}{2\sqrt{2}d^{3/4}(cd^2-bed+ae^2)}-\frac{\log\left(\sqrt{e}x^2-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}\right)e^{7/4}}{4\sqrt{2}d^{3/4}(cd^2-bed+ae^2)}+\frac{\log\left(\sqrt{e}x^2+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}\right)e}{4\sqrt{2}d^{3/4}(cd^2-bed+ae^2)}\right.$$

input
```
Int[x^4/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(-1/4*(x*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^4))/((b^2 - 4*a*c)*(
a + b*x^4 + c*x^8)) + (-1/2*(c*(3*(2*c*d - b*e) + (8*b*c*d + 3*b^2*e - 28*
a*c*e)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4
*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[
(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b -
 Sqrt[b^2 - 4*a*c])^(3/4)))) - (c*(3*(2*c*d - b*e) - (8*b*c*d + 3*b^2*e -
28*a*c*e)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2
- 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTa
nh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-
b + Sqrt[b^2 - 4*a*c])^(3/4))))/2)/(4*(b^2 - 4*a*c)))/(c*d^2 - b*d*e + a*e
^2) - (d*e*((c^(3/4)*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(-b - Sqrt[b^2 - 4
*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)) + (c^(3/4)*(e - (2*c*d - b*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(
2*2^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)) - (e^(7/
4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*d^(3/4)*(c*d^2 - b*
d*e + a*e^2)) + (e^(7/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[
2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)) + (c^(3/4)*(e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2
*2^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)) + (c^(...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1754
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

rule 1760
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/
(a*n*(p + 1)*(b^2 - 4*a*c))   Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*
d*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n
+ c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n
] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

rule 1862
```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_))
/((d_.) + (e_.)*(x_)^(n_)), x_Symbol] :> Simp[f^n/(c*d^2 - b*d*e + a*e^2)
 Int[(f*x)^(m - n)*(a*e + c*d*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] - Simp
[d*e*(f^n/(c*d^2 - b*d*e + a*e^2))   Int[(f*x)^(m - n)*((a + b*x^n + c*x^(2
*n))^(p + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.52 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.36

method	result
default	$\frac{\frac{-\frac{c\left(ab\,e^{3}-2acd\,e^{2}-b^{2}d\,e^{2}+3bc\,d^{2}e-2c^{2}d^{3}\right)x^{5}}{4\left(4ac-b^{2}\right)}+\frac{\left(2a^{2}c\,e^{3}-a\,b^{2}e^{3}-abcd\,e^{2}+2a\,c^{2}d^{2}e+b^{3}d\,e^{2}-2b^{2}c\,d^{2}e+b\,c^{2}d^{3}\right)x}{16ac-4b^{2}}}{c\,x^{8}+b\,x^{4}+a}+\frac{\sum_{\_R=\text{RootOf}\left(\_Z^{8}c+\_Z^{4}\right.}}{}}{\left(a\,e^{2}-bde\right.}$
risch	Expression too large to display

input
```
int(x^4/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2-b*d*e+c*d^2)^2*((-1/4*c*(a*b*e^3-2*a*c*d*e^2-b^2*d*e^2+3*b*c*d^2*
e-2*c^2*d^3)/(4*a*c-b^2)*x^5+1/4*(2*a^2*c*e^3-a*b^2*e^3-a*b*c*d*e^2+2*a*c^
2*d^2*e+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)/(4*a*c-b^2)*x)/(c*x^8+b*x^4+a)+
1/16/(4*a*c-b^2)*sum((c*(-3*a*b*e^3+22*a*c*d*e^2-b^2*d*e^2-9*b*c*d^2*e+6*c
^2*d^3)*_R^4+14*a^2*c*e^3-3*a*b^2*e^3+a*b*c*d*e^2-2*a*c^2*d^2*e-b^3*d*e^2+
2*b^2*c*d^2*e-b*c^2*d^3)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*
b+a))-1/8*e^3/(a*e^2-b*d*e+c*d^2)^2*(d/e)^(1/4)*2^(1/2)*(ln((x^2+(d/e)^(1
/4)*x*2^(1/2)+(d/e)^(1/2))/(x^2-(d/e)^(1/4)*x*2^(1/2)+(d/e)^(1/2)))+2*arct
an(2^(1/2)/(d/e)^(1/4)*x+1)+2*arctan(2^(1/2)/(d/e)^(1/4)*x-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \int \frac{x^4}{(cx^8+bx^4+a)^2(ex^4+d)}\,dx$$

input
```
integrate(x^4/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/8*(sqrt(2)*e^(11/4)*log(sqrt(e)*x^2 + sqrt(2)*d^(1/4)*e^(1/4)*x + sqrt(
d))/d^(3/4) - sqrt(2)*e^(11/4)*log(sqrt(e)*x^2 - sqrt(2)*d^(1/4)*e^(1/4)*x
 + sqrt(d))/d^(3/4) + sqrt(2)*e^3*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)
*sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*
sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4)))/(sqrt(d)*sqrt(-sqrt(d)*sqrt(e))) + sq
rt(2)*e^3*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1
/4)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/
4)*e^(1/4)))/(sqrt(d)*sqrt(-sqrt(d)*sqrt(e))))*d/(c^2*d^4 - 2*b*c*d^3*e -
2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2) - 1/4*((2*c^2*d - b*c*e)*x^
5 + (b*c*d - (b^2 - 2*a*c)*e)*x)/(((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*c - 4*a*
b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^8 + ((b^3*c - 4*a*b*c^2)*d^2 - (
b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^2*b*c)*e^2)*x^4 + (a*b^2*c - 4*a^2*c^2
)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2) + 1/4*integrate
((b*c^2*d^3 - (6*c^3*d^3 - 9*b*c^2*d^2*e - 3*a*b*c*e^3 - (b^2*c - 22*a*c^2
)*d*e^2)*x^4 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 + (3*a*b^2 -
14*a^2*c)*e^3)/(c*x^8 + b*x^4 + a), x)/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c
 - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4
*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \text{Hanged}$$

