Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.4-Nested-quadratic-trinomial/136-1.2.4.2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**62**]. This is test number [136].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (62)	0.00 (0)
Mathematica	100.00 (62)	0.00 (0)
Fricas	100.00 (62)	0.00 (0)
Maple	95.16 (59)	4.84 (3)
Giac	93.55 (58)	6.45 (4)
Reduce	35.48 (22)	64.52 (40)
Mupad	0.00 (0)	100.00 (62)
Maxima	0.00 (0)	100.00 (62)
Sympy	0.00 (0)	100.00 (62)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

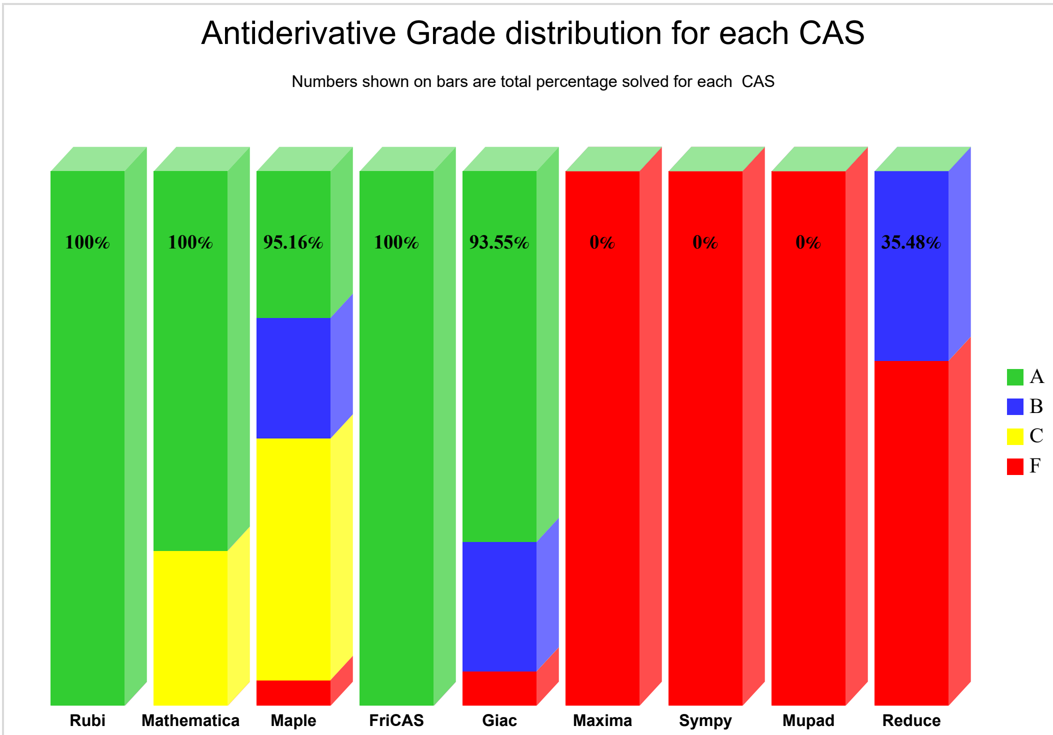
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

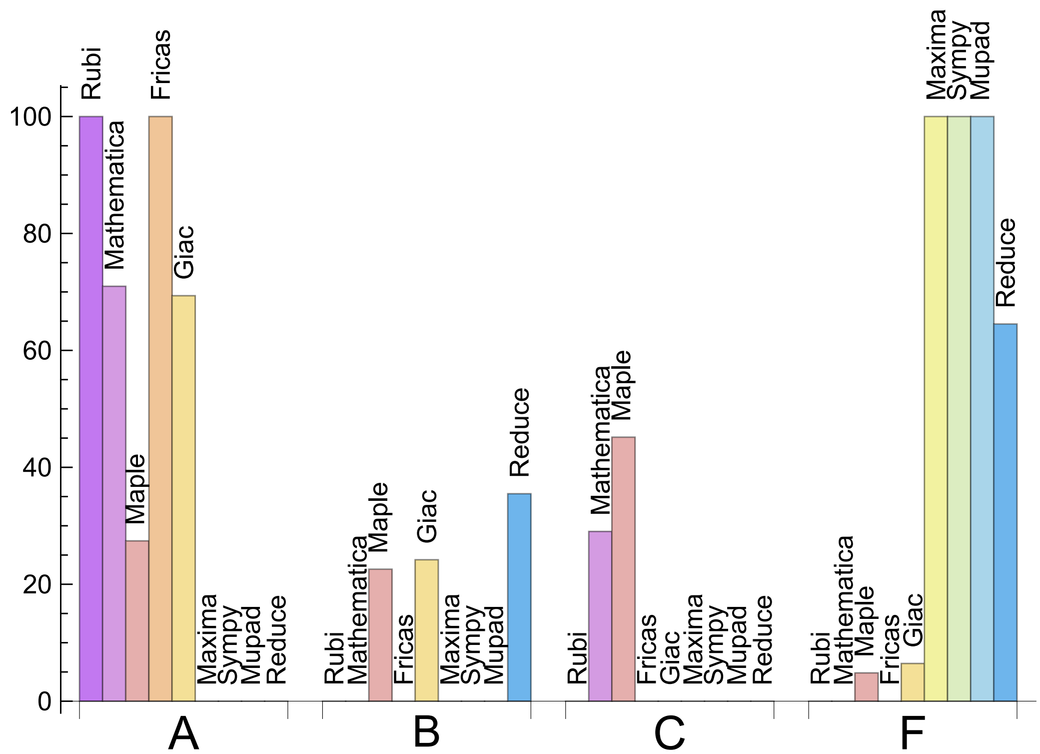
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Fricas	100.000	0.000	0.000	0.000
Mathematica	70.968	0.000	29.032	0.000
Giac	69.355	24.194	0.000	6.452
Maple	27.419	22.581	45.161	4.839
Mupad	0.000	0.000	0.000	100.000
Maxima	0.000	0.000	0.000	100.000
Reduce	0.000	35.484	0.000	64.516
Sympy	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	3	33.33	66.67	0.00
Giac	4	25.00	0.00	75.00
Reduce	40	100.00	0.00	0.00
Mupad	62	0.00	100.00	0.00
Maxima	62	100.00	0.00	0.00
Sympy	62	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.09
Giac	0.16
Reduce	0.37
Rubi	0.68
Maple	2.72
Mathematica	4.07
Sympy	-nan(ind)
Maxima	-nan(ind)
Mupad	-nan(ind)

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	214.06	0.87	184.50	0.87
Fricas	246.69	0.96	229.00	0.98
Reduce	282.86	1.48	159.50	1.12
Giac	384.40	1.43	339.50	1.40
Mathematica	418.89	1.25	167.50	0.83
Maple	886.25	3.20	486.00	1.81
Sympy	-nan(ind)	-nan(ind)	nan	nan
Maxima	-nan(ind)	-nan(ind)	nan	nan
Mupad	-nan(ind)	-nan(ind)	nan	nan

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

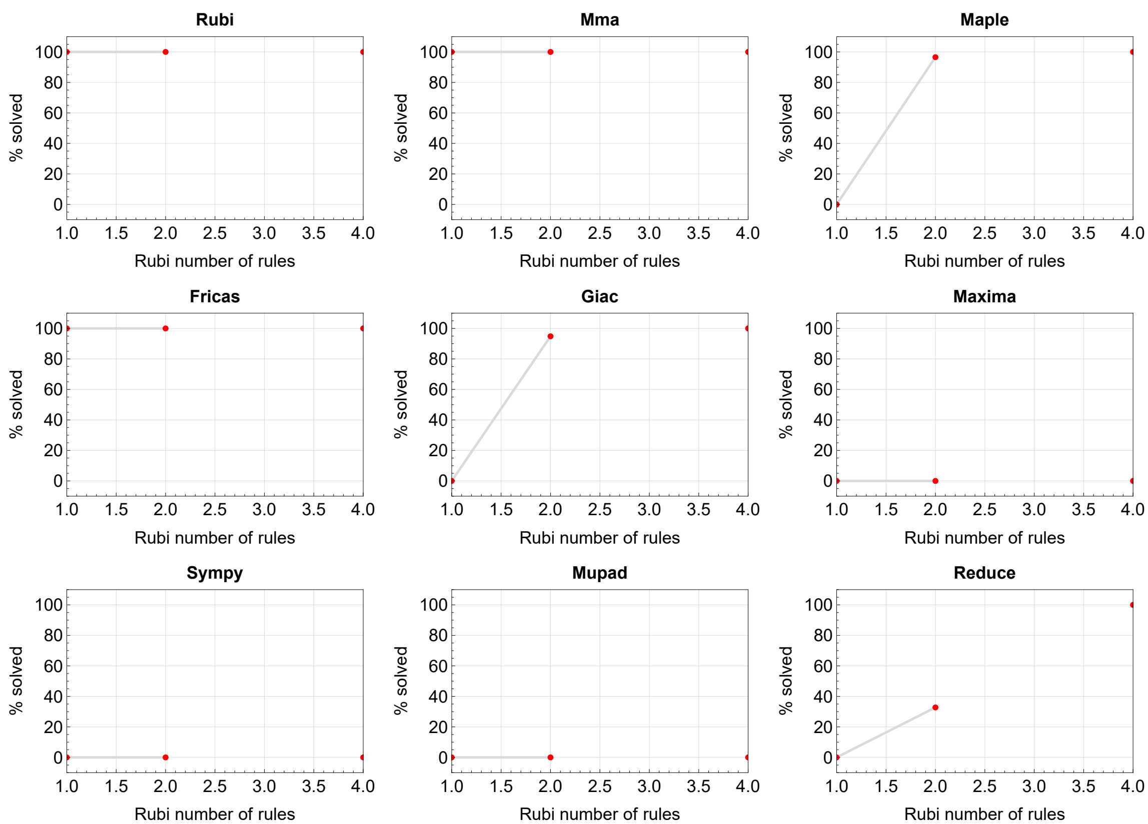


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

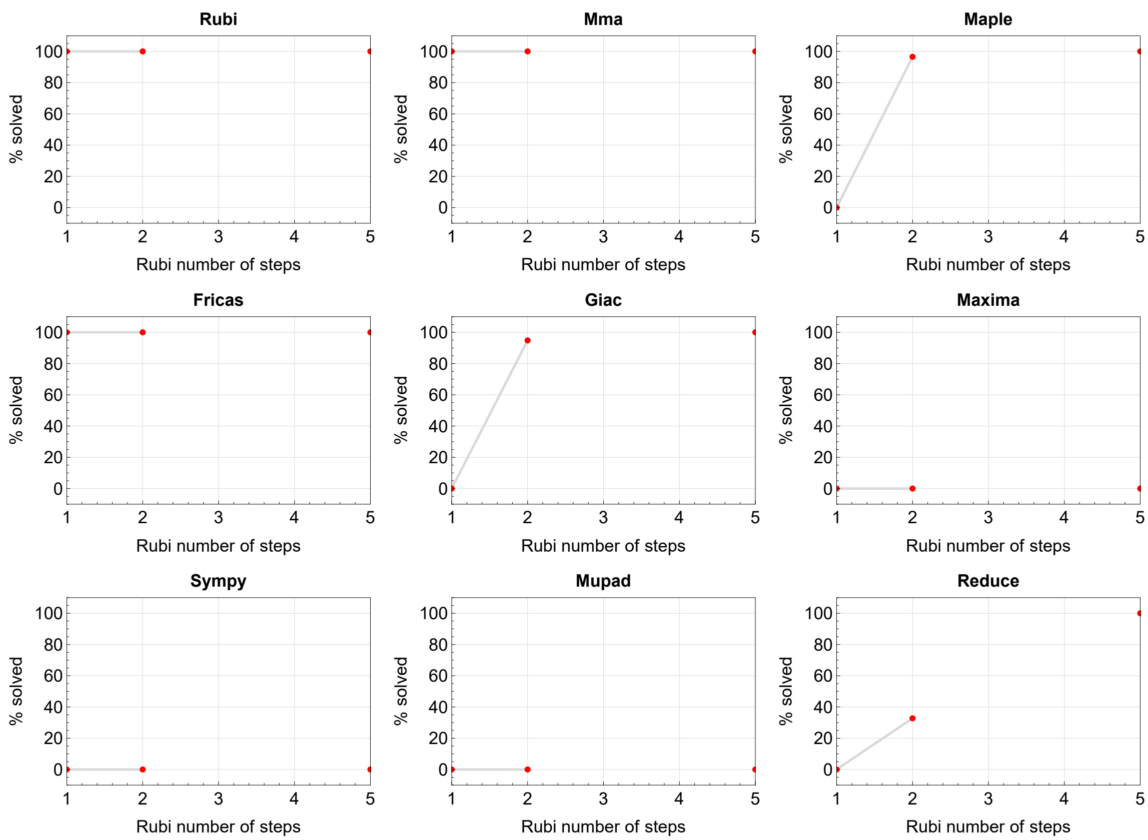


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

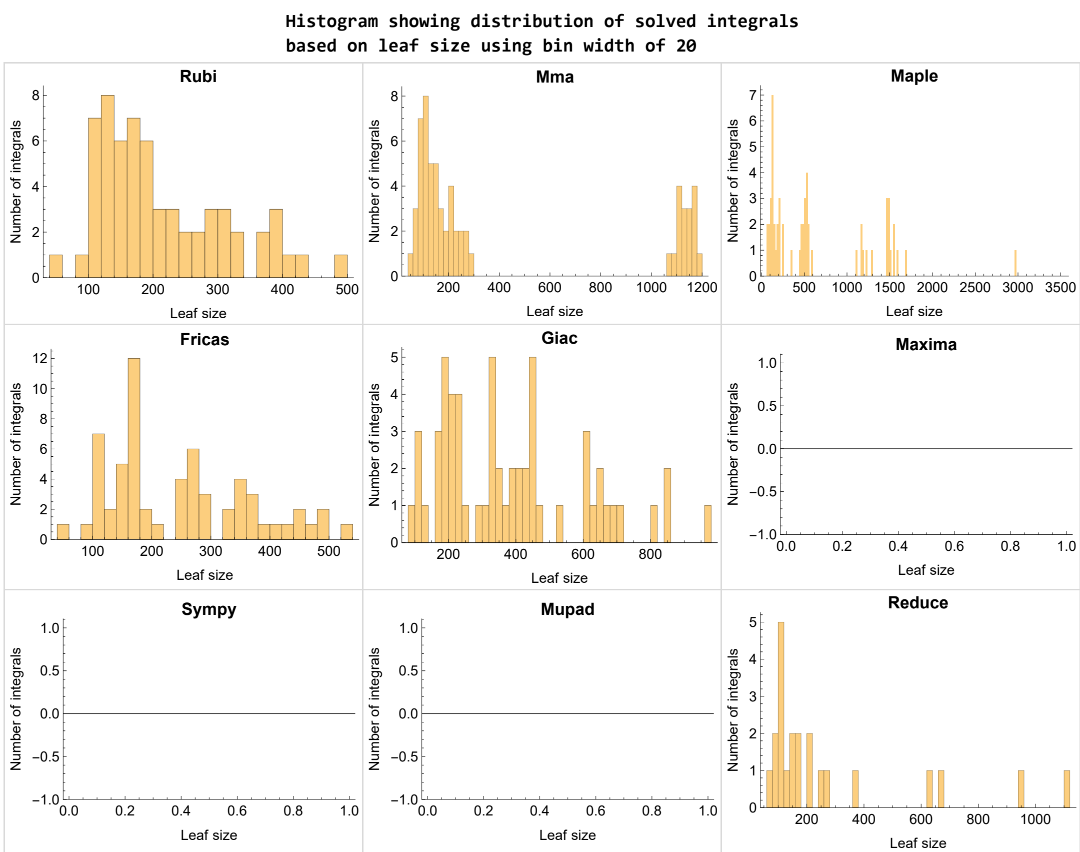


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

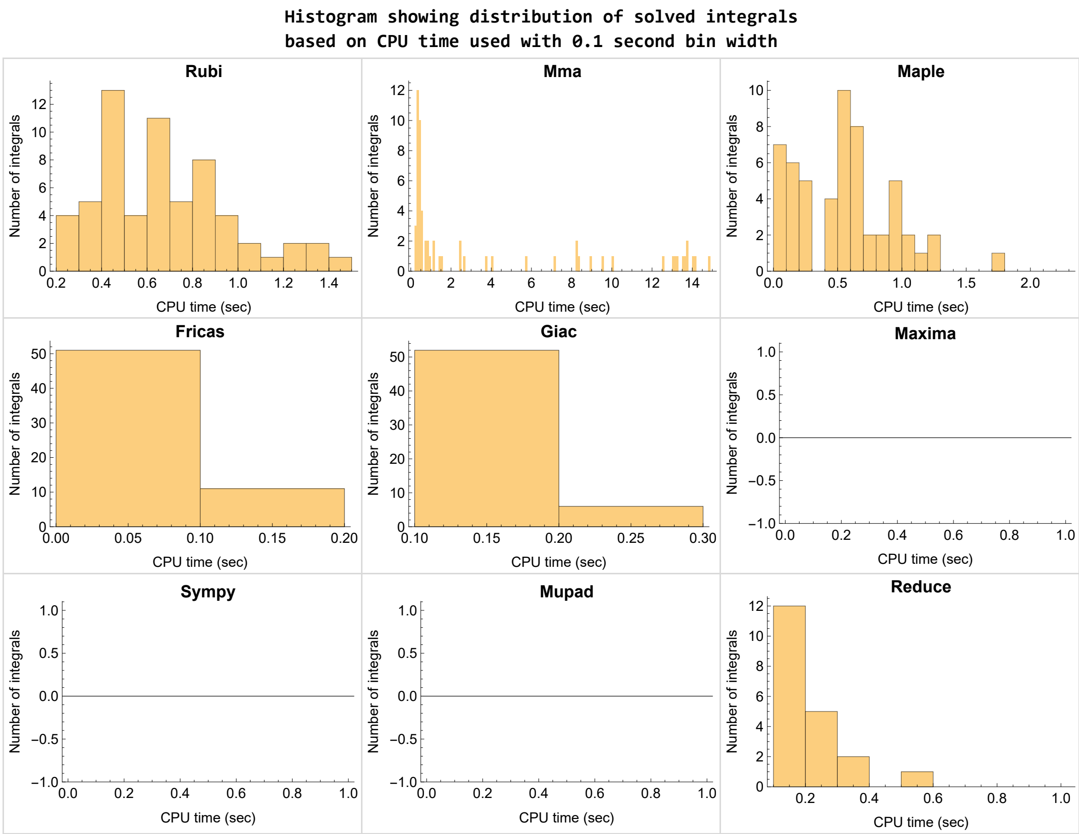


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

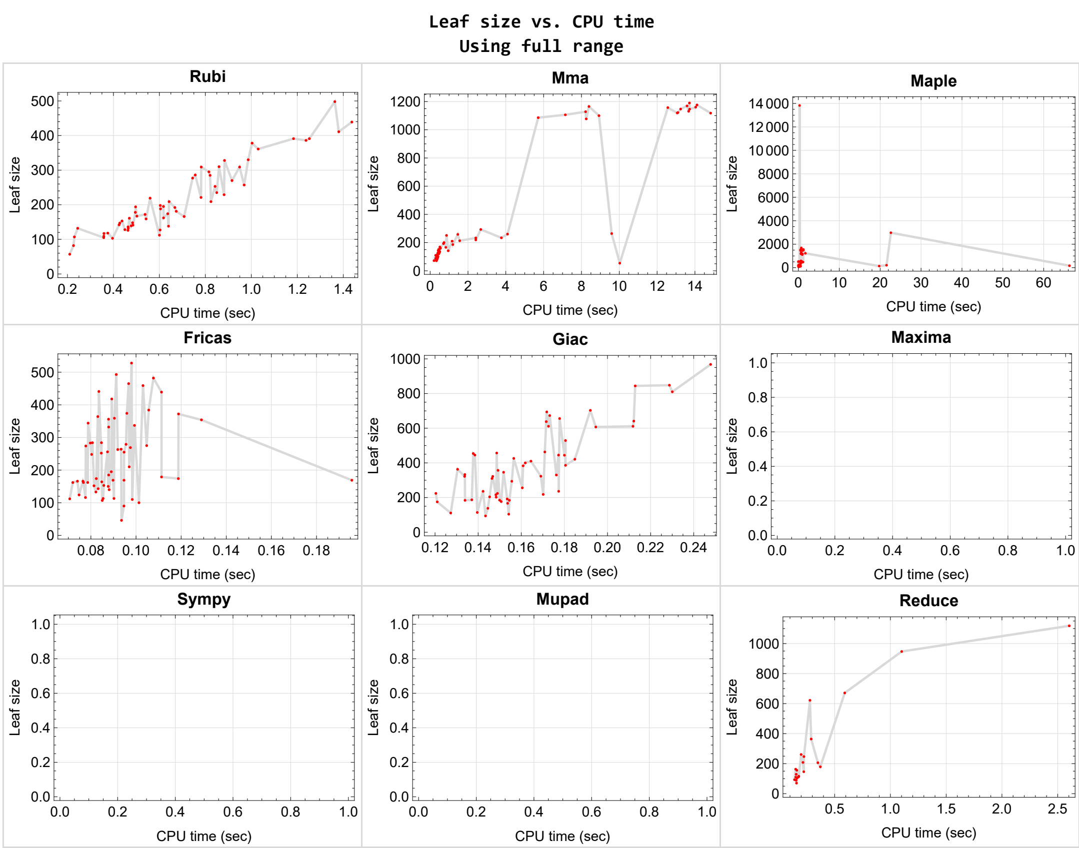


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

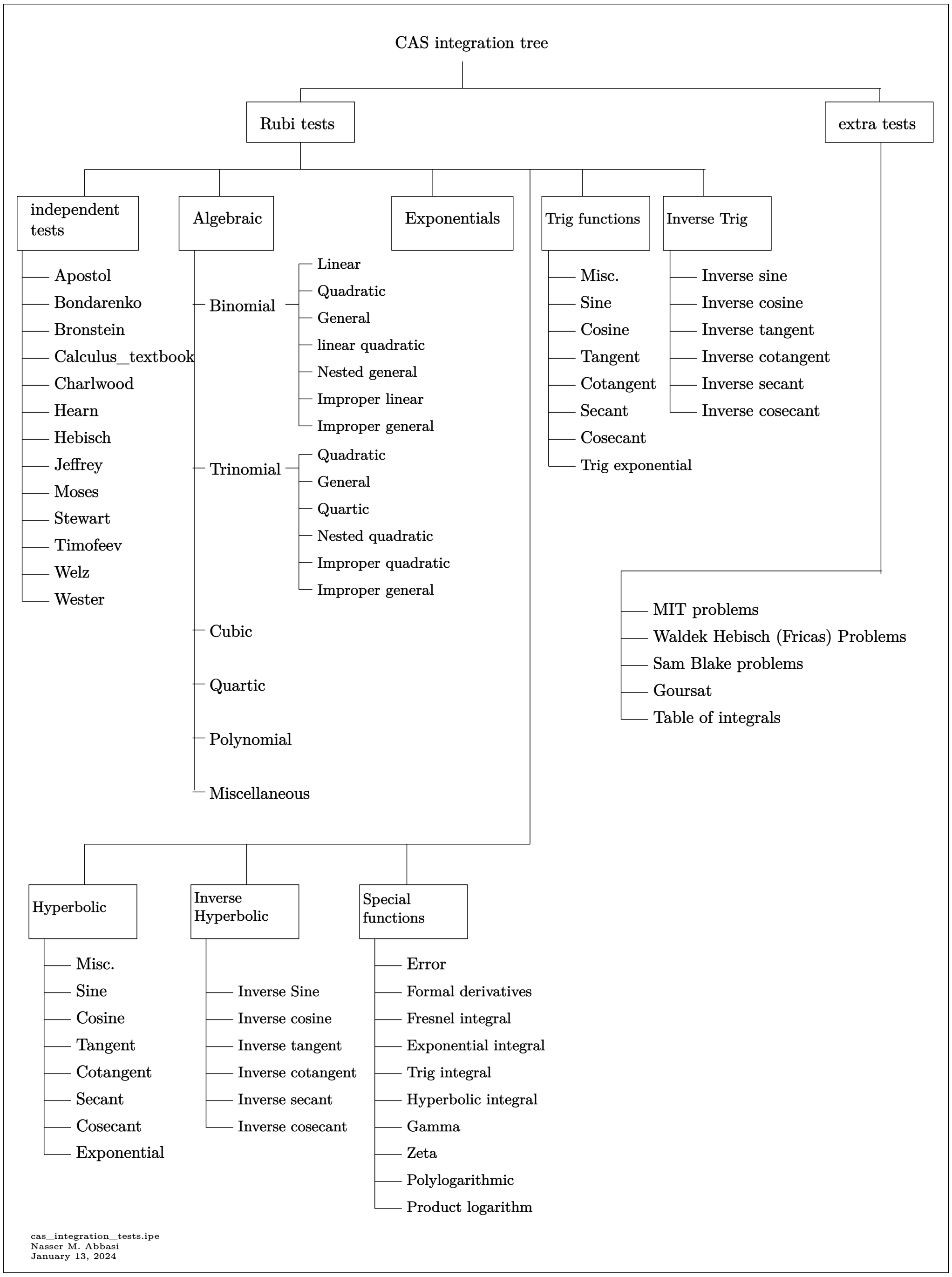
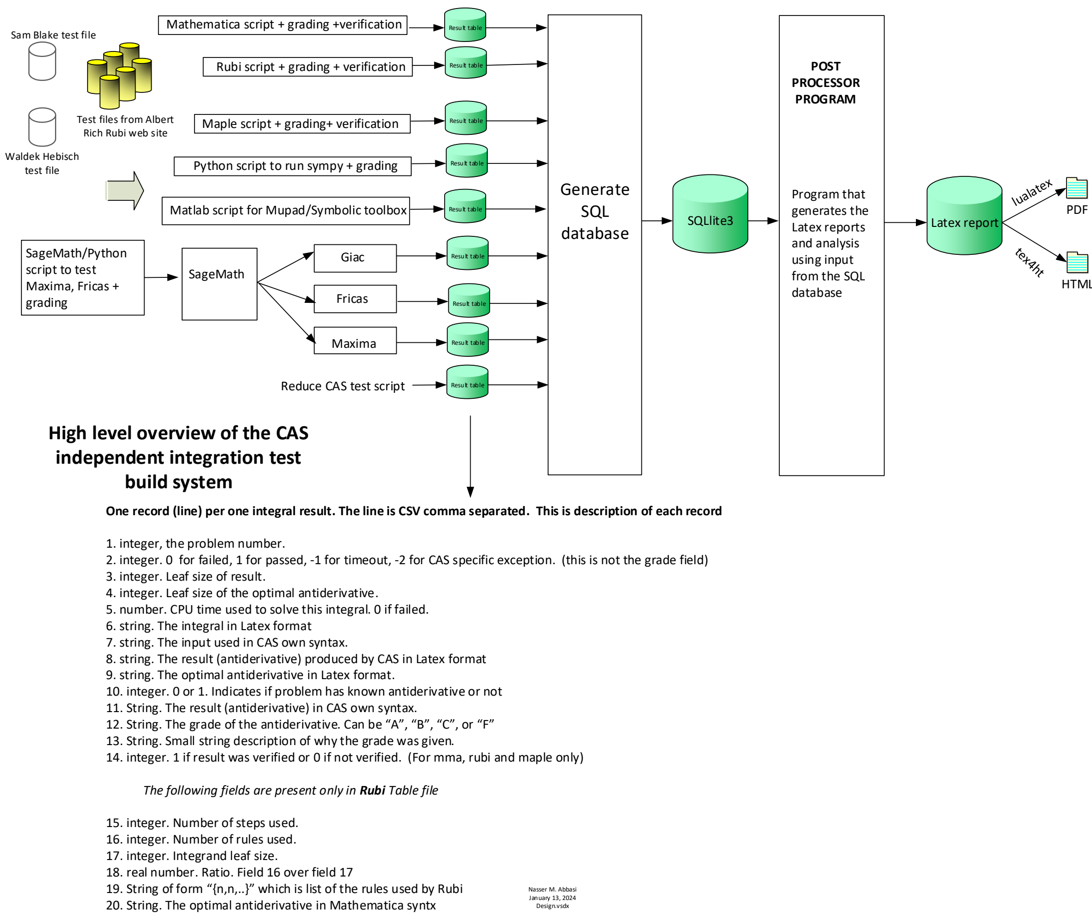


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 29, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 62 }

B grade { }

C grade { 22, 23, 25, 26, 27, 28, 30, 31, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 47, 48, 51 }

B grade { 5, 6, 18, 19, 20, 21, 32, 33, 34, 35, 36, 42, 49, 50 }

C grade { 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 43, 44, 45, 46, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61 }

F normal fail { 62 }

F(-1) timedout fail { 12, 37 }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 40, 41, 45, 46, 47, 48, 52, 56, 57, 60, 61 }

B grade { 20, 24, 34, 38, 39, 42, 43, 44, 49, 50, 53, 54, 55, 58, 59 }

C grade { }

F normal fail { 62 }

F(-1) timedout fail { }

F(-2) exception fail { 22, 23, 51 }

Mupad

A grade { }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 13, 14, 16, 32, 33, 34, 35, 36, 39, 40, 41, 45, 46 }

C grade { }

F normal fail { 7, 8, 12, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 37, 38, 42, 43, 44, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	142	92	156	0	110	0	114	110	0
N.S.	1	1.15	0.74	1.26	0.00	0.89	0.00	0.92	0.89	0.00
time (sec)	N/A	0.425	0.395	0.009	0.000	0.098	0.000	0.140	0.174	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	117	82	137	0	100	0	104	89	0
N.S.	1	1.18	0.83	1.38	0.00	1.01	0.00	1.05	0.90	0.00
time (sec)	N/A	0.359	0.312	19.764	0.000	0.101	0.000	0.154	0.155	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	82	72	92	0	90	0	94	70	0
N.S.	1	0.94	0.83	1.06	0.00	1.03	0.00	1.08	0.80	0.00
time (sec)	N/A	0.226	0.205	0.145	0.000	0.095	0.000	0.143	0.158	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	87	156	0	107	0	111	116	0
N.S.	1	1.00	0.84	1.51	0.00	1.04	0.00	1.08	1.13	0.00
time (sec)	N/A	0.396	0.293	0.407	0.000	0.085	0.000	0.127	0.179	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	136	110	197	0	113	0	138	114	0
N.S.	1	1.32	1.07	1.91	0.00	1.10	0.00	1.34	1.11	0.00
time (sec)	N/A	0.464	0.292	0.170	0.000	0.085	0.000	0.145	0.178	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	167	119	219	0	133	0	185	146	0
N.S.	1	1.27	0.91	1.67	0.00	1.02	0.00	1.41	1.11	0.00
time (sec)	N/A	0.502	0.424	0.180	0.000	0.082	0.000	0.150	0.223	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	178	120	207	0	150	0	187	25	0
N.S.	1	0.97	0.66	1.13	0.00	0.82	0.00	1.02	0.14	0.00
time (sec)	N/A	0.495	0.457	0.021	0.000	0.088	0.000	0.137	200.021	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	153	110	188	0	140	0	177	23	0
N.S.	1	1.15	0.83	1.41	0.00	1.05	0.00	1.33	0.17	0.00
time (sec)	N/A	0.437	0.381	0.181	0.000	0.088	0.000	0.151	200.025	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	71	74	0	113	0	167	104	0
N.S.	1	1.00	0.66	0.69	0.00	1.06	0.00	1.56	0.97	0.00
time (sec)	N/A	0.230	0.333	0.065	0.000	0.090	0.000	0.154	0.163	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	139	126	207	0	164	0	184	261	0
N.S.	1	0.99	0.90	1.48	0.00	1.17	0.00	1.31	1.86	0.00
time (sec)	N/A	0.477	0.405	21.560	0.000	0.085	0.000	0.134	0.200	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	172	133	248	0	169	0	256	247	0
N.S.	1	1.08	0.83	1.55	0.00	1.06	0.00	1.60	1.54	0.00
time (sec)	N/A	0.538	0.470	0.179	0.000	0.095	0.000	0.160	0.225	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	219	130	0	0	185	0	236	25	0
N.S.	1	1.18	0.70	0.00	0.00	0.99	0.00	1.27	0.13	0.00
time (sec)	N/A	0.560	0.495	180.000	0.000	0.088	0.000	0.177	200.013	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	93	85	0	153	0	224	156	0
N.S.	1	1.00	0.48	0.44	0.00	0.79	0.00	1.15	0.80	0.00
time (sec)	N/A	0.496	0.399	0.085	0.000	0.086	0.000	0.149	0.161	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	83	75	0	143	0	216	163	0
N.S.	1	1.00	0.63	0.57	0.00	1.08	0.00	1.64	1.23	0.00
time (sec)	N/A	0.245	0.367	0.067	0.000	0.083	0.000	0.148	0.151	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	147	97	166	0	169	0	204	25	0
N.S.	1	1.12	0.74	1.27	0.00	1.29	0.00	1.56	0.19	0.00
time (sec)	N/A	0.428	0.385	66.283	0.000	0.196	0.000	0.145	200.017	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	159	137	242	0	210	0	218	364	0
N.S.	1	0.86	0.74	1.32	0.00	1.14	0.00	1.18	1.98	0.00
time (sec)	N/A	0.542	0.508	0.186	0.000	0.097	0.000	0.170	0.290	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	174	258	549	0	275	0	323	238	0
N.S.	1	0.46	0.68	1.44	0.00	0.72	0.00	0.85	0.62	0.00
time (sec)	N/A	0.636	1.463	0.644	0.000	0.105	0.000	0.169	0.337	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	126	210	525	0	263	0	310	219	0
N.S.	1	0.42	0.71	1.77	0.00	0.89	0.00	1.04	0.74	0.00
time (sec)	N/A	0.463	1.162	0.589	0.000	0.092	0.000	0.146	0.356	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	105	166	509	0	248	0	294	124	0
N.S.	1	0.48	0.76	2.35	0.00	1.14	0.00	1.35	0.57	0.00
time (sec)	N/A	0.358	0.829	0.511	0.000	0.080	0.000	0.156	0.412	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	127	251	528	0	256	0	330	24	0
N.S.	1	0.73	1.45	3.05	0.00	1.48	0.00	1.91	0.14	0.00
time (sec)	N/A	0.603	0.865	0.517	0.000	0.087	0.000	0.176	0.257	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	188	293	581	0	284	0	386	28	0
N.S.	1	0.69	1.08	2.14	0.00	1.05	0.00	1.42	0.10	0.00
time (sec)	N/A	0.605	2.677	0.615	0.000	0.085	0.000	0.181	2.449	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	285	1100	1280	0	354	0	0	25	0
N.S.	1	0.55	2.13	2.48	0.00	0.69	0.00	0.00	0.05	0.00
time (sec)	N/A	0.821	8.927	0.683	0.000	0.129	0.000	0.000	200.018	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	195	1086	1195	0	337	0	0	43	0
N.S.	1	0.55	3.09	3.39	0.00	0.96	0.00	0.00	0.12	0.00
time (sec)	N/A	0.618	5.713	0.654	0.000	0.099	0.000	0.000	0.324	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	161	213	106	0	116	0	333	41	0
N.S.	1	0.79	1.04	0.52	0.00	0.57	0.00	1.62	0.20	0.00
time (sec)	N/A	0.469	1.562	0.221	0.000	0.078	0.000	0.134	0.234	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	235	1106	1515	0	356	0	529	25	0
N.S.	1	0.71	3.35	4.59	0.00	1.08	0.00	1.60	0.08	0.00
time (sec)	N/A	0.850	7.148	0.522	0.000	0.088	0.000	0.181	200.013	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	309	1119	1598	0	384	0	638	25	0
N.S.	1	0.75	2.71	3.87	0.00	0.93	0.00	1.54	0.06	0.00
time (sec)	N/A	0.949	13.065	0.680	0.000	0.106	0.000	0.172	200.024	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	361	1118	1178	0	439	0	607	66	0
N.S.	1	0.74	2.31	2.43	0.00	0.91	0.00	1.25	0.14	0.00
time (sec)	N/A	1.030	14.830	0.619	0.000	0.111	0.000	0.195	0.573	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	328	1077	126	0	166	0	445	64	0
N.S.	1	1.02	3.34	0.39	0.00	0.52	0.00	1.38	0.20	0.00
time (sec)	N/A	0.883	8.252	0.253	0.000	0.074	0.000	0.177	0.558	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	309	233	125	0	166	0	445	62	0
N.S.	1	0.92	0.70	0.37	0.00	0.50	0.00	1.33	0.19	0.00
time (sec)	N/A	0.782	2.412	0.260	0.000	0.076	0.000	0.138	0.539	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	446	386	1146	1480	0	465	0	641	25	0
N.S.	1	0.87	2.57	3.32	0.00	1.04	0.00	1.44	0.06	0.00
time (sec)	N/A	1.238	13.718	0.596	0.000	0.097	0.000	0.212	200.017	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	620	498	1169	1548	0	493	0	703	25	0
N.S.	1	0.80	1.89	2.50	0.00	0.80	0.00	1.13	0.04	0.00
time (sec)	N/A	1.363	13.582	0.631	0.000	0.091	0.000	0.192	200.025	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	162	117	528	0	179	0	202	129	0
N.S.	1	0.78	0.56	2.53	0.00	0.86	0.00	0.97	0.62	0.00
time (sec)	N/A	0.618	0.394	0.024	0.000	0.111	0.000	0.148	0.155	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	132	105	502	0	169	0	192	110	0
N.S.	1	0.84	0.67	3.20	0.00	1.08	0.00	1.22	0.70	0.00
time (sec)	N/A	0.465	0.331	0.022	0.000	0.090	0.000	0.153	0.151	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	109	90	485	0	152	0	175	93	0
N.S.	1	1.04	0.86	4.62	0.00	1.45	0.00	1.67	0.89	0.00
time (sec)	N/A	0.358	0.304	0.928	0.000	0.081	0.000	0.121	0.140	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	112	119	486	0	162	0	184	179	0
N.S.	1	0.79	0.84	3.45	0.00	1.15	0.00	1.30	1.27	0.00
time (sec)	N/A	0.600	0.398	0.541	0.000	0.077	0.000	0.154	0.372	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	181	143	545	0	195	0	236	208	0
N.S.	1	0.94	0.74	2.82	0.00	1.01	0.00	1.22	1.08	0.00
time (sec)	N/A	0.673	0.423	0.646	0.000	0.089	0.000	0.142	0.216	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	221	152	0	0	274	0	363	25	0
N.S.	1	0.97	0.67	0.00	0.00	1.21	0.00	1.60	0.11	0.00
time (sec)	N/A	0.781	0.450	180.000	0.000	0.078	0.000	0.130	200.014	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	198	145	2977	0	264	0	346	23	0
N.S.	1	1.12	0.82	16.82	0.00	1.49	0.00	1.95	0.13	0.00
time (sec)	N/A	0.604	0.451	22.622	0.000	0.093	0.000	0.152	200.014	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	128	78	111	0	112	0	224	206	0
N.S.	1	1.23	0.75	1.07	0.00	1.08	0.00	2.15	1.98	0.00
time (sec)	N/A	0.450	0.354	0.211	0.000	0.071	0.000	0.120	0.350	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	209	169	1460	0	284	0	357	622	0
N.S.	1	0.97	0.79	6.79	0.00	1.32	0.00	1.66	2.89	0.00
time (sec)	N/A	0.824	0.514	0.532	0.000	0.081	0.000	0.149	0.279	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	270	193	1687	0	332	0	454	671	0
N.S.	1	0.88	0.63	5.48	0.00	1.08	0.00	1.47	2.18	0.00
time (sec)	N/A	0.916	0.703	0.723	0.000	0.088	0.000	0.138	0.590	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	378	160	13828	0	364	0	444	25	0
N.S.	1	1.52	0.64	55.53	0.00	1.46	0.00	1.78	0.10	0.00
time (sec)	N/A	1.003	0.543	0.293	0.000	0.083	0.000	0.180	200.012	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	310	100	131	0	162	0	322	23	0
N.S.	1	1.76	0.57	0.74	0.00	0.92	0.00	1.83	0.13	0.00
time (sec)	N/A	0.860	0.406	0.464	0.000	0.079	0.000	0.147	200.017	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	277	100	130	0	162	0	322	21	0
N.S.	1	1.57	0.57	0.74	0.00	0.92	0.00	1.83	0.12	0.00
time (sec)	N/A	0.745	0.413	0.537	0.000	0.072	0.000	0.134	200.016	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	286	158	341	0	344	0	426	947	0
N.S.	1	1.13	0.62	1.35	0.00	1.36	0.00	1.68	3.74	0.00
time (sec)	N/A	0.756	0.531	0.550	0.000	0.079	0.000	0.157	1.101	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	391	200	1489	0	441	0	463	1118	0
N.S.	1	1.03	0.52	3.91	0.00	1.16	0.00	1.22	2.93	0.00
time (sec)	N/A	1.253	0.732	1.088	0.000	0.084	0.000	0.171	2.603	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	192	260	500	0	279	0	410	273	0
N.S.	1	0.47	0.63	1.22	0.00	0.68	0.00	1.00	0.66	0.00
time (sec)	N/A	0.667	4.090	0.842	0.000	0.096	0.000	0.164	0.198	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	141	234	476	0	269	0	400	254	0
N.S.	1	0.43	0.72	1.46	0.00	0.83	0.00	1.23	0.78	0.00
time (sec)	N/A	0.485	3.766	1.142	0.000	0.098	0.000	0.162	0.198	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	118	186	459	0	252	0	383	156	0
N.S.	1	0.47	0.74	1.81	0.00	1.00	0.00	1.51	0.62	0.00
time (sec)	N/A	0.376	1.198	0.966	0.000	0.085	0.000	0.161	0.202	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	138	219	477	0	255	0	421	0	0
N.S.	1	0.53	0.84	1.82	0.00	0.97	0.00	1.61	0.00	0.00
time (sec)	N/A	0.640	2.414	0.768	0.000	0.095	0.000	0.185	1.434	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	166	264	532	0	283	0	0	0	0
N.S.	1	0.48	0.76	1.53	0.00	0.82	0.00	0.00	0.00	0.00
time (sec)	N/A	0.708	9.598	0.857	0.000	0.080	0.000	0.000	1.127	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	253	1128	1172	0	374	0	673	0	0
N.S.	1	0.46	2.04	2.12	0.00	0.68	0.00	1.21	0.00	0.00
time (sec)	N/A	0.842	8.218	0.991	0.000	0.096	0.000	0.173	0.922	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	209	1122	1113	0	359	0	656	0	0
N.S.	1	0.51	2.76	2.73	0.00	0.88	0.00	1.61	0.00	0.00
time (sec)	N/A	0.642	13.101	0.950	0.000	0.090	0.000	0.178	0.843	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	148	142	110	0	124	0	457	0	0
N.S.	1	0.77	0.74	0.58	0.00	0.65	0.00	2.39	0.00	0.00
time (sec)	N/A	0.483	0.955	0.404	0.000	0.075	0.000	0.149	0.834	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	229	1157	1463	0	372	0	694	0	0
N.S.	1	0.55	2.75	3.48	0.00	0.89	0.00	1.65	0.00	0.00
time (sec)	N/A	0.882	12.567	1.006	0.000	0.119	0.000	0.172	1.335	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	257	1190	1479	0	418	0	844	0	0
N.S.	1	0.43	2.00	2.48	0.00	0.70	0.00	1.42	0.00	0.00
time (sec)	N/A	0.969	13.710	0.991	0.000	0.089	0.000	0.213	2.160	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	391	1165	1222	0	459	0	810	0	0
N.S.	1	0.65	1.94	2.03	0.00	0.76	0.00	1.35	0.00	0.00
time (sec)	N/A	1.184	8.397	1.717	0.000	0.103	0.000	0.230	1.724	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	330	1147	130	0	174	0	611	0	0
N.S.	1	0.97	3.37	0.38	0.00	0.51	0.00	1.80	0.00	0.00
time (sec)	N/A	0.987	13.243	0.512	0.000	0.119	0.000	0.212	1.827	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	295	1131	130	0	174	0	611	0	0
N.S.	1	0.85	3.27	0.38	0.00	0.50	0.00	1.77	0.00	0.00
time (sec)	N/A	0.816	13.668	0.467	0.000	0.082	0.000	0.173	1.914	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	635	411	1160	1483	0	482	0	848	0	0
N.S.	1	0.65	1.83	2.34	0.00	0.76	0.00	1.34	0.00	0.00
time (sec)	N/A	1.381	14.039	1.204	0.000	0.108	0.000	0.229	2.790	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	822	439	1176	1551	0	528	0	968	0	0
N.S.	1	0.53	1.43	1.89	0.00	0.64	0.00	1.18	0.00	0.00
time (sec)	N/A	1.437	14.110	1.227	0.000	0.098	0.000	0.248	13.202	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	0	0	46	0	0	19	0
N.S.	1	1.00	0.95	0.00	0.00	0.81	0.00	0.00	0.33	0.00
time (sec)	N/A	0.210	10.026	0.000	0.000	0.094	0.000	0.000	0.161	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [3] had the largest ratio of [.1904760000000000006]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.15	25	0.080
2	A	2	2	1.18	23	0.087
3	A	5	4	0.94	21	0.190
4	A	2	2	1.00	25	0.080
5	A	2	2	1.32	25	0.080
6	A	2	2	1.27	25	0.080
7	A	2	2	0.97	25	0.080
8	A	2	2	1.15	23	0.087
9	A	5	4	1.00	21	0.190
10	A	2	2	0.99	25	0.080
11	A	2	2	1.08	25	0.080
12	A	2	2	1.18	25	0.080
13	A	2	2	1.00	23	0.087
14	A	5	4	1.00	21	0.190
15	A	2	2	1.12	25	0.080
16	A	2	2	0.86	25	0.080
17	A	2	2	0.46	25	0.080
18	A	2	2	0.42	23	0.087
19	A	2	2	0.48	21	0.095
20	A	2	2	0.73	25	0.080
21	A	2	2	0.69	25	0.080

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	0.55	25	0.080
23	A	2	2	0.55	23	0.087
24	A	2	2	0.79	21	0.095
25	A	2	2	0.71	25	0.080
26	A	2	2	0.75	25	0.080
27	A	2	2	0.74	25	0.080
28	A	2	2	1.02	23	0.087
29	A	2	2	0.92	21	0.095
30	A	2	2	0.87	25	0.080
31	A	2	2	0.80	25	0.080
32	A	2	2	0.78	25	0.080
33	A	2	2	0.84	23	0.087
34	A	2	2	1.04	21	0.095
35	A	2	2	0.79	25	0.080
36	A	2	2	0.94	25	0.080
37	A	2	2	0.97	25	0.080
38	A	2	2	1.12	23	0.087
39	A	2	2	1.23	21	0.095
40	A	2	2	0.97	25	0.080
41	A	2	2	0.88	25	0.080
42	A	2	2	1.52	25	0.080
43	A	2	2	1.76	23	0.087
44	A	2	2	1.57	21	0.095
45	A	2	2	1.13	25	0.080
46	A	2	2	1.03	25	0.080
47	A	2	2	0.47	25	0.080
48	A	2	2	0.43	23	0.087
49	A	2	2	0.47	21	0.095
50	A	2	2	0.53	25	0.080
51	A	2	2	0.48	25	0.080
52	A	2	2	0.46	25	0.080
53	A	2	2	0.51	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	0.77	21	0.095
55	A	2	2	0.55	25	0.080
56	A	2	2	0.43	25	0.080
57	A	2	2	0.65	25	0.080
58	A	2	2	0.97	23	0.087
59	A	2	2	0.85	21	0.095
60	A	2	2	0.65	25	0.080
61	A	2	2	0.53	25	0.080
62	A	1	1	1.00	23	0.043

CHAPTER 3

Listing of integrals

3.1 $\int \frac{x^2}{1+2x+\sqrt{-3-2x+4x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{x^2}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \frac{x}{27}-\frac{x^2}{36}+\frac{x^3}{9}-\frac{(143-60x)\sqrt{-3-2x+4x^2}}{1728} -\frac{1}{72}\left(-3-2x+4x^2\right)^{3/2}-\frac{823\text{arctanh}\left(\frac{1-4x}{2\sqrt{-3-2x+4x^2}}\right)}{10368} -\frac{2}{81}\text{arctanh}\left(\frac{7+11x}{\sqrt{-3-2x+4x^2}}\right)-\frac{2}{81}\log(2+3x)$$

output
```
1/27*x-1/36*x^2+1/9*x^3-1/1728*(143-60*x)*(4*x^2-2*x-3)^(1/2)-1/72*(4*x^2-
2*x-3)^(3/2)-823/10368*arctanh(1/2*(1-4*x)/(4*x^2-2*x-3)^(1/2))-2/81*arcta
nh((7+11*x)/(4*x^2-2*x-3)^(1/2))-2/81*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \frac{96x(4-3x+12x^2)-6\sqrt{-3-2x+4x^2}(71-108x+96x^2)-567\log\left(1-4x+2\sqrt{-3-2x+4x^2}\right)-}{10368}$$

input
```
Integrate[x^2/(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2]),x]
```

output
```
(96*x*(4 - 3*x + 12*x^2) - 6*Sqrt[-3 - 2*x + 4*x^2]*(71 - 108*x + 96*x^2)
- 567*Log[1 - 4*x + 2*Sqrt[-3 - 2*x + 4*x^2]] - 512*Log[-5 - 6*x + 3*Sqrt[
-3 - 2*x + 4*x^2]])/10368
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{4x^2-2x-3}+2x+1}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{x^2}{3}-\frac{1}{6}\sqrt{4x^2-2x-3}x-\frac{2\sqrt{4x^2-2x-3}}{9(3x+2)}+\frac{1}{9}\sqrt{4x^2-2x-3}-\frac{x}{18}-\frac{2}{27(3x+2)}+\frac{1}{27}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{823\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)}{10368}-\frac{2}{81}\text{arctanh}\left(\frac{11x+7}{\sqrt{4x^2-2x-3}}\right)+\frac{x^3}{9}-\frac{x^2}{36}-\frac{1}{72}\left(4x^2-2x-3\right)^{3/2}-\frac{5}{576}(1-4x)\sqrt{4x^2-2x-3}-\frac{2}{27}\sqrt{4x^2-2x-3}+\frac{x}{27}-\frac{2}{81}\log(3x+2)$$

input
```
Int[x^2/(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2]),x]
```

output
```
x/27 - x^2/36 + x^3/9 - (2*Sqrt[-3 - 2*x + 4*x^2])/27 - (5*(1 - 4*x)*Sqrt[
-3 - 2*x + 4*x^2])/576 - (-3 - 2*x + 4*x^2)^(3/2)/72 - (823*ArcTanh[(1 - 4
*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/10368 - (2*ArcTanh[(7 + 11*x)/Sqrt[-3 - 2
*x + 4*x^2]])/81 - (2*Log[2 + 3*x])/81
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26

$$\frac{5(8x-2)\sqrt{4x^2-2x-3}}{1152}-\frac{65\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4x^2-2x-3}\right)\sqrt{4}}{2304}-\frac{(4x^2-2x-3)^{\frac{3}{2}}}{72}-\frac{2\sqrt{36\left(x+\frac{2}{3}\right)^2-66}}{81}$$

input
```
int(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2)),x)
```

output
```
5/1152*(8*x-2)*(4*x^2-2*x-3)^(1/2)-65/2304*ln(1/4*(4*x-1)*4^(1/2)+(4*x^2-2
*x-3)^(1/2))*4^(1/2)-1/72*(4*x^2-2*x-3)^(3/2)-2/81*(36*(x+2/3)^2-66*x-43)^
(1/2)+11/162*ln(1/4*(4*x-1)*4^(1/2)+(4*(x+2/3)^2-22/3*x-43/9)^(1/2))*4^(1/
2)+2/81*arctanh(9/2*(-14/3-22/3*x)/(36*(x+2/3)^2-66*x-43)^(1/2))-1/36*x^2+
1/27*x-2/81*ln(2+3*x)+1/9*x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \frac{1}{9}\,x^3 - \frac{1}{36}\,x^2 - \frac{1}{1728}\,\left(96\,x^2 - 108\,x + 71\right)\sqrt{4\,x^2 - 2\,x - 3} + \frac{1}{27}\,x - \frac{2}{81}\,\log\left(3\,x + 2\right) + \frac{2}{81}\,\log\left(-2\,x + \sqrt{4\,x^2 - 2\,x - 3} - 1\right) - \frac{823}{10368}\,\log\left(-4\,x + 2\,\sqrt{4\,x^2 - 2\,x - 3} + 1\right) - \frac{2}{81}\,\log\left(-6\,x + 3\,\sqrt{4\,x^2 - 2\,x - 3} - 5\right)$$

input
```
integrate(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

output
```
1/9*x^3 - 1/36*x^2 - 1/1728*(96*x^2 - 108*x + 71)*sqrt(4*x^2 - 2*x - 3) +
1/27*x - 2/81*log(3*x + 2) + 2/81*log(-2*x + sqrt(4*x^2 - 2*x - 3) - 1) -
823/10368*log(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1) - 2/81*log(-6*x + 3*sqrt
(4*x^2 - 2*x - 3) - 5)
```

Sympy [F]

$$\int \frac{x^2}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \int \frac{x^2}{2x+\sqrt{4x^2-2x-3}+1}\,dx$$

input
```
integrate(x**2/(1+2*x+(4*x**2-2*x-3)**(1/2)),x)
```

output
```
Integral(x**2/(2*x + sqrt(4*x**2 - 2*x - 3) + 1), x)
```

Maxima [F]

$$\int \frac{x^2}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \int \frac{x^2}{2\,x+\sqrt{4\,x^2-2\,x-3}+1}\,dx$$

input
```
integrate(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(2*x + sqrt(4*x^2 - 2*x - 3) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{x^2}{1+2x+\sqrt{-3-2x+4x^2}}\,dx &= \frac{1}{9}\,x^3-\frac{1}{36}\,x^2 \\ &\quad -\frac{1}{1728}\,(12\,(8\,x-9)x+71)\sqrt{4\,x^2-2\,x-3} \\ &\quad +\frac{1}{27}\,x-\frac{2}{81}\,\log\left(|3\,x+2|\right) \\ &\quad +\frac{2}{81}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right) \\ &\quad -\frac{823}{10368}\,\log\left(\left|-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right|\right) \\ &\quad -\frac{2}{81}\,\log\left(\left|-6\,x+3\,\sqrt{4\,x^2-2\,x-3}-5\right|\right)\end{aligned}$$

input
```
integrate(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

output
```
1/9*x^3 - 1/36*x^2 - 1/1728*(12*(8*x - 9)*x + 71)*sqrt(4*x^2 - 2*x - 3) +
1/27*x - 2/81*log(abs(3*x + 2)) + 2/81*log(abs(-2*x + sqrt(4*x^2 - 2*x - 3
) - 1)) - 823/10368*log(abs(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1)) - 2/81*lo
g(abs(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \frac{x}{27} - \frac{2\,\ln\left(x+\frac{2}{3}\right)}{81} - \int \frac{x^2\,\sqrt{4\,x^2-2\,x-3}}{2\,(3\,x+2)}\,dx - \frac{x^2}{36} + \frac{x^3}{9}$$

input
```
int(x^2/(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1),x)
```

output
```
x/27 - (2*log(x + 2/3))/81 - int((x^2*(4*x^2 - 2*x - 3)^(1/2))/(2*(3*x + 2
)), x) - x^2/36 + x^3/9
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = -\frac{\sqrt{4x^2-2x-3}\,x^2}{18} + \frac{\sqrt{4x^2-2x-3}\,x}{16} - \frac{71\sqrt{4x^2-2x-3}}{1728} - \frac{4\log\left(\frac{26\sqrt{4x^2-2x-3}+52x+26}{\sqrt{13}}\right)}{81} + \frac{1079\log\left(\frac{2\sqrt{4x^2-2x-3}+4x-1}{\sqrt{13}}\right)}{10368} + \frac{x^3}{9} - \frac{x^2}{36} + \frac{x}{27} - \frac{55}{1728}$$

input
```
int(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2)),x)
```

output
```
( - 576*sqrt(4*x**2 - 2*x - 3)*x**2 + 648*sqrt(4*x**2 - 2*x - 3)*x - 426*s
qrt(4*x**2 - 2*x - 3) - 512*log((26*sqrt(4*x**2 - 2*x - 3) + 52*x + 26)/sq
rt(13)) + 1079*log((2*sqrt(4*x**2 - 2*x - 3) + 4*x - 1)/sqrt(13)) + 1152*x
**3 - 288*x**2 + 384*x - 330)/10368
```

3.2 $\int \frac{x}{1+2x+\sqrt{-3-2x+4x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 99

$$\int \frac{x}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = -\frac{x}{18}+\frac{x^2}{6}+\frac{1}{144}(19-12x)\sqrt{-3-2x+4x^2} + \frac{59}{864}\text{arctanh}\left(\frac{1-4x}{2\sqrt{-3-2x+4x^2}}\right) + \frac{1}{27}\text{arctanh}\left(\frac{7+11x}{\sqrt{-3-2x+4x^2}}\right)+\frac{1}{27}\log(2+3x)$$

output
```
-1/18*x+1/6*x^2+1/144*(19-12*x)*(4*x^2-2*x-3)^(1/2)+59/864*arctanh(1/2*(1-
4*x)/(4*x^2-2*x-3)^(1/2))+1/27*arctanh((7+11*x)/(4*x^2-2*x-3)^(1/2))+1/27*
ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \frac{1}{864}\Big(48x(-1+3x)+6(19-12x)\sqrt{-3-2x+4x^2} + 27\log\left(1-4x+2\sqrt{-3-2x+4x^2}\right) + 64\log\left(-5-6x+3\sqrt{-3-2x+4x^2}\right)\Big)$$

input
```
Integrate[x/(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2]),x]
```

output
```
(48*x*(-1 + 3*x) + 6*(19 - 12*x)*Sqrt[-3 - 2*x + 4*x^2] + 27*Log[1 - 4*x +
 2*Sqrt[-3 - 2*x + 4*x^2]] + 64*Log[-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x^2]])/
864
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{4x^2-2x-3}+2x+1}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{\sqrt{4x^2-2x-3}}{3(3x+2)}-\frac{1}{6}\sqrt{4x^2-2x-3}+\frac{x}{3}+\frac{1}{9(3x+2)}-\frac{1}{18}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{59}{864}\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)+\frac{1}{27}\text{arctanh}\left(\frac{11x+7}{\sqrt{4x^2-2x-3}}\right)+\frac{x^2}{6}+\frac{1}{48}(1-4x)\sqrt{4x^2-2x-3}+\frac{1}{9}\sqrt{4x^2-2x-3}-\frac{x}{18}+\frac{1}{27}\log(3x+2)$$

input
```
Int[x/(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2]),x]
```

output
```
-1/18*x + x^2/6 + Sqrt[-3 - 2*x + 4*x^2]/9 + ((1 - 4*x)*Sqrt[-3 - 2*x + 4*
x^2])/48 + (59*ArcTanh[(1 - 4*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/864 + ArcTan
h[(7 + 11*x)/Sqrt[-3 - 2*x + 4*x^2]]/27 + Log[2 + 3*x]/27
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 19.76 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.38

method	result
default	$-\frac{(8x-2)\sqrt{4x^2-2x-3}}{96}+\frac{13\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4x^2-2x-3}\right)\sqrt{4}}{192}+\frac{\sqrt{36\left(x+\frac{2}{3}\right)^2-66x-43}}{27}-\frac{11\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4\left(x+\frac{2}{3}\right)^2-\frac{22x}{3}}\right.}{108}$
trager	Expression too large to display

input
```
int(x/(1+2*x+(4*x^2-2*x-3)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/96*(8*x-2)*(4*x^2-2*x-3)^(1/2)+13/192*ln(1/4*(4*x-1)*4^(1/2)+(4*x^2-2*x
-3)^(1/2))*4^(1/2)+1/27*(36*(x+2/3)^2-66*x-43)^(1/2)-11/108*ln(1/4*(4*x-1)
*4^(1/2)+(4*(x+2/3)^2-22/3*x-43/9)^(1/2))*4^(1/2)-1/27*arctanh(9/2*(-14/3-
22/3*x)/(36*(x+2/3)^2-66*x-43)^(1/2))-1/18*x+1/27*ln(2+3*x)+1/6*x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{x}{1+2x+\sqrt{-3-2x+4x^2}}\,dx &= \frac{1}{6}\,x^2-\frac{1}{144}\,\sqrt{4\,x^2-2\,x-3}(12\,x-19)\\ &\quad-\frac{1}{18}\,x+\frac{1}{27}\,\log\left(3\,x+2\right)\\ &\quad-\frac{1}{27}\,\log\left(-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right)\\ &\quad+\frac{59}{864}\,\log\left(-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right)\\ &\quad+\frac{1}{27}\,\log\left(-6\,x+3\,\sqrt{4\,x^2-2\,x-3}-5\right)\end{aligned}$$

input
```
integrate(x/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

output
```
1/6*x^2 - 1/144*sqrt(4*x^2 - 2*x - 3)*(12*x - 19) - 1/18*x + 1/27*log(3*x
+ 2) - 1/27*log(-2*x + sqrt(4*x^2 - 2*x - 3) - 1) + 59/864*log(-4*x + 2*sq
rt(4*x^2 - 2*x - 3) + 1) + 1/27*log(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5)
```

Sympy [F]

$$\int \frac{x}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \int \frac{x}{2x+\sqrt{4x^2-2x-3}+1}\,dx$$

input
```
integrate(x/(1+2*x+(4*x**2-2*x-3)**(1/2)),x)
```

output
```
Integral(x/(2*x + sqrt(4*x**2 - 2*x - 3) + 1), x)
```

Maxima [F]

$$\int \frac{x}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \int \frac{x}{2\,x+\sqrt{4\,x^2-2\,x-3}+1}\,dx$$

input
```
integrate(x/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(2*x + sqrt(4*x^2 - 2*x - 3) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

$$\int \frac{x}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \frac{1}{6}\,x^2 - \frac{1}{144}\,\sqrt{4\,x^2-2\,x-3}(12\,x-19) - \frac{1}{18}\,x + \frac{1}{27}\,\log\left(|3\,x+2|\right) - \frac{1}{27}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right) + \frac{59}{864}\,\log\left(\left|-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right|\right) + \frac{1}{27}\,\log\left(\left|-6\,x+3\,\sqrt{4\,x^2-2\,x-3}-5\right|\right)$$

input
```
integrate(x/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

output
```
1/6*x^2 - 1/144*sqrt(4*x^2 - 2*x - 3)*(12*x - 19) - 1/18*x + 1/27*log(abs(
3*x + 2)) - 1/27*log(abs(-2*x + sqrt(4*x^2 - 2*x - 3) - 1)) + 59/864*log(a
bs(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1)) + 1/27*log(abs(-6*x + 3*sqrt(4*x^2
 - 2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \frac{\ln\left(x+\frac{2}{3}\right)}{27} - \frac{x}{18} - \int \frac{x\,\sqrt{4\,x^2-2\,x-3}}{2\,(3\,x+2)}\,dx + \frac{x^2}{6}$$

input
```
int(x/(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1),x)
```

output
```
log(x + 2/3)/27 - x/18 - int((x*(4*x^2 - 2*x - 3)^(1/2))/(2*(3*x + 2)), x)
 + x^2/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

$$\int \frac{x}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = -\frac{\sqrt{4x^2-2x-3}\,x}{12}+\frac{19\sqrt{4x^2-2x-3}}{144}+\frac{2\log\left(\frac{26\sqrt{4x^2-2x-3}+52x+26}{\sqrt{13}}\right)}{27}-\frac{91\log\left(\frac{2\sqrt{4x^2-2x-3}+4x-1}{\sqrt{13}}\right)}{864}+\frac{x^2}{6}-\frac{x}{18}-\frac{37}{576}$$

input

```
int(x/(1+2*x+(4*x^2-2*x-3)^(1/2)),x)
```

output

```
( - 144*sqrt(4*x**2 - 2*x - 3)*x + 228*sqrt(4*x**2 - 2*x - 3) + 128*log((2
6*sqrt(4*x**2 - 2*x - 3) + 52*x + 26)/sqrt(13)) - 182*log((2*sqrt(4*x**2 -
 2*x - 3) + 4*x - 1)/sqrt(13)) + 288*x**2 - 96*x - 111)/1728
```

3.3 $\int \frac{1}{1+2x+\sqrt{-3-2x+4x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{1}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \frac{x}{3}-\frac{1}{6}\sqrt{-3-2x+4x^2} - \frac{11}{36}\text{arctanh}\left(\frac{1-4x}{2\sqrt{-3-2x+4x^2}}\right) - \frac{1}{18}\text{arctanh}\left(\frac{7+11x}{\sqrt{-3-2x+4x^2}}\right)-\frac{1}{18}\log(2+3x)$$

output
```
1/3*x-1/6*(4*x^2-2*x-3)^(1/2)-11/36*arctanh(1/2*(1-4*x)/(4*x^2-2*x-3)^(1/2
))-1/18*arctanh((7+11*x)/(4*x^2-2*x-3)^(1/2))-1/18*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \frac{1}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \frac{1}{36}\Big(12x-6\sqrt{-3-2x+4x^2} - 9\log\left(1-4x+2\sqrt{-3-2x+4x^2}\right) - 4\log\left(-5-6x+3\sqrt{-3-2x+4x^2}\right)\Big)$$

input
```
Integrate[(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^(-1),x]
```

output
```
(12*x - 6*Sqrt[-3 - 2*x + 4*x^2] - 9*Log[1 - 4*x + 2*Sqrt[-3 - 2*x + 4*x^2
]] - 4*Log[-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x^2]])/36
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2541, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4x^2-2x-3}+2x+1}dx$$

$$\Big\downarrow \text{2541}$$

$$2\int -\frac{-\left(2x+\sqrt{4x^2-2x-3}+1\right)^2+3\left(2x+\sqrt{4x^2-2x-3}+1\right)+1}{2\left(2x+\sqrt{4x^2-2x-3}+1\right)\left(3-2\left(2x+\sqrt{4x^2-2x-3}+1\right)\right)^2}d\left(2x+\sqrt{4x^2-2x-3}+1\right)$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{-\left(2x+\sqrt{4x^2-2x-3}+1\right)^2+3\left(2x+\sqrt{4x^2-2x-3}+1\right)+1}{\left(2x+\sqrt{4x^2-2x-3}+1\right)\left(3-2\left(2x+\sqrt{4x^2-2x-3}+1\right)\right)^2}d\left(2x+\sqrt{4x^2-2x-3}+1\right)$$

$$\Big\downarrow \text{1195}$$

$$-\int \left(-\frac{13}{18\left(2\left(2x+\sqrt{4x^2-2x-3}+1\right)-3\right)}+\frac{13}{6\left(2\left(2x+\sqrt{4x^2-2x-3}+1\right)-3\right)^2}+\frac{1}{9\left(2x+\sqrt{4x^2-2x\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{13}{12\left(3-2\left(\sqrt{4x^2-2x-3}+2x+1\right)\right)}-\frac{1}{9}\log\left(\sqrt{4x^2-2x-3}+2x+1\right)+\frac{13}{36}\log\left(3-2\left(\sqrt{4x^2-2x-3}+2x+1\right)\right)$$

input
```
Int[(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^(-1),x]
```

output
```
-13/(12*(3 - 2*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2]))) - Log[1 + 2*x + Sqrt[-
3 - 2*x + 4*x^2]]/9 + (13*Log[3 - 2*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])])/3
6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06

method	result
default	$-\frac{\sqrt{36\left(x+\frac{2}{3}\right)^{2}-66x-43}}{18}+\frac{11\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4\left(x+\frac{2}{3}\right)^{2}-\frac{22x}{3}-\frac{43}{9}}\right)\sqrt{4}}{72}+\frac{\operatorname{arctanh}\left(\frac{-21-33x}{\sqrt{36\left(x+\frac{2}{3}\right)^{2}-66x-43}}\right)}{18}+\frac{\ln(-2-3x)}{6}-$
trager	$\frac{x}{3}-\frac{\sqrt{4x^{2}-2x-3}}{6}+\frac{\ln\left(\frac{35511830+832470756x-3099091026x^{2}\sqrt{4x^{2}-2x-3}-376883380224x^{12}+780593528832x^{10}+962873786368x^{9}-23414}{}\right.}{}$

input
```
int(1/(1+2*x+(4*x^2-2*x-3)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/18*(36*(x+2/3)^2-66*x-43)^(1/2)+11/72*ln(1/4*(4*x-1)*4^(1/2)+(4*(x+2/3)
^2-22/3*x-43/9)^(1/2))*4^(1/2)+1/18*arctanh(9/2*(-14/3-22/3*x)/(36*(x+2/3)
^2-66*x-43)^(1/2))+1/6*ln(-2-3*x)+1/3*x-2/9*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{1}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \frac{1}{3}\,x-\frac{1}{6}\,\sqrt{4\,x^{2}-2\,x-3}-\frac{1}{18}\,\log\left(3\,x+2\right)+\frac{1}{18}\,\log\left(-2\,x+\sqrt{4\,x^{2}-2\,x-3}-1\right)-\frac{11}{36}\,\log\left(-4\,x+2\,\sqrt{4\,x^{2}-2\,x-3}+1\right)-\frac{1}{18}\,\log\left(-6\,x+3\,\sqrt{4\,x^{2}-2\,x-3}-5\right)$$

input
```
integrate(1/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

output
```
1/3*x - 1/6*sqrt(4*x^2 - 2*x - 3) - 1/18*log(3*x + 2) + 1/18*log(-2*x + sq
rt(4*x^2 - 2*x - 3) - 1) - 11/36*log(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1) -
 1/18*log(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5)
```

Sympy [F]

$$\int \frac{1}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \int \frac{1}{2x+\sqrt{4x^2-2x-3}+1}\,dx$$

input
```
integrate(1/(1+2*x+(4*x**2-2*x-3)**(1/2)),x)
```

output
```
Integral(1/(2*x + sqrt(4*x**2 - 2*x - 3) + 1), x)
```

Maxima [F]

$$\int \frac{1}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \int \frac{1}{2\,x+\sqrt{4\,x^2-2\,x-3}+1}\,dx$$

input
```
integrate(1/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(2*x + sqrt(4*x^2 - 2*x - 3) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{1}{1+2x+\sqrt{-3-2x+4x^2}}\,dx &= \frac{1}{3}\,x-\frac{1}{6}\,\sqrt{4\,x^2-2\,x-3}-\frac{1}{18}\,\log\left(|3\,x+2|\right)\\ &\quad+\frac{1}{18}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right)\\ &\quad-\frac{11}{36}\,\log\left(\left|-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right|\right)\\ &\quad-\frac{1}{18}\,\log\left(\left|-6\,x+3\,\sqrt{4\,x^2-2\,x-3}-5\right|\right)\end{aligned}$$

input
```
integrate(1/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

output
```
1/3*x - 1/6*sqrt(4*x^2 - 2*x - 3) - 1/18*log(abs(3*x + 2)) + 1/18*log(abs(
-2*x + sqrt(4*x^2 - 2*x - 3) - 1)) - 11/36*log(abs(-4*x + 2*sqrt(4*x^2 - 2
*x - 3) + 1)) - 1/18*log(abs(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = \frac{x}{3}-\frac{\ln\left(x+\frac{2}{3}\right)}{18}-\int \frac{\sqrt{4\,x^2-2\,x-3}}{6\,x+4}\,dx$$

input
```
int(1/(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1),x)
```

output
```
x/3 - log(x + 2/3)/18 - int((4*x^2 - 2*x - 3)^(1/2)/(6*x + 4), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{1}{1+2x+\sqrt{-3-2x+4x^2}}\,dx = -\frac{\sqrt{4x^2-2x-3}}{6}-\frac{\log\left(\frac{26\sqrt{4x^2-2x-3}+52x+26}{\sqrt{13}}\right)}{9}+\frac{13\log\left(\frac{2\sqrt{4x^2-2x-3}+4x-1}{\sqrt{13}}\right)}{36}+\frac{x}{3}-\frac{1}{12}$$

input
```
int(1/(1+2*x+(4*x^2-2*x-3)^(1/2)),x)
```

output
```
( - 6*sqrt(4*x**2 - 2*x - 3) - 4*log((26*sqrt(4*x**2 - 2*x - 3) + 52*x + 2
6)/sqrt(13)) + 13*log((2*sqrt(4*x**2 - 2*x - 3) + 4*x - 1)/sqrt(13)) + 12*
x - 3)/36
```

3.4 $\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\begin{aligned}\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx = & -\frac{1}{4}\sqrt{3}\arctan\left(\frac{3+x}{\sqrt{3}\sqrt{-3-2x+4x^2}}\right) \\ & +\frac{1}{3}\text{arctanh}\left(\frac{1-4x}{2\sqrt{-3-2x+4x^2}}\right) \\ & +\frac{1}{12}\text{arctanh}\left(\frac{7+11x}{\sqrt{-3-2x+4x^2}}\right) \\ & +\frac{\log(x)}{4}+\frac{1}{12}\log(2+3x)\end{aligned}$$

output
```
-1/4*3^(1/2)*arctan(1/3*(3+x)*3^(1/2)/(4*x^2-2*x-3)^(1/2))+1/3*arctanh(1/2
*(1-4*x)/(4*x^2-2*x-3)^(1/2))+1/12*arctanh((7+11*x)/(4*x^2-2*x-3)^(1/2))+1
/4*ln(x)+1/12*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx=\frac{1}{12}\left(6\sqrt{3}\arctan\left(\frac{-2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)+3\log\left(x\left(-1+4x-2\sqrt{-3-2x+4x^2}\right)\right)+2\log\left(-5-6x+3\sqrt{-3-2x+4x^2}\right)\right)$$

input
```
Integrate[1/(x*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])),x]
```

output
```
(6*Sqrt[3]*ArcTan[(-2*x + Sqrt[-3 - 2*x + 4*x^2])/Sqrt[3]] + 3*Log[x*(-1 +
 4*x - 2*Sqrt[-3 - 2*x + 4*x^2])] + 2*Log[-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x
^2]])/12
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{4x^2-2x-3}+2x+1\right)}\,dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(-\frac{\sqrt{4x^2-2x-3}}{4x}+\frac{3\sqrt{4x^2-2x-3}}{4(3x+2)}+\frac{1}{4x}+\frac{1}{4(3x+2)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)+\frac{1}{3}\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)+\frac{1}{12}\text{arctanh}\left(\frac{11x+7}{\sqrt{4x^2-2x-3}}\right)+\frac{\log(x)}{4}+\frac{1}{12}\log(3x+2)$$

input
```
Int[1/(x*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])),x]
```

output
```
-1/4*(Sqrt[3]*ArcTan[(3 + x)/(Sqrt[3]*Sqrt[-3 - 2*x + 4*x^2])]) + ArcTanh[
(1 - 4*x)/(2*Sqrt[-3 - 2*x + 4*x^2])]/3 + ArcTanh[(7 + 11*x)/Sqrt[-3 - 2*x
 + 4*x^2]]/12 + Log[x]/4 + Log[2 + 3*x]/12
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.51

method	result
default	$\frac{\ln(2+3x)}{12}+\frac{\ln(x)}{4}-\frac{\sqrt{4x^2-2x-3}}{4}+\frac{\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4x^2-2x-3}\right)\sqrt{4}}{16}+\frac{\sqrt{3}\,\arctan\left(\frac{(-6-2x)\sqrt{3}}{6\sqrt{4x^2-2x-3}}\right)}{4}+\frac{\sqrt{36\left(x+\frac{2}{3}\right)^2-66x-}}{12}$
trager	Expression too large to display

input
```
int(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(2+3*x)+1/4*ln(x)-1/4*(4*x^2-2*x-3)^(1/2)+1/16*ln(1/4*(4*x-1)*4^(1/
2)+(4*x^2-2*x-3)^(1/2))*4^(1/2)+1/4*3^(1/2)*arctan(1/6*(-6-2*x)*3^(1/2)/(4
*x^2-2*x-3)^(1/2))+1/12*(36*(x+2/3)^2-66*x-43)^(1/2)-11/48*ln(1/4*(4*x-1)*
4^(1/2)+(4*(x+2/3)^2-22/3*x-43/9)^(1/2))*4^(1/2)-1/12*arctanh(9/2*(-14/3-2
2/3*x)/(36*(x+2/3)^2-66*x-43)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(-\frac{2}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{4x^2-2x-3}\right) + \frac{1}{12}\log\left(3x+2\right)+\frac{1}{4}\log\left(x\right) - \frac{1}{12}\log\left(-2x+\sqrt{4x^2-2x-3}-1\right) + \frac{1}{3}\log\left(-4x+2\sqrt{4x^2-2x-3}+1\right) + \frac{1}{12}\log\left(-6x+3\sqrt{4x^2-2x-3}-5\right)$$

input
```
integrate(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*arctan(-2/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(4*x^2 - 2*x - 3)) + 1
/12*log(3*x + 2) + 1/4*log(x) - 1/12*log(-2*x + sqrt(4*x^2 - 2*x - 3) - 1)
 + 1/3*log(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1) + 1/12*log(-6*x + 3*sqrt(4*
x^2 - 2*x - 3) - 5)
```

Sympy [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx = \int \frac{1}{x\left(2x+\sqrt{4x^2-2x-3}+1\right)}\,dx$$

input
```
integrate(1/x/(1+2*x+(4*x**2-2*x-3)**(1/2)),x)
```

output
```
Integral(1/(x*(2*x + sqrt(4*x**2 - 2*x - 3) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx = \int \frac{1}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)x}\,dx$$

input
```
integrate(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(4*x^2 - 2*x - 3) + 1)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx &= \frac{1}{2}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)\right)\\ &\quad+\frac{1}{12}\,\log\left(|3\,x+2|\right)+\frac{1}{4}\,\log\left(|x|\right)\\ &\quad-\frac{1}{12}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right)\\ &\quad+\frac{1}{3}\,\log\left(\left|-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right|\right)\\ &\quad+\frac{1}{12}\,\log\left(\left|-6\,x+3\,\sqrt{4\,x^2-2\,x-3}-5\right|\right)\end{aligned}$$

input
```
integrate(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

output
```
1/2*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - sqrt(4*x^2 - 2*x - 3))) + 1/12*log(
abs(3*x + 2)) + 1/4*log(abs(x)) - 1/12*log(abs(-2*x + sqrt(4*x^2 - 2*x - 3
) - 1)) + 1/3*log(abs(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1)) + 1/12*log(abs(
-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx = \frac{\ln\left(x+\frac{2}{3}\right)}{12} + \frac{\ln(x)}{4} - \int \frac{\sqrt{4\,x^2-2\,x-3}}{2\,\left(3\,x^2+2\,x\right)}\,dx$$

input
```
int(1/(x*(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)),x)
```

output
```
log(x + 2/3)/12 + log(x)/4 - int((4*x^2 - 2*x - 3)^(1/2)/(2*(2*x + 3*x^2))
, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx = \frac{\sqrt{3}\,atan\left(\frac{\sqrt{4x^2-2x-3}+2x}{\sqrt{3}}\right)}{2} + \frac{\log(3x+2)}{12} + \frac{\log\left(\frac{26\sqrt{4x^2-2x-3}+52x+26}{\sqrt{13}}\right)}{12} - \frac{\log\left(\frac{6\sqrt{4x^2-2x-3}+12x+10}{\sqrt{13}}\right)}{12} - \frac{\log\left(\frac{2\sqrt{4x^2-2x-3}+4x-1}{\sqrt{13}}\right)}{3} + \frac{\log(x)}{4}$$

input
```
int(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2)),x)
```

output
```
(6*sqrt(3)*atan((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3)) + log(3*x + 2) + l
og((26*sqrt(4*x**2 - 2*x - 3) + 52*x + 26)/sqrt(13)) - log((6*sqrt(4*x**2
- 2*x - 3) + 12*x + 10)/sqrt(13)) - 4*log((2*sqrt(4*x**2 - 2*x - 3) + 4*x
- 1)/sqrt(13)) + 3*log(x))/12
```

3.5 $\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\begin{aligned}\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx &= -\frac{1}{4x}+\frac{\sqrt{-3-2x+4x^2}}{4x} \\ &\quad+\frac{7\arctan\left(\frac{3+x}{\sqrt{3}\sqrt{-3-2x+4x^2}}\right)}{8\sqrt{3}} \\ &\quad-\frac{1}{8}\operatorname{arctanh}\left(\frac{7+11x}{\sqrt{-3-2x+4x^2}}\right) \\ &\quad+\frac{\log(x)}{8}-\frac{1}{8}\log(2+3x)\end{aligned}$$

output
```
-1/4/x+1/4*(4*x^2-2*x-3)^(1/2)/x+7/24*3^(1/2)*arctan(1/3*(3+x)*3^(1/2)/(4*
x^2-2*x-3)^(1/2))-1/8*arctanh((7+11*x)/(4*x^2-2*x-3)^(1/2))+1/8*ln(x)-1/8*
ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx = -\frac{6-6\sqrt{-3-2x+4x^2}+14\sqrt{3}x\arctan\left(\frac{-2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)-3x\log\left(x\left(-1+4x-2\sqrt{-3-2x+4x^2}\right)\right.}{24x}$$

input
```
Integrate[1/(x^2*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])),x]
```

output
```
-1/24*(6 - 6*Sqrt[-3 - 2*x + 4*x^2] + 14*Sqrt[3]*x*ArcTan[(-2*x + Sqrt[-3
- 2*x + 4*x^2])/Sqrt[3]] - 3*x*Log[x*(-1 + 4*x - 2*Sqrt[-3 - 2*x + 4*x^2])
] + 6*x*Log[-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x^2]])/x
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{4x^2-2x-3}+2x+1\right)}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{3\sqrt{4x^2-2x-3}}{8x}-\frac{9\sqrt{4x^2-2x-3}}{8(3x+2)}-\frac{\sqrt{4x^2-2x-3}}{4x^2}+\frac{1}{4x^2}+\frac{1}{8x}-\frac{3}{8(3x+2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{8}\sqrt{3}\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)-\frac{\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)}{4\sqrt{3}}-\frac{1}{8}\text{arctanh}\left(\frac{11x+7}{\sqrt{4x^2-2x-3}}\right)+\frac{\sqrt{4x^2-2x-3}}{4x}-\frac{1}{4x}+\frac{\log(x)}{8}-\frac{1}{8}\log(3x+2)$$

input
```
Int[1/(x^2*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])),x]
```

output
```
-1/4*1/x + Sqrt[-3 - 2*x + 4*x^2]/(4*x) - ArcTan[(3 + x)/(Sqrt[3]*Sqrt[-3
- 2*x + 4*x^2])]/(4*Sqrt[3]) + (3*Sqrt[3]*ArcTan[(3 + x)/(Sqrt[3]*Sqrt[-3
- 2*x + 4*x^2])])/8 - ArcTanh[(7 + 11*x)/Sqrt[-3 - 2*x + 4*x^2]]/8 + Log[x
]/8 - Log[2 + 3*x]/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(82) = 164$.

Time = 0.17 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.91

method	result
default	$\frac{\ln(x)}{8} - \frac{\ln(2+3x)}{8} - \frac{1}{4x} - \frac{(4x^2-2x-3)^{\frac{3}{2}}}{12x} + \frac{7\sqrt{4x^2-2x-3}}{24} - \frac{11\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4x^2-2x-3}\right)\sqrt{4}}{32} - \frac{7\sqrt{3}\ \arctan\left(\frac{(-6-2}{6\sqrt{4x^2}}\right.}{24}$
trager	$\frac{x-1}{4x} + \frac{\sqrt{4x^2-2x-3}}{4x} - \frac{\ln\left(\frac{3\,\mathrm{RootOf}\left(3\_Z^2-3\_Z+13\right)^2 x-3\,\mathrm{RootOf}\left(3\_Z^2-3\_Z+13\right)^2-259\,\mathrm{RootOf}\left(3\_Z^2-3\_Z+13\right)\sqrt{4x^2}}{x}\right.}{4}$

input
```
int(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(x)-1/8*ln(2+3*x)-1/4/x-1/12/x*(4*x^2-2*x-3)^(3/2)+7/24*(4*x^2-2*x-3
)^(1/2)-11/32*ln(1/4*(4*x-1)*4^(1/2)+(4*x^2-2*x-3)^(1/2))*4^(1/2)-7/24*3^(
1/2)*arctan(1/6*(-6-2*x)*3^(1/2)/(4*x^2-2*x-3)^(1/2))+1/24*(8*x-2)*(4*x^2-
2*x-3)^(1/2)-1/8*(36*(x+2/3)^2-66*x-43)^(1/2)+11/32*ln(1/4*(4*x-1)*4^(1/2)
+(4*(x+2/3)^2-22/3*x-43/9)^(1/2))*4^(1/2)+1/8*arctanh(9/2*(-14/3-22/3*x)/(
36*(x+2/3)^2-66*x-43)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx = -\frac{14\sqrt{3}x\arctan\left(-\frac{2}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{4x^2-2x-3}\right)+3x\log(3x+2)-3x\log(x)-3x\log\left(-2x+\sqrt{4\cdots}\right.}{24x}$$

input
```
integrate(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

output
```
-1/24*(14*sqrt(3)*x*arctan(-2/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(4*x^2 - 2*x -
 3)) + 3*x*log(3*x + 2) - 3*x*log(x) - 3*x*log(-2*x + sqrt(4*x^2 - 2*x - 3
) - 1) + 3*x*log(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5) - 12*x - 6*sqrt(4*x^2
 - 2*x - 3) + 6)/x
```

Sympy [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx = \int \frac{1}{x^2\cdot\left(2x+\sqrt{4x^2-2x-3}+1\right)}\,dx$$

input
```
integrate(1/x**2/(1+2*x+(4*x**2-2*x-3)**(1/2)),x)
```

output
```
Integral(1/(x**2*(2*x + sqrt(4*x**2 - 2*x - 3) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx = \int \frac{1}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)x^2}\,dx$$

input
```
integrate(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(4*x^2 - 2*x - 3) + 1)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\begin{aligned}\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx &= -\frac{7}{12}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)\right) \\ &\quad+\frac{2\,x-\sqrt{4\,x^2-2\,x-3}+6}{2\left(\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+3\right)}-\frac{1}{4\,x}-\frac{1}{8}\log\left(|3\,x+2|\right)+\frac{1}{8}\log\left(|x|\right) \\ &\quad+\frac{1}{8}\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right)-\frac{1}{8}\log\left(\left|-6\,x+3\,\sqrt{4\,x^2-2\,x-3}-5\right|\right)\end{aligned}$$

input
```
integrate(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

output
```
-7/12*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - sqrt(4*x^2 - 2*x - 3))) + 1/2*(2*
x - sqrt(4*x^2 - 2*x - 3) + 6)/((2*x - sqrt(4*x^2 - 2*x - 3))^2 + 3) - 1/4
/x - 1/8*log(abs(3*x + 2)) + 1/8*log(abs(x)) + 1/8*log(abs(-2*x + sqrt(4*x
^2 - 2*x - 3) - 1)) - 1/8*log(abs(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx$$
$$= -\int \frac{\sqrt{4\,x^2-2\,x-3}}{2\,\left(3\,x^3+2\,x^2\right)}\,dx - \frac{1}{4\,x} + \frac{\mathrm{atan}(x\,3\mathrm{i}+1\mathrm{i})\,1\mathrm{i}}{4}$$

input
```
int(1/(x^2*(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)),x)
```

output
```
(atan(x*3i + 1i)*1i)/4 - int((4*x^2 - 2*x - 3)^(1/2)/(2*(2*x^2 + 3*x^3)),
x) - 1/(4*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx$$
$$= \frac{-14\sqrt{3}\,atan\left(\frac{\sqrt{4x^2-2x-3}+2x}{\sqrt{3}}\right)x + 6\sqrt{4x^2-2x-3} - 3\log(3x+2)\,x - 3\log\left(\frac{26\sqrt{4x^2-2x-3}+52x+26}{\sqrt{13}}\right)x + 3\,\mathrm{lo}}{24x}$$

input
```
int(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2)),x)
```

output
```
( - 14*sqrt(3)*atan((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x + 6*sqrt(4*x
**2 - 2*x - 3) - 3*log(3*x + 2)*x - 3*log((26*sqrt(4*x**2 - 2*x - 3) + 52*
x + 26)/sqrt(13))*x + 3*log((6*sqrt(4*x**2 - 2*x - 3) + 12*x + 10)/sqrt(13
))*x + 3*log(x)*x - 6)/(24*x)
```

3.6 $\int \frac{1}{x^3\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\begin{aligned}\int \frac{1}{x^3\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx =& -\frac{1}{8x^2}-\frac{1}{8x}+\frac{\sqrt{-3-2x+4x^2}}{8x^2} \\ &-\frac{\sqrt{-3-2x+4x^2}}{3x} \\ &-\frac{37\arctan\left(\frac{3+x}{\sqrt{3}\sqrt{-3-2x+4x^2}}\right)}{48\sqrt{3}} \\ &+\frac{3}{16}\text{arctanh}\left(\frac{7+11x}{\sqrt{-3-2x+4x^2}}\right) \\ &-\frac{3\log(x)}{16}+\frac{3}{16}\log(2+3x)\end{aligned}$$

output
```
-1/8/x^2-1/8/x+1/8*(4*x^2-2*x-3)^(1/2)/x^2-1/3*(4*x^2-2*x-3)^(1/2)/x-37/14
4*3^(1/2)*arctan(1/3*(3+x)*3^(1/2)/(4*x^2-2*x-3)^(1/2))+3/16*arctanh((7+11
*x)/(4*x^2-2*x-3)^(1/2))-3/16*ln(x)+3/16*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^3\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx = \frac{1}{144}\left(-\frac{18(1+x)}{x^2}+\frac{6(3-8x)\sqrt{-3-2x+4x^2}}{x^2} + 74\sqrt{3}\arctan\left(\frac{-2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right) - 27\log\left(x\left(-1+4x-2\sqrt{-3-2x+4x^2}\right)\right) + 54\log\left(-5-6x+3\sqrt{-3-2x+4x^2}\right)\right)$$

input
```
Integrate[1/(x^3*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])),x]
```

output
```
((-18*(1 + x))/x^2 + (6*(3 - 8*x)*Sqrt[-3 - 2*x + 4*x^2])/x^2 + 74*Sqrt[3]
*ArcTan[(-2*x + Sqrt[-3 - 2*x + 4*x^2])/Sqrt[3]] - 27*Log[x*(-1 + 4*x - 2*
Sqrt[-3 - 2*x + 4*x^2])] + 54*Log[-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x^2]])/14
4
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(\sqrt{4x^2-2x-3}+2x+1\right)}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{1}{4x^3}-\frac{9\sqrt{4x^2-2x-3}}{16x}+\frac{27\sqrt{4x^2-2x-3}}{16(3x+2)}+\frac{3\sqrt{4x^2-2x-3}}{8x^2}+\frac{1}{8x^2}-\frac{\sqrt{4x^2-2x-3}}{4x^3}-\frac{3}{16x}+\frac{9}{16(3x+\right.$$

$$\big\downarrow \text{2009}$$

$$-\frac{7}{16}\sqrt{3}\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)+\frac{13\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)}{24\sqrt{3}}+\frac{3}{16}\text{arctanh}\left(\frac{11x+7}{\sqrt{4x^2-2x-3}}\right)+\frac{\sqrt{4x^2-2x-3}(x+3)}{24x^2}-\frac{3\sqrt{4x^2-2x-3}}{8x}-\frac{1}{8x^2}-\frac{1}{8x}-\frac{3\log(x)}{16}+\frac{3}{16}\log(3x+2)$$

input
```
Int[1/(x^3*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])),x]
```

output
```
-1/8*1/x^2 - 1/(8*x) - (3*Sqrt[-3 - 2*x + 4*x^2])/(8*x) + ((3 + x)*Sqrt[-3
 - 2*x + 4*x^2])/(24*x^2) + (13*ArcTan[(3 + x)/(Sqrt[3]*Sqrt[-3 - 2*x + 4*
x^2])])/(24*Sqrt[3]) - (7*Sqrt[3]*ArcTan[(3 + x)/(Sqrt[3]*Sqrt[-3 - 2*x +
4*x^2])])/16 + (3*ArcTanh[(7 + 11*x)/Sqrt[-3 - 2*x + 4*x^2]])/16 - (3*Log[
x])/16 + (3*Log[2 + 3*x])/16
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(104) = 208$.

Time = 0.18 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.67

method	result
default	$-\frac{1}{8x}-\frac{3\ln(x)}{16}+\frac{3\ln(2+3x)}{16}-\frac{1}{8x^2}-\frac{(4x^2-2x-3)^{\frac{3}{2}}}{24x^2}+\frac{5(4x^2-2x-3)^{\frac{3}{2}}}{36x}-\frac{37\sqrt{4x^2-2x-3}}{144}+\frac{37\sqrt{3}\arctan\left(\frac{(-6-2x)}{6\sqrt{4x^2-2x}}\right.}{144}$
trager	$\frac{(x-1)(2x+1)}{8x^2}-\frac{(8x-3)\sqrt{4x^2-2x-3}}{24x^2}+\frac{\operatorname{RootOf}\left(3\_Z^2+27\_Z+403\right)\ln\left(-\frac{3\operatorname{RootOf}\left(3\_Z^2+27\_Z+403\right)^2x-3\operatorname{RootOf}\left(3\_Z\right.}{}\right.}{}$

input
```
int(1/x^3/(1+2*x+(4*x^2-2*x-3)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/8/x-3/16*ln(x)+3/16*ln(2+3*x)-1/8/x^2-1/24/x^2*(4*x^2-2*x-3)^(3/2)+5/36
/x*(4*x^2-2*x-3)^(3/2)-37/144*(4*x^2-2*x-3)^(1/2)+37/144*3^(1/2)*arctan(1/
6*(-6-2*x)*3^(1/2)/(4*x^2-2*x-3)^(1/2))-5/72*(8*x-2)*(4*x^2-2*x-3)^(1/2)+3
3/64*ln(1/4*(4*x-1)*4^(1/2)+(4*x^2-2*x-3)^(1/2))*4^(1/2)+3/16*(36*(x+2/3)^
2-66*x-43)^(1/2)-33/64*ln(1/4*(4*x-1)*4^(1/2)+(4*(x+2/3)^2-22/3*x-43/9)^(1
/2))*4^(1/2)-3/16*arctanh(9/2*(-14/3-22/3*x)/(36*(x+2/3)^2-66*x-43)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^3\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx$$
$$=\frac{74\sqrt{3}x^2\arctan\left(-\frac{2}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{4x^2-2x-3}\right)+27x^2\log(3x+2)-27x^2\log(x)-27x^2\log\left(-2x\right.}{}$$

input
```
integrate(1/x^3/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

output
```
1/144*(74*sqrt(3)*x^2*arctan(-2/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(4*x^2 - 2*x
 - 3)) + 27*x^2*log(3*x + 2) - 27*x^2*log(x) - 27*x^2*log(-2*x + sqrt(4*x^
2 - 2*x - 3) - 1) + 27*x^2*log(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5) - 96*x^
2 - 6*sqrt(4*x^2 - 2*x - 3)*(8*x - 3) - 18*x - 18)/x^2
```

Sympy [F]

$$\int \frac{1}{x^3\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx=\int \frac{1}{x^3\cdot\left(2x+\sqrt{4x^2-2x-3}+1\right)}\,dx$$

input
```
integrate(1/x**3/(1+2*x+(4*x**2-2*x-3)**(1/2)),x)
```

output
```
Integral(1/(x**3*(2*x + sqrt(4*x**2 - 2*x - 3) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx=\int \frac{1}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)x^3}\,dx$$

input
```
integrate(1/x^3/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(4*x^2 - 2*x - 3) + 1)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^3\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx = \frac{37}{72}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-\sqrt{4x^2-2x-3}\right)\right) - \frac{10\left(2x-\sqrt{4x^2-2x-3}\right)^3+15\left(2x-\sqrt{4x^2-2x-3}\right)^2-6x+3\sqrt{4x^2-2x-3}+81}{6\left(\left(2x-\sqrt{4x^2-2x-3}\right)^2+3\right)^2} - \frac{x+1}{8x^2} + \frac{3}{16}\log\left(|3x+2|\right) - \frac{3}{16}\log\left(|x|\right) - \frac{3}{16}\log\left(\left|-2x+\sqrt{4x^2-2x-3}-1\right|\right) + \frac{3}{16}\log\left(\left|-6x+3\sqrt{4x^2-2x-3}-5\right|\right)$$

input
```
integrate(1/x^3/(1+2*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

output
```
37/72*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - sqrt(4*x^2 - 2*x - 3))) - 1/6*(10
*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 15*(2*x - sqrt(4*x^2 - 2*x - 3))^2 - 6*
x + 3*sqrt(4*x^2 - 2*x - 3) + 81)/((2*x - sqrt(4*x^2 - 2*x - 3))^2 + 3)^2
- 1/8*(x + 1)/x^2 + 3/16*log(abs(3*x + 2)) - 3/16*log(abs(x)) - 3/16*log(a
bs(-2*x + sqrt(4*x^2 - 2*x - 3) - 1)) + 3/16*log(abs(-6*x + 3*sqrt(4*x^2 -
 2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx = -\int \frac{\sqrt{4x^2-2x-3}}{2\left(3x^4+2x^3\right)}\,dx - \frac{\frac{x}{8}+\frac{1}{8}}{x^2} - \frac{\mathrm{atan}(x\,3\mathrm{i}+1\mathrm{i})\,3\mathrm{i}}{8}$$

input
```
int(1/(x^3*(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)),x)
```

output
```
- (atan(x*3i + 1i)*3i)/8 - int((4*x^2 - 2*x - 3)^(1/2)/(2*(2*x^3 + 3*x^4))
, x) - (x/8 + 1/8)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\,dx$$
$$= \frac{4070\sqrt{3}\,atan\left(\frac{\sqrt{4x^2-2x-3}+2x}{\sqrt{3}}\right)x^2 - 2640\sqrt{4x^2-2x-3}\,x + 990\sqrt{4x^2-2x-3} + 1485\log(3x+2)\,x^2 + 1}{79}$$

input
```
int(1/x^3/(1+2*x+(4*x^2-2*x-3)^(1/2)),x)
```

output
```
(4070*sqrt(3)*atan((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x**2 - 2640*sqr
t(4*x**2 - 2*x - 3)*x + 990*sqrt(4*x**2 - 2*x - 3) + 1485*log(3*x + 2)*x**
2 + 1485*log((26*sqrt(4*x**2 - 2*x - 3) + 52*x + 26)/sqrt(13))*x**2 - 1485
*log((6*sqrt(4*x**2 - 2*x - 3) + 12*x + 10)/sqrt(13))*x**2 - 1485*log(x)*x
**2 + 3184*x**2 - 990*x - 990)/(7920*x**2)
```