input
```
int(x^4/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x^4}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \int \frac{x^4}{(e\,x^4+d)\,(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^4/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^4/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.117 $\int \frac{x^2}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 1702

$$\int \frac{x^2}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

output
```
-1/5*e^2*(-5*b*e+c*d)*x^3/(a*e^2-b*d*e+c*d^2)^2/(c*x^8+b*x^4+a)+c*e^3*x^7/
(a*e^2-b*d*e+c*d^2)^2/(c*x^8+b*x^4+a)+1/20*x^3*(a*b*c*e*(5*c*d^2-9*e*(-5*a
*e+b*d))+(-2*a*c+b^2)*(5*c^2*d^3-c*d*e*(-13*a*e+10*b*d)+5*b*e^2*(-5*a*e+b*
d))+5*c*(b^3*d*e^2+b*c*d*(-a*e^2+c*d^2)+2*a*c*e*(9*a*e^2+c*d^2)-b^2*(5*a*e
^3+2*c*d^2*e))*x^4)/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x^8+b*x^4+a)-1
/16*c^(1/4)*(b^4*d*e^2+b^2*(c^2*d^3+5*a*(-4*a*c+b^2)^(1/2)*e^3+c*d*e*(2*(-
4*a*c+b^2)^(1/2)*d-3*a*e))-b^3*e*(2*c*d^2+e*((-4*a*c+b^2)^(1/2)*d+5*a*e))-
2*a*c*(10*c^2*d^3+9*a*(-4*a*c+b^2)^(1/2)*e^3+c*d*e*((-4*a*c+b^2)^(1/2)*d+2
6*a*e))-b*c*(c*d^2*((-4*a*c+b^2)^(1/2)*d-32*a*e)-a*e^2*((-4*a*c+b^2)^(1/2)
*d+28*a*e)))*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/
4)/a/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(c*d^2-e*(-a*e+b*d))
^2+1/16*c^(1/4)*(b^4*d*e^2-2*a*c*(10*c^2*d^3-9*a*(-4*a*c+b^2)^(1/2)*e^3-c*
d*e*((-4*a*c+b^2)^(1/2)*d-26*a*e))-b^3*e*(2*c*d^2-e*((-4*a*c+b^2)^(1/2)*d-
5*a*e))+b^2*(c^2*d^3-5*a*(-4*a*c+b^2)^(1/2)*e^3-c*d*e*(2*(-4*a*c+b^2)^(1/2
)*d+3*a*e))-b*c*(a*e^2*((-4*a*c+b^2)^(1/2)*d-28*a*e)-c*d^2*((-4*a*c+b^2)^(
1/2)*d+32*a*e)))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2
^(1/4)/a/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)/(a*e^2-b*d*e+c*d
^2)^2+1/4*e^(13/4)*arctan(-1+2^(1/2)*e^(1/4)*x/d^(1/4))*2^(1/2)/d^(1/4)/(a
*e^2-b*d*e+c*d^2)^2+1/4*e^(13/4)*arctan(1+2^(1/2)*e^(1/4)*x/d^(1/4))*2^(1/
2)/d^(1/4)/(a*e^2-b*d*e+c*d^2)^2+1/16*c^(1/4)*(b^4*d*e^2+b^2*(c^2*d^3+5...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 8.58 (sec) , antiderivative size = 645, normalized size of antiderivative = 0.38

$$\int \frac{x^2}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$$

$$= \frac{-\frac{4(cd^2+e(-bd+ae))x^3(-b^3e+bc(3ae+cdx^4)+b^2c(d-ex^4)-2ac^2(d-ex^4))}{a(-b^2+4ac)(a+bx^4+cx^8)} - \frac{4\sqrt{2}e^{13/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{\sqrt[4]{d}} + \frac{4\sqrt{2}e^{13/4}\arctan\left(1+\frac{\sqrt{}}{}\right.}{\sqrt[4]{d}}}{}$$

input
```
Integrate[x^2/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((-4*(c*d^2 + e*(-(b*d) + a*e))*x^3*(-(b^3*e) + b*c*(3*a*e + c*d*x^4) + b^
2*c*(d - e*x^4) - 2*a*c^2*(d - e*x^4)))/(a*(-b^2 + 4*a*c)*(a + b*x^4 + c*x
^8)) - (4*Sqrt[2]*e^(13/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/d^(1/4
) + (4*Sqrt[2]*e^(13/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/d^(1/4) +
 (2*Sqrt[2]*e^(13/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2
])/d^(1/4) - (2*Sqrt[2]*e^(13/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*x +
 Sqrt[e]*x^2])/d^(1/4) + RootSum[a + b*#1^4 + c*#1^8 & , (b^2*c^2*d^3*Log[
x - #1] - 10*a*c^3*d^3*Log[x - #1] - 2*b^3*c*d^2*e*Log[x - #1] + 17*a*b*c^
2*d^2*e*Log[x - #1] + b^4*d*e^2*Log[x - #1] - 2*a*b^2*c*d*e^2*Log[x - #1]
- 26*a^2*c^2*d*e^2*Log[x - #1] - 5*a*b^3*e^3*Log[x - #1] + 23*a^2*b*c*e^3*
Log[x - #1] + b*c^3*d^3*Log[x - #1]*#1^4 - 2*b^2*c^2*d^2*e*Log[x - #1]*#1^
4 + 2*a*c^3*d^2*e*Log[x - #1]*#1^4 + b^3*c*d*e^2*Log[x - #1]*#1^4 - a*b*c^
2*d*e^2*Log[x - #1]*#1^4 - 5*a*b^2*c*e^3*Log[x - #1]*#1^4 + 18*a^2*c^2*e^3
*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(a*(b^2 - 4*a*c)))/(16*(c*d^2 + e
*(-(b*d) + a*e))^2)
```

Rubi [A] (verified)

Time = 2.53 (sec) , antiderivative size = 1320, normalized size of antiderivative = 0.78, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {1862, 1824, 27, 1828, 1834, 27, 827, 218, 221, 1836, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$$

$$\Big\downarrow \text{1862}$$

$$\frac{\int \frac{cdx^4+ae}{x^2(cx^8+bx^4+a)^2}dx}{ae^2-bde+cd^2}-\frac{de\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \text{1824}$$

$$\frac{-\frac{\int \frac{a\left(5c(2cd-be)x^4+bcd-5b^2e+18ace\right)}{x^2\left(cx^8+bx^4+a\right)}dx}{4a(b^2-4ac)}-\frac{-e(b^2-2ac)+cx^4(2cd-be)+bcd}{4x(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}-\frac{de\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{5c(2cd-be)x^4+bcd-5b^2e+18ace}{x^2\left(cx^8+bx^4+a\right)}dx}{4(b^2-4ac)}-\frac{-e(b^2-2ac)+cx^4(2cd-be)+bcd}{4x(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}-\frac{de\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}$$