3.7 $\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 183

$$\begin{aligned}\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx &= \frac{1}{64}\left(2x+\sqrt{-3-2x+4x^2}\right)\\ &+\frac{4}{81\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\\ &-\frac{2197}{3456\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)^3}\\ &-\frac{1183}{3456\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)^2}\\ &-\frac{377}{864\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)}\\ &-\frac{56}{243}\log\left(1+2x+\sqrt{-3-2x+4x^2}\right)\\ &+\frac{3341\log\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)}{15552}\end{aligned}$$

output
```
1/32*x+1/64*(4*x^2-2*x-3)^(1/2)+4/(81+162*x+81*(4*x^2-2*x-3)^(1/2))-2197/3
456/(1-4*x-2*(4*x^2-2*x-3)^(1/2))^3-1183/3456/(1-4*x-2*(4*x^2-2*x-3)^(1/2)
)^2-377/(864-3456*x-1728*(4*x^2-2*x-3)^(1/2))-56/243*ln(1+2*x+(4*x^2-2*x-3
)^(1/2))+3341/15552*ln(1-4*x-2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$= \frac{-\frac{6\sqrt{-3-2x+4x^2}(346+159x-204x^2+288x^3)}{2+3x}+\frac{16(-8+324x+252x^2-207x^3+216x^4)}{2+3x}+243\log\left(1-4x+2\sqrt{-3-2x+4x^2}\right.}{15552}$$

input
```
Integrate[x^2/(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^2,x]
```

output
```
((-6*Sqrt[-3 - 2*x + 4*x^2]*(346 + 159*x - 204*x^2 + 288*x^3))/(2 + 3*x) +
 (16*(-8 + 324*x + 252*x^2 - 207*x^3 + 216*x^4))/(2 + 3*x) + 243*Log[1 - 4
*x + 2*Sqrt[-3 - 2*x + 4*x^2]] - 3584*Log[-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x
^2]])/15552
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{4x^2-2x-3}+2x+1\right)^2}dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{2x^2}{9}-\frac{1}{9}\sqrt{4x^2-2x-3}x-\frac{2\sqrt{4x^2-2x-3}}{9(3x+2)}+\frac{2\sqrt{4x^2-2x-3}}{27(3x+2)^2}+\frac{5}{54}\sqrt{4x^2-2x-3}-\frac{13x}{54}-\frac{28}{81(3x+2)}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{1549\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)}{15552}-\frac{28}{243}\text{arctanh}\left(\frac{11x+7}{\sqrt{4x^2-2x-3}}\right)+\frac{2x^3}{27}-\frac{13x^2}{108}-\frac{1}{108}\left(4x^2-2x-3\right)^{3/2}-\frac{7}{864}(1-4x)\sqrt{4x^2-2x-3}-\frac{2\sqrt{4x^2-2x-3}}{81(3x+2)}-\frac{2}{27}\sqrt{4x^2-2x-3}+\frac{x}{6}-\frac{2}{243(3x+2)}-\frac{28}{243}\log(3x+2)$$

input
```
Int[x^2/(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^2,x]
```

output
```
x/6 - (13*x^2)/108 + (2*x^3)/27 - 2/(243*(2 + 3*x)) - (2*Sqrt[-3 - 2*x + 4
*x^2])/27 - (7*(1 - 4*x)*Sqrt[-3 - 2*x + 4*x^2])/864 - (2*Sqrt[-3 - 2*x +
4*x^2])/(81*(2 + 3*x)) - (-3 - 2*x + 4*x^2)^(3/2)/108 - (1549*ArcTanh[(1 -
 4*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/15552 - (28*ArcTanh[(7 + 11*x)/Sqrt[-3
- 2*x + 4*x^2]])/243 - (28*Log[2 + 3*x])/243
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.13

$$\frac{x}{6}-\frac{2}{243\left(2+3x\right)}-\frac{28\ln\left(2+3x\right)}{243}-\frac{13x^2}{108}+\frac{2x^3}{27}+\frac{7(8x-2)\sqrt{4x^2-2x-3}}{1728}-\frac{91\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4x^2-2x-\right.}{3456}$$

input
```
int(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

output
```
1/6*x-2/243/(2+3*x)-28/243*ln(2+3*x)-13/108*x^2+2/27*x^3+7/1728*(8*x-2)*(4
*x^2-2*x-3)^(1/2)-91/3456*ln(1/4*(4*x-1)*4^(1/2)+(4*x^2-2*x-3)^(1/2))*4^(1
/2)-2/27/(x+2/3)*(4*(x+2/3)^2-22/3*x-43/9)^(3/2)-28/243*(36*(x+2/3)^2-66*x
-43)^(1/2)+37/486*ln(1/4*(4*x-1)*4^(1/2)+(4*(x+2/3)^2-22/3*x-43/9)^(1/2))*
4^(1/2)+28/243*arctanh(9/2*(-14/3-22/3*x)/(36*(x+2/3)^2-66*x-43)^(1/2))+1/
27*(8*x-2)*(4*(x+2/3)^2-22/3*x-43/9)^(1/2)-1/108*(4*x^2-2*x-3)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$= \frac{13824\,x^4 - 13248\,x^3 + 16128\,x^2 - 7168\,(3\,x+2)\log\left(3\,x+2\right) + 7168\,(3\,x+2)\log\left(-2\,x+\sqrt{4\,x^2-2\,x}\right.}{}$$

input
```
integrate(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/62208*(13824*x^4 - 13248*x^3 + 16128*x^2 - 7168*(3*x + 2)*log(3*x + 2) +
 7168*(3*x + 2)*log(-2*x + sqrt(4*x^2 - 2*x - 3) - 1) - 6196*(3*x + 2)*log
(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1) - 7168*(3*x + 2)*log(-6*x + 3*sqrt(4*
x^2 - 2*x - 3) - 5) - 24*(288*x^3 - 204*x^2 + 159*x + 346)*sqrt(4*x^2 - 2*
x - 3) + 20283*x - 814)/(3*x + 2)
```

Sympy [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{x^2}{\left(2x+\sqrt{4x^2-2x-3}+1\right)^2}\,dx$$

input
```
integrate(x**2/(1+2*x+(4*x**2-2*x-3)**(1/2))**2,x)
```

output
```
Integral(x**2/(2*x + sqrt(4*x**2 - 2*x - 3) + 1)**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{x^2}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2}\,dx$$

input
```
integrate(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x^2/(2*x + sqrt(4*x^2 - 2*x - 3) + 1)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx &= \frac{2}{27}\,x^3-\frac{13}{108}\,x^2-\frac{1}{864}\,(4\,(8\,x-11)x+47)\sqrt{4\,x^2-2\,x-3}+\frac{1}{6}\,x \\ &\quad-\frac{4\,\left(22\,x-11\,\sqrt{4\,x^2-2\,x-3}+14\right)}{243\,\left(3\,\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+16\,x-8\,\sqrt{4\,x^2-2\,x-3}+5\right)} \\ &\quad-\frac{2}{243\,(3\,x+2)}-\frac{28}{243}\,\log\left(|3\,x+2|\right)+\frac{28}{243}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right) \\ &\quad-\frac{1549}{15552}\,\log\left(\left|-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right|\right) \\ &\quad-\frac{28}{243}\,\log\left(\left|-6\,x+3\,\sqrt{4\,x^2-2\,x-3}-5\right|\right)\end{aligned}$$

input
```
integrate(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

output
```
2/27*x^3 - 13/108*x^2 - 1/864*(4*(8*x - 11)*x + 47)*sqrt(4*x^2 - 2*x - 3)
+ 1/6*x - 4/243*(22*x - 11*sqrt(4*x^2 - 2*x - 3) + 14)/(3*(2*x - sqrt(4*x^
2 - 2*x - 3))^2 + 16*x - 8*sqrt(4*x^2 - 2*x - 3) + 5) - 2/243/(3*x + 2) -
28/243*log(abs(3*x + 2)) + 28/243*log(abs(-2*x + sqrt(4*x^2 - 2*x - 3) - 1
)) - 1549/15552*log(abs(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1)) - 28/243*log(
abs(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{x}{6}-\frac{28\,\ln\left(x+\frac{2}{3}\right)}{243}-\frac{2}{729\,\left(x+\frac{2}{3}\right)}-\frac{13\,x^2}{108}+\frac{2\,x^3}{27}+\int \frac{x^2\,\left(-8\,x^3+8\,x+3\right)}{2\,{\left(3\,x+2\right)}^2\,\sqrt{4\,x^2-2\,x-3}}\,dx$$

input
```
int(x^2/(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2,x)
```

output
```
x/6 - (28*log(x + 2/3))/243 - 2/(729*(x + 2/3)) - (13*x^2)/108 + (2*x^3)/2
7 + int((x^2*(8*x - 8*x^3 + 3))/(2*(3*x + 2)^2*(4*x^2 - 2*x - 3)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{x^2}{\left(1+2x+\sqrt{4x^2-2x-3}\right)^2}dx$$

input
```
int(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

output
```
int(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

3.8 $\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 133

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2} \, dx = -\frac{2}{27\left(1+2x+\sqrt{-3-2x+4x^2}\right)} + \frac{169}{288\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)^2} + \frac{65}{108\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)} + \frac{1}{3}\log\left(1+2x+\sqrt{-3-2x+4x^2}\right) - \frac{13}{48}\log\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)$$

output
```
-2/(27+54*x+27*(4*x^2-2*x-3)^(1/2))+169/288/(1-4*x-2*(4*x^2-2*x-3)^(1/2))^
2+65/(108-432*x-216*(4*x^2-2*x-3)^(1/2))+1/3*ln(1+2*x+(4*x^2-2*x-3)^(1/2))
-13/48*ln(1-4*x-2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{\frac{54(6+5x-4x^2)\sqrt{-3-2x+4x^2}}{2+3x}+\frac{8(2-78x-81x^2+54x^3)}{2+3x}-81\log\left(1-4x+2\sqrt{-3-2x+4x^2}\right)+432\log\left(-5-6x+\right.}{1296}$$

input
```
Integrate[x/(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^2,x]
```

output
```
((54*(6 + 5*x - 4*x^2)*Sqrt[-3 - 2*x + 4*x^2])/(2 + 3*x) + (8*(2 - 78*x -
81*x^2 + 54*x^3))/(2 + 3*x) - 81*Log[1 - 4*x + 2*Sqrt[-3 - 2*x + 4*x^2]] +
 432*Log[-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x^2]])/1296
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{4x^2-2x-3}+2x+1\right)^2}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{5\sqrt{4x^2-2x-3}}{18(3x+2)}-\frac{\sqrt{4x^2-2x-3}}{9(3x+2)^2}-\frac{1}{9}\sqrt{4x^2-2x-3}+\frac{2x}{9}+\frac{1}{2(3x+2)}-\frac{1}{27(3x+2)^2}-\frac{13}{54}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{48}\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)+\frac{1}{6}\text{arctanh}\left(\frac{11x+7}{\sqrt{4x^2-2x-3}}\right)+\frac{x^2}{9}+\frac{1}{72}(1-4x)\sqrt{4x^2-2x-3}+\frac{\sqrt{4x^2-2x-3}}{27(3x+2)}+\frac{5}{54}\sqrt{4x^2-2x-3}-\frac{13x}{54}+\frac{1}{81(3x+2)}+\frac{1}{6}\log(3x+2)$$

input
```
Int[x/(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^2,x]
```

output
```
(-13*x)/54 + x^2/9 + 1/(81*(2 + 3*x)) + (5*Sqrt[-3 - 2*x + 4*x^2])/54 + ((
1 - 4*x)*Sqrt[-3 - 2*x + 4*x^2])/72 + Sqrt[-3 - 2*x + 4*x^2]/(27*(2 + 3*x)
) + (5*ArcTanh[(1 - 4*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/48 + ArcTanh[(7 + 11
*x)/Sqrt[-3 - 2*x + 4*x^2]]/6 + Log[2 + 3*x]/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.41

method	result
default	$\frac{1}{162+243x}+\frac{\ln(2+3x)}{6}-\frac{13x}{54}+\frac{x^2}{9}+\frac{\left(4\left(x+\frac{2}{3}\right)^2-\frac{22x}{3}-\frac{43}{9}\right)^{\frac{3}{2}}}{9x+6}+\frac{\sqrt{36\left(x+\frac{2}{3}\right)^2-66x-43}}{6}-\frac{7\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4\left(x+\frac{2}{3}\right)^2-\right.}{72}$
trager	$\frac{(2x^2-3x-3)x}{18x+12}-\frac{(4x^2-5x-6)\sqrt{4x^2-2x-3}}{24(2+3x)}-\frac{\ln\left(\frac{2272757120+40547330304x+64553303872x^2\sqrt{4x^2-2x-3}+3820252999680x^{12}+676635\right.}{}$

input
```
int(x/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/81/(2+3*x)+1/6*ln(2+3*x)-13/54*x+1/9*x^2+1/9/(x+2/3)*(4*(x+2/3)^2-22/3*x
-43/9)^(3/2)+1/6*(36*(x+2/3)^2-66*x-43)^(1/2)-7/72*ln(1/4*(4*x-1)*4^(1/2)+
(4*(x+2/3)^2-22/3*x-43/9)^(1/2))*4^(1/2)-1/6*arctanh(9/2*(-14/3-22/3*x)/(3
6*(x+2/3)^2-66*x-43)^(1/2))-1/18*(8*x-2)*(4*(x+2/3)^2-22/3*x-43/9)^(1/2)-1
/144*(8*x-2)*(4*x^2-2*x-3)^(1/2)+13/288*ln(1/4*(4*x-1)*4^(1/2)+(4*x^2-2*x-
3)^(1/2))*4^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.05

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$= \frac{1728\,x^3 - 2592\,x^2 + 864\,(3\,x+2)\log\left(3\,x+2\right) - 864\,(3\,x+2)\log\left(-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right) + 540\,(}{}$$

input
```
integrate(x/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/5184*(1728*x^3 - 2592*x^2 + 864*(3*x + 2)*log(3*x + 2) - 864*(3*x + 2)*l
og(-2*x + sqrt(4*x^2 - 2*x - 3) - 1) + 540*(3*x + 2)*log(-4*x + 2*sqrt(4*x
^2 - 2*x - 3) + 1) + 864*(3*x + 2)*log(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5)
 - 216*sqrt(4*x^2 - 2*x - 3)*(4*x^2 - 5*x - 6) - 2499*x + 62)/(3*x + 2)
```

Sympy [F]

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{x}{\left(2x+\sqrt{4x^2-2x-3}+1\right)^2}\,dx$$

input
```
integrate(x/(1+2*x+(4*x**2-2*x-3)**(1/2))**2,x)
```

output
```
Integral(x/(2*x + sqrt(4*x**2 - 2*x - 3) + 1)**2, x)
```

Maxima [F]

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{x}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2}\,dx$$

input
```
integrate(x/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x/(2*x + sqrt(4*x^2 - 2*x - 3) + 1)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.33

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{1}{9}\,x^2 - \frac{1}{216}\,\sqrt{4\,x^2-2\,x-3}(12\,x-23) - \frac{13}{54}\,x + \frac{2\,\left(22\,x-11\,\sqrt{4\,x^2-2\,x-3}+14\right)}{81\,\left(3\,\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+16\,x-8\,\sqrt{4\,x^2-2\,x-3}+5\right)} + \frac{1}{81\,(3\,x+2)} + \frac{1}{6}\,\log\left(|3\,x+2|\right) - \frac{1}{6}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right) + \frac{5}{48}\,\log\left(\left|-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right|\right) + \frac{1}{6}\,\log\left(\left|-6\,x+3\,\sqrt{4\,x^2-2\,x-3}-5\right|\right)$$

input
```
integrate(x/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

output
```
1/9*x^2 - 1/216*sqrt(4*x^2 - 2*x - 3)*(12*x - 23) - 13/54*x + 2/81*(22*x -
 11*sqrt(4*x^2 - 2*x - 3) + 14)/(3*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 16*x
- 8*sqrt(4*x^2 - 2*x - 3) + 5) + 1/81/(3*x + 2) + 1/6*log(abs(3*x + 2)) -
1/6*log(abs(-2*x + sqrt(4*x^2 - 2*x - 3) - 1)) + 5/48*log(abs(-4*x + 2*sqr
t(4*x^2 - 2*x - 3) + 1)) + 1/6*log(abs(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5)
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{\ln\left(x+\frac{2}{3}\right)}{6}-\frac{13\,x}{54}+\frac{1}{243\,\left(x+\frac{2}{3}\right)} + \int \frac{x\,\left(-8\,x^3+8\,x+3\right)}{2\,{\left(3\,x+2\right)}^2\,\sqrt{4\,x^2-2\,x-3}}\,dx+\frac{x^2}{9}$$

input
```
int(x/(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2,x)
```

output
```
log(x + 2/3)/6 - (13*x)/54 + 1/(243*(x + 2/3)) + int((x*(8*x - 8*x^3 + 3))
/(2*(3*x + 2)^2*(4*x^2 - 2*x - 3)^(1/2)), x) + x^2/9
```

Reduce [F]

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{x}{\left(1+2x+\sqrt{4x^2-2x-3}\right)^2}dx$$

input
```
int(x/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

output
```
int(x/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

3.9 $\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2} \, dx = \frac{1}{9\left(1+2x+\sqrt{-3-2x+4x^2}\right)} - \frac{13}{18\left(3-2\left(1+2x+\sqrt{-3-2x+4x^2}\right)\right)} - \frac{13}{27}\log\left(1+2x+\sqrt{-3-2x+4x^2}\right) + \frac{13}{27}\log\left(3-2\left(1+2x+\sqrt{-3-2x+4x^2}\right)\right)$$

output
```
1/(9+18*x+9*(4*x^2-2*x-3)^(1/2))-13/(18-72*x-36*(4*x^2-2*x-3)^(1/2))-13/27
*ln(1+2*x+(4*x^2-2*x-3)^(1/2))+13/27*ln(1-4*x-2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.66

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{-1+24x+36x^2-3(5+6x)\sqrt{-3-2x+4x^2}-26(2+3x)\log\left(-5-6x+3\sqrt{-3-2x+4x^2}\right)}{54(2+3x)}$$

input
```
Integrate[(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^(-2),x]
```

output
```
(-1 + 24*x + 36*x^2 - 3*(5 + 6*x)*Sqrt[-3 - 2*x + 4*x^2] - 26*(2 + 3*x)*Lo
g[-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x^2]])/(54*(2 + 3*x))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2541, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{4x^2-2x-3}+2x+1\right)^2}dx$$

$$\Big\downarrow \text{ 2541}$$

$$2\int -\frac{-\left(2x+\sqrt{4x^2-2x-3}+1\right)^2+3\left(2x+\sqrt{4x^2-2x-3}+1\right)+1}{2\left(2x+\sqrt{4x^2-2x-3}+1\right)^2\left(3-2\left(2x+\sqrt{4x^2-2x-3}+1\right)\right)^2}d\left(2x+\sqrt{4x^2-2x-3}+1\right)$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{-\left(2x+\sqrt{4x^2-2x-3}+1\right)^2+3\left(2x+\sqrt{4x^2-2x-3}+1\right)+1}{\left(2x+\sqrt{4x^2-2x-3}+1\right)^2\left(3-2\left(2x+\sqrt{4x^2-2x-3}+1\right)\right)^2}d\left(2x+\sqrt{4x^2-2x-3}+1\right)$$

$$\Big\downarrow \text{1195}$$

$$-\int \left(-\frac{26}{27\left(2\left(2x+\sqrt{4x^2-2x-3}+1\right)-3\right)}+\frac{13}{9\left(2\left(2x+\sqrt{4x^2-2x-3}+1\right)-3\right)^2}+\frac{13}{27\left(2x+\sqrt{4x^2-2x\right.}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9\left(\sqrt{4x^2-2x-3}+2x+1\right)}-\frac{13}{18\left(3-2\left(\sqrt{4x^2-2x-3}+2x+1\right)\right)}-\frac{13}{27}\log\left(\sqrt{4x^2-2x-3}+2x+1\right)+\frac{13}{27}\log\left(3-2\left(\sqrt{4x^2-2x-3}+2x+1\right)\right)$$

input
```
Int[(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^(-2),x]
```

output
```
1/(9*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])) - 13/(18*(3 - 2*(1 + 2*x + Sqrt[-
3 - 2*x + 4*x^2]))) - (13*Log[1 + 2*x + Sqrt[-3 - 2*x + 4*x^2]])/27 + (13*
Log[3 - 2*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])])/27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.69

method	result
trager	$\frac{(17+24x)x}{72+108x}-\frac{(5+6x)\sqrt{4x^2-2x-3}}{18(2+3x)}+\frac{13\ln\left(-\frac{5+6x+3\sqrt{4x^2-2x-3}}{2+3x}\right)}{27}$
default	$-\frac{1}{54(2+3x)}-\frac{13\ln(2+3x)}{54}+\frac{2x}{9}-\frac{\left(4\left(x+\frac{2}{3}\right)^2-\frac{22x}{3}-\frac{43}{9}\right)^{\frac{3}{2}}}{6\left(x+\frac{2}{3}\right)}-\frac{13\sqrt{36\left(x+\frac{2}{3}\right)^2-66x-43}}{54}+\frac{13\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4\left(x+\frac{2}{3}\right)^2-\cdots}\right.}{108}$

input
```
int(1/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/36*(17+24*x)*x/(2+3*x)-1/18*(5+6*x)/(2+3*x)*(4*x^2-2*x-3)^(1/2)+13/27*ln
(-(5+6*x+3*(4*x^2-2*x-3)^(1/2))/(2+3*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$=\frac{72\,x^2-26\,(3\,x+2)\log\left(48\,x^2-\sqrt{4\,x^2-2\,x-3}(24\,x+7)+2\,x-23\right)-26\,(3\,x+2)\log\left(3\,x+2\right)+26\cdots}{108\,(3\,x+2)}$$

input
```
integrate(1/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/108*(72*x^2 - 26*(3*x + 2)*log(48*x^2 - sqrt(4*x^2 - 2*x - 3)*(24*x + 7)
 + 2*x - 23) - 26*(3*x + 2)*log(3*x + 2) + 26*(3*x + 2)*log(-2*x + sqrt(4*
x^2 - 2*x - 3) - 1) - 6*sqrt(4*x^2 - 2*x - 3)*(6*x + 5) + 45*x - 4)/(3*x +
 2)
```

Sympy [F]

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{1}{\left(2x+\sqrt{4x^2-2x-3}+1\right)^2}\,dx$$

input
```
integrate(1/(1+2*x+(4*x**2-2*x-3)**(1/2))**2,x)
```

output
```
Integral((2*x + sqrt(4*x**2 - 2*x - 3) + 1)**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{1}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2}\,dx$$

input
```
integrate(1/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((2*x + sqrt(4*x^2 - 2*x - 3) + 1)^(-2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.56

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{2}{9}\,x - \frac{1}{9}\sqrt{4\,x^2-2\,x-3} - \frac{22\,x - 11\sqrt{4\,x^2-2\,x-3}+14}{27\left(3\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+16\,x-8\sqrt{4\,x^2-2\,x-3}+5\right)} - \frac{1}{54\,(3\,x+2)} - \frac{13}{54}\log\left(|3\,x+2|\right) + \frac{13}{54}\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right) - \frac{13}{54}\log\left(\left|-4\,x+2\sqrt{4\,x^2-2\,x-3}+1\right|\right) - \frac{13}{54}\log\left(\left|-6\,x+3\sqrt{4\,x^2-2\,x-3}-5\right|\right)$$

input
```
integrate(1/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

output
```
2/9*x - 1/9*sqrt(4*x^2 - 2*x - 3) - 1/27*(22*x - 11*sqrt(4*x^2 - 2*x - 3)
+ 14)/(3*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 16*x - 8*sqrt(4*x^2 - 2*x - 3)
+ 5) - 1/54/(3*x + 2) - 13/54*log(abs(3*x + 2)) + 13/54*log(abs(-2*x + sqr
t(4*x^2 - 2*x - 3) - 1)) - 13/54*log(abs(-4*x + 2*sqrt(4*x^2 - 2*x - 3) +
1)) - 13/54*log(abs(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{2\,x}{9} - \frac{13\,\ln\left(x+\frac{2}{3}\right)}{54} + \int \frac{-8\,x^3+8\,x+3}{2\,(3\,x+2)^2\,\sqrt{4\,x^2-2\,x-3}}\,dx - \frac{1}{162\,\left(x+\frac{2}{3}\right)}$$

input
```
int(1/(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2,x)
```

output
```
(2*x)/9 - (13*log(x + 2/3))/54 + int((8*x - 8*x^3 + 3)/(2*(3*x + 2)^2*(4*x
^2 - 2*x - 3)^(1/2)), x) - 1/(162*(x + 2/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$= \frac{-36\sqrt{4x^2-2x-3}\,x-30\sqrt{4x^2-2x-3}+156\log\left(3\sqrt{4x^2-2x-3}+6x+5\right)x+104\log\left(3\sqrt{4x^2-2x}\right.}{324x+216}$$

input
```
int(1/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

output
```
( - 36*sqrt(4*x**2 - 2*x - 3)*x - 30*sqrt(4*x**2 - 2*x - 3) + 156*log(3*sq
rt(4*x**2 - 2*x - 3) + 6*x + 5)*x + 104*log(3*sqrt(4*x**2 - 2*x - 3) + 6*x
 + 5) - 156*log(3*x + 2)*x - 104*log(3*x + 2) + 72*x**2 + 51*x)/(108*(3*x
+ 2))
```

3.10 $\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\begin{aligned}\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = &-\frac{1}{6\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\\ &+\frac{1}{4}\sqrt{3}\arctan\left(\frac{2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)\\ &+\frac{25}{36}\log\left(1+2x+\sqrt{-3-2x+4x^2}\right)\\ &-\frac{1}{8}\log\left(x-4x^2-2x\sqrt{-3-2x+4x^2}\right)\\ &-\frac{4}{9}\log\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\end{aligned}$$

output
```
-1/6/(1+2*x+(4*x^2-2*x-3)^(1/2))+1/4*3^(1/2)*arctan(1/3*(2*x+(4*x^2-2*x-3)
^(1/2))*3^(1/2))+25/36*ln(1+2*x+(4*x^2-2*x-3)^(1/2))-1/8*ln(x-4*x^2-2*x*(4
*x^2-2*x-3)^(1/2))-4/9*ln(1-4*x-2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$= \frac{2+6\sqrt{-3-2x+4x^2}+18\sqrt{3}(2+3x)\arctan\left(\frac{-2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)-9(2+3x)\log\left(x\left(-1+4x-2\sqrt{-3-\cdots}\right.\right.}{72(2+3x)}$$

input
```
Integrate[1/(x*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^2),x]
```

output
```
(2 + 6*Sqrt[-3 - 2*x + 4*x^2] + 18*Sqrt[3]*(2 + 3*x)*ArcTan[(-2*x + Sqrt[-
3 - 2*x + 4*x^2])/Sqrt[3]] - 9*(2 + 3*x)*Log[x*(-1 + 4*x - 2*Sqrt[-3 - 2*x
 + 4*x^2])] + 50*(2 + 3*x)*Log[-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x^2]])/(72*(
2 + 3*x))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{4x^2-2x-3}+2x+1\right)^2}dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(-\frac{\sqrt{4x^2-2x-3}}{8x}+\frac{3\sqrt{4x^2-2x-3}}{8(3x+2)}-\frac{\sqrt{4x^2-2x-3}}{4(3x+2)^2}-\frac{1}{8x}+\frac{25}{24(3x+2)}-\frac{1}{12(3x+2)^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{8}\sqrt{3}\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)+\frac{2}{9}\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)+\frac{25}{72}\text{arctanh}\left(\frac{11x+7}{\sqrt{4x^2-2x-3}}\right)+\frac{\sqrt{4x^2-2x-3}}{12(3x+2)}+\frac{1}{36(3x+2)}-\frac{\log(x)}{8}+\frac{25}{72}\log(3x+2)$$

input
```
Int[1/(x*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^2),x]
```

output
```
1/(36*(2 + 3*x)) + Sqrt[-3 - 2*x + 4*x^2]/(12*(2 + 3*x)) - (Sqrt[3]*ArcTan
[(3 + x)/(Sqrt[3]*Sqrt[-3 - 2*x + 4*x^2])])/8 + (2*ArcTanh[(1 - 4*x)/(2*Sq
rt[-3 - 2*x + 4*x^2])])/9 + (25*ArcTanh[(7 + 11*x)/Sqrt[-3 - 2*x + 4*x^2]]
)/72 - Log[x]/8 + (25*Log[2 + 3*x])/72
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 21.56 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.48

method	result
default	$\frac{1}{72+108x}-\frac{\ln(x)}{8}+\frac{25\ln(2+3x)}{72}+\frac{\left(4\left(x+\frac{2}{3}\right)^2-\frac{22x}{3}-\frac{43}{9}\right)^{\frac{3}{2}}}{4x+\frac{8}{3}}+\frac{25\sqrt{36\left(x+\frac{2}{3}\right)^2-66x-43}}{72}-\frac{41\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4\left(x+\frac{2}{3}\right)^2-\right.}{288}$
trager	Expression too large to display

input
```
int(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/36/(2+3*x)-1/8*ln(x)+25/72*ln(2+3*x)+1/4/(x+2/3)*(4*(x+2/3)^2-22/3*x-43/
9)^(3/2)+25/72*(36*(x+2/3)^2-66*x-43)^(1/2)-41/288*ln(1/4*(4*x-1)*4^(1/2)+
(4*(x+2/3)^2-22/3*x-43/9)^(1/2))*4^(1/2)-25/72*arctanh(9/2*(-14/3-22/3*x)/
(36*(x+2/3)^2-66*x-43)^(1/2))-1/8*(8*x-2)*(4*(x+2/3)^2-22/3*x-43/9)^(1/2)-
1/8*(4*x^2-2*x-3)^(1/2)+1/32*ln(1/4*(4*x-1)*4^(1/2)+(4*x^2-2*x-3)^(1/2))*4
^(1/2)+1/8*3^(1/2)*arctan(1/6*(-6-2*x)*3^(1/2)/(4*x^2-2*x-3)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$= \frac{18\sqrt{3}(3x+2)\arctan\left(-\frac{2}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{4x^2-2x-3}\right)+25(3x+2)\log(3x+2)-9(3x+2)\log(x)$$

input
```
integrate(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/72*(18*sqrt(3)*(3*x + 2)*arctan(-2/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(4*x^2
- 2*x - 3)) + 25*(3*x + 2)*log(3*x + 2) - 9*(3*x + 2)*log(x) - 25*(3*x + 2
)*log(-2*x + sqrt(4*x^2 - 2*x - 3) - 1) + 16*(3*x + 2)*log(-4*x + 2*sqrt(4
*x^2 - 2*x - 3) + 1) + 25*(3*x + 2)*log(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5
) + 12*x + 6*sqrt(4*x^2 - 2*x - 3) + 10)/(3*x + 2)
```

Sympy [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{1}{x\left(2x+\sqrt{4x^2-2x-3}+1\right)^2}\,dx$$

input
```
integrate(1/x/(1+2*x+(4*x**2-2*x-3)**(1/2))**2,x)
```

output
```
Integral(1/(x*(2*x + sqrt(4*x**2 - 2*x - 3) + 1)**2), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx=\int \frac{1}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2x}\,dx$$

input
```
integrate(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(4*x^2 - 2*x - 3) + 1)^2*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.31

$$\begin{aligned}&\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx\\&=\frac{1}{4}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)\right)\\&\quad+\frac{22\,x-11\,\sqrt{4\,x^2-2\,x-3}+14}{18\left(3\,\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+16\,x-8\,\sqrt{4\,x^2-2\,x-3}+5\right)}\\&\quad+\frac{1}{36\,(3\,x+2)}+\frac{25}{72}\,\log\left(|3\,x+2|\right)-\frac{1}{8}\,\log\left(|x|\right)\\&\quad-\frac{25}{72}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right)+\frac{2}{9}\,\log\left(\left|-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right|\right)\\&\quad+\frac{25}{72}\,\log\left(\left|-6\,x+3\,\sqrt{4\,x^2-2\,x-3}-5\right|\right)\end{aligned}$$

input
```
integrate(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

output
```
1/4*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - sqrt(4*x^2 - 2*x - 3))) + 1/18*(22*
x - 11*sqrt(4*x^2 - 2*x - 3) + 14)/(3*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 16
*x - 8*sqrt(4*x^2 - 2*x - 3) + 5) + 1/36/(3*x + 2) + 25/72*log(abs(3*x + 2
)) - 1/8*log(abs(x)) - 25/72*log(abs(-2*x + sqrt(4*x^2 - 2*x - 3) - 1)) +
2/9*log(abs(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1)) + 25/72*log(abs(-6*x + 3*
sqrt(4*x^2 - 2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{25\,\ln\left(x+\frac{2}{3}\right)}{72} - \frac{\ln\left(x\right)}{8} + \frac{1}{108\,\left(x+\frac{2}{3}\right)} + \int \frac{-8\,x^3+8\,x+3}{2\,x\,{\left(3\,x+2\right)}^2\,\sqrt{4\,x^2-2\,x-3}}\,dx$$

input
```
int(1/(x*(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2),x)
```

output
```
(25*log(x + 2/3))/72 - log(x)/8 + 1/(108*(x + 2/3)) + int((8*x - 8*x^3 + 3
)/(2*x*(3*x + 2)^2*(4*x^2 - 2*x - 3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.86

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{54\sqrt{3}\,atan\left(\frac{\sqrt{4x^2-2x-3}+2x}{\sqrt{3}}\right)x+36\sqrt{3}\,atan\left(\frac{\sqrt{4x^2-2x-3}+2x}{\sqrt{3}}\right)+6\sqrt{4x^2-2x-3}+75\log(3x+2)x+50\log}{}$$

input
```
int(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

output
```
(54*sqrt(3)*atan((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x + 36*sqrt(3)*at
an((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3)) + 6*sqrt(4*x**2 - 2*x - 3) + 75
*log(3*x + 2)*x + 50*log(3*x + 2) + 75*log((26*sqrt(4*x**2 - 2*x - 3) + 52
*x + 26)/sqrt(13))*x + 50*log((26*sqrt(4*x**2 - 2*x - 3) + 52*x + 26)/sqrt
(13)) - 75*log((6*sqrt(4*x**2 - 2*x - 3) + 12*x + 10)/sqrt(13))*x - 50*log
((6*sqrt(4*x**2 - 2*x - 3) + 12*x + 10)/sqrt(13)) - 48*log((2*sqrt(4*x**2
- 2*x - 3) + 4*x - 1)/sqrt(13))*x - 32*log((2*sqrt(4*x**2 - 2*x - 3) + 4*x
 - 1)/sqrt(13)) - 27*log(x)*x - 18*log(x) - 3*x)/(72*(3*x + 2))
```

3.11 $\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{1}{4\left(1+2x+\sqrt{-3-2x+4x^2}\right)} - \frac{7-2x-\sqrt{-3-2x+4x^2}}{4\left(3+\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} - \frac{\arctan\left(\frac{2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)}{2\sqrt{3}} - \log\left(1+2x+\sqrt{-3-2x+4x^2}\right) + \frac{1}{2}\log\left(x-4x^2-2x\sqrt{-3-2x+4x^2}\right)$$

output

```
1/(4+8*x+4*(4*x^2-2*x-3)^(1/2))-(7-2*x-(4*x^2-2*x-3)^(1/2))/(12+4*(2*x+(4*
x^2-2*x-3)^(1/2))^2)-1/6*3^(1/2)*arctan(1/3*(2*x+(4*x^2-2*x-3)^(1/2))*3^(1
/2))-ln(1+2*x+(4*x^2-2*x-3)^(1/2))+1/2*ln(x-4*x^2-2*x*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{1}{12}\left(\frac{3+4x}{2x+3x^2}+\frac{3(1+x)\sqrt{-3-2x+4x^2}}{x(2+3x)} - 2\sqrt{3}\arctan\left(\frac{-2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right) + 6\log\left(x\left(-1+4x-2\sqrt{-3-2x+4x^2}\right)\right) - 12\log\left(-5-6x+3\sqrt{-3-2x+4x^2}\right)\right)$$

input
```
Integrate[1/(x^2*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^2),x]
```

output
```
((3 + 4*x)/(2*x + 3*x^2) + (3*(1 + x)*Sqrt[-3 - 2*x + 4*x^2])/(x*(2 + 3*x)
) - 2*Sqrt[3]*ArcTan[(-2*x + Sqrt[-3 - 2*x + 4*x^2])/Sqrt[3]] + 6*Log[x*(-
1 + 4*x - 2*Sqrt[-3 - 2*x + 4*x^2])] - 12*Log[-5 - 6*x + 3*Sqrt[-3 - 2*x +
 4*x^2]])/12
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{4x^2-2x-3}+2x+1\right)^2}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\sqrt{4x^2-2x-3}}{8x}-\frac{3\sqrt{4x^2-2x-3}}{8(3x+2)}-\frac{\sqrt{4x^2-2x-3}}{8x^2}+\frac{3\sqrt{4x^2-2x-3}}{8(3x+2)^2}-\frac{1}{8x^2}+\frac{1}{2x}-\frac{3}{2(3x+2)}+\frac{1}{8(3x\ldots}\right.$$

$\downarrow$ 2009

$$\frac{1}{8}\sqrt{3}\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)-\frac{\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)}{8\sqrt{3}}-\frac{1}{2}\text{arctanh}\left(\frac{11x+7}{\sqrt{4x^2-2x-3}}\right)+\frac{\sqrt{4x^2-2x-3}}{8x}-\frac{\sqrt{4x^2-2x-3}}{8(3x+2)}+\frac{1}{8x}-\frac{1}{24(3x+2)}+\frac{\log(x)}{2}-\frac{1}{2}\log(3x+2)$$

input
```
Int[1/(x^2*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^2),x]
```

output
```
1/(8*x) - 1/(24*(2 + 3*x)) + Sqrt[-3 - 2*x + 4*x^2]/(8*x) - Sqrt[-3 - 2*x
+ 4*x^2]/(8*(2 + 3*x)) - ArcTan[(3 + x)/(Sqrt[3]*Sqrt[-3 - 2*x + 4*x^2])]/
(8*Sqrt[3]) + (Sqrt[3]*ArcTan[(3 + x)/(Sqrt[3]*Sqrt[-3 - 2*x + 4*x^2])])/8
 - ArcTanh[(7 + 11*x)/Sqrt[-3 - 2*x + 4*x^2]]/2 + Log[x]/2 - Log[2 + 3*x]/
2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.55

method	result
default	$-\frac{1}{24(2+3x)}+\frac{1}{8x}+\frac{\ln(x)}{2}-\frac{\ln(2+3x)}{2}-\frac{(4x^2-2x-3)^{\frac{3}{2}}}{24x}+\frac{\sqrt{4x^2-2x-3}}{12}-\frac{5\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4x^2-2x-3}\right)\sqrt{4}}{32}-\frac{\sqrt{3}\ \text{ar}}{}$
trager	$-\frac{(x-1)(7x+5)}{20x(2+3x)}+\frac{(x+1)\sqrt{4x^2-2x-3}}{4x(2+3x)}+\ln\left(-\frac{75\,\text{RootOf}\left(75\_Z^2-60\_Z+13\right)^2x-75\,\text{RootOf}\left(75\_Z^2-60\_Z+13\right)^2+3}{}\right.$

input
```
int(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24/(2+3*x)+1/8/x+1/2*ln(x)-1/2*ln(2+3*x)-1/24/x*(4*x^2-2*x-3)^(3/2)+1/1
2*(4*x^2-2*x-3)^(1/2)-5/32*ln(1/4*(4*x-1)*4^(1/2)+(4*x^2-2*x-3)^(1/2))*4^(
1/2)-1/12*3^(1/2)*arctan(1/6*(-6-2*x)*3^(1/2)/(4*x^2-2*x-3)^(1/2))+1/48*(8
*x-2)*(4*x^2-2*x-3)^(1/2)-3/8/(x+2/3)*(4*(x+2/3)^2-22/3*x-43/9)^(3/2)-1/2*
(36*(x+2/3)^2-66*x-43)^(1/2)+5/32*ln(1/4*(4*x-1)*4^(1/2)+(4*(x+2/3)^2-22/3
*x-43/9)^(1/2))*4^(1/2)+1/2*arctanh(9/2*(-14/3-22/3*x)/(36*(x+2/3)^2-66*x-
43)^(1/2))+3/16*(8*x-2)*(4*(x+2/3)^2-22/3*x-43/9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = -\frac{2\sqrt{3}(3\,x^2+2\,x)\arctan\left(-\frac{2}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{4\,x^2-2\,x-3}\right)-6\,x^2+6\,(3\,x^2+2\,x)\log(3\,x+2)-6\,(3\,x$$

input
```
integrate(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

output
```
-1/12*(2*sqrt(3)*(3*x^2 + 2*x)*arctan(-2/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(4*
x^2 - 2*x - 3)) - 6*x^2 + 6*(3*x^2 + 2*x)*log(3*x + 2) - 6*(3*x^2 + 2*x)*l
og(x) - 6*(3*x^2 + 2*x)*log(-2*x + sqrt(4*x^2 - 2*x - 3) - 1) + 6*(3*x^2 +
 2*x)*log(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5) - 3*sqrt(4*x^2 - 2*x - 3)*(x
 + 1) - 8*x - 3)/(3*x^2 + 2*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{1}{x^2\left(2x+\sqrt{4x^2-2x-3}+1\right)^2}\,dx$$

input
```
integrate(1/x**2/(1+2*x+(4*x**2-2*x-3)**(1/2))**2,x)
```

output
```
Integral(1/(x**2*(2*x + sqrt(4*x**2 - 2*x - 3) + 1)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx=\int \frac{1}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(4*x^2 - 2*x - 3) + 1)^2*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.60

$$\begin{aligned}&\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx\\&=-\frac{1}{6}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)\right)\\&-\frac{\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^3-32\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2-126\,x+63\,\sqrt{4\,x^2-2\,x-}}{6\left(3\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^4+8\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^3+14\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+48\,x-24\right.}\\&+\frac{4\,x+3}{12\,(3\,x^2+2\,x)}-\frac{1}{2}\,\log\left(|3\,x+2|\right)+\frac{1}{2}\,\log\left(|x|\right)\\&+\frac{1}{2}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right)-\frac{1}{2}\,\log\left(\left|-6\,x+3\,\sqrt{4\,x^2-2\,x-3}-5\right|\right)\end{aligned}$$

input
```
integrate(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - sqrt(4*x^2 - 2*x - 3))) - 1/6*((2*
x - sqrt(4*x^2 - 2*x - 3))^3 - 32*(2*x - sqrt(4*x^2 - 2*x - 3))^2 - 126*x
+ 63*sqrt(4*x^2 - 2*x - 3) - 24)/(3*(2*x - sqrt(4*x^2 - 2*x - 3))^4 + 8*(2
*x - sqrt(4*x^2 - 2*x - 3))^3 + 14*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 48*x
- 24*sqrt(4*x^2 - 2*x - 3) + 15) + 1/12*(4*x + 3)/(3*x^2 + 2*x) - 1/2*log(
abs(3*x + 2)) + 1/2*log(abs(x)) + 1/2*log(abs(-2*x + sqrt(4*x^2 - 2*x - 3)
 - 1)) - 1/2*log(abs(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{\frac{x}{9}+\frac{1}{12}}{x^2+\frac{2\,x}{3}}+\int \frac{-8\,x^3+8\,x+3}{2\,x^2\,{\left(3\,x+2\right)}^2\,\sqrt{4\,x^2-2\,x-3}}\,dx + \mathrm{atan}(x\,3\mathrm{i}+1\mathrm{i})\,1\mathrm{i}$$

input
```
int(1/(x^2*(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2),x)
```

output
```
atan(x*3i + 1i)*1i + (x/9 + 1/12)/((2*x)/3 + x^2) + int((8*x - 8*x^3 + 3)/
(2*x^2*(3*x + 2)^2*(4*x^2 - 2*x - 3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{-4758\sqrt{3}\,atan\left(\frac{\sqrt{4x^2-2x-3}+2x}{\sqrt{3}}\right)x^2-3172\sqrt{3}\,atan\left(\frac{\sqrt{4x^2-2x-3}+2x}{\sqrt{3}}\right)x+2379\sqrt{4x^2-2x-3}\,x+2379\sqrt{4x}}{}$$

input
```
int(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

output
```
( - 4758*sqrt(3)*atan((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x**2 - 3172*
sqrt(3)*atan((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x + 2379*sqrt(4*x**2
- 2*x - 3)*x + 2379*sqrt(4*x**2 - 2*x - 3) - 14274*log(3*x + 2)*x**2 - 951
6*log(3*x + 2)*x - 14274*log((26*sqrt(4*x**2 - 2*x - 3) + 52*x + 26)/sqrt(
13))*x**2 - 9516*log((26*sqrt(4*x**2 - 2*x - 3) + 52*x + 26)/sqrt(13))*x +
 14274*log((6*sqrt(4*x**2 - 2*x - 3) + 12*x + 10)/sqrt(13))*x**2 + 9516*lo
g((6*sqrt(4*x**2 - 2*x - 3) + 12*x + 10)/sqrt(13))*x + 14274*log(x)*x**2 +
 9516*log(x)*x + 3759*x**2 + 5678*x + 2379)/(9516*x*(3*x + 2))
```

3.12 $\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 186

$$\begin{aligned}\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx &= \frac{2}{81\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2}\\ &\quad+\frac{56}{243\left(1+2x+\sqrt{-3-2x+4x^2}\right)}\\ &\quad-\frac{2197}{5184\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)^3}\\ &\quad-\frac{169}{384\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)^2}\\ &\quad-\frac{3029}{5184\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)}\\ &\quad-\frac{235}{729}\log\left(1+2x+\sqrt{-3-2x+4x^2}\right)\\ &\quad+\frac{15769\log\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)}{46656}\end{aligned}$$

output
```
2/81/(1+2*x+(4*x^2-2*x-3)^(1/2))^2+56/(243+486*x+243*(4*x^2-2*x-3)^(1/2))-
2197/5184/(1-4*x-2*(4*x^2-2*x-3)^(1/2))^3-169/384/(1-4*x-2*(4*x^2-2*x-3)^(
1/2))^2-3029/(5184-20736*x-10368*(4*x^2-2*x-3)^(1/2))-235/729*ln(1+2*x+(4*
x^2-2*x-3)^(1/2))+15769/46656*ln(1-4*x-2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \frac{-\frac{6\sqrt{-3-2x+4x^2}(4900+10236x+2505x^2-1476x^3+1728x^4)}{(2+3x)^2}+\frac{48(-108+616x+1872x^2+543x^3-477x^4+432x^5)}{(2+3x)^2}-729\log\left(1-4x\right.}{46656}$$

input
```
Integrate[x^2/(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^3,x]
```

output
```
((-6*Sqrt[-3 - 2*x + 4*x^2]*(4900 + 10236*x + 2505*x^2 - 1476*x^3 + 1728*x
^4))/(2 + 3*x)^2 + (48*(-108 + 616*x + 1872*x^2 + 543*x^3 - 477*x^4 + 432*
x^5))/(2 + 3*x)^2 - 729*Log[1 - 4*x + 2*Sqrt[-3 - 2*x + 4*x^2]] - 15040*Lo
g[-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x^2]])/46656
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{4x^2-2x-3}+2x+1\right)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{4x^2}{27}-\frac{2}{27}\sqrt{4x^2-2x-3}x-\frac{17\sqrt{4x^2-2x-3}}{54(3x+2)}+\frac{19\sqrt{4x^2-2x-3}}{81(3x+2)^2}-\frac{2\sqrt{4x^2-2x-3}}{81(3x+2)^3}+\frac{11}{108}\sqrt{4x^2-2x-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8249\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)}{46656}-\frac{235\text{arctanh}\left(\frac{11x+7}{\sqrt{4x^2-2x-3}}\right)}{1458}+\frac{4x^3}{81}-\frac{13x^2}{108}-\frac{1}{162}\left(4x^2-2x-3\right)^{3/2}-\frac{1}{96}(1-4x)\sqrt{4x^2-2x-3}-\frac{(11x+7)\sqrt{4x^2-2x-3}}{81(3x+2)^2}-\frac{19\sqrt{4x^2-2x-3}}{243(3x+2)}-\frac{17}{162}\sqrt{4x^2-2x-3}+\frac{65x}{324}-\frac{41}{729(3x+2)}+\frac{1}{729(3x+2)^2}-\frac{235\log(3x+2)}{1458}$$

input
```
Int[x^2/(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^3,x]
```

output
```
(65*x)/324 - (13*x^2)/108 + (4*x^3)/81 + 1/(729*(2 + 3*x)^2) - 41/(729*(2
+ 3*x)) - (17*Sqrt[-3 - 2*x + 4*x^2])/162 - ((1 - 4*x)*Sqrt[-3 - 2*x + 4*x
^2])/96 - (19*Sqrt[-3 - 2*x + 4*x^2])/(243*(2 + 3*x)) - ((7 + 11*x)*Sqrt[-
3 - 2*x + 4*x^2])/(81*(2 + 3*x)^2) - (-3 - 2*x + 4*x^2)^(3/2)/162 - (8249*
ArcTanh[(1 - 4*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/46656 - (235*ArcTanh[(7 + 1
1*x)/Sqrt[-3 - 2*x + 4*x^2]])/1458 - (235*Log[2 + 3*x])/1458
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

output
```
int(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \frac{82944\,x^5 - 91584\,x^4 + 104256\,x^3 + 261279\,x^2 - 30080\,(9\,x^2 + 12\,x + 4)\log(3\,x + 2) + 30080\,(9\,x^2 + 12$$

input
```
integrate(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/186624*(82944*x^5 - 91584*x^4 + 104256*x^3 + 261279*x^2 - 30080*(9*x^2 +
 12*x + 4)*log(3*x + 2) + 30080*(9*x^2 + 12*x + 4)*log(-2*x + sqrt(4*x^2 -
 2*x - 3) - 1) - 32996*(9*x^2 + 12*x + 4)*log(-4*x + 2*sqrt(4*x^2 - 2*x -
3) + 1) - 30080*(9*x^2 + 12*x + 4)*log(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5)
 - 24*(1728*x^4 - 1476*x^3 + 2505*x^2 + 10236*x + 4900)*sqrt(4*x^2 - 2*x -
 3) - 12588*x - 64356)/(9*x^2 + 12*x + 4)
```

Sympy [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{x^2}{\left(2x+\sqrt{4x^2-2x-3}+1\right)^3}\,dx$$

input
```
integrate(x**2/(1+2*x+(4*x**2-2*x-3)**(1/2))**3,x)
```

output
```
Integral(x**2/(2*x + sqrt(4*x**2 - 2*x - 3) + 1)**3, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{x^2}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3}\,dx$$

input
```
integrate(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x^2/(2*x + sqrt(4*x^2 - 2*x - 3) + 1)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$= \frac{4}{81}\,x^3 - \frac{13}{108}\,x^2 - \frac{1}{2592}\,(4\,(16\,x-35)x+251)\sqrt{4\,x^2-2\,x-3} + \frac{65}{324}\,x$$
$$- \frac{4\left(167\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^3 + 647\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2 + 1660\,x - 830\,\sqrt{4\,x^2-2\,x-3} + 35\right.}{243\left(3\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2 + 16\,x - 8\,\sqrt{4\,x^2-2\,x-3} + 5\right)^2}$$
$$- \frac{41\,x+27}{243\,(3\,x+2)^2} - \frac{235}{1458}\,\log\left(|3\,x+2|\right) + \frac{235}{1458}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right)$$
$$- \frac{8249}{46656}\,\log\left(\left|-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right|\right)$$
$$- \frac{235}{1458}\,\log\left(\left|-6\,x+3\,\sqrt{4\,x^2-2\,x-3}-5\right|\right)$$

input
```
integrate(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

output
```
4/81*x^3 - 13/108*x^2 - 1/2592*(4*(16*x - 35)*x + 251)*sqrt(4*x^2 - 2*x -
3) + 65/324*x - 4/243*(167*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 647*(2*x - sq
rt(4*x^2 - 2*x - 3))^2 + 1660*x - 830*sqrt(4*x^2 - 2*x - 3) + 353)/(3*(2*x
 - sqrt(4*x^2 - 2*x - 3))^2 + 16*x - 8*sqrt(4*x^2 - 2*x - 3) + 5)^2 - 1/24
3*(41*x + 27)/(3*x + 2)^2 - 235/1458*log(abs(3*x + 2)) + 235/1458*log(abs(
-2*x + sqrt(4*x^2 - 2*x - 3) - 1)) - 8249/46656*log(abs(-4*x + 2*sqrt(4*x^
2 - 2*x - 3) + 1)) - 235/1458*log(abs(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{x^2}{{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)}^3} \,d x$$

input
```
int(x^2/(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3,x)
```

output
```
int(x^2/(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3, x)
```

Reduce [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{x^2}{\left(1+2x+\sqrt{4x^2-2x-3}\right)^3}dx$$

input
```
int(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

output
```
int(x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

3.13 $\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 194

$$\begin{aligned}\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx &= \frac{137+233\left(2x+\sqrt{-3-2x+4x^2}\right)}{288\left(1-2x-\sqrt{-3-2x+4x^2}-2\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)^2} \\ &\quad+\frac{179+1148\left(2x+\sqrt{-3-2x+4x^2}\right)}{864\left(1-2x-\sqrt{-3-2x+4x^2}-2\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} \\ &\quad+\frac{65}{162}\log\left(1+2x+\sqrt{-3-2x+4x^2}\right)-\frac{65}{162}\log\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\end{aligned}$$

output

```
1/288*(137+466*x+233*(4*x^2-2*x-3)^(1/2))/(1-2*x-(4*x^2-2*x-3)^(1/2)-2*(2*
x+(4*x^2-2*x-3)^(1/2))^2)^2+(179+2296*x+1148*(4*x^2-2*x-3)^(1/2))/(864-172
8*x-864*(4*x^2-2*x-3)^(1/2)-1728*(2*x+(4*x^2-2*x-3)^(1/2))^2)+65/162*ln(1+
2*x+(4*x^2-2*x-3)^(1/2))-65/162*ln(1-4*x-2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.48

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \frac{79-348x-1260x^2-621x^3+324x^4+9\sqrt{-3-2x+4x^2}(43+93x+30x^2-18x^3)+195(2+3x)^2\log$$

input
```
Integrate[x/(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^3,x]
```

output
```
(79 - 348*x - 1260*x^2 - 621*x^3 + 324*x^4 + 9*Sqrt[-3 - 2*x + 4*x^2]*(43
+ 93*x + 30*x^2 - 18*x^3) + 195*(2 + 3*x)^2*Log[-5 - 6*x + 3*Sqrt[-3 - 2*x
 + 4*x^2]])/(486*(2 + 3*x)^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{4x^2-2x-3}+2x+1\right)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{11\sqrt{4x^2-2x-3}}{36(3x+2)}-\frac{\sqrt{4x^2-2x-3}}{3(3x+2)^2}+\frac{\sqrt{4x^2-2x-3}}{27(3x+2)^3}-\frac{2}{27}\sqrt{4x^2-2x-3}+\frac{4x}{27}+\frac{65}{108(3x+2)}-\frac{20}{81(3x+}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{65}{324}\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)+\frac{65}{324}\text{arctanh}\left(\frac{11x+7}{\sqrt{4x^2-2x-3}}\right)+\frac{2x^2}{27}+\frac{1}{108}(1-4x)\sqrt{4x^2-2x-3}+\frac{(11x+7)\sqrt{4x^2-2x-3}}{54(3x+2)^2}+\frac{\sqrt{4x^2-2x-3}}{9(3x+2)}+\frac{11}{108}\sqrt{4x^2-2x-3}-\frac{13x}{54}+\frac{20}{243(3x+2)}-\frac{1}{486(3x+2)^2}+\frac{65}{324}\log(3x+2)$$

input
```
Int[x/(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^3,x]
```

output
```
(-13*x)/54 + (2*x^2)/27 - 1/(486*(2 + 3*x)^2) + 20/(243*(2 + 3*x)) + (11*S
qrt[-3 - 2*x + 4*x^2])/108 + ((1 - 4*x)*Sqrt[-3 - 2*x + 4*x^2])/108 + Sqrt
[-3 - 2*x + 4*x^2]/(9*(2 + 3*x)) + ((7 + 11*x)*Sqrt[-3 - 2*x + 4*x^2])/(54
*(2 + 3*x)^2) + (65*ArcTanh[(1 - 4*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/324 + (
65*ArcTanh[(7 + 11*x)/Sqrt[-3 - 2*x + 4*x^2]])/324 + (65*Log[2 + 3*x])/324
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.44

method	result
trager	$\frac{(144x^3-276x^2-639x-260)x}{216(2+3x)^2}-\frac{(18x^3-30x^2-93x-43)\sqrt{4x^2-2x-3}}{54(2+3x)^2}+\frac{65\ln\left(-3\sqrt{4x^2-2x-3}+5+6x\right)}{162}$
default	$\frac{20}{243(2+3x)}-\frac{1}{486(2+3x)^2}+\frac{65\ln(2+3x)}{324}-\frac{13x}{54}+\frac{2x^2}{27}-\frac{\left(4\left(x+\frac{2}{3}\right)^2-\frac{22x}{3}-\frac{43}{9}\right)^{\frac{3}{2}}}{162\left(x+\frac{2}{3}\right)^2}+\frac{7\left(4\left(x+\frac{2}{3}\right)^2-\frac{22x}{3}-\frac{43}{9}\right)^{\frac{3}{2}}}{54\left(x+\frac{2}{3}\right)}+65\sqrt{\ldots}$

input
```
int(x/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/216*(144*x^3-276*x^2-639*x-260)*x/(2+3*x)^2-1/54*(18*x^3-30*x^2-93*x-43)
/(2+3*x)^2*(4*x^2-2*x-3)^(1/2)+65/162*ln(-3*(4*x^2-2*x-3)^(1/2)+5+6*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.79

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$= \frac{5184\,x^4-9936\,x^3-13671\,x^2+1560\,(9\,x^2+12\,x+4)\log\left(48\,x^2-\sqrt{4\,x^2-2\,x-3}(24\,x+7)+2\,x-23\right.}{}$$

input
```
integrate(x/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/7776*(5184*x^4 - 9936*x^3 - 13671*x^2 + 1560*(9*x^2 + 12*x + 4)*log(48*x
^2 - sqrt(4*x^2 - 2*x - 3)*(24*x + 7) + 2*x - 23) + 1560*(9*x^2 + 12*x + 4
)*log(3*x + 2) - 1560*(9*x^2 + 12*x + 4)*log(-2*x + sqrt(4*x^2 - 2*x - 3)
- 1) - 144*(18*x^3 - 30*x^2 - 93*x - 43)*sqrt(4*x^2 - 2*x - 3) + 3084*x +
4148)/(9*x^2 + 12*x + 4)
```

Sympy [F]

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{x}{\left(2x+\sqrt{4x^2-2x-3}+1\right)^3}\,dx$$

input
```
integrate(x/(1+2*x+(4*x**2-2*x-3)**(1/2))**3,x)
```

output
```
Integral(x/(2*x + sqrt(4*x**2 - 2*x - 3) + 1)**3, x)
```

Maxima [F]

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{x}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3}\,dx$$

input
```
integrate(x/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x/(2*x + sqrt(4*x^2 - 2*x - 3) + 1)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.15

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\frac{2}{27}\,x^2-\frac{1}{27}\,\sqrt{4\,x^2-2\,x-3}(x-3)-\frac{13}{54}\,x$$
$$+\frac{969\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^3+3752\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+9626\,x-4813\,\sqrt{4\,x^2-2\,x-3}+204}{243\left(3\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+16\,x-8\,\sqrt{4\,x^2-2\,x-3}+5\right)^2}$$
$$+\frac{120\,x+79}{486\,(3\,x+2)^2}+\frac{65}{324}\,\log\left(|3\,x+2|\right)-\frac{65}{324}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right)$$
$$+\frac{65}{324}\,\log\left(\left|-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right|\right)$$
$$+\frac{65}{324}\,\log\left(\left|-6\,x+3\,\sqrt{4\,x^2-2\,x-3}-5\right|\right)$$

input
```
integrate(x/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

output
```
2/27*x^2 - 1/27*sqrt(4*x^2 - 2*x - 3)*(x - 3) - 13/54*x + 1/243*(969*(2*x
- sqrt(4*x^2 - 2*x - 3))^3 + 3752*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 9626*x
 - 4813*sqrt(4*x^2 - 2*x - 3) + 2048)/(3*(2*x - sqrt(4*x^2 - 2*x - 3))^2 +
 16*x - 8*sqrt(4*x^2 - 2*x - 3) + 5)^2 + 1/486*(120*x + 79)/(3*x + 2)^2 +
65/324*log(abs(3*x + 2)) - 65/324*log(abs(-2*x + sqrt(4*x^2 - 2*x - 3) - 1
)) + 65/324*log(abs(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1)) + 65/324*log(abs(
-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{x}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3}\,dx$$

input
```
int(x/(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3,x)
```

output
```
int(x/(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

$$\int \frac{x}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$=\frac{-54\sqrt{4x^2-2x-3}\,x^3+90\sqrt{4x^2-2x-3}\,x^2+279\sqrt{4x^2-2x-3}\,x+129\sqrt{4x^2-2x-3}+585\log(-3}{}$$

input
```
int(x/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

output
```
( - 54*sqrt(4*x**2 - 2*x - 3)*x**3 + 90*sqrt(4*x**2 - 2*x - 3)*x**2 + 279*
sqrt(4*x**2 - 2*x - 3)*x + 129*sqrt(4*x**2 - 2*x - 3) + 585*log( - 3*sqrt(
4*x**2 - 2*x - 3) + 6*x + 5)*x**2 + 780*log( - 3*sqrt(4*x**2 - 2*x - 3) +
6*x + 5)*x + 260*log( - 3*sqrt(4*x**2 - 2*x - 3) + 6*x + 5) + 108*x**4 - 2
07*x**3 - 333*x**2 + 65)/(162*(9*x**2 + 12*x + 4))
```

3.14 $\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \frac{1}{18\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2} + \frac{13}{27\left(1+2x+\sqrt{-3-2x+4x^2}\right)} - \frac{13}{27\left(3-2\left(1+2x+\sqrt{-3-2x+4x^2}\right)\right)} - \frac{13}{27}\log\left(1+2x+\sqrt{-3-2x+4x^2}\right) + \frac{13}{27}\log\left(3-2\left(1+2x+\sqrt{-3-2x+4x^2}\right)\right)$$

output
```
1/18/(1+2*x+(4*x^2-2*x-3)^(1/2))^2+13/(27+54*x+27*(4*x^2-2*x-3)^(1/2))-13/
(27-108*x-54*(4*x^2-2*x-3)^(1/2))-13/27*ln(1+2*x+(4*x^2-2*x-3)^(1/2))+13/2
7*ln(1-4*x-2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.63

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \frac{-77+75x+576x^2+432x^3-9\sqrt{-3-2x+4x^2}(29+60x+24x^2)-156(2+3x)^2\log\left(-5-6x+3\sqrt{\cdot}\right.}{324(2+3x)^2}$$

input
```
Integrate[(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^(-3),x]
```

output
```
(-77 + 75*x + 576*x^2 + 432*x^3 - 9*Sqrt[-3 - 2*x + 4*x^2]*(29 + 60*x + 24
*x^2) - 156*(2 + 3*x)^2*Log[-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x^2]])/(324*(2
+ 3*x)^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2541, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{4x^2-2x-3}+2x+1\right)^3}\,dx$$

$$\Big\downarrow \text{ 2541}$$

$$2\int -\frac{-\left(2x+\sqrt{4x^2-2x-3}+1\right)^2+3\left(2x+\sqrt{4x^2-2x-3}+1\right)+1}{2\left(2x+\sqrt{4x^2-2x-3}+1\right)^3\left(3-2\left(2x+\sqrt{4x^2-2x-3}+1\right)\right)^2}d\left(2x+\sqrt{4x^2-2x-3}+1\right)$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{-\left(2x+\sqrt{4x^2-2x-3}+1\right)^2+3\left(2x+\sqrt{4x^2-2x-3}+1\right)+1}{\left(2x+\sqrt{4x^2-2x-3}+1\right)^3\left(3-2\left(2x+\sqrt{4x^2-2x-3}+1\right)\right)^2}d\left(2x+\sqrt{4x^2-2x-3}+1\right)$$

$$\Big\downarrow \text{1195}$$

$$-\int \left(-\frac{26}{27\left(2\left(2x+\sqrt{4x^2-2x-3}+1\right)-3\right)}+\frac{26}{27\left(2\left(2x+\sqrt{4x^2-2x-3}+1\right)-3\right)^2}+\frac{13}{27\left(2x+\sqrt{4x^2-2\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{13}{27\left(\sqrt{4x^2-2x-3}+2x+1\right)}-\frac{13}{27\left(3-2\left(\sqrt{4x^2-2x-3}+2x+1\right)\right)}+\frac{1}{18\left(\sqrt{4x^2-2x-3}+2x+1\right)^2}-\frac{13}{27}\log\left(\sqrt{4x^2-2x-3}+2x+1\right)+\frac{13}{27}\log\left(3-2\left(\sqrt{4x^2-2x-3}+2x+1\right)\right)$$

input
```
Int[(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^(-3),x]
```

output
```
1/(18*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^2) + 13/(27*(1 + 2*x + Sqrt[-3 -
2*x + 4*x^2])) - 13/(27*(3 - 2*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2]))) - (13*
Log[1 + 2*x + Sqrt[-3 - 2*x + 4*x^2]])/27 + (13*Log[3 - 2*(1 + 2*x + Sqrt[
-3 - 2*x + 4*x^2])])/27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.57

method	result
trager	$\frac{(192x^2+333x+136)x}{144(2+3x)^2}-\frac{(24x^2+60x+29)\sqrt{4x^2-2x-3}}{36(2+3x)^2}-\frac{13\ln\left(-3\sqrt{4x^2-2x-3}+5+6x\right)}{27}$
default	$\frac{1}{324(2+3x)^2}-\frac{13}{108(2+3x)}-\frac{13\ln(2+3x)}{54}+\frac{4x}{27}+\frac{\left(4\left(x+\frac{2}{3}\right)^2-\frac{22x}{3}-\frac{43}{9}\right)^{\frac{3}{2}}}{108\left(x+\frac{2}{3}\right)^2}-\frac{\left(4\left(x+\frac{2}{3}\right)^2-\frac{22x}{3}-\frac{43}{9}\right)^{\frac{3}{2}}}{6\left(x+\frac{2}{3}\right)}-\frac{13\sqrt{36\left(x+\frac{2}{3}\right)^2-}}{54}$

input
```
int(1/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/144*(192*x^2+333*x+136)*x/(2+3*x)^2-1/36*(24*x^2+60*x+29)/(2+3*x)^2*(4*x
^2-2*x-3)^(1/2)-13/27*ln(-3*(4*x^2-2*x-3)^(1/2)+5+6*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$= \frac{432\,x^3+126\,x^2-78\left(9\,x^2+12\,x+4\right)\log\left(48\,x^2-\sqrt{4\,x^2-2\,x-3}(24\,x+7)+2\,x-23\right)-78\left(9\,x^2+1\right.}{}$$

input
```
integrate(1/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/324*(432*x^3 + 126*x^2 - 78*(9*x^2 + 12*x + 4)*log(48*x^2 - sqrt(4*x^2 -
 2*x - 3)*(24*x + 7) + 2*x - 23) - 78*(9*x^2 + 12*x + 4)*log(3*x + 2) + 78
*(9*x^2 + 12*x + 4)*log(-2*x + sqrt(4*x^2 - 2*x - 3) - 1) - 9*(24*x^2 + 60
*x + 29)*sqrt(4*x^2 - 2*x - 3) - 525*x - 277)/(9*x^2 + 12*x + 4)
```

Sympy [F]

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{\left(2x+\sqrt{4x^2-2x-3}+1\right)^3}\,dx$$

input
```
integrate(1/(1+2*x+(4*x**2-2*x-3)**(1/2))**3,x)
```

output
```
Integral((2*x + sqrt(4*x**2 - 2*x - 3) + 1)**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3}\,dx$$

input
```
integrate(1/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate((2*x + sqrt(4*x^2 - 2*x - 3) + 1)^(-3), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.64

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \frac{4}{27}\,x - \frac{2}{27}\,\sqrt{4\,x^2-2\,x-3} - \frac{468\,\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^3 + 1811\,\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2 + 4646\,x - 2323\,\sqrt{4\,x^2-2\,x-3} + 989}{81\,\left(3\,\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2 + 16\,x - 8\,\sqrt{4\,x^2-2\,x-3} + 5\right)^2} - \frac{117\,x+77}{324\,(3\,x+2)^2} - \frac{13}{54}\,\log\left(|3\,x+2|\right) + \frac{13}{54}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right) - \frac{13}{54}\,\log\left(\left|-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right|\right) - \frac{13}{54}\,\log\left(\left|-6\,x+3\,\sqrt{4\,x^2-2\,x-3}-5\right|\right)$$

input
```
integrate(1/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

output
```
4/27*x - 2/27*sqrt(4*x^2 - 2*x - 3) - 1/81*(468*(2*x - sqrt(4*x^2 - 2*x -
3))^3 + 1811*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 4646*x - 2323*sqrt(4*x^2 -
2*x - 3) + 989)/(3*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 16*x - 8*sqrt(4*x^2 -
 2*x - 3) + 5)^2 - 1/324*(117*x + 77)/(3*x + 2)^2 - 13/54*log(abs(3*x + 2)
) + 13/54*log(abs(-2*x + sqrt(4*x^2 - 2*x - 3) - 1)) - 13/54*log(abs(-4*x
+ 2*sqrt(4*x^2 - 2*x - 3) + 1)) - 13/54*log(abs(-6*x + 3*sqrt(4*x^2 - 2*x
- 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3}\,dx$$

input
```
int(1/(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3,x)
```

output
```
int(1/(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$= \frac{-288\sqrt{4x^2-2x-3}\,x^2-720\sqrt{4x^2-2x-3}\,x-348\sqrt{4x^2-2x-3}+1872\log\left(3\sqrt{4x^2-2x-3}+6x+\right.}{}$$

input
```
int(1/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

output
```
( - 288*sqrt(4*x**2 - 2*x - 3)*x**2 - 720*sqrt(4*x**2 - 2*x - 3)*x - 348*s
qrt(4*x**2 - 2*x - 3) + 1872*log(3*sqrt(4*x**2 - 2*x - 3) + 6*x + 5)*x**2
+ 2496*log(3*sqrt(4*x**2 - 2*x - 3) + 6*x + 5)*x + 832*log(3*sqrt(4*x**2 -
 2*x - 3) + 6*x + 5) - 1872*log(3*x + 2)*x**2 - 2496*log(3*x + 2)*x - 832*
log(3*x + 2) + 576*x**3 + 693*x**2 - 136)/(432*(9*x**2 + 12*x + 4))
```

3.15 $\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = -\frac{1}{12\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2} - \frac{25}{36\left(1+2x+\sqrt{-3-2x+4x^2}\right)} + \frac{59}{108}\log\left(1+2x+\sqrt{-3-2x+4x^2}\right) - \frac{1}{8}\log\left(x-4x^2-2x\sqrt{-3-2x+4x^2}\right) - \frac{8}{27}\log\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)$$

output
```
-1/12/(1+2*x+(4*x^2-2*x-3)^(1/2))^2-25/(36+72*x+36*(4*x^2-2*x-3)^(1/2))+59
/108*ln(1+2*x+(4*x^2-2*x-3)^(1/2))-1/8*ln(x-4*x^2-2*x*(4*x^2-2*x-3)^(1/2))
-8/27*ln(1-4*x-2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \frac{1}{216}\left(\frac{3(25+38x)}{(2+3x)^2} + \frac{3(53+81x)\sqrt{-3-2x+4x^2}}{(2+3x)^2} - 27\log\left(x\left(-1+4x-2\sqrt{-3-2x+4x^2}\right)\right) + 118\log\left(-5-6x+3\sqrt{-3-2x+4x^2}\right)\right)$$

input
```
Integrate[1/(x*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^3),x]
```

output
```
((3*(25 + 38*x))/(2 + 3*x)^2 + (3*(53 + 81*x)*Sqrt[-3 - 2*x + 4*x^2])/(2 +
 3*x)^2 - 27*Log[x*(-1 + 4*x - 2*Sqrt[-3 - 2*x + 4*x^2])] + 118*Log[-5 - 6
*x + 3*Sqrt[-3 - 2*x + 4*x^2]])/216
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{4x^2-2x-3}+2x+1\right)^3}dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(-\frac{2\sqrt{4x^2-2x-3}}{3(3x+2)^2}+\frac{\sqrt{4x^2-2x-3}}{12(3x+2)^3}-\frac{1}{8x}+\frac{59}{72(3x+2)}-\frac{19}{36(3x+2)^2}+\frac{1}{36(3x+2)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{27}\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)+\frac{59}{216}\text{arctanh}\left(\frac{11x+7}{\sqrt{4x^2-2x-3}}\right)+\frac{\sqrt{4x^2-2x-3}(11x+7)}{24(3x+2)^2}+\frac{2\sqrt{4x^2-2x-3}}{9(3x+2)}+\frac{19}{108(3x+2)}-\frac{1}{216(3x+2)^2}-\frac{\log(x)}{8}+\frac{59}{216}\log(3x+2)$$

input
```
Int[1/(x*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^3),x]
```

output
```
-1/216*1/(2 + 3*x)^2 + 19/(108*(2 + 3*x)) + (2*Sqrt[-3 - 2*x + 4*x^2])/(9*
(2 + 3*x)) + ((7 + 11*x)*Sqrt[-3 - 2*x + 4*x^2])/(24*(2 + 3*x)^2) + (4*Arc
Tanh[(1 - 4*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/27 + (59*ArcTanh[(7 + 11*x)/Sq
rt[-3 - 2*x + 4*x^2]])/216 - Log[x]/8 + (59*Log[2 + 3*x])/216
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 66.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.27

method	result
default	$-\frac{1}{216(2+3x)^2}-\frac{\ln(x)}{8}+\frac{19}{108(2+3x)}+\frac{59\ln(2+3x)}{216}-\frac{\left(4(x+\frac{2}{3})^2-\frac{22x}{3}-\frac{43}{9}\right)^{\frac{3}{2}}}{72(x+\frac{2}{3})^2}+\frac{5\left(4(x+\frac{2}{3})^2-\frac{22x}{3}-\frac{43}{9}\right)^{\frac{3}{2}}}{24(x+\frac{2}{3})}+\frac{59\sqrt{36(x}$
trager	Expression too large to display

input
```
int(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/216/(2+3*x)^2-1/8*ln(x)+19/108/(2+3*x)+59/216*ln(2+3*x)-1/72/(x+2/3)^2*
(4*(x+2/3)^2-22/3*x-43/9)^(3/2)+5/24/(x+2/3)*(4*(x+2/3)^2-22/3*x-43/9)^(3/
2)+59/216*(36*(x+2/3)^2-66*x-43)^(1/2)-59/216*arctanh(9/2*(-14/3-22/3*x)/(
36*(x+2/3)^2-66*x-43)^(1/2))-5/48*(8*x-2)*(4*(x+2/3)^2-22/3*x-43/9)^(1/2)-
2/27*ln(1/4*(4*x-1)*4^(1/2)+(4*(x+2/3)^2-22/3*x-43/9)^(1/2))*4^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$= \frac{486\,x^2+59\,(9\,x^2+12\,x+4)\log{(3\,x+2)}-27\,(9\,x^2+12\,x+4)\log{(x)}-59\,(9\,x^2+12\,x+4)\log{(-2\,x}$$

input
```
integrate(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/216*(486*x^2 + 59*(9*x^2 + 12*x + 4)*log(3*x + 2) - 27*(9*x^2 + 12*x + 4
)*log(x) - 59*(9*x^2 + 12*x + 4)*log(-2*x + sqrt(4*x^2 - 2*x - 3) - 1) + 3
2*(9*x^2 + 12*x + 4)*log(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1) + 59*(9*x^2 +
 12*x + 4)*log(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5) + 3*sqrt(4*x^2 - 2*x -
3)*(81*x + 53) + 762*x + 291)/(9*x^2 + 12*x + 4)
```

Sympy [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{1}{x\left(2x+\sqrt{4x^2-2x-3}+1\right)^3}\,dx$$

input
```
integrate(1/x/(1+2*x+(4*x**2-2*x-3)**(1/2))**3,x)
```

output
```
Integral(1/(x*(2*x + sqrt(4*x**2 - 2*x - 3) + 1)**3), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3 x}\,dx$$

input
```
integrate(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(4*x^2 - 2*x - 3) + 1)^3*x), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.56

$$\begin{aligned}&\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx\\&=\frac{301\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^3+1164\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+2986\,x-1493\sqrt{4\,x^2-2\,x-3}+636}{36\left(3\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+16\,x-8\sqrt{4\,x^2-2\,x-3}+5\right)^2}\\&\quad+\frac{38\,x+25}{72\,(3\,x+2)^2}+\frac{59}{216}\log\left(|3\,x+2|\right)-\frac{1}{8}\log\left(|x|\right)\\&\quad-\frac{59}{216}\log\left(\left|-2\,x+\sqrt{4\,x^2-2\,x-3}-1\right|\right)+\frac{4}{27}\log\left(\left|-4\,x+2\sqrt{4\,x^2-2\,x-3}+1\right|\right)\\&\quad+\frac{59}{216}\log\left(\left|-6\,x+3\sqrt{4\,x^2-2\,x-3}-5\right|\right)\end{aligned}$$

input
```
integrate(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

output
```
1/36*(301*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 1164*(2*x - sqrt(4*x^2 - 2*x -
 3))^2 + 2986*x - 1493*sqrt(4*x^2 - 2*x - 3) + 636)/(3*(2*x - sqrt(4*x^2 -
 2*x - 3))^2 + 16*x - 8*sqrt(4*x^2 - 2*x - 3) + 5)^2 + 1/72*(38*x + 25)/(3
*x + 2)^2 + 59/216*log(abs(3*x + 2)) - 1/8*log(abs(x)) - 59/216*log(abs(-2
*x + sqrt(4*x^2 - 2*x - 3) - 1)) + 4/27*log(abs(-4*x + 2*sqrt(4*x^2 - 2*x
- 3) + 1)) + 59/216*log(abs(-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{x\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3}\,dx$$

input
```
int(1/(x*(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3),x)
```

output
```
int(1/(x*(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3), x)
```

Reduce [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{x\left(1+2x+\sqrt{4x^2-2x-3}\right)^3}dx$$

input
```
int(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

output
```
int(1/x/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

3.16 $\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\begin{aligned}\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx &= \frac{1}{8\left(1+2x+\sqrt{-3-2x+4x^2}\right)^2} \\ &\quad+\frac{1}{1+2x+\sqrt{-3-2x+4x^2}} \\ &\quad-\frac{1-2\left(2x+\sqrt{-3-2x+4x^2}\right)}{4\left(3+\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} \\ &\quad+\frac{5}{16}\sqrt{3}\arctan\left(\frac{2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right) \\ &\quad-\frac{9}{16}\log\left(1+2x+\sqrt{-3-2x+4x^2}\right) \\ &\quad+\frac{9}{32}\log\left(x-4x^2-2x\sqrt{-3-2x+4x^2}\right)\end{aligned}$$

output
```
1/8/(1+2*x+(4*x^2-2*x-3)^(1/2))^2+1/(1+2*x+(4*x^2-2*x-3)^(1/2))-(1-4*x-2*(
4*x^2-2*x-3)^(1/2))/(12+4*(2*x+(4*x^2-2*x-3)^(1/2))^2)+5/16*3^(1/2)*arctan
(1/3*(2*x+(4*x^2-2*x-3)^(1/2))*3^(1/2))-9/16*ln(1+2*x+(4*x^2-2*x-3)^(1/2))
+9/32*ln(x-4*x^2-2*x*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\frac{1}{288}\left(-\frac{18(17+26x)\sqrt{-3-2x+4x^2}}{(2+3x)^2}+\frac{2(72+143x+51x^2)}{x(2+3x)^2}+90\sqrt{3}\arctan\left(\frac{-2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)+81\log\left(x\left(-1+4x-2\sqrt{-3-2x+4x^2}\right)\right)-162\log\left(-5-6x+3\sqrt{-3-2x+4x^2}\right)\right)$$

input
```
Integrate[1/(x^2*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^3),x]
```

output
```
((-18*(17 + 26*x)*Sqrt[-3 - 2*x + 4*x^2])/(2 + 3*x)^2 + (2*(72 + 143*x + 5
1*x^2))/(x*(2 + 3*x)^2) + 90*Sqrt[3]*ArcTan[(-2*x + Sqrt[-3 - 2*x + 4*x^2]
)/Sqrt[3]] + 81*Log[x*(-1 + 4*x - 2*Sqrt[-3 - 2*x + 4*x^2])] - 162*Log[-5
- 6*x + 3*Sqrt[-3 - 2*x + 4*x^2]])/288
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{4x^2-2x-3}+2x+1\right)^3}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(-\frac{5\sqrt{4x^2-2x-3}}{32x}+\frac{15\sqrt{4x^2-2x-3}}{32(3x+2)}+\frac{15\sqrt{4x^2-2x-3}}{16(3x+2)^2}-\frac{\sqrt{4x^2-2x-3}}{8(3x+2)^3}-\frac{1}{8x^2}+\frac{9}{32x}-\frac{27}{32(3x+2)}\right.$$

$\big\downarrow$ 2009

$$-\frac{5}{32}\sqrt{3}\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)-\frac{9}{32}\text{arctanh}\left(\frac{11x+7}{\sqrt{4x^2-2x-3}}\right)-\frac{\sqrt{4x^2-2x-3}(11x+7)}{16(3x+2)^2}-\frac{5\sqrt{4x^2-2x-3}}{16(3x+2)}+\frac{1}{8x}-\frac{37}{144(3x+2)}+\frac{1}{144(3x+2)^2}+\frac{9\log(x)}{32}-\frac{9}{32}\log(3x+2)$$

input
```
Int[1/(x^2*(1 + 2*x + Sqrt[-3 - 2*x + 4*x^2])^3),x]
```

output
```
1/(8*x) + 1/(144*(2 + 3*x)^2) - 37/(144*(2 + 3*x)) - (5*Sqrt[-3 - 2*x + 4*
x^2])/(16*(2 + 3*x)) - ((7 + 11*x)*Sqrt[-3 - 2*x + 4*x^2])/(16*(2 + 3*x)^2
) - (5*Sqrt[3]*ArcTan[(3 + x)/(Sqrt[3]*Sqrt[-3 - 2*x + 4*x^2])])/32 - (9*A
rcTanh[(7 + 11*x)/Sqrt[-3 - 2*x + 4*x^2]])/32 + (9*Log[x])/32 - (9*Log[2 +
 3*x])/32
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.32

method	result
default	$\frac{1}{144(2+3x)^{2}}+\frac{1}{8x}+\frac{9\ln(x)}{32}-\frac{37}{144(2+3x)}-\frac{9\ln(2+3x)}{32}+\frac{\left(4\left(x+\frac{2}{3}\right)^{2}-\frac{22x}{3}-\frac{43}{9}\right)^{\frac{3}{2}}}{48\left(x+\frac{2}{3}\right)^{2}}-\frac{\left(4\left(x+\frac{2}{3}\right)^{2}-\frac{22x}{3}-\frac{43}{9}\right)^{\frac{3}{2}}}{4\left(x+\frac{2}{3}\right)}-\frac{9\sqrt{36}$
trager	$-\frac{(x-1)(266x^{2}+479x+200)}{400x(2+3x)^{2}}-\frac{(17+26x)\sqrt{4x^{2}-2x-3}}{16(2+3x)^{2}}+\frac{9\ln\left(-\frac{1875\,\mathrm{RootOf}\left(1875\_Z^{2}-225\_Z+13\right)^{2}x-1875\,\mathrm{RootOf}\left(1875\_Z\right.}{}\right.}{}$

input
```
int(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/144/(2+3*x)^2+1/8/x+9/32*ln(x)-37/144/(2+3*x)-9/32*ln(2+3*x)+1/48/(x+2/3
)^2*(4*(x+2/3)^2-22/3*x-43/9)^(3/2)-1/4/(x+2/3)*(4*(x+2/3)^2-22/3*x-43/9)^
(3/2)-9/32*(36*(x+2/3)^2-66*x-43)^(1/2)+9/32*arctanh(9/2*(-14/3-22/3*x)/(3
6*(x+2/3)^2-66*x-43)^(1/2))+1/8*(8*x-2)*(4*(x+2/3)^2-22/3*x-43/9)^(1/2)-5/
32*(4*x^2-2*x-3)^(1/2)+5/128*ln(1/4*(4*x-1)*4^(1/2)+(4*x^2-2*x-3)^(1/2))*4
^(1/2)+5/32*3^(1/2)*arctan(1/6*(-6-2*x)*3^(1/2)/(4*x^2-2*x-3)^(1/2))-5/128
*ln(1/4*(4*x-1)*4^(1/2)+(4*(x+2/3)^2-22/3*x-43/9)^(1/2))*4^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^{2}\left(1+2x+\sqrt{-3-2x+4x^{2}}\right)^{3}}\,dx = -\frac{936\,x^{3}-90\,\sqrt{3}(9\,x^{3}+12\,x^{2}+4\,x)\arctan\left(-\frac{2}{3}\,\sqrt{3}x+\frac{1}{3}\,\sqrt{3}\sqrt{4\,x^{2}-2\,x-3}\right)+1146\,x^{2}+81\,(9\,x^{3}+1}{}$$

input
```
integrate(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

output
```
-1/288*(936*x^3 - 90*sqrt(3)*(9*x^3 + 12*x^2 + 4*x)*arctan(-2/3*sqrt(3)*x
+ 1/3*sqrt(3)*sqrt(4*x^2 - 2*x - 3)) + 1146*x^2 + 81*(9*x^3 + 12*x^2 + 4*x
)*log(3*x + 2) - 81*(9*x^3 + 12*x^2 + 4*x)*log(x) - 81*(9*x^3 + 12*x^2 + 4
*x)*log(-2*x + sqrt(4*x^2 - 2*x - 3) - 1) + 81*(9*x^3 + 12*x^2 + 4*x)*log(
-6*x + 3*sqrt(4*x^2 - 2*x - 3) - 5) + 18*(26*x^2 + 17*x)*sqrt(4*x^2 - 2*x
- 3) + 130*x - 144)/(9*x^3 + 12*x^2 + 4*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{x^2\left(2x+\sqrt{4x^2-2x-3}+1\right)^3}\,dx$$

input
```
integrate(1/x**2/(1+2*x+(4*x**2-2*x-3)**(1/2))**3,x)
```

output
```
Integral(1/(x**2*(2*x + sqrt(4*x**2 - 2*x - 3) + 1)**3), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{\left(2\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3x^2}\,dx$$

input
```
integrate(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(4*x^2 - 2*x - 3) + 1)^3*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$= \frac{5}{16}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-\sqrt{4x^2-2x-3}\right)\right)$$
$$-\frac{435\left(2x-\sqrt{4x^2-2x-3}\right)^3+1681\left(2x-\sqrt{4x^2-2x-3}\right)^2+4312x-2156\sqrt{4x^2-2x-3}+919}{36\left(3\left(2x-\sqrt{4x^2-2x-3}\right)^2+16x-8\sqrt{4x^2-2x-3}+5\right)^2}$$
$$+\frac{51x^2+143x+72}{144(3x+2)^2x}-\frac{9}{32}\log\left(|3x+2|\right)+\frac{9}{32}\log\left(|x|\right)$$
$$+\frac{9}{32}\log\left(\left|-2x+\sqrt{4x^2-2x-3}-1\right|\right)-\frac{9}{32}\log\left(\left|-6x+3\sqrt{4x^2-2x-3}-5\right|\right)$$

input
```
integrate(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

output
```
5/16*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - sqrt(4*x^2 - 2*x - 3))) - 1/36*(43
5*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 1681*(2*x - sqrt(4*x^2 - 2*x - 3))^2 +
 4312*x - 2156*sqrt(4*x^2 - 2*x - 3) + 919)/(3*(2*x - sqrt(4*x^2 - 2*x - 3
))^2 + 16*x - 8*sqrt(4*x^2 - 2*x - 3) + 5)^2 + 1/144*(51*x^2 + 143*x + 72)
/((3*x + 2)^2*x) - 9/32*log(abs(3*x + 2)) + 9/32*log(abs(x)) + 9/32*log(ab
s(-2*x + sqrt(4*x^2 - 2*x - 3) - 1)) - 9/32*log(abs(-6*x + 3*sqrt(4*x^2 -
2*x - 3) - 5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{x^2\left(2x+\sqrt{4x^2-2x-3}+1\right)^3}\,dx$$

input
```
int(1/(x^2*(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3),x)
```

output
```
int(1/(x^2*(2*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.98

$$\int \frac{1}{x^2 \left(1+2x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$

$$= \frac{33078060\sqrt{3}\,atan\left(\frac{\sqrt{4x^2-2x-3}+2x}{\sqrt{3}}\right)x^3 + 44104080\sqrt{3}\,atan\left(\frac{\sqrt{4x^2-2x-3}+2x}{\sqrt{3}}\right)x^2 + 14701360\sqrt{3}\,atan\left(\frac{\sqrt{4x^2-2}}{\sqrt{}}\right.}{}$$

input
```
int(1/x^2/(1+2*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

output
```
(33078060*sqrt(3)*atan((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x**3 + 4410
4080*sqrt(3)*atan((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x**2 + 14701360*
sqrt(3)*atan((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x - 19111768*sqrt(4*x
**2 - 2*x - 3)*x**2 - 12496156*sqrt(4*x**2 - 2*x - 3)*x - 29770254*log(3*x
 + 2)*x**3 - 39693672*log(3*x + 2)*x**2 - 13231224*log(3*x + 2)*x - 297702
54*log((26*sqrt(4*x**2 - 2*x - 3) + 52*x + 26)/sqrt(13))*x**3 - 39693672*l
og((26*sqrt(4*x**2 - 2*x - 3) + 52*x + 26)/sqrt(13))*x**2 - 13231224*log((
26*sqrt(4*x**2 - 2*x - 3) + 52*x + 26)/sqrt(13))*x + 29770254*log((6*sqrt(
4*x**2 - 2*x - 3) + 12*x + 10)/sqrt(13))*x**3 + 39693672*log((6*sqrt(4*x**
2 - 2*x - 3) + 12*x + 10)/sqrt(13))*x**2 + 13231224*log((6*sqrt(4*x**2 - 2
*x - 3) + 12*x + 10)/sqrt(13))*x + 29770254*log(x)*x**3 + 39693672*log(x)*
x**2 + 13231224*log(x)*x + 27414617*x**3 + 40718208*x**2 + 23863688*x + 58
80544)/(11761088*x*(9*x**2 + 12*x + 4))
```

3.17 $\int \frac{x^2}{1+2x+\sqrt{2+3x+5x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 381

$$\begin{aligned}\int \frac{x^2}{1+2x+\sqrt{2+3x+5x^2}}\,dx = &-\frac{x^4\left(2\left(49+30\sqrt{2}\right)+\frac{\left(49+24\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}\right)}{2\left(4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}\right)^2}\\
&-\frac{x^2\left(20\left(17+16\sqrt{2}\right)+\frac{\left(109+120\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}\right)}{20\left(4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}\right)}\\
&-\frac{10\arctan\left(\frac{3-4\sqrt{2}-\frac{2\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3}}\right)}{\sqrt{3}}\\
&-\frac{91\text{arctanh}\left(\frac{\sqrt{2}-\sqrt{2+3x+5x^2}}{\sqrt{5}x}\right)}{20\sqrt{5}}\\
&+\log\left(\frac{4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}}{x^2}\right)\\
&-\log\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}\right.\\
&\left.\quad+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)\end{aligned}$$

output
```
-1/2*x^4*(98+60*2^(1/2)+(49+24*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)/(
4+3*x-2*2^(1/2)*(5*x^2+3*x+2)^(1/2))^2-x^2*(340+320*2^(1/2)+(109+120*2^(1/
2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)/(80+60*x-40*2^(1/2)*(5*x^2+3*x+2)^(1/
2))-10/3*arctan(1/3*(3-4*2^(1/2)-2*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2
))/x)*3^(1/2))*3^(1/2)-91/100*arctanh(1/5*(2^(1/2)-(5*x^2+3*x+2)^(1/2))*5^
(1/2)/x)*5^(1/2)+ln((4+3*x-2*2^(1/2)*(5*x^2+3*x+2)^(1/2))/x^2)-ln(1-5*2^(1
/2)-(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+(1-2^(1/2))*(2^(1/2)-(5*
x^2+3*x+2)^(1/2))^2/x^2)
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{1+2x+\sqrt{2+3x+5x^2}}\,dx = -x(3+x)+\frac{1}{20}(23+10x)\sqrt{2+3x+5x^2} - \frac{10\arctan\left(\sqrt{3+\frac{4\sqrt{5}}{3}}\left(16-7\sqrt{5}+\left(40-18\sqrt{5}\right)x+18\sqrt{2+3x+5x^2}-8\sqrt{5}\sqrt{2+3x+5x^2}\right)\right)}{\sqrt{3}} - \log\left(-55+26\sqrt{5}-140x+62\sqrt{5}x-200x^2+90\sqrt{5}x^2+8\sqrt{5}(2+5x)\sqrt{2+3x+5x^2} - 5(7+18x)\sqrt{2+3x+5x^2}\right)+\frac{1}{200}\left(200-91\sqrt{5}\right)\log\left(-6-20x - 10\sqrt{2+3x+5x^2}+\sqrt{5}\left(3+10x+4\sqrt{2+3x+5x^2}\right)\right)$$

input
```
Integrate[x^2/(1 + 2*x + Sqrt[2 + 3*x + 5*x^2]),x]
```

output
```
-(x*(3 + x)) + ((23 + 10*x)*Sqrt[2 + 3*x + 5*x^2])/20 - (10*ArcTan[Sqrt[3
+ (4*Sqrt[5])/3]*(16 - 7*Sqrt[5] + (40 - 18*Sqrt[5])*x + 18*Sqrt[2 + 3*x +
 5*x^2] - 8*Sqrt[5]*Sqrt[2 + 3*x + 5*x^2])])/Sqrt[3] - Log[-55 + 26*Sqrt[5
] - 140*x + 62*Sqrt[5]*x - 200*x^2 + 90*Sqrt[5]*x^2 + 8*Sqrt[5]*(2 + 5*x)*
Sqrt[2 + 3*x + 5*x^2] - 5*(7 + 18*x)*Sqrt[2 + 3*x + 5*x^2]] + ((200 - 91*S
qrt[5])*Log[-6 - 20*x - 10*Sqrt[2 + 3*x + 5*x^2] + Sqrt[5]*(3 + 10*x + 4*S
qrt[2 + 3*x + 5*x^2])])/200
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{5x^2+3x+2}+2x+1}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{3-x}{x^2-x+1}+\frac{x\sqrt{5x^2+3x+2}}{x^2-x+1}-\frac{\sqrt{5x^2+3x+2}}{x^2-x+1}+\sqrt{5x^2+3x+2}-2x-3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\sqrt{5}\text{arcsinh}\left(\frac{10x+3}{\sqrt{31}}\right)+\frac{291\text{arcsinh}\left(\frac{10x+3}{\sqrt{31}}\right)}{40\sqrt{5}}+\frac{5\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{\sqrt{3}}-\frac{5\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\text{arctanh}\left(\frac{2x+1}{\sqrt{5x^2+3x+2}}\right)-x^2+\frac{1}{20}(10x+3)\sqrt{5x^2+3x+2}+\sqrt{5x^2+3x+2}-\frac{1}{2}\log\left(x^2-x+1\right)-3x$$

input
```
Int[x^2/(1 + 2*x + Sqrt[2 + 3*x + 5*x^2]),x]
```

output
```
-3*x - x^2 + Sqrt[2 + 3*x + 5*x^2] + ((3 + 10*x)*Sqrt[2 + 3*x + 5*x^2])/20
 + (291*ArcSinh[(3 + 10*x)/Sqrt[31]])/(40*Sqrt[5]) - Sqrt[5]*ArcSinh[(3 +
10*x)/Sqrt[31]] - (5*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + (5*ArcTan[(5 - 4
*x)/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])])/Sqrt[3] - ArcTanh[(1 + 2*x)/Sqrt[2 +
 3*x + 5*x^2]] - Log[1 - x + x^2]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.44

method	result
default	$\frac{(10x+3)\sqrt{5x^2+3x+2}}{20}+\frac{91\sqrt{5}\,\operatorname{arcsinh}\left(\frac{10\sqrt{31}\left(x+\frac{3}{10}\right)}{31}\right)}{200}+\sqrt{5x^2+3x+2}+\frac{5\sqrt{7}\sqrt{16}\sqrt{\frac{28\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}+217}\left(5\sqrt{3}\,\text{arctan}\ldots\right.}{58\ldots}$
trager	Expression too large to display

input
```
int(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/20*(10*x+3)*(5*x^2+3*x+2)^(1/2)+91/200*5^(1/2)*arcsinh(10/31*31^(1/2)*(x
+3/10))+(5*x^2+3*x+2)^(1/2)+5/588*7^(1/2)*16^(1/2)*(28*(-5/4+x)^2/(-1/2-x)
^2+217)^(1/2)*(5*3^(1/2)*arctan(4/3*3^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)
^(1/2)/(4*(-5/4+x)^2/(-1/2-x)^2+31)*(-5/4+x)/(-1/2-x))+3*arctanh(1/14*(28*
(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)))/((4*(-5/4+x)^2/(-1/2-x)^2+31)/((-5/4+x)
/(-1/2-x)+1)^2)^(1/2)/((-5/4+x)/(-1/2-x)+1)+1/98*7^(1/2)*16^(1/2)*(28*(-5/
4+x)^2/(-1/2-x)^2+217)^(1/2)*(2*3^(1/2)*arctan(4/3*3^(1/2)*(28*(-5/4+x)^2/
(-1/2-x)^2+217)^(1/2)/(4*(-5/4+x)^2/(-1/2-x)^2+31)*(-5/4+x)/(-1/2-x))-3*ar
ctanh(1/14*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)))/((4*(-5/4+x)^2/(-1/2-x)^
2+31)/((-5/4+x)/(-1/2-x)+1)^2)^(1/2)/((-5/4+x)/(-1/2-x)+1)-1/294*7^(1/2)*1
6^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)*(3^(1/2)*arctan(4/3*3^(1/2)*(
28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)/(4*(-5/4+x)^2/(-1/2-x)^2+31)*(-5/4+x)/
(-1/2-x))+9*arctanh(1/14*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)))/((4*(-5/4+
x)^2/(-1/2-x)^2+31)/((-5/4+x)/(-1/2-x)+1)^2)^(1/2)/((-5/4+x)/(-1/2-x)+1)-3
*x-1/2*ln(x^2-x+1)+5/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.72

$$\begin{aligned}\int \frac{x^2}{1+2x+\sqrt{2+3x+5x^2}}\,dx &= -x^2+\frac{1}{20}\sqrt{5x^2+3x+2}(10x+23)+\frac{5}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\\ &\quad+\frac{5}{6}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\sqrt{5x^2+3x+2}(4x-5)+31\sqrt{3}(x^2-2x)}{3(11x^2-12x-8)}\right)\\ &\quad+\frac{5}{6}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\sqrt{5x^2+3x+2}(4x-5)-31\sqrt{3}(x^2-2x)}{3(11x^2-12x-8)}\right)\\ &\quad+\frac{91}{400}\sqrt{5}\log\left(-4\sqrt{5}\sqrt{5x^2+3x+2}(10x+3)-200x^2-120x-49\right)-3x\\ &\quad-\frac{1}{2}\log\left(x^2-x+1\right)-\frac{1}{4}\log\left(\frac{9x^2+2\sqrt{5x^2+3x+2}(2x+1)+7x+3}{x^2}\right)\\ &\quad+\frac{1}{4}\log\left(\frac{9x^2-2\sqrt{5x^2+3x+2}(2x+1)+7x+3}{x^2}\right)\end{aligned}$$

input
```
integrate(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="fricas")
```

output
```
-x^2 + 1/20*sqrt(5*x^2 + 3*x + 2)*(10*x + 23) + 5/3*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x - 1)) + 5/6*sqrt(3)*arctan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*
(4*x - 5) + 31*sqrt(3)*(x^2 - 2*x))/(11*x^2 - 12*x - 8)) + 5/6*sqrt(3)*arc
tan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) - 31*sqrt(3)*(x^2 - 2*x
))/(11*x^2 - 12*x - 8)) + 91/400*sqrt(5)*log(-4*sqrt(5)*sqrt(5*x^2 + 3*x +
 2)*(10*x + 3) - 200*x^2 - 120*x - 49) - 3*x - 1/2*log(x^2 - x + 1) - 1/4*
log((9*x^2 + 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^2) + 1/4*log((
9*x^2 - 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^2)
```

Sympy [F]

$$\int \frac{x^2}{1+2x+\sqrt{2+3x+5x^2}}\,dx = \int \frac{x^2}{2x+\sqrt{5x^2+3x+2}+1}\,dx$$

input
```
integrate(x**2/(1+2*x+(5*x**2+3*x+2)**(1/2)),x)
```

output
```
Integral(x**2/(2*x + sqrt(5*x**2 + 3*x + 2) + 1), x)
```

Maxima [F]

$$\int \frac{x^2}{1+2x+\sqrt{2+3x+5x^2}}\,dx = \int \frac{x^2}{2\,x+\sqrt{5\,x^2+3\,x+2}+1}\,dx$$

input
```
integrate(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(2*x + sqrt(5*x^2 + 3*x + 2) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{1+2x+\sqrt{2+3x+5x^2}}\,dx = -x^2 + \frac{1}{20}\sqrt{5\,x^2+3\,x+2}(10\,x+23) + \frac{5}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{91}{200}\sqrt{5}\log\left(-10\sqrt{5}x-3\sqrt{5}+10\sqrt{5\,x^2+3\,x+2}\right) - 3\,x - \frac{5\left(\sqrt{5}+2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5\,x^2+3\,x+2}-4}{\sqrt{15}+2\sqrt{3}}\right)}{\sqrt{15}+2\sqrt{3}} + \frac{5\left(\sqrt{5}-2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5\,x^2+3\,x+2}+4}{\sqrt{15}-2\sqrt{3}}\right)}{\sqrt{15}-2\sqrt{3}} - \frac{1}{2}\log\left(\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^2 - \left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)\left(\sqrt{5}+4\right)+5\sqrt{5}+12\right) + \frac{1}{2}\log\left(\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^2 - \left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)\left(\sqrt{5}-4\right)-5\sqrt{5}+12\right) - \frac{1}{2}\log\left(x^2-x+1\right)$$

input
```
integrate(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="giac")
```

output
```
-x^2 + 1/20*sqrt(5*x^2 + 3*x + 2)*(10*x + 23) + 5/3*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x - 1)) - 91/200*sqrt(5)*log(-10*sqrt(5)*x - 3*sqrt(5) + 10*sqrt(5
*x^2 + 3*x + 2)) - 3*x - 5*(sqrt(5) + 2)*arctan(-(2*sqrt(5)*x - sqrt(5) -
2*sqrt(5*x^2 + 3*x + 2) - 4)/(sqrt(15) + 2*sqrt(3)))/(sqrt(15) + 2*sqrt(3)
) + 5*(sqrt(5) - 2)*arctan(-(2*sqrt(5)*x - sqrt(5) - 2*sqrt(5*x^2 + 3*x +
2) + 4)/(sqrt(15) - 2*sqrt(3)))/(sqrt(15) - 2*sqrt(3)) - 1/2*log((sqrt(5)*
x - sqrt(5*x^2 + 3*x + 2))^2 - (sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))*(sqrt(5
) + 4) + 5*sqrt(5) + 12) + 1/2*log((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 -
 (sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))*(sqrt(5) - 4) - 5*sqrt(5) + 12) - 1/2
*log(x^2 - x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{1+2x+\sqrt{2+3x+5x^2}}\,dx = \int \frac{x^2}{2\,x+\sqrt{5\,x^2+3\,x+2}+1}\,dx$$

input
```
int(x^2/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1),x)
```

output
```
int(x^2/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{x^2}{1+2x+\sqrt{2+3x+5x^2}}\,dx &= \frac{5\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3}+\frac{\sqrt{5x^2+3x+2}\,x}{2} \\ &\quad -\frac{239\sqrt{5x^2+3x+2}}{140} \\ &\quad +\frac{91\sqrt{5}\log\left(-2\sqrt{5x^2+3x+2}\,\sqrt{5}-10x-3\right)}{200} \\ &\quad -\frac{5\left(\int \frac{\sqrt{5x^2+3x+2}}{5x^4-2x^3+4x^2+x+2}dx\right)}{7} \\ &\quad +\frac{100\left(\int \frac{\sqrt{5x^2+3x+2}\,x^3}{5x^4-2x^3+4x^2+x+2}dx\right)}{7} \\ &\quad -10\left(\int \frac{\sqrt{5x^2+3x+2}\,x^2}{5x^4-2x^3+4x^2+x+2}dx\right) \\ &\quad +7\left(\int \frac{\sqrt{5x^2+3x+2}\,x}{5x^4-2x^3+4x^2+x+2}dx\right) \\ &\quad -\frac{\log(x^2-x+1)}{2}-x^2-3x\end{aligned}$$

input
```
int(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2)),x)
```

output
```
(7000*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 2100*sqrt(5*x**2 + 3*x + 2)*x - 71
70*sqrt(5*x**2 + 3*x + 2) + 1911*sqrt(5)*log( - 2*sqrt(5*x**2 + 3*x + 2)*s
qrt(5) - 10*x - 3) - 3000*int(sqrt(5*x**2 + 3*x + 2)/(5*x**4 - 2*x**3 + 4*
x**2 + x + 2),x) + 60000*int((sqrt(5*x**2 + 3*x + 2)*x**3)/(5*x**4 - 2*x**
3 + 4*x**2 + x + 2),x) - 42000*int((sqrt(5*x**2 + 3*x + 2)*x**2)/(5*x**4 -
 2*x**3 + 4*x**2 + x + 2),x) + 29400*int((sqrt(5*x**2 + 3*x + 2)*x)/(5*x**
4 - 2*x**3 + 4*x**2 + x + 2),x) - 2100*log(x**2 - x + 1) - 4200*x**2 - 126
00*x)/4200
```

3.18 $\int \frac{x}{1+2x+\sqrt{2+3x+5x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 297

$$\int \frac{x}{1+2x+\sqrt{2+3x+5x^2}}\,dx = -\frac{x^2\left(2\left(3+5\sqrt{2}\right)+\frac{\left(3+4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}\right)}{4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}} - \frac{2\arctan\left(\frac{3-4\sqrt{2}-\frac{2\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{13\operatorname{arctanh}\left(\frac{\sqrt{2}-\sqrt{2+3x+5x^2}}{\sqrt{5}x}\right)}{\sqrt{5}} + 3\log\left(\frac{4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}}{x^2}\right) - 3\log\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x} + \frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)$$

output
```
-x^2*(6+10*2^(1/2)+(3+4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)/(4+3*x-2
*2^(1/2)*(5*x^2+3*x+2)^(1/2))-2/3*arctan(1/3*(3-4*2^(1/2)-2*(1-2^(1/2))*(2
^(1/2)-(5*x^2+3*x+2)^(1/2))/x)*3^(1/2))*3^(1/2)-13/5*arctanh(1/5*(2^(1/2)-
(5*x^2+3*x+2)^(1/2))*5^(1/2)/x)*5^(1/2)+3*ln((4+3*x-2*2^(1/2)*(5*x^2+3*x+2
)^(1/2))/x^2)-3*ln(1-5*2^(1/2)-(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))
/x+(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.71

$$\int \frac{x}{1+2x+\sqrt{2+3x+5x^2}}\,dx = -2x+\sqrt{2+3x+5x^2} - \frac{2\arctan\left(\frac{4+\sqrt{5}-2\sqrt{5}x+2\sqrt{2+3x+5x^2}}{\sqrt{3\left(9+4\sqrt{5}\right)}}\right)}{\sqrt{3}} - 3\log\left(25+14\sqrt{5}-20x-2\sqrt{5}x+10\sqrt{5}x^2 + 4\sqrt{5}\sqrt{2+3x+5x^2}+(5-10x)\sqrt{2+3x+5x^2}\right) + \frac{1}{10}\left(30-13\sqrt{5}\right)\log\left(-15-50x+20\sqrt{2+3x+5x^2} + 2\sqrt{5}\left(-3-10x+5\sqrt{2+3x+5x^2}\right)\right)$$

input
```
Integrate[x/(1 + 2*x + Sqrt[2 + 3*x + 5*x^2]),x]
```

output
```
-2*x + Sqrt[2 + 3*x + 5*x^2] - (2*ArcTan[(4 + Sqrt[5] - 2*Sqrt[5]*x + 2*Sq
rt[2 + 3*x + 5*x^2])/Sqrt[3*(9 + 4*Sqrt[5])]])/Sqrt[3] - 3*Log[25 + 14*Sqr
t[5] - 20*x - 2*Sqrt[5]*x + 10*Sqrt[5]*x^2 + 4*Sqrt[5]*Sqrt[2 + 3*x + 5*x^
2] + (5 - 10*x)*Sqrt[2 + 3*x + 5*x^2]] + ((30 - 13*Sqrt[5])*Log[-15 - 50*x
 + 20*Sqrt[2 + 3*x + 5*x^2] + 2*Sqrt[5]*(-3 - 10*x + 5*Sqrt[2 + 3*x + 5*x^
2])])/10
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.42, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{5x^2+3x+2}+2x+1}dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{2-3x}{x^2-x+1}+\frac{x\sqrt{5x^2+3x+2}}{x^2-x+1}-2\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{13\text{arcsinh}\left(\frac{10x+3}{\sqrt{31}}\right)}{2\sqrt{5}}+\frac{\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-3\text{arctanh}\left(\frac{2x+1}{\sqrt{5x^2+3x+2}}\right)+\sqrt{5x^2+3x+2}-\frac{3}{2}\log\left(x^2-x+1\right)-2x$$

input
```
Int[x/(1 + 2*x + Sqrt[2 + 3*x + 5*x^2]),x]
```

output
```
-2*x + Sqrt[2 + 3*x + 5*x^2] + (13*ArcSinh[(3 + 10*x)/Sqrt[31]])/(2*Sqrt[5
]) - ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(5 - 4*x)/(Sqrt[3]*Sqrt[2
+ 3*x + 5*x^2])]/Sqrt[3] - 3*ArcTanh[(1 + 2*x)/Sqrt[2 + 3*x + 5*x^2]] - (3
*Log[1 - x + x^2])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(246) = 492$.

Time = 0.59 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.77

method	result
default	$\frac{13\sqrt{5}\,\operatorname{arcsinh}\left(\frac{10\sqrt{31}\left(x+\frac{3}{10}\right)}{31}\right)}{10}+\sqrt{5x^2+3x+2}+\frac{5\sqrt{7}\sqrt{16}\sqrt{\frac{28\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}+217}\,\Bigg(2\sqrt{3}\arctan\Bigg(\frac{4\sqrt{3}\sqrt{\frac{28\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}+217}}{3\left(\frac{4\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}+31\right)(\cdots}\cdots}{294\sqrt{\frac{\frac{4\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}+31}{\left(\frac{-\frac{5}{4}+x}{-\frac{1}{2}-x}+1\right)^2}}\,\Big(\frac{-\frac{5}{4}\cdots}{-\frac{1}{2}\cdots}\cdots}$
trager	Expression too large to display

input
```
int(x/(1+2*x+(5*x^2+3*x+2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
13/10*5^(1/2)*arcsinh(10/31*31^(1/2)*(x+3/10))+(5*x^2+3*x+2)^(1/2)+5/294*7
^(1/2)*16^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)*(2*3^(1/2)*arctan(4/3
*3^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)/(4*(-5/4+x)^2/(-1/2-x)^2+31)
*(-5/4+x)/(-1/2-x))-3*arctanh(1/14*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)))/
((4*(-5/4+x)^2/(-1/2-x)^2+31)/((-5/4+x)/(-1/2-x)+1)^2)^(1/2)/((-5/4+x)/(-1
/2-x)+1)-1/196*7^(1/2)*16^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)*(3^(1
/2)*arctan(4/3*3^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)/(4*(-5/4+x)^2/
(-1/2-x)^2+31)*(-5/4+x)/(-1/2-x))+9*arctanh(1/14*(28*(-5/4+x)^2/(-1/2-x)^2
+217)^(1/2)))/((4*(-5/4+x)^2/(-1/2-x)^2+31)/((-5/4+x)/(-1/2-x)+1)^2)^(1/2)
/((-5/4+x)/(-1/2-x)+1)-1/294*7^(1/2)*16^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+21
7)^(1/2)*(5*3^(1/2)*arctan(4/3*3^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2
)/(4*(-5/4+x)^2/(-1/2-x)^2+31)*(-5/4+x)/(-1/2-x))+3*arctanh(1/14*(28*(-5/4
+x)^2/(-1/2-x)^2+217)^(1/2)))/((4*(-5/4+x)^2/(-1/2-x)^2+31)/((-5/4+x)/(-1/
2-x)+1)^2)^(1/2)/((-5/4+x)/(-1/2-x)+1)-2*x-3/2*ln(x^2-x+1)+1/3*3^(1/2)*arc
tan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.89

$$
\begin{aligned}
&\int \frac{x}{1+2x+\sqrt{2+3x+5x^2}}\,dx \\
&= \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) \\
&\quad + \frac{1}{6}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\sqrt{5x^2+3x+2}(4x-5)+31\sqrt{3}(x^2-2x)}{3(11x^2-12x-8)}\right) \\
&\quad + \frac{1}{6}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\sqrt{5x^2+3x+2}(4x-5)-31\sqrt{3}(x^2-2x)}{3(11x^2-12x-8)}\right) \\
&\quad + \frac{13}{20}\sqrt{5}\log\left(-4\sqrt{5}\sqrt{5x^2+3x+2}(10x+3)-200x^2-120x-49\right) \\
&\quad - 2x+\sqrt{5x^2+3x+2}-\frac{3}{2}\log\left(x^2-x+1\right) \\
&\quad - \frac{3}{4}\log\left(\frac{9x^2+2\sqrt{5x^2+3x+2}(2x+1)+7x+3}{x^2}\right) \\
&\quad + \frac{3}{4}\log\left(\frac{9x^2-2\sqrt{5x^2+3x+2}(2x+1)+7x+3}{x^2}\right)
\end{aligned}
$$

input
```
integrate(x/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*sqrt(3)*arctan(1/3*(4*sqrt
(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) + 31*sqrt(3)*(x^2 - 2*x))/(11*x^2 - 12
*x - 8)) + 1/6*sqrt(3)*arctan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x -
5) - 31*sqrt(3)*(x^2 - 2*x))/(11*x^2 - 12*x - 8)) + 13/20*sqrt(5)*log(-4*s
qrt(5)*sqrt(5*x^2 + 3*x + 2)*(10*x + 3) - 200*x^2 - 120*x - 49) - 2*x + sq
rt(5*x^2 + 3*x + 2) - 3/2*log(x^2 - x + 1) - 3/4*log((9*x^2 + 2*sqrt(5*x^2
 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^2) + 3/4*log((9*x^2 - 2*sqrt(5*x^2 + 3*
x + 2)*(2*x + 1) + 7*x + 3)/x^2)
```

Sympy [F]

$$\int \frac{x}{1+2x+\sqrt{2+3x+5x^2}}\,dx = \int \frac{x}{2x+\sqrt{5x^2+3x+2}+1}\,dx$$

input
```
integrate(x/(1+2*x+(5*x**2+3*x+2)**(1/2)),x)
```

output
```
Integral(x/(2*x + sqrt(5*x**2 + 3*x + 2) + 1), x)
```

Maxima [F]

$$\int \frac{x}{1+2x+\sqrt{2+3x+5x^2}}\,dx = \int \frac{x}{2\,x+\sqrt{5\,x^2+3\,x+2}+1}\,dx$$

input
```
integrate(x/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(2*x + sqrt(5*x^2 + 3*x + 2) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.04

$$\int \frac{x}{1+2x+\sqrt{2+3x+5x^2}}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{13}{10}\,\sqrt{5}\log\left(-10\,\sqrt{5}x-3\,\sqrt{5}+10\,\sqrt{5\,x^2+3\,x+2}\right) - 2\,x - \frac{\left(\sqrt{5}+2\right)\arctan\left(-\frac{2\,\sqrt{5}x-\sqrt{5}-2\,\sqrt{5\,x^2+3\,x+2}-4}{\sqrt{15}+2\,\sqrt{3}}\right)}{\sqrt{15}+2\,\sqrt{3}} + \frac{\left(\sqrt{5}-2\right)\arctan\left(-\frac{2\,\sqrt{5}x-\sqrt{5}-2\,\sqrt{5\,x^2+3\,x+2}+4}{\sqrt{15}-2\,\sqrt{3}}\right)}{\sqrt{15}-2\,\sqrt{3}} + \sqrt{5\,x^2+3\,x+2} - \frac{3}{2}\log\left(\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^2 - \left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)\left(\sqrt{5}+4\right)+5\,\sqrt{5}+12\right) + \frac{3}{2}\log\left(\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^2 - \left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)\left(\sqrt{5}-4\right)-5\,\sqrt{5}+12\right) - \frac{3}{2}\log\left(x^2-x+1\right)$$

input
```
integrate(x/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 13/10*sqrt(5)*log(-10*sqrt(5)*
x - 3*sqrt(5) + 10*sqrt(5*x^2 + 3*x + 2)) - 2*x - (sqrt(5) + 2)*arctan(-(2
*sqrt(5)*x - sqrt(5) - 2*sqrt(5*x^2 + 3*x + 2) - 4)/(sqrt(15) + 2*sqrt(3))
)/(sqrt(15) + 2*sqrt(3)) + (sqrt(5) - 2)*arctan(-(2*sqrt(5)*x - sqrt(5) -
2*sqrt(5*x^2 + 3*x + 2) + 4)/(sqrt(15) - 2*sqrt(3)))/(sqrt(15) - 2*sqrt(3)
) + sqrt(5*x^2 + 3*x + 2) - 3/2*log((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2
- (sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))*(sqrt(5) + 4) + 5*sqrt(5) + 12) + 3/
2*log((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - (sqrt(5)*x - sqrt(5*x^2 + 3*
x + 2))*(sqrt(5) - 4) - 5*sqrt(5) + 12) - 3/2*log(x^2 - x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{1+2x+\sqrt{2+3x+5x^2}}\,dx = \int \frac{x\sqrt{5\,x^2+3\,x+2}}{x^2-x+1}\,dx - 2\,x + \frac{\sqrt{3}\,\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\,1\mathrm{i}}{3} + \frac{\sqrt{3}\,\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\,1\mathrm{i}}{3}$$

input
```
int(x/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1),x)
```

output
```
int((x*(3*x + 5*x^2 + 2)^(1/2))/(x^2 - x + 1), x) - 2*x + (3^(1/2)*log(x -
 (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*3i)/2 - 1/2)*1i)/3 + (3^(1/2)*log(x + (3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*3i)/2 + 1/2)*1i)/3
```

Reduce [F]

$$\int \frac{x}{1+2x+\sqrt{2+3x+5x^2}}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3} + \frac{121\sqrt{5x^2+3x+2}}{21} + \frac{13\sqrt{5}\log\left(-2\sqrt{5x^2+3x+2}\,\sqrt{5}-10x-3\right)}{10} - \frac{106\left(\int \frac{\sqrt{5x^2+3x+2}}{5x^4-2x^3+4x^2+x+2}dx\right)}{7} - \frac{500\left(\int \frac{\sqrt{5x^2+3x+2}\,x^3}{5x^4-2x^3+4x^2+x+2}dx\right)}{21} + \frac{50\left(\int \frac{\sqrt{5x^2+3x+2}\,x^2}{5x^4-2x^3+4x^2+x+2}dx\right)}{3} - \frac{35\left(\int \frac{\sqrt{5x^2+3x+2}\,x}{5x^4-2x^3+4x^2+x+2}dx\right)}{3} - \frac{3\log(x^2-x+1)}{2} - 2x$$

input
```
int(x/(1+2*x+(5*x^2+3*x+2)^(1/2)),x)
```

output
```
(70*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 1210*sqrt(5*x**2 + 3*x + 2) + 273*sq
rt(5)*log( - 2*sqrt(5*x**2 + 3*x + 2)*sqrt(5) - 10*x - 3) - 3180*int(sqrt(
5*x**2 + 3*x + 2)/(5*x**4 - 2*x**3 + 4*x**2 + x + 2),x) - 5000*int((sqrt(5
*x**2 + 3*x + 2)*x**3)/(5*x**4 - 2*x**3 + 4*x**2 + x + 2),x) + 3500*int((s
qrt(5*x**2 + 3*x + 2)*x**2)/(5*x**4 - 2*x**3 + 4*x**2 + x + 2),x) - 2450*i
nt((sqrt(5*x**2 + 3*x + 2)*x)/(5*x**4 - 2*x**3 + 4*x**2 + x + 2),x) - 315*
log(x**2 - x + 1) - 420*x)/210
```

3.19 $\int \frac{1}{1+2x+\sqrt{2+3x+5x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 217

$$\begin{aligned}\int \frac{1}{1+2x+\sqrt{2+3x+5x^2}}\,dx &= \frac{8\arctan\left(\frac{3-4\sqrt{2}-\frac{2\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3}}\right)}{\sqrt{3}}\\
&\quad -2\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{2}-\sqrt{2+3x+5x^2}}{\sqrt{5}x}\right)\\
&\quad +2\log\left(\frac{4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}}{x^2}\right)\\
&\quad -2\log\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}\right.\\
&\qquad \left.+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)\end{aligned}$$

output
```
8/3*arctan(1/3*(3-4*2^(1/2)-2*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)
*3^(1/2))*3^(1/2)-2*arctanh(1/5*(2^(1/2)-(5*x^2+3*x+2)^(1/2))*5^(1/2)/x)*5
^(1/2)+2*ln((4+3*x-2*2^(1/2)*(5*x^2+3*x+2)^(1/2))/x^2)-2*ln(1-5*2^(1/2)-(3
-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+(1-2^(1/2))*(2^(1/2)-(5*x^2+3*
x+2)^(1/2))^2/x^2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+2x+\sqrt{2+3x+5x^2}}\,dx = \frac{8\arctan\left(\frac{4+\sqrt{5}-2\sqrt{5}x+2\sqrt{2+3x+5x^2}}{\sqrt{3\left(9+4\sqrt{5}\right)}}\right)}{\sqrt{3}} - \left(-2+\sqrt{5}\right)\log\left(-3-10x+2\sqrt{5}\sqrt{2+3x+5x^2}\right) - 2\log\left(25+14\sqrt{5}-20x-2\sqrt{5}x+10\sqrt{5}x^2 + 4\sqrt{5}\sqrt{2+3x+5x^2}+(5-10x)\sqrt{2+3x+5x^2}\right)$$

input
```
Integrate[(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^(-1),x]
```

output
```
(8*ArcTan[(4 + Sqrt[5] - 2*Sqrt[5]*x + 2*Sqrt[2 + 3*x + 5*x^2])/Sqrt[3*(9
+ 4*Sqrt[5])]])/Sqrt[3] - (-2 + Sqrt[5])*Log[-3 - 10*x + 2*Sqrt[5]*Sqrt[2
+ 3*x + 5*x^2]] - 2*Log[25 + 14*Sqrt[5] - 20*x - 2*Sqrt[5]*x + 10*Sqrt[5]*
x^2 + 4*Sqrt[5]*Sqrt[2 + 3*x + 5*x^2] + (5 - 10*x)*Sqrt[2 + 3*x + 5*x^2]]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.48, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{5x^2+3x+2}+2x+1}\,dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{-2x-1}{x^2-x+1}+\frac{\sqrt{5x^2+3x+2}}{x^2-x+1}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\sqrt{5}\operatorname{arcsinh}\left(\frac{10x+3}{\sqrt{31}}\right)-\frac{4\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{\sqrt{3}}+\frac{4\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-2\operatorname{arctanh}\left(\frac{2x+1}{\sqrt{5x^2+3x+2}}\right)-\log\left(x^2-x+1\right)$$

input
```
Int[(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^(-1),x]
```

output
```
Sqrt[5]*ArcSinh[(3 + 10*x)/Sqrt[31]] + (4*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[
3] - (4*ArcTan[(5 - 4*x)/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])])/Sqrt[3] - 2*Arc
Tanh[(1 + 2*x)/Sqrt[2 + 3*x + 5*x^2]] - Log[1 - x + x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(181) = 362$.

Time = 0.51 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.35

method	result
default	$\sqrt{5}\,\operatorname{arcsinh}\left(\frac{10\sqrt{31}\,\left(x+\frac{3}{10}\right)}{31}\right)-\frac{5\sqrt{7}\,\sqrt{16}\,\sqrt{\frac{28\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}+217}\,\left(\sqrt{3}\,\arctan\left(\frac{4\sqrt{3}\,\sqrt{\frac{28\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}}+217\left(-\frac{5}{4}+x\right)}{3\left(\frac{4\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}+31\right)\left(-\frac{1}{2}-x\right)}\right)+9\operatorname{arctanh}}{588\sqrt{\frac{\frac{4\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}+31}{\left(\frac{-\frac{5}{4}+x}{-\frac{1}{2}-x}+1\right)^2}}\,\left(\frac{-\frac{5}{4}+x}{-\frac{1}{2}-x}+1\right)}$
trager	Expression too large to display

input
```
int(1/(1+2*x+(5*x^2+3*x+2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
5^(1/2)*arcsinh(10/31*31^(1/2)*(x+3/10))-5/588*7^(1/2)*16^(1/2)*(28*(-5/4+
x)^2/(-1/2-x)^2+217)^(1/2)*(3^(1/2)*arctan(4/3*3^(1/2)*(28*(-5/4+x)^2/(-1/
2-x)^2+217)^(1/2)/(4*(-5/4+x)^2/(-1/2-x)^2+31)*(-5/4+x)/(-1/2-x))+9*arctan
h(1/14*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)))/((4*(-5/4+x)^2/(-1/2-x)^2+31
)/((-5/4+x)/(-1/2-x)+1)^2)^(1/2)/((-5/4+x)/(-1/2-x)+1)-1/196*7^(1/2)*16^(1
/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)*(5*3^(1/2)*arctan(4/3*3^(1/2)*(28
*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)/(4*(-5/4+x)^2/(-1/2-x)^2+31)*(-5/4+x)/(-
1/2-x))+3*arctanh(1/14*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)))/((4*(-5/4+x)
^2/(-1/2-x)^2+31)/((-5/4+x)/(-1/2-x)+1)^2)^(1/2)/((-5/4+x)/(-1/2-x)+1)-1/1
47*7^(1/2)*16^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)*(2*3^(1/2)*arctan
(4/3*3^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)/(4*(-5/4+x)^2/(-1/2-x)^2
+31)*(-5/4+x)/(-1/2-x))-3*arctanh(1/14*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2
)))/((4*(-5/4+x)^2/(-1/2-x)^2+31)/((-5/4+x)/(-1/2-x)+1)^2)^(1/2)/((-5/4+x)
/(-1/2-x)+1)-4/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-ln(x^2-x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.14

$$\begin{aligned}
&\int \frac{1}{1+2x+\sqrt{2+3x+5x^2}}\,dx \\
&= -\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) \\
&\quad -\frac{2}{3}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\sqrt{5x^2+3x+2}(4x-5)+31\sqrt{3}(x^2-2x)}{3(11x^2-12x-8)}\right) \\
&\quad -\frac{2}{3}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\sqrt{5x^2+3x+2}(4x-5)-31\sqrt{3}(x^2-2x)}{3(11x^2-12x-8)}\right) \\
&\quad +\frac{1}{2}\sqrt{5}\log\left(-4\sqrt{5}\sqrt{5x^2+3x+2}(10x+3)-200x^2-120x-49\right) \\
&\quad -\log\left(x^2-x+1\right)-\frac{1}{2}\log\left(\frac{9x^2+2\sqrt{5x^2+3x+2}(2x+1)+7x+3}{x^2}\right) \\
&\quad +\frac{1}{2}\log\left(\frac{9x^2-2\sqrt{5x^2+3x+2}(2x+1)+7x+3}{x^2}\right)
\end{aligned}$$

input
```
integrate(1/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="fricas")
```

output
```
-4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*sqrt(3)*arctan(1/3*(4*sqr
t(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) + 31*sqrt(3)*(x^2 - 2*x))/(11*x^2 - 1
2*x - 8)) - 2/3*sqrt(3)*arctan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x -
 5) - 31*sqrt(3)*(x^2 - 2*x))/(11*x^2 - 12*x - 8)) + 1/2*sqrt(5)*log(-4*sq
rt(5)*sqrt(5*x^2 + 3*x + 2)*(10*x + 3) - 200*x^2 - 120*x - 49) - log(x^2 -
 x + 1) - 1/2*log((9*x^2 + 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^
2) + 1/2*log((9*x^2 - 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^2)
```

Sympy [F]

$$\int \frac{1}{1+2x+\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{2x+\sqrt{5x^2+3x+2}+1}\,dx$$

input
```
integrate(1/(1+2*x+(5*x**2+3*x+2)**(1/2)),x)
```

output
```
Integral(1/(2*x + sqrt(5*x**2 + 3*x + 2) + 1), x)
```

Maxima [F]

$$\int \frac{1}{1+2x+\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{2\,x+\sqrt{5\,x^2+3\,x+2}+1}\,dx$$

input
```
integrate(1/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(2*x + sqrt(5*x^2 + 3*x + 2) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.35

$$\int \frac{1}{1+2x+\sqrt{2+3x+5x^2}}\,dx = -\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \sqrt{5}\log\left(-10\sqrt{5}x - 3\sqrt{5} + 10\sqrt{5x^2+3x+2}\right) + \frac{4\left(\sqrt{5}+2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5x^2+3x+2}-4}{\sqrt{15}+2\sqrt{3}}\right)}{\sqrt{15}+2\sqrt{3}} - \frac{4\left(\sqrt{5}-2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5x^2+3x+2}+4}{\sqrt{15}-2\sqrt{3}}\right)}{\sqrt{15}-2\sqrt{3}} - \log\left(\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^2 - \left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)\left(\sqrt{5}+4\right) + 5\sqrt{5} + 12\right) + \log\left(\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^2 - \left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)\left(\sqrt{5}-4\right) - 5\sqrt{5} + 12\right) - \log\left(x^2-x+1\right)$$

input

```
integrate(1/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="giac")
```

output

```
-4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - sqrt(5)*log(-10*sqrt(5)*x - 3
*sqrt(5) + 10*sqrt(5*x^2 + 3*x + 2)) + 4*(sqrt(5) + 2)*arctan(-(2*sqrt(5)*
x - sqrt(5) - 2*sqrt(5*x^2 + 3*x + 2) - 4)/(sqrt(15) + 2*sqrt(3)))/(sqrt(1
5) + 2*sqrt(3)) - 4*(sqrt(5) - 2)*arctan(-(2*sqrt(5)*x - sqrt(5) - 2*sqrt(
5*x^2 + 3*x + 2) + 4)/(sqrt(15) - 2*sqrt(3)))/(sqrt(15) - 2*sqrt(3)) - log
((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - (sqrt(5)*x - sqrt(5*x^2 + 3*x + 2
))*(sqrt(5) + 4) + 5*sqrt(5) + 12) + log((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2
))^2 - (sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))*(sqrt(5) - 4) - 5*sqrt(5) + 12)
 - log(x^2 - x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+2x+\sqrt{2+3x+5x^2}}\,dx = \int \frac{\sqrt{5\,x^2+3\,x+2}}{x^2-x+1}\,dx + \frac{\sqrt{3}\,\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{3} + \frac{\sqrt{3}\,\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-2+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{3}$$

input

```
int(1/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1),x)
```

output

```
int((3*x + 5*x^2 + 2)^(1/2)/(x^2 - x + 1), x) + (3^(1/2)*log(x - (3^(1/2)*
1i)/2 - 1/2)*(3^(1/2)*1i + 2)*1i)/3 + (3^(1/2)*log(x + (3^(1/2)*1i)/2 - 1/
2)*(3^(1/2)*1i - 2)*1i)/3
```

Reduce [F]

$$\int \frac{1}{1+2x+\sqrt{2+3x+5x^2}}\,dx = -\frac{4\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3} + \sqrt{5}\log\left(-2\sqrt{5x^2+3x+2}\,\sqrt{5}-10x-3\right) - 3\left(\int \frac{\sqrt{5x^2+3x+2}}{5x^4-2x^3+4x^2+x+2}dx\right) + 8\left(\int \frac{\sqrt{5x^2+3x+2}\,x}{5x^4-2x^3+4x^2+x+2}dx\right) - \log\left(x^2-x+1\right)$$

input

```
int(1/(1+2*x+(5*x^2+3*x+2)^(1/2)),x)
```

output

```
( - 4*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 3*sqrt(5)*log( - 2*sqrt(5*x**2 + 3
*x + 2)*sqrt(5) - 10*x - 3) - 9*int(sqrt(5*x**2 + 3*x + 2)/(5*x**4 - 2*x**
3 + 4*x**2 + x + 2),x) + 24*int((sqrt(5*x**2 + 3*x + 2)*x)/(5*x**4 - 2*x**
3 + 4*x**2 + x + 2),x) - 3*log(x**2 - x + 1))/3
```

3.20 $\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 173

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx = -\frac{10\arctan\left(\frac{5+\sqrt{2}-\frac{2\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3\left(3+2\sqrt{2}\right)}}\right)}{\sqrt{3}} - \left(1-\sqrt{2}\right)\log\left(\frac{4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}}{x}\right) + \log\left(9+4\sqrt{2} - \frac{\left(5+\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x} + \frac{\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)$$

output
```
-10/3*arctan((5+2^(1/2)-2*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)/(6^(1/2)+3^(1/2
)))*3^(1/2)-(1-2^(1/2))*ln((4+3*x-2*2^(1/2)*(5*x^2+3*x+2)^(1/2))/x)+ln(9+4
*2^(1/2)-(5+2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+(2^(1/2)-(5*x^2+3*x+2
)^(1/2))^2/x^2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.45

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx = \frac{10\arctan\left(\frac{\sqrt{29-12\sqrt{5}}+2\left(-5+2\sqrt{5}\right)x+2\left(-2+\sqrt{5}\right)\sqrt{2+3x+5x^2}}{\sqrt{3}}\right)}{\sqrt{3}} + \left(-1+\sqrt{2}\right)\log\left(\sqrt{2}+\sqrt{5}x-\sqrt{2+3x+5x^2}\right)-\log\left(\sqrt{2}-\sqrt{5}x+\sqrt{2+3x+5x^2}\right) - \sqrt{2}\log\left(\sqrt{2}-\sqrt{5}x+\sqrt{2+3x+5x^2}\right)+\log\left(-3+4\sqrt{5}-16x+6\sqrt{5}x-20x^2 + 10\sqrt{5}x^2+\sqrt{5}(2+4x)\sqrt{2+3x+5x^2}-(3+10x)\sqrt{2+3x+5x^2}\right)$$

input
```
Integrate[1/(x*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])),x]
```

output
```
(10*ArcTan[(Sqrt[29 - 12*Sqrt[5]] + 2*(-5 + 2*Sqrt[5])*x + 2*(-2 + Sqrt[5]
)*Sqrt[2 + 3*x + 5*x^2])/Sqrt[3]])/Sqrt[3] + (-1 + Sqrt[2])*Log[Sqrt[2] +
Sqrt[5]*x - Sqrt[2 + 3*x + 5*x^2]] - Log[Sqrt[2] - Sqrt[5]*x + Sqrt[2 + 3*
x + 5*x^2]] - Sqrt[2]*Log[Sqrt[2] - Sqrt[5]*x + Sqrt[2 + 3*x + 5*x^2]] + L
og[-3 + 4*Sqrt[5] - 16*x + 6*Sqrt[5]*x - 20*x^2 + 10*Sqrt[5]*x^2 + Sqrt[5]
*(2 + 4*x)*Sqrt[2 + 3*x + 5*x^2] - (3 + 10*x)*Sqrt[2 + 3*x + 5*x^2]]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.73, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{5x^2+3x+2}+2x+1\right)}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{x-3}{x^2-x+1}+\frac{\sqrt{5x^2+3x+2}}{x}-\frac{x\sqrt{5x^2+3x+2}}{x^2-x+1}+\frac{\sqrt{5x^2+3x+2}}{x^2-x+1}-\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{\sqrt{3}}+\frac{5\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\text{arctanh}\left(\frac{2x+1}{\sqrt{5x^2+3x+2}}\right)-\sqrt{2}\text{arctanh}\left(\frac{3x+4}{2\sqrt{2}\sqrt{5x^2+3x+2}}\right)+\frac{1}{2}\log\left(x^2-x+1\right)-\log(x)$$

input
```
Int[1/(x*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])),x]
```

output
```
(5*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] - (5*ArcTan[(5 - 4*x)/(Sqrt[3]*Sqrt[
2 + 3*x + 5*x^2])])/Sqrt[3] + ArcTanh[(1 + 2*x)/Sqrt[2 + 3*x + 5*x^2]] - S
qrt[2]*ArcTanh[(4 + 3*x)/(2*Sqrt[2]*Sqrt[2 + 3*x + 5*x^2])] - Log[x] + Log
[1 - x + x^2]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(141) = 282$.

Time = 0.52 (sec) , antiderivative size = 528, normalized size of antiderivative = 3.05

method	result
default	$-\sqrt{2}\,\operatorname{arctanh}\left(\frac{(4+3x)\sqrt{2}}{4\sqrt{5x^2+3x+2}}\right)-\frac{5\sqrt{7}\,\sqrt{16}\,\sqrt{\frac{28\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}+217}\left(5\sqrt{3}\arctan\left(\frac{4\sqrt{3}\sqrt{\frac{28\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}+217}\left(-\frac{5}{4}+x\right)}{3\left(\frac{4\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}+31\right)\left(-\frac{1}{2}-x\right)}\right)+3\,\mathrm{arct}\right.}{588\sqrt{\frac{\frac{4\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}+31}{\left(\frac{-\frac{5}{4}+x}{-\frac{1}{2}-x}+1\right)^2}}\left(\frac{-\frac{5}{4}+x}{-\frac{1}{2}-x}+1\right)}$
trager	Expression too large to display

input
```
int(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2^(1/2)*arctanh(1/4*(4+3*x)*2^(1/2)/(5*x^2+3*x+2)^(1/2))-5/588*7^(1/2)*16
^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)*(5*3^(1/2)*arctan(4/3*3^(1/2)*
(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)/(4*(-5/4+x)^2/(-1/2-x)^2+31)*(-5/4+x)
/(-1/2-x))+3*arctanh(1/14*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)))/((4*(-5/4
+x)^2/(-1/2-x)^2+31)/((-5/4+x)/(-1/2-x)+1)^2)^(1/2)/((-5/4+x)/(-1/2-x)+1)-
1/98*7^(1/2)*16^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)*(2*3^(1/2)*arct
an(4/3*3^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)/(4*(-5/4+x)^2/(-1/2-x)
^2+31)*(-5/4+x)/(-1/2-x))-3*arctanh(1/14*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1
/2)))/((4*(-5/4+x)^2/(-1/2-x)^2+31)/((-5/4+x)/(-1/2-x)+1)^2)^(1/2)/((-5/4+
x)/(-1/2-x)+1)+1/294*7^(1/2)*16^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)
*(3^(1/2)*arctan(4/3*3^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)/(4*(-5/4
+x)^2/(-1/2-x)^2+31)*(-5/4+x)/(-1/2-x))+9*arctanh(1/14*(28*(-5/4+x)^2/(-1/
2-x)^2+217)^(1/2)))/((4*(-5/4+x)^2/(-1/2-x)^2+31)/((-5/4+x)/(-1/2-x)+1)^2)
^(1/2)/((-5/4+x)/(-1/2-x)+1)-5/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/2*l
n(x^2-x+1)-ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.48

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx$$
$$= -\frac{5}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{5}{6}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\sqrt{5x^2+3x+2}(4x-5)+31\sqrt{3}(x^2-2x)}{3(11x^2-12x-8)}\right) - \frac{5}{6}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\sqrt{5x^2+3x+2}(4x-5)-31\sqrt{3}(x^2-2x)}{3(11x^2-12x-8)}\right) + \frac{1}{2}\sqrt{2}\log\left(\frac{4\sqrt{2}\sqrt{5x^2+3x+2}(3x+4)-49x^2-48x-32}{x^2}\right) + \frac{1}{2}\log\left(x^2-x+1\right) - \log(x) + \frac{1}{4}\log\left(\frac{9x^2+2\sqrt{5x^2+3x+2}(2x+1)+7x+3}{x^2}\right) - \frac{1}{4}\log\left(\frac{9x^2-2\sqrt{5x^2+3x+2}(2x+1)+7x+3}{x^2}\right)$$

input
```
integrate(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="fricas")
```

output
```
-5/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 5/6*sqrt(3)*arctan(1/3*(4*sqr
t(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) + 31*sqrt(3)*(x^2 - 2*x))/(11*x^2 - 1
2*x - 8)) - 5/6*sqrt(3)*arctan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x -
 5) - 31*sqrt(3)*(x^2 - 2*x))/(11*x^2 - 12*x - 8)) + 1/2*sqrt(2)*log((4*sq
rt(2)*sqrt(5*x^2 + 3*x + 2)*(3*x + 4) - 49*x^2 - 48*x - 32)/x^2) + 1/2*log
(x^2 - x + 1) - log(x) + 1/4*log((9*x^2 + 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1
) + 7*x + 3)/x^2) - 1/4*log((9*x^2 - 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7
*x + 3)/x^2)
```

Sympy [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx=\int \frac{1}{x\left(2x+\sqrt{5x^2+3x+2}+1\right)}\,dx$$

input
```
integrate(1/x/(1+2*x+(5*x**2+3*x+2)**(1/2)),x)
```

output
```
Integral(1/(x*(2*x + sqrt(5*x**2 + 3*x + 2) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx=\int \frac{1}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)x}\,dx$$

input
```
integrate(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(5*x^2 + 3*x + 2) + 1)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(139) = 278$.

Time = 0.18 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.91

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx$$
$$= -\frac{5}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \sqrt{2}\log\left(-\frac{\left|-2\sqrt{5}x-2\sqrt{2}+2\sqrt{5x^2+3x+2}\right|}{2\left(\sqrt{5}x-\sqrt{2}-\sqrt{5x^2+3x+2}\right)}\right) + \frac{5\left(\sqrt{5}+2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5x^2+3x+2}-4}{\sqrt{15}+2\sqrt{3}}\right)}{\sqrt{15}+2\sqrt{3}} - \frac{5\left(\sqrt{5}-2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5x^2+3x+2}+4}{\sqrt{15}-2\sqrt{3}}\right)}{\sqrt{15}-2\sqrt{3}} + \frac{1}{2}\log\left(\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^2-\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)\left(\sqrt{5}+4\right)+5\sqrt{5}+12\right) - \frac{1}{2}\log\left(\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^2-\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)\left(\sqrt{5}-4\right)-5\sqrt{5}+12\right) + \frac{1}{2}\log\left(x^2-x+1\right) - \log\left(|x|\right)$$

input
```
integrate(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="giac")
```

output
```
-5/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + sqrt(2)*log(-1/2*abs(-2*sqrt(
5)*x - 2*sqrt(2) + 2*sqrt(5*x^2 + 3*x + 2))/(sqrt(5)*x - sqrt(2) - sqrt(5*
x^2 + 3*x + 2))) + 5*(sqrt(5) + 2)*arctan(-(2*sqrt(5)*x - sqrt(5) - 2*sqrt
(5*x^2 + 3*x + 2) - 4)/(sqrt(15) + 2*sqrt(3)))/(sqrt(15) + 2*sqrt(3)) - 5*
(sqrt(5) - 2)*arctan(-(2*sqrt(5)*x - sqrt(5) - 2*sqrt(5*x^2 + 3*x + 2) + 4
)/(sqrt(15) - 2*sqrt(3)))/(sqrt(15) - 2*sqrt(3)) + 1/2*log((sqrt(5)*x - sq
rt(5*x^2 + 3*x + 2))^2 - (sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))*(sqrt(5) + 4)
 + 5*sqrt(5) + 12) - 1/2*log((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - (sqrt
(5)*x - sqrt(5*x^2 + 3*x + 2))*(sqrt(5) - 4) - 5*sqrt(5) + 12) + 1/2*log(x
^2 - x + 1) - log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx = \int \frac{1}{x\,\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)}\,dx$$

input
```
int(1/(x*(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)),x)
```

output
```
int(1/(x*(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx = \int \frac{1}{\sqrt{5x^2+3x+2}\,x+2x^2+x}dx$$

input
```
int(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2)),x)
```

output
```
int(1/(sqrt(5*x**2 + 3*x + 2)*x + 2*x**2 + x),x)
```

3.21 $\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 271

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx = \frac{\sqrt{2}-\sqrt{2+3x+5x^2}}{2\left(2-\sqrt{2}\right)x} - \frac{31\left(1-\sqrt{2}\right)x}{8\left(4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}\right)} + \frac{2\arctan\left(\frac{3-4\sqrt{2}-\frac{2\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{4}\left(12-7\sqrt{2}\right)\log\left(\frac{4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}}{x}\right) + 3\log\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)$$

output
```
1/2*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/(2-2^(1/2))/x-31*(1-2^(1/2))*x/(32+24*x-
16*2^(1/2)*(5*x^2+3*x+2)^(1/2))+2/3*arctan(1/3*(3-4*2^(1/2)-2*(1-2^(1/2))*
(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)*3^(1/2))*3^(1/2)-1/4*(12-7*2^(1/2))*ln((4
+3*x-2*2^(1/2)*(5*x^2+3*x+2)^(1/2))/x)+3*ln(1-5*2^(1/2)-(3-4*2^(1/2))*(2^(
1/2)-(5*x^2+3*x+2)^(1/2))/x+(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))^2/x^
2)
```

Mathematica [A] (verified)

Time = 2.68 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx = \frac{1}{x}-\frac{\sqrt{2+3x+5x^2}}{x} + \frac{2\arctan\left(\frac{\sqrt{29-12\sqrt{5}}+2\left(-5+2\sqrt{5}\right)x+2\left(-2+\sqrt{5}\right)\sqrt{2+3x+5x^2}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{7\text{arctanh}\left(\frac{\sqrt{5}x-\sqrt{2+3x+5x^2}}{\sqrt{2}}\right)}{\sqrt{2}} + 6\text{arctanh}\left(\frac{-20974964966+9380288779\sqrt{5}-51657237985x+23101819444\sqrt{5}x-74388271700x^2+}{-20974964966+9380288779\sqrt{5}-51657237979x+23101819444\sqrt{5}x-74388271680x^2+}\right.$$

input
```
Integrate[1/(x^2*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])),x]
```

output
```
x^(-1) - Sqrt[2 + 3*x + 5*x^2]/x + (2*ArcTan[(Sqrt[29 - 12*Sqrt[5]] + 2*(-
5 + 2*Sqrt[5])*x + 2*(-2 + Sqrt[5])*Sqrt[2 + 3*x + 5*x^2])/Sqrt[3]])/Sqrt[
3] + (7*ArcTanh[(Sqrt[5]*x - Sqrt[2 + 3*x + 5*x^2])/Sqrt[2]])/Sqrt[2] + 6*
ArcTanh[(-20974964966 + 9380288779*Sqrt[5] - 51657237985*x + 23101819444*S
qrt[5]*x - 74388271700*x^2 + 33267446160*Sqrt[5]*x^2 + 5*Sqrt[5]*(11736302
59 + 2975530868*x)*Sqrt[2 + 3*x + 5*x^2] - 4*(3280396399 + 8316861540*x)*S
qrt[2 + 3*x + 5*x^2])/(-20974964966 + 9380288779*Sqrt[5] - 51657237979*x +
 23101819444*Sqrt[5]*x - 74388271680*x^2 + 33267446160*Sqrt[5]*x^2 - 4*(32
80396399 + 8316861540*x)*Sqrt[2 + 3*x + 5*x^2] + Sqrt[5]*(5868151295 + 148
77654336*x)*Sqrt[2 + 3*x + 5*x^2])]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.69, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{5x^2+3x+2}+2x+1\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{\sqrt{5x^2+3x+2}x}{x^2-x+1}+\frac{3x-2}{x^2-x+1}+\frac{\sqrt{5x^2+3x+2}}{x}+\frac{\sqrt{5x^2+3x+2}}{x^2}-\frac{1}{x^2}-\frac{3}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+3\text{arctanh}\left(\frac{2x+1}{\sqrt{5x^2+3x+2}}\right)-\sqrt{2}\text{arctanh}\left(\frac{3x+4}{2\sqrt{2}\sqrt{5x^2+3x+2}}\right)-\frac{3\text{arctanh}\left(\frac{3x+4}{2\sqrt{2}\sqrt{5x^2+3x+2}}\right)}{2\sqrt{2}}-\frac{\sqrt{5x^2+3x+2}}{x}+\frac{3}{2}\log\left(x^2-x+1\right)+\frac{1}{x}-3\log(x)$$

input
```
Int[1/(x^2*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])),x]
```

output
```
x^(-1) - Sqrt[2 + 3*x + 5*x^2]/x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Arc
Tan[(5 - 4*x)/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])]/Sqrt[3] + 3*ArcTanh[(1 + 2*
x)/Sqrt[2 + 3*x + 5*x^2]] - (3*ArcTanh[(4 + 3*x)/(2*Sqrt[2]*Sqrt[2 + 3*x +
 5*x^2])])/(2*Sqrt[2]) - Sqrt[2]*ArcTanh[(4 + 3*x)/(2*Sqrt[2]*Sqrt[2 + 3*x
 + 5*x^2])] - 3*Log[x] + (3*Log[1 - x + x^2])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(222) = 444$.

Time = 0.62 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.14

method	result
default	$-\frac{(5x^2+3x+2)^{\frac{3}{2}}}{2x}+\frac{3\sqrt{5x^2+3x+2}}{4}-\frac{7\sqrt{2}\,\text{arctanh}\left(\frac{(4+3x)\sqrt{2}}{4\sqrt{5x^2+3x+2}}\right)}{4}+\frac{(10x+3)\sqrt{5x^2+3x+2}}{4}-\frac{5\sqrt{7}\sqrt{16}\sqrt{\frac{28\left(-\frac{5}{4}+x\right)^2}{\left(-\frac{1}{2}-x\right)^2}+21}}{}$
trager	Expression too large to display

input
```
int(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x*(5*x^2+3*x+2)^(3/2)+3/4*(5*x^2+3*x+2)^(1/2)-7/4*2^(1/2)*arctanh(1/4
*(4+3*x)*2^(1/2)/(5*x^2+3*x+2)^(1/2))+1/4*(10*x+3)*(5*x^2+3*x+2)^(1/2)-5/2
94*7^(1/2)*16^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)*(2*3^(1/2)*arctan
(4/3*3^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)/(4*(-5/4+x)^2/(-1/2-x)^2
+31)*(-5/4+x)/(-1/2-x))-3*arctanh(1/14*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2
)))/((4*(-5/4+x)^2/(-1/2-x)^2+31)/((-5/4+x)/(-1/2-x)+1)^2)^(1/2)/((-5/4+x)
/(-1/2-x)+1)+1/196*7^(1/2)*16^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)*(
3^(1/2)*arctan(4/3*3^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^(1/2)/(4*(-5/4+x
)^2/(-1/2-x)^2+31)*(-5/4+x)/(-1/2-x))+9*arctanh(1/14*(28*(-5/4+x)^2/(-1/2-
x)^2+217)^(1/2)))/((4*(-5/4+x)^2/(-1/2-x)^2+31)/((-5/4+x)/(-1/2-x)+1)^2)^(
1/2)/((-5/4+x)/(-1/2-x)+1)+1/294*7^(1/2)*16^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^
2+217)^(1/2)*(5*3^(1/2)*arctan(4/3*3^(1/2)*(28*(-5/4+x)^2/(-1/2-x)^2+217)^
(1/2)/(4*(-5/4+x)^2/(-1/2-x)^2+31)*(-5/4+x)/(-1/2-x))+3*arctanh(1/14*(28*(
-5/4+x)^2/(-1/2-x)^2+217)^(1/2)))/((4*(-5/4+x)^2/(-1/2-x)^2+31)/((-5/4+x)/
(-1/2-x)+1)^2)^(1/2)/((-5/4+x)/(-1/2-x)+1)+3/2*ln(x^2-x+1)-1/3*3^(1/2)*arc
tan(1/3*(2*x-1)*3^(1/2))+1/x-3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx = -\frac{8\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+4\sqrt{3}x\arctan\left(\frac{4\sqrt{3}\sqrt{5x^2+3x+2}(4x-5)+31\sqrt{3}(x^2-2x)}{3(11x^2-12x-8)}\right)+4\sqrt{3}x\arctan\left(\frac{4\sqrt{}}{}\right.}{}$$

input
```
integrate(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="fricas")
```

output
```
-1/24*(8*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*sqrt(3)*x*arctan(1/3*
(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) + 31*sqrt(3)*(x^2 - 2*x))/(11*x
^2 - 12*x - 8)) + 4*sqrt(3)*x*arctan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*
(4*x - 5) - 31*sqrt(3)*(x^2 - 2*x))/(11*x^2 - 12*x - 8)) - 21*sqrt(2)*x*lo
g((4*sqrt(2)*sqrt(5*x^2 + 3*x + 2)*(3*x + 4) - 49*x^2 - 48*x - 32)/x^2) -
36*x*log(x^2 - x + 1) + 72*x*log(x) - 18*x*log((9*x^2 + 2*sqrt(5*x^2 + 3*x
 + 2)*(2*x + 1) + 7*x + 3)/x^2) + 18*x*log((9*x^2 - 2*sqrt(5*x^2 + 3*x + 2
)*(2*x + 1) + 7*x + 3)/x^2) + 24*sqrt(5*x^2 + 3*x + 2) - 24)/x
```

Sympy [F]

$$\int \frac{1}{x^2 \left(1+2x+\sqrt{2+3x+5x^2}\right)} \, dx = \int \frac{1}{x^2 \cdot \left(2x+\sqrt{5x^2+3x+2}+1\right)} \, dx$$

input
```
integrate(1/x**2/(1+2*x+(5*x**2+3*x+2)**(1/2)),x)
```

output
```
Integral(1/(x**2*(2*x + sqrt(5*x**2 + 3*x + 2) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(1+2x+\sqrt{2+3x+5x^2}\right)} \, dx = \int \frac{1}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)x^2} \, dx$$

input
```
integrate(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(5*x^2 + 3*x + 2) + 1)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx$$
$$= -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{7}{4}\sqrt{2}\log\left(-\frac{\left|-2\sqrt{5}x-2\sqrt{2}+2\sqrt{5x^2+3x+2}\right|}{2\left(\sqrt{5}x-\sqrt{2}-\sqrt{5x^2+3x+2}\right)}\right) + \frac{\left(\sqrt{5}+2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5x^2+3x+2}-4}{\sqrt{15}+2\sqrt{3}}\right)}{\sqrt{15}+2\sqrt{3}} - \frac{\left(\sqrt{5}-2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5x^2+3x+2}+4}{\sqrt{15}-2\sqrt{3}}\right)}{\sqrt{15}-2\sqrt{3}} + \frac{3\sqrt{5}x+4\sqrt{5}-3\sqrt{5x^2+3x+2}}{\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^2-2} + \frac{1}{x} + \frac{3}{2}\log\left(\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^2-\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)\left(\sqrt{5}+4\right)+5\sqrt{5}+12\right) - \frac{3}{2}\log\left(\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^2-\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)\left(\sqrt{5}-4\right)-5\sqrt{5}+12\right) + \frac{3}{2}\log\left(x^2-x+1\right) - 3\log\left(|x|\right)$$

input

```
integrate(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2)),x, algorithm="giac")
```

output

```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 7/4*sqrt(2)*log(-1/2*abs(-2*s
qrt(5)*x - 2*sqrt(2) + 2*sqrt(5*x^2 + 3*x + 2))/(sqrt(5)*x - sqrt(2) - sqr
t(5*x^2 + 3*x + 2))) + (sqrt(5) + 2)*arctan(-(2*sqrt(5)*x - sqrt(5) - 2*sq
rt(5*x^2 + 3*x + 2) - 4)/(sqrt(15) + 2*sqrt(3)))/(sqrt(15) + 2*sqrt(3)) -
(sqrt(5) - 2)*arctan(-(2*sqrt(5)*x - sqrt(5) - 2*sqrt(5*x^2 + 3*x + 2) + 4
)/(sqrt(15) - 2*sqrt(3)))/(sqrt(15) - 2*sqrt(3)) + (3*sqrt(5)*x + 4*sqrt(5
) - 3*sqrt(5*x^2 + 3*x + 2))/((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - 2) +
 1/x + 3/2*log((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - (sqrt(5)*x - sqrt(5
*x^2 + 3*x + 2))*(sqrt(5) + 4) + 5*sqrt(5) + 12) - 3/2*log((sqrt(5)*x - sq
rt(5*x^2 + 3*x + 2))^2 - (sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))*(sqrt(5) - 4)
 - 5*sqrt(5) + 12) + 3/2*log(x^2 - x + 1) - 3*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx = \int \frac{1}{x^2\,\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)}\,dx$$

input
```
int(1/(x^2*(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)),x)
```

output
```
int(1/(x^2*(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 \left(1+2x+\sqrt{2+3x+5x^2}\right)}\,dx = \int \frac{1}{\sqrt{5x^2+3x+2}\,x^2+2x^3+x^2}dx$$

input
```
int(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2)),x)
```

output
```
int(1/(sqrt(5*x**2 + 3*x + 2)*x**2 + 2*x**3 + x**2),x)
```

3.22 $\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 516

$$\begin{aligned}
&\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx \\
&= \frac{223+29\sqrt{2}-\frac{2\left(41-18\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)} \\
&\quad-\frac{x^2\left(2\left(49+30\sqrt{2}\right)+\frac{\left(49+24\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}\right)}{\left(4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}\right)\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)} \\
&\quad-\frac{58\arctan\left(\frac{3-4\sqrt{2}-\frac{2\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{112\operatorname{arctanh}\left(\frac{\sqrt{2}-\sqrt{2+3x+5x^2}}{\sqrt{5}x}\right)}{\sqrt{5}} \\
&\quad-25\log\left(\frac{4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}}{x^2}\right)+25\log\left(1-5\sqrt{2}\right. \\
&\qquad\left.-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)
\end{aligned}$$

output
```
(223+29*2^(1/2)-2*(41-18*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)/(3-15*2
^(1/2)-3*(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+3*(1-2^(1/2))*(2^(1
/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)-x^2*(98+60*2^(1/2)+(49+24*2^(1/2))*(2^(1/2
)-(5*x^2+3*x+2)^(1/2))/x)/(4+3*x-2*2^(1/2)*(5*x^2+3*x+2)^(1/2))/(1-5*2^(1/
2)-(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+(1-2^(1/2))*(2^(1/2)-(5*x
^2+3*x+2)^(1/2))^2/x^2)-58/9*arctan(1/3*(3-4*2^(1/2)-2*(1-2^(1/2))*(2^(1/2
)-(5*x^2+3*x+2)^(1/2))/x)*3^(1/2))*3^(1/2)+112/5*arctanh(1/5*(2^(1/2)-(5*x
^2+3*x+2)^(1/2))*5^(1/2)/x)*5^(1/2)-25*ln((4+3*x-2*2^(1/2)*(5*x^2+3*x+2)^(
1/2))/x^2)+25*ln(1-5*2^(1/2)-(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x
+(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.93 (sec) , antiderivative size = 1100, normalized size of antiderivative = 2.13

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[x^2/(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^2,x]
```

output
```
9*x + (22 - 26*x)/(3 - 3*x + 3*x^2) + Sqrt[2 + 3*x + 5*x^2]*(-4 + (2*(-1 +
 5*x))/(3*(1 - x + x^2))) - (56*ArcSinh[(3 + 10*x)/Sqrt[31]])/Sqrt[5] + (2
9*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - ((I/6)*(-167*I + 179*Sqrt[3])*
ArcTan[(3*(9*(34937 + (128*I)*Sqrt[3]) + (659658 - (67008*I)*Sqrt[3])*x +
(-2446517 + (925088*I)*Sqrt[3])*x^2 + (6*I)*(489919*I + 227688*Sqrt[3])*x^
3 + (4498469 + (2563280*I)*Sqrt[3])*x^4))/((7174320*I - 3656043*Sqrt[3])*x
^4 + 3*(67584*I + 351351*Sqrt[3] + 124012*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2
+ 3*x + 5*x^2]) + 2*x^3*(-5528904*I - 2114605*Sqrt[3] + 620060*Sqrt[3 - (1
2*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) - x^2*(5840832*I + 1586929*Sqrt[3] +
868084*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + x*(-5840832*I + 3
046486*Sqrt[3] + 868084*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]))])
/Sqrt[3 - (12*I)*Sqrt[3]] + ((167 - (179*I)*Sqrt[3])*ArcTan[(3*(-314433 +
(1152*I)*Sqrt[3] + (-659658 - (67008*I)*Sqrt[3])*x + (2446517 + (925088*I)
*Sqrt[3])*x^2 + 6*(489919 + (227688*I)*Sqrt[3])*x^3 + (-4498469 + (2563280
*I)*Sqrt[3])*x^4))/(-3*(2391440*I + 1218681*Sqrt[3])*x^4 + x^2*(5840832*I
- 1586929*Sqrt[3] - 868084*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2])
 + 3*(-67584*I + 351351*Sqrt[3] + 124012*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 +
 3*x + 5*x^2]) + 2*x^3*(5528904*I - 2114605*Sqrt[3] + 620060*Sqrt[3 + (12*
I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + x*(5840832*I + 3046486*Sqrt[3] + 8680
84*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]))])/(6*Sqrt[3 + (12*I...
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.55, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{5x^2+3x+2}+2x+1\right)^2}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{4\sqrt{5x^2+3x+2}x}{x^2-x+1}-\frac{2\sqrt{5x^2+3x+2}x}{(x^2-x+1)^2}-\frac{6\sqrt{5x^2+3x+2}}{x^2-x+1}+\frac{6\sqrt{5x^2+3x+2}}{(x^2-x+1)^2}+\frac{25x+1}{x^2-x+1}-\frac{2(3x+5}{(x^2-x+$$

$$\Big\downarrow \text{2009}$$

$$-6\sqrt{5}\text{arcsinh}\left(\frac{10x+3}{\sqrt{31}}\right)-\frac{26\text{arcsinh}\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{5}}+8\sqrt{3}\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)-\frac{43\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{3\sqrt{3}}-9\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}}\right)+\frac{52\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+25\text{arctanh}\left(\frac{2x+1}{\sqrt{5x^2+3x+2}}\right)+\frac{2(11-13x)}{3(x^2-x+1)}-\frac{2(1-2x)\sqrt{5x^2+3x+2}}{x^2-x+1}+\frac{2(2-x)\sqrt{5x^2+3x+2}}{3(x^2-x+1)}-4\sqrt{5x^2+3x+2}+\frac{25}{2}\log\left(x^2-x+1\right)+9x$$

input
```
Int[x^2/(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^2,x]
```

output
```
9*x + (2*(11 - 13*x))/(3*(1 - x + x^2)) - 4*Sqrt[2 + 3*x + 5*x^2] - (2*(1
- 2*x)*Sqrt[2 + 3*x + 5*x^2])/(1 - x + x^2) + (2*(2 - x)*Sqrt[2 + 3*x + 5*
x^2])/(3*(1 - x + x^2)) - (26*ArcSinh[(3 + 10*x)/Sqrt[31]])/Sqrt[5] - 6*Sq
rt[5]*ArcSinh[(3 + 10*x)/Sqrt[31]] + (52*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqr
t[3]) - 9*Sqrt[3]*ArcTan[(1 - 2*x)/Sqrt[3]] - (43*ArcTan[(5 - 4*x)/(Sqrt[3
]*Sqrt[2 + 3*x + 5*x^2])])/(3*Sqrt[3]) + 8*Sqrt[3]*ArcTan[(5 - 4*x)/(Sqrt[
3]*Sqrt[2 + 3*x + 5*x^2])] + 25*ArcTanh[(1 + 2*x)/Sqrt[2 + 3*x + 5*x^2]] +
 (25*Log[1 - x + x^2])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 1280, normalized size of antiderivative = 2.48

method	result	size
trager	Expression too large to display	1280
default	Expression too large to display	3216

input
```
int(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(27*x^2-49*x+23)*x/(x^2-x+1)-2/3*(6*x^2-11*x+7)/(x^2-x+1)*(5*x^2+3*x+2
)^(1/2)+2/3*RootOf(20*_Z^2+1500*_Z-99)*ln((1620*RootOf(3*_Z^2-225*_Z+4429)
^2*RootOf(20*_Z^2+1500*_Z-99)^2*x-250260*RootOf(3*_Z^2-225*_Z+4429)*RootOf
(20*_Z^2+1500*_Z-99)^2*x+378540*RootOf(3*_Z^2-225*_Z+4429)^2*RootOf(20*_Z^
2+1500*_Z-99)*x+1400440*RootOf(20*_Z^2+1500*_Z-99)^2*x-1783152*RootOf(3*_Z
^2-225*_Z+4429)*RootOf(20*_Z^2+1500*_Z-99)*(5*x^2+3*x+2)^(1/2)-31324140*Ro
otOf(3*_Z^2-225*_Z+4429)*RootOf(20*_Z^2+1500*_Z-99)*x+9376965*RootOf(3*_Z^
2-225*_Z+4429)^2*x-4998672*RootOf(3*_Z^2-225*_Z+4429)*RootOf(20*_Z^2+1500*
_Z-99)+227512656*RootOf(20*_Z^2+1500*_Z-99)*(5*x^2+3*x+2)^(1/2)-139203720*
RootOf(20*_Z^2+1500*_Z-99)*x-179820648*(5*x^2+3*x+2)^(1/2)*RootOf(3*_Z^2-2
25*_Z+4429)-300724245*RootOf(3*_Z^2-225*_Z+4429)*x+29066352*RootOf(20*_Z^2
+1500*_Z-99)-140795928*RootOf(3*_Z^2-225*_Z+4429)+19010110392*(5*x^2+3*x+2
)^(1/2)-28651884570*x-5845946904)/(3*RootOf(3*_Z^2-225*_Z+4429)*x-98*x-29)
)-2/3*ln((1620*RootOf(3*_Z^2-225*_Z+4429)^2*RootOf(20*_Z^2+1500*_Z-99)^2*x
-250260*RootOf(3*_Z^2-225*_Z+4429)*RootOf(20*_Z^2+1500*_Z-99)^2*x-135540*R
ootOf(3*_Z^2-225*_Z+4429)^2*RootOf(20*_Z^2+1500*_Z-99)*x+1400440*RootOf(20
*_Z^2+1500*_Z-99)^2*x+1783152*RootOf(3*_Z^2-225*_Z+4429)*RootOf(20*_Z^2+15
00*_Z-99)*(5*x^2+3*x+2)^(1/2)-6214860*RootOf(3*_Z^2-225*_Z+4429)*RootOf(20
*_Z^2+1500*_Z-99)*x-9901035*RootOf(3*_Z^2-225*_Z+4429)^2*x+4998672*RootOf(
3*_Z^2-225*_Z+4429)*RootOf(20*_Z^2+1500*_Z-99)-227512656*RootOf(20*_Z^2...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \frac{1620\,x^3+580\,\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+290\,\sqrt{3}(x^2-x+1)\arctan\left(\frac{4\,\sqrt{3}\sqrt{5\,x^2+3\,x+2}(4\,x-5)}{3\,(11\,x^2-12\,x}\right.}{}$$

input
```
integrate(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/180*(1620*x^3 + 580*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1))
+ 290*sqrt(3)*(x^2 - x + 1)*arctan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4
*x - 5) + 31*sqrt(3)*(x^2 - 2*x))/(11*x^2 - 12*x - 8)) + 290*sqrt(3)*(x^2
- x + 1)*arctan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) - 31*sqrt(3
)*(x^2 - 2*x))/(11*x^2 - 12*x - 8)) + 1008*sqrt(5)*(x^2 - x + 1)*log(4*sqr
t(5)*sqrt(5*x^2 + 3*x + 2)*(10*x + 3) - 200*x^2 - 120*x - 49) - 1620*x^2 +
 2250*(x^2 - x + 1)*log(x^2 - x + 1) + 1125*(x^2 - x + 1)*log((9*x^2 + 2*s
qrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^2) - 1125*(x^2 - x + 1)*log((9
*x^2 - 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^2) - 120*(6*x^2 - 11
*x + 7)*sqrt(5*x^2 + 3*x + 2) + 60*x + 1320)/(x^2 - x + 1)
```

Sympy [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{x^2}{\left(2x+\sqrt{5x^2+3x+2}+1\right)^2}\,dx$$

input
```
integrate(x**2/(1+2*x+(5*x**2+3*x+2)**(1/2))**2,x)
```

output
```
Integral(x**2/(2*x + sqrt(5*x**2 + 3*x + 2) + 1)**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{x^2}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^2}\,dx$$

input
```
integrate(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x^2/(2*x + sqrt(5*x^2 + 3*x + 2) + 1)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{923521,[8]%%%}+%%%{%%{[3694084,0]:[1,0,-5]%%},[7]%%%}+%%%{
42481966,
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{x^2}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^2}\,dx$$

input
```
int(x^2/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^2,x)
```

output
```
int(x^2/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^2, x)
```

Reduce [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{x^2}{\left(1+2x+\sqrt{5x^2+3x+2}\right)^2}dx$$

input
```
int(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x)
```

output
```
int(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x)
```

3.23 $\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 352

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx$$
$$= -\frac{2\left(53+16\sqrt{2}-\frac{2\left(10+3\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}\right)}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)}$$
$$-\frac{158\arctan\left(\frac{3-4\sqrt{2}-\frac{2\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3}}\right)}{3\sqrt{3}}+8\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{2}-\sqrt{2+3x+5x^2}}{\sqrt{5}x}\right)$$
$$-9\log\left(\frac{4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}}{x^2}\right)+9\log\left(1-5\sqrt{2}\right.$$
$$\left.-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)$$

output
```
(-106-32*2^(1/2)+4*(10+3*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)/(3-15*2
^(1/2)-3*(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+3*(1-2^(1/2))*(2^(1
/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)-158/9*arctan(1/3*(3-4*2^(1/2)-2*(1-2^(1/2)
)*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)*3^(1/2))*3^(1/2)+8*arctanh(1/5*(2^(1/2)
-(5*x^2+3*x+2)^(1/2))*5^(1/2)/x)*5^(1/2)-9*ln((4+3*x-2*2^(1/2)*(5*x^2+3*x+
2)^(1/2))/x^2)+9*ln(1-5*2^(1/2)-(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2)
)/x+(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.71 (sec) , antiderivative size = 1086, normalized size of antiderivative = 3.09

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[x/(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^2,x]
```

output
```
-((4 + 22*x)/(3 - 3*x + 3*x^2)) + (2*(4 + x)*Sqrt[2 + 3*x + 5*x^2])/(3*(1
- x + x^2)) - 4*Sqrt[5]*ArcSinh[(3 + 10*x)/Sqrt[31]] + (79*ArcTan[(-1 + 2*
x)/Sqrt[3]])/(3*Sqrt[3]) + (7*(11 - (19*I)*Sqrt[3])*ArcTan[(3*(9*(1321 + (
800*I)*Sqrt[3]) + 6*(6043 - (1240*I)*Sqrt[3])*x + (-1637 + (2192*I)*Sqrt[3
])*x^2 + (-21450 - (28272*I)*Sqrt[3])*x^3 + (2189 + (28880*I)*Sqrt[3])*x^4
))/(33120*I + 4143*Sqrt[3] - 3*(16720*I + 10667*Sqrt[3])*x^4 + 3612*Sqrt[3
 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2] + x^2*(23568*I + 51389*Sqrt[3] -
8428*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + 2*x*(11784*I + 9497
*Sqrt[3] + 4214*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + 2*x^3*(-
47688*I + 26297*Sqrt[3] + 6020*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x
^2]))])/(6*Sqrt[3 - (12*I)*Sqrt[3]]) - (((7*I)/6)*(-11*I + 19*Sqrt[3])*Arc
Tan[(3*(-11889 + (7200*I)*Sqrt[3] + (-36258 - (7440*I)*Sqrt[3])*x + (1637
+ (2192*I)*Sqrt[3])*x^2 + 6*(3575 - (4712*I)*Sqrt[3])*x^3 + (-2189 + (2888
0*I)*Sqrt[3])*x^4))/(-33120*I + 4143*Sqrt[3] + (50160*I - 32001*Sqrt[3])*x
^4 + 3612*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2] + x^2*(-23568*I +
 51389*Sqrt[3] - 8428*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + 2*
x*(-11784*I + 9497*Sqrt[3] + 4214*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x +
5*x^2]) + 2*x^3*(47688*I + 26297*Sqrt[3] + 6020*Sqrt[3 + (12*I)*Sqrt[3]]*S
qrt[2 + 3*x + 5*x^2]))])/Sqrt[3 + (12*I)*Sqrt[3]] + (9*Log[1 - x + x^2])/2
 - (7*(-11*I + 19*Sqrt[3])*Log[16*(1 - x + x^2)^2])/(12*Sqrt[3 + (12*I)...
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.55, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{5x^2+3x+2}+2x+1\right)^2}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{6\sqrt{5x^2+3x+2}x}{\left(x^2-x+1\right)^2}-\frac{4\sqrt{5x^2+3x+2}}{x^2-x+1}+\frac{4\sqrt{5x^2+3x+2}}{\left(x^2-x+1\right)^2}+\frac{9x+16}{x^2-x+1}+\frac{2(5x-8)}{\left(x^2-x+1\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-4\sqrt{5}\text{arcsinh}\left(\frac{10x+3}{\sqrt{31}}\right)+\frac{79\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{3\sqrt{3}}-\frac{79\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+9\text{arctanh}\left(\frac{2x+1}{\sqrt{5x^2+3x+2}}\right)-\frac{4\sqrt{5x^2+3x+2}(1-2x)}{3(x^2-x+1)}+\frac{2(2-x)\sqrt{5x^2+3x+2}}{x^2-x+1}-\frac{2(11x+2)}{3(x^2-x+1)}+\frac{9}{2}\log(x^2-x+1)$$

input
```
Int[x/(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^2,x]
```

output
```
(-2*(2 + 11*x))/(3*(1 - x + x^2)) - (4*(1 - 2*x)*Sqrt[2 + 3*x + 5*x^2])/(3
*(1 - x + x^2)) + (2*(2 - x)*Sqrt[2 + 3*x + 5*x^2])/(1 - x + x^2) - 4*Sqrt
[5]*ArcSinh[(3 + 10*x)/Sqrt[31]] - (79*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[
3]) + (79*ArcTan[(5 - 4*x)/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])])/(3*Sqrt[3]) +
 9*ArcTanh[(1 + 2*x)/Sqrt[2 + 3*x + 5*x^2]] + (9*Log[1 - x + x^2])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.65 (sec) , antiderivative size = 1195, normalized size of antiderivative = 3.39

method	result	size
trager	Expression too large to display	1195
default	Expression too large to display	2886

input
```
int(x/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(2*x-13)*x/(x^2-x+1)+2/3*(4+x)/(x^2-x+1)*(5*x^2+3*x+2)^(1/2)+2/3*RootO
f(4*_Z^2+108*_Z+9)*ln((324*RootOf(3*_Z^2-81*_Z+2107)^2*RootOf(4*_Z^2+108*_
Z+9)^2*x+27108*RootOf(3*_Z^2-81*_Z+2107)^2*RootOf(4*_Z^2+108*_Z+9)*x+61404
*RootOf(3*_Z^2-81*_Z+2107)*RootOf(4*_Z^2+108*_Z+9)^2*x+242109*RootOf(3*_Z^
2-81*_Z+2107)^2*x+346968*RootOf(3*_Z^2-81*_Z+2107)*RootOf(4*_Z^2+108*_Z+9)
*(5*x^2+3*x+2)^(1/2)-146052*RootOf(3*_Z^2-81*_Z+2107)*RootOf(4*_Z^2+108*_Z
+9)*x-9357040*RootOf(4*_Z^2+108*_Z+9)^2*x+12533508*(5*x^2+3*x+2)^(1/2)*Roo
tOf(3*_Z^2-81*_Z+2107)+972648*RootOf(3*_Z^2-81*_Z+2107)*RootOf(4*_Z^2+108*
_Z+9)-34449381*RootOf(3*_Z^2-81*_Z+2107)*x+80468136*RootOf(4*_Z^2+108*_Z+9
)*(5*x^2+3*x+2)^(1/2)-419049960*RootOf(4*_Z^2+108*_Z+9)*x+9888588*RootOf(3
*_Z^2-81*_Z+2107)+2162148156*(5*x^2+3*x+2)^(1/2)-97084680*RootOf(4*_Z^2+10
8*_Z+9)-4686181920*x-2248708140)/(3*RootOf(3*_Z^2-81*_Z+2107)*x-80*x+79))-
2/3*ln((324*RootOf(3*_Z^2-81*_Z+2107)^2*RootOf(4*_Z^2+108*_Z+9)^2*x-9612*R
ootOf(3*_Z^2-81*_Z+2107)^2*RootOf(4*_Z^2+108*_Z+9)*x+61404*RootOf(3*_Z^2-8
1*_Z+2107)*RootOf(4*_Z^2+108*_Z+9)^2*x-253611*RootOf(3*_Z^2-81*_Z+2107)^2*
x-346968*RootOf(3*_Z^2-81*_Z+2107)*RootOf(4*_Z^2+108*_Z+9)*(5*x^2+3*x+2)^(
1/2)+3461868*RootOf(3*_Z^2-81*_Z+2107)*RootOf(4*_Z^2+108*_Z+9)*x-9357040*R
ootOf(4*_Z^2+108*_Z+9)^2*x+3165372*(5*x^2+3*x+2)^(1/2)*RootOf(3*_Z^2-81*_Z
+2107)-972648*RootOf(3*_Z^2-81*_Z+2107)*RootOf(4*_Z^2+108*_Z+9)+14257539*R
ootOf(3*_Z^2-81*_Z+2107)*x-80468136*RootOf(4*_Z^2+108*_Z+9)*(5*x^2+3*x+...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.96

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx$$

$$= \frac{316\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+158\sqrt{3}(x^2-x+1)\arctan\left(\frac{4\sqrt{3}\sqrt{5x^2+3x+2}(4x-5)+31\sqrt{3}(x^2-\dots}{3(11x^2-12x-8)}\right.}{\dots}$$

input
```
integrate(x/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/36*(316*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 158*sqrt(3
)*(x^2 - x + 1)*arctan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) + 31
*sqrt(3)*(x^2 - 2*x))/(11*x^2 - 12*x - 8)) + 158*sqrt(3)*(x^2 - x + 1)*arc
tan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) - 31*sqrt(3)*(x^2 - 2*x
))/(11*x^2 - 12*x - 8)) + 72*sqrt(5)*(x^2 - x + 1)*log(4*sqrt(5)*sqrt(5*x^
2 + 3*x + 2)*(10*x + 3) - 200*x^2 - 120*x - 49) + 162*(x^2 - x + 1)*log(x^
2 - x + 1) + 81*(x^2 - x + 1)*log((9*x^2 + 2*sqrt(5*x^2 + 3*x + 2)*(2*x +
1) + 7*x + 3)/x^2) - 81*(x^2 - x + 1)*log((9*x^2 - 2*sqrt(5*x^2 + 3*x + 2)
*(2*x + 1) + 7*x + 3)/x^2) + 24*sqrt(5*x^2 + 3*x + 2)*(x + 4) - 264*x - 48
)/(x^2 - x + 1)
```

Sympy [F]

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{x}{\left(2x+\sqrt{5x^2+3x+2}+1\right)^2}\,dx$$

input
```
integrate(x/(1+2*x+(5*x**2+3*x+2)**(1/2))**2,x)
```

output
```
Integral(x/(2*x + sqrt(5*x**2 + 3*x + 2) + 1)**2, x)
```

Maxima [F]

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{x}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^2}\,dx$$

input
```
integrate(x/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x/(2*x + sqrt(5*x^2 + 3*x + 2) + 1)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{923521,[8]%%%}+%%%{%%{[3694084,0]:[1,0,-5]%%},[7]%%%}+%%%{
42481966,
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{x}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^2}\,dx$$

input
```
int(x/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^2,x)
```

output
```
int(x/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^2, x)
```

Reduce [F]

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{x}{4\sqrt{5x^2+3x+2}\,x+2\sqrt{5x^2+3x+2}+9x^2+7x+3}dx$$

input
```
int(x/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x)
```

output
```
int(x/(4*sqrt(5*x**2 + 3*x + 2)*x + 2*sqrt(5*x**2 + 3*x + 2) + 9*x**2 + 7*
x + 3),x)
```

3.24 $\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \frac{2(1+\sqrt{2})\left(2(21-31\sqrt{2})-\frac{(31-28\sqrt{2})\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}\right)}{3\left(1-5\sqrt{2}-\frac{(3-4\sqrt{2})\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{(1-\sqrt{2})\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)} - \frac{124\arctan\left(\frac{3-4\sqrt{2}-\frac{2(1-\sqrt{2})\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
2*(1+2^(1/2))*(42-62*2^(1/2)-(31-28*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))
/x)/(3-15*2^(1/2)-3*(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+3*(1-2^(
1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)-124/9*arctan(1/3*(3-4*2^(1/2)-2
*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \frac{2}{9}\left(\frac{-39+6x}{1-x+x^2}-\frac{3(-5+4x)\sqrt{2+3x+5x^2}}{1-x+x^2}\right.$$
$$+6\sqrt{15\left(9+4\sqrt{5}\right)}\arctan\left(\sqrt{3+\frac{4\sqrt{5}}{3}}\left(-16+7\sqrt{5}+2\left(-20+9\sqrt{5}\right)x-18\sqrt{2+3x+5x^2}+8\sqrt{5}\sqrt{2+\right.\right.$$
$$\left.\left.+4\sqrt{327-144\sqrt{5}}\arctan\left(\frac{3+10x+4\sqrt{2+3x+5x^2}-2\sqrt{5}\left(1+2x+\sqrt{2+3x+5x^2}\right)}{\sqrt{3}}\right)\right)$$

input
```
Integrate[(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^(-2),x]
```

output
```
(2*((-39 + 6*x)/(1 - x + x^2) - (3*(-5 + 4*x)*Sqrt[2 + 3*x + 5*x^2])/(1 -
x + x^2) + 6*Sqrt[15*(9 + 4*Sqrt[5])]*ArcTan[Sqrt[3 + (4*Sqrt[5])/3]*(-16
+ 7*Sqrt[5] + 2*(-20 + 9*Sqrt[5])*x - 18*Sqrt[2 + 3*x + 5*x^2] + 8*Sqrt[5]
*Sqrt[2 + 3*x + 5*x^2])] + 4*Sqrt[327 - 144*Sqrt[5]]*ArcTan[(3 + 10*x + 4*
Sqrt[2 + 3*x + 5*x^2] - 2*Sqrt[5]*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2]))/Sqrt[
3]]))/9
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.79, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{5x^2+3x+2}+2x+1\right)^2}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(-\frac{4\sqrt{5x^2+3x+2}x}{(x^2-x+1)^2}-\frac{2\sqrt{5x^2+3x+2}}{(x^2-x+1)^2}+\frac{9}{x^2-x+1}+\frac{2(8x-3)}{(x^2-x+1)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{62\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{3\sqrt{3}}-6\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}}\right)-\frac{8\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{5x^2+3x+2}(1-2x)}{3(x^2-x+1)}+\frac{4(2-x)\sqrt{5x^2+3x+2}}{3(x^2-x+1)}-\frac{2(13-2x)}{3(x^2-x+1)}$$

input
```
Int[(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^(-2),x]
```

output
```
(-2*(13 - 2*x))/(3*(1 - x + x^2)) + (2*(1 - 2*x)*Sqrt[2 + 3*x + 5*x^2])/(3
*(1 - x + x^2)) + (4*(2 - x)*Sqrt[2 + 3*x + 5*x^2])/(3*(1 - x + x^2)) - (8
*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3]) - 6*Sqrt[3]*ArcTan[(1 - 2*x)/Sqrt[
3]] + (62*ArcTan[(5 - 4*x)/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])])/(3*Sqrt[3])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.52

method	result
trager	$\frac{2(-11+13x)x}{3(x^2-x+1)}-\frac{2(4x-5)\sqrt{5x^2+3x+2}}{3(x^2-x+1)}+\frac{62\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{4\,\mathrm{RootOf}\left(\_Z^2+3\right)x-5\,\mathrm{RootOf}\left(\_Z^2+3\right)+3\sqrt{5x^2+3x+2}}{\mathrm{RootOf}\left(\_Z^2+3\right)x-x+2}\right)}{9}$
default	Expression too large to display

input
```
int(1/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(-11+13*x)*x/(x^2-x+1)-2/3*(4*x-5)/(x^2-x+1)*(5*x^2+3*x+2)^(1/2)+62/9*
RootOf(_Z^2+3)*ln((4*RootOf(_Z^2+3)*x-5*RootOf(_Z^2+3)+3*(5*x^2+3*x+2)^(1/
2))/(RootOf(_Z^2+3)*x-x+2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \frac{62\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - 31\sqrt{3}(x^2-x+1)\arctan\left(\frac{\sqrt{3}\sqrt{5x^2+3x+2}(x^2-49x+19)}{6(20x^3-13x^2-7x-10)}\right) - 6\sqrt{}}{9(x^2-x+1)}$$

input
```
integrate(1/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/9*(62*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 31*sqrt(3)*(
x^2 - x + 1)*arctan(1/6*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(x^2 - 49*x + 19)/(2
0*x^3 - 13*x^2 - 7*x - 10)) - 6*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) + 12*x - 7
8)/(x^2 - x + 1)
```

Sympy [F]

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{1}{\left(2x+\sqrt{5x^2+3x+2}+1\right)^2}\,dx$$

input
```
integrate(1/(1+2*x+(5*x**2+3*x+2)**(1/2))**2,x)
```

output
```
Integral((2*x + sqrt(5*x**2 + 3*x + 2) + 1)**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{1}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^2}\,dx$$

input
```
integrate(1/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((2*x + sqrt(5*x^2 + 3*x + 2) + 1)^(-2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(158) = 316$.

Time = 0.13 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.62

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx$$
$$= \frac{62}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{62\left(\sqrt{5}+2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5\,x^2+3\,x+2}-4}{\sqrt{15}+2\sqrt{3}}\right)}{3\left(\sqrt{15}+2\sqrt{3}\right)}$$
$$+ \frac{62\left(\sqrt{5}-2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5\,x^2+3\,x+2}+4}{\sqrt{15}-2\sqrt{3}}\right)}{3\left(\sqrt{15}-2\sqrt{3}\right)}$$
$$+ \frac{2\left(2\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^3 - 97\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^2 - 489\sqrt{5}x - 122\sqrt{5}\right.}{3\left(\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^4 - 2\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^3 + 13\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^2 + 16\right.}$$
$$+ \frac{2\,(2\,x-13)}{3\,(x^2-x+1)}$$

input
```
integrate(1/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="giac")
```

output
```
62/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 62/3*(sqrt(5) + 2)*arctan(-(2
*sqrt(5)*x - sqrt(5) - 2*sqrt(5*x^2 + 3*x + 2) - 4)/(sqrt(15) + 2*sqrt(3))
)/(sqrt(15) + 2*sqrt(3)) + 62/3*(sqrt(5) - 2)*arctan(-(2*sqrt(5)*x - sqrt(
5) - 2*sqrt(5*x^2 + 3*x + 2) + 4)/(sqrt(15) - 2*sqrt(3)))/(sqrt(15) - 2*sq
rt(3)) + 2/3*(2*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^3 - 97*sqrt(5)*(sqrt(5
)*x - sqrt(5*x^2 + 3*x + 2))^2 - 489*sqrt(5)*x - 122*sqrt(5) + 489*sqrt(5*
x^2 + 3*x + 2))/((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^4 - 2*sqrt(5)*(sqrt(5
)*x - sqrt(5*x^2 + 3*x + 2))^3 + 13*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2
+ 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2)) + 19) + 2/3*(2*x - 13)/(x
^2 - x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx=\int \frac{1}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^2}\,dx$$

input
```
int(1/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^2,x)
```

output
```
int(1/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^2, x)
```

Reduce [F]

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx =\int \frac{1}{4\sqrt{5x^2+3x+2}\,x+2\sqrt{5x^2+3x+2}+9x^2+7x+3}dx$$

input
```
int(1/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x)
```

output
```
int(1/(4*sqrt(5*x**2 + 3*x + 2)*x + 2*sqrt(5*x**2 + 3*x + 2) + 9*x**2 + 7*
x + 3),x)
```

3.25 $\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 330

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = -\frac{2\left(99+70\sqrt{2}\right)\left(2311-1634\sqrt{2}-\frac{\left(1755-1241\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}\right)}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)} - \frac{122\arctan\left(\frac{3-4\sqrt{2}-\frac{2\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3}}\right)}{3\sqrt{3}} + \left(3-2\sqrt{2}\right)\log\left(\frac{4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}}{x}\right) - 3\log\left(1-5\sqrt{2} - \frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)$$

output
```
-2*(99+70*2^(1/2))*(2311-1634*2^(1/2)-(1755-1241*2^(1/2))*(2^(1/2)-(5*x^2+
3*x+2)^(1/2))/x)/(3-15*2^(1/2)-3*(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2
))/x+3*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)-122/9*arctan(1/3*(
3-4*2^(1/2)-2*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)*3^(1/2))*3^(1/2
)+(3-2*2^(1/2))*ln((4+3*x-2*2^(1/2)*(5*x^2+3*x+2)^(1/2))/x)-3*ln(1-5*2^(1/
2)-(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+(1-2^(1/2))*(2^(1/2)-(5*x
^2+3*x+2)^(1/2))^2/x^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.15 (sec) , antiderivative size = 1106, normalized size of antiderivative = 3.35

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(x*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^2),x]
```

output
```
((24*(-11 + 13*x))/(1 - x + x^2) - (24*(-1 + 5*x)*Sqrt[2 + 3*x + 5*x^2])/(
1 - x + x^2) + 244*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + (2*(149 - (43*I)*S
qrt[3])*ArcTan[(3*(9*(21049 + (32768*I)*Sqrt[3]) + 6*(113131 + (29696*I)*S
qrt[3])*x + (808411 + (400160*I)*Sqrt[3])*x^2 + 6*(118057 - (95288*I)*Sqrt
[3])*x^3 + (123821 + (147920*I)*Sqrt[3])*x^4))/(-92160*I - 242151*Sqrt[3]
+ 3*(-512560*I + 288259*Sqrt[3])*x^4 + 83244*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt
[2 + 3*x + 5*x^2] + x^2*(1276224*I + 283307*Sqrt[3] - 194236*Sqrt[3 - (12*
I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + 2*x*(638112*I - 354889*Sqrt[3] + 9711
8*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + x^3*(2613552*I + 87398
2*Sqrt[3] + 277480*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]))])/Sqrt
[1/3 - (4*I)/Sqrt[3]] + (2*(-149*I + 43*Sqrt[3])*ArcTanh[(3*(9*(21049*I +
32768*Sqrt[3]) + 6*(113131*I + 29696*Sqrt[3])*x + (808411*I + 400160*Sqrt[
3])*x^2 + (708342*I - 571728*Sqrt[3])*x^3 + (123821*I + 147920*Sqrt[3])*x^
4))/(92160*I - 242151*Sqrt[3] + 3*(512560*I + 288259*Sqrt[3])*x^4 + 83244*
Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2] + x^2*(-1276224*I + 283307*
Sqrt[3] - 194236*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + 2*x*(-6
38112*I - 354889*Sqrt[3] + 97118*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5
*x^2]) + x^3*(-2613552*I + 873982*Sqrt[3] + 277480*Sqrt[3 + (12*I)*Sqrt[3]
]*Sqrt[2 + 3*x + 5*x^2]))])/Sqrt[1/3 + (4*I)/Sqrt[3]] + 108*Log[x] - 72*Sq
rt[2]*Log[x] - 54*Log[1 - x + x^2] - ((-149*I + 43*Sqrt[3])*Log[16*(1 -...
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.71, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{5x^2+3x+2}+2x+1\right)^2}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{3(x-1)}{x^2-x+1}-\frac{2\sqrt{5x^2+3x+2}}{x}+\frac{2x\sqrt{5x^2+3x+2}}{x^2-x+1}-\frac{2\sqrt{5x^2+3x+2}}{x^2-x+1}+\frac{2x\sqrt{5x^2+3x+2}}{(x^2-x+1)^2}-\frac{6\sqrt{5x^2+3}}{(x^2-x+}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{61\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{3\sqrt{3}}-\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}}\right)-\frac{52\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}-3\text{arctanh}\left(\frac{2x+1}{\sqrt{5x^2+3x+2}}\right)+2\sqrt{2}\text{arctanh}\left(\frac{3x+4}{2\sqrt{2}\sqrt{5x^2+3x+2}}\right)-\frac{2(11-13x)}{3\left(x^2-x+1\right)}+\frac{2(1-2x)\sqrt{5x^2+3x+2}}{x^2-x+1}-\frac{2(2-x)\sqrt{5x^2+3x+2}}{3\left(x^2-x+1\right)}-\frac{3}{2}\log\left(x^2-x+1\right)+3\log(x)$$

input
```
Int[1/(x*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^2),x]
```

output
```
(-2*(11 - 13*x))/(3*(1 - x + x^2)) + (2*(1 - 2*x)*Sqrt[2 + 3*x + 5*x^2])/(
1 - x + x^2) - (2*(2 - x)*Sqrt[2 + 3*x + 5*x^2])/(3*(1 - x + x^2)) - (52*A
rcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3]) - Sqrt[3]*ArcTan[(1 - 2*x)/Sqrt[3]]
+ (61*ArcTan[(5 - 4*x)/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])])/(3*Sqrt[3]) - 3*A
rcTanh[(1 + 2*x)/Sqrt[2 + 3*x + 5*x^2]] + 2*Sqrt[2]*ArcTanh[(4 + 3*x)/(2*S
qrt[2]*Sqrt[2 + 3*x + 5*x^2])] + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.52 (sec) , antiderivative size = 1515, normalized size of antiderivative = 4.59

method	result	size
trager	Expression too large to display	1515
default	Expression too large to display	2593

input
```
int(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(x-1)*(2*x-13)/(x^2-x+1)-2/3*(5*x-1)/(x^2-x+1)*(5*x^2+3*x+2)^(1/2)-2/
3*ln(-(23364*RootOf(3*_Z^2+27*_Z+991)^2*RootOf(4*_Z^2-36*_Z+9)^2*x-23364*R
ootOf(3*_Z^2+27*_Z+991)^2*RootOf(4*_Z^2-36*_Z+9)^2-108972*RootOf(3*_Z^2+27
*_Z+991)^2*RootOf(4*_Z^2-36*_Z+9)*x+852972*RootOf(3*_Z^2+27*_Z+991)*RootOf
(4*_Z^2-36*_Z+9)^2*x+108972*RootOf(3*_Z^2+27*_Z+991)^2*RootOf(4*_Z^2-36*_Z
+9)-125307*RootOf(3*_Z^2+27*_Z+991)^2*x-852972*RootOf(3*_Z^2+27*_Z+991)*Ro
otOf(4*_Z^2-36*_Z+9)^2+1080432*RootOf(3*_Z^2+27*_Z+991)*RootOf(4*_Z^2-36*_
Z+9)*(5*x^2+3*x+2)^(1/2)-684288*RootOf(3*_Z^2+27*_Z+991)*RootOf(4*_Z^2-36*
_Z+9)*x-7994960*RootOf(4*_Z^2-36*_Z+9)^2*x+125307*RootOf(3*_Z^2+27*_Z+991)
^2+10919844*RootOf(3*_Z^2+27*_Z+991)*RootOf(4*_Z^2-36*_Z+9)-17326440*(5*x^
2+3*x+2)^(1/2)*RootOf(3*_Z^2+27*_Z+991)-32012307*RootOf(3*_Z^2+27*_Z+991)*
x+7994960*RootOf(4*_Z^2-36*_Z+9)^2-39745404*RootOf(4*_Z^2-36*_Z+9)*(5*x^2+
3*x+2)^(1/2)-58797420*RootOf(4*_Z^2-36*_Z+9)*x-25497639*RootOf(3*_Z^2+27*_
Z+991)-20369112*RootOf(4*_Z^2-36*_Z+9)-76294530*(5*x^2+3*x+2)^(1/2)-828200
70*x-67503816)/x)*RootOf(3*_Z^2+27*_Z+991)-6*ln(-(23364*RootOf(3*_Z^2+27*_
Z+991)^2*RootOf(4*_Z^2-36*_Z+9)^2*x-23364*RootOf(3*_Z^2+27*_Z+991)^2*RootO
f(4*_Z^2-36*_Z+9)^2-108972*RootOf(3*_Z^2+27*_Z+991)^2*RootOf(4*_Z^2-36*_Z+
9)*x+852972*RootOf(3*_Z^2+27*_Z+991)*RootOf(4*_Z^2-36*_Z+9)^2*x+108972*Roo
tOf(3*_Z^2+27*_Z+991)^2*RootOf(4*_Z^2-36*_Z+9)-125307*RootOf(3*_Z^2+27*_Z+
991)^2*x-852972*RootOf(3*_Z^2+27*_Z+991)*RootOf(4*_Z^2-36*_Z+9)^2+10804...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx$$
$$= \frac{244\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+122\sqrt{3}(x^2-x+1)\arctan\left(\frac{4\sqrt{3}\sqrt{5x^2+3x+2}(4x-5)+31\sqrt{3}(x^2-}{3(11x^2-12x-8)}\right.}{}$$

input
```
integrate(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/36*(244*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 122*sqrt(3
)*(x^2 - x + 1)*arctan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) + 31
*sqrt(3)*(x^2 - 2*x))/(11*x^2 - 12*x - 8)) + 122*sqrt(3)*(x^2 - x + 1)*arc
tan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) - 31*sqrt(3)*(x^2 - 2*x
))/(11*x^2 - 12*x - 8)) + 36*sqrt(2)*(x^2 - x + 1)*log(-(4*sqrt(2)*sqrt(5*
x^2 + 3*x + 2)*(3*x + 4) + 49*x^2 + 48*x + 32)/x^2) - 54*(x^2 - x + 1)*log
(x^2 - x + 1) + 108*(x^2 - x + 1)*log(x) - 27*(x^2 - x + 1)*log((9*x^2 + 2
*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^2) + 27*(x^2 - x + 1)*log((9
*x^2 - 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^2) - 24*sqrt(5*x^2 +
 3*x + 2)*(5*x - 1) + 312*x - 264)/(x^2 - x + 1)
```

Sympy [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx=\int \frac{1}{x\left(2x+\sqrt{5x^2+3x+2}+1\right)^2}\,dx$$

input
```
integrate(1/x/(1+2*x+(5*x**2+3*x+2)**(1/2))**2,x)
```

output
```
Integral(1/(x*(2*x + sqrt(5*x**2 + 3*x + 2) + 1)**2), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx=\int \frac{1}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^2x}\,dx$$

input
```
integrate(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(5*x^2 + 3*x + 2) + 1)^2*x), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.60