$\Big\downarrow$ 1828

$$-\frac{-\frac{-\frac{\int \frac{x^2\left(c\left(-5eb^2+cdb+18ace\right)x^4-10ac^2d+b^2cd-5b^3e+23abce\right)}{cx^8+bx^4+a}dx}{a}-\frac{18ace-5b^2e+bcd}{ax}}{4(b^2-4ac)}-\frac{-e(b^2-2ac)+cx^4(2cd-be)+bcd}{4x(b^2-4ac)(a+bx^4+cx^8)}}{ae^2-bde+cd^2}-\frac{de\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}$$

$\Big\downarrow$ 1834

$$-\frac{-\frac{\frac{1}{2}c\left(\frac{28abce-20ac^2d-5b^3e+b^2cd}{\sqrt{b^2-4ac}}+18ace-5b^2e+bcd\right)\int \frac{2x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}c\left(-\frac{28abce-20ac^2d-5b^3e+b^2cd}{\sqrt{b^2-4ac}}+18ace-5b^2e+bcd\right)\int \frac{2x^2}{2cx^4+b+\sqrt{b^2-4a\cdots}}\cdots}{a}\cdots}{4(b^2-4ac)}\cdots$$

$$ae^2-bde+cd^2$$

$$\frac{de\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{c\left(\frac{28abce-20ac^2d-5b^3e+b^2cd}{\sqrt{b^2-4ac}}+18ace-5b^2e+bcd\right)\int \frac{x^2}{2cx^4+b-\sqrt{b^2-4ac}}dx+c\left(-\frac{28abce-20ac^2d-5b^3e+b^2cd}{\sqrt{b^2-4ac}}+18ace-5b^2e+bcd\right)\int \frac{x^2}{2cx^4+b+\sqrt{b^2-4ac}}dx\cdots}{a}\cdots}{4(b^2-4ac)}\cdots$$

$$ae^2-bde+cd^2$$

$$\frac{de\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}$$

$\Big\downarrow$ 827

$$-\frac{-\frac{c\left(-\frac{28abce-20ac^2d-5b^3e+b^2cd}{\sqrt{b^2-4ac}}+18ace-5b^2e+bcd\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{28abce-20ac^2d-5b^3e+b^2c\cdots}{\sqrt{b^2-4ac}}\cdots\right.}{a}\cdots}{4(b^2-4ac)}\cdots$$

$$ae^2-bde+\cdots$$

$$\frac{de\int \frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}$$

$\Big\downarrow$ 218

$$-\frac{-\frac{c\left(-\frac{28abce-20ac^2d-5b^3e+b^2cd}{\sqrt{b^2-4ac}}+18ace-5b^2e+bcd\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x^2}}dx}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{28abce-20ac^2d-5b^3\cdots}{\sqrt{b^2-4ac}}\cdots\right.}{a}}{4(b^2-4ac)}\cdots}{ae^2-b\cdots}$$

$$\frac{de\int\frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}$$

$\downarrow$ 221

$$-\frac{-\frac{c\left(-\frac{28abce-20ac^2d-5b^3e+b^2cd}{\sqrt{b^2-4ac}}+18ace-5b^2e+bcd\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(\frac{28abce-\cdots}{}\right.}{a}}{4(b^2-4ac)}\cdots}{\cdots}$$

$$\frac{de\int\frac{1}{x^2(ex^4+d)(cx^8+bx^4+a)}dx}{ae^2-bde+cd^2}$$

$\downarrow$ 1836

$$-\frac{-\frac{c\left(-\frac{28abce-20ac^2d-5b^3e+b^2cd}{\sqrt{b^2-4ac}}+18ace-5b^2e+bcd\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(\frac{28abce-\cdots}{}\right.}{a}}{4(b^2-4ac)}\cdots}{\cdots}$$

$$\frac{de\int\left(-\frac{x^2e^3}{d(cd^2-bed+ae^2)(ex^4+d)}+\frac{x^2\left(-c(cd-be)x^4-bcd+b^2e-ace\right)}{a(cd^2-bed+ae^2)(cx^8+bx^4+a)}+\frac{1}{adx^2}\right)dx}{ae^2-bde+cd^2}$$

$\downarrow$ 2009

$$-\frac{c(2cd-be)x^4+bcd-(b^2-2ac)e}{4(b^2-4ac)x(cx^8+bx^4+a)}-\frac{-\frac{-5eb^2+cdb+18ace}{ax}-\frac{c\left(-5eb^2+cdb+18ace-\frac{-5eb^3+cdb^2+28aceb-20ac^2d}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}}\cdots\right.}{\cdots}}{\cdots}$$

$$de\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)e^{9/4}}{2\sqrt{2}d^{5/4}(cd^2-bed+ae^2)}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}+1\right)e^{9/4}}{2\sqrt{2}d^{5/4}(cd^2-bed+ae^2)}-\frac{\log\left(\sqrt{e}x^2-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}\right)e^{9/4}}{4\sqrt{2}d^{5/4}(cd^2-bed+ae^2)}+\frac{\log\left(\sqrt{e}x^2+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{d}\right)e^{9/4}}{4\sqrt{2}d^{5/4}(cd^2-bed+ae^2)}\cdots\right.$$

input
```
Int[x^2/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(-1/4*(b*c*d - (b^2 - 2*a*c)*e + c*(2*c*d - b*e)*x^4)/((b^2 - 4*a*c)*x*(a
+ b*x^4 + c*x^8)) - (-((b*c*d - 5*b^2*e + 18*a*c*e)/(a*x)) - (c*(b*c*d - 5
*b^2*e + 18*a*c*e - (b^2*c*d - 20*a*c^2*d - 5*b^3*e + 28*a*b*c*e)/Sqrt[b^2
 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*
2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)
*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*
a*c])^(1/4))) + c*(b*c*d - 5*b^2*e + 18*a*c*e + (b^2*c*d - 20*a*c^2*d - 5*
b^3*e + 28*a*b*c*e)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + S
qrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)
) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)
*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/a)/(4*(b^2 - 4*a*c)))/(c*d^2 -
b*d*e + a*e^2) - (d*e*(-(1/(a*d*x)) - (c^(1/4)*(c*d - b*e - (b*c*d - b^2*e
 + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 -
 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)*(c*d^2 - b*d*
e + a*e^2)) - (c^(1/4)*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4
*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3
/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)*(c*d^2 - b*d*e + a*e^2)) + (e^(9/4)*A
rcTan[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*d^(5/4)*(c*d^2 - b*d*e
+ a*e^2)) - (e^(9/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*d
^(5/4)*(c*d^2 - b*d*e + a*e^2)) + (c^(1/4)*(c*d - b*e - (b*c*d - b^2*e ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1824
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m
+ 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

rule 1836
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.))/((a_) + (c_.)*(x_)^
(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e
*x^n)^q/(a + b*x^n + c*x^(2*n))), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[q] && Int
egerQ[m]
```