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \frac{61}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)$$
$$-2\,\sqrt{2}\log\left(-\frac{\left|-2\,\sqrt{5}x-2\,\sqrt{2}+2\,\sqrt{5\,x^2+3\,x+2}\right|}{2\left(\sqrt{5}x-\sqrt{2}-\sqrt{5\,x^2+3\,x+2}\right)}\right)$$
$$-\frac{61\left(\sqrt{5}+2\right)\arctan\left(-\frac{2\,\sqrt{5}x-\sqrt{5}-2\,\sqrt{5\,x^2+3\,x+2}-4}{\sqrt{15}+2\,\sqrt{3}}\right)}{3\left(\sqrt{15}+2\,\sqrt{3}\right)}$$
$$+\frac{61\left(\sqrt{5}-2\right)\arctan\left(-\frac{2\,\sqrt{5}x-\sqrt{5}-2\,\sqrt{5\,x^2+3\,x+2}+4}{\sqrt{15}-2\,\sqrt{3}}\right)}{3\left(\sqrt{15}-2\,\sqrt{3}\right)}$$
$$+\frac{2\left(55\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^3-74\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^2-459\,\sqrt{5}x-121\,\sqrt{5}}{3\left(\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^4-2\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^3+13\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^2+16}$$
$$+\frac{2\,(13\,x-11)}{3\,(x^2-x+1)}-\frac{3}{2}\,\log\left(\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^2\right.$$
$$\left.-\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)\left(\sqrt{5}+4\right)+5\,\sqrt{5}+12\right)$$
$$+\frac{3}{2}\,\log\left(\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)^2-\left(\sqrt{5}x-\sqrt{5\,x^2+3\,x+2}\right)\left(\sqrt{5}-4\right)-5\,\sqrt{5}\right.$$
$$\left.+12\right)-\frac{3}{2}\,\log\left(x^2-x+1\right)+3\,\log\left(|x|\right)$$

input
```
integrate(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="giac")
```

output
```
61/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2*sqrt(2)*log(-1/2*abs(-2*sqr
t(5)*x - 2*sqrt(2) + 2*sqrt(5*x^2 + 3*x + 2))/(sqrt(5)*x - sqrt(2) - sqrt(
5*x^2 + 3*x + 2))) - 61/3*(sqrt(5) + 2)*arctan(-(2*sqrt(5)*x - sqrt(5) - 2
*sqrt(5*x^2 + 3*x + 2) - 4)/(sqrt(15) + 2*sqrt(3)))/(sqrt(15) + 2*sqrt(3))
 + 61/3*(sqrt(5) - 2)*arctan(-(2*sqrt(5)*x - sqrt(5) - 2*sqrt(5*x^2 + 3*x
+ 2) + 4)/(sqrt(15) - 2*sqrt(3)))/(sqrt(15) - 2*sqrt(3)) + 2/3*(55*(sqrt(5
)*x - sqrt(5*x^2 + 3*x + 2))^3 - 74*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x
+ 2))^2 - 459*sqrt(5)*x - 121*sqrt(5) + 459*sqrt(5*x^2 + 3*x + 2))/((sqrt(
5)*x - sqrt(5*x^2 + 3*x + 2))^4 - 2*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x
+ 2))^3 + 13*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 + 16*sqrt(5)*(sqrt(5)*x
 - sqrt(5*x^2 + 3*x + 2)) + 19) + 2/3*(13*x - 11)/(x^2 - x + 1) - 3/2*log(
(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - (sqrt(5)*x - sqrt(5*x^2 + 3*x + 2)
)*(sqrt(5) + 4) + 5*sqrt(5) + 12) + 3/2*log((sqrt(5)*x - sqrt(5*x^2 + 3*x
+ 2))^2 - (sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))*(sqrt(5) - 4) - 5*sqrt(5) +
12) - 3/2*log(x^2 - x + 1) + 3*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{1}{x\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^2}\,dx$$

input
```
int(1/(x*(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^2),x)
```

output
```
int(1/(x*(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{1}{x\left(1+2x+\sqrt{5x^2+3x+2}\right)^2}dx$$

input
```
int(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x)
```

output
```
int(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x)
```

3.26 $\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 413

$$\begin{aligned}
\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx
&= -\frac{\left(4+3\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{4x}+\frac{31\left(3-2\sqrt{2}\right)x}{8\left(4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}\right)}\\
&\quad+\frac{2\left(577+408\sqrt{2}\right)\left(17525-12392\sqrt{2}-\frac{2\left(3322-2349\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}\right)}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)}\\
&\quad-\frac{130\arctan\left(\frac{3-4\sqrt{2}-\frac{2\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3}}\right)}{3\sqrt{3}}\\
&\quad+\frac{1}{2}\left(26-19\sqrt{2}\right)\log\left(\frac{4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}}{x}\right)-13\log\left(1-5\sqrt{2}\right.\\
&\quad\left.-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)
\end{aligned}$$

output
```
-1/4*(4+3*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+31*(3-2*2^(1/2))*x/(32+
24*x-16*2^(1/2)*(5*x^2+3*x+2)^(1/2))+2*(577+408*2^(1/2))*(17525-12392*2^(1
/2)-2*(3322-2349*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)/(3-15*2^(1/2)-3
*(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+3*(1-2^(1/2))*(2^(1/2)-(5*x
^2+3*x+2)^(1/2))^2/x^2)-130/9*arctan(1/3*(3-4*2^(1/2)-2*(1-2^(1/2))*(2^(1/
2)-(5*x^2+3*x+2)^(1/2))/x)*3^(1/2))*3^(1/2)+1/2*(26-19*2^(1/2))*ln((4+3*x-
2*2^(1/2)*(5*x^2+3*x+2)^(1/2))/x)-13*ln(1-5*2^(1/2)-(3-4*2^(1/2))*(2^(1/2)
-(5*x^2+3*x+2)^(1/2))/x+(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.07 (sec) , antiderivative size = 1119, normalized size of antiderivative = 2.71

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(x^2*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^2),x]
```

output
```
-3/x + (4 + 22*x)/(3 - 3*x + 3*x^2) + Sqrt[2 + 3*x + 5*x^2]*(2/x - (2*(4 +
 x))/(3*(1 - x + x^2))) + (65*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (1
3*(19 + I*Sqrt[3])*ArcTan[(3*(9*(49 + (288*I)*Sqrt[3]) + 18*(125 + (248*I)
*Sqrt[3])*x + (7555 + (7376*I)*Sqrt[3])*x^2 + 6*(1689 + (248*I)*Sqrt[3])*x
^3 + (10589 + (80*I)*Sqrt[3])*x^4))/(3*(1520*I + 4577*Sqrt[3])*x^4 + x^2*(
1104*I - 12247*Sqrt[3] - 2548*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^
2]) + 3*(-3168*I - 847*Sqrt[3] + 364*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x
 + 5*x^2]) + 2*x*(552*I - 4651*Sqrt[3] + 1274*Sqrt[3 - (12*I)*Sqrt[3]]*Sqr
t[2 + 3*x + 5*x^2]) + x^3*(41136*I - 9110*Sqrt[3] + 3640*Sqrt[3 - (12*I)*S
qrt[3]]*Sqrt[2 + 3*x + 5*x^2]))])/(6*Sqrt[3 - (12*I)*Sqrt[3]]) - (13*(19*I
 + Sqrt[3])*ArcTanh[(3*(9*(49*I + 288*Sqrt[3]) + 18*(125*I + 248*Sqrt[3])*
x + (7555*I + 7376*Sqrt[3])*x^2 + 6*(1689*I + 248*Sqrt[3])*x^3 + (10589*I
+ 80*Sqrt[3])*x^4))/(3*(-1520*I + 4577*Sqrt[3])*x^4 + 3*(3168*I - 847*Sqrt
[3] + 364*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + 2*x*(-552*I -
4651*Sqrt[3] + 1274*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) - x^2*
(1104*I + 12247*Sqrt[3] + 2548*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x
^2]) + x^3*(-41136*I - 9110*Sqrt[3] + 3640*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2
 + 3*x + 5*x^2]))])/(6*Sqrt[3 + (12*I)*Sqrt[3]]) + 13*Log[x] - (19*Log[x])
/Sqrt[2] - (13*Log[1 - x + x^2])/2 + (13*(-19*I + Sqrt[3])*Log[16*(1 - x +
 x^2)^2])/(12*Sqrt[3 - (12*I)*Sqrt[3]]) + (13*(19*I + Sqrt[3])*Log[16*(...
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.75, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(\sqrt{5x^2+3x+2}+2x+1\right)^2} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{10-13x}{x^2-x+1} - \frac{8\sqrt{5x^2+3x+2}}{x} + \frac{8x\sqrt{5x^2+3x+2}}{x^2-x+1} - \frac{6\sqrt{5x^2+3x+2}}{x^2-x+1} - \frac{2\sqrt{5x^2+3x+2}}{x^2} + \frac{6x\sqrt{5x^2+3x}}{(x^2-x+1}\right.$$

$$\Big\downarrow \text{2009}$$

$$8\sqrt{3}\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)-\frac{7\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{3\sqrt{3}}-\frac{65\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}-13\text{arctanh}\left(\frac{2x+1}{\sqrt{5x^2+3x+2}}\right)+8\sqrt{2}\text{arctanh}\left(\frac{3x+4}{2\sqrt{2}\sqrt{5x^2+3x+2}}\right)+\frac{3\text{arctanh}\left(\frac{3x+4}{2\sqrt{2}\sqrt{5x^2+3x+2}}\right)}{\sqrt{2}}+\frac{4\sqrt{5x^2+3x+2}(1-2x)}{3(x^2-x+1)}+\frac{2\sqrt{5x^2+3x+2}}{x}-\frac{2(2-x)\sqrt{5x^2+3x+2}}{x^2-x+1}+\frac{2(11x+2)}{3(x^2-x+1)}-\frac{13}{2}\log\left(x^2-x+1\right)-\frac{3}{x}+13\log(x)$$

input
```
Int[1/(x^2*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^2),x]
```

output
```
-3/x + (2*(2 + 11*x))/(3*(1 - x + x^2)) + (2*Sqrt[2 + 3*x + 5*x^2])/x + (4
*(1 - 2*x)*Sqrt[2 + 3*x + 5*x^2])/(3*(1 - x + x^2)) - (2*(2 - x)*Sqrt[2 +
3*x + 5*x^2])/(1 - x + x^2) - (65*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3]) -
 (7*ArcTan[(5 - 4*x)/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])])/(3*Sqrt[3]) + 8*Sqr
t[3]*ArcTan[(5 - 4*x)/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])] - 13*ArcTanh[(1 + 2
*x)/Sqrt[2 + 3*x + 5*x^2]] + (3*ArcTanh[(4 + 3*x)/(2*Sqrt[2]*Sqrt[2 + 3*x
+ 5*x^2])])/Sqrt[2] + 8*Sqrt[2]*ArcTanh[(4 + 3*x)/(2*Sqrt[2]*Sqrt[2 + 3*x
+ 5*x^2])] + 13*Log[x] - (13*Log[1 - x + x^2])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 1598, normalized size of antiderivative = 3.87

method	result	size
trager	Expression too large to display	1598
default	Expression too large to display	2959

input
```
int(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(x-1)*(17*x^2-13*x-9)/x/(x^2-x+1)+2/3*(2*x^2-7*x+3)/x/(x^2-x+1)*(5*x^
2+3*x+2)^(1/2)+2/3*RootOf(8*_Z^2-312*_Z-207)*ln(-(-46728*RootOf(3*_Z^2+117
*_Z+2197)^2*RootOf(8*_Z^2-312*_Z-207)^2*x+46728*RootOf(3*_Z^2+117*_Z+2197)
^2*RootOf(8*_Z^2-312*_Z-207)^2+860004*RootOf(3*_Z^2+117*_Z+2197)^2*RootOf(
8*_Z^2-312*_Z-207)*x-3192072*RootOf(3*_Z^2+117*_Z+2197)*RootOf(8*_Z^2-312*
_Z-207)^2*x+10937160*RootOf(3*_Z^2+117*_Z+2197)*RootOf(8*_Z^2-312*_Z-207)*
(5*x^2+3*x+2)^(1/2)-860004*RootOf(3*_Z^2+117*_Z+2197)^2*RootOf(8*_Z^2-312*
_Z-207)+7431264*RootOf(3*_Z^2+117*_Z+2197)^2*x+3192072*RootOf(3*_Z^2+117*_
Z+2197)*RootOf(8*_Z^2-312*_Z-207)^2+25402806*RootOf(3*_Z^2+117*_Z+2197)*Ro
otOf(8*_Z^2-312*_Z-207)*x-18679232*RootOf(8*_Z^2-312*_Z-207)^2*x-812617650
*(5*x^2+3*x+2)^(1/2)*RootOf(3*_Z^2+117*_Z+2197)-267893730*RootOf(8*_Z^2-31
2*_Z-207)*(5*x^2+3*x+2)^(1/2)-7431264*RootOf(3*_Z^2+117*_Z+2197)^2-1290168
36*RootOf(3*_Z^2+117*_Z+2197)*RootOf(8*_Z^2-312*_Z-207)+1764429966*RootOf(
3*_Z^2+117*_Z+2197)*x+18679232*RootOf(8*_Z^2-312*_Z-207)^2+889900596*RootO
f(8*_Z^2-312*_Z-207)*x-16662440925*(5*x^2+3*x+2)^(1/2)+806602914*RootOf(3*
_Z^2+117*_Z+2197)-1559586756*RootOf(8*_Z^2-312*_Z-207)+29835374751*x+17274
763584)/(3*RootOf(3*_Z^2+117*_Z+2197)*x-3*RootOf(3*_Z^2+117*_Z+2197)+91*x-
26))-2/3*ln((46728*RootOf(3*_Z^2+117*_Z+2197)^2*RootOf(8*_Z^2-312*_Z-207)^
2*x-46728*RootOf(3*_Z^2+117*_Z+2197)^2*RootOf(8*_Z^2-312*_Z-207)^2-2784780
*RootOf(3*_Z^2+117*_Z+2197)^2*RootOf(8*_Z^2-312*_Z-207)*x+3192072*RootO...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx$$
$$= \frac{260\sqrt{3}(x^3-x^2+x)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+130\sqrt{3}(x^3-x^2+x)\arctan\left(\frac{4\sqrt{3}\sqrt{5x^2+3x+2}(4x-5)+31\sqrt{3}(x^2}{3(11x^2-12x-8)}\right.}{}$$

input
```
integrate(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/36*(260*sqrt(3)*(x^3 - x^2 + x)*arctan(1/3*sqrt(3)*(2*x - 1)) + 130*sqrt
(3)*(x^3 - x^2 + x)*arctan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5)
+ 31*sqrt(3)*(x^2 - 2*x))/(11*x^2 - 12*x - 8)) + 130*sqrt(3)*(x^3 - x^2 +
x)*arctan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) - 31*sqrt(3)*(x^2
 - 2*x))/(11*x^2 - 12*x - 8)) + 171*sqrt(2)*(x^3 - x^2 + x)*log(-(4*sqrt(2
)*sqrt(5*x^2 + 3*x + 2)*(3*x + 4) + 49*x^2 + 48*x + 32)/x^2) + 156*x^2 - 2
34*(x^3 - x^2 + x)*log(x^2 - x + 1) + 468*(x^3 - x^2 + x)*log(x) - 117*(x^
3 - x^2 + x)*log((9*x^2 + 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^2
) + 117*(x^3 - x^2 + x)*log((9*x^2 - 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7
*x + 3)/x^2) + 24*sqrt(5*x^2 + 3*x + 2)*(2*x^2 - 7*x + 3) + 156*x - 108)/(
x^3 - x^2 + x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{1}{x^2\left(2x+\sqrt{5x^2+3x+2}+1\right)^2}\,dx$$

input
```
integrate(1/x**2/(1+2*x+(5*x**2+3*x+2)**(1/2))**2,x)
```

output
```
Integral(1/(x**2*(2*x + sqrt(5*x**2 + 3*x + 2) + 1)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \int \frac{1}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(5*x^2 + 3*x + 2) + 1)^2*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x, algorithm="giac")
```

output
```
65/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 19/2*sqrt(2)*log(-1/2*abs(-2*
sqrt(5)*x - 2*sqrt(2) + 2*sqrt(5*x^2 + 3*x + 2))/(sqrt(5)*x - sqrt(2) - sq
rt(5*x^2 + 3*x + 2))) - 65/3*(sqrt(5) + 2)*arctan(-(2*sqrt(5)*x - sqrt(5)
- 2*sqrt(5*x^2 + 3*x + 2) - 4)/(sqrt(15) + 2*sqrt(3)))/(sqrt(15) + 2*sqrt(
3)) + 65/3*(sqrt(5) - 2)*arctan(-(2*sqrt(5)*x - sqrt(5) - 2*sqrt(5*x^2 + 3
*x + 2) + 4)/(sqrt(15) - 2*sqrt(3)))/(sqrt(15) - 2*sqrt(3)) + 2/3*(44*(sqr
t(5)*x - sqrt(5*x^2 + 3*x + 2))^5 + 29*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3
*x + 2))^4 - 73*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^3 - 345*sqrt(5)*(sqrt(
5)*x - sqrt(5*x^2 + 3*x + 2))^2 - 1191*sqrt(5)*x - 230*sqrt(5) + 1191*sqrt
(5*x^2 + 3*x + 2))/((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^6 - 2*sqrt(5)*(sqr
t(5)*x - sqrt(5*x^2 + 3*x + 2))^5 + 11*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))
^4 + 20*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^3 - 7*(sqrt(5)*x - sqr
t(5*x^2 + 3*x + 2))^2 - 32*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2)) - 3
8) + 1/3*(13*x^2 + 13*x - 9)/(x^3 - x^2 + x) - 13/2*log((sqrt(5)*x - sqrt(
5*x^2 + 3*x + 2))^2 - (sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))*(sqrt(5) + 4) +
5*sqrt(5) + 12) + 13/2*log((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - (sqrt(5
)*x - sqrt(5*x^2 + 3*x + 2))*(sqrt(5) - 4) - 5*sqrt(5) + 12) - 13/2*log(x^
2 - x + 1) + 13*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx=\int \frac{1}{x^2\,{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)}^2} \,d x$$

input
```
int(1/(x^2*(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^2),x)
```

output
```
int(1/(x^2*(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^2}\,dx=\int \frac{1}{x^2\left(1+2x+\sqrt{5x^2+3x+2}\right)^2}dx$$

input
```
int(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x)
```

output
```
int(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^2,x)
```

3.27 $\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 485

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx$$
$$= -\frac{695+504\sqrt{2}-\frac{4\left(49+34\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)^2}$$
$$+\frac{2\left(419+94\sqrt{2}-\frac{\left(173+27\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}\right)}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)}$$
$$+\frac{1028\arctan\left(\frac{3-4\sqrt{2}-\frac{2\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3}}\right)}{3\sqrt{3}}$$
$$-34\sqrt{5}\text{arctanh}\left(\frac{\sqrt{2}-\sqrt{2+3x+5x^2}}{\sqrt{5}x}\right)+38\log\left(\frac{4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}}{x^2}\right)$$
$$-38\log\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)$$

output
```
-1/3*(695+504*2^(1/2)-4*(49+34*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)/(
1-5*2^(1/2)-(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+(1-2^(1/2))*(2^(
1/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)^2+2*(419+94*2^(1/2)-(173+27*2^(1/2))*(2^(
1/2)-(5*x^2+3*x+2)^(1/2))/x)/(3-15*2^(1/2)-3*(3-4*2^(1/2))*(2^(1/2)-(5*x^2
+3*x+2)^(1/2))/x+3*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)+1028/9
*arctan(1/3*(3-4*2^(1/2)-2*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)*3^
(1/2))*3^(1/2)-34*arctanh(1/5*(2^(1/2)-(5*x^2+3*x+2)^(1/2))*5^(1/2)/x)*5^(
1/2)+38*ln((4+3*x-2*2^(1/2)*(5*x^2+3*x+2)^(1/2))/x^2)-38*ln(1-5*2^(1/2)-(3
-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+(1-2^(1/2))*(2^(1/2)-(5*x^2+3*
x+2)^(1/2))^2/x^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.83 (sec) , antiderivative size = 1118, normalized size of antiderivative = 2.31

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[x^2/(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^3,x]
```

output
```
(2*(-37 + 17*x))/(3*(1 - x + x^2)^2) + (151 + 163*x)/(3 - 3*x + 3*x^2) + (
2*Sqrt[2 + 3*x + 5*x^2]*(-33 + 34*x - 47*x^2 + 3*x^3))/(3*(1 - x + x^2)^2)
 + 17*Sqrt[5]*ArcSinh[(3 + 10*x)/Sqrt[31]] - (514*ArcTan[(-1 + 2*x)/Sqrt[3
]])/(3*Sqrt[3]) + (((7*I)/3)*(49*I + 53*Sqrt[3])*ArcTan[(3*(9*(12329 + (92
48*I)*Sqrt[3]) + 6*(58271 - (8024*I)*Sqrt[3])*x + (76651 + (38480*I)*Sqrt[
3])*x^2 + (-92058 - (297648*I)*Sqrt[3])*x^3 + (-24139 + (224720*I)*Sqrt[3]
)*x^4))/(99*(2720*I + 41*Sqrt[3]) - 3*(207760*I + 53511*Sqrt[3])*x^4 + 324
84*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2] + x^2*(363984*I + 472577
*Sqrt[3] - 75796*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + 2*x^3*(
-236904*I + 279341*Sqrt[3] + 54140*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x +
 5*x^2]) + x*(363984*I + 70042*Sqrt[3] + 75796*Sqrt[3 - (12*I)*Sqrt[3]]*Sq
rt[2 + 3*x + 5*x^2]))])/Sqrt[3 - (12*I)*Sqrt[3]] + (7*(49 + (53*I)*Sqrt[3]
)*ArcTan[(3*(-110961 + (83232*I)*Sqrt[3] + (-349626 - (48144*I)*Sqrt[3])*x
 + (-76651 + (38480*I)*Sqrt[3])*x^2 + 6*(15343 - (49608*I)*Sqrt[3])*x^3 +
(24139 + (224720*I)*Sqrt[3])*x^4))/(99*(-2720*I + 41*Sqrt[3]) + (623280*I
- 160533*Sqrt[3])*x^4 + 32484*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^
2] + x^2*(-363984*I + 472577*Sqrt[3] - 75796*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt
[2 + 3*x + 5*x^2]) + 2*x^3*(236904*I + 279341*Sqrt[3] + 54140*Sqrt[3 + (12
*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + x*(-363984*I + 70042*Sqrt[3] + 75796
*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]))])/(3*Sqrt[3 + (12*I)*...
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{5x^2+3x+2}+2x+1\right)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{-38x-121}{x^2-x+1}+\frac{17\sqrt{5x^2+3x+2}}{x^2-x+1}+\frac{49x\sqrt{5x^2+3x+2}}{(x^2-x+1)^2}+\frac{3\sqrt{5x^2+3x+2}}{(x^2-x+1)^2}-\frac{12x\sqrt{5x^2+3x+2}}{(x^2-x+1)^3}-\frac{20\sqrt{5x^2}}{(x^2-}\right.$$

$$\Big\downarrow \text{2009}$$

$$17\sqrt{5}\text{arcsinh}\left(\frac{10x+3}{\sqrt{31}}\right)-\frac{514\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{3\sqrt{3}}+\frac{514\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}-38\text{arctanh}\left(\frac{2x+1}{\sqrt{5x^2+3x+2}}\right)+\frac{5\sqrt{5x^2+3x+2}(101-338x)}{147\left(x^2-x+1\right)}+\frac{(44-237x)\sqrt{5x^2+3x+2}}{49\left(x^2-x+1\right)}-\frac{(1-2x)\sqrt{5x^2+3x+2}}{x^2-x+1}-\frac{49(2-x)\sqrt{5x^2+3x+2}}{3\left(x^2-x+1\right)}+\frac{10(1-2x)\sqrt{5x^2+3x+2}}{3\left(x^2-x+1\right)^2}+\frac{2(2-x)\sqrt{5x^2+3x+2}}{\left(x^2-x+1\right)^2}-\frac{34(1-2x)}{3\left(x^2-x+1\right)}+\frac{5(19x+37)}{3\left(x^2-x+1\right)}-\frac{2(37-17x)}{3\left(x^2-x+1\right)^2}-19\log\left(x^2-x+1\right)$$

input
```
Int[x^2/(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^3,x]
```

output
```
(-2*(37 - 17*x))/(3*(1 - x + x^2)^2) - (34*(1 - 2*x))/(3*(1 - x + x^2)) +
(5*(37 + 19*x))/(3*(1 - x + x^2)) + (10*(1 - 2*x)*Sqrt[2 + 3*x + 5*x^2])/(
3*(1 - x + x^2)^2) + (2*(2 - x)*Sqrt[2 + 3*x + 5*x^2])/(1 - x + x^2)^2 + (
5*(101 - 338*x)*Sqrt[2 + 3*x + 5*x^2])/(147*(1 - x + x^2)) + ((44 - 237*x)
*Sqrt[2 + 3*x + 5*x^2])/(49*(1 - x + x^2)) - ((1 - 2*x)*Sqrt[2 + 3*x + 5*x
^2])/(1 - x + x^2) - (49*(2 - x)*Sqrt[2 + 3*x + 5*x^2])/(3*(1 - x + x^2))
+ 17*Sqrt[5]*ArcSinh[(3 + 10*x)/Sqrt[31]] + (514*ArcTan[(1 - 2*x)/Sqrt[3]]
)/(3*Sqrt[3]) - (514*ArcTan[(5 - 4*x)/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])])/(3
*Sqrt[3]) - 38*ArcTanh[(1 + 2*x)/Sqrt[2 + 3*x + 5*x^2]] - 19*Log[1 - x + x
^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 1178, normalized size of antiderivative = 2.43

method	result	size
trager	Expression too large to display	1178
default	Expression too large to display	10342

input
```
int(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(77*x^3-317*x^2+243*x-200)*x/(x^2-x+1)^2+2/3*(3*x^3-47*x^2+34*x-33)/(
x^2-x+1)^2*(5*x^2+3*x+2)^(1/2)-2/3*ln(-162*RootOf(3*_Z^2+342*_Z+75796)^2*R
ootOf(4*_Z^2-456*_Z-9)^2*x+57483*RootOf(3*_Z^2+342*_Z+75796)^2*RootOf(4*_Z
^2-456*_Z-9)*x+209748*RootOf(3*_Z^2+342*_Z+75796)*RootOf(4*_Z^2-456*_Z-9)^
2*x-2164968*RootOf(3*_Z^2+342*_Z+75796)^2*x+4797162*RootOf(3*_Z^2+342*_Z+7
5796)*RootOf(4*_Z^2-456*_Z-9)*(5*x^2+3*x+2)^(1/2)-1375902*RootOf(3*_Z^2+34
2*_Z+75796)*RootOf(4*_Z^2-456*_Z-9)*x+191738960*RootOf(4*_Z^2-456*_Z-9)^2*
x-13447782*RootOf(3*_Z^2+342*_Z+75796)*RootOf(4*_Z^2-456*_Z-9)-734673564*(
5*x^2+3*x+2)^(1/2)*RootOf(3*_Z^2+342*_Z+75796)-1889954613*RootOf(3*_Z^2+34
2*_Z+75796)*x-7386528492*RootOf(4*_Z^2-456*_Z-9)*(5*x^2+3*x+2)^(1/2)-37559
822580*RootOf(4*_Z^2-456*_Z-9)*x+576013329*RootOf(3*_Z^2+342*_Z+75796)-831
8698332*RootOf(4*_Z^2-456*_Z-9)+846558481104*(5*x^2+3*x+2)^(1/2)+183781968
5520*x+838675407024)*RootOf(3*_Z^2+342*_Z+75796)-76*ln(-162*RootOf(3*_Z^2+
342*_Z+75796)^2*RootOf(4*_Z^2-456*_Z-9)^2*x+57483*RootOf(3*_Z^2+342*_Z+757
96)^2*RootOf(4*_Z^2-456*_Z-9)*x+209748*RootOf(3*_Z^2+342*_Z+75796)*RootOf(
4*_Z^2-456*_Z-9)^2*x-2164968*RootOf(3*_Z^2+342*_Z+75796)^2*x+4797162*RootO
f(3*_Z^2+342*_Z+75796)*RootOf(4*_Z^2-456*_Z-9)*(5*x^2+3*x+2)^(1/2)-1375902
*RootOf(3*_Z^2+342*_Z+75796)*RootOf(4*_Z^2-456*_Z-9)*x+191738960*RootOf(4*
_Z^2-456*_Z-9)^2*x-13447782*RootOf(3*_Z^2+342*_Z+75796)*RootOf(4*_Z^2-456*
_Z-9)-734673564*(5*x^2+3*x+2)^(1/2)*RootOf(3*_Z^2+342*_Z+75796)-1889954...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \frac{978\,x^3 - 1028\,\sqrt{3}(x^4 - 2\,x^3 + 3\,x^2 - 2\,x + 1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x - 1)\right) - 514\,\sqrt{3}(x^4 - 2\,x^3 + 3\,x^2 - 2\,x +}{}$$

input
```
integrate(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/18*(978*x^3 - 1028*sqrt(3)*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*arctan(1/3*sq
rt(3)*(2*x - 1)) - 514*sqrt(3)*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*arctan(1/3*
(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) + 31*sqrt(3)*(x^2 - 2*x))/(11*x
^2 - 12*x - 8)) - 514*sqrt(3)*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*arctan(1/3*(
4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) - 31*sqrt(3)*(x^2 - 2*x))/(11*x^
2 - 12*x - 8)) + 153*sqrt(5)*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*log(-4*sqrt(5
)*sqrt(5*x^2 + 3*x + 2)*(10*x + 3) - 200*x^2 - 120*x - 49) - 72*x^2 - 342*
(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*log(x^2 - x + 1) - 171*(x^4 - 2*x^3 + 3*x^
2 - 2*x + 1)*log((9*x^2 + 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^2
) + 171*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*log((9*x^2 - 2*sqrt(5*x^2 + 3*x +
2)*(2*x + 1) + 7*x + 3)/x^2) + 12*(3*x^3 - 47*x^2 + 34*x - 33)*sqrt(5*x^2
+ 3*x + 2) + 276*x + 462)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)
```

Sympy [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx=\int \frac{x^2}{\left(2x+\sqrt{5x^2+3x+2}+1\right)^3}\,dx$$

input
```
integrate(x**2/(1+2*x+(5*x**2+3*x+2)**(1/2))**3,x)
```

output
```
Integral(x**2/(2*x + sqrt(5*x**2 + 3*x + 2) + 1)**3, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx=\int \frac{x^2}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^3}\,dx$$

input
```
integrate(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x^2/(2*x + sqrt(5*x^2 + 3*x + 2) + 1)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="giac")
```

output
```
-514/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 17*sqrt(5)*log(-2*sqrt(5)*(
sqrt(5)*x - sqrt(5*x^2 + 3*x + 2)) - 3) + 514/3*(sqrt(5) + 2)*arctan(-(2*s
qrt(5)*x - sqrt(5) - 2*sqrt(5*x^2 + 3*x + 2) - 4)/(sqrt(15) + 2*sqrt(3)))/
(sqrt(15) + 2*sqrt(3)) - 514/3*(sqrt(5) - 2)*arctan(-(2*sqrt(5)*x - sqrt(5
) - 2*sqrt(5*x^2 + 3*x + 2) + 4)/(sqrt(15) - 2*sqrt(3)))/(sqrt(15) - 2*sqr
t(3)) + 2/3*(401*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^7 - 107*sqrt(5)*(sqrt
(5)*x - sqrt(5*x^2 + 3*x + 2))^6 + 1757*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2)
)^5 + 6621*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^4 + 44512*(sqrt(5)*
x - sqrt(5*x^2 + 3*x + 2))^3 + 29123*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x
 + 2))^2 + 45133*sqrt(5)*x + 5265*sqrt(5) - 45133*sqrt(5*x^2 + 3*x + 2))/(
(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^4 - 2*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2
+ 3*x + 2))^3 + 13*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 + 16*sqrt(5)*(sqr
t(5)*x - sqrt(5*x^2 + 3*x + 2)) + 19)^2 + 1/3*(163*x^3 - 12*x^2 + 46*x + 7
7)/(x^2 - x + 1)^2 - 19*log((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - (sqrt(
5)*x - sqrt(5*x^2 + 3*x + 2))*(sqrt(5) + 4) + 5*sqrt(5) + 12) + 19*log((sq
rt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - (sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))*(
sqrt(5) - 4) - 5*sqrt(5) + 12) - 19*log(x^2 - x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \int \frac{x^2}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^3}\,dx$$

input
```
int(x^2/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^3,x)
```

output
```
int(x^2/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^3, x)
```

Reduce [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx$$
$$=\int \frac{x^2}{17\sqrt{5x^2+3x+2}\,x^2+15\sqrt{5x^2+3x+2}\,x+5\sqrt{5x^2+3x+2}+38x^3+45x^2+27x+7}dx$$

input
```
int(x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x)
```

output
```
int(x**2/(17*sqrt(5*x**2 + 3*x + 2)*x**2 + 15*sqrt(5*x**2 + 3*x + 2)*x + 5
*sqrt(5*x**2 + 3*x + 2) + 38*x**3 + 45*x**2 + 27*x + 7),x)
```

3.28 $\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 322

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = -\frac{2\left(269+231\sqrt{2}\right)-\frac{5\left(7+13\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)^2}+\frac{811+179\sqrt{2}-\frac{310\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)}+\frac{620\arctan\left(\frac{3-4\sqrt{2}-\frac{2\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
-1/3*(538+462*2^(1/2)-5*(7+13*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)/(1
-5*2^(1/2)-(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+(1-2^(1/2))*(2^(1
/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)^2+(811+179*2^(1/2)-310*(2^(1/2)-(5*x^2+3*x
+2)^(1/2))/x)/(3-15*2^(1/2)-3*(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/
x+3*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)+620/9*arctan(1/3*(3-4
*2^(1/2)-2*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.25 (sec) , antiderivative size = 1077, normalized size of antiderivative = 3.34

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[x/(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^3,x]
```

output
```
(2*(-20 + 37*x))/(3*(1 - x + x^2)^2) + (202 - 41*x)/(3 - 3*x + 3*x^2) + (2
*Sqrt[2 + 3*x + 5*x^2]*(-29 + 41*x - 52*x^2 + 25*x^3))/(3*(1 - x + x^2)^2)
 - (310*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (((155*I)/3)*(2*I + Sqrt
[3])*ArcTan[(3*(9*(7 + (8*I)*Sqrt[3]) + 3*(71 + (4*I)*Sqrt[3])*x + (163 +
(68*I)*Sqrt[3])*x^2 + 96*(1 - (2*I)*Sqrt[3])*x^3 + (-16 + (80*I)*Sqrt[3])*
x^4))/(72*I - 42*Sqrt[3] + 6*(-80*I + 17*Sqrt[3])*x^4 + 21*Sqrt[3 - (12*I)
*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2] + x^2*(348*I + 206*Sqrt[3] - 49*Sqrt[3 - (
12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + x*(348*I - 104*Sqrt[3] + 49*Sqrt[3
 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + x^3*(312*I + 340*Sqrt[3] + 70*
Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]))])/Sqrt[3 - (12*I)*Sqrt[3]
] + (155*(2 + I*Sqrt[3])*ArcTan[(3*(-63 + (72*I)*Sqrt[3] + (3*I)*(71*I + 4
*Sqrt[3])*x + (-163 + (68*I)*Sqrt[3])*x^2 + (-96 - (192*I)*Sqrt[3])*x^3 +
16*(1 + (5*I)*Sqrt[3])*x^4))/(-72*I - 42*Sqrt[3] - 4*(87*I + 26*Sqrt[3])*x
 + 6*(80*I + 17*Sqrt[3])*x^4 + 21*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x +
5*x^2] + 49*Sqrt[3 + (12*I)*Sqrt[3]]*x*Sqrt[2 + 3*x + 5*x^2] + x^2*(-348*I
 + 206*Sqrt[3] - 49*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + x^3*
(-312*I + 340*Sqrt[3] + 70*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2])
)])/(3*Sqrt[3 + (12*I)*Sqrt[3]]) + (155*(-2*I + Sqrt[3])*Log[16*(1 - x + x
^2)^2])/(6*Sqrt[3 + (12*I)*Sqrt[3]]) + (155*(2*I + Sqrt[3])*Log[16*(1 - x
+ x^2)^2])/(6*Sqrt[3 - (12*I)*Sqrt[3]]) - (155*(2*I + Sqrt[3])*Log[(1 -...
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{5x^2+3x+2}+2x+1\right)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{-121x-34}{(x^2-x+1)^2}+\frac{17x\sqrt{5x^2+3x+2}}{(x^2-x+1)^2}+\frac{32\sqrt{5x^2+3x+2}}{(x^2-x+1)^2}+\frac{20x\sqrt{5x^2+3x+2}}{(x^2-x+1)^3}-\frac{32\sqrt{5x^2+3x+2}}{(x^2-x+1)^3}-\frac{3}{x^2-}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{310\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{3\sqrt{3}}+42\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}}\right)-\frac{68\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{8\sqrt{5x^2+3x+2}(101-338x)}{147\left(x^2-x+1\right)}-\frac{5(44-237x)\sqrt{5x^2+3x+2}}{147\left(x^2-x+1\right)}-\frac{32(1-2x)\sqrt{5x^2+3x+2}}{3\left(x^2-x+1\right)}-\frac{17(2-x)\sqrt{5x^2+3x+2}}{3\left(x^2-x+1\right)}+\frac{16(1-2x)\sqrt{5x^2+3x+2}}{3\left(x^2-x+1\right)^2}-\frac{10(2-x)\sqrt{5x^2+3x+2}}{3\left(x^2-x+1\right)^2}+\frac{92-63x}{x^2-x+1}-\frac{74(1-2x)}{3\left(x^2-x+1\right)}-\frac{2(20-37x)}{3\left(x^2-x+1\right)^2}$$

input
```
Int[x/(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^3,x]
```

output
```
(-2*(20 - 37*x))/(3*(1 - x + x^2)^2) + (92 - 63*x)/(1 - x + x^2) - (74*(1
 - 2*x))/(3*(1 - x + x^2)) + (16*(1 - 2*x)*Sqrt[2 + 3*x + 5*x^2])/(3*(1 - x
 + x^2)^2) - (10*(2 - x)*Sqrt[2 + 3*x + 5*x^2])/(3*(1 - x + x^2)^2) + (8*(
101 - 338*x)*Sqrt[2 + 3*x + 5*x^2])/(147*(1 - x + x^2)) - (5*(44 - 237*x)*
Sqrt[2 + 3*x + 5*x^2])/(147*(1 - x + x^2)) - (32*(1 - 2*x)*Sqrt[2 + 3*x +
5*x^2])/(3*(1 - x + x^2)) - (17*(2 - x)*Sqrt[2 + 3*x + 5*x^2])/(3*(1 - x +
 x^2)) - (68*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3]) + 42*Sqrt[3]*ArcTan[(1
 - 2*x)/Sqrt[3]] - (310*ArcTan[(5 - 4*x)/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])])
/(3*Sqrt[3])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.39

method	result
trager	$-\frac{(162x^3-283x^2+243x-155)x}{3(x^2-x+1)^2}+\frac{2(25x^3-52x^2+41x-29)\sqrt{5x^2+3x+2}}{3(x^2-x+1)^2}-\frac{310\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{4\,\mathrm{RootOf}\left(\_Z^2+3\right)x-5}{\mathrm{RootOf}\left(\right.}\right.}{9}$
default	Expression too large to display

input
```
int(x/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(162*x^3-283*x^2+243*x-155)*x/(x^2-x+1)^2+2/3*(25*x^3-52*x^2+41*x-29)
/(x^2-x+1)^2*(5*x^2+3*x+2)^(1/2)-310/9*RootOf(_Z^2+3)*ln((4*RootOf(_Z^2+3)
*x-5*RootOf(_Z^2+3)+3*(5*x^2+3*x+2)^(1/2))/(RootOf(_Z^2+3)*x-x+2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.52

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx =$$
$$-\frac{123\,x^3+310\,\sqrt{3}(x^4-2\,x^3+3\,x^2-2\,x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)-155\,\sqrt{3}(x^4-2\,x^3+3\,x^2-2\,x}{9\,(x^4-2\,x}$$

input
```
integrate(x/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="fricas")
```

output
```
-1/9*(123*x^3 + 310*sqrt(3)*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*arctan(1/3*sqr
t(3)*(2*x - 1)) - 155*sqrt(3)*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*arctan(1/6*s
qrt(3)*sqrt(5*x^2 + 3*x + 2)*(x^2 - 49*x + 19)/(20*x^3 - 13*x^2 - 7*x - 10
)) - 729*x^2 - 6*(25*x^3 - 52*x^2 + 41*x - 29)*sqrt(5*x^2 + 3*x + 2) + 507
*x - 486)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)
```

Sympy [F]

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \int \frac{x}{\left(2x+\sqrt{5x^2+3x+2}+1\right)^3}\,dx$$

input
```
integrate(x/(1+2*x+(5*x**2+3*x+2)**(1/2))**3,x)
```

output
```
Integral(x/(2*x + sqrt(5*x**2 + 3*x + 2) + 1)**3, x)
```

Maxima [F]

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \int \frac{x}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^3}\,dx$$

input
```
integrate(x/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x/(2*x + sqrt(5*x^2 + 3*x + 2) + 1)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.38

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx$$
$$= -\frac{310}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{310\left(\sqrt{5}+2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5x^2+3x+2}-4}{\sqrt{15}+2\sqrt{3}}\right)}{3\left(\sqrt{15}+2\sqrt{3}\right)}$$
$$- \frac{310\left(\sqrt{5}-2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5x^2+3x+2}+4}{\sqrt{15}-2\sqrt{3}}\right)}{3\left(\sqrt{15}-2\sqrt{3}\right)}$$
$$- \frac{2\left(55\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^7 - 898\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^6 + 1742\left(\sqrt{5}x-\sqrt{5x^2+3x+2\right.}\right.}{3\left(\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^4 - 2\right.}$$
$$- \frac{41x^3-243x^2+169x-162}{3\left(x^2-x+1\right)^2}$$

input
```
integrate(x/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="giac")
```

output
```
-310/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 310/3*(sqrt(5) + 2)*arctan(
-(2*sqrt(5)*x - sqrt(5) - 2*sqrt(5*x^2 + 3*x + 2) - 4)/(sqrt(15) + 2*sqrt(
3)))/(sqrt(15) + 2*sqrt(3)) - 310/3*(sqrt(5) - 2)*arctan(-(2*sqrt(5)*x - s
qrt(5) - 2*sqrt(5*x^2 + 3*x + 2) + 4)/(sqrt(15) - 2*sqrt(3)))/(sqrt(15) -
2*sqrt(3)) - 2/3*(55*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^7 - 898*sqrt(5)*(
sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^6 + 1742*(sqrt(5)*x - sqrt(5*x^2 + 3*x
+ 2))^5 - 2470*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^4 - 76551*(sqrt
(5)*x - sqrt(5*x^2 + 3*x + 2))^3 - 66727*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 +
 3*x + 2))^2 - 112695*sqrt(5)*x - 13715*sqrt(5) + 112695*sqrt(5*x^2 + 3*x
+ 2))/((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^4 - 2*sqrt(5)*(sqrt(5)*x - sqrt
(5*x^2 + 3*x + 2))^3 + 13*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 + 16*sqrt(
5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2)) + 19)^2 - 1/3*(41*x^3 - 243*x^2 + 1
69*x - 162)/(x^2 - x + 1)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \int \frac{x\,\sqrt{5\,x^2+3\,x+2}\,\left(17\,x^2+15\,x+5\right)}{{\left(x^2-x+1\right)}^3}\,dx - \frac{310\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9} - \frac{\frac{41\,x^3}{3}-81\,x^2+\frac{169\,x}{3}-54}{x^4-2\,x^3+3\,x^2-2\,x+1}$$

input
```
int(x/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^3,x)
```

output
```
int((x*(3*x + 5*x^2 + 2)^(1/2)*(15*x + 17*x^2 + 5))/(x^2 - x + 1)^3, x) -
(310*3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9 - ((169*x)/3 - 81*x^2 +
(41*x^3)/3 - 54)/(3*x^2 - 2*x - 2*x^3 + x^4 + 1)
```

Reduce [F]

$$\int \frac{x}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \int \frac{x}{17\sqrt{5x^2+3x+2}\,x^2+15\sqrt{5x^2+3x+2}\,x+5\sqrt{5x^2+3x+2}+38x^3+45x^2+27x+7}dx$$

input
```
int(x/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x)
```

output
```
int(x/(17*sqrt(5*x**2 + 3*x + 2)*x**2 + 15*sqrt(5*x**2 + 3*x + 2)*x + 5*sq
rt(5*x**2 + 3*x + 2) + 38*x**3 + 45*x**2 + 27*x + 7),x)
```

3.29 $\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 335

$$\begin{aligned}\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3} dx &= \frac{157+42\sqrt{2}-\frac{\left(161+71\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)^2} \\ &\quad -\frac{\left(1+\sqrt{2}\right)\left(327-526\sqrt{2}-\frac{2\left(130-121\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}\right)}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)} \\ &\quad +\frac{496\arctan\left(\frac{3-4\sqrt{2}-\frac{2\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3}}\right)}{3\sqrt{3}}\end{aligned}$$

output
```
1/3*(157+42*2^(1/2)-(161+71*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)/(1-5
*2^(1/2)-(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+(1-2^(1/2))*(2^(1/2
)-(5*x^2+3*x+2)^(1/2))^2/x^2)^2-(1+2^(1/2))*(327-526*2^(1/2)-2*(130-121*2^
(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)/(3-15*2^(1/2)-3*(3-4*2^(1/2))*(2^(
1/2)-(5*x^2+3*x+2)^(1/2))/x+3*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))^2/
x^2)+496/9*arctan(1/3*(3-4*2^(1/2)-2*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1
/2))/x)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx$$
$$= \frac{1}{9}\Bigg(\frac{459-609x+729x^2-372x^3}{(1-x+x^2)^2}+\frac{6\sqrt{2+3x+5x^2}(-22+37x-35x^2+26x^3)}{(1-x+x^2)^2}$$
$$-96\sqrt{15\left(9+4\sqrt{5}\right)}\arctan\left(\sqrt{3+\frac{4\sqrt{5}}{3}}\left(-16+7\sqrt{5}+2\left(-20+9\sqrt{5}\right)x-18\sqrt{2+3x+5x^2}+8\sqrt{5}\sqrt{2+\right.\right.$$
$$+16\sqrt{2163-72\sqrt{5}}\arctan\left(\frac{3+10x+4\sqrt{2+3x+5x^2}-2\sqrt{5}\left(1+2x+\sqrt{2+3x+5x^2}\right)}{\sqrt{3}}\right)\Bigg)$$

input
```
Integrate[(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^(-3),x]
```

output
```
((459 - 609*x + 729*x^2 - 372*x^3)/(1 - x + x^2)^2 + (6*Sqrt[2 + 3*x + 5*x
^2]*(-22 + 37*x - 35*x^2 + 26*x^3))/(1 - x + x^2)^2 - 96*Sqrt[15*(9 + 4*Sq
rt[5])]*ArcTan[Sqrt[3 + (4*Sqrt[5])/3]*(-16 + 7*Sqrt[5] + 2*(-20 + 9*Sqrt[
5])*x - 18*Sqrt[2 + 3*x + 5*x^2] + 8*Sqrt[5]*Sqrt[2 + 3*x + 5*x^2])] + 16*
Sqrt[2163 - 72*Sqrt[5]]*ArcTan[(3 + 10*x + 4*Sqrt[2 + 3*x + 5*x^2] - 2*Sqr
t[5]*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2]))/Sqrt[3]])/9
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{5x^2+3x+2}+2x+1\right)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{-38x-83}{(x^2-x+1)^2}+\frac{17\sqrt{5x^2+3x+2}}{(x^2-x+1)^2}+\frac{32x\sqrt{5x^2+3x+2}}{(x^2-x+1)^3}-\frac{12\sqrt{5x^2+3x+2}}{(x^2-x+1)^3}-\frac{4(18x-19)}{(x^2-x+1)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4154}{343}\sqrt{3}\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)-\frac{122450\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{1029\sqrt{3}}+\frac{248\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\sqrt{5x^2+3x+2}(101-338x)}{49(x^2-x+1)}-\frac{8(44-237x)\sqrt{5x^2+3x+2}}{147(x^2-x+1)}-\frac{17(1-2x)\sqrt{5x^2+3x+2}}{3(x^2-x+1)}+\frac{2(1-2x)\sqrt{5x^2+3x+2}}{(x^2-x+1)^2}-\frac{16(2-x)\sqrt{5x^2+3x+2}}{3(x^2-x+1)^2}+\frac{53-68x}{x^2-x+1}-\frac{40(1-2x)}{3(x^2-x+1)}+\frac{2(20x+17)}{3(x^2-x+1)^2}$$

input
```
Int[(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^(-3),x]
```

output
```
(2*(17 + 20*x))/(3*(1 - x + x^2)^2) + (53 - 68*x)/(1 - x + x^2) - (40*(1 -
 2*x))/(3*(1 - x + x^2)) + (2*(1 - 2*x)*Sqrt[2 + 3*x + 5*x^2])/(1 - x + x^
2)^2 - (16*(2 - x)*Sqrt[2 + 3*x + 5*x^2])/(3*(1 - x + x^2)^2) + ((101 - 33
8*x)*Sqrt[2 + 3*x + 5*x^2])/(49*(1 - x + x^2)) - (8*(44 - 237*x)*Sqrt[2 +
3*x + 5*x^2])/(147*(1 - x + x^2)) - (17*(1 - 2*x)*Sqrt[2 + 3*x + 5*x^2])/(
3*(1 - x + x^2)) + (248*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (122450*A
rcTan[(5 - 4*x)/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])])/(1029*Sqrt[3]) + (4154*S
qrt[3]*ArcTan[(5 - 4*x)/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])])/343
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.37

method	result
trager	$-\frac{(153x^3-182x^2+216x-103)x}{3(x^2-x+1)^2}+\frac{2(26x^3-35x^2+37x-22)\sqrt{5x^2+3x+2}}{3(x^2-x+1)^2}+\frac{248\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-\frac{4\,\mathrm{RootOf}\left(\_Z^2+3\right)x-}{\mathrm{RootO}}\right.}{9}$
default	Expression too large to display

input
```
int(1/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(153*x^3-182*x^2+216*x-103)*x/(x^2-x+1)^2+2/3*(26*x^3-35*x^2+37*x-22)
/(x^2-x+1)^2*(5*x^2+3*x+2)^(1/2)+248/9*RootOf(_Z^2+3)*ln(-(4*RootOf(_Z^2+3
)*x-5*RootOf(_Z^2+3)-3*(5*x^2+3*x+2)^(1/2))/(RootOf(_Z^2+3)*x+x-2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.50

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx =$$

$$-\frac{372\,x^3+248\,\sqrt{3}(x^4-2\,x^3+3\,x^2-2\,x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-124\,\sqrt{3}(x^4-2\,x^3+3\,x^2-2\,x}{9\,(x^4-2\,x}$$

input
```
integrate(1/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="fricas")
```

output
```
-1/9*(372*x^3 + 248*sqrt(3)*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*arctan(1/3*sqr
t(3)*(2*x - 1)) - 124*sqrt(3)*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*arctan(1/6*s
qrt(3)*sqrt(5*x^2 + 3*x + 2)*(x^2 - 49*x + 19)/(20*x^3 - 13*x^2 - 7*x - 10
)) - 729*x^2 - 6*(26*x^3 - 35*x^2 + 37*x - 22)*sqrt(5*x^2 + 3*x + 2) + 609
*x - 459)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)
```

Sympy [F]

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \int \frac{1}{\left(2x+\sqrt{5x^2+3x+2}+1\right)^3}\,dx$$

input
```
integrate(1/(1+2*x+(5*x**2+3*x+2)**(1/2))**3,x)
```

output
```
Integral((2*x + sqrt(5*x**2 + 3*x + 2) + 1)**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \int \frac{1}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^3}\,dx$$

input
```
integrate(1/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate((2*x + sqrt(5*x^2 + 3*x + 2) + 1)^(-3), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx$$

$$= -\frac{248}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{248\left(\sqrt{5}+2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5x^2+3x+2}-4}{\sqrt{15}+2\sqrt{3}}\right)}{3\left(\sqrt{15}+2\sqrt{3}\right)}$$

$$- \frac{248\left(\sqrt{5}-2\right)\arctan\left(-\frac{2\sqrt{5}x-\sqrt{5}-2\sqrt{5x^2+3x+2}+4}{\sqrt{15}-2\sqrt{3}}\right)}{3\left(\sqrt{15}-2\sqrt{3}\right)}$$

$$- \frac{2\left(248\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^7 - 875\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^6 + 2557\left(\sqrt{5}x-\sqrt{5x^2+3x+\cdots\right.\right.}{3\left(\left(\sqrt{5}x-\sqrt{5x^2+3x+2}\right)^4 - 2\cdots\right.}$$

$$- \frac{124x^3-243x^2+203x-153}{3\left(x^2-x+1\right)^2}$$

input
```
integrate(1/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="giac")
```

output
```
-248/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 248/3*(sqrt(5) + 2)*arctan(
-(2*sqrt(5)*x - sqrt(5) - 2*sqrt(5*x^2 + 3*x + 2) - 4)/(sqrt(15) + 2*sqrt(
3)))/(sqrt(15) + 2*sqrt(3)) - 248/3*(sqrt(5) - 2)*arctan(-(2*sqrt(5)*x - s
qrt(5) - 2*sqrt(5*x^2 + 3*x + 2) + 4)/(sqrt(15) - 2*sqrt(3)))/(sqrt(15) -
2*sqrt(3)) - 2/3*(248*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^7 - 875*sqrt(5)*
(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^6 + 2557*(sqrt(5)*x - sqrt(5*x^2 + 3*x
 + 2))^5 - 947*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^4 - 70665*(sqrt
(5)*x - sqrt(5*x^2 + 3*x + 2))^3 - 63830*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 +
 3*x + 2))^2 - 108264*sqrt(5)*x - 13162*sqrt(5) + 108264*sqrt(5*x^2 + 3*x
+ 2))/((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^4 - 2*sqrt(5)*(sqrt(5)*x - sqrt
(5*x^2 + 3*x + 2))^3 + 13*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 + 16*sqrt(
5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2)) + 19)^2 - 1/3*(124*x^3 - 243*x^2 +
203*x - 153)/(x^2 - x + 1)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \int \frac{1}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^3}\,dx$$

input
```
int(1/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^3,x)
```

output
```
int(1/(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^3, x)
```

Reduce [F]

$$\int \frac{1}{\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx$$
$$=\int \frac{1}{17\sqrt{5x^2+3x+2}\,x^2+15\sqrt{5x^2+3x+2}\,x+5\sqrt{5x^2+3x+2}+38x^3+45x^2+27x+7}dx$$

input
```
int(1/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x)
```

output
```
int(1/(17*sqrt(5*x**2 + 3*x + 2)*x**2 + 15*sqrt(5*x**2 + 3*x + 2)*x + 5*sq
rt(5*x**2 + 3*x + 2) + 38*x**3 + 45*x**2 + 27*x + 7),x)
```

3.30 $\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 446

$$\begin{aligned}\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx &= \frac{695+504\sqrt{2}-\frac{4\left(49+34\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)^2} \\ &+\frac{2\left(49\left(7+2\sqrt{2}\right)-\frac{\left(67-39\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}\right)}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)} \\ &-\frac{646\arctan\left(\frac{5+\sqrt{2}-\frac{2\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3\left(3+2\sqrt{2}\right)}}\right)}{3\sqrt{3}} \\ &-\left(7-5\sqrt{2}\right)\log\left(\frac{4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}}{x}\right) \\ &+7\log\left(9+4\sqrt{2}-\frac{\left(5+\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)\end{aligned}$$

output
```
1/3*(695+504*2^(1/2)-4*(49+34*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)/(1
-5*2^(1/2)-(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+(1-2^(1/2))*(2^(1
/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)^2+2*(343+98*2^(1/2)-(67-39*2^(1/2))*(2^(1/
2)-(5*x^2+3*x+2)^(1/2))/x)/(3-15*2^(1/2)-3*(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3
*x+2)^(1/2))/x+3*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)-646/9*ar
ctan((5+2^(1/2)-2*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)/(6^(1/2)+3^(1/2)))*3^(1
/2)-(7-5*2^(1/2))*ln((4+3*x-2*2^(1/2)*(5*x^2+3*x+2)^(1/2))/x)+7*ln(9+4*2^(
1/2)-(5+2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+(2^(1/2)-(5*x^2+3*x+2)^(1
/2))^2/x^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.72 (sec) , antiderivative size = 1146, normalized size of antiderivative = 2.57

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(x*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^3),x]
```

output
```
(74 - 34*x)/(3*(1 - x + x^2)^2) + (65 - 151*x)/(3 - 3*x + 3*x^2) + (2*Sqrt
[2 + 3*x + 5*x^2]*(-15 + 44*x - 31*x^2 + 27*x^3))/(3*(1 - x + x^2)^2) - (3
23*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + ((I/6)*(709*I + 281*Sqrt[3])*
ArcTan[(3*(9*(667681 + (871200*I)*Sqrt[3]) + 18*(1156301 + (158840*I)*Sqrt
[3])*x + (19667923 + (8775632*I)*Sqrt[3])*x^2 + 6*(2417865 - (3077512*I)*S
qrt[3])*x^3 + (102989 + (6316880*I)*Sqrt[3])*x^4))/(3183840*I - 5513277*Sq
rt[3] + 3*(-15938320*I + 5602673*Sqrt[3])*x^4 + 2218692*Sqrt[3 - (12*I)*Sq
rt[3]]*Sqrt[2 + 3*x + 5*x^2] + x^2*(36660048*I + 15867449*Sqrt[3] - 517694
8*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + 2*x*(18330024*I - 7506
523*Sqrt[3] + 2588474*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + x^
3*(51021744*I + 31209994*Sqrt[3] + 7395640*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2
 + 3*x + 5*x^2]))])/Sqrt[3 - (12*I)*Sqrt[3]] - ((-709*I + 281*Sqrt[3])*Arc
Tanh[(3*(9*(667681*I + 871200*Sqrt[3]) + 18*(1156301*I + 158840*Sqrt[3])*x
 + (19667923*I + 8775632*Sqrt[3])*x^2 - 6*(-2417865*I + 3077512*Sqrt[3])*x
^3 + (102989*I + 6316880*Sqrt[3])*x^4))/(-33*(96480*I + 167069*Sqrt[3]) +
3*(15938320*I + 5602673*Sqrt[3])*x^4 + 2218692*Sqrt[3 + (12*I)*Sqrt[3]]*Sq
rt[2 + 3*x + 5*x^2] + x^2*(-36660048*I + 15867449*Sqrt[3] - 5176948*Sqrt[3
 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + 2*x*(-18330024*I - 7506523*Sqr
t[3] + 2588474*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + x^3*(-510
21744*I + 31209994*Sqrt[3] + 7395640*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + ...
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.87, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{5x^2+3x+2}+2x+1\right)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{7(x-1)}{x^2-x+1}+\frac{5\sqrt{5x^2+3x+2}}{x}-\frac{5x\sqrt{5x^2+3x+2}}{x^2-x+1}+\frac{5\sqrt{5x^2+3x+2}}{x^2-x+1}-\frac{5x\sqrt{5x^2+3x+2}}{\left(x^2-x+1\right)^2}+\frac{5\sqrt{5x^2+3x}}{(x^2-x+1}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{323\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{3\sqrt{3}}+\frac{323\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+7\text{arctanh}\left(\frac{2x+1}{\sqrt{5x^2+3x+2}}\right)-5\sqrt{2}\text{arctanh}\left(\frac{3x+4}{2\sqrt{2}\sqrt{5x^2+3x+2}}\right)-\frac{5\sqrt{5x^2+3x+2}(101-338x)}{147\left(x^2-x+1\right)}-\frac{(44-237x)\sqrt{5x^2+3x+2}}{49\left(x^2-x+1\right)}-\frac{5(1-2x)\sqrt{5x^2+3x+2}}{3\left(x^2-x+1\right)}+\frac{5(2-x)\sqrt{5x^2+3x+2}}{3\left(x^2-x+1\right)}-\frac{10(1-2x)\sqrt{5x^2+3x+2}}{3\left(x^2-x+1\right)^2}-\frac{2(2-x)\sqrt{5x^2+3x+2}}{\left(x^2-x+1\right)^2}+\frac{31-83x}{3\left(x^2-x+1\right)}+\frac{34(1-2x)}{3\left(x^2-x+1\right)}+\frac{2(37-17x)}{3\left(x^2-x+1\right)^2}+\frac{7}{2}\log\left(x^2-x+1\right)-7\log(x)$$

input
```
Int[1/(x*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^3),x]
```

output
```
(2*(37 - 17*x))/(3*(1 - x + x^2)^2) + (31 - 83*x)/(3*(1 - x + x^2)) + (34*
(1 - 2*x))/(3*(1 - x + x^2)) - (10*(1 - 2*x)*Sqrt[2 + 3*x + 5*x^2])/(3*(1
- x + x^2)^2) - (2*(2 - x)*Sqrt[2 + 3*x + 5*x^2])/(1 - x + x^2)^2 - (5*(10
1 - 338*x)*Sqrt[2 + 3*x + 5*x^2])/(147*(1 - x + x^2)) - ((44 - 237*x)*Sqrt
[2 + 3*x + 5*x^2])/(49*(1 - x + x^2)) - (5*(1 - 2*x)*Sqrt[2 + 3*x + 5*x^2]
)/(3*(1 - x + x^2)) + (5*(2 - x)*Sqrt[2 + 3*x + 5*x^2])/(3*(1 - x + x^2))
+ (323*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (323*ArcTan[(5 - 4*x)/(Sqr
t[3]*Sqrt[2 + 3*x + 5*x^2])])/(3*Sqrt[3]) + 7*ArcTanh[(1 + 2*x)/Sqrt[2 + 3
*x + 5*x^2]] - 5*Sqrt[2]*ArcTanh[(4 + 3*x)/(2*Sqrt[2]*Sqrt[2 + 3*x + 5*x^2
])] - 7*Log[x] + (7*Log[1 - x + x^2])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 1480, normalized size of antiderivative = 3.32

method	result	size
trager	Expression too large to display	1480
default	Expression too large to display	7585

input
```
int(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(x-1)*(46*x^3-197*x^2+157*x-185)/(x^2-x+1)^2+2/3*(27*x^3-31*x^2+44*x-1
5)/(x^2-x+1)^2*(5*x^2+3*x+2)^(1/2)-2/3*ln(-(23364*RootOf(3*_Z^2-63*_Z+2641
3)^2*RootOf(4*_Z^2+84*_Z-9)^2*x-23364*RootOf(3*_Z^2-63*_Z+26413)^2*RootOf(
4*_Z^2+84*_Z-9)^2+743904*RootOf(3*_Z^2-63*_Z+26413)^2*RootOf(4*_Z^2+84*_Z-
9)*x-3893772*RootOf(3*_Z^2-63*_Z+26413)*RootOf(4*_Z^2+84*_Z-9)^2*x-743904*
RootOf(3*_Z^2-63*_Z+26413)^2*RootOf(4*_Z^2+84*_Z-9)+4344111*RootOf(3*_Z^2-
63*_Z+26413)^2*x-14302440*RootOf(3*_Z^2-63*_Z+26413)*RootOf(4*_Z^2+84*_Z-9
)*(5*x^2+3*x+2)^(1/2)+3893772*RootOf(3*_Z^2-63*_Z+26413)*RootOf(4*_Z^2+84*
_Z-9)^2-167582502*RootOf(3*_Z^2-63*_Z+26413)*RootOf(4*_Z^2+84*_Z-9)*x-2802
07712*RootOf(4*_Z^2+84*_Z-9)^2*x-4344111*RootOf(3*_Z^2-63*_Z+26413)^2+2623
27680*(5*x^2+3*x+2)^(1/2)*RootOf(3*_Z^2-63*_Z+26413)+32087232*RootOf(3*_Z^
2-63*_Z+26413)*RootOf(4*_Z^2+84*_Z-9)-764374158*RootOf(3*_Z^2-63*_Z+26413)
*x-2976564510*RootOf(4*_Z^2+84*_Z-9)*(5*x^2+3*x+2)^(1/2)+280207712*RootOf(
4*_Z^2+84*_Z-9)^2-16238088012*RootOf(4*_Z^2+84*_Z-9)*x-279398727*RootOf(3*
_Z^2-63*_Z+26413)-70467613155*(5*x^2+3*x+2)^(1/2)+8883067932*RootOf(4*_Z^2
+84*_Z-9)-221155036128*x+60207324138)/x)*RootOf(3*_Z^2-63*_Z+26413)+14*ln(
-(23364*RootOf(3*_Z^2-63*_Z+26413)^2*RootOf(4*_Z^2+84*_Z-9)^2*x-23364*Root
Of(3*_Z^2-63*_Z+26413)^2*RootOf(4*_Z^2+84*_Z-9)^2+743904*RootOf(3*_Z^2-63*
_Z+26413)^2*RootOf(4*_Z^2+84*_Z-9)*x-3893772*RootOf(3*_Z^2-63*_Z+26413)*Ro
otOf(4*_Z^2+84*_Z-9)^2*x-743904*RootOf(3*_Z^2-63*_Z+26413)^2*RootOf(4*_...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = -\frac{1812\,x^3+1292\,\sqrt{3}(x^4-2\,x^3+3\,x^2-2\,x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+646\,\sqrt{3}(x^4-2\,x^3+3\,x^2-2}{}$$

input
```
integrate(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="fricas")
```

output
```
-1/36*(1812*x^3 + 1292*sqrt(3)*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 646*sqrt(3)*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*arctan(1/
3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) + 31*sqrt(3)*(x^2 - 2*x))/(11
*x^2 - 12*x - 8)) + 646*sqrt(3)*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*arctan(1/3
*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) - 31*sqrt(3)*(x^2 - 2*x))/(11*
x^2 - 12*x - 8)) - 90*sqrt(2)*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*log((4*sqrt(
2)*sqrt(5*x^2 + 3*x + 2)*(3*x + 4) - 49*x^2 - 48*x - 32)/x^2) - 2592*x^2 -
 126*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*log(x^2 - x + 1) + 252*(x^4 - 2*x^3 +
 3*x^2 - 2*x + 1)*log(x) - 63*(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)*log((9*x^2 +
 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^2) + 63*(x^4 - 2*x^3 + 3*x
^2 - 2*x + 1)*log((9*x^2 - 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^
2) - 24*(27*x^3 - 31*x^2 + 44*x - 15)*sqrt(5*x^2 + 3*x + 2) + 3000*x - 166
8)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)
```

Sympy [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \int \frac{1}{x\left(2x+\sqrt{5x^2+3x+2}+1\right)^3}\,dx$$

input
```
integrate(1/x/(1+2*x+(5*x**2+3*x+2)**(1/2))**3,x)
```

output
```
Integral(1/(x*(2*x + sqrt(5*x**2 + 3*x + 2) + 1)**3), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \int \frac{1}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^3 x}\,dx$$

input
```
integrate(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(5*x^2 + 3*x + 2) + 1)^3*x), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.44

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="giac")
```

output
```
-323/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5*sqrt(2)*log(-1/2*abs(-2*s
qrt(5)*x - 2*sqrt(2) + 2*sqrt(5*x^2 + 3*x + 2))/(sqrt(5)*x - sqrt(2) - sqr
t(5*x^2 + 3*x + 2))) + 323/3*(sqrt(5) + 2)*arctan(-(2*sqrt(5)*x - sqrt(5)
- 2*sqrt(5*x^2 + 3*x + 2) - 4)/(sqrt(15) + 2*sqrt(3)))/(sqrt(15) + 2*sqrt(
3)) - 323/3*(sqrt(5) - 2)*arctan(-(2*sqrt(5)*x - sqrt(5) - 2*sqrt(5*x^2 +
3*x + 2) + 4)/(sqrt(15) - 2*sqrt(3)))/(sqrt(15) - 2*sqrt(3)) - 2/3*(311*(s
qrt(5)*x - sqrt(5*x^2 + 3*x + 2))^7 - 749*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2
+ 3*x + 2))^6 + 4835*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^5 + 1461*sqrt(5)*
(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^4 - 64814*(sqrt(5)*x - sqrt(5*x^2 + 3*
x + 2))^3 - 62761*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - 108575*s
qrt(5)*x - 13317*sqrt(5) + 108575*sqrt(5*x^2 + 3*x + 2))/((sqrt(5)*x - sqr
t(5*x^2 + 3*x + 2))^4 - 2*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^3 +
13*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*
x^2 + 3*x + 2)) + 19)^2 - 1/3*(151*x^3 - 216*x^2 + 250*x - 139)/(x^2 - x +
 1)^2 + 7/2*log((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - (sqrt(5)*x - sqrt(
5*x^2 + 3*x + 2))*(sqrt(5) + 4) + 5*sqrt(5) + 12) - 7/2*log((sqrt(5)*x - s
qrt(5*x^2 + 3*x + 2))^2 - (sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))*(sqrt(5) - 4
) - 5*sqrt(5) + 12) + 7/2*log(x^2 - x + 1) - 7*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx=\int \frac{1}{x\,{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)}^3}\,dx$$

input
```
int(1/(x*(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^3),x)
```

output
```
int(1/(x*(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^3), x)
```

Reduce [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx=\int \frac{1}{x\left(1+2x+\sqrt{5x^2+3x+2}\right)^3}dx$$

input
```
int(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x)
```

output
```
int(1/x/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x)
```

3.31 $\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 620

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx$$

$$= \frac{\left(10+7\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{4x} - \frac{31\left(7-5\sqrt{2}\right)x}{8\left(4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}\right)}$$

$$- \frac{\left(3880899+2744210\sqrt{2}\right)\left(2\left(223863189-158295179\sqrt{2}\right)-\frac{5\left(1500927047-1061315693\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{\left(1-\sqrt{2}\right)^4 x}\right)}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)^2}$$

$$+ \frac{\left(3880899+2744210\sqrt{2}\right)\left(118357619-83691475\sqrt{2}-\frac{26\left(2678084635-1893691806\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{\left(1-\sqrt{2}\right)^6 x}\right)}{3\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)^2}{x^2}\right)}$$

$$+ \frac{904\left(367296043199-259717522849\sqrt{2}\right)\arctan\left(\frac{3-4\sqrt{2}-\frac{2\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}}{\sqrt{3}}\right)}{3\sqrt{3}\left(1-\sqrt{2}\right)^{31}}$$

$$- \frac{3}{4}\left(64-45\sqrt{2}\right)\log\left(\frac{4+3x-2\sqrt{2}\sqrt{2+3x+5x^2}}{x}\right)$$

$$+ \frac{48\left(367296043199-259717522849\sqrt{2}\right)\log\left(1-5\sqrt{2}-\frac{\left(3-4\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5x^2}\right)}{x}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{2+3x+5 \cdots}\right)}{x^2} \cdots\right)}{\left(1-\sqrt{2}\right)^{31}}$$

output
```
1/4*(10+7*2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x-31*(7-5*2^(1/2))*x/(32+
24*x-16*2^(1/2)*(5*x^2+3*x+2)^(1/2))-1/3*(3880899+2744210*2^(1/2))*(447726
378-316590358*2^(1/2)-5*(1500927047-1061315693*2^(1/2))*(2^(1/2)-(5*x^2+3*
x+2)^(1/2))/(1-2^(1/2))^4/x)/(1-5*2^(1/2)-(3-4*2^(1/2))*(2^(1/2)-(5*x^2+3*
x+2)^(1/2))/x+(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))^2/x^2)^2+(3880899+
2744210*2^(1/2))*(118357619-83691475*2^(1/2)-26*(2678084635-1893691806*2^(
1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/(1-2^(1/2))^6/x)/(3-15*2^(1/2)-3*(3-4*
2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+3*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x
+2)^(1/2))^2/x^2)+904/9*(367296043199-259717522849*2^(1/2))*arctan(1/3*(3-
4*2^(1/2)-2*(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))/x)*3^(1/2))*3^(1/2)/
(1-2^(1/2))^31-3/4*(64-45*2^(1/2))*ln((4+3*x-2*2^(1/2)*(5*x^2+3*x+2)^(1/2)
)/x)+48*(367296043199-259717522849*2^(1/2))*ln(1-5*2^(1/2)-(3-4*2^(1/2))*(
2^(1/2)-(5*x^2+3*x+2)^(1/2))/x+(1-2^(1/2))*(2^(1/2)-(5*x^2+3*x+2)^(1/2))^2
/x^2)/(1-2^(1/2))^31
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.58 (sec) , antiderivative size = 1169, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(x^2*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^3),x]
```

output
```
7/x + (40 - 74*x)/(3*(1 - x + x^2)^2) + (26 - 175*x)/(3 - 3*x + 3*x^2) + (
Sqrt[2 + 3*x + 5*x^2]*(-15 + 52*x + 5*x^2 + 2*x^3 + 31*x^4))/(3*x*(1 - x +
 x^2)^2) - (452*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (((2*I)/3)*(334*
I + 41*Sqrt[3])*ArcTan[(3*(9*(53431 + (125000*I)*Sqrt[3]) + 3*(626711 + (3
98500*I)*Sqrt[3])*x + (3368563 + (2128292*I)*Sqrt[3])*x^2 + 96*(37885 - (1
1972*I)*Sqrt[3])*x^3 + 16*(110459 + (8405*I)*Sqrt[3])*x^4))/(6*(-547760*I
+ 780839*Sqrt[3])*x^4 + x^2*(3578748*I - 1380166*Sqrt[3] - 816193*Sqrt[3 -
 (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + 3*(-633000*I - 361634*Sqrt[3] +
116599*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) + 2*x^3*(7048092*I
+ 366662*Sqrt[3] + 582995*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2])
+ x*(3578748*I - 3502736*Sqrt[3] + 816193*Sqrt[3 - (12*I)*Sqrt[3]]*Sqrt[2
+ 3*x + 5*x^2]))])/Sqrt[3 - (12*I)*Sqrt[3]] - (2*(-334*I + 41*Sqrt[3])*Arc
Tanh[(3*(9*(53431*I + 125000*Sqrt[3]) + 3*(626711*I + 398500*Sqrt[3])*x +
(3368563*I + 2128292*Sqrt[3])*x^2 - 96*(-37885*I + 11972*Sqrt[3])*x^3 + 16
*(110459*I + 8405*Sqrt[3])*x^4))/(-4*(894687*I + 875684*Sqrt[3])*x + 6*(54
7760*I + 780839*Sqrt[3])*x^4 + 816193*Sqrt[3 + (12*I)*Sqrt[3]]*x*Sqrt[2 +
3*x + 5*x^2] + 3*(633000*I - 361634*Sqrt[3] + 116599*Sqrt[3 + (12*I)*Sqrt[
3]]*Sqrt[2 + 3*x + 5*x^2]) + 2*x^3*(-7048092*I + 366662*Sqrt[3] + 582995*S
qrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]) - x^2*(3578748*I + 1380166*
Sqrt[3] + 816193*Sqrt[3 + (12*I)*Sqrt[3]]*Sqrt[2 + 3*x + 5*x^2]))])/(3*...
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{5x^2+3x+2}+2x+1\right)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{30\sqrt{5x^2+3x+2}x}{x^2-x+1}-\frac{25\sqrt{5x^2+3x+2}x}{(x^2-x+1)^2}-\frac{20\sqrt{5x^2+3x+2}x}{(x^2-x+1)^3}+\frac{25\sqrt{5x^2+3x+2}}{x^2-x+1}+\frac{20\sqrt{5x^2+3x+2}}{(x^2-x+1)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-10\sqrt{3}\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)-\frac{362\arctan\left(\frac{5-4x}{\sqrt{3}\sqrt{5x^2+3x+2}}\right)}{3\sqrt{3}}+6\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}}\right)+\frac{398\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+48\text{arctanh}\left(\frac{2x+1}{\sqrt{5x^2+3x+2}}\right)-30\sqrt{2}\text{arctanh}\left(\frac{3x+4}{2\sqrt{2}\sqrt{5x^2+3x+2}}\right)-\frac{15\text{arctanh}\left(\frac{3x+4}{2\sqrt{2}\sqrt{5x^2+3x+2}}\right)}{2\sqrt{2}}-\frac{8\sqrt{5x^2+3x+2}(101-338x)}{147(x^2-x+1)}-\frac{5\sqrt{5x^2+3x+2}}{x}+\frac{5(44-237x)\sqrt{5x^2+3x+2}}{147(x^2-x+1)}-\frac{20(1-2x)\sqrt{5x^2+3x+2}}{3(x^2-x+1)}+\frac{25(2-x)\sqrt{5x^2+3x+2}}{3(x^2-x+1)}-\frac{16(1-2x)\sqrt{5x^2+3x+2}}{3(x^2-x+1)^2}+\frac{10(2-x)\sqrt{5x^2+3x+2}}{3(x^2-x+1)^2}+\frac{74(1-2x)}{3(x^2-x+1)}-\frac{9x+16}{x^2-x+1}+\frac{2(20-37x)}{3(x^2-x+1)^2}+24\log\left(x^2-x+1\right)+\frac{7}{x}-48\log(x)$$

input
```
Int[1/(x^2*(1 + 2*x + Sqrt[2 + 3*x + 5*x^2])^3),x]
```

output
```
7/x + (2*(20 - 37*x))/(3*(1 - x + x^2)^2) + (74*(1 - 2*x))/(3*(1 - x + x^2
)) - (16 + 9*x)/(1 - x + x^2) - (5*Sqrt[2 + 3*x + 5*x^2])/x - (16*(1 - 2*x
)*Sqrt[2 + 3*x + 5*x^2])/(3*(1 - x + x^2)^2) + (10*(2 - x)*Sqrt[2 + 3*x +
5*x^2])/(3*(1 - x + x^2)^2) - (8*(101 - 338*x)*Sqrt[2 + 3*x + 5*x^2])/(147
*(1 - x + x^2)) + (5*(44 - 237*x)*Sqrt[2 + 3*x + 5*x^2])/(147*(1 - x + x^2
)) - (20*(1 - 2*x)*Sqrt[2 + 3*x + 5*x^2])/(3*(1 - x + x^2)) + (25*(2 - x)*
Sqrt[2 + 3*x + 5*x^2])/(3*(1 - x + x^2)) + (398*ArcTan[(1 - 2*x)/Sqrt[3]])
/(3*Sqrt[3]) + 6*Sqrt[3]*ArcTan[(1 - 2*x)/Sqrt[3]] - (362*ArcTan[(5 - 4*x)
/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])])/(3*Sqrt[3]) - 10*Sqrt[3]*ArcTan[(5 - 4*
x)/(Sqrt[3]*Sqrt[2 + 3*x + 5*x^2])] + 48*ArcTanh[(1 + 2*x)/Sqrt[2 + 3*x +
5*x^2]] - (15*ArcTanh[(4 + 3*x)/(2*Sqrt[2]*Sqrt[2 + 3*x + 5*x^2])])/(2*Sqr
t[2]) - 30*Sqrt[2]*ArcTanh[(4 + 3*x)/(2*Sqrt[2]*Sqrt[2 + 3*x + 5*x^2])] -
48*Log[x] + 24*Log[1 - x + x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 1548, normalized size of antiderivative = 2.50

method	result	size
trager	Expression too large to display	1548
default	Expression too large to display	8830

input

```
int(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output

```
1/3*(x-1)*(162*x^4-316*x^3+329*x^2-207*x-21)/x/(x^2-x+1)^2+1/3*(31*x^4+2*x
^3+5*x^2+52*x-15)/x/(x^2-x+1)^2*(5*x^2+3*x+2)^(1/2)+1/3*RootOf(3*_Z^2-864*
_Z+266512)*ln((-11682*RootOf(3*_Z^2-864*_Z+266512)^2*RootOf(8*_Z^2+2304*_Z
+1863)^2*x+11682*RootOf(3*_Z^2-864*_Z+266512)^2*RootOf(8*_Z^2+2304*_Z+1863
)^2-1654911*RootOf(3*_Z^2-864*_Z+266512)^2*RootOf(8*_Z^2+2304*_Z+1863)*x-1
397856*RootOf(3*_Z^2-864*_Z+266512)*RootOf(8*_Z^2+2304*_Z+1863)^2*x+270196
560*RootOf(3*_Z^2-864*_Z+266512)*RootOf(8*_Z^2+2304*_Z+1863)*(5*x^2+3*x+2)
^(1/2)+1654911*RootOf(3*_Z^2-864*_Z+266512)^2*RootOf(8*_Z^2+2304*_Z+1863)+
85123143*RootOf(3*_Z^2-864*_Z+266512)^2*x+1397856*RootOf(3*_Z^2-864*_Z+266
512)*RootOf(8*_Z^2+2304*_Z+1863)^2+625759452*RootOf(3*_Z^2-864*_Z+266512)*
RootOf(8*_Z^2+2304*_Z+1863)*x+1691009440*RootOf(8*_Z^2+2304*_Z+1863)^2*x+1
44111971340*(5*x^2+3*x+2)^(1/2)*RootOf(3*_Z^2-864*_Z+266512)-204229058400*
RootOf(8*_Z^2+2304*_Z+1863)*(5*x^2+3*x+2)^(1/2)-85123143*RootOf(3*_Z^2-864
*_Z+266512)^2+1933968528*RootOf(3*_Z^2-864*_Z+266512)*RootOf(8*_Z^2+2304*_
Z+1863)+235279131354*RootOf(3*_Z^2-864*_Z+266512)*x-1691009440*RootOf(8*_Z
^2+2304*_Z+1863)^2-290117964960*RootOf(8*_Z^2+2304*_Z+1863)*x-196596664596
00*(5*x^2+3*x+2)^(1/2)+229962275316*RootOf(3*_Z^2-864*_Z+266512)-542589024
240*RootOf(8*_Z^2+2304*_Z+1863)-41219406253080*x-35688271684320)/x)-1/3*ln
(-(11682*RootOf(3*_Z^2-864*_Z+266512)^2*RootOf(8*_Z^2+2304*_Z+1863)^2*x-11
682*RootOf(3*_Z^2-864*_Z+266512)^2*RootOf(8*_Z^2+2304*_Z+1863)^2+165491...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 493, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = -\frac{3696\,x^4 - 3816\,x^3 + 3616\,\sqrt{3}(x^5 - 2\,x^4 + 3\,x^3 - 2\,x^2 + x)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + 1808\,\sqrt{3}(x^5 - 2\,x}{}$$

input
```
integrate(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="fricas")
```

output
```
-1/72*(3696*x^4 - 3816*x^3 + 3616*sqrt(3)*(x^5 - 2*x^4 + 3*x^3 - 2*x^2 + x
)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1808*sqrt(3)*(x^5 - 2*x^4 + 3*x^3 - 2*x^
2 + x)*arctan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) + 31*sqrt(3)*
(x^2 - 2*x))/(11*x^2 - 12*x - 8)) + 1808*sqrt(3)*(x^5 - 2*x^4 + 3*x^3 - 2*
x^2 + x)*arctan(1/3*(4*sqrt(3)*sqrt(5*x^2 + 3*x + 2)*(4*x - 5) - 31*sqrt(3
)*(x^2 - 2*x))/(11*x^2 - 12*x - 8)) - 1215*sqrt(2)*(x^5 - 2*x^4 + 3*x^3 -
2*x^2 + x)*log((4*sqrt(2)*sqrt(5*x^2 + 3*x + 2)*(3*x + 4) - 49*x^2 - 48*x
- 32)/x^2) + 5088*x^2 - 1728*(x^5 - 2*x^4 + 3*x^3 - 2*x^2 + x)*log(x^2 - x
 + 1) + 3456*(x^5 - 2*x^4 + 3*x^3 - 2*x^2 + x)*log(x) - 864*(x^5 - 2*x^4 +
 3*x^3 - 2*x^2 + x)*log((9*x^2 + 2*sqrt(5*x^2 + 3*x + 2)*(2*x + 1) + 7*x +
 3)/x^2) + 864*(x^5 - 2*x^4 + 3*x^3 - 2*x^2 + x)*log((9*x^2 - 2*sqrt(5*x^2
 + 3*x + 2)*(2*x + 1) + 7*x + 3)/x^2) - 24*(31*x^4 + 2*x^3 + 5*x^2 + 52*x
- 15)*sqrt(5*x^2 + 3*x + 2) - 576*x - 504)/(x^5 - 2*x^4 + 3*x^3 - 2*x^2 +
x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \int \frac{1}{x^2\left(2x+\sqrt{5x^2+3x+2}+1\right)^3}\,dx$$

input
```
integrate(1/x**2/(1+2*x+(5*x**2+3*x+2)**(1/2))**3,x)
```

output
```
Integral(1/(x**2*(2*x + sqrt(5*x**2 + 3*x + 2) + 1)**3), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \int \frac{1}{\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^3x^2}\,dx$$

input
```
integrate(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(5*x^2 + 3*x + 2) + 1)^3*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x, algorithm="giac")
```

output
```
-452/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 135/4*sqrt(2)*log(-1/2*abs(
-2*sqrt(5)*x - 2*sqrt(2) + 2*sqrt(5*x^2 + 3*x + 2))/(sqrt(5)*x - sqrt(2) -
 sqrt(5*x^2 + 3*x + 2))) + 452/3*(sqrt(5) + 2)*arctan(-(2*sqrt(5)*x - sqrt
(5) - 2*sqrt(5*x^2 + 3*x + 2) - 4)/(sqrt(15) + 2*sqrt(3)))/(sqrt(15) + 2*s
qrt(3)) - 452/3*(sqrt(5) - 2)*arctan(-(2*sqrt(5)*x - sqrt(5) - 2*sqrt(5*x^
2 + 3*x + 2) + 4)/(sqrt(15) - 2*sqrt(3)))/(sqrt(15) - 2*sqrt(3)) + 5*(3*sq
rt(5)*x + 4*sqrt(5) - 3*sqrt(5*x^2 + 3*x + 2))/((sqrt(5)*x - sqrt(5*x^2 +
3*x + 2))^2 - 2) - 2/3*(389*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^7 - 776*sq
rt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^6 + 7240*(sqrt(5)*x - sqrt(5*x^2
 + 3*x + 2))^5 + 7444*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^4 - 2622
3*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^3 - 38363*sqrt(5)*(sqrt(5)*x - sqrt(
5*x^2 + 3*x + 2))^2 - 72615*sqrt(5)*x - 9313*sqrt(5) + 72615*sqrt(5*x^2 +
3*x + 2))/((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^4 - 2*sqrt(5)*(sqrt(5)*x -
sqrt(5*x^2 + 3*x + 2))^3 + 13*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 + 16*s
qrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 3*x + 2)) + 19)^2 - 1/3*(154*x^4 - 159*x^
3 + 212*x^2 - 24*x - 21)/((x^2 - x + 1)^2*x) + 24*log((sqrt(5)*x - sqrt(5*
x^2 + 3*x + 2))^2 - (sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))*(sqrt(5) + 4) + 5*
sqrt(5) + 12) - 24*log((sqrt(5)*x - sqrt(5*x^2 + 3*x + 2))^2 - (sqrt(5)*x
- sqrt(5*x^2 + 3*x + 2))*(sqrt(5) - 4) - 5*sqrt(5) + 12) + 24*log(x^2 - x
+ 1) - 48*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx=\int \frac{1}{x^2\left(2\,x+\sqrt{5\,x^2+3\,x+2}+1\right)^3}\,dx$$

input
```
int(1/(x^2*(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^3),x)
```

output
```
int(1/(x^2*(2*x + (3*x + 5*x^2 + 2)^(1/2) + 1)^3), x)
```

Reduce [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{2+3x+5x^2}\right)^3}\,dx=\int \frac{1}{x^2\left(1+2x+\sqrt{5x^2+3x+2}\right)^3}dx$$

input
```
int(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x)
```

output
```
int(1/x^2/(1+2*x+(5*x^2+3*x+2)^(1/2))^3,x)
```

3.32 $\int \frac{x^2}{1+3x+\sqrt{-3-2x+4x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 209

$$\int \frac{x^2}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = \frac{1}{800}\left(2x+\sqrt{-3-2x+4x^2}\right) + \frac{1}{160}\left(2x+\sqrt{-3-2x+4x^2}\right)^2 + \frac{169}{128\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)^2} + \frac{65}{32\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)} - \frac{12}{125}\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right) + \frac{92}{125}\log\left(7+3x-20x^2-\sqrt{-3-2x+4x^2} - 10x\sqrt{-3-2x+4x^2}\right) - \frac{23}{16}\log\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)$$

output
```
1/400*x+1/800*(4*x^2-2*x-3)^(1/2)+1/160*(2*x+(4*x^2-2*x-3)^(1/2))^2+169/12
8/(1-4*x-2*(4*x^2-2*x-3)^(1/2))^2+65/(32-128*x-64*(4*x^2-2*x-3)^(1/2))-12/
125*arctan(1/2+5*x+5/2*(4*x^2-2*x-3)^(1/2))+92/125*ln(7+3*x-20*x^2-(4*x^2-
2*x-3)^(1/2)-10*x*(4*x^2-2*x-3)^(1/2))-23/16*ln(1-4*x-2*(4*x^2-2*x-3)^(1/2
))
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{1+3x+\sqrt{-3-2x+4x^2}}\,dx$$
$$= \frac{40x(-38+15x)+10(69-20x)\sqrt{-3-2x+4x^2}+192\arctan\left(\frac{3}{2}+x-\frac{1}{2}\sqrt{-3-2x+4x^2}\right)-69\log\left(1\right.}{2000}$$

input
```
Integrate[x^2/(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2]),x]
```

output
```
(40*x*(-38 + 15*x) + 10*(69 - 20*x)*Sqrt[-3 - 2*x + 4*x^2] + 192*ArcTan[3/
2 + x - Sqrt[-3 - 2*x + 4*x^2]/2] - 69*Log[1 - 4*x + 2*Sqrt[-3 - 2*x + 4*x
^2]] + 1472*Log[-5 - 5*x - 4*x^2 + (3 + 2*x)*Sqrt[-3 - 2*x + 4*x^2]])/2000
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{4x^2-2x-3}+3x+1}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{8\sqrt{4x^2-2x-3}x}{5(5x^2+8x+4)}-\frac{1}{5}\sqrt{4x^2-2x-3}+\frac{4(23x+19)}{25(5x^2+8x+4)}+\frac{4\sqrt{4x^2-2x-3}}{5(5x^2+8x+4)}+\frac{3x}{5}-\frac{19}{25}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{6}{125}\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)+\frac{6}{125}\arctan\left(\frac{5x}{2}+2\right)+\frac{1403\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)}{2000}+\frac{92}{125}\text{arctanh}\left(\frac{3x+1}{\sqrt{4x^2-2x-3}}\right)+\frac{3x^2}{10}+\frac{1}{40}(1-4x)\sqrt{4x^2-2x-3}+\frac{8}{25}\sqrt{4x^2-2x-3}+\frac{46}{125}\log\left(5x^2+8x+4\right)-\frac{19x}{25}$$

input
```
Int[x^2/(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2]),x]
```

output
```
(-19*x)/25 + (3*x^2)/10 + (8*Sqrt[-3 - 2*x + 4*x^2])/25 + ((1 - 4*x)*Sqrt[
-3 - 2*x + 4*x^2])/40 + (6*ArcTan[2 + (5*x)/2])/125 + (6*ArcTan[(8 + 7*x)/
(2*Sqrt[-3 - 2*x + 4*x^2])])/125 + (1403*ArcTanh[(1 - 4*x)/(2*Sqrt[-3 - 2*
x + 4*x^2])])/2000 + (92*ArcTanh[(1 + 3*x)/Sqrt[-3 - 2*x + 4*x^2]])/125 +
(46*Log[4 + 8*x + 5*x^2])/125
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(170) = 340$.

Time = 0.02 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.53

$$-\frac{(8x-2)\sqrt{4x^2-2x-3}}{80}-\frac{1403\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4x^2-2x-3}\right)\sqrt{4}}{4000}+\frac{8\sqrt{4x^2-2x-3}}{25}-\frac{32\sqrt{-\frac{833\left(\frac{8}{7}+x\right)^2}{\left(-\frac{1}{3}-x\right)^2}}}{}$$

input
```
int(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2)),x)
```

output
```
-1/80*(8*x-2)*(4*x^2-2*x-3)^(1/2)-1403/4000*ln(1/4*(4*x-1)*4^(1/2)+(4*x^2-
2*x-3)^(1/2))*4^(1/2)+8/25*(4*x^2-2*x-3)^(1/2)-32/2125*(-833*(8/7+x)^2/(-1
/3-x)^2+1989)^(1/2)*(42*arctanh(1/51*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2
))-19*arctan(7/2/(49*(8/7+x)^2/(-1/3-x)^2-117)*(-833*(8/7+x)^2/(-1/3-x)^2+
1989)^(1/2)*(8/7+x)/(-1/3-x)))/(-17*(49*(8/7+x)^2/(-1/3-x)^2-117)/((8/7+x)
/(-1/3-x)+1)^2)^(1/2)/((8/7+x)/(-1/3-x)+1)-8/425*(-833*(8/7+x)^2/(-1/3-x)^
2+1989)^(1/2)*(13*arctanh(1/51*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2))-16*
arctan(7/2/(49*(8/7+x)^2/(-1/3-x)^2-117)*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^
(1/2)*(8/7+x)/(-1/3-x)))/(-17*(49*(8/7+x)^2/(-1/3-x)^2-117)/((8/7+x)/(-1/3
-x)+1)^2)^(1/2)/((8/7+x)/(-1/3-x)+1)+6/85*(-833*(8/7+x)^2/(-1/3-x)^2+1989)
^(1/2)*(2*arctanh(1/51*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2))-9*arctan(7/
2/(49*(8/7+x)^2/(-1/3-x)^2-117)*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2)*(8/
7+x)/(-1/3-x)))/(-17*(49*(8/7+x)^2/(-1/3-x)^2-117)/((8/7+x)/(-1/3-x)+1)^2)
^(1/2)/((8/7+x)/(-1/3-x)+1)-19/25*x+46/125*ln(5*x^2+8*x+4)+6/125*arctan(5/
2*x+2)+3/10*x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{x^2}{1+3x+\sqrt{-3-2x+4x^2}}\,dx &= \frac{3}{10}\,x^2-\frac{1}{200}\,\sqrt{4\,x^2-2\,x-3}(20\,x-69)\\&-\frac{19}{25}\,x+\frac{6}{125}\,\arctan\left(\frac{5}{2}\,x+2\right)\\&-\frac{6}{125}\,\arctan\left(-x+\frac{1}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{3}{2}\right)\\&-\frac{6}{125}\,\arctan\left(-5\,x+\frac{5}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{1}{2}\right)\\&-\frac{46}{125}\,\log\left(20\,x^2-\sqrt{4\,x^2-2\,x-3}(10\,x+1)-3\,x\right.\\&\left.-7\right)+\frac{46}{125}\,\log\left(5\,x^2+8\,x+4\right)\\&+\frac{46}{125}\,\log\left(4\,x^2-\sqrt{4\,x^2-2\,x-3}(2\,x+3)+5\,x+5\right)\\&+\frac{1403}{2000}\,\log\left(-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right)\end{aligned}$$

input
```
integrate(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

output
```
3/10*x^2 - 1/200*sqrt(4*x^2 - 2*x - 3)*(20*x - 69) - 19/25*x + 6/125*arcta
n(5/2*x + 2) - 6/125*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - 6/125*
arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) - 46/125*log(20*x^2 - sqrt(
4*x^2 - 2*x - 3)*(10*x + 1) - 3*x - 7) + 46/125*log(5*x^2 + 8*x + 4) + 46/
125*log(4*x^2 - sqrt(4*x^2 - 2*x - 3)*(2*x + 3) + 5*x + 5) + 1403/2000*log
(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1)
```

Sympy [F]

$$\int \frac{x^2}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = \int \frac{x^2}{3x+\sqrt{4x^2-2x-3}+1}\,dx$$

input
```
integrate(x**2/(1+3*x+(4*x**2-2*x-3)**(1/2)),x)
```

output
```
Integral(x**2/(3*x + sqrt(4*x**2 - 2*x - 3) + 1), x)
```

Maxima [F]

$$\int \frac{x^2}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = \int \frac{x^2}{3\,x+\sqrt{4\,x^2-2\,x-3}+1}\,dx$$

input
```
integrate(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(3*x + sqrt(4*x^2 - 2*x - 3) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = \frac{3}{10}\,x^2 - \frac{1}{200}\,\sqrt{4\,x^2-2\,x-3}(20\,x-69) - \frac{19}{25}\,x + \frac{6}{125}\,\arctan\left(\frac{5}{2}\,x+2\right) - \frac{6}{125}\,\arctan\left(-x+\frac{1}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{3}{2}\right) - \frac{6}{125}\,\arctan\left(-5\,x+\frac{5}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{1}{2}\right) - \frac{46}{125}\,\log\left(5\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+4\,x-2\,\sqrt{4\,x^2-2\,x-3}+1\right) + \frac{46}{125}\,\log\left(\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+12\,x-6\,\sqrt{4\,x^2-2\,x-3}+13\right) + \frac{46}{125}\,\log\left(5\,x^2+8\,x+4\right) + \frac{1403}{2000}\,\log\left(\left|-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right|\right)$$

input
```
integrate(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

output
```
3/10*x^2 - 1/200*sqrt(4*x^2 - 2*x - 3)*(20*x - 69) - 19/25*x + 6/125*arcta
n(5/2*x + 2) - 6/125*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - 6/125*
arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) - 46/125*log(5*(2*x - sqrt(
4*x^2 - 2*x - 3))^2 + 4*x - 2*sqrt(4*x^2 - 2*x - 3) + 1) + 46/125*log((2*x
 - sqrt(4*x^2 - 2*x - 3))^2 + 12*x - 6*sqrt(4*x^2 - 2*x - 3) + 13) + 46/12
5*log(5*x^2 + 8*x + 4) + 1403/2000*log(abs(-4*x + 2*sqrt(4*x^2 - 2*x - 3)
+ 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{1+3x+\sqrt{-3-2x+4x^2}}\,dx=\int \frac{x^2}{3\,x+\sqrt{4\,x^2-2\,x-3}+1}\,dx$$

input
```
int(x^2/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1),x)
```

output
```
int(x^2/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.62

$$\begin{aligned}\int \frac{x^2}{1+3x+\sqrt{-3-2x+4x^2}}\,dx&=-\frac{12atan\left(\frac{5\sqrt{4x^2-2x-3}}{2}+5x+\frac{1}{2}\right)}{125}\\&\quad-\frac{\sqrt{4x^2-2x-3}\,x}{10}+\frac{69\sqrt{4x^2-2x-3}}{200}\\&\quad+\frac{92\log\left(\frac{80\sqrt{4x^2-2x-3}\,x+8\sqrt{4x^2-2x-3}+160x^2-24x-56}{\sqrt{13}}\right)}{125}\\&\quad-\frac{23\log\left(\frac{2\sqrt{4x^2-2x-3}+4x-1}{\sqrt{13}}\right)}{16}+\frac{3x^2}{10}-\frac{19x}{25}+\frac{79}{1600}\end{aligned}$$

input
```
int(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2)),x)
```

output
```
( - 768*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x + 1)/2) - 800*sqrt(4*x**2 -
2*x - 3)*x + 2760*sqrt(4*x**2 - 2*x - 3) + 5888*log((80*sqrt(4*x**2 - 2*x
- 3)*x + 8*sqrt(4*x**2 - 2*x - 3) + 160*x**2 - 24*x - 56)/sqrt(13)) - 1150
0*log((2*sqrt(4*x**2 - 2*x - 3) + 4*x - 1)/sqrt(13)) + 2400*x**2 - 6080*x
+ 395)/8000
```

3.33 $\int \frac{x}{1+3x+\sqrt{-3-2x+4x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{x}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = \frac{1}{20}\left(2x+\sqrt{-3-2x+4x^2}\right) - \frac{13}{8\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)} - \frac{16}{25}\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right) - \frac{19}{25}\log\left(7+3x-20x^2-\sqrt{-3-2x+4x^2}-10x\sqrt{-3-2x+4x^2}\right) + \frac{3}{2}\log\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)$$

output
```
1/10*x+1/20*(4*x^2-2*x-3)^(1/2)-13/(8-32*x-16*(4*x^2-2*x-3)^(1/2))-16/25*a
rctan(1/2+5*x+5/2*(4*x^2-2*x-3)^(1/2))-19/25*ln(7+3*x-20*x^2-(4*x^2-2*x-3)
^(1/2)-10*x*(4*x^2-2*x-3)^(1/2))+3/2*ln(1-4*x-2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.67

$$\int \frac{x}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = \frac{1}{50}\left(30x-10\sqrt{-3-2x+4x^2}+32\arctan\left(\frac{3}{2}+x-\frac{1}{2}\sqrt{-3-2x+4x^2}\right)+\log\left(1-4x+2\sqrt{-3-2x+4x^2}\right)-38\log\left(-5-5x-4x^2+(3+2x)\sqrt{-3-2x+4x^2}\right)\right)$$

input

```
Integrate[x/(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2]),x]
```

output

```
(30*x - 10*Sqrt[-3 - 2*x + 4*x^2] + 32*ArcTan[3/2 + x - Sqrt[-3 - 2*x + 4*
x^2]/2] + Log[1 - 4*x + 2*Sqrt[-3 - 2*x + 4*x^2]] - 38*Log[-5 - 5*x - 4*x^
2 + (3 + 2*x)*Sqrt[-3 - 2*x + 4*x^2]])/50
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{4x^2-2x-3}+3x+1}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{-19x-12}{5(5x^2+8x+4)}-\frac{x\sqrt{4x^2-2x-3}}{5x^2+8x+4}+\frac{3}{5}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{8}{25}\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)+\frac{8}{25}\arctan\left(\frac{5x}{2}+2\right)-\frac{37}{50}\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)-\frac{19}{25}\text{arctanh}\left(\frac{3x+1}{\sqrt{4x^2-2x-3}}\right)-\frac{1}{5}\sqrt{4x^2-2x-3}-\frac{19}{50}\log\left(5x^2+8x+4\right)+\frac{3x}{5}$$

input
```
Int[x/(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2]),x]
```

output
```
(3*x)/5 - Sqrt[-3 - 2*x + 4*x^2]/5 + (8*ArcTan[2 + (5*x)/2])/25 + (8*ArcTa
n[(8 + 7*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/25 - (37*ArcTanh[(1 - 4*x)/(2*Sqr
t[-3 - 2*x + 4*x^2])])/50 - (19*ArcTanh[(1 + 3*x)/Sqrt[-3 - 2*x + 4*x^2]])
/25 - (19*Log[4 + 8*x + 5*x^2])/50
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(127) = 254$.