rule 1862
```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_))
/((d_.) + (e_.)*(x_)^(n_)), x_Symbol] :> Simp[f^n/(c*d^2 - b*d*e + a*e^2)
 Int[(f*x)^(m - n)*(a*e + c*d*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] - Simp
[d*e*(f^n/(c*d^2 - b*d*e + a*e^2))   Int[(f*x)^(m - n)*((a + b*x^n + c*x^(2
*n))^(p + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.77 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.34

method	result
default	$-\frac{\frac{c\left(2a^{2}c\,e^{3}-a\,b^{2}e^{3}-abcd\,e^{2}+2a\,c^{2}d^{2}e+b^{3}d\,e^{2}-2b^{2}c\,d^{2}e+b\,c^{2}d^{3}\right)x^{7}}{4a\left(4ac-b^{2}\right)}+\frac{\left(3a^{2}bc\,e^{3}-2a^{2}c^{2}e^{2}d-a\,b^{3}e^{3}-2a\,b^{2}cd\,e^{2}+5ab\,c^{2}d^{2}e-2a\,c^{3}d^{3}+b^{4}d\,e^{2}-\right.}{4a\left(4ac-b^{2}\right)}}{c\,x^{8}+b\,x^{4}+a}$
risch	Expression too large to display

input
```
int(x^2/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*e^2-b*d*e+c*d^2)^2*((1/4*c*(2*a^2*c*e^3-a*b^2*e^3-a*b*c*d*e^2+2*a*c^
2*d^2*e+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)/a/(4*a*c-b^2)*x^7+1/4*(3*a^2*b*
c*e^3-2*a^2*c^2*d*e^2-a*b^3*e^3-2*a*b^2*c*d*e^2+5*a*b*c^2*d^2*e-2*a*c^3*d^
3+b^4*d*e^2-2*b^3*c*d^2*e+b^2*c^2*d^3)/a/(4*a*c-b^2)*x^3)/(c*x^8+b*x^4+a)+
1/16/a/(4*a*c-b^2)*sum((c*(18*a^2*c*e^3-5*a*b^2*e^3-a*b*c*d*e^2+2*a*c^2*d^
2*e+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)*_R^6+(23*a^2*b*c*e^3-26*a^2*c^2*d*e
^2-5*a*b^3*e^3-2*a*b^2*c*d*e^2+17*a*b*c^2*d^2*e-10*a*c^3*d^3+b^4*d*e^2-2*b
^3*c*d^2*e+b^2*c^2*d^3)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+
_Z^4*b+a)))+1/8*e^3/(a*e^2-b*d*e+c*d^2)^2/(d/e)^(1/4)*2^(1/2)*(ln((x^2-(d/
e)^(1/4)*x*2^(1/2)+(d/e)^(1/2))/(x^2+(d/e)^(1/4)*x*2^(1/2)+(d/e)^(1/2)))+2
*arctan(2^(1/2)/(d/e)^(1/4)*x+1)+2*arctan(2^(1/2)/(d/e)^(1/4)*x-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^4)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^4)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2}{(d+ex^4)\left(a+bx^4+cx^8\right)^2}\,dx=\int \frac{x^2}{(cx^8+bx^4+a)^2(ex^4+d)}\,dx$$

input
```
integrate(x^2/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/8*e^4*(sqrt(2)*log(sqrt(e)*x^2 + sqrt(2)*d^(1/4)*e^(1/4)*x + sqrt(d))/(
d^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*x^2 - sqrt(2)*d^(1/4)*e^(1/4)*x + s
qrt(d))/(d^(1/4)*e^(3/4)) - sqrt(2)*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(
d)*sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d
)*sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4)))/(sqrt(-sqrt(d)*sqrt(e))*sqrt(e)) -
sqrt(2)*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4
)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4)
*e^(1/4)))/(sqrt(-sqrt(d)*sqrt(e))*sqrt(e)))/(c^2*d^4 - 2*b*c*d^3*e - 2*a*
b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2) + 1/4*((b*c^2*d - (b^2*c - 2*a*
c^2)*e)*x^7 + ((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*x^3)/(((a*b^2*c^2
- 4*a^2*c^3)*d^2 - (a*b^3*c - 4*a^2*b*c^2)*d*e + (a^2*b^2*c - 4*a^3*c^2)*e
^2)*x^8 + ((a*b^3*c - 4*a^2*b*c^2)*d^2 - (a*b^4 - 4*a^2*b^2*c)*d*e + (a^2*
b^3 - 4*a^3*b*c)*e^2)*x^4 + (a^2*b^2*c - 4*a^3*c^2)*d^2 - (a^2*b^3 - 4*a^3
*b*c)*d*e + (a^3*b^2 - 4*a^4*c)*e^2) + 1/4*integrate(((b*c^3*d^3 - 2*(b^2*
c^2 - a*c^3)*d^2*e + (b^3*c - a*b*c^2)*d*e^2 - (5*a*b^2*c - 18*a^2*c^2)*e^
3)*x^6 + ((b^2*c^2 - 10*a*c^3)*d^3 - (2*b^3*c - 17*a*b*c^2)*d^2*e + (b^4 -
 2*a*b^2*c - 26*a^2*c^2)*d*e^2 - (5*a*b^3 - 23*a^2*b*c)*e^3)*x^2)/(c*x^8 +
 b*x^4 + a), x)/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d
^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)
*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^4)\left(a+bx^4+cx^8\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^2/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \text{Hanged}$$

input
```
int(x^2/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x^2}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \int \frac{x^2}{(e\,x^4+d)\,(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(x^2/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(x^2/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.118 $\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 1534

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)^2} dx = \text{Too large to display}$$

output
```
1/4*x*(c*(-2*a*c+b^2)*d-b*(-3*a*c+b^2)*e+c*(2*a*c*e-b^2*e+b*c*d)*x^4)/a/(-
4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^8+b*x^4+a)+1/16*c^(3/4)*(3*b^4*d*e^2+b
^2*(3*c^2*d^3+7*a*(-4*a*c+b^2)^(1/2)*e^3+3*c*d*e*(2*(-4*a*c+b^2)^(1/2)*d-3
*a*e))-b^3*e*(6*c*d^2+e*(3*(-4*a*c+b^2)^(1/2)*d+7*a*e))-2*a*c*(14*c^2*d^3+
11*a*(-4*a*c+b^2)^(1/2)*e^3+3*c*d*e*((-4*a*c+b^2)^(1/2)*d+10*a*e))-3*b*c*(
c*d^2*((-4*a*c+b^2)^(1/2)*d-16*a*e)-a*e^2*((-4*a*c+b^2)^(1/2)*d+12*a*e)))*
arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-4*a*c+
b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(c*d^2-e*(-a*e+b*d))^2-1/16*c^(3/
4)*(3*b^4*d*e^2-2*a*c*(14*c^2*d^3-11*a*(-4*a*c+b^2)^(1/2)*e^3-3*c*d*e*((-4
*a*c+b^2)^(1/2)*d-10*a*e))-b^3*e*(6*c*d^2-e*(3*(-4*a*c+b^2)^(1/2)*d-7*a*e)
)+b^2*(3*c^2*d^3-7*a*(-4*a*c+b^2)^(1/2)*e^3-3*c*d*e*(2*(-4*a*c+b^2)^(1/2)*
d+3*a*e))-3*b*c*(a*e^2*((-4*a*c+b^2)^(1/2)*d-12*a*e)-c*d^2*((-4*a*c+b^2)^(
1/2)*d+16*a*e)))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2
^(3/4)/a/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)/(a*e^2-b*d*e+c*d
^2)^2+1/4*e^(15/4)*arctan(-1+2^(1/2)*e^(1/4)*x/d^(1/4))*2^(1/2)/d^(3/4)/(a
*e^2-b*d*e+c*d^2)^2+1/4*e^(15/4)*arctan(1+2^(1/2)*e^(1/4)*x/d^(1/4))*2^(1/
2)/d^(3/4)/(a*e^2-b*d*e+c*d^2)^2+1/16*c^(3/4)*(3*b^4*d*e^2+b^2*(3*c^2*d^3+
7*a*(-4*a*c+b^2)^(1/2)*e^3+3*c*d*e*(2*(-4*a*c+b^2)^(1/2)*d-3*a*e))-b^3*e*(
6*c*d^2+e*(3*(-4*a*c+b^2)^(1/2)*d+7*a*e))-2*a*c*(14*c^2*d^3+11*a*(-4*a*c+b
^2)^(1/2)*e^3+3*c*d*e*((-4*a*c+b^2)^(1/2)*d+10*a*e))-3*b*c*(c*d^2*((-4*...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 10.07 (sec) , antiderivative size = 649, normalized size of antiderivative = 0.42

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$$

$$= \frac{-\frac{4(cd^2+e(-bd+ae))x(-b^3e+bc(3ae+cdx^4)+b^2c(d-ex^4)-2ac^2(d-ex^4))}{a(-b^2+4ac)(a+bx^4+cx^8)} - \frac{4\sqrt{2}e^{15/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{d^{3/4}} + \frac{4\sqrt{2}e^{15/4}\arctan\left(1+\frac{\sqrt{2}\ldots}{\ldots}\right)}{d^{3/4}} \ldots}{\ldots}$$

input
```
Integrate[1/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
((-4*(c*d^2 + e*(-(b*d) + a*e))*x*(-(b^3*e) + b*c*(3*a*e + c*d*x^4) + b^2*
c*(d - e*x^4) - 2*a*c^2*(d - e*x^4)))/(a*(-b^2 + 4*a*c)*(a + b*x^4 + c*x^8
)) - (4*Sqrt[2]*e^(15/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/d^(3/4)
+ (4*Sqrt[2]*e^(15/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/d^(3/4) - (
2*Sqrt[2]*e^(15/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2])
/d^(3/4) + (2*Sqrt[2]*e^(15/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*x + S
qrt[e]*x^2])/d^(3/4) + RootSum[a + b*#1^4 + c*#1^8 & , (3*b^2*c^2*d^3*Log[
x - #1] - 14*a*c^3*d^3*Log[x - #1] - 6*b^3*c*d^2*e*Log[x - #1] + 27*a*b*c^
2*d^2*e*Log[x - #1] + 3*b^4*d*e^2*Log[x - #1] - 6*a*b^2*c*d*e^2*Log[x - #1
] - 30*a^2*c^2*d*e^2*Log[x - #1] - 7*a*b^3*e^3*Log[x - #1] + 29*a^2*b*c*e^
3*Log[x - #1] + 3*b*c^3*d^3*Log[x - #1]*#1^4 - 6*b^2*c^2*d^2*e*Log[x - #1]
*#1^4 + 6*a*c^3*d^2*e*Log[x - #1]*#1^4 + 3*b^3*c*d*e^2*Log[x - #1]*#1^4 -
3*a*b*c^2*d*e^2*Log[x - #1]*#1^4 - 7*a*b^2*c*e^3*Log[x - #1]*#1^4 + 22*a^2
*c^2*e^3*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(a*(b^2 - 4*a*c)))/(16*
(c*d^2 + e*(-(b*d) + a*e))^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx$$