Time = 0.02 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.20

$$\frac{37\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4x^2-2x-3}\right)\sqrt{4}}{100}-\frac{\sqrt{4x^2-2x-3}}{5}+\frac{16\sqrt{-\frac{833\left(\frac{8}{7}+x\right)^2}{\left(-\frac{1}{3}-x\right)^2}+1989}\left(13\,\text{arctanh}\left(\frac{\sqrt{-\frac{833\left(\frac{8}{7}+\right.}{\left(-\frac{1}{3}-x\right.}}}{51}\right.\right.}{425\sqrt{-\frac{17\left(\frac{49\left(\frac{8}{7}+x\right.}{\left(-\frac{1}{3}-x\right.}\right.}{\left(\frac{\frac{8}{7}+x}{-\frac{1}{3}-x}\right.}}}$$

input
```
int(x/(1+3*x+(4*x^2-2*x-3)^(1/2)),x)
```

output
```
37/100*ln(1/4*(4*x-1)*4^(1/2)+(4*x^2-2*x-3)^(1/2))*4^(1/2)-1/5*(4*x^2-2*x-
3)^(1/2)+16/425*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2)*(13*arctanh(1/51*(-
833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2))-16*arctan(7/2/(49*(8/7+x)^2/(-1/3-x)
^2-117)*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2)*(8/7+x)/(-1/3-x)))/(-17*(49
*(8/7+x)^2/(-1/3-x)^2-117)/((8/7+x)/(-1/3-x)+1)^2)^(1/2)/((8/7+x)/(-1/3-x)
+1)+4/85*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2)*(2*arctanh(1/51*(-833*(8/7
+x)^2/(-1/3-x)^2+1989)^(1/2))-9*arctan(7/2/(49*(8/7+x)^2/(-1/3-x)^2-117)*(
-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2)*(8/7+x)/(-1/3-x)))/(-17*(49*(8/7+x)^
2/(-1/3-x)^2-117)/((8/7+x)/(-1/3-x)+1)^2)^(1/2)/((8/7+x)/(-1/3-x)+1)+3/17*
(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2)*(arctanh(1/51*(-833*(8/7+x)^2/(-1/3
-x)^2+1989)^(1/2))+4*arctan(7/2/(49*(8/7+x)^2/(-1/3-x)^2-117)*(-833*(8/7+x
)^2/(-1/3-x)^2+1989)^(1/2)*(8/7+x)/(-1/3-x)))/(-17*(49*(8/7+x)^2/(-1/3-x)^
2-117)/((8/7+x)/(-1/3-x)+1)^2)^(1/2)/((8/7+x)/(-1/3-x)+1)-19/50*ln(5*x^2+8
*x+4)+8/25*arctan(5/2*x+2)+3/5*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08

$$\int \frac{x}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = \frac{3}{5}\,x - \frac{1}{5}\,\sqrt{4\,x^2-2\,x-3} + \frac{8}{25}\,\arctan\left(\frac{5}{2}\,x+2\right) - \frac{8}{25}\,\arctan\left(-x+\frac{1}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{3}{2}\right) - \frac{8}{25}\,\arctan\left(-5\,x+\frac{5}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{1}{2}\right) + \frac{19}{50}\,\log\left(20\,x^2-\sqrt{4\,x^2-2\,x-3}(10\,x+1)-3\,x-7\right) - \frac{19}{50}\,\log\left(5\,x^2+8\,x+4\right) - \frac{19}{50}\,\log\left(4\,x^2-\sqrt{4\,x^2-2\,x-3}(2\,x+3)+5\,x+5\right) - \frac{37}{50}\,\log\left(-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right)$$

input
```
integrate(x/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

output
```
3/5*x - 1/5*sqrt(4*x^2 - 2*x - 3) + 8/25*arctan(5/2*x + 2) - 8/25*arctan(-
x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - 8/25*arctan(-5*x + 5/2*sqrt(4*x^2 -
 2*x - 3) - 1/2) + 19/50*log(20*x^2 - sqrt(4*x^2 - 2*x - 3)*(10*x + 1) - 3
*x - 7) - 19/50*log(5*x^2 + 8*x + 4) - 19/50*log(4*x^2 - sqrt(4*x^2 - 2*x
- 3)*(2*x + 3) + 5*x + 5) - 37/50*log(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1)
```

Sympy [F]

$$\int \frac{x}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = \int \frac{x}{3x+\sqrt{4x^2-2x-3}+1}\,dx$$

input
```
integrate(x/(1+3*x+(4*x**2-2*x-3)**(1/2)),x)
```

output
```
Integral(x/(3*x + sqrt(4*x**2 - 2*x - 3) + 1), x)
```

Maxima [F]

$$\int \frac{x}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = \int \frac{x}{3\,x+\sqrt{4\,x^2-2\,x-3}+1}\,dx$$

input
```
integrate(x/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(3*x + sqrt(4*x^2 - 2*x - 3) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.22

$$\int \frac{x}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = \frac{3}{5}\,x - \frac{1}{5}\,\sqrt{4\,x^2-2\,x-3} + \frac{8}{25}\,\arctan\left(\frac{5}{2}\,x+2\right) - \frac{8}{25}\,\arctan\left(-x+\frac{1}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{3}{2}\right) - \frac{8}{25}\,\arctan\left(-5\,x+\frac{5}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{1}{2}\right) + \frac{19}{50}\,\log\left(5\,\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+4\,x-2\,\sqrt{4\,x^2-2\,x-3}+1\right) - \frac{19}{50}\,\log\left(\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+12\,x-6\,\sqrt{4\,x^2-2\,x-3}+13\right) - \frac{19}{50}\,\log\left(5\,x^2+8\,x+4\right) - \frac{37}{50}\,\log\left(\left|-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right|\right)$$

input

```
integrate(x/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

output

```
3/5*x - 1/5*sqrt(4*x^2 - 2*x - 3) + 8/25*arctan(5/2*x + 2) - 8/25*arctan(-
x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - 8/25*arctan(-5*x + 5/2*sqrt(4*x^2 -
 2*x - 3) - 1/2) + 19/50*log(5*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 4*x - 2*s
qrt(4*x^2 - 2*x - 3) + 1) - 19/50*log((2*x - sqrt(4*x^2 - 2*x - 3))^2 + 12
*x - 6*sqrt(4*x^2 - 2*x - 3) + 13) - 19/50*log(5*x^2 + 8*x + 4) - 37/50*lo
g(abs(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = \frac{3\,x}{5} - \int \frac{x\,\sqrt{4\,x^2-2\,x-3}}{5\,x^2+8\,x+4}\,dx + \ln\left(x+\frac{4}{5}-\frac{2}{5}\mathrm{i}\right)\left(-\frac{19}{50}-\frac{4}{25}\mathrm{i}\right) + \ln\left(x+\frac{4}{5}+\frac{2}{5}\mathrm{i}\right)\left(-\frac{19}{50}+\frac{4}{25}\mathrm{i}\right)$$

input
```
int(x/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1),x)
```

output
```
(3*x)/5 - log(x + (4/5 - 2i/5))*(19/50 + 4i/25) - log(x + (4/5 + 2i/5))*(1
9/50 - 4i/25) - int((x*(4*x^2 - 2*x - 3)^(1/2))/(8*x + 5*x^2 + 4), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.70

$$\int \frac{x}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = -\frac{16 atan\left(\frac{5\sqrt{4x^2-2x-3}}{2}+5x+\frac{1}{2}\right)}{25} - \frac{\sqrt{4x^2-2x-3}}{5} - \frac{19\log\left(\frac{80\sqrt{4x^2-2x-3}\,x+8\sqrt{4x^2-2x-3}+160x^2-24x-56}{\sqrt{13}}\right)}{25} + \frac{3\log\left(\frac{2\sqrt{4x^2-2x-3}+4x-1}{\sqrt{13}}\right)}{2} + \frac{3x}{5} - \frac{3}{20}$$

input
```
int(x/(1+3*x+(4*x^2-2*x-3)^(1/2)),x)
```

output
```
( - 64*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x + 1)/2) - 20*sqrt(4*x**2 - 2*
x - 3) - 76*log((80*sqrt(4*x**2 - 2*x - 3)*x + 8*sqrt(4*x**2 - 2*x - 3) +
160*x**2 - 24*x - 56)/sqrt(13)) + 150*log((2*sqrt(4*x**2 - 2*x - 3) + 4*x
- 1)/sqrt(13)) + 60*x - 15)/100
```

3.34 $\int \frac{1}{1+3x+\sqrt{-3-2x+4x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{1}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = \frac{7}{5}\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right) + \frac{3}{5}\log\left(7+3x-20x^2-\sqrt{-3-2x+4x^2} - 10x\sqrt{-3-2x+4x^2}\right) - \log\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)$$

output
```
7/5*arctan(1/2+5*x+5/2*(4*x^2-2*x-3)^(1/2))+3/5*ln(7+3*x-20*x^2-(4*x^2-2*x
-3)^(1/2)-10*x*(4*x^2-2*x-3)^(1/2))-ln(1-4*x-2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

$$\int \frac{1}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = -\frac{7}{5}\arctan\left(\frac{3}{2}+x-\frac{1}{2}\sqrt{-3-2x+4x^2}\right) - \frac{1}{5}\log\left(1-4x+2\sqrt{-3-2x+4x^2}\right) + \frac{3}{5}\log\left(-5-5x-4x^2+(3+2x)\sqrt{-3-2x+4x^2}\right)$$

input
```
Integrate[(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^(-1),x]
```

output
```
(-7*ArcTan[3/2 + x - Sqrt[-3 - 2*x + 4*x^2]/2])/5 - Log[1 - 4*x + 2*Sqrt[-
3 - 2*x + 4*x^2]]/5 + (3*Log[-5 - 5*x - 4*x^2 + (3 + 2*x)*Sqrt[-3 - 2*x +
4*x^2]])/5
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4x^2-2x-3}+3x+1}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{3x+1}{5x^2+8x+4}-\frac{\sqrt{4x^2-2x-3}}{5x^2+8x+4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{7}{10}\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)-\frac{7}{10}\arctan\left(\frac{5x}{2}+2\right)+\frac{2}{5}\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)+\frac{3}{5}\text{arctanh}\left(\frac{3x+1}{\sqrt{4x^2-2x-3}}\right)+\frac{3}{10}\log\left(5x^2+8x+4\right)$$

input
```
Int[(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^(-1),x]
```

output
```
(-7*ArcTan[2 + (5*x)/2])/10 - (7*ArcTan[(8 + 7*x)/(2*Sqrt[-3 - 2*x + 4*x^2
])])/10 + (2*ArcTanh[(1 - 4*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/5 + (3*ArcTanh
[(1 + 3*x)/Sqrt[-3 - 2*x + 4*x^2]])/5 + (3*Log[4 + 8*x + 5*x^2])/10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(87) = 174$.

Time = 0.93 (sec) , antiderivative size = 485, normalized size of antiderivative = 4.62

method	result
default	$-\frac{\ln\left(\frac{(4x-1)\sqrt{4}}{4}+\sqrt{4x^2-2x-3}\right)\sqrt{4}}{5}-\frac{8\sqrt{-\frac{833\left(\frac{8}{7}+x\right)^2}{\left(-\frac{1}{3}-x\right)^2}+1989}\left(2\,\text{arctanh}\left(\frac{\sqrt{-\frac{833\left(\frac{8}{7}+x\right)^2}{\left(-\frac{1}{3}-x\right)^2}+1989}}{51}\right)-9\arctan\left(\frac{7\sqrt{-\frac{833\left(\frac{8}{7}+x\right)^2}{\left(-\frac{1}{3}-x\right)^2}}}{2\left(\frac{49\left(\frac{8}{7}+x\right)^2}{\left(-\frac{1}{3}-x\right)^2}-1\right.}\right.\right.}{85\sqrt{-\frac{17\left(\frac{49\left(\frac{8}{7}+x\right)^2}{\left(-\frac{1}{3}-x\right)^2}-117\right)}{\left(\frac{\frac{8}{7}+x}{-\frac{1}{3}-x}+1\right)^2}}\left(\frac{\frac{8}{7}+x}{-\frac{1}{3}-x}+1\right)}$
trager	Expression too large to display

input
```
int(1/(1+3*x+(4*x^2-2*x-3)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/5*ln(1/4*(4*x-1)*4^(1/2)+(4*x^2-2*x-3)^(1/2))*4^(1/2)-8/85*(-833*(8/7+x
)^2/(-1/3-x)^2+1989)^(1/2)*(2*arctanh(1/51*(-833*(8/7+x)^2/(-1/3-x)^2+1989
)^(1/2))-9*arctan(7/2/(49*(8/7+x)^2/(-1/3-x)^2-117)*(-833*(8/7+x)^2/(-1/3-
x)^2+1989)^(1/2)*(8/7+x)/(-1/3-x)))/(-17*(49*(8/7+x)^2/(-1/3-x)^2-117)/((8
/7+x)/(-1/3-x)+1)^2)^(1/2)/((8/7+x)/(-1/3-x)+1)+2/17*(-833*(8/7+x)^2/(-1/3
-x)^2+1989)^(1/2)*(arctanh(1/51*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2))+4*
arctan(7/2/(49*(8/7+x)^2/(-1/3-x)^2-117)*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^
(1/2)*(8/7+x)/(-1/3-x)))/(-17*(49*(8/7+x)^2/(-1/3-x)^2-117)/((8/7+x)/(-1/3
-x)+1)^2)^(1/2)/((8/7+x)/(-1/3-x)+1)-3/34*(-833*(8/7+x)^2/(-1/3-x)^2+1989)
^(1/2)*(6*arctanh(1/51*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2))+7*arctan(7/
2/(49*(8/7+x)^2/(-1/3-x)^2-117)*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2)*(8/
7+x)/(-1/3-x)))/(-17*(49*(8/7+x)^2/(-1/3-x)^2-117)/((8/7+x)/(-1/3-x)+1)^2)
^(1/2)/((8/7+x)/(-1/3-x)+1)-7/10*arctan(5/2*x+2)+3/10*ln(5*x^2+8*x+4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{1}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = & -\frac{7}{10}\arctan\left(\frac{5}{2}x+2\right) \\ & + \frac{7}{10}\arctan\left(-x+\frac{1}{2}\sqrt{4x^2-2x-3}-\frac{3}{2}\right) \\ & + \frac{7}{10}\arctan\left(-5x+\frac{5}{2}\sqrt{4x^2-2x-3}-\frac{1}{2}\right) \\ & - \frac{3}{10}\log\left(20x^2-\sqrt{4x^2-2x-3}(10x+1)-3x\right. \\ & \left.-7\right)+\frac{3}{10}\log\left(5x^2+8x+4\right) \\ & + \frac{3}{10}\log\left(4x^2-\sqrt{4x^2-2x-3}(2x+3)+5x+5\right) \\ & + \frac{2}{5}\log\left(-4x+2\sqrt{4x^2-2x-3}+1\right)\end{aligned}$$

input
```
integrate(1/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

output
```
-7/10*arctan(5/2*x + 2) + 7/10*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2
) + 7/10*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) - 3/10*log(20*x^2
- sqrt(4*x^2 - 2*x - 3)*(10*x + 1) - 3*x - 7) + 3/10*log(5*x^2 + 8*x + 4)
+ 3/10*log(4*x^2 - sqrt(4*x^2 - 2*x - 3)*(2*x + 3) + 5*x + 5) + 2/5*log(-4
*x + 2*sqrt(4*x^2 - 2*x - 3) + 1)
```

Sympy [F]

$$\int \frac{1}{1+3x+\sqrt{-3-2x+4x^2}}\,dx=\int \frac{1}{3x+\sqrt{4x^2-2x-3}+1}\,dx$$

input
```
integrate(1/(1+3*x+(4*x**2-2*x-3)**(1/2)),x)
```

output
```
Integral(1/(3*x + sqrt(4*x**2 - 2*x - 3) + 1), x)
```

Maxima [F]

$$\int \frac{1}{1+3x+\sqrt{-3-2x+4x^2}}\,dx=\int \frac{1}{3\,x+\sqrt{4\,x^2-2\,x-3}+1}\,dx$$

input
```
integrate(1/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(3*x + sqrt(4*x^2 - 2*x - 3) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(87) = 174$.

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.67

$$\int \frac{1}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = -\frac{7}{10}\arctan\left(\frac{5}{2}x+2\right) + \frac{7}{10}\arctan\left(-x+\frac{1}{2}\sqrt{4x^2-2x-3}-\frac{3}{2}\right) + \frac{7}{10}\arctan\left(-5x+\frac{5}{2}\sqrt{4x^2-2x-3}-\frac{1}{2}\right) - \frac{3}{10}\log\left(5\left(2x-\sqrt{4x^2-2x-3}\right)^2+4x - 2\sqrt{4x^2-2x-3}+1\right) + \frac{3}{10}\log\left(\left(2x-\sqrt{4x^2-2x-3}\right)^2+12x - 6\sqrt{4x^2-2x-3}+13\right) + \frac{3}{10}\log\left(5x^2+8x+4\right) + \frac{2}{5}\log\left(\left|-4x+2\sqrt{4x^2-2x-3}+1\right|\right)$$

input
```
integrate(1/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

output
```
-7/10*arctan(5/2*x + 2) + 7/10*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2
) + 7/10*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) - 3/10*log(5*(2*x
- sqrt(4*x^2 - 2*x - 3))^2 + 4*x - 2*sqrt(4*x^2 - 2*x - 3) + 1) + 3/10*log
((2*x - sqrt(4*x^2 - 2*x - 3))^2 + 12*x - 6*sqrt(4*x^2 - 2*x - 3) + 13) +
3/10*log(5*x^2 + 8*x + 4) + 2/5*log(abs(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = -\int \frac{\sqrt{4\,x^2-2\,x-3}}{5\,x^2+8\,x+4}\,dx + \ln\left(x+\frac{4}{5}-\frac{2}{5}\mathrm{i}\right)\left(\frac{3}{10}+\frac{7}{20}\mathrm{i}\right) + \ln\left(x+\frac{4}{5}+\frac{2}{5}\mathrm{i}\right)\left(\frac{3}{10}-\frac{7}{20}\mathrm{i}\right)$$

input
```
int(1/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1),x)
```

output
```
log(x + (4/5 - 2i/5))*(3/10 + 7i/20) + log(x + (4/5 + 2i/5))*(3/10 - 7i/20
) - int((4*x^2 - 2*x - 3)^(1/2)/(8*x + 5*x^2 + 4), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{1}{1+3x+\sqrt{-3-2x+4x^2}}\,dx = \frac{7atan\left(\frac{5\sqrt{4x^2-2x-3}}{2}+5x+\frac{1}{2}\right)}{5} + \frac{3\log\left(\frac{80\sqrt{4x^2-2x-3}\,x+8\sqrt{4x^2-2x-3}+160x^2-24x-56}{\sqrt{13}}\right)}{5} - \log\left(\frac{2\sqrt{4x^2-2x-3}+4x-1}{\sqrt{13}}\right)$$

input
```
int(1/(1+3*x+(4*x^2-2*x-3)^(1/2)),x)
```

output
```
(7*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x + 1)/2) + 3*log((80*sqrt(4*x**2 -
 2*x - 3)*x + 8*sqrt(4*x**2 - 2*x - 3) + 160*x**2 - 24*x - 56)/sqrt(13)) -
 5*log((2*sqrt(4*x**2 - 2*x - 3) + 4*x - 1)/sqrt(13)))/5
```

3.35 $\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\begin{aligned}\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx &= \frac{1}{2}\sqrt{3}\arctan\left(\frac{2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)\\&\quad-2\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right)\\&\quad-\frac{1}{4}\log\left(7+3x-20x^2-\sqrt{-3-2x+4x^2}\right.\\&\qquad\left.-10x\sqrt{-3-2x+4x^2}\right)\\&\quad+\frac{1}{4}\log\left(x-4x^2-2x\sqrt{-3-2x+4x^2}\right)\end{aligned}$$

output
```
1/2*3^(1/2)*arctan(1/3*(2*x+(4*x^2-2*x-3)^(1/2))*3^(1/2))-2*arctan(1/2+5*x
+5/2*(4*x^2-2*x-3)^(1/2))-1/4*ln(7+3*x-20*x^2-(4*x^2-2*x-3)^(1/2)-10*x*(4*
x^2-2*x-3)^(1/2))+1/4*ln(x-4*x^2-2*x*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx$$
$$=\frac{1}{2}\left(4\arctan\left(\frac{3}{2}+x-\frac{1}{2}\sqrt{-3-2x+4x^2}\right)+\sqrt{3}\arctan\left(\frac{-2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)\right.$$
$$\left.-\operatorname{arctanh}\left(\frac{-5-6x+3\sqrt{-3-2x+4x^2}}{-5-4x-8x^2+(3+4x)\sqrt{-3-2x+4x^2}}\right)\right)$$

input
```
Integrate[1/(x*(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])),x]
```

output
```
(4*ArcTan[3/2 + x - Sqrt[-3 - 2*x + 4*x^2]/2] + Sqrt[3]*ArcTan[(-2*x + Sqr
t[-3 - 2*x + 4*x^2])/Sqrt[3]] - ArcTanh[(-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x^
2])/(-5 - 4*x - 8*x^2 + (3 + 4*x)*Sqrt[-3 - 2*x + 4*x^2])])/2
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{4x^2-2x-3}+3x+1\right)}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{4-5x}{4(5x^2+8x+4)}-\frac{\sqrt{4x^2-2x-3}}{4x}+\frac{5x\sqrt{4x^2-2x-3}}{4(5x^2+8x+4)}+\frac{2\sqrt{4x^2-2x-3}}{5x^2+8x+4}+\frac{1}{4x}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)+\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)+\arctan\left(\frac{5x}{2}+2\right)-\frac{1}{4}\text{arctanh}\left(\frac{3x+1}{\sqrt{4x^2-2x-3}}\right)-\frac{1}{8}\log\left(5x^2+8x+4\right)+\frac{\log(x)}{4}$$

input
```
Int[1/(x*(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])),x]
```

output
```
ArcTan[2 + (5*x)/2] - (Sqrt[3]*ArcTan[(3 + x)/(Sqrt[3]*Sqrt[-3 - 2*x + 4*x
^2])])/4 + ArcTan[(8 + 7*x)/(2*Sqrt[-3 - 2*x + 4*x^2])] - ArcTanh[(1 + 3*x
)/Sqrt[-3 - 2*x + 4*x^2]]/4 + Log[x]/4 - Log[4 + 8*x + 5*x^2]/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. 2(117) = 234.

Time = 0.54 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.45

method	result
default	$-\frac{\ln(5x^2+8x+4)}{8}+\arctan\left(\frac{5x}{2}+2\right)+\frac{\ln(x)}{4}+\frac{\sqrt{3}\arctan\left(\frac{(-6-2x)\sqrt{3}}{6\sqrt{4x^2-2x-3}}\right)}{4}-\frac{4\sqrt{-\frac{833\left(\frac{8}{7}+x\right)^2}{\left(-\frac{1}{3}-x\right)^2}+1989}\left(\text{arctanh}\left(\ldots\right.\right.}{17\sqrt{\ldots}}$
trager	Expression too large to display

input
```
int(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(5*x^2+8*x+4)+arctan(5/2*x+2)+1/4*ln(x)+1/4*3^(1/2)*arctan(1/6*(-6-
2*x)*3^(1/2)/(4*x^2-2*x-3)^(1/2))-4/17*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1
/2)*(arctanh(1/51*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2))+4*arctan(7/2/(49
*(8/7+x)^2/(-1/3-x)^2-117)*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2)*(8/7+x)/
(-1/3-x)))/(-17*(49*(8/7+x)^2/(-1/3-x)^2-117)/((8/7+x)/(-1/3-x)+1)^2)^(1/2
)/((8/7+x)/(-1/3-x)+1)-1/17*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2)*(6*arct
anh(1/51*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2))+7*arctan(7/2/(49*(8/7+x)^
2/(-1/3-x)^2-117)*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2)*(8/7+x)/(-1/3-x))
)/(-17*(49*(8/7+x)^2/(-1/3-x)^2-117)/((8/7+x)/(-1/3-x)+1)^2)^(1/2)/((8/7+x
)/(-1/3-x)+1)+3/68*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2)*(19*arctanh(1/51
*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2))+8*arctan(7/2/(49*(8/7+x)^2/(-1/3-
x)^2-117)*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2)*(8/7+x)/(-1/3-x)))/(-17*(
49*(8/7+x)^2/(-1/3-x)^2-117)/((8/7+x)/(-1/3-x)+1)^2)^(1/2)/((8/7+x)/(-1/3-
x)+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx = {} & \frac{1}{2}\,\sqrt{3}\arctan\left(-\frac{2}{3}\,\sqrt{3}x+\frac{1}{3}\,\sqrt{3}\sqrt{4\,x^2-2\,x-3}\right)\\ & +\arctan\left(\frac{5}{2}\,x+2\right)\\ & -\arctan\left(-x+\frac{1}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{3}{2}\right)\\ & -\arctan\left(-5\,x+\frac{5}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{1}{2}\right)\\ & +\frac{1}{8}\,\log\left(20\,x^2-\sqrt{4\,x^2-2\,x-3}(10\,x+1)\right.\\ & \left.-3\,x-7\right)-\frac{1}{8}\,\log\left(5\,x^2+8\,x+4\right)-\frac{1}{8}\,\log\left(4\,x^2\right.\\ & \left.-\sqrt{4\,x^2-2\,x-3}(2\,x+3)+5\,x+5\right)+\frac{1}{4}\,\log\left(x\right)\end{aligned}$$

input
```
integrate(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*arctan(-2/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(4*x^2 - 2*x - 3)) + a
rctan(5/2*x + 2) - arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - arctan(-
5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) + 1/8*log(20*x^2 - sqrt(4*x^2 - 2*x
 - 3)*(10*x + 1) - 3*x - 7) - 1/8*log(5*x^2 + 8*x + 4) - 1/8*log(4*x^2 - s
qrt(4*x^2 - 2*x - 3)*(2*x + 3) + 5*x + 5) + 1/4*log(x)
```

Sympy [F]

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx=\int \frac{1}{x\left(3x+\sqrt{4x^2-2x-3}+1\right)}\,dx$$

input
```
integrate(1/x/(1+3*x+(4*x**2-2*x-3)**(1/2)),x)
```

output
```
Integral(1/(x*(3*x + sqrt(4*x**2 - 2*x - 3) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx=\int \frac{1}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)x}\,dx$$

input
```
integrate(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((3*x + sqrt(4*x^2 - 2*x - 3) + 1)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.30

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-\sqrt{4x^2-2x-3}\right)\right) + \arctan\left(\frac{5}{2}x+2\right) - \arctan\left(-x+\frac{1}{2}\sqrt{4x^2-2x-3}-\frac{3}{2}\right) - \arctan\left(-5x+\frac{5}{2}\sqrt{4x^2-2x-3}-\frac{1}{2}\right) + \frac{1}{8}\log\left(5\left(2x-\sqrt{4x^2-2x-3}\right)^2+4x-2\sqrt{4x^2-2x-3}+1\right) - \frac{1}{8}\log\left(\left(2x-\sqrt{4x^2-2x-3}\right)^2+12x-6\sqrt{4x^2-2x-3}+13\right) - \frac{1}{8}\log\left(5x^2+8x+4\right) + \frac{1}{4}\log\left(|x|\right)$$

input
```
integrate(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

output
```
1/2*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - sqrt(4*x^2 - 2*x - 3))) + arctan(5/
2*x + 2) - arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - arctan(-5*x + 5/
2*sqrt(4*x^2 - 2*x - 3) - 1/2) + 1/8*log(5*(2*x - sqrt(4*x^2 - 2*x - 3))^2
 + 4*x - 2*sqrt(4*x^2 - 2*x - 3) + 1) - 1/8*log((2*x - sqrt(4*x^2 - 2*x -
3))^2 + 12*x - 6*sqrt(4*x^2 - 2*x - 3) + 13) - 1/8*log(5*x^2 + 8*x + 4) +
1/4*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx=\int \frac{1}{x\,\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)}\,dx$$

input
```
int(1/(x*(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)),x)
```

output
```
int(1/(x*(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx=&-atan\left(\frac{\sqrt{4x^2-2x-3}}{2}+x+\frac{3}{2}\right)\\&+\frac{\sqrt{3}\,atan\left(\frac{\sqrt{4x^2-2x-3}+2x}{\sqrt{3}}\right)}{2}\\&-atan\left(\frac{5\sqrt{4x^2-2x-3}}{2}+5x+\frac{1}{2}\right)\\&+atan\left(\frac{5x}{2}+2\right)-\frac{\log(5x^2+8x+4)}{8}\\&-\frac{\log\left(\frac{80\sqrt{4x^2-2x-3}\,x+8\sqrt{4x^2-2x-3}+160x^2-24x-56}{\sqrt{13}}\right)}{8}\\&+\frac{\log\left(\frac{16\sqrt{4x^2-2x-3}\,x+24\sqrt{4x^2-2x-3}+32x^2+40x+40}{\sqrt{13}}\right)}{8}\\&+\frac{\log(x)}{4}\end{aligned}$$

input
```
int(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2)),x)
```

output
```
( - 8*atan((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2) + 4*sqrt(3)*atan((sqrt(4*
x**2 - 2*x - 3) + 2*x)/sqrt(3)) - 8*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x
+ 1)/2) + 8*atan((5*x + 4)/2) - log(5*x**2 + 8*x + 4) - log((80*sqrt(4*x**
2 - 2*x - 3)*x + 8*sqrt(4*x**2 - 2*x - 3) + 160*x**2 - 24*x - 56)/sqrt(13)
) + log((16*sqrt(4*x**2 - 2*x - 3)*x + 24*sqrt(4*x**2 - 2*x - 3) + 32*x**2
 + 40*x + 40)/sqrt(13)) + 2*log(x))/8
```

3.36 $\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx = -\frac{5+3\left(2x+\sqrt{-3-2x+4x^2}\right)}{2\left(3+\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} - \frac{5\arctan\left(\frac{2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{9}{4}\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right) - \frac{1}{4}\log\left(7+3x-20x^2-\sqrt{-3-2x+4x^2}-10x\sqrt{-3-2x+4x^2}\right) + \frac{1}{4}\log\left(x-4x^2-2x\sqrt{-3-2x+4x^2}\right)$$

output

```
-1/2*(5+6*x+3*(4*x^2-2*x-3)^(1/2))/(3+(2*x+(4*x^2-2*x-3)^(1/2))^2)-5/6*3^(
1/2)*arctan(1/3*(2*x+(4*x^2-2*x-3)^(1/2))*3^(1/2))+9/4*arctan(1/2+5*x+5/2*
(4*x^2-2*x-3)^(1/2))-1/4*ln(7+3*x-20*x^2-(4*x^2-2*x-3)^(1/2)-10*x*(4*x^2-2
*x-3)^(1/2))+1/4*ln(x-4*x^2-2*x*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx =$$
$$-\frac{3-3\sqrt{-3-2x+4x^2}+27x\arctan\left(\frac{3}{2}+x-\frac{1}{2}\sqrt{-3-2x+4x^2}\right)+10\sqrt{3}x\arctan\left(\frac{-2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)}{12x}$$

input
```
Integrate[1/(x^2*(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])),x]
```

output
```
-1/12*(3 - 3*Sqrt[-3 - 2*x + 4*x^2] + 27*x*ArcTan[3/2 + x - Sqrt[-3 - 2*x
+ 4*x^2]/2] + 10*Sqrt[3]*x*ArcTan[(-2*x + Sqrt[-3 - 2*x + 4*x^2])/Sqrt[3]]
 + 6*x*ArcTanh[(-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x^2])/(-5 - 4*x - 8*x^2 + (
3 + 4*x)*Sqrt[-3 - 2*x + 4*x^2])])/x
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{4x^2-2x-3}+3x+1\right)}dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{-5x-13}{4\left(5x^2+8x+4\right)}+\frac{\sqrt{4x^2-2x-3}}{2x}-\frac{\sqrt{4x^2-2x-3}}{4x^2}-\frac{5x\sqrt{4x^2-2x-3}}{2\left(5x^2+8x+4\right)}-\frac{11\sqrt{4x^2-2x-3}}{4\left(5x^2+8x+4\right)}+\frac{1}{4x^2}+\frac{1}{4x}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)-\frac{\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)}{4\sqrt{3}}-\frac{9}{8}\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)-\frac{9}{8}\arctan\left(\frac{5x}{2}+2\right)-\frac{1}{4}\text{arctanh}\left(\frac{3x+1}{\sqrt{4x^2-2x-3}}\right)+\frac{\sqrt{4x^2-2x-3}}{4x}-\frac{1}{8}\log\left(5x^2+8x+4\right)-\frac{1}{4x}+\frac{\log(x)}{4}$$

input
```
Int[1/(x^2*(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])),x]
```

output
```
-1/4*1/x + Sqrt[-3 - 2*x + 4*x^2]/(4*x) - (9*ArcTan[2 + (5*x)/2])/8 - ArcT
an[(3 + x)/(Sqrt[3]*Sqrt[-3 - 2*x + 4*x^2])]/(4*Sqrt[3]) + (Sqrt[3]*ArcTan
[(3 + x)/(Sqrt[3]*Sqrt[-3 - 2*x + 4*x^2])])/2 - (9*ArcTan[(8 + 7*x)/(2*Sqr
t[-3 - 2*x + 4*x^2])])/8 - ArcTanh[(1 + 3*x)/Sqrt[-3 - 2*x + 4*x^2]]/4 + L
og[x]/4 - Log[4 + 8*x + 5*x^2]/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(160) = 320.

Time = 0.65 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.82

method	result
default	$-\frac{\ln(5x^2+8x+4)}{8}-\frac{9\arctan(\frac{5x}{2}+2)}{8}-\frac{1}{4x}+\frac{\ln(x)}{4}-\frac{(4x^2-2x-3)^{\frac{3}{2}}}{12x}-\frac{\sqrt{4x^2-2x-3}}{12}-\frac{5\sqrt{3}\arctan\left(\frac{(-6-2x)\sqrt{3}}{6\sqrt{4x^2-2x-3}}\right)}{12}+$
trager	Expression too large to display

input
```
int(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(5*x^2+8*x+4)-9/8*arctan(5/2*x+2)-1/4/x+1/4*ln(x)-1/12/x*(4*x^2-2*x
-3)^(3/2)-1/12*(4*x^2-2*x-3)^(1/2)-5/12*3^(1/2)*arctan(1/6*(-6-2*x)*3^(1/2
)/(4*x^2-2*x-3)^(1/2))+1/24*(8*x-2)*(4*x^2-2*x-3)^(1/2)+2/17*(-833*(8/7+x)
^2/(-1/3-x)^2+1989)^(1/2)*(6*arctanh(1/51*(-833*(8/7+x)^2/(-1/3-x)^2+1989)
^(1/2))+7*arctan(7/2/(49*(8/7+x)^2/(-1/3-x)^2-117)*(-833*(8/7+x)^2/(-1/3-x
)^2+1989)^(1/2)*(8/7+x)/(-1/3-x)))/(-17*(49*(8/7+x)^2/(-1/3-x)^2-117)/((8/
7+x)/(-1/3-x)+1)^2)^(1/2)/((8/7+x)/(-1/3-x)+1)+1/34*(-833*(8/7+x)^2/(-1/3-
x)^2+1989)^(1/2)*(19*arctanh(1/51*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2))+
8*arctan(7/2/(49*(8/7+x)^2/(-1/3-x)^2-117)*(-833*(8/7+x)^2/(-1/3-x)^2+1989
)^(1/2)*(8/7+x)/(-1/3-x)))/(-17*(49*(8/7+x)^2/(-1/3-x)^2-117)/((8/7+x)/(-1
/3-x)+1)^2)^(1/2)/((8/7+x)/(-1/3-x)+1)-3/136*(-833*(8/7+x)^2/(-1/3-x)^2+19
89)^(1/2)*(46*arctanh(1/51*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2))-3*arcta
n(7/2/(49*(8/7+x)^2/(-1/3-x)^2-117)*(-833*(8/7+x)^2/(-1/3-x)^2+1989)^(1/2)
*(8/7+x)/(-1/3-x)))/(-17*(49*(8/7+x)^2/(-1/3-x)^2-117)/((8/7+x)/(-1/3-x)+1
)^2)^(1/2)/((8/7+x)/(-1/3-x)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx = -\frac{20\sqrt{3}x\arctan\left(-\frac{2}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{4\,x^2-2\,x-3}\right)+27\,x\arctan\left(\frac{5}{2}\,x+2\right)-27\,x\arctan\left(-x+\frac{1}{2}\sqrt{4\,x^2}\right.}{}$$

input
```
integrate(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

output
```
-1/24*(20*sqrt(3)*x*arctan(-2/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(4*x^2 - 2*x -
 3)) + 27*x*arctan(5/2*x + 2) - 27*x*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3)
 - 3/2) - 27*x*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) - 3*x*log(20
*x^2 - sqrt(4*x^2 - 2*x - 3)*(10*x + 1) - 3*x - 7) + 3*x*log(5*x^2 + 8*x +
 4) + 3*x*log(4*x^2 - sqrt(4*x^2 - 2*x - 3)*(2*x + 3) + 5*x + 5) - 6*x*log
(x) - 12*x - 6*sqrt(4*x^2 - 2*x - 3) + 6)/x
```

Sympy [F]

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx = \int \frac{1}{x^2\cdot\left(3x+\sqrt{4x^2-2x-3}+1\right)}\,dx$$

input
```
integrate(1/x**2/(1+3*x+(4*x**2-2*x-3)**(1/2)),x)
```

output
```
Integral(1/(x**2*(3*x + sqrt(4*x**2 - 2*x - 3) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx=\int \frac{1}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)x^2}\,dx$$

input
```
integrate(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((3*x + sqrt(4*x^2 - 2*x - 3) + 1)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.22

$$\begin{aligned}
&\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx \\
&= -\frac{5}{6}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)\right) \\
&\quad+\frac{2\,x-\sqrt{4\,x^2-2\,x-3}+6}{2\left(\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+3\right)}-\frac{1}{4\,x}-\frac{9}{8}\arctan\left(\frac{5}{2}\,x+2\right) \\
&\quad+\frac{9}{8}\arctan\left(-x+\frac{1}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{3}{2}\right) \\
&\quad+\frac{9}{8}\arctan\left(-5\,x+\frac{5}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{1}{2}\right) \\
&\quad+\frac{1}{8}\log\left(5\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+4\,x-2\,\sqrt{4\,x^2-2\,x-3}+1\right) \\
&\quad-\frac{1}{8}\log\left(\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+12\,x-6\,\sqrt{4\,x^2-2\,x-3}+13\right) \\
&\quad-\frac{1}{8}\log\left(5\,x^2+8\,x+4\right)+\frac{1}{4}\log\left(|x|\right)
\end{aligned}$$

input
```
integrate(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

output
```
-5/6*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - sqrt(4*x^2 - 2*x - 3))) + 1/2*(2*x
 - sqrt(4*x^2 - 2*x - 3) + 6)/((2*x - sqrt(4*x^2 - 2*x - 3))^2 + 3) - 1/4/
x - 9/8*arctan(5/2*x + 2) + 9/8*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/
2) + 9/8*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) + 1/8*log(5*(2*x -
 sqrt(4*x^2 - 2*x - 3))^2 + 4*x - 2*sqrt(4*x^2 - 2*x - 3) + 1) - 1/8*log((
2*x - sqrt(4*x^2 - 2*x - 3))^2 + 12*x - 6*sqrt(4*x^2 - 2*x - 3) + 13) - 1/
8*log(5*x^2 + 8*x + 4) + 1/4*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx=\int \frac{1}{x^2\,\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)}\,dx$$

input
```
int(1/(x^2*(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)),x)
```

output
```
int(1/(x^2*(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)}\,dx$$
$$=\frac{27atan\left(\frac{\sqrt{4x^2-2x-3}}{2}+x+\frac{3}{2}\right)x-20\sqrt{3}\,atan\left(\frac{\sqrt{4x^2-2x-3}+2x}{\sqrt{3}}\right)x+27atan\left(\frac{5\sqrt{4x^2-2x-3}}{2}+5x+\frac{1}{2}\right)x-27at}{}$$

input
```
int(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2)),x)
```

output
```
(27*atan((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x - 20*sqrt(3)*atan((sqrt(4
*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x + 27*atan((5*sqrt(4*x**2 - 2*x - 3) + 1
0*x + 1)/2)*x - 27*atan((5*x + 4)/2)*x + 6*sqrt(4*x**2 - 2*x - 3) - 3*log(
5*x**2 + 8*x + 4)*x - 3*log((80*sqrt(4*x**2 - 2*x - 3)*x + 8*sqrt(4*x**2 -
 2*x - 3) + 160*x**2 - 24*x - 56)/sqrt(13))*x + 3*log((16*sqrt(4*x**2 - 2*
x - 3)*x + 24*sqrt(4*x**2 - 2*x - 3) + 32*x**2 + 40*x + 40)/sqrt(13))*x +
6*log(x)*x - 6)/(24*x)
```

3.37 $\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 227

$$\begin{aligned}\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2} dx &= \frac{1}{100}\left(2x+\sqrt{-3-2x+4x^2}\right)-\frac{13}{8\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)} \\ &+\frac{2\left(101-163\left(2x+\sqrt{-3-2x+4x^2}\right)\right)}{125\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} \\ &-\frac{557}{125}\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right) \\ &-\frac{188}{125}\log\left(7+3x-20x^2-\sqrt{-3-2x+4x^2}-10x\sqrt{-3-2x+4x^2}\right) \\ &+3\log\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\end{aligned}$$

output

```
1/50*x+1/100*(4*x^2-2*x-3)^(1/2)-13/(8-32*x-16*(4*x^2-2*x-3)^(1/2))+2*(101
-326*x-163*(4*x^2-2*x-3)^(1/2))/(125+500*x+250*(4*x^2-2*x-3)^(1/2)+625*(2*
x+(4*x^2-2*x-3)^(1/2))^2)-557/125*arctan(1/2+5*x+5/2*(4*x^2-2*x-3)^(1/2))-
188/125*ln(7+3*x-20*x^2-(4*x^2-2*x-3)^(1/2)-10*x*(4*x^2-2*x-3)^(1/2))+3*ln
(1-4*x-2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{1}{125}\left(-\frac{5\sqrt{-3-2x+4x^2}(8+51x+30x^2)}{4+8x+5x^2} + \frac{-116-37x+520x^2+325x^3}{4+8x+5x^2} + 557\arctan\left(\frac{3}{2}+x-\frac{1}{2}\sqrt{-3-2x+4x^2}\right) + \log\left(1-4x+2\sqrt{-3-2x+4x^2}\right) - 188\log\left(-5-5x-4x^2+(3+2x)\sqrt{-3-2x+4x^2}\right)\right)$$

input
```
Integrate[x^2/(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^2,x]
```

output
```
((-5*Sqrt[-3 - 2*x + 4*x^2]*(8 + 51*x + 30*x^2))/(4 + 8*x + 5*x^2) + (-116
 - 37*x + 520*x^2 + 325*x^3)/(4 + 8*x + 5*x^2) + 557*ArcTan[3/2 + x - Sqrt
[-3 - 2*x + 4*x^2]/2] + Log[1 - 4*x + 2*Sqrt[-3 - 2*x + 4*x^2]] - 188*Log[
-5 - 5*x - 4*x^2 + (3 + 2*x)*Sqrt[-3 - 2*x + 4*x^2]])/125
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{4x^2-2x-3}+3x+1\right)^2}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{6\sqrt{4x^2-2x-3}x}{5(5x^2+8x+4)}-\frac{184\sqrt{4x^2-2x-3}x}{25(5x^2+8x+4)^2}-\frac{2(470x-51)}{125(5x^2+8x+4)}+\frac{38\sqrt{4x^2-2x-3}}{25(5x^2+8x+4)}-\frac{8(152x+181)}{125(5x^2+8x+4)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{557}{250}\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)+\frac{557}{250}\arctan\left(\frac{5x}{2}+2\right)-\frac{187}{125}\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)-\frac{188}{125}\text{arctanh}\left(\frac{3x+1}{\sqrt{4x^2-2x-3}}\right)-\frac{6}{25}\sqrt{4x^2-2x-3}-\frac{297x+116}{125(5x^2+8x+4)}+\frac{92(x+1)\sqrt{4x^2-2x-3}}{25(5x^2+8x+4)}-\frac{19(5x+4)\sqrt{4x^2-2x-3}}{25(5x^2+8x+4)}-\frac{94}{125}\log\left(5x^2+8x+4\right)+\frac{13x}{25}$$

input
```
Int[x^2/(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^2,x]
```

output
```
(13*x)/25 - (6*Sqrt[-3 - 2*x + 4*x^2])/25 - (116 + 297*x)/(125*(4 + 8*x +
5*x^2)) + (92*(1 + x)*Sqrt[-3 - 2*x + 4*x^2])/(25*(4 + 8*x + 5*x^2)) - (19
*(4 + 5*x)*Sqrt[-3 - 2*x + 4*x^2])/(25*(4 + 8*x + 5*x^2)) + (557*ArcTan[2
+ (5*x)/2])/250 + (557*ArcTan[(8 + 7*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/250 -
 (187*ArcTanh[(1 - 4*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/125 - (188*ArcTanh[(1
 + 3*x)/Sqrt[-3 - 2*x + 4*x^2]])/125 - (94*Log[4 + 8*x + 5*x^2])/125
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

output
```
int(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{650\,x^3+1055\,x^2+557\,(5\,x^2+8\,x+4)\arctan\left(\frac{5}{2}\,x+2\right)-557\,(5\,x^2+8\,x+4)\arctan\left(-x+\frac{1}{2}\,\sqrt{4\,x^2-\right.}{}$$

input
```
integrate(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/250*(650*x^3 + 1055*x^2 + 557*(5*x^2 + 8*x + 4)*arctan(5/2*x + 2) - 557*
(5*x^2 + 8*x + 4)*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - 557*(5*x^
2 + 8*x + 4)*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) + 188*(5*x^2 +
 8*x + 4)*log(20*x^2 - sqrt(4*x^2 - 2*x - 3)*(10*x + 1) - 3*x - 7) - 188*(
5*x^2 + 8*x + 4)*log(5*x^2 + 8*x + 4) - 188*(5*x^2 + 8*x + 4)*log(4*x^2 -
sqrt(4*x^2 - 2*x - 3)*(2*x + 3) + 5*x + 5) - 374*(5*x^2 + 8*x + 4)*log(-4*
x + 2*sqrt(4*x^2 - 2*x - 3) + 1) - 10*(30*x^2 + 51*x + 8)*sqrt(4*x^2 - 2*x
 - 3) - 50*x - 220)/(5*x^2 + 8*x + 4)
```

Sympy [F]

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx=\int \frac{x^2}{\left(3x+\sqrt{4x^2-2x-3}+1\right)^2}\,dx$$

input
```
integrate(x**2/(1+3*x+(4*x**2-2*x-3)**(1/2))**2,x)
```

output
```
Integral(x**2/(3*x + sqrt(4*x**2 - 2*x - 3) + 1)**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx=\int \frac{x^2}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2}\,dx$$

input
```
integrate(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x^2/(3*x + sqrt(4*x^2 - 2*x - 3) + 1)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.60

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{13}{25}\,x - \frac{6}{25}\,\sqrt{4\,x^2-2\,x-3}$$
$$-\frac{2\left(431\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^3 - 246\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2 - 1318\,x + 659\,\sqrt{4\,x^2-}\right.}{125\left(5\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^4 + 32\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^3 + 78\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2 + 64\,x\right.}$$
$$-\frac{297\,x+116}{125\left(5\,x^2+8\,x+4\right)} + \frac{557}{250}\,\arctan\left(\frac{5}{2}\,x+2\right)$$
$$-\frac{557}{250}\,\arctan\left(-x+\frac{1}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{3}{2}\right)$$
$$-\frac{557}{250}\,\arctan\left(-5\,x+\frac{5}{2}\,\sqrt{4\,x^2-2\,x-3}-\frac{1}{2}\right)$$
$$+\frac{94}{125}\,\log\left(5\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+4\,x-2\,\sqrt{4\,x^2-2\,x-3}+1\right)$$
$$-\frac{94}{125}\,\log\left(\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+12\,x-6\,\sqrt{4\,x^2-2\,x-3}+13\right)$$
$$-\frac{94}{125}\,\log\left(5\,x^2+8\,x+4\right) - \frac{187}{125}\,\log\left(\left|-4\,x+2\,\sqrt{4\,x^2-2\,x-3}+1\right|\right)$$

input
```
integrate(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

output
```
13/25*x - 6/25*sqrt(4*x^2 - 2*x - 3) - 2/125*(431*(2*x - sqrt(4*x^2 - 2*x
- 3))^3 - 246*(2*x - sqrt(4*x^2 - 2*x - 3))^2 - 1318*x + 659*sqrt(4*x^2 -
2*x - 3) + 654)/(5*(2*x - sqrt(4*x^2 - 2*x - 3))^4 + 32*(2*x - sqrt(4*x^2
- 2*x - 3))^3 + 78*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 64*x - 32*sqrt(4*x^2
- 2*x - 3) + 13) - 1/125*(297*x + 116)/(5*x^2 + 8*x + 4) + 557/250*arctan(
5/2*x + 2) - 557/250*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - 557/25
0*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) + 94/125*log(5*(2*x - sqr
t(4*x^2 - 2*x - 3))^2 + 4*x - 2*sqrt(4*x^2 - 2*x - 3) + 1) - 94/125*log((2
*x - sqrt(4*x^2 - 2*x - 3))^2 + 12*x - 6*sqrt(4*x^2 - 2*x - 3) + 13) - 94/
125*log(5*x^2 + 8*x + 4) - 187/125*log(abs(-4*x + 2*sqrt(4*x^2 - 2*x - 3)
+ 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx=\int \frac{x^2}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2}\,dx$$

input
```
int(x^2/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2,x)
```

output
```
int(x^2/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2, x)
```

Reduce [F]

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx=\int \frac{x^2}{\left(1+3x+\sqrt{4x^2-2x-3}\right)^2}dx$$

input
```
int(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

output
```
int(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

3.38 $\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 177

$$\begin{aligned}
\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx
&= -\frac{2\left(7-66\left(2x+\sqrt{-3-2x+4x^2}\right)\right)}{25\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} \\
&\quad+\frac{107}{25}\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right) \\
&\quad+\frac{13}{25}\log\left(7+3x-20x^2-\sqrt{-3-2x+4x^2}-10x\sqrt{-3-2x+4x^2}\right) \\
&\quad-\log\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)
\end{aligned}$$

output

```
(-14+264*x+132*(4*x^2-2*x-3)^(1/2))/(25+100*x+50*(4*x^2-2*x-3)^(1/2)+125*(
2*x+(4*x^2-2*x-3)^(1/2))^2)+107/25*arctan(1/2+5*x+5/2*(4*x^2-2*x-3)^(1/2))
+13/25*ln(7+3*x-20*x^2-(4*x^2-2*x-3)^(1/2)-10*x*(4*x^2-2*x-3)^(1/2))-ln(1-
4*x-2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.82

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{1}{25}\left(-\frac{5(7+4x)\sqrt{-3-2x+4x^2}}{4+8x+5x^2} - \frac{5(9+7x+8x^2)}{4+8x+5x^2} - 107\arctan\left(\frac{3}{2}+x-\frac{1}{2}\sqrt{-3-2x+4x^2}\right) - \log\left(1-4x+2\sqrt{-3-2x+4x^2}\right) + 13\log\left(-5-5x-4x^2+(3+2x)\sqrt{-3-2x+4x^2}\right)\right)$$

input
```
Integrate[x/(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^2,x]
```

output
```
((-5*(7 + 4*x)*Sqrt[-3 - 2*x + 4*x^2])/(4 + 8*x + 5*x^2) - (5*(9 + 7*x + 8
*x^2))/(4 + 8*x + 5*x^2) - 107*ArcTan[3/2 + x - Sqrt[-3 - 2*x + 4*x^2]/2]
- Log[1 - 4*x + 2*Sqrt[-3 - 2*x + 4*x^2]] + 13*Log[-5 - 5*x - 4*x^2 + (3 +
 2*x)*Sqrt[-3 - 2*x + 4*x^2]])/25
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{4x^2-2x-3}+3x+1\right)^2}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{38\sqrt{4x^2-2x-3}x}{5\left(5x^2+8x+4\right)^2}+\frac{65x-84}{25\left(5x^2+8x+4\right)}-\frac{6\sqrt{4x^2-2x-3}}{5\left(5x^2+8x+4\right)}+\frac{2(181x+168)}{25\left(5x^2+8x+4\right)^2}+\frac{24\sqrt{4x^2-2x-3}}{5\left(5x^2+8x+4\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{107}{50}\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)-\frac{107}{50}\arctan\left(\frac{5x}{2}+2\right)+\frac{12}{25}\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)+\frac{13}{25}\text{arctanh}\left(\frac{3x+1}{\sqrt{4x^2-2x-3}}\right)-\frac{13-29x}{25\left(5x^2+8x+4\right)}-\frac{19(x+1)\sqrt{4x^2-2x-3}}{5\left(5x^2+8x+4\right)}+\frac{3(5x+4)\sqrt{4x^2-2x-3}}{5\left(5x^2+8x+4\right)}+\frac{13}{50}\log\left(5x^2+8x+4\right)$$

input
```
Int[x/(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^2,x]
```

output
```
-1/25*(13 - 29*x)/(4 + 8*x + 5*x^2) - (19*(1 + x)*Sqrt[-3 - 2*x + 4*x^2])/
(5*(4 + 8*x + 5*x^2)) + (3*(4 + 5*x)*Sqrt[-3 - 2*x + 4*x^2])/(5*(4 + 8*x +
 5*x^2)) - (107*ArcTan[2 + (5*x)/2])/50 - (107*ArcTan[(8 + 7*x)/(2*Sqrt[-3
 - 2*x + 4*x^2])])/50 + (12*ArcTanh[(1 - 4*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])
/25 + (13*ArcTanh[(1 + 3*x)/Sqrt[-3 - 2*x + 4*x^2]])/25 + (13*Log[4 + 8*x
+ 5*x^2])/50
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 22.62 (sec) , antiderivative size = 2977, normalized size of antiderivative = 16.82

method	result	size
trager	Expression too large to display	2977
default	Expression too large to display	3632

input

```
int(x/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output

```
1/20*(44+13*x)*x/(5*x^2+8*x+4)-1/5*(7+4*x)/(5*x^2+8*x+4)*(4*x^2-2*x-3)^(1/
2)-1/25*ln((211150175041091137382344334016733937182896946236658706237128587
766882112650845749099827200+2226806844516425181530528408044034509299660801
7167626731792413870575707344308026274560000*x-7152654474746606884072463350
1022379770286077542151896982189798876201111483704110215157760000*x^2*(4*x^2
-2*x-3)^(1/2)-6050281592314954717716441582668912735085903486104625069043211
1374160300623367797473280000*x^25-1395808099409202433073363573324512899691100
32743876576335401651237332413021362792793083527937500*x^12+1013693199476489880
0801189553105795451120249763115734870597849273230911661512125376021195260000
00*x^14-41062219832349518548462439991893412672111010188893909948244639357114
932813618974243659881850000*x^10+104562755038177213998622096026308268715342
628673794186854323379401604908597478291804305148000*x^9+683285461552861774
83447847953317528728234776769647099436938607204131079281121971683255040000*
x^16-379391631734948762294764848393824659434301008249986667135358802430453
96132223978391449600000*x^3+7363574328262871510905596076608758309983967494019
91978737900131156540220807730380228608000*x^2-920795352249327176489945643
68306494969530541769844218641485097388384183148522122977837440000*x^4+12039
63739172869121952854189471385576250876699292725473698197070995594954663612
71877632000*x^22+492327469066405320077383732658507697211326966532195201194
48372955254583623982002680627200000*x^21+52003114909598848976945734957649...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.49

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = -\frac{80\,x^2+107\,(5\,x^2+8\,x+4)\arctan\left(\frac{5}{2}\,x+2\right)-107\,(5\,x^2+8\,x+4)\arctan\left(-x+\frac{1}{2}\,\sqrt{4\,x^2-2\,x-3}-\right.}{}$$

input
```
integrate(x/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

output
```
-1/50*(80*x^2 + 107*(5*x^2 + 8*x + 4)*arctan(5/2*x + 2) - 107*(5*x^2 + 8*x
 + 4)*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - 107*(5*x^2 + 8*x + 4)
*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) + 13*(5*x^2 + 8*x + 4)*log
(20*x^2 - sqrt(4*x^2 - 2*x - 3)*(10*x + 1) - 3*x - 7) - 13*(5*x^2 + 8*x +
4)*log(5*x^2 + 8*x + 4) - 13*(5*x^2 + 8*x + 4)*log(4*x^2 - sqrt(4*x^2 - 2*
x - 3)*(2*x + 3) + 5*x + 5) - 24*(5*x^2 + 8*x + 4)*log(-4*x + 2*sqrt(4*x^2
 - 2*x - 3) + 1) + 10*sqrt(4*x^2 - 2*x - 3)*(4*x + 7) + 70*x + 90)/(5*x^2
+ 8*x + 4)
```

Sympy [F]

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{x}{\left(3x+\sqrt{4x^2-2x-3}+1\right)^2}\,dx$$

input
```
integrate(x/(1+3*x+(4*x**2-2*x-3)**(1/2))**2,x)
```

output
```
Integral(x/(3*x + sqrt(4*x**2 - 2*x - 3) + 1)**2, x)
```

Maxima [F]

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{x}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2}\,dx$$

input
```
integrate(x/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x/(3*x + sqrt(4*x^2 - 2*x - 3) + 1)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(149) = 298.

Time = 0.15 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.95

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx =$$

$$-\frac{4\left(4\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^3+261\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+538\,x-269\sqrt{4\,x^2-2\,\cdots}\right.}{25\left(5\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^4+32\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^3+78\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+64\,x-\cdots\right.}$$

$$+\frac{29\,x-13}{25\,(5\,x^2+8\,x+4)}-\frac{107}{50}\arctan\left(\frac{5}{2}\,x+2\right)$$

$$+\frac{107}{50}\arctan\left(-x+\frac{1}{2}\sqrt{4\,x^2-2\,x-3}-\frac{3}{2}\right)$$

$$+\frac{107}{50}\arctan\left(-5\,x+\frac{5}{2}\sqrt{4\,x^2-2\,x-3}-\frac{1}{2}\right)$$

$$-\frac{13}{50}\log\left(5\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+4\,x-2\sqrt{4\,x^2-2\,x-3}+1\right)$$

$$+\frac{13}{50}\log\left(\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+12\,x-6\sqrt{4\,x^2-2\,x-3}+13\right)$$

$$+\frac{13}{50}\log\left(5\,x^2+8\,x+4\right)+\frac{12}{25}\log\left(\left|-4\,x+2\sqrt{4\,x^2-2\,x-3}+1\right|\right)$$

input
```
integrate(x/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

output
```
-4/25*(4*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 261*(2*x - sqrt(4*x^2 - 2*x - 3
))^2 + 538*x - 269*sqrt(4*x^2 - 2*x - 3) + 11)/(5*(2*x - sqrt(4*x^2 - 2*x
- 3))^4 + 32*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 78*(2*x - sqrt(4*x^2 - 2*x
- 3))^2 + 64*x - 32*sqrt(4*x^2 - 2*x - 3) + 13) + 1/25*(29*x - 13)/(5*x^2
+ 8*x + 4) - 107/50*arctan(5/2*x + 2) + 107/50*arctan(-x + 1/2*sqrt(4*x^2
- 2*x - 3) - 3/2) + 107/50*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2)
- 13/50*log(5*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 4*x - 2*sqrt(4*x^2 - 2*x -
 3) + 1) + 13/50*log((2*x - sqrt(4*x^2 - 2*x - 3))^2 + 12*x - 6*sqrt(4*x^2
 - 2*x - 3) + 13) + 13/50*log(5*x^2 + 8*x + 4) + 12/25*log(abs(-4*x + 2*sq
rt(4*x^2 - 2*x - 3) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx=\int \frac{x}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2}\,dx$$

input
```
int(x/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2,x)
```

output
```
int(x/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2, x)
```

Reduce [F]

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx=\int \frac{x}{\left(1+3x+\sqrt{4x^2-2x-3}\right)^2}dx$$

input
```
int(x/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

output
```
int(x/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

3.39 $\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = -\frac{9+73\left(2x+\sqrt{-3-2x+4x^2}\right)}{10\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} - \frac{13}{4}\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right)$$

output
```
-1/10*(9+146*x+73*(4*x^2-2*x-3)^(1/2))/(1+4*x+2*(4*x^2-2*x-3)^(1/2)+5*(2*x
+(4*x^2-2*x-3)^(1/2))^2)-13/4*arctan(1/2+5*x+5/2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{44+13x+5(8+7x)\sqrt{-3-2x+4x^2}+65(4+8x+5x^2)\arctan\left(\frac{3}{2}+x-\frac{1}{2}\sqrt{-3-2x+4x^2}\right)}{20\,(4+8x+5x^2)}$$

input
```
Integrate[(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^(-2),x]
```

output
```
(44 + 13*x + 5*(8 + 7*x)*Sqrt[-3 - 2*x + 4*x^2] + 65*(4 + 8*x + 5*x^2)*Arc
Tan[3/2 + x - Sqrt[-3 - 2*x + 4*x^2]/2])/(20*(4 + 8*x + 5*x^2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{4x^2-2x-3}+3x+1\right)^2}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(-\frac{6\sqrt{4x^2-2x-3}x}{(5x^2+8x+4)^2}+\frac{13}{5\,(5x^2+8x+4)}-\frac{2(42x+31)}{5\,(5x^2+8x+4)^2}-\frac{2\sqrt{4x^2-2x-3}}{(5x^2+8x+4)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{13}{8}\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)+\frac{13}{8}\arctan\left(\frac{5x}{2}+2\right)+\frac{3\sqrt{4x^2-2x-3}(x+1)}{5x^2+8x+4}+\frac{13x+44}{20\,(5x^2+8x+4)}-\frac{(5x+4)\sqrt{4x^2-2x-3}}{4\,(5x^2+8x+4)}$$

input
```
Int[(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^(-2),x]
```

output
```
(44 + 13*x)/(20*(4 + 8*x + 5*x^2)) + (3*(1 + x)*Sqrt[-3 - 2*x + 4*x^2])/(4
 + 8*x + 5*x^2) - ((4 + 5*x)*Sqrt[-3 - 2*x + 4*x^2])/(4*(4 + 8*x + 5*x^2))
 + (13*ArcTan[2 + (5*x)/2])/8 + (13*ArcTan[(8 + 7*x)/(2*Sqrt[-3 - 2*x + 4*
x^2])])/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07

method	result
trager	$-\frac{(15+11x)x}{4(5x^2+8x+4)}+\frac{(8+7x)\sqrt{4x^2-2x-3}}{20x^2+32x+16}+\frac{13\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{7\,\mathrm{RootOf}\left(\_Z^2+1\right)x+8\,\mathrm{RootOf}\left(\_Z^2+1\right)-2\sqrt{4x^2-2x-3}}{\mathrm{RootOf}\left(\_Z^2+1\right)x+2x+2}\right)}{8}$
default	Expression too large to display

input
```
int(1/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(15+11*x)*x/(5*x^2+8*x+4)+1/4*(8+7*x)/(5*x^2+8*x+4)*(4*x^2-2*x-3)^(1/
2)+13/8*RootOf(_Z^2+1)*ln(-(7*RootOf(_Z^2+1)*x+8*RootOf(_Z^2+1)-2*(4*x^2-2
*x-3)^(1/2))/(RootOf(_Z^2+1)*x+2*x+2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$= \frac{28\,x^2+13\,(5\,x^2+8\,x+4)\arctan\left(\frac{5}{2}\,x+2\right)-13\,(5\,x^2+8\,x+4)\arctan\left(-\frac{70\,x^2-5\,\sqrt{4\,x^2-2\,x-3}(7\,x+8)+112\,x+}{2\,(42\,x+31)}\right.}{8\,(5\,x^2+8\,x+4)}$$

input
```
integrate(1/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/8*(28*x^2 + 13*(5*x^2 + 8*x + 4)*arctan(5/2*x + 2) - 13*(5*x^2 + 8*x + 4
)*arctan(-1/2*(70*x^2 - 5*sqrt(4*x^2 - 2*x - 3)*(7*x + 8) + 112*x + 56)/(4
2*x + 31)) + 2*sqrt(4*x^2 - 2*x - 3)*(7*x + 8) + 50*x + 40)/(5*x^2 + 8*x +
 4)
```

Sympy [F]

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{1}{\left(3x+\sqrt{4x^2-2x-3}+1\right)^2}\,dx$$

input
```
integrate(1/(1+3*x+(4*x**2-2*x-3)**(1/2))**2,x)
```

output
```
Integral((3*x + sqrt(4*x**2 - 2*x - 3) + 1)**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{1}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2}\,dx$$

input
```
integrate(1/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((3*x + sqrt(4*x^2 - 2*x - 3) + 1)^(-2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(85) = 170.

Time = 0.12 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.15

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$= \frac{99\left(2x-\sqrt{4x^2-2x-3}\right)^3+786\left(2x-\sqrt{4x^2-2x-3}\right)^2+1458x-729\sqrt{4x^2-2x-}}{10\left(5\left(2x-\sqrt{4x^2-2x-3}\right)^4+32\left(2x-\sqrt{4x^2-2x-3}\right)^3+78\left(2x-\sqrt{4x^2-2x-3}\right)^2+64x-32\right.}$$
$$+\frac{13x+44}{20(5x^2+8x+4)}+\frac{13}{8}\arctan\left(\frac{5}{2}x+2\right)$$
$$-\frac{13}{8}\arctan\left(-x+\frac{1}{2}\sqrt{4x^2-2x-3}-\frac{3}{2}\right)$$
$$-\frac{13}{8}\arctan\left(-5x+\frac{5}{2}\sqrt{4x^2-2x-3}-\frac{1}{2}\right)$$

input
```
integrate(1/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

output
```
1/10*(99*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 786*(2*x - sqrt(4*x^2 - 2*x - 3
))^2 + 1458*x - 729*sqrt(4*x^2 - 2*x - 3) + 166)/(5*(2*x - sqrt(4*x^2 - 2*
x - 3))^4 + 32*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 78*(2*x - sqrt(4*x^2 - 2*
x - 3))^2 + 64*x - 32*sqrt(4*x^2 - 2*x - 3) + 13) + 1/20*(13*x + 44)/(5*x^
2 + 8*x + 4) + 13/8*arctan(5/2*x + 2) - 13/8*arctan(-x + 1/2*sqrt(4*x^2 -
2*x - 3) - 3/2) - 13/8*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{1}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2}\,dx$$

input
```
int(1/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2,x)
```

output
```
int(1/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.98

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$= \frac{-3818100atan\left(\frac{\sqrt{4x^2-2x-3}}{2}+x+\frac{3}{2}\right)x^2-6108960atan\left(\frac{\sqrt{4x^2-2x-3}}{2}+x+\frac{3}{2}\right)x-3054480atan\left(\frac{\sqrt{4x^2-2x-}}{2}\right.}{}$$

input
```
int(1/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

output
```
( - 3818100*atan((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x**2 - 6108960*atan
((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x - 3054480*atan((sqrt(4*x**2 - 2*x
 - 3) + 2*x + 3)/2) - 3818100*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x + 1)/2
)*x**2 - 6108960*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x + 1)/2)*x - 3054480
*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x + 1)/2) + 3818100*atan((5*x + 4)/2)
*x**2 + 6108960*atan((5*x + 4)/2)*x + 3054480*atan((5*x + 4)/2) + 822360*s
qrt(4*x**2 - 2*x - 3)*x + 939840*sqrt(4*x**2 - 2*x - 3) - 412945*x**2 - 35
5264*x + 703468)/(469920*(5*x**2 + 8*x + 4))
```

3.40 $\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 215

$$\begin{aligned}\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx &= \frac{5+16\left(2x+\sqrt{-3-2x+4x^2}\right)}{2\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} \\ &\quad+\frac{1}{4}\sqrt{3}\arctan\left(\frac{2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right) \\ &\quad+\frac{9}{4}\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right) \\ &\quad+\frac{1}{8}\log\left(7+3x-20x^2-\sqrt{-3-2x+4x^2}-10x\sqrt{-3-2x+4x^2}\right) \\ &\quad-\frac{1}{8}\log\left(x-4x^2-2x\sqrt{-3-2x+4x^2}\right)\end{aligned}$$

output
```
(5+32*x+16*(4*x^2-2*x-3)^(1/2))/(2+8*x+4*(4*x^2-2*x-3)^(1/2)+10*(2*x+(4*x^
2-2*x-3)^(1/2))^2)+1/4*3^(1/2)*arctan(1/3*(2*x+(4*x^2-2*x-3)^(1/2))*3^(1/2
))+9/4*arctan(1/2+5*x+5/2*(4*x^2-2*x-3)^(1/2))+1/8*ln(7+3*x-20*x^2-(4*x^2-
2*x-3)^(1/2)-10*x*(4*x^2-2*x-3)^(1/2))-1/8*ln(x-4*x^2-2*x*(4*x^2-2*x-3)^(1
/2))
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$=\frac{1}{4}\left(-\frac{15+11x}{4+8x+5x^2}-\frac{(9+10x)\sqrt{-3-2x+4x^2}}{4+8x+5x^2}-9\arctan\left(\frac{3}{2}+x-\frac{1}{2}\sqrt{-3-2x+4x^2}\right)+\sqrt{3}\arctan\left(\frac{-2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)+\operatorname{arctanh}\left(\frac{-5-6x+3\sqrt{-3-2x+4x^2}}{-5-4x-8x^2+(3+4x)\sqrt{-3-2x+4x^2}}\right)\right)$$

input
```
Integrate[1/(x*(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^2),x]
```

output
```
(-((15 + 11*x)/(4 + 8*x + 5*x^2)) - ((9 + 10*x)*Sqrt[-3 - 2*x + 4*x^2])/(4
 + 8*x + 5*x^2) - 9*ArcTan[3/2 + x - Sqrt[-3 - 2*x + 4*x^2]/2] + Sqrt[3]*A
rcTan[(-2*x + Sqrt[-3 - 2*x + 4*x^2])/Sqrt[3]] + ArcTanh[(-5 - 6*x + 3*Sqr
t[-3 - 2*x + 4*x^2])/(-5 - 4*x - 8*x^2 + (3 + 4*x)*Sqrt[-3 - 2*x + 4*x^2])
])/4
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{4x^2-2x-3}+3x+1\right)^2}dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{5\sqrt{4x^2-2x-3}x}{8\left(5x^2+8x+4\right)}+\frac{5\sqrt{4x^2-2x-3}x}{2\left(5x^2+8x+4\right)^2}+\frac{5x+8}{8\left(5x^2+8x+4\right)}+\frac{\sqrt{4x^2-2x-3}}{5x^2+8x+4}+\frac{31x+16}{2\left(5x^2+8x+4\right)^2}-\frac{2\sqrt{4x^2}}{(5x^2+}\right.$$

$\big\downarrow$ 2009

$$-\frac{1}{8}\sqrt{3}\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)-\frac{9}{8}\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)-\frac{9}{8}\arctan\left(\frac{5x}{2}+2\right)+\frac{1}{8}\text{arctanh}\left(\frac{3x+1}{\sqrt{4x^2-2x-3}}\right)-\frac{5\sqrt{4x^2-2x-3}(x+1)}{4\left(5x^2+8x+4\right)}-\frac{11x+15}{4\left(5x^2+8x+4\right)}-\frac{(5x+4)\sqrt{4x^2-2x-3}}{4\left(5x^2+8x+4\right)}+\frac{1}{16}\log\left(5x^2+8x+4\right)-\frac{\log(x)}{8}$$

input
```
Int[1/(x*(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^2),x]
```

output
```
-1/4*(15 + 11*x)/(4 + 8*x + 5*x^2) - (5*(1 + x)*Sqrt[-3 - 2*x + 4*x^2])/(4
*(4 + 8*x + 5*x^2)) - ((4 + 5*x)*Sqrt[-3 - 2*x + 4*x^2])/(4*(4 + 8*x + 5*x
^2)) - (9*ArcTan[2 + (5*x)/2])/8 - (Sqrt[3]*ArcTan[(3 + x)/(Sqrt[3]*Sqrt[-
3 - 2*x + 4*x^2])])/8 - (9*ArcTan[(8 + 7*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/8
 + ArcTanh[(1 + 3*x)/Sqrt[-3 - 2*x + 4*x^2]]/8 - Log[x]/8 + Log[4 + 8*x +
5*x^2]/16
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 1460, normalized size of antiderivative = 6.79

method	result	size
trager	Expression too large to display	1460
default	Expression too large to display	3204

input
```
int(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/68*(x-1)*(151+130*x)/(5*x^2+8*x+4)-1/4*(9+10*x)/(5*x^2+8*x+4)*(4*x^2-2*x
-3)^(1/2)+17/4*RootOf(578*_Z^2-34*_Z+41)*ln(-(-9455579452*RootOf(578*_Z^2-
34*_Z+41)^2*RootOf(289*_Z^2+17*_Z+1)^2*x+9455579452*RootOf(578*_Z^2-34*_Z+
41)^2*RootOf(289*_Z^2+17*_Z+1)^2+6700604876*RootOf(578*_Z^2-34*_Z+41)*Root
Of(289*_Z^2+17*_Z+1)^2*x-1064706056*RootOf(578*_Z^2-34*_Z+41)^2*RootOf(289
*_Z^2+17*_Z+1)*x+89890560*RootOf(289*_Z^2+17*_Z+1)*RootOf(578*_Z^2-34*_Z+4
1)*(4*x^2-2*x-3)^(1/2)-6700604876*RootOf(578*_Z^2-34*_Z+41)*RootOf(289*_Z^
2+17*_Z+1)^2-964952215*RootOf(289*_Z^2+17*_Z+1)^2*x+1064706056*RootOf(289*
_Z^2+17*_Z+1)*RootOf(578*_Z^2-34*_Z+41)^2+200581606*RootOf(289*_Z^2+17*_Z+
1)*RootOf(578*_Z^2-34*_Z+41)*x-24506044*RootOf(578*_Z^2-34*_Z+41)^2*x-1240
63569*RootOf(289*_Z^2+17*_Z+1)*(4*x^2-2*x-3)^(1/2)-24448176*(4*x^2-2*x-3)^
(1/2)*RootOf(578*_Z^2-34*_Z+41)+964952215*RootOf(289*_Z^2+17*_Z+1)^2-13451
93272*RootOf(289*_Z^2+17*_Z+1)*RootOf(578*_Z^2-34*_Z+41)-42542993*RootOf(2
89*_Z^2+17*_Z+1)*x+24506044*RootOf(578*_Z^2-34*_Z+41)^2-34235314*RootOf(57
8*_Z^2-34*_Z+41)*x+1640358*(4*x^2-2*x-3)^(1/2)+272667386*RootOf(289*_Z^2+1
7*_Z+1)-57725948*RootOf(578*_Z^2-34*_Z+41)+17366468*x+18992479)/x)+1/4*ln(
(9455579452*RootOf(578*_Z^2-34*_Z+41)^2*RootOf(289*_Z^2+17*_Z+1)^2*x-94555
79452*RootOf(578*_Z^2-34*_Z+41)^2*RootOf(289*_Z^2+17*_Z+1)^2+5588183764*Ro
otOf(578*_Z^2-34*_Z+41)*RootOf(289*_Z^2+17*_Z+1)^2*x+1064706056*RootOf(578
*_Z^2-34*_Z+41)^2*RootOf(289*_Z^2+17*_Z+1)*x+89890560*RootOf(289*_Z^2+1...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.32

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$= \frac{4\sqrt{3}(5x^2+8x+4)\arctan\left(-\frac{2}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{4x^2-2x-3}\right)-80x^2-18(5x^2+8x+4)\arctan\left(\frac{5}{2}x\right.}{}$$

input
```
integrate(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/16*(4*sqrt(3)*(5*x^2 + 8*x + 4)*arctan(-2/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt
(4*x^2 - 2*x - 3)) - 80*x^2 - 18*(5*x^2 + 8*x + 4)*arctan(5/2*x + 2) + 18*
(5*x^2 + 8*x + 4)*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) + 18*(5*x^2
 + 8*x + 4)*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) - (5*x^2 + 8*x
+ 4)*log(20*x^2 - sqrt(4*x^2 - 2*x - 3)*(10*x + 1) - 3*x - 7) + (5*x^2 + 8
*x + 4)*log(5*x^2 + 8*x + 4) + (5*x^2 + 8*x + 4)*log(4*x^2 - sqrt(4*x^2 -
2*x - 3)*(2*x + 3) + 5*x + 5) - 2*(5*x^2 + 8*x + 4)*log(x) - 4*sqrt(4*x^2
- 2*x - 3)*(10*x + 9) - 172*x - 124)/(5*x^2 + 8*x + 4)
```

Sympy [F]

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{1}{x\left(3x+\sqrt{4x^2-2x-3}+1\right)^2}\,dx$$

input
```
integrate(1/x/(1+3*x+(4*x**2-2*x-3)**(1/2))**2,x)
```

output
```
Integral(1/(x*(3*x + sqrt(4*x**2 - 2*x - 3) + 1)**2), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{1}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2 x}\,dx$$

input
```
integrate(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/((3*x + sqrt(4*x^2 - 2*x - 3) + 1)^2*x), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.66

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \frac{1}{4}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)\right)$$
$$-\frac{19\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^3+105\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+184\,x-92\sqrt{4\,x^2-2\,x-}}{5\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^4+32\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^3+78\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+64\,x-32\sqrt{}}$$
$$-\frac{11\,x+15}{4\left(5\,x^2+8\,x+4\right)}-\frac{9}{8}\arctan\left(\frac{5}{2}\,x+2\right)+\frac{9}{8}\arctan\left(-x+\frac{1}{2}\sqrt{4\,x^2-2\,x-3}-\frac{3}{2}\right)$$
$$+\frac{9}{8}\arctan\left(-5\,x+\frac{5}{2}\sqrt{4\,x^2-2\,x-3}-\frac{1}{2}\right)$$
$$-\frac{1}{16}\log\left(5\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+4\,x-2\sqrt{4\,x^2-2\,x-3}+1\right)$$
$$+\frac{1}{16}\log\left(\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+12\,x-6\sqrt{4\,x^2-2\,x-3}+13\right)$$
$$+\frac{1}{16}\log\left(5\,x^2+8\,x+4\right)-\frac{1}{8}\log\left(|x|\right)$$

input
```
integrate(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

output
```
1/4*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - sqrt(4*x^2 - 2*x - 3))) - (19*(2*x
- sqrt(4*x^2 - 2*x - 3))^3 + 105*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 184*x -
 92*sqrt(4*x^2 - 2*x - 3) + 31)/(5*(2*x - sqrt(4*x^2 - 2*x - 3))^4 + 32*(2
*x - sqrt(4*x^2 - 2*x - 3))^3 + 78*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 64*x
- 32*sqrt(4*x^2 - 2*x - 3) + 13) - 1/4*(11*x + 15)/(5*x^2 + 8*x + 4) - 9/8
*arctan(5/2*x + 2) + 9/8*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) + 9/
8*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) - 1/16*log(5*(2*x - sqrt(
4*x^2 - 2*x - 3))^2 + 4*x - 2*sqrt(4*x^2 - 2*x - 3) + 1) + 1/16*log((2*x -
 sqrt(4*x^2 - 2*x - 3))^2 + 12*x - 6*sqrt(4*x^2 - 2*x - 3) + 13) + 1/16*lo
g(5*x^2 + 8*x + 4) - 1/8*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{1}{x\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2}\,dx$$

input
```
int(1/(x*(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2),x)
```

output
```
int(1/(x*(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.89

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

output

```
(11454300*atan((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x**2 + 18326880*atan(
(sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x + 9163440*atan((sqrt(4*x**2 - 2*x
- 3) + 2*x + 3)/2) + 2545400*sqrt(3)*atan((sqrt(4*x**2 - 2*x - 3) + 2*x)/s
qrt(3))*x**2 + 4072640*sqrt(3)*atan((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3)
)*x + 2036320*sqrt(3)*atan((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3)) + 11454
300*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x + 1)/2)*x**2 + 18326880*atan((5*
sqrt(4*x**2 - 2*x - 3) + 10*x + 1)/2)*x + 9163440*atan((5*sqrt(4*x**2 - 2*
x - 3) + 10*x + 1)/2) - 11454300*atan((5*x + 4)/2)*x**2 - 18326880*atan((5
*x + 4)/2)*x - 9163440*atan((5*x + 4)/2) - 5090800*sqrt(4*x**2 - 2*x - 3)*
x - 4581720*sqrt(4*x**2 - 2*x - 3) + 636350*log(5*x**2 + 8*x + 4)*x**2 + 1
018160*log(5*x**2 + 8*x + 4)*x + 509080*log(5*x**2 + 8*x + 4) + 636350*log
((80*sqrt(4*x**2 - 2*x - 3)*x + 8*sqrt(4*x**2 - 2*x - 3) + 160*x**2 - 24*x
 - 56)/sqrt(13))*x**2 + 1018160*log((80*sqrt(4*x**2 - 2*x - 3)*x + 8*sqrt(
4*x**2 - 2*x - 3) + 160*x**2 - 24*x - 56)/sqrt(13))*x + 509080*log((80*sqr
t(4*x**2 - 2*x - 3)*x + 8*sqrt(4*x**2 - 2*x - 3) + 160*x**2 - 24*x - 56)/s
qrt(13)) - 636350*log((16*sqrt(4*x**2 - 2*x - 3)*x + 24*sqrt(4*x**2 - 2*x
- 3) + 32*x**2 + 40*x + 40)/sqrt(13))*x**2 - 1018160*log((16*sqrt(4*x**2 -
 2*x - 3)*x + 24*sqrt(4*x**2 - 2*x - 3) + 32*x**2 + 40*x + 40)/sqrt(13))*x
 - 509080*log((16*sqrt(4*x**2 - 2*x - 3)*x + 24*sqrt(4*x**2 - 2*x - 3) + 3
2*x**2 + 40*x + 40)/sqrt(13)) - 1272700*log(x)*x**2 - 2036320*log(x)*x ...
```

3.41 $\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 308

$$\begin{aligned}
&\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx \\
&= -\frac{97+45\left(2x+\sqrt{-3-2x+4x^2}\right)}{8\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} \\
&\quad+\frac{23-7\left(2x+\sqrt{-3-2x+4x^2}\right)}{\left(3+\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} \\
&\quad-\frac{\arctan\left(\frac{2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{37}{16}\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right) \\
&\quad-\frac{3}{4}\log\left(7+3x-20x^2-\sqrt{-3-2x+4x^2}-10x\sqrt{-3-2x+4x^2}\right) \\
&\quad+\frac{3}{4}\log\left(x-4x^2-2x\sqrt{-3-2x+4x^2}\right)
\end{aligned}$$

output
```
-1/8*(97+90*x+45*(4*x^2-2*x-3)^(1/2))/(1+4*x+2*(4*x^2-2*x-3)^(1/2)+5*(2*x+
(4*x^2-2*x-3)^(1/2))^2)+(23-14*x-7*(4*x^2-2*x-3)^(1/2))/(3+(2*x+(4*x^2-2*x
-3)^(1/2))^2)/(1+4*x+2*(4*x^2-2*x-3)^(1/2)+5*(2*x+(4*x^2-2*x-3)^(1/2))^2)-
1/6*3^(1/2)*arctan(1/3*(2*x+(4*x^2-2*x-3)^(1/2))*3^(1/2))-37/16*arctan(1/2
+5*x+5/2*(4*x^2-2*x-3)^(1/2))-3/4*ln(7+3*x-20*x^2-(4*x^2-2*x-3)^(1/2)-10*x
*(4*x^2-2*x-3)^(1/2))+3/4*ln(x-4*x^2-2*x*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$= \frac{1}{48}\left(\frac{3\sqrt{-3-2x+4x^2}(8+48x+55x^2)}{x\,(4+8x+5x^2)}+\frac{24+540x+783x^2+330x^3}{4x+8x^2+5x^3}\right.$$
$$+111\arctan\left(\frac{3}{2}+x-\frac{1}{2}\sqrt{-3-2x+4x^2}\right)$$
$$-8\sqrt{3}\arctan\left(\frac{-2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)$$
$$\left.-72\text{arctanh}\left(\frac{-5-6x+3\sqrt{-3-2x+4x^2}}{-5-4x-8x^2+(3+4x)\sqrt{-3-2x+4x^2}}\right)\right)$$

input
```
Integrate[1/(x^2*(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^2),x]
```

output
```
((3*Sqrt[-3 - 2*x + 4*x^2]*(8 + 48*x + 55*x^2))/(x*(4 + 8*x + 5*x^2)) + (2
4 + 540*x + 783*x^2 + 330*x^3)/(4*x + 8*x^2 + 5*x^3) + 111*ArcTan[3/2 + x
- Sqrt[-3 - 2*x + 4*x^2]/2] - 8*Sqrt[3]*ArcTan[(-2*x + Sqrt[-3 - 2*x + 4*x
^2])/Sqrt[3]] - 72*ArcTanh[(-5 - 6*x + 3*Sqrt[-3 - 2*x + 4*x^2])/(-5 - 4*x
 - 8*x^2 + (3 + 4*x)*Sqrt[-3 - 2*x + 4*x^2])])/48
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.88, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{4x^2-2x-3}+3x+1\right)^2}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{-30x-43}{8(5x^2+8x+4)}+\frac{\sqrt{4x^2-2x-3}}{8x}-\frac{\sqrt{4x^2-2x-3}}{8x^2}-\frac{5x\sqrt{4x^2-2x-3}}{8(5x^2+8x+4)}-\frac{3\sqrt{4x^2-2x-3}}{8(5x^2+8x+4)}-\frac{1}{8x^2}+\frac{}{2(5}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}\sqrt{3}\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)-\frac{\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)}{8\sqrt{3}}+\frac{37}{32}\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)+\frac{37}{32}\arctan\left(\frac{5x}{2}+2\right)-\frac{3}{4}\text{arctanh}\left(\frac{3x+1}{\sqrt{4x^2-2x-3}}\right)-\frac{5\sqrt{4x^2-2x-3}(x+1)}{4(5x^2+8x+4)}+\frac{\sqrt{4x^2-2x-3}}{8x}+\frac{75x+76}{16(5x^2+8x+4)}+\frac{13(5x+4)\sqrt{4x^2-2x-3}}{16(5x^2+8x+4)}-\frac{3}{8}\log\left(5x^2+8x+4\right)+\frac{1}{8x}+\frac{3\log(x)}{4}$$

input
```
Int[1/(x^2*(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^2),x]
```

output
```
1/(8*x) + Sqrt[-3 - 2*x + 4*x^2]/(8*x) + (76 + 75*x)/(16*(4 + 8*x + 5*x^2)
) - (5*(1 + x)*Sqrt[-3 - 2*x + 4*x^2])/(4*(4 + 8*x + 5*x^2)) + (13*(4 + 5*
x)*Sqrt[-3 - 2*x + 4*x^2])/(16*(4 + 8*x + 5*x^2)) + (37*ArcTan[2 + (5*x)/2
])/32 - ArcTan[(3 + x)/(Sqrt[3]*Sqrt[-3 - 2*x + 4*x^2])]/(8*Sqrt[3]) + (Sq
rt[3]*ArcTan[(3 + x)/(Sqrt[3]*Sqrt[-3 - 2*x + 4*x^2])])/8 + (37*ArcTan[(8
+ 7*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/32 - (3*ArcTanh[(1 + 3*x)/Sqrt[-3 - 2*
x + 4*x^2]])/4 + (3*Log[x])/4 - (3*Log[4 + 8*x + 5*x^2])/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.72 (sec) , antiderivative size = 1687, normalized size of antiderivative = 5.48

method	result	size
trager	Expression too large to display	1687
default	Expression too large to display	3547

input
```
int(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/272*(x-1)*(925*x^2+960*x+136)/x/(5*x^2+8*x+4)+1/16*(55*x^2+48*x+8)/x/(5
*x^2+8*x+4)*(4*x^2-2*x-3)^(1/2)+17/16*RootOf(867*_Z^2-1224*_Z+448)*ln((-70
91684589*RootOf(867*_Z^2-1224*_Z+448)^2*RootOf(1156*_Z^2+1632*_Z+1945)^2*x
+7091684589*RootOf(867*_Z^2-1224*_Z+448)^2*RootOf(1156*_Z^2+1632*_Z+1945)^
2-28957005828*RootOf(867*_Z^2-1224*_Z+448)^2*RootOf(1156*_Z^2+1632*_Z+1945
)*x+8994799008*RootOf(867*_Z^2-1224*_Z+448)*RootOf(1156*_Z^2+1632*_Z+1945)
^2*x+739100160*RootOf(867*_Z^2-1224*_Z+448)*RootOf(1156*_Z^2+1632*_Z+1945)
*(4*x^2-2*x-3)^(1/2)+28957005828*RootOf(867*_Z^2-1224*_Z+448)^2*RootOf(115
6*_Z^2+1632*_Z+1945)-26743884288*RootOf(867*_Z^2-1224*_Z+448)^2*x-89947990
08*RootOf(867*_Z^2-1224*_Z+448)*RootOf(1156*_Z^2+1632*_Z+1945)^2+412822547
08*RootOf(867*_Z^2-1224*_Z+448)*RootOf(1156*_Z^2+1632*_Z+1945)*x-282300748
8*RootOf(1156*_Z^2+1632*_Z+1945)^2*x+4626004402*RootOf(867*_Z^2-1224*_Z+44
8)*(4*x^2-2*x-3)^(1/2)-1115735296*(4*x^2-2*x-3)^(1/2)*RootOf(1156*_Z^2+163
2*_Z+1945)+26743884288*RootOf(867*_Z^2-1224*_Z+448)^2-31871003232*RootOf(8
67*_Z^2-1224*_Z+448)*RootOf(1156*_Z^2+1632*_Z+1945)+38819732560*RootOf(867
*_Z^2-1224*_Z+448)*x+2823007488*RootOf(1156*_Z^2+1632*_Z+1945)^2-139676624
00*RootOf(1156*_Z^2+1632*_Z+1945)*x-4089671568*(4*x^2-2*x-3)^(1/2)-2553568
5312*RootOf(867*_Z^2-1224*_Z+448)+8602624512*RootOf(1156*_Z^2+1632*_Z+1945
)-12035612608*x+6073635072)/(68*RootOf(1156*_Z^2+1632*_Z+1945)*x-68*RootOf
(1156*_Z^2+1632*_Z+1945)-285*x-344))+3/2*ln(-(7091684589*RootOf(867*_Z^...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$= \frac{660\,x^3 - 16\,\sqrt{3}(5\,x^3 + 8\,x^2 + 4\,x)\arctan\left(-\frac{2}{3}\,\sqrt{3}x + \frac{1}{3}\,\sqrt{3}\sqrt{4\,x^2 - 2\,x - 3}\right) + 1566\,x^2 + 111\,(5\,x^3 + 8\,x^2}{}$$

input
```
integrate(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/96*(660*x^3 - 16*sqrt(3)*(5*x^3 + 8*x^2 + 4*x)*arctan(-2/3*sqrt(3)*x + 1
/3*sqrt(3)*sqrt(4*x^2 - 2*x - 3)) + 1566*x^2 + 111*(5*x^3 + 8*x^2 + 4*x)*a
rctan(5/2*x + 2) - 111*(5*x^3 + 8*x^2 + 4*x)*arctan(-x + 1/2*sqrt(4*x^2 -
2*x - 3) - 3/2) - 111*(5*x^3 + 8*x^2 + 4*x)*arctan(-5*x + 5/2*sqrt(4*x^2 -
 2*x - 3) - 1/2) + 36*(5*x^3 + 8*x^2 + 4*x)*log(20*x^2 - sqrt(4*x^2 - 2*x
- 3)*(10*x + 1) - 3*x - 7) - 36*(5*x^3 + 8*x^2 + 4*x)*log(5*x^2 + 8*x + 4)
 - 36*(5*x^3 + 8*x^2 + 4*x)*log(4*x^2 - sqrt(4*x^2 - 2*x - 3)*(2*x + 3) +
5*x + 5) + 72*(5*x^3 + 8*x^2 + 4*x)*log(x) + 6*(55*x^2 + 48*x + 8)*sqrt(4*
x^2 - 2*x - 3) + 1080*x + 48)/(5*x^3 + 8*x^2 + 4*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{1}{x^2\left(3x+\sqrt{4x^2-2x-3}+1\right)^2}\,dx$$

input
```
integrate(1/x**2/(1+3*x+(4*x**2-2*x-3)**(1/2))**2,x)
```

output
```
Integral(1/(x**2*(3*x + sqrt(4*x**2 - 2*x - 3) + 1)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx = \int \frac{1}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/((3*x + sqrt(4*x^2 - 2*x - 3) + 1)^2*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx$$
$$= -\frac{1}{6}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-\sqrt{4x^2-2x-3}\right)\right)$$
$$+\frac{215\left(2x-\sqrt{4x^2-2x-3}\right)^5+1018\left(2x-\sqrt{4x^2-2x-3}\right)^4+1898\left(2x-\sqrt{4x^2-2x-\cdots\right.}{8\left(5\left(2x-\sqrt{4x^2-2x-3}\right)^6+32\left(2x-\sqrt{4x^2-2x-3}\right)^5+93\left(2x-\sqrt{4x^2-2x-3}\right)^4+128\left(2x\cdots\right.\right.}$$
$$+\frac{85x^2+92x+8}{16(5x^3+8x^2+4x)}+\frac{37}{32}\arctan\left(\frac{5}{2}x+2\right)$$
$$-\frac{37}{32}\arctan\left(-x+\frac{1}{2}\sqrt{4x^2-2x-3}-\frac{3}{2}\right)$$
$$-\frac{37}{32}\arctan\left(-5x+\frac{5}{2}\sqrt{4x^2-2x-3}-\frac{1}{2}\right)$$
$$+\frac{3}{8}\log\left(5\left(2x-\sqrt{4x^2-2x-3}\right)^2+4x-2\sqrt{4x^2-2x-3}+1\right)$$
$$-\frac{3}{8}\log\left(\left(2x-\sqrt{4x^2-2x-3}\right)^2+12x-6\sqrt{4x^2-2x-3}+13\right)$$
$$-\frac{3}{8}\log\left(5x^2+8x+4\right)+\frac{3}{4}\log\left(|x|\right)$$

input
```
integrate(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - sqrt(4*x^2 - 2*x - 3))) + 1/8*(215
*(2*x - sqrt(4*x^2 - 2*x - 3))^5 + 1018*(2*x - sqrt(4*x^2 - 2*x - 3))^4 +
1898*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 4012*(2*x - sqrt(4*x^2 - 2*x - 3))^
2 + 5278*x - 2639*sqrt(4*x^2 - 2*x - 3) + 1146)/(5*(2*x - sqrt(4*x^2 - 2*x
 - 3))^6 + 32*(2*x - sqrt(4*x^2 - 2*x - 3))^5 + 93*(2*x - sqrt(4*x^2 - 2*x
 - 3))^4 + 128*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 247*(2*x - sqrt(4*x^2 - 2
*x - 3))^2 + 192*x - 96*sqrt(4*x^2 - 2*x - 3) + 39) + 1/16*(85*x^2 + 92*x
+ 8)/(5*x^3 + 8*x^2 + 4*x) + 37/32*arctan(5/2*x + 2) - 37/32*arctan(-x + 1
/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - 37/32*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x
 - 3) - 1/2) + 3/8*log(5*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 4*x - 2*sqrt(4*
x^2 - 2*x - 3) + 1) - 3/8*log((2*x - sqrt(4*x^2 - 2*x - 3))^2 + 12*x - 6*s
qrt(4*x^2 - 2*x - 3) + 13) - 3/8*log(5*x^2 + 8*x + 4) + 3/4*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx=\int \frac{1}{x^2\,{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)}^2} \,dx$$

input
```
int(1/(x^2*(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2),x)
```

output
```
int(1/(x^2*(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^2}\,dx=\text{Too large to display}$$

input
```
int(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^2,x)
```

output

```
( - 1753722300*atan((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x**3 - 280595568
0*atan((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x**2 - 1402977840*atan((sqrt(
4*x**2 - 2*x - 3) + 2*x + 3)/2)*x - 252788800*sqrt(3)*atan((sqrt(4*x**2 -
2*x - 3) + 2*x)/sqrt(3))*x**3 - 404462080*sqrt(3)*atan((sqrt(4*x**2 - 2*x
- 3) + 2*x)/sqrt(3))*x**2 - 202231040*sqrt(3)*atan((sqrt(4*x**2 - 2*x - 3)
 + 2*x)/sqrt(3))*x - 1753722300*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x + 1)
/2)*x**3 - 2805955680*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x + 1)/2)*x**2 -
 1402977840*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x + 1)/2)*x + 1753722300*a
tan((5*x + 4)/2)*x**3 + 2805955680*atan((5*x + 4)/2)*x**2 + 1402977840*ata
n((5*x + 4)/2)*x + 1042753800*sqrt(4*x**2 - 2*x - 3)*x**2 + 910039680*sqrt
(4*x**2 - 2*x - 3)*x + 151673280*sqrt(4*x**2 - 2*x - 3) - 568774800*log(5*
x**2 + 8*x + 4)*x**3 - 910039680*log(5*x**2 + 8*x + 4)*x**2 - 455019840*lo
g(5*x**2 + 8*x + 4)*x - 568774800*log((80*sqrt(4*x**2 - 2*x - 3)*x + 8*sqr
t(4*x**2 - 2*x - 3) + 160*x**2 - 24*x - 56)/sqrt(13))*x**3 - 910039680*log
((80*sqrt(4*x**2 - 2*x - 3)*x + 8*sqrt(4*x**2 - 2*x - 3) + 160*x**2 - 24*x
 - 56)/sqrt(13))*x**2 - 455019840*log((80*sqrt(4*x**2 - 2*x - 3)*x + 8*sqr
t(4*x**2 - 2*x - 3) + 160*x**2 - 24*x - 56)/sqrt(13))*x + 568774800*log((1
6*sqrt(4*x**2 - 2*x - 3)*x + 24*sqrt(4*x**2 - 2*x - 3) + 32*x**2 + 40*x +
40)/sqrt(13))*x**3 + 910039680*log((16*sqrt(4*x**2 - 2*x - 3)*x + 24*sqrt(
4*x**2 - 2*x - 3) + 32*x**2 + 40*x + 40)/sqrt(13))*x**2 + 455019840*log...
```

3.42 $\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 249

$$\begin{aligned}
\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx
&= -\frac{2\left(179-2477\left(2x+\sqrt{-3-2x+4x^2}\right)\right)}{625\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)^2} \\
&\quad+\frac{3\left(2129+17685\left(2x+\sqrt{-3-2x+4x^2}\right)\right)}{2500\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} \\
&\quad+\frac{12331\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right)}{1000} \\
&\quad+\frac{63}{125}\log\left(7+3x-20x^2-\sqrt{-3-2x+4x^2}-10x\sqrt{-3-2x+4x^2}\right) \\
&\quad-\log\left(1-2\left(2x+\sqrt{-3-2x+4x^2}\right)\right)
\end{aligned}$$

output
```
1/625*(-358+9908*x+4954*(4*x^2-2*x-3)^(1/2))/(1+4*x+2*(4*x^2-2*x-3)^(1/2)+
5*(2*x+(4*x^2-2*x-3)^(1/2))^2)^2+3*(2129+35370*x+17685*(4*x^2-2*x-3)^(1/2)
)/(2500+10000*x+5000*(4*x^2-2*x-3)^(1/2)+12500*(2*x+(4*x^2-2*x-3)^(1/2))^2
)+12331/1000*arctan(1/2+5*x+5/2*(4*x^2-2*x-3)^(1/2))+63/125*ln(7+3*x-20*x^
2-(4*x^2-2*x-3)^(1/2)-10*x*(4*x^2-2*x-3)^(1/2))-ln(1-4*x-2*(4*x^2-2*x-3)^(
1/2))
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.64

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$= \frac{-\frac{5\sqrt{-3-2x+4x^2}(4544+13356x+14656x^2+5465x^3)}{(4+8x+5x^2)^2} - \frac{28464+72596x+66276x^2+15455x^3}{(4+8x+5x^2)^2} - 12331\arctan\left(\frac{3}{2}+x-\frac{1}{2}\sqrt{-3-\right.}{10}$$

input
```
Integrate[x^2/(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^3,x]
```

output
```
((-5*Sqrt[-3 - 2*x + 4*x^2]*(4544 + 13356*x + 14656*x^2 + 5465*x^3))/(4 +
8*x + 5*x^2)^2 - (28464 + 72596*x + 66276*x^2 + 15455*x^3)/(4 + 8*x + 5*x^
2)^2 - 12331*ArcTan[3/2 + x - Sqrt[-3 - 2*x + 4*x^2]/2] - 8*Log[1 - 4*x +
2*Sqrt[-3 - 2*x + 4*x^2]] + 504*Log[-5 - 5*x - 4*x^2 + (3 + 2*x)*Sqrt[-3 -
 2*x + 4*x^2]])/1000
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{4x^2-2x-3}+3x+1\right)^3}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{416\sqrt{4x^2-2x-3}x}{25(5x^2+8x+4)^2}+\frac{2432\sqrt{4x^2-2x-3}x}{125(5x^2+8x+4)^3}+\frac{21(15x-43)}{125(5x^2+8x+4)}-\frac{31\sqrt{4x^2-2x-3}}{25(5x^2+8x+4)}+\frac{16(2280x+89}{625(5x^2+8x+}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{12331 \arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)}{2000}-\frac{12331 \arctan\left(\frac{5x}{2}+2\right)}{2000}+\frac{62}{125}\text{arctanh}\left(\frac{1-4x}{2\sqrt{4x^2-2x-3}}\right)+\frac{63}{125}\text{arctanh}\left(\frac{3x+1}{\sqrt{4x^2-2x-3}}\right)-\frac{208\sqrt{4x^2-2x-3}(x+1)}{25\,(5x^2+8x+4)}-\frac{608\sqrt{4x^2-2x-3}(x+1)}{125\,(5x^2+8x+4)^2}+\frac{11709(5x+4)}{5000\,(5x^2+8x+4)}-\frac{2(4625x+5524)}{625\,(5x^2+8x+4)}-\frac{13(5x+4)\sqrt{4x^2-2x-3}}{125\,(5x^2+8x+4)}-\frac{304(965x+701)\sqrt{4x^2-2x-3}}{36125\,(5x^2+8x+4)}+\frac{181(18355x+13928)\sqrt{4x^2-2x-3}}{289000\,(5x^2+8x+4)}+\frac{3903x+2984}{625\,(5x^2+8x+4)^2}+\frac{181(5x+4)\sqrt{4x^2-2x-3}}{125\,(5x^2+8x+4)^2}+\frac{63}{250}\log\left(5x^2+8x+4\right)$$

input
```
Int[x^2/(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^3,x]
```

output
```
(2984 + 3903*x)/(625*(4 + 8*x + 5*x^2)^2) - (608*(1 + x)*Sqrt[-3 - 2*x + 4
*x^2])/(125*(4 + 8*x + 5*x^2)^2) + (181*(4 + 5*x)*Sqrt[-3 - 2*x + 4*x^2])/
(125*(4 + 8*x + 5*x^2)^2) + (11709*(4 + 5*x))/(5000*(4 + 8*x + 5*x^2)) - (
2*(5524 + 4625*x))/(625*(4 + 8*x + 5*x^2)) - (208*(1 + x)*Sqrt[-3 - 2*x +
4*x^2])/(25*(4 + 8*x + 5*x^2)) - (13*(4 + 5*x)*Sqrt[-3 - 2*x + 4*x^2])/(12
5*(4 + 8*x + 5*x^2)) - (304*(701 + 965*x)*Sqrt[-3 - 2*x + 4*x^2])/(36125*(
4 + 8*x + 5*x^2)) + (181*(13928 + 18355*x)*Sqrt[-3 - 2*x + 4*x^2])/(289000
*(4 + 8*x + 5*x^2)) - (12331*ArcTan[2 + (5*x)/2])/2000 - (12331*ArcTan[(8
+ 7*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/2000 + (62*ArcTanh[(1 - 4*x)/(2*Sqrt[-
3 - 2*x + 4*x^2])])/125 + (63*ArcTanh[(1 + 3*x)/Sqrt[-3 - 2*x + 4*x^2]])/1
25 + (63*Log[4 + 8*x + 5*x^2])/250
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13827 vs. 2(211) = 422.

Time = 0.29 (sec) , antiderivative size = 13828, normalized size of antiderivative = 55.53

output too large to display

input
```
int(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.46

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = -\frac{109300\,x^4+380670\,x^3+587240\,x^2+12331\,(25\,x^4+80\,x^3+104\,x^2+64\,x+16)\arctan\left(\frac{5}{2}\,x+2\right)-1}{}$$

input
```
integrate(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

output
```
-1/2000*(109300*x^4 + 380670*x^3 + 587240*x^2 + 12331*(25*x^4 + 80*x^3 + 1
04*x^2 + 64*x + 16)*arctan(5/2*x + 2) - 12331*(25*x^4 + 80*x^3 + 104*x^2 +
 64*x + 16)*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - 12331*(25*x^4 +
 80*x^3 + 104*x^2 + 64*x + 16)*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1
/2) + 504*(25*x^4 + 80*x^3 + 104*x^2 + 64*x + 16)*log(20*x^2 - sqrt(4*x^2
- 2*x - 3)*(10*x + 1) - 3*x - 7) - 504*(25*x^4 + 80*x^3 + 104*x^2 + 64*x +
 16)*log(5*x^2 + 8*x + 4) - 504*(25*x^4 + 80*x^3 + 104*x^2 + 64*x + 16)*lo
g(4*x^2 - sqrt(4*x^2 - 2*x - 3)*(2*x + 3) + 5*x + 5) - 992*(25*x^4 + 80*x^
3 + 104*x^2 + 64*x + 16)*log(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1) + 10*(546
5*x^3 + 14656*x^2 + 13356*x + 4544)*sqrt(4*x^2 - 2*x - 3) + 425000*x + 126
880)/(25*x^4 + 80*x^3 + 104*x^2 + 64*x + 16)
```

Sympy [F]

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{x^2}{\left(3x+\sqrt{4x^2-2x-3}+1\right)^3}\,dx$$

input
```
integrate(x**2/(1+3*x+(4*x**2-2*x-3)**(1/2))**3,x)
```

output
```
Integral(x**2/(3*x + sqrt(4*x**2 - 2*x - 3) + 1)**3, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{x^2}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3}\,dx$$

input
```
integrate(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x^2/(3*x + sqrt(4*x^2 - 2*x - 3) + 1)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(211) = 422$.

Time = 0.18 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.78

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = -\frac{83965\left(2x-\sqrt{4x^2-2x-3}\right)^7+1179022\left(2x-\sqrt{4x^2-2x-3}\right)^6+5708901\left(2x-\sqrt{4x^2-2x-\cdots}\right.}{500\left(5\left(2x-\sqrt{4x^2-2x-3}\right)^4\cdots\right.}$$
$$-\frac{15455x^3+66276x^2+72596x+28464}{1000\left(5x^2+8x+4\right)^2}-\frac{12331}{2000}\arctan\left(\frac{5}{2}x+2\right)$$
$$+\frac{12331}{2000}\arctan\left(-x+\frac{1}{2}\sqrt{4x^2-2x-3}-\frac{3}{2}\right)$$
$$+\frac{12331}{2000}\arctan\left(-5x+\frac{5}{2}\sqrt{4x^2-2x-3}-\frac{1}{2}\right)$$
$$-\frac{63}{250}\log\left(5\left(2x-\sqrt{4x^2-2x-3}\right)^2+4x-2\sqrt{4x^2-2x-3}+1\right)$$
$$+\frac{63}{250}\log\left(\left(2x-\sqrt{4x^2-2x-3}\right)^2+12x-6\sqrt{4x^2-2x-3}+13\right)$$
$$+\frac{63}{250}\log\left(5x^2+8x+4\right)+\frac{62}{125}\log\left(\left|-4x+2\sqrt{4x^2-2x-3}+1\right|\right)$$

input

```
integrate(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

output

```
-1/500*(83965*(2*x - sqrt(4*x^2 - 2*x - 3))^7 + 1179022*(2*x - sqrt(4*x^2
- 2*x - 3))^6 + 5708901*(2*x - sqrt(4*x^2 - 2*x - 3))^5 + 13528366*(2*x -
sqrt(4*x^2 - 2*x - 3))^4 + 13425247*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 7270
266*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 5034622*x - 2517311*sqrt(4*x^2 - 2*x
 - 3) + 299722)/(5*(2*x - sqrt(4*x^2 - 2*x - 3))^4 + 32*(2*x - sqrt(4*x^2
- 2*x - 3))^3 + 78*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 64*x - 32*sqrt(4*x^2
- 2*x - 3) + 13)^2 - 1/1000*(15455*x^3 + 66276*x^2 + 72596*x + 28464)/(5*x
^2 + 8*x + 4)^2 - 12331/2000*arctan(5/2*x + 2) + 12331/2000*arctan(-x + 1/
2*sqrt(4*x^2 - 2*x - 3) - 3/2) + 12331/2000*arctan(-5*x + 5/2*sqrt(4*x^2 -
 2*x - 3) - 1/2) - 63/250*log(5*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 4*x - 2*
sqrt(4*x^2 - 2*x - 3) + 1) + 63/250*log((2*x - sqrt(4*x^2 - 2*x - 3))^2 +
12*x - 6*sqrt(4*x^2 - 2*x - 3) + 13) + 63/250*log(5*x^2 + 8*x + 4) + 62/12
5*log(abs(-4*x + 2*sqrt(4*x^2 - 2*x - 3) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{x^2}{{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)}^3}\,dx$$

input
```
int(x^2/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3,x)
```

output
```
int(x^2/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3, x)
```

Reduce [F]

$$\int \frac{x^2}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{x^2}{\left(1+3x+\sqrt{4x^2-2x-3}\right)^3}dx$$

input
```
int(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

output
```
int(x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

3.43 $\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 176

$$\begin{aligned}
\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx &= -\frac{2\left(97+664\left(2x+\sqrt{-3-2x+4x^2}\right)\right)}{125\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)^2} \\
&\quad -\frac{947+4915\left(2x+\sqrt{-3-2x+4x^2}\right)}{250\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} \\
&\quad -\frac{39}{4}\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right)
\end{aligned}$$

output
```
1/125*(-194-2656*x-1328*(4*x^2-2*x-3)^(1/2))/(1+4*x+2*(4*x^2-2*x-3)^(1/2)+
5*(2*x+(4*x^2-2*x-3)^(1/2))^2)^2-(947+9830*x+4915*(4*x^2-2*x-3)^(1/2))/(25
0+1000*x+500*(4*x^2-2*x-3)^(1/2)+1250*(2*x+(4*x^2-2*x-3)^(1/2))^2)-39/4*ar
ctan(1/2+5*x+5/2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.57

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$=\frac{3428+9812x+10482x^2+3615x^3+25\sqrt{-3-2x+4x^2}(84+256x+296x^2+121x^3)+975(4+8x+5}{100\left(4+8x+5x^2\right)^2}$$

input
```
Integrate[x/(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^3,x]
```

output
```
(3428 + 9812*x + 10482*x^2 + 3615*x^3 + 25*Sqrt[-3 - 2*x + 4*x^2]*(84 + 25
6*x + 296*x^2 + 121*x^3) + 975*(4 + 8*x + 5*x^2)^2*ArcTan[3/2 + x - Sqrt[-
3 - 2*x + 4*x^2]/2])/(100*(4 + 8*x + 5*x^2)^2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{4x^2-2x-3}+3x+1\right)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{31\sqrt{4x^2-2x-3}x}{5\left(5x^2+8x+4\right)^2}-\frac{724\sqrt{4x^2-2x-3}x}{25\left(5x^2+8x+4\right)^3}+\frac{63}{25\left(5x^2+8x+4\right)}-\frac{3(1505x-24)}{125\left(5x^2+8x+4\right)^2}+\frac{168\sqrt{4x^2-2x-}}{25\left(5x^2+8x+4\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{39}{8}\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)+\frac{39}{8}\arctan\left(\frac{5x}{2}+2\right)+\frac{31\sqrt{4x^2-2x-3}(x+1)}{10\left(5x^2+8x+4\right)}+\frac{181\sqrt{4x^2-2x-3}(x+1)}{25\left(5x^2+8x+4\right)^2}-\frac{1119(5x+4)}{500\left(5x^2+8x+4\right)}+\frac{3(1535x+1529)}{250\left(5x^2+8x+4\right)}+\frac{21(5x+4)\sqrt{4x^2-2x-3}}{25\left(5x^2+8x+4\right)}+\frac{181(965x+701)\sqrt{4x^2-2x-3}}{14450\left(5x^2+8x+4\right)}-\frac{21(18355x+13928)\sqrt{4x^2-2x-3}}{28900\left(5x^2+8x+4\right)}-\frac{746x+413}{125\left(5x^2+8x+4\right)^2}-\frac{42(5x+4)\sqrt{4x^2-2x-3}}{25\left(5x^2+8x+4\right)^2}$$

input
```
Int[x/(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^3,x]
```

output
```
-1/125*(413 + 746*x)/(4 + 8*x + 5*x^2)^2 + (181*(1 + x)*Sqrt[-3 - 2*x + 4*
x^2])/(25*(4 + 8*x + 5*x^2)^2) - (42*(4 + 5*x)*Sqrt[-3 - 2*x + 4*x^2])/(25
*(4 + 8*x + 5*x^2)^2) - (1119*(4 + 5*x))/(500*(4 + 8*x + 5*x^2)) + (3*(152
9 + 1535*x))/(250*(4 + 8*x + 5*x^2)) + (31*(1 + x)*Sqrt[-3 - 2*x + 4*x^2])
/(10*(4 + 8*x + 5*x^2)) + (21*(4 + 5*x)*Sqrt[-3 - 2*x + 4*x^2])/(25*(4 + 8
*x + 5*x^2)) + (181*(701 + 965*x)*Sqrt[-3 - 2*x + 4*x^2])/(14450*(4 + 8*x
+ 5*x^2)) - (21*(13928 + 18355*x)*Sqrt[-3 - 2*x + 4*x^2])/(28900*(4 + 8*x
+ 5*x^2)) + (39*ArcTan[2 + (5*x)/2])/8 + (39*ArcTan[(8 + 7*x)/(2*Sqrt[-3 -
 2*x + 4*x^2])])/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.74

method	result
trager	$-\frac{(857x^3+2164x^2+1888x+624)x}{16(5x^2+8x+4)^2}+\frac{(121x^3+296x^2+256x+84)\sqrt{4x^2-2x-3}}{4(5x^2+8x+4)^2}+\frac{39\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{7\,\mathrm{RootOf}\left(\_Z^2+1\right.}{\mathrm{Roo}}\right.}{8}$
default	Expression too large to display

input
```
int(x/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(857*x^3+2164*x^2+1888*x+624)*x/(5*x^2+8*x+4)^2+1/4*(121*x^3+296*x^2
+256*x+84)/(5*x^2+8*x+4)^2*(4*x^2-2*x-3)^(1/2)+39/8*RootOf(_Z^2+1)*ln(-(7*
RootOf(_Z^2+1)*x+8*RootOf(_Z^2+1)-2*(4*x^2-2*x-3)^(1/2))/(RootOf(_Z^2+1)*x
+2*x+2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$= \frac{484\,x^4+1838\,x^3+2852\,x^2+39\,(25\,x^4+80\,x^3+104\,x^2+64\,x+16)\arctan\left(\frac{5}{2}\,x+2\right)-39\,(25\,x^4+80\,x}{8\,(25}$$

input
```
integrate(x/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/8*(484*x^4 + 1838*x^3 + 2852*x^2 + 39*(25*x^4 + 80*x^3 + 104*x^2 + 64*x
+ 16)*arctan(5/2*x + 2) - 39*(25*x^4 + 80*x^3 + 104*x^2 + 64*x + 16)*arcta
n(-1/2*(70*x^2 - 5*sqrt(4*x^2 - 2*x - 3)*(7*x + 8) + 112*x + 56)/(42*x + 3
1)) + 2*(121*x^3 + 296*x^2 + 256*x + 84)*sqrt(4*x^2 - 2*x - 3) + 2024*x +
584)/(25*x^4 + 80*x^3 + 104*x^2 + 64*x + 16)
```

Sympy [F]

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{x}{\left(3x+\sqrt{4x^2-2x-3}+1\right)^3}\,dx$$

input
```
integrate(x/(1+3*x+(4*x**2-2*x-3)**(1/2))**3,x)
```

output
```
Integral(x/(3*x + sqrt(4*x**2 - 2*x - 3) + 1)**3, x)
```

Maxima [F]

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{x}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3}\,dx$$

input
```
integrate(x/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x/(3*x + sqrt(4*x^2 - 2*x - 3) + 1)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(147) = 294$.

Time = 0.15 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.83

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$= \frac{12145\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^7+142134\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^6+636777\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^5+}{50\left(5\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^4+32\,(2\right.}$$
$$+\frac{3615\,x^3+10482\,x^2+9812\,x+3428}{100\,(5\,x^2+8\,x+4)^2}+\frac{39}{8}\arctan\left(\frac{5}{2}\,x+2\right)$$
$$-\frac{39}{8}\arctan\left(-x+\frac{1}{2}\sqrt{4\,x^2-2\,x-3}-\frac{3}{2}\right)$$
$$-\frac{39}{8}\arctan\left(-5\,x+\frac{5}{2}\sqrt{4\,x^2-2\,x-3}-\frac{1}{2}\right)$$

input
```
integrate(x/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

output
```
1/50*(12145*(2*x - sqrt(4*x^2 - 2*x - 3))^7 + 142134*(2*x - sqrt(4*x^2 - 2
*x - 3))^6 + 636777*(2*x - sqrt(4*x^2 - 2*x - 3))^5 + 1429702*(2*x - sqrt(
4*x^2 - 2*x - 3))^4 + 1397579*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 793042*(2*
x - sqrt(4*x^2 - 2*x - 3))^2 + 578294*x - 289147*sqrt(4*x^2 - 2*x - 3) + 4
5874)/(5*(2*x - sqrt(4*x^2 - 2*x - 3))^4 + 32*(2*x - sqrt(4*x^2 - 2*x - 3)
)^3 + 78*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 64*x - 32*sqrt(4*x^2 - 2*x - 3)
 + 13)^2 + 1/100*(3615*x^3 + 10482*x^2 + 9812*x + 3428)/(5*x^2 + 8*x + 4)^
2 + 39/8*arctan(5/2*x + 2) - 39/8*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) -
3/2) - 39/8*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{x}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3}\,dx$$

input
```
int(x/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3,x)
```

output
```
int(x/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3, x)
```

Reduce [F]

$$\int \frac{x}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{x}{\left(1+3x+\sqrt{4x^2-2x-3}\right)^3}dx$$

input
```
int(x/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

output
```
int(x/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

3.44 $\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 176

$$\begin{aligned}\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx &= \frac{191+567\left(2x+\sqrt{-3-2x+4x^2}\right)}{50\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)^2} \\ &\quad + \frac{1237+6825\left(2x+\sqrt{-3-2x+4x^2}\right)}{400\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} \\ &\quad + \frac{273}{32}\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right)\end{aligned}$$

output
```
1/50*(191+1134*x+567*(4*x^2-2*x-3)^(1/2))/(1+4*x+2*(4*x^2-2*x-3)^(1/2)+5*(
2*x+(4*x^2-2*x-3)^(1/2))^2)^2+(1237+13650*x+6825*(4*x^2-2*x-3)^(1/2))/(400
+1600*x+800*(4*x^2-2*x-3)^(1/2)+2000*(2*x+(4*x^2-2*x-3)^(1/2))^2)+273/32*a
rctan(1/2+5*x+5/2*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = -\frac{5072+15148x+17388x^2+6825x^3+5\sqrt{-3-2x+4x^2}(576+1748x+2064x^2+895x^3)+1365(4+}{160\left(4+8x+5x^2\right)^2}$$

input
```
Integrate[(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^(-3),x]
```

output
```
-1/160*(5072 + 15148*x + 17388*x^2 + 6825*x^3 + 5*Sqrt[-3 - 2*x + 4*x^2]*(
576 + 1748*x + 2064*x^2 + 895*x^3) + 1365*(4 + 8*x + 5*x^2)^2*ArcTan[3/2 +
 x - Sqrt[-3 - 2*x + 4*x^2]/2])/(4 + 8*x + 5*x^2)^2
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.57, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{4x^2-2x-3}+3x+1\right)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{168\sqrt{4x^2-2x-3}x}{5\left(5x^2+8x+4\right)^3}+\frac{21(15x-19)}{25\left(5x^2+8x+4\right)^2}-\frac{31\sqrt{4x^2-2x-3}}{5\left(5x^2+8x+4\right)^2}+\frac{4(333x+349)}{25\left(5x^2+8x+4\right)^3}+\frac{124\sqrt{4x^2-2x-3}}{5\left(5x^2+8x+4\right)^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{273}{64}\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)-\frac{273}{64}\arctan\left(\frac{5x}{2}+2\right)-\frac{42\sqrt{4x^2-2x-3}(x+1)}{5\left(5x^2+8x+4\right)^2}+\frac{1239(5x+4)}{800\left(5x^2+8x+4\right)}-\frac{21(155x+136)}{200\left(5x^2+8x+4\right)}-\frac{31(5x+4)\sqrt{4x^2-2x-3}}{40\left(5x^2+8x+4\right)}-\frac{21(965x+701)\sqrt{4x^2-2x-3}}{1445\left(5x^2+8x+4\right)}+\frac{31(18355x+13928)\sqrt{4x^2-2x-3}}{46240\left(5x^2+8x+4\right)}+\frac{413x+64}{100\left(5x^2+8x+4\right)^2}+\frac{31(5x+4)\sqrt{4x^2-2x-3}}{20\left(5x^2+8x+4\right)^2}$$

input
```
Int[(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^(-3),x]
```

output
```
(64 + 413*x)/(100*(4 + 8*x + 5*x^2)^2) - (42*(1 + x)*Sqrt[-3 - 2*x + 4*x^2
])/(5*(4 + 8*x + 5*x^2)^2) + (31*(4 + 5*x)*Sqrt[-3 - 2*x + 4*x^2])/(20*(4
+ 8*x + 5*x^2)^2) + (1239*(4 + 5*x))/(800*(4 + 8*x + 5*x^2)) - (21*(136 +
155*x))/(200*(4 + 8*x + 5*x^2)) - (31*(4 + 5*x)*Sqrt[-3 - 2*x + 4*x^2])/(4
0*(4 + 8*x + 5*x^2)) - (21*(701 + 965*x)*Sqrt[-3 - 2*x + 4*x^2])/(1445*(4
+ 8*x + 5*x^2)) + (31*(13928 + 18355*x)*Sqrt[-3 - 2*x + 4*x^2])/(46240*(4
+ 8*x + 5*x^2)) - (273*ArcTan[2 + (5*x)/2])/64 - (273*ArcTan[(8 + 7*x)/(2*
Sqrt[-3 - 2*x + 4*x^2])])/64
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.74

method	result
trager	$\frac{(1585x^3+3707x^2+3116x+1028)x}{32(5x^2+8x+4)^2} - \frac{(895x^3+2064x^2+1748x+576)\sqrt{4x^2-2x-3}}{32(5x^2+8x+4)^2} + \frac{273\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{7\,\mathrm{RootOf}\left(\_Z^2\right.}{}\right.}{64}$
default	Expression too large to display

input
```
int(1/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(1585*x^3+3707*x^2+3116*x+1028)*x/(5*x^2+8*x+4)^2-1/32*(895*x^3+2064*
x^2+1748*x+576)/(5*x^2+8*x+4)^2*(4*x^2-2*x-3)^(1/2)+273/64*RootOf(_Z^2+1)*
ln((7*RootOf(_Z^2+1)*x+2*(4*x^2-2*x-3)^(1/2)+8*RootOf(_Z^2+1))/(RootOf(_Z^
2+1)*x-2*x-2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = -\frac{3580\,x^4+14186\,x^3+21848\,x^2+273\,(25\,x^4+80\,x^3+104\,x^2+64\,x+16)\arctan\left(\frac{5}{2}\,x+2\right)-273\,(25\,}{}$$

input
```
integrate(1/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

output
```
-1/64*(3580*x^4 + 14186*x^3 + 21848*x^2 + 273*(25*x^4 + 80*x^3 + 104*x^2 +
 64*x + 16)*arctan(5/2*x + 2) - 273*(25*x^4 + 80*x^3 + 104*x^2 + 64*x + 16
)*arctan(-1/2*(70*x^2 - 5*sqrt(4*x^2 - 2*x - 3)*(7*x + 8) + 112*x + 56)/(4
2*x + 31)) + 2*(895*x^3 + 2064*x^2 + 1748*x + 576)*sqrt(4*x^2 - 2*x - 3) +
 15224*x + 4320)/(25*x^4 + 80*x^3 + 104*x^2 + 64*x + 16)
```

Sympy [F]

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{\left(3x+\sqrt{4x^2-2x-3}+1\right)^3}\,dx$$

input
```
integrate(1/(1+3*x+(4*x**2-2*x-3)**(1/2))**3,x)
```

output
```
Integral((3*x + sqrt(4*x**2 - 2*x - 3) + 1)**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3}\,dx$$

input
```
integrate(1/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate((3*x + sqrt(4*x^2 - 2*x - 3) + 1)^(-3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(147) = 294.

Time = 0.13 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.83

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx =$$

$$-\frac{20475\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^7+218146\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^6+942003\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)}{80\left(5\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^4+32\right.}$$

$$-\frac{6825\,x^3+17388\,x^2+15148\,x+5072}{160\left(5\,x^2+8\,x+4\right)^2}-\frac{273}{64}\arctan\left(\frac{5}{2}\,x+2\right)$$

$$+\frac{273}{64}\arctan\left(-x+\frac{1}{2}\sqrt{4\,x^2-2\,x-3}-\frac{3}{2}\right)$$

$$+\frac{273}{64}\arctan\left(-5\,x+\frac{5}{2}\sqrt{4\,x^2-2\,x-3}-\frac{1}{2}\right)$$

input
```
integrate(1/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

output
```
-1/80*(20475*(2*x - sqrt(4*x^2 - 2*x - 3))^7 + 218146*(2*x - sqrt(4*x^2 -
2*x - 3))^6 + 942003*(2*x - sqrt(4*x^2 - 2*x - 3))^5 + 2074498*(2*x - sqrt
(4*x^2 - 2*x - 3))^4 + 2003721*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 1144598*(
2*x - sqrt(4*x^2 - 2*x - 3))^2 + 873106*x - 436553*sqrt(4*x^2 - 2*x - 3) +
 80966)/(5*(2*x - sqrt(4*x^2 - 2*x - 3))^4 + 32*(2*x - sqrt(4*x^2 - 2*x -
3))^3 + 78*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 64*x - 32*sqrt(4*x^2 - 2*x -
3) + 13)^2 - 1/160*(6825*x^3 + 17388*x^2 + 15148*x + 5072)/(5*x^2 + 8*x +
4)^2 - 273/64*arctan(5/2*x + 2) + 273/64*arctan(-x + 1/2*sqrt(4*x^2 - 2*x
- 3) - 3/2) + 273/64*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{1}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3}\,dx$$

input
```
int(1/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3,x)
```

output
```
int(1/(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3, x)
```

Reduce [F]

$$\int \frac{1}{\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{1}{\left(1+3x+\sqrt{4x^2-2x-3}\right)^3}dx$$

input
```
int(1/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

output
```
int(1/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

3.45 $\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 253

$$\begin{aligned}
\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3} \, dx =& -\frac{57+94\left(2x+\sqrt{-3-2x+4x^2}\right)}{10\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)^2} \\
& -\frac{31+330\left(2x+\sqrt{-3-2x+4x^2}\right)}{20\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)} \\
& -\frac{65}{8}\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right) \\
& +\frac{1}{8}\log\left(7+3x-20x^2-\sqrt{-3-2x+4x^2}-10x\sqrt{-3-2x+4x^2}\right) \\
& -\frac{1}{8}\log\left(x-4x^2-2x\sqrt{-3-2x+4x^2}\right)
\end{aligned}$$

output

```
-1/10*(57+188*x+94*(4*x^2-2*x-3)^(1/2))/(1+4*x+2*(4*x^2-2*x-3)^(1/2)+5*(2*
x+(4*x^2-2*x-3)^(1/2))^2)^2-(31+660*x+330*(4*x^2-2*x-3)^(1/2))/(20+80*x+40
*(4*x^2-2*x-3)^(1/2)+100*(2*x+(4*x^2-2*x-3)^(1/2))^2)-65/8*arctan(1/2+5*x+
5/2*(4*x^2-2*x-3)^(1/2))+1/8*ln(7+3*x-20*x^2-(4*x^2-2*x-3)^(1/2)-10*x*(4*x
^2-2*x-3)^(1/2))-1/8*ln(x-4*x^2-2*x*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.62

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx =\frac{1}{8}\left(\frac{\sqrt{-3-2x+4x^2}(138+398x+455x^2+200x^3)}{(4+8x+5x^2)^2}+\frac{210+612x+722x^2+305x^3}{(4+8x+5x^2)^2}+65\arctan\left(\frac{3}{2}+x-\frac{1}{2}\sqrt{-3-2x+4x^2}\right)+2\text{arctanh}\left(\frac{-5-6x+3\sqrt{-3-2x+4x^2}}{-5-4x-8x^2+(3+4x)\sqrt{-3-2x+4x^2}}\right)\right)$$

input
```
Integrate[1/(x*(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^3),x]
```

output
```
((Sqrt[-3 - 2*x + 4*x^2]*(138 + 398*x + 455*x^2 + 200*x^3))/(4 + 8*x + 5*x
^2)^2 + (210 + 612*x + 722*x^2 + 305*x^3)/(4 + 8*x + 5*x^2)^2 + 65*ArcTan[
3/2 + x - Sqrt[-3 - 2*x + 4*x^2]/2] + 2*ArcTanh[(-5 - 6*x + 3*Sqrt[-3 - 2*
x + 4*x^2])/(-5 - 4*x - 8*x^2 + (3 + 4*x)*Sqrt[-3 - 2*x + 4*x^2])])/8
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{4x^2-2x-3}+3x+1\right)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{-349x-292}{5\left(5x^2+8x+4\right)^3}+\frac{5x+8}{8\left(5x^2+8x+4\right)}+\frac{25x+166}{10\left(5x^2+8x+4\right)^2}-\frac{31x\sqrt{4x^2-2x-3}}{\left(5x^2+8x+4\right)^3}-\frac{16\sqrt{4x^2-2x-3}}{\left(5x^2+8x+4\right)^3}-\frac{1}{8x}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{65}{16}\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)+\frac{65}{16}\arctan\left(\frac{5x}{2}+2\right)+\frac{1}{8}\text{arctanh}\left(\frac{3x+1}{\sqrt{4x^2-2x-3}}\right)+\frac{57-16x}{20\left(5x^2+8x+4\right)^2}-\frac{3(5x+4)}{10\left(5x^2+8x+4\right)}+\frac{365x+282}{40\left(5x^2+8x+4\right)}+\frac{31(965x+701)\sqrt{4x^2-2x-3}}{2312\left(5x^2+8x+4\right)}-\frac{(18355x+13928)\sqrt{4x^2-2x-3}}{2312\left(5x^2+8x+4\right)}+\frac{31(x+1)\sqrt{4x^2-2x-3}}{4\left(5x^2+8x+4\right)^2}-\frac{(5x+4)\sqrt{4x^2-2x-3}}{\left(5x^2+8x+4\right)^2}+\frac{1}{16}\log\left(5x^2+8x+4\right)-\frac{\log(x)}{8}$$

input
```
Int[1/(x*(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^3),x]
```

output
```
(57 - 16*x)/(20*(4 + 8*x + 5*x^2)^2) + (31*(1 + x)*Sqrt[-3 - 2*x + 4*x^2])
/(4*(4 + 8*x + 5*x^2)^2) - ((4 + 5*x)*Sqrt[-3 - 2*x + 4*x^2])/(4 + 8*x + 5
*x^2)^2 - (3*(4 + 5*x))/(10*(4 + 8*x + 5*x^2)) + (282 + 365*x)/(40*(4 + 8*
x + 5*x^2)) + (31*(701 + 965*x)*Sqrt[-3 - 2*x + 4*x^2])/(2312*(4 + 8*x + 5
*x^2)) - ((13928 + 18355*x)*Sqrt[-3 - 2*x + 4*x^2])/(2312*(4 + 8*x + 5*x^2
)) + (65*ArcTan[2 + (5*x)/2])/16 + (65*ArcTan[(8 + 7*x)/(2*Sqrt[-3 - 2*x +
 4*x^2])])/16 + ArcTanh[(1 + 3*x)/Sqrt[-3 - 2*x + 4*x^2]]/8 - Log[x]/8 + L
og[4 + 8*x + 5*x^2]/16
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.55 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.35

method	result
trager	$-\frac{(x-1)(46225x^3+106000x^2+89638x+31106)}{2312(5x^2+8x+4)^2}+\frac{(200x^3+455x^2+398x+138)\sqrt{4x^2-2x-3}}{8(5x^2+8x+4)^2}+\frac{289\,\mathrm{RootOf}\left(334084\_Z^2-2312\right.}{}$
default	Expression too large to display

input
```
int(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2312*(x-1)*(46225*x^3+106000*x^2+89638*x+31106)/(5*x^2+8*x+4)^2+1/8*(20
0*x^3+455*x^2+398*x+138)/(5*x^2+8*x+4)^2*(4*x^2-2*x-3)^(1/2)+289/8*RootOf(
334084*_Z^2-2312*_Z+4229)*ln((1+3*x+(4*x^2-2*x-3)^(1/2))/x)+1/8*ln(-(-5782
99404*RootOf(334084*_Z^2-2312*_Z+4229)^2*x+135147960*(4*x^2-2*x-3)^(1/2)*R
ootOf(334084*_Z^2-2312*_Z+4229)+578299404*RootOf(334084*_Z^2-2312*_Z+4229)
^2+368591756*RootOf(334084*_Z^2-2312*_Z+4229)*x+6980645*(4*x^2-2*x-3)^(1/2
)+161682784*RootOf(334084*_Z^2-2312*_Z+4229)-4010960*x-1861440)/x)-289/8*l
n(-(-578299404*RootOf(334084*_Z^2-2312*_Z+4229)^2*x+135147960*(4*x^2-2*x-3
)^(1/2)*RootOf(334084*_Z^2-2312*_Z+4229)+578299404*RootOf(334084*_Z^2-2312
*_Z+4229)^2+368591756*RootOf(334084*_Z^2-2312*_Z+4229)*x+6980645*(4*x^2-2*
x-3)^(1/2)+161682784*RootOf(334084*_Z^2-2312*_Z+4229)-4010960*x-1861440)/x
)*RootOf(334084*_Z^2-2312*_Z+4229)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$=\frac{800\,x^4+3170\,x^3+4772\,x^2+65\,(25\,x^4+80\,x^3+104\,x^2+64\,x+16)\arctan\left(\frac{5}{2}\,x+2\right)-65\,(25\,x^4+80\,x}{}$$

input
```
integrate(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/16*(800*x^4 + 3170*x^3 + 4772*x^2 + 65*(25*x^4 + 80*x^3 + 104*x^2 + 64*x
 + 16)*arctan(5/2*x + 2) - 65*(25*x^4 + 80*x^3 + 104*x^2 + 64*x + 16)*arct
an(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - 65*(25*x^4 + 80*x^3 + 104*x^2 +
 64*x + 16)*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) - (25*x^4 + 80*
x^3 + 104*x^2 + 64*x + 16)*log(20*x^2 - sqrt(4*x^2 - 2*x - 3)*(10*x + 1) -
 3*x - 7) + (25*x^4 + 80*x^3 + 104*x^2 + 64*x + 16)*log(5*x^2 + 8*x + 4) +
 (25*x^4 + 80*x^3 + 104*x^2 + 64*x + 16)*log(4*x^2 - sqrt(4*x^2 - 2*x - 3)
*(2*x + 3) + 5*x + 5) - 2*(25*x^4 + 80*x^3 + 104*x^2 + 64*x + 16)*log(x) +
 2*(200*x^3 + 455*x^2 + 398*x + 138)*sqrt(4*x^2 - 2*x - 3) + 3272*x + 932)
/(25*x^4 + 80*x^3 + 104*x^2 + 64*x + 16)
```

Sympy [F]

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{x\left(3x+\sqrt{4x^2-2x-3}+1\right)^3}\,dx$$

input
```
integrate(1/x/(1+3*x+(4*x**2-2*x-3)**(1/2))**3,x)
```

output
```
Integral(1/(x*(3*x + sqrt(4*x**2 - 2*x - 3) + 1)**3), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3 x}\,dx$$

input
```
integrate(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(1/((3*x + sqrt(4*x^2 - 2*x - 3) + 1)^3*x), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.68

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$= \frac{470\left(2x-\sqrt{4x^2-2x-3}\right)^7+4797\left(2x-\sqrt{4x^2-2x-3}\right)^6+20915\left(2x-\sqrt{4x^2-2x-3}\right)^5+4693}{2\left(5\left(2x-\sqrt{4x^2-2x-3}\right)^4+32\left(2x-\sqrt{4x\right.\right.}}$$
$$+\frac{305x^3+722x^2+612x+210}{8\left(5x^2+8x+4\right)^2}+\frac{65}{16}\arctan\left(\frac{5}{2}x+2\right)$$
$$-\frac{65}{16}\arctan\left(-x+\frac{1}{2}\sqrt{4x^2-2x-3}-\frac{3}{2}\right)$$
$$-\frac{65}{16}\arctan\left(-5x+\frac{5}{2}\sqrt{4x^2-2x-3}-\frac{1}{2}\right)$$
$$-\frac{1}{16}\log\left(5\left(2x-\sqrt{4x^2-2x-3}\right)^2+4x-2\sqrt{4x^2-2x-3}+1\right)$$
$$+\frac{1}{16}\log\left(\left(2x-\sqrt{4x^2-2x-3}\right)^2+12x-6\sqrt{4x^2-2x-3}+13\right)$$
$$+\frac{1}{16}\log\left(5x^2+8x+4\right)-\frac{1}{8}\log\left(|x|\right)$$

input
```
integrate(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

output
```
1/2*(470*(2*x - sqrt(4*x^2 - 2*x - 3))^7 + 4797*(2*x - sqrt(4*x^2 - 2*x -
3))^6 + 20915*(2*x - sqrt(4*x^2 - 2*x - 3))^5 + 46933*(2*x - sqrt(4*x^2 -
2*x - 3))^4 + 45116*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 24799*(2*x - sqrt(4*
x^2 - 2*x - 3))^2 + 19238*x - 9619*sqrt(4*x^2 - 2*x - 3) + 1999)/(5*(2*x -
 sqrt(4*x^2 - 2*x - 3))^4 + 32*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 78*(2*x -
 sqrt(4*x^2 - 2*x - 3))^2 + 64*x - 32*sqrt(4*x^2 - 2*x - 3) + 13)^2 + 1/8*
(305*x^3 + 722*x^2 + 612*x + 210)/(5*x^2 + 8*x + 4)^2 + 65/16*arctan(5/2*x
 + 2) - 65/16*arctan(-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - 65/16*arctan(
-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) - 1/16*log(5*(2*x - sqrt(4*x^2 - 2
*x - 3))^2 + 4*x - 2*sqrt(4*x^2 - 2*x - 3) + 1) + 1/16*log((2*x - sqrt(4*x
^2 - 2*x - 3))^2 + 12*x - 6*sqrt(4*x^2 - 2*x - 3) + 13) + 1/16*log(5*x^2 +
 8*x + 4) - 1/8*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{x\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3}\,dx$$

input
```
int(1/(x*(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3),x)
```

output
```
int(1/(x*(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3), x)
```

Reduce [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 947, normalized size of antiderivative = 3.74

$$\int \frac{1}{x\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/x/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

output

```
( - 33257184675000*atan((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x**4 - 10642
2990960000*atan((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x**3 - 1383498882480
00*atan((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x**2 - 8513839276800*atan((
sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x - 21284598192000*atan((sqrt(4*x**2
- 2*x - 3) + 2*x + 3)/2) - 33257184675000*atan((5*sqrt(4*x**2 - 2*x - 3) +
 10*x + 1)/2)*x**4 - 106422990960000*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x
 + 1)/2)*x**3 - 138349888248000*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x + 1)
/2)*x**2 - 8513839276800*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x + 1)/2)*x
- 21284598192000*atan((5*sqrt(4*x**2 - 2*x - 3) + 10*x + 1)/2) + 332571846
75000*atan((5*x + 4)/2)*x**4 + 106422990960000*atan((5*x + 4)/2)*x**3 + 13
8349888248000*atan((5*x + 4)/2)*x**2 + 8513839276800*atan((5*x + 4)/2)*x
+ 21284598192000*atan((5*x + 4)/2) + 8186383920000*sqrt(4*x**2 - 2*x - 3)*
x**3 + 18624023418000*sqrt(4*x**2 - 2*x - 3)*x**2 + 16290904000800*sqrt(4*
x**2 - 2*x - 3)*x + 5648604904800*sqrt(4*x**2 - 2*x - 3) + 511648995000*lo
g(5*x**2 + 8*x + 4)*x**4 + 1637276784000*log(5*x**2 + 8*x + 4)*x**3 + 2128
459819200*log(5*x**2 + 8*x + 4)*x**2 + 1309821427200*log(5*x**2 + 8*x + 4)
*x + 327455356800*log(5*x**2 + 8*x + 4) + 511648995000*log((80*sqrt(4*x**2
 - 2*x - 3)*x + 8*sqrt(4*x**2 - 2*x - 3) + 160*x**2 - 24*x - 56)/sqrt(13))
*x**4 + 1637276784000*log((80*sqrt(4*x**2 - 2*x - 3)*x + 8*sqrt(4*x**2 - 2
*x - 3) + 160*x**2 - 24*x - 56)/sqrt(13))*x**3 + 2128459819200*log((80*...
```

3.46 $\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 381

$$
\begin{aligned}
&\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx\\
&=-\frac{47+263\left(2x+\sqrt{-3-2x+4x^2}\right)}{8\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)^2}\\
&\quad+\frac{2\left(19+53\left(2x+\sqrt{-3-2x+4x^2}\right)\right)}{\left(3+\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)^2}\\
&\quad+\frac{3\left(17+485\left(2x+\sqrt{-3-2x+4x^2}\right)\right)}{64\left(1+2\left(2x+\sqrt{-3-2x+4x^2}\right)+5\left(2x+\sqrt{-3-2x+4x^2}\right)^2\right)}\\
&\quad+\frac{1}{2}\sqrt{3}\arctan\left(\frac{2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right)\\
&\quad+\frac{927}{128}\arctan\left(\frac{1}{2}\left(1+5\left(2x+\sqrt{-3-2x+4x^2}\right)\right)\right)\\
&\quad-\frac{3}{8}\log\left(7+3x-20x^2-\sqrt{-3-2x+4x^2}-10x\sqrt{-3-2x+4x^2}\right)\\
&\quad+\frac{3}{8}\log\left(x-4x^2-2x\sqrt{-3-2x+4x^2}\right)
\end{aligned}
$$

output
```
-1/8*(47+526*x+263*(4*x^2-2*x-3)^(1/2))/(1+4*x+2*(4*x^2-2*x-3)^(1/2)+5*(2*
x+(4*x^2-2*x-3)^(1/2))^2)^2+2*(19+106*x+53*(4*x^2-2*x-3)^(1/2))/(3+(2*x+(4
*x^2-2*x-3)^(1/2))^2)/(1+4*x+2*(4*x^2-2*x-3)^(1/2)+5*(2*x+(4*x^2-2*x-3)^(1
/2))^2)^2+3*(17+970*x+485*(4*x^2-2*x-3)^(1/2))/(64+256*x+128*(4*x^2-2*x-3)
^(1/2)+320*(2*x+(4*x^2-2*x-3)^(1/2))^2)+1/2*3^(1/2)*arctan(1/3*(2*x+(4*x^2
-2*x-3)^(1/2))*3^(1/2))+927/128*arctan(1/2+5*x+5/2*(4*x^2-2*x-3)^(1/2))-3/
8*ln(7+3*x-20*x^2-(4*x^2-2*x-3)^(1/2)-10*x*(4*x^2-2*x-3)^(1/2))+3/8*ln(x-4
*x^2-2*x*(4*x^2-2*x-3)^(1/2))
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \frac{1}{128}\left(-\frac{\sqrt{-3-2x+4x^2}(2752+7628x+8160x^2+3425x^3)}{(4+8x+5x^2)^2} - \frac{-256+2288x+7012x^2+8340x^3+3675x^4}{x(4+8x+5x^2)^2} - 927\arctan\left(\frac{3}{2}+x-\frac{1}{2}\sqrt{-3-2x+4x^2}\right) + 64\sqrt{3}\arctan\left(\frac{-2x+\sqrt{-3-2x+4x^2}}{\sqrt{3}}\right) - 96\operatorname{arctanh}\left(\frac{-5-6x+3\sqrt{-3-2x+4x^2}}{-5-4x-8x^2+(3+4x)\sqrt{-3-2x+4x^2}}\right)\right)$$

input
```
Integrate[1/(x^2*(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^3),x]
```

output
```
(-((Sqrt[-3 - 2*x + 4*x^2]*(2752 + 7628*x + 8160*x^2 + 3425*x^3))/(4 + 8*x
 + 5*x^2)^2) - (-256 + 2288*x + 7012*x^2 + 8340*x^3 + 3675*x^4)/(x*(4 + 8*
x + 5*x^2)^2) - 927*ArcTan[3/2 + x - Sqrt[-3 - 2*x + 4*x^2]/2] + 64*Sqrt[3
]*ArcTan[(-2*x + Sqrt[-3 - 2*x + 4*x^2])/Sqrt[3]] - 96*ArcTanh[(-5 - 6*x +
 3*Sqrt[-3 - 2*x + 4*x^2])/(-5 - 4*x - 8*x^2 + (3 + 4*x)*Sqrt[-3 - 2*x + 4
*x^2])])/128
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{4x^2-2x-3}+3x+1\right)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{-15x-19}{8\left(5x^2+8x+4\right)}-\frac{\sqrt{4x^2-2x-3}}{4x}+\frac{5x\sqrt{4x^2-2x-3}}{4\left(5x^2+8x+4\right)}+\frac{2\sqrt{4x^2-2x-3}}{5x^2+8x+4}-\frac{1}{8x^2}+\frac{-5x-3}{2\left(5x^2+8x+4\right)^2}+\frac{5}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{x+3}{\sqrt{3}\sqrt{4x^2-2x-3}}\right)-\frac{927}{256}\arctan\left(\frac{7x+8}{2\sqrt{4x^2-2x-3}}\right)-\frac{927}{256}\arctan\left(\frac{5x}{2}+2\right)-\frac{3}{8}\text{arctanh}\left(\frac{3x+1}{\sqrt{4x^2-2x-3}}\right)-\frac{5\sqrt{4x^2-2x-3}(x+1)}{2\left(5x^2+8x+4\right)}-\frac{5\sqrt{4x^2-2x-3}(x+1)}{\left(5x^2+8x+4\right)^2}-\frac{171(5x+4)}{128\left(5x^2+8x+4\right)}+\frac{5x+8}{16\left(5x^2+8x+4\right)}+\frac{(5x+4)\sqrt{4x^2-2x-3}}{5x^2+8x+4}-\frac{5(965x+701)\sqrt{4x^2-2x-3}}{578\left(5x^2+8x+4\right)}+\frac{(18355x+13928)\sqrt{4x^2-2x-3}}{36992\left(5x^2+8x+4\right)}-\frac{57x+104}{16\left(5x^2+8x+4\right)^2}+\frac{(5x+4)\sqrt{4x^2-2x-3}}{16\left(5x^2+8x+4\right)^2}-\frac{3}{16}\log\left(5x^2+8x+4\right)+\frac{1}{8x}+\frac{3\log(x)}{8}$$

input
```
Int[1/(x^2*(1 + 3*x + Sqrt[-3 - 2*x + 4*x^2])^3),x]
```

output
```
1/(8*x) - (104 + 57*x)/(16*(4 + 8*x + 5*x^2)^2) - (5*(1 + x)*Sqrt[-3 - 2*x
 + 4*x^2])/(4 + 8*x + 5*x^2)^2 + ((4 + 5*x)*Sqrt[-3 - 2*x + 4*x^2])/(16*(4
 + 8*x + 5*x^2)^2) - (171*(4 + 5*x))/(128*(4 + 8*x + 5*x^2)) + (8 + 5*x)/(
16*(4 + 8*x + 5*x^2)) - (5*(1 + x)*Sqrt[-3 - 2*x + 4*x^2])/(2*(4 + 8*x + 5
*x^2)) + ((4 + 5*x)*Sqrt[-3 - 2*x + 4*x^2])/(4 + 8*x + 5*x^2) - (5*(701 +
965*x)*Sqrt[-3 - 2*x + 4*x^2])/(578*(4 + 8*x + 5*x^2)) + ((13928 + 18355*x
)*Sqrt[-3 - 2*x + 4*x^2])/(36992*(4 + 8*x + 5*x^2)) - (927*ArcTan[2 + (5*x
)/2])/256 - (Sqrt[3]*ArcTan[(3 + x)/(Sqrt[3]*Sqrt[-3 - 2*x + 4*x^2])])/4 -
 (927*ArcTan[(8 + 7*x)/(2*Sqrt[-3 - 2*x + 4*x^2])])/256 - (3*ArcTanh[(1 +
3*x)/Sqrt[-3 - 2*x + 4*x^2]])/8 + (3*Log[x])/8 - (3*Log[4 + 8*x + 5*x^2])/
16
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.09 (sec) , antiderivative size = 1489, normalized size of antiderivative = 3.91

method	result	size
trager	Expression too large to display	1489
default	Expression too large to display	8350

input
```
int(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/36992*(x-1)*(526475*x^4+1149120*x^3+928996*x^2+250304*x-73984)/x/(5*x^2+
8*x+4)^2-1/128*(3425*x^3+8160*x^2+7628*x+2752)/(5*x^2+8*x+4)^2*(4*x^2-2*x-
3)^(1/2)+867/128*RootOf(334084*_Z^2+36992*_Z+96505)*ln((197434862852623*Ro
otOf(250563*_Z^2-27744*_Z+1792)^2*RootOf(334084*_Z^2+36992*_Z+96505)^2*x-1
97434862852623*RootOf(250563*_Z^2-27744*_Z+1792)^2*RootOf(334084*_Z^2+3699
2*_Z+96505)^2-237247296748516*RootOf(250563*_Z^2-27744*_Z+1792)^2*RootOf(3
34084*_Z^2+36992*_Z+96505)*x-8537361605024*RootOf(250563*_Z^2-27744*_Z+179
2)*RootOf(334084*_Z^2+36992*_Z+96505)^2*x+237247296748516*RootOf(250563*_Z
^2-27744*_Z+1792)^2*RootOf(334084*_Z^2+36992*_Z+96505)+52354097844256*Root
Of(250563*_Z^2-27744*_Z+1792)^2*x+8537361605024*RootOf(250563*_Z^2-27744*_
Z+1792)*RootOf(334084*_Z^2+36992*_Z+96505)^2-4756928532480*RootOf(250563*_
Z^2-27744*_Z+1792)*RootOf(334084*_Z^2+36992*_Z+96505)*(4*x^2-2*x-3)^(1/2)+
39571439980384*RootOf(250563*_Z^2-27744*_Z+1792)*RootOf(334084*_Z^2+36992*
_Z+96505)*x-87428446464*RootOf(334084*_Z^2+36992*_Z+96505)^2*x-52354097844
256*RootOf(250563*_Z^2-27744*_Z+1792)^2+21000391951744*RootOf(250563*_Z^2-
27744*_Z+1792)*RootOf(334084*_Z^2+36992*_Z+96505)+12713477603376*RootOf(25
0563*_Z^2-27744*_Z+1792)*(4*x^2-2*x-3)^(1/2)-5965569258240*RootOf(250563*_
Z^2-27744*_Z+1792)*x+87428446464*RootOf(334084*_Z^2+36992*_Z+96505)^2+2062
493990912*(4*x^2-2*x-3)^(1/2)*RootOf(334084*_Z^2+36992*_Z+96505)+827079713
280*RootOf(334084*_Z^2+36992*_Z+96505)*x-1167773262784*RootOf(250563*_...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx =$$
$$-\frac{13700\,x^5+51190\,x^4+73672\,x^3-128\,\sqrt{3}(25\,x^5+80\,x^4+104\,x^3+64\,x^2+16\,x)\arctan\left(-\frac{2}{3}\,\sqrt{3}x+\frac{1}{3}\right.}{}$$

input
```
integrate(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

output
```
-1/256*(13700*x^5 + 51190*x^4 + 73672*x^3 - 128*sqrt(3)*(25*x^5 + 80*x^4 +
 104*x^3 + 64*x^2 + 16*x)*arctan(-2/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(4*x^2 -
 2*x - 3)) + 49096*x^2 + 927*(25*x^5 + 80*x^4 + 104*x^3 + 64*x^2 + 16*x)*a
rctan(5/2*x + 2) - 927*(25*x^5 + 80*x^4 + 104*x^3 + 64*x^2 + 16*x)*arctan(
-x + 1/2*sqrt(4*x^2 - 2*x - 3) - 3/2) - 927*(25*x^5 + 80*x^4 + 104*x^3 + 6
4*x^2 + 16*x)*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) - 48*(25*x^5
+ 80*x^4 + 104*x^3 + 64*x^2 + 16*x)*log(20*x^2 - sqrt(4*x^2 - 2*x - 3)*(10
*x + 1) - 3*x - 7) + 48*(25*x^5 + 80*x^4 + 104*x^3 + 64*x^2 + 16*x)*log(5*
x^2 + 8*x + 4) + 48*(25*x^5 + 80*x^4 + 104*x^3 + 64*x^2 + 16*x)*log(4*x^2
- sqrt(4*x^2 - 2*x - 3)*(2*x + 3) + 5*x + 5) - 96*(25*x^5 + 80*x^4 + 104*x
^3 + 64*x^2 + 16*x)*log(x) + 2*(3425*x^4 + 8160*x^3 + 7628*x^2 + 2752*x)*s
qrt(4*x^2 - 2*x - 3) + 13344*x - 512)/(25*x^5 + 80*x^4 + 104*x^3 + 64*x^2
+ 16*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{1}{x^2\left(3x+\sqrt{4x^2-2x-3}+1\right)^3}\,dx$$

input
```
integrate(1/x**2/(1+3*x+(4*x**2-2*x-3)**(1/2))**3,x)
```

output
```
Integral(1/(x**2*(3*x + sqrt(4*x**2 - 2*x - 3) + 1)**3), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx=\int \frac{1}{\left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3x^2}\,dx$$

input
```
integrate(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(1/((3*x + sqrt(4*x^2 - 2*x - 3) + 1)^3*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^2\left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx$$
$$= \frac{1}{2}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)\right)$$
$$-\frac{14625\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^7+153510\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^6+712297\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^5}{64\left(5\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^4+32\right.}$$
$$-\frac{3675\,x^4+8340\,x^3+7012\,x^2+2288\,x-256}{128\left(5\,x^2+8\,x+4\right)^2x}-\frac{927}{256}\arctan\left(\frac{5}{2}\,x+2\right)$$
$$+\frac{927}{256}\arctan\left(-x+\frac{1}{2}\sqrt{4\,x^2-2\,x-3}-\frac{3}{2}\right)$$
$$+\frac{927}{256}\arctan\left(-5\,x+\frac{5}{2}\sqrt{4\,x^2-2\,x-3}-\frac{1}{2}\right)$$
$$+\frac{3}{16}\log\left(5\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+4\,x-2\sqrt{4\,x^2-2\,x-3}+1\right)$$
$$-\frac{3}{16}\log\left(\left(2\,x-\sqrt{4\,x^2-2\,x-3}\right)^2+12\,x-6\sqrt{4\,x^2-2\,x-3}+13\right)$$
$$-\frac{3}{16}\log\left(5\,x^2+8\,x+4\right)+\frac{3}{8}\log\left(|x|\right)$$

input
```
integrate(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

output
```
1/2*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - sqrt(4*x^2 - 2*x - 3))) - 1/64*(146
25*(2*x - sqrt(4*x^2 - 2*x - 3))^7 + 153510*(2*x - sqrt(4*x^2 - 2*x - 3))^
6 + 712297*(2*x - sqrt(4*x^2 - 2*x - 3))^5 + 1664454*(2*x - sqrt(4*x^2 - 2
*x - 3))^4 + 1614955*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 843778*(2*x - sqrt(
4*x^2 - 2*x - 3))^2 + 616470*x - 308235*sqrt(4*x^2 - 2*x - 3) + 68498)/(5*
(2*x - sqrt(4*x^2 - 2*x - 3))^4 + 32*(2*x - sqrt(4*x^2 - 2*x - 3))^3 + 78*
(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 64*x - 32*sqrt(4*x^2 - 2*x - 3) + 13)^2
 - 1/128*(3675*x^4 + 8340*x^3 + 7012*x^2 + 2288*x - 256)/((5*x^2 + 8*x + 4)
^2*x) - 927/256*arctan(5/2*x + 2) + 927/256*arctan(-x + 1/2*sqrt(4*x^2 - 2
*x - 3) - 3/2) + 927/256*arctan(-5*x + 5/2*sqrt(4*x^2 - 2*x - 3) - 1/2) +
3/16*log(5*(2*x - sqrt(4*x^2 - 2*x - 3))^2 + 4*x - 2*sqrt(4*x^2 - 2*x - 3)
 + 1) - 3/16*log((2*x - sqrt(4*x^2 - 2*x - 3))^2 + 12*x - 6*sqrt(4*x^2 - 2
*x - 3) + 13) - 3/16*log(5*x^2 + 8*x + 4) + 3/8*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \int \frac{1}{x^2 \left(3\,x+\sqrt{4\,x^2-2\,x-3}+1\right)^3}\,dx$$

input
```
int(1/(x^2*(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3),x)
```

output
```
int(1/(x^2*(3*x + (4*x^2 - 2*x - 3)^(1/2) + 1)^3), x)
```

Reduce [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 1118, normalized size of antiderivative = 2.93

$$\int \frac{1}{x^2 \left(1+3x+\sqrt{-3-2x+4x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(1+3*x+(4*x^2-2*x-3)^(1/2))^3,x)
```

output
```
(2709985056075000*atan((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x**5 + 867195
2179440000*atan((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x**4 + 1127353783327
2000*atan((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x**3 + 6937561743552000*at
an((sqrt(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x**2 + 1734390435888000*atan((sqr
t(4*x**2 - 2*x - 3) + 2*x + 3)/2)*x + 374194268800000*sqrt(3)*atan((sqrt(4
*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x**5 + 1197421660160000*sqrt(3)*atan((sqr
t(4*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x**4 + 1556648158208000*sqrt(3)*atan((
sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x**3 + 957937328128000*sqrt(3)*atan
((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x**2 + 239484332032000*sqrt(3)*at
an((sqrt(4*x**2 - 2*x - 3) + 2*x)/sqrt(3))*x + 2709985056075000*atan((5*sq
rt(4*x**2 - 2*x - 3) + 10*x + 1)/2)*x**5 + 8671952179440000*atan((5*sqrt(4
*x**2 - 2*x - 3) + 10*x + 1)/2)*x**4 + 11273537833272000*atan((5*sqrt(4*x*
*2 - 2*x - 3) + 10*x + 1)/2)*x**3 + 6937561743552000*atan((5*sqrt(4*x**2 -
 2*x - 3) + 10*x + 1)/2)*x**2 + 1734390435888000*atan((5*sqrt(4*x**2 - 2*x
 - 3) + 10*x + 1)/2)*x - 2709985056075000*atan((5*x + 4)/2)*x**5 - 8671952
179440000*atan((5*x + 4)/2)*x**4 - 11273537833272000*atan((5*x + 4)/2)*x**
3 - 6937561743552000*atan((5*x + 4)/2)*x**2 - 1734390435888000*atan((5*x +
 4)/2)*x - 801009606650000*sqrt(4*x**2 - 2*x - 3)*x**4 - 1908390770880000*
sqrt(4*x**2 - 2*x - 3)*x**3 - 1783971176504000*sqrt(4*x**2 - 2*x - 3)*x**2
 - 643614142336000*sqrt(4*x**2 - 2*x - 3)*x - 140322850800000*log(5*x**...
```

3.47 $\int \frac{x^2}{1+2x+\sqrt{-2+8x-5x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 411

$$\begin{aligned}\int \frac{x^2}{1+2x+\sqrt{-2+8x-5x^2}}\,dx &= \frac{4\left(2+\frac{5\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)}{75\left(1-\frac{5(2-8x+5x^2)}{\left(4-\sqrt{6}-5x\right)^2}\right)^2} \\ &+ \frac{\sqrt{\frac{2}{3}}\left(2\left(157-18\sqrt{6}\right)+\frac{5\left(40-9\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)}{675\left(1-\frac{5(2-8x+5x^2)}{\left(4-\sqrt{6}-5x\right)^2}\right)} - \frac{2476\arctan\left(\frac{\sqrt{5}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)}{3645\sqrt{5}} \\ &- \frac{862\arctan\left(\frac{6+\frac{\left(12-13\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{729\sqrt{23}} - \frac{14}{729}\log\left(\frac{2\left(3-2\sqrt{6}\right)+5\sqrt{6}x}{\left(4-\sqrt{6}-5x\right)^2}\right) \\ &+ \frac{14}{729}\log\left(\frac{2\left(3-2\sqrt{6}\right)+12x-3\sqrt{6}x+10\sqrt{6}x^2+6\sqrt{-2+8x-5x^2}-4\sqrt{6}\sqrt{-2+8x-5x^2}+5\sqrt{6}x}{\left(4-\sqrt{6}-5x\right)^2}\right.\end{aligned}$$

output
```
4/75*(2+5*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))/(1-5*(5*x^2-8*x+2)/(4-6^(1
/2)-5*x)^2)^2+1/3*6^(1/2)*(314-36*6^(1/2)+5*(40-9*6^(1/2))*(-5*x^2+8*x-2)^
(1/2)/(4-6^(1/2)-5*x))/(675-3375*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2)-2476/182
25*arctan(5^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))*5^(1/2)-862/16767*
arctan(1/138*(6+(12-13*6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))*138^
(1/2))*23^(1/2)-14/729*ln((6-4*6^(1/2)+5*x*6^(1/2))/(4-6^(1/2)-5*x)^2)+14/
729*ln((6-4*6^(1/2)+12*x-3*x*6^(1/2)+10*6^(1/2)*x^2+6*(-5*x^2+8*x-2)^(1/2)
-4*6^(1/2)*(-5*x^2+8*x-2)^(1/2)+5*6^(1/2)*x*(-5*x^2+8*x-2)^(1/2))/(4-6^(1/
2)-5*x)^2)
```

Mathematica [A] (verified)

Time = 4.09 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \frac{113896\sqrt{5}\arctan\left(\frac{4+\sqrt{6}-5x}{\sqrt{5}\sqrt{-2+8x-5x^2}}\right)-5\left(8620\sqrt{23}\arctan\left(\frac{\sqrt{46\left(1429012686912434410345 1+58338302532324994644 6\right.}}{292002624366+119872147024\sqrt{6}-5\left(44877761532+18748596\right.}\right.\right.}{}$$

input
```
Integrate[x^2/(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2]),x]
```

output
```
(113896*Sqrt[5]*ArcTan[(4 + Sqrt[6] - 5*x)/(Sqrt[5]*Sqrt[-2 + 8*x - 5*x^2]
)] - 5*(8620*Sqrt[23]*ArcTan[(Sqrt[46*(14290126869124344103451 + 583383025
3232499464464*Sqrt[6])] - 5*Sqrt[23*(687178779311511919633 + 2804650123624
04707812*Sqrt[6])]*x)/(292002624366 + 119872147024*Sqrt[6] - 5*(4487776153
2 + 18748596373*Sqrt[6])*x + 333487275913*Sqrt[-2 + 8*x - 5*x^2] + 1347323
42732*Sqrt[6]*Sqrt[-2 + 8*x - 5*x^2])] + 23*(9*(-90*x^2 + 4*Sqrt[-2 + 8*x
- 5*x^2] + 5*x*(-34 + 9*Sqrt[-2 + 8*x - 5*x^2])) + 140*Log[-6 - 4*Sqrt[6]
+ 5*Sqrt[6]*x] - 140*Log[(-292002624366 - 119872147024*Sqrt[6] + 224388807
660*x + 93742981865*Sqrt[6]*x)*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])])))/8383
50
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.47, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{-5x^2+8x-2}+2x+1}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{4\sqrt{-5x^2+8x-2}x}{9\left(9x^2-4x+3\right)}-\frac{1}{9}\sqrt{-5x^2+8x-2}+\frac{14x-51}{81\left(9x^2-4x+3\right)}+\frac{\sqrt{-5x^2+8x-2}}{3\left(9x^2-4x+3\right)}+\frac{2x}{9}+\frac{17}{81}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{27}\sqrt{5}\arcsin\left(\frac{4-5x}{\sqrt{6}}\right)+\frac{563\arcsin\left(\frac{4-5x}{\sqrt{6}}\right)}{3645\sqrt{5}}-\frac{431\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{729\sqrt{23}}+\frac{431\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{729\sqrt{23}}+\frac{14}{729}\text{arctanh}\left(\frac{2x+1}{\sqrt{-5x^2+8x-2}}\right)+\frac{x^2}{9}+\frac{1}{90}(4-5x)\sqrt{-5x^2+8x-2}-\frac{4}{81}\sqrt{-5x^2+8x-2}+\frac{7}{729}\log\left(9x^2-4x+3\right)+\frac{17x}{81}$$

input
```
Int[x^2/(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2]),x]
```

output
```
(17*x)/81 + x^2/9 - (4*Sqrt[-2 + 8*x - 5*x^2])/81 + ((4 - 5*x)*Sqrt[-2 + 8
*x - 5*x^2])/90 + (563*ArcSin[(4 - 5*x)/Sqrt[6]])/(3645*Sqrt[5]) + (Sqrt[5
]*ArcSin[(4 - 5*x)/Sqrt[6]])/27 + (431*ArcTan[(2 - 9*x)/Sqrt[23]])/(729*Sq
rt[23]) - (431*ArcTan[(8 - 13*x)/(Sqrt[23]*Sqrt[-2 + 8*x - 5*x^2])])/(729*
Sqrt[23]) + (14*ArcTanh[(1 + 2*x)/Sqrt[-2 + 8*x - 5*x^2]])/729 + (7*Log[3
- 4*x + 9*x^2])/729
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.22

method	result
default	$\frac{(-10x+8)\sqrt{-5x^2+8x-2}}{180}-\frac{1238\sqrt{5}\,\arcsin\left(\frac{5\sqrt{6}\left(x-\frac{4}{5}\right)}{6}\right)}{18225}-\frac{4\sqrt{-5x^2+8x-2}}{81}-\frac{5\sqrt{29}\,\sqrt{676}\,\sqrt{\frac{696\left(x+\frac{1}{2}\right)^2}{\left(\frac{8}{13}-x\right)^2}-4901}\left(787\sqrt{23}\ \mathrm{a}\right.}{36}$
trager	Expression too large to display

input
```
int(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/180*(-10*x+8)*(-5*x^2+8*x-2)^(1/2)-1238/18225*5^(1/2)*arcsin(5/6*6^(1/2)
*(x-4/5))-4/81*(-5*x^2+8*x-2)^(1/2)-5/366627222*29^(1/2)*676^(1/2)*(696*(x
+1/2)^2/(8/13-x)^2-4901)^(1/2)*(787*23^(1/2)*arctan(1/377*(696*(x+1/2)^2/(
8/13-x)^2-4901)^(1/2)*23^(1/2))-5014*arctanh(58*(x+1/2)/(8/13-x)/(696*(x+1
/2)^2/(8/13-x)^2-4901)^(1/2)))/((24*(x+1/2)^2/(8/13-x)^2-169)/((x+1/2)/(8/
13-x)+1)^2)^(1/2)/((x+1/2)/(8/13-x)+1)+4/20368179*29^(1/2)*676^(1/2)*(696*
(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*(244*23^(1/2)*arctan(1/377*(696*(x+1/2)^2
/(8/13-x)^2-4901)^(1/2)*23^(1/2))+299*arctanh(58*(x+1/2)/(8/13-x)/(696*(x+
1/2)^2/(8/13-x)^2-4901)^(1/2)))/((24*(x+1/2)^2/(8/13-x)^2-169)/((x+1/2)/(8
/13-x)+1)^2)^(1/2)/((x+1/2)/(8/13-x)+1)-1/2263131*29^(1/2)*676^(1/2)*(696*
(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*(7*23^(1/2)*arctan(1/377*(696*(x+1/2)^2/(
8/13-x)^2-4901)^(1/2)*23^(1/2))+230*arctanh(58*(x+1/2)/(8/13-x)/(696*(x+1/
2)^2/(8/13-x)^2-4901)^(1/2)))/((24*(x+1/2)^2/(8/13-x)^2-169)/((x+1/2)/(8/1
3-x)+1)^2)^(1/2)/((x+1/2)/(8/13-x)+1)+17/81*x+7/729*ln(9*x^2-4*x+3)-431/16
767*23^(1/2)*arctan(1/46*(18*x-4)*23^(1/2))+1/9*x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.68

$$
\begin{aligned}
\int \frac{x^2}{1+2x+\sqrt{-2+8x-5x^2}}\,dx
&= \frac{1}{9}\,x^2 - \frac{1}{810}\,\sqrt{-5\,x^2+8\,x-2}(45\,x+4) - \frac{431}{16767}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(9\,x-2)\right) \\
&\quad + \frac{1238}{18225}\,\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-5\,x^2+8\,x-2}(5\,x-4)}{5\,(5\,x^2-8\,x+2)}\right) \\
&\quad - \frac{431}{33534}\,\sqrt{23}\arctan\left(\frac{\sqrt{23}\sqrt{-5\,x^2+8\,x-2}(13\,x-8)+2\,\sqrt{23}(2\,x^2-3\,x)}{23\,(7\,x^2-8\,x+2)}\right) \\
&\quad - \frac{431}{33534}\,\sqrt{23}\arctan\left(\frac{\sqrt{23}\sqrt{-5\,x^2+8\,x-2}(13\,x-8)-2\,\sqrt{23}(2\,x^2-3\,x)}{23\,(7\,x^2-8\,x+2)}\right) \\
&\quad + \frac{17}{81}\,x + \frac{7}{729}\,\log\left(9\,x^2-4\,x+3\right) \\
&\quad - \frac{7}{1458}\,\log\left(-\frac{x^2+2\,\sqrt{-5\,x^2+8\,x-2}(2\,x+1)-12\,x+1}{x^2}\right) \\
&\quad + \frac{7}{1458}\,\log\left(-\frac{x^2-2\,\sqrt{-5\,x^2+8\,x-2}(2\,x+1)-12\,x+1}{x^2}\right)
\end{aligned}
$$

input
```
integrate(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="fricas")
```

output
```
1/9*x^2 - 1/810*sqrt(-5*x^2 + 8*x - 2)*(45*x + 4) - 431/16767*sqrt(23)*arc
tan(1/23*sqrt(23)*(9*x - 2)) + 1238/18225*sqrt(5)*arctan(1/5*sqrt(5)*sqrt(
-5*x^2 + 8*x - 2)*(5*x - 4)/(5*x^2 - 8*x + 2)) - 431/33534*sqrt(23)*arctan
(1/23*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) + 2*sqrt(23)*(2*x^2 - 3*
x))/(7*x^2 - 8*x + 2)) - 431/33534*sqrt(23)*arctan(1/23*(sqrt(23)*sqrt(-5*
x^2 + 8*x - 2)*(13*x - 8) - 2*sqrt(23)*(2*x^2 - 3*x))/(7*x^2 - 8*x + 2)) +
 17/81*x + 7/729*log(9*x^2 - 4*x + 3) - 7/1458*log(-(x^2 + 2*sqrt(-5*x^2 +
 8*x - 2)*(2*x + 1) - 12*x + 1)/x^2) + 7/1458*log(-(x^2 - 2*sqrt(-5*x^2 +
8*x - 2)*(2*x + 1) - 12*x + 1)/x^2)
```

Sympy [F]

$$\int \frac{x^2}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \int \frac{x^2}{2x+\sqrt{-5x^2+8x-2}+1}\,dx$$

input
```
integrate(x**2/(1+2*x+(-5*x**2+8*x-2)**(1/2)),x)
```

output
```
Integral(x**2/(2*x + sqrt(-5*x**2 + 8*x - 2) + 1), x)
```

Maxima [F]

$$\int \frac{x^2}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \int \frac{x^2}{2\,x+\sqrt{-5\,x^2+8\,x-2}+1}\,dx$$

input
```
integrate(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(2*x + sqrt(-5*x^2 + 8*x - 2) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \frac{1}{9}\,x^2-\frac{1}{810}\,\sqrt{-5\,x^2+8\,x-2}(45\,x+4)-\frac{1238}{18225}\,\sqrt{5}\arcsin\left(\frac{1}{6}\,\sqrt{6}(5\,x-4)\right) - \frac{431}{16767}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(9\,x-2)\right)+\frac{17}{81}\,x - \frac{431\,\left(5\,\sqrt{6}+13\,\sqrt{5}\right)\arctan\left(-\frac{26\,\sqrt{6}+12\,\sqrt{5}-\frac{139\,\left(\sqrt{5}\sqrt{-5\,x^2+8\,x-2}-\sqrt{6}\right)}{5\,x-4}}{5\,\sqrt{138}+13\,\sqrt{115}}\right)}{729\,\left(5\,\sqrt{138}+13\,\sqrt{115}\right)} - \frac{431\,\left(5\,\sqrt{6}-13\,\sqrt{5}\right)\arctan\left(\frac{26\,\sqrt{6}-12\,\sqrt{5}-\frac{139\,\left(\sqrt{5}\sqrt{-5\,x^2+8\,x-2}-\sqrt{6}\right)}{5\,x-4}}{5\,\sqrt{138}-13\,\sqrt{115}}\right)}{729\,\left(5\,\sqrt{138}-13\,\sqrt{115}\right)} + \frac{7}{729}\,\log\left(9\,x^2-4\,x+3\right) + \frac{7}{729}\,\log\left(-\frac{4\,\left(\sqrt{5}\sqrt{-5\,x^2+8\,x-2}-\sqrt{6}\right)\left(13\,\sqrt{6}+6\,\sqrt{5}\right)}{5\,x-4}+26\,\sqrt{30} + \frac{139\,\left(\sqrt{5}\sqrt{-5\,x^2+8\,x-2}-\sqrt{6}\right)^2}{(5\,x-4)^2}+199\right) - \frac{7}{729}\,\log\left(-\frac{4\,\left(\sqrt{5}\sqrt{-5\,x^2+8\,x-2}-\sqrt{6}\right)\left(13\,\sqrt{6}-6\,\sqrt{5}\right)}{5\,x-4}-26\,\sqrt{30} + \frac{139\,\left(\sqrt{5}\sqrt{-5\,x^2+8\,x-2}-\sqrt{6}\right)^2}{(5\,x-4)^2}+199\right)$$

input
```
integrate(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="giac")
```

output
```
1/9*x^2 - 1/810*sqrt(-5*x^2 + 8*x - 2)*(45*x + 4) - 1238/18225*sqrt(5)*arc
sin(1/6*sqrt(6)*(5*x - 4)) - 431/16767*sqrt(23)*arctan(1/23*sqrt(23)*(9*x
- 2)) + 17/81*x - 431/729*(5*sqrt(6) + 13*sqrt(5))*arctan(-(26*sqrt(6) + 1
2*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*s
qrt(138) + 13*sqrt(115)))/(5*sqrt(138) + 13*sqrt(115)) - 431/729*(5*sqrt(6
) - 13*sqrt(5))*arctan((26*sqrt(6) - 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2
 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138) - 13*sqrt(115)))/(5*sqrt(13
8) - 13*sqrt(115)) + 7/729*log(9*x^2 - 4*x + 3) + 7/729*log(-4*(sqrt(5)*sq
rt(-5*x^2 + 8*x - 2) - sqrt(6))*(13*sqrt(6) + 6*sqrt(5))/(5*x - 4) + 26*sq
rt(30) + 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 + 19
9) - 7/729*log(-4*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))*(13*sqrt(6) -
 6*sqrt(5))/(5*x - 4) - 26*sqrt(30) + 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2)
- sqrt(6))^2/(5*x - 4)^2 + 199)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \int \frac{x^2}{2\,x+\sqrt{-5\,x^2+8\,x-2}+1}\,dx$$

input
```
int(x^2/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1),x)
```

output
```
int(x^2/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1), x)
```

Reduce [F]

$$\int \frac{x^2}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = -\frac{431\sqrt{23}\,atan\left(\frac{9x-2}{\sqrt{23}}\right)}{16767} + \frac{1238\sqrt{5}\,atan\left(\frac{5\sqrt{-5x^2+8x-2}\,\sqrt{5}\,x-4\sqrt{-5x^2+8x-2}\,\sqrt{5}}{25x^2-40x+10}\right)}{18225} - \frac{\sqrt{-5x^2+8x-2}\,x}{18} - \frac{342682\sqrt{-5x^2+8x-2}}{31525605} - \frac{1715069\left(\int \frac{\sqrt{-5x^2+8x-2}}{45x^4-92x^3+65x^2-32x+6}dx\right)}{2101707} + \frac{187000\left(\int \frac{\sqrt{-5x^2+8x-2}\,x^3}{45x^4-92x^3+65x^2-32x+6}dx\right)}{700569} - \frac{2094400\left(\int \frac{\sqrt{-5x^2+8x-2}\,x^2}{45x^4-92x^3+65x^2-32x+6}dx\right)}{6305121} - \frac{152320\left(\int \frac{\sqrt{-5x^2+8x-2}\,x}{45x^4-92x^3+65x^2-32x+6}dx\right)}{203391} + \frac{7\log(9x^2-4x+3)}{729} + \frac{x^2}{9} + \frac{17x}{81}$$

input
```
int(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x)
```

output
```
( - 186385950*sqrt(23)*atan((9*x - 2)/sqrt(23)) + 492543252*sqrt(5)*atan((
5*sqrt( - 5*x**2 + 8*x - 2)*sqrt(5)*x - 4*sqrt( - 5*x**2 + 8*x - 2)*sqrt(5
))/(25*x**2 - 40*x + 10)) - 402827175*sqrt( - 5*x**2 + 8*x - 2)*x - 788168
60*sqrt( - 5*x**2 + 8*x - 2) - 5916988050*int(sqrt( - 5*x**2 + 8*x - 2)/(4
5*x**4 - 92*x**3 + 65*x**2 - 32*x + 6),x) + 1935450000*int((sqrt( - 5*x**2
 + 8*x - 2)*x**3)/(45*x**4 - 92*x**3 + 65*x**2 - 32*x + 6),x) - 2408560000
*int((sqrt( - 5*x**2 + 8*x - 2)*x**2)/(45*x**4 - 92*x**3 + 65*x**2 - 32*x
+ 6),x) - 5430208000*int((sqrt( - 5*x**2 + 8*x - 2)*x)/(45*x**4 - 92*x**3
+ 65*x**2 - 32*x + 6),x) + 69624450*log(9*x**2 - 4*x + 3) + 805654350*x**2
 + 1521791550*x)/7250889150
```

3.48 $\int \frac{x}{1+2x+\sqrt{-2+8x-5x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 326

$$\int \frac{x}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \frac{2\sqrt{\frac{2}{3}}\left(2+\frac{5\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)}{15\left(1-\frac{5(2-8x+5x^2)}{\left(4-\sqrt{6}-5x\right)^2}\right)}-\frac{32\arctan\left(\frac{\sqrt{5}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)}{81\sqrt{5}}$$
$$-\frac{40\arctan\left(\frac{6+\frac{\left(12-13\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{81\sqrt{23}}-\frac{17}{81}\log\left(\frac{2(3-2\sqrt{6})+5\sqrt{6}x}{\left(4-\sqrt{6}-5x\right)^2}\right)$$
$$+\frac{17}{81}\log\left(\frac{2(3-2\sqrt{6})+12x-3\sqrt{6}x+10\sqrt{6}x^2+6\sqrt{-2+8x-5x^2}-4\sqrt{6}\sqrt{-2+8x-5x^2}+5\sqrt{6}x}{\left(4-\sqrt{6}-5x\right)^2}\right.$$

output
```
2/3*6^(1/2)*(2+5*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))/(15-75*(5*x^2-8*x+2
)/(4-6^(1/2)-5*x)^2)-32/405*arctan(5^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)
-5*x))*5^(1/2)-40/1863*arctan(1/138*(6+(12-13*6^(1/2))*(-5*x^2+8*x-2)^(1/2
)/(4-6^(1/2)-5*x))*138^(1/2))*23^(1/2)-17/81*ln((6-4*6^(1/2)+5*x*6^(1/2))/
(4-6^(1/2)-5*x)^2)+17/81*ln((6-4*6^(1/2)+12*x-3*x*6^(1/2)+10*6^(1/2)*x^2+6
*(-5*x^2+8*x-2)^(1/2)-4*6^(1/2)*(-5*x^2+8*x-2)^(1/2)+5*6^(1/2)*x*(-5*x^2+8
*x-2)^(1/2))/(4-6^(1/2)-5*x)^2)
```

Mathematica [A] (verified)

Time = 3.77 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.72

$$\int \frac{x}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \frac{32\arctan\left(\frac{4+\sqrt{6}-5x}{\sqrt{5}\sqrt{-2+8x-5x^2}}\right)}{81\sqrt{5}}$$
$$+\frac{40\arctan\left(\frac{-\sqrt{46\left(5327843406035187907
31+2175043291949393833
84\sqrt{6}\right)}+5\sqrt{589281938283348577079+240497438031837409956\cdots}}{56368423806+23151720184\sqrt{6}-5\left(8656337412+3623845693\sqrt{6}\right)x+64422668833\sqrt{-2+8x-5x^2}+26003089112\sqrt{6}\sqrt{-2+8\cdots}}\right.}{81\sqrt{23}}$$
$$+\frac{1}{81}\left(18x-9\sqrt{-2+8x-5x^2}-17\log\left(-6-4\sqrt{6}+5\sqrt{6}x\right)\right.$$
$$+17\log\left(\left(-56368423806-23151720184\sqrt{6}+5\left(8656337412+3623845693\sqrt{6}\right)x\right)\left(1+2x+\sqrt{-2+8x\cdots}\right.\right.$$

input
```
Integrate[x/(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2]),x]
```

output
```
(32*ArcTan[(4 + Sqrt[6] - 5*x)/(Sqrt[5]*Sqrt[-2 + 8*x - 5*x^2])])/(81*Sqrt
[5]) + (40*ArcTan[(-Sqrt[46*(532784340603518790731 + 217504329194939383384
*Sqrt[6])] + 5*Sqrt[589281938283348577079 + 240497438031837409956*Sqrt[6]]
*x)/(56368423806 + 23151720184*Sqrt[6] - 5*(8656337412 + 3623845693*Sqrt[6
])*x + 64422668833*Sqrt[-2 + 8*x - 5*x^2] + 26003089112*Sqrt[6]*Sqrt[-2 +
8*x - 5*x^2])])/(81*Sqrt[23]) + (18*x - 9*Sqrt[-2 + 8*x - 5*x^2] - 17*Log[
-6 - 4*Sqrt[6] + 5*Sqrt[6]*x] + 17*Log[(-56368423806 - 23151720184*Sqrt[6]
 + 5*(8656337412 + 3623845693*Sqrt[6])*x)*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2
])]])/81
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.43, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-5x^2+8x-2}+2x+1}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{\sqrt{-5x^2+8x-2}x}{9x^2-4x+3}+\frac{17x-6}{9(9x^2-4x+3)}+\frac{2}{9}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{16\arcsin\left(\frac{4-5x}{\sqrt{6}}\right)}{81\sqrt{5}}-\frac{20\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{81\sqrt{23}}+\frac{20\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{81\sqrt{23}}+\frac{17}{81}\text{arctanh}\left(\frac{2x+1}{\sqrt{-5x^2+8x-2}}\right)-\frac{1}{9}\sqrt{-5x^2+8x-2}+\frac{17}{162}\log\left(9x^2-4x+3\right)+\frac{2x}{9}$$

input
```
Int[x/(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2]),x]
```

output
```
(2*x)/9 - Sqrt[-2 + 8*x - 5*x^2]/9 + (16*ArcSin[(4 - 5*x)/Sqrt[6]])/(81*Sq
rt[5]) + (20*ArcTan[(2 - 9*x)/Sqrt[23]])/(81*Sqrt[23]) - (20*ArcTan[(8 - 1
3*x)/(Sqrt[23]*Sqrt[-2 + 8*x - 5*x^2])])/(81*Sqrt[23]) + (17*ArcTanh[(1 +
2*x)/Sqrt[-2 + 8*x - 5*x^2]])/81 + (17*Log[3 - 4*x + 9*x^2])/162
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.46

method	result
default	$-\frac{16\sqrt{5}\,\arcsin\left(\frac{5\sqrt{6}\left(x-\frac{4}{5}\right)}{6}\right)}{405}-\frac{\sqrt{-5x^2+8x-2}}{9}-\frac{5\sqrt{29}\,\sqrt{676}\,\sqrt{\frac{696\left(x+\frac{1}{2}\right)^2}{\left(\frac{8}{13}-x\right)^2}-4901}\left(244\sqrt{23}\,\arctan\left(\frac{\sqrt{\frac{696\left(x+\frac{1}{2}\right)^2}{\left(\frac{8}{13}-x\right)^2}-4901}\,\sqrt{23}}{377}\right.\right.}{40736358\sqrt{\frac{\frac{24\left(x+\frac{1}{2}\right)^2}{\left(\frac{8}{13}-x\right)^2}-169}{\left(\frac{x+\frac{1}{2}}{\frac{8}{13}-x}+1\right)^2}}\left(\frac{x+}{\frac{8}{13}}\right.}$
trager	Expression too large to display

input
```
int(x/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-16/405*5^(1/2)*arcsin(5/6*6^(1/2)*(x-4/5))-1/9*(-5*x^2+8*x-2)^(1/2)-5/407
36358*29^(1/2)*676^(1/2)*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*(244*23^(1/
2)*arctan(1/377*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*23^(1/2))+299*arctan
h(58*(x+1/2)/(8/13-x)/(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)))/((24*(x+1/2)
^2/(8/13-x)^2-169)/((x+1/2)/(8/13-x)+1)^2)^(1/2)/((x+1/2)/(8/13-x)+1)+4/22
63131*29^(1/2)*676^(1/2)*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*(7*23^(1/2)
*arctan(1/377*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*23^(1/2))+230*arctanh(
58*(x+1/2)/(8/13-x)/(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)))/((24*(x+1/2)^2
/(8/13-x)^2-169)/((x+1/2)/(8/13-x)+1)^2)^(1/2)/((x+1/2)/(8/13-x)+1)+1/2514
59*29^(1/2)*676^(1/2)*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*(8*23^(1/2)*ar
ctan(1/377*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*23^(1/2))-23*arctanh(58*(
x+1/2)/(8/13-x)/(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)))/((24*(x+1/2)^2/(8/
13-x)^2-169)/((x+1/2)/(8/13-x)+1)^2)^(1/2)/((x+1/2)/(8/13-x)+1)+17/162*ln(
9*x^2-4*x+3)-20/1863*23^(1/2)*arctan(1/46*(18*x-4)*23^(1/2))+2/9*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.83

$$
\begin{aligned}
&\int \frac{x}{1+2x+\sqrt{-2+8x-5x^2}}\,dx \\
&= -\frac{20}{1863}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(9x-2)\right) \\
&\quad+\frac{16}{405}\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-5x^2+8x-2}(5x-4)}{5(5x^2-8x+2)}\right) \\
&\quad-\frac{10}{1863}\sqrt{23}\arctan\left(\frac{\sqrt{23}\sqrt{-5x^2+8x-2}(13x-8)+2\sqrt{23}(2x^2-3x)}{23(7x^2-8x+2)}\right) \\
&\quad-\frac{10}{1863}\sqrt{23}\arctan\left(\frac{\sqrt{23}\sqrt{-5x^2+8x-2}(13x-8)-2\sqrt{23}(2x^2-3x)}{23(7x^2-8x+2)}\right) \\
&\quad+\frac{2}{9}x-\frac{1}{9}\sqrt{-5x^2+8x-2}+\frac{17}{162}\log\left(9x^2-4x+3\right) \\
&\quad-\frac{17}{324}\log\left(-\frac{x^2+2\sqrt{-5x^2+8x-2}(2x+1)-12x+1}{x^2}\right) \\
&\quad+\frac{17}{324}\log\left(-\frac{x^2-2\sqrt{-5x^2+8x-2}(2x+1)-12x+1}{x^2}\right)
\end{aligned}
$$

input
```
integrate(x/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="fricas")
```

output
```
-20/1863*sqrt(23)*arctan(1/23*sqrt(23)*(9*x - 2)) + 16/405*sqrt(5)*arctan(
1/5*sqrt(5)*sqrt(-5*x^2 + 8*x - 2)*(5*x - 4)/(5*x^2 - 8*x + 2)) - 10/1863*
sqrt(23)*arctan(1/23*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) + 2*sqrt(
23)*(2*x^2 - 3*x))/(7*x^2 - 8*x + 2)) - 10/1863*sqrt(23)*arctan(1/23*(sqrt
(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) - 2*sqrt(23)*(2*x^2 - 3*x))/(7*x^2
- 8*x + 2)) + 2/9*x - 1/9*sqrt(-5*x^2 + 8*x - 2) + 17/162*log(9*x^2 - 4*x
+ 3) - 17/324*log(-(x^2 + 2*sqrt(-5*x^2 + 8*x - 2)*(2*x + 1) - 12*x + 1)/x
^2) + 17/324*log(-(x^2 - 2*sqrt(-5*x^2 + 8*x - 2)*(2*x + 1) - 12*x + 1)/x^
2)
```

Sympy [F]

$$\int \frac{x}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \int \frac{x}{2x+\sqrt{-5x^2+8x-2}+1}\,dx$$

input
```
integrate(x/(1+2*x+(-5*x**2+8*x-2)**(1/2)),x)
```

output
```
Integral(x/(2*x + sqrt(-5*x**2 + 8*x - 2) + 1), x)
```

Maxima [F]

$$\int \frac{x}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \int \frac{x}{2\,x+\sqrt{-5\,x^2+8\,x-2}+1}\,dx$$

input
```
integrate(x/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(2*x + sqrt(-5*x^2 + 8*x - 2) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.23

$$\int \frac{x}{1+2x+\sqrt{-2+8x-5x^2}}\,dx$$
$$= -\frac{16}{405}\sqrt{5}\arcsin\left(\frac{1}{6}\sqrt{6}(5x-4)\right) - \frac{20}{1863}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(9x-2)\right) + \frac{2}{9}x - \frac{20\left(5\sqrt{6}+13\sqrt{5}\right)\arctan\left(-\frac{26\sqrt{6}+12\sqrt{5}-\frac{139\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)}{5x-4}}{5\sqrt{138}+13\sqrt{115}}\right)}{81\left(5\sqrt{138}+13\sqrt{115}\right)} - \frac{20\left(5\sqrt{6}-13\sqrt{5}\right)\arctan\left(\frac{26\sqrt{6}-12\sqrt{5}-\frac{139\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)}{5x-4}}{5\sqrt{138}-13\sqrt{115}}\right)}{81\left(5\sqrt{138}-13\sqrt{115}\right)} - \frac{1}{9}\sqrt{-5x^2+8x-2} + \frac{17}{162}\log\left(9x^2-4x+3\right) + \frac{17}{162}\log\left(-\frac{4\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)\left(13\sqrt{6}+6\sqrt{5}\right)}{5x-4} + 26\sqrt{30} + \frac{139\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)^2}{(5x-4)^2} + 199\right) - \frac{17}{162}\log\left(-\frac{4\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)\left(13\sqrt{6}-6\sqrt{5}\right)}{5x-4} - 26\sqrt{30} + \frac{139\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)^2}{(5x-4)^2} + 199\right)$$

input
```
integrate(x/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="giac")
```

output
```
-16/405*sqrt(5)*arcsin(1/6*sqrt(6)*(5*x - 4)) - 20/1863*sqrt(23)*arctan(1/
23*sqrt(23)*(9*x - 2)) + 2/9*x - 20/81*(5*sqrt(6) + 13*sqrt(5))*arctan(-(2
6*sqrt(6) + 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5
*x - 4))/(5*sqrt(138) + 13*sqrt(115)))/(5*sqrt(138) + 13*sqrt(115)) - 20/8
1*(5*sqrt(6) - 13*sqrt(5))*arctan((26*sqrt(6) - 12*sqrt(5) - 139*(sqrt(5)*
sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138) - 13*sqrt(115)))
/(5*sqrt(138) - 13*sqrt(115)) - 1/9*sqrt(-5*x^2 + 8*x - 2) + 17/162*log(9*
x^2 - 4*x + 3) + 17/162*log(-4*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))*
(13*sqrt(6) + 6*sqrt(5))/(5*x - 4) + 26*sqrt(30) + 139*(sqrt(5)*sqrt(-5*x^
2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 + 199) - 17/162*log(-4*(sqrt(5)*sqrt
(-5*x^2 + 8*x - 2) - sqrt(6))*(13*sqrt(6) - 6*sqrt(5))/(5*x - 4) - 26*sqrt
(30) + 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 + 199)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \int \frac{x}{2\,x+\sqrt{-5\,x^2+8\,x-2}+1}\,dx$$

input
```
int(x/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1),x)
```

output
```
int(x/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1), x)
```

Reduce [F]

$$\int \frac{x}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = -\frac{20\sqrt{23}\, atan\left(\frac{9x-2}{\sqrt{23}}\right)}{1863} + \frac{16\sqrt{5}\, atan\left(\frac{5\sqrt{-5x^2+8x-2}\,\sqrt{5}\,x-4\sqrt{-5x^2+8x-2}\,\sqrt{5}}{25x^2-40x+10}\right)}{405} - \frac{67886\sqrt{-5x^2+8x-2}}{700569} - \frac{409928\left(\int \frac{\sqrt{-5x^2+8x-2}}{45x^4-92x^3+65x^2-32x+6}dx\right)}{233523} - \frac{49775\left(\int \frac{\sqrt{-5x^2+8x-2}\,x^3}{45x^4-92x^3+65x^2-32x+6}dx\right)}{77841} + \frac{557480\left(\int \frac{\sqrt{-5x^2+8x-2}\,x^2}{45x^4-92x^3+65x^2-32x+6}dx\right)}{700569} + \frac{40544\left(\int \frac{\sqrt{-5x^2+8x-2}\,x}{45x^4-92x^3+65x^2-32x+6}dx\right)}{22599} + \frac{17\log(9x^2-4x+3)}{162} + \frac{2x}{9}$$

input

```
int(x/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x)
```

output

```
( - 1729800*sqrt(23)*atan((9*x - 2)/sqrt(23)) + 6365664*sqrt(5)*atan((5*sq
rt( - 5*x**2 + 8*x - 2)*sqrt(5)*x - 4*sqrt( - 5*x**2 + 8*x - 2)*sqrt(5))/(
25*x**2 - 40*x + 10)) - 15613780*sqrt( - 5*x**2 + 8*x - 2) - 282850320*int
(sqrt( - 5*x**2 + 8*x - 2)/(45*x**4 - 92*x**3 + 65*x**2 - 32*x + 6),x) - 1
03034250*int((sqrt( - 5*x**2 + 8*x - 2)*x**3)/(45*x**4 - 92*x**3 + 65*x**2
 - 32*x + 6),x) + 128220400*int((sqrt( - 5*x**2 + 8*x - 2)*x**2)/(45*x**4
- 92*x**3 + 65*x**2 - 32*x + 6),x) + 289078720*int((sqrt( - 5*x**2 + 8*x -
 2)*x)/(45*x**4 - 92*x**3 + 65*x**2 - 32*x + 6),x) + 16908795*log(9*x**2 -
 4*x + 3) + 35806860*x)/161130870
```

3.49 $\int \frac{1}{1+2x+\sqrt{-2+8x-5x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 253

$$\int \frac{1}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \frac{2}{9}\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right) + \frac{26\arctan\left(\frac{6+\frac{(12-13\sqrt{6})\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{9\sqrt{23}} - \frac{2}{9}\log\left(\frac{2(3-2\sqrt{6})+5\sqrt{6}x}{(4-\sqrt{6}-5x)^2}\right) + \frac{2}{9}\log\left(\frac{2(3-2\sqrt{6})+12x-3\sqrt{6}x+10\sqrt{6}x^2+6\sqrt{-2+8x-5x^2}-4\sqrt{6}\sqrt{-2+8x-5x^2}+5\sqrt{6}x\sqrt{}}{(4-\sqrt{6}-5x)^2}\right)$$

output
```
2/9*arctan(5^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))*5^(1/2)+26/207*ar
ctan(1/138*(6+(12-13*6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))*138^(1
/2))*23^(1/2)-2/9*ln((6-4*6^(1/2)+5*x*6^(1/2))/(4-6^(1/2)-5*x)^2)+2/9*ln((
6-4*6^(1/2)+12*x-3*x*6^(1/2)+10*6^(1/2)*x^2+6*(-5*x^2+8*x-2)^(1/2)-4*6^(1/
2)*(-5*x^2+8*x-2)^(1/2)+5*6^(1/2)*x*(-5*x^2+8*x-2)^(1/2))/(4-6^(1/2)-5*x)^
2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.74

$$\int \frac{1}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = -\frac{2}{9}\sqrt{5}\arctan\left(\frac{4+\sqrt{6}-5x}{\sqrt{5}\sqrt{-2+8x-5x^2}}\right) + \frac{26\arctan\left(\frac{\sqrt{23}\left(4+\sqrt{6}-5x\right)}{6+4\sqrt{6}-5\sqrt{6}x+13\sqrt{-2+8x-5x^2}+2\sqrt{6}\sqrt{-2+8x-5x^2}}\right)}{9\sqrt{23}} + \frac{2}{9}\left(-\log\left(-6-4\sqrt{6}+5\sqrt{6}x\right) + \log\left(\left(-6-4\sqrt{6}+5\sqrt{6}x\right)\left(1+2x+\sqrt{-2+8x-5x^2}\right)\right)\right)$$

input
```
Integrate[(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^(-1),x]
```

output
```
(-2*Sqrt[5]*ArcTan[(4 + Sqrt[6] - 5*x)/(Sqrt[5]*Sqrt[-2 + 8*x - 5*x^2])])/
9 + (26*ArcTan[(Sqrt[23]*(4 + Sqrt[6] - 5*x))/(6 + 4*Sqrt[6] - 5*Sqrt[6]*x
 + 13*Sqrt[-2 + 8*x - 5*x^2] + 2*Sqrt[6]*Sqrt[-2 + 8*x - 5*x^2])])/(9*Sqrt
[23]) + (2*(-Log[-6 - 4*Sqrt[6] + 5*Sqrt[6]*x] + Log[(-6 - 4*Sqrt[6] + 5*S
qrt[6]*x)*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])]))/9
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.47, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-5x^2+8x-2}+2x+1}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{2x+1}{9x^2-4x+3}-\frac{\sqrt{-5x^2+8x-2}}{9x^2-4x+3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{9}\sqrt{5}\arcsin\left(\frac{4-5x}{\sqrt{6}}\right)+\frac{13\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{9\sqrt{23}}-\frac{13\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{9\sqrt{23}}+\frac{2}{9}\text{arctanh}\left(\frac{2x+1}{\sqrt{-5x^2+8x-2}}\right)+\frac{1}{9}\log\left(9x^2-4x+3\right)$$

input
```
Int[(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^(-1),x]
```

output
```
-1/9*(Sqrt[5]*ArcSin[(4 - 5*x)/Sqrt[6]]) - (13*ArcTan[(2 - 9*x)/Sqrt[23]])
/(9*Sqrt[23]) + (13*ArcTan[(8 - 13*x)/(Sqrt[23]*Sqrt[-2 + 8*x - 5*x^2])])/
(9*Sqrt[23]) + (2*ArcTanh[(1 + 2*x)/Sqrt[-2 + 8*x - 5*x^2]])/9 + Log[3 - 4
*x + 9*x^2]/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(198) = 396$.

Time = 0.97 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.81

method	result
default	$\frac{\sqrt{5}\,\arcsin\left(\frac{5\sqrt{6}\left(x-\frac{4}{5}\right)}{6}\right)}{9}-\frac{5\sqrt{29}\,\sqrt{676}\,\sqrt{\frac{696\left(x+\frac{1}{2}\right)^{2}}{\left(\frac{8}{13}-x\right)^{2}}-4901}\,\left(7\sqrt{23}\,\arctan\left(\frac{\sqrt{\frac{696\left(x+\frac{1}{2}\right)^{2}}{\left(\frac{8}{13}-x\right)^{2}}-4901}\,\sqrt{23}}{377}\right)+230\,\mathrm{arctanh}\left(\frac{\cdots}{\left(\frac{8}{13}-x\right)\cdots}\right.\right.}{4526262\sqrt{\frac{\frac{24\left(x+\frac{1}{2}\right)^{2}}{\left(\frac{8}{13}-x\right)^{2}}-169}{\left(\frac{x+\frac{1}{2}}{\frac{8}{13}-x}+1\right)^{2}}}\,\left(\frac{x+\frac{1}{2}}{\frac{8}{13}-x}+1\right)}$
trager	Expression too large to display

input
```
int(1/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/9*5^(1/2)*arcsin(5/6*6^(1/2)*(x-4/5))-5/4526262*29^(1/2)*676^(1/2)*(696*
(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*(7*23^(1/2)*arctan(1/377*(696*(x+1/2)^2/(
8/13-x)^2-4901)^(1/2)*23^(1/2))+230*arctanh(58*(x+1/2)/(8/13-x)/(696*(x+1/
2)^2/(8/13-x)^2-4901)^(1/2)))/((24*(x+1/2)^2/(8/13-x)^2-169)/((x+1/2)/(8/1
3-x)+1)^2)^(1/2)/((x+1/2)/(8/13-x)+1)-4/251459*29^(1/2)*676^(1/2)*(696*(x+
1/2)^2/(8/13-x)^2-4901)^(1/2)*(8*23^(1/2)*arctan(1/377*(696*(x+1/2)^2/(8/1
3-x)^2-4901)^(1/2)*23^(1/2))-23*arctanh(58*(x+1/2)/(8/13-x)/(696*(x+1/2)^2
/(8/13-x)^2-4901)^(1/2)))/((24*(x+1/2)^2/(8/13-x)^2-169)/((x+1/2)/(8/13-x)
+1)^2)^(1/2)/((x+1/2)/(8/13-x)+1)+1/251459*29^(1/2)*676^(1/2)*(696*(x+1/2)
^2/(8/13-x)^2-4901)^(1/2)*(13*23^(1/2)*arctan(1/377*(696*(x+1/2)^2/(8/13-x
)^2-4901)^(1/2)*23^(1/2))+46*arctanh(58*(x+1/2)/(8/13-x)/(696*(x+1/2)^2/(8
/13-x)^2-4901)^(1/2)))/((24*(x+1/2)^2/(8/13-x)^2-169)/((x+1/2)/(8/13-x)+1)
^2)^(1/2)/((x+1/2)/(8/13-x)+1)+13/207*23^(1/2)*arctan(1/46*(18*x-4)*23^(1/
2))+1/9*ln(9*x^2-4*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+2x+\sqrt{-2+8x-5x^2}}\,dx$$
$$= \frac{13}{207}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(9x-2)\right) - \frac{1}{9}\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-5x^2+8x-2}(5x-4)}{5(5x^2-8x+2)}\right) + \frac{13}{414}\sqrt{23}\arctan\left(\frac{\sqrt{23}\sqrt{-5x^2+8x-2}(13x-8)+2\sqrt{23}(2x^2-3x)}{23(7x^2-8x+2)}\right) + \frac{13}{414}\sqrt{23}\arctan\left(\frac{\sqrt{23}\sqrt{-5x^2+8x-2}(13x-8)-2\sqrt{23}(2x^2-3x)}{23(7x^2-8x+2)}\right) + \frac{1}{9}\log\left(9x^2-4x+3\right) - \frac{1}{18}\log\left(-\frac{x^2+2\sqrt{-5x^2+8x-2}(2x+1)-12x+1}{x^2}\right) + \frac{1}{18}\log\left(-\frac{x^2-2\sqrt{-5x^2+8x-2}(2x+1)-12x+1}{x^2}\right)$$

input
```
integrate(1/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="fricas")
```

output
```
13/207*sqrt(23)*arctan(1/23*sqrt(23)*(9*x - 2)) - 1/9*sqrt(5)*arctan(1/5*s
qrt(5)*sqrt(-5*x^2 + 8*x - 2)*(5*x - 4)/(5*x^2 - 8*x + 2)) + 13/414*sqrt(2
3)*arctan(1/23*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) + 2*sqrt(23)*(2
*x^2 - 3*x))/(7*x^2 - 8*x + 2)) + 13/414*sqrt(23)*arctan(1/23*(sqrt(23)*sq
rt(-5*x^2 + 8*x - 2)*(13*x - 8) - 2*sqrt(23)*(2*x^2 - 3*x))/(7*x^2 - 8*x +
 2)) + 1/9*log(9*x^2 - 4*x + 3) - 1/18*log(-(x^2 + 2*sqrt(-5*x^2 + 8*x - 2
)*(2*x + 1) - 12*x + 1)/x^2) + 1/18*log(-(x^2 - 2*sqrt(-5*x^2 + 8*x - 2)*(
2*x + 1) - 12*x + 1)/x^2)
```

Sympy [F]

$$\int \frac{1}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \int \frac{1}{2x+\sqrt{-5x^2+8x-2}+1}\,dx$$

input
```
integrate(1/(1+2*x+(-5*x**2+8*x-2)**(1/2)),x)
```

output
```
Integral(1/(2*x + sqrt(-5*x**2 + 8*x - 2) + 1), x)
```

Maxima [F]

$$\int \frac{1}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \int \frac{1}{2\,x+\sqrt{-5\,x^2+8\,x-2}+1}\,dx$$

input
```
integrate(1/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(2*x + sqrt(-5*x^2 + 8*x - 2) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(191) = 382.

Time = 0.16 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.51

$$\int \frac{1}{1+2x+\sqrt{-2+8x-5x^2}}\,dx$$
$$= \frac{1}{9}\sqrt{5}\arcsin\left(\frac{1}{6}\sqrt{6}(5x-4)\right) + \frac{13}{207}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(9x-2)\right)$$
$$+ \frac{13\left(5\sqrt{6}+13\sqrt{5}\right)\arctan\left(-\frac{26\sqrt{6}+12\sqrt{5}-\frac{139\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)}{5x-4}}{5\sqrt{138}+13\sqrt{115}}\right)}{9\left(5\sqrt{138}+13\sqrt{115}\right)}$$
$$+ \frac{13\left(5\sqrt{6}-13\sqrt{5}\right)\arctan\left(\frac{26\sqrt{6}-12\sqrt{5}-\frac{139\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)}{5x-4}}{5\sqrt{138}-13\sqrt{115}}\right)}{9\left(5\sqrt{138}-13\sqrt{115}\right)}$$
$$+ \frac{1}{9}\log\left(9x^2-4x+3\right) + \frac{1}{9}\log\left(-\frac{4\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)\left(13\sqrt{6}+6\sqrt{5}\right)}{5x-4} + 26\sqrt{30} + \frac{139\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)^2}{(5x-4)^2} + 199\right)$$
$$- \frac{1}{9}\log\left(-\frac{4\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)\left(13\sqrt{6}-6\sqrt{5}\right)}{5x-4} - 26\sqrt{30} + \frac{139\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)^2}{(5x-4)^2} + 199\right)$$

input
```
integrate(1/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="giac")
```

output
```
1/9*sqrt(5)*arcsin(1/6*sqrt(6)*(5*x - 4)) + 13/207*sqrt(23)*arctan(1/23*sq
rt(23)*(9*x - 2)) + 13/9*(5*sqrt(6) + 13*sqrt(5))*arctan(-(26*sqrt(6) + 12
*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sq
rt(138) + 13*sqrt(115)))/(5*sqrt(138) + 13*sqrt(115)) + 13/9*(5*sqrt(6) -
13*sqrt(5))*arctan((26*sqrt(6) - 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8
*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138) - 13*sqrt(115)))/(5*sqrt(138) -
 13*sqrt(115)) + 1/9*log(9*x^2 - 4*x + 3) + 1/9*log(-4*(sqrt(5)*sqrt(-5*x^
2 + 8*x - 2) - sqrt(6))*(13*sqrt(6) + 6*sqrt(5))/(5*x - 4) + 26*sqrt(30) +
 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 + 199) - 1/9
*log(-4*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))*(13*sqrt(6) - 6*sqrt(5)
)/(5*x - 4) - 26*sqrt(30) + 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))
^2/(5*x - 4)^2 + 199)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \int \frac{1}{2\,x+\sqrt{-5\,x^2+8\,x-2}+1}\,dx$$

input
```
int(1/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1),x)
```

output
```
int(1/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1), x)
```

Reduce [F]

$$\int \frac{1}{1+2x+\sqrt{-2+8x-5x^2}}\,dx = \frac{13\sqrt{23}\,atan\left(\frac{9x-2}{\sqrt{23}}\right)}{207} - \frac{\sqrt{5}\,atan\left(\frac{5\sqrt{-5x^2+8x-2}\,\sqrt{5}\,x-4\sqrt{-5x^2+8x-2}\,\sqrt{5}}{25x^2-40x+10}\right)}{9} - \frac{\left(\int \frac{\sqrt{-5x^2+8x-2}}{45x^4-92x^3+65x^2-32x+6}dx\right)}{3} + \frac{52\left(\int \frac{\sqrt{-5x^2+8x-2}\,x}{45x^4-92x^3+65x^2-32x+6}dx\right)}{9} + \frac{\log(9x^2-4x+3)}{9}$$

input
```
int(1/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x)
```

output
```
(13*sqrt(23)*atan((9*x - 2)/sqrt(23)) - 23*sqrt(5)*atan((5*sqrt( - 5*x**2
+ 8*x - 2)*sqrt(5)*x - 4*sqrt( - 5*x**2 + 8*x - 2)*sqrt(5))/(25*x**2 - 40*
x + 10)) - 69*int(sqrt( - 5*x**2 + 8*x - 2)/(45*x**4 - 92*x**3 + 65*x**2 -
 32*x + 6),x) + 1196*int((sqrt( - 5*x**2 + 8*x - 2)*x)/(45*x**4 - 92*x**3
+ 65*x**2 - 32*x + 6),x) + 23*log(9*x**2 - 4*x + 3))/207
```

3.50 $\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 262

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx = \frac{2}{3}\sqrt{2}\arctan\left(\frac{\sqrt{11-4\sqrt{6}}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)$$
$$+\frac{16\arctan\left(\frac{6+\frac{(12-13\sqrt{6})\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{3\sqrt{23}}+\frac{1}{3}\log\left(\frac{x(2(3-2\sqrt{6})+5\sqrt{6}x)}{\left(4-\sqrt{6}-5x\right)^2}\right)$$
$$-\frac{1}{3}\log\left(\frac{2(3-2\sqrt{6})+12x-3\sqrt{6}x+10\sqrt{6}x^2+6\sqrt{-2+8x-5x^2}-4\sqrt{6}\sqrt{-2+8x-5x^2}+5\sqrt{6}x\sqrt{}}{\left(4-\sqrt{6}-5x\right)^2}\right.$$

output
```
2/3*2^(1/2)*arctan((2*2^(1/2)-3^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x
))+16/69*arctan(1/138*(6+(12-13*6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5
*x))*138^(1/2))*23^(1/2)+1/3*ln(x*(6-4*6^(1/2)+5*x*6^(1/2))/(4-6^(1/2)-5*x
)^2)-1/3*ln((6-4*6^(1/2)+12*x-3*x*6^(1/2)+10*6^(1/2)*x^2+6*(-5*x^2+8*x-2)^
(1/2)-4*6^(1/2)*(-5*x^2+8*x-2)^(1/2)+5*6^(1/2)*x*(-5*x^2+8*x-2)^(1/2))/(4-
6^(1/2)-5*x)^2)
```

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx = \frac{1}{69}\left(-46\sqrt{2}\arctan\left(\frac{4+\sqrt{6}-5x}{5\sqrt{\frac{2-8x+5x^2}{-11+4\sqrt{6}}}}\right)\right.$$
$$-16\sqrt{23}\arctan\left(\frac{\sqrt{23}\left(-22-8\sqrt{6}+5\sqrt{22+8\sqrt{6}}x\right)}{48+22\sqrt{6}-10\left(3+2\sqrt{6}\right)x+64\sqrt{-2+8x-5x^2}+21\sqrt{6}\sqrt{-2+8x-5x^2}}\right)$$
$$+23\left(\log\left(x\left(-6-4\sqrt{6}+5\sqrt{6}x\right)\right)\right.$$
$$\left.\left.-\log\left(\left(-24-11\sqrt{6}+5\left(3+2\sqrt{6}\right)x\right)\left(1+2x+\sqrt{-2+8x-5x^2}\right)\right)\right)\right)$$

input
```
Integrate[1/(x*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])),x]
```

output
```
(-46*Sqrt[2]*ArcTan[(4 + Sqrt[6] - 5*x)/(5*Sqrt[(2 - 8*x + 5*x^2)/(-11 + 4
*Sqrt[6])])] - 16*Sqrt[23]*ArcTan[(Sqrt[23]*(-22 - 8*Sqrt[6] + 5*Sqrt[22 +
 8*Sqrt[6]]*x))/(48 + 22*Sqrt[6] - 10*(3 + 2*Sqrt[6])*x + 64*Sqrt[-2 + 8*x
 - 5*x^2] + 21*Sqrt[6]*Sqrt[-2 + 8*x - 5*x^2])] + 23*(Log[x*(-6 - 4*Sqrt[6
] + 5*Sqrt[6]*x)] - Log[(-24 - 11*Sqrt[6] + 5*(3 + 2*Sqrt[6])*x)*(1 + 2*x
+ Sqrt[-2 + 8*x - 5*x^2])]))/69
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.53, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{-5x^2+8x-2}+2x+1\right)}dx$$
$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{10-9x}{3(9x^2-4x+3)}-\frac{\sqrt{-5x^2+8x-2}}{3x}+\frac{3x\sqrt{-5x^2+8x-2}}{9x^2-4x+3}-\frac{4\sqrt{-5x^2+8x-2}}{3(9x^2-4x+3)}+\frac{1}{3x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{8\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{3\sqrt{23}}-\frac{1}{3}\sqrt{2}\arctan\left(\frac{\sqrt{2}(1-2x)}{\sqrt{-5x^2+8x-2}}\right)-\frac{8\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{3\sqrt{23}}-\frac{1}{3}\text{arctanh}\left(\frac{2x+1}{\sqrt{-5x^2+8x-2}}\right)-\frac{1}{6}\log\left(9x^2-4x+3\right)+\frac{\log(x)}{3}$$

input
```
Int[1/(x*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])),x]
```

output
```
(-8*ArcTan[(2 - 9*x)/Sqrt[23]])/(3*Sqrt[23]) + (8*ArcTan[(8 - 13*x)/(Sqrt[
23]*Sqrt[-2 + 8*x - 5*x^2])])/(3*Sqrt[23]) - (Sqrt[2]*ArcTan[(Sqrt[2]*(1 -
 2*x))/Sqrt[-2 + 8*x - 5*x^2]])/3 - ArcTanh[(1 + 2*x)/Sqrt[-2 + 8*x - 5*x^
2]]/3 + Log[x]/3 - Log[3 - 4*x + 9*x^2]/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(207) = 414$.