$$\Big\downarrow \; 1769$$

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)^2}dx$$

input
```
Int[1/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1769
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
))^(p_), x_Symbol] :> Unintegrable[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p,
 x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.41 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.37

method	result
default	$-\frac{\frac{\frac{c\left(2a^2c\,e^3-a\,b^2e^3-abcd\,e^2+2a\,c^2d^2e+b^3d\,e^2-2b^2c\,d^2e+b\,c^2d^3\right)x^5}{4a\left(4ac-b^2\right)}+\frac{\left(3a^2bc\,e^3-2a^2c^2e^2d-a\,b^3e^3-2a\,b^2cd\,e^2+5ab\,c^2d^2e-2a\,c^3d^3+b^4d\,e^2-\right.}{4a\left(4ac-b^2\right)}}{c\,x^8+b\,x^4+a}}{}$
risch	Expression too large to display

input
```
int(1/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*e^2-b*d*e+c*d^2)^2*((1/4*c*(2*a^2*c*e^3-a*b^2*e^3-a*b*c*d*e^2+2*a*c^
2*d^2*e+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)/a/(4*a*c-b^2)*x^5+1/4*(3*a^2*b*
c*e^3-2*a^2*c^2*d*e^2-a*b^3*e^3-2*a*b^2*c*d*e^2+5*a*b*c^2*d^2*e-2*a*c^3*d^
3+b^4*d*e^2-2*b^3*c*d^2*e+b^2*c^2*d^3)/a/(4*a*c-b^2)*x)/(c*x^8+b*x^4+a)+1/
16/a/(4*a*c-b^2)*sum((c*(22*a^2*c*e^3-7*a*b^2*e^3-3*a*b*c*d*e^2+6*a*c^2*d^
2*e+3*b^3*d*e^2-6*b^2*c*d^2*e+3*b*c^2*d^3)*_R^4+29*a^2*b*c*e^3-30*a^2*c^2*
e^2*d-7*a*b^3*e^3-6*a*b^2*c*d*e^2+27*a*b*c^2*d^2*e-14*a*c^3*d^3+3*b^4*d*e^
2-6*b^3*c*d^2*e+3*b^2*c^2*d^3)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c
+_Z^4*b+a)))+1/8*e^4/(a*e^2-b*d*e+c*d^2)^2*(d/e)^(1/4)/d*2^(1/2)*(ln((x^2+
(d/e)^(1/4)*x*2^(1/2)+(d/e)^(1/2))/(x^2-(d/e)^(1/4)*x*2^(1/2)+(d/e)^(1/2))
)+2*arctan(2^(1/2)/(d/e)^(1/4)*x+1)+2*arctan(2^(1/2)/(d/e)^(1/4)*x-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx=\int \frac{1}{\left(cx^8+bx^4+a\right)^2\left(ex^4+d\right)}\,dx$$

input
```
integrate(1/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*((b*c^2*d - (b^2*c - 2*a*c^2)*e)*x^5 + ((b^2*c - 2*a*c^2)*d - (b^3 - 3
*a*b*c)*e)*x)/(((a*b^2*c^2 - 4*a^2*c^3)*d^2 - (a*b^3*c - 4*a^2*b*c^2)*d*e
+ (a^2*b^2*c - 4*a^3*c^2)*e^2)*x^8 + ((a*b^3*c - 4*a^2*b*c^2)*d^2 - (a*b^4
 - 4*a^2*b^2*c)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x^4 + (a^2*b^2*c - 4*a^3*
c^2)*d^2 - (a^2*b^3 - 4*a^3*b*c)*d*e + (a^3*b^2 - 4*a^4*c)*e^2) + 1/8*(sqr
t(2)*e^(15/4)*log(sqrt(e)*x^2 + sqrt(2)*d^(1/4)*e^(1/4)*x + sqrt(d))/d^(3/
4) - sqrt(2)*e^(15/4)*log(sqrt(e)*x^2 - sqrt(2)*d^(1/4)*e^(1/4)*x + sqrt(d
))/d^(3/4) + sqrt(2)*e^4*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)*sqrt(e))
 + sqrt(2)*d^(1/4)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*sqrt(e))
+ sqrt(2)*d^(1/4)*e^(1/4)))/(sqrt(d)*sqrt(-sqrt(d)*sqrt(e))) + sqrt(2)*e^4
*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4)*e^(1/
4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4)*e^(1/4
)))/(sqrt(d)*sqrt(-sqrt(d)*sqrt(e))))/(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3
 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2) + 1/4*integrate(((3*b*c^3*d^3 - 6*(b^2
*c^2 - a*c^3)*d^2*e + 3*(b^3*c - a*b*c^2)*d*e^2 - (7*a*b^2*c - 22*a^2*c^2)
*e^3)*x^4 + (3*b^2*c^2 - 14*a*c^3)*d^3 - 3*(2*b^3*c - 9*a*b*c^2)*d^2*e + 3
*(b^4 - 2*a*b^2*c - 10*a^2*c^2)*d*e^2 - (7*a*b^3 - 29*a^2*b*c)*e^3)/(c*x^8
 + b*x^4 + a), x)/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)
*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*
c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \text{Hanged}$$

input
```
int(1/((d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{(d+ex^4)\,(a+bx^4+cx^8)^2}\,dx = \int \frac{1}{(e\,x^4+d)\,(c\,x^8+b\,x^4+a)^2}dx$$