Time = 0.77 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.82

method	result
default	$\frac{\ln(x)}{3} - \frac{\ln(9x^2-4x+3)}{6} + \frac{8\sqrt{23}\,\arctan\left(\frac{(18x-4)\sqrt{23}}{46}\right)}{69} + \frac{\sqrt{2}\,\arctan\left(\frac{(8x-4)\sqrt{2}}{4\sqrt{-5x^2+8x-2}}\right)}{3} + \frac{5\sqrt{29}\,\sqrt{676}\,\sqrt{\frac{696\left(x+\frac{1}{2}\right)^2}{\left(\frac{8}{13}-x\right)^2}-4901}\,\left(}{}$
trager	Expression too large to display

input

```
int(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x,method=_RETURNVERBOSE)
```

output

```
1/3*ln(x)-1/6*ln(9*x^2-4*x+3)+8/69*23^(1/2)*arctan(1/46*(18*x-4)*23^(1/2))
+1/3*2^(1/2)*arctan(1/4*(8*x-4)*2^(1/2)/(-5*x^2+8*x-2)^(1/2))+5/502918*29^
(1/2)*676^(1/2)*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*(8*23^(1/2)*arctan(1
/377*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*23^(1/2))-23*arctanh(58*(x+1/2)
/(8/13-x)/(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)))/((24*(x+1/2)^2/(8/13-x)^
2-169)/((x+1/2)/(8/13-x)+1)^2)^(1/2)/((x+1/2)/(8/13-x)+1)-4/251459*29^(1/2
)*676^(1/2)*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*(13*23^(1/2)*arctan(1/37
7*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*23^(1/2))+46*arctanh(58*(x+1/2)/(8
/13-x)/(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)))/((24*(x+1/2)^2/(8/13-x)^2-1
69)/((x+1/2)/(8/13-x)+1)^2)^(1/2)/((x+1/2)/(8/13-x)+1)-1/754377*29^(1/2)*6
76^(1/2)*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*(20*23^(1/2)*arctan(1/377*(
696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*23^(1/2))-391*arctanh(58*(x+1/2)/(8/1
3-x)/(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)))/((24*(x+1/2)^2/(8/13-x)^2-169
)/((x+1/2)/(8/13-x)+1)^2)^(1/2)/((x+1/2)/(8/13-x)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx$$
$$= \frac{8}{69}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(9x-2)\right) - \frac{1}{3}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-5x^2+8x-2}(2x-1)}{5x^2-8x+2}\right)$$
$$+ \frac{4}{69}\sqrt{23}\arctan\left(\frac{\sqrt{23}\sqrt{-5x^2+8x-2}(13x-8)+2\sqrt{23}(2x^2-3x)}{23(7x^2-8x+2)}\right)$$
$$+ \frac{4}{69}\sqrt{23}\arctan\left(\frac{\sqrt{23}\sqrt{-5x^2+8x-2}(13x-8)-2\sqrt{23}(2x^2-3x)}{23(7x^2-8x+2)}\right)$$
$$- \frac{1}{6}\log\left(9x^2-4x+3\right) + \frac{1}{3}\log(x)$$
$$+ \frac{1}{12}\log\left(-\frac{x^2+2\sqrt{-5x^2+8x-2}(2x+1)-12x+1}{x^2}\right)$$
$$- \frac{1}{12}\log\left(-\frac{x^2-2\sqrt{-5x^2+8x-2}(2x+1)-12x+1}{x^2}\right)$$

input
```
integrate(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="fricas")
```

output
```
8/69*sqrt(23)*arctan(1/23*sqrt(23)*(9*x - 2)) - 1/3*sqrt(2)*arctan(sqrt(2)
*sqrt(-5*x^2 + 8*x - 2)*(2*x - 1)/(5*x^2 - 8*x + 2)) + 4/69*sqrt(23)*arcta
n(1/23*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) + 2*sqrt(23)*(2*x^2 - 3
*x))/(7*x^2 - 8*x + 2)) + 4/69*sqrt(23)*arctan(1/23*(sqrt(23)*sqrt(-5*x^2
+ 8*x - 2)*(13*x - 8) - 2*sqrt(23)*(2*x^2 - 3*x))/(7*x^2 - 8*x + 2)) - 1/6
*log(9*x^2 - 4*x + 3) + 1/3*log(x) + 1/12*log(-(x^2 + 2*sqrt(-5*x^2 + 8*x
- 2)*(2*x + 1) - 12*x + 1)/x^2) - 1/12*log(-(x^2 - 2*sqrt(-5*x^2 + 8*x - 2
)*(2*x + 1) - 12*x + 1)/x^2)
```

Sympy [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx = \int \frac{1}{x\left(2x+\sqrt{-5x^2+8x-2}+1\right)}\,dx$$

input
```
integrate(1/x/(1+2*x+(-5*x**2+8*x-2)**(1/2)),x)
```

output
```
Integral(1/(x*(2*x + sqrt(-5*x**2 + 8*x - 2) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx = \int \frac{1}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)x}\,dx$$

input
```
integrate(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(-5*x^2 + 8*x - 2) + 1)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(197) = 394$.

Time = 0.18 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.61

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx$$
$$= -\frac{2}{15}\sqrt{10}\sqrt{5}\arctan\left(-\frac{1}{10}\sqrt{10}\left(\sqrt{6}-\frac{4\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)}{5x-4}\right)\right)$$
$$+\frac{8}{69}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(9x-2)\right)$$
$$+\frac{8\left(5\sqrt{6}+13\sqrt{5}\right)\arctan\left(-\frac{26\sqrt{6}+12\sqrt{5}-\frac{139\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)}{5x-4}}{5\sqrt{138}+13\sqrt{115}}\right)}{3\left(5\sqrt{138}+13\sqrt{115}\right)}$$
$$+\frac{8\left(5\sqrt{6}-13\sqrt{5}\right)\arctan\left(\frac{26\sqrt{6}-12\sqrt{5}-\frac{139\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)}{5x-4}}{5\sqrt{138}-13\sqrt{115}}\right)}{3\left(5\sqrt{138}-13\sqrt{115}\right)}$$
$$-\frac{1}{6}\log\left(9x^2-4x+3\right)-\frac{1}{6}\log\left(-\frac{4\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)\left(13\sqrt{6}+6\sqrt{5}\right)}{5x-4}\right.$$
$$\left.+26\sqrt{30}+\frac{139\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)^2}{(5x-4)^2}+199\right)$$
$$+\frac{1}{6}\log\left(-\frac{4\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)\left(13\sqrt{6}-6\sqrt{5}\right)}{5x-4}-26\sqrt{30}\right.$$
$$\left.+\frac{139\left(\sqrt{5}\sqrt{-5x^2+8x-2}-\sqrt{6}\right)^2}{(5x-4)^2}+199\right)+\frac{1}{3}\log\left(|x|\right)$$

input
```
integrate(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="giac")
```

output
```
-2/15*sqrt(10)*sqrt(5)*arctan(-1/10*sqrt(10)*(sqrt(6) - 4*(sqrt(5)*sqrt(-5
*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))) + 8/69*sqrt(23)*arctan(1/23*sqrt(23
)*(9*x - 2)) + 8/3*(5*sqrt(6) + 13*sqrt(5))*arctan(-(26*sqrt(6) + 12*sqrt(
5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138
) + 13*sqrt(115)))/(5*sqrt(138) + 13*sqrt(115)) + 8/3*(5*sqrt(6) - 13*sqrt
(5))*arctan((26*sqrt(6) - 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2)
 - sqrt(6))/(5*x - 4))/(5*sqrt(138) - 13*sqrt(115)))/(5*sqrt(138) - 13*sqr
t(115)) - 1/6*log(9*x^2 - 4*x + 3) - 1/6*log(-4*(sqrt(5)*sqrt(-5*x^2 + 8*x
 - 2) - sqrt(6))*(13*sqrt(6) + 6*sqrt(5))/(5*x - 4) + 26*sqrt(30) + 139*(s
qrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 + 199) + 1/6*log(-4
*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))*(13*sqrt(6) - 6*sqrt(5))/(5*x
- 4) - 26*sqrt(30) + 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x
 - 4)^2 + 199) + 1/3*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx = \int \frac{1}{x\,\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)}\,dx$$

input
```
int(1/(x*(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)),x)
```

output
```
int(1/(x*(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx = \text{too large to display}$$

input
```
int(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x)
```

output

```
(329045635848810631332960*sqrt(5)*asin((5*x - 4)/sqrt(6)) + 34933931535011
9874328080*sqrt(6)*asin((5*x - 4)/sqrt(6)) + 141840387171668727314208*sqrt
(15)*atan((sqrt(3) - 2*sqrt(2)*tan(asin((5*x - 4)/sqrt(6))/2))/sqrt(5)) +
344805731627951929750560*sqrt(2)*atan((sqrt(3) - 2*sqrt(2)*tan(asin((5*x -
 4)/sqrt(6))/2))/sqrt(5)) - 82501749357835074762528*sqrt(15)*atan((sqrt(3)
 + 2*sqrt(2)*tan(asin((5*x - 4)/sqrt(6))/2))/sqrt(5)) - 988895003621265596
45520*sqrt(2)*atan((sqrt(3) + 2*sqrt(2)*tan(asin((5*x - 4)/sqrt(6))/2))/sq
rt(5)) - 28255013935671420745002277920*sqrt(30)*int(tan(asin((5*x - 4)/sqr
t(6))/2)**6/(38642*sqrt( - 5*x**2 + 8*x - 2)*tan(asin((5*x - 4)/sqrt(6))/2
)**12 + 164193*sqrt( - 5*x**2 + 8*x - 2)*tan(asin((5*x - 4)/sqrt(6))/2)**1
0 + 416850*sqrt( - 5*x**2 + 8*x - 2)*tan(asin((5*x - 4)/sqrt(6))/2)**8 + 4
42630*sqrt( - 5*x**2 + 8*x - 2)*tan(asin((5*x - 4)/sqrt(6))/2)**6 + 416850
*sqrt( - 5*x**2 + 8*x - 2)*tan(asin((5*x - 4)/sqrt(6))/2)**4 + 164193*sqrt
( - 5*x**2 + 8*x - 2)*tan(asin((5*x - 4)/sqrt(6))/2)**2 + 38642*sqrt( - 5*
x**2 + 8*x - 2)),x) - 113408736435798599080596860640*sqrt(30)*int(tan(asin
((5*x - 4)/sqrt(6))/2)**4/(38642*sqrt( - 5*x**2 + 8*x - 2)*tan(asin((5*x -
 4)/sqrt(6))/2)**12 + 164193*sqrt( - 5*x**2 + 8*x - 2)*tan(asin((5*x - 4)/
sqrt(6))/2)**10 + 416850*sqrt( - 5*x**2 + 8*x - 2)*tan(asin((5*x - 4)/sqrt
(6))/2)**8 + 442630*sqrt( - 5*x**2 + 8*x - 2)*tan(asin((5*x - 4)/sqrt(6))/
2)**6 + 416850*sqrt( - 5*x**2 + 8*x - 2)*tan(asin((5*x - 4)/sqrt(6))/2)...
```

3.51 $\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 347

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)} \, dx = \frac{\left(4-\sqrt{6}-5x\right)^2\left(4+\sqrt{6}-\frac{10\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)}{5\sqrt{6}x\left(2\left(3-2\sqrt{6}\right)+5\sqrt{6}x\right)} - \frac{4}{9}\sqrt{2}\arctan\left(\frac{\sqrt{11-4\sqrt{6}}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right) - \frac{14\arctan\left(\frac{6+\frac{\left(12-13\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{9\sqrt{23}} + \frac{10}{9}\log\left(\frac{x\left(2\left(3-2\sqrt{6}\right)+5\sqrt{6}x\right)}{\left(4-\sqrt{6}-5x\right)^2}\right) - \frac{10}{9}\log\left(\frac{2\left(3-2\sqrt{6}\right)+12x-3\sqrt{6}x+10\sqrt{6}x^2+6\sqrt{-2+8x-5x^2}-4\sqrt{6}\sqrt{-2+8x-5x^2}+5\sqrt{6}x\sqrt{}}{\left(4-\sqrt{6}-5x\right)^2}\right)$$

output
```
1/30*(4-6^(1/2)-5*x)^2*(4+6^(1/2)-10*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))
*6^(1/2)/x/(6-4*6^(1/2)+5*x*6^(1/2))-4/9*2^(1/2)*arctan((2*2^(1/2)-3^(1/2)
)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))-14/207*arctan(1/138*(6+(12-13*6^(1
/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))*138^(1/2))*23^(1/2)+10/9*ln(x*(
6-4*6^(1/2)+5*x*6^(1/2))/(4-6^(1/2)-5*x)^2)-10/9*ln((6-4*6^(1/2)+12*x-3*x*
6^(1/2)+10*6^(1/2)*x^2+6*(-5*x^2+8*x-2)^(1/2)-4*6^(1/2)*(-5*x^2+8*x-2)^(1/
2)+5*6^(1/2)*x*(-5*x^2+8*x-2)^(1/2))/(4-6^(1/2)-5*x)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.60 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx$$
$$= \frac{126\sqrt{23}x\arctan\left(\frac{2-9x}{\sqrt{23}}\right)+\left(92+19i\sqrt{23}\right)\sqrt{77-52i\sqrt{23}}x\,\text{arctanh}\left(\frac{10+4i\sqrt{23}+\left(-26-5i\sqrt{23}\right)x}{\sqrt{77-52i\sqrt{23}}\sqrt{-2+8x-5x^2}}\right)+\left(92-19\right.}{}$$

input
```
Integrate[1/(x^2*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])),x]
```

output
```
(126*Sqrt[23]*x*ArcTan[(2 - 9*x)/Sqrt[23]] + (92 + (19*I)*Sqrt[23])*Sqrt[7
7 - (52*I)*Sqrt[23]]*x*ArcTanh[(10 + (4*I)*Sqrt[23] + (-26 - (5*I)*Sqrt[23
])*x)/(Sqrt[77 - (52*I)*Sqrt[23]]*Sqrt[-2 + 8*x - 5*x^2])] + (92 - (19*I)*
Sqrt[23])*Sqrt[77 + (52*I)*Sqrt[23]]*x*ArcTanh[(10 - (4*I)*Sqrt[23] + (-26
 + (5*I)*Sqrt[23])*x)/(Sqrt[77 + (52*I)*Sqrt[23]]*Sqrt[-2 + 8*x - 5*x^2])]
 + 414*(-3 + 3*Sqrt[-2 + 8*x - 5*x^2] + 2*Sqrt[2]*x*ArcTan[(1 - 2*x)/Sqrt[
-1 + 4*x - (5*x^2)/2]] + 10*x*Log[x] - 5*x*Log[3 - 4*x + 9*x^2]))/(3726*x)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.48, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{-5x^2+8x-2}+2x+1\right)}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{13-90x}{9\left(9x^2-4x+3\right)}-\frac{4\sqrt{-5x^2+8x-2}}{9x}-\frac{\sqrt{-5x^2+8x-2}}{3x^2}+\frac{4x\sqrt{-5x^2+8x-2}}{9x^2-4x+3}+\frac{11\sqrt{-5x^2+8x-2}}{9\left(9x^2-4x+3\right)}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{7\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{9\sqrt{23}}+\frac{2}{9}\sqrt{2}\arctan\left(\frac{\sqrt{2}(1-2x)}{\sqrt{-5x^2+8x-2}}\right)+\frac{7\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{9\sqrt{23}}-\frac{10}{9}\text{arctanh}\left(\frac{2x+1}{\sqrt{-5x^2+8x-2}}\right)+\frac{\sqrt{-5x^2+8x-2}}{3x}-\frac{5}{9}\log\left(9x^2-4x+3\right)-\frac{1}{3x}+\frac{10\log(x)}{9}$$

input
```
Int[1/(x^2*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])),x]
```

output
```
-1/3*1/x + Sqrt[-2 + 8*x - 5*x^2]/(3*x) + (7*ArcTan[(2 - 9*x)/Sqrt[23]])/(
9*Sqrt[23]) - (7*ArcTan[(8 - 13*x)/(Sqrt[23]*Sqrt[-2 + 8*x - 5*x^2])])/(9*
Sqrt[23]) + (2*Sqrt[2]*ArcTan[(Sqrt[2]*(1 - 2*x))/Sqrt[-2 + 8*x - 5*x^2]])
/9 - (10*ArcTanh[(1 + 2*x)/Sqrt[-2 + 8*x - 5*x^2]])/9 + (10*Log[x])/9 - (5
*Log[3 - 4*x + 9*x^2])/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.53

method	result
default	$-\frac{1}{3x}+\frac{10\ln(x)}{9}-\frac{5\ln(9x^2-4x+3)}{9}-\frac{7\sqrt{23}\,\arctan\left(\frac{(18x-4)\sqrt{23}}{46}\right)}{207}-\frac{(-5x^2+8x-2)^{\frac{3}{2}}}{6x}+\frac{2\sqrt{-5x^2+8x-2}}{3}-\frac{2\sqrt{2}\,\arctan}{}$
trager	Expression too large to display

input
```
int(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x+10/9*ln(x)-5/9*ln(9*x^2-4*x+3)-7/207*23^(1/2)*arctan(1/46*(18*x-4)*
23^(1/2))-1/6/x*(-5*x^2+8*x-2)^(3/2)+2/3*(-5*x^2+8*x-2)^(1/2)-2/9*2^(1/2)*
arctan(1/4*(8*x-4)*2^(1/2)/(-5*x^2+8*x-2)^(1/2))+1/12*(-10*x+8)*(-5*x^2+8*
x-2)^(1/2)+5/502918*29^(1/2)*676^(1/2)*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/
2)*(13*23^(1/2)*arctan(1/377*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*23^(1/2
))+46*arctanh(58*(x+1/2)/(8/13-x)/(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)))/
((24*(x+1/2)^2/(8/13-x)^2-169)/((x+1/2)/(8/13-x)+1)^2)^(1/2)/((x+1/2)/(8/1
3-x)+1)+4/754377*29^(1/2)*676^(1/2)*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*
(20*23^(1/2)*arctan(1/377*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*23^(1/2))-
391*arctanh(58*(x+1/2)/(8/13-x)/(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)))/((
24*(x+1/2)^2/(8/13-x)^2-169)/((x+1/2)/(8/13-x)+1)^2)^(1/2)/((x+1/2)/(8/13-
x)+1)-1/2263131*29^(1/2)*676^(1/2)*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*(
431*23^(1/2)*arctan(1/377*(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)*23^(1/2))-
322*arctanh(58*(x+1/2)/(8/13-x)/(696*(x+1/2)^2/(8/13-x)^2-4901)^(1/2)))/((
24*(x+1/2)^2/(8/13-x)^2-169)/((x+1/2)/(8/13-x)+1)^2)^(1/2)/((x+1/2)/(8/13-
x)+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 \left(1+2x+\sqrt{-2+8x-5x^2}\right)} \, dx =$$
$$-\frac{14\sqrt{23}x\arctan\left(\frac{1}{23}\sqrt{23}(9x-2)\right)-92\sqrt{2}x\arctan\left(\frac{\sqrt{2}\sqrt{-5x^2+8x-2}(2x-1)}{5x^2-8x+2}\right)+7\sqrt{23}x\arctan\left(\frac{\sqrt{23}\sqrt{-5x}}{}\right.}{}$$

input
```
integrate(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="fricas")
```

output
```
-1/414*(14*sqrt(23)*x*arctan(1/23*sqrt(23)*(9*x - 2)) - 92*sqrt(2)*x*arcta
n(sqrt(2)*sqrt(-5*x^2 + 8*x - 2)*(2*x - 1)/(5*x^2 - 8*x + 2)) + 7*sqrt(23)
*x*arctan(1/23*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) + 2*sqrt(23)*(2
*x^2 - 3*x))/(7*x^2 - 8*x + 2)) + 7*sqrt(23)*x*arctan(1/23*(sqrt(23)*sqrt(
-5*x^2 + 8*x - 2)*(13*x - 8) - 2*sqrt(23)*(2*x^2 - 3*x))/(7*x^2 - 8*x + 2)
) + 230*x*log(9*x^2 - 4*x + 3) - 460*x*log(x) - 115*x*log(-(x^2 + 2*sqrt(-
5*x^2 + 8*x - 2)*(2*x + 1) - 12*x + 1)/x^2) + 115*x*log(-(x^2 - 2*sqrt(-5*
x^2 + 8*x - 2)*(2*x + 1) - 12*x + 1)/x^2) - 138*sqrt(-5*x^2 + 8*x - 2) + 1
38)/x
```

Sympy [F]

$$\int \frac{1}{x^2 \left(1+2x+\sqrt{-2+8x-5x^2}\right)} \, dx = \int \frac{1}{x^2 \cdot \left(2x+\sqrt{-5x^2+8x-2}+1\right)} \, dx$$

input
```
integrate(1/x**2/(1+2*x+(-5*x**2+8*x-2)**(1/2)),x)
```

output
```
Integral(1/(x**2*(2*x + sqrt(-5*x**2 + 8*x - 2) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx=\int \frac{1}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)x^2}\,dx$$

input
```
integrate(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(-5*x^2 + 8*x - 2) + 1)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1117030881589892578125000000000000000000000000000000
000000000
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx=\int \frac{1}{x^2\,\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)}\,dx$$

input
```
int(1/(x^2*(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)),x)
```

output
```
int(1/(x^2*(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 \left(1+2x+\sqrt{-2+8x-5x^2}\right)}\,dx = \text{too large to display}$$

input
```
int(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2)),x)
```

output
```
( - 2545573874008441663996268271111119030275000*sqrt(5)*asin((5*x - 4)/sqrt(
6))*x**2 + 4072918198413506662394029233777904484400 0*sqrt(5)*asin((5*x - 4
)/sqrt(6))*x - 2302649256466878981862163535103421425950 0*sqrt(6)*asin((5*x
 - 4)/sqrt(6))*x**2 + 3684238810347006370979461656165474281520 0*sqrt(6)*as
in((5*x - 4)/sqrt(6))*x - 2820291702348357303516381161730250574235 0*sqrt(1
5)*atan((sqrt(3) - 2*sqrt(2)*tan(asin((5*x - 4)/sqrt(6))/2))/sqrt(5))*x**2
 + 4512466723757371685626209858768400918776 0*sqrt(15)*atan((sqrt(3) - 2*sq
rt(2)*tan(asin((5*x - 4)/sqrt(6))/2))/sqrt(5))*x - 892459533412394976878 92
5547206697209864 00*sqrt(2)*atan((sqrt(3) - 2*sqrt(2)*tan(asin((5*x - 4)/sq
rt(6))/2))/sqrt(5))*x**2 + 1427935253459831963006280875530715535782 40*sqrt
(2)*atan((sqrt(3) - 2*sqrt(2)*tan(asin((5*x - 4)/sqrt(6))/2))/sqrt(5))*x -
 2222737779752986757850022952510766168165 0*sqrt(15)*atan((sqrt(3) + 2*sqrt
(2)*tan(asin((5*x - 4)/sqrt(6))/2))/sqrt(5))*x**2 + 3556380447604778812560
0367240172258690640*sqrt(15)*atan((sqrt(3) + 2*sqrt(2)*tan(asin((5*x - 4)/
sqrt(6))/2))/sqrt(5))*x - 7594060624451751387928942151177783458680 0*sqrt(2
)*atan((sqrt(3) + 2*sqrt(2)*tan(asin((5*x - 4)/sqrt(6))/2))/sqrt(5))*x**2
+ 1215049699912280222068630744188445353888 0*sqrt(2)*atan((sqrt(3) + 2*sqr
t(2)*tan(asin((5*x - 4)/sqrt(6))/2))/sqrt(5))*x + 3419762554605723656373 32
361351410943 9900*sqrt( - 5*x**2 + 8*x - 2)*sqrt(30)*x - 6825516131964476 54
4157606698928994508720*sqrt( - 5*x**2 + 8*x - 2)*sqrt(30) + 21066050253...
```

3.52 $\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 554

$$
\begin{aligned}
&\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx \\
&= -\frac{2\sqrt{\frac{2}{3}}\left(2\left(767+532\sqrt{6}\right)-\frac{5\left(10636-2629\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{\left(13-2\sqrt{6}\right)\left(4-\sqrt{6}-5x\right)}\right)}{3105\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)(2-8x+5x^2)}{\left(4-\sqrt{6}-5x\right)^2}\right)} \\
&\quad+\frac{8\left(2+\frac{5\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)}{15\left(1-\frac{5(2-8x+5x^2)}{\left(4-\sqrt{6}-5x\right)^2}\right)\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)(2-8x+5x^2)}{\left(4-\sqrt{6}-5x\right)^2}\right)} \\
&\quad+\frac{212\arctan\left(\frac{\sqrt{5}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)}{729\sqrt{5}}+\frac{14600\arctan\left(\frac{6+\frac{\left(12-13\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{16767\sqrt{23}} \\
&\quad-\frac{100}{729}\log\left(\frac{2\left(3-2\sqrt{6}\right)+5\sqrt{6}x}{\left(4-\sqrt{6}-5x\right)^2}\right) \\
&\quad+\frac{100}{729}\log\left(\frac{2\left(3-2\sqrt{6}\right)+12x-3\sqrt{6}x+10\sqrt{6}x^2+6\sqrt{-2+8x-5x^2}-4\sqrt{6}\sqrt{-2+8x-5x^2}+5\sqrt{6}x}{\left(4-\sqrt{6}-5x\right)^2}\right.
\end{aligned}
$$

output
```
-2/3*6^(1/2)*(1534+1064*6^(1/2)-5*(10636-2629*6^(1/2))*(-5*x^2+8*x-2)^(1/2
)/(13-2*6^(1/2))/(4-6^(1/2)-5*x))/(40365+6210*6^(1/2)-31050*6^(1/2)*(-5*x^
2+8*x-2)^(1/2)/(4-6^(1/2)-5*x)-15525*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/
2)-5*x)^2)+8/15*(2+5*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))/(1-5*(5*x^2-8*x
+2)/(4-6^(1/2)-5*x)^2)/(13+2*6^(1/2)-10*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^
(1/2)-5*x)-5*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2)+212/3645*arct
an(5^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))*5^(1/2)+14600/385641*arct
an(1/138*(6+(12-13*6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))*138^(1/2
))*23^(1/2)-100/729*ln((6-4*6^(1/2)+5*x*6^(1/2))/(4-6^(1/2)-5*x)^2)+100/72
9*ln((6-4*6^(1/2)+12*x-3*x*6^(1/2)+10*6^(1/2)*x^2+6*(-5*x^2+8*x-2)^(1/2)-4
*6^(1/2)*(-5*x^2+8*x-2)^(1/2)+5*6^(1/2)*x*(-5*x^2+8*x-2)^(1/2))/(4-6^(1/2)
-5*x)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.22 (sec) , antiderivative size = 1128, normalized size of antiderivative = 2.04

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[x^2/(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^2,x]
```

output
```
(-23805*x + (115*(3126 - 6247*x))/(3 - 4*x + 9*x^2) - (1035*Sqrt[-2 + 8*x
- 5*x^2]*(336 - 799*x + 828*x^2))/(3 - 4*x + 9*x^2) - 56074*Sqrt[5]*ArcSin
[(4 - 5*x)/Sqrt[6]] + 36500*Sqrt[23]*ArcTan[(-2 + 9*x)/Sqrt[23]] - ((250*I
)*(-109*I + 445*Sqrt[23])*ArcTan[(23*(-625034999 + (31717972*I)*Sqrt[23] +
 8*(65984878 - (56521751*I)*Sqrt[23])*x + (85877608 + (2214584710*I)*Sqrt[
23])*x^2 + 900*(7286713 - (4610645*I)*Sqrt[23])*x^3 + (10125*I)*(791161*I
+ 205946*Sqrt[23])*x^4))/(9763896428*I + 2370058321*Sqrt[23] - 2025*(-5801
198*I + 12282473*Sqrt[23])*x^4 - 246588624*Sqrt[23*(77 - (52*I)*Sqrt[23])]
*Sqrt[-2 + 8*x - 5*x^2] + 36*x^3*(-4062801535*I + 1309323961*Sqrt[23] + 25
686315*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) - 2*x^2*(-1
16526334717*I + 8516310250*Sqrt[23] + 575373456*Sqrt[23*(77 - (52*I)*Sqrt[
23])]*Sqrt[-2 + 8*x - 5*x^2]) + x*(-90041575208*I - 6413927920*Sqrt[23] +
637020612*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]))])/Sqrt[
77/23 - (52*I)/Sqrt[23]] + (250*(109 - (445*I)*Sqrt[23])*ArcTan[(23*(-6250
34999 - (31717972*I)*Sqrt[23] + 8*(65984878 + (56521751*I)*Sqrt[23])*x + (
85877608 - (2214584710*I)*Sqrt[23])*x^2 + 900*(7286713 + (4610645*I)*Sqrt[
23])*x^3 - (10125*I)*(-791161*I + 205946*Sqrt[23])*x^4))/(9763896428*I - 2
370058321*Sqrt[23] + 2025*(5801198*I + 12282473*Sqrt[23])*x^4 + 246588624*
Sqrt[23*(77 + (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2] + x*(-90041575208*I
 + 6413927920*Sqrt[23] - 637020612*Sqrt[23*(77 + (52*I)*Sqrt[23])]*Sqrt...
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{-5x^2+8x-2}+2x+1\right)^2}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{4\sqrt{-5x^2+8x-2x}}{9\left(9x^2-4x+3\right)}-\frac{28\sqrt{-5x^2+8x-2x}}{81\left(9x^2-4x+3\right)^2}+\frac{900x+389}{729\left(9x^2-4x+3\right)}-\frac{34\sqrt{-5x^2+8x-2}}{81\left(9x^2-4x+3\right)}-\frac{2(680x+5\right.}{729\left(9x^2-4x\right.}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{34}{729}\sqrt{5}\arcsin\left(\frac{4-5x}{\sqrt{6}}\right)+\frac{64\arcsin\left(\frac{4-5x}{\sqrt{6}}\right)}{729\sqrt{5}}+\frac{7300\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{16767\sqrt{23}}-\frac{7300\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{16767\sqrt{23}}+\frac{100}{729}\text{arctanh}\left(\frac{2x+1}{\sqrt{-5x^2+8x-2}}\right)+\frac{3126-6247x}{16767\left(9x^2-4x+3\right)}-\frac{4}{81}\sqrt{-5x^2+8x-2}-\frac{17(2-9x)\sqrt{-5x^2+8x-2}}{621\left(9x^2-4x+3\right)}+\frac{14(3-2x)\sqrt{-5x^2+8x-2}}{1863\left(9x^2-4x+3\right)}+\frac{50}{729}\log\left(9x^2-4x+3\right)-\frac{x}{81}$$

input

```
Int[x^2/(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^2,x]
```

output

```
-1/81*x - (4*Sqrt[-2 + 8*x - 5*x^2])/81 + (3126 - 6247*x)/(16767*(3 - 4*x
+ 9*x^2)) - (17*(2 - 9*x)*Sqrt[-2 + 8*x - 5*x^2])/(621*(3 - 4*x + 9*x^2))
+ (14*(3 - 2*x)*Sqrt[-2 + 8*x - 5*x^2])/(1863*(3 - 4*x + 9*x^2)) + (64*Arc
Sin[(4 - 5*x)/Sqrt[6]])/(729*Sqrt[5]) - (34*Sqrt[5]*ArcSin[(4 - 5*x)/Sqrt[
6]])/729 - (7300*ArcTan[(2 - 9*x)/Sqrt[23]])/(16767*Sqrt[23]) + (7300*ArcT
an[(8 - 13*x)/(Sqrt[23]*Sqrt[-2 + 8*x - 5*x^2])])/(16767*Sqrt[23]) + (100*
ArcTanh[(1 + 2*x)/Sqrt[-2 + 8*x - 5*x^2]])/729 + (50*Log[3 - 4*x + 9*x^2])
/729
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.99 (sec) , antiderivative size = 1172, normalized size of antiderivative = 2.12

method	result	size
trager	Expression too large to display	1172
default	Expression too large to display	6561

input
```
int(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/1863*(207*x^2+950*x+300)*x/(9*x^2-4*x+3)-1/1863*(828*x^2-799*x+336)/(9*
x^2-4*x+3)*(-5*x^2+8*x-2)^(1/2)+200/729*ln(37671726726*RootOf(135*_Z^2+115
00*_Z+299943)^2*RootOf(621*_Z^2-52900*_Z+1620000)^2*x+2468671220475*RootOf
(135*_Z^2+11500*_Z+299943)*RootOf(621*_Z^2-52900*_Z+1620000)^2*x-395650312
1550*RootOf(135*_Z^2+11500*_Z+299943)^2*RootOf(621*_Z^2-52900*_Z+1620000)*
x-855439149960*RootOf(135*_Z^2+11500*_Z+299943)*RootOf(621*_Z^2-52900*_Z+1
620000)*(-5*x^2+8*x-2)^(1/2)+40244022639756*RootOf(621*_Z^2-52900*_Z+16200
00)^2*x-239945560251300*RootOf(621*_Z^2-52900*_Z+1620000)*RootOf(135*_Z^2+
11500*_Z+299943)*x+101037495487500*RootOf(135*_Z^2+11500*_Z+299943)^2*x+27
0740228057280*(-5*x^2+8*x-2)^(1/2)*RootOf(621*_Z^2-52900*_Z+1620000)-24887
1188718000*RootOf(135*_Z^2+11500*_Z+299943)*(-5*x^2+8*x-2)^(1/2)-273243179
64420*RootOf(135*_Z^2+11500*_Z+299943)*RootOf(621*_Z^2-52900*_Z+1620000)-3
601271727150750*RootOf(621*_Z^2-52900*_Z+1620000)*x+5636016844605000*RootO
f(135*_Z^2+11500*_Z+299943)*x-19985447861813000*(-5*x^2+8*x-2)^(1/2)-83238
7238465040*RootOf(621*_Z^2-52900*_Z+1620000)+1197372142395000*RootOf(135*_
Z^2+11500*_Z+299943)+78089605249347500*x+30714221756597000)-2/621*ln(37671
726726*RootOf(135*_Z^2+11500*_Z+299943)^2*RootOf(621*_Z^2-52900*_Z+1620000
)^2*x+2468671220475*RootOf(135*_Z^2+11500*_Z+299943)*RootOf(621*_Z^2-52900
*_Z+1620000)^2*x-3956503121550*RootOf(135*_Z^2+11500*_Z+299943)^2*RootOf(6
21*_Z^2-52900*_Z+1620000)*x-855439149960*RootOf(135*_Z^2+11500*_Z+29994...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx =$$

$$-\frac{214245\,x^3 - 36500\,\sqrt{23}(9\,x^2-4\,x+3)\arctan\left(\frac{1}{23}\,\sqrt{23}(9\,x-2)\right) + 56074\,\sqrt{5}(9\,x^2-4\,x+3)\arctan\Big(}{}$$

input
```
integrate(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="fricas")
```

output
```
-1/1928205*(214245*x^3 - 36500*sqrt(23)*(9*x^2 - 4*x + 3)*arctan(1/23*sqrt
(23)*(9*x - 2)) + 56074*sqrt(5)*(9*x^2 - 4*x + 3)*arctan(1/5*sqrt(5)*sqrt(
-5*x^2 + 8*x - 2)*(5*x - 4)/(5*x^2 - 8*x + 2)) - 18250*sqrt(23)*(9*x^2 - 4
*x + 3)*arctan(1/23*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) + 2*sqrt(2
3)*(2*x^2 - 3*x))/(7*x^2 - 8*x + 2)) - 18250*sqrt(23)*(9*x^2 - 4*x + 3)*ar
ctan(1/23*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) - 2*sqrt(23)*(2*x^2
- 3*x))/(7*x^2 - 8*x + 2)) - 95220*x^2 - 132250*(9*x^2 - 4*x + 3)*log(9*x^
2 - 4*x + 3) + 66125*(9*x^2 - 4*x + 3)*log(-(x^2 + 2*sqrt(-5*x^2 + 8*x - 2
)*(2*x + 1) - 12*x + 1)/x^2) - 66125*(9*x^2 - 4*x + 3)*log(-(x^2 - 2*sqrt(
-5*x^2 + 8*x - 2)*(2*x + 1) - 12*x + 1)/x^2) + 1035*(828*x^2 - 799*x + 336
)*sqrt(-5*x^2 + 8*x - 2) + 789820*x - 359490)/(9*x^2 - 4*x + 3)
```

Sympy [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \int \frac{x^2}{\left(2x+\sqrt{-5x^2+8x-2}+1\right)^2}\,dx$$

input
```
integrate(x**2/(1+2*x+(-5*x**2+8*x-2)**(1/2))**2,x)
```

output
```
Integral(x**2/(2*x + sqrt(-5*x**2 + 8*x - 2) + 1)**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \int \frac{x^2}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^2}\,dx$$

input
```
integrate(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x^2/(2*x + sqrt(-5*x^2 + 8*x - 2) + 1)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="giac")
```

output
```
106/3645*sqrt(5)*arcsin(1/6*sqrt(6)*(5*x - 4)) + 7300/385641*sqrt(23)*arct
an(1/23*sqrt(23)*(9*x - 2)) - 1/81*x + 7300/16767*(5*sqrt(6) + 13*sqrt(5))
*arctan(-(26*sqrt(6) + 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) -
sqrt(6))/(5*x - 4))/(5*sqrt(138) + 13*sqrt(115)))/(5*sqrt(138) + 13*sqrt(1
15)) + 7300/16767*(5*sqrt(6) - 13*sqrt(5))*arctan((26*sqrt(6) - 12*sqrt(5)
 - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138)
- 13*sqrt(115)))/(5*sqrt(138) - 13*sqrt(115)) - 4/81*sqrt(-5*x^2 + 8*x - 2
) - 1/16767*(6247*x - 3126)/(9*x^2 - 4*x + 3) - 4/258957*(98968*sqrt(30) -
 42417*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^3/(5*x - 4)^3 +
175864*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 -
 401871*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(104
*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^3/(5*x - 4)^3 + 104*sq
rt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4) - 139*(sqrt(5)*
sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^4/(5*x - 4)^4 - 494*(sqrt(5)*sqrt(-5*x^2
 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 - 139) + 50/729*log(9*x^2 - 4*x + 3)
+ 50/729*log(-4*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))*(13*sqrt(6) + 6
*sqrt(5))/(5*x - 4) + 26*sqrt(30) + 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) -
sqrt(6))^2/(5*x - 4)^2 + 199) - 50/729*log(-4*(sqrt(5)*sqrt(-5*x^2 + 8*x -
 2) - sqrt(6))*(13*sqrt(6) - 6*sqrt(5))/(5*x - 4) - 26*sqrt(30) + 139*(sqr
t(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 + 199)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \int \frac{x^2}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^2}\,dx$$

input
```
int(x^2/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^2,x)
```

output
```
int(x^2/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^2, x)
```

Reduce [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \text{too large to display}$$

input
```
int(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x)
```

output
```
( - 882880448836923249918688055703371482829611835517465 0*sqrt(30)*asin((5*
x - 4)/sqrt(6))*x**4 + 28252174362781543997398017782507887450547578736558 8
80*sqrt(30)*asin((5*x - 4)/sqrt(6))*x**3 - 330219087382264528142426831303 5
72128935875557639964 44*sqrt(30)*asin((5*x - 4)/sqrt(6))*x**2 + 16672270796
80194818611883024696144469303918843959894 4*sqrt(30)*asin((5*x - 4)/sqrt(6)
)*x - 642213778339154541792704941259785774917539868502333 8*sqrt(30)*asin((
5*x - 4)/sqrt(6)) - 500016121127901730694632421816922495231477788472000 00*
asin((5*x - 4)/sqrt(6))*x**4 + 160005158760928553822282374981415198474072 8
92311040000*asin((5*x - 4)/sqrt(6))*x**3 - 187018375378900133742278812979 8
207890806524824635520 00*asin((5*x - 4)/sqrt(6))*x**2 + 944227973922516650 9
5124660791501808482230669783552000*asin((5*x - 4)/sqrt(6))*x - 36371543033
155518484602150979571695579060088020704000*asin((5*x - 4)/sqrt(6)) - 50230
824275776406582148781106045073567080592000000 00*sqrt( - 5*x**2 + 8*x - 2)*
sqrt(5)*x**4 + 105744677907144505827975271885379061086308690773600 00*sqrt(
 - 5*x**2 + 8*x - 2)*sqrt(5)*x**3 + 10149621181905563417624564385032288228
48446566336000*sqrt( - 5*x**2 + 8*x - 2)*sqrt(5)*x**2 - 255858000475527431
386470982563045505018403593329504 00*sqrt( - 5*x**2 + 8*x - 2)*sqrt(5)*x +
147353091320252248134667759668211882801370593343769 60*sqrt( - 5*x**2 + 8*x
 - 2)*sqrt(5) - 592935059021780170110069823198250540272666499958550 0*sqrt(
 - 5*x**2 + 8*x - 2)*sqrt(6)*x**4 + 1304798122055149909124702449000341 2...
```

3.53 $\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 407

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2} \, dx = \frac{2\sqrt{\frac{2}{3}}\left(2\left(13+2\sqrt{6}\right)-\frac{25\left(55-4\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{\left(13-2\sqrt{6}\right)\left(4-\sqrt{6}-5x\right)}\right)}{69\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)\left(2-8x+5x^2\right)}{\left(4-\sqrt{6}-5x\right)^2}\right)} + \frac{8}{81}\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right) + \frac{2608\arctan\left(\frac{6+\frac{\left(12-13\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{1863\sqrt{23}} + \frac{1}{81}\log\left(\frac{2\left(3-2\sqrt{6}\right)+5\sqrt{6}x}{\left(4-\sqrt{6}-5x\right)^2}\right) - \frac{1}{81}\log\left(\frac{2\left(3-2\sqrt{6}\right)+12x-3\sqrt{6}x+10\sqrt{6}x^2+6\sqrt{-2+8x-5x^2}-4\sqrt{6}\sqrt{-2+8x-5x^2}+5\sqrt{6}x\sqrt{}}{\left(4-\sqrt{6}-5x\right)^2}\right.$$

output
```
2/3*6^(1/2)*(26+4*6^(1/2)-25*(55-4*6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(13-2*6^(
1/2))/(4-6^(1/2)-5*x))/(897+138*6^(1/2)-690*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/(
4-6^(1/2)-5*x)-345*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2)+8/81*ar
ctan(5^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))*5^(1/2)+2608/42849*arct
an(1/138*(6+(12-13*6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))*138^(1/2
))*23^(1/2)+1/81*ln((6-4*6^(1/2)+5*x*6^(1/2))/(4-6^(1/2)-5*x)^2)-1/81*ln((
6-4*6^(1/2)+12*x-3*x*6^(1/2)+10*6^(1/2)*x^2+6*(-5*x^2+8*x-2)^(1/2)-4*6^(1/
2)*(-5*x^2+8*x-2)^(1/2)+5*6^(1/2)*x*(-5*x^2+8*x-2)^(1/2))/(4-6^(1/2)-5*x)^
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.10 (sec) , antiderivative size = 1122, normalized size of antiderivative = 2.76

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2} \, dx = \text{Too large to display}$$

input
```
Integrate[x/(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^2,x]
```

output
```
((-92*(231 + 1042*x))/(3 - 4*x + 9*x^2) + (828*(39 + 20*x)*Sqrt[-2 + 8*x -
 5*x^2])/(3 - 4*x + 9*x^2) - 8464*Sqrt[5]*ArcSin[(4 - 5*x)/Sqrt[6]] + 5216
*Sqrt[23]*ArcTan[(-2 + 9*x)/Sqrt[23]] + (2*(18010 - (2309*I)*Sqrt[23])*Arc
Tan[(23*(-103794579760 + (53250743936*I)*Sqrt[23] + 8*(111189866015 - (388
51416136*I)*Sqrt[23])*x + (-3116668883563 + (569272669160*I)*Sqrt[23])*x^2
 + 72*(59716994161 - (5891256488*I)*Sqrt[23])*x^3 + (81*I)*(24577827241*I
+ 1386185060*Sqrt[23])*x^4))/(924853969312*I - 415308162016*Sqrt[23] + 405
0*(-4973576764*I + 928720771*Sqrt[23])*x^4 - 48274289604*Sqrt[23*(77 - (52
*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2] + 9*x^3*(5782072124140*I - 146877754
024*Sqrt[23] + 20114287335*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x -
 5*x^2]) - 2*x^2*(18720427066432*I + 2439958754825*Sqrt[23] + 112640009076
*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) + x*(422453279736
8*I + 3316594666720*Sqrt[23] + 124708581477*Sqrt[23*(77 - (52*I)*Sqrt[23])
]*Sqrt[-2 + 8*x - 5*x^2]))])/Sqrt[77/23 - (52*I)/Sqrt[23]] - ((2*I)*(-1801
0*I + 2309*Sqrt[23])*ArcTan[(23*(16*(6487161235 + (3328171496*I)*Sqrt[23])
 + (-889518928120 - (310811329088*I)*Sqrt[23])*x + (3116668883563 + (56927
2669160*I)*Sqrt[23])*x^2 - (72*I)*(-59716994161*I + 5891256488*Sqrt[23])*x
^3 + 81*(24577827241 + (1386185060*I)*Sqrt[23])*x^4))/(4050*(4973576764*I
+ 928720771*Sqrt[23])*x^4 + x^2*(37440854132864*I - 4879917509650*Sqrt[23]
 - 225280018152*Sqrt[23*(77 + (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2])...
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.51, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{-5x^2+8x-2}+2x+1\right)^2} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{104-9x}{81\left(9x^2-4x+3\right)}-\frac{4\sqrt{-5x^2+8x-2}}{9\left(9x^2-4x+3\right)}+\frac{2(181x-156)}{81\left(9x^2-4x+3\right)^2}-\frac{34x\sqrt{-5x^2+8x-2}}{9\left(9x^2-4x+3\right)^2}+\frac{4\sqrt{-5x^2+8x-2}}{3\left(9x^2-4x+3\right)^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4}{81}\sqrt{5}\arcsin\left(\frac{4-5x}{\sqrt{6}}\right)+\frac{1304\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{1863\sqrt{23}}-\frac{1304\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{1863\sqrt{23}}-\frac{1}{81}\text{arctanh}\left(\frac{2x+1}{\sqrt{-5x^2+8x-2}}\right)-\frac{2\sqrt{-5x^2+8x-2}(2-9x)}{69\left(9x^2-4x+3\right)}-\frac{1042x+231}{1863\left(9x^2-4x+3\right)}+\frac{17(3-2x)\sqrt{-5x^2+8x-2}}{207\left(9x^2-4x+3\right)}-\frac{1}{162}\log\left(9x^2-4x+3\right)$$

input
```
Int[x/(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^2,x]
```

output
```
-1/1863*(231 + 1042*x)/(3 - 4*x + 9*x^2) - (2*(2 - 9*x)*Sqrt[-2 + 8*x - 5*
x^2])/(69*(3 - 4*x + 9*x^2)) + (17*(3 - 2*x)*Sqrt[-2 + 8*x - 5*x^2])/(207*
(3 - 4*x + 9*x^2)) - (4*Sqrt[5]*ArcSin[(4 - 5*x)/Sqrt[6]])/81 - (1304*ArcT
an[(2 - 9*x)/Sqrt[23]])/(1863*Sqrt[23]) + (1304*ArcTan[(8 - 13*x)/(Sqrt[23
]*Sqrt[-2 + 8*x - 5*x^2])])/(1863*Sqrt[23]) - ArcTanh[(1 + 2*x)/Sqrt[-2 +
8*x - 5*x^2]]/81 - Log[3 - 4*x + 9*x^2]/162
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.95 (sec) , antiderivative size = 1113, normalized size of antiderivative = 2.73

method	result	size
trager	Expression too large to display	1113
default	Expression too large to display	6132

input
```
int(x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/207*(-150+77*x)*x/(9*x^2-4*x+3)+1/207*(39+20*x)/(9*x^2-4*x+3)*(-5*x^2+8*
x-2)^(1/2)-2/81*ln(2092873707*RootOf(27*_Z^2-46*_Z+1587)^2*RootOf(621*_Z^2
+1058*_Z+63429)^2*x+11956413564*RootOf(27*_Z^2-46*_Z+1587)*RootOf(621*_Z^2
+1058*_Z+63429)^2*x-11269201302*RootOf(27*_Z^2-46*_Z+1587)^2*RootOf(621*_Z
^2+1058*_Z+63429)*x+6407011008*RootOf(27*_Z^2-46*_Z+1587)*RootOf(621*_Z^2+
1058*_Z+63429)*(-5*x^2+8*x-2)^(1/2)+15496755057*RootOf(621*_Z^2+1058*_Z+63
429)^2*x-209146083264*RootOf(621*_Z^2+1058*_Z+63429)*RootOf(27*_Z^2-46*_Z+
1587)*x-5012193249*RootOf(27*_Z^2-46*_Z+1587)^2*x+862717000768*(-5*x^2+8*x
-2)^(1/2)*RootOf(621*_Z^2+1058*_Z+63429)+768876591552*RootOf(27*_Z^2-46*_Z
+1587)*(-5*x^2+8*x-2)^(1/2)+204651851616*RootOf(27*_Z^2-46*_Z+1587)*RootOf
(621*_Z^2+1058*_Z+63429)-519356741562*RootOf(621*_Z^2+1058*_Z+63429)*x+281
069202852*RootOf(27*_Z^2-46*_Z+1587)*x+3818595116352*(-5*x^2+8*x-2)^(1/2)+
762381883936*RootOf(621*_Z^2+1058*_Z+63429)+84537490656*RootOf(27*_Z^2-46*
_Z+1587)+2941295052501*x-5502747769184)-1/69*ln(2092873707*RootOf(27*_Z^2-
46*_Z+1587)^2*RootOf(621*_Z^2+1058*_Z+63429)^2*x+11956413564*RootOf(27*_Z^
2-46*_Z+1587)*RootOf(621*_Z^2+1058*_Z+63429)^2*x-11269201302*RootOf(27*_Z^
2-46*_Z+1587)^2*RootOf(621*_Z^2+1058*_Z+63429)*x+6407011008*RootOf(27*_Z^2
-46*_Z+1587)*RootOf(621*_Z^2+1058*_Z+63429)*(-5*x^2+8*x-2)^(1/2)+154967550
57*RootOf(621*_Z^2+1058*_Z+63429)^2*x-209146083264*RootOf(621*_Z^2+1058*_Z
+63429)*RootOf(27*_Z^2-46*_Z+1587)*x-5012193249*RootOf(27*_Z^2-46*_Z+15...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.88

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \frac{5216\sqrt{23}(9\,x^2-4\,x+3)\arctan\left(\frac{1}{23}\sqrt{23}(9\,x-2)\right)-8464\sqrt{5}(9\,x^2-4\,x+3)\arctan\left(\frac{\sqrt{5}\sqrt{-5\,x^2+8\,x-2}(5\,x-}{5\,(5\,x^2-8\,x+2)}\right.}{}$$

input
```
integrate(x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/171396*(5216*sqrt(23)*(9*x^2 - 4*x + 3)*arctan(1/23*sqrt(23)*(9*x - 2))
- 8464*sqrt(5)*(9*x^2 - 4*x + 3)*arctan(1/5*sqrt(5)*sqrt(-5*x^2 + 8*x - 2)
*(5*x - 4)/(5*x^2 - 8*x + 2)) + 2608*sqrt(23)*(9*x^2 - 4*x + 3)*arctan(1/2
3*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) + 2*sqrt(23)*(2*x^2 - 3*x))/
(7*x^2 - 8*x + 2)) + 2608*sqrt(23)*(9*x^2 - 4*x + 3)*arctan(1/23*(sqrt(23)
*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) - 2*sqrt(23)*(2*x^2 - 3*x))/(7*x^2 - 8*
x + 2)) - 1058*(9*x^2 - 4*x + 3)*log(9*x^2 - 4*x + 3) + 529*(9*x^2 - 4*x +
 3)*log(-(x^2 + 2*sqrt(-5*x^2 + 8*x - 2)*(2*x + 1) - 12*x + 1)/x^2) - 529*
(9*x^2 - 4*x + 3)*log(-(x^2 - 2*sqrt(-5*x^2 + 8*x - 2)*(2*x + 1) - 12*x +
1)/x^2) + 828*sqrt(-5*x^2 + 8*x - 2)*(20*x + 39) - 95864*x - 21252)/(9*x^2
 - 4*x + 3)
```

Sympy [F]

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx=\int \frac{x}{\left(2x+\sqrt{-5x^2+8x-2}+1\right)^2}\,dx$$

input
```
integrate(x/(1+2*x+(-5*x**2+8*x-2)**(1/2))**2,x)
```

output
```
Integral(x/(2*x + sqrt(-5*x**2 + 8*x - 2) + 1)**2, x)
```

Maxima [F]

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx=\int \frac{x}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^2}\,dx$$

input
```
integrate(x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x/(2*x + sqrt(-5*x^2 + 8*x - 2) + 1)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(308) = 616$.

Time = 0.18 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.61

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="giac")
```

output
```
4/81*sqrt(5)*arcsin(1/6*sqrt(6)*(5*x - 4)) + 1304/42849*sqrt(23)*arctan(1/
23*sqrt(23)*(9*x - 2)) + 1304/1863*(5*sqrt(6) + 13*sqrt(5))*arctan(-(26*sq
rt(6) + 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x -
 4))/(5*sqrt(138) + 13*sqrt(115)))/(5*sqrt(138) + 13*sqrt(115)) + 1304/186
3*(5*sqrt(6) - 13*sqrt(5))*arctan((26*sqrt(6) - 12*sqrt(5) - 139*(sqrt(5)*
sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138) - 13*sqrt(115)))
/(5*sqrt(138) - 13*sqrt(115)) - 1/1863*(1042*x + 231)/(9*x^2 - 4*x + 3) -
10/28773*(7645*sqrt(30) - 13824*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) -
sqrt(6))^3/(5*x - 4)^3 + 13585*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) -
sqrt(6))^2/(5*x - 4)^2 - 20496*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - s
qrt(6))/(5*x - 4))/(104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))
^3/(5*x - 4)^3 + 104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5
*x - 4) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^4/(5*x - 4)^4 - 4
94*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 - 139) - 1/162
*log(9*x^2 - 4*x + 3) - 1/162*log(-4*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqr
t(6))*(13*sqrt(6) + 6*sqrt(5))/(5*x - 4) + 26*sqrt(30) + 139*(sqrt(5)*sqrt
(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 + 199) + 1/162*log(-4*(sqrt(5)
*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))*(13*sqrt(6) - 6*sqrt(5))/(5*x - 4) - 26
*sqrt(30) + 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 +
 199)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \int \frac{x}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^2}\,dx$$

input
```
int(x/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^2,x)
```

output
```
int(x/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^2, x)
```

Reduce [F]

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \text{too large to display}$$

input
```
int(x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x)
```

output

```
(5753782891093977142291906340375831611077000*sqrt(5)*asin((5*x - 4)/sqrt(
6))*x**4 - 18412105251500726855334100289202661155446400*sqrt(5)*asin((5*x
 - 4)/sqrt(6))*x**3 + 21520568699825077963317974628151382035702320*sqrt(5
)*asin((5*x - 4)/sqrt(6))*x**2 - 108654151977991943664811110348628049781523200*sqrt(5)*asin((5*x - 4)/sqrt(6))*x + 4185344295595767076837520019443753068205640*sqrt(5)*asin((5*x - 4)/sqrt(6)) + 36212952971294073941184415266485991664320000*sqrt(6)*asin((5*x - 4)/sqrt(6))*x**4 - 1158814495081410366117901288527551733258240000*sqrt(6)*asin((5*x - 4)/sqrt(6))*x**3 + 13544538558250929186692877097697032141509120*sqrt(6)*asin((5*x - 4)/sqrt(6))*x**
2 - 6838436156159434012399466863162589240709120*sqrt(6)*asin((5*x - 4)/sq
rt(6))*x + 263415702354153930446245154012512917143424*sqrt(6)*asin((5*x
- 4)/sqrt(6)) + 262314816114550368473005483375538836737600*sqrt( - 5*x**
2 + 8*x - 2)*sqrt(30)*x**3 - 82029413591937213584595010429005651210624000*
sqrt( - 5*x**2 + 8*x - 2)*sqrt(30)*x**2 + 82028927218021985128126033845566553669062400*sqrt( - 5*x**2 + 8*x - 2)*sqrt(30)*x - 19879019074458708917442825419381399826631680*sqrt( - 5*x**2 + 8*x - 2)*sqrt(30) + 141132756217060539890354686339193419413315000*sqrt( - 5*x**2 + 8*x - 2)*x**3 - 4507198203452295593629675614919304027877630000*sqrt( - 5*x**2 + 8*x - 2)*x**2 + 48177760123097933922131557031774600557303140*sqrt( - 5*x**2 + 8*x - 2)*x - 12951774638683179118710728426039654668153320*sqrt( - 5*x**2 + 8*x - 2) -...
```

3.54 $\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 191

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \frac{2\sqrt{6}\left(13+2\sqrt{6}-\frac{5\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)}{23\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)(2-8x+5x^2)}{\left(4-\sqrt{6}-5x\right)^2}\right)} + \frac{12\arctan\left(\frac{6+\frac{\left(12-13\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{23\sqrt{23}}$$

output
```
2*6^(1/2)*(13+2*6^(1/2)-5*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))/(2
99+46*6^(1/2)-230*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x)-115*(13-2*6
^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2)+12/529*arctan(1/138*(6+(12-13*6^(
1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))*138^(1/2))*23^(1/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx$$
$$= \frac{23(-150+77x)+207(8-13x)\sqrt{-2+8x-5x^2}+108\sqrt{23}(3-4x+9x^2)\arctan\left(\frac{\sqrt{23}\left(\sqrt{22\ldots}\right.}{6+4\sqrt{6}-5\sqrt{6}x+13\sqrt{-2+8\ldots}}\right.}{4761\,(3-4x+9x^2)}$$

input
```
Integrate[(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^(-2),x]
```

output
```
(23*(-150 + 77*x) + 207*(8 - 13*x)*Sqrt[-2 + 8*x - 5*x^2] + 108*Sqrt[23]*(
3 - 4*x + 9*x^2)*ArcTan[(Sqrt[23]*(Sqrt[22 + 8*Sqrt[6]] - 5*x))/(6 + 4*Sqr
t[6] - 5*Sqrt[6]*x + 13*Sqrt[-2 + 8*x - 5*x^2] + 2*Sqrt[6]*Sqrt[-2 + 8*x -
 5*x^2])])/(4761*(3 - 4*x + 9*x^2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{-5x^2+8x-2}+2x+1\right)^2}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{4\sqrt{-5x^2+8x-2}x}{(9x^2-4x+3)^2}-\frac{1}{9\,(9x^2-4x+3)}+\frac{2(52x-3)}{9\,(9x^2-4x+3)^2}-\frac{2\sqrt{-5x^2+8x-2}}{(9x^2-4x+3)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{23\sqrt{23}}-\frac{6\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{23\sqrt{23}}-\frac{150-77x}{207\left(9x^2-4x+3\right)}+\frac{(2-9x)\sqrt{-5x^2+8x-2}}{23\left(9x^2-4x+3\right)}+\frac{2(3-2x)\sqrt{-5x^2+8x-2}}{23\left(9x^2-4x+3\right)}$$

input
```
Int[(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^(-2),x]
```

output
```
-1/207*(150 - 77*x)/(3 - 4*x + 9*x^2) + ((2 - 9*x)*Sqrt[-2 + 8*x - 5*x^2])
/(23*(3 - 4*x + 9*x^2)) + (2*(3 - 2*x)*Sqrt[-2 + 8*x - 5*x^2])/(23*(3 - 4*
x + 9*x^2)) - (6*ArcTan[(2 - 9*x)/Sqrt[23]])/(23*Sqrt[23]) + (6*ArcTan[(8
- 13*x)/(Sqrt[23]*Sqrt[-2 + 8*x - 5*x^2])])/(23*Sqrt[23])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.58

method	result
trager	$\frac{(-41+150x)x}{621x^2-276x+207}-\frac{(-8+13x)\sqrt{-5x^2+8x-2}}{23(9x^2-4x+3)}+\frac{6\,\mathrm{RootOf}\left(\_Z^2+23\right)\ln\left(\frac{13\,\mathrm{RootOf}\left(\_Z^2+23\right)x-8\,\mathrm{RootOf}\left(\_Z^2+23\right)+23\sqrt{-}}{\mathrm{RootOf}\left(\_Z^2+23\right)x-2x+3}\right.}{529}$
default	Expression too large to display

input
```
int(1/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/69*(-41+150*x)*x/(9*x^2-4*x+3)-1/23*(-8+13*x)/(9*x^2-4*x+3)*(-5*x^2+8*x-
2)^(1/2)+6/529*RootOf(_Z^2+23)*ln((13*RootOf(_Z^2+23)*x-8*RootOf(_Z^2+23)+
23*(-5*x^2+8*x-2)^(1/2))/(RootOf(_Z^2+23)*x-2*x+3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx$$
$$= \frac{54\sqrt{23}(9x^2-4x+3)\arctan\left(\frac{1}{23}\sqrt{23}(9x-2)\right)+27\sqrt{23}(9x^2-4x+3)\arctan\left(\frac{\sqrt{23}(142x^2-196x+55)\sqrt{-5}}{23(65x^3-144x^2+90x}\right.}{4761\,(9x^2-4x+3)}$$

input
```
integrate(1/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/4761*(54*sqrt(23)*(9*x^2 - 4*x + 3)*arctan(1/23*sqrt(23)*(9*x - 2)) + 27
*sqrt(23)*(9*x^2 - 4*x + 3)*arctan(1/23*sqrt(23)*(142*x^2 - 196*x + 55)*sq
rt(-5*x^2 + 8*x - 2)/(65*x^3 - 144*x^2 + 90*x - 16)) - 207*sqrt(-5*x^2 + 8
*x - 2)*(13*x - 8) + 1771*x - 3450)/(9*x^2 - 4*x + 3)
```

Sympy [F]

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \int \frac{1}{\left(2x+\sqrt{-5x^2+8x-2}+1\right)^2}\,dx$$

input
```
integrate(1/(1+2*x+(-5*x**2+8*x-2)**(1/2))**2,x)
```

output
```
Integral((2*x + sqrt(-5*x**2 + 8*x - 2) + 1)**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \int \frac{1}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^2}\,dx$$

input
```
integrate(1/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((2*x + sqrt(-5*x^2 + 8*x - 2) + 1)^(-2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(148) = 296$.

Time = 0.15 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.39

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \frac{6}{529}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(9\,x-2)\right) + \frac{6\left(5\sqrt{6}+13\sqrt{5}\right)\arctan\left(-\frac{26\sqrt{6}+12\sqrt{5}-\frac{139\left(\sqrt{5}\sqrt{-5\,x^2+8\,x-2}-\sqrt{6}\right)}{5\,x-4}}{5\sqrt{138}+13\sqrt{115}}\right)}{23\left(5\sqrt{138}+13\sqrt{115}\right)} + \frac{6\left(5\sqrt{6}-13\sqrt{5}\right)\arctan\left(\frac{26\sqrt{6}-12\sqrt{5}-\frac{139\left(\sqrt{5}\sqrt{-5\,x^2+8\,x-2}-\sqrt{6}\right)}{5\,x-4}}{5\sqrt{138}-13\sqrt{115}}\right)}{23\left(5\sqrt{138}-13\sqrt{115}\right)} + \frac{77\,x-150}{207\left(9\,x^2-4\,x+3\right)} + \frac{12\left(278\sqrt{30}+\frac{1183\sqrt{5}\left(\sqrt{5}\sqrt{-5\,x^2+8\,x-2}-\sqrt{6}\right)^3}{(5\,x-4)^3}+\frac{494\sqrt{30}\left(\sqrt{5}\sqrt{-5\,x^2+8\,x-2}-\sqrt{6}\right)^2}{(5\,x-4)^2}-\frac{2431\sqrt{5}\left(\sqrt{5}\sqrt{-5\,x^2+8\right.}{5\,x-4}\right.}{3197\left(\frac{104\sqrt{6}\left(\sqrt{5}\sqrt{-5\,x^2+8\,x-2}-\sqrt{6}\right)^3}{(5\,x-4)^3}+\frac{104\sqrt{6}\left(\sqrt{5}\sqrt{-5\,x^2+8\,x-2}-\sqrt{6}\right)}{5\,x-4}-\frac{139\left(\sqrt{5}\sqrt{-5\,x^2+8\,x-2}-\sqrt{6}\right)^4}{(5\,x-4)^4}-\frac{494\left(\sqrt{5}\sqrt{-5\,x^2\right.}{(5\,x\right.}$$

input
```
integrate(1/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="giac")
```

output
```
6/529*sqrt(23)*arctan(1/23*sqrt(23)*(9*x - 2)) + 6/23*(5*sqrt(6) + 13*sqrt
(5))*arctan(-(26*sqrt(6) + 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2
) - sqrt(6))/(5*x - 4))/(5*sqrt(138) + 13*sqrt(115)))/(5*sqrt(138) + 13*sq
rt(115)) + 6/23*(5*sqrt(6) - 13*sqrt(5))*arctan((26*sqrt(6) - 12*sqrt(5) -
 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138) -
13*sqrt(115)))/(5*sqrt(138) - 13*sqrt(115)) + 1/207*(77*x - 150)/(9*x^2 -
4*x + 3) + 12/3197*(278*sqrt(30) + 1183*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x
 - 2) - sqrt(6))^3/(5*x - 4)^3 + 494*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x -
 2) - sqrt(6))^2/(5*x - 4)^2 - 2431*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2
) - sqrt(6))/(5*x - 4))/(104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqr
t(6))^3/(5*x - 4)^3 + 104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6
))/(5*x - 4) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^4/(5*x - 4)^
4 - 494*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 - 139)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx=\int \frac{1}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^2}\,dx$$

input
```
int(1/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^2,x)
```

output
```
int(1/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^2, x)
```

Reduce [F]

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx=\text{too large to display}$$

input
```
int(1/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x)
```

output

```
(3337104171021340901699934769453582072500*sqrt(5)*asin((5*x - 4)/sqrt(6))*
x**4 - 10678733347268290885439791262251462632000*sqrt(5)*asin((5*x - 4)/sq
rt(6))*x**3 + 12481593575958338760234718984613052156600*sqrt(5)*asin((5*x
- 4)/sqrt(6))*x**2 - 6301771037029929683012617559699011281600*sqrt(5)*asin
((5*x - 4)/sqrt(6))*x + 1142743429329107908553284143970623525570 0*sqrt(5)*a
sin((5*x - 4)/sqrt(6)) + 3577938426615033236385763861986775125000 0*sqrt(6)
*asin((5*x - 4)/sqrt(6))*x**4 - 1144940296516810635643444435835768040000 00
*sqrt(6)*asin((5*x - 4)/sqrt(6))*x**3 + 1338237315712704283080779036577177
02700000*sqrt(6)*asin((5*x - 4)/sqrt(6))*x**2 - 6756561255988092393056375 8
065370015200000*sqrt(6)*asin((5*x - 4)/sqrt(6))*x + 260261891476737973194 8
7556388674171650000*sqrt(6)*asin((5*x - 4)/sqrt(6)) + 304762895008942115 71
53340969599997213500 0*sqrt( - 5*x**2 + 8*x - 2)*sqrt(30)*x**3 - 9683914504
263775098083615333066040493200 0*sqrt( - 5*x**2 + 8*x - 2)*sqrt(30)*x**2 +
956933827775894094160160104009945752260 0*sqrt( - 5*x**2 + 8*x - 2)*sqrt(3
0)*x - 2188793779911751326772429855318004295120 0*sqrt( - 5*x**2 + 8*x - 2)
*sqrt(30) - 5195425750977909830096479800050530062412 5*sqrt( - 5*x**2 + 8*x
 - 2)*x**3 + 1687179834192236965520914523177316056838 00*sqrt( - 5*x**2 + 8
*x - 2)*x**2 - 191453453983614797943154461052618366322580*sqrt( - 5*x**2 +
 8*x - 2)*x + 766763916550132094762120878199592700698 96*sqrt( - 5*x**2 + 8
*x - 2) - 503943787523333270567506417940353427767468857600000*sqrt(30)*...
```

3.55 $\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 420

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \frac{2\sqrt{\frac{2}{3}}\left(2\left(13+2\sqrt{6}\right)+\frac{5\left(323-72\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{\left(13-2\sqrt{6}\right)\left(4-\sqrt{6}-5x\right)}\right)}{23\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)\left(2-8x+5x^2\right)}{\left(4-\sqrt{6}-5x\right)^2}\right)}$$
$$+\frac{4}{9}\sqrt{2}\arctan\left(\frac{\sqrt{11-4\sqrt{6}}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)+\frac{808\arctan\left(\frac{6+\frac{\left(12-13\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{207\sqrt{23}}$$
$$-\frac{1}{9}\log\left(\frac{x\left(2\left(1203-542\sqrt{6}\right)-5\left(312-193\sqrt{6}\right)x\right)}{\left(4-\sqrt{6}-5x\right)^2}\right)$$
$$+\frac{1}{9}\log\left(\frac{2\left(3-2\sqrt{6}\right)+12x-3\sqrt{6}x+10\sqrt{6}x^2+6\sqrt{-2+8x-5x^2}-4\sqrt{6}\sqrt{-2+8x-5x^2}+5\sqrt{6}x\sqrt{}}{\left(4-\sqrt{6}-5x\right)^2}\right.$$

output
```
2/3*6^(1/2)*(26+4*6^(1/2)+5*(323-72*6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(13-2*6^
(1/2))/(4-6^(1/2)-5*x))/(299+46*6^(1/2)-230*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/(
4-6^(1/2)-5*x)-115*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2)+4/9*2^(
1/2)*arctan((2*2^(1/2)-3^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))+808/
4761*arctan(1/138*(6+(12-13*6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))
*138^(1/2))*23^(1/2)-1/9*ln(x*(2406-1084*6^(1/2)-5*(312-193*6^(1/2))*x)/(4
-6^(1/2)-5*x)^2)+1/9*ln((6-4*6^(1/2)+12*x-3*x*6^(1/2)+10*6^(1/2)*x^2+6*(-5
*x^2+8*x-2)^(1/2)-4*6^(1/2)*(-5*x^2+8*x-2)^(1/2)+5*6^(1/2)*x*(-5*x^2+8*x-2
)^(1/2))/(4-6^(1/2)-5*x)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.57 (sec) , antiderivative size = 1157, normalized size of antiderivative = 2.75

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(x*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^2),x]
```

output
```
(-41 + 150*x)/(69*(3 - 4*x + 9*x^2)) - ((7 + 72*x)*Sqrt[-2 + 8*x - 5*x^2])
/(69*(3 - 4*x + 9*x^2)) + (404*ArcTan[(-2 + 9*x)/Sqrt[23]])/(207*Sqrt[23])
 - (2*Sqrt[2]*ArcTan[(1 - 2*x)/Sqrt[-1 + 4*x - (5*x^2)/2]])/9 + ((466 - (1
23*I)*Sqrt[23])*ArcTan[(23*(-78288400 + (52722176*I)*Sqrt[23] + 8*(6506563
1 - (46853872*I)*Sqrt[23])*x + (-2667985531 + (915021224*I)*Sqrt[23])*x^2
+ 216*(23290691 - (4257604*I)*Sqrt[23])*x^3 + (81*I)*(36383449*I + 3933540
*Sqrt[23])*x^4))/(162*(-171380820*I + 5425559*Sqrt[23])*x^4 - 4*(-80769707
2*I + 42117106*Sqrt[23] + 15258321*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2
 + 8*x - 5*x^2]) + 9*x^3*(5524564396*I + 775570808*Sqrt[23] + 25430535*Sqr
t[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) - 2*x^2*(5753590024*I
 + 5250004481*Sqrt[23] + 142410996*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2
 + 8*x - 5*x^2]) + x*(-10803484744*I + 3720914272*Sqrt[23] + 157669317*Sqr
t[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]))])/(46*Sqrt[23*(77 -
(52*I)*Sqrt[23])]) - ((I/46)*(-466*I + 123*Sqrt[23])*ArcTan[(23*(78288400
+ (52722176*I)*Sqrt[23] + (-520525048 - (374830976*I)*Sqrt[23])*x + (26679
85531 + (915021224*I)*Sqrt[23])*x^2 + (-5030789256 - (919642464*I)*Sqrt[23
])*x^3 + 81*(36383449 + (3933540*I)*Sqrt[23])*x^4))/(162*(171380820*I + 54
25559*Sqrt[23])*x^4 - 4*(807697072*I + 42117106*Sqrt[23] + 15258321*Sqrt[2
3*(77 + (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) + 9*x^3*(-5524564396*I +
 775570808*Sqrt[23] + 25430535*Sqrt[23*(77 + (52*I)*Sqrt[23])]*Sqrt[-2 ...
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.55, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{-5x^2+8x-2}+2x+1\right)^2}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{2\sqrt{-5x^2+8x-2}x}{9x^2-4x+3}+\frac{6\sqrt{-5x^2+8x-2}x}{(9x^2-4x+3)^2}+\frac{9x-4}{9(9x^2-4x+3)}-\frac{8\sqrt{-5x^2+8x-2}}{9(9x^2-4x+3)}+\frac{2(3x+16)}{3(9x^2-4x+3)^2}-\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{404\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{207\sqrt{23}}-\frac{2}{9}\sqrt{2}\arctan\left(\frac{\sqrt{2}(1-2x)}{\sqrt{-5x^2+8x-2}}\right)-\frac{404\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{207\sqrt{23}}+\frac{1}{9}\text{arctanh}\left(\frac{2x+1}{\sqrt{-5x^2+8x-2}}\right)-\frac{41-150x}{69\left(9x^2-4x+3\right)}+\frac{10(2-9x)\sqrt{-5x^2+8x-2}}{69\left(9x^2-4x+3\right)}-\frac{3(3-2x)\sqrt{-5x^2+8x-2}}{23\left(9x^2-4x+3\right)}+\frac{1}{18}\log\left(9x^2-4x+3\right)-\frac{\log(x)}{9}$$

input
```
Int[1/(x*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^2),x]
```

output
```
-1/69*(41 - 150*x)/(3 - 4*x + 9*x^2) + (10*(2 - 9*x)*Sqrt[-2 + 8*x - 5*x^2
])/(69*(3 - 4*x + 9*x^2)) - (3*(3 - 2*x)*Sqrt[-2 + 8*x - 5*x^2])/(23*(3 -
4*x + 9*x^2)) - (404*ArcTan[(2 - 9*x)/Sqrt[23]])/(207*Sqrt[23]) + (404*Arc
Tan[(8 - 13*x)/(Sqrt[23]*Sqrt[-2 + 8*x - 5*x^2])])/(207*Sqrt[23]) - (2*Sqr
t[2]*ArcTan[(Sqrt[2]*(1 - 2*x))/Sqrt[-2 + 8*x - 5*x^2]])/9 + ArcTanh[(1 +
2*x)/Sqrt[-2 + 8*x - 5*x^2]]/9 - Log[x]/9 + Log[3 - 4*x + 9*x^2]/18
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.01 (sec) , antiderivative size = 1463, normalized size of antiderivative = 3.48

method	result	size
trager	Expression too large to display	1463
default	Expression too large to display	5738

input
```
int(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/552*(x-1)*(-655+981*x)/(9*x^2-4*x+3)-1/69*(7+72*x)/(9*x^2-4*x+3)*(-5*x^
2+8*x-2)^(1/2)+8/69*RootOf(4416*_Z^2-8464*_Z+58461)*ln((-89451343872*RootO
f(192*_Z^2+368*_Z+1587)^2*RootOf(4416*_Z^2-8464*_Z+58461)^2*x+89451343872*
RootOf(192*_Z^2+368*_Z+1587)^2*RootOf(4416*_Z^2-8464*_Z+58461)^2+465496565
76*RootOf(192*_Z^2+368*_Z+1587)^2*RootOf(4416*_Z^2-8464*_Z+58461)*x+144649
08288*RootOf(192*_Z^2+368*_Z+1587)*RootOf(4416*_Z^2-8464*_Z+58461)^2*x+650
380531200*RootOf(192*_Z^2+368*_Z+1587)*RootOf(4416*_Z^2-8464*_Z+58461)*(-5
*x^2+8*x-2)^(1/2)-46549656576*RootOf(192*_Z^2+368*_Z+1587)^2*RootOf(4416*_
Z^2-8464*_Z+58461)+580683339840*RootOf(192*_Z^2+368*_Z+1587)^2*x-144649082
88*RootOf(192*_Z^2+368*_Z+1587)*RootOf(4416*_Z^2-8464*_Z+58461)^2-32879158
24896*RootOf(192*_Z^2+368*_Z+1587)*RootOf(4416*_Z^2-8464*_Z+58461)*x+17166
5663040*RootOf(4416*_Z^2-8464*_Z+58461)^2*x-4853853119680*RootOf(192*_Z^2+
368*_Z+1587)*(-5*x^2+8*x-2)^(1/2)+3723879909312*(-5*x^2+8*x-2)^(1/2)*RootO
f(4416*_Z^2-8464*_Z+58461)-580683339840*RootOf(192*_Z^2+368*_Z+1587)^2+188
4817283328*RootOf(192*_Z^2+368*_Z+1587)*RootOf(4416*_Z^2-8464*_Z+58461)-19
46078840592*RootOf(192*_Z^2+368*_Z+1587)*x-171665663040*RootOf(4416*_Z^2-8
464*_Z+58461)^2+1641919518864*RootOf(4416*_Z^2-8464*_Z+58461)*x-2589122322
48*(-5*x^2+8*x-2)^(1/2)-1282338842928*RootOf(192*_Z^2+368*_Z+1587)+4185660
19248*RootOf(4416*_Z^2-8464*_Z+58461)-16643164132803*x+3942356932059)/x)+2
/9*ln(-(29817114624*RootOf(192*_Z^2+368*_Z+1587)^2*RootOf(4416*_Z^2-846...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx$$
$$= \frac{1616\,\sqrt{23}(9\,x^2-4\,x+3)\arctan\left(\frac{1}{23}\,\sqrt{23}(9\,x-2)\right)-4232\,\sqrt{2}(9\,x^2-4\,x+3)\arctan\left(\frac{\sqrt{2}\sqrt{-5\,x^2+8\,x-2}(2\,x-}{5\,x^2-8\,x+2}\right.}{}$$

input
```
integrate(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/19044*(1616*sqrt(23)*(9*x^2 - 4*x + 3)*arctan(1/23*sqrt(23)*(9*x - 2)) -
 4232*sqrt(2)*(9*x^2 - 4*x + 3)*arctan(sqrt(2)*sqrt(-5*x^2 + 8*x - 2)*(2*x
 - 1)/(5*x^2 - 8*x + 2)) + 808*sqrt(23)*(9*x^2 - 4*x + 3)*arctan(1/23*(sqr
t(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) + 2*sqrt(23)*(2*x^2 - 3*x))/(7*x^2
 - 8*x + 2)) + 808*sqrt(23)*(9*x^2 - 4*x + 3)*arctan(1/23*(sqrt(23)*sqrt(-
5*x^2 + 8*x - 2)*(13*x - 8) - 2*sqrt(23)*(2*x^2 - 3*x))/(7*x^2 - 8*x + 2))
 + 1058*(9*x^2 - 4*x + 3)*log(9*x^2 - 4*x + 3) - 2116*(9*x^2 - 4*x + 3)*lo
g(x) - 529*(9*x^2 - 4*x + 3)*log(-(x^2 + 2*sqrt(-5*x^2 + 8*x - 2)*(2*x + 1
) - 12*x + 1)/x^2) + 529*(9*x^2 - 4*x + 3)*log(-(x^2 - 2*sqrt(-5*x^2 + 8*x
 - 2)*(2*x + 1) - 12*x + 1)/x^2) - 276*sqrt(-5*x^2 + 8*x - 2)*(72*x + 7) +
 41400*x - 11316)/(9*x^2 - 4*x + 3)
```

Sympy [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx=\int \frac{1}{x\left(2x+\sqrt{-5x^2+8x-2}+1\right)^2}\,dx$$

input
```
integrate(1/x/(1+2*x+(-5*x**2+8*x-2)**(1/2))**2,x)
```

output
```
Integral(1/(x*(2*x + sqrt(-5*x**2 + 8*x - 2) + 1)**2), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx=\int \frac{1}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^2x}\,dx$$

input
```
integrate(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(-5*x^2 + 8*x - 2) + 1)^2*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(318) = 636$.