input
```
int(1/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(1/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

3.119 $\int \frac{1}{x^2(d+ex^4)(a+bx^4+cx^8)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 2110

$$\int \frac{1}{x^2(d+ex^4)(a+bx^4+cx^8)^2}\,dx = \text{Too large to display}$$

output
```
-1/a^2/d/x+1/5*(5*b^3*d*e^2+a^2*c*e^3+5*b*c*d*(2*a*e^2+c*d^2)-10*b^2*(a*e^
3+c*d^2*e))*x^3/a^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^8+b*x^4+a)+c*(-b*e+c*d)*(c*
d^2-e*(-2*a*e+b*d))*x^7/a^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^8+b*x^4+a)-1/20*x^3
*(25*b^5*d*e^2+5*b^3*c*d*(-11*a*e^2+5*c*d^2)-2*a^2*c^2*e*(13*a*e^2+5*c*d^2
)-5*a*b*c^2*d*(33*a*e^2+19*c*d^2)+a*b^2*c*e*(184*a*e^2+195*c*d^2)-5*b^4*(9
*a*e^3+10*c*d^2*e)+5*c*(5*b^4*d*e^2+5*b^2*c*d*(-2*a*e^2+c*d^2)-2*a*c^2*d*(
17*a*e^2+9*c*d^2)+a*b*c*e*(35*a*e^2+37*c*d^2)-b^3*(9*a*e^3+10*c*d^2*e))*x^
4)/a^2/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x^8+b*x^4+a)+1/16*c^(1/4)*(5*
b^5*d*e^2+2*a*c^2*(a*e^2*(17*(-4*a*c+b^2)^(1/2)*d-26*a*e)+c*d^2*(9*(-4*a*c
+b^2)^(1/2)*d-10*a*e))+b^3*(5*c^2*d^3+9*a*(-4*a*c+b^2)^(1/2)*e^3+10*c*d*e*
((-4*a*c+b^2)^(1/2)*d-2*a*e))-b^4*e*(10*c*d^2+e*(5*(-4*a*c+b^2)^(1/2)*d+9*
a*e))-a*b*c*(28*c^2*d^3+35*a*(-4*a*c+b^2)^(1/2)*e^3+c*d*e*(37*(-4*a*c+b^2)
^(1/2)*d+24*a*e))-b^2*c*(c*d^2*(5*(-4*a*c+b^2)^(1/2)*d-57*a*e)-a*e^2*(10*(
-4*a*c+b^2)^(1/2)*d+53*a*e)))*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1
/2))^(1/4))*2^(1/4)/a^2/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(
a*e^2-b*d*e+c*d^2)^2-1/16*c^(1/4)*(5*b^5*d*e^2-a*b*c*(28*c^2*d^3-35*a*(-4*
a*c+b^2)^(1/2)*e^3-c*d*e*(37*(-4*a*c+b^2)^(1/2)*d-24*a*e))-b^4*e*(10*c*d^2
-e*(5*(-4*a*c+b^2)^(1/2)*d-9*a*e))+b^3*(5*c^2*d^3-9*a*(-4*a*c+b^2)^(1/2)*e
^3-10*c*d*e*((-4*a*c+b^2)^(1/2)*d+2*a*e))-2*a*c^2*(c*d^2*(9*(-4*a*c+b^2)^(
1/2)*d+10*a*e)+a*e^2*(17*(-4*a*c+b^2)^(1/2)*d+26*a*e))-b^2*c*(a*e^2*(10...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 10.69 (sec) , antiderivative size = 847, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^2 (d+ex^4) (a+bx^4+cx^8)^2} \, dx = \frac{1}{16}\left(-\frac{16}{a^2dx} - \frac{4x^3(b^4e+2ac^2(ae+cdx^4)-b^2c(4ae+cdx^4)+3abc^2(d-ex^4)+b^3c(-d+ex^4))}{a^2(b^2-4ac)(-cd^2+e(bd-ae))(a+bx^4+cx^8)} + \frac{4\sqrt{2}e^{17/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{d^{5/4}(cd^2+e(-bd+ae))^2} - \frac{4\sqrt{2}e^{17/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}x}{\sqrt[4]{d}}\right)}{d^{5/4}(cd^2+e(-bd+ae))^2} - \frac{2\sqrt{2}e^{17/4}\log\left(\sqrt{d}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{e}x^2\right)}{d^{5/4}(cd^2+e(-bd+ae))^2} + \frac{2\sqrt{2}e^{17/4}\log\left(\sqrt{d}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}x+\sqrt{e}x^2\right)}{d^{5/4}(cd^2+e(-bd+ae))^2} + \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{5b^3c^2d^3\log(x-\#1)-23abc^3d^3\log(x-\#1)-10b^4cd^2e\log(x-\#1)+47ab^2c^2d^2e\log(x-\#1)-}{}\right.}{}$$

input
```
Integrate[1/(x^2*(d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
(-16/(a^2*d*x) - (4*x^3*(b^4*e + 2*a*c^2*(a*e + c*d*x^4) - b^2*c*(4*a*e +
c*d*x^4) + 3*a*b*c^2*(d - e*x^4) + b^3*c*(-d + e*x^4)))/(a^2*(b^2 - 4*a*c)
*(-(c*d^2) + e*(b*d - a*e))*(a + b*x^4 + c*x^8)) + (4*Sqrt[2]*e^(17/4)*Arc
Tan[1 - (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(d^(5/4)*(c*d^2 + e*(-(b*d) + a*e))^
2) - (4*Sqrt[2]*e^(17/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*x)/d^(1/4)])/(d^(5/4)
*(c*d^2 + e*(-(b*d) + a*e))^2) - (2*Sqrt[2]*e^(17/4)*Log[Sqrt[d] - Sqrt[2]
*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^2])/(d^(5/4)*(c*d^2 + e*(-(b*d) + a*e))^2)
+ (2*Sqrt[2]*e^(17/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*x + Sqrt[e]*x^
2])/(d^(5/4)*(c*d^2 + e*(-(b*d) + a*e))^2) + RootSum[a + b*#1^4 + c*#1^8 &
 , (5*b^3*c^2*d^3*Log[x - #1] - 23*a*b*c^3*d^3*Log[x - #1] - 10*b^4*c*d^2*
e*Log[x - #1] + 47*a*b^2*c^2*d^2*e*Log[x - #1] - 10*a^2*c^3*d^2*e*Log[x -
#1] + 5*b^5*d*e^2*Log[x - #1] - 15*a*b^3*c*d*e^2*Log[x - #1] - 29*a^2*b*c^
2*d*e^2*Log[x - #1] - 9*a*b^4*e^3*Log[x - #1] + 44*a^2*b^2*c*e^3*Log[x - #
1] - 26*a^3*c^2*e^3*Log[x - #1] + 5*b^2*c^3*d^3*Log[x - #1]*#1^4 - 18*a*c^
4*d^3*Log[x - #1]*#1^4 - 10*b^3*c^2*d^2*e*Log[x - #1]*#1^4 + 37*a*b*c^3*d^
2*e*Log[x - #1]*#1^4 + 5*b^4*c*d*e^2*Log[x - #1]*#1^4 - 10*a*b^2*c^2*d*e^2
*Log[x - #1]*#1^4 - 34*a^2*c^3*d*e^2*Log[x - #1]*#1^4 - 9*a*b^3*c*e^3*Log[
x - #1]*#1^4 + 35*a^2*b*c^2*e^3*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(a
^2*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2))/16
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (d+ex^4) (a+bx^4+cx^8)^2} \, dx$$
$$\Big\downarrow \text{ 1887}$$
$$\int \frac{1}{x^2 (d+ex^4) (a+bx^4+cx^8)^2} dx$$

input
```
Int[1/(x^2*(d + e*x^4)*(a + b*x^4 + c*x^8)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1887
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^n
)^q*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q},
x] && EqQ[n2, 2*n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 8.98 (sec) , antiderivative size = 697, normalized size of antiderivative = 0.33

method	result
default	$-\frac{-\frac{c\left(3a^{2}bce^{3}-2a^{2}c^{2}e^{2}d-a\,b^{3}e^{3}-2a\,b^{2}cd\,e^{2}+5ab\,c^{2}d^{2}e-2a\,c^{3}d^{3}+b^{4}d\,e^{2}-2b^{3}c\,d^{2}e+b^{2}c^{2}d^{3}\right)x^{7}}{4\left(4ac-b^{2}\right)}+\frac{\left(2a^{3}c^{2}e^{3}-4a^{2}b^{2}c\,e^{3}+a^{2}b\,c^{2}d\,e^{2}+2a^{2}c^{3}d^{2} \ldots}{\ldots}}{c\,x^{8}+b\,x^{4}+a} \ldots$
risch	Expression too large to display

input
```
int(1/x^2/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*e^2-b*d*e+c*d^2)^2/a^2*((-1/4*c*(3*a^2*b*c*e^3-2*a^2*c^2*d*e^2-a*b^3
*e^3-2*a*b^2*c*d*e^2+5*a*b*c^2*d^2*e-2*a*c^3*d^3+b^4*d*e^2-2*b^3*c*d^2*e+b
^2*c^2*d^3)/(4*a*c-b^2)*x^7+1/4*(2*a^3*c^2*e^3-4*a^2*b^2*c*e^3+a^2*b*c^2*d
*e^2+2*a^2*c^3*d^2*e+a*b^4*e^3+3*a*b^3*c*d*e^2-7*a*b^2*c^2*d^2*e+3*a*b*c^3
*d^3-b^5*d*e^2+2*b^4*c*d^2*e-b^3*c^2*d^3)/(4*a*c-b^2)*x^3)/(c*x^8+b*x^4+a)
+1/16/(4*a*c-b^2)*sum((c*(-35*a^2*b*c*e^3+34*a^2*c^2*d*e^2+9*a*b^3*e^3+10*
a*b^2*c*d*e^2-37*a*b*c^2*d^2*e+18*a*c^3*d^3-5*b^4*d*e^2+10*b^3*c*d^2*e-5*b
^2*c^2*d^3)*_R^6+(26*a^3*c^2*e^3-44*a^2*b^2*c*e^3+29*a^2*b*c^2*d*e^2+10*a^
2*c^3*d^2*e+9*a*b^4*e^3+15*a*b^3*c*d*e^2-47*a*b^2*c^2*d^2*e+23*a*b*c^3*d^3
-5*b^5*d*e^2+10*b^4*c*d^2*e-5*b^3*c^2*d^3)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x-_R
),_R=RootOf(_Z^8*c+_Z^4*b+a)))-1/8*e^4/d/(a*e^2-b*d*e+c*d^2)^2/(d/e)^(1/4)
*2^(1/2)*(ln((x^2-(d/e)^(1/4)*x*2^(1/2)+(d/e)^(1/2))/(x^2+(d/e)^(1/4)*x*2^
(1/2)+(d/e)^(1/2)))+2*arctan(2^(1/2)/(d/e)^(1/4)*x+1)+2*arctan(2^(1/2)/(d/
e)^(1/4)*x-1))-1/a^2/d/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d+ex^4) (a+bx^4+cx^8)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d+ex^4) (a+bx^4+cx^8)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*x**4+d)/(c*x**8+b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 (d+ex^4) (a+bx^4+cx^8)^2} \, dx = \int \frac{1}{(cx^8+bx^4+a)^2 (ex^4+d)x^2} \, dx$$

input
```
integrate(1/x^2/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/8*e^5*(sqrt(2)*log(sqrt(e)*x^2 + sqrt(2)*d^(1/4)*e^(1/4)*x + sqrt(d))/(d
^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*x^2 - sqrt(2)*d^(1/4)*e^(1/4)*x + sq
rt(d))/(d^(1/4)*e^(3/4)) - sqrt(2)*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d
)*sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)
*sqrt(e)) + sqrt(2)*d^(1/4)*e^(1/4)))/(sqrt(-sqrt(d)*sqrt(e))*sqrt(e)) - s
qrt(2)*log((2*sqrt(e)*x - sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4)
*e^(1/4))/(2*sqrt(e)*x + sqrt(2)*sqrt(-sqrt(d)*sqrt(e)) - sqrt(2)*d^(1/4)*
e^(1/4)))/(sqrt(-sqrt(d)*sqrt(e))*sqrt(e)))/(c^2*d^5 - 2*b*c*d^4*e - 2*a*b
*d^2*e^3 + a^2*d*e^4 + (b^2 + 2*a*c)*d^3*e^2) - 1/4*(((5*b^2*c^2 - 18*a*c^
3)*d^2 - (5*b^3*c - 19*a*b*c^2)*d*e + 4*(a*b^2*c - 4*a^2*c^2)*e^2)*x^8 + (
(5*b^3*c - 19*a*b*c^2)*d^2 - (5*b^4 - 20*a*b^2*c + 2*a^2*c^2)*d*e + 4*(a*b
^3 - 4*a^2*b*c)*e^2)*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d^2 - 4*(a*b^3 - 4*a^2*
b*c)*d*e + 4*(a^2*b^2 - 4*a^3*c)*e^2)/(((a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - (a
^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^3*b^2*c - 4*a^4*c^2)*d*e^2)*x^9 + ((a^2
*b^3*c - 4*a^3*b*c^2)*d^3 - (a^2*b^4 - 4*a^3*b^2*c)*d^2*e + (a^3*b^3 - 4*a
^4*b*c)*d*e^2)*x^5 + ((a^3*b^2*c - 4*a^4*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*
d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x) - 1/4*integrate((((5*b^2*c^3 - 18*a*
c^4)*d^3 - (10*b^3*c^2 - 37*a*b*c^3)*d^2*e + (5*b^4*c - 10*a*b^2*c^2 - 34*
a^2*c^3)*d*e^2 - (9*a*b^3*c - 35*a^2*b*c^2)*e^3)*x^6 + ((5*b^3*c^2 - 23*a*
b*c^3)*d^3 - (10*b^4*c - 47*a*b^2*c^2 + 10*a^2*c^3)*d^2*e + (5*b^5 - 15...
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d+ex^4) (a+bx^4+cx^8)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx = \text{Hanged}$$

input
```
int(1/(x^2*(d + e*x^4)*(a + b*x^4 + c*x^8)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{x^2\left(d+ex^4\right)\left(a+bx^4+cx^8\right)^2}\,dx = \int \frac{1}{x^2\left(e\,x^4+d\right)\left(c\,x^8+b\,x^4+a\right)^2}dx$$

input
```
int(1/x^2/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

output
```
int(1/x^2/(e*x^4+d)/(c*x^8+b*x^4+a)^2,x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
           return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
           if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
              return 1
           else:
              return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
           return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral',
        'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
        'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
             #is checked before calling the grading function that is passed.
             #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file