Time = 0.17 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.65

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="giac")
```

output
```
-4/45*sqrt(10)*sqrt(5)*arctan(-1/10*sqrt(10)*(sqrt(6) - 4*(sqrt(5)*sqrt(-5
*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))) + 404/4761*sqrt(23)*arctan(1/23*sqr
t(23)*(9*x - 2)) + 404/207*(5*sqrt(6) + 13*sqrt(5))*arctan(-(26*sqrt(6) +
12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*
sqrt(138) + 13*sqrt(115)))/(5*sqrt(138) + 13*sqrt(115)) + 404/207*(5*sqrt(
6) - 13*sqrt(5))*arctan((26*sqrt(6) - 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^
2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138) - 13*sqrt(115)))/(5*sqrt(1
38) - 13*sqrt(115)) + 1/69*(150*x - 41)/(9*x^2 - 4*x + 3) + 2/9591*(44897*
sqrt(30) - 40728*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^3/(5*x
 - 4)^3 + 79781*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x
 - 4)^2 - 160824*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x -
 4))/(104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^3/(5*x - 4)^3
 + 104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4) - 139*
(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^4/(5*x - 4)^4 - 494*(sqrt(5)*sq
rt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 - 139) + 1/18*log(9*x^2 - 4*
x + 3) + 1/18*log(-4*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))*(13*sqrt(6
) + 6*sqrt(5))/(5*x - 4) + 26*sqrt(30) + 139*(sqrt(5)*sqrt(-5*x^2 + 8*x -
2) - sqrt(6))^2/(5*x - 4)^2 + 199) - 1/18*log(-4*(sqrt(5)*sqrt(-5*x^2 + 8*
x - 2) - sqrt(6))*(13*sqrt(6) - 6*sqrt(5))/(5*x - 4) - 26*sqrt(30) + 139*(
sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 + 199) - 1/9*lo...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \int \frac{1}{x\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^2}\,dx$$

input
```
int(1/(x*(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^2),x)
```

output
```
int(1/(x*(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \text{too large to display}$$

input
```
int(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x)
```

output

```
(12435021403323020576110302650812338394467617187095009398523815504572570000
00*sqrt(5)*asin((5*x - 4)/sqrt(6))*x**4 - 397920684906336658435529684825994
8286229637499870403007527620961463222400000*sqrt(5)*asin((5*x - 4)/sqrt(6)
)*x**3 + 4651005042408323844121453445790254370355443583953461786884907586
74729112000*sqrt(5)*asin((5*x - 4)/sqrt(6))*x**2 - 2348223301052208922619545
5474916731861947737344914230093804973081227411 2000*sqrt(5)*asin((5*x - 4)
/sqrt(6))*x + 904532668004681941165949422451682689138311168686888318037323
5744372324000*sqrt(5)*asin((5*x - 4)/sqrt(6)) + 19999021601815816282142150
998856122542718051056448303738088371884986280 0000*sqrt(6)*asin((5*x - 4)/
sqrt(6))*x**4 - 63996869125810612102854883196339592136697763380634571961882
7900319560960000*sqrt(6)*asin((5*x - 4)/sqrt(6))*x**3 + 74801278820816292
6029356304520179121671834660254515814875463351392030048000*sqrt(6)*asin((5
*x - 4)/sqrt(6))*x**2 - 377660536322684846730427582319139815325204085135102
78268864164981821004 8000*sqrt(6)*asin((5*x - 4)/sqrt(6))*x + 145474364540
61578954861920208056823953295648990690543904298356437819649 6000*sqrt(6)*as
in((5*x - 4)/sqrt(6)) - 172244156981241162954596028314135108750395880542921
4789889245708224524288 00*sqrt(15)*atan((sqrt(3) - 2*sqrt(2)*tan(asin((5*x
 - 4)/sqrt(6))/2))/sqrt(5))*x**4 + 55118130233997172145470729060523234800126
68177373487327645586266318477721 60*sqrt(15)*atan((sqrt(3) - 2*sqrt(2)*ta
n(asin((5*x - 4)/sqrt(6))/2))/sqrt(5))*x**3 - 6442356765313187991200544...
```

3.56 $\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 596

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx$$

$$= -\frac{\sqrt{\frac{2}{3}}\left(379-154\sqrt{6}+\frac{40\left(3949-1511\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{\left(191-74\sqrt{6}\right)\left(4-\sqrt{6}-5x\right)}\right)}{69\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)\left(2-8x+5x^2\right)}{\left(4-\sqrt{6}-5x\right)^2}\right)}$$

$$-\frac{\sqrt{\frac{2}{3}}\left(4-\sqrt{6}-5x\right)^2\left(24+11\sqrt{6}-\frac{10\left(3+2\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)}{5x\left(2\left(3-2\sqrt{6}\right)+5\sqrt{6}x\right)\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)\left(2-8x+5x^2\right)}{\left(4-\sqrt{6}-5x\right)^2}\right)}$$

$$+\frac{32}{27}\sqrt{2}\arctan\left(\frac{\sqrt{11-4\sqrt{6}}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)$$

$$+\frac{6032\arctan\left(\frac{6+\frac{\left(12-13\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{621\sqrt{23}}$$

$$+\frac{28}{27}\log\left(\frac{x\left(2\left(26241-10774\sqrt{6}\right)-5\left(8064-3371\sqrt{6}\right)x\right)}{\left(4-\sqrt{6}-5x\right)^2}\right)$$

$$-\frac{28}{27}\log\left(\frac{2\left(3-2\sqrt{6}\right)+12x-3\sqrt{6}x+10\sqrt{6}x^2+6\sqrt{-2+8x-5x^2}-4\sqrt{6}\sqrt{-2+8x-5x^2}+5\sqrt{6}x\sqrt{}}{\left(4-\sqrt{6}-5x\right)^2}\right.$$

output

```
-1/207*6^(1/2)*(379-154*6^(1/2)+40*(3949-1511*6^(1/2))*(-5*x^2+8*x-2)^(1/2
)/(191-74*6^(1/2))/(4-6^(1/2)-5*x))/(13+2*6^(1/2)-10*6^(1/2)*(-5*x^2+8*x-2
)^(1/2)/(4-6^(1/2)-5*x)-5*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2)-
1/15*6^(1/2)*(4-6^(1/2)-5*x)^2*(24+11*6^(1/2)-10*(3+2*6^(1/2))*(-5*x^2+8*x
-2)^(1/2)/(4-6^(1/2)-5*x))/x/(6-4*6^(1/2)+5*x*6^(1/2))/(13+2*6^(1/2)-10*6^
(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x)-5*(13-2*6^(1/2))*(5*x^2-8*x+2)/
(4-6^(1/2)-5*x)^2)+32/27*2^(1/2)*arctan((2*2^(1/2)-3^(1/2))*(-5*x^2+8*x-2)
^(1/2)/(4-6^(1/2)-5*x))+6032/14283*arctan(1/138*(6+(12-13*6^(1/2))*(-5*x^2
+8*x-2)^(1/2)/(4-6^(1/2)-5*x))*138^(1/2))*23^(1/2)+28/27*ln(x*(52482-21548
*6^(1/2)-5*(8064-3371*6^(1/2))*x)/(4-6^(1/2)-5*x)^2)-28/27*ln((6-4*6^(1/2)
+12*x-3*x*6^(1/2)+10*6^(1/2)*x^2+6*(-5*x^2+8*x-2)^(1/2)-4*6^(1/2)*(-5*x^2+
8*x-2)^(1/2)+5*6^(1/2)*x*(-5*x^2+8*x-2)^(1/2))/(4-6^(1/2)-5*x)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.71 (sec) , antiderivative size = 1190, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^2 \left(1+2x+\sqrt{-2+8x-5x^2}\right)^2} \, dx = \text{Too large to display}$$

input
```
Integrate[1/(x^2*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^2),x]
```

output
```
1/(9*x) + (286 + 369*x)/(207*(3 - 4*x + 9*x^2)) + Sqrt[-2 + 8*x - 5*x^2]*(
2/(9*x) + (-244 + 63*x)/(207*(3 - 4*x + 9*x^2))) + (3016*ArcTan[(-2 + 9*x)
/Sqrt[23]])/(621*Sqrt[23]) - (16*Sqrt[2]*ArcTan[(1 - 2*x)/Sqrt[-1 + 4*x -
(5*x^2)/2]])/27 + (2*(1912 + (65*I)*Sqrt[23])*ArcTan[(23*(72*(-345941 + (5
6628*I)*Sqrt[23]) + 24*(6307633 - (567996*I)*Sqrt[23])*x + (-296303111 - (
75400*I)*Sqrt[23])*x^2 + 40*(5759561 + (123370*I)*Sqrt[23])*x^3 + (25*I)*(
2542541*I + 43940*Sqrt[23])*x^4))/(130*(5716880*I + 3093953*Sqrt[23])*x^4
+ x^2*(-3657150328*I + 654251390*Sqrt[23] - 23351496*Sqrt[23*(77 - (52*I)*
Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) - 36*(7775196*I + 984347*Sqrt[23] + 138
997*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) + 3*x*(6444311
12*I - 5168800*Sqrt[23] + 4308907*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2
+ 8*x - 5*x^2]) + x^3*(845051740*I - 1057253912*Sqrt[23] + 18764595*Sqrt[2
3*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]))])/(69*Sqrt[23*(77 - (52
*I)*Sqrt[23])]) + (((2*I)/69)*(1912*I + 65*Sqrt[23])*ArcTan[(23*(72*(34594
1 + (56628*I)*Sqrt[23]) - (24*I)*(-6307633*I + 567996*Sqrt[23])*x + 13*(22
792547 - (5800*I)*Sqrt[23])*x^2 + (40*I)*(5759561*I + 123370*Sqrt[23])*x^3
 + 25*(2542541 + (43940*I)*Sqrt[23])*x^4))/(130*(-5716880*I + 3093953*Sqrt
[23])*x^4 + x^2*(3657150328*I + 654251390*Sqrt[23] - 23351496*Sqrt[23*(77
+ (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) - 36*(-7775196*I + 984347*Sqrt
[23] + 138997*Sqrt[23*(77 + (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) +...
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.43, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{-5x^2+8x-2}+2x+1\right)^2}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{139-252x}{27\left(9x^2-4x+3\right)}-\frac{28\sqrt{-5x^2+8x-2}}{27x}-\frac{2\sqrt{-5x^2+8x-2}}{9x^2}+\frac{28x\sqrt{-5x^2+8x-2}}{3\left(9x^2-4x+3\right)}-\frac{58\sqrt{-5x^2+8x-2}}{27\left(9x^2-4x+3\right)}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3016\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{621\sqrt{23}}-\frac{16}{27}\sqrt{2}\arctan\left(\frac{\sqrt{2}(1-2x)}{\sqrt{-5x^2+8x-2}}\right)-\frac{3016\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{621\sqrt{23}}-\frac{28}{27}\text{arctanh}\left(\frac{2x+1}{\sqrt{-5x^2+8x-2}}\right)+\frac{13\sqrt{-5x^2+8x-2}(2-9x)}{207\left(9x^2-4x+3\right)}+\frac{2\sqrt{-5x^2+8x-2}}{9x}+\frac{369x+286}{207\left(9x^2-4x+3\right)}-\frac{10(3-2x)\sqrt{-5x^2+8x-2}}{23\left(9x^2-4x+3\right)}-\frac{14}{27}\log\left(9x^2-4x+3\right)+\frac{1}{9x}+\frac{28\log(x)}{27}$$

input
```
Int[1/(x^2*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^2),x]
```

output
```
1/(9*x) + (2*Sqrt[-2 + 8*x - 5*x^2])/(9*x) + (286 + 369*x)/(207*(3 - 4*x +
 9*x^2)) + (13*(2 - 9*x)*Sqrt[-2 + 8*x - 5*x^2])/(207*(3 - 4*x + 9*x^2)) -
 (10*(3 - 2*x)*Sqrt[-2 + 8*x - 5*x^2])/(23*(3 - 4*x + 9*x^2)) - (3016*ArcT
an[(2 - 9*x)/Sqrt[23]])/(621*Sqrt[23]) + (3016*ArcTan[(8 - 13*x)/(Sqrt[23]
*Sqrt[-2 + 8*x - 5*x^2])])/(621*Sqrt[23]) - (16*Sqrt[2]*ArcTan[(Sqrt[2]*(1
 - 2*x))/Sqrt[-2 + 8*x - 5*x^2]])/27 - (28*ArcTanh[(1 + 2*x)/Sqrt[-2 + 8*x
 - 5*x^2]])/27 + (28*Log[x])/27 - (14*Log[3 - 4*x + 9*x^2])/27
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.99 (sec) , antiderivative size = 1479, normalized size of antiderivative = 2.48

method	result	size
trager	Expression too large to display	1479
default	Expression too large to display	6067

input
```
int(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/1656*(x-1)*(7551*x^2-413*x+552)/x/(9*x^2-4*x+3)+1/207*(477*x^2-428*x+13
8)/x/(9*x^2-4*x+3)*(-5*x^2+8*x-2)^(1/2)-56/27*ln(-(29817114624*RootOf(192*
_Z^2-2576*_Z+14283)^2*RootOf(4416*_Z^2+59248*_Z+388233)^2*x-29817114624*Ro
otOf(192*_Z^2-2576*_Z+14283)^2*RootOf(4416*_Z^2+59248*_Z+388233)^2-5239884
99456*RootOf(192*_Z^2-2576*_Z+14283)*RootOf(4416*_Z^2+59248*_Z+388233)^2*x
+322345248768*RootOf(192*_Z^2-2576*_Z+14283)^2*RootOf(4416*_Z^2+59248*_Z+3
88233)*x+809219340800*RootOf(192*_Z^2-2576*_Z+14283)*RootOf(4416*_Z^2+5924
8*_Z+388233)*(-5*x^2+8*x-2)^(1/2)+523988499456*RootOf(192*_Z^2-2576*_Z+142
83)*RootOf(4416*_Z^2+59248*_Z+388233)^2+2072399464512*RootOf(4416*_Z^2+592
48*_Z+388233)^2*x-322345248768*RootOf(192*_Z^2-2576*_Z+14283)^2*RootOf(441
6*_Z^2+59248*_Z+388233)-9746245893120*RootOf(4416*_Z^2+59248*_Z+388233)*Ro
otOf(192*_Z^2-2576*_Z+14283)*x+189952217280*RootOf(192*_Z^2-2576*_Z+14283)
^2*x+2287167844416*(-5*x^2+8*x-2)^(1/2)*RootOf(4416*_Z^2+59248*_Z+388233)+
15252499878720*RootOf(192*_Z^2-2576*_Z+14283)*(-5*x^2+8*x-2)^(1/2)-2072399
464512*RootOf(4416*_Z^2+59248*_Z+388233)^2+8000476420608*RootOf(192*_Z^2-2
576*_Z+14283)*RootOf(4416*_Z^2+59248*_Z+388233)+61915595071200*RootOf(4416
*_Z^2+59248*_Z+388233)*x-189952217280*RootOf(192*_Z^2-2576*_Z+14283)^2-191
17285857168*RootOf(192*_Z^2-2576*_Z+14283)*x-84760478467704*(-5*x^2+8*x-2)
^(1/2)-41730920513568*RootOf(4416*_Z^2+59248*_Z+388233)+18025360839504*Roo
tOf(192*_Z^2-2576*_Z+14283)+242612809284027*x-148983165585891)/x)-32/20...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx$$

$$= \frac{3016\sqrt{23}(9\,x^3-4\,x^2+3\,x)\arctan\left(\frac{1}{23}\sqrt{23}(9\,x-2)\right)-8464\sqrt{2}(9\,x^3-4\,x^2+3\,x)\arctan\left(\frac{\sqrt{2}\sqrt{-5\,x^2+8\,x}}{5\,x^2-8\,x}\right.}{}$$

input
```
integrate(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/14283*(3016*sqrt(23)*(9*x^3 - 4*x^2 + 3*x)*arctan(1/23*sqrt(23)*(9*x - 2
)) - 8464*sqrt(2)*(9*x^3 - 4*x^2 + 3*x)*arctan(sqrt(2)*sqrt(-5*x^2 + 8*x -
 2)*(2*x - 1)/(5*x^2 - 8*x + 2)) + 1508*sqrt(23)*(9*x^3 - 4*x^2 + 3*x)*arc
tan(1/23*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) + 2*sqrt(23)*(2*x^2 -
 3*x))/(7*x^2 - 8*x + 2)) + 1508*sqrt(23)*(9*x^3 - 4*x^2 + 3*x)*arctan(1/2
3*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) - 2*sqrt(23)*(2*x^2 - 3*x))/
(7*x^2 - 8*x + 2)) + 39744*x^2 - 7406*(9*x^3 - 4*x^2 + 3*x)*log(9*x^2 - 4*
x + 3) + 14812*(9*x^3 - 4*x^2 + 3*x)*log(x) + 3703*(9*x^3 - 4*x^2 + 3*x)*l
og(-(x^2 + 2*sqrt(-5*x^2 + 8*x - 2)*(2*x + 1) - 12*x + 1)/x^2) - 3703*(9*x
^3 - 4*x^2 + 3*x)*log(-(x^2 - 2*sqrt(-5*x^2 + 8*x - 2)*(2*x + 1) - 12*x +
1)/x^2) + 69*(477*x^2 - 428*x + 138)*sqrt(-5*x^2 + 8*x - 2) + 13386*x + 47
61)/(9*x^3 - 4*x^2 + 3*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx=\int \frac{1}{x^2\left(2x+\sqrt{-5x^2+8x-2}+1\right)^2}\,dx$$

input
```
integrate(1/x**2/(1+2*x+(-5*x**2+8*x-2)**(1/2))**2,x)
```

output
```
Integral(1/(x**2*(2*x + sqrt(-5*x**2 + 8*x - 2) + 1)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx=\int \frac{1}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(-5*x^2 + 8*x - 2) + 1)^2*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x, algorithm="giac")
```

output
```
-32/135*sqrt(10)*sqrt(5)*arctan(-1/10*sqrt(10)*(sqrt(6) - 4*(sqrt(5)*sqrt(
-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))) + 3016/14283*sqrt(23)*arctan(1/23
*sqrt(23)*(9*x - 2)) + 3016/621*(5*sqrt(6) + 13*sqrt(5))*arctan(-(26*sqrt(
6) + 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4)
)/(5*sqrt(138) + 13*sqrt(115)))/(5*sqrt(138) + 13*sqrt(115)) + 3016/621*(5
*sqrt(6) - 13*sqrt(5))*arctan((26*sqrt(6) - 12*sqrt(5) - 139*(sqrt(5)*sqrt
(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138) - 13*sqrt(115)))/(5*
sqrt(138) - 13*sqrt(115)) + 1/207*(576*x^2 + 194*x + 69)/(9*x^3 - 4*x^2 +
3*x) - 1/28773*(350558*sqrt(30) + 85083*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x
 - 2) - sqrt(6))^5/(5*x - 4)^5 + 220730*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*
x - 2) - sqrt(6))^4/(5*x - 4)^4 - 3442530*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8
*x - 2) - sqrt(6))^3/(5*x - 4)^3 + 2162560*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 +
 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 - 2710485*sqrt(5)*(sqrt(5)*sqrt(-5*x^2
+ 8*x - 2) - sqrt(6))/(5*x - 4))/(347*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x -
 2) - sqrt(6))^5/(5*x - 4)^5 + 910*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2)
 - sqrt(6))^3/(5*x - 4)^3 + 347*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) -
sqrt(6))/(5*x - 4) - 278*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^6/(5*x
 - 4)^6 - 1890*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^4/(5*x - 4)^4 -
1890*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 - 278) - 14/
27*log(9*x^2 - 4*x + 3) - 14/27*log(-4*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx = \int \frac{1}{x^2\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^2}\,dx$$

input
```
int(1/(x^2*(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^2),x)
```

output
```
int(1/(x^2*(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^2}\,dx=\text{too large to display}$$

input
```
int(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^2,x)
```

output
```
(8240432737453414511572801174788082190003102897077594247620834070055876085
4743928363011500000*sqrt(5)*asin((5*x - 4)/sqrt(6))*x**6 - 39554077139776
389655549445638982794512014893905972452388580003536268205210277085614245520
000000*sqrt(5)*asin((5*x - 4)/sqrt(6))*x**5 + 7301226872858724106994528329
780136822372131959474873876089531105429260677806496605872357200000*sqrt(5)
*asin((5*x - 4)/sqrt(6))*x**4 - 648751974684332422268139384933044550646446
7537104683384359031361899199945929232610046787840000*sqrt(5)*asin((5*x - 4
)/sqrt(6))*x**3 + 308920597941253584805697594263141961171306446088270859117
969381528010754149482991493810124000*sqrt(5)*asin((5*x - 4)/sqrt(6))*x**
2 - 9590642901545781393024581633945951656987315016213567468045818141386512
967035085943997308800000*sqrt(5)*asin((5*x - 4)/sqrt(6))*x + 76702891293469
387430982650804714224197347324159192889639965318213619474195695126391918000
0000*sqrt(6)*asin((5*x - 4)/sqrt(6))*x**6 - 368173878208653059668716723862
628276147267155964125870271833527425373476139336606681206400000*sqrt(6)*
asin((5*x - 4)/sqrt(6))*x**5 + 679606555845643332981081837401571659530274414
84653769181247295772284474371316639887594704000000*sqrt(6)*asin((5*x - 4)
/sqrt(6))*x**4 - 603865765015805610952937191204481998935204928728650233973
42374769817634835429628789654988800000*sqrt(6)*asin((5*x - 4)/sqrt(6))*x**3
 + 287546829118638769256079849540201264973699988455356495018682823055244877
2391111024649907680000*sqrt(6)*asin((5*x - 4)/sqrt(6))*x**2 - 89270802...
```

3.57 $\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 601

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx$$
$$= \frac{2\sqrt{\frac{2}{3}}\left(2\left(31827+21103\sqrt{6}\right)-\frac{21025\left(17088-4157\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{\left(13-2\sqrt{6}\right)^3\left(4-\sqrt{6}-5x\right)}\right)}{58029\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)\left(2-8x+5x^2\right)}{\left(4-\sqrt{6}-5x\right)^2}\right)^2}$$
$$+\frac{\sqrt{\frac{2}{3}}\left(4\left(2659802+143683\sqrt{6}\right)-\frac{4205\left(1738463972-614311533\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{\left(13-2\sqrt{6}\right)^5\left(4-\sqrt{6}-5x\right)}\right)}{12012003\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)\left(2-8x+5x^2\right)}{\left(4-\sqrt{6}-5x\right)^2}\right)}$$
$$+\frac{14}{729}\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)$$
$$+\frac{150656\left(295429693-119673902\sqrt{6}\right)\arctan\left(\frac{6+\frac{\left(12-13\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{385641\sqrt{23}\left(13-2\sqrt{6}\right)^7}$$
$$+\frac{22}{729}\log\left(\frac{2\left(3-2\sqrt{6}\right)+5\sqrt{6}x}{\left(4-\sqrt{6}-5x\right)^2}\right)$$
$$-\frac{22\left(295429693-119673902\sqrt{6}\right)\log\left(\frac{2\left(3-2\sqrt{6}\right)+12x-3\sqrt{6}x+10\sqrt{6}x^2+6\sqrt{-2+8x-5x^2}-4\sqrt{6}\sqrt{-2+8x-5x^2}+5\sqrt{6}x\sqrt{-2+}}{\left(4-\sqrt{6}-5x\right)^2}\right.}{729\left(13-2\sqrt{6}\right)^7}$$

output
```
2/174087*6^(1/2)*(63654+42206*6^(1/2)-21025*(17088-4157*6^(1/2))*(-5*x^2+8
*x-2)^(1/2)/(13-2*6^(1/2))^3/(4-6^(1/2)-5*x))/(13+2*6^(1/2)-10*6^(1/2)*(-5
*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x)-5*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2
)-5*x)^2)^2+1/3*6^(1/2)*(10639208+574732*6^(1/2)-4205*(1738463972-61431153
3*6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(13-2*6^(1/2))^5/(4-6^(1/2)-5*x))/(1561560
39+24024006*6^(1/2)-120120030*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x)
-60060015*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2)+14/729*arctan(5^
(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))*5^(1/2)+150656/8869743*(295429
693-119673902*6^(1/2))*arctan(1/138*(6+(12-13*6^(1/2))*(-5*x^2+8*x-2)^(1/2
)/(4-6^(1/2)-5*x))*138^(1/2))*23^(1/2)/(13-2*6^(1/2))^7+22/729*ln((6-4*6^(
1/2)+5*x*6^(1/2))/(4-6^(1/2)-5*x)^2)-22/729*(295429693-119673902*6^(1/2))*
ln((6-4*6^(1/2)+12*x-3*x*6^(1/2)+10*6^(1/2)*x^2+6*(-5*x^2+8*x-2)^(1/2)-4*6
^(1/2)*(-5*x^2+8*x-2)^(1/2)+5*6^(1/2)*x*(-5*x^2+8*x-2)^(1/2))/(4-6^(1/2)-5
*x)^2)/(13-2*6^(1/2))^7
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.40 (sec) , antiderivative size = 1165, normalized size of antiderivative = 1.94

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[x^2/(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^3,x]
```

output
```
((1058*(65616 - 49931*x))/(3 - 4*x + 9*x^2)^2 + (9522*(-3126 + 6247*x)*Sqr
t[-2 + 8*x - 5*x^2])/(3 - 4*x + 9*x^2)^2 - (46*(1232461 + 202833*x))/(3 -
4*x + 9*x^2) + (207*(143620 - 80973*x)*Sqrt[-2 + 8*x - 5*x^2])/(3 - 4*x +
9*x^2) - 1533042*Sqrt[5]*ArcSin[(4 - 5*x)/Sqrt[6]] + 1355904*Sqrt[23]*ArcT
an[(-2 + 9*x)/Sqrt[23]] + (5742*(2374 + I*Sqrt[23])*ArcTan[(23*(-269864555
2 + (585143936*I)*Sqrt[23] + 8*(2261221271 - (292016920*I)*Sqrt[23])*x + (
-40472491747 + (1445106728*I)*Sqrt[23])*x^2 + 72*(505810249 + (153712*I)*S
qrt[23])*x^3 + (81*I)*(140280721*I + 260*Sqrt[23])*x^4))/(162*(7098260*I +
 365753563*Sqrt[23])*x^4 + x^2*(-551711422496*I + 52403805566*Sqrt[23] - 2
840493096*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) - 4*(676
7453576*I + 1388973052*Sqrt[23] + 152169273*Sqrt[23*(77 - (52*I)*Sqrt[23])
]*Sqrt[-2 + 8*x - 5*x^2]) + 9*x^3*(33441184972*I - 13500123304*Sqrt[23] +
253615455*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) + x*(229
403470712*I + 11557191904*Sqrt[23] + 1572415821*Sqrt[23*(77 - (52*I)*Sqrt[
23])]*Sqrt[-2 + 8*x - 5*x^2]))])/Sqrt[77/23 - (52*I)/Sqrt[23]] + ((5742*I)
*(2374*I + Sqrt[23])*ArcTan[(23*(2698645552 + (585143936*I)*Sqrt[23] + (-1
8089770168 - (2336135360*I)*Sqrt[23])*x + (40472491747 + (1445106728*I)*Sq
rt[23])*x^2 + (72*I)*(505810249*I + 153712*Sqrt[23])*x^3 + 81*(140280721 +
 (260*I)*Sqrt[23])*x^4))/(27069814304*I - 5555892208*Sqrt[23] + 162*(-7098
260*I + 365753563*Sqrt[23])*x^4 - 608677092*Sqrt[23*(77 + (52*I)*Sqrt[2...
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.65, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{-5x^2+8x-2}+2x+1\right)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{229-198x}{729\left(9x^2-4x+3\right)}-\frac{7\sqrt{-5x^2+8x-2}}{81\left(9x^2-4x+3\right)}+\frac{43470x-5107}{6561\left(9x^2-4x+3\right)^2}-\frac{236x\sqrt{-5x^2+8x-2}}{81\left(9x^2-4x+3\right)^2}-\frac{535\sqrt{-5x^2+8}}{729\left(9x^2-4x\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{7}{729}\sqrt{5}\arcsin\left(\frac{4-5x}{\sqrt{6}}\right)+\frac{75328\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{385641\sqrt{23}}-\frac{75328\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{385641\sqrt{23}}-\frac{22}{729}\text{arctanh}\left(\frac{2x+1}{\sqrt{-5x^2+8x-2}}\right)-\frac{181\sqrt{-5x^2+8x-2}(23351-82206x)}{108108027\,(9x^2-4x+3)}-\frac{120196-40977x}{301806\,(9x^2-4x+3)}+\frac{49931(2-9x)}{2313846\,(9x^2-4x+3)}+\frac{535(2-9x)\sqrt{-5x^2+8x-2}}{33534\,(9x^2-4x+3)}+\frac{118(3-2x)\sqrt{-5x^2+8x-2}}{1863\,(9x^2-4x+3)}+\frac{1700(1758x+215)\sqrt{-5x^2+8x-2}}{108108027\,(9x^2-4x+3)}+\frac{65616-49931x}{150903\,(9x^2-4x+3)^2}-\frac{181(2-9x)\sqrt{-5x^2+8x-2}}{5589\,(9x^2-4x+3)^2}-\frac{680(3-2x)\sqrt{-5x^2+8x-2}}{16767\,(9x^2-4x+3)^2}-\frac{11}{729}\log\left(9x^2-4x+3\right)$$

input
```
Int[x^2/(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^3,x]
```

output
```
(65616 - 49931*x)/(150903*(3 - 4*x + 9*x^2)^2) - (181*(2 - 9*x)*Sqrt[-2 +
8*x - 5*x^2])/(5589*(3 - 4*x + 9*x^2)^2) - (680*(3 - 2*x)*Sqrt[-2 + 8*x -
5*x^2])/(16767*(3 - 4*x + 9*x^2)^2) - (120196 - 40977*x)/(301806*(3 - 4*x
+ 9*x^2)) + (49931*(2 - 9*x))/(2313846*(3 - 4*x + 9*x^2)) - (181*(23351 -
82206*x)*Sqrt[-2 + 8*x - 5*x^2])/(108108027*(3 - 4*x + 9*x^2)) + (535*(2 -
 9*x)*Sqrt[-2 + 8*x - 5*x^2])/(33534*(3 - 4*x + 9*x^2)) + (118*(3 - 2*x)*S
qrt[-2 + 8*x - 5*x^2])/(1863*(3 - 4*x + 9*x^2)) + (1700*(215 + 1758*x)*Sqr
t[-2 + 8*x - 5*x^2])/(108108027*(3 - 4*x + 9*x^2)) - (7*Sqrt[5]*ArcSin[(4
- 5*x)/Sqrt[6]])/729 - (75328*ArcTan[(2 - 9*x)/Sqrt[23]])/(385641*Sqrt[23]
) + (75328*ArcTan[(8 - 13*x)/(Sqrt[23]*Sqrt[-2 + 8*x - 5*x^2])])/(385641*S
qrt[23]) - (22*ArcTanh[(1 + 2*x)/Sqrt[-2 + 8*x - 5*x^2]])/729 - (11*Log[3
- 4*x + 9*x^2])/729
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.72 (sec) , antiderivative size = 1222, normalized size of antiderivative = 2.03

method	result	size
trager	Expression too large to display	1222
default	Expression too large to display	29653

input

```
int(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output

```
1/42849*(243135*x^3-238657*x^2+83193*x-32868)*x/(9*x^2-4*x+3)^2-1/85698*(8
0973*x^3-179608*x^2+58893*x-31896)/(9*x^2-4*x+3)^2*(-5*x^2+8*x-2)^(1/2)+44
/729*ln((-339045540534*RootOf(81*_Z^2-23276*_Z+2518569)^2*RootOf(1863*_Z^2
+535348*_Z+108512316)^2*x-51151499368200*RootOf(81*_Z^2-23276*_Z+2518569)^
2*RootOf(1863*_Z^2+535348*_Z+108512316)*x+63690280589889*RootOf(81*_Z^2-23
276*_Z+2518569)*RootOf(1863*_Z^2+535348*_Z+108512316)^2*x+268146853615872*
RootOf(81*_Z^2-23276*_Z+2518569)*RootOf(1863*_Z^2+535348*_Z+108512316)*(-5
*x^2+8*x-2)^(1/2)-717032418056856*RootOf(81*_Z^2-23276*_Z+2518569)^2*x+156
67659912906540*RootOf(81*_Z^2-23276*_Z+2518569)*RootOf(1863*_Z^2+535348*_Z
+108512316)*x-2945014725038436*RootOf(1863*_Z^2+535348*_Z+108512316)^2*x+6
53757292689532416*RootOf(81*_Z^2-23276*_Z+2518569)*(-5*x^2+8*x-2)^(1/2)+44
8966676326232064*(-5*x^2+8*x-2)^(1/2)*RootOf(1863*_Z^2+535348*_Z+108512316
)-8565109382358144*RootOf(81*_Z^2-23276*_Z+2518569)*RootOf(1863*_Z^2+53534
8*_Z+108512316)+656245066192931556*RootOf(81*_Z^2-23276*_Z+2518569)*x-1000
810995046798320*RootOf(1863*_Z^2+535348*_Z+108512316)*x+164847576462722334
72*(-5*x^2+8*x-2)^(1/2)-1158261263062573824*RootOf(81*_Z^2-23276*_Z+251856
9)+704646356476457472*RootOf(1863*_Z^2+535348*_Z+108512316)-76253064761928
971664*x+158192066442615073792)/(621*RootOf(1863*_Z^2+535348*_Z+108512316)
*x+39006*x+75328))-1/4761*ln((-339045540534*RootOf(81*_Z^2-23276*_Z+251856
9)^2*RootOf(1863*_Z^2+535348*_Z+108512316)^2*x-51151499368200*RootOf(81...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 459, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = -\frac{9330318\,x^3 - 150656\,\sqrt{23}(81\,x^4 - 72\,x^3 + 70\,x^2 - 24\,x + 9)\arctan\left(\frac{1}{23}\sqrt{23}(9\,x - 2)\right) + 170338\,\sqrt{5}(8}{}$$

input
```
integrate(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="fricas")
```

output
```
-1/17739486*(9330318*x^3 - 150656*sqrt(23)*(81*x^4 - 72*x^3 + 70*x^2 - 24*
x + 9)*arctan(1/23*sqrt(23)*(9*x - 2)) + 170338*sqrt(5)*(81*x^4 - 72*x^3 +
 70*x^2 - 24*x + 9)*arctan(1/5*sqrt(5)*sqrt(-5*x^2 + 8*x - 2)*(5*x - 4)/(5
*x^2 - 8*x + 2)) - 75328*sqrt(23)*(81*x^4 - 72*x^3 + 70*x^2 - 24*x + 9)*ar
ctan(1/23*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) + 2*sqrt(23)*(2*x^2
- 3*x))/(7*x^2 - 8*x + 2)) - 75328*sqrt(23)*(81*x^4 - 72*x^3 + 70*x^2 - 24
*x + 9)*arctan(1/23*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) - 2*sqrt(2
3)*(2*x^2 - 3*x))/(7*x^2 - 8*x + 2)) + 52546398*x^2 + 267674*(81*x^4 - 72*
x^3 + 70*x^2 - 24*x + 9)*log(9*x^2 - 4*x + 3) - 133837*(81*x^4 - 72*x^3 +
70*x^2 - 24*x + 9)*log(-(x^2 + 2*sqrt(-5*x^2 + 8*x - 2)*(2*x + 1) - 12*x +
 1)/x^2) + 133837*(81*x^4 - 72*x^3 + 70*x^2 - 24*x + 9)*log(-(x^2 - 2*sqrt
(-5*x^2 + 8*x - 2)*(2*x + 1) - 12*x + 1)/x^2) + 207*(80973*x^3 - 179608*x^
2 + 58893*x - 31896)*sqrt(-5*x^2 + 8*x - 2) - 16217208*x + 11184210)/(81*x
^4 - 72*x^3 + 70*x^2 - 24*x + 9)
```

Sympy [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \int \frac{x^2}{\left(2x+\sqrt{-5x^2+8x-2}+1\right)^3}\,dx$$

input
```
integrate(x**2/(1+2*x+(-5*x**2+8*x-2)**(1/2))**3,x)
```

output
```
Integral(x**2/(2*x + sqrt(-5*x**2 + 8*x - 2) + 1)**3, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \int \frac{x^2}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^3}\,dx$$

input
```
integrate(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x^2/(2*x + sqrt(-5*x^2 + 8*x - 2) + 1)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 810, normalized size of antiderivative = 1.35

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="giac")
```

output
```
7/729*sqrt(5)*arcsin(1/6*sqrt(6)*(5*x - 4)) + 75328/8869743*sqrt(23)*arcta
n(1/23*sqrt(23)*(9*x - 2)) + 75328/385641*(5*sqrt(6) + 13*sqrt(5))*arctan(
-(26*sqrt(6) + 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))
/(5*x - 4))/(5*sqrt(138) + 13*sqrt(115)))/(5*sqrt(138) + 13*sqrt(115)) + 7
5328/385641*(5*sqrt(6) - 13*sqrt(5))*arctan((26*sqrt(6) - 12*sqrt(5) - 139
*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138) - 13*s
qrt(115)))/(5*sqrt(138) - 13*sqrt(115)) - 1/128547*(67611*x^3 + 380771*x^2
 - 117516*x + 81045)/(9*x^2 - 4*x + 3)^2 + 2/827885529*(70367738914*sqrt(3
0) - 210096118791*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^7/(5*
x - 4)^7 + 306429213898*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6)
)^6/(5*x - 4)^6 - 1445417463471*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) -
sqrt(6))^5/(5*x - 4)^5 + 751114881990*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x
- 2) - sqrt(6))^4/(5*x - 4)^4 - 1431731123985*sqrt(5)*(sqrt(5)*sqrt(-5*x^2
 + 8*x - 2) - sqrt(6))^3/(5*x - 4)^3 + 430092037854*sqrt(30)*(sqrt(5)*sqrt
(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 - 421694803257*sqrt(5)*(sqrt(5
)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(104*sqrt(6)*(sqrt(5)*sqrt(
-5*x^2 + 8*x - 2) - sqrt(6))^3/(5*x - 4)^3 + 104*sqrt(6)*(sqrt(5)*sqrt(-5*
x^2 + 8*x - 2) - sqrt(6))/(5*x - 4) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2)
- sqrt(6))^4/(5*x - 4)^4 - 494*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^
2/(5*x - 4)^2 - 139)^2 - 11/729*log(9*x^2 - 4*x + 3) - 11/729*log(-4*(s...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx=\int \frac{x^2}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^3}\,dx$$

input
```
int(x^2/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^3,x)
```

output
```
int(x^2/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^3, x)
```

Reduce [F]

$$\int \frac{x^2}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{too large to display}$$

input

```
int(x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x)
```

output

```
(8680157081254855858890436136590657128938785104811113310469138659992828079
4697564124521099746515498596093750*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(asi
n((5*x - 4)/sqrt(6))/2)**16*x**8 - 555530053200310774968987912741802056252
082246707911251870024874239540997086064410396935038377699191015000000*sqrt
(5)*asin((5*x - 4)/sqrt(6))*tan(asin((5*x - 4)/sqrt(6))/2)**16*x**7 + 1538
16669977160122291567610037244019957945365284465180263147936629719823606761
8464713242046076030692159125000*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(asin((
5*x - 4)/sqrt(6))/2)**16*x**6 - 240565088222890132627311358362116653396272
0603151443835875678085040264199233431513467023877300762570869400000*sqrt(5
)*asin((5*x - 4)/sqrt(6))*tan(asin((5*x - 4)/sqrt(6))/2)**16*x**5 + 238964
63924619690076908127694488473277487769652685968121046684376250155054431994
96128719797808064028797022500*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(asin((5*
x - 4)/sqrt(6))/2)**16*x**4 - 16302667411308046431548686212214538523843989
62775811142630006689347220892276034270126180521933322244382920000*sqrt(5)*
asin((5*x - 4)/sqrt(6))*tan(asin((5*x - 4)/sqrt(6))/2)**16*x**3 + 78185668
85537880954899801160369206642922490787516566011601204600828456591049738355
86533479607450455926925000*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(asin((5*x -
 4)/sqrt(6))/2)**16*x**2 - 23846692716233477505929862852249741014769309398
7843356179956283373153713642624865096186300205156368967240000*sqrt(5)*asin
((5*x - 4)/sqrt(6))*tan(asin((5*x - 4)/sqrt(6))/2)**16*x + 459285805060...
```

3.58 $\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 340

$$\begin{aligned}
&\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3} dx \\
&= \frac{10\sqrt{6}\left(3288+1387\sqrt{6}-\frac{84100\left(9-4\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{\left(193-52\sqrt{6}\right)\left(4-\sqrt{6}-5x\right)}\right)}{19343\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)(2-8x+5x^2)}{\left(4-\sqrt{6}-5x\right)^2}\right)^2} \\
&+ \frac{2\sqrt{6}\left(75812+5763\sqrt{6}-\frac{50460\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)}{444889\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)(2-8x+5x^2)}{\left(4-\sqrt{6}-5x\right)^2}\right)} \\
&+ \frac{144\arctan\left(\frac{6+\frac{\left(12-13\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{529\sqrt{23}}
\end{aligned}$$

output
```
10/19343*6^(1/2)*(3288+1387*6^(1/2)-84100*(9-4*6^(1/2))*(-5*x^2+8*x-2)^(1/
2)/(193-52*6^(1/2))/(4-6^(1/2)-5*x))/(13+2*6^(1/2)-10*6^(1/2)*(-5*x^2+8*x-
2)^(1/2)/(4-6^(1/2)-5*x)-5*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2)
^2+2*6^(1/2)*(75812+5763*6^(1/2)-50460*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(
1/2)-5*x))/(5783557+889778*6^(1/2)-4448890*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4
-6^(1/2)-5*x)-2224445*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2)+144/
12167*arctan(1/138*(6+(12-13*6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x)
)*138^(1/2))*23^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.24 (sec) , antiderivative size = 1147, normalized size of antiderivative = 3.37

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[x/(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^3,x]
```

output
```
(4173 - 21872*x)/(16767*(3 - 4*x + 9*x^2)^2) + (-191021 + 314460*x)/(77128
2*(3 - 4*x + 9*x^2)) + (Sqrt[-2 + 8*x - 5*x^2]*(150 - 208*x + 775*x^2 - 11
74*x^3))/(529*(3 - 4*x + 9*x^2)^2) + (72*ArcTan[(-2 + 9*x)/Sqrt[23]])/(529
*Sqrt[23]) + (36*(13 - (2*I)*Sqrt[23])*ArcTan[(23*(-52606 + (30056*I)*Sqrt
[23] + 8*(56309 - (22984*I)*Sqrt[23])*x + (-1707793 + (363428*I)*Sqrt[23])
*x^2 + 144*(17501 - (2041*I)*Sqrt[23])*x^3 + (81*I)*(15229*I + 1040*Sqrt[2
3])*x^4))/(27378*(-460*I + 67*Sqrt[23])*x^4 - 4*(260*(-782*I + 203*Sqrt[23
]) + 7047*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) + 9*x^3*
(52*(63871*I + 494*Sqrt[23]) + 11745*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[
-2 + 8*x - 5*x^2]) - 2*x^2*(9295312*I + 1752257*Sqrt[23] + 65772*Sqrt[23*(
77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) + x*(643448*I + 1971424*Sqr
t[23] + 72819*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]))])/(
529*Sqrt[23*(77 - (52*I)*Sqrt[23])]) - (((36*I)/529)*(-13*I + 2*Sqrt[23])*
ArcTan[(23*(52606 + (30056*I)*Sqrt[23] + (-450472 - (183872*I)*Sqrt[23])*x
 + (1707793 + (363428*I)*Sqrt[23])*x^2 + (-2520144 - (293904*I)*Sqrt[23])*
x^3 + 81*(15229 + (1040*I)*Sqrt[23])*x^4))/(27378*(460*I + 67*Sqrt[23])*x^
4 - 4*(260*(782*I + 203*Sqrt[23]) + 7047*Sqrt[23*(77 + (52*I)*Sqrt[23])]*S
qrt[-2 + 8*x - 5*x^2]) + 9*x^3*(52*(-63871*I + 494*Sqrt[23]) + 11745*Sqrt[
23*(77 + (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) - 2*x^2*(-9295312*I + 1
752257*Sqrt[23] + 65772*Sqrt[23*(77 + (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x -...
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{-5x^2+8x-2}+2x+1\right)^3}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{7\sqrt{-5x^2+8x-2}x}{9\left(9x^2-4x+3\right)^2}-\frac{724\sqrt{-5x^2+8x-2}x}{81\left(9x^2-4x+3\right)^3}-\frac{22}{81\left(9x^2-4x+3\right)}+\frac{2061x+3914}{729\left(9x^2-4x+3\right)^2}-\frac{208\sqrt{-5x^2+\right.}{81\left(9x^2-4x\right.}$$

$$\Big\downarrow \text{2009}$$

$$\frac{72\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{529\sqrt{23}}-\frac{72\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{529\sqrt{23}}-\frac{52\sqrt{-5x^2+8x-2}(23351-82206x)}{12012003\,(9x^2-4x+3)}-\frac{14011-39348x}{33534\,(9x^2-4x+3)}+\frac{10936(2-9x)}{128547\,(9x^2-4x+3)}+\frac{104(2-9x)\sqrt{-5x^2+8x-2}}{1863\,(9x^2-4x+3)}+\frac{7(3-2x)\sqrt{-5x^2+8x-2}}{414\,(9x^2-4x+3)}-\frac{905(1758x+215)\sqrt{-5x^2+8x-2}}{24024006\,(9x^2-4x+3)}+\frac{4173-21872x}{16767\,(9x^2-4x+3)^2}-\frac{52(2-9x)\sqrt{-5x^2+8x-2}}{621\,(9x^2-4x+3)^2}+\frac{181(3-2x)\sqrt{-5x^2+8x-2}}{1863\,(9x^2-4x+3)^2}$$

input
```
Int[x/(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^3,x]
```

output
```
(4173 - 21872*x)/(16767*(3 - 4*x + 9*x^2)^2) - (52*(2 - 9*x)*Sqrt[-2 + 8*x
 - 5*x^2])/(621*(3 - 4*x + 9*x^2)^2) + (181*(3 - 2*x)*Sqrt[-2 + 8*x - 5*x^
2])/(1863*(3 - 4*x + 9*x^2)^2) - (14011 - 39348*x)/(33534*(3 - 4*x + 9*x^2
)) + (10936*(2 - 9*x))/(128547*(3 - 4*x + 9*x^2)) - (52*(23351 - 82206*x)*
Sqrt[-2 + 8*x - 5*x^2])/(12012003*(3 - 4*x + 9*x^2)) + (104*(2 - 9*x)*Sqrt
[-2 + 8*x - 5*x^2])/(1863*(3 - 4*x + 9*x^2)) + (7*(3 - 2*x)*Sqrt[-2 + 8*x
- 5*x^2])/(414*(3 - 4*x + 9*x^2)) - (905*(215 + 1758*x)*Sqrt[-2 + 8*x - 5*
x^2])/(24024006*(3 - 4*x + 9*x^2)) - (72*ArcTan[(2 - 9*x)/Sqrt[23]])/(529*
Sqrt[23]) + (72*ArcTan[(8 - 13*x)/(Sqrt[23]*Sqrt[-2 + 8*x - 5*x^2])])/(529
*Sqrt[23])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.38

method	result
trager	$\frac{(14115x^3-900x^2-53x-1296)x}{3174(9x^2-4x+3)^2} - \frac{(1174x^3-775x^2+208x-150)\sqrt{-5x^2+8x-2}}{529(9x^2-4x+3)^2} + \frac{72\,\mathrm{RootOf}\left(\_Z^2+23\right)\ln\left(\frac{13\,\mathrm{RootOf}\left(\_Z^2 \ldots\right.}{\ldots}\right.}{12\ldots}$
default	Expression too large to display

input
```
int(x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3174*(14115*x^3-900*x^2-53*x-1296)*x/(9*x^2-4*x+3)^2-1/529*(1174*x^3-775
*x^2+208*x-150)/(9*x^2-4*x+3)^2*(-5*x^2+8*x-2)^(1/2)+72/12167*RootOf(_Z^2+
23)*ln((13*RootOf(_Z^2+23)*x-8*RootOf(_Z^2+23)+23*(-5*x^2+8*x-2)^(1/2))/(R
ootOf(_Z^2+23)*x-2*x+3))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.51

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx$$
$$= \frac{7232580\,x^3 + 11664\,\sqrt{23}(81\,x^4 - 72\,x^3 + 70\,x^2 - 24\,x + 9)\arctan\left(\frac{1}{23}\sqrt{23}(9\,x-2)\right) + 5832\,\sqrt{23}(81\,x^4 \ldots}{\ldots}$$

input
```
integrate(x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/1971054*(7232580*x^3 + 11664*sqrt(23)*(81*x^4 - 72*x^3 + 70*x^2 - 24*x +
 9)*arctan(1/23*sqrt(23)*(9*x - 2)) + 5832*sqrt(23)*(81*x^4 - 72*x^3 + 70*
x^2 - 24*x + 9)*arctan(1/23*sqrt(23)*(142*x^2 - 196*x + 55)*sqrt(-5*x^2 +
8*x - 2)/(65*x^3 - 144*x^2 + 90*x - 16)) - 7607963*x^2 - 3726*(1174*x^3 -
775*x^2 + 208*x - 150)*sqrt(-5*x^2 + 8*x - 2) + 1792344*x - 973935)/(81*x^
4 - 72*x^3 + 70*x^2 - 24*x + 9)
```

Sympy [F]

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \int \frac{x}{\left(2x+\sqrt{-5x^2+8x-2}+1\right)^3}\,dx$$

input
```
integrate(x/(1+2*x+(-5*x**2+8*x-2)**(1/2))**3,x)
```

output
```
Integral(x/(2*x + sqrt(-5*x**2 + 8*x - 2) + 1)**3, x)
```

Maxima [F]

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \int \frac{x}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^3}\,dx$$

input
```
integrate(x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x/(2*x + sqrt(-5*x^2 + 8*x - 2) + 1)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(265) = 530$.

Time = 0.21 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.80

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="giac")
```

output
```
72/12167*sqrt(23)*arctan(1/23*sqrt(23)*(9*x - 2)) + 72/529*(5*sqrt(6) + 13
*sqrt(5))*arctan(-(26*sqrt(6) + 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*
x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138) + 13*sqrt(115)))/(5*sqrt(138) +
13*sqrt(115)) + 72/529*(5*sqrt(6) - 13*sqrt(5))*arctan((26*sqrt(6) - 12*sq
rt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(
138) - 13*sqrt(115)))/(5*sqrt(138) - 13*sqrt(115)) + 1/85698*(314460*x^3 -
 330781*x^2 + 77928*x - 42345)/(9*x^2 - 4*x + 3)^2 - 12/10220809*(48901451
*sqrt(30) + 370766376*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^7
/(5*x - 4)^7 - 187632663*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6
))^6/(5*x - 4)^6 + 134958936*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqr
t(6))^5/(5*x - 4)^5 + 521980785*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) -
 sqrt(6))^4/(5*x - 4)^4 - 2134408440*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x -
2) - sqrt(6))^3/(5*x - 4)^3 + 699471731*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*
x - 2) - sqrt(6))^2/(5*x - 4)^2 - 809824008*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 +
 8*x - 2) - sqrt(6))/(5*x - 4))/(104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x -
2) - sqrt(6))^3/(5*x - 4)^3 + 104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2)
- sqrt(6))/(5*x - 4) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^4/(5
*x - 4)^4 - 494*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 -
 139)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx=\int \frac{x}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^3}\,dx$$

input
```
int(x/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^3,x)
```

output
```
int(x/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^3, x)
```

Reduce [F]

$$\int \frac{x}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{too large to display}$$

input

```
int(x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x)
```

output

```
(3880064717747954795119000528147445517011040829409477705730943362699581805
139332547888566461313825000000*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(asin((
5*x - 4)/sqrt(6))/2)**16*x**8 - 248324141935869106887616033801436513088706
61308220657316678037521277323552891728306486825352408480000000*sqrt(5)*as
in((5*x - 4)/sqrt(6))*tan(asin((5*x - 4)/sqrt(6))/2)**16*x**7 + 6875666287
98358567249630691121199678678637970975455298728144057417836017262320045177
4484883841249200000000*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(asin((5*x - 4)/s
qrt(6))/2)**16*x**6 - 1075335506486733925085187654520887300382651007465614
538320739313699164692520037360590533192667999808000000*sqrt(5)*asin((5*x -
 4)/sqrt(6))*tan(asin((5*x - 4)/sqrt(6))/2)**16*x**5 + 1068181435945276056
65351003658382026371914579622573051711206303320043651422102461078577830774
5161487200000*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(asin((5*x - 4)/sqrt(6))/
2)**16*x**4 - 728735713370882376309558636753715749809794518813856122146894
479285129946511150373337789646745733414400000*sqrt(5)*asin((5*x - 4)/sqrt(
6))*tan(asin((5*x - 4)/sqrt(6))/2)**16*x**3 + 3494930475560289062116616396
95518328711865785957632995822158840239730527817077358212860559167829816000
000*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(asin((5*x - 4)/sqrt(6))/2)**16*x**
2 - 1065956643147043578387008832210874197510622124868179575840823420241197
12541684185643819751926100876800000*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(as
in((5*x - 4)/sqrt(6))/2)**16*x + 20530258045967510342103347409082344311...
```

3.59 $\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 346

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \frac{2\sqrt{6}\left(2\left(351-2411\sqrt{6}\right)+\frac{21025\left(156+61\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{\left(193-52\sqrt{6}\right)\left(4-\sqrt{6}-5x\right)}\right)}{19343\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)(2-8x+5x^2)}{\left(4-\sqrt{6}-5x\right)^2}\right)^2} + \frac{\sqrt{6}\left(4\left(73103+17147\sqrt{6}\right)-\frac{163995\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)}{444889\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)(2-8x+5x^2)}{\left(4-\sqrt{6}-5x\right)^2}\right)} + \frac{234\arctan\left(\frac{6+\frac{\left(12-13\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{529\sqrt{23}}$$

output
```
2/19343*6^(1/2)*(702-4822*6^(1/2)+21025*(156+61*6^(1/2))*(-5*x^2+8*x-2)^(1
/2)/(193-52*6^(1/2))/(4-6^(1/2)-5*x))/(13+2*6^(1/2)-10*6^(1/2)*(-5*x^2+8*x
-2)^(1/2)/(4-6^(1/2)-5*x)-5*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2
)^2+6^(1/2)*(292412+68588*6^(1/2)-163995*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6
^(1/2)-5*x))/(5783557+889778*6^(1/2)-4448890*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/
(4-6^(1/2)-5*x)-2224445*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2)+23
4/12167*arctan(1/138*(6+(12-13*6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*
x))*138^(1/2))*23^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.67 (sec) , antiderivative size = 1131, normalized size of antiderivative = 3.27

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^(-3),x]
```

output
```
((-2116*(1812 + 1391*x))/(3 - 4*x + 9*x^2)^2 + (92*(3713 + 9477*x))/(3 - 4
*x + 9*x^2) - (3726*Sqrt[-2 + 8*x - 5*x^2]*(-752 + 1205*x - 1064*x^2 + 249
3*x^3))/(3 - 4*x + 9*x^2)^2 + 37908*Sqrt[23]*ArcTan[(-2 + 9*x)/Sqrt[23]] +
 (18954*(13 - (2*I)*Sqrt[23])*ArcTan[(23*(-52606 + (30056*I)*Sqrt[23] + 8*
(56309 - (22984*I)*Sqrt[23])*x + (-1707793 + (363428*I)*Sqrt[23])*x^2 + 14
4*(17501 - (2041*I)*Sqrt[23])*x^3 + (81*I)*(15229*I + 1040*Sqrt[23])*x^4))
/(27378*(-460*I + 67*Sqrt[23])*x^4 - 4*(260*(-782*I + 203*Sqrt[23]) + 7047
*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) + 9*x^3*(52*(6387
1*I + 494*Sqrt[23]) + 11745*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x
- 5*x^2]) - 2*x^2*(9295312*I + 1752257*Sqrt[23] + 65772*Sqrt[23*(77 - (52*
I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) + x*(643448*I + 1971424*Sqrt[23] + 7
2819*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]))])/Sqrt[77/23
 - (52*I)/Sqrt[23]] - ((18954*I)*(-13*I + 2*Sqrt[23])*ArcTan[(23*(52606 +
(30056*I)*Sqrt[23] + (-450472 - (183872*I)*Sqrt[23])*x + (1707793 + (36342
8*I)*Sqrt[23])*x^2 + (-2520144 - (293904*I)*Sqrt[23])*x^3 + 81*(15229 + (1
040*I)*Sqrt[23])*x^4))/(27378*(460*I + 67*Sqrt[23])*x^4 - 4*(260*(782*I +
203*Sqrt[23]) + 7047*Sqrt[23*(77 + (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2
]) + 9*x^3*(52*(-63871*I + 494*Sqrt[23]) + 11745*Sqrt[23*(77 + (52*I)*Sqrt
[23])]*Sqrt[-2 + 8*x - 5*x^2]) - 2*x^2*(-9295312*I + 1752257*Sqrt[23] + 65
772*Sqrt[23*(77 + (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) + x*(-64344...
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.85, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{-5x^2+8x-2}+2x+1\right)^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{317-198x}{81\left(9x^2-4x+3\right)^2}-\frac{7\sqrt{-5x^2+8x-2}}{9\left(9x^2-4x+3\right)^2}+\frac{4(830x-339)}{81\left(9x^2-4x+3\right)^3}-\frac{208x\sqrt{-5x^2+8x-2}}{9\left(9x^2-4x+3\right)^3}+\frac{4\sqrt{-5x^2+8x-}}{3\left(9x^2-4x+3\right)^{}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{117\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{529\sqrt{23}}-\frac{117\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{529\sqrt{23}}-\frac{\sqrt{-5x^2+8x-2}(23351-82206x)}{1334667\,(9x^2-4x+3)}-\frac{40-2457x}{3726\,(9x^2-4x+3)}+\frac{1391(2-9x)}{28566\,(9x^2-4x+3)}+\frac{7(2-9x)\sqrt{-5x^2+8x-2}}{414\,(9x^2-4x+3)}-\frac{130(1758x+215)\sqrt{-5x^2+8x-2}}{1334667\,(9x^2-4x+3)}-\frac{1391x+1812}{1863\,(9x^2-4x+3)^2}-\frac{(2-9x)\sqrt{-5x^2+8x-2}}{69\,(9x^2-4x+3)^2}+\frac{52(3-2x)\sqrt{-5x^2+8x-2}}{207\,(9x^2-4x+3)^2}$$

input
```
Int[(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^(-3),x]
```

output
```
-1/1863*(1812 + 1391*x)/(3 - 4*x + 9*x^2)^2 - ((2 - 9*x)*Sqrt[-2 + 8*x - 5
*x^2])/(69*(3 - 4*x + 9*x^2)^2) + (52*(3 - 2*x)*Sqrt[-2 + 8*x - 5*x^2])/(2
07*(3 - 4*x + 9*x^2)^2) - (40 - 2457*x)/(3726*(3 - 4*x + 9*x^2)) + (1391*(
2 - 9*x))/(28566*(3 - 4*x + 9*x^2)) - ((23351 - 82206*x)*Sqrt[-2 + 8*x - 5
*x^2])/(1334667*(3 - 4*x + 9*x^2)) + (7*(2 - 9*x)*Sqrt[-2 + 8*x - 5*x^2])/
(414*(3 - 4*x + 9*x^2)) - (130*(215 + 1758*x)*Sqrt[-2 + 8*x - 5*x^2])/(133
4667*(3 - 4*x + 9*x^2)) - (117*ArcTan[(2 - 9*x)/Sqrt[23]])/(529*Sqrt[23])
+ (117*ArcTan[(8 - 13*x)/(Sqrt[23]*Sqrt[-2 + 8*x - 5*x^2])])/(529*Sqrt[23]
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.38

method	result
trager	$\frac{(30537x^3-17667x^2+25891x-11094)x}{4761(9x^2-4x+3)^2}-\frac{(2493x^3-1064x^2+1205x-752)\sqrt{-5x^2+8x-2}}{1058(9x^2-4x+3)^2}+\frac{117\,\mathrm{RootOf}\left(\_Z^2+23\right)\ln\left(\frac{13\,\mathrm{Root}}{}\right.}{}$
default	Expression too large to display

input
```
int(1/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4761*(30537*x^3-17667*x^2+25891*x-11094)*x/(9*x^2-4*x+3)^2-1/1058*(2493*
x^3-1064*x^2+1205*x-752)/(9*x^2-4*x+3)^2*(-5*x^2+8*x-2)^(1/2)+117/12167*Ro
otOf(_Z^2+23)*ln((13*RootOf(_Z^2+23)*x-8*RootOf(_Z^2+23)+23*(-5*x^2+8*x-2)
^(1/2))/(RootOf(_Z^2+23)*x-2*x+3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.50

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx$$
$$=\frac{435942\,x^3+2106\,\sqrt{23}(81\,x^4-72\,x^3+70\,x^2-24\,x+9)\arctan\left(\frac{1}{23}\sqrt{23}(9\,x-2)\right)+1053\,\sqrt{23}(81\,x^4-7}{}$$

input
```
integrate(1/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/219006*(435942*x^3 + 2106*sqrt(23)*(81*x^4 - 72*x^3 + 70*x^2 - 24*x + 9)
*arctan(1/23*sqrt(23)*(9*x - 2)) + 1053*sqrt(23)*(81*x^4 - 72*x^3 + 70*x^2
 - 24*x + 9)*arctan(1/23*sqrt(23)*(142*x^2 - 196*x + 55)*sqrt(-5*x^2 + 8*x
 - 2)/(65*x^3 - 144*x^2 + 90*x - 16)) - 22954*x^2 - 207*(2493*x^3 - 1064*x
^2 + 1205*x - 752)*sqrt(-5*x^2 + 8*x - 2) - 94116*x - 156078)/(81*x^4 - 72
*x^3 + 70*x^2 - 24*x + 9)
```

Sympy [F]

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \int \frac{1}{\left(2x+\sqrt{-5x^2+8x-2}+1\right)^3}\,dx$$

input
```
integrate(1/(1+2*x+(-5*x**2+8*x-2)**(1/2))**3,x)
```

output
```
Integral((2*x + sqrt(-5*x**2 + 8*x - 2) + 1)**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \int \frac{1}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^3}\,dx$$

input
```
integrate(1/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate((2*x + sqrt(-5*x^2 + 8*x - 2) + 1)^(-3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(265) = 530$.

Time $= 0.17$ (sec) , antiderivative size $= 611$, normalized size of antiderivative $= 1.77$

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="giac")
```

output
```
117/12167*sqrt(23)*arctan(1/23*sqrt(23)*(9*x - 2)) + 117/529*(5*sqrt(6) +
13*sqrt(5))*arctan(-(26*sqrt(6) + 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 +
8*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138) + 13*sqrt(115)))/(5*sqrt(138)
+ 13*sqrt(115)) + 117/529*(5*sqrt(6) - 13*sqrt(5))*arctan((26*sqrt(6) - 12
*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sq
rt(138) - 13*sqrt(115)))/(5*sqrt(138) - 13*sqrt(115)) + 1/4761*(9477*x^3 -
 499*x^2 - 2046*x - 3393)/(9*x^2 - 4*x + 3)^2 - 6/10220809*(325017862*sqrt
(30) + 612698517*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^7/(5*x
 - 4)^7 + 425483214*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^6/
(5*x - 4)^6 - 3416769843*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6)
)^5/(5*x - 4)^5 + 3469285170*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sq
rt(6))^4/(5*x - 4)^4 - 9872341005*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2)
- sqrt(6))^3/(5*x - 4)^3 + 2976396202*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x
- 2) - sqrt(6))^2/(5*x - 4)^2 - 3530844501*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 +
8*x - 2) - sqrt(6))/(5*x - 4))/(104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2
) - sqrt(6))^3/(5*x - 4)^3 + 104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) -
 sqrt(6))/(5*x - 4) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^4/(5*
x - 4)^4 - 494*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 -
139)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx=\int \frac{1}{{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)}^3}\,dx$$

input
```
int(1/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^3,x)
```

output
```
int(1/(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^3, x)
```

Reduce [F]

$$\int \frac{1}{\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{too large to display}$$

input
```
int(1/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x)
```

output
```
(63051051663404265420683758582395989651429413477904012718127829643868204333
35141539031892049963496562500000*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(asin((
5*x - 4)/sqrt(6))/2)**16*x**8 - 403526730645787298692376054927334333769148
2462585856813960181097207565077344905849804109119766378000000000*sqrt(5)*as
in((5*x - 4)/sqrt(6))*tan(asin((5*x - 4)/sqrt(6))/2)**16*x**7 + 1117295771
797332671780649873071949477852786702835114860433234093303983528051270073411
33537936242029950000000*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(asin((5*x - 4)/
sqrt(6))/2)**16*x**6 - 1747420198040942628263429938596441863121807887131622
36247712013847611426253450607109596164380854996880000000*sqrt(5)*asin((5*x
- 4)/sqrt(6))*tan(asin((5*x - 4)/sqrt(6))/2)**16*x**5 + 173579483341107359
20619538094487079285436119188668120903071024289507093356091649925268897500
8588741670000000*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(asin((5*x - 4)/sqrt(6))
/2)**16*x**4 - 118419553422768386150303278472478809344091609307251619848870
03528838336163080619356673908175961816798400000*sqrt(5)*asin((5*x - 4)/sqr
t(6))*tan(asin((5*x - 4)/sqrt(6))/2)**16*x**3 + 567926202278546972593950164
505217284156781902181153618211008115389562107702750707095898408647723451000
00000*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(asin((5*x - 4)/sqrt(6))/2)**16*x
**2 - 17321795451139458148788893523426705709547609529107918107413380578919
4532880236801671207096879913924800000*sqrt(5)*asin((5*x - 4)/sqrt(6))*tan(
asin((5*x - 4)/sqrt(6))/2)**16*x + 3336166932469720430591793953975880 95...
```

3.60 $\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 635

$$
\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx
$$

$$
= -\frac{2\sqrt{\frac{2}{3}}\left(145152+81703\sqrt{6}-\frac{1429700\left(513-157\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{\left(13-2\sqrt{6}\right)^3\left(4-\sqrt{6}-5x\right)}\right)}{19343\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)(2-8x+5x^2)}{\left(4-\sqrt{6}-5x\right)^2}\right)^2}
$$

$$
+\frac{2\sqrt{\frac{2}{3}}\left(3730892+874903\sqrt{6}+\frac{42050\left(160392893-64943077\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{\left(13-2\sqrt{6}\right)^5\left(4-\sqrt{6}-5x\right)}\right)}{1334667\left(13+2\sqrt{6}-\frac{10\sqrt{6}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}-\frac{5\left(13-2\sqrt{6}\right)(2-8x+5x^2)}{\left(4-\sqrt{6}-5x\right)^2}\right)}
$$

$$
+\frac{2}{27}\sqrt{2}\arctan\left(\frac{\sqrt{11-4\sqrt{6}}\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}\right)
$$

$$
+\frac{18472\left(295429693-119673902\sqrt{6}\right)\arctan\left(\frac{6+\frac{\left(12-13\sqrt{6}\right)\sqrt{-2+8x-5x^2}}{4-\sqrt{6}-5x}}{\sqrt{138}}\right)}{14283\sqrt{23}\left(13-2\sqrt{6}\right)^7}
$$

$$
-\frac{5\left(106257964-43200721\sqrt{6}\right)\log\left(\frac{x\left(2\left(3-2\sqrt{6}\right)+5\sqrt{6}x\right)}{\left(4-\sqrt{6}-5x\right)^2}\right)}{27\left(13-2\sqrt{6}\right)^6\left(4-\sqrt{6}\right)}
$$

$$
+\frac{5\left(295429693-119673902\sqrt{6}\right)\log\left(\frac{2\left(3-2\sqrt{6}\right)+12x-3\sqrt{6}x+10\sqrt{6}x^2+6\sqrt{-2+8x-5x^2}-4\sqrt{6}\sqrt{-2+8x-5x^2}+5\sqrt{6}x\sqrt{-2+8x}}{\left(4-\sqrt{6}-5x\right)^2}\right.}{27\left(13-2\sqrt{6}\right)^7}
$$

output
```
-2/58029*6^(1/2)*(145152+81703*6^(1/2)-1429700*(513-157*6^(1/2))*(-5*x^2+8
*x-2)^(1/2)/(13-2*6^(1/2))^3/(4-6^(1/2)-5*x))/(13+2*6^(1/2)-10*6^(1/2)*(-5
*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x)-5*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2
)-5*x)^2)^2+2/3*6^(1/2)*(3730892+874903*6^(1/2)+42050*(160392893-64943077*
6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(13-2*6^(1/2))^5/(4-6^(1/2)-5*x))/(17350671+
2669334*6^(1/2)-13346670*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x)-6673
335*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2)+2/27*2^(1/2)*arctan((2
*2^(1/2)-3^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x))+18472/328509*(2954
29693-119673902*6^(1/2))*arctan(1/138*(6+(12-13*6^(1/2))*(-5*x^2+8*x-2)^(1
/2)/(4-6^(1/2)-5*x))*138^(1/2))*23^(1/2)/(13-2*6^(1/2))^7-5/27*(106257964-
43200721*6^(1/2))*ln(x*(6-4*6^(1/2)+5*x*6^(1/2))/(4-6^(1/2)-5*x)^2)/(13-2*
6^(1/2))^6/(4-6^(1/2))+5/27*(295429693-119673902*6^(1/2))*ln((6-4*6^(1/2)+
12*x-3*x*6^(1/2)+10*6^(1/2)*x^2+6*(-5*x^2+8*x-2)^(1/2)-4*6^(1/2)*(-5*x^2+8
*x-2)^(1/2)+5*6^(1/2)*x*(-5*x^2+8*x-2)^(1/2))/(4-6^(1/2)-5*x)^2)/(13-2*6^(
1/2))^7
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.04 (sec) , antiderivative size = 1160, normalized size of antiderivative = 1.83

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(x*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^3),x]
```

output
```
((6348*(-423 + 604*x))/(3 - 4*x + 9*x^2)^2 + (1242*(-533 + 1076*x))/(3 - 4
*x + 9*x^2) - (276*Sqrt[-2 + 8*x - 5*x^2]*(-6120 + 29858*x - 30033*x^2 + 4
5360*x^3))/(3 - 4*x + 9*x^2)^2 + 36944*Sqrt[23]*ArcTan[(-2 + 9*x)/Sqrt[23]
] - 48668*Sqrt[2]*ArcTan[(1 - 2*x)/Sqrt[-1 + 4*x - (5*x^2)/2]] - ((18*I)*(
-178*I + 5873*Sqrt[23])*ArcTan[(23*(-22459465744 + (1546348544*I)*Sqrt[23]
 + 8*(2035528511 - (2539584736*I)*Sqrt[23])*x + (-10233103867 + (926756389
52*I)*Sqrt[23])*x^2 + 24*(11991284939 - (6852780844*I)*Sqrt[23])*x^3 + (9*
I)*(36325138529*I + 8967953540*Sqrt[23])*x^4))/(416397255040*I + 876886235
60*Sqrt[23] - 8*(446641816361*I + 23928664948*Sqrt[23])*x + (56263105080*I
 - 932718674466*Sqrt[23])*x^4 - 9520207812*Sqrt[23*(77 - (52*I)*Sqrt[23])]
*Sqrt[-2 + 8*x - 5*x^2] + 24593870181*Sqrt[23*(77 - (52*I)*Sqrt[23])]*x*Sq
rt[-2 + 8*x - 5*x^2] + x^2*(8693680422544*I - 794728685026*Sqrt[23] - 4442
7636456*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) + 9*x^3*(-
539876082772*I + 209900528952*Sqrt[23] + 3966753255*Sqrt[23*(77 - (52*I)*S
qrt[23])]*Sqrt[-2 + 8*x - 5*x^2]))])/Sqrt[77/23 - (52*I)/Sqrt[23]] + (18*(
178 - (5873*I)*Sqrt[23])*ArcTan[(23*(-22459465744 - (1546348544*I)*Sqrt[23
] + 8*(2035528511 + (2539584736*I)*Sqrt[23])*x + (-10233103867 - (92675638
952*I)*Sqrt[23])*x^2 + 24*(11991284939 + (6852780844*I)*Sqrt[23])*x^3 + (-
326926246761 - (80711581860*I)*Sqrt[23])*x^4))/(416397255040*I - 876886235
60*Sqrt[23] + 18*(3125728060*I + 51817704137*Sqrt[23])*x^4 + 9520207812...
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.65, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{-5x^2+8x-2}+2x+1\right)^3}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\sqrt{-5x^2+8x-2}x}{3\left(9x^2-4x+3\right)}+\frac{\sqrt{-5x^2+8x-2}x}{\left(9x^2-4x+3\right)^2}-\frac{4\sqrt{-5x^2+8x-2}x}{\left(9x^2-4x+3\right)^3}+\frac{5(9x-4)}{27\left(9x^2-4x+3\right)}-\frac{4\sqrt{-5x^2+8x-2}}{27\left(9x^2-4x+3\right)}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{9236\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{14283\sqrt{23}}-\frac{1}{27}\sqrt{2}\arctan\left(\frac{\sqrt{2}(1-2x)}{\sqrt{-5x^2+8x-2}}\right)-\frac{9236\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{14283\sqrt{23}}+\frac{5}{27}\text{arctanh}\left(\frac{2x+1}{\sqrt{-5x^2+8x-2}}\right)+\frac{16\sqrt{-5x^2+8x-2}(23351-82206x)}{1334667\,(9x^2-4x+3)}-\frac{302(2-9x)}{1587\,(9x^2-4x+3)}-\frac{96x+17}{138\,(9x^2-4x+3)}+\frac{2(2-9x)\sqrt{-5x^2+8x-2}}{207\,(9x^2-4x+3)}-\frac{(3-2x)\sqrt{-5x^2+8x-2}}{46\,(9x^2-4x+3)}-\frac{15(1758x+215)\sqrt{-5x^2+8x-2}}{889778\,(9x^2-4x+3)}-\frac{423-604x}{207\,(9x^2-4x+3)^2}+\frac{16(2-9x)\sqrt{-5x^2+8x-2}}{69\,(9x^2-4x+3)^2}+\frac{(3-2x)\sqrt{-5x^2+8x-2}}{23\,(9x^2-4x+3)^2}+\frac{5}{54}\log\left(9x^2-4x+3\right)-\frac{5\log(x)}{27}$$

input
```
Int[1/(x*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^3),x]
```

output
```
-1/207*(423 - 604*x)/(3 - 4*x + 9*x^2)^2 + (16*(2 - 9*x)*Sqrt[-2 + 8*x - 5
*x^2])/(69*(3 - 4*x + 9*x^2)^2) + ((3 - 2*x)*Sqrt[-2 + 8*x - 5*x^2])/(23*(
3 - 4*x + 9*x^2)^2) - (302*(2 - 9*x))/(1587*(3 - 4*x + 9*x^2)) - (17 + 96*
x)/(138*(3 - 4*x + 9*x^2)) + (16*(23351 - 82206*x)*Sqrt[-2 + 8*x - 5*x^2])
/(1334667*(3 - 4*x + 9*x^2)) + (2*(2 - 9*x)*Sqrt[-2 + 8*x - 5*x^2])/(207*(
3 - 4*x + 9*x^2)) - ((3 - 2*x)*Sqrt[-2 + 8*x - 5*x^2])/(46*(3 - 4*x + 9*x^
2)) - (15*(215 + 1758*x)*Sqrt[-2 + 8*x - 5*x^2])/(889778*(3 - 4*x + 9*x^2)
) - (9236*ArcTan[(2 - 9*x)/Sqrt[23]])/(14283*Sqrt[23]) + (9236*ArcTan[(8 -
 13*x)/(Sqrt[23]*Sqrt[-2 + 8*x - 5*x^2])])/(14283*Sqrt[23]) - (Sqrt[2]*Arc
Tan[(Sqrt[2]*(1 - 2*x))/Sqrt[-2 + 8*x - 5*x^2]])/27 + (5*ArcTanh[(1 + 2*x)
/Sqrt[-2 + 8*x - 5*x^2]])/27 - (5*Log[x])/27 + (5*Log[3 - 4*x + 9*x^2])/54
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.20 (sec) , antiderivative size = 1483, normalized size of antiderivative = 2.34

method	result	size
trager	Expression too large to display	1483
default	Expression too large to display	17094

input
```
int(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/304704*(x-1)*(1920591*x^3-2575593*x^2+1705265*x-1296567)/(9*x^2-4*x+3)^
2-1/4761*(45360*x^3-30033*x^2+29858*x-6120)/(9*x^2-4*x+3)^2*(-5*x^2+8*x-2)
^(1/2)+10/27*ln(-(122130901499904*RootOf(12288*_Z^2+338560*_Z+2518569)^2*R
ootOf(282624*_Z^2-7786880*_Z+82070757)^2*x-122130901499904*RootOf(12288*_Z
^2+338560*_Z+2518569)^2*RootOf(282624*_Z^2-7786880*_Z+82070757)^2+30000749
04797184*RootOf(12288*_Z^2+338560*_Z+2518569)*RootOf(282624*_Z^2-7786880*_
Z+82070757)^2*x-3852275311706112*RootOf(12288*_Z^2+338560*_Z+2518569)^2*Ro
otOf(282624*_Z^2-7786880*_Z+82070757)*x+3647763313049600*RootOf(12288*_Z^2
+338560*_Z+2518569)*RootOf(282624*_Z^2-7786880*_Z+82070757)*(-5*x^2+8*x-2)
^(1/2)-3000074904797184*RootOf(12288*_Z^2+338560*_Z+2518569)*RootOf(282624
*_Z^2-7786880*_Z+82070757)^2+17937800004759552*RootOf(282624*_Z^2-7786880*
_Z+82070757)^2*x+3852275311706112*RootOf(12288*_Z^2+338560*_Z+2518569)^2*R
ootOf(282624*_Z^2-7786880*_Z+82070757)-113027496828223488*RootOf(282624*_Z
^2-7786880*_Z+82070757)*RootOf(12288*_Z^2+338560*_Z+2518569)*x+23835316211
601408*RootOf(12288*_Z^2+338560*_Z+2518569)^2*x+75250156724959488*(-5*x^2+
8*x-2)^(1/2)*RootOf(282624*_Z^2-7786880*_Z+82070757)+17554671764171520*Roo
tOf(12288*_Z^2+338560*_Z+2518569)*(-5*x^2+8*x-2)^(1/2)-17937800004759552*R
ootOf(282624*_Z^2-7786880*_Z+82070757)^2+105157994069458944*RootOf(12288*_
Z^2+338560*_Z+2518569)*RootOf(282624*_Z^2-7786880*_Z+82070757)-77995293140
3654784*RootOf(282624*_Z^2-7786880*_Z+82070757)*x-23835316211601408*Roo...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx$$
$$= \frac{12027528\,x^3+36944\,\sqrt{23}(81\,x^4-72\,x^3+70\,x^2-24\,x+9)\arctan\left(\frac{1}{23}\,\sqrt{23}(9\,x-2)\right)-48668\,\sqrt{2}(81\,x^4}{}$$

input
```
integrate(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/1314036*(12027528*x^3 + 36944*sqrt(23)*(81*x^4 - 72*x^3 + 70*x^2 - 24*x
+ 9)*arctan(1/23*sqrt(23)*(9*x - 2)) - 48668*sqrt(2)*(81*x^4 - 72*x^3 + 70
*x^2 - 24*x + 9)*arctan(sqrt(2)*sqrt(-5*x^2 + 8*x - 2)*(2*x - 1)/(5*x^2 -
8*x + 2)) + 18472*sqrt(23)*(81*x^4 - 72*x^3 + 70*x^2 - 24*x + 9)*arctan(1/
23*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) + 2*sqrt(23)*(2*x^2 - 3*x))
/(7*x^2 - 8*x + 2)) + 18472*sqrt(23)*(81*x^4 - 72*x^3 + 70*x^2 - 24*x + 9)
*arctan(1/23*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8) - 2*sqrt(23)*(2*x
^2 - 3*x))/(7*x^2 - 8*x + 2)) - 11303442*x^2 + 121670*(81*x^4 - 72*x^3 + 7
0*x^2 - 24*x + 9)*log(9*x^2 - 4*x + 3) - 243340*(81*x^4 - 72*x^3 + 70*x^2
- 24*x + 9)*log(x) - 60835*(81*x^4 - 72*x^3 + 70*x^2 - 24*x + 9)*log(-(x^2
 + 2*sqrt(-5*x^2 + 8*x - 2)*(2*x + 1) - 12*x + 1)/x^2) + 60835*(81*x^4 - 7
2*x^3 + 70*x^2 - 24*x + 9)*log(-(x^2 - 2*sqrt(-5*x^2 + 8*x - 2)*(2*x + 1)
- 12*x + 1)/x^2) - 276*(45360*x^3 - 30033*x^2 + 29858*x - 6120)*sqrt(-5*x^
2 + 8*x - 2) + 10491312*x - 4671162)/(81*x^4 - 72*x^3 + 70*x^2 - 24*x + 9)
```

Sympy [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx=\int \frac{1}{x\left(2x+\sqrt{-5x^2+8x-2}+1\right)^3}\,dx$$

input
```
integrate(1/x/(1+2*x+(-5*x**2+8*x-2)**(1/2))**3,x)
```

output
```
Integral(1/(x*(2*x + sqrt(-5*x**2 + 8*x - 2) + 1)**3), x)
```

Maxima [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \int \frac{1}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^3 x}\,dx$$

input
```
integrate(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(-5*x^2 + 8*x - 2) + 1)^3*x), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/135*sqrt(10)*sqrt(5)*arctan(-1/10*sqrt(10)*(sqrt(6) - 4*(sqrt(5)*sqrt(-
5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))) + 9236/328509*sqrt(23)*arctan(1/23
*sqrt(23)*(9*x - 2)) + 9236/14283*(5*sqrt(6) + 13*sqrt(5))*arctan(-(26*sqr
t(6) + 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x -
4))/(5*sqrt(138) + 13*sqrt(115)))/(5*sqrt(138) + 13*sqrt(115)) + 9236/1428
3*(5*sqrt(6) - 13*sqrt(5))*arctan((26*sqrt(6) - 12*sqrt(5) - 139*(sqrt(5)*
sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138) - 13*sqrt(115)))
/(5*sqrt(138) - 13*sqrt(115)) + 1/9522*(87156*x^3 - 81909*x^2 + 76024*x -
33849)/(9*x^2 - 4*x + 3)^2 - 40/91987281*(2628815260*sqrt(30) - 1817695077
*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^7/(5*x - 4)^7 + 655947
4327*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^6/(5*x - 4)^6 - 3
7448922405*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^5/(5*x - 4)^
5 + 28060334100*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^4/(5*x
 - 4)^4 - 70036300635*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^3
/(5*x - 4)^3 + 20955667353*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt
(6))^2/(5*x - 4)^2 - 21784905243*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) -
 sqrt(6))/(5*x - 4))/(104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6
))^3/(5*x - 4)^3 + 104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/
(5*x - 4) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^4/(5*x - 4)^4 -
 494*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^2/(5*x - 4)^2 - 139)^2 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \int \frac{1}{x\,\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^3}\,dx$$

input
```
int(1/(x*(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^3),x)
```

output
```
int(1/(x*(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^3), x)
```

Reduce [F]

$$\int \frac{1}{x\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{too large to display}$$

input
```
int(1/x/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x)
```

output
```
(32810812991591157189470328679356930270475589964802315711857226834420107396
51320316139367899717057239269758110158676379417500000000*sqrt(5)*asin((5*x
 - 4)/sqrt(6))*x**8 - 2099892031461834060126101035478843537310437757747348
20555886251740288687337684500232919545581891663313264519050155288282720000
0000*sqrt(5)*asin((5*x - 4)/sqrt(6))*x**7 + 58142380908160843683306334534
8169918926294355702205578767548013692015374724187546786889272658996948019535
8025922663861679880000000*sqrt(5)*asin((5*x - 4)/sqrt(6))*x**6 - 90933102
340191717892571752988217180141606067642896278590348965012868716013044710249
0109380349436047206921761753487270593171200000*sqrt(5)*asin((5*x - 4)/sq
rt(6))*x**5 + 903281359601446691480298456134653773872132723688332285418922
8485017138698930536993949598447410576309042273112895211408327280800000*sqr
t(5)*asin((5*x - 4)/sqrt(6))*x**4 - 616237432905081613905508769180276441062
69573344260790014656277062910109866015486495360993659352335628210662336653
5942971772160000*sqrt(5)*asin((5*x - 4)/sqrt(6))*x**3 + 2955401999551656
51608506574167146225107558233544432626023312127593136265421847582186904118
1687943468337704622140738730721224000000*sqrt(5)*asin((5*x - 4)/sqrt(6))*x
**2 - 9014000182899488436094662292177661097753728776731360386799340011146
27788583079594616657549318313847211642543978662018278515200000*sqrt(5)*asin
((5*x - 4)/sqrt(6))*x + 1736090777904150950100344868773234152326260785581
2361205224147275651761670801206104476158082451311414000385293940496246 39...
```

3.61 $\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 822

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{Too large to display}$$

output
```
-2/58029*6^(1/2)*(1070289+323446*6^(1/2)+42050*(3464517-1461538*6^(1/2))*(
-5*x^2+8*x-2)^(1/2)/(385088-156807*6^(1/2))/(4-6^(1/2)-5*x))/(13+2*6^(1/2)
-10*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x)-5*(13-2*6^(1/2))*(5*x^2-8
*x+2)/(4-6^(1/2)-5*x)^2)^2+2/5*6^(1/2)*(4-6^(1/2)-5*x)^2*(68+27*6^(1/2)-50
*(4+6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2))/(4-6^(1/2)-5*x))/x/(6-4*6^(1
/2)+5*x*6^(1/2))/(13+2*6^(1/2)-10*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-
5*x)-5*(13-2*6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2)^2-5/3*6^(1/2)*(2520
325+628478*6^(1/2)-6728*(234386673413-95626095232*6^(1/2))*(-5*x^2+8*x-2)^
(1/2)/(13-2*6^(1/2))^5/(4-6^(1/2))^3/(4-6^(1/2)-5*x))/(17350671+2669334*6^
(1/2)-13346670*6^(1/2)*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x)-6673335*(13-2*
6^(1/2))*(5*x^2-8*x+2)/(4-6^(1/2)-5*x)^2)+44*(2700719361*2^(1/2)-220565490
8*3^(1/2))*arctan((2*2^(1/2)-3^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)-5*x)
)/(72919422747-29776341258*6^(1/2))+3598160/328509*(39476391248-1611442704
7*6^(1/2))*arctan(1/138*(6+(12-13*6^(1/2))*(-5*x^2+8*x-2)^(1/2)/(4-6^(1/2)
-5*x))*138^(1/2))*23^(1/2)/(13-2*6^(1/2))^7/(4-6^(1/2))^4/(4+6^(1/2))-2/27
*ln(x*(6-4*6^(1/2)+5*x*6^(1/2))/(4-6^(1/2)-5*x)^2)+2/27*ln((6-4*6^(1/2)+12
*x-3*x*6^(1/2)+10*6^(1/2)*x^2+6*(-5*x^2+8*x-2)^(1/2)-4*6^(1/2)*(-5*x^2+8*x
-2)^(1/2)+5*6^(1/2)*x*(-5*x^2+8*x-2)^(1/2))/(4-6^(1/2)-5*x)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.11 (sec) , antiderivative size = 1176, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(x^2*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^3),x]
```

output
```
(121670/x + (3174*(40 + 1269*x))/(3 - 4*x + 9*x^2)^2 + (46*(-11467 + 68265
*x))/(3 - 4*x + 9*x^2) - (69*Sqrt[-2 + 8*x - 5*x^2]*(-3174 + 38760*x + 212
65*x^2 + 7704*x^3 + 126927*x^4))/(x*(3 - 4*x + 9*x^2)^2) + 179908*Sqrt[23]
*ArcTan[(-2 + 9*x)/Sqrt[23]] - 535348*Sqrt[2]*ArcTan[(1 - 2*x)/Sqrt[-1 + 4
*x - (5*x^2)/2]] + (522*(2147 - (371*I)*Sqrt[23])*ArcTan[(23*(-8813295 + (
5358132*I)*Sqrt[23] + 24*(3096994 - (1409083*I)*Sqrt[23])*x + (-298062772
+ (70094518*I)*Sqrt[23])*x^2 + (460991572 - (59592988*I)*Sqrt[23])*x^3 + (
-234188473 + (17893330*I)*Sqrt[23])*x^4))/((-2381645630*I + 287409781*Sqrt
[23])*x^4 - 9*(-20091604*I + 3816017*Sqrt[23] + 575952*Sqrt[23*(77 - (52*I
)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) + 12*x*(-9001694*I + 30089092*Sqrt[23
] + 1115907*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) + 4*x^
3*(1337206637*I + 41332211*Sqrt[23] + 4859595*Sqrt[23*(77 - (52*I)*Sqrt[23
])]*Sqrt[-2 + 8*x - 5*x^2]) - 2*x^2*(1480853413*I + 357440992*Sqrt[23] + 1
2094992*Sqrt[23*(77 - (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]))])/Sqrt[77
/23 - (52*I)/Sqrt[23]] - ((522*I)*(-2147*I + 371*Sqrt[23])*ArcTan[(23*(9*(
979255 + (595348*I)*Sqrt[23]) + (-74327856 - (33817992*I)*Sqrt[23])*x + (2
98062772 + (70094518*I)*Sqrt[23])*x^2 + (-460991572 - (59592988*I)*Sqrt[23
])*x^3 + (234188473 + (17893330*I)*Sqrt[23])*x^4))/((2381645630*I + 287409
781*Sqrt[23])*x^4 + x^2*(2961706826*I - 714881984*Sqrt[23] - 24189984*Sqrt
[23*(77 + (52*I)*Sqrt[23])]*Sqrt[-2 + 8*x - 5*x^2]) - 9*(20091604*I + 3...
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.53, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{-5x^2+8x-2}+2x+1\right)^3}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{191-126x}{27\left(9x^2-4x+3\right)^2}-\frac{8\sqrt{-5x^2+8x-2}}{9x}-\frac{\sqrt{-5x^2+8x-2}}{27x^2}+\frac{18x+37}{27\left(9x^2-4x+3\right)}+\frac{8x\sqrt{-5x^2+8x-2}}{9x^2-4x+3}-\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{89954\arctan\left(\frac{8-13x}{\sqrt{23}\sqrt{-5x^2+8x-2}}\right)}{14283\sqrt{23}}-\frac{22}{27}\sqrt{2}\arctan\left(\frac{\sqrt{2}(1-2x)}{\sqrt{-5x^2+8x-2}}\right)-\frac{89954\arctan\left(\frac{2-9x}{\sqrt{23}}\right)}{14283\sqrt{23}}+\frac{2}{27}\text{arctanh}\left(\frac{2x+1}{\sqrt{-5x^2+8x-2}}\right)+\frac{73\sqrt{-5x^2+8x-2}(23351-82206x)}{4004001\left(9x^2-4x+3\right)}+\frac{\sqrt{-5x^2+8x-2}}{27x}-\frac{4-1467x}{1242\left(9x^2-4x+3\right)}-\frac{423(2-9x)}{1058\left(9x^2-4x+3\right)}+\frac{245(2-9x)\sqrt{-5x^2+8x-2}}{1242\left(9x^2-4x+3\right)}-\frac{34(3-2x)\sqrt{-5x^2+8x-2}}{69\left(9x^2-4x+3\right)}+\frac{120(1758x+215)\sqrt{-5x^2+8x-2}}{444889\left(9x^2-4x+3\right)}+\frac{1269x+40}{207\left(9x^2-4x+3\right)^2}+\frac{73(2-9x)\sqrt{-5x^2+8x-2}}{207\left(9x^2-4x+3\right)^2}-\frac{16(3-2x)\sqrt{-5x^2+8x-2}}{23\left(9x^2-4x+3\right)^2}+\frac{1}{27}\log\left(9x^2-4x+3\right)+\frac{5}{27x}-\frac{2\log(x)}{27}$$

input
```
Int[1/(x^2*(1 + 2*x + Sqrt[-2 + 8*x - 5*x^2])^3),x]
```

output
```
5/(27*x) + Sqrt[-2 + 8*x - 5*x^2]/(27*x) + (40 + 1269*x)/(207*(3 - 4*x + 9
*x^2)^2) + (73*(2 - 9*x)*Sqrt[-2 + 8*x - 5*x^2])/(207*(3 - 4*x + 9*x^2)^2)
 - (16*(3 - 2*x)*Sqrt[-2 + 8*x - 5*x^2])/(23*(3 - 4*x + 9*x^2)^2) - (4 - 1
467*x)/(1242*(3 - 4*x + 9*x^2)) - (423*(2 - 9*x))/(1058*(3 - 4*x + 9*x^2))
 + (73*(23351 - 82206*x)*Sqrt[-2 + 8*x - 5*x^2])/(4004001*(3 - 4*x + 9*x^2
)) + (245*(2 - 9*x)*Sqrt[-2 + 8*x - 5*x^2])/(1242*(3 - 4*x + 9*x^2)) - (34
*(3 - 2*x)*Sqrt[-2 + 8*x - 5*x^2])/(69*(3 - 4*x + 9*x^2)) + (120*(215 + 17
58*x)*Sqrt[-2 + 8*x - 5*x^2])/(444889*(3 - 4*x + 9*x^2)) - (89954*ArcTan[(
2 - 9*x)/Sqrt[23]])/(14283*Sqrt[23]) + (89954*ArcTan[(8 - 13*x)/(Sqrt[23]*
Sqrt[-2 + 8*x - 5*x^2])])/(14283*Sqrt[23]) - (22*Sqrt[2]*ArcTan[(Sqrt[2]*(
1 - 2*x))/Sqrt[-2 + 8*x - 5*x^2]])/27 + (2*ArcTanh[(1 + 2*x)/Sqrt[-2 + 8*x
 - 5*x^2]])/27 - (2*Log[x])/27 + Log[3 - 4*x + 9*x^2]/27
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.23 (sec) , antiderivative size = 1551, normalized size of antiderivative = 1.89

method	result	size
trager	Expression too large to display	1551
default	Expression too large to display	18899

input
```
int(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/304704*(x-1)*(19277595*x^4-15535485*x^3+13213829*x^2-3663363*x+507840)/
x/(9*x^2-4*x+3)^2-1/9522*(126927*x^4+7704*x^3+21265*x^2+38760*x-3174)/x/(9
*x^2-4*x+3)^2*(-5*x^2+8*x-2)^(1/2)+128/4761*RootOf(35328*_Z^2-194672*_Z+84
556953)*ln(-(549589056749568*RootOf(12288*_Z^2+67712*_Z+22667121)^2*RootOf
(35328*_Z^2-194672*_Z+84556953)^2*x-549589056749568*RootOf(12288*_Z^2+6771
2*_Z+22667121)^2*RootOf(35328*_Z^2-194672*_Z+84556953)^2+21090315307843584
*RootOf(12288*_Z^2+67712*_Z+22667121)*RootOf(35328*_Z^2-194672*_Z+84556953
)^2*x+7650482517835776*RootOf(12288*_Z^2+67712*_Z+22667121)^2*RootOf(35328
*_Z^2-194672*_Z+84556953)*x+879302706938726400*RootOf(12288*_Z^2+67712*_Z+
22667121)*RootOf(35328*_Z^2-194672*_Z+84556953)*(-5*x^2+8*x-2)^(1/2)-21090
315307843584*RootOf(12288*_Z^2+67712*_Z+22667121)*RootOf(35328*_Z^2-194672
*_Z+84556953)^2-62280238075545600*RootOf(35328*_Z^2-194672*_Z+84556953)^2*
x-7650482517835776*RootOf(12288*_Z^2+67712*_Z+22667121)^2*RootOf(35328*_Z^
2-194672*_Z+84556953)-4141440798301403136*RootOf(35328*_Z^2-194672*_Z+8455
6953)*RootOf(12288*_Z^2+67712*_Z+22667121)*x-671503012082380800*RootOf(122
88*_Z^2+67712*_Z+22667121)^2*x-6386259695490225091 2*(-5*x^2+8*x-2)^(1/2)*R
ootOf(35328*_Z^2-194672*_Z+84556953)-8201826524549381856 0*RootOf(12288*_Z^
2+67712*_Z+22667121)*(-5*x^2+8*x-2)^(1/2)+62280238075545600*RootOf(35328*_
Z^2-194672*_Z+84556953)^2+2244476964795863040*RootOf(12288*_Z^2+67712*_Z+2
2667121)*RootOf(35328*_Z^2-194672*_Z+84556953)-117304285137685971456*Ro...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 528, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \frac{38116980\,x^4-26068338\,x^3+179908\,\sqrt{23}(81\,x^5-72\,x^4+70\,x^3-24\,x^2+9\,x)\arctan\left(\frac{1}{23}\sqrt{23}(9\,x-2)\right)}{}$$

input
```
integrate(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/657018*(38116980*x^4 - 26068338*x^3 + 179908*sqrt(23)*(81*x^5 - 72*x^4 +
 70*x^3 - 24*x^2 + 9*x)*arctan(1/23*sqrt(23)*(9*x - 2)) - 535348*sqrt(2)*(
81*x^5 - 72*x^4 + 70*x^3 - 24*x^2 + 9*x)*arctan(sqrt(2)*sqrt(-5*x^2 + 8*x
- 2)*(2*x - 1)/(5*x^2 - 8*x + 2)) + 89954*sqrt(23)*(81*x^5 - 72*x^4 + 70*x
^3 - 24*x^2 + 9*x)*arctan(1/23*(sqrt(23)*sqrt(-5*x^2 + 8*x - 2)*(13*x - 8)
 + 2*sqrt(23)*(2*x^2 - 3*x))/(7*x^2 - 8*x + 2)) + 89954*sqrt(23)*(81*x^5 -
 72*x^4 + 70*x^3 - 24*x^2 + 9*x)*arctan(1/23*(sqrt(23)*sqrt(-5*x^2 + 8*x -
 2)*(13*x - 8) - 2*sqrt(23)*(2*x^2 - 3*x))/(7*x^2 - 8*x + 2)) + 24075204*x
^2 + 24334*(81*x^5 - 72*x^4 + 70*x^3 - 24*x^2 + 9*x)*log(9*x^2 - 4*x + 3)
- 48668*(81*x^5 - 72*x^4 + 70*x^3 - 24*x^2 + 9*x)*log(x) - 12167*(81*x^5 -
 72*x^4 + 70*x^3 - 24*x^2 + 9*x)*log(-(x^2 + 2*sqrt(-5*x^2 + 8*x - 2)*(2*x
 + 1) - 12*x + 1)/x^2) + 12167*(81*x^5 - 72*x^4 + 70*x^3 - 24*x^2 + 9*x)*l
og(-(x^2 - 2*sqrt(-5*x^2 + 8*x - 2)*(2*x + 1) - 12*x + 1)/x^2) - 69*(12692
7*x^4 + 7704*x^3 + 21265*x^2 + 38760*x - 3174)*sqrt(-5*x^2 + 8*x - 2) - 43
75566*x + 1095030)/(81*x^5 - 72*x^4 + 70*x^3 - 24*x^2 + 9*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \int \frac{1}{x^2\left(2x+\sqrt{-5x^2+8x-2}+1\right)^3}\,dx$$

input
```
integrate(1/x**2/(1+2*x+(-5*x**2+8*x-2)**(1/2))**3,x)
```

output
```
Integral(1/(x**2*(2*x + sqrt(-5*x**2 + 8*x - 2) + 1)**3), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \int \frac{1}{\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^3x^2}\,dx$$

input
```
integrate(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(1/((2*x + sqrt(-5*x^2 + 8*x - 2) + 1)^3*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 968, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x, algorithm="giac")
```

output
```
-44/135*sqrt(10)*sqrt(5)*arctan(-1/10*sqrt(10)*(sqrt(6) - 4*(sqrt(5)*sqrt(
-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))) + 89954/328509*sqrt(23)*arctan(1/
23*sqrt(23)*(9*x - 2)) + 89954/14283*(5*sqrt(6) + 13*sqrt(5))*arctan(-(26*
sqrt(6) + 12*sqrt(5) - 139*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x
 - 4))/(5*sqrt(138) + 13*sqrt(115)))/(5*sqrt(138) + 13*sqrt(115)) + 89954/
14283*(5*sqrt(6) - 13*sqrt(5))*arctan((26*sqrt(6) - 12*sqrt(5) - 139*(sqrt
(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(5*sqrt(138) - 13*sqrt(11
5)))/(5*sqrt(138) - 13*sqrt(115)) - 1/54*(2*sqrt(30) - 3*sqrt(5)*(sqrt(5)*
sqrt(-5*x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(sqrt(6)*(sqrt(5)*sqrt(-5*x^2
 + 8*x - 2) - sqrt(6))/(5*x - 4) - 2*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqr
t(6))^2/(5*x - 4)^2 - 2) - 2/275961843*(490328685778*sqrt(30) - 8266064111
07*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^7/(5*x - 4)^7 + 1457
577177946*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^6/(5*x - 4)^
6 - 8000608193067*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6))^5/(5*
x - 4)^5 + 5233835542230*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2) - sqrt(6
))^4/(5*x - 4)^4 - 12047619978645*sqrt(5)*(sqrt(5)*sqrt(-5*x^2 + 8*x - 2)
 - sqrt(6))^3/(5*x - 4)^3 + 3674568646158*sqrt(30)*(sqrt(5)*sqrt(-5*x^2 + 8
*x - 2) - sqrt(6))^2/(5*x - 4)^2 - 3575769127389*sqrt(5)*(sqrt(5)*sqrt(-5*
x^2 + 8*x - 2) - sqrt(6))/(5*x - 4))/(104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8
*x - 2) - sqrt(6))^3/(5*x - 4)^3 + 104*sqrt(6)*(sqrt(5)*sqrt(-5*x^2 + 8...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx=\int \frac{1}{x^2\left(2\,x+\sqrt{-5\,x^2+8\,x-2}+1\right)^3}\,dx$$

input
```
int(1/(x^2*(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^3),x)
```

output
```
int(1/(x^2*(2*x + (8*x - 5*x^2 - 2)^(1/2) + 1)^3), x)
```

Reduce [F]

$$\int \frac{1}{x^2\left(1+2x+\sqrt{-2+8x-5x^2}\right)^3}\,dx = \text{too large to display}$$

input

```
int(1/x^2/(1+2*x+(-5*x^2+8*x-2)^(1/2))^3,x)
```

output

```
( - 67875259749493905645120103163531569915201118316275683853067686302135512392607893794563481597956313943014967464385244350786337522183501562500000
*sqrt(5)*asin((5*x - 4)/sqrt(6))*x**10 + 5430020779959512451609608253082525593216089465302054708245414904170840991408631503565078527836505115441197397150819548062907001774680125000000000*sqrt(5)*asin((5*x - 4)/sqrt(6))*x**
9 - 1897825781242639475371209995614398759801820898304952701116391307865142079836769850196634236087055059483213559423946930934085150867175238750000000
00*sqrt(5)*asin((5*x - 4)/sqrt(6))*x**8 + 380557308193508151670338569519739275155174033044774125527994707753049754815166905178249849220028052337488757828960153215351981082401285600000000000*sqrt(5)*asin((5*x - 4)/sqrt(6))*x
**7 - 487839737496084237681773487036345750743440857791526464606349001380607397036447560600957091540882581654948568162623902791106359587590639037500000000*sqrt(5)*asin((5*x - 4)/sqrt(6))*x**6 + 426456994276163055626765758068464566741597265949899786289842972030498690525468332201567533266016479831181970035751422175529603249002382426400000000*sqrt(5)*asin((5*x - 4)/sqrt(6))
*x**5 - 2651062294065574711801572697475018220470161691351662498961212440093868546440577983144921787454318750788619060536105749838750146679268598318
0000000*sqrt(5)*asin((5*x - 4)/sqrt(6))*x**4 + 1164678991317748305799431127582502576611206872749550303868956325474915363447757719575728935039823239342043237987915594328174930331278602080000000
0*sqrt(5)*asin((5*x - 4)/sqr...
```

3.62 $\int x\sqrt{1+x+\sqrt{1+2x+2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int x\sqrt{1+x+\sqrt{1+2x+2x^2}}\,dx = \frac{\sqrt{1+x+\sqrt{1+2x+2x^2}}\left(2+x+6x^3-(2-x)\sqrt{1+2x+2x^2}\right)}{15x}$$

output
```
1/15*(1+x+(2*x^2+2*x+1)^(1/2))^(1/2)*(2+x+6*x^3-(2-x)*(2*x^2+2*x+1)^(1/2))
/x
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int x\sqrt{1+x+\sqrt{1+2x+2x^2}}\,dx = \frac{\sqrt{1+x+\sqrt{1+2x+2x^2}}\left(2+x+6x^3+(-2+x)\sqrt{1+2x+2x^2}\right)}{15x}$$

input
```
Integrate[x*Sqrt[1 + x + Sqrt[1 + 2*x + 2*x^2]],x]
```

output
```
(Sqrt[1 + x + Sqrt[1 + 2*x + 2*x^2]]*(2 + x + 6*x^3 + (-2 + x)*Sqrt[1 + 2*
x + 2*x^2]))/(15*x)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{2539\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\sqrt{2x^2+2x+1}+x+1}\,dx$$

$$\Big\downarrow \text{2539}$$

$$\frac{\sqrt{\sqrt{2x^2+2x+1}+x+1}\left(6x^3-(2-x)\sqrt{2x^2+2x+1}+x+2\right)}{15x}$$

input
```
Int[x*Sqrt[1 + x + Sqrt[1 + 2*x + 2*x^2]],x]
```

output
```
(Sqrt[1 + x + Sqrt[1 + 2*x + 2*x^2]]*(2 + x + 6*x^3 - (2 - x)*Sqrt[1 + 2*x
 + 2*x^2]))/(15*x)
```

Defintions of rubi rules used

rule 2539
```
Int[((g_.) + (h_.)*(x_))*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]], x_Symbol] :> Simp[2*((f*(5*b*c*g^2 - 2*b^2*g*h - 3*
a*c*g*h + 2*a*b*h^2) + c*f*(10*c*g^2 - b*g*h + a*h^2)*x + 9*c^2*f*g*h*x^2 +
 3*c^2*f*h^2*x^3 - (e*g - d*h)*(5*c*g - 2*b*h + c*h*x)*Sqrt[a + b*x + c*x^2
])/(15*c^2*f*(g + h*x)))*Sqrt[d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && EqQ[(e*g - d*h)^2 - f^2*(c*g^2 - b*g*h +
 a*h^2), 0] && EqQ[2*e^2*g - 2*d*e*h - f^2*(2*c*g - b*h), 0]
```

Maple [F]

$$\int x\sqrt{1+x+\sqrt{2x^2+2x+1}}dx$$

input
```
int(x*(1+x+(2*x^2+2*x+1)^(1/2))^(1/2),x)
```

output
```
int(x*(1+x+(2*x^2+2*x+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int x\sqrt{1+x+\sqrt{1+2x+2x^2}}\,dx = \frac{\left(6\,x^3+\sqrt{2\,x^2+2\,x+1}(x-2)+x+2\right)\sqrt{x+\sqrt{2\,x^2+2\,x+1}+1}}{15\,x}$$

input
```
integrate(x*(1+x+(2*x^2+2*x+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/15*(6*x^3 + sqrt(2*x^2 + 2*x + 1)*(x - 2) + x + 2)*sqrt(x + sqrt(2*x^2 +
 2*x + 1) + 1)/x
```

Sympy [F]

$$\int x\sqrt{1+x+\sqrt{1+2x+2x^2}}\,dx = \int x\sqrt{x+\sqrt{2x^2+2x+1}+1}\,dx$$

input
```
integrate(x*(1+x+(2*x**2+2*x+1)**(1/2))**(1/2),x)
```

output
```
Integral(x*sqrt(x + sqrt(2*x**2 + 2*x + 1) + 1), x)
```

Maxima [F]

$$\int x\sqrt{1+x+\sqrt{1+2x+2x^2}}\,dx=\int \sqrt{x+\sqrt{2\,x^2+2\,x+1}+1}x\,dx$$

input
```
integrate(x*(1+x+(2*x^2+2*x+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + sqrt(2*x^2 + 2*x + 1) + 1)*x, x)
```

Giac [F]

$$\int x\sqrt{1+x+\sqrt{1+2x+2x^2}}\,dx=\int \sqrt{x+\sqrt{2\,x^2+2\,x+1}+1}x\,dx$$

input
```
integrate(x*(1+x+(2*x^2+2*x+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x + sqrt(2*x^2 + 2*x + 1) + 1)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{1+x+\sqrt{1+2x+2x^2}}\,dx=\int x\,\sqrt{x+\sqrt{2\,x^2+2\,x+1}+1}\,dx$$

input
```
int(x*(x + (2*x + 2*x^2 + 1)^(1/2) + 1)^(1/2),x)
```

output
```
int(x*(x + (2*x + 2*x^2 + 1)^(1/2) + 1)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{1+x+\sqrt{1+2x+2x^2}}\,dx = \int \sqrt{\sqrt{2x^2+2x+1}+x+1}\,xdx$$

input
```
int(x*(1+x+(2*x^2+2*x+1)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(2*x**2 + 2*x + 1) + x + 1)*x,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